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ABSTRACT

This issue of the journal contains abstracts and critical comments for ten published reports of research in mathematics education. The reports concern teaching probability and estimation of measurements through microcomputer games, mental addition, use of calculators at the intermediate level, logical reasoning hierarchies, two approaches to the development phase of instruction, subject matter knowledge of teachers and their classroom behaviors, verbal disagreements during small-group learning, eighth-grade achievement in the Second International Mathematics Study, high school achievement and its relationship to teaching in earlier grades, and the acquisition of basic multiplication skills. Research references from the Current Index to Journals in Education (CIJE) and Resources in Education (RIE) for January through March, 1986 are also listed. (MNS)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 19, Number 3 - Summer 1986

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Summer 1986

Bright, George N. WHAT RESEARCH SAYS: TEACHING PROBABILITY AND ESTIMATION OF LENGTH AND ANGLE MEASUREMENTS THROUGH MICROCOMPUTER INSTRUCTIONAL GAMES. School Science and Mathematics 85: 513-522; October 1985.

Abstract and comments prepared for I.M.E. by TERRY GOODMAN, Central Missouri State University, Warrensburg, Missouri.

1. Purpose

The purpose of the study was to determine the effectiveness of two microcomputer instructional games in teaching probability and estimation of length and angle measurements to preservice elementary school teachers.

2. Rationale

Previous research reported that a set of fair/unfair games effectively teaches knowledge of probability to sixth- and eighth-grade students. It was also reported that a game had been used effectively to teach tenth-grade geometry students to estimate length and angle measurements. A question for further study was whether non-computer instructional games can be adapted to microcomputer format. It was suggested that the primary difference in the two formats is the manner of presentation of content.

3. Research Design and Procedures

Two games, Jar Game and Golf Classic, were used in this study. In Golf Classic, students choose angles for shots of a particular distance and the computer draws the shots. In Jar Game the computer graphically displays the sample space for the experiment.

The subjects for the study were 78 students enrolled in mathematics methods courses for prospective elementary teachers.

Since these students were taught by two different instructors, the data were analyzed by instructor and considered as two distinct experiments. The students of the two different instructors were referred to as Group 1 and Group 2.

Within each class subjects were randomly assigned to play one of the two microcomputer games. Each subject played that game twice, the first time alone and the second time with someone else who had been assigned to the same game. Each student played a game for a total of 40 minutes over a period of five weeks.

The subjects who played one game served as the control group for the other game. Since all subjects completed all of the tests, there were built-in controls for learning due to testing and for non-computer, classroom learning of content.

It was noted that the classroom instruction on probability and measurement might be different. Since probability was an optional topic, both instructors indicated that they did not explicitly teach probability. Measurement was not an optional topic in the course and since a common final examination was used, it was expected that differences in classroom instruction between the two instructors was minimal. For both topics, no measure of the nature of instruction was made.

A test of probability was adapted from instruments reported at the First International Conference on Teaching Statistics in August 1982. The tests of length and angle measurement were adapted from the tests that accompanied the games. In these tests students had to estimate lengths and angles. Length estimates were given relative to an arbitrary unit, while angle estimates were made in degrees. All tests were given to all subjects prior to the beginning of the game playing period and after the completion of all the game playing.

The results of each posttest were analyzed using an analysis of variance with membership in the appropriate game-playing group as the classification variable. Following this, regression analysis was applied, with each posttest as the dependent variable and with the corresponding pretest and the appropriate group membership variable as the independent variables.

4. Findings

For the probability posttest there was no significant F-statistic. For the estimation tests, there was one significant F-statistic, for the posttest on angle estimation for Group 2. The Golf Classic game players appeared to score higher. One of the regression coefficients on group membership was significant, namely, that on the posttest for angle estimation for Group 2.

5. Interpretations

Jar Game does not seem to have been effective in promoting learning of probability for the sample in this study. It was suggested that there may have been some "interaction" between the instruction provided by the game and the instruction provided by the two instructors. In Group 1, the control group improved while the experimental group declined. In Group 2, the experimental group improved more than the control group.

Golf Classic was marginally effective at promoting learning of angle estimation skills. For length estimation, in Group 1 the experimental group improved while the control group declined. For Group 2, these trends were reversed. Since the instructor for Group 2 did spend more time on measurement in class, it was suggested that there could have been interaction between classroom instruction and microcomputer game instruction.

It was further suggested that for the Golf Classic game students may have simply played it as a number game rather than a measurement game. Also, there is some distortion on the monitor screen due to the graphics characteristics of the Apple, particularly for length estimation. This may have been a factor in the study.

In a previous study, both of these games (non-computer versions) had produced rather strong positive effects. It was suggested that for this current sample, the games may have been too elementary or the tests may not have measured the appropriate learning outcomes.

Finally, the researcher concluded that expectations should not be too high when attempts are made to translate effective non-computer instructional techniques into computer formats. It is possible that students may not process information presented in a computer environment in the same way that they process information in a non-computer environment. More study is needed to develop appropriate instructional uses of computers.

Abstractor's Comments

This study was well designed and carried out. Studies that attempt to build upon previous studies are important and useful. There has been and continues to be considerable interest in using the computer as an instructional tool. This study investigates some important questions relative to the instructional uses of microcomputers.

The fact that the microcomputer versions of the instructional games were not effective in promoting learning should encourage researchers, software developers, and teachers to look carefully at the use of the microcomputer as an instructional tool. As suggested by this study we need to be careful not to simply adapt effective non-computer instructional techniques for computer formats.

There are several questions raised by this study that could be the focus of future research.

1. In the earlier study, the non-computer versions of the games were used with sixth-, eighth-, and tenth-grade students. It would be desirable to investigate the effects of the computer versions of the games with respect to these same age groups rather than elementary school teachers.

2. Each subject in this study spent 40 minutes on the computer games. Further, this was over a five-week period. It may be that for instruction using a computer to be effective, students need more time with the computer activities.

3. The author suggests that future studies consider the interaction between instruction provided by the instructor and instruction provided by the computer. This could be an important consideration since in the classroom students will not be working with the computer in isolation. Computer activities will most often be a part of an overall instructional setting. Knowing how computer instruction interacts with various other types/formats of instruction may help us develop specific computer activities to best support and supplement other instruction.

4. The author's comment, "...the possibility that students may not process information presented in a computer environment in the same way that they process information in a non-computer environment," is important. Future research must look carefully at this possibility. Computer activities and materials will be much more useful and effective if they reflect how students process information in computer environment.

Hamann, Mary Sue and Ashcraft, Mark H. SIMPLE AND COMPLEX MENTAL ADDITION ACROSS DEVELOPMENT. Journal of Experimental Child Psychology 40: 49-72; August 1985.

Abstract and comments prepared for I.M.E. by GLEN BLUME, University of Iowa.

1. Purpose

The purpose of this study was to determine the cognitive processes used by first-, fourth-, seventh- and tenth-graders on simple and complex mental addition exercises. The study attempted to provide evidence for whether children use counting or fact retrieval processes for such exercises.

2. Rationale

Previous research that has used response latencies to examine children's processing of mental arithmetic problems has given rise to two models, the reconstructive (mental counting) model suggested by Groen and Parkman (1972) and the reproductive (fact retrieval) model described by Ashcraft and Battaglia (1978). Studies by Ashcraft and his colleagues suggest that children shift from counting to memory-based processes quite early, and that mature processing of mental arithmetic problems "consists largely of a fact retrieval process" (p. 52).

3. Research Design and Procedures

This study was a cross-sectional investigation in which 13 subjects at each of Grades 1, 4, 7, and 10 were randomly selected from a pool of volunteers from one middle-class school system.

In two half-hour sessions on separate days each subject was individually presented, via microcomputer, 160 addition sentences (half false, e.g., 9 + 12 = 23, and half true) for verification. Reaction time necessary to determine whether each addition sentence was true was measured. Problem sizes were small (sum < 10), medium ($10 \leq$ sum ≤ 18), and large ($18 <$ sum ≤ 30). The split (difference between presented sum and correct sum) for the false sentences was either $\pm$ 1 or 2, $\pm$ 6 or 7, or $\pm$ 12 or 13.

Of the 80 true sentences, 55 used small and medium sums and 25 had large sums, some of which required regrouping. Forty-eight of the true sentences were paired with false versions that had the same addends but were randomly assigned to one of the three split conditions. An additional 16 true sentences were each paired with two false (multiplication confusion and control) versions, one containing the product of the two addends rather than the sum, and the other containing the addends in the commuted form with an incorrect sum within two of the product, e.g., 3 + 4 = 12 and 4 + 3 = 10. The final 16 true sentences included the ties 0 + 0 through 15 + 15; these were not paired with false sentences.

Following the second half-hour session approximately 20 additional items were administered in an exploratory tape-recorded "postexperimental interview" (p. 55). One fourth of these were verification tasks and the others were production tasks, ones in which the subject was asked to produce the correct sum for the given addends. These tasks were administered because the authors expected that "other strategies, in addition to fact retrieval and counting, could be identified" (p. 65).

