DOCUMENT RESUME

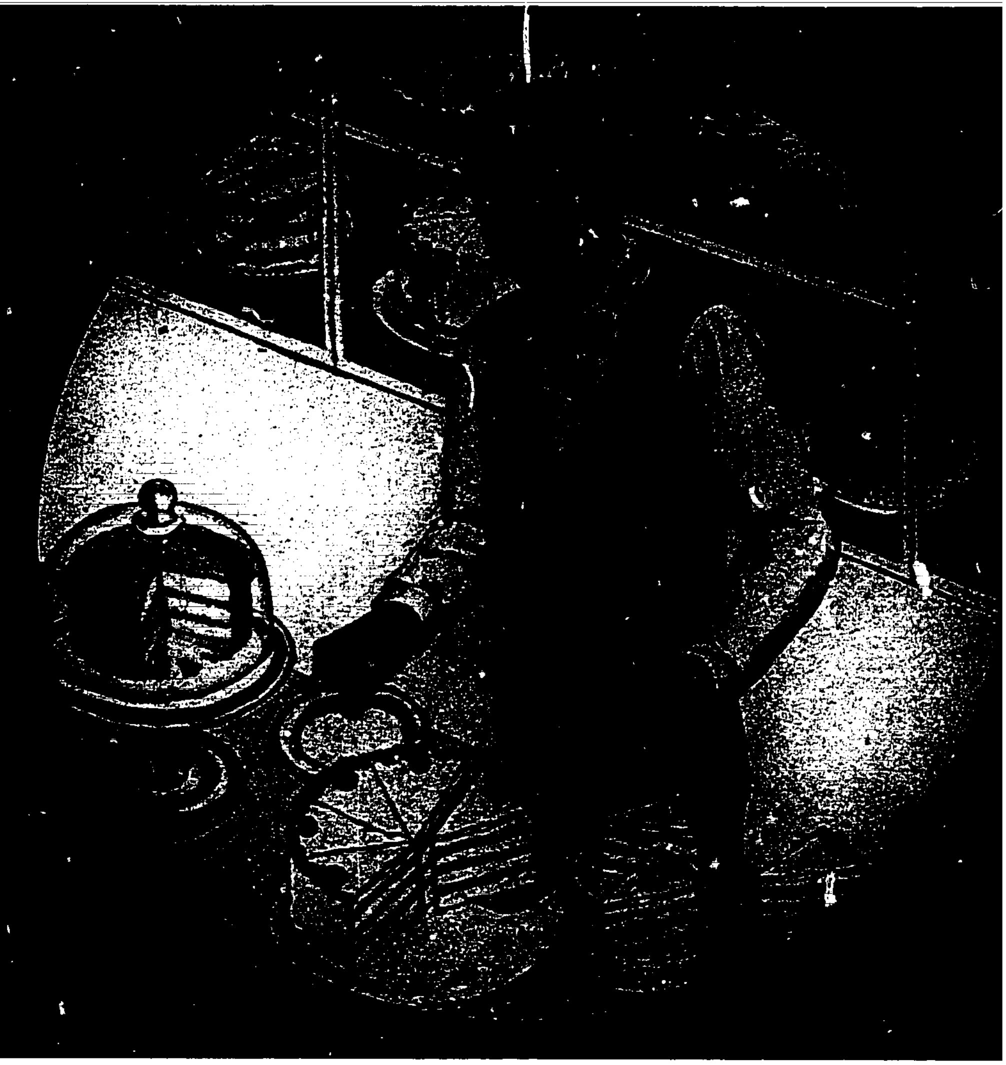
ED 276 581 SE 047 556

AUTHOR	Lock, Rose; And Others
TITLE	Simple Fractions. Math in Action. Workbook.
REPORT NO	ISBN-0-88102-044-3
PUB DATE	86
NOTE	67p.; For the teacher's guide, see SE 047 555.
AVAILABLE FROM	Janus Book Publishers, Inc., 2501 Industrial Parkway West, Hayward, CA 94545 ($4.65).
PUB TYPE	Guides - Classroom Use - Materials (For Learner) (051)
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	*Elementary School Mathematics; Elementary Secondary Education; *Fractions; *Instructional Materials; Low Achievement; Mathematics Curriculum; *Mathematics Instruction; Secondary School Students; *Slow Learners; Textbooks; Workbooks

ABSTRACT

This workbook is designed as an easy-to-read, slower-paced mathematics text for students who have learning, reading, and language problems. Helping students fulfill mathematics requirements for graduation is a goal; the book can be used as the core or supplement to the mathematics curriculum in mainstreamed or special education classrooms, in mathematics laboratories, or as part of sheltered workshop and vocational training programs. The eight units begin with a brief discussion of how fractions are used in real life. Each lesson focuses on only one major concept, and includes comprehension exercises with an answer key. Most lessons are reinforced with at least one worksheet, available in the teacher's guide. The units are titled: Fractions All Around You, Naming Fractions, Different Names for the Same Amount, Renaming Fractions, Comparing Fractions, Multiplying Fractions, Adding and Subtracting Fractions, and Dividing Fractions. A brief glossary is included. (MNS)

SIMPLE FRACTIONS



Math in Action: Simple Fractions

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2

Math in Action

SIMPLE FRACTIONS

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Typographer: Typothetae
Printer: WCP, Inc.

International Standard Book Number 0-88102-044-3

Printed in the United States of America.

6 7 8 9 0 1 2 3 4 5 D—P 0 9 8 7 6 5 4 3 2 1

Contents

Introduction

Where would we be without fractions?

Without fractions, runners couldn't break records. A runner's time is measured in fractions of a minute. Some races are won by a fraction of a second!

Without fractions, we wouldn't have the music we love to hear. In music, there are half notes, quarter notes, and eighth notes. Music is played in half time and three-quarter time. Music and fractions go together.

Without fractions, we couldn't measure amounts less than an inch or a pound. We couldn't buy $\frac{3}{4}$ inch tape or a $\frac{1}{2}$ pound hamburger.

No, life wouldn't be the same without fractions. We couldn't get along without them. And we can get along better when we know how to figure with them.

Some people think it's hard to figure with fractions. But figuring with fractions can be pretty easy. This book can show you how. It can help you learn to add, subtract, multiply, and divide fractions. You will also learn some other things about fractions that can help you pass the fraction part of math tests.

So if you're ready, let's begin.

Unit 1

Fractions All Around You

Suppose you are making a pizza for yourself and two friends. You will use fractions when you make the pizza and when you serve it.

The first time you will use fractions is when you make the crust. Look at the recipe below. Notice that the amounts for four of the ingredients are given in fractions. Which ingredients are they?

Pizza Crust

- 2 cups flour
- $\frac{1}{2}$ teaspoon salt
- $\frac{3}{4}$ teaspoon sugar
- $\frac{2}{3}$ cup water
- $\frac{1}{3}$ cup milk
- 2 tablespoons oil
- 1 package yeast

In the pizza crust recipe, the amounts for salt, sugar, water, and milk are all fractions. Recipes are full of fractions like that. To use recipes, you need to understand what the fractions mean.

The second time you will use fractions is when you slice the baked pizza. How can you make sure each person gets the *same* amount?

Right. You can cut the pizza into *equal* slices. You're using fractions when you do that too.

Fractions are parts of a whole thing, like the slices of a pizza. And they are parts of a whole group of things, like the cans in a six-pack of soft drinks.

Making and sharing pizzas is just one of many ways to use fractions. What are some other ways that you use fractions at school, work, and home?

Math Words

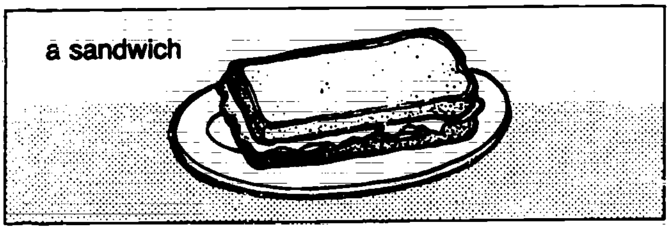
Look up these words in the glossary at the back of the book. Find out what they mean.

