DOCUMENT RESUME

ED 285 746 SE 048 334

AUTHOR	Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed.
TITLE	Investigations in Mathematics Education. Volume 20, Number 1.
INSTITUTION	Ohio State Univ., Columbus. Center for Science and Mathematics Education.
PUB DATE	87
NOTE	75p.
AVAILABLE FROM	SMEAC Information Reference Center, The Ohio State University, 1200 Chambers Road, 3rd Floor, Columbus, OH 43212 (U.S. Subscription $8.00; $2.75 single copy).
PUB TYPE	Collected Works - Serials (022) -- Reports - Research/Technical (143)
JOURNAL CIT	Investigations in Mathematics Education; v20 n1 Win 1987
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	*College Mathematics; Computer Simulation; *Elementary School Mathematics; Elementary Secondary Education; Geometric Constructions; Gifted; Higher Education; *Mathematical Concepts; Mathematics Anxiety; Mathematics Education; *Mathematics Instruction; Mathematics Skills; Preservice Teacher Education; *Secondary School Mathematics; Spatial Ability
IDENTIFIERS	*Mathematics Education Research

ABSTRACT

This issue of the journal contains abstracts and critical comments for 13 published reports of research in mathematics education. Topics include: (1) effects of mathematics anxiety on preservice elementary teachers; (2) visualizing rectangular solids; (3) computer simulations and abstract thinking; (4) the association of foldout shapes with polyhedra by third graders; (5) written and oral mathematics; (6) spacial representations in gifted children; (7) selection of mathematically talented students; (8) sex and ethnic group differences in mathematics achievement; (9) students' distortions of theorems; (10) an empirical classification model for errors in high school mathematics; (11) cooperative versus individualistic classrooms; (12) information processing by intellectually gifted pupils; and (13) feature frequency and the use of negative instances in a geometric task. Research references from the "Current Index to Journals in Education" (CIJE) and "Resources in Education" (RIE) for July through September 1986 are also listed. (TW)

INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 20, Number 1 - Winter 1987

INVESTIGATIONS IN MATHEMATICS EDUCATION

Published quarterly by

SMEAC Information Reference Center
The Ohio State University
1200 Chambers Road, Room 310
Columbus, Ohio 43212

Telephone: (614) 292-6717

With the cooperation of the ERIC Clearinghouse for Science, Mathematics and Environmental Education

Volume 20, Number 1 - Winter 1987

Subscription Price: $8.00 per year. Single Copy Price: $2.75
$9.00 for Canadian mailings and $11.00 for foreign mailings.

INVESTIGATIONS IN MATHEMATICS EDUCATION

Winter 1987

Battista, Michael T. THE RELATIONSHIP OF MATHEMATICS ANXIETY AND MATHEMATICAL KNOWLEDGE TO THE LEARNING OF MATHEMATICAL PEDAGOGY BY PRESERVICE ELEMENTARY TEACHERS. School Science and Mathematics 86: 10-19; January 1986.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

The study examined how preservice elementary teachers' mathematical knowledge and mathematics anxiety affect success in a methods course. It also hypothesized that a methods course could reduce the mathematics anxiety of preservice elementary teachers.

2. Rationale

Within the mathematics education community, there has been growing concern about the low level of mathematical knowledge and the negative attitudes toward mathematics possessed by pre- and inservice elementary school teachers. "This concern is based on the belief that the lack of knowledge of and poor attitudes toward mathematics exhibited by many preservice elementary teachers may inhibit their learning and later use of effective methods for teaching mathematics. It is also based on the belief that teachers' negative attitudes towards mathematics may be transmitted to their pupils (Larson, 1983), or may negatively affect their pupils' mathematics achivement (Schofield, 1981)."

3. Research Design and Procedures

Thirty-eight preservice elementary teachers from two methods courses taught be the researcher were used on subjects in the study. Subjects had completed either one or two college mathematics courses.

The methods course incorporated lecture hands-on experiences and a field experience component which included small group teaching. Students wrote lesson plans and had a midterm and final exam.

Scores for each subject included (EXAMS), (MARS 1 and MARS 2), (TEACHING), (MATH T), (L PLANS), and (F GRADE). (F GRADE), the final grade, included (EXAMS), (L PLANS), and (TEACHING). There was also a grade (MATH C) which was the average of grades from previously taken college mathematics courses. A (MATH) score was the average of the (MATH T) competency score and (MATH C) score. There was also a special ability score (SA). Detailed descriptions of the instruments may be found in the report.

In addition, partial data were collected for two methods courses not taught by the experimenter. As well as examining the pre-to-post test decreases on the MARS, scores from all tests are intercorrelated to help determine how different measures of achievement in methods courses were related to mathematical knowledge and mathematica anxiety. Subjects taught by the experimenter were labeled Group 1 and the others were labeled Group 2.

4. Findings

a. There was a significant negative correlation (p .05) between the entry level of mathematical anxiety and all measures of mathematical knowledge.
b. Pre-test anxiety was not significantly related to the final grade in the methods course.
c. Pre-test anxiety was not significantly related to any of the measures of achievement in the methods courses (EXAMS, TEACHING, and L PLANS).
d. Subjects' EXAMS scores were significantly correlated with lesson plan scores.
e. There was a significant relationship between final grade and previous grades in college mathematics courses.

f. Spatial ability is positively related to mathematical knowledge and negatively related to anxiety measures.
g. There was a significant decrease in anxiety.
h. Partial data collected for Group 2 showed somewhat similar results.

5. Interpretations

One purpose of the study was to determine whether or not preservice elementary teachers' mathematical knowledge would affect performance in a methods course. As expected, knowledge was related to methods course performance. It had not been expected that this knowledge would not relate to field experiences.

Data collected in the study did not support the conjecture that mathematics anxiety would inhibit the learning of methodology. Also, the study indicated that mathematics anxiety can be reduced by taking a methods course. This result may be explained partially by the fact that the methods course points out the usefulness of mathematics as well as helps to build self-confidence for the preservice teacher.

The author suggests further studies aimed at showing the relationship between mathematical knowledge and teaching performance.

Abstractor's Comments

1. The study addresses a very significant issue in mathematics education.
2. The study was a very ambitious one which attempted to consider several variables which might be related to achievement and anxiety.
3. Because of the complexity of the study, the description was hard to follow at times.
4. The author's findings are of particular relevance to mathematics educators who teach courses for preservice elementary teachers.

5. As the author suggests, more research is needed to further clarify the relationship between mathematical knowledge and teaching performance.

References

Larson, C. N. (1983). Techniques for developing positive attitudes in preservice elementary teachers. Arithmetic Teacher, 8-9.

Schofield, H. (1981). Teacher effects on cognitive and affective pupil outcomes in elementary school mathematics. Journal of Educational Psychology, 462-471.

Ben-Haim, D.; Lappan, G.; and Houang, R.T. VISUALIZING RECTANGULAR SOLIDS MADE OF SMALL CUBES: ANALYZING AND EFFECTING STUDENTS' PERFORMANCE. Educational Studies in Mathematics 16: 389-409; November 1985.

Abstract and comments prepared for I.M.E. by DOUGLAS T. OWENS, University of British Columbia, Vancouver.

1. Purpose

This paper focuses on two-dimensional, isometric drawings of three-dimensional figures. Two items of the type "How many cubes does it take to build a given (pictorially presented) rectangular solid?" were included in a spatial visualization test. The first purpose for including these items "was to determine whether students' performance would be affected by instruction in spatial visualization activities, and if so, whether the effect would vary by grade and by sex. The second [purpose] was to study strategies used by students in attempting to answer this type of item" (p. 395).

2. Rationale

In the study of volumes, children from grade three are asked to visualize information about solid objects from pictures. These representations of solids are also included in many large-scale educational assessments around the world. The question is raised by the literature as to whether students can benefit from training in spatial visualization. The mixed results of various training studies leave the field open for further study. The reports that male performance on spatial visualization is superior to female performance also raise questions regarding the interaction of sex and instruction in affecting spatial ability.

3. Research Design and Procedures

A pilot study was conducted to develop test items for a spatial visualization test. Open-ended items were used to elicit distractors for use in a multiple-choice format. Interviews revealed four counting strategies were dominant among the incorrect responses to the "How many cubes to build...?" items: The number of faces pictured, double the number of faces pictured, the number of cubes pictured and double the number of cubes pictured. The two items constructed with these distractors were a part of the 32-item spatial visualization test, each with five options.

The training unit involved ten carefully sequenced activities, requiring 12-15 class periods. The activities involved representing three-dimensional objects in two-dimensional drawings, and vice-versa. The students were asked to mentally rotate a "building" and draw flat views of the other sides or isometric drawings on dot paper. "It should be noted that none of the activities, posed to the students during the instruction, specifically involved counting or evaluating the number of cubes needed to build rectangular solids" (p. 400).

Students in grades five through eight were chosen from three different sites in and around a major city in the United States. Approximately 1000 students representing a broad range of socio-economic status were included in the sample.

The students were given the Spatial Visualization Test before and after three weeks of instruction in the spatial visualization unit.

4. Findings

Means and standard deviations for the Spatial Visualization Test by site, grade (not all grades at all sites), sex, and time (pretest, posttest) are presented. "Within each site, on both pretest and posttest, the means increase as grade level increases and the boys' means are generally higher than the girls' means. The findings also indicate that fifth, sixth, seventh, and eighth grade students in all sites, regardless of sex, profited considerably from instruction in activities involving spatial visualization tasks. ... Retention tests administered at Site II four weeks after the end of the instruction and a year later showed that the effects of the training were persistent over time" (p. 401).

Percentages of students correctly responding to the two "How many cubes to build...?" (items 10 and 12), are presented by grade, site, and sex. Site differences existed at grade six, the only grade at all three sites. Boys generally performed better than girls on the pretest and the posttest. While not at all grades, there were statistically significant differences at all sites (McNemar Test) between pretest and posttest of those students who got both items 10 and 12 correct. "Except for boys in grade 6 of site II, both girls and boys in each site and grade showed considerable improvement from pretest to posttest" (p. 403).

The percentages of students selecting the various options for items 10 and 12 in the pretest and posttest are presented by site and grade. The most frequently chosen distractor for both items corresponded to counting the number of faces and doubling. The next most frequent error was counting the number of faces showing.

5. Interpretations

The paper gives conclusions for before and after instruction. Prior to instruction there is a general increase in the level of performance with increase in grade level—five to eight on items of the type "How many cubes to build...?". About half of the students in middle grades cannot correctly answer this type of item. Students who miss the item are using faulty counting strategies. Interpretation regarding instruction is that instruction on spatial visualization significantly improved performance regardless of grade level.

Results of this study are consistent with that of previous studies with regard to sex differences on spatial visualization in favor of boys. Boys and girls did not respond differently to instruction. "The present finding lends support to the conjecture that part of the differences in performance between boys and girls is the result of different environmental influences such as toys. ... This strongly suggests that such concrete experience should be provided for children in the middle grades" (p. 407). In summary, "the evidence from the present study strongly suggests that students in grades 5 through 8 have difficulty relating isometric type drawings to the rectangular solids they represent" (p. 407).

