DOCUMENT RESUME

ED 285 721 SE 048 309

AUTHOR Carter, Betsy Y.
TITLE Grade 4 Mastery Test in Mathematics. Handbook for Teachers.
INSTITUTION Connecticut State Dept. of Education, Hartford.
PUB DATE 86
NOTE 96p.; For the other handbooks in this series, see SE 048 310-311.
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052) -- Tests/Evaluation Instruments (160)

EDRS PRICE MF01/PC04 Plus Postage.
DESCRIPTORS Achievement Tests; Behavioral Objectives; Computation; *Elementary School Mathematics; Geometry; *Grade 4; Intermediate Grades; *Mastery Tests; Mathematical Applications; Mathematical Concepts; *Mathematics Curriculum; *Mathematics Tests; Problem Solving; *State Programs; Testing Programs
IDENTIFIERS *Connecticut

ABSTRACT

This handbook is part of a series of three, corresponding to the three grades (4, 6, and 8) at which Mastery Tests are administered. This publication was written as a resource for teachers for developing mathematics programs for students in grades 3 and 4. The Mastery Test in Mathematics at Grade 4 assesses student performance for 25 instructional objectives. The following information is provided for each objective: (1) Appropriate Materials (manipulative materials to use for exploring the concept); (2) Enabling Skills and Activities (a description of prerequisite skills and activities); (3) Sample Lessons (student activities that build toward mastery); (4) Teacher Resource Materials; and (5) Mathematics Objectives and Sample Test Items. The instructional objectives are grouped into four categories: (1) conceptual understandings; (2) computational skills; (3) problem solving and applications; and (4) measurement and geometry. Appendixes provide the sample test items for each objective, a list of suppliers of math manipulatives and resource materials, a list, by learning objective, of related computer software, and a list of addresses of software publishers. (RH)

Handbook for Teachers

GRADE 4 MASTERY TEST IN MATHEMATICS

STATE OF CONNECTICUT DEPARTMENT OF EDUCATION — 1986

Connecticut State Board of Education

Abraham Glassman, Chairman
James J. Szerejko, Vice Chairman
A. Walter Esdaile
Warren J. Foley
Dorothy C. Goodwin
Rita L. Hendel
John F. Mannix
Julia S. Rankin
Humberto Solano

Norma Foreman Glasgow (ex officio)
Commissioner of Higher Education

Gerald N. Tirozzi
Commissioner of Education

Frank A. Altieri
Deputy Commissioner
Finance and Operations

Lorraine M. Aronson
Deputy Commissioner
Program and Support Services

Handbook for Teachers

GRADE 4 MASTERY TEST IN MATHEMATICS

This handbook was prepared by Betsy Y. Carter, mathematics consultant with the Connecticut State Department of Education. A special note of thanks is extended to Clare Clark for her critical review and helpful suggestions.

CONTENTS

GUIDE TO INSTRUCTIONAL OBJECTIVES

FOREWORD

One of my highest priorities and a very central aspect of Connecticut's Challenge: An Agenda for Educational Equity and Excellence is the implementation of the statewide mastery testing program in reading, mathematics, and language arts for grades 4, 6, and 8.

The primary focus of this testing program is instructional improvement for Connecticut's students. Once teachers and administrators have reviewed class, school, and district test results, the task of using these results to improve student mastery of basic skills moves directly into the classroom. It is there -- in thousands of Connecticut classrooms -- where high quality instruction translates test results into meaningful instructional activities.

For these reasons, I am pleased to provide you with the Handbook for Teachers for the Grade 4 Mastery Test in Mathematics. This practical resource includes specific instructional strategies and sample lessons that are keyed to each of the test objectives. You will also find sample test items and resource lists in the handbook.

I encourage you to carefully review the mastery test results for your class and your school and to use this handbook as a sourcebook for planning lessons and units that address your students' needs.

Gerald N. Tirozzi
Commissioner of Education

The Handbook for Teachers, Grade 4 Mastery Test in Mathematics is written as a resource for teachers as they develop a meaningful mathematics program for students in Grades 3 and 4. Although the Mastery Test in Mathematics at Grade 4 assesses student performance on 25 objectives, mathematics instruction should NOT be reduced to providing lessons that are focused only on those objectives. Rather, instruction on the objectives should be integrated into a mathematics program that has as its primary goal the understanding of basic mathematical concepts and the use of mathematical ideas to solve problems.

The Mastery Test objectives were chosen as significant benchmarks of growth. The role of this handbook is to place the objectives in the perspective of a mathematics curriculum that is based on the way children develop mathematical skills and concepts. This handbook is designed also to assist teachers in providing a mathematics program that continually moves through a sequence of concrete, pictorial, and abstract experiences as concepts are explored and objectives are mastered.

The limitations of a multiple-choice, paper-and-pencil test have inevitably influenced the final list of objectives. For example, we were able to address the development of skills in solving story problems, but could not provide significant opportunities to test beyond the translation level to the problem-solving process itself.

The ideas, activities, and sample lessons found in this handbook were designed to be a part of a mathematics curriculum that has problem solving as the central focus. The process of problem solving is enhanced by using such strategies as:

guessing, testing, revising
acting out
building a model, diagram, or picture
breaking the problem into smaller, simpler problems
organizing information into tables, charts, and graphs
using patterns
classifying the problem by its underlying structure

Significant strands in the curriculum, which deserve equal time in the classroom include:

spatial relationships (measurement, estimation, geometry)
the organization of data (classification, patterns, probability, statistics, graphing)
the development of number sense (estimation and computation including mental computation, place value)

Handbook Components

The following information is provided for each objective:

- Appropriate Materials. Exploration of concepts through the daily use of manipulative materials promotes understanding. Appendix B contains a list of suppliers of math manipulatives and resource materials.
- Enabling Skills and Activities. Students should NOT be asked to practice examples at the same level of difficulty as the test items without experiences at earlier stages. Therefore, this handbook provides a description of prerequisite skills and activities suitable earlier in the sequence of instruction.
- Sample Lessons. Included in the sample lessons are ideas that involve students directly in activities that build toward mastery. Each lesson has been designed to develop or reinforce several skills and concepts. This presents an opportunity for students to engage in the process of problem solving in the various strands of the curriculum. Rather than teach each skill in isolation, the ideas are explored together so that students may begin to discover the structure of mathematics.

- **Teacher Resource Materials.** Information about suppliers can be found in Appendix B. A list, by learning objective, of related computer software is included in Appendix C. Addresses of software publishers appear in Appendix D.

OBJECTIVE 1 Identify the number one more, one less, ten more, or ten less than a given number.

APPROPRIATE MATERIALS

Beansticks and loose beans
Dienes Blocks (or Powers of Ten Blocks)
Place value dice
Number lines
Numeral cards
Counting objects such as buttons, macaroni, etc.
Counting cups or Set Boards
Multilinks or Unifix cubes

ENABLING SKILLS AND ACTIVITIES

Before working with numerals only, students need to compare groups of counting objects for sets that contain more, less, or the same number of objects. Students should organize a large amount of objects, such as a jarful of buttons, into smaller cups that contain ten buttons each. Students must recognize that when you say "ten more," or move up the number line by tens, you include all the objects that came before: twenty includes the first ten plus ten more; thirty includes the first ten, the second ten, plus a third ten.

One way to illustrate the idea of inclusion is to use cupfuls of ten objects to make a number line. Draw a line segment on a large piece of paper and mark the left end zero. Place a cupful of ten buttons a short distance to the right. Make a mark and label the spot 10. Make a mark to the right the same distance as before. Slide the cupful of ten over to the new mark. Take a second cupful of ten buttons and make ten more by pouring its contents into the first cup. Label the mark 20. Now move the cupful of 20 buttons over to another mark. Pour in another cupful of ten buttons. What should you label the mark? Continue to slide the cup to the right. Pour and label to 100 and beyond. The result will be a number line labeled by tens. Students observe that the numbers grow larger as you move from left to right on the number

line, even though when you write the numerals your place value holders increase in value from right to left. Students need such exploratory activities on the concrete level in order to understand what the numerals represent.

Counting and skip counting (such as 13, 18, 23, 28...) that combine oral recitation with the recording of the numerals on a number line, help students see a pattern. Students may also label number lines, working forward or backward from a few labeled points.

Constructing and reading simple pictographs and bar graphs develops comparison skills. In order to accurately produce or reproduce a graph, students must count and compare the items in each row. When reading graphs, students may be asked to point out a row that has one more, one less, ten more, or ten less.

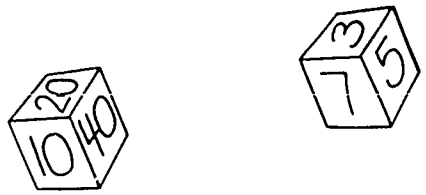
Never introduce the inequality symbols $<$ and $>$ together. Many students become confused. Pick one symbol and work with it for several weeks. Only when students use the first symbol as a matter of routine should the second symbol be introduced. Once students are using both symbols, it may help them use the symbols correctly if they remember that Pacman and alligators only eat the bigger numbers!

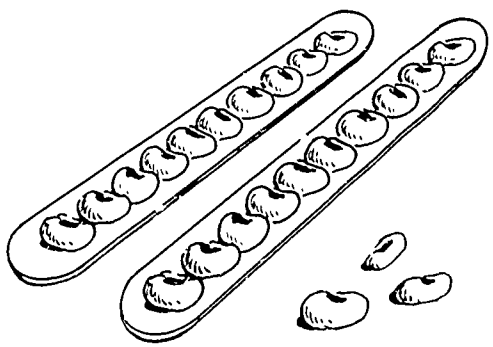
SAMPLE LESSON Building and Recording Sets that are Ten More or Ten Less

MATERIALS NEEDED

Beansticks and loose beans (or Dienes Blocks or Powers of Ten Blocks)
Place value dice

Place value dice can be made by finding a few cubes such as building blocks or alphabet blocks, 2" wooden cubes made from scrap lumber, or a set of large dice. Cover each side with a piece of nonpatterned contact paper or a plain sticker. Label the sides, one number per side, to make some cubes with single digits only (0-5, 1-6, or 4-9 are possible combinations); and other cubes that are labeled with multiples of ten (10-60, 40-90, or any other combination).





Students work in pairs or small groups at this game-like activity. One student should take the pair of place value dice (one single digit and one multiple of ten cube) and roll them. Another student must use the beansticks and beans (or other place value materials) to build the number. The first student or a third student must then build a set that is ten more or ten less than the first set. Another student may record on the score sheet the first set and the second set. The recording should be done in expanded notation to reinforce the place value concept. When rewritten in standard form, a pattern emerges which shows the ones place staying the same when ten more or ten less is achieved.

The "score" for a round of three to five tosses could be found by adding the tens from each set.

First Set	Second Set Ten More
30+3=33	40+3=43
70+5=75	80+5=85
⋮	⋮

Conceptual Understandings, continued

TEACHER RESOURCE MATERIALS

Arithmetic Teacher
The Mathworks
Mathematics: A Good Beginning
Math for Girls and Other Problem Solvers
Math in Stride, Books 2 and 3
Unimath

OBJECTIVE 2 Extend patterns involving numbers and attributes.

APPROPRIATE MATERIALS

Pattern blocks
Buttons
Dry pasta shapes dyed with food coloring
Bakery tabs, corks, nuts and bolts, leaves from several kinds of trees, or other "found" objects
Multilinks or Unifix cubes
Assorted colors of 1" construction paper squares
Colored toothpicks or straws
Cuisenaire rods
Attribute blocks
Geoboards
Centimeter graph paper and crayons or markers in several colors
Design cards -- made with stickers, wallpaper, or wrapping paper
Numeral cards

ENABLING SKILLS AND ACTIVITIES

Patterning is an important area of the mathematics curriculum. The ability to classify and organize information and identify patterns is useful in developing good problem-solving skills.

Before students can extend a pattern, they need time to explore with materials that lend themselves to being sorted by one or more attributes. Students should have experience in developing their own ways of classifying materials. For example, students can be given cupfuls of buttons and allowed to examine them and then sort them in any appropriate way. They may arrange them in piles according to the number of holes in each button or they may sort them by

color or size. Other students may sort the buttons into piles by more than one attribute. For example, piles of large red, small red, large blue, or small blue buttons, etc., may be produced.

Work on patterning may begin once students have had experience in classification. Students should be allowed to make their own patterns. Often this happens spontaneously when students are given materials, such as Pattern Blocks, that readily lend themselves to patterning. Students also enjoy reproducing and extending each others' patterns. Interesting patterns can be made using materials such as dry pasta of different shapes, lengths, and colors. The patterns may be made on strips of oaktag. Since the materials are relatively inexpensive, the objects may be glued down and the pattern strips put on display. Interesting patterns may also be made by a mix of several kinds of "found" junk.

Spatial relationships are explored during patterning activities. Patterns that involve the rotation or flipping of shape pieces are possible. Patterns need not be horizontal; explore vertical arrangements and try building some two-dimensional symmetrical "tiling" patterns. Materials like multilinks may be used to build three-dimensional patterns. Encourage students to use their imagination and creativity in the selection of materials and pattern "rules."

Once such "concrete" patterns have been produced, students may begin to classify the patterns by their rules: OoOo .. is an ABAB pattern; △OO△O.. is an ABBABB pattern; OO△△□OO△△□... is an AABBCAABBC pattern. Since patterns can become rather complex, establish some rules that limit patterns to only one or two attributes repeated no more than three times per segment. It is interesting to discover how many different patterns can be made under such a limit. After some experience in working on a symbolic level, students are ready to explore number patterns with various "rules."

