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ABSTRACT

This project was designed to study the effects of an inservice model on the quantity and quality of developmental mathematics instruction provided by participating teachers. Participants were seven teachers of students in grades from 2 to 6 who had volunteered for the project after being selected as potential lead teachers in mathematics. The inservice consisted of teacher observations, group inservice on effective teaching, individual planning and recommendations for becoming more effective, demonstration teaching, and more observations together with feedback and discussion. The results indicate that these seven elementary teachers increased the quantity of their developmental instruction to about half of their class period, from about 11 minutes each day to about 20 minutes each day. The inservice model also had a positive effect on the quality of the developmental instruction provided: teachers attended to more details related to instruction, used models more often, and used process questions to assess student understanding more effectively. As an inservice model, the one used in this project has the potential of effectively changing the behavior of teachers and improving instruction. Appendices contain samples of postobservation feedback for grades 4 and 6, papers on teaching mathematics concepts and algorithms, and an administrator/teacher feedback questionnaire. (Author/MNS)

THE FINAL REPORT OF THE COOPERATIVE GRINNELL-NEWBURG COMMUNITY SCHOOL DISTRICT AND UNIVERSITY OF NORTHERN IOWA MATHEMATICS PROJECT

AN IOWA TITLE II
EDUCATION FOR ECONOMIC SECURITY ACT OF 1984

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ABSTRACT OF THE GRINNELL-NEWBURG MATHEMATICS PROJECT

This project was designed to study the effects of an inservice model on the quantity and quality of developmental mathematics instruction provided by participating teachers. These teachers had volunteered for the project after being selected by the school administrators as potential lead teachers in mathematics. There were seven participants teaching students in grades from two to six.

The inservice consisted of (1) teacher observations, (2) group inservice on effective teaching, (3) individual planning and recommendations for becoming more effective, (4) demonstration teaching and (5) more observations together with feedback and discussion. The initial observations provided the opportunity to collect valuable information about typical patterns of behavior for each teacher that could possibly be changed to make their mathematics instruction more meaningful. This enabled the researcher to jointly plan a unit of instruction with each of the individual teachers and include some specific suggestions for being more effective. The demonstration classes allowed the researcher to model some of the teaching behaviors that had been recommended. The final phase of observations and feedback enabled each of the teachers to practice some of the suggested changes and discuss the results of their efforts.

The results of the study indicate that these teachers did increase the quantity of their developmental instruction to about half of their class period, from about 11 minutes each day to about 20 minutes each day. This increase made their instruction consistent with recommended time allotments for developmental instruction, as suggested by researchers investigating effective instruction in mathematics.

The inservice model also had a positive effect of the quality of the developmental instruction provided. The teachers began attending to more of the details related to instruction. They began using models more often to illustrate ideas and procedures. And they began using process questions to assess student understanding more effectively. After the inservice, there were only about half as many specific instances that led to or had the potential to lead to confusion and misunderstanding. A questionnaire also confirmed that the teachers changed their behavior. Each of them indicated some ways in which they were altering the behaviors they previously had used for instruction in mathematics.

The effect that the participating teachers can have on other teachers in the Grinnell-Newburg School District is yet to be determined. There are plans for them to share information with the other teachers in the school district through mathematics meetings, grade level meetings, teaching demonstration lessons and informal sharing.

As an inservice model, the one used in this project has the potential of effectively changing the behavior of teachers. Furthermore, the changes in behavior can have a positive effect on the quantity and quality of developmental mathematics instruction. Further investigation of this inservice model is needed.

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INTRODUCTION TO THE GRINNELL-NEWBURG MATHEMATICS PROJECT

In recent years there has been a growing concern about mathematics instruction in our schools. While efforts to stop the continuing decline in test scores for computation have been successful the past two or three years, Dr. H. D. Hoover with the Iowa Test of Basic Skills reported in 1985 that we have still not been universally successful at stopping the decline in test scores for problem solving. Test scores are significantly below what they were twenty years ago. There is concern for the improvement of mathematics instruction in many schools.

One of the most promising indications that educators can have a positive impact on student achievement has come from recent research on effective teaching. There are many general studies such as those reported by Gage in the book Hard Gains in the Soft Sciences and by Berliner in his article, "The Half-Full Glass: A Review of Research on Teaching;" however, the most relevant and convincing evidence as it relates to this project is presented by Good, Grouws, and Ebmeier in their book Active Mathematics Teaching.

In a series of studies on effective teaching practices, Good and Grouws have been able to identify a number of specific teacher behaviors that improve student achievement in mathematics. They concluded that teachers who use effective teaching practices positively influence student learning and that teachers can be trained to use these effective teaching behaviors.

Good and Grouws also identified several problems that need to be resolved as researchers work towards the development of more effective inservice programs for mathematics teaching. Two of these problems are directly related to this project. First, they noted the importance of providing quality instruction. Many of the behaviors that were implemented by teachers in these studies involved time schedule and effective management practices. The evidence indicates that it is more difficult to help teachers improve the quality of their developmental instruction, that is, to provide meaningful presentations and demonstrations and to lead discussions that promote better understanding. One of their conclusions dealt with the need to develop more adequate procedures for communicating to teachers criteria by which they can determine the quality of their developmental instruction.

A second problem that is directly related to this project involves the format of the inservice model. They noted that inservice programs that do not account for differences in instructional needs of children based on the content topic will be less effective than other programs that plan developmental experiences for specific content. Instructional demands are different for different topics. The use of models and the verbalization of thinking skills related to one topic may not be appropriate for a different topic. This greatly complicates the design of inservice programs if they are to be effective over a broad range of topics.

Another direction that has been suggested for improving mathematics instruction is currently being stated by the major professional organization for mathematics teachers, the National Council of Teachers of Mathematics. In a recent position statement, the National Council has taken the stance that lead statement, the National Council has taken the stance that lead teachers should be trained as resource teachers so that others will have ready access to information and ideas about providing more effective instruction. Research to support the effectiveness of this effort to provide better instruction in mathematics is lacking.

The administrators and teachers at the Grinnell-Newburg Community School District are also concerned about mathematics instruction. They have formed a mathematics committee to study the curriculum and to determine what they can do to improve instruction in mathematics. The principal investigator has developed a research interest in effective teaching practices and the related design of inservice programs. Dr. Barbara Wickless, mathematics consultant with the Department of Public Instruction, being aware of the efforts of both parties, suggested that the principal investigator and the Grinnell-Newburg Community School District might be able to work cooperatively on their efforts. Her suggestion led to the development of this project.

The project was designed (1) to provide inservice on effective instruction in mathematics to lead teachers from the elementary and middle schools at the Grinnell-Newburg Community School District and (2) to provide the opportunity to determine the effects of this inservice on the quantity and quality of the developmental instruction provided by these teachers as well as determine the ability of these teachers to transfer knowledge about effectively teaching one topic to different topics. The inservice included both general knowledge about accepted practices in effective instruction and planning for the use of specific teaching strategies related to a content topic that was taught soon after the inservice. The inservice was designed to provide the Grinnell-Newburg Community School District with trained lead teachers representing each grade level from two through six. These lead teachers in turn will act as resource teachers for other mathematics teachers in the district after the completion of the project.

DESCRIPTION OF THE PROCEDURES

Procedures

The general procedures including a timetable follow. The principal investigator directed these procedures and was responsible to see they were carried out.

Preparation: November-January 1985

(1) The administrators from the Grinnell-Newburg Community School District selected seven teachers with the potential to become lead teachers. They were asked to volunteer for the project. All of them accepted. Collectively, they represented grades two through six and each of the three elementary schools as well as the middle school in the Grinnell-Newburg Community School District.

(2) The researcher met with the seven teachers to provide clarification about the purposes and procedures of the project and to answer questions.