Abstractor's Comments

This paper was written well and is a pleasure to read. The review of the literature and rationale are thorough. The design is straightforward and clearly presented. Of particular note is the fact that the instruction on spatial visualization did not include tasks of the type of particular interest as a criterion measure. It is a significant finding that performance improved following instruction under these conditions. An even more significant finding is that girls and boys improved alike. The tests and instructional materials were developed by the Middle Grades Mathematics Project at Michigan State University, but it is not clear from the article whether these data were collected as part of that study or subsequently.

Berlin, Donna and White, Arthur. COMPUTER SIMULATIONS AND THE TRANSITION FROM CONCRETE MANIPULATION OF OBJECTS TO ABSTRACT THINKING IN ELEMENTARY SCHOOL MATHEMATICS. School Science and Mathematics 86: 468-479; October 1986.

Abstract and comments prepared for I.M.E. by LOYE Y. "MICKEY" HOLLIS, The University of Houston, Houston, Texas.

1. Purpose

The purpose of the study was to investigate the effects of combining interactive microcomputer simulations and concrete activities on the development of abstract thinking in elementary school mathematics.

2. Rationale

The literature supports the importance of the initial, concrete manipulation of objects in the development of mathematical concepts. The use of multi-embodiments, concepts presented in as many different ways as possible, has been suggested; however, this procedure does not have the support of research. An alternative to this procedure could be the use of mixed-mode multi-embodiments or the interplay of concrete, manipulative objects and representational forms such as the computer to facilitate the development of abstract understandings. The computer could serve as an interactive link between the concrete and abstract levels of thought.

3. Research Design and Procedures

Students were randomly elected from two elementary schools. The subjects were from two different socio-cultural backgrounds: suburban, all black (n = 57) and rural, all white (n = 56). There were 28 second-grade boys; 30 second-grade girls; 26 fourth-grade boys and 29 fourth-grade girls. Students within each site were randomly assigned by gender and grade level to the treatment conditions.

There were three levels of treatment conditions:

a. Concrete-only activities.

b. Combination of concrete and computer simulation activities.

c. Computer simulation-only activities.

Twenty task cards were research-developed and produced instruction in:

a. Recognition and duplication of design.

b. Recognition and extension of pattern.

c. Spatial orientation and discrimination.

These same twenty task cards were used with researcher-developed, computer simulations of pegboard and colored cube activities. Students completed two-to-three task cards during a thirty-minute period.

At the end of the three-week treatment period, two paper-and-pencil instruments requiring reflective abstract thought were administered to all subjects. One instrument required design recognition and is a test of the specific skills required in the concrete and computer activities. The second instrument required pattern extension and spatial orientation discrimination and is a test of the general transfer and extension of these skills.

4. Findings

The following effects were found:

a. When using concrete-only activities, the rural-white girls scored higher on Pattern Extension than did the suburban-black girls.

b. The rural-white boys, when using concrete-computer on the computer-only activities scored higher on Pattern Extension that the suburban-black girls using concrete-only activities.

c. The rural-white fourth-grade children scored higher on the Pattern Extension than the rural-white second-grade children.

d. The suburban-black fourth-grade children and the suburban-black second-grade children did not score differently on Pattern Extension.

e. Generally, the fourth-grade children scored higher on Pattern Extension than the second-grade children.

f. Generally, the rural-white children scored higher on Pattern Extension than the suburban-black children.

The influence that the concrete manipulation of objects and computer activities have on the students' transition from concrete understanding to abstract thinking in elementary school mathematics is not clear at this point. The evidence from this study suggests that not all students are influenced in the same manner.

5. Interpretations

The concrete and computer activities have different effects on children depending upon their socio-cultural background and their gender. It is apparent that some children do better with activities involving computer use.

Abstractor's Comments

This study, like so many others involving the use of manipulatives, raises more questions than it answers. The central question, "What is the effect of combining interactive micro-computer simulations and concrete activities on the development of abstract thinking in elementary school mathematics?", was not anywhere near answered.

One troublesome aspect of the study was the subjects. For one thing, they were too few. For another, the fact that the black students came from one school and the white students came from another school does not provide an equal basis for comparison. This was further confounded by one school being rural and the other school being suburban. This type of sample does not permit reliable conclusions.

The authors do bring to the front an important issue. What is the relationship, if any, between concrete manipulative activities and computer activities?

Bourgeois, Roger. THIRD GRADERS' ABILITY TO ASSOCIATE FOLDOUT SHAPES WITH POLYHEDRA. Journal for Research in Mathematics Education 17: 222-230; May 1986.

Abstract and comments prepared for I.M.E. by J. LARRY MARTIN, Missouri Southern State College.

1. Purpose

The purposes of the study were (1) to identify which common polyhedra are more easily associated with their foldout shapes, (2) to determine whether different foldout representations of the same polyhedron present different degress of difficulty, and (3) to measure the effect of a short-term experience with solids on the child's ability to associate polyhedra with corresponding foldout shapes.

2. Rationale

The role of geometry in the elementary school curriculum continues to grow in importance. Activities cover a wide range of topics including transformations, work with solids, and tessellations. Much research on the child's construction of space during the 1960's and 1970's focusd on Piaget's topological primacy thesis. Bourgeois notes that more recent writings indicate that the complexity of geometric figures used may influence children's responses as much as topological or Euclidean characteristics. Several studies have shown that children have more difficulty with some solids than with others. If the figures or solids used in investigations can influence results, then what are the specific factors which affect children's responses?

3. Research Design and Procedures

The sample consisted of two third-grade classes in a "medium-sized rural school". There was a total of 34 subjects, 11 girls and 23 boys. At the beginning of the study the mean age of the sample was 8:8. Previous research by other investigators indicated that children begin to have success with tasks similar to ones used in this study at about eight or nine years of age. The design was test-retest, with intervening instruction.

During the test, children were asked to match 14 foldout shapes (nets), presented one at a time, with 7 polyhedra. Each polyhedron had two correct foldout representations. The polyhedra used were: square pyramid, triangular prism, rectangular prism, octahedron, cuboid, cube, and tetrahedron. They ranged in height from 4 cm to 8.5 cm.

A warm-up task consisting of two items oriented the subjects to the test. During the warm-up, each child was shown a rectangular prism and a triangular prism (different from those on the test) and a foldout shape of each. The interviewer explained that the foldout was a "picture" of the solid. The child was then shown how the foldout of the rectangular prism could be folded to obtain the solid and invited to fold the triangular prism foldout into its corresponding solid.

Instructional time between the pretest and posttest consisted of one hour per week for 10 weeks. The regular classroom teachers did the teaching. Activities emphasized finding faces, vertices, and edges of solids, tracing faces of solids and cutting them out, and identifying congruent regions. The teachers had no knowledge of the test content.

4. Findings

Results are given as frequencies of associations of each net with each solid. The first purpose of the study was to identify which polyhedra are more easily associated with their nets. The greatest success was with the octahedron. Among the three rectangular prisms, subjects had most success with the cuboid. They were also more successful with the nets containing triangular regions than with those composed entirely of rectangular regions.

The second purpose was to determine whether some nets of a solid might be easier than others for children to identify. Almost every solid had one net that could be identified as the easier of two. A pattern was that when lateral faces were distributed around the base (e.g. square pyramid, tetrahedron, triangular prism), nets were more often associated correctly with their corresponding solid than when the lateral faces were arranged otherwise.

The third purpose was to measure the effect of a short-term experience on the children's success rate. The number of correct responses increased for every net except one, that of the octahedron. The total percentage of correct responses rose from 41% to 61%. The greatest changes were for the nets composed of rectangular regions only. Significant increases in success rates were reported for both nets for the cube and the cuboid. For the two nets of the rectangular prism one net showed a significant increase. Only one net of one solid having some triangular regions showed a significant success rate increase. That was for the triangular prism.

5. Interpretations

Previous experience of the children apparently had no effect on the results. The only solid pointed out as unfamiliar by students was the octahedron. Over 95% indicated they had not seen it before, but it was the easiest solid for them to associate correctly with its nets. One would expect the cube to have been familiar to subjects, yet it was one of the most difficult to associate with its nets. Both the octahedron and the cube had all congruent faces, so evidently it was not congruent faces which made the octahedron the easiest solid.

Generally, children were more successful on the posttest than on the pretest. There were more significant changes for solids having only rectangular faces. However, children were quite successful with the nets of some solids with triangular regions on the pretest, leaving little room for improvement on the posttest.

A strategy which led to correct associations was to count the faces. However, only a few used the strategy. Another characteristic of successful students was that they took more time to make their associations. In general, boys took more time than girls and the only subjects to use the "counting faces" strategy were boys.

Bourgeois states that some of his results confirm those of earlier findings. Not all nets are of the same difficulty. Nets with triangular regions are easier than those without. Authors who include nets of triangular prisms and pyramids among their topics also are supported by Bourgeois' results.

Abstractor's Comments

This investigation has clearly stated purposes. The tasks are designed to provide appropriate data. And the results are linked to the purposes nicely. A control group could have strengthened the claims for the instruction. Only three months intervened between pretest and posttest. However, the sample age was selected because previous research had indicated that it was a transitional age. Could some of the improvement in success rates be attributed to the intervening time rather than to instruction?

Although the purposes of the study were clearly stated, I was not completely satisfied with why I wanted the related questions answered. What are the underlying mental constructions of interest? Are we interested in foldouts (nets) per se or are we interested in them because of what they can reveal about other, maybe broader, spatial representations? Why are nets included in textbooks anyway? Is it for the sake of the content or are there other, more fundamental, justifications? For example, when I learn that nets whose lateral faces are distributed around the base are easier for children to identify than those whose lateral faces are not so distributed, what does that tell me about children's processes in their construction of space? A little more rationale, psychological framework, and follow-up discussion would have been helpful to me.

Bourgeois recommends further investigations into learning styles and strategies. Some of his observations about successful strategies are interesting and deserve follow-up.

Carraher, Terizinha Nunes; Carraher, David William; and Schliemann, Analucia Dias. WRITTEN AND ORAL MATHEMATICS. Journal for Research in Mathematics Education 18: 83-97; March 1987.

Abstract and comments prepared for I.M.E. by JUDITH T. SOWDER, San Diego State University.

1. Purpose

The purpose of the study was to systematically investigate children's choice of computational procedures and the efficiency with which they solved problems presented in a school context and problems presented in a simulated market context. Oral (informal) and written (formal) procedures were to be contrasted.

2. Rationale

There is evidence that people prefer informal procedures to school-learned algorithms when solving non-classroom-related problems. A previous study by the authors (1985) showed this preference to be true for children working as market or street vendors in Brazil. Moreover, the children performed significantly better in a natural setting than in a school-like setting. There are various reasons why this might have happened, and further investigation is needed to determine the effect of the situation, or problem context, on problem-solving procedures.

3. Research Design and Procedures

Sixteen Brazilian third graders were the subjects of this experiment. They ranged in age from eight to thirteen years, as is typical in the Brazilian school system. They were randomly selected from two schools. All had received instruction on algorithms for all four arithmetic operations.

Arithmetic problems were presented in three situations: computation exercises, story problems, and simulated store problems. One, two, and three digit numbers were presented in computation problems, each using one operation. Division problems were limited to divisors of three, four, and five, with no remainders. A second matched set of combinations were used in story problems, and a third set in the simulated store problems. In order to prevent performance differences explainable by problem differences, the three sets of combinations were matched across subjects for the three contexts. Additionally, a Latin Square design was employed to order the contexts. Students were individually interviewed and, when necessary, asked to work aloud and to justify answers. All questions were presented orally.