SAMPLE LESSON Exploring Number Patterns on a 100-grid

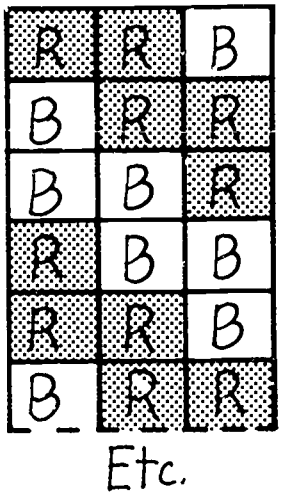
MATERIALS NEEDED

Centimeter grid paper
Crayons or markers in several colors

Mark off a 10" x 10" grid area. Beginning with the first square as 1, write a number from 1 to 100 in each square.

Color every third square red. Color every fifth square blue. List all the numerals that are in the red squares. What is the rule for extending this list of numerals? Do the same for the blue squares. Which squares are both red and blue? What is the rule for finding these numbers? Try this with other color patterns on the grid. Try numbering the grid beginning with a different numeral, such as 4. How do these patterns relate to skip counting or to multiplication?

Mark off a 3" x 20" grid. Starting with the first square, color in a pattern - red, red, blue, blue, red, red, blue, blue, etc., moving across each row and then on to the next row. What kind of a pattern appears when you examine each of the three columns?



TEACHER RESOURCE MATERIALS

Mathematics Their Way
Math in Stride, Book 3
Arithmetic Teacher
Let's Pattern Block It
Problem-mathics
Pattern Blocks Activities

OBJECTIVE 3 Order whole numbers.

APPROPRIATE MATERIALS

Number lines
Numeral cards
Picture cards
Rack-O
Cuisenaire rods
Dienes Blocks (or Powers of Ten Blocks)
Numbers Up (a game by Milton Bradley)
Try-A-Tiles
Money
Rummifun (a game from New England School Supply)

ENABLING SKILLS AND ACTIVITIES

Sorting, comparing, and ordering objects by size (length, weight, and volume) all reinforce the idea of order on the concrete level. Estimating and measuring with nonstandard and standard units also help develop the concept of order. Students should always be encouraged to make comparisons and rearrange sets of objects in some order.

Sets of picture cards, dominoes, classmates, or jars (filled to different levels with tinted water or beans) can all be lined up in ascending or descending order. Students enjoy placing extra items into an already organized series.

Counting and skip counting activities (forward and in reverse) aid in sequencing numbers. Students also benefit from activities such as filling in the numerals on number lines which have only a few points labeled.

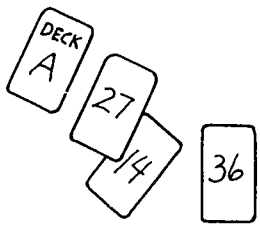
Students can play many games in which they must select the larger or smaller number. Begin with comparing two numeral cards, then try ordering three cards, then more.

SAMPLE LESSON A Secret Number Game Using Order on the Number Line

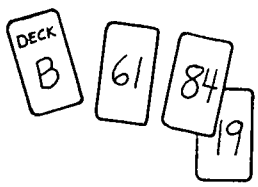
MATERIALS NEEDED

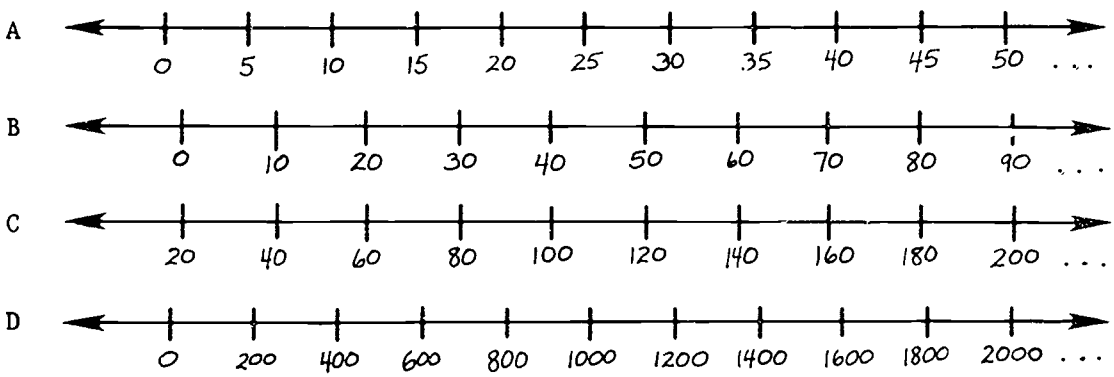
A set of 20 numeral cards
Number line sheets

Use the information below to make the numeral cards and number line sheets. Numeral card sets used will depend on students' skills. A first set, Set A, could include any 20 numbers from 1 to 50. Set B might include numbers from 1 to 100; Set C might include numbers from 1 to 200; Set D might include numbers from 1 to 1000.



The number line sheets should be labeled to correspond to the kinds of numbers in each set of cards. For example, if students are working with Card Deck A, they would record their guesses on a number line sheet that contained four or five number lines all like Number Line A as shown below. For Deck B, use number lines like B below, etc.





Have students work in pairs or small groups. Give each group a set of numeral cards and some number line sheets.

Have one student select a numeral card from the pile - this is the secret number. Other students in turn guess what the number is. The keeper of the secret number may only say, "Yes," "It's larger," or "It's smaller."

Students should record each guess on the number line and use this record to make further guesses. The student who guesses the secret number first wins the round and becomes the next secret card holder.

TEACHER RESOURCE MATERIALS

Math in Stride, Book 3
Unimath
Math for Girls and Other Problem Solvers
Mathematics: A Good Beginning

OBJECTIVE 4 Rewrite numbers using expanded notation.

APPROPRIATE MATERIALS

Beansticks and loose beans
Dienes Blocks (or Powers of Ten Blocks)
Place value dice
Money
Counting objects
Place value boards
Place value charts
Place value stamps

ENABLING SKILLS AND ACTIVITIES

Activities organizing large sets of counting objects into groups of ten and groups of one hundred will reinforce the concept of place value. Patterned number sequences for adding a ten or multiples of ten, such as 24, 44, 64, -, -, -, ..., also reinforce place value while establishing patterns for mental computation.

Use Dienes Blocks or Powers of Ten Blocks to represent large numbers. Roll place value dice and build that number. First use dice for tens and ones, then hundreds. (See Objective 1 for a description of place value dice.)

Write the number in expanded form. Rewrite the number in standard form. Reverse the process and pick a large number in standard form. Build it with the blocks and write it in expanded form.

Work on the pictorial level by using worksheets that depict various combinations of beansticks and beans, or Dienes Blocks. Label the pictures.

Students enjoy using place value stamps to make pictures that illustrate numbers. Select numbers by rolling the place value dice or select numbers from a pile of numeral cards.

Once students have progressed through the above sequence and have developed a sense of place value, you may reinforce the skill at the abstract level by rewriting large numbers in expanded and standard form.

SAMPLE LESSON Using Place Value to Organize and Record a Large Number of Objects

PLACE VALUE BOARD

HUNDREDS	TENS	ONES

MATERIALS NEEDED

A quart jar of large lima beans
A large (18" x 24") place value board
Smaller place value sheets
Counting cups (4-oz. and 8-oz., a dozen of each size)

This lesson may be done as a whole class activity and later repeated (with other jarfuls) with small groups of students at math stations.

Display the jarful of large lima beans. Ask the students to guess how many beans are in the jar.

Each student may wish to write his/her guess on a slip of paper. The guesses may then be put in order, placed on a number line, or graphed. In this way the lesson will help to develop more than one concept.

Now it is time to find out how many beans are in the jar. Have three or four students work together to count the beans into the smaller cups - ten beans to a cup. Have the students help decide that once ten small cups are full they may pour the beans into a larger cup to make one hundred.

Put the cupful of one hundred beans onto the large place value board in the hundreds area. Continue counting, filling, and pouring until all the beans are used. Put the smaller cups of ten beans in the tens area on the place value board. Put the leftover beans in the ones area. How many hundreds do you have? How many tens? How many ones?

Record the amount on the smaller place value chart...

HUNDREDS	TENS	ONES
5 large cups	3 small cups	7 loose beans
5 Hundreds	3 Tens	7 Ones
500 +	30 +	7

Write this number in standard form. 537

Repeat the activity with different jars and/or different counting items.

TEACHER RESOURCE MATERIALS

Math in Stride, Book 3
The Mathworks
Mathematics: A Good Beginning
The Arithmetic Teacher
Unimath
Developing Number Concepts Using Unifix Cubes

OBJECTIVE 5 Rewrite numbers by regrouping tens and ones.

APPROPRIATE MATERIALS

Beansticks and loose beans
Dienes Blocks (or Powers of Ten Blocks)
Money
Place value dice

ENABLING SKILLS AND ACTIVITIES

Students develop a sense of place value and flexibility using numbers when they are provided with many opportunities to explore ways to write equivalent expressions. Students should practice the rewriting of numbers in two directions. They need to be able to write a number such as 74 in the form 60 + 14. They also need to have experience in changing 60 + 14 to 74.

The following lesson explores a way to practice doing one of the directions--74 = 60 + 14. It also encourages the exploration of other ways of expressing the number by regrouping more than one ten. Such exploration will provide the flexibility needed to do mental computation.

Students will later need to work under a rule that limits them to regrouping only one ten. That is the precursor to regrouping under the standard algorithm for addition and subtraction.

SAMPLE LESSON Exploring Regrouping of Tens and Ones

MATERIALS NEEDED

Place value dice
Dienes Blocks (Powers of Ten Blocks, beansticks, or beans)
Record Sheets

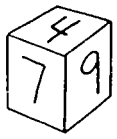
Roll a set of place value dice. (See Objective 1 for a description of place value dice.) Build the number using the rods and unit cubes from the Dienes Blocks set. Record the number on the Record Sheet using expanded notation. Now try to build the same number in a different way. Record the new way using expanded notation. How many ways can you build this number? List them on the Record Sheet. Can you see a pattern? Does the pattern help you find all the ways you can write the number?

To do the same lesson at an earlier phase, you may ask the student to build the number and record it, and its variations, using place value stamps. Such pictorial recording is necessary for some students before they can understand the recording done with expanded notation.

RECORD SHEET

Roll	7 tens	and	4 ones
	70	+	4
Other ways:	6 tens	and	14 ones
	60	+	14
	4 tens	and	34 ones
	40	+	34
	0 tens	and	74 ones
	0	+	74

etc.



Conceptual Understandings, continued

TEACHER RESOURCE MATERIALS

Developing Number Concepts Using Unifix Cubes
Unimath
Math in Stride, Book 3
Mathematics: A Good Beginning
The Mathworks

OBJECTIVE 6 Identify fractional parts of regions and sets from pictures for halves, thirds, fourths, and sixths.

APPROPRIATE MATERIALS

Counting objects
Set boards for fractions
Construction paper, assorted colors
Fraction action game
Fit-a-fraction circles
Square parts fraction game
Proportional fraction blocks
Hands-on fractions strips
Pattern blocks
Multilinks

ENABLING SKILLS AND ACTIVITIES

Students need a variety of concrete and pictorial activities with many kinds of materials. These activities will enable them to grasp the concept of fractions as a relationship of one or more equal parts of a particular whole object. All halves are not the same. Half of an apple, for example, is not the same size as half a grape. It is not size, but proportional part that determines the relationship. Students need activities with many models to gain the concept of ratio.

Students als need to develop an understanding of the fraction symbols. This understanding comes from relating concrete and pictorial examples to the symbols.

A further idea that students must see is that not only may 1/2 represent one of two equal parts of a whole object, but it may also represent one of two equal subsets of a larger set. Finding fractional parts of whole number sets is actually multiplication of a whole number by a fraction. It is easier to grasp than addition and subtraction of fractions because multiplication of whole number sets requires only the idea of parts of the same size; addition and subtraction of fractions requires not only the part-whole relationship,

but also the idea of equivalence. Activities that explore fractional parts of sets should precede computation with fractions.

Equivalence is an idea about fractions that must be explored at the concrete and pictorial levels. How many ways can 1/2 be made? Students will feel more comfortable about equivalent fractions if they have many opportunities to identify fractional parts in more than one way.

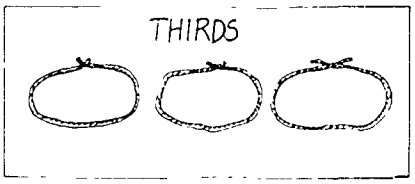
SAMPLE LESSON Exploring Fractional Parts of Whole Number Sets

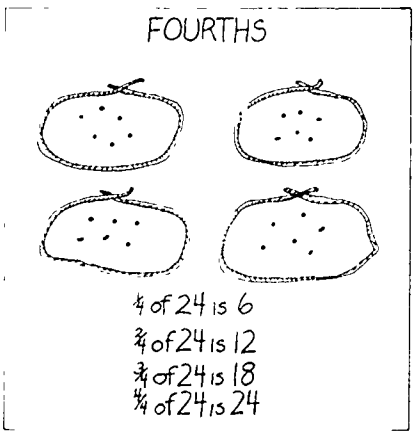
MATERIALS NEEDED

Set boards for fractions
Counting objects

To make set boards for fractions, use large sheets (18" x 24") of oaktag. With a marker, draw two or more rings of about the same size. A very attractive board can be made by gluing yarn onto the outline of each ring. The yarn will help keep small counting objects in place. Make several set boards - the one for 1/2 has two rings, the one for 1/3 has three rings, and the one for 1/4 has four rings. It is also possible to place two set boards next to each other - 1/7 could be made from a board with three rings and a board with four rings.