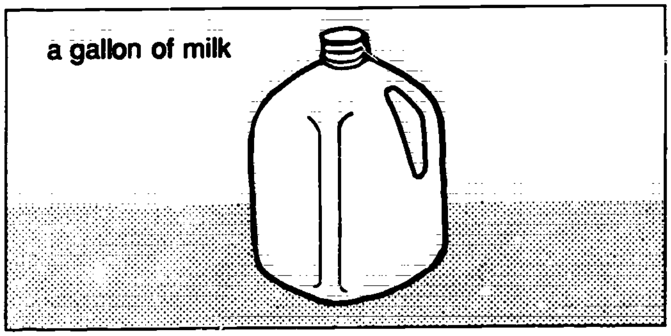
equal **fraction** **unequal**

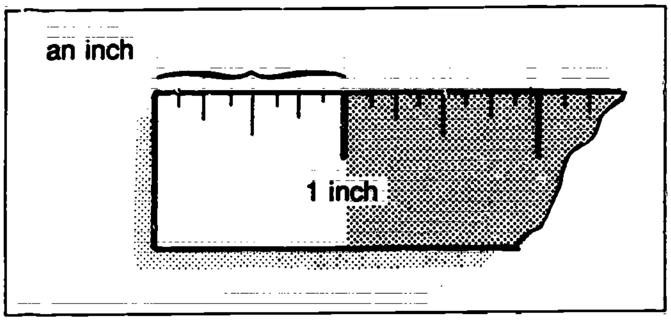
Whole Things and Whole Groups

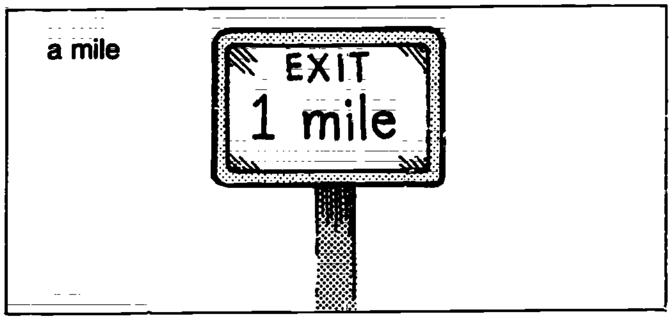
We say a fraction is a part of a whole thing or whole group. But what do we mean by *whole?* Something that is whole has no part missing.

One *whole thing* can be:


a sandwich


a gallon of milk

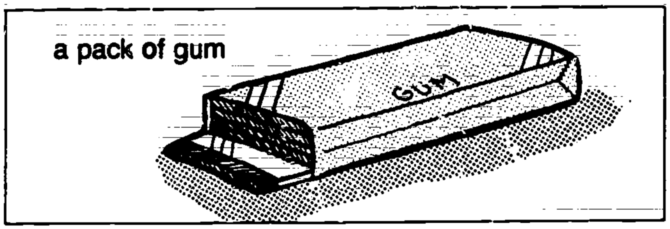

an inch

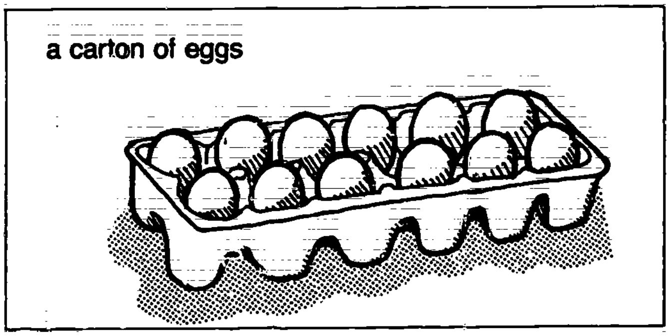

a mile

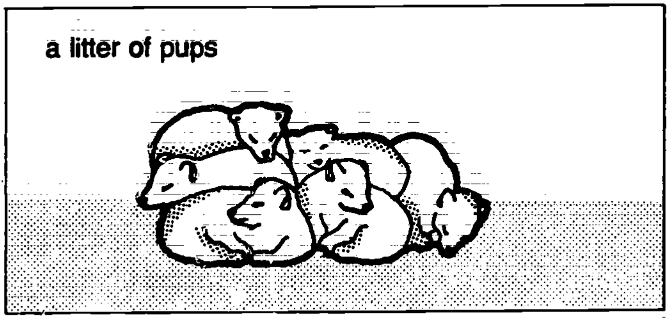
No part is missing from the sandwich, gallon of milk, inch, or mile, so each is an example of a *whole thing.*

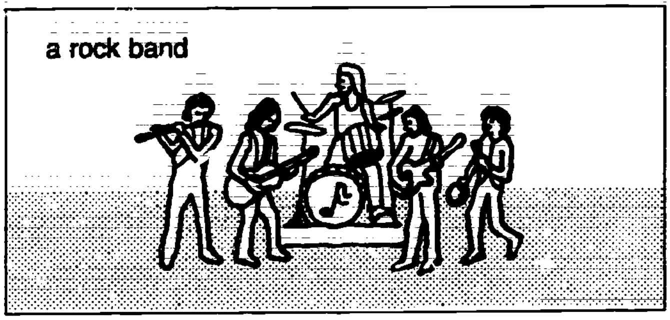
What other examples of whole things can you think of?

One *whole group* can be:


a pack of gum


a carton of eggs


a litter of pups


a rock band

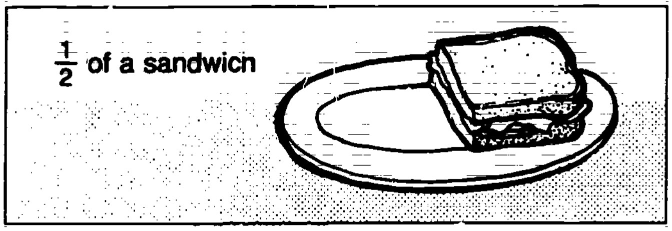
Nothing or no one is missing from the pack, carton, litter, or band, so each is an example of a *whole group.*

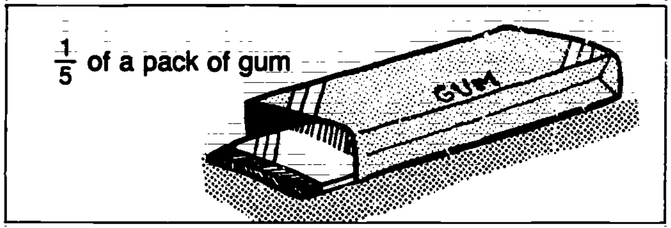
What other examples of whole groups can you think of?

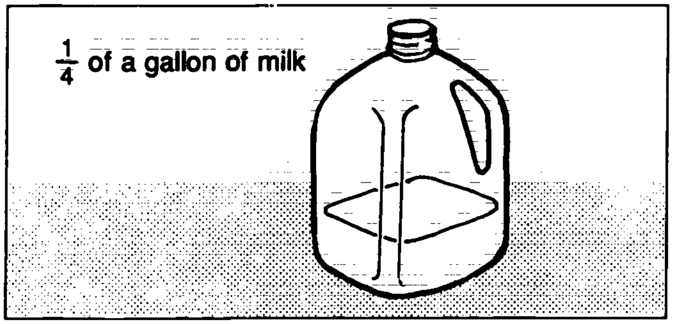
What's a Fraction?

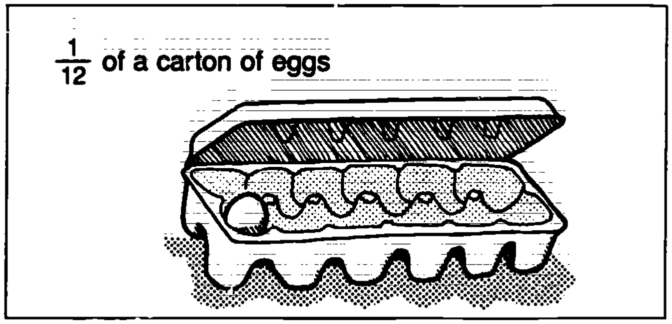
Now you know what a whole thing and a whole group are. But what's a **fraction?**

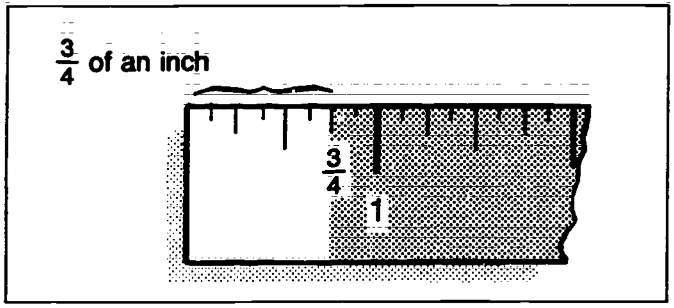
A fraction is a *part of a whole thing or a whole group*. For example, these are fractions:

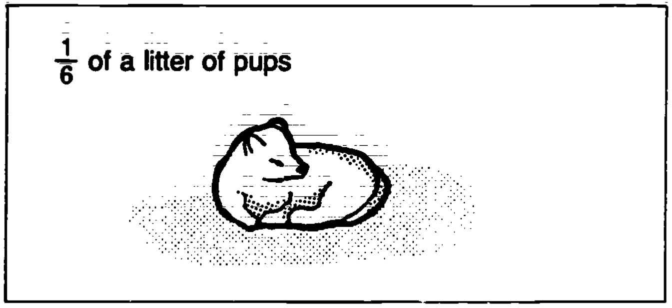

$\frac{1}{2}$ of a sandwich

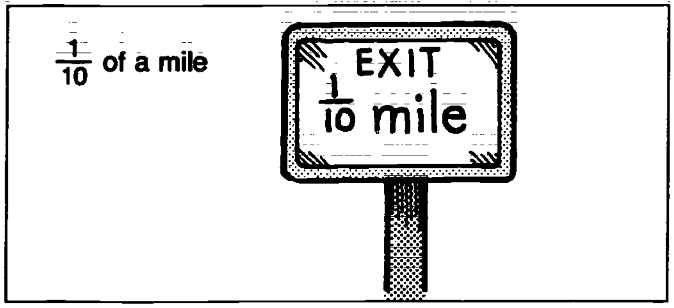

$\frac{1}{5}$ of a pack of gum

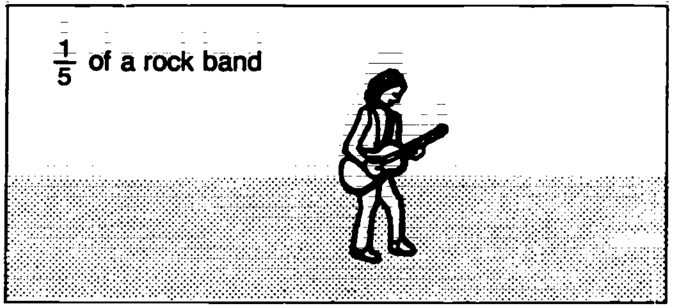

$\frac{1}{4}$ of a gallon of milk


$\frac{1}{12}$ of a carton of eggs


$\frac{3}{4}$ of an inch


$\frac{1}{6}$ of a litter of pups


$\frac{1}{10}$ of a mile


$\frac{1}{5}$ of a rock band

Today, you probably used fractions of several things. For example, you may have used a fraction of a tube of toothpaste, a pound of butter, or a dollar.