(3) The researcher met with Doug Grouws, an expert on effective instruction in mathematics, and discussed the observations. In particular, the type of data that was to be collected and the value of different types of information as it relates to meaningful development of the lessons was discussed. After this two-day meeting, the researcher planned the types of data that were to be recorded during the observations.

Initial Observations: January-March 1986

(4) The researcher observed each of the seven lead teachers three times, noting characteristics of their teaching behaviors as they related to accepted knowledge about effective teaching in mathematics. Meticulous notes conerning the behaviors of the teachers, the questions that were asked and the models that were used to promote meaningful instruction were taken. A summary of what happened in the class and suggestions for increasing the meaningfulness of the instruction were prepared for each of the twenty-one lessons that was observed.

Inservice: April-May 1986

(5) The researcher provided and discussed the implications of knowledge about the most recent findings related to research on effective instruction in mathematics during three one-half day inservice sessions. Also included were suggestions for teaching mathematics concepts and algorithms and the implications they had for using the adopted mathematics textbook of the Grinnell-Newburg Community School District.

(6) The researcher then met individually with each of the seven teachers to provide and discuss the implications of specific suggestions for helping them become more effective in their instruction. This was based on the characteristics of their teaching behaviors during observation and how it related to research findings about effective instruction in mathematics.

(7) The researcher and each of the seven teachers cooperatively planned instructional activities for a major topic they taught shortly after this inservice.

(8) The researcher then taught at least one demonstration

class for each of the seven teachers. This was done to specifically illustrate a behavior for that teacher to practice throughout the unit that had been cooperatively planned.

Post-Inservice Observations: April-May 1986

(9) The researcher then observed each of the lead teachers as they taught the topic for which joint planning took place.

(10) The researcher provided feedback to each of the lead teachers at the conclusion of each of these post-inservice observations. In particular, feedback on the specific behavior that related to the change being practiced was provided immediately at the conclusion of the lesson. In two cases the researcher returned for additional demonstration lessons and for additional observations at the request of the teachers.

(11) The project called for additional observations while the teachers were providing instruction for a different topic than the one jointly planned. Some of these were conducted; but, because of scheduling difficulties, it was impractical to collect enough data regarding the transfer of effective teaching practices to other topics to reach any research conclusions.

Evaluation: May-June 1986

(12) A survey was then used to collect data from each of the seven teachers and each of the school administrators involved to help characterize the efforts to incorporate effective teaching practices into daily mathematics instruction and note any changes in common teaching practices and beliefs about mathematics instruction.

(13) The data was then used to identify any changes in the characteristics of teacher behavior and to answer the research questions:

(a) Did this inservice procedure have a significant effect on the quantity of time devoted to meaningful instruction during the mathematics lessons?

(b) Did this inservice procedure have a significant effect on the quality (based on accepted research findings of effective teaching in mathematics) of the instruction provided during the mathematics lessons?

Report: August-September 1986

(14) The researcher prepared a written report of the project including the purposes, the procedures, the data, evaluations based on the evidence collected during observations before and after inservice and the surveys of both the teachers and the administrators and recommendations.

Inservice for all Grinnell-Newburg teachers: August 1986

(15) With another investigator of effective teaching practices in mathematics instruction, the researcher provided inservice for all the mathematics teachers at all grade levels in the Grinnell-Newburg Community Schools.

In a project such as this it was very important that the integrity of all persons involved in the project as well as the other teachers in the Grinnell-Newburg Community Schools be respected at all times. The lead teachers were in the awkward position of having another professional observe and comment on their teaching. The principal investigator was aware that the situation was sensitive and acted accordingly. Many years of experience in working with teachers in the classroom, including the responsibility of being a classroom mathematics consultant for AEA 7, gave the principal investigator confidence in his ability to create a non-threatening attitude that helped promote professional growth on the the part of all the teachers in the Grinnell-Newburg Community School District.

Participants

There were five groups of persons involved in this project. First, the director of the project was also the principal investigator, Professor Edward C. Rathmell from the University of Northern Iowa. The responsiblilities of the director were to see that each of the procedures described above was completed in cooperation with the Grinnell-Newburg Community School District in a competent and professional manner and within the general guidelines of the timetable provided. It was also the responsiblity of the director to cooperate with the Grant and Contracts Officer at the University of Northern Iowa in the management of the budget for the project.

The second group of persons involved was the administrative team at the Grinnell-Newburg Community School District. This group included the superintendent, the principals of each of the three elementary schools and the principal of their middle school. Their initial responsibilities included participating in preliminary discussions about the project with the director, deciding if the project as planned had merit for the Grinnell-Newburg Community Schools, making a commitment to become involved, identifying potential lead teachers and determining if these teachers were willing to make a commitment to be involved in the project. Additional responsibilities during the project included cooperation with all the parties involved to facilitate scheduling of the events described in the procedures above, participating in the evaluation of the project and scheduling an inservice session for all of the mathematics teachers in the school district in August 1986. After this project has been completed the responsibility of the administrators continues in the form of providing appropriate encouragement and conditions for using these lead teachers as models and resource teachers for all of the math teachers in the school district. It is anticipated that the director and the Grinnell-Newburg Community School District might continue a cooperative effort to help these lead teachers in this role; however, the director would be involved in only a very limited way after the project is completed.

The third group of persons involved in the project were the lead teachers. These teachers were identified by their administrators as having the potential for successfully participating in the project and successfully filling the role as a resource teacher. Their responsibilities included making a commitment to participate in the project, agreeing to cooperate in arranging schedules so the principal investigator was able to observe their classes and meet with them for inservice and planning, completing surveys and acting as a resource teacher for the other math teachers in the school district after the project is completed. They were all Caucasian. Five of the participating teachers were female and two were male. The Grinnell-Newburg administrators decided which teachers to invite. The researcher only suggested that they have the potential to become effective lead teachers in the area of mathematics and that they be voluntary participants.

There were two consultants for the project. Dr. Douglas Grouws is a researcher at the University of Missouri. He has a long and impressive history of research on effective teaching in mathematics. He acted as a consultant in the sense that the principal investigator visited the University of Missouri to discuss the project with him and to incorporate some of the observational techniques he has developed into the observation procedures for this project. Some additional techniques were used in this project because there was a particular emphasis on describing the teaching behaviors during that portion of the lesson when the teacher is presenting material to the students, the developmental instruction portion of the lesson. A second consultant, Dr. Diane Thiessen from the University of Northern Iowa, was involved in the project at two levels. The principal investigator and Dr. Thiessen discussed the project at regular intervals. Dr. Thiessen also assisted with the inservice for the entire group of mathematics teachers at the end of the project. Dr. Thiessen was on a Professional Development Leave to continue her study of effective instruction in mathematics during the same time period that the project was completed. Furthermore, she was working with Dr. Thomas Good and Dr. Douglas Grouws at the University of Missouri during a portion of that time.

Finally the Grants and Contracts Officer at the University of Northern Iowa was responsible for managing the budget. He has prepared the final budget for this project and the final budget report.

TEACHER BEHAVIORS BEFORE INSERVICE

The primary purposes of this project were to investigate the developmental instruction that was provided for mathematics in the elementary and middle school and to determine the effect that an inservice program of this type has on the quantity and quality of that developmental instruction. It should be noted again that the teachers participating in this project were selected because of their potential as lead teachers in mathematics. They were considered to be among the best mathematics teachers in the school district by the administrators. The observations confirmed that they were indeed providing good instruction and they were all particularly effective in their management techniques. In general, their classes performed well, were well organized, were on task almost all of the time and pupils with learning problems received a considerable amount of individual assistance.

Quantity of Developmental Instruction

One of the interests of the researcher was the amount of time that teachers actually spent on developmental instruction. Recommendations from research on the effects of different amounts of time spent on instruction generally suggest that about half of the mathematics class period be devoted to development. Good, Grouws and Ebmeier suggest about 20 minutes of development each day. They found that the teachers in their studies generally spent less than half as much time on development of the lesson as research suggests.