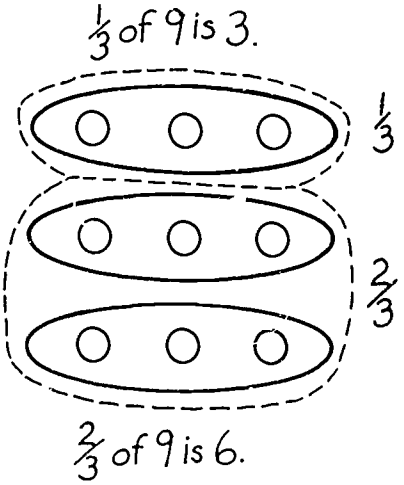
Use a set board to organize the counting of objects. To find 1/4 of 24 the student will use a set board with four rings. The student should count out twenty-four objects, such as multilinks, into a pile. The multilinks are then dealt out one to each ring; the student then must deal out a second multilink to each ring, then a third multilink, then a fourth multilink, etc.





The student may also record the result by drawing a picture of the board with the distributed objects shown. The picture could also be labeled. How many objects are in 1/4 of 24? How many are in 2/4 of 24? How many are in 3/4 or 4/4 of 24?

The action should be repeated many times. Later, students can be given pictures of sets of objects and asked to draw a ring around each unit fraction. They can list the number of objects in 1/3 of 9, 2/3 of 9, and 3/3 of 9.



TEACHER RESOURCE MATERIALS

Beginning Fractions
Math in Stride, Book 3
Mathematics: A Good Beginning
Fractions with Pattern Blocks

OBJECTIVE 7 Relate multiplication and division facts to rectangular arrays.

APPROPRIATE MATERIALS

Counting objects - such as buttons, lima beans, paper clips, multilinks, etc.
Egg cartons - sections used to hold groups of objects
Ceramic tiles
One-inch construction paper squares
Centimeter graph paper

ENABLING SKILLS AND ACTIVITIES

Before using rectangular arrays, students should have experienced multiplication as addition of sets that contain the same number of objects. Division should be explored as dealing out objects into equal subsets or into groups the same size.

The meaning of multiplication can best be developed by first exploring multiplication as repeated addition. At concrete and pictorial levels students should observe sets of the same number of objects being combined one after the other. For example:

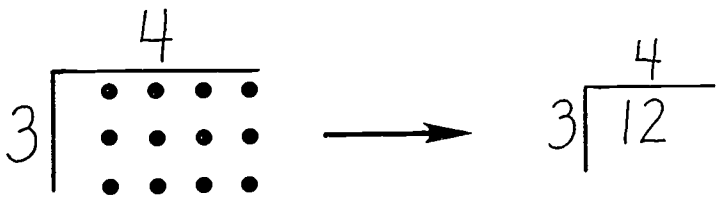
***** + ***** + ***** + ***** is 5 + 5 + 5 + 5 = 20;

so four groups of five or 4 times 5 = 20.

The basic multiplication facts can also be established by giving a student a few sections of an egg carton and asking him/her to place the same number of counting objects in each section.

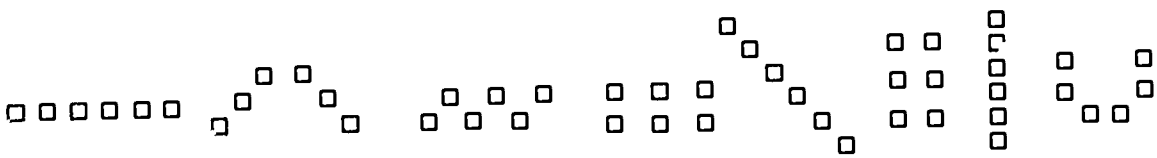
For example, an egg carton cut down to six sections with each section filled with three objects shows six groups of three, or 6 times 3. The student should count the number of objects used in all and record the fact sentence. The task can be repeated with another amount of objects placed in each section until the "six table" has been recorded.

The use of arrays to develop multiplication and division not only presents a clear, concrete and pictorial model, but also relates multiplication to division. The array, when labeled with its factors (the product is inside), is consistent with the standard symbol used for division.



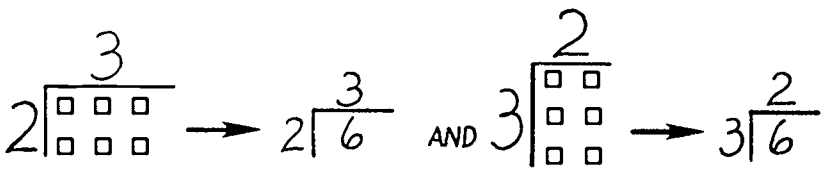
It is also very helpful to use the factor-factor-product terminology rather than to introduce more terms, like dividend and quotient for division. This promotes the idea that multiplication and division are related. When students see the reversibility of the procedures they find there are fewer facts to remember.

Before using rectangular arrays as computational models, give students the opportunity to explore all the different ways that a set of objects may be arranged. For example, here are some ways to arrange six tiles:



Conceptual Understandings, continued

Students then pick out only the rectangular arrays, label them, and then write them using the multiplication/division frame as shown. They then relate the arrays to particular symbolic sentences and the multiplication/division frame.



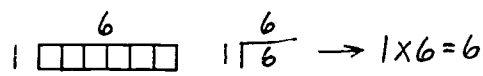
SAMPLE LESSON Exploring Six as a Factor

MATERIALS NEEDED

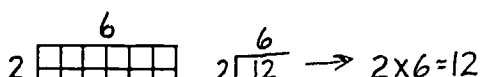
Centimeter graph paper
A crayon or marker in a favorite color

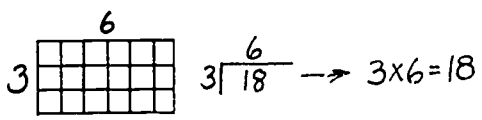
Ask the student to pick a favorite color marker or crayon.

Color a row of six squares. Label the array, write it using the multiplication/division frame, and write it as a multiplication sentence.



Color two rows of six. Label the array, write it using the multiplication/division frame, and write it as a multiplication sentence.





Color three rows of six. Label the array, write it using the multiplication/division frame, and write it as a multiplication sentence.

Continue to color and label.

Try this with other factors.

Students can bind the pages together to make their own multiplication booklets. They may refer to the booklets on those occasions when a multiplication fact is temporarily forgotten.

TEACHER RESOURCE MATERIALS

Beginning Multiplication and Division
Math in Stride, Book 3
Teacher Handbook for Sequencing Math Skills, K-4
Developing Number Concepts Using Unifix Cubes

OBJECTIVE 8 Know addition and subtraction facts to 18.

APPROPRIATE MATERIALS

Cuisenaire rods
Multilinks
Numeral cards
Sets of counting objects
Set boards
Pan balances

ENABLING SKILLS AND ACTIVITIES

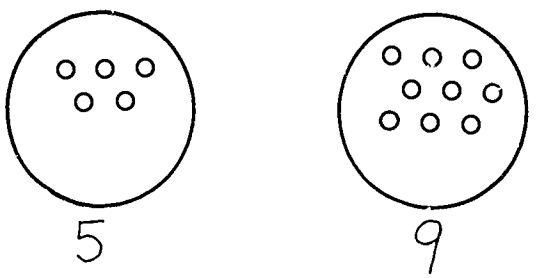
The mastery of basic addition and subtraction facts is simplified when students understand that two addends and their sum can be used to find four basic facts. For example, the number sentence 7 + 6 = 13 can be restated as:

6 + 7 = 13,
13 - 7 = 6, and
13 - 6 = 7.

The set of four such number sentences is a fact family.

If students are not able to state all the addition and subtraction facts, the use of fact families and number patterns can help to organize the facts into those they know, and those facts that they may now acquire by linking to known number facts. Students will be reassured to see that they have mastered many facts.

Fact families can be established by having the students arrange counters into two piles, with no more than nine counters in each pile. With 14 counters, one such arrangement might be:



Students would record this combination as 5 + 9 = 14. Next, each student should list the other three number sentences that belong in the fact family. Students should continue to select combinations of counters until they have found all the pairs possible and have generated all the fact families. Using a systematic method to find the pairs of addends makes the task simpler.

Students who have mastered the facts may find it interesting to explore the combinations without concrete models. As in the previous activity, they should be asked to find all the possible pairs for two addends, each addend less than ten, as well as the related fact family for each pair. These pupils will usually discover some interesting number patterns as they establish a method for listing all the pairs and the related fact family sentences.

Additional exploration of number patterns will reinforce the facts and also provide ways for students to reestablish temporarily forgotten facts.

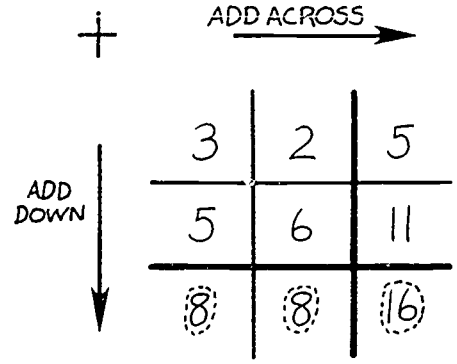
SAMPLE LESSON Basic Addition and Subtraction Facts with Number Grids

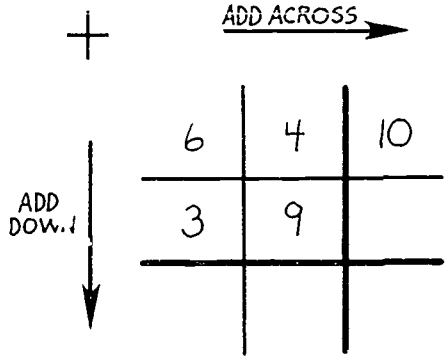
MATERIALS NEEDED

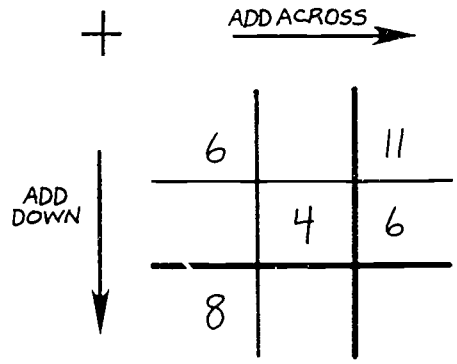
Number grid worksheets for addition and subtraction

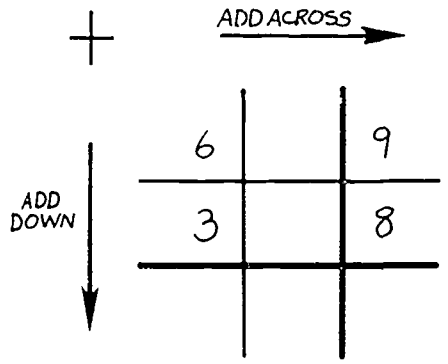
Make up some number grid "puzzle" sheets. Several puzzles are shown on the next page. Do all of ONE type first. Later mix the addition, missing addend, and subtraction grids.

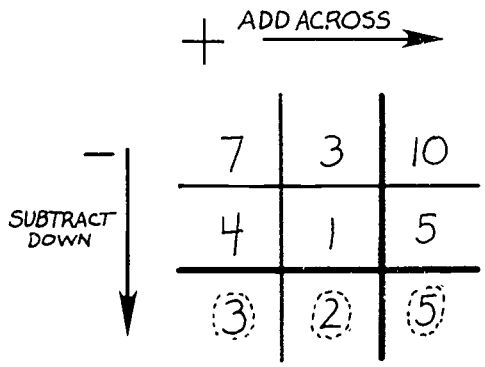
Ask students to complete the number grids.

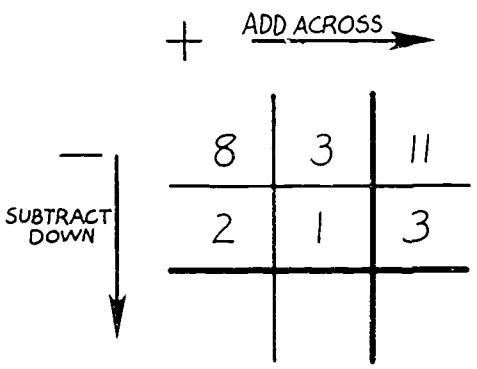


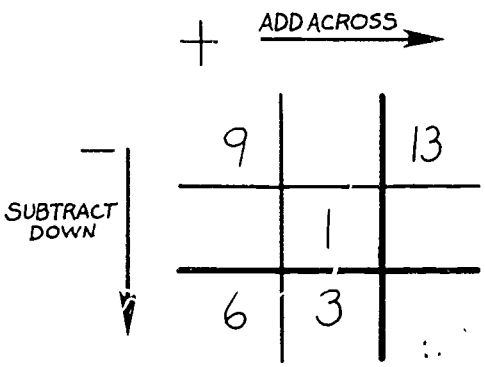


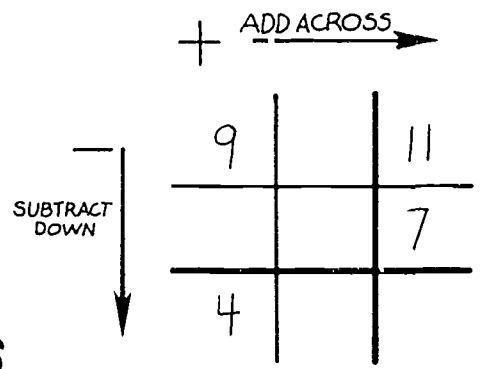












Watch how students do the puzzles. Some students may count on or reverse count; some may make pictures or counting marks; others will refer to the fact families; while others will simply put in the known facts.

For further reinforcement have students make up and solve each others' number grid puzzles.