4. Findings

Computation problems were significantly less likely to be solved correctly than problems presented in the other two contexts, but results were not significantly different between story problems and simulated store problems. Both story problems and simulated store problems were more likely to be solved by oral procedures, while computation problems were more likely to be solved by traditional written procedures. Moreover, children were significantly more successful when they used oral computation procedures than when they used written procedures. It appeared that the number of correct responses was related to the procedure selected rather than problem context.

Oral calculation methods used by children were not as idiosyncratic as other literature might suggest. Two general oral procedures, referred to as heuristics, were identified. The first, a decomposing heuristic, exemplified the child's knowledge of the number system and ability to decompose numbers into smaller numbers easier to use mentally. For example, 252 - 57 was orally calculated 252 - 52 = 200, then 200 - 5 = 195. The second heuristic, used for multiplication and division, used a repeated grouping with continuous monitoring of subtotals. For example, 100 ÷ 4 was done by halving, twice. Generally speaking, oral procedures allowed children to alter problems in a mannner that made quantities easier to manipulate mentally. Although no uniform strategy emerged, children worked from left to right rather than the traditional right to left demanded by most written algorithms, and had less difficulty with zeros. Also, they monitored themselves more effectively when using oral procedures.

5. Interpretations

The context in which arithmetic problems are presented has a strong impact on the choice of solution method. School-taught algorithms have little carry-over to real-life problems, particularly whenever prices are involved. The superior success rate with oral procedures as compared to written procedures needs to be given more consideration as a way of promoting development of understanding and skill.

Abstractor's Comments

The research described here is extremely relevant to current questions regarding the role of traditional algorithms and the place of algorithmic thinking in the elementary school curriculum, and regarding the influence of mental computation on the understanding of numbers and number properties. Reports of children's arithmetic competence are usually cause for dismay, but this evidence of the real understanding of number and place value children exhibited in non-school contexts is a hopeful sign that children, after all, can function mathematically when the context is one that matters to them. How we tap and advance this knowledge in school settings is an important research topic.

Another result that both researchers and practioners will find to be particularly interesting is the somewhat serendipitous finding concerning the metacognitive aspects of children's performance within different contexts. Oral procedures, to be successful, call for continuous monitoring, and this was exhibited by the children in this study. Attempts by others to teach monitoring have been relatively unsuccessful. Perhaps teachers can profit from studying and understanding the monitoring procedures used by children in oral arithmetic, and learn how to teach transfer of such skills in other contexts.

In some of my own work with children of this age, in which I used story problems and simulated store settings, I found them much less likely to use informal methods than the children described in this study. Is this due to differences in the questions asked, or is it due to cultural differences? It would be valuable to have a replication of this study done elsewhere.

One final question. Why do the authors refer to Plunkett's (1979) treatment of informal procedures as idiosyncratic? My interpretation of his article was quite different. If he believed mental methods were truly idiosyncratic, would he have proposed such a structured approach to teaching mental algorithms as he did?

References

Carraher, T. N., Carraher, D. W., and Schliemann, A. D. Mathematics in the Streets and in the Schools. British Journal of Developmental Psychology 3: 21-29; 1985.

Plunkett, S. Decomposition and All That Rot. Mathematics in Schools 8 (3): 2-7, 1979.

Hermelin, B. and O'Connor, N. SPATIAL REPRESENTATIONS IN MATHEMATICALLY AND IN ARTISTICALLY GIFTED CHILDREN. British Journal of Educational Psychology 56: 150-157; 1986.

Abstract and comments prepared for I.M.E. by DAIYO SAWADA, University of Alberta, Edmonton.

1. Purpose

This study is concerned with identifying commonalities across and differences between mathematically gifted and artistically gifted children with regard to spatial representation in verbal and non-verbal contexts while controlling for IQ.

> Both artistically and mathematically gifted children may process some visual-spatial data more efficiently than do children of similar general intelligence but without these gifts. The question asked in the present context is which particular components of spatial ability these two groups of gifted children might share, and which spatial operations might distinguish them. (p. 151)

2. Research Design and Procedures

Two groups of children, one mathematically gifted and the other artistically gifted, were selected from a "mixed ability school in Southern England" by teachers and by the headmaster as falling in the top 2 per cent of "gifted artists or mathematicians" in the 12-14 year-old group. On the basis of the English Progress Test, which measures "verbal ability and scholastic achievement," IQ-matched control subjects who were neither mathematically nor artistically gifted were identified, resulting in four groups of students: (1) 10 mathematically gifted students with mean IQ of 117 (SD6); (2) an IQ-matched control group of 7 students; (3) 9 artistically gifted children with mean IQ of 107 (SD6); and (4) an IQ-matched control group of 8 students. Four types of tasks, described in the next section, were administered.

3. Findings

Verbally Presented Spatial Problems

The 12 tasks in this condition were designed to test "the subject's ability to translate verbally formulated problems into spatial terms, manipulate spatial images and recode them into verbal responses" (p. 152). An example task: "How many diagonals are there on the surface of a cube?" The mean score (maximum 12) for each group is shown in Table 1.

TABLE I
VERBALLY PRESENTED SPATIAL PROBLEMS

Groups	Mean Score	SD
Mathematical Children	8·30	1·83
Mathematical Controls	6·85	2·67
Artistic Children	5·90	1·17
Artistic Controls	6·00	1·60

Mann-Whitney U tests indicated that the mathematically gifted children scored significantly higher than their IQ controls, while the artistically gifted children did not score significantly higher than their IQ controls. "This result indicates that a factor other than intelligence is affecting the results" (p. 152). As well, the mathematically gifted children scored significantly higher than the artistically gifted; the mathematics controls likewise did better than the arts controls. These differences "reflect the IQ difference between the groups" (p. 152).

Non-Spatial Verbal Reasoning Tasks

The 16 tasks in this condition were designed to test verbal reasoning of four kinds: (1) Three Term Series Problems, e.g., "The red car is faster than the blue car, the green car is faster than the red car. Which is the slowest?" (p. 153); (2) Semantic Identity, e.g., "John is more industrious than Peter. Peter is less lazy than John. Do these sentences have the same or different meanings?" (p. 153); (3) Analogies, e.g., "A palm tree is to a flagpole as a snake is to a zoo, or a garden hose. Which?" (p. 153); and (4) Syllogisms. Mean scores (maximum 16) are shown in Table 2.

TABLE 2

NON-SPATIAL VERBAL REASONING TASK

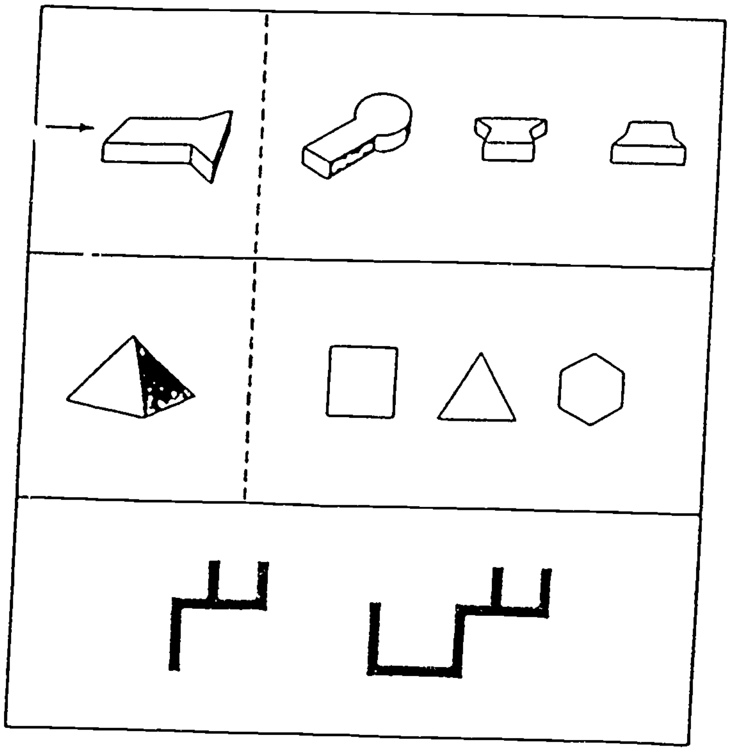
Groups	Mean Score	SD
Mathematical Children	14·20	1·23
Mathematical Controls	13·00	2·08
Artistic Children	11·44	2·24
Artistic Controls	12·13	2·80

Mann-Whitney U tests were used again and indicated that the mathematically gifted did significantly better than both the artistically gifted and the art controls but not better than their own IQ controls. "These results reflect IQ levels" (p. 153).

Visually Presented Spatial Problems

The three tasks in this condition were designed to test ability to manipulate (mentally rotate) spatial configurations visually presented (see row 1 of Figure 1); to identify a cross-section parallel to the base of a simple solid visually presented (see row 2 of Figure 2); lastly, to reconstruct from memory visually presented "Persian letters" (see row 3 of Figure 1).

FIGURE 1
EXAMPLES FOR VISUAL-SPATIAL TASKS
(top = Rotation, middle = Sectioning, bottom = Memory)



Mann-Whitney U tests revealed no significant group differences on the Rotation and Sectioning tasks (rows 1 and 2 of Figure 1). "This could well be because the tests were too easy" (p. 154).

On the Memory task, however, both the mathematically and artistically gifted children differed significantly from their IQ Controls but not from each other. "Thus the results in this task did not reflect IQ level but ability in both the mathematicians and the artists to remember non-verbalisable shapes" (p. 154).

Constructive Imagination: Identifying Incomplete Pictures

Tasks in this condition tested the child's ability to identify drawings on the basis of minimal necessary information: The child looked into a display box in which only a small portion of the display was illuminated; on subsequent trials more and more of the display was lit. Table 3 shows the mean number of trials required by each group in order to identify the figure.

TABLE 3
IDENTIFICATION OF INCOMPLETE PICTURES

Groups	Mean No. of Exposures	SD
Mathematical Children	23·60	5·60
Mathematical Controls	29·00	4·28
Artistic Children	17·11	6·64
Artistic Controls	24·88	6·81

Mann-Whitney tests were used once again and showed that the artistically gifted were superior to all other groups.

4. Interpretations

The authors summarize: "Verbally presented tasks requiring spatial reasoning and the mental manipulation of spatial images were better solved by the mathematically gifted than by either the IQ-matched control group, or by those gifted for the visual arts" (p. 155). More generally, the authors claim that their study confirms that "those who are gifted for mathematics can convert verbal codes into spatial images, are able to operate on these images, and can then retranslate their solutions into verbal form" (p. 155). As well, "it was found that artistically gifted children could identify incomplete pictures on the basis of less information than was required by any of the other groups. This finding confirmed a previous result ... and might not be regarded as reliably established" (p. 155). The authors go on to interpret this finding further:

> The ability to arrive at conclusions on the basis of insufficient information has previously been regarded as an indicator of high general intelligence. It could be assumed that with minimal cues an individual has to draw on information represented in some internal "lexical" system which can be accessed to supplement incoming data. It might be argued that a marked facility with this operation could be a component feature of what is often vaguely referred to in the literature as "creativity" ... What seems to distinguish children gifted for the visual arts appears to be not so much an outstanding ability to process and mentally operate on incoming perceptual data, but rather the ready generation of stored visual images. (p. 156)

Abstractor's Comments

This study deals with an age-old phenomenon: how artistically and mathematically talented individuals are both alike and different from each other as well as alike and different from others not similarly gifted yet of equivalent intelligence. The authors come to some interesting conclusions (see Interpretations above) which, while provocative, need to be judged with some caution. I suggest three such cautions and offer an alternate explanation for the results on task four.