Reaction times to the 48 true and false items were analyzed in a Grade x Split x Problem Size x True/False mixed design, with each subject's mean reaction time to the appropriate items serving as the dependent variable for the ANOVA. Other ANOVAs were performed for the multiplication confusion condition and the tie problems. Multiple regression analyses also were conducted to determine predictors of item difficulty. The verbally-reported interview data were analyzed descriptively.

4. Findings

Missing or deleted data were caused by inattention (less than 1% of the first-graders' responses), extreme reaction times (3.4% overall), and incorrect responses (10% in Grade 1 and less than 5% for the other three grades). Error rates exceeded 10% only for the medium and large problems in Grade 1. Expected results were obtained for the split condition, namely, the greatest error rates occurred on large false problems with a small split.

The True/False ANOVA indicated that reaction time increased significantly with number size and decreased with grade level. A significant Grade x Problem Size interaction was attributed to first-graders' greater increase in reaction time from the small to the medium and large problems than that at other grade levels.

The multiplication confusion ANOVA yielded significant main effects for Grade and Problem Size (small and medium) and a Grade x Problem Size x Confusion Condition interaction. However, only the tenth-graders exhibited a slowing of reaction time to multiplication confusion items.

The Tie ANOVA yielded significant main effects for Grade and Problem Size and a significant two-way interaction. Differences in small and medium item reaction times were quite large in Grades 1 and 4, but not in Grades 7 and 10.

The regression analyses for true problems identified problem size, difficulty as measured by Wheeler's (1939) ranking and the existence of a carry as significant predictors of reaction time. Other regression analyses identified the correct sum squared as a predictor for reaction time on tie problems in Grades 4, 7, and 10. No predictors were found for the non-tie, large items.

The interview data indicated that first-graders used magnitude comparisons for verification problems and first attended to the digits in one column of the two-digit items. As with first-graders, the fourth-graders relied on counting infrequently (4% of the time). The interviews at the upper two grade levels were largely uninformative.

5. Interpretations

The primary result from the reaction time analyses was that even first-graders rely heavily on fact recall (for small problems) rather than counting for verification items. The lack of associative confusions from products rather than sums was attributed to the rather large splits inherent in many of the items (e.g., 9 + 3 = 27).

The authors note that the emergence of the "Wheeler difficulty" variable as a predictor of reaction time is a significant new finding. Also, results from the ties indicated that the "ties effect, a flat reaction time pattern across problem size, is clearly not limitless" (p. 65). The sophistication of solution methods revealed by the interviews led the authors to conclude that children's performance becomes more rapid with age, but both fact retrieval and conscious problem-solving through carrying and estimating exist. Each of these is elaborated further beyond first grade, accounting for speeding of performance on both types of processing across grades.

Abstractor's Comments

Considering the complexity of the item types and data analyses, the study is fairly well-reported. The authors are to be commended for having a theoretical basis for their study and for linking the study to a chain of inquiry. The use of the computer to control various presentation variables was another strength of the study. Given the variety of items used, the researchers did a good job of balancing the occurrence of the various problem conditions.

The finding that counting alone cannot account for performance on addition problems and that fact retrieval is a frequent process even for first-graders is not surprising, given the abundant evidence for young children's propensity to employ a variety of strategies on simple addition and subtraction word problems. However, I believe that recall of facts most likely will occur more frequently on verification than production tasks because verification items "suggest" recalling a fact to which the given sentence can be compared. To some extent, I found the implicit and explicit references to simple and complex "addition problems" somewhat misleading, since determining whether an addition sentence is true is not what one usually thinks of in reference to addition problems.

I found it surprising that no reference was made to the substantial body of mathematics education research on children's solution processes for simple addition problems. In fact, it occasionally appeared that reference was being made to a false dichotomy, namely, that children either use fact retrieval or that "all of a first-grader's performance depends on counting or incrementing processes" (p. 58). Quite a bit of research has been conducted subsequent to Groen and Parkman's (1972) study to which the foregoing quote refers; children's eclecticism in using solution processes should be expected on the basis of recent research.

The authors use descriptions of solution strategies that do not reflect those used by other researchers who have studied simple addition problems. For example, categories such as "ten's referent" describe what other researchers term "derived facts" or "thinking strategies." I also found it difficult to distinguish between the memory ("I know in my head that three plus six is nine.") and fact ("Six plus three is nine.") strategies coded from the interview data.

Although the study should be of interest both to psychologists and mathematics educators, I think it has more to offer as description of cognitive processing than as a description of performance on addition problems.

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Ashcraft, M.H., & Battaglia, J. (1978). Cognitive arithmetic: Evidence for retrieval and decision processes in mental addition. Journal of Experimental Psychology: Human Learning and Memory, 4, 527-538.

Groen, G.J., & Parkman, J.M. (1972). A chronometric analysis of simple addition. Psychological Review, 79, 329-343.

Wheeler, L.R. (1939). A comparative study of the difficulty of the 100 addition combinations. Journal of Genetic Psychology, 54, 295-312.

Hedren, Rolf. THE HAND-HELD CALCULATOR AT THE INTERMEDIATE LEVEL. Educational Studies in Mathematics 16: 163-179; May 1985.

Abstract and comments prepared for I.M.E by JAMES F. VANCE, University of Victoria, British Columbia.

1. Purpose

The project was a three-year longitudinal study designed to investigate the effects of the consistent use of hand-held calculators, during mathematics instruction, by intermediate level (age 10-12) pupils on (a) their conception of numbers; (b) their ability to work with algorithms, word problems, and estimation; and (c) their motivation for mathematics.

2. Rationale

A large body of research indicates that the use of calculators in mathematics instruction will result in equal or higher achievement compared to non-use. Many educators feel that calculators can be an effective aid in problem solving; they relieve students of the burden of computation, freeing them to develop strategies in solving problems. Because calculators are used extensively in society, it follows that less instructional time should be devoted to teaching paper-and-pencil algorithms. If algorithms are not taught, however, it is possible that positive side effects such as the development of mental arithmetic and estimation might be lost. While some researchers believe that calculator-related activities can enhance understanding of number and provide training in mental arithmetic and estimation, there is also the danger that students will perceive checking calculator results through estimation procedures as meaningless.

3. Research Design and Procedures

Experimental texts for each of the three forms of the intermediate level in Sweden (ages 10-12) were produced. The materials, which were designed to complement the regular textbooks and to encourage the use of the hand-held calculator as a natural teaching aid in mathematics, contained sections on mental arithmetic, estimations, algorithms, word problems, and "project tasks".

The experimental group consisted of eight classes, six from two small towns and two from Gothenberg. Choice of classes was limited to classes of teachers who "had at least a little interest in using calculators in their mathematics instruction" (p. 166). These classes, in form 4 in 1979-80, were taught using the experimental materials for three years (forms 4, 5, and 6). Except when working with algorithms, the pupils had access to calculators and were encouraged (but not forced) to use them whenever they could be of value. Uses of calculators included discovering rules for mental computation, comparing computational estimations with exact answers, solving word problems, carrying out calculations on data collected for project tasks, and working on "free discovery" exercises. Work on algorithms was substantially reduced compared to the regular program. In form 6, students were required only to multiply or divide multidigit numbers by single-digit numbers or products of single-digit numbers and a power of ten.

A pretest on mental arithmetic, algorithms, and word problems was administered to the experimental classes and three control classes at the beginning of form 4. At the end of form 6, all classes were given another battery of tests dealing with non-algorithmic basic skills, algorithms, mental arithmetic, and word problem solving. The tests contained 23 clusters with five problems in each cluster. All of the test items except the algorithm questions could be solved without computation or with simple mental arithmetic. These tests were produced by another set of researchers and neither group of students had practiced the types of problems in the tests. The tests were also given to a "representative sample" of form 6 Swedish pupils. For each item the results of experimental classes and control classes were compared statistically with the results of the representative sample.

To further evaluate the project, pupil questionnnaires dealing with motivation for mathematics and calculator use in learning mathematics were given to pupils in both groups at the beginning of form 4 and at the end of form 6. The investigator also observed in four of the experimental classes (each for five consecutive mathematics periods) and interviewed teachers of the experimental classes.

4. Findings

Tests. Pretest results indicated "very small" differences between experimental and control classes. On the posttest, the control group performed significantly ($p < .05$) better than the representative group on 12 of the 115 items and worse on 12 items. The experimental group scored significantly higher than the representative group on 47 items and lower on 9 items. Clusters where the experimental group performed particularly well included understanding of number, word problems (especially in choosing the operation and utilizing correct information), and estimation. They did worse than the representative group on two algorithm items not taught in their curriculum, but better on two other algorithm items. In mental arithmetic the experimental classes had better results on eight of the 15 items and worse on four.

Questionnaires. The experimental classes had a more negative attitude toward estimations than the control classes but were more optimistic that one will learn more (rather than less) mathematics by using a calculator at school.