What else have you used a fraction of today?

Equal Parts of Whole Things

Suppose you cut a pizza into three parts. Each part is a different size: one is very large, one is very small, and one is between those two sizes.

Is each of those parts $\frac{1}{3}$ (one third) of the whole pizza?

The answer is *no*. A part can be $\frac{1}{3}$ of the whole pizza only if all the parts are the same size. Parts that are *not* the same size are **unequal**. They are *not* the same amounts. So, each part of that pizza cannot possibly be $\frac{1}{3}$ of the whole amount.

When all the parts are the same size, each is exactly the same amount. Each is *equal* to the other parts. And each is an *equal fraction*.

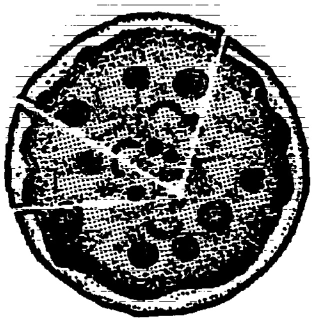
You'll sometimes be asked to draw a picture showing a whole thing divided into equal fractions. Remember: Each part (each fraction of the whole) must be the *same size* as the other parts.

Now look at the pictures of the four pizzas. All the pizzas are divided into three parts. But one pizza is divided into equal parts. And three pizzas are divided into unequal parts.

Which pizza is divided into $\frac{1}{3}$ parts?

Which pizzas are *not* divided into $\frac{1}{3}$ parts?

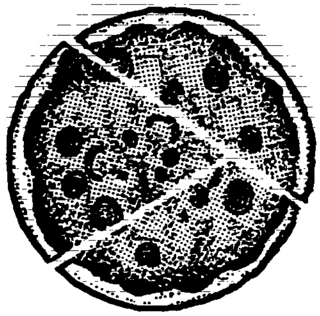
Make your choice. Then check your answers. The right answers are printed upside down.



Pizza 1

Pizza 2

Pizza 3



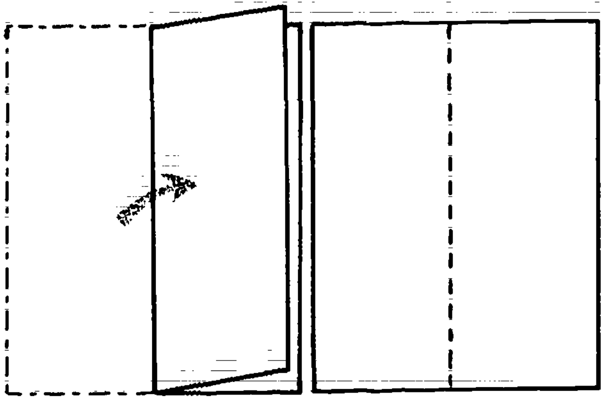
Pizza 4

Answers: Pizza 3 is divided into equal parts.
Pizzas 1, 2, and 4 are divided into unequal parts.

Make Some Equal Parts

You can divide a whole thing into 2 or more equal parts. Each of those equal parts will be an equal fraction of the whole thing. To see what we mean, do this:

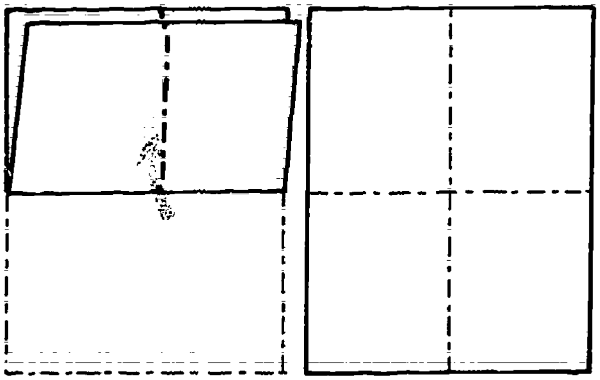
1. Get a piece of paper.
2. Fold the paper *exactly* in half, making sure all the edges line up. Press down the folded edge with your finger.
3. Open the paper.



Fold side to side.

You now have 2 equal parts, right?

4. Fold the paper *exactly* in half again.
5. Fold it *exactly* in half one more time.
6. Open the paper.



Fold bottom to top.

You now have 4 equal parts. Each part is a fraction of the whole paper.

Equal Parts of a Whole Group

You learned that you can divide a whole thing into equal fractions. You can also divide a whole group into equal fractions.

Remember: A whole group is made up of single things. Any number of single things can make up a whole group. For example: A **pair** of shoes (**2** shoes) is a whole group. A box of **10** pencils is a whole group. A class of **30** students is a whole group.

Now think of a volleyball team. It is made up of 6 people. Those 6 people make up 6 equal parts of that whole group. Each person is $\frac{1}{6}$ of the whole group.

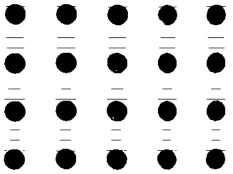
Think of a baseball team. There are 9 people on a baseball team. Each person is $\frac{1}{9}$ of that whole group.

Look at the picture of the pack of juice. Each can is 1 part of a whole group of six cans. Suppose you drink 1 can of juice from that pack. What fraction of that pack do you drink?

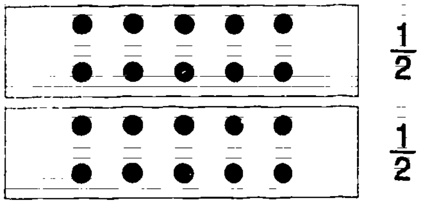
Answer: You drink $\frac{1}{6}$ of the pack.

Dividing a Whole Group into Equal Parts

You can divide one whole group into smaller equal groups. For example, suppose you're part of a group of 20 people at a picnic. The dots below stand for all the people in the group.



After lunch, you all decide to have a tug-of-war. You want 2 equal teams. What fraction of the group will be on each team?



Right. $\frac{1}{2}$ (one half) of the group will be on one team. The other $\frac{1}{2}$ will be on the other team.

How many players will be on each team?

One way to find the answer is to count the number of dots in each box.

Another way is to divide 20 by 2.

$$\begin{array}{r}10\\2\overline{)20}\end{array}$$

Either way, there will be 10 players on each team.

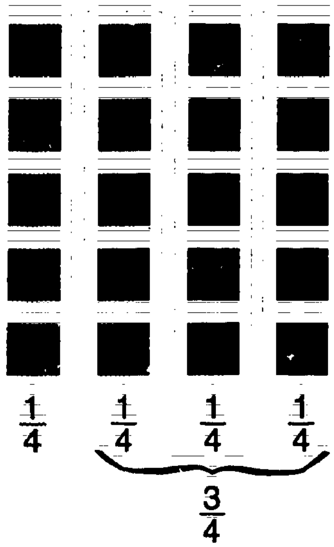
Here's another example of dividing whole groups into smaller equal groups.

Suppose you work as a stock clerk in a supermarket. A shipment of 20 cases of canned peaches arrives.

Your boss wants you to stack $\frac{3}{4}$ (three fourths) of the shipment in a display at the front of the store. She wants you to put the rest of the shipment on the shelves.

How will you divide the shipment?

Whole Shipment



Since you have to divide the shipment into fourths to get $\frac{3}{4}$, you will divide the cases into 4 equal groups.

How many cases will you put in each group?

$$4\overline{)20}$$

Right. Each group gets 5 cases.

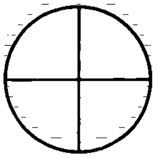
Now you can separate $\frac{3}{4}$ of the shipment for the display at the front of the store. What fraction of the shipment will go on the shelves?

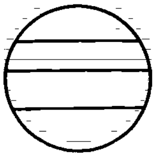
Right. $\frac{1}{4}$ (one fourth) of the shipment will go on the shelves.