1. Many of the conclusions stated in this study derive from comparisons with the IQ-matched control groups. However, the nature of the IQ measure used in this study is only sketchily presented. It is identified as the "English Progress Test" and is said to measure "verbal ability and scholastic attainment." What does "scholastic attainment" entail? Could it involve mathematics achievement, visual ability, memory? More confoundingly, could it be that this test has more in common with mathematics giftedness than it does with artistic giftedness? Assuming that it does (and this assumption is likely valid given that most school systems place higher curricular priority on mathematics than on art), then the use of the IQ-matched controls is not equivalent for the mathematically gifted and the artistically gifted. Confounding interpretations could arise.

2. The process of identifying the gifted children was done by the teachers and the headmaster. How valid is such a procedure? Is it reasonable to expect teachers of 12-14 year-old children to be expert at assessing mathematical giftedness? If it is, then does this not imply that mathematical giftedness is not a particularly complex construct, and therefore that studies such as the present are probing into commonsensical relationships? I suggest that the answer to this last question is no, and therefore, on the basis of contrapositional logic, it is not reasonable to presume that ordinary teachers are expert assessors of mathematical giftedness. Similar comments could be made about the headmaster and about the process used to identify the artistically gifted children.

3. The authors spend much of their discussion focusing upon the superior performance of the artistically gifted children on the fourth task: identifying drawings on the basis of minimal information. They speculate that artistic children have the ability to "draw on

information represented in some internal "lexical" system which can be accessed to supplement incoming data" (p. 156). Since the mathematically gifted children did significantly poorer on these tasks, it might be concluded that they have less facility in drawing on this internal "lexical" system. Furthermore, the authors suggest that such facility could be a "component feature" of creativity, and in this sense the mathematically gifted would be less creative. Let me offer an alternate explanation. I would suggest that children who are identified as mathematically gifted by their teacher and principal are likely to be those who do well on mathematics achievement tests and who are "good" pupils in general. Such children are likely clever and careful analysts of mathematical procedures, particularly those likely to appear on tests. In contrast, students who are not so careful, who may often go off on a tangent wasting time thinking about topics that may not be on the test, would not likely be in the "top 2 per cent". However, may not these students be precisely the ones who would do well on task four? Since they are not likely to be in the sample of "mathematically gifted" students as chosen in this study, the differences between the mathematically gifted and the artistically gifted in regard to "creativity" obtained in this study may be aritifactual.

A Closing Comment

Despite the cautions described above (it is easy to identify cautions for any study), this study was not only well worth doing, it is well worth reading. It sensitizes us to the artistic nature of mathematics as well as to its verbal-spatial articulation in thought and in language. The authors offer some intriguing explanations of differences between the artistically gifted and the mathematically gifted. Nevertheless, I would be cautious in accepting the authors' interpretation that artistic giftedness has more to do with creativity than does mathematical giftedness. Their interpretation, I submit, is an artifact of the design of their study.

Kissane, Barry V. SELECTION OF MATHEMATICALLY TALENTED STUDENTS. Educational Studies in Mathematics 17: 221-241; August 1986.

Abstract and comments prepared for I.M.E. by THOMAS E. ROWAN, Montogomery County Public Schools, Rockville, Maryland.

1. Purpose

This study had as its first purpose the investigation of the use of the Scholastic Aptitude Test (SAT) for the selection of mathematically talented students at the beginning of secondary school (approximately 13 years of age) in Australia. It also studied two other questions, the extent to which teachers correctly nominated their most able mathematically talented students and the degree to which these younger students responded differently from the older student population which usually takes the SAT (approximate age 17).

2. Rationale

The Study of Mathematically Precocious Youth (SMPY) at the Johns Hopkins University and various other studies have supported the use of the SAT as a screening instrument for young, mathematically talented students. It appears to have several advantages. These include its difficulty -- which would eliminate any ceiling effect, its careful vertical equating -- which permits comparisons across forms and administrations, its focus on mathematical reasoning rather than achievement, and its validity for predicting the likely ability of young students. This study was intended to collect further evidence related to the previous studies which support this use of the SAT. In addition to validating prior work on the use of the SAT for this purpose, the study investigated whether there were differences in response patterns between younger and older students which could further reinforce its value when used for this type of selection process. In fact, young students may be using their reasoning powers more than older ones on the SAT because they do not have the background knowledge of the older students. Finally, if teacher judgment is not very reliable as a method of selecting talented students, then the SAT is all the more attractive.

3. Research Design and Procedures

Teachers from five high schools in Perth, Australia were asked to nominate year 8 students (approximately 13 years of age) for special extension classes in mathematics, English, science, and social science. The nominations were to be based on either current high performance or potential that the teachers felt the students had in the subject. Ninety-eight students were nominated altogether. These students were then administered the mathematics and verbal sections of the SAT. The test was administered to all recommended students, not just those who were recommended in mathematics. Students were told that the results of the test would be used to select participants in a special extension program. A comparison group was obtained by administering the SAT to 70 year 11 students at two other metropolitan high schools in Perth. These 70 students were in approximately the upper 7-8% of the age cohort in terms of mathematical achievement. All students who were administered the test were prepared in a similar manner with practice items and test-taking information to correct for possible pre-test differences. All scores were corrected for guessing according to the scoring procedures specified by the College Board. Additional comparisons were made with United States national data showing the performance of USA juniors and seniors. These data were obtained from the College Board.

The data for all groups compared are presented graphically and in tables and are studied both visually and through statistical analyses. Comparisons are made between the scores of year 8 and year 11 students. Male versus female performance is also studied. The pattern of student responses is studied to determine whether certain items are missed more frequently by year 11 students when compared to year 8 students. The results of the teacher selection process are compared with two approaches which used the SAT scores. One of these simply identified the top 25 students, the other identified all students who scored over 400 on the mathematics section of the SAT.

4. Findings

By presenting and commenting on the graphic display of the SAT results for the various groups of the students, the author drew several conclusions. It was noted that many year 8 students obtained very low scores, while others obtained scores which exceeded the means of all of the comparison groups. This was cited as evidence that the test has sufficient ceiling for the talented year 8 students. It was noted that mean scores of males were higher than females' scores ($p < 0.05$) and that this appeared to be caused by a lack of very high-scoring females. A substantial portion of the paper was allotted to the discussion of this finding.

Teacher nomination data were presented graphically and in tables, and comparisons were made to similarly presented data for the SAT. Sex differences were also considered in this comparison. If the goal had been to select the top 25 students for the special program, then 14 of them would have been correctly selected by the teachers, 12 males and 2 females. Another 11 not nominated by the teachers would have rounded out the top 25, 6 males and 5 females. Similarly, if all students who scored over 400 on the mathematics section of the SAT were to be selected, then the teacher nominees would have been accurate for 23 students, 18 males and 5 females. The remaining 21 who scored over 400 included 9 males and 12 females who were not selected by the teachers.

Direct observation of the data was used along with several statistical procedures to determine whether there are differences in response patterns on the SAT between younger and older students. A number of items were identified as exhibiting differential patterns of response. The nature of the items was examined and discussed, but no conclusions were reached as to the cause of the differences in response pattern. It was suggested that older students might be performing better on items which required learned knowledge and younger students performing better on items which were dependent upon reasoning.

5. Interpretation

The author found that the SAT is a useful means of selecting young Australian students for special mathematics programs. He also found that the test does not have a ceiling problem. Marked sex differences were observed in the test results and were "not readily explained". Teacher nomination was found to be a poor method of selecting students who are mathematically talented. Finally, differences in item response patterns were found between younger and older students, although acceptable explanations for these differences were not found.

Abstractor's Comments

This was a very interesting study which investigated current and pertinent issues in mathematics education. The findings which were reported seemed to be supported by the data and were in agreement with other studies of the same questions. The use of the SAT as a selection instrument for mathematically talented young students may well appeal to many school systems which are seeking better ways of making such selections. If the SAT is a reliable instrument for this purpose, and is relatively independent of previous achievement, then it would help to balance out differences which are created by the varied strengths of elementary school teachers with respect to mathematics knowledge and teaching strategies. Many school systems rely on achievement tests or teacher judgment, both of which seem unlikely to provide equality of opportunity for selection. This study clearly indicated that the SAT identified more students than teacher nomination. The SAT also yielded a better balance of males and females. Despite this better balance of males and females in the group selected, the study still pointed up marked differences in performance on the SAT between males and females, with males scoring significantly higher. No clear reason was found for the different performance of males and females. However, it was suggested that the lowest scores of females might have been on items which were dependent less on background knowledge and more on reasoning of a type which would have been strengthened by certain kinds of out-of-school experiences not usually afforded females.

A great deal of space in the article was devoted to exploring and discussing the existence of, and possible causes for, different item response patterns between younger and older students. It was concluded that differences exist, but no explanation for them was found to be satisfactory. The importance of this issue apparently relates to the nature of the SAT and its potential in reliably identifying talented young students. If young students score better on items which are more dependent on reasoning and less dependent on background knowledge, then the accuracy of the selection may be considered to be further reinforced, since the aim is to get the students with the best mathematical reasoning abilities into the program for talented students.

While this abstractor found the article interesting and pertinent, several questions arose with regard to the implementation of the study and its findings. The paper does not present a clear statement of the questions or hypotheses being investigated. Similarly, the findings are presented in a rather loose narrative fashion which is not explicitly related to the issues under investigation. In fact, one issue which is discussed extensively, and about which findings are presented, is not mentioned at all in the initial presentation of the questions being investigated. That is the differential performance between males and females. In identifying the hypotheses, this writer really inferred the first one, since it was not explicitly stated. In general, the presentation is rambling and somewhat hard to follow. Very little effort is made to point out some of the possible weaknesses in the overall study. For example, United States performance data are used, but there is no discussion of the potentially wide differences between the population producing those data and the Australian population under study. It is not even stated why the US data were deemed important to the study. And, in fact, these data are not used extensively. Even if such population differences are relatively obvious, it would be better if the writer stated them. At one point it is stated that the year 8 Australian students had scores comparable to the US students "presumably with a good deal more mathematics coursework". It is not discussed why the latter is a valid assumption.

A discussion of the "speededness" of the test is presented in the "Findings" section of the paper, rather than in the research methodology section where it might be more logically placed.

In the discussion of male-female performance, it is pointed out that more boys than girls were correctly nominated by the teachers. It is not noted that the numbers of students who were not nominated by the teachers and then appeared to be acceptable via the SAT scores were nearly equally distributed between males and females.

Despite these many questions about the study and its presentation, the paper is interesting nonetheless. Good means of selecting young talented students are needed, especially if those means prove to be somewhat independent of previously learned content and less biased against females. It might be well to have a followup study of females selected by the SAT to determine their future success in advanced mathematics courses. Also, would a subset of the SAT items produce an instrument which would be even better for identifying young, mathematically talented students? This seems possible if one looks at the differential scoring of younger and older students. On the other hand, would such an instrument discriminate more against females, who are purported by the writer to do better on knowledge items and less well on reasoning items?