Observations. Initial pupil enthusiasm for being allowed to use calculators gradually faded, and some pupils indicated a preference to learning and using algorithms. Having access to a calculator did not seem to help pupils better learn to solve problems from everyday life or to enable them to handle complicated numbers in word problems. Pupils found it difficult and boring to do estimations.

Interviews. All the teachers had a positive attitude toward the calculator work. They reported that teaching word problems required careful planning and was more work than having pupils do algorithm exercises. Although teachers considered estimation essential in working with calculators, they felt that pupils did not understand or appreciate the significance of the exercise.

5. Interpretations

Using hand-held calculators regularly made the experimental classes as competent as the control classes at mental arithmetic and calculations with simple algorithms. Experimental classes had better quantitative understanding of numbers and were better able to choose the correct operation and make use of information in problem solving. These positive results were attributed to "our pupils' greater opportunities to concentrate on the process of problem solving when they used their hand-held calculators for calculations" (p. 175).

Abstractor's Comments

In my opinion the project is significant not as (still) another study comparing the achievement of students who had access to calculators during instruction with that of a control group, but as a major effort to field-test a mathematics curriculum specifically designed to reflect the reality of calculators in our society and to use the calculator as an instructional aid. The major variable in the study is not the calculator but the experimental calculator-based curriculum. The article contains a term syllabus for one of the experimental classes and illustrative activities from the experimental materials in the areas of problem solving, mental arithmetic, estimation, and projects (data collection and interpretation tasks). Sample items from the "non-algorithmic basic skills" test used in the investigation are also presented. The sections of the report dealing with the results of the pupil questionnaire, classroom observations, and teacher interviews contain interesting information and point out some of the difficulties of implementing a calculator-based curriculum (such as pupil resistance to checking calculator answers through estimation). Anecdotal data are particularly valuable in this respect.

There are a number of difficulties and unanswered questions relating to the experimental aspect of the study. The author acknowledges that the experimental classes were not chosen randomly and that it is not known what really happened in these classes. No mention is made of how the control classes were chosen or what actually happened in them; one can only assume that calculators were not permitted and that the standard curriculum was different with respect to problem solving and estimation. The actual number of students in each group at the beginning and end of the experiment is never given. Again no information is provided about the "representative sample." And why were the experimental and control groups compared to the representative group rather than to each other?

The lack of information in the report regarding statistical procedures and results makes it impossible to judge how well the experimental group actually did, either relative to the other groups or in absolute terms. The number of subjects in each group and the optimistic that one will learn more (rather than less) mathematics by using a calculator at school.

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The lack of information in the report regarding statistical procedures and results makes it impossible to judge how well the experimental group actually did, either relative to the other groups or in absolute terms. The number of subjects in each group and the group means and standard deviations on the total test and subtests should have been reported. In addition, tables showing the performance of each group on selected items would be helpful. The educational significance as well as the statistical significance of observed differences should have been discussed. Even where it is found that one group scores significantly higher than another, it is possible that the performance of both groups might be considered either excellent or very poor.

Jansson, Lars C. LOGICAL REASONING HIERARCHIES IN MATHEMATICS. Journal for Research in Mathematics Education 17: [illegible]; January 19 6.

Abstract and comments prepared for I.M.E. by THEODORE EISENBERG, Ben-Gurion University, Beer-Sheva, Israel.

1. Purpose

Piaget suggested that the elementary forms of formal logical reasoning can be described in terms of 16 binary combinations of the simple propositions p and q, and their negations not-p and not-q ($\bar{p}$, $\bar{q}$). E.g., if v and . represent disjunction and conjunction respectively, then "complete affirmation" would be represented as: $p \cdot q \vee p \cdot \bar{q} \vee \bar{p} \cdot q \vee \bar{p} \cdot \bar{q}$; the formal logical form of "implication" would be $p \cdot q \vee \bar{p} \cdot q \vee \bar{p} \cdot q$; "equivalence" would be: $p \cdot q \vee \bar{p} \cdot \bar{q}$; and disjunction: $p \cdot q \vee p \cdot \bar{q} \vee \bar{p} \cdot q$, etc. (These are easily checked with truth tables or operations on set-theoretic equivalents.) However, Piaget did not hierarchically order these statements. Moreover, acquisition of them at a mastery level seems dependent upon linguistic form and content setting. Hence, the purposes of this study were: 1) to empirically derive a hierarchical ordering of these statements vis-a-vis a mathematical context, 2) to relate the developed hierarchy to others constructed in earlier investigations and other content settings, and 3) to investigate the effects of different linguist forms on logical reasoning performance.

2. Rationale

Empirical evidence suggests that logical reasoning develops with chronological age. Hence, having an understanding of this development process is essential for mathematics educators. Having an understanding of this development in a hierarchical way is even better.

> Task hierarchies generated by ordering theoretic methods provide more information than simple ranking of tasks by

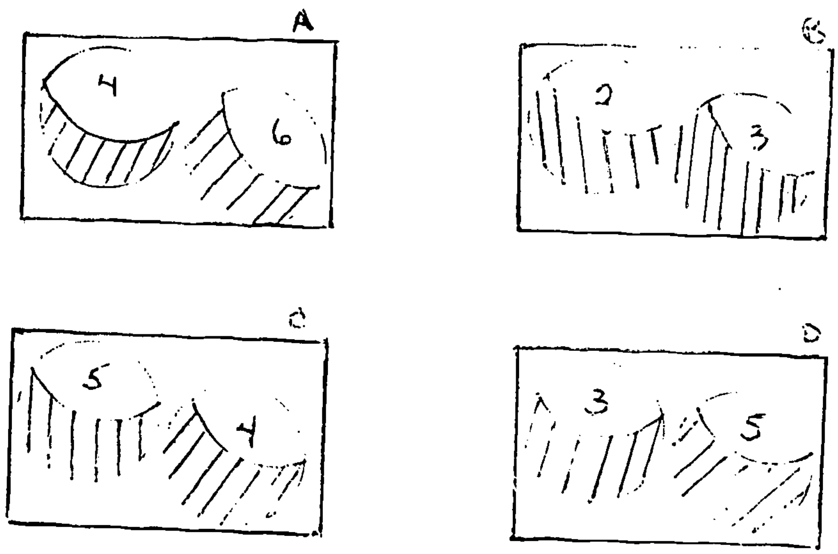
> difficulty, because the hierarchies can depict non-linear, branched sequences among tasks (Bart et al., 1979). If, as Inhelder and Piaget (1958) suggested, formal operational abilities are tightly interrelated and develop together, then their interrelationships would be represented more accurately by a branched hierarchy or network than by a simple linear diagram (p. 4).

Ordering theory suggests procedures for constructing a measure of precedence for two tasks i and j. The basic idea is that if 1 and 0 represent acquisition and non-acquisition of a specific task, then task i can be judged to be precedent to task j, at some pre-set tolerance level, by focusing upon the proportion (P) of (0, 1) and (1, 0) counts obtained from administering the tasks to a sample of students. Specifically, the ordering of two items (i, j) with respect to a tolerance level t can be summarized as follows:

Criterion	Relationship
$P(0, 1) \leq t$ and $P(1, 0) \geq t$	i precedes j
$P(0, 1) \leq t$ and $P(1, 0) < t$	i equivalent to j
$P(0, 1) > t$ and $P(1, 0) \geq t$	i independent of j
$P(0, 1) > t$ and $P(1, 0) < t$	j precedes i

3. Research Design and Procedures

Two studies were carried out in the Netherlands using adaptations of well-known games designed to measure logical reasoning performance. In the billiard-ball game, four cards were placed on a table, each illustrating two billiard-balls, with even-even, even-odd, odd-even or odd-odd numbers showing on them. The cards were labeled A, B, C, and D. The subject was presented with a statement and had to decide which of the cards verified the statement: "If the second number is odd, then the first number is even" is verified by cards A, B, and C below, and represents disjunction. There were 20 such statements for the billiard-ball game, which comprised the first study. Another game was similarly constructed for the second study, but placed into a geometric setting. Each of the 20 statements in the billiard-ball setting had its dual in the geometric setting, and vice versa.



Ninety-four students participated in the first study. They were about evenly divided between boys and girls with a mean age of 13.6. In the second study 30 children participated (10 girls). They were chosen from the same population as those in the first study, but students participated in only one of the two studies, not in both. None of the subjects had previous training in logical reasoning. The game was individually administered to each of the subjects in interview form. The subject had to select the exact set of cards to be awarded credit for the statement.

4. Findings

The sixteen logical binary operations were hierarchically organized from the data obtained from each study. The two hierarchies were then compared and contrasted. From the first study, nine of the sixteen logical statements turned out to be "equivalent" (one not being precedent to the other) and formed the base of the hierarchy.

However, from the second study there were only five equivalents forming the base of the hierarchy, and these five were not completely embedded in the base of the hierarchy derived from the first study. The author then argued that these results are compatible with those from previous studies and that there is "...evidence of a certain stability of precedence relations across contexts." However, as the author stated at the end of his first study,..." it is clear that the logical operation itself is not the sole structural variable of importance. In each case a change in the vocabulary and the associated linguistic structure led to a change in performance."