TEACHER RESOURCE MATERIALS

The Piaget Primer
Math in Stride, Book 3
Teacher Handbook for Sequencing Math Skills, K-4
Mathematics: A Good Beginning

OBJECTIVE 9 Add and subtract one- and two-digit numbers without regrouping.

APPROPRIATE MATERIALS

Numeral cards
Number lines
Dienes Blocks (or Powers of Ten Blocks)
Place value stamps

ENABLING SKILLS AND ACTIVITIES

Objectives 1,3,4,5,8, and 11 must be fairly well developed before students can effectively explore computation with two-digit numbers. Students should be able to consistently count on from any number and skip count by 2's and 10's (forward and in reverse) in order to explore ways to add and subtract.

Each student should be given the freedom to try various methods to add and subtract. Students should be allowed to spend time on solving computational problems through the logical application of counting, order, place value, expanded notation, estimation, and basic computational facts.

SAMPLE LESSON Exploring Nonroutine Algorithms for Addition

MATERIALS NEEDED

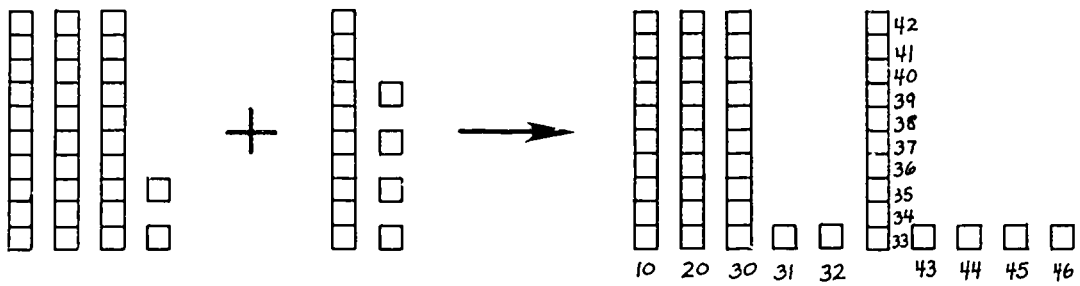
Paper and pencil
Base ten materials such as Dienes Blocks (Powers of Ten Blocks) or beansticks and loose beans

Present students with a problem such as 32 + 14.

Have counting and base ten materials available. How many ways can this problem be solved? Suggest that students build models and record their steps pictorially or symbolically.

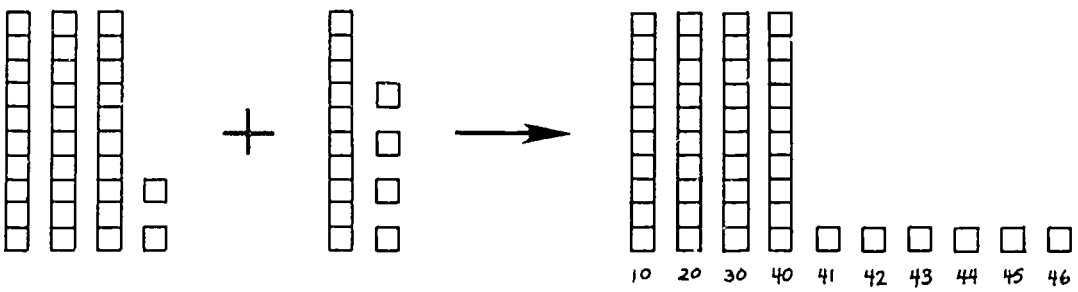
Some model or pictorial solutions that might be "discovered" are shown below. However, students will come up with others.

Some possible solutions that might appear include:



in which counting on from 32 is employed.

Or



in which the tens are dealt with first. As a written method, this becomes

$$30+2+10+4 = \begin{array}{r} 30 \\ +10 \\ \hline 40 \end{array} + \begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array} \quad \text{OR} \quad \begin{array}{r} 32 \\ +14 \\ \hline 40 \text{ (ADD TENS)} \\ +\ 6 \text{ (ADD ONES)} \\ \hline 46 \end{array}$$

and is useful in developing front-end strategies for mental computation.

TEACHER RESOURCE MATERIAL

Learning from Children
Teacher Handbook for Sequencing Math Skills, K-4

OBJECTIVE 10 Add one- and two-digit numbers with regrouping.

APPROPRIATE MATERIALS

Beansticks and loose beans
Dienes Blocks (or Powers of Ten Blocks)
Money
Place value dice
Place value stamps
Number lines

ENABLING SKILLS AND ACTIVITIES

Before students are ready to add and subtract with regrouping they must have a great deal of experience working on activities that develop the concepts of order and place value. Concept objectives 1, 3, 4, and 5 as well as skill objectives 8 and 9 must be fairly well developed.

Too often students are asked to compute prematurely with the standard algorithms of regrouping. Such students cope by doing the exercises in a rote manner, and then often take much longer to understand what they are really doing with place value. If students are given frequent opportunities to investigate place value at concrete and pictorial levels, and freely move materials around, they will gain insights into computation that will allow them to develop a real number sense. They will also be able to invent nonstandard algorithms that provide them with the ability to estimate answers and to do mental arithmetic.

SAMPLE LESSON Exploring Regrouping

MATERIALS NEEDED

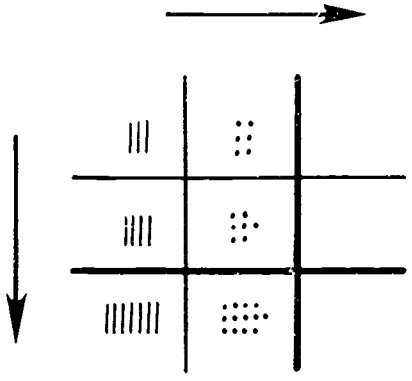
Paper and pencil
Base ten materials such as Dienes Blocks (Powers of Ten Blocks) or beansticks and loose beans

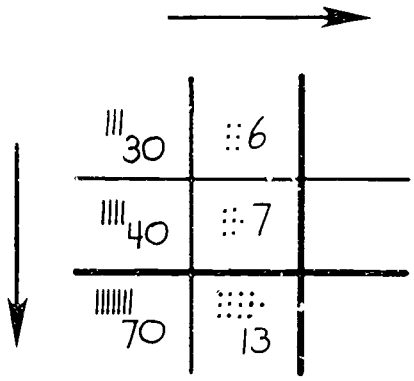
Students' efforts to compute usually go through different levels of development. At the concrete, pictorial, and symbolic levels they can often construct efficient ways to do computational tasks that require "regrouping." The methods employed are based on their understanding of number and place value. In this lesson an opportunity is provided for students to explore such computational problems.

Begin by presenting a problem such as:

$$\begin{array}{r} 36 \\ +47 \\ \hline \end{array}$$

Allow students to use base ten materials if they wish, or let them represent the numbers pictorially using place value stamps. Build the picture inside a number grid, as shown below. Label the gric sections.

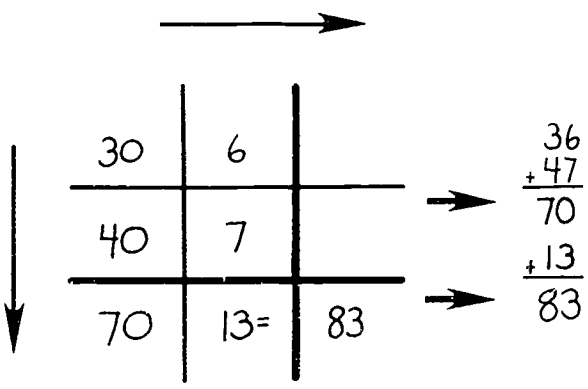




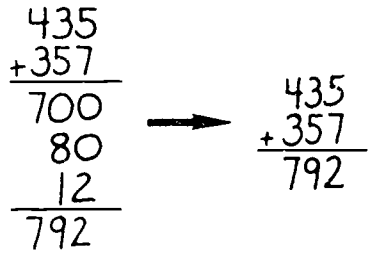
Build another grid using expanded notation only. On this grid record the sum of the tens and then the sum of the ones. Use the last section to record the total.

Notice that the sum in expanded form shows the "extra ten" in the units position. It becomes combined with the rest of the tens.

Rewrite the sum in its usual combined form. Continue doing more computational problems using these steps. The lesson, when repeated many times, will move the students from the pictorial level to an understanding of the expanded form, which in turn will provide a foundation for the use of the long and short computational forms. This may seem like a great deal of work, but it does pay off with student understanding of advanced computation.



This "front end" approach is easily extended to hundreds. Students would use the expanded form to get intermediate sums and then record the work in long, and then later, short forms.



TEACHER RESOURCE MATERIALS

Learning from Children
Math in Stride, Book 3

OBJECTIVE 11. Estimate sums and differences to 100.

APPROPRIATE MATERIALS

Counting objects such as buttons, beans, macaroni, etc.
Number lines

ENABLING SKILLS AND ACTIVITIES

Estimation is a very important prerequisite for all advanced computation. Before students are expected to make estimates of computations on the abstract level, they should have opportunities estimating on the concrete level. Estimation activities include making estimates of length, area, and volume, often with nonstandard units.

Students should have many chances "guessing" how many objects are in a container (jelly beans in a bowl, macaroni in a jar, lima beans in a box, etc.). Once a guess is made, the objects should be counted. Watch how each student does the counting task. Is the count organized into groups of two, five, ten, or some other number of objects? Students who count large groups of objects by ones are operating at an earlier level than those who use grouping strategies. Do not impose strategies on the students; their strategies for finding the total allows them to apply their own computational skills. During successive estimation activities, students should be allowed to think through some of their previous ways of estimating and counting in order to arrive at increasingly efficient strategies.

Students need exercises rounding numbers to the nearest ten and nearest hundred before doing written estimation work. Students should also be able to use the and | symbols. Lessons that have students locate the nearest ten or nearest hundred on a number line are also helpful.

Once students begin to work on estimation at the abstract level, much time should be spent exploring the many ways to estimate answers to computational problems rather than mechanically applying algorithms. Too often students are expected to use algorithms without exercises in rounding and estimating. This results in an inability to do mental computations and prevents many students from developing ways to deal with computations with large numbers.

SAMPLE LESSON Estimating Experiences for Addition

MATERIALS NEEDED

Number lines
Paper and pencil

Estimation includes rounding to arrive quickly at a reasonable answer. Addition examples can be estimated, and often solved more efficiently by applying rounding strategies rather than by applying the regrouping algorithm. This lesson builds on estimation and rounding skills, and allows students to extend the skills in order to discover some alternative computational strategies. This lesson also helps students to master Objective 10.

Ask students to consider a sample problem such as 42 + 19.

What are the closest multiples of ten? Students may need to locate 42 on the number line and find the closest multiple of ten. Do the same for 19.

So 42 + 19 is close to 40 + 20, which is 60.

Some of the estimation strategies and skills can be applied to solving computations mentally. For example, in the above case rather than rounding both the 42 and the 19, just the 19 might be rounded to 20. So

$42 + 19 \rightarrow 42 + 20$ (less one) = 62 (less one)

or

$42 + 20 = 62$; so $42 + 19 = 61$.

Likewise, $34 + 57 \rightarrow 34 + 60$ (less three) = 94 (less three)

$34 + 60 = 94$; so $34 + 57 = 91$.

This approach assumes that students have had experience adding and subtracting multiples of ten.

TEACHER RESOURCE MATERIAL

Learning from Children
Math in Stride, Book 3

OBJECTIVE 12 Multiply and divide by 2, 5, and 10.

APPROPRIATE MATERIALS

Number puzzles
Games
Multiplication and division story problems

ENABLING SKILLS AND ACTIVITIES

The multiplication array and the multiplication/division frame are bridges from the pictorial to the abstract recording of multiplication and division facts. (See Objective 7.) Students who use factor-factor-product terminology can explore division as a missing factor situation and understand that multiplication and division are related.

Once students have developed an understanding of the factor-factor-product relationship, skills may be reinforced by playing games such as Multiplication Bingo.

SAMPLE LESSON Multiplication Bingo

MATERIALS NEEDED

A pair of numeral dice
Chips or beans as playing markers
Multiplication bingo cards

Make dice by putting factors that are being studied on the faces of each die. Die A might have 6,3,7,5,2, and 4 on its faces and Die B might have 9,8,6,3,2, and 5.

Make multiplication bingo cards by writing a product in each space. Each player uses a different card.

[BINGO CARD]

B	I	N	G	O
36	8	12	10	45
9	56	4	27	6
63	21	20	18	14
40	25	15	24	16
54	42	48	35	30

Students play in pairs. They take turns rolling the dice. They multiply the two numbers that are rolled and then cover the product with a marker if it is on the card. If the card does not have the product, the other student must roll the dice. The first student with five markers in a row (vertically, horizontally, or diagonally) wins the game.

TEACHER RESOURCE MATERIALS

The Mathworks
Try-A-Tile, Set C

OBJECTIVE 13 Identify objects or numbers that do or do not belong in a collection, matrix, or array.

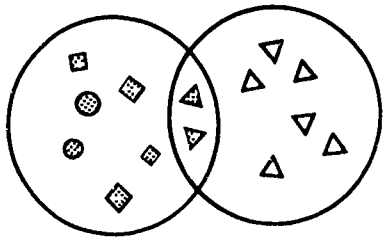
APPROPRIATE MATERIALS

Attribute blocks (developed by ESS - Elementary Science Study)
Junk boxes (boxes of buttons, nuts and bolts, dry pasta shapes, flat plastic bakery tab seals, or other items that can be sorted by one or more attribute)
Numeral cards
Attribute block templates and colored markers
Construction paper attribute shapes

ENABLING SKILLS AND ACTIVITIES

The ability to classify objects, or ideas such as numbers, into sets is an important problem-solving skill. Before students solve problems that require them to place an item into a collection, they should have had many opportunities exploring attributes by sorting objects into a set based on their own, or someone else's, "rules" for inclusion.