In summary, this was an interesting study on an issue of continuing importance in mathematics education. This abstractor would have liked to have seen a better-organized presentation of the study and its findings. However, and despite these reservations, the study seems to have been reasonably thorough, and the findings support existing information as well as suggest some new avenues for investigation.

Moore, Elsie G. J. and Smith, A. Wade. SEX AND ETHNIC GROUP DIFFERENCES IN MATHEMATICS ACHIEVEMENT: RESULTS FROM THE NATIONAL LONGITUDINAL STUDY. Journal for Research in Mathematics Education 18: 25-36; January 1987.

Abstract and comments prepared for I.M.E. by ALBA G. THOMPSON, Illinois State University, Normal, Illinois.

1. Purpose

This study compared the mathematics competence of young people who varied in ethnic background and amount of schooling. The results describe differences in the average performance of various youth groups in two mathematics tests administered in 1980 as part of the National Longitudinal Study of Labor Force Behavior (NLS). Data were obtained from youth between the ages of 15 and 22 in a national probability sample of nearly 12,000 men and women. The study does not address the causes of the differences in mathematics competence observed among the youth groups.

2. Rationale

Although there has been a considerable amount of research on sex and ethnic group differences in mathematics achievement, most of this research has been limited to youth who were enrolled in school at the time of the assessment (e.g., NAEP and SAT). Using data from a national probability sample, the authors sought to provide a "broader view of mathematics competence of the general population of youth than results from surveys of schoolchildren or those obtained from college-bound high school seniors." In addition to sex and ethnic group differences the study examined the effects of amount of schooling on mathematical competence.

3. Research Design and Procedures

Data for the study were obtained from a sample of 11,914 young men and women who participated in the NLS. The NLS used three independent samples: (a) a full probability sample of the non-institutionalized civilian segment of the American population, aged 14 to 21 years as of January 1979; (b) a supplemental oversample of civilian black, Hispanic, and economically disadvantaged white youth of the same age range; and (c) a probability sample of youth aged 17 to 21 and serving in the military as of 30 September 1978. Complete data were available for 11,496 of the NLS respondents.

The study used a multivariate analysis of variance to examine the differences in the performance of various youth groups on the two mathematics tests of the NLS: Arithmetic Reasoning and Mathematics Knowledge. A 2 x 3 x 4 design resulted by crossing sex, ethnic group (black, white, Hispanic), and highest grade completed (K-8, 9-11, 12, 13 and above).

4. Findings

The performance of all groups on both tests, Mathematics Knowledge and Arithmetic Reasoning, improved with increased education. However, large differences in average performance were observed between ethnic groups with the same amount of formal education. Whites had the highest scores on both tests in all categories. Generally, Hispanics scored higher than blacks, and males outperformed their female counterparts. The analysis showed no significant three-way interaction. Significant two-way interactions resulted between education and sex, education and ethnicity, and sex and ethnicity on both tests.

On the Mathematics Knowledge test, the education-by-sex interaction was due to the combination of: (a) significantly higher scores of females in the Grades K-8 group compared to their male counterparts; (b) no significant sex differences in the Grades 9-11 group; and, (c) significantly higher scores of males over their female counterparts in the other two education groups (Grade 12, Grades 13 and over). The education-by-ethnicity interaction was due to the combination of: (a) no significant difference between Hispanics and blacks in the Grades K-8 group; (b) significant difference in favor of Hispanics over blacks in the remaining three groups, with the greatest difference at the highest educational level; and (c) greatest differences in favor of whites over Hispanics at the lowest and the highest educational levels, with a nearly constant difference in favor of whites at the other levels. The sex-by-ethnicity interaction was due to a variation in the magnitude of the difference between male and female scores across each ethnic group, with a greater male advantage among whites and Hispanics than among blacks.

On the Arithmetic Reasoning test, the education-by-sex interaction was due to a pattern of increasing sex difference in favor of males with an increase in educational level. The education-by-ethnicity interaction was due to a combination of: (a) whites outperforming Hispanics, with a significantly greater difference in their average performance at the highest educational level than at the lowest educational level and an almost constant difference at the other levels; and (b) a pattern of increasing black-white differences and black-Hispanic differences, in favor of Hispanics, with an increase in educational level. The sex-by-ethnicity interaction was due to significantly greater sex differences among Hispanics than among the whites and the blacks, whose average performance difference in favor of males was similar.

5. Interpretations

The NLS data reveal the same general pattern of effects of sex, ethnic group, and education on mathematics test performance as those from previous investigations. Performance improves with increased education, males generally outperform females, and whites perform at a higher level than Hispanics, who perform higher than blacks.

Differential course-taking in high school appears to be a significant factor in the intergroup sex differences observed. Sex differences in favor of males are first observed in the Grade 9-11 group, the years when differentiated programs of study are common, and those differences increase with further education. In the case of ethnic groups, large differences in performance between whites and Hispanics and between whites and blacks were observed in all groups, including the Grades K-8, the years when students generally experience a uniform curriculum. This finding raises questions about the quality of the mathematics education that minority youth experience in the elementary and junior high school years.

The greatest differences between ethnic groups, favoring whites, were observed among youth with some college background. One would not anticipate this finding in view of the selection procedures involved in college admissions. However, this may reflect differences in college majors pursued by youth in different ethnic groups.

The factors that contribute to the lower average performance of blacks, in general, may function to suppress sex differentiation of performance in this group. However, this was not the case among Hispanics, who also show a general pattern of lower performance than whites, and for whom sex differentiation of performance was even larger than that observed among whites.

The authors conclude with a call for more research to identify factors that contribute to the low mean performance of blacks and Hispanics. Although studies point to differential course taking as an important factor, the causes of the underrepresentation of these minorities in academic track mathematics courses have not been determined, nor does course-taking account for all the variance. Therefore, there is a need for research to examine other factors that may contribute to the differential in performance, such as the influence of instructional practices and personal characteristics of black and Hispanic students on mathematics achievement.

Abstractor's Comments

The contribution of the study to the already existing large body of research on the effects of sex, ethnic group, and education on mathematics achievement lies in the sample used. Most of the previous research sampled the "in-school" youth population. Using data from the National Longitudinal Study of Labor Force Behavior, the authors studied a sample representative of the national youth population. Therefore this study provides a broader view of the mathematics competence of the general youth population than previous investigations. The size of the sample (11,496) is also considerably larger than the samples that typically have been studied.

The findings are in line with those from previous investigations. In this sense, the study adds little to what we already know about the effects of sex, ethnicity, and amount of schooling on mathematics achievement. The interesting interactions observed between education and sex, education and ethnicity, and sex and ethnicity suggest important questions that should stimulate research into the social and cultural factors contributing to these effects. It is not until this kind of research is undertaken that we will begin to gain insights into the factors that contribute to the low performance of females and members of minority groups in mathematics.

Movshovitz-Hadar; Inbar, S.; and Zaslavsky, O. STUDENTS' DISTORTIONS OF THEOREMS. Focus on Learning Problems in Mathematics 8: 49-57; Winter 1986.

Abstract and comments prepared for I.M.E. by WILLIAM E. GEESLIN, University of New Hampshire.

1. Purpose

The purpose of the study was to analyze student errors in applying mathematical theorems to open-ended test problems. Written solutions to the Israeli mathematics matriculation exam were subjected to a content-oriented analysis.

2. Rationale

High school students are expected to learn a large number of varied theorems in their mathematics classes. Matriculation exams require the accurate recall and application of many of these in order to solve test problems. Our knowledge of students' errors (distortions) in recalling and/or applying these theorems may be useful in designing, constructing, and implementing the mathematics curriculum and associated classroom teaching practices.

3. Research Design and Procedures

The authors examined 820 eleventh-grade (age 17) students' answers to 18 open-ended test items from the Israeli mathematics matriculation exam. Subjects came from two successive years of examinees. The exam covered typical high school college preparation mathematical topics from algebra, geometry, probability and statistics, and trigonometry. No examples from probability and statistics were reported in this article. The authors outline examples of theorem distortions and list categories of errors.

In order to learn more about procedures, the reader must consult Movshovitz-Hadar, Zaslavsky, and Inbar (1987).

4. Findings

An analysis reported elsewhere (dated 1984 in this article but not listed in the reference section; probably the 1987 JRME article noted above) developed the classification system consisting "of the following categories: (1) misused data; (2) misinterpreted language; (3) logically invalid inference; (4) distorted theorem or definition; (5) unverified solution; [and] (6) technical errors" (p. 49). This article gives eleven examples of distorted theorems or definitions, which was the category containing the largest percentage of errors (32%).

5. Interpretations

The authors grouped the errors into (1) distortions of the antecedent and (2) distortions of the consequent. In essence, error type (1) is misapplying the theorem by applying it to a problem situation which does not satisfy the condition stated in the theorem. Error type (2) is altering the claim made by the theorem and using the altered claim to work the problem. The authors explain these errors as resulting from rote memorization of abbreviated forms and overlearning of rules while ignoring conditions, respectively. The authors recommend that teachers examine distorted theorems with students and spend more class time focusing on exact statements and the precise meanings of symbols. They also recommend not allowing shortcuts, abbreviations, or incomplete statements in solving problems.

Abstractor's Comments

In this article Movshovitz-Hadar and Zaslavsky primarily give examples of student mistakes and make teaching recommendations they feel will reduce the future incidence of similar errors. As such they do not present a "research study," but rather interpretations based on a study reported elsewhere. Editing and/or writing problems make it impossible to be certain that the study on which this article is based is the one reported by Movshovitz-Hadar et at. (1987). However, since it is likely that the 1987 article is a report of the study, some comments are in order. It may well be that the number of subjects noted above (820) is the number of student re ponses (or possibly erroneous responses) rather than the number of subjects analyzed. The adequacy of the error classification can be judged only by reading the 1987 article. The actual procedures are not described in this article being abstracted. As a result the authors do a disservice to the novice or careless reader by not clearly distinguishing "research" from "interpretation and extension." The research was aimed at developing an error classification scheme based solely on student written responses. The researchers did not observe or interview students. The Focus article abstracted here generalizes well beyond any data obtained from research or even psychological theory. Stating "further research is needed in order to find out the extent to which the preventive treatments and remediation measures recommended above actually contribute to students ability and to avoid (sic) distortions of theorems" (p. 57) is probably an insufficient warning to the classroom teacher. The authors at least should establish that some students do, in fact, use "distorted theorems" in the manner suggested. That is, the authors' explanations for each type of error may not model the students' thought processes. No research was noted on the causes of these errors, how they are formed, etc. Placed in the proper context, the authors do provide hypotheses for future research and important reflections for the classroom teacher. The researcher would need to read the 1987 article. I have avoided commenting in depth on the study since little information was given in the article abstracted here.

Reference

Movshovitz-Hadar, N., Zaslavsky, O., and Inbar, S. An Empirical Model for Errors in High School Mathematics. Journal of Research in Mathematics Education 18: 3-14; 1987.

Movshovitz-Hadour, Nitsa; Zaslavsky, Orit; and Inbar, Shlomo. AN EMPIRICAL CLASSIFICATION MODEL FOR ERRORS IN HIGH SCHOOL MATHEMATICS. Journal for Research in Mathematics Education 18: 3-14; January 1987.

Abstract and comments prepared for I.M.E. by JON M. ENGELHARDT, Arizona State University.