Abstractor's Comments

Methodologically, this study is well thought out and carefully done. But the underlying motivation for doing it in the first place seems to be another matter. The author argues that because logical forms are inherently tied to mathematics itself, it is crucial for mathematics educators to understand how logical reasoning develops in children. I thoroughly endorse this statement; but translating the forms of logical reasoning which are necessary for mastering school mathematics into Piaget's taxonomy, and then to go still further by translating them int only conjunctions and disjunctions, is just too much. Consider the author's statement from the billiard-ball game. "The first number is odd or even." The correct response is cards A, B, C, and D. Fine. But does this really mean the student is thinking in terms of complete affirmation? $p \cdot q \vee p \cdot \bar{q} \vee \bar{p} \cdot q \vee \bar{p} \cdot \bar{q}$? It is this latter interpretation of the translation process which bothers me. True, this is the symbolic abstraction of the complete affirmation form, but how does this help us understand what the child is thinking when confronted with the problem? I certainly doubt if the child is thinking in disjuncture forms. The author is well aware that student performance on logical reasoning structures which are contextually based in school mathematics, and those which are extensively used in logical reasoning tests, are not significantly correlated. He addressed a series of studies done by Hadar et al., which essentially prove this point, but the author then interprets their findings in an interesting way. "This (Hadar's zero correlation) does not imply that there is no relationship between the learning of logic and the learning of at least some aspects of mathematics. It does suggest that any such relationship is highly complex and multivariate." Perhaps so, but formal logic per se has slipped from the school curriculum, and I'm not sure we're any worse off for it.

Although this study is methodologically sound, I question whether or not it is telling us anything.

Kameenui, Edward J.; Carnine, Douglas W.; Darch, Craig B.; and Stein, Marcy. TWO APPROACHES TO THE DEVELOPMENT PHASE OF MATHEMATICS INSTRUCTION. Elementary School Journal 86: 633-650; May 1986.

Abstract and comments prepared for I.M.E. by BOYD HOLTAN, West Virginia University.

1. Purpose

The article reports the results of three studies of approaches to the development phase of mathematics instruction. The developmental phase is described as the collection of acts controlled by the teacher which facilitate a meaningful acquisition of an idea by the learner. Two methods are identified. The first is a "Project Follow Through" model in which the instruction uses clearly articulated teaching sequences that contain explicit, step-by-step teacher modeling and a means of assessing student mastery at each step of development. The second instructional method is the common basal approach with an open-ended, personalized approach that allows the teacher great flexibility in demonstrating, explaining, and illustrating concepts. These two methods are compared in three studies using different grade levels, mathematics content, and student ability levels.

2. Rationale

Researchers have suggested that the developmental phase of instruction needs to be studied in more detail. Results of the evaluation of Project Follow Through have suggested that a "direct instruction" or "active teaching" approach may be more effective than the traditional basal approach. Gersten and Carnine (1984) describe this approach and point out that in the model "nothing is left to chance."

3. Research Design and Procedures

The three research activities compared the achievement and retention of children after they had received one of the two developmental instruction methods, "Project Follow Through" or "Basal Comparison." The first study compared 23 first-grade students who could not perform single-digit subtraction problems and were randomly assigned to the two treatments. The second study randomly assigned to two groups of 17 students from a middle-skill-level first-grade classroom and 27 students from a high-skill-level second-grade classroom. The third study randomly assigned 24 children who did not score above 15 out of 18 on a pretest of simple division problems involving one-digit quotients and no remainders to a direct instruction group or a comparison group. The mathematical topics were subtraction in the first-grade study, introducing fraction concepts in the first- and second-grade study, and division with single-digit divisors and no remainders in the fourth-grade study.

The treatment lengths were 25 minutes per day for six days and 10-12 minutes on the seventh day for the first-grade study. The first-second-grade study consisted of five lessons for each group. The fourth-grade treatment was conducted once a day for 11 consecutive school days with each session 35 minutes long. The Project Follow Through group in the fourth-grade study used the first three training sessions to teach basic multiplication facts. The comparison group integrated the basic fact study with presentation of division concepts through use of pictures and concrete objects.

The Project Follow Through approach used a carefully scripted development sequence and modeled the mathematical concepts. They used marks on the chalkboard and crossed them off for subtraction. Pictures and parts of pictures were used to show the fraction as telling how many parts in each whole and how many parts are used.

Lines or marks were used again for the division as students were asked to group the marks in sets the size of the divisor and count the number of sets.

The comparison basal method used pictures and experiences to illustrate the mathematical concepts. The students were required to model the ideas with physical concrete objects.

The measures were experimenter-contrived training probes consisting of problems of the mathematical content of the treatment. The number of problems ranged from two to twelve at the end of selected sessions. Transfer tests were administered, usually the following day, but in one case a month later. On the posttest, both groups were required to demonstrate the concepts using concrete materials.

4. Findings

The means for the Project Follow Through instructional method measures are consistently higher than the comparison basal method. This is true for all three experiments over the abilities, content, and grade level. The mean scores on the seventh day for the first-grade group on nine numerical problems were 8.2 for the direct instruction group and 7.0 for the comparison group, and on two picture problems was 1.6 for the direct instruction group and 1.2 for the comparison group. A maintenance test of four numerical problems was given one month later. The mean for the direct instruction group was 2.2 and for the comparison group, .7.

The results of the first-second-grade study were analyzed separately. On the posttest of the middle-ability first graders, the mean percentage of correct responses was 48 for the comparison group and 86 for the Project Follow Through direct instruction group. On a transfer test the mean percentage of correct responses was 31 for the comparison group and 46 for the direct instruction group. The high-ability second-grade group had a mean percentage of correct responses on the posttest of 83 for the comparison group and 94 for the direct instruction group. Mean percentages on a transfer test were 57 for the comparison group and 67 for the direct instruction group.

On a posttest of twelve items in the fourth-grade study, means were 10.00 for the direct instruction group and 8.92 for the comparison group. Means on a maintenance test of six items were 5.00 for the direct instruction group and 4.36 for the comparison group. As the time of the studies increased, the difference between the two group means decreased.

5. Interpretations

The authors concluded that the Project Follow Through direct instruction approach to development was superior to the comparison strategy. They replicated the results across three skill areas (subtraction, fractions, and division), three age groups (first, second, and fourth grades), three ability levels, and on post and transfer tests. They suggest the achievement effectiveness of a highly structured approach to development. They also reported a lack of statistically significant differences between groups on the manipulative transfer measures and suggested that no advantage was gained by extensive training with manipulatives. They stated that the advantage of direct instruction decreased over time.

Abstractor's Comments

The three studies are a very interesting pilot series of instructional investigations. I presume that the three replications were an attempt to bolster confidence in results since the studies were a classroom action type of research and the quality of the measuring instruments, sample selection, and retention times do not seem as "rigorous" as in other experimental research.

Although the Project Follow Through approach is based on a carefully scripted series of questions from the teacher and student responses, the "comparison basal" approach does somewhat appear to be a "straw man" in the comparisons. I would expect a good flexible teacher with a basal program to include the sharp questioning that the other treatment used.

The direct approach appears to produce the more immediate results. The difference between the two methods appears to decrease with more training sessions and over retention time. It may be that the direct instruction model provides an immediate rote method which may not be recalled as well at a later time.

The Project Follow Through treatment suggested that although they did not use manipulatives, they did as well on the posttests with the manipulatives as the comparison group. Modeling is the key idea, not whether it is a physical concrete object that the student manipulates. In this study, both groups used models of the concepts, but I had the feeling that more effort was made to connect the model and the concept in the Project Follow Through approach. Apparently the investigators served as the instructors for both the direct instruction and the comparison groups.

I would conclude that the three studies are like the story of the blind Indians who were learning the meaning of "elephantness." It may be the case that two different methods were not compared, but different parts of a good instructional system were compared.

Reference

Gersten, R. & Carnine, D.W. (1984). Direct instruction mathematics: A longitudinal evaluation of low-income elementary students. Elementary School Journal, 84, 395-407.

Leinhardt, Gaea and Smith, Donald A. EXPERTISE IN MATHEMATICS INSTRUCTION: SUBJECT MATTER KNOWLEDGE. *Journal of Educational Psychology* 77: 247-271; June 1985.

Abstract and comments prepared for I.M.E. by WALTER SZETELA, University of British Columbia.

1. Purpose

The purpose of this study was to examine the relationship between expert teachers' classroom behavior and their subject matter knowledge. The focus of this study was upon grade four teachers and the topic, fractions.