Prior to inservice, the teachers in this study also did not spend as much time on development as research suggests. The mean number of minutes of developmental instruction that each child received for each lesson was 11. That is well below the 20 minutes recommended by Good, Grouws and Ebmeier. However, three of the seven teachers did provide 15 minutes or more of development.

The number of minutes that the teachers in this study spent on development for each lesson and the mean scores are shown below in Table 1. Although there were only seven teachers involved in the study, there are eight entries in the table because one of the teachers had two math groups. The amount of time that was spent with each group was considered as separate entry because that was the amount of developmental instruction that each child in that group received. Note that the number of minutes that were spent on development varied from an average of 2 minutes per day to 17 minutes per day. The mean for all of the groups for each lesson was 11 minutes. That amount of time spent on development compares favorably with the amount of time that teachers in the Good, Grouws and Ebmeier studies, but not favorably with the recommended times.

It is interesting to note the decrease in amount of time spent on development from the first observation to the third observation. The teachers only spent about half as much time on development during the third observation as they did during the first. The teachers were aware that the purpose of the project was to study the developmental instruction they provided. That fact alone might have made them spend slightly more time on that part of the lesson initially. They were also quite nervous, especially during the first visit. These two factors may have contributed to extra time being spent on development.

TABLE 1

NUMBER OF MINUTES OF CLASS TIME DEVOTED TO DEVELOPMENTAL INSTRUCTION PRIOR TO THE INSERVICE

teacher	observations			
	first	second	third	$\bar{x}$
a	15	2	0	6
b	19	16	15	17
c	13	3	14	10
d	0	0	7	2
e	14	4	5	8
f	20	16	15	17
g	28	6	10	15
h	13	25	0	13
totals	15	9	8	11

On four occasions out of the 24 lessons for children, there was no time devoted to the development of mathematics ideas. These lessons consisted of review followed by seatwork. Only one of these lessons was legitimately a review lesson. It was the day prior to the chapter test. The others should have had some meaningful instruction.

On a total of nine occasions out of the 24 lessons, there was less than 10 minutes spent on development. This amounts to a little over a third of the lessons. This lack of adequate time spent on development was demonstrated when many of the students worked mechanically through the seatwork without sufficient understanding. That lack of understanding was demonstrated because they often had questions about the procedures involved.

Quality of Developmental Instruction

Assessing the quality of the developmental instruction provided by a teacher necessarily involves some subjective judgements. The researcher was well aware of that fact when collecting information about the teacher behaviors in the classroom. The decisions that were made about the quality of instruction were research based as much as possible. For example, the use of models to illustrate mathematical ideas has been shown to be effective in many studies. Consequently, appropriate use of a model to develop meaning for a topic was judged to positively affect the quality of a lesson.

The teachers were already familiar with and using some of the effective teaching behaviors suggested by Good, Grouws and Ebmeier. They all conducted their classrooms efficiently and had good management techniques. In no instance did a discipline problem, a lack of organization or failure to use management skills interfere with the quality of the developmental instruction provided. The teachers were all found to be sensitive and caring and had developed an open comfortable atmosphere in which the children were willing to ask questions and share their ideas. The teachers involved in this project were good teachers who have the potential to become good lead teachers.

Nearly every lesson that was observed had parts that were conducted in a meaningful way. The students were able to successfully complete that part of the seatwork with understanding. However, because the teacher does many different things to conduct a lesson, it was often difficult to identify the precise behaviors that were responsible for that learning. It was much easier to identify specific situations that led to confusion and questions on the part of the students. For that reason, the situations described below are negative rather than positive instances of teacher behavior. That in no way is intended to imply that no positive teacher behaviors were identified. Every teacher exhibited many positive behaviors.

Whenever possible, situations that led to confusion were documented. This was often demonstrated by procedural or conceptual questions asked by the students. However, a few of the situations recorded below could not be directly linked to pupil misunderstandings. It was inferred that later confusion and pupil questions were at least indirectly related to these teacher behaviors. These situations reflect the beliefs of the researcher concerning the types of experiences that children need in order to understand and use mathematics meaningfully.

The types of situations that were recorded included failure to check for prerequisite understanding, failure to use diagrams, concrete models or even concrete examples to illustrate and provide meaning for a topic, situations that obviously lacked clarity and failure to check for student understanding. The different types of situations and the number of occasions that each was noted is reported in Table 2. The situations are listed under primary topic and review topic depending on whether the instruction was intended for the primary objective of the lesson or simply to review a topic that had been previously taught.

Failure to deal with Prerequisites There were several instances when the teachers did not check for understanding of the prerequisite skills needed during the lesson. Three times this oversight caused confusion on the part of the students and cost extra time because the teacher had to stop the class and provide unanticipated instruction for these prerequisites. In two instances the teacher failed to attend to prerequisite skills

TABLE 2

THE TYPES AND NUMBER OF SITUATIONS THAT LED TO CONFUSION PRIOR TO THE INSERVICE

situation	primary topic	review topic
failure to check understanding of prerequisites	2	1
rule-example-practice (no explanation)	5	1
failure to use models (symbolic explanation only)	9	3
lack of clarity (some children were confused)	17	1
— used model without clear explanation	2	
— used model without correct thinking	3	
— used model without connecting to symbolic work	4	
— used inappropriate numbers for examples	2	
— asked questions but failed to answer or explain	2	
— failed to clearly explain how to write the algorithm	1	
— developed an idea, but failed to relate it to topic		1
— failed to prepare students for transition to seatwork	3	
failed to ask questions to check for understanding	3	

that were essential to the lesson. In one case, the lesson dealt with two types of numbers. Since only one type had been discussed during the lesson, the students had a great deal of difficulty working with the other problems during the seatwork. In the other instance, the order of the lessons had been changed from that presented in the textbook. The seatwork assignment was not adjusted to exclude those problems that had not been taught. A third instance is mentioned because it caused the teacher to use about five extra minutes of class time for explanation, although it involved only a review topic. The teacher presented a problem for review. It turned out that because many of the students did not understand all of the prerequisites for this review topic, too much extra time was spent on a topic that was not even essential for the lesson. Because a spontaneous decision had to be made, the extra time was used for this instruction. Since that ended up taking valuable time away from the instruction for the primary objective of the lesson, perhaps the difficulty should have been noted and the instruction delayed until later.

Rule-Example-Practice In six instances, no explanations were given for a topic. In each case, the teacher presented a rule or told how to complete the procedure, did one or two examples and assigned seatwork. This could be described as rule-example-practice instruction. In one instance this involved only a review topic and no evidence of confusion was found; however, with little additional effort, the teacher could have explained this procedure and avoided potential trouble. In the other five instances the teachers simply did not explain the primary objective of the lesson. They only showed how to perform the procedure. In each of these cases, at least one student was not sure how to do the seatwork. The teacher had to reteach the topic individually at that time.

Failure to use Concrete Models There were twelve instances when the teacher failed to use a concrete model or a diagram when it might have helped prevent some misunderstanding. In each of these cases, an explanation was presented, but without the benefit of a visual model to help the children understand. Nine of these occurred for the primary objective of the lesson. In five of these cases, the teacher did provide a concrete example in the setting of a story problem that involved the same procedure. In two instances concrete models were used, but not to help illustrate the procedures involved. In one instance, children were used to show the problem. In the other a place value chart was used to show the answer. In neither case did these models help explain the procedures that were being taught.

It is difficult to determine exactly how much misunderstanding arises from instruction without illustration. In many cases the children can complete the assigned seatwork. A lack of understanding often does not become evident until later when children overgeneralize the procedure to other inappropriate situations, can not transfer the basic idea to another similar situation or simply forget the procedure and can not reconstruct it. Although the researcher did not collect evidence that each of these twelve situations caused some confusion, it is reasonable to assume that models should be used to illustrate an idea whenever easily possible.