1. Purpose

The purpose of this study was to develop an empirical classification model for errors in high school mathematics and to demonstrate its reliability. 'Empirical' was taken to mean that the data for generating the model was students' documented performance (in this case, performance on a national comprehensive examination).

2. Rationale

Much of the research on error analysis has been concerned with errors in arithmetic. The researchers argue that in order to better understand the mathematics difficulties of high school students, their erroneous mathematical behavior has become increasingly of interest in the last decade. Thus a qualitative analysis of high school mathematics errors is appropriate.

The researchers professed to work purely from an analysis of students' written work without any preset theoretical orientation. Furthermore, they sought to develop a classification scheme in which error categories were mutually exclusive and a given error could be clearly classified by a single category.

3. Research Design and Procedures

The study was conducted over a two-year period. One hundred ten answer books, selected at random from the eleventh-grade comprehensive examination given throughout academic high schools in Israel, were selected during each of two successive years. Each year the solution to 18 open-ended items were analyzed, covering the following topics: linear and quadratic functions, linear and quadratic equations, powers and logarithims, arithmetical and geometrical series, plane and solid geometry, elementary statistics, probability, and trigonometry.

During the first year, errors occurring in five or more answer books were analyzed. The researchers professed to look for the "logic" the students seemed to follow in coming up with incorrect answers (also incorrect procedures that netted "correct" solutions). Five error clusters (categories) were initially identified. To judge the reliability of these, a set of four raters (experienced high school mathematics teachers) were asked to classify the 150 identified errors; 75% were identically categorized. Based upon interviews with coders and an analysis of the items where disagreements occurred, the category descriptions were refined, and new raters applied the category system -- this time with 88% agreement.

During the second year, the category system was reconfirmed. A sixth category was found to be needed, however; it was added to the devised scheme.

4. Findings

Based upon the above analysis six categories of errors in high school mathematics thus were identified and defined:

1. Misused data
2. Misinterpreted language
3. Logically invalid inference
4. Distorted theorem or definition
5. Unverified solution
6. Technical error

These categories were shown to be inclusive and mutually exclusive. The researchers presented the proportion of each type of error in each year's error analysis.

5. Interpretations

The investigators felt that their classification system was appropriately operational for high school mathematics and serves as a model for classifying errors according to student logic, rather than according to a projected cause. Despite suggesting various cautions with regard to generalizing the study, they did pose possibilities of the model and data for designating test item difficulty on the basis of possible error types provoked, for forseeing possible difficulties as a consideration in planning instruction, and for generating a predictive model of high school mathematics errors.

Abstractor's Comments

This study was both an attempt to extend earlier arithmetic error analysis efforts to high school mathematics and to generate a model that focuses on students' logic for erroneous behavior. The researchers were successful on both counts. While some of the criticism of earlier error analysis efforts may be misplaced, the results of this study clearly advance the knowledge base concerning high school mathematics difficulties. Error categories were sufficiently defined to permit the reliable classification of "logic" errors, seemingly with a minimum of speculation.

Considering all the effort the researchers went to in identifying, defining, redefining, etc. the error categories, it seems strange that, even on a small pilot basis, they didn't take a sample of students who exhibited a given category of error and interview them to confirm the erroneous "logic" as projected from the error category. Such an attempt at a validity check would seem to have added greater credibility to the findings.

A second concern, while not related to the conduct of the study, relates to the researchers' awareness of their own biases and related influence on the results. The researchers made a big point of trying to avoid preconceived bias by identifying/classifying students' mathematical errors "assuming no theory to begin with". Yet they seem to have failed to realize that defining "constructive analysis" of errors as looking for the erroneous "logic" of students created just that very bias, no less than Radatz' erroneous information processing mechanisms orientation.

Researchers, curriculum writers, and teachers of high school mathematics will find the error categories helpful in thinking about mathematics students, their difficulties, and educational practice.

Sherman, Lawrence W. and Thomas, Mary. MATHEMATICS ACHIEVEMENT IN COOPERATIVE VERSUS INDIVIDUALISTIC GOAL-STRUCTURED HIGH SCHOOL CLASSROOMS. Journal of Educational Research 79: 169-172; January/February 1987.

Abstract and comments prepared for I.M.E. by J. DAN KNIFONG, Marymount University, Arlington, Virginia.

1. Purpose

The purpose was to discover whether mathematics achievement was better in a "cooperative" classroom versus an individualistic classroom as defined by Slavin.

2. Rationale

Most of us can recall times during our student days when we were motivated to study mathematics by a sense of competitiveness (often with a friend) and other times when we were motivated by a sense of cooperation (sometimes with the same friend). For some time now there have been a string of studies (Slavin is reported to have listed 46 such studies) which define two social "climates" as possible for mathematics classrooms: 1. small-group cooperation and 2. individualist competition. This replication study is another in that tradition.

3. Research Design and Procedures

Two midwestern, middle-class, general mathematics classrooms were chosen. The students were predominately low-achieving freshmen and sophomores trying to complete the minimum state requirement in mathematics. The "cooperative" class was taught by a teacher trained in cooperative techniques. The "individualistic" class was taught by a faculty member who volunteered to teach using her usual class structure. The treatment for both groups was 25 days of instruction on computation and interpretation of percentages. A 30-item pre/post test was given before instruction began and again at the end of instruction. ANOVA was used to analyze the data.

4. Findings

The two class means on the pretest were not significantly different (both were just a little more than three correct out of 30). The means on the posttest were significantly different, favoring the cooperative class. Both classes showed substantial gains. The individualistic class gained about 10 points and the cooperative class gained about 17.

5. Interpretations

The findings are interpreted as supporting Slavin's (1980, 1983, and 1984) and Deutch's (1949) position that group study results in higher achievement than individual study. It is concluded that "teachers of general mathematics and other disciplines should give this approach serious and favorable consideration."

Abstractor's Comments

Intuitively as a teacher, I am drawn to the belief that cooperative, small-group, mathematics classrooms will result in high mathematics achievement (and probably low classroom disturbances as well). Professionally as a researcher, I remain skeptical that "classroom structure" can be such a decisive factor in mathematics achievement and that cooperative structure is the best. I agree with Sherman and Thomas when they point out that a serious challenge to their interpretation (that cooperative classroom structures are best) is the uncontrolled influence of the teachers used in the two classrooms. It is this influence which leaves me so skeptical of their findings.

Sherman and Thomas suggest that one method of remediating this problem of teacher influence (which plagues many education studies) is to use the same teacher in both classrooms or possibly by randomly assigning the teachers. I have never been convinced by studies which use the same teacher for two different treatments. It reminds me of doctors dispensing two different kinds of pills to patients. In the case of doctors it is well known that the mere knowledge on the part of the doctor of which pills are which will prevent the comparative measurement of a drug's effectiveness. For this reason the medical community insists on "double blind" studies where the dispensing doctor does not know which of the two drugs he is dispensing. It would seem to me that there is no way to design a "double blind" teacher treatment study. Since I see no way of controlling the teacher effect directly, I can only suggest randomizing it by choosing 30 teacher/classrooms for one treatment and 30 teacher/classrooms for a second treatment. This is seldom done because of the expense and trouble.

Span, Pieter and Overtoom-Corsmit, Ruth. INFORMATION PROCESSING BY INTELLECTUALLY GIFTED PUPILS SOLVING MATHEMATICAL PROBLEMS. Educational Studies in Mathematics 17: 273-295; August 1986.

Abstract and comments prepared for I.M.E. by PEGGY A. HOUSE, University of Minnesota.

1. Purpose

The study sought to investigate differences in the ways gifted students process information when solving mathematics problems as compared to the approaches employed by "averagely gifted" pupils.

2. Rationale

The investigation was rooted in an information processing approach to intelligence applied to the mathematical activity in which pupils solve problems while in dialog with teachers or other observers. In particular, the investigators were interested in the psychological processes employed by gifted children when they solve mathematical problems. In this regard, "giftedness" was considered to be general intellectual ability rather than specific mathematical ability. It was hypothesized that knowledge of the differences in the ways that gifted children process information when solving mathematical problems, if such differences could be found, would not only contribute to an understanding of giftedness, but also would suggest ways in which teachers might adapt classroom instruction.

3. Research Design and Procedures

Subjects of the research were reported to be "14 relatively highly gifted pupils (7 boys and 7 girls) and 14 averagely gifted pupils (7 boys and 7 girls)" from the second form of lower secondary education in the Netherlands. The independent variable of general intellectual ability was defined as the average of the scores on three standardized tests: a test of general information, the Raven Progressive Matrices test, and a test of creativity. A composite standardized test score above 94% was considered to indicate high intellectual ability. In addition, teachers were surveyed for their judgments about the pupils.

From a po... of 399 pupils tested, ten were nominated both by test score and by teacher judgment; these ten became part of the experimental group. Seventeen others who scored high on the tests but were not nominated by teachers also were added to the experimental group. (No information is given to explain how the 14 gifted subjects referred to above were selected from among these 27.) The control group of averagely gifted pupils was selected randomly from those with test scores between 33% and 66%.

Subjects were presented individually with seven problems and requested to think aloud while solving each. Interviews were tape recorded. The problems were assumed to be "difficult or very difficult" so that the pupils could not respond merely by recalling previously learned information or algorithms, although at least one item, a straightforward age problem, appears to have been too easy for the pupils. A pupil's initial response, following the reading of the problem by the experimenter, was termed the "first production." It was followed by an intervention from the experimenter who, depending on the pupil's reply, asked attention-steering questions like, "What are you thinking now?", suggested an alternative point of view, or requested the pupil first solve a less complex subproblem. Follow-up questions probed for reasons why the pupil solved the problem a particular way or whether the pupil could find an alternative solution method.

Tapes of the interviews were analyzed and first scored on the basis of correctness of solutions. An initial four-point scale ranging from perfect (3 points) to not solved (0 points) was collapsed into a binary score of correct or incorrect, and a chi-square analysis was performed. Subsequent scorings of the protocols focused on three parts of the problem-solving process: orientation (analysis of means and objectives, planning, and reflection); execution, and evaluation. Each of these was quantified in a manner similar to the correctness scale using a scheme of 0-3 or 0-2.

4. Findings

The chi-squared analysis indicated that the experimental group solved five of the seven problems more successfully than the control group. The gifted students reached a solution faster, using a smaller number of productions, and requiring fewer interventions than did the average pupils.

Comparison of the subjects' scores on a 4-point orientation scale showed that gifted students took more time to orient themselves to the problems, reflecting on what to do and planning their approaches. By contrast, the average pupils immediately tried to solve the problem before they understood what was required. In execution, average pupils preferred a trial-and-error approach, while gifted pupils attempted other strategies. Gifted pupils also were more likely to evaluate their solutions and to do so without prompting from the experimenter. Finally, the gifted pupils were more successful in responding to a request to "construct a problem like this last one," and in their ability to discuss the strategies they employed.

5. Interpretations

The investigators liken the behavior of the gifted students to the behavior of "experts", while the behavior of the average pupils is compared to that of "novices" in problem solving. They conclude that gifted pupils process information at a different ("higher") level than average pupils and that average students can be taught to process information differently: to orient themselves to the problem, to proceed systematically, and to become conscious of their actions.