2. Rationale

The authors state that teaching includes two knowledge bases, one consisting of general teaching skills and strategies, the other comprising domain specific information or the knowledge component. Most studies of teaching have attempted to analyze structural or procedural aspects of teaching performance, neglecting the exploration of the content knowledge of teachers. Recent studies in cognitive psychology have explored the significance of content knowledge in expert performance. In mathematics teaching, lesson segments can be analyzed in terms of the goals and subgoals that influence teachers' actions. These goals and actions can be represented in "planning nets" which use links to show the relationships between actions and goals. For actions such as stating an algorithm or selecting a particular demonstration, subject matter knowledge must be activated. The purpose of the planning net is to provide a context for examining the lesson circumstances where subject matter knowledge is needed. The topic of fractions encompasses many complex relationships, concepts, and conceptual meanings and is therefore appropriate for study of teachers' subject matter knowledge and its relationship to expert teaching.

3. Research Design and Procedures

Eight fourth-grade teachers, four experts and four novices (student teachers), were selected for analysis. Two of the experts "seemed" to have high subject matter knowledge, one moderate, and the other low. The four novices had low to moderate subject matter knowledge. The expert teachers had unusual and consistent growth scores of their students over a five-year period. In the first two years the expert teachers were observed for three months each year while teaching mathematics, videotaped for 10 hours, and interviewed on several topics. Teachers and novices were also given a card-sort task of 40 problems randomly selected from grade 4 texts. Interviews of the experts and novices focused upon teachers' knowledge and understanding of fraction concepts and algorithms. Videotapes of the three teachers with high or moderate knowledge teaching a lesson on reducing fractions were examined in detail, including the construction of several "semantic nets" for each teacher. The semantic nets were used to demonstrate differences and similarities of knowledge bases of the teachers.

4. Findings

The fraction knowledge interviews indicated that two of the expert teachers had high mathematics knowledge, one had middle level knowledge, and one had "barely sufficient knowledge for classroom instruction." Novices generally had low knowledge. High knowledge experts sorted 45 mathematics topic cards into about 10 categories and ordered topics by difficulty to teach. Novices had categories for every one or two problems and noted little difference in problem difficulty. All except the two high knowledge teachers had difficulty explaining the equivalence of the fractions 3/7 and 243/567. All teachers had some difficulty with the concept of ratio. One teacher consistently used the number line to explain fraction concepts. The interviews revealed that teachers were able to express an algorithm but lacked understanding of the underlying concepts and relationships.

Expert teacher Konrad presented an 11-minute lesson on Day 1 with a 7-minute follow-up on Day 2. Guided and individual practice that followed contained six segments. First, the fact that a/a = 1 was reviewed. Second, any number multiplied by 1 "yields that same number." Third, whole numbers and fractions divided by one yield the same number. This idea was supported pictorially. Fourth, one can multiply a fraction by any other name for one, but dividing by fractions equivalent to one is restricted. Fifth, iterative divisions by one were used as a "precursor to lowest terms." Sixth, the iterative division process was labeled "reducing to lowest terms." The identity element is "the key to Konrad's method of teaching the concept of equivalence." In addition to regions, Konrad made extensive use of vertically aligned number line families to show equivalence of fractions. The semantic net for division/equivalence is shown in Figure 1 (Figure 6 of article) along with the authors' explanatory remarks. The label "isa" designates an instance of a concept situated at a higher level of the net. The label "has-prop" designates one node as being the property of another.

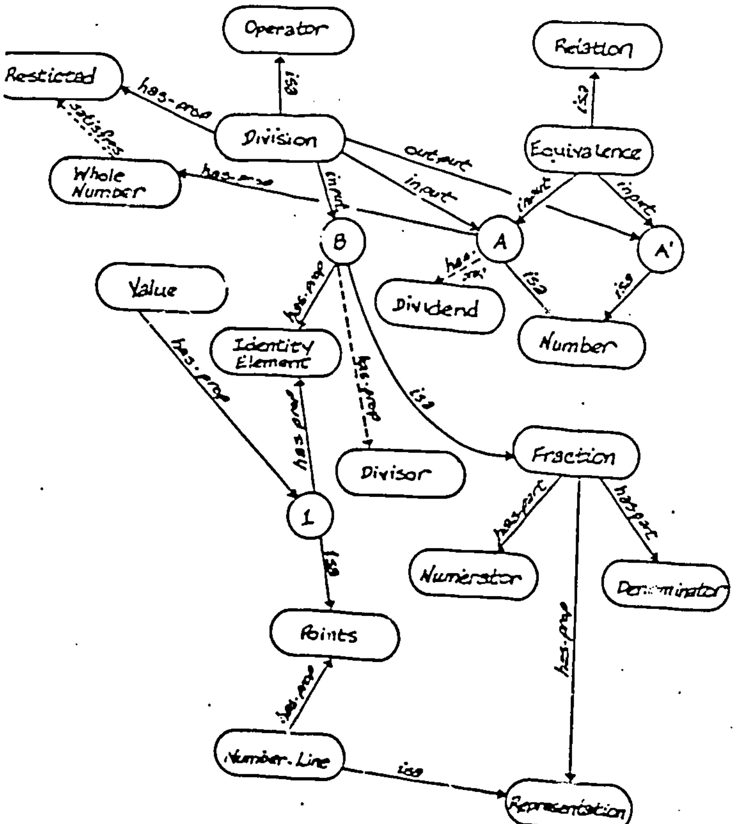


Figure 6. Semantic net for division/equivalence—Konrad. (p. 262)

Figure 6 shows the next step in Konrad's lesson, that is, the division of a whole number by the identity element. The main point of focus in this figure is the relationship among division and its inputs. One of the inputs to division is implicitly specified as the dividend, while the other input, the identity element, is implicitly specified as the divisor. These relationships are implicit because Konrad does not use these terms but emphasizes that *A* is divided by *B*. Notice that the inputs to equivalence are numbers, but they are not identified as fractions. Instead, *A* is defined as a whole number, which satisfies the restriction that is placed on the division of fractions. (p. 259)

Figure 1

The authors characterize Konrad's lesson as "providing a richness of representation systems, general heuristics for solutions, and linkages to basic math principles." By contrast Wall's lesson, presented in a single 8-minute presentation, is "clean and sparse," but little time is given to conceptual development. Wall's algorithm for reducing fractions is very detailed and efficient and is shown in Figure 2 along with the authors' explanatory remarks (Figure 9 of article).

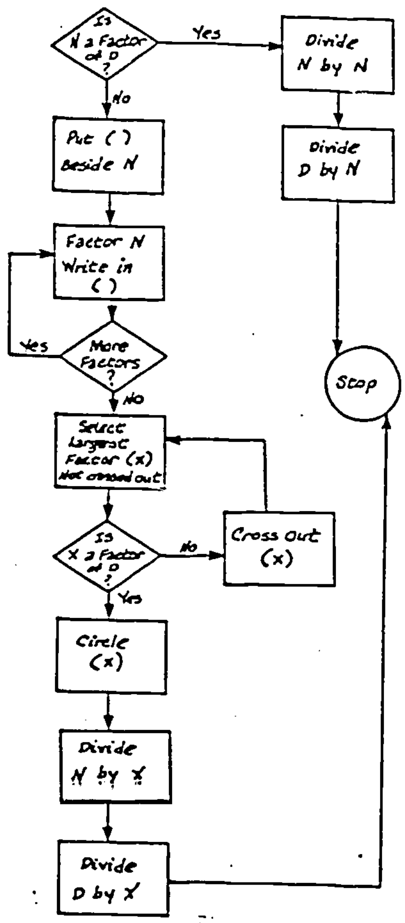


Figure 9. Algorithm for reducing fractions—Wall. (p. 265)

The major part of Wall's lesson is captured in her very detailed presentation of an algorithm for reducing fractions. This procedure is explicitly taught and very efficient. Figure 9 shows the procedure in a flow chart that reads from top to bottom. Given the objective of reducing a fraction, examine the numerator and see if it is a factor of the denominator; if it is, divide both the numerator and the denominator by the numerator and the new fraction is in lowest terms. If the numerator is not a factor of the denominator, then list all factors of the numerator, select the largest, and test to see if it is also a factor of the denominator. Proceed down the list of factors, and when the largest common one is found, divide by it; then the fraction will be in lowest terms. In contrast to Konrad's procedure, Wall's is noniterative and requires no checks for reducibility. (pp. 264-5)

Figure 2

Yoda gave two lessons on reducing fractions, one for 2-1/2 minutes, the other for 10-1/2 minutes. The majority of Yoda's lesson describes an algorithm for reducing fractions. The algorithm includes a first step to check if both the numerator and the denominator are even. The algorithm is "neither as succinct nor reliable" as Wall's. Both Wall's and Yoda's lessons involve misconceptions about the relationship between reducing and "making smaller."

5. Interpretations

The detailed analyses of three expert teachers with moderate or high knowledge of fraction concepts and relationships as indicated by interviews showed considerable differences in the level of conceptual information presented and also in the degree of procedural algorithmic details. Teachers and textbooks often provide incomplete descriptions of concepts and relationships. Teachers and texts fail to note the symmetry of multiplication and division in "reducing or raising" a fraction. Children confuse concepts of smaller and larger with respect to fractions. Current efforts to improve mathematics competency are aimed at improvement of reasoning and understanding of conceptual aspects as opposed to mere skill development. By using semantic nets to display the system of knowledge it can be determined if teachers have conceptual understandings themselves. Where such understanding is lacking, remediation can be undertaken; for example, through in-service support and more challenging examples that expand the application of principles. Planning nets permit both a behavioral and goal-driven description of actions which build upon the use of task analysis in educational research.