Assigning an object to a set based on one attribute is the first stage in developing classification strategies. Later students should explore sets of objects, such as Attribute Blocks, for two attributes (shape and color, perhaps). Some of the objects may then fit into more than one set - a red triangle, for instance, could be placed into a set of red objects and into a set of three-sided objects. Inclusion into both sets develops the idea of intersection. Students must reason logically in order to deal with the classification of objects by attributes.



SAMPLE LESSON Attribute Block Matrices

MATERIALS NEEDED

Attribute blocks - Use the ESS blocks, which are a set of 32 blocks with four shapes (square, triangle, diamond, and circle); four colors (red, green, blue, and yellow); and two sizes (large and small)
A large piece of oaktag (18" x 24") for each student
Magic markers

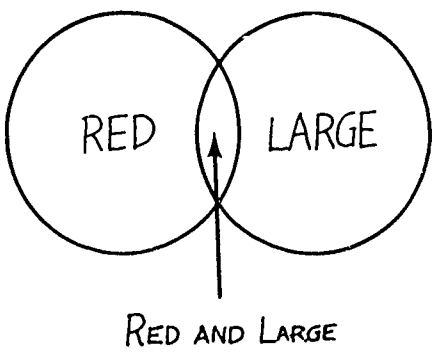
Seat students in small groups (3-4) and give each student a set of Attribute Blocks. Have each student use a sheet of oaktag as a work space. Ask them to keep their blocks on the oaktag so that sets do not get mixed up during the activities.

Allow the students some time to become familiar with the Attribute Blocks. Encourage them to build, describe, and list different sets of blocks, such as:

- the set of all blocks that are red
- the set of all large blocks
- the set of all blocks that are not triangles.

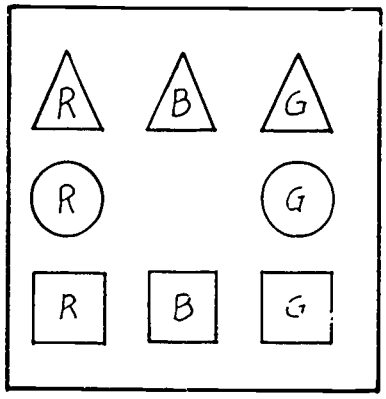
How many sets of blocks can be defined using one attribute?

Try building sets such as a set of all large, red blocks. Build a set of red blocks, then build a set of large blocks. Show the intersection using overlapping yarn rings.



Build a set of all green blocks. Have each student secretly remove one piece. Ask the students in each group to look at each other's green sets and determine what piece is missing. Repeat the "secret shape" activity with other sets, including sets sorted by two attributes. Encourage students to rearrange the pieces in order to help spot the piece that is missing. Discuss the arrangements.

Next build a sample matrix, such as:



What is missing?
How can you tell?

Have students build their own 3" x 3" matrices. They should build complete ones first. Discuss the "rules" or attributes used to assign pieces to rows and columns. Students may remove a piece and try to guess what is missing in each other's matrix. As students become more familiar with the strategy, they may not need to build complete matrices before they assign a missing piece.

This lesson may take several days. As an extension to the lesson, students may record their Missing Piece Matrix Puzzles. Use markers and templates or construction paper pieces and draw the puzzles on oaktag (9" x 12"). Combine the puzzles into a special deck, or make a matrix puzzle book by placing them in a 3-ring binder. Share the puzzles in a math activity center.

TEACHER RESOURCE MATERIALS

Attribute Acrobatics
The Fun World of Relationshapes: Patches
Attribute Games and Activities

OBJECTIVE 14 Read and interpret bar graphs and pictographs.

APPROPRIATE MATERIALS

Graph paper (inch and centimeter ruled)
Sorting objects such as buttons, M&M's, mixed nuts in the shell, etc.
Stickers
Large sheets of newsprint or fingerpaint paper
Newspapers

ENABLING SKILLS AND ACTIVITIES

Graphing is an important topic that deserves increased emphasis in the classroom. The organization of information is part of good problem-solving strategy. Students should be permitted many opportunities to make graphs. The classroom routine generates many pieces of information that could be graphed, and many graphs could be added to on a daily basis. Weather (sunny, rainy, cloudy, and snowy); temperature (hot, warm, and cold); beverage count (milk, chocolate milk, and fruit drink); and daily attendance are all excellent topics for graphs.

The classification of objects into sets, the exploration of different shapes made from a certain number of tangram pieces, the estimates of beans in a jar, or the results of a simple class survey are examples of math lessons that can be recorded graphically.

Graphing begins with the construction of "real" graphs - graphs in which the actual objects are arranged in rows by category. This allows students to develop an understanding of what a graph represents. Pictographs (picture graphs) are easily made by drawing the corresponding real graph. Later students will be able to represent the information pictorially without first

making a real graph. Bar graphs are more symbolic than pictographs and should only be constructed when it is clear that students can read and make comparisons and predictions from real graphs and pictographs. Students should be working with graphs in which one symbol (picture or square) represents one object. Reading graphs and comparing data helps students to develop the organizational and language skills needed to interpret and solve problems.

SAMPLE LESSON Making and Reading Graphs--The M&M Investigation

MATERIALS NEEDED

Graph paper (1-inch ruled)
Small bags of M&M's
Markers - brown, green, yellow, orange, tan

Do the first graph as a whole class activity. Open a bag of M&M's and sort by color. Count the number of M&M's in each pile. Plan the graph together. Discuss how it will be organized and labeled. Use your colored markers (with colors matching each pile of M&M's) to shade in one box on the graph for each pile of M&M's. What information does the graph show?

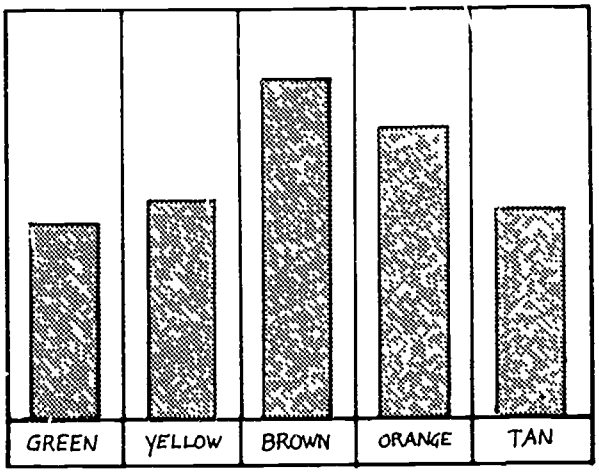
How many pieces are green?

Which color is most frequent?

How many M&M's are in the bag?

List all the possible questions that can be answered from the graph.

What questions about the M&M's can't be answered from the graph?



Problem Solving and Applications, continued

Next, work in small groups of 3-4 students. Give each group a small bag of M&M's. Will the distribution of colors be the same? Ask each group to make a graph of their bag's contents.

Compare the graphs. Is the same color most frequent on each graph? Did each bag have the same number of M&M's? If another bag is opened, which color might appear most frequently? Which least?

TEACHER RESOURCE MATERIALS

Mathematics Their Way
The Primary Math Lab
Math in Stride, Book 3

OBJECTIVE 15 Read and interpret data from tables and charts.

APPROPRIATE MATERIALS

Graph paper (1-inch ruled)
Newspapers
Menus
Pictographs and bar graphs

ENABLING SKILLS AND ACTIVITIES

Like graphs, tables also organize information. However, tables are more abstract than graphs. A good way to help students understand what a table represents is to have them collect information, organize it into graphs, and then reorganize it into a table.

The table depicts how many "events" are in each category numerically rather than pictorially. Comparisons can be made from the table and double-checked on the graphs.

SAMPLE LESSON Making and Reading Charts - How Many Questions Can We Answer?

MATERIALS NEEDED

A large sheet of newsprint or oaktag (18" x 24" or larger)
Graph paper (1-inch ruled)
Markers

Complete the graphing activities in the Sample Lesson for Objective 14 - The M&M Investigation.

Problem Solving and Applications, continued

Use the graphs to make a chart that summarizes all the important information found on the graphs. Discuss how the chart might be organized and labeled. What labels from the graph will be used on the chart? What should the title of the chart be?

Build a chart such as the one below.

Color Distribution in Small Bags of M&M's

M+M's BAG	GREEN	YELLOW	ORANGE	BROWN	TAN	TOTAL IN BAG
1.	8	9	7	25	6	55
2.	7	5	8			
3.						
4						
5.						
etc.						

List all the questions that can be answered from the chart.

- Which is the most frequent color?
- How many green ones are in the first bag?
- Does each bag contain the same number of M&M's?

Which questions about M&M's can't be answered from the chart?

TEACHER RESOURCE MATERIALS

The Good Time Math Event Book
Measurement and the Child's Environment

OBJECTIVE 16 Identify or write number sentences from pictures.

APPROPRIATE MATERIALS

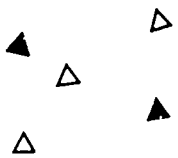
Sets of counting objects
Simple pictures of sets of objects

ENABLING SKILLS AND ACTIVITIES

Students need the opportunity to match number sentences to collections of objects and then to pictures of objects. The matching activities enable them to see that a number sentence provides a specific description of the information in the picture.

Which number sentence matches the picture?

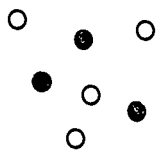
$5 + 2 = 7$
$2 + 3 = 5$
$10 - 2 = 5$



Try to use some open sentences.

Complete these number sentences. Circle the one that matches the picture.

$7 - \square = 3$
$6 + 2 = \square$
$\square - 4 = 10$



Later, students should write their own number sentences for pictures. Several number sentences may be possible, and students should be encouraged to interpret pictures in more than one way.

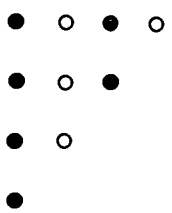
SAMPLE LESSON Writing Number Sentences to Describe Pictures

MATERIALS NEEDED

Worksheets of pictures and number sentences
Pencils, crayons, or markers

A sample of the picture/number sentence work is shown below.

Here is a picture.

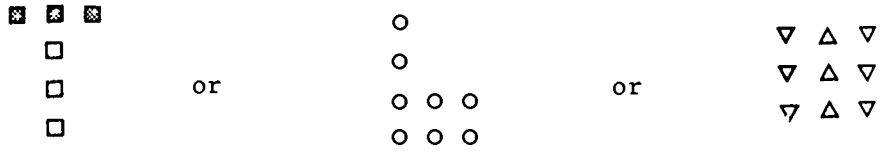


Here are some number sentences that describe the picture. Complete the number sentences.

4 + ☐ = 10
4 + 3 + 2 + 1 = ☐
10 - 6 = ☐

Now write some more number sentences that describe the picture. The answers will vary.

Provide a variety of pictures using different shapes, layouts, and shadings, such as:



Notice the different ways that each picture can be perceived.

As an extension to the lesson, you may give students a number sentence and ask them to draw a picture to fit it. What meaning did each student assign to the number sentence as he/she translated the idea from symbols to pictures?

TEACHER RESOURCE MATERIALS

Math in Stride, Book 3
Problem Solving in School Mathematics: 1980 NCTM Yearbook

OBJECTIVE 17 Identify number sentences from addition or subtraction story problems.

APPROPRIATE MATERIALS

Simple addition and subtraction story problems
Pictures and matching number sentences

ENABLING SKILLS AND ACTIVITIES

Story problems present one kind of problem-solving experience. Students learn to "translate" sentences from English to open number sentences that use mathematical symbols. They then apply computational skills to solve the open sentence and check for reasonableness of the answer. Story problems provide additional opportunities to practice computational skills and are often used in basal texts to reinforce lessons on computation.

To promote an understanding of the special syntax of story problems, students should write their own. By solving their own problems they will learn strategies for decoding the component parts of story problems. They will also learn to write number sentences that describe problems.

SAMPLE LESSON Linking Number Sentences to Story Problems

MATERIALS NEEDED

Paper and pencil
Worksheets of number sentences
Oaktag
Markers or crayons

Give each student a worksheet with five different number sentences, such as 10 - 3 = ☐ , each with a corresponding story problem.

Ask the student to write a new and different story problem for each number sentence. For example:

> 10 - 3 = ☐
>
> The story below fits the number sentence:
>
> Bob had 10 stickers. He gave 3 of them to Alida. How many stickers did Bob have left?
>
> Write a different story to fit the number sentence.

Share the story problems. Can more stories be written for these number sentences?

As an extension of this lesson, students can try to match number sentences to story problems. Write the number sentences, one per card, on 3" x 9" oaktag strips. Write the students' corresponding story problems on 4" x 9" oaktag strips. (The larger cards provide more writing space and also keep the two decks from getting mixed up.) Shuffle the cards in each deck. Ask students to place the number sentences in a row. Then place story problem cards that fit the sentence next to each sentence card.

TEACHER RESOURCE MATERIALS

Arithmetic Teacher, Focus Issue on Problem Solving, February, 1982

OBJECTIVE 18 Solve simple story problems involving addition or subtraction.

APPROPRIATE MATERIALS

Addition and subtraction story problems
Paper and pencil

ENABLING SKILLS AND ACTIVITIES

When students have had practice identifying number sentences from pictures and from story problems (as described in activities for Objectives 16 and 17) they will have developed the ability to identify the key parts of the typical story problem.

Students must now be able to write a number sentence for the story problem and use it to solve the problem. Some students may still wish to use counting objects and "act out" the story. "Acting it out" is a good problem-solving strategy. Other students may wish to draw a picture in order to organize the information in a way that helps write the number sentence. Both strategies should be encouraged. Students should have daily practice in writing and solving story problems.