Lack of Clarity Eighteen situations were observed when the researcher made a judgment that the instruction lacked clarity. In each case there was some evidence that at least one student did not clearly understand. In this study lack of clarity is defined by the categories of situations that arose in the classroom.

It is not always the case that using a concrete model will clarify instruction. If models are used, they must be used in a way that helps illustrate the procedures or conceptual bases for the work. Three different types of situations arose in this study where a concrete model was used, but it was not particularly helpful to the children.

In two instances the teacher used a concrete model without clearly explaining how to use it. A symbolic explanation was also given. The children were given their choice as to which procedure they could use to complete the seatwork. Later in the class period it became obvious that some children did not understand the symbolic explanation well enough to use it and could not fall back on concrete procedures because they did not understand how to use the materials to derive the answers to the problems.

On three occasions a model was used, but the thinking that was used was inconsistent with the model. In one instance part of the model was removed and most of the children in the class misinterpreted the remaining visual illustration. In the other two instances, the thinking that was verbalized was inconsistent with what was illustrated by the model. In effect, the possible benefit of the model was removed because the children could not use the thinkng that had been expressed and the model at the same time. They solved the problems during the seatwork by using the thinking they had heard, not by using what had been illustrated.

In four instances the teacher used a model to illustrate a procedure and did it well. The instruction fell short of providing appropriate meaning because it was not related to the symbol work that children were asked to complete for their seatwork. In all of these cases the students could use the models to derive the answers to assigned problems. But since the work with models had not been related to the work with symbols, they had to rely on other procedures to complete the symbol work. It meant that the students had to learn two procedures that day, one for the model and one for the symbols. When appropriate connections are made between model work and symbol work, the two support and reinforce each other.

In two instances the teacher chose inappropriate numbers for the examples used during the explanations. In each case the procedure that was intended to be shown was explained. However, because of the choice of the numbers involved, some children saw patterns other than the intended procedures and overgeneralized. In each instance this caused unnecessary confusion and the teacher spent an additional five to ten minutes reexplaining the procedure. Both of these situations could have been avoided had different numbers been used in the sample problems.

Twice a teacher asked good questions that focused on the meaning behind the procedures only to leave the question unanswered. In one case it involved understanding why the children were supposed to use the procedures that were explained. In the other case, the students were repeatedly asked if their answers were reasonable. Never did the pupils hear an explanation about how to make such a decision.

In one instance the teacher provided a clear explanation of the procedures involved in the lesson, but only did one example during the development. The children generally knew what they were supposed to do and why, but later during the seatwork it became obvious that they did not know where to write the symbols in the algorithm. This is a problem similar to that of not relating model work to symbol work, except that no models were used. The explanation had been strictly symbolic.

In one review session a teacher asked the children to do a problem that they had previously learned. It turned out that knowledge about that problem related quite closely to the primary objective for that day's lesson. The pupils could have used that same idea to help them solve several of the problems during the seatwork. However, the pupils did not discover the relationship and the information, that they had and could have used, was not seen as being relevant.

In three instances the teacher did not prepare the students for the transition from the instructional part of the lesson to the seatwork. In each case the lesson was well taught and understood, but the format of the problems in the textbook assignment was different than the format of the symbolic work during the development. Failing to discuss these changes in format caused enough difficulties during the seatwork to make the teacher interrupt the practice and reteach that aspect of the lesson.

Lack of Assessment Questions During three of the lessons the teacher did not ask any questions to determine the level of understanding of the students. By careful monitoring during seatwork, they were able to determine that most of the children could complete the problems. But it was not at all clear that the children had developed meaning for the material. The evidence is not clear that many students did not understand, but the teacher did not ask any questions during the entire mathematics lesson to determine that. In general, there were too few questions to assess understanding. Often it was the case that a question was asked concerning the answer to a problem, but no follow-up questions were asked to determine the level of understanding of the students. It is relatively easy to ask these questions on a regular basis and the information gained can help teachers decide what further instruction is needed.

Summary

Overall the teachers did provide good instruction for their classes. The exceptions that were noted above often did not involve misconceptions on the part of very many students, but they did require the teachers to spend additional time with individual students. If two general suggestions for improvement were to be made, they would be (1) to focus on the mental processes that children are using to a greater extent than just the answers they have derived and (2) to make more effective use of models to illustrate the mathematical ideas and procedures to be learned. While most of the teachers did ask students to explain their thinking processes, only two used this technique of assessing student understanding on a consistent daily basis. Five of the seven teachers involved used concrete models on at least one occasion, but only one of them consistently developed meaning from these representations and related the models to the symbol work that children were asked to complete.

The researcher also rated the quality of each lesson on a 1 to 5 scale. These ratings are necessarily subjective; however, evidence based on the number of students who could successfully complete the seatwork and the number of students who did or did not understand the lesson was used as a guide. An explanation of the rating scale is listed below. The ratings of the quality of each lesson is shown in Table 3. Overall, the mean score was about 3 which indicates that most of the students successfully completed their seatwork assignment, but there was either no evidence of understanding or some children were having difficulties.

5 All of the students in the class can successfully complete the seatwork assignment and there is evidence that most of the children understand.

4 Most of the students in the class can successfully complete the seatwork assignment but there is evidence that a few children do not understand.

3 Most of the students in the class can successfully complete the seatwork assignment but there is evidence that several children do not understand or assessment of the students did not provide evidence of understanding.

2 Many of the students in the class had difficulties with the seatwork assignment and there is evidence that several children do not understand.

1 Most of the students in the class had difficulties with the seatwork assignment and there is evidence that many of the students are confused.

TABLE 3

RANKING OF THE QUALITY OF DEVELOPMENTAL INSTRUCTION PRIOR TO INSERVICE

teacher	lesson 1	2	3	$\bar{x}$
a	2	3	3	2.7
b	4	4	5	4.3
c	5	3	3	3.7
d	2	2	2	2.0
e	3	2	2	2.3
f	2	2	2	2.0
g	3	3	2	2.3
h	2	2	2	2.0
mean				2.7

DESCRIPTION OF THE INSERVICE PROGRAM

The inservice program for the participating teachers consisted of two parts. First, there was one and a half days of meeting together as a group. That was followed by individual meetings with each of the teachers for the purposes of discussing feedback from the pre-inservice observations, selecting one or two behaviors that each teacher could try to change and cooperatively planning a unit of instruction with those changes in behavior built into the plans.

The group inservice provided the opportunity for the researcher to present some information about effective instruction, to discuss the implications of these ideas for teaching mathematics and for using some specific examples to illustrate how these ideas could be implemented. The focus for the first day of the inservice was on the research and writings by Good, Grouws and Ebmeier (1983), Hunter (1982), Gage (1985), Berliner (1984) and Johnson (1982). This information about the effects of various teacher behaviors on student achievement provided a sound basis from which the participating teachers could make instructional decisions. The participants also viewed a video tape by Good that illustrated many of these teaching actions that have been shown to be effective.

The extra half day of inservice was devoted to discussion and examples of using effective teaching behaviors for various topics of importance to the participants. Two position papers by the researcher provided a framework from which the teachers could make decisions about appropriate models and thinking skills for a topic and how these could be related to the symbol work that children need to learn. These two papers were titled, "Teaching Mathematics Concepts" and "Teaching Mathematics Algorithms." They are included in the appendix. Opportunities were provided for the participants to raise questions about teaching topics that were of importance to them. In each case, the researcher attempted to relate what could be done back to the framework for using models and developing thinking skills and to the effective teaching behaviors that had been presented earlier.

Each of the teachers also decided what unit of instruction they wanted to jointly plan with the researcher. They had been aware of this responsibility for some time, but were asked to make a commitment to a specific topic at the time of the inservice. Tentative schedules also had to be coordinated so the researcher could schedule observations of these lessons.