They make four recommendations for adjusting instruction so that the gifted are confronted with challenging problems and are made aware of the necessity of analysis and evalution, and they argue for the importance of average pupils having "lively contact with gifted pupils" so that the gifted serve as role models for the average.

Abstractor's Comments

The findings of this study, that gifted children process information differently than do average pupils when solving problems, is consistent with the extensive research of Krutetskii (1976) and with the informal observations of teachers who work with the gifted. There are no surprises here. In general, the study has more to offer by way of support for what is already known than in the addition of new information.

There are some questions to be raised about the study, however. As noted above, it was not clear how the experimental sample was derived since the numbers reported do not seem to coincide. Also, the following statement perplexes the reader: "Because of the intricacies of the raw material we will confine ourselves in this article to all pupils of only one of the experimenters" (p. 276). Does this mean that there were more than 28 subjects and that only some of the data are being reported?

There are other questions that, if answered, would help to interpret the results: For one thing, both experimental and control groups consisted of seven boys and seven girls, yet no mention is made of sex differences in performances. Since gender has been shown to be a factor in so many other studies, it would be interesting to know its effect in this one. If there was no sex difference in performance, that information, too, would be instructive.

There is no description given of the prior mathematical instruction of the pupils vis-a-vis problem-solving strategies.

The question arises as to whether the mathematical training and experience of the experimental and control subjects were comparable, or whether something in that background would contribute to the greater facility with problem-solving strategies on the part of the gifted pupils.

It is also hard to evaluate the interview process. The information is given that the investigator made interventions that included suggesting alternate strategies or requesting the solution of subproblems. Not enough information is presented, however, to assure the reader that these interventions were controlled and consistent, or that they did not constitute "giving away" the solution to some of the subjects.

With regard to the conclusions, other considerations arise. For one, care must be taken in generalizing too much on the basis of so small a sample. Also, there are some "leaps" apparent in the conclusions that go beyond the reported data. For example, three of the four recommendations for adjusting instruction for the gifted suggest that gifted pupils must be confronted regularly with difficult problems, must be made to recognize the importance of conscious processing of information, and must be stimulated to persist in their search for a solution in the face of difficulties. These suggestions have not been "put to the test" in the sense of demonstrating that overt efforts to adhere to them do enhance problem-solving facility, although the "common sense" appeal of the recommendations is compelling. The fourth recommendation states that, "It is necessary for the teacher as compared to the pupil to be an expert in the field." The validity of this statement as a conclusion derived from the experiment is highly speculative since the role and influence of the teacher was not a dimension in this investigation.

The claim also is made that "average pupils may be instructed how to process information differently." This statement would more properly be offered as a hypothesis needing verification.

To recognize that there is a difference in the ways in which gifted and average pupils process information is not equivalent to stating that one can be taught to perform like the other. It also does not address the question of how to undertake such an instructional task. The questions of whether and how average pupils can be taught to emulate the problem-solving behaviors of the gifted are open to investigation. The answers to those questions could significantly further the goals of enabling all pupils to become better problem solvers.

Reference

Krutetskii, V. A. The Psychology of Mathematical Abilities in Schoolchildren. (Translated from the Russian by Joan Teller; edited by Jeremy Kilpatrick and Izaak Wirszup.) Chicago: University of Chicago Press, 1976.

Wilson, Patricia S. FEATURE FREQUENCY AND THE USE OF NEGATIVE INSTANCES IN A GEOMETRIC TASK. Journal for Research in Mathematics Education 17: 130-139; March 1986.

Abstract and comments prepared for I.M.E. by MORRIS K. LAI, University of Hawaii.

1. Purpose

The study was designed to investigate the effect of frequency of irrelevant features in learning a conjunctive (two or more relevant features) concept. In particular, the concept of altitude of a triangle was used.

2. Rationale

Some previous psychological studies have found sequences of positive instances to be more useful to a learner than sequences of positive and negative instances. Other studies using classroom-related content have found mixed sequences to be more useful than sequences of all positive instances. A variable that might help explain these differences in findings is the frequency of irrelevant features. A learner's need for a negative instance may be influenced by the frequency of an irrelevant feature.

The concept of altitude of a triangle was chosen because it is a common classroom concept that is often misunderstood. Relevant features (dimensions) are (a) originating at a vertex and (b) terminating perpendicular to a line containing the side opposite the vertex. Irrelevant dimensions selected were (a) orientation of the triangle, (b) slope of the proposed altitude (vertical/not vertical), and (c) location of the proposed altitude (inside/outside).

3. Research Design and Procedures

The initial sample consisted of 120 algebra students from five classes in a suburban junior high school. Students took a pretest designed to determine (a) their ability to identify the features of a binary conjunctive concept and (b) their ability to use negative instances.

A set of 20 five-sequence instruments, each containing instances of triangles with dashed line segments representing proposed altitudes, was used as follows: Each instance was accompanied by feedback indicating whether the figure was a positive or negative instance of the concept. After each instance, the student was asked to identify the two features relevant to the concept. The dependent variable was the number of trials before the correct pair of critical features was reported consistently for all the remaining instances.

Over a period of two class days, students received instruction on the rule of conjunction, the use of negative instances, and the five dimensions mentioned previously.

Each of 100 students was randomly assigned to one of four treatment groups formed by crossing two levels of feature frequency--(a) the two irrelevant features were equally likely or (b) one feature appeared 90% and the other appeared 10% of the time with two levels of sequence type--(1) contained only positive instances or (2) contained alternating positive and negative instances.

4. Findings

There were no statistically significant differences ($\underline{p} < .05$) among treatment groups on the pretest. An analysis of variance found a statistically significant interaction between the sequence condition and the feature frequency condition. The analysis also showed that the students in the unequal-feature frequency groups had more trials to criterion than the students in the equal-feature-frequency treatment. The treatments having one feature occurring mu i more frequently than the other feature were more difficult.

5. Interpretations

The interaction found confirms and potentially explains the conflicting results between the psychological research favoring positive instances and the educational research favoring mixed instances. It suggests that the mixture is helpful when some of the irrelevant features appear often.

In a sequence where every feature of every irrelevant dimension has the same probability of appearing, positive instances may be sufficient to identify the relevant features. Such a variety, however, is often not feasible in a classroom.

Abstractor's Comments

Overall I found the article to be well-written and based on a sound research design. In my abstracting I sometimes chose to use much of the author's own words inasmuch as the writing was often so precise. Some confusion might have been promoted, however, by the author's use of "features" and "dimensions" somewhat inconsistently. For example, she lists three "irrelevant dimensions" right after listing the two "relevant features." Then later she mentions "the five dimensions and the two levels or feature of each dimension."

The major criticism I have deals with the use of analysis of variance without any explicit mention of size of effects or measure of association. Since no statistical power analysis was performed and no post hoc contrasts were presented, it is difficult to determine whether the results had any practical significance in addition to the reported statistical significance.

The author was appropriately cautious in warning that "further extensions are necessary before generalizations can be made about learning in the classroom." I would go even further in noting that the author has shown rather clearly that feature frequency and type can affect learning about the concept of altitude of a triangle. An enormous amount of additional research would be necessary before any useful generalizations could be made even within the domain of mathematics education at the junior high school level.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
July - September 1986

EJ 333 173 Hollinger, Constance L. Career Aspirations as a Function of Holland Personality Type among Mathematically Talented Female Adolescents. Journal for the Education of the Gifted, v9 n2, 133-45, Winter 1986.

EJ 333 498 Cobb, Paul. Concrete Can Be Abstract: A Case Study. Educational Studies in Mathematics, v17 n1, 37-48, February 1986.

EJ 333 499 Hunting, Robert P. Rachel's Schemes for Constructing Fraction Knowledge. Educational Studies in Mathematics, v17 n1, 49-66, February 1986.

EJ 333 500 Nesher, Pearla; Peled, Irit. Shifts in Reasoning. Educational Studies in Mathematics, v17 n1, 67-79, February 1986.

EJ 333 501 Markovits, Henry. The Curious Effect of Using Drawings in Conditional Reasoning Problems. Educational Studies in Mathematics, v17 n1, 81-87, February 1986.

EJ 333 509 Exploiting Present Opportunities of Computers in Science and Mathematics Education. Journal of Computers in Mathematics and Science Teaching, v5 n1, 15-26, Fall 1985.

EJ 333 514 Janke, Robert W.; Pilkey, Peter J. Microcomputer Diagnosis of Whole Number Computational Errors. Journal of Computers in Mathematics and Science Teaching, v5 n1, 45-51, Fall 1985.

EJ 333 587 Denvir, B.; Brown, M. Understanding of Number Concepts in Low Attaining 7-9 Year Olds: Part I. Development of Descriptive Framework and Diagnostic Instrument. Educational Studies in Mathematics, v17 n1, 15-36, February 1986.

EJ 333 778 Raphael, Dennis; And Others. Debunking the Semestering Myth. Canadian Journal of Education, v11 n1, 36-52, Winter 1986.

EJ 334 316 Messerly, Carol. The Use of Computer-Assisted Instruction in Facilitating the Acquisition of Math Skills with Hearing-Impaired High School Students. Volta Review, v88 n2, 67-77, February-March 1986.

EJ 334 381 Van Erp, J. W. M.; Heshusius, L. Action Psychology: Learning as the Interiorization of Action in Early Instruction of Mathematically Disabled Learners. Journal of Learning Disabilities, v19 n5, 274-79, May 1986.

EJ 335 309 Ismail, Ainon. Applying Prescriptive Test Theory to the Assessment of the Basic Skills Mathematics Curriculum of Singapore. Studies in Educational Evaluation, v12 n1, 31-58, 71-87, 1986.

EJ 335 440 Schmidt, June. Performance on the High School and Beyond Data Set Vocabulary, Reading, and Mathematics Tests Related to Vocational Business Preparation. Journal of Vocational Education Research, v10 n4, 49-62, Fall 1985.

EJ 336 307 Bass, George; And Others. Teaching Basic Skills through Microcomputer Assisted Instruction. Journal of Educational Computing Research, v2 n2, 207-19, 1986.

EJ 336 458 Stanic, George M. A.; Reyes, Laurie Hart. Minorities and Mathematics, Arithmetic Teacher, v33 n7, 12, March 1986.

EJ 336 469 Hembree, Ray; Dessart, Donald J. Effects of Hand-Held Calculators in Precollege Mathematics Education: A Meta-Analysis. Journal for Research in Mathematics Education, v17 n2, 83-99, March 1986.

EJ 336 470 Nesher, Pearla; Katriel, Tamar. Learning Numbers: A Linguistic Perspective. Journal for Research in Mathematics Education, v17 n2, 100-11, March 1986.

EJ 336 471 Close, John S.; Murtagh, Fionn. An Analysis of the Relationships among Computation-Related Skills Using a Hierarchical-Clustering Technique. Journal for Research in Mathematics Education, v17 n2, 112-29, March 1986.

EJ 336 472 Wilson, Patricia S. Feature Frequency and the Use of Negative Instances in a Geometric Task. Journal for Research in Mathematics Education, v17 n2, 130-39, March 1986.

EJ 336 473 Dickey, Edwin M. A Comparison of Advanced Placement and College Students on a Calculus Achievement Test. Journal for Research in Mathematics Education, v17 n2, 140-44, March 1986.

EJ 336 474 Ferguson, Ronald D. Abstraction Anxiety: A Factor of Mathematics Anxiety. Journal for Research in Mathematics Education, v17 n2, 145-50, March 1986.

