percentages of F- and S-strategies on problems starting with the smaller and the larger given number during the collective and the individual testing situations.

Table 7

The results are in line with the hypothesis in both the collective and the individual tests. Children reported much more frequently that they had solved a problem with a S-strategy when it started with the smaller number than when the larger addend was given first. This finding suggests that children's solution strategies are indeed strongly influenced by the location of the smaller and the larger given number. Interestingly, the obvious tendency to begin the solution process with the first given number was much greater in the collective than in the individual test.

Hypothesis A2. It was expected that addition problems involving a large difference between the given numbers would evoke more S-strategies than those in which both addends have almost the same size, when the smaller number is given first (Hypothesis A2a); similarly, we hypothesized that problems in which the first given number is considerably larger than the second one, would elicit less S-strategies than problems in which the difference between both numbers is almost non-existant (Hypothesis A2b). Table 8 shows the percentages of F- and S-strategies for items with a small and a large difference between the two given numbers.

Table 8

Although the results are generally consistent with the predictions, they are not very convincing: for the collective test the observed difference in the amount of S-strategies between the small- and the large-difference condition is only 6 and 2 percent for problems starting with the smaller and the larger given number respectively. For the individual test the differences are somewhat larger, especially for problems starting with the larger given number (12 %). These data suggest that with respect to young problem solvers no considerable differences

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in solution strategies occur depending on the size of the difference between the two given numbers.

Hypothesis A3. Finally, it was predicted that Combine 1 problems would elicit more S-strategies than normal Change 1 problems when the smaller number is given first (Hypothesis A3a), and also that inversed Change 1 problems would provoke more S-strategies both when the smaller and the larger number is given first as compared to normal Change 1 problems (Hypothesis A3b).

Table 9

The results in Table 9 are in line with both hypotheses. With respect to problems starting with the smaller given number, children seemed to find it easier to use the more efficient S-strategy in the context of Combine 1 and inversed Change 1 problems, than when the problem had a normal Change 1 structure, especially on the individual test; the same trend occurs in the collective test results, although less strong. With respect to the problems in which the larger number is given first, inversed Change 1 problems obviously elicited the highest percentage of S-strategies in the individual test situation; the difference is again in the predicted direction on the collective test, but very small.

7.2 Results for subtraction problems

The most remarkable finding for the subtraction problems was children's apparently very strong tendency to use IA-strategies, especially during the individual tests: on a total of 78 appropriate solution strategies, only four DS-strategies were observed. Consequently, the further discusssion is restricted to the data of the collective test.

Hypothesis S1. It was hypothesized that the choice of either a direct subtractive (DS) or an indirect additive (IA) strategy would be influenced by the relative size of the first given number: problems starting with the smaller

number would elicit more IA-strategies and less DS-strategies than problems in which the larger number is given first.
The results shown in Table 10, are in line with this prediction: we observed considerably more IA-strategies for problems starting with the smaller given number than for problems in which the larger number was given first. This finding supports the hypothesis that the order of presentation of the two given numbers has an influence on the kind of strategies children use to solve subtraction problems.

Table 10

Hypothesis S2. According to the second hypothesis, children would choose different strategies for problems with a small and a large difference between the two given numbers. More specifically, it was predicted that more DS- and less IA-strategies would be observed for problems with a small difference than for problems with a large difference.

Table 11

As Table 11 shows, there was no difference in the amount of DS- and IA-strategies for both types of problems. This result suggests that the relative size of the two given numbers has no significant influence on the strategies children use to solve subtraction problems. This conclusion complements the similar finding for addition problems (Hypothesis A2).

Hypothesis S3. Finally, it was expected that the effects of the order of presentation and the relative size of the numbers would interact with the semantic structure of the problem. More specifically, we assumed that the influence of these task characteristics will be greater for Combine 2 problems than for Change 3 problems.