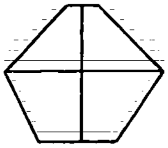
Unit Review

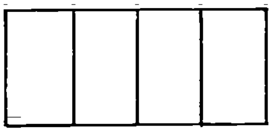
Check Yourself

1. Look at the drawings below. Decide if the parts in each drawing are equal or unequal. Copy the letters on notepaper. Then write **equal** or **unequal** after each letter.

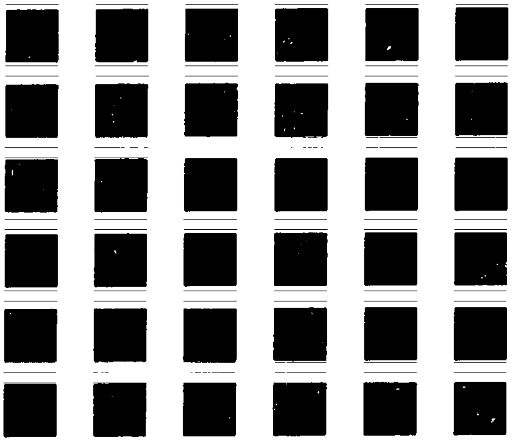
a.


b.


c.


d.


2. Trace or draw this group of boxes. Circle as many smaller *equal* groups as you can.



3. Choose the right word to complete each sentence below.

amount **equal** **group** **part**

a. A fraction is a ________ of a whole thing or whole group.
b. $\frac{1}{2}$ and $\frac{1}{2}$ are ________ fractions.
c. Parts of a *whole thing* that have the same size and shape also have the same ________.
d. A whole ________ is made up of several things.

Bonus Work

1. Fold a piece of paper exactly in half as many times as you can. Then open the paper. Count the number of equal parts. Tell what fraction of the whole paper 1 part is.
2. Draw 24 dots on a piece of paper. Arrange them in rows like the ones on page 10. Then divide the dots into as many smaller equal groups as you can.
3. Look in newspapers and magazines for pictures of:
   - whole things and fractions of whole things;
   - whole groups and fractions of whole groups.

   Make a poster with the pictures to show the difference between fractions and whole things or whole groups.

Unit 2

Naming Fractions

Sale!

$\frac{1}{2}$ OFF
all tapes

$\frac{1}{3}$ OFF
all records

Real Pro Wrench Kits

Each kit contains these sizes:

$\frac{3}{16}$ $\frac{7}{32}$ $\frac{1}{4}$ $\frac{9}{32}$ $\frac{5}{16}$ $\frac{11}{32}$

$\frac{3}{8}$ $\frac{7}{16}$ $\frac{1}{2}$

Can you read
and say all
these fractions?

FAST TAXI
Rates

$1.00 first $\frac{1}{9}$ mile
10¢ each additional $\frac{1}{9}$ mile

Best service in town!

Fractions! Fractions! Fractions! Look around. You will probably see and hear fractions everywhere.

- A store has a sale offering $\frac{1}{2}$ off the usual price.
- You feel OK when your gas tank is $\frac{1}{2}$ full. You start to worry when it gets below the $\frac{1}{4}$ mark.
- You are at the deli. You hear someone ask for $\frac{3}{4}$ (three quarters) of a pound of ham and $\frac{1}{8}$ (one eighth) of a pound of cheese.
- You and your three friends eat out. You divide the bill 4 ways. Each person pays $\frac{1}{4}$ of the bill.
- You fix a bike with a $\frac{3}{8}$ inch wrench.

It's really helpful to know how to read and say the fractions you see everywhere. For many things you do, it's also helpful to know how to **name** fractions. When you name a fraction, you say what numbers are in that fraction. Cooks, carpenters, clothing makers, repair people, and other workers have to name fractions all the time.

By the end of this unit, you'll be naming fractions. Meanwhile, test yourself with the fractions above. Read and say each one.

Math Words

Look up these words in the glossary. What do they mean?

denominator **numerator**
name **terms**

Denominators and Numerators

Suppose there are 8 equal slices in a pizza. You eat 3 of those slices. You'd be eating a fraction of the whole pizza. To write that fraction, you'd do this:

- First, write the number of *all* the slices in the pizza (the *total* number of slices):

$$8$$

- Next, draw a bar (a line) over the number:

$$\overline{8}$$

- Then, count the slices you eat. Write that number over the bar.

$$\frac{3}{8}$$

All fractions are written in that way: They have a bottom number, a bar, and a top number.

The bottom number stands for all the equal parts in a whole thing or whole group. That number is called the **denominator.**

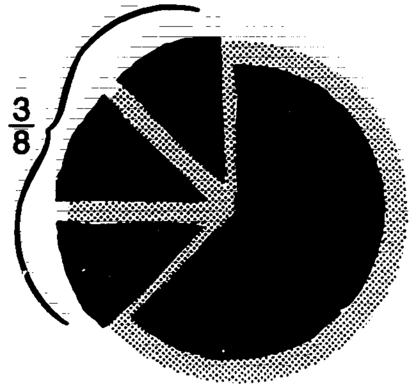
The top number stands for the parts that you are counting. That number is called the **numerator.**

Now suppose you are 1 member of a baseball team. You are $\frac{1}{9}$ of that team.

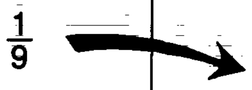
Which number in that fraction is the denominator?

Which number in that fraction is the numerator?

Answers: The numerator is 1.
The denominator is 9.



$$\frac{3}{8} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{equal parts you count}}{\text{equal parts in all}}$$



Today's Batting Order

1 You
2 Garcia
3 Brown
4 Wong
5 Tallchief
6 Nasser
7 Stallone
8 Abrams
9 Thomas

Each person is $\frac{1}{9}$ of the whole group.

Exercise

On a sheet of paper write these fractions. Then check your answers.

1. denominator 10, numerator 3
2. numerator 2, denominator 5
3. denominator 6, numerator 1
4. numerator 4, denominator 9
5. numerator 10, denominator 11
6. denominator 8, numerator 7

Answers: 1. $\frac{3}{10}$ 2. $\frac{2}{5}$ 3. $\frac{1}{6}$
4. $\frac{4}{9}$ 5. $\frac{10}{11}$ 6. $\frac{7}{8}$

Reading and Saying Fractions

Suppose you are in an auto shop. You need some $\frac{7}{16}$ inch bolts. How will you say $\frac{7}{16}$ when you ask for the bolts?

If you ask for "seven sixteen" or "seven over sixteen," you are not saying fractions correctly. You must say the denominator—*sixteen*—a certain way.

Look at the fractions below. Each is a fraction for one equal part of something. Say the fraction out loud. How do you say the denominator?

Read:	**Say:**
$\frac{1}{2}$	one half
$\frac{1}{3}$	one third
$\frac{1}{4}$	one fourth (or one quarter)
$\frac{1}{5}$	one fifth

Read:	**Say:**
$\frac{1}{6}$	one sixth
$\frac{1}{7}$	one seventh
$\frac{1}{8}$	one eighth
$\frac{1}{9}$	one ninth
$\frac{1}{10}$	one tenth

Read:	**Say:**
$\frac{1}{11}$	one eleventh
$\frac{1}{12}$	one twelfth
$\frac{1}{13}$	one thirteenth
$\frac{1}{14}$	one fourteenth
$\frac{1}{15}$	one fifteenth

Read:	**Say:**
$\frac{1}{16}$	one sixteenth
$\frac{1}{17}$	one seventeenth
$\frac{1}{18}$	one eighteenth
$\frac{1}{19}$	one nineteenth
$\frac{1}{20}$	one twentieth

Suppose the numerator is a number that is *more* than 1. For example: $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{7}{16}$. Fractions with numerators like that are *plurals*—they show more than one equal part. So, use their plural forms when you say them.

Look at these plural fractions. Say them out loud.

Read:	**Say:**
$\frac{2}{2}$	two halves
$\frac{2}{3}$	two thirds
$\frac{3}{4}$	three fourths (or three quarters)
	and so on

Read:	**Say:**
$\frac{3}{12}$	three twelfths
$\frac{5}{15}$	five fifteenths
$\frac{7}{19}$	seven nineteenths
	and so on

Now, let's go back to the $\frac{7}{16}$ inch bolts. How would you say that fraction?

Answer: $\frac{7}{16}$ is seven sixteenths.

Exercise

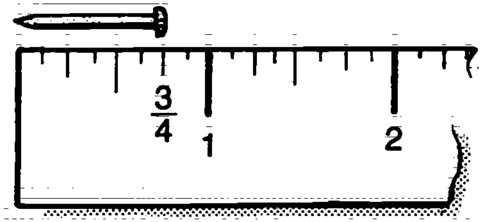
On a sheet of paper, write two rules that tell:

1. How to say fractions for one equal part, starting with $\frac{1}{2}$.
2. How to turn those fractions into plurals.

Naming the Terms

Let's say you are measuring the nail shown here. The nail is less than an inch long. So, it is a fraction of an inch long. To write that fraction, you must decide what number is the denominator. And you must decide what number is the numerator.

(Remember: The denominator is the bottom number of a fraction. The numerator is the top number.)