The individual meetings with each of the participants consisted of a discussion of the pre-inservice observations and any suggestions that the researcher had that might have made these lessons more meaningful to children followed by joint planning of a new unit of instruction for which some suggested changes of teacher behavior were included. Samples of the feedback that teachers received from the pre-inservice observations are included in the appendix. Typical suggestions for change in teacher behaviors included the use of models to illustrate some ideas that had previously been explained symbolically and better use of questions to assess student understanding. After the lessons for the new topic were planned, the researcher offered to teach a demonstration class to illustrate the changes in teacher behavior if the participant wanted that. In every case, the participants not only wanted to observe that lesson, but felt some relief that they would have at least one example to follow. It turned out that the researcher taught more than one demonstration class for some of the participants. In two instances, after trying a new idea, neither of the participants felt comfortable using it and asked the researcher to return for further demonstrations.

TEACHER BEHAVIORS AFTER INSERVICE

After the inservice there was evidence of change in the teachers both in the quantity and the quality of the developmental instruction provided. This evidence was collected during observations of lessons for which joint planning had taken place. Each teacher was observed at least once during this phase of the study. Since scheduling difficulties did not permit as many observations as had been planned and it was not possible to observe each of the teachers the same number of times, only the first observation during this phase of the study has been analyzed. Not enough evidence was collected when teachers were teaching a topic that had not been jointly planned to warrant inclusion.

Quantity of Developmental Instruction

After the inservice the teachers in this study spent much more time developing the lesson in a meaninful way. Two of the teachers were preparing their students for an end-of-the-unit test. Consequently, they included a considerable amount of time on review during these lessons. Even that was done in a meaningful instructional way. One of them drew many diagrams and related the written work to those diagrams. The other used concrete examples in the form of story problems together with good symbolic explanations. Both teachers used follow-up questions to determine the understanding of their students.

Table 4

NUMBER OF MINUTES OF CLASS TIME DEVOTED TO DEVELOPMENTAL INSTRUCTION AFTER INSERVICE

teacher	number of minutes including developmental review	number of minutes excluding developmental review
a	26	11
b	17	17
c	26	26
d	11	11
e	33	12
f	35	35
g	29	29
mean	25	20

Since these reviews were done with a developmental flavor, the number of minutes of time spent on development after the inservice is shown both including that time for review and excluding it. These times are shown in Table 3. There are only seven scores in this table because the teacher who had previously been separating the class into two groups did not do so for this unit of instruction.

Excluding the developmental review, the mean number of minutes spent on development was 20. Including the review it was 25 minutes. This is about twice as much time spent on development as before the inservice. If the review is included, six out of the seven teachers spent over 15 minutes teaching the lesson meaningfully.

Quality of Developmental Instruction

During the joint planning for these lessons, the teachers and the researcher discussed the models that were appropriate for the topic, how they could be used to represent this idea or procedure and relate it to the symbol work, any language that would be helpful and questions that could be used to assess student understanding. Each teacher was encouraged to used models and to ask follow-up questions to check for understanding. The situations that either led to or had the potential to lead to some confusion are recorded in Table 4.

TABLE 5

THE TYPES AND NUMBER OF SITUATIONS THAT LED TO CONFUSION AFTER THE INSERVICE

situation	primary topic	review topic
failure to use models (symbolic explanation only)	4	1
lack of clarity (some children were confused)	3	
used model without clear explanation	1	
used inappropriate numbers for examples	1	
failed to prepare students for transition to seatwork	1	

<u>Failure to use Concrete Models</u> While six of the teachers used models during at least part of the lesson, five times there were strictly symbolic explanations. On at least four of these occasions there was evidence that the students understood. While only one of the situations occurred during review, each of the other topics had also been developed earlier and the students appeared to understand these symbolic explanations. There was one

instance where a few of the students could perform the procedures, but there was some question about their understanding.

Lack of Clarity A lack of clarity in the instruction was evident on only three occasions. Once the teacher introduced a slightly different version of a model than what had been used previously. Some children had difficulty making the transfer to this new form of the model. Because the main idea of the lesson had been well learned, they were able to use the new model after the teacher stopped the seatwork and gave a brief explanation.

On one occasion the teacher used an inappropriate number that could have encouraged some students to overgeneralize the procedure being learned. There was no evidence that any child was misled, but a different number would have been a safer choice.

One teacher carefully developed the major objective of the lesson, but did not help the children make the transition to the format of the symbolic work in the textbook. The children had worked with this format earlier, but an example or two just prior to the seatwork would have eliminated confusion on the part of several pupils.

Summary There was very little confusion on the part of students in any of these lessons. Six of the seven teachers used models and the other used concrete examples for a review lesson. Overall, there was very little evidence of confusion or misunderstanding. Four of the seven teachers asked such good assessment questions that there was little doubt that their students understood the procedures and could perform them with confidence.

The researcher also rated the quality of each lesson on a 1 to 5 scale. These ratings are necessarily subjective; however, evidence based on the number of students who could successfully complete the seatwork and the number of students who did or did not understand the lesson was used as a guide. An explanation of the rating scale is listed below. The rating of these lessons on the basis of meaningfulness is shown in Table 6.

Overall, the quality of the developmental instruction after the inservice improved. The mean score on this scale improved from a rating of about 3 before the inservice to a rating of about 4 after the inservice. In general, the differences were due to fewer students having difficulties with the lesson and better assessment techniques for determining the understanding of the students. For most of these lessons, the teachers had assessed student comprehension and knew before the seatwork was assigned that most of the children understood the lesson. That had not been as typical prior to the inservice.

An explanation of the rating scale is shown below:

5 All of the students in the class can successfully complete the seatwork assignment and there is evidence that most of the children understand.

4 Most of the students in the class can successfully complete the seatwork assignment but there is evidence that a few children do not understand.

3 Most of the students in the class can successfully

complete the seatwork assignment but there is evidence that several children do not understand or assessment of the students did not provide evidence of understanding.

2 Many of the students in the class had difficulties with the seatwork assignment and there is evidence that several children do not understand.

1 Most of the students in the class had difficulties with the seatwork assignment and there is evidence that many of the students are confused.

TABLE 6

RANKING OF THE QUALITY OF DEVELOPMENTAL INSTRUCTION AFTER INSERVICE

teacher	rank (maximum of 5)
a	4
b	4
c	5
d	3
e	5
f	3
g	4
mean	4.0

EVALUATION OF THE PROJECT

The effectiveness of the project was determined on several different bases. First, the quantity of developmental instruction changed from a mean of about 11 minutes to a mean of 20 minutes, or more, if developmental review is also included (Refer to Table 1 and Table 4). According to numerous research studies and implications from summaries of these studies, an increase in the amount of developmental instruction, up to about half of the class period, is accompanied by increased student achievement.

In this study the increase in the amount of time spent on development appeared to be due to more careful attention to details such as prerequisites and different formats for presenting problems, more time spent with models to illustrate the lesson and more time spent on questions to assess student understanding. Prior to the inservice there were more symbolic explanations and fewer examples to help children learn the procedures or ideas. It takes more time to illustrate all of the different aspects of the topic more carefully, to do more examples with models and to ask more explanatory questions rather than answer-oriented questions.

A second criteria for evaluating the project involved the quality of the instruction. Although this is necessarily more subjective, decisions about the quality of these lessons were related to instances of teacher behavior that led to confusion on the part of some children. In some cases there was no evidence of misunderstanding, but the potential for confusion was noted and there was no also evidence of understanding. A comparison of Table 2 and Table 5 indicates that nearly twice as many situations that led to confusion occurred in each lesson prior to the inservice. After the inservice, there were no instances of rule-example-practice instruction with no explanation and there were fewer instances of symbolic explanations. Furthermore, the symbolic explanations that did occur, were more appropriate for the lessons in which they were used. They all occurred in the last few days of the unit of instruction. Prior to the inservice, several of the symbolic explanations were the basis of the initial instruction for the topic.