Table 12

The results appear in Table 12 and show that Combine 2 problems starting with the smaller and the larger given number elicited indeed considerably different percentages of DS- and IA-strategies: while Combine 2 problems beginning with the smaller given number were solved much more frequently with IA- than with DS-strategies, the percentages of IA- and DS-strategies were much closer when the larger number was given first. For Change 3 problems starting with the smaller and the larger given number, the distribution of DS- and IA-strategies was almost alike: most children continued to apply IA-strategies even when the larger number was given first. These findings confirm the hypothesis that the influence of the order of presentation of the given numbers is not alike for all semantic problem types.

6. Discussion

Over the past few years a substantial body of research has yielded evidence that the semantic structure of simple addition and subtraction word problems seriously influence children's solution processes (Briars & Larkin, 1984; Carpenter & Moser, 1982, 1984; De Corte & Verschaffel, 1987; Nesher, 1982; Riley et al., 1983). The results of this study are certainly not in conflict with this well-documented finding but rather complementary. Indeed, our data show that with respect to young problem solvers considerable differences in solution strategies can occur within a given semantic problem type, depending on other task characteristics, i.e. the position of the two given numbers and the order of presentation of the sets in the problem text. Moreover, the findings reveal that the effects of these two additional task characteristics on children's solution strategies are not alike for all semantic problem types. These findings are not only helpful in explaining apparently conflicting results from different previous empirical studies involving the same types of word problems, but they also provide guidelines for improving and elaborating the available theoretical (computer) models of young children's skill in solving elementary arithmetic word problems.

Our data do not provide evidence supporting the impact on children's strategy choice of the third additional task variable involved in this study, namely the

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size of the difference between the two given numbers. The importance of this task characteristic has originally been raised in the context of children solving numerical problems using verbal counting strategies. However, verbal counting is not the only way of solving elementary addition and subtraction problems; as we have seen, there are also material and mental solution strategies. Furthermore, the available research has not produced strong empirical evidence in favour of the influence of the size of the difference between the given numbers on children's strategy choice. Finally, it is plausible to assume that the impact of this task characteristic is less important with respect to word problems as compared to numerical problems.

Considerable differences were found between the collective and the individual tests. Several factors may contribute to explain this fact, such as the age of the children (first versus second graders), and the mathematics program used in each group. Probably a more important factor relates to the way in which we tried to identify children's solution strategies. In the individual interviews observational data together with thinking-aloud and retrospective data were used to unravel the child's solution strategy; however, in the collective test we could only rely on the child's written response to the instruction to write down his solution strategy. The latter approach is subjected to several problems. First, many young children have considerable difficulties in connecting their informal solution strategies to the formal-mathematical symbols and rules involved in number sentences. Consequently, a child may fail to write down an appropriate number sentence for a word problem he was able to solve (Carpenter, Hiebert & Moser, 1983; De Corte & Verschaffel, 1985; Lindvall & Ibarra, 1980). A second problem relates to the ambiguity of the sentence-writing task. Indeed, number sentences can fulfil two different functions with respect to word problems: they can be used either as a formal-mathematical representation of the semantic relations between the known and the unknown quantities in the word problem, or as the mathematical notation of the operation that has been performed with the two given numbers to find the solution of the problem. Sometimes, the same number sentence can fulfil both functions; in other cases, both aspects have to be expressed by different number sentences (De Corte & Verschaffel, 1985; Vergnaud, 1982). Although the children were instructed to write down the numerical sentence that shows how they solved the problem, some

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may have preferred to state the one that represents the problem structure. This may explain why we found so many strategies beginning with the first given number in the collective as compared to the individual test.

It is clear that more theoretical and empirical work is required to clarify the relationship between the different task characteristics of word problems and children's solution strategies. First, more empirical research is needed in which the results of the present investigation are replicated with more tasks and larger samples of subjects. Second, the present study provides a relatively static view of children's solution processes. Further research is needed focusing at the effects of different task characteristics on children's strategy choice in different developmental stages. Third, one should investigate to what extent children are aware of the factors that determine their strategy choice and how this relates to their knowledge of mathematical principles such as the commutativity principle and the complementarity of addition and subtraction.

Note

(1) We point to the fact that in that longitudinal investigation a slightly different terminology was used, namely S(maller)- versus L(arger)-strategies, as in all eight problems the smaller number was always given first.