The numbers that make up the denominator and numerator of a fraction are called the **terms** of that fraction. When you decide what those numbers are, you are *naming the terms of the fraction.*

Let's name the terms of the fraction that describes the nail. Look at the ruler under the nail. Notice this: an inch can be divided into 4 equal parts. Now look at the nail. It is as long as 3 of those 4 parts. So the nail is $\frac{3}{4}$ of an inch long. The number **3** (the numerator) and the number **4** (the denominator) are the terms of that fraction.

Remember:

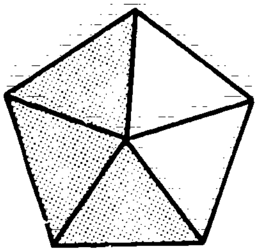
To name the terms of any fraction, do this:

1. Find the number of all the equal parts in the whole thing or whole group. That number is the denominator.
2. Find the number of the parts that you are counting. That number is the numerator.

Look at the shape below. How many parts are there altogether? That number becomes the denominator.

How many parts are shaded? That number becomes the numerator.

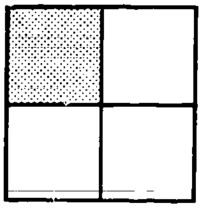
$\frac{3}{5}$



Exercise

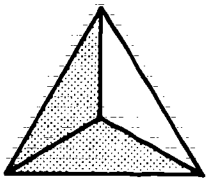
Now name fractions for the shaded parts of these shapes. Then check your answers.

1.



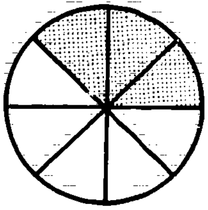
$\frac{[illegible]}{4}$

2.



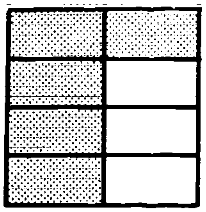
[illegible]

3.



$\frac{3}{[illegible]}$

4.



[illegible]

Answers: 1. $\frac{1}{4}$ 2. $\frac{2}{3}$ 3. $\frac{3}{8}$ 4. $\frac{5}{8}$

Fractions in Word Problems

Sometimes you must find the terms of a fraction in word problems. Here's an example of such a problem.

> Cora has $5. She lends $2 to Kim. What fraction of her money does she lend to her friend?

What is the denominator?

What is the numerator?

The fraction is $\frac{2}{5}$. The 5 stands for the number of dollars that Cora has to start with. It is the *whole amount*. The 2 stands for the number of dollars she lends to her friend. It is the number of the parts you count.

Naming the terms of fractions in word problems is like naming the terms of fractions shown in pictures.

1. Find the total number of equal parts that are in a whole amount. That number is your denominator.
2. Find the number of equal parts you are counting. That number is your numerator.

Exercise

Name the terms of the fractions for the word problems below. Then check your answers.

1. Kim, Martin, and Cora have lunch at a restaurant. The 3 friends decide to divide the bill equally. What fraction of the bill will 1 person pay?

   1 person will pay $\frac{\blacksquare}{3}$ of the bill.

2. Kim ran 2 miles on a trail that is 3 miles long. What fraction of the trail did she run?

   Kim ran $\frac{2}{\blacksquare}$ of the trail.

3. Cora makes 4 out of 7 free throws. What fraction of her free throws does she make?

   Cora makes $\frac{\blacksquare}{\blacksquare}$ of her free throws.

4. Martin is at bat 5 times. He gets 3 hits. What fraction of his times at bat does Martin get hits?

   Martin gets hits $\frac{\blacksquare}{\blacksquare}$ of his times at bat.

Answers: 1. $\frac{1}{3}$ 2. $\frac{2}{3}$ 3. $\frac{4}{7}$ 4. $\frac{3}{5}$

Unit Review

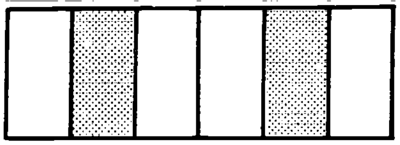
Check Yourself

1. Copy each fraction. Next to the fraction, write the way to say it—for example: $\frac{2}{4}$ two fourths.

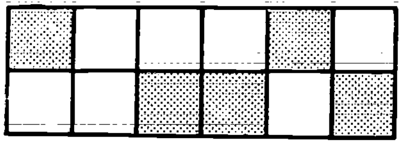
   a. $\frac{2}{4}$
   b. $\frac{6}{8}$
   c. $\frac{1}{2}$
   d. $\frac{2}{2}$
   e. $\frac{3}{9}$
   f. $\frac{1}{16}$
   g. $\frac{3}{15}$
   h. $\frac{6}{17}$
   i. $\frac{1}{20}$
   j. $\frac{7}{20}$
   k. $\frac{4}{5}$
   l. $\frac{8}{11}$

2. Name fractions for the shaded parts of the shapes below.

   a.



   b.



3. Name the terms of the fractions in these word problems.

   a. You have 5 yards of cloth. You use 2 yards to make a pillow. What fraction of the whole cloth do you use?

   You use $\frac{■}{■}$ of the whole cloth.

   b. There are 10 questions on a test. You answer 9 of them correctly. What fraction of the questions do you answer correctly?

   You answer $\frac{■}{■}$ of the questions correctly.

4. Choose the right words to complete these sentences about fractions.

   bottom
   denominator
   numerator
   terms
   top

   a. The number that tells how many equal parts there are in the whole amount is the ________. It is the number on the ________ of the fraction.

   b. The number that tells how many equal parts you are counting is the ________. It is the number on the ________ of the fraction.

   c. The numbers in a fraction are called its ________.

Bonus Work

1. Figure out how to say these fractions:

   a. $\frac{2}{21}$
   b. $\frac{5}{32}$
   c. $\frac{7}{55}$
   d. $\frac{20}{100}$

2. Make a poster about fractions. On a large piece of paper, write some fractions. Label the numerators and the denominators. Tell what each term stands for. Add pictures that help explain the fractions.
3. Draw or trace some shapes. Then divide them into equal parts. Be sure the parts in each shape are the same size. Shade some of the parts. Ask your classmates to name fractions for each shape.
4. Write some word problems like those on this page. Ask your classma to solve them.

Unit 3

Different Names for the Same Amount

Suppose you want to buy a snack from a machine. The snack costs 50¢. You can put only quarters, dimes, or nickels into the machine.

All you have is a half dollar. You must get change for that half dollar. How many quarters can you get for a half dollar? How many dimes? How many nickels? Here's what a half dollar ($\frac{1}{2}$ dollar) is the same as:

$\frac{1}{2}$ dollar = **2 quarters**
$\frac{1}{2}$ dollar = **5 dimes**
$\frac{1}{2}$ dollar = **10 nickels**

Let's turn all those amounts into fractions:

1 **half dollar** is $\frac{1}{2}$ of a dollar
($1 = 2 halves)
2 **quarters** are $\frac{2}{4}$ of a dollar
($1 = 4 quarters)
5 **dimes** are $\frac{5}{10}$ of a dollar
($1 = 10 dimes)
10 **nickels** are $\frac{10}{20}$ of a dollar
($1 = 20 nickels)

2 quarters, 5 dimes, and 10 nickels are all equal to a same amount. They are different names for a same amount.

$\frac{1}{2}$, $\frac{2}{4}$, $\frac{5}{10}$, and $\frac{10}{20}$ all equal a same amount, too. They are also different names for a same amount.

Different numbers, such as those fractions, that equal the same amount are called **equivalents.** They are equal to each other.

When you figure with fractions, you often have to use equivalents for a same amount—for example, $\frac{2}{4}$ instead of $\frac{1}{2}$. In this unit, you'll learn about equivalents.

Math Words

Look up these words in the glossary. What do they mean?

equivalents
improper fraction
mixed number
proper fraction
whole number

Fractions That Equal 1

You learned how to write fractions that stand for *parts* of 1 whole amount. You can also write fractions that stand for *1 whole amount*.

Think of 4 quarters. Those quarters make up $1. So, each quarter is an equal part of $1.

4 quarters = $1

If you count just 1 of those quarters, you are counting 1 quarter of 4 quarters. In other words: $\frac{1}{4}$ of $1.

If you count 2 of those quarters, you are counting 2 quarters of 4 quarters. In other words: $\frac{2}{4}$ of $1.

If you count 3 of those quarters, you are counting 3 quarters of 4 quarters. In other words: $\frac{3}{4}$ of $1.

And if you count 4 of those quarters, you are counting 4 quarters of 4 quarters. In other words: $\frac{4}{4}$ of $1.

You know that 4 quarters are the same as 1 whole amount—$1. Why is $\frac{4}{4}$ also the same as 1 whole amount?

Notice this about the fraction $\frac{4}{4}$: its numerator is the same as its denominator. That means you are counting *all* the parts that make up 1 whole amount. So:

$\frac{4}{4}$ = 1 and 1 = $\frac{4}{4}$ (in this case)

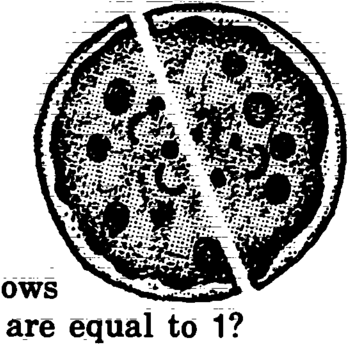
A Rule to Remember

Here's a rule to remember when you are writing fractions equal to 1:
☞ When the numerator of a fraction is the same as its denominator, that fraction is equal to 1.