SAMPLE LESSON Writing Number Sentences to Solve Story Problems - Bulletin Board Center

MATERIALS NEEDED

Pencils
Paper cut into 3" x 9" strips

Give each student a strip of paper. Assign each strip a number by counting around the room. Ask each student to write an addition story problem on the slip. Ask each student to write a number sentence that fits the story on the back of the slip.

Collect the strips, check the work, and place the problem strips on the bulletin board. Allow students time each day to visit the bulletin board. During these visits the students should write a number sentence that fits a problem, and then solve the problem. Since some problems may be more difficult than others, let students select the problems they will do. Allow students to check their work by looking at the number sentences on the reverse side of the slips, and by talking with the author of the story problem.

Repeat this activity for subtraction story problems. Later mix addition and subtraction problems. Keep a story problem bulletin board as an ongoing activity all year.

TEACHER RESOURCE MATERIALS

Problem Solving in School Mathematics, the 1980 NCTM Yearbook
Arithmetic Teacher, Focus Issue on Problem Solving, February, 1982

OBJECTIVE 19 Solve and identify number sentences in simple story problems involving addition and subtraction with extraneous information.

APPROPRIATE MATERIALS

Addition and subtraction story problems with extraneous information
Paper and pencil

ENABLING SKILLS AND ACTIVITIES

Extraneous information can distract students. Continue to have students write their own story problems. Ask them to add a piece of extra information to their own story problems. Trade stories and help each other find the unnecessary information. Cross it out and solve the problem. Students gain practice focusing on needed information if they write their own questions for given information.

SAMPLE LESSON Writing Questions From Story Problem Information

MATERIALS NEEDED

Paper and pencil
Worksheet of story problem information

Prepare a worksheet with a completed sample problem and six similar examples.

Here is some information:

Our class has 9 boys and 15 girls. We also have five visiting parents and our teacher in the room today.

Here are some questions that could be asked:
How many children are in the class?
How many adults are in the class?

Note that the questions will vary and that more than one is possible.

This activity is open-ended. Advanced students will write more interesting questions and will discover more answerable questions.

TEACHER RESOURCE MATERIALS

Math in Stride, Book 3
Basal Textbook series

OBJECTIVE 20 Identify needed information in problem situations.

APPROPRIATE MATERIALS

Addition and subtraction story problems that cannot be solved due to insufficient information
Paper and pencil

ENABLING SKILLS AND ACTIVITIES

Some time each day should be devoted to solving problems. Students should have worked their way through the sequence of activities for Objectives 16, 17, 18, and 19. They will have developed a sense of the component parts of story problems. Now they will need to identify a missing component. The sample lesson below provides practice in determining when enough information is available. It also simulates having extraneous information.

SAMPLE LESSON Story Problem Mix-Up - A Game for Finding Needed Information

MATERIALS NEEDED

Student-written addition and subtraction story problems
Oaktag strips (3" x 5") or 3" x 5" cards
Markers
Worksheet for recording solutions and keeping score

Have students help make up the game cards. Use 20 or more of their story problems. Break up each story problem and question into separate pieces of information. Put each piece on a separate card. Be certain to add needed nouns or adjectives.

For example, if a student's problem said:

"Sue had 16 red ribbons and Jill had 14. How many more did Sue have?"

Then make three cards that say:

Sue had 16 red ribbons.

Jill had 14 red ribbons.

How many more ribbons did Sue have than Jill?

Also include in the deck six cards that say:

Put one of your cards back in the pile. Put this card back in, too. Shuffle the pile and try again.

This will provide more opportunity to match cards. Students may return an information card or a question card.

Allow 2 to 4 students to play the game. Each student in turn takes a card from the top of the pile. Whenever a student gets enough information to answer a question on one of his/her cards, he/she says, "Solved it!" and writes the number sentence and the answer on his/her record sheet. One point is received for each correct solution.

The student who gets 3 points wins.

TEACHER RESOURCE MATERIALS

Problem Solving in School Mathematics, The 1980 NCTM Handbook

OBJECTIVE 21 Measure length and identify appropriate units for measuring length and distance.

APPROPRIATE MATERIALS

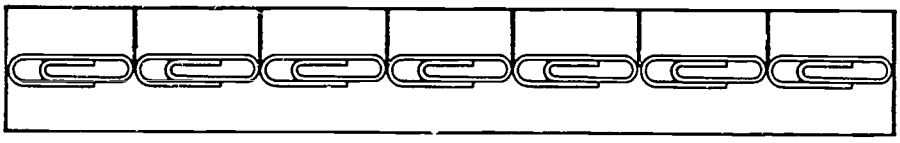
Dry pasta
Construction paper squares
Ceramic tiles
Paper clips
Oaktag
Centimeter cubes
Decimeter rods (orange Powers of Ten Rods with lines)
Rulers marked in whole inches or whole centimeters (no fractional parts)

ENABLING SKILLS AND ACTIVITIES

Students need to explore measurement with nonstandard units when they begin to compare lengths or areas. They need to check how many fairly uniform objects are needed to span a length or cover a shape. The use of familiar items helps them focus on the idea that they measure by finding out how many of the same item is used. It is the number of items or units that gives them the measure. Continued use of nonstandard units provides a variety of estimation (guess how many) and measurement (now check by covering and counting) activities. A rich discussion can occur as students think about why it takes more handspans when a student measures the length of a table than when the teacher measures it.

The introduction of standard units too early often results in students making conceptual errors about measurement, such as looking at the lines rather than the spaces on a ruler. Students who are confused may even measure from the "1" mark on a ruler, rather than from the end or zero mark.

You can help prevent such problems by having students construct their own rulers. Use a nonstandard unit, such as a large paper clip. Stretch a chain of large paper clips onto a strip of paper and glue them down. Draw lines from the ends of each clip to the edge of the paper.



Use these "rulers" to measure many objects. Make another "ruler" using smaller paper clips. Measure objects with one, then the other ruler. Record the measures of each. What is happening? Draw some line segments that are a certain number of clips long. Guess which objects in the classroom might be a certain number of clips long. Measure the objects. How many clips off was the estimate? These activities will help students understand why they have many standard units for measuring length (centimeter, meter, inch, foot, yard, and mile) and when each might be an appropriate unit to use.

SAMPLE LESSON Guess, Measure, Record - Using a Centimeter Ruler

MATERIALS NEEDED

A centimeter ruler (30-cm long - whole centimeters only)
Note: If you do not have these you can make them by marking strips of paper, by using Powers of Ten Rods as a guide, or by cutting strips of centimeter graph paper.
Various objects to measure, such as pencils, shoes, handspans, etc.
Paper and pencil for recording

Give each student a centimeter ruler, a Record Sheet, and some objects to measure. Ask each student to guess how long the object might be in centimeters. Record the guess. Then measure the object. Record the measure.

OBJECT MEASURED	GUESS	MEASURE
My Pencil	9	11
My Shoe		
Billy's Shoe		
ETC.		
"		
"		
"		

Continue to guess and record. Allow students to select a few more objects to estimate and measure.

Compare measures. Discuss the results of the guesses.

TEACHER RESOURCE MATERIALS

Measurement and the Child's Environment
The Primary Math Lab

OBJECTIVE 22 Estimate lengths and areas.

APPROPRIATE MATERIALS

Straws, toothpicks, pencils, paper clips, hands, ans, or other nonstandard units of measure
Objects for covering such as lima beans, buttons, stickers, centimeter cubes, etc.
Tangrams
Geoboards
Ceramic tiles
Pattern blocks
Transparent grids
Graph paper
Construction paper squares

ENABLING SKILLS AND ACTIVITIES

As with Objective 21, students need practice with familiar objects used as nonstandard units of measure. Often, real life situations which require students to estimate area or length do not require them to use standard units. They may need to estimate how many tiles will cover a floor, or how many pieces of border trim will outline a bulletin board. Whether it is with standard or nonstandard units, students must develop their spatial skills to improve estimating length and area.

SAMPLE LESSON Estimating and Checking Area with Tangram Pieces

MATERIALS NEEDED

Tangrams
Tangram templates
Construction paper tangram pieces
Oaktag (9" x 12")

Have students help make tangram puzzles. Place several tangram pieces together on top of oaktag so that sides meet and there are no holes in the design. (You may use more than one set.) Trace the outline of the large shape.

Guess how many of the smallest triangle pieces it would take to cover the shape.

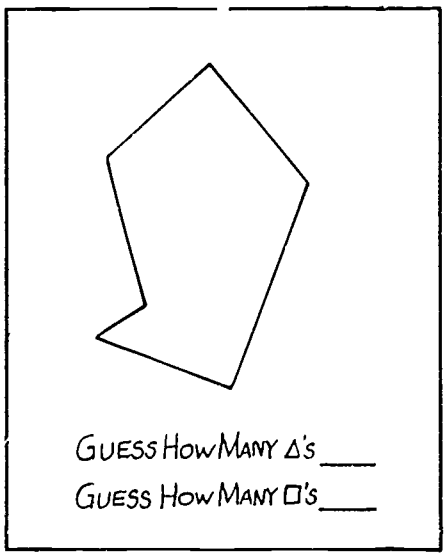
Find out by using construction paper pieces. Cover the shape with as many small tangram triangles as needed.

Guess how many tangram squares it might take. Cover and find out.

Repeat with other tangram pieces.

Make more puzzle shapes. Use only tangram squares to make the shapes.

What other pieces can cover the shape?



TEACHER RESOURCE MATERIALS

Tangram Diary

OBJECTIVE 23 Tell time to the nearest hour, half hour, and quarter hour using analog and digital clocks.

APPROPRIATE MATERIALS

Clock faces
Analog and digital clocks
Timers - minute, egg, oven, stopwatch, etc.

ENABLING SKILLS AND ACTIVITIES

Students must have opportunities learning to sequence events (before, after, later, etc.) and learning to determine "about what time" certain things occur; (at 12:00 I eat lunch, at 6:00 I eat dinner, at 9:00 I go to bed, etc.). They also need to measure "how long it takes" to do everyday tasks, like walk to school or wash all the chalkboards. What task might take 5 minutes, 15 minutes, 30 minutes, an hour, a day, etc.? Then time will have some meaning.

Students should also gain familiarity with both digital and analog clocks. They need to develop the ability to sense that 6:45 is closer to 7:00 than to 6:00 when reading a digital clock.

SAMPLE LESSON Matching Time

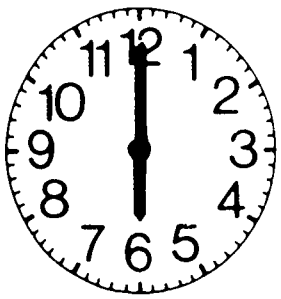
MATERIALS NEEDED

Oaktag squares (4" x 4")
Clock face stamps or template

Cut up about 50 oaktag squares. Make a clock face on half of the oaktag squares and mark each one with a time to the hour, half hour, or quarter hour. Make digital clock readings on other cards so that they match the clock face times.

Then shuffle each deck. Pull five cards out of one deck. Ask students to find the cards from the other deck with the same time.

Write a story about the five times - put them in sequence in the story. Illustrate the story with clock faces.



6:00

TEACHER RESOURCE MATERIALS

Measurement and the Child's Environment

OBJECTIVE 24 Determine the value of a set of coins.

APPROPRIATE MATERIALS

Real coins
Realistic play money
Coin stamps
Objects for trading

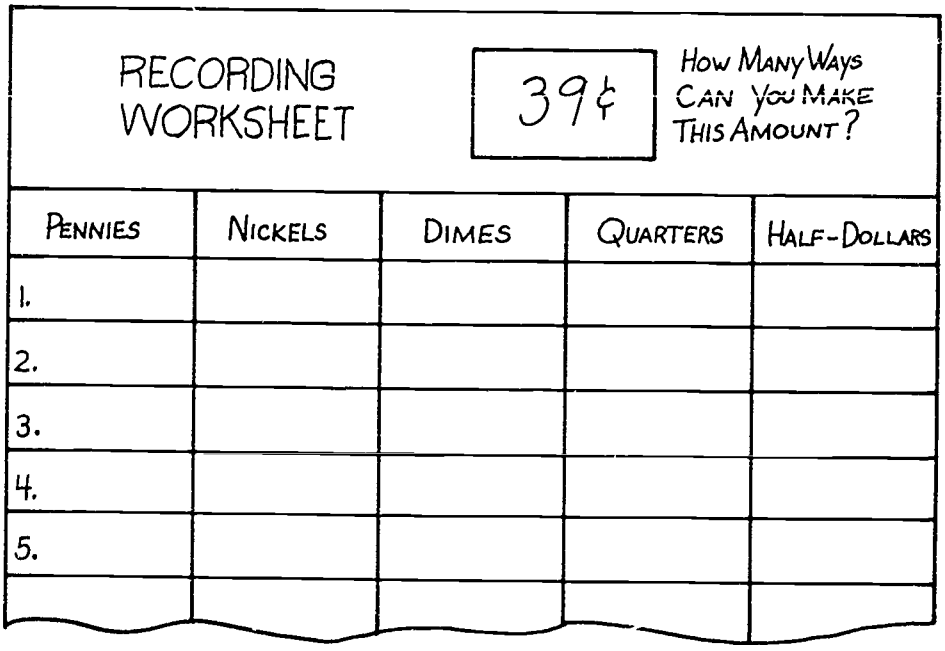
ENABLING SKILLS AND ACTIVITIES

Students should have the opportunity to trade objects for coins. "Play" store activities demonstrate how money is used. Students will need to be familiar with the names and monetary values of each coin - penny, nickel, dime, quarter, and half-dollar. They need to understand the use of the notation for money - 50¢ or $.50. They also should have some practice making change.

