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ABSTRACT

It is generally accepted that mathematical reasoning, like language acquisition, is part of normal cognitive development. This paper proposes that other variables must be considered when explaining the differences in the acquisition of mathematical reasoning skills in young children. Considered is some of the evidence that suggests that certain patterns of affective and motivational characteristics relate to the quality of children's mathematical thinking; this includes not only such thinking as measured by academic achievement tests, but also strategies individuals use to obtain their answers. Also discussed is the nature of certain patterns of affective, motivational, or personal characteristics that are related to, yet go beyond, the development of purely cognitive differences. Based on the observation of videotapes of teachers and young children engaged in mathematical activities, it is suggested that different types of non-cognitive behaviors are associated with the development of certain kinds of mathematical thinking. The results indicated a difference between learners who were classified as pro-mathematical thinkers with those categorized as anti-mathematical thinkers. (TW)

THE DEVELOPMENT OF MATHEMATICAL THINKING AS A FUNCTION OF THE INTERACTION BETWEEN AFFECTIVE AND COGNITIVE FACTORS

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Introduction

It is generally accepted, based on many years of research beginning with Piaget and extended by investigators such as Gelman, Ginsburg, and Saxe, that mathematical reasoning, like language acquisition, is part of normal cognitive development. While controversy may exist about the ages at which children develop skills and reasoning abilities in the area of mathematics and about the significance for cognitive development of children's "precoscious" performance on tasks such as conservation, counting and simple arithmetic, it is also generally accepted that all children will develop essentially the same set of basic mathematical understandings. That is, all children will wind up able to count conventionally, classify groups of objects into categories and subcategories, seriate a series of objects no matter how large the collection, demonstrate an understading of reversibility through conservation, and so on.

We find, however, that despite the universal development of these broad categories of mathematically related cognitive capacities, as children enter the world of formal mathematics during the elementary school years, many more individual differences in the quality of the development of mathematical thinking seem to emerge. These differences can be found among children who have had the same basic educational and cultural experiences (Ginsburg, 1980; K. Sowder, personal communication, September, 1986; Benbow, 1986; Carpenter, Matthews, Lindquist, & Silver, 1984). Moreover, there does not seem to be any evidence to suggest that differences in intellectual ability alone can account for this observed variability in the quality of mathematical thinking, since such differences can be observed among children who seem to have good general intellectual ability (Stevenson, Lee, & Stigler, 1986). Therefore, other explanatory factors must be considered. One such factor for which evidence seems to be accumulating is that certain patterns of affective and motivational charactristics impact upon the quality of development of children's mathematical thinking (Cadigan, Entwisle, Alexander, & Pallas, in press; Keogh, 1986; McLeod, 1986; Stevenson & Newman, in press).

When I speak about the quality of mathematical thinking, I refer not to the usual criteria for mathematics success as measured by standardized achievement tests, but to a complicated constellation of skills that manifest themselves in the processes rather than the products of mathematical activity. That is, mathematical thinking is not defined by whether or not an answer is correct, but by the strategies that the individual uses to come to that answer. In elementary mathematical thinking this might refer the child's ability to recombine a forgotten number combination fact by breaking apart the problem into two known facts and then combining the totals of these facts. For example, if the child forgets or does not know how much 4 x 7 is, he/she can rely on the knowledge that 2 x 7 = 14 and so then figures out that 14 + 14 = 28. In doing this, the child is utilzing the relationship between addition and multiplication operations and also is constucting new information based on existing knowledge. Alternatively, another child might not know the same combination, but instead of using the deductive approach, takes a guess that the answer is 24 because the number is remembered as being 4 times something in the Times Table and so it sounds like it could be the right answer. This child then is relying primarily on his/her memory of answers rather than on the operations and relationships among numbers for deriving answers. In this formulation, then, children may develop different approaches for doing mathematics, some of which reflect more abstract and flexible reasoning than others. The purpose of this paper is to discuss the nature of these specific differences in children's mathematical thinking and then to examine these differences in the context of certain patterns of affective, motivational, or personal characteristics that are related to, yet go beyond, the development of purely cognitive differences.

Nature of Cognitive Differences

Our knowledge of the nature of these cognitive differences comes from extensive interviewing and observations of videotapes of interviews of preschool and elementary school-age children engaged in doing mathematical activities. The particular interviews upon which this paper is based were part of a project intended to introduce teachers to the psychology of children's mathematical thinking through their participation in videotape workshops. Careful observation of these tapes has led us to several conclusions about the nature of individual cognitive differences in the development of children's mathematical thinking during the school years. On the one hand,

1) Some children, without the benefit of explicit instruction, seem to be able to make a leap from reliance on perceptual cues and concrete representations of quantity to abstract numerosity. These children are able to mentally manipulate both whole and

fractional parts of numbers in all kinds of combinations.

2) Some children spontaneously use and apply mathematical principles to problems presented to them. For example, they intuitively recognize and/or invent distributive rules and understand the complementary and commutative relationships between arithmetic operations. That is, they are able to generate new mathematical terms and ideas based on known concepts or rules in contrast to simply increasing the number of memorized facts in their repetoires.