Exercise

Suppose you have 1 whole pizza.

1. You first cut the whole pizza into 2 equal parts.

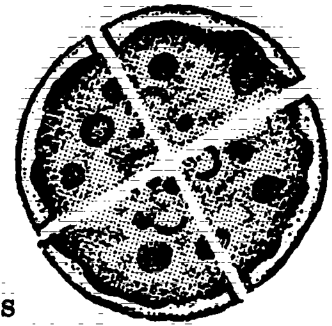


What fraction shows how those parts are equal to 1?

2 halves = 1 whole pizza

$\frac{\blacksquare}{\blacksquare}$ = 1 and 1 = $\frac{\blacksquare}{\blacksquare}$

2. You next cut the 2 pieces into 4 equal parts.

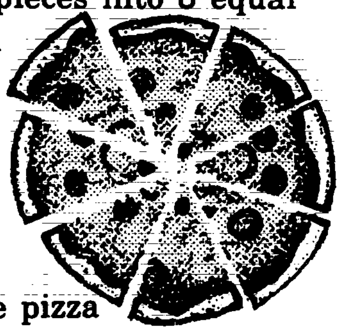


What fraction shows how those parts are equal to 1?

4 fourths = 1 whole pizza

$\frac{\blacksquare}{\blacksquare}$ = 1 and 1 = $\frac{\blacksquare}{\blacksquare}$

3. You then cut the 4 pieces into 8 equal parts. What fraction shows how those parts are equal to 1?



8 eighths = 1 whole pizza

$\frac{\blacksquare}{\blacksquare}$ = 1 and 1 = $\frac{\blacksquare}{\blacksquare}$

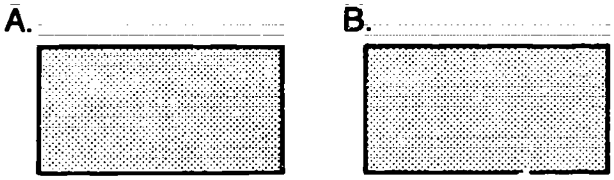
Answers: 1. $\frac{2}{2}$ = 1 and 1 = $\frac{2}{2}$ 2. $\frac{4}{4}$ = 1 and 1 = $\frac{4}{4}$ 3. $\frac{8}{8}$ = 1 and 1 = $\frac{8}{8}$

Fractions That Equal Many Whole Things

You learned how to write fractions that are equivalent to 1 whole thing. You can also write fractions that are equivalent to many whole things.

When a number stands for a whole thing or many equal whole things it is called a **whole number.** The numbers 1, 10, 100, and 1,000 are examples of whole numbers. 1 stands for one whole thing; 10 stands for ten equal whole things; 100 stands for one hundred equal whole things; and 1,000 stands for one thousand equal whole things.

The picture shows two equal whole things, box A and box B. Box A is exactly the same size as box B, so they both stand for the same amount.



You learned that the denominator of a fraction stands for the total number of parts in *one whole thing*. How many parts make up any one of the boxes? The number is 1, so 1 is the denominator.

$$\frac{\ }{1}$$

You also learned that the numerator stands for the parts you are counting. When we talk about two whole things we are counting 2 parts. So 2 is the numerator.

$$\frac{2}{1}$$

The fraction $\frac{2}{1}$ is the equivalent of the whole number 2. In other words, $\frac{2}{1}$ is the same as 2 whole things that are each made up of 1 part. So:

$$\frac{2}{1} = 2 \text{ and } 2 = \frac{2}{1}$$

A Rule to Remember

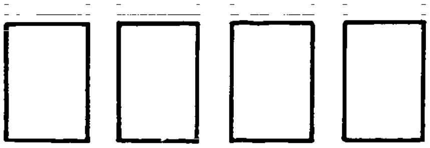
Here's a rule to remember when you write a whole number as a fraction:

- write 1 as the denominator;
- write the amount of the whole number as the numerator.

Exercise

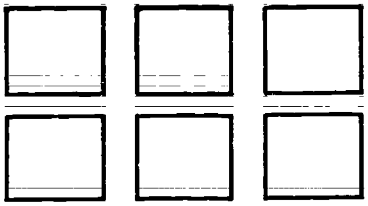
1. Write fractions that stand for these shapes. Then check your answers.

a.



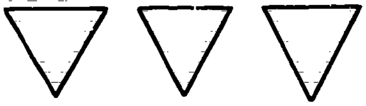
= $\frac{\square}{\square}$

b.



= $\frac{\square}{\square}$

c.



= $\frac{\square}{\square}$

2. Write fractions that stand for these whole numbers.

a. 9 = $\frac{\square}{\square}$

b. 24 = $\frac{\square}{\square}$

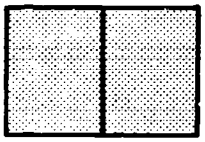
c. 126 = $\frac{\square}{\square}$

Answers:
1. a. $\frac{4}{1}$ b. $\frac{6}{1}$ c. $\frac{3}{1}$
2. a. $\frac{9}{1}$ b. $\frac{24}{1}$ c. $\frac{126}{1}$

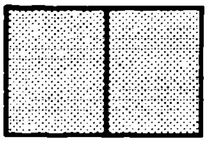
Fractions That Equal More Than Whole Things

Look at the shapes below. They show this amount: 2 equal whole things plus part of another equal whole thing. To write that amount you'd write a **mixed number**—a number that is made up of a whole number and a fraction. What mixed number is that?

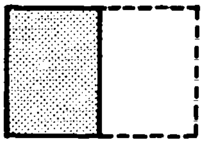
A.



B.



C.



The mixed number is $2\frac{1}{2}$. (You'd say: *two and a half*, or *two and one half.*)

You can write a fraction that is equal to that mixed number. Look at box A or B. You learned that the denominator of a fraction stands for the total number of parts in *one whole thing*. How many parts are in any one whole thing? That number is the denominator.

$\frac{}{2}$

Now look at the parts that are shaded in boxes A, B, and C. Those are the parts you are counting. How many parts are shaded? That number is the numerator.

$\frac{5}{2}$

You can see that the fraction $\frac{5}{2}$ is equivalent to $2\frac{1}{2}$. In other words, $2\frac{1}{2}$ is the same as 5 parts of whole things that are made up of 2 total parts.

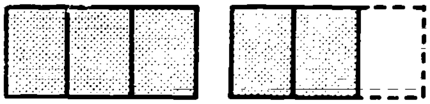
So:

$$2\frac{1}{2} = \frac{5}{2} \text{ and } \frac{5}{2} = 2\frac{1}{2}$$

Exercise

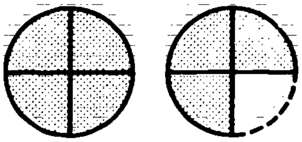
Write mixed numbers for these shapes. Then write fractions that are equivalents to those mixed numbers. Check your answers.

1.

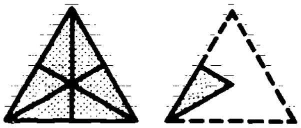


(mixed number) = = (fraction)

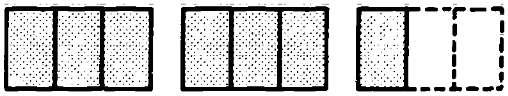
2.



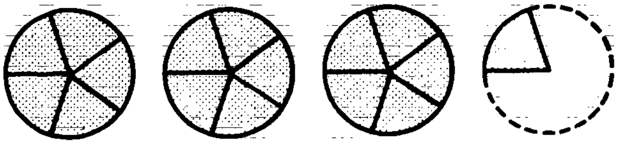
3.



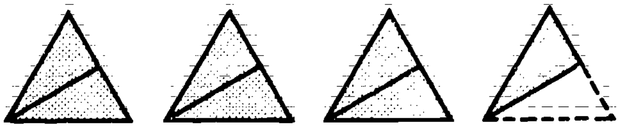
4.



5.



6.