Table 3 and Table 6 show a quantification of the quality of the lessons before and after the inservice. They indicate that there were fewer children with confusion and more evidence of understanding after the inservice. The reasons for a judgement of improved quality are similar to the reasons listed for an increase in the amount of time spent on development. In general, the teachers were more careful about the details of the lessons, spent more time illustrating the topic with models and asked more explanatory questions.

A third criteria for evaluating the project came from questionnaires that were completed by the participating teachers and their administrators. The questionnaire asked for evaluations of each phase of the study. Copies of these questionnaires are included in the appendix.

The teachers all were extremely nervous during the initial observations. As they became better acquainted with the researcher this anxiety was reduced. Six of the seven teachers felt that they did not change from their typical behavior during these recommended that others, who were about to be observed, should just be themselves, be positive and open to suggestions. Suggestions to the researcher about ways that could be used to help make other teachers more comfortable in similar observations included the use of more regularly scheduled observations and a brief feedback session immediately after each observation. The administrators responding to similar questions about the observations prior to the inservice gave nearly the same feedback except that they did not generally feel that the teachers were as nervous as the teachers indicated and two of the four felt that the teachers probably did change their behavior during the initial observations.

All of the teachers indicated a positive feeling about the overall effectiveness of the group inservice. They all felt the information about research on effective instruction was helpful and evaluations of the discussions about using models and verbalizing the thinking that children use ranged from helpful to extremely helpful. The suggestions for improving this phase of the study focused on providing more examples at their own grade levels. Recommendations included spending more time with lower or upper grade teachers and providing more video tapes showing good instruction on important topics at the grade level they teach. This latter suggestion about video tapes is one that has the possibility of relatively inexpensive and yet potentially widespread influence on the way that important topics are presented to children. The administrators also gave positive evaluations of the group inservice with no additional recommendations.

All of the teachers indicated that the feedback they received accurately reflected what had actually happened in the classroom. They all responded that the suggestions for improving instruction had been fair and helpful, but three of them felt that the researcher had been too positive. All of the teachers also felt that it had been helpful to cooperatively plan a unit of instruction and that the practical suggestions had been helpful. Recommendations for change included conducting this much earlier in the school year and cooperatively planning other units of instruction. One teacher wrote: "I wish I had the opportunity to plan other units of instruction in this manner. I found the discussion valuable in focusing my lesson on the unit objectives." The administrators were apparently not well informed about the feedback to the teachers, but they were very positive about the cooperative planning.

Initially, demonstration teaching by the researcher to illustrate some of the specific behaviors that promote student achievement had not been included as part of the study. When they were asked, the teachers all indicated that they would like to observe the researcher demonstrate some of the teaching behaviors that had been recommended for them. The evaluations also indicated the importance of this part of the project. All of the teachers felt that the demonstration teaching had made them feel more comfortable about trying new techniques that had been suggested. Their comments included statements like those below. "Your techniques with the manipulatives gave me new ideas for instruction." "This is definitely an important part of the procedure. Just as we provide a model for students to use, you provide me with a model to observe." The administrators also felt provide me with a model to observe." The administrators also felt that the demonstration lessons had been very helpful to their teachers.

Reactions to suggestions for teaching the planned unit were also positive, but several teachers would have liked more observations and more time for feedback and discussion after each lesson. The administrators were also positive; however, two of the four felt that the principal needs to be a part of this entire process so they can provide the day-to-day support and encouragement that the researcher was not able to provide. In this particular study, the researcher let each teacher decide how much information was provided to the principal. Apparently, in some cases, the principals were not as well informed as they would like to have been.

Overall, the teachers all indicated that the project had a positive effect on their teaching and had changed their teaching behavior. Four of the teachers specifically indicated an observance of effective time management procedures. Four of them wrote that they were now using manipulatives more. Two indicated an increased emphasis on review. One told how they now tried to verbalize the thinking that children can use more often. One noted the importance of being familiar with an appropriate teaching sequence. The responses indicated that all of them felt that inservice projects designed like this one, with observing, inservice, demonstrating and planning then observing again, could have a real affect on the teaching behaviors of the participants. Some of their comments are included below: "I spend more time on the lesson with the whole group. The assignments are shorter and the homework assignments are carefully planned and monitored. I plan to increase 'review time' and application discussion." "I increased the use of pictures and now include more manipulative models." "I have made greater use of manipulatives." "I sincerely feel it was one of the best learning experiences I have had since entering teaching. I wish I could do the same in other subjects. I now use the time schedule from the Missouri model (except for the homework) for my math class. I also have added more manipulatives and review." "It was one of the best long-term inservices I have been involved with. I am more aware of the need for modeling and sequencing math teaching." The administrators were just as positive. Three of them indicated that there were no administrative problems associated with the project. One said that finding substitute teachers was the only problem.

Both the teachers and the administrators indicated that the information and expertise that the participants gained could be shared with other teachers in the district through math meetings, grade level meetings and informal exchanges. Apparently there has already been a great deal of informal sharing. In addition to that, four of the teachers said they would be willing to teach demonstration classes for other teachers in their building if the grade level and the topic were both appropriate.

All of the teachers and administrators indicated that they were glad they participated in the project and would participate again if given the opportunity. Some additional suggestions were presented by the teachers. First, there should be time for discussing feedback after the observations. Second, the planning should be done for other topics and other subject matters as well. One of the administrators summarized it by saying, the "individual observations, conferences, substitute teaching and continued discussion worked very well to change behaviors. Without this close attachment, I don't think changes will occur."

The evidence that was collected to evaluate this project indicates that the teachers did change their behaviors in some ways. They began to provide a greater quantity of developmental instruction. They also improved the quality of their developmental instruction by using more models, fewer symbolic explanations, no longer using rule-example-practice lessons, attending to details, helping children make a better transition to the seatwork assignments and by using better questioning techniques to assess student understanding. The questionnaires also provided evidence that both the teachers and the administrators were convinced that the project had a positive effect on the instructional behaviors of these teachers.

DISCUSSION

In recent years there has been a growing concern about the quality of mathematics instruction in our schools. Several projects studying effective teaching have identified some teacher behaviors that do enhance student achievement. Many of the conclusions that have been drawn from these studies involve time schedules and effective management practices. Nearly all of the researchers also note the importance of providing quality developmental instruction; however, few specific recommendations had been made to help teachers provide meaningful presentations, demonstrations and to lead discussions that promote better understanding.

The results of this study indicate that teacher behaviors that affect the quantity and quality of instruction can be changed by using this inservice model. By noting patterns of teaching behaviors during the initial observations, the researcher was able to suggest specific changes in behavior for each individual teacher. The researcher suggested behaviors, such as, to check understanding of prerequisites, to use models to illustrate ideas and procedures, to relate the use of models to the symbol work in the assignments, to use appropriate numbers for examples, to prepare students for the transition to the seatwork and to check for understanding before assigning the seatwork. The teachers were able to change many of these behaviors and more effectively provide instruction for their students. Most of these changes involved using models more effectively and asking better questions to assess understanding of the students.

Also, by planning a topic jointly, the researcher was able to familiarize each teacher with appropriate models and language for that topic. The specificity of the inservice for a given topic and the specific suggestions for change were probably significant factors in enabling the teachers to change their behaviors. Unfortunately, due to scheduling difficulties, the researcher was not able to observe as many lessons after the inservice as had originally been planned. Not enough data was collected to confirm or deny the hypothesis that the teachers would be able to transfer the knowledge they had gained to other topics for which joint planning had not taken place. However, the results from the questionnaire indicate that the teachers did not have as much confidence in teaching another topic other than the one planned jointly. Since the models and language development are very often topic specific, it seems reasonable that a teacher who knows about effective instruction for one topic, may not know about effective instruction for another topic. Further study needs to be completed to determine the ability of teachers to transfer knowledge about effective instruction to various topics.