SAMPLE LESSON How Many Ways Can You Make This Change?

MATERIALS NEEDED

Recording worksheet
Paper and pencil
Play coins



Give each student a recording worksheet. Think of an amount of money less than fifty cents. Put it in the box at the top of the worksheet. List all the ways you can make this amount of money using pennies, nickels, dimes, and/or quarters. Students may need to rearrange the play money to help them find all the combinations. With some practice, a pattern of listing the ways may develop. Repeat for other amounts less than a dollar.

As an extension, discuss values that can be made with one coin, two coins, three coins, etc.

TEACHER RESOURCE MATERIALS

Solving Problems Kids Care About

OBJECTIVE 25 Identify shapes, angles, and sides.

APPROPRIATE MATERIALS

Tangrams
Pattern blocks
Cuisenaire rods
Geoboards
Geodot paper
Graph paper
Mirrors

ENABLING SKILLS AND ACTIVITIES

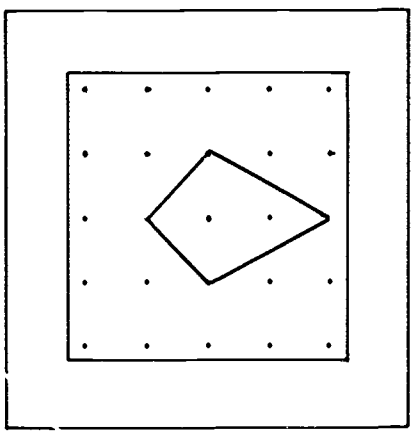
Geometry topics provide a rich strand in the mathematics curriculum. Geometry should not be restricted to memorization of a vocabulary. When students work with manipulative materials they can construct various shapes. The shapes can be sorted and compared by sizes, number of sides, number of angles, types of angles, or number of equal sides. Vocabulary can be built as a natural extension of the activity.

Geometrical ideas such as shape, size, length, area, volume, congruence, and similarity can all be discovered during exploration activities with manipulative materials such as pattern blocks, tangrams, and geoboards. Spatial relationships can also be investigated by considering the transformation of shape pieces as they are rotated or flipped.

SAMPLE LESSON Geodot Shape Cards

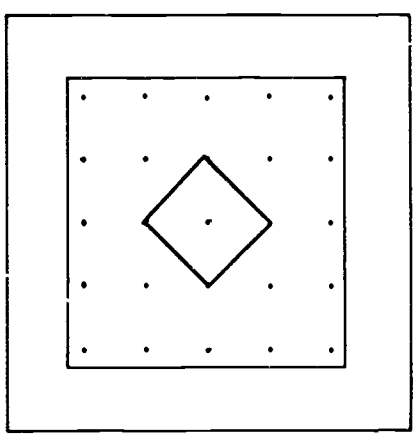
MATERIALS NEEDED

Geoboards and geobands
Geodot paper (3 x 3 inch scale)
Oaktag (3 x 3 inch)
Pencils and markers



Have students make a shape on the geoboard. You may wish to make this interesting by establishing a rule, such as - the shape can only touch four pegs.

Ask students to copy each design onto separate small pieces of geodot paper. Paste each onto an oaktag card.



Change the rule and make more shapes on the geoboard. Copy onto the geodot paper and paste onto the oaktag squares.

Now sort the cards. Have the students help decide the categories - triangles, right triangles, quadrilaterals, or parallelograms.

TEACHER RESOURCE MATERIALS

The Mathworks
Pattern Block Activities
MIRA Math
Geoboard Activity Cards
Spatial Problem Solving with Cuisenaire Rods

OBJECTIVE	SAMPLE TEST ITEM

CONCEPTUAL UNDERSTANDINGS

1. Identify the number one more, one less, ten more, or ten less than a given number.

What number is ten more than 52?

a. 42
b 51
c. 62
d. 53

2. Extend patterns involving numbers and attributes.

Which shape should come next?

a. 
b. 
c. 
d. 

3. Order whole numbers.

After school, Jason earned 85¢ shoveling snow. Thomas made 60¢ and Caron received 45¢.

Which of the following lists the children from lowest to highest in their earnings?

a. Thomas, Jason, Caron
b. Jason, Caron, Thomas
c. Caron, Thomas, Jason
d. Caron, Jason, Thomas

4. Rewrite numbers using expanded notation.

Which numeral means the same as 400 + 60 + 7?

a. 467
b. 476
c. 764
d. 400607

5. Rewrite numbers by regrouping tens and ones.

Which means the same as 83?

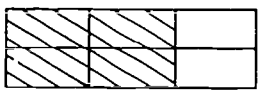
a. 7 tens and 13 ones
b. 6 tens and 13 ones
c. 7 tens and 3 ones
d. 3 tens and 8 ones

OBJECTIVE	SAMPLE TEST ITEM

CONCEPTUAL UNDERSTANDINGS, continued

6. Identify fractional parts of regions and sets from pictures for halves, thirds, fourths, and sixths.

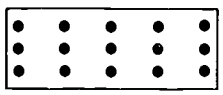
What fraction of the rectangle is shaded?



a. 1/6
b. 2/6
b. 4/6
d. 5/6

7. Relate multiplication and division facts to rectangular arrays.

Which number sentence goes with the picture?



a. 15 - 5 =
b. 5 x 3 =
c. 4 x 3 =
d. 3 + 5 =

COMPUTATIONAL SKILLS

8. Know addition and subtraction facts to 18.

17 - 9 =

a. 6
b. 8
c. 9
d. 12

9. Add and subtract one- and two-digit numbers without regrouping.

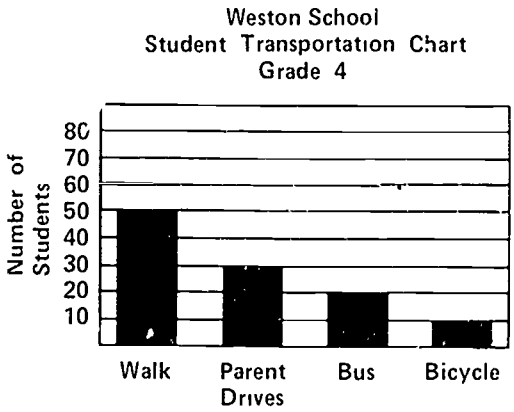
$$\begin{array}{r} 57 \\ -32 \\ \hline \end{array}$$

a. 35
b. 25
c. 89
d. 26

10. Add one- and two-digit numbers with regrouping.

$$\begin{array}{r} 54 \\ +37 \\ \hline \end{array}$$

a. 81
b. 811
b. 92
d. 91

OBJECTIVE	SAMPLE TEST ITEM
11. Estimate sums and differences to 100.	50 + 40 is a good estimate for which of the following? a. 53 + 37 c. 51 + 29 b. 49 + 61 d. 63 + 42
12. Multiply and divide by 2, 5, and 10.	Exactly how many 5's are in 45?
PROBLEM SOLVING AND APPLICATIONS	
13. Identify objects or numbers that do or do not belong in a collection, matrix, or array.	Which number belongs in the missing space? 5 13 21 10 ? 26 15 23 31 a. 15 c. 23 b. 17 d. 18
14. Read and interpret bar graphs and pictographs.	 How many of the 4th grade students at Weston ride the bus to and from school each day? a. 30 c. 50 b. 20 d. 10

OBJECTIVE	SAMPLE TEST ITEM

PROBLEM SOLVING AND APPLICATIONS, continued

15. Read and interpret data from tables and charts.

BICYCLE RODEO SCORES

STUDENT	BIKE SAFETY TEST	BIKE CHECK	SKILL IN RIDING EVENTS
Jason	44	25	25
Dawn	47	21	21
Amy	49	24	23
Daniel	42	22	24
Scott	45	23	23

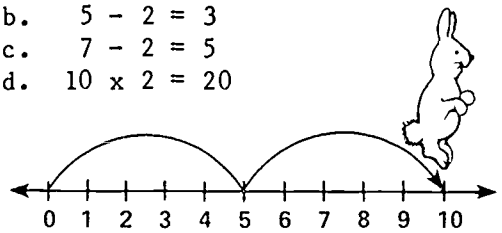
Who scored highest on the Bike Safety Test?

a. Daniel
b. Dawn
c. Amy
d. Scott

16. Identify or write number sentences from pictures.

The rabbit made 2 hops along the number line to get to 10. Which number sentence shows how long each hop was?

a. $10 \div 2 = 5$
b. $5 - 2 = 3$
c. $7 - 2 = 5$
d. $10 \times 2 = 20$

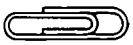
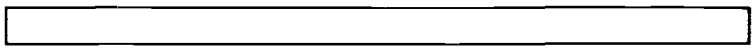
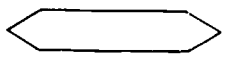


17. Identify number sentences from addition or subtraction story problems.

Mara read 16 books. Steven read 7. Which number sentence can you use to find out how many more books Mara read than Steven?

a. $16 \div 7 = \square$
b. $7 + \square = 16$
c. $9 + \square = 16$
d. $16 + 7 = \square$

OBJECTIVE	SAMPLE TEST ITEM
18. Solve simple story problems involving addition or subtraction.	Brian planted 17 bean seeds in his garden. Only 9 plants came up. How many seeds did not grow? a. 26 c. 9 b. 8 d. 12
19. Solve and identify number sentences in simple story problems involving addition and subtraction with extraneous information.	Kate found 4 hats and 2 scarves in the classroom closet. Then she found 1 hat and 2 more scarves on a desk. How many scarves did she find altogether? a. 4 c. 5 b. 9 d. 6
20. Identify needed information in problem situations.	Monica has a dollar. She wants to buy 1 hamburger and 1 order of french fries. What do you need to know to figure out if Monica has enough money to buy her lunch? a. How much allowance she gets b. How many hamburgers and french fries she will buy c. How much a shake costs d. The cost of the hamburger and french fries
MEASUREMENT AND GEOMETRY	
21. Measure length and identify appropriate units for measuring length and distance.	Which of the following is the best unit for measuring the length of a pencil? a. Yards c. Miles b. Feet d. Inches

OBJECTIVE	SAMPLE TEST ITEM
MEASUREMENT AND GEOMETRY, continued	
22. Estimate lengths and areas.	About how many paper clips are needed to cover the ribbon?   a. 10 b. 3 c. 6 d. 9
23. Tell time to the nearest hour, half hour, and quarter hour using analog and digital clocks.	The clock shows 6:30. What time is it? a. Half past twelve b. Quarter past six c. Six o'clock d. Half past six
24. Determine the value of a set of coins.	What is the total value of these coins?  a. 80¢ c. 75¢ b. 70¢ d. 65¢
25. Identify shapes, angles, and sides.	How many sides does this figure have?  a. 5 c. 7 b. 8 d. 6

APPENDIX B

SUPPLIERS OF MANIPULATIVES AND TEACHER RESOURCE MATERALS

SUPPLIER	MATERIALS
Burt Harrison & Co. P.O. Box 732 Weston, MA 02193-0732 (617) 647-0674	Manipulatives
Constructive Educational Concepts P.O. Box 667 Bloomfield, CT 06002	Set Boards Counting/Sorting Trays
Creative Publications 5005 West 110th Street Oak Lawn, IL 60453 (800) 624-0822	Manipulatives Resource Materials
Cuisenaire Co. 12 Church Street New Rochelle, NY 10805 (914) 235-0900	Manipulatives Resource Materials
Dale Seymour Publications P.O. Box 10888 Palo Alto, CA 94303 (800) 872-1100	Resource Materials
Dick Blick/Hortoncraft P.O. Box 330 Farmington, CT 06032 (800) 447-8192	Felt, Railboard Art Supplies
Didax 6 Doulton Place Peabody, MA 01960 (617) 535-4757	Manipulatives Resource Materials
Economy Handicrafts, Inc. 50-21 69th Street Woodside, NY 11377 (718) 426-1600	Tongue Depressors, Yarn, Craft Supplies
National Council of Teachers of Mathematics 1906 Association Drive Reston, VA 22091 (703) 620-9840	Professional Publications Resource Materials

Note: The above list contains suppliers of materials mentioned in the handbook. The list is NOT exhaustive, but does include many regional suppliers. Your school office may have their catalogs, as well as those of other suppliers. The addresses provided above are for your convenience in acquiring information.

APPENDIX C
SOFTWARE

The software listed below provides opportunities to explore and reinforce the skills and concepts of particular objectives. Publisher's name appears in parentheses.