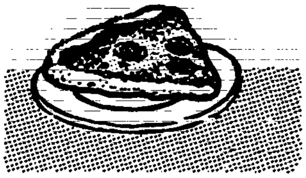


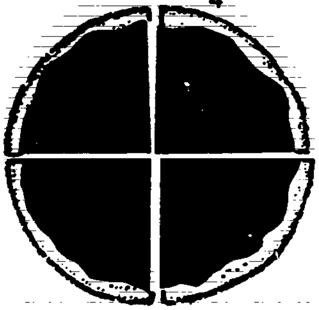
Answers: 1. $1\frac{2}{3} = \frac{5}{3}$ 2. $1\frac{3}{4} = \frac{7}{4}$ 3. $1\frac{1}{6} = \frac{7}{6}$

4. $2\frac{1}{3} = \frac{7}{3}$ 5. $3\frac{1}{5} = \frac{16}{5}$ 6. $3\frac{1}{2} = \frac{7}{2}$

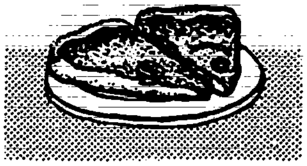
Fractions That Equal the Same Amount

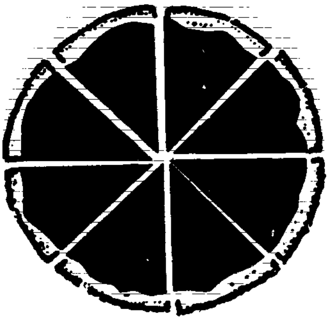
Suppose you are dividing a pizza equally among four people. You can cut the pizza into 4 equal slices and give each person 1 large slice. That's $\frac{1}{4}$ of the whole pizza.



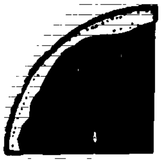


Or you can divide the pizza like this: Cut the pizza into 8 equal slices and give each person 2 smaller slices. That's $\frac{2}{8}$ of the whole pizza.

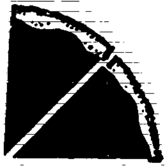




Either way, each person gets the same amount. That's because $\frac{1}{4}$ and $\frac{2}{8}$ equal the same amount:



$\frac{1}{4} = \frac{2}{8}$



$\frac{1}{4}$ and $\frac{2}{8}$ have different terms but they equal the same amount. They are *equivalent fractions*. You can name another equivalent fraction for $\frac{1}{4}$ and $\frac{2}{8}$. Do this: Divide *each* $\frac{1}{8}$ slice in half. Then count the new number of *total* slices in the pizza. That number is the denominator.

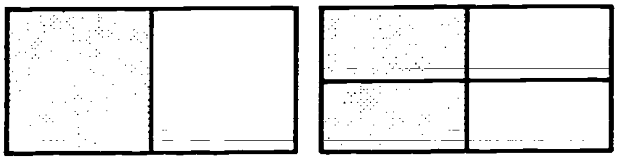
Next count the number of slices in a part that's the same size as $\frac{1}{4}$ of the pizza. That number is the numerator. What is the other equivalent of $\frac{1}{4}$ and $\frac{2}{8}$?

Answer: The other equivalent is $\frac{4}{16}$.

Exercise

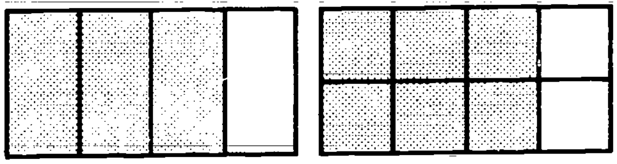
Find the equivalents shown in these pictures. To do that: Count the total number of parts in each box to find the denominator. Then count the number of shaded parts to find the numerator.

1.



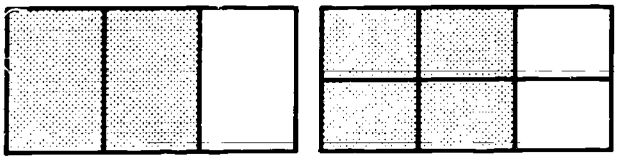
$\frac{1}{2} = \frac{\blacksquare}{4}$

2.



$\frac{\blacksquare}{4} = \frac{6}{\blacksquare}$

3.



$\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

Answers:

1. $\frac{1}{2} = \frac{2}{4}$ 2. $\frac{3}{4} = \frac{6}{8}$ 3. $\frac{2}{3} = \frac{4}{6}$

Names to Know

So far, you've seen numbers like these:

$\frac{1}{2}$ 1 $\frac{2}{2}$ $\frac{2}{1}$ $1\frac{1}{2}$

Those kinds of numbers have certain names. Get to know their names. You'll be hearing and reading those names when you solve fraction problems.

Proper fractions are fractions that have a small numerator and a large denominator. Examples are:

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

What are some other examples? Complete these numbers to make proper fractions:

$\frac{\blacksquare}{3}$ $\frac{\blacksquare}{5}$ $\frac{3}{\blacksquare}$ $\frac{6}{\blacksquare}$

Improper fractions are two kinds of fractions:

- fractions that have the same numerator and denominator. For example:

  $\frac{1}{1}$ $\frac{2}{2}$ $\frac{3}{3}$ $\frac{4}{4}$

- fractions that have a large numerator and a small denominator. For example:

  $\frac{2}{1}$ $\frac{4}{3}$ $\frac{5}{2}$ $\frac{5}{4}$

What are some other examples of improper fractions? Complete these numbers to make improper fractions:

$\frac{\blacksquare}{2}$ $\frac{3}{\blacksquare}$ $\frac{\blacksquare}{6}$ $\frac{1}{\blacksquare}$

Whole numbers are numbers that stand for a whole thing or several equal whole things. For example:

1 2 15 25 300

What are some other examples?

Mixed numbers are numbers that are made up of whole numbers and fractions. For example:

$1\frac{1}{3}$ $2\frac{1}{2}$ $11\frac{3}{4}$ $54\frac{1}{4}$

What are some other examples? Complete the numbers below to make mixed numbers:

$\blacksquare\frac{1}{7}$ $3\frac{\blacksquare}{2}$ $\blacksquare\frac{1}{\blacksquare}$ $5\frac{2}{\blacksquare}$

Exercise

Number a sheet of paper. Then write the answers to these questions.

1. Which of these numbers are proper fractions?

   a. 2 **b.** $\frac{2}{1}$ **c.** $\frac{3}{4}$
   d. $\frac{6}{15}$ **e.** $\frac{21}{22}$ **f.** $3\frac{1}{6}$

2. Which of these are improper fractions?

   a. $\frac{3}{3}$ **b.** $1\frac{1}{12}$ **c.** $\frac{5}{2}$
   d. $\frac{33}{32}$ **e.** $7\frac{1}{8}$ **f.** $\frac{8}{1}$

3. Which of these are whole numbers?

   a. $2\frac{1}{3}$ **b.** 1 **c.** $\frac{20}{9}$
   d. 10 **e.** 24 **f.** 8

4. Which of these are mixed numbers?

   a. $1\frac{1}{2}$ **b.** 125 **c.** $2\frac{1}{3}$
   d. $4\frac{1}{4}$ **e.** $\frac{12}{8}$ **f.** $15\frac{1}{6}$

Answers:
1. c. $\frac{3}{4}$ d. $\frac{6}{15}$ e. $\frac{21}{22}$
2. a. $\frac{3}{3}$ c. $\frac{5}{2}$ d. $\frac{33}{32}$ f. $\frac{8}{1}$
3. b. 1 d. 10 e. 24 f. 8
4. a. $1\frac{1}{2}$ c. $2\frac{1}{3}$ d. $4\frac{1}{4}$ f. $15\frac{1}{6}$

Unit Review

Check Yourself

1. Copy each problem. Fill in the missing numbers to make all the fractions equal to 1.

a. $\frac{2}{\blacksquare} = 1$

b. $\frac{\blacksquare}{4} = 1$

c. $\frac{8}{\blacksquare} = 1$

d. $\frac{\blacksquare}{15} = 1$

e. $\frac{27}{\blacksquare} = 1$

f. $\frac{\blacksquare}{32} = 1$

2. Copy the whole numbers. Then rename them as equivalent fractions.

a. $3 = \frac{\blacksquare}{\blacksquare}$

b. $10 = \frac{\blacksquare}{\blacksquare}$

c. $7 = \frac{\blacksquare}{\blacksquare}$

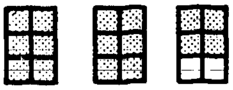
d. $25 = \frac{\blacksquare}{\blacksquare}$

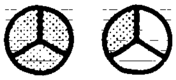
e. $469 = \frac{\blacksquare}{\blacksquare}$

f. $8 = \frac{\blacksquare}{\blacksquare}$

3. Write a mixed number for each shape. Then rename that mixed number as an improper fraction.

a.  $= \blacksquare\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

b.  $= \blacksquare\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

c.  $= \blacksquare\frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

4. Write a proper fraction for each shape. Then reduce the fraction to its lowest terms.

a.  $= \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

b.  $= \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

c.  $= \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

5. Choose the right word to complete each sentence.

equivalents **proper**
improper **whole**
mixed

a. The numbers 2, 5, 10 and 125 are all ________ numbers.

b. Fractions that stand for the same amount are called ________.

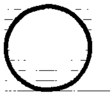
c. A ________ fraction has a numerator that is smaller than its denominator.

d. A ________ number is made up of a whole amount and a fraction.

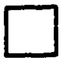
e. ________ fractions have denominators that are smaller than their numerators.

Bonus Work

Copy each fraction and shape below. Divide the shape into the right number of equal parts. Then shade some of those parts to show the fraction.