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Table 1. Examples of F- and S-strategies for addition problems (Problem : "Pete had 3 apples; Ann gave him 8 more apples; how many apples does Pete have now ?")

F-strategies	S-strategies
Material	
The child makes a set of 3 blocks; then makes a set of 8 blocks and adds this set to the first; finally the union is counted starting with one.	The child makes a set of 8 blocks; then makes a set of 3 blocks and adds this set to the first; finally the union is counted starting with one.
Verbal	
Counting-on-from-first: the child begins a counting sequence starting with the first given number and continuing the number of units represented by the larger number (3,... 4,5,6,7,8,9,10,11)	Counting-on-from-second: the child starts a counting sequence beginning with the second given number and continuing the number of units represented by the smaller number (8,... 9,10,11).
Mental	
The answer is derived from a recalled number fact beginning with the second given number (3+8=3+7+1=10+1=11).	The answer is derived from a recalled number fact beginning with the second given number (3+8=8+2+1=10+1=11).

Table 2. Examples of DS- and IA-strategies for subtraction problems (Problem : "Joe has 3 balloons; Connie has 11 balloons; how many more balloons does Connie have than Joe ?")

Direct subtractive	Indirect additive
Material	
Separating from : a set of 11 objects is constructed; 3 objects are removed; the answer is the number of remaining objects.	Adding on : A set of 3 objects is constructed; objects are added to this set until there is a total of 11 objects; the answer is the number of elements added.
Verbal	
Counting-down-from-given : a backward counting sequence is initiated starting with 11; the sequence contains 3 counting words (11,...10,9,8); the last number in the counting sequence is given as the answer.	Counting-up-from-given : a forward counting sequence is initiated starting with 3 and continues until 11 is reached (3,...4,5,6,7,8,9,10,11); the answer is the number of counting words in the sequence.
Mental	
The answer is derived from a subtraction fact recalled from memory (11-3=11-1-2=10-2=8).	The answer is derived from an addition fact recalled from memory (3+7+1=11 thus 3+8=11).

Table 3. Overview of the addition problems used in the study

Structure	Sequence	First	Difference	Problem
Change 1	Normal	Larger	Large	Pete had 8 apples; Ann gave Pete 4 more apples; how many apples does Pete have now ?
Change 1	Normal	Larger	Small	Pete had 6 apples; Ann gave Pete 5 more apples; how many apples does Pete have now ?
Change 1	Normal	Smaller	Large	Pete had 4 apples; Ann gave Pete 8 more apples; how many apples does Pete have now ?
Change 1	Normal	Smaller	Small	Pete had 5 apples; Ann gave Pete 6 more apples; how many apples does Pete have now ?
Change 1	Inversed	Larger	Large	Ann gave Pete 8 more apples; Pete started with 4; how many apples does Pete have now ?
Change 1	Inversed	Larger	Small	Ann gave Pete 6 more apples; Pete started with 5; how many apples does Pete have now ?
Change 1	Inversed	Smaller	Large	Ann gave Pete 4 more apples; Pete started with 8; how many apples does Pete have now ?
Change 1	Inversed	Smaller	Small	Ann gave Pete 5 more apples; Pete started with 6; how many apples does Pete have now ?
Combine 1	-	Larger	Large	Pete has 8 apples; Ann has 4 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Larger	Small	Pete has 6 apples; Ann has 5 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Smaller	Large	Pete has 4 apples; Ann has 8 apples; How many apples do Pete and Ann have altogether ?
Combine 1	-	Smaller	Small	Pete has 5 apples; Ann has 6 apples; How many apples do Pete and Ann have altogether ?