EJ 336 509 Balka, Don S. Characteristics of Outstanding Secondary School Mathematics Teachers. School Science and Mathematics, v86 n4, 322-26, April 1986.

EJ 336 571 Borasi, Raffaella. Using Errors as Springboards for the Learning of Mathematics: An Introduction. Focus on Learning Problems in Mathematics, v7 n3-4, 1-14, Summer-Fall 1985.

EJ 336 572 Johnson, Ellie. Algebraic and Numerical Explorations Inspired by the Simplification: 16/641/4. Focus on Learning Problems in Mathematics, v7 n3-4, 15-28, Summer-Fall 1985.

EJ 336 573 Stepus, George Wayne. Problems Reducing Fractions: A Case Study. Focus on Learning Problems in Mathematics, v7 n3-4, 29-33, Summer-Fall 1985.

EJ 336 574 Borasi, Raffealla; And Others. Alternative Rules to Operate With Fractions. Focus on Learning Problems in Mathematics, v7 n3-4, 35-51, Summer-Fall 1985.

EJ 336 575 Borasi, Raffaella; Michaelsen, Jean. Discovering the Difference between Fractions and Ratios. Focus on Learning Problems in Mathematics, v7 n3-4, 53-63, Summer-Fall 1985.

EJ 336 576 Borasi, Raffaella. Intuition and Rigor in the Evaluation of Infinite Expressions. Focus on Learning Problems in Mathematics, v7 n3-4, 65-75, Summer-Fall 1985.

EJ 336 577 Borasi, Raffaella. Errors in the Enumeration of Infinite Sets. Focus on Learning Problems in Mathematics, v7 n3-4, 77-89, Summer-Fall 1985.

EJ 336 657 Paul, Douglas J.; And Others. The Effects of Adjusting Readability on the Difficulty of Mathematics Story Problems. Journal for Research in Mathematics Education, v17 n3, 163-71, May 1986.

EJ 336 658 Fuson, Karen C. Teaching Children to Subtract by Counting Up. Journal for Research in Mathematics Education, v17 n3, 172-89, May 1986.

EJ 336 660 Collis, Kevin F.; And Others. A Technique for Assessing Mathematical Problem-Solving Ability. Journal for Research in Mathematics Education, v17 n3, 206-21, May 1986.

EJ 336 661 Bourgeois, Roger D. Third Graders' Ability to Associate Foldout Shapes with Polyhedra. Journal for Research in Mathematics Education, v17 n3, 222-30, May 1986.

EJ 336 662 Hanna, Gila. Sex Differences in the Mathematics Achievement of Eighth Graders in Ontario. Journal for Research in Mathematics Education, v17 n3, 231-37, May 1986.

EJ 336 885 Brody, Linda E.; Benbow, Camilla Persson. Social and Emotional Adjustment of Adolescents Extremely Talented in Verbal or Mathematical Reasoning. Journal of Youth and Adolescence, v15 n1, 1-18, February 1986.

EJ 336 924 Wahlstrom, Merlin; And Others. Comparative Analysis of Ontario Senior Division Mathematics Achievement 1968-82. Canadian Journal of Education, v11 n2, 174-79, Spring 1986.

EJ 336 934 Fong, Geoffrey T.; And Others. The Effects of Statistical Training on Thinking about Everyday Problems. Cognitive Psychology, v18 n3, 253-92, July 1986.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN
RESOURCES IN EDUCATION
July - September 1986

ED 266 932 Culler, Katharyn; And Others. University of California, Berkeley's Undergraduate Honors Program for Minority Mathematics and Science Students-The Math/Science Workshop Program. Final Report. 30p. MF01/PC02 Plus Postage. Available from EDRS.

ED 266 935 Morris, Robert, Ed. Studies in Mathematics Education, Volume 4. The Education of Secondary School Teachers of Mathematics. 187p. Document Not Available from EDRS.

ED 266 940 Mathematics. Grade 8. Oregon Statewide Assessment, 1985. 32p. MF01/PC02 Plus Postage. Available from EDRS.

ED 266 963 Martel, Henry J., Jr.; Mehallis, George. An Investigation of the Success Rates of Older Versus Younger Students Placed into College-Preparatory Mathematics on the Basis of ACT Scores: Politics, Law and Economics of Higher Education. 25p. MF01/PC01 Plus Postage. Available from EDRS.

ED 267 089 McBride, James R. Computerized Adaptive Testing: An Overview and an Example. 11p. MF01/PC01 Plus Postage. Available from EDRS.

ED 267 096 Greeno, James G.; And Others. Investigations of a Cognitive Skill. 40p. MF01/PC02 Plus Postage. Available from EDRS.

ED 267 111 Yagi, Kan; And Others. ECIA, Chapter 1: Disadvantaged Child Project. Portland Public Schools Year-End Evaluation Report, 1984-85. 37p. MF01/PC02 Plus Postage. Available from EDRS.

ED 267 959 Ewert, William B.; Prevost, Fernand J. Mathematics, Science, and Computer Education, Grades 7-12. A Report on the New Hampshire Mathematics and Science Education 1983 Status Survey. 35p. MF01/PC02 Plus Postage. Available from EDRS.

ED 267 960 Beluschak, Elaine Burns. Providing Human and Material Resources to Remediate and Enrich Math Skills of Second Grade Students. 76p. MF01/PC04 Plus Postage. Available from EDRS.

ED 267 973 Wearne, Diana; Hiebert, James. Learning Decimal Numbers: A Study of Knowledge Acquisition. Final Report. 107p. MF01/PC05 Plus Postage. Available from EDRS.

ED 267 976 Brenner, Mary E. Arithmetic Achievement at Ka Na'i Pono, 1984. Results from Standardized Testing. Technical Report No. 126. 40p. MF01/PC02 Plus Postage. Available from EDRS.

ED 267 977 Anderson, John R.; And Others. Cognitive Modelling and Intelligent Tutoring. 70p. MF01/PC03 Plus Postage. Available from EDRS.

ED 267 995 Kessler, Carolyn. Mathematics and Language Intersections for Hispanic Billingual Students. 24p. MF01/PC01 Plus Postage. Available from EDRS.

ED 268 154 Rock, Donald A.; And Others. The Internal Construct Validity of the SAT Across Handicapped and Nonhandicapped Populations. 43p. MF01/PC02 Plus Postage. Available from EDRS.

ED 268 329 Loadman, William E.; And Others. Comparison Study of Vocational and Traditional Students on Mathematics and Science Achievement. Technical Report. 136p. MF01/PC06 Plus Postage. Available from EDRS.

ED 268 473 Van Blerkom, Malcolm L. Mathematical Ability in College Students: A Causal Analysis. 9p. MF01/PC01 Plus Postage. Available from EDRS.

ED 268 474 Van Blerkom, Malcolm L. Cognitive Style Correlates of Mathematical Ability among College Students. 10p. MF01/PC01 Plus Postage. Available from EDRS.

ED 268 997 Formative Evaluation of "Measurement: The Difference Between Perimeter and Area" and "Measurement: Dividing Regions into Subregions for Finding Area" from Math Works. Research Report 94. 31p. MF01/PC02 Plus Postage. Available from EDRS.

ED 269 231 Handel, Ruth D. Achievement Attitudes in Mathematics and Science: Relationships between Self-Perceptions, Aspirations, and Extra-Curricular Activities. 17p. MF01/PC01 Plus Postage. Available from EDRS.

ED 269 244 Marshall, Sandra P. Errors in Processing Mathematical Information: A Cross-Sectional and Longitudinal Study of Individual and Sex Differences. Final Report. 50p. MF01/PC02 Plus Postage. Available from EDRS.

ED 269 251 Mathematics Instruction in Tokyo's and Hawaii's Junior High Schools. Final Report. 59p. MF01/PC03 Plus Postage. Available from EDRS.

ED 269 252 Blank, Rolf K. Teacher Quality in Science and Mathematics Education: Recommendations for Research from a National Research Council Conference. 11p. MF01/PC01 Plus Postage. Available from EDRS.

ED 269 253 Raizen, Senta A. Report on the Work of the Committee on Indicators of Precollege Education in Science and Mathematics. 25p. MF01/PC01 Plus Postage. Available from EDRS.

ED 269 261 Stanic, George M. A.; Reyes, Laurie Hart. Gender and Race Differences in Mathematics: A Case Study of a Seventh-Grade Classroom. 32p. MF01/PC02 Plus Postage. Available from EDRS.

ED 269 263 Romberg, Thomas A. The Effect of Overt Verbalization on the Learning and Retention of Geometry Concepts. Working Paper No. 189. 129p. MF01/PC06 Plus Postage. Available from EDRS.

ED 269 264 Kouba, Vicky L.; And Others. Results from First Individual Interview (October 1978), Coordinated Study No. 1. Working Paper No. 281. 144p. MF01/PC06 Plus Postage. Available from EDRS.

ED 269 265 Kouba, Vicky L.; Moser, James M. Results from Second Individual Interview (January 1979), Coordinated Study No. 1. Working Paper No. 285. 148p. MF01/PC06 Plus Postage. Available from EDRS.

ED 269 266 Kouba, Vicky L.; Moser, James M. Results From Third Individual Interview (May 1979), Coordinated Study No. 1. Working Paper No. 289. 149p. MF01/PC06 Plus Postage. Available from EDRS.

ED 269 267 Cookson, Connie; Moser, James M. Coordinated Study Individual Interview Procedures. Working Paper No. 290. 37p. MF01/PC02 Plus Postage. Available from EDRS.

ED 269 268 Kouba, Vicky L.; Moser, James M. Results from Fourth Individual Interview (September 1979), Coordinated Study No. 1. Working Paper No. 292. 123p. MF01/PC05 Plus Postage. Available from EDRS.

ED 269 269 Kouba, Vicky L.; Moser, James M. Results from Fifth Individual Interview (January 1980), Coordinated Study No. 1. Working Paper No. 293. 120p. MF01/PC05 Plus Postage. Available from EDRS.

ED 269 270 Anick, Constance M.; And Others. Results from First and Second Individual Interviews (February and May 1980) of the Second Phase of the Longitudinal Study, Working Paper No. 310. 135p. MF01/PC06 Plus Postage. Available from EDRS.

ED 269 271 Anick, Constance M.; And Others. Results from Eighth Individual Interview (January 1981), Coordinated Study No. 1. Working Paper No. 313. 157p. MF01/PC07 Plus Postage. Available from EDRS.

ED 269 272 Romberg, Thomas A.; And Others. Classroom Learning: Results from Observations - Sandy Bay Study. Working Paper No. 326. 54p. MF01/PC03 Plus Postage. Available from EDRS.

ED 269 273 Romberg, Thomas A., Ed. Using Research in the Professional Life of Mathematics Teachers. International Congress on Mathematical Education (5th, Adelaide, Australia, August 24-30, 1984). 226p. MF01/PC10 Plus Postage. Available from EDRS.

ED 269 276 Sophian, Catherine. Early Developments in Children's Use of Counting to Solve Quantitative Problems. 19p. MF01/PC01 Plus Postage. Available from EDRS.

ED 269 439 Gramenz, Gary W.; And Others. The Effect of Sex-Dependent Norms on Aggregated Reading and Mathematics Test Scores. 26p. MF01/PC02 Plus Postage. Available from EDRS.