If teachers are unable to transfer knowledge about instruction for one topic to another, there are important implications for inservice. It will be important for teachers to become effective at teaching the key topics for their students. For example, it will be important for a second grade teacher to be able to provide effective instruction for addition and subtraction facts, numeration through three-digit numbers and two-digit addition and subtraction. Each mathematics teacher will have four or five similar key topics for which they must become proficient at providing instruction. This idea is not consistent with the general pattern of inservice that is now provided for teachers. Grade level meetings will replace many of the general inservice meetings that now often include teachers from kindergarten to grade eight. The use of video tapes to illustrate effective instruction for key topics at each grade level will perhaps become a necessity.

APPENDIX A

SAMPLE GRADE 6 FEEDBACK AFTER OBSERVATIONS

topic: problem solving (routine story problems) involving multiplication and division of decimals

context: this was a lesson near the end of the unit

mental arithmetic (2 minutes)

You started this lesson with some mental arithmetic. The problems were all chains of basic facts. The kids did quite well at this.

development (15 minutes)

The problems of this lesson involved figuring the amount of pay earned when there is overtime. You started this lesson with two good concrete examples. One of them involved walking beans for $3.50 per hour for 45 hours with time and a half for overtime. These examples did a nice job of relating the task to the real world that the kids know. On the second example you set up a chart to help them get the rate of pay for the overtime and the number of hours at regular pay and overtime pay. During these examples, decimals were consistently read "one point five" rather than one and five tenths. I could not tell whether or not the kids understood the place value or not. They may have, but without knowing their background I couldn't tell. Two or three times when an error was made, you asked if the answer was reasonable.

seatwork (40 minutes)

You then assigned a set of problems that involved payroll activities. Three minutes into the assignment you stopped them and discussed the meaning of the numbers in the charts that were presented. After that brief delay, you began monitoring like I have never seen before. You almost wore me out getting around to every one of the kids several times each. During this time you gave lots of positive reinforcement, much more than in most classes. When an error was made you often said things like: (1) I don't think that looks logical. (2) Double check your addition in this column. (3) Check the overtime pay. (4) Check your multiplication by 7. (5) I think you wrote that in the wrong spot on the chart. After fifteen minutes you gave another similar assignment. Some of the kids were making errors when multiplying. They were lining up the decimal points. You told them not to line up the decimal points, but to line up the numbers on the right. A couple of kids also made errors in placing the decimal point in multiplication problems. You told them to count the number of decimal places. Overall, the kids were on task almost all of the time and you were almost making more contacts with students than is humanly possible.

my comments

The class started well. The mental arithmetic was handled well. The only comment that I can make about this part of the lesson is that you might consider having them do some problems that go beyond basic facts. Problems that involve multiplication by 10 and using multiples of ten (4 x 70 and 37 + 40) are certainly appropriate for kids this age. This thinking could have been used to help with checking the reasonableness of results (see below). The development made use of two good concrete examples. I would have preferred that the decimals were read using place value names rather than one point five. That way I can get a better feeling about the understanding of the kids. You may already know that these kids understand, and if so, then what you did is fine. Several times during the lesson you asked about the reasonableness of an answer. At no time did I hear any example of how the kids could determine what is reasonable. When you ask if an answer is logical, it would probably help the kids more if you followed that by some mental computation as an estimate. For example, when the kids misplaced the decimal point in some of the problems, their answers were not even close. For example, how much is 40 hrs at $7 per hour? 7 x 4 is 28, so 7 x 4 tens is 28 tens or 280. Verbalizing that thinking will be helpful to the kids because it helps them see why an answer is not reasonable. The monitoring that you did when the kids were doing seatwork was incredible. You gave lots and lots of positive feedback. Overall, the class was on task and managed very well.

APPENDIX B

SAMPLE GRADE 4 FEEDBACK AFTER OBSERVATIONS

Topic: 2-digit multiplication

context: first lesson for this type of algorithm, just completed multiplication by multiples of ten

mental arithmetic (15 minutes)

You started this lesson with a series of mental arithmetic problems of the form (6 x 5) + 4. This was a good choice because they were going to have to do this same type of problem in the algorithm that was going to be taught. After presenting the problems orally and having the students write the answers, you corrected these problems. Then you did some other mental arithmetic problems that involved chains of operations. Again you had the students write the answers and then checked them.

check homework (5 minutes)

You had the answers to the first 6 problems on the board and spot checked others. I really like the way that you handle this part of the lesson. It is done very efficiently. You didn't process as many answers in this lesson as you did in the first one that I observed, but there may not have been much need to at this point in the unit.

development (4 minutes)

You did one example, 29 x 368. First, you talked through the procedure as you multiplied by 9. Then you noted the need to cross out the carry numbers. Then you put a zero in the ones place and multiplied by the 20. After getting the two partial products, you wrote 9 x 368 and 20 x 368 as two separate problems off to the side. The discussion that followed included the idea that by adding the answer to those two problems you would have multiplied 29 x 368. Then you added the partial products.

seatwork (19 minutes)

After you gave the assignment and the students had started work, you called each of them to the board to show you they knew the procedure. The monitoring that went on during this time included (1) asking about the rule for multiplying by 100, (2) asking about dropping the zero when you are multiplying by tens, (3) noting the importance of lining up the digits in the place value columns and (4) asking about the first thing that you do.

My comments:

The mental arithmetic that was used to start the class was good. It involved the same type of problems that the students needed to do the assignment. The next set of mental arithmetic problems were probably not needed in this lesson. They were ok, but they took time from your discussion about the new algorithm. They were also similar to the ones that you had already practiced. The development for this lesson was short. You only used one example. I'm sure you were thinking about the time and you wanted to get the kids started on the assignment and that affected your decision, but I think some more examples would have helped in this case. The reason that I state this is simply because several of the students made procedural errors in their work. Perhaps not so many corrections would have had to be made during the seatwork if a few more examples had been done earlier. The way you had each student show you how to do a problem during the seatwork was an excellent way to monitor their work. The students obviously enjoyed that and it worked quite well.

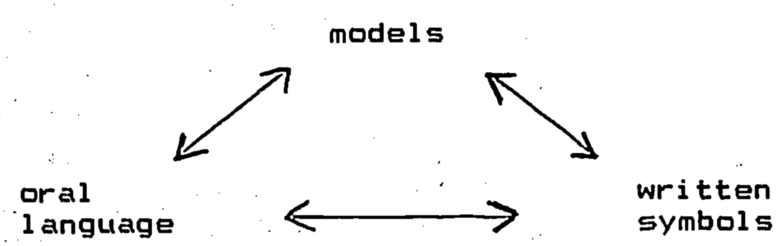
APPENDIX C

TEACHING MATHEMATICS CONCEPTS

Concept learning involves several different components. Included are (1) generating the idea, (2) recognizing instances of the idea, (3) representing the idea, (4) translating from one form of representation to another and (5) learning the properties of the specific concept.

In order to help children generate the idea for a specific concept, teachers need to present examples of the concept and identify them by using the name of the concept. It should be noted that the language that is used at this stage not only helps children learn the name, but also helps them focus on the particular attributes that are relevant characteristics in the recognition of the instances of the concept. Soon after children can recognize instances of the concept, counterexamples and examples need to be mixed so that children are forced to focus on only the relevant attributes. Teachers need to supplement the textbook in order to provide sufficient experience for children to recognize examples of a new concept.

Next, children need to learn to represent the concept using models, oral language and written symbols. Note that the use of models and the oral language have already been presented to the students as they were generating the idea and learning to recognize instances of the concept. Research indicates that children who can represent a concept with models and oral language and very fluently translate from one type of representation to the other are better prepared to learn to use written symbols as a means of representing the concept. Many of the tasks that children are asked to perform in the exercises in elementary mathematics textbooks while they are initially learning the concept involve these translations. The six translations that children need to master are shown in the diagram below.