Table 4. Overview of the subtraction problems used in the study

Structure	Sequence	First	Difference	Problem
Change 3	Normal	Smaller	Large	First Pete had 4 marbles; now Pete has 13 marbles; how many marbles did Pete win ?
Change 3	Normal	Smaller	Small	First Pete had 9 marbles; now Pete has 13 marbles; how many marbles did Pete win ?
Change 3	Inversed	Larger	Large	Now Pete has 13 marbles; first Pete had 9 marbles; how many marbles did Pete win ?
Change 3	Inversed	Larger	Small	Now Pete has 13 marbles; first Pete had 4 marbles; how many marbles did Pete win ?
Combine 2	-	Smaller	Large	Pete has 4 car toys; Pete and Ann have 13 toy cars together; how many toy cars does Ann have ?
Combine 2	-	Smaller	Small	Pete has 9 car toys; Pete and Ann have 13 toy cars together; how many toy cars does Ann have ?
Combine 2	-	Larger	Large	Pete and Ann have 13 toy cars; Pete has 4 toy cars; how many toy cars does Ann have ?
Combine 2	-	Larger	Small	Pete and Ann have 13 toy cars; Pete has 9 toy cars; how many toy cars does Ann have ?

Table 5. Overview of the level of difficulty of the addition problems used in the study

Structure	Sequence	First	Difference	Problem difficulty	
				Collective test	Individual test
Change 1	Normal	Larger	Large	95	100
Change 1	Normal	Larger	Small	95	95
Change 1	Normal	Smaller	Large	93	95
Change 1	Normal	Smaller	Small	95	95
Change 1	Inversed	Larger	Large	91	95
Change 1	Inversed	Larger	Small	95	95
Change 1	Inversed	Smaller	Large	99	100
Change 1	Inversed	Smaller	Small	93	85
Combine 1	-	Larger	Large	96	95
Combine 1	-	Larger	Small	93	95
Combine 1	-	Smaller	Large	98	95
Combine 1	-	Smaller	Small	99	100

Table 6. Overview of the level of difficulty of the subtraction problems used in the study

Structure	Sequence	First	Difference	Problem difficulty	
				Collective test	Individual test
Change 3	Normal	Smaller	Large	53	45
Change 3	Normal	Smaller	Small	54	65
Change 3	Inversed	Larger	Large	46	40
Change 3	Inversed	Larger	Small	53	75
Combine 2	-	Smaller	Large	53	55
Combine 2	-	Smaller	Small	58	50
Combine 2	-	Larger	Large	68	35
Combine 2	-	Larger	Small	69	40

Table 7. Percentage of F- and S-strategies on addition problems starting with the smaller and with the larger given number during the collective and the individual test

First given number	Collective test Strategies		Individual test Strategies	
	F	S	F	S
Smaller	85	15	43	57
Larger	96	4	78	22

Table 8. Percentage of F- and S-strategies on addition problems with a small and a large difference between the two given numbers during the collective and the individual tests

First given number	Difference between given numbers	Collective test Strategies		Individual test Strategies	
		F	S	F	S
Smaller	Large	82	18	40	60
	Small	88	12	45	55
Larger	Large	95	5	84	16
	Small	97	3	72	28

Table 9. Percentage of F- and S-strategies on normal Change 1, inversed Change 1 and Combine 1 problems during the collective and the individual tests

First given number	Structure	Collective test Strategies F	S	Individual test Strategies F	S
Smaller	Change 1 normal	91	9	52	48
	Change 1 inversed	77	23	39	61
	Combine 1	86	14	37	63
Larger	Change 1 normal	97	3	86	14
	Change 1 inversed	92	8	66	34
	Combine 1	99	1	83	17

Table 10. Percentages of DS- and IA-strategies on subtraction problems starting with the larger and with the smaller given number during the collective test

First given number	DS-strategies	IA-strategies
Larger	33	67
Smaller	17	83

Table 11. Percentages of DS- and IA-strategies on subtraction problems with a large and a small difference between the two given numbers during the collective test

Difference between given numbers	DS-strategies	IA-strategies
Large	26	74
Small	25	75

Table 12. Percentages of DS- and IA-strategies on subtraction problems with a Combine 2 and a Change 3 structure during the collective test

Structure	First given number	DS-strategies	IA-strategies
Change 3	Smaller	16	84
	Larger	22	78
Combine 2	Smaller	18	82
	Larger	43	57