Abstractor's Comments

There is no doubt that the relationship between teachers' mathematics knowledge and their teaching of mathematics is an important question for study. As the authors report, subject matter knowledge influences the selection of examples, the explanations, and the demonstrations that teachers use in their lesson presentations. The authors recognize the scarcity of research on this topic and are commended for their attempt to explore a small part of nearly uncharted territory in mathematics education research.

The title of the article is somewhat misleading. The subject matter in question is quite narrow, dealing exclusively with the topic of fractions. The report attempts to show how knowledge of fraction concepts and relationships influences instruction in simplifying fractions. The detailed accounts of three expert teachers' lessons are interesting and revealing. The elements of strengths and weaknesses and errors in the lessons are clearly discussed. The fact that the three teachers carefully studied were rated as moderate or high in fraction knowledge and yet made significant instructional errors is disconcerting. How much more severe are the errors made by teachers with less than moderate knowledge? The four novices had low to moderate subject matter knowledge. Are universities failing to provide adequate knowledge for those who are entering the teaching profession? What are the consequences of inadequate knowledge of mathematics? This study would have been more useful if it had included careful analyses of lessons by teachers who were low in subject matter knowledge.

The article contains a full page table, nine planning and semantic nets, and three flow charts. The authors claim that the nets are valuable in "graphically demonstrating differences in content and organization in teachers' lessons that would not otherwise have been easily detected." The unsuspecting reader is likely to disagree. How could the nets have been constructed without first detecting differences in teachers' lessons and describing them verbally? The complex nets require extraordinary and excessive attention if the reader desires to understand the sequences of concepts and relationships embedded in the nets. Two of the nets, occupying a full page of cryptic labels and connections, boggle the reader. Diagrams and tables should be used to improve clarity of the text, provide visual enrichment, or summarize data. Indeed, the table that summarizes the interviews on knowledge of fractions with eight teachers provides an excellent, compact, and clear summary. The three flow charts are also clear and helpful. However, the semantic nets confuse rather than clarify.

By contrast, for the most part the descriptions of the three expert teachers' lessons are clear and well detailed. One has a much better sense of what is happening in the lessons by the verbal descriptions than by tediously trying to follow the semantic nets. Does one really need two very similar semantic nets to see the differences in one teacher's presentations of finding equivalent fractions, one by using multiplication, the other by division? The main difference in these two nets seemed to be that by using multiplication any fraction name for one could be used to find equivalent fractions, but when division is used, the fraction names for one are restricted to numbers that are factors of both the numerator and the denominator. This point was clearly and adequately presented in the written text as were most of the descriptions of the teachers' lessons. I question the reviewers' and editors' judgment in accepting the article with such an oppressive saturation of ungainly and unsuitable diagrams. Finally, I must express my skepticism about the potential of planning and semantic nets for showing differences in teachers' lessons as compared to verbally descriptive accounts. The authors themselves close with the statements that "the techniques are limited" and that semantic nets "show no order, emphasis, repetition, or entrance," and that such data must be included descriptively.

Additional studies involving detailed observations of mathematics lessons by teachers with both low and high mathematics knowledge are certainly needed. As calculators and computers become more common, and as curriculum changes including topics less familiar to teachers occur, the problems of inadequate subject matter knowledge will become more severe. Documentation of the negative effects of teachers' mathematical weaknesses may help to provide support for better preparation of mathematics teachers and remedial or in-service programs that will improve mathematics teaching.

Lindow, Janet A.; Wilkinson, Louise C.; and Peterson, Penelope L. ANTECEDENTS AND CONSEQUENCES OF VERBAL DISAGREEMENTS DURING SMALL-GROUP LEARNING. Journal of Educational Psychology 77: 658-667; December 1985.

Abstract and comments prepared for I.M.E. by KAREN SCHULTZ, Georgia State University.

1. Purpose

The purpose of this investigation was to study controversy, an aspect of small-group interactions. Of special interest were the effects of student characteristics on expression and resolution of verbal disagreements about mathematics answers and the relation between these interactional variables associated with disagreements and two student outcomes. Specifically, sex and ability were studied in relation to the four dissension variables of initiation, participation, demonstration, and prevailing answers. These four dissension variables were then related to two outcome measures of adjusted achievement and peer nominations of competence.

2. Rationale

One reason given to conduct the present study is Piagetian-based. Children's participation in social conflict enhances the development of perspective-taking skills, therefore decreasing egocentric thought and stimulating cognitive growth. A second reason was that learning in groups structured by controversy has been shown to be superior to learning individually or in groups structured to seek concurrence. It appears that conflict leads the adversaries to seek more information in order to clarify their respective positions. Heretofore, controversial exchanges have not been examined during small-group learning.

3. Research Design and Procedures

Subjects were 40 second- and third-grade mathematics students from two mixed-grade mathematics classes. Ten mixed-sex, mixed-ability groups of four students each were studied, where the group was the unit of analysis.

Various measures of mathematical ability were taken at the beginning of the study and two weeks later after a unit on money and time. A sociometric questionnaire was administered to determine students' perceptions of the most "competent" group member.

The stratified ability groups formed in each classroom were videotaped three or four times over the course of a two-week unit on time and money while they worked on booklets of seatwork problems. Children were instructed by the teacher to seek help from other group members before seeking help from her. They were responsible for turning in their own assignments.

Complete transcriptions in print were then made of the videotaped sessions and were reviewed for three dissension episodes: (a) the antecedent position, an assertion about the answer or step in arriving at the answer to a mathematics problem; (b) the dissension move, which expressed a position that contrasted with the antecedent position; and (c) resolution moves, which followed the dissension move related to the dissented issue and were characterized by the participants' attempts to convince each other of the correctness of their respective positions. This last episode ended when consensus was reached or when the interaction changed to a new topic.

Dissension episodes were then coded along the following dimensions: (a) initiator, person who initiated the dissension; (b) participants, persons who made at least one verbal move related to the dissented issue; (c) type of demonstration, specific type of resolution move that provided justification, evidence, or reason for one's position; and (d) prevailing answer, an answer proposed by the initiator or other episode participants that was eventually accepted as the correct answer by the participants.

4. Findings

One hundred forty-nine dissension episodes were identified and analyzed yielding the following results: (a) total seatwork time spent in task-related interaction ranged from 24% to 79%; (b) frequency of episodes between groups ranged from 8 to 21; (c) the percentage of on-task interaction time engaged in dissension episodes ranged from 10.39% to 49.47%; (d) of the two antecedent variables, neither sex nor ability was significantly related to initiation or overall participation; (e) however, both the sex and ability variables were significantly related to the frequency of demonstrations and to prevailing-answer scores; (f) boys and high-ability students gave more demonstrations and had more prevailing answers than did girls and low-ability students; (g) initiation was not related to either of the two outcome variables of adjusted achievement and competence nominations; (h) participation was significantly positively related to competence nominations; (i) students who provided more demonstrations were also nominated more often as competent than were students who gave fewer demonstrations; and (j) prevailing-answer scores were significantly positively related to both adjusted achievement and competence nominations.

5. Interpretations

Regarding the relation of antecedent variables of sex and ability to process variables of initiation, etc., it was predicted that the high-ability students' answers would prevail more often than the low-ability students' answers. Sex differences were not predicted. Both sex and ability were significantly positively related to providing demonstrations and to the prevailing-answer variable. It should be noted that of the 92 episodes in which consensus was reached, the answer was correct 79 times (86%). Interpretations of the sex differences in providing demonstrations and prevailing answers were attempted. The "expectation states" theory, that boys exert more social influence than girls because of their higher status, might account for the differences found. It is difficult to argue that sex operated solely as a status-organizing process during dissension episodes. Perhaps boys' greater frequency of demonstrations influenced the prevalence of answers. Other possible interpretations are that the boys were superior in mathematics to the girls or that the boys simply learned time and money concepts more quickly.

Regarding the relation between process variables and the two outcomes of adjusted achievement and competence nominations, it was expected that students who participated in more episodes relative to other group members would attain higher achievement scores. This was not found. The intragroup analysis showed a trend toward higher participators attaining higher achievement, but it was impossible to detect a statistically significant relationship. Perhaps the sample was too small, or perhaps other communicative variables interacted with participation in dissension episodes to influence achievement. Another unexpected finding was that groups that had higher rates of participation among group members and spent more seatwork time engaged in dissension episodes performed no better on the achievement test than did groups ranked lower on these variables. Apparently other group dynamics besides dissension influenced outcomes. The expected relation between providing demonstrations and achievement did not occur. Perhaps this was due to the nature of the demonstrations. Although demonstration moves could include explanations, they were not necessarily explanations. Lack of effective explanatory behavior might have been a function of the young age of the subjects or of the nature of time and money as objects of study.