OBJECTIVE 1

Fish-Metic (Commodore)
Fundamental Math Levels 1-2 (Random House)
Number Stumper (Learning Company)
Math Concepts (Hartley)
Math Sequences 1-4 (Milliken)
Aliencounter and Face Flash (Milliken)
Math Ideas with Base 10 Blocks (Cuisenaire)

OBJECTIVE 2

Flip Flop (Edufun/Milliken)
Logic Builders (Scholastic)
The King's Rule (Sunburst)
Early Games Matchmaker (Springboard)
Tangrams Puzzler (Edufun/Milliken)
Odd One Out (Sunburst)
Discrimination, Attributes, and Rules (Sunburst)
The Pond (Sunburst)

OBJECTIVE 3

Learning About Numbers Vol. 1 (C & C Software)
Whole Numbers (Control Data)

OBJECTIVE 4

Expanded Notation (Hartley)

OBJECTIVE 5

None listed

OBJECTIVE 6

Elementary Math Classroom Learning (Sterling Swift)
Fractions 1 (Scott, Foresman)

OBJECTIVE 7

Addition Table/Multiplication Table (SVE)

OBJECTIVE 8

Teasers by Tobbs (Sunburst)
Math Maze (DesignWare)
Piece of Cake Math (Springboard)
Learning About Numbers, Vol. 1 (C & C Software)
Addition and Subtraction 1-2 (Scott, Foresman)
Arith-Magic (Quality Education Design)
Basic Math System H/S (Mathware)
Basic Number Facts (Control Data)
Stickybear Math (Xerox-Weekly Reader)
Beginning Mathematics Series (SVE)
Milliken Basic Math Facts (Milliken)
Number Builder (Commodore)
Shipshape Math (Data Command)
Targets-A-Number Game (Sunburst)
Math Blaster (Davidson)
Challenge Math (Sunburst)
Monkey Up A Tree (Atari)
SRA Computer Math A,B,C (SRA)
Alien Action (DTM)
Gulp Arrow Graphics (Edufun)
Math Invaders (Winners Circle)
Addition Magician (Learning Company)
Math Concepts I/II (Hartley)

OBJECTIVE 9

Math Man (Scholastic)
Teasers by Tobbs (Sunburst)
Arith-Magic (Quality Education Design)
Basic Math System H/S (Mathware)

OBJECTIVE 10

Teasers by Tobbs (Sunburst)
Math Man (Scholastic)
Basic Math System H/S (Mathware)
Prescriptive Math Drill (Hartley)

OBJECTIVE 11

Challenge Math (Sunburst)
Power Drill (Sunburst)
Elementary Vol. 4 - Math/Science (MECC)
Building Estimating Skills (Cuisenaire)

OBJECTIVE 12

Teasers by Tobbs (Sunburst)
Math Maze (DesignWare)
Math Man (Scholastic)
Piece of Cake Math (Springboard)
Basic Math System H/S (Mathware)
Basic Number Facts (Control Data)
Milliken Basic Math Facts (Milliken)
Number Builder (Commodore)
Shipshape Math (Data Command)
Math Blaster (Davidson)
Division Skills (Milton Bradley)
Gulp Arrow Graphics (Edufun/Milliken)
Math Invaders (Winners Circle)

OBJECTIVE 13

Bumble Games (Learning Company)
Discrimination, Attributes, etc. (Sunburst)
Logic Tools (Learning Company)
Moptown (Advanced Learning Technology)
Gertrude's Puzzles (Learning Company)
Gertrude's Secrets (Learning Company)

OBJECTIVE 14

Exploring Tables and Graphs (Xerox-Weekly Reader)
Easy Graph (Grolier)

OBJECTIVE 15

Exploring Tables and Graphs (Xerox-Weekly Reader)

OBJECTIVE 16

None listed

OBJECTIVE 17

None listed

OBJECTIVE 18

Piece of Cake Math (Springboard)

OBJECTIVE 19

Read and Solve Story Problems (Educational Activities)

OBJECTIVE 20

None listed

OBJECTIVE 21

Survival Math (Sunburst)
Delta Drawing (Spinaker)

OBJECTIVE 22

Survival Math (Sunburst)

OBJECTIVE 23

Learning About Numbers, Vol. 1 (C & C Software)
Money and Time (SVE)
Clock (Hartley)

OBJECTIVE 24

Magic Cash Register (Avant-Garde)
Money and Time (SVE)

OBJECTIVE 25

Trap-A-Zoid (DesignWare)
Flip Flop (Edufun/Milliken)
Delta Drawing (Spinaker)

APPENDIX D
SOFTWARE PUBLISHERS

Below are the addresses of software publishers whose programs are included in Appendix C.

Atari Program Exchange
P.O. Box 3705
Santa Clara, CA 95055

Avant-Garde
P.O. Box 30160
1907 Garden Avenue
Eugene, OR 97403

C & C Software
5713 Kentford Circle
Wichita, KS 67220

Commodore Software
1200 Wilson Drive
West Chester, PA 19380

Control Data
P.O. Box 261127
San Diego, CA 92126

Cuisenaire Company of America
12 Church Street
New Rochelle, NY 10805

Data Command
P.O. Box 548
Kankakee, IL 60901

Davidson and Associates
6069 Groveoak Place #12
Rancho Palos Verdes, CA 90274

DesignWare
10 Berry Street
San Francisco, CA 94107

DLM Learning Materials
One DLM Park
Allen, TX 75002

Educational Activities
P.O. Box 392
Freeport, NY 11520

Edufun (See Milliken)

Grolier Electronic Publishing
Dept. 336 Sherman Turnpike
Danbury, CT 06816

Hartley Courseware
P.O. Box 431
Dimondale, MI 48821

The Learning Company
545 Middlefield Road, Suite 170
Menlo Park, CA 94025

Mathware
919 14th Street
Hermosa Beach, CA 90254

MECC Distribution Center
2520 Broadway Drive
St. Paul, MN 55113

Milliken Publishing Co.
(Edufun)
1100 Research Boulevard
St. Louis, MO 63132

Milton Bradley (Media Materials)
2936 Remington Avenue
Baltimore, MD 21211

Quality Educational Design
P.O. Box 12486
Portland, OR 97212

Random House School Division
Dept. 9305 400 Hahn Road
Westminster, MD 21157

Scholastic Software
730 Broadway
New York, NY 10003

Scott, Foresman and Co.
Electronic Publishing
1900 East Lake Avenue
Glenview, IL 60025

Spinaker Software
215 First Street
Cambridge, MA 02142

Springboard Software
7807 Creekridge Circle
Minneapolis, MN 55435

SRA
Science Research Associates
Box 1000
Chicago, IL 60601

Sterling Swift
1600 Fortview Road
Austin, TX 78704

Sunburst Communications, Inc.
39 Washington Avenue
Pleasantville, NY 10570

SVE
1345 Diversey Parkway
Chicago, IL 60614

Winners Circle Education Co.
1308 Temple Building
Rochester, NY 14604

Xerox Education Publications
245 Long Hill Road
Middletown, CT 06457

Ashlock, Robert B. Error Patterns in Computation: A Semi-Programmed Approach. 3rd ed. Columbus, Ohio: Charles E. Merrill Publishing Co., 1982.

Baratta-Lorton, Mary. Workjobs: Activity-Centered Learning for Early Childhood Education. Menlo Park, California: Addison-Wesley Publishing Co., 1972.

Baratta-Lorton, Mary. Workjobs...For Parents: Activity-Centered Learning in the Home. Menlo Park, California: Addison-Wesley Publishing Co., 1975.

Baratta-Lorton, Mary. Mathematics Their Way. Menlo Park, California: Addison-Wesley Publishing Co., 1976.

Baratta-Lorton, Mary. Workjobs II: Number Activities for Early Childhood. Menlo Park, California: Addison-Wesley Publishing Co., 1979.

Barson, Alan. Geoboard Activity Cards. Palo Alto, California: Creative Publications, 1979.

Baur, Gregory P. and Linda O. George. Helping Children Learn Mathematics: A Competency-Based Approach. Menlo Park, California: Cummings Publishing Co., 1976.

Burns, Marilyn. The Good Time Math Event Book. Palo Alto, California: Creative Publications, Inc., 1977.

Burton, Grace M. Towards a Good Beginning: Teaching Early Childhood Mathematics. Menlo Park, California: Addison-Wesley Publishing Co., 1985.

Castellano, Janet and Matthew Scaffa. Unimath: Mathematics Activities for the Primary Grades Using Interlocking Cubes. Silver Springs, Maryland: Unimath Corp., 1982.

Clark, Clara. Beginning Fractions. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1979.

Clark, Clara. Beginning Multiplication and Division. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1979.

Clark, Clara. Teacher Handbook for Sequencing Math Skills Grades K-4. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1979.

Clark, Clara. Tangram Diary. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1980.

Clark, Clara and Betty Sternberg. Math in Stride, Book 1. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1980.

Clark, Clara and Betty Sternberg. Math in Stride, Book 2. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1980.

Bibliography, continued

Clark, Clara and Betty Sternberg. Math in Stride, Book 3. Bloomfield, Connecticut: Constructive Educational Concepts, Inc., 1980.

Cook, Marcy. Try-A-Tile, Set A: Count-a-Tile, Add-a-Tile, Take Away Tiles, Sum Tiles. Palo Alto, California: Creative Publications, Inc., 1979.

Cook, Marcy. Try-A-Tile, Set B: Subtract-a-Tile, Carry-a-Tile, Difference Tiles, Borrow-a-Tile. Palo Alto, California: Creative Publications, Inc., 1979.

Cook, Marcy. Try-A-Tile, Set C: Multi-Tiles, Product Tiles, Divide-a-Tile, Remainder Tiles. Palo Alto, California: Creative Publications, Inc., 1979.

Copeland, Richard W. Math Activities for Children: A Diagnostic and Developmental Approach. Columbus, Ohio: Charles E. Merrill Publishing Co., 1979.

Copeland, Richard W. How Children Learn Mathematics: Teaching Implications of Piaget's Research. 4th edition. New York: Macmillan Publishing Co., 1984.

Creative Publications. MIRA Math for Elementary School. Palo Alto, California: Creative Publications, 1981.

Crosswhite, Joe and Robert Reys, editors. Organizing for Mathematics Instruction: 1977 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1977.

Cruikshank, Douglas; David Fitzgerald; and Linda Jensen. Young Children Learning Mathematics. Boston: Allyn and Bacon, Inc., 1980.

Davidson, Patricia and Robert Willcutt. Spatial Problem Solving with Cuisenaire Rods. New Rochelle, New York: Cuisenaire Company of America, 1983.

Downie, Diane; Twila Slesnick; and Jean Kern Stenmark. Math for Girls and Other Problem Solvers. Berkeley, California: The Regents, University of California, 1981.

Driscoll, Mark. Research Within Reach: Elementary School Mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 1981.

Greenes, Carole; Robert Willcutt; and Mark Spikell. Problem Solving in the Mathematics Laboratory: How to Do It. Boston: Prindle, Weber, and Schmidt, Inc., 1972.

Greenes, Carole; Rita Spungin; and Justine Dombrowski. Problem-mathics: Mathematical Challenge Problems with Solution Strategies. Palo Alto, California: Creative Publications, 1977.

Greenes, Carole, et al. The Mathworks: Handbook of Activities for Helping Students Learn Mathematics. Palo Alto, California: Creative Publications, Inc., 1979.

Keyser, Tamara and Randall Souviney. Measurement and the Child's Environment. Santa Monica, California: Goodyear Publishing Co., Inc., 1980.

Krulik, Stephen and Robert Reys, editors. Problem Solving in School Mathematics: 1980 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1980.

Labinowicz, Ed. The Piaget Primer: Thinking, Learning, Teaching. Menlo Park, California: Addison-Wesley Pub.ishing Co., 1980.

Labinowicz, Ed. Learning from Children: New Beginnings for Teaching Numerical Thinking. Menlo Park, California: Addison-Wesley Publishing Co., 1985.

Marolda, Maria. Attribute Games and Activities. Palo Alto, California: Creative Publications, 1979.

McLaughlin, Jack. People Piece Puzzles. Hayward, California: Activity Resources Co., 1973.

McLean, Peggy; Lee Jenkins; and Jack McLaughlin. Let's Pattern Block It. Hayward, California: Activity Resources Co., 1973.

National Council of Teachers of Mathematics. The Arithmetic Teacher. Reston, Virginia: National Council of Teachers of Mathematics.

Pasternack, Marion and Linda Silvey. Pattern Blocks Activities A. Palo Alto, California: Creative Publications, 1975.

Perl, Teri. The Fun World of Relationshapes: Patches. New Rochelle, New York: Cuisenaire Company of America, 1975.

Price, Shirley and Merle Price. The Primary Math Lab: An Active Learning Approach. Santa Monica, California: Goodyear Publishing Co., 1978.

Reys, Robert and Thomas Post. The Mathematics Laboratory: Theory to Practice. Boston: Prindle, Weber, and Schmidt, Inc., 1973.

Richardson, Kathy. Developing Number Concepts Using Unifix Cubes. Menlo Park, California: Addison-Wesley Publishing Co., 1984.

Riedesel, C. Alan. Teaching Elementary School Mathematics. 3rd edition. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1980.

Roper, Ann and Linda Harvey. The Pattern Factory: Elementary Problem Solving Through Patterning. Palo Alto, California: Creative Publications, 1980.

Shufelt, Gwen and James Smart. The Agenda in Action: 1983 Yearbook. Reston, Virginia: National Council of Teachers of Mathematics, 1983.

Sternberg, Betty. Attribute Acrobatics. Hayward, California: Activity Resources Co., 1974.

Bibliography, continued

Souviney, Randall. Solving Problems Kids Care About. Santa Monica, California: Goodyear Publishing Co., 1981.

Souviney, Randall; Tamara Keyser; and Alan Sarver. Mathmatters. Santa Monica, California: Goodyear Publishing Co., 1978.

Troutman, Andria and Betty Lichtenberg. Mathematics: A Good Beginning. Belmont, California: Wadsworth Publishing Co., 1977.

Williams, Elizabeth and Hilary Shuard. Elementary Mathematics Today: A Resource for Teachers Grades 1-8. 2nd edition. Menlo Park, California: Addison-Wesley Publishing Co., 1976.

Zullie, Mathew. Fractions with Pattern Blocks. Palo Alto, California: Creative Publications, 1975.

Connecticut State Department of Education

Program and Support Services
Lorraine M. Aronson
Deputy Commissioner

Office of Research and Evaluation
Pascal D. Forgione, Jr., Chief

Douglas A. Rindone
Education Consultant

Division of Curriculum and Professional Development
Betty J. Sternberg, Director

Betsy Y. Carter
Steven Leinwand
Mathematics Consultants

Velma A. Adams, Editor

"END OF DOCUMENT"