It should be noted that children who work through textbooks without interaction with models and appropriate language are not learning all of the six translations. Teachers need to supplement the textbook to insure that children can fluently deal with all the different types of representations for a concept and the translations among them. Otherwise, the student's application of the concept to real world situations will be limited.

If there are different types of models that can be used to represent a concept, the children need to learn to use each different type. The purpose of studying these concepts is to be able to use them in the real world. That can not happen unless children recognize it. Since we can not predict how the children will encouter this concept in real world settings, we need to prepare them to recognize the concept in any form that it might appear. To facilitate learning, children should become familiar and develop competence with one model before another model is introduced. However, each of the different types of models needs to be included in the elementary curriculum at some stage.

In order to make effective instructional decisions, teachers need to know to what extent children understand a new concept. Teachers need to monitor the language and thinking that children are using as well as the written work in order to get adequate feedback for making these instructional decisions.

Finally, after children have learned to recognize and represent a new concept and they can easily make translations from one type of representation to the others, they need to begin learning the properties of that concept. They also need to learn which transformations can be applied to an instance of the concept without changing it and which transformations do change it. These ideas enable children to broaden their understanding of the concept and apply it in new and varied situations.

APPENDIX D

TEACHING MATHEMATICS ALGORITHMS

Helping children learn algorithms is an important component of the elementary school mathematics curriculum. The focus on learning an algorithm should be on the step-by-step procedure that is involved.

First, the teacher needs to determine which algorithm to help children learn for performing a particular skill. Often there are more than one to choose from. The decision should depend on factors such as prerequisites, difficulty, generalizability, ease of modeling, appropriateness of the thinking skills involved and, if the algorithm is written rather than mental, ease of recording the steps. After the algorithm is chosen, the thinking that children need to use with that algorithm should be identified and a model selected that can clearly illustrate that thinking.

The initial activities with the children should focus on using the model to illustrate the step-by-step procedure for the algorithm. At first this should be an oral-manipulative activity. No recording is necessary until the children can perform the algorithm by manipulating the model. Verbalization of the thinking that is used in the step-by-step procedure is crucial at this stage. It helps children focus on the steps involved and the children's language provides feedback to the teacher so better instructional decisions can be made.

For written algorithms, the next stage of learning involves recording the steps of the algorithm. At this point, the students already have learned the steps, can manipulate the model to show them and can verbalize the thinking that is being used at each step of the procedure. The written algorithm should simply become a written record of the steps that the students already know. It should be noted that it may take children a few examples each day for as much as two weeks before these ideas are meaningfully integrated into the children's cognitive structure. Even then, they will have to practice the procedure many times and it will need to be maintained on a regular schedule before the skill becomes mastered.

Elementary mathematics textbooks do not generally provide enough of the meaningful development that children need. First, the oral-manipulative activities are not usually done from the page in the book. The teacher will need to structure those activities. Also, the verbalization of the thinking must be part of the classroom environment. It does not occur in the textbook. Second, relating models to the written algorithm takes a considerable amount of space in the textbook. Publishers feel they need to have a reasonable number of computational exercises. A compromise between having a meaningful development and having enough practice exercises often eliminates most references to the model after the first two or three lessons. Unfortunately, most children need to see and hear the connection between what is written and the manipulation of the model for more days than that. The teacher needs to supplement each of the next few lessons with at least one or two examples where the children see the model again and hear the thinking verbalized. Regular practice in recording the algorithm is also needed during this same time period. After the first two weeks, distributed practice should help children maintain this newly learned skill. In order to help the students maintain the connection between the model manipulation and the written algorithm, it is helpful to occasionally ask them to explain their thinking as they perform the operation. Ideally, they should be able to refer to the manipulation of the model as they explain why they are recording digits in the algorithm.

There is another feature that needs attention immediately after children have learned a new algorithm. In order for the algorithm to be useful to them, they must learn when to apply it and when it is not appropriate. Teachers can help children learn this by providing discrimination activities where the purpose is not to derive the answer to the problem, but only to decide whether or not the algorithm is appropriate. Again, these activities are often not found in elementary school mathematics textbooks, so the teacher needs to supplement the text in order to provide this experience.

APPENDIX E

ADMINISTRATOR FEEDBACK

(The teacher feedback questionnaire was similar, but reworded to be appropriate for teachers.)

This project has been particularly rewarding to me. It is one of the first projects on effective teaching of mathematics where there has been a specific focus on what teachers do to help children understand. Your teachers have done some interesting activities in the classroom and their students have learned well. They have also made me give serious thought to what makes an activity meaningful. I have enjoyed observing their instruction. Thanks for letting me be a part of your school this year.

One of the conditions of the grant that I had to work with your teachers was to get some feedback from you about the effectiveness of the project. In addition to that, I would like you to respond to questions that may help others plan inservice and may help your teachers share some of the information they have gained with other teachers in your school next year. Thank you for taking the time to complete this form. I appreciate it. Please return this form within the next two weeks. A complete report will be written during the summer. It will be available for you to see next fall.

Ed Rathmell

Directions: There is no need to put your name on this response. Complete only those questions for which you have pertinent comments. If there is not enough room on the front of the sheet, please continue on the back. Please mail it back in the envelope that is attached.

Part I. Observations

A. How did you feel when I observed your math lessons?

B. Did you become more comfortable with the initial observations after the first time?

C. Did the observations affect how you taught your class or did you teach differently because you were being observed?

D. What advice would you give to a colleague who was about to be observed?

E. Please write any other comments that you have about how the observations were handled. Also give me any advice that you can about how an observer could be more sensitive during observations and how they might be more effective in using observations to help teachers. I am aware that too much time elapsed between the observations and the feedback. That was done because of some research conditions. Perhaps it was a mistake.

Part II. Group Inservice

F. What is your overall reaction to the effectiveness of the group inservice?

G. Was the information about research on effective teaching helpful?

H. Was the discussion about using models and verbalization to teach concepts and algorithms helpful?

I. Please give any comments or suggestions about how this inservice could have been more effective. Is there any other type of information that would have interested you? Was some of the information not particularly useful or interesting? Were you familiar with some of the information to the extent that it was a waste of your time?

Part III. Individual Feedback and Planning

J. Did you feel that the written comments you received about your teaching accurately described what happened?

K. Were the suggestions about your teaching fair and helpful?

L. Was it helpful for us to plan ahead for a unit of instruction?

M. Did you get enough practical suggestions to help you try new ideas during this portion of the inservice for it to be helpful?

N. Please give any comments or suggestions about how this portion of the inservice could have been more helpful to you.

Part IV. Demonstration Teaching and Follow-up Observations

O. Was it helpful for you to observe me teach a lesson?

P. Did I provide enough support and on-the-spot suggestions during this phase of the project?

Q. Please give any comments or suggestions about how this could be handled more effectively.

Part V. General Evaluation

R. Was the project helpful to you as a mathematics teacher?

S. Have you changed your mathematics instruction because of the project? If so, how?

T. Is this general procedure of (1) observing, (2) providing inservice, (3) planning together and (4) observing again a viable method of inservice that can have a real effect on what teachers do in the classroom?

U. Please give any other comments or suggestions about the feasibility of using an inservice plan similar to the one used in this project.

Part VI. Planning for Next Year

V. Would you feel comfortable sharing some of the information you have gained with other teachers in your building or district next year?

W. Would you feel comfortable teaching demonstration classes for other teachers in your building? Would the answer to this question depend on the topic or the grade level?

X. Please give any suggestions that you have about how you might share ideas with your colleagues next year.

Part VII. The Bottom Line

Y. Are you glad you participated in this project?

Z. Would you do this again if given the opportunity? A brief explanation would be helpful.

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