The prevailing answer was related both to adjusted achievement and competence nominations. An interesting interpretation might be that since participation was also related to competence nominations, perhaps verbally active children are perceived as more competent by their peers regardless of what they say!

Abstractor's Comments

This was an impressive study from at least two perspectives. First, the procedures for parsing of seatwork assignment protocols into dissension episodes and the subsequent coding of these episodes was impressive--making a contribution to the whole area of protocol analysis. Protocol analysis in the area of problem solving has been troubled by the difficulty in quantifying such qualitative data. Here, the reader should carefully note the algorithms followed to form the five individual scores. The adjusted achievement scores were obtained through a pooled within-group regression of the achievement test scores on the ability test scores, then four scores determined by the dissension variables were constructed for each student. Procedures outlined in the manuscript for these four scores are extremely useful for replication. Because of the nonindependence of individual scores within groups, intragroup analyses compared individuals' scores only with other group members' scores. These individual comparisons provided group indices, which were then compared across groups. The intergroup analyses compared group means across all groups, therefore both intra- and intergroup comparisons had an n of 10. The episodic structure applied in this study is similar to that of Schoenfeld's model (Schoenfeld, 1983) of parsing protocols for identification of metacognitive behaviors during problem-solving sessions. The procedures used to transform protocol data in the present study to a quantitative form could be combined with Schoenfeld's episodic parsing for intra- and intergroup comparisons of metacognitive behaviors and effects of metacognition.

Second, this is a timely area of investigation because of the recent emphasis on group work in the mathematics classroom. It has been stated (e.g., California State Department of Education, 1985) that working in small groups of four increases each student's opportunity to interact with materials and with other students while learning. As students work together in small groups, the differences will be less significant than the task at hand. Studies such as the present one should greatly influence the directions teachers give students when assigning group work, and on the beliefs and attitudes of dissension within the group.

For those reviewing this piece in detail, there will be some frustration as the jargon at times interferes with the flow of ideas. Labels are seemingly relabeled and unlabeled items aren't labeled until late in the manuscript. These reporting problems will be reduced as researchers continue to develop analysis techniques where initial data are qualitative, such as with protocols. An acceptable uniform language will be developed as in the world of experimental research where well-established statistical procedures are couched in a universal language.

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McKnight, Curtis C.; Travers, Kenneth J.; Crosswhite, F. Joe; and Swafford, Jane O. EIGHTH-GRADE MATHEMATICS IN U.S. SCHOOLS: A REPORT FROM THE SECOND INTERNATIONAL MATHEMATICS STUDY. Arithmetic Teacher 32: 20-26; April 1985.

Abstract and comments prepared for I.M.E. by LELAND F. WEBB, California State College, Bakersfield.

1. Purpose

The purpose of the Second International Mathematics Study (SIMS) was to provide "detailed information for each of more than 20 countries about the content of the mathematics curriculum, how mathematics is taught, and how much mathematics students learn."

2. Rationale

Information obtained from studies such as these will assist individual countries to analyze their school programs' strengths and weaknesses as they plan for future directions. The mathematics education community is interested in this type of information as they plan for the improvement of mathematics education in the schools.

3. Research Design and Procedures

Information provided in this article is descriptive in nature. SIMS students in 236 eighth-grade classes in the U.S. completed internationally developed achievement tests at the beginning and end of the 1981-82 school year. A representative sample of eighth-grade students in the U.S. was chosen because "The concern was to study students at the last point in their schooling in which virtually all students were still studying mathematics in most of the countries participating in the study." The reason for choosing this grade level was so that comparisons could be made of "how various educational systems performed in offering mathematics to the majority of students and not just to the select few intensely interested in mathematics and science." Nearly 250 teachers and 7500 students in the U.S. participated in the study.

4. Findings

An overview of selected results is presented in the article. The following questions were addressed:

a. "What was Taught in the Eighth-Grade Classrooms?" Four groups of classrooms were sampled: 24 remedial, 26 enriched, 31 algebra, and 155 typical classes. Teacher reports and type of textbooks used comprised the method of classification. The median class size was 26, with mathematics being taught 5 days a week for a median of 145 hours during the school year.

   With respect to the 180-item achievement test used in the study, teachers were asked to determine whether the content needed to respond to that item had been taught during the school year. Five areas of content were defined: arithmetic, algebra, geometry, measurement, and statistics. The general finding was that all areas were reasonably covered with the exception of geometry, and as the class type increased from remedial to typical to enriched to algebra, the amount of arithmetic instruction generally decreased while the amount of algebra instruction increased. In addition, little of the geometry that was included on the international tests was covered in the classes.

b. "What were the Teachers Like?" The data showed that teachers were evenly divided between male and female, and were well trained and experienced. When asked to rate the relative importance of various goals for teaching mathematics, those of highest priority included developing a systematic approach to problem solving and developing students' awareness of the importance of mathematics in everyday life. The understanding of the nature of proof was rated the lowest.

c. "What was the Instruction Like?" The textbook was the most commonly and consistently used resource, with 75-80 percent of the teachers rarely or never using additional resources. About 4 percent of the teachers used calculators two or more periods per week, but approximately two-thirds of the teachers reported that their students never used calculators.

Overall, the most common teaching approach was the formal lecture, with emphasis on rules, computation, and formulas. The "tell and show" method was generally used, despite the teacher's belief in the importance of problem solving as a teaching strategy.

d. "How Much Mathematics do Students Know?" The 180-item achievement test administered to all students was again subdivided into the five categories mentioned earlier. In general, sizable gains were made in the areas of arithmetic and algebra (when comparing pretest to posttest percentages) and modest gains were made in all areas. Since the test was an international test and "sought to include items relevant to the curricula of many participating countries," the test did not match the curriculum of any one country. Thus, there was also a calculation of an "opportunity to learn" percentage, which indicated the maximum percentage of the items in that category to which the students had been exposed. This represented a "ceiling" of expectation for the students. Only in the area of geometry (in whi... the ...ing" percentage was quite low) did the students' mean achievement ..me close to the ceiling.

In addition, there was a large difference between class types. Students' achievement in remedial classes at the end of the year was not as great as students' achievement in typical classes at the beginning of the year. This pattern continued for comparisons of the typical versus the enriched classes and the enriched versus the algebra classes. Thus, for the U.S. students there was a dramatic difference in achievement between class types.

e. "How did the Students Compare with Those in Other Countries?" The data indicate that the U.S. students performed at the median with respect to arithmetic, algebra, and statistics, while they performed at approximately the 25th percentile in geometry and below the 25th percentile in measurement.

f. "Have U.S. Students Gotten Better or Worse?" The First International Mathematics Study (FIMS) was conducted in 1964, in which 12 countries participated. There was some repetition of items (36) between the FIMS and SIMS. On these items U.S. students' achievement declined "modestly" from 1964 to 1982, particularly in the areas of arithmetic and geometry. The decline was more evident in more difficult comprehension and application than on computational items.

5. <u>Interpretations</u>

The authors provide the following conclusions:

a. The results are "disturbingly mixed." The U.S. eighth-grade sample never achieved better than the international median and sometimes at or below the 25th percentile.

b. There were "modest" declines on higher level tasks between the 1964 and 1982 studies.

c. Students' attitudes were positive when considering the importance of mathematics, but neutral when considering mathematics as process-oriented.

d. Although the teachers were experienced and well trained and spent time preparing class presentations, instruction was invariably tied to the textbook and little use was made of calculators and other resources.

e. The U.S. curriculum is dominated by arithmetic when compared to other countries, but despite this fact, the U.S. scores were not outstanding in this area. Scores in the area of measurement were "truly disturbing."

f. The strong tradition of local control of schools and curriculum in the U.S. resulted in a wide variation in the coverage of topics, and there appears to be "little evidence that the majority of teachers are committed to a core of shared curricular goals in eighth-grade mathematics."

Abstractor's Comments

Studies of this type are very important for the improvement of mathematics education in all countries. Despite the numerous criticisms that will inevitably occur with this study (as occurred with the First International Mathematics Study), it is important for the mathematics education community, and even more important for the lay population, to realize that there are serious problems with the achievement of the "average" U.S. student in mathematics. To highlight the eighth-grade population was a very wise action on the part of the authors of this article, since they were able to study students of all ability levels and not just students with above average ability in mathematics, a criticism which was made of some of the results of the FIMS.

What is disturbing to the abstractor is the decline in scores of items that are classified as higher-order thinking skill items. Although teachers rate a systematic approach to problem solving as their highest priority, it appears as though this is not being taught in very many classrooms. Until teachers are willing to investigate alternative methods of teaching and move away from the sole use of a textbook as a resource, the teaching of problem solving will continue to be a problem. It is obvious from the comments made in the article that the authors are aware of this situation.