3) Some children seem to spontaneously create mathematical problems for themselves and know when to apply existing mathematical knowledge in their everyday lives.

4) Finally some children seem to spontaneously engage in metacognitive activities about their mathematical thinking, i.e., they naturally reflect upon their own thinking processes.

In contrast,

1) Other children remain at the counting level of mathematical thinking for a much longer period of time and seem to have little grasp of cardinality, resulting in early failure to spontaneously use counting-on techniques and later, in a lack of facility for remembering addition or multiplication number combinations.

2) These children move through the mathematics curriculum as though it were a series of unrelated bits of information, governed by arbitrary rules. They fail to see the relationships between different mathematical principles and/or procedures and tend to rely on the mechanical and arbitrary application of memorized or familiar procedures to new situations and problems.

3) Some children seem to develop a sharp distinction between

mathematics as a school-learned or school-only activity and the practical problems of their everyday life.

4) These children also seem to have difficulty thinking about or describing how they figure out a math problem.

Why Cognitive Differences Develop: The Effect of Non-cognitive Factors

There are at least two possible approaches to viewing the relationship between cognitive and non-cognitive or affective domains in the development of mathematical thinking. The first view suggests that particular cognitive experiences evoke certain emotional states. For example, a difficult problem may evoke feelings of frustration. The way in which this feeling is dealt with will determine the type of cognitive approach that will be applied next (McLeod, 1986). This view has limitations for explaining why some children consistently respond in one way to frustration and others in another way.

An alternative viewpoint, one that we find more compelling and which is consistent with Piaget's (1978) description of affect as the energy behind cognitive development, views affect not as a transitory response in relation to cognitive activity, but as a characteristic set of responses that impacts upon the way in which cognition develops. Our observations of children have encouraged us to support the position that different types of non-cognitive behaviors seem to be associated with the development of certain kinds of mathematical thinking. That is, children display individual differences in affect and motivation in a mathematics situation, at least on the tasks that we have observed children doing, and these differences seem to be associated with particular expressions of mathematical thinking as outlined above.

The particular non-cognitive variables that we are currently focusing on have been selected from a wide range of possible choices. The two main criteria for their selection were that they must be readily and reliably identifiable by many observers and that they must also be reasonably independent of the specific cognitive functioning variables discussed earlier. For example, "confidence" had been suggested as an affective variable, but was rejected because even though it can be easily identifed, it is also just as easily confounded with cognitive ability. We have, for example, a tape of a 6-year old first-grade boy who when asked about whether he was good at math when he was 3-years old, exclaimed, "Good? I was great!" This child was described by his teachers as a superior math student and during his interview demonstrated some very clever mental calculation strategies. Clearly his confidence was a reaction to and a reflection of intellectual competence and the two variables could not be separated. Similarly, "rigidity" or "flexibility" in the selection of solution strategies must be dealt with cautiously because the extent of flexibility is clearly limited by the number of possible strategies that a child knows. On the other hand, "risk-taking" is an example of another variable that had been suggested as a likely candidate for looking at non-cognitive factors in development. The problem here was that the variable was hard to identify and even if it could be identified, it might vary with individual concerns about what might be at risk (i.e., self esteem, chance of failure, possible criticism).

Given the above selection criteria of reliability and factor independence, and consistent with previous research, our observations have led us to focus on the following non-cognitive variables:

1)How children respond to their own errors (including whether they monitor themselves or wait for others to point them out)

2)How, when and if children indicate awareness of the limits of their knowledge (including how and if they ask questions)

3)Related to the above, the extent to which children seem to enjoy reflecting upon or describing their own thinking processes, i.e., willingness to share their thoughts with another

4)The extent to which children are involved, engaged, or seem to be "at one" with a task in terms of whether their involvement is in the solution process ("deep approach" as termed by Marton) or in obtaining an answer in order to meet a task requirement ("surface approach")

5)The extent of persistence in the face of frustration or a difficult task in contrast to the extent to which energy is expended to avoid doing a task

6)The dominant mood and/or shift in moods of children from the beginning to the end of the task and across tasks

Method of Investigation

To date we have looked at these variables within the context of our existing tapes of children. These tapes include a variety of mathematical tasks carried out during individual clinical interviews of children between 3 and 9-years of age and include mental and written calculation as well as some spatial and measurement tasks. Observation of these tapes has led us to distinguish two general profiles of mathematical thinkers that can be described in terms of an association between particular cognitive abilites and particular affective or motivational approaches.