There are portions of the article which could benefit from additional elaboration. Questions such as the length of the school year in months or days, rather than merely in hours spent, could have been more fully explained. For instance, the abstractor has had experience working in several other countries and is aware that students in these countries spend much more than 180 days per year in school. In addition, more information could have been provided on the selection of the U.S. students to participate in the study, as well as the selection process in the other countries that participated. These are small points and, in all fairness, this article is merely a summary of a part of the data that is now available. The authors acknowledge this and provide the interested reader with an address to obtain more complete reports.

This is a very well written article and hopefully, it will tantalize most mathematics educators to obtain and read the additional reports that are available.

Schimizzi, Ned V. MATHEMATICS ACHIEVEMENT SCORES IN HIGH SCHOOL: ARE THEY RELATED TO THE WAY WE TEACH IN THE EARLIER GRADES? ERIC: SE 046 301.

Abstract and comments prepared for I.M.E. by DOUGLAS E. SCOTT, Amphitheater High School, Tucson, Arizona.

1. Purpose

Schimizzi's paper is not a research report, but rather a survey of some of the literature dealing with the problems of mathematics avoidance and poor mathematics performance by high school and college students. The purpose of the paper is to put forth some solutions to those problems as seen from the author's point of view.

2. Rationale

The rationale underlying the author's selection of studies to be reported is two-fold: a belief that teachers in the earlier grades mistake correct conditioned responses for internalization of a concept, and that teachers at all levels should give greater attention to Piagetian levels of cognitive development.

3. Research Design and Procedures

(Not applicable)

4. Findings

The author summarizes the findings and conclusions from several articles as follows:

a. Grouping of students for learning mathematics should be done after Piagetian-based testing; the premature presentation of topics (whether base ten concepts in second grade or algebra in the seventh) will produce little more than severe cases of math anxiety. "Knowing how to" is not the same as "knowing."

b. The "concrete-operational" stage is also characterized by a limited memory. Thus, when junior high school students finally reach the "formal operation" stage of development, with its concomitant improvement in long-term memory, they have often forgotton the number facts and algorithms which they supposedly "memorized" in the lower grades.

c. A too-rapid passage from the concrete to the abstract may be one of the factors producing lack of success for the average student. Concrete materials, particularly three-dimensional ones, should be used over long periods of time.

d. Play -- particularly outdoor play in a rural environment -- may have provided earlier generations of children with the opportunity to classify, label, imagine, and think before and along with formal schooling. These opportunities have been lost; passive television viewing is not a replacement.

e. Programs and courses in problem-solving skills at the college level not only significantly improved those skills, but also resulted in a gain of 6 to 10 IQ points.

5. Interpretations

The schools need to revise their objectives for teaching mathematics The most important objective should be strengthening the thinking/reasoning processes, not the short-term production of correct answers. This would include conscious use of logic and logical processes.

Concepts, skills and algorithms should not be presented to learners too immature (in Piagetian development) to learn them. All elementary school mathematics concepts should be taught using practical applications, such as computing utility bills, writing checks, planning trips, and budgeting.

Problem-solving skills should be taught from the intermediate grades onward; these skills should involve both left- and right-brain activities, and should encourage sketching or drawing solutions as well as calculating them. Diagnostic methods should provide indicators of the student's psychological level.

The home and school environments should encourage playing.

Logic for elementary school learners must be structured in physical form.

Abstractor's Comments

Nearly all of the references cited in this paper are themselves summaries, essays, surveys of the literature, or some other form of research-once-removed. This paper does an adequate job of pulling together articles that make the author's point, and few educators would argue against the dangers of excessively abstract teaching. Some might, however, question the value of explicit teaching of logic, in "physical form" or otherwise, at anything less than about age 14.

By its very nature, then, this paper adds nothing new concerning math anxiety and math failure; it amounts to a useful annotated bibliography of one segment of a large field of inquiry.

Ter Heege, Hans. THE ACQUISITION OF BASIC MULTIPLICATION SKILLS. Educational Studies in Mathematics 16: 375-388; November 1985.

Abstract and comments prepared for I.M.E. by ROBERT KALIN, Florida State University.

1. Purpose

The author wondered why it was that "some children easily learn the multiplication facts by heart, while for others it is a matter of extreme effort." Assuming that children eventually need to have the basic multiplication skills memorized, with that process having started by the beginning of the third grade, he questioned how this mastery should take place. The purpose of his research was to determine whether children did or did not have this memorization done, and if not, whether and by what strategy they would calculate the products.

2. Rationale

Although some have advocated paying less attention to memorized mastery of the 100 basic multiplication facts, the author claims that there is general consensus that they must be memorized for use in the algorism and in estimation. He sees prior research as concluding that there are three stages in learning these basic facts: 1) ample time to become acquainted with problem situations requiring multiplication, 2) development of multiplicative thinking strategies, and 3) memorization. The second stage is crucial in providing support for mastery. Cramming through rote learning too soon only leads to inefficiency. A positive attitude is necessary for memorization; students must consciously want to do it. The issue is not whether the facts should be memorized, but how it should be done. [illegible] current tendency to memorize by reciting rows of a table "can [illegible]ruct the mastery of individual multiplication facts."

3. Research Design and Procedures

Location for the research was a small provincial school (apparently in the Netherlands). Using the principal's office, the author interviewed "all the second- and third-graders (9 and 5 children, respectively), with many fourth-graders (11 children) and with some fifth- and sixth-graders (3 children)." Where less than all children in a grade were interviewed, the selection was done by the teacher from among those having trouble with "column arithmetic" (algorism?).

These interviews (an unreported number) were conducted from October until May, following "no standard procedure" beyond: 1) some introductory questions to establish a relationship, 2) asking whether the student knew multiplications right away, 3) then presenting some facts for him or her to answer. Answers were recorded by students in one of two categories: a column headed "knows right away," or one headed "figure out." This decision was made by students, although (occasionally) changed by the experimenter in agreement with the student. Criteria for whether a student knew a fact were two: promptness and his or her saying so. The experimenter tabulated the results of each interview by noting in the "figure out" column how any calculating was done. Immediately following or on the same day of an interview, the experimenter made a record of results that could include some interpretations.

4. Findings

The author claimed that "Some of the children I interviewed had learned the multiplication facts by continually using the tables as they are taught at school. Most children, however, made use of more or less developed strategies which they had not been taught." He claimed that these strategies could be categorized into six general types:

1. some kind of application of the commutative property (e.g., answering 8 x 7 by calculating 7 x 8 in some fashion);

2. using multiplication by 10 (e.g., finding 5 x 6 as half of 10 x 6);

3. doubling (e.g., getting 4 x 7 by doubling a known 2 x 7);

4. a limited use of halving known multiplications (mostly restricted to strategy 2. above);

5. increasing a known fact by adding multiplicand once (e.g., getting 6 x 7 from 5 x 7 by calculating 35 + 7);

6. decreasing a known fact by subtracting the multiplicand once (e.g., doing 9 x 7 from 10 x 7).

The author illustrates how these strategies can lead to a kind of complete calculation-mastery of the 8-row of the time table, normally regarded as difficult. He claims that exercises that focused on memorization were not effective with students who used such calculations, but that, given permission to use their strategies openly, they were able to improve their calculating skills. This led to his theorizing that there is a "borderline area between 'figure out' and 'know by heart'" which, if true, should lead to instruction that seeks improvement in their use of their strategies.

5. Interpretations

The experimenter feels that there is a need to extend his work to a more intensive study of the relationship between students' calculation of products and their memorization of the basic facts. He claims that the two are different cognitive structures, and theorizes that they "develop reciprocally."

Abstractor's Comments

Treated as an informal, though careful and thoughtful report of an informally constructed research, this article has much merit and contains a great deal of information and wisdom useful to both teachers and other researchers. The findings are largely consistent with the much earlier multiplication findings of Brownell and Carper (1943) and the related recent division findings of Anderson (1981) and Kalin (1983).

Although reference is made to other Brownell work and theories, no mention is made of his classic multiplication findings, which would have provided a firm foundation for the current study through an outline of six even more inclusive student thinking strategies.

Looking at this research as formally constructed, the abstractor suggests that there are several other concerns that make objective generalization difficult:

1) Only one school was sampled. Even more important, it was only from a certain kind of student at three grade levels that subjects were selected.

2) The particular facts used doing each interview were not reported. It is known that some facts are handled differently

by different students (Anderson, 1981; Kalin, 1983), which might lead to different conclusions.

3) The finding on the central point (did students have facts memorized? if not, did they use strategies?) should have been supported by some kind of numerical statement.

Certainly research should be carried out to investigate this experimenter's propositions that calculating products and memorizing basic facts develop reciprocally, that children be given instructional opportunities to develop their own strategies, that an "approach based on the use of supports and strategies would seem to be much more flexible than an approach based on learning the tables."

But thought should be given and care taken to consider alternative ways of constructing a direct memorization program intervening at some point. One based upon prior meanings, meaningful problem-solving experiences, and an organization of facts different from the conventional down-the-rows-from-the-twos-on, may be helpful or indeed even essential. Such seems to be suggested by some of the research referred to in this article itself as well as those by Brownell and others the author did not mention.

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