The mathematical thinking types can be considered as:

1. The pro-mathematical thinker
2. The anti-mathematical thinker

In defining these types, a few images come to mind. First, the anti-mathematical thinker may be seen as analogous to a person who is wearing very heavy lead-soled shoes. This person uses up a lot of effort to take each step and his/her gait is slow, plodding, and graceless. Because it takes so much effort to lift each leg, he/she can only focus on taking one step at a time and so the flow of movement is always interrupted. The pro-mathematical thinker, on the other hand, brings to mind an image of a coordiated, graceful dancer who not only moves both feet at the same time, but whose movement has a rhythm and fluidity that involves the entire body. Although these images do not directly address the issue of different types of mathematical thinking, they do reflect the quality of the kind of distinctions that are intended to be conveyed by the use of the terms pro- and anti-matheatical thinkers, i.e., the difference between a "plodding or piecemeal" approach as compared to a "fluid or soaring" approach to mathematics.

Specifically in terms of the association between cognitive and non-cognitive factors in the development of children's mathematical thinking, the following descriptions would apply.

The pro-mathematical thinker

a. On the cognitive side, the child, without explicit instruction, develops procedures for mentally manipulating and recombining numbers and on the non-cognitive side, is persistent in the face of error, challenge, or difficult problems.

b. On the cognitive side, the child can make explicit in words or pictures the way in which he/she thinks about a problem and on the non-cognitive side, tends to take pleasure in spontaneously describing these solution strategies.

c. On the cognitive side, the child makes connections between and applies known principles or procedures to new problems while flexibly varying the principle or procedure used depending upon the structure of the problem. On the non-cognitive side, the child does not wait to be told what to do and upon spotting an error, spontaneously attempts to rework the problem. One third grade youngster, for example, had been working with calculation of fractions and although he could not quite remember the appropriate algorithm for finding the "least common denominator," refused to be stumped. First he started to draw a diagram to illustrate the relationship between fractional parts and then when that didn't quite come off, he proceded to use other numbers in order to try and explain the problem as a division problem, using multiplication of whole numbers to justify his answer. He finally was stopped by the interviewer, but it seemed that he could go on tirelessly forever.

d. On the cognitive side, the child constructs new mathematical problems for him/herself beyond the task requirements. On the non-cognitive side, the child exhibits focused concentration, seems to be personally invested in the process of task solution and does not seem to want to "let go" of the problem (i.e., the child takes pleaure in trying to generate new approaches even after the initial problem is solved). An example, of this was a second grade boy who was quite advanced in his ability to work written calculational problems, but who had a bit of a bug in his procedure for doing subtraction with borrowing across zero. Because of some technical video problems, the child's interview was not usable. However, he had such a nice calculational error, that we decided to redo the interview. At the next taping, the first thing he mentioned, was that he had been thinking about one of the problems from the previous day and that he had figured out his mistake. He then reworked the problem, explaining what he had done wrong. So this time the problem, and others like it, was solved without "the bug" to the child's credit, but at the expense of a very good tape segment.

The_anti-mathematical_thinker

a. On the cognitive side, the child remains at the counting level of mathematical thinking, rigidly applying the same solution strategies to all kinds of problems and on the non-cognitive side, gives up easily in the face of errors, frustration, and difficult or challenging tasks.

b. On the cognitive side, the child maintains multiple sets of mathematical principles and procedures as discrete and unrelated units and on the non-cognitive side, takes only a "surface approach" to task solution (i.e., comes up with some perfunctory answer as a means of meeting some external obligation and ending the activity with noticeable relief). Children like this tend to write out calculations in silence and then at the end, sigh and emphatically put down their pencils. They also typically will get two different answers when directed by the interviewer to use an alternatetwo procedure and accept the correctness of both answers because "when you count you get 53, but when you add, you get 39."

c. On the cognitive side, the child seems unable or unwilling to describe his/her solution procedures and on the non-cognitive side, spends most of his/her energy in avoiding tasks. Some children, for example, can maintain complete silence and not move, rather than attempt a solution. Alternatively, other children will kid around and make jokes and just get generally silly. The effect of both is the same: task avoidance.

d. On the cognitive side, the child does not apply mathematics outside of a school context and on the non-cognitive side, waits for instructions and judgment by others before completing or evaluating a task.

The formulation of these tentative hypothetical patterns, based on observations of tapes intended for a different purpose, however, has led to additional questions about the relationship between cognition and affect in the development of mathematical thinking. For example, do individual children exhibit patterns of either type of mathematical thinking depending upon the type of task employed or do affect and motivation vary consistently across all kinds of problems in mathematics? What happens in a learning situation? Are children's styles unique to mathematics or do they carry over into other areas as well? We are now interested in putting our initial hypotheses to a more formal test and are in the process of developing and trying out a series of tasks to be used in a case study approach. This more systematic and controlled approach to the study of cognitive and non-cognitive factors upon the development of mathematical thinking should lead us to a better understanding of the relationship between them.

I believe that this research will demonstrate that the affective domain has a significant impact on the course of development of mathematical thinking and that if the development of the affective and motivational systems associated with pro-mathematical thinking are encouraged in children with anti-mathematical patterns, that these children can be helped to develop a richer knowledge of mathematical concepts. To do this we need to stick to simple broad catagories of individual affective differences that are easily observed and dealt with by teachers while maintaining a sufficiently comprehensive model from which we can select those variables that have the most potential value for applied areas.

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