1. $\frac{1}{3} =$ 

2. $\frac{2}{7} =$ 

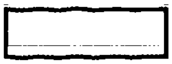
3. $\frac{4}{6} =$ 

4. $\frac{7}{8} =$ 

5. $\frac{1}{2} =$ 

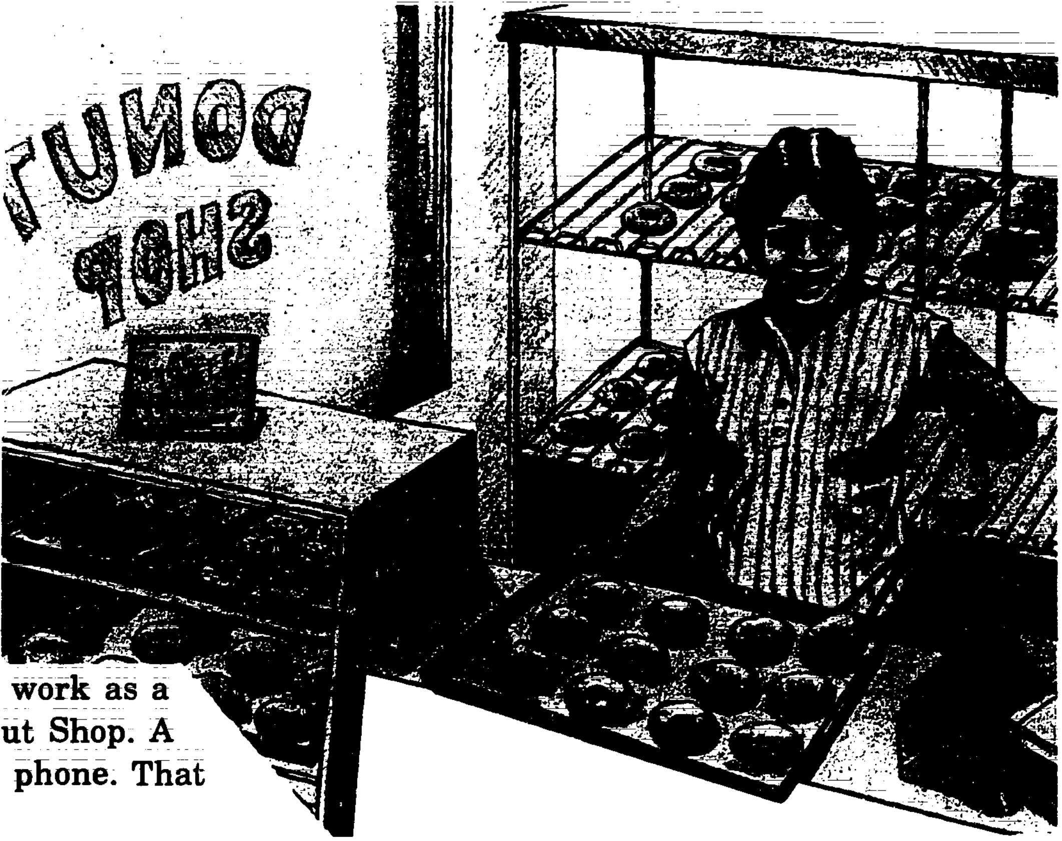
6. $\frac{4}{4} =$ 

7. $\frac{1}{1} =$ 

8. $\frac{9}{10} =$ 

Unit 4

Renaming Fractions



Let's say that you work as a salesclerk in the Donut Shop. A customer calls on the phone. That person wants:

$\frac{1}{4}$ dozen raised doughnuts
$\frac{1}{2}$ dozen lemon doughnuts
$\frac{3}{4}$ dozen crumb doughnuts

You know that 12 is a dozen and 6 is $\frac{1}{2}$ dozen. But how many doughnuts are in $\frac{1}{4}$ dozen? How many are in $\frac{3}{4}$ dozen? You can use equivalents to figure that out.

The customer wants amounts that are parts of a dozen. So you count out 12 doughnuts. Next you divide the 12 doughnuts into 4 equal groups, the way they are shown in the picture.

Now you can see how many doughnuts are in $\frac{1}{4}$ dozen and $\frac{3}{4}$ dozen and you can fill your order.

How many are in $\frac{1}{4}$ dozen? (Count the doughnuts in 1 group.)

How many are in $\frac{3}{4}$ dozen? (Count the doughnuts in 3 groups.)

Now you can fill your order:

$\frac{1}{4}$ dozen = 3 doughnuts (of 12 doughnuts)
$\frac{1}{2}$ dozen = 6 doughnuts (of 12 doughnuts)
$\frac{3}{4}$ dozen = 9 doughnuts (of 12 doughnuts)

You are using equivalents to help you do your job:

$\frac{1}{4} = \frac{3}{12} \quad \frac{1}{2} = \frac{6}{12} \quad \frac{3}{4} = \frac{9}{12}$

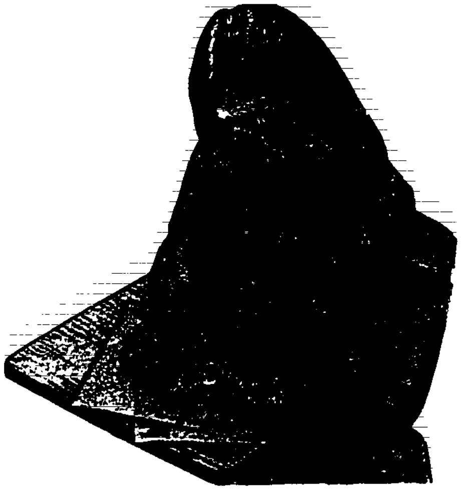
Notice this: You can turn each set around and the fractions still are equivalents. Why is that so?

$\frac{3}{12} = \frac{1}{4} \quad \frac{6}{12} = \frac{1}{2} \quad \frac{9}{12} = \frac{3}{4}$

Math Words

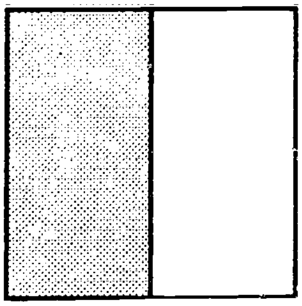
Look up these words in the glossary. What do they mean?

factors
raising terms
reducing terms
rename
simplest form
simplify

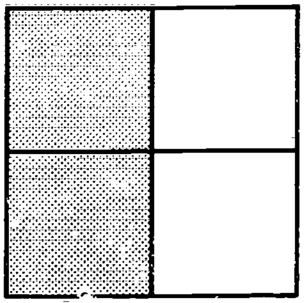


Lower and Higher Terms

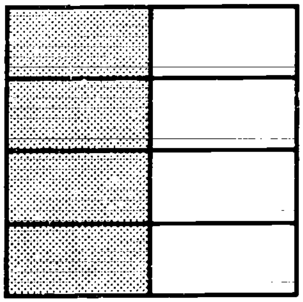
Sometimes we must change a fraction to an equivalent. We do that by **renaming** its terms. We change the terms so they are *lower* or *higher*. (Remember: the terms of a fraction are the numbers in its numerator and denominator.)



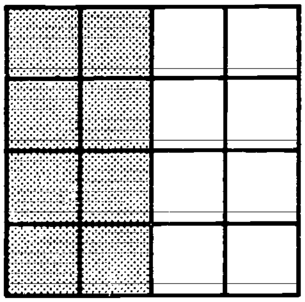
A. $\frac{1}{2}$



B. $\frac{2}{4}$



C. $\frac{4}{8}$



D. $\frac{8}{16}$

Look at the boxes. They are all the same size. And $\frac{1}{2}$ of each box is shaded. Notice this: Each box is made up of a different number of equal parts.

The shaded parts in box A show 1 part of 2 parts. In other words: $\frac{1}{2}$ of a whole. The shaded parts in box B show 2 parts of 4 parts. In other words: $\frac{2}{4}$ of a whole. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalents.

$$\frac{1}{2} = \frac{2}{4}$$

Look at the numerators of $\frac{1}{2}$ and $\frac{2}{4}$: 2 is higher than 1. Look at the denominators of $\frac{1}{2}$ and $\frac{2}{4}$: 4 is higher than 2. So, the terms of $\frac{2}{4}$ are *higher* than the terms of $\frac{1}{2}$. And the terms of $\frac{1}{2}$ are *lower* than the terms of $\frac{2}{4}$.

Now look at box C. It is divided into 8 equal parts. 4 of those parts are shaded. What fraction shows that the shaded parts are equivalent to $\frac{1}{2}$?

$$\frac{1}{2} = \frac{2}{4} = \frac{■}{8}$$

Look at box D. It is divided into 16 equal parts. What fraction shows that the shaded parts are equivalent to $\frac{1}{2}$?

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{■}{■}$$

The fractions are all equivalents. They are all equal to $\frac{1}{2}$ of a whole amount. Notice this: Each equivalent after $\frac{1}{2}$ has higher terms than the equivalent before it.

Exercise

On a sheet of paper, write these sets. In each set, draw a circle around the equivalent that has higher terms.

1. $\frac{8}{16}$ and $\frac{1}{2}$
2. $\frac{3}{12}$ and $\frac{1}{4}$
3. $\frac{2}{8}$ and $\frac{4}{16}$
4. $\frac{2}{10}$ and $\frac{4}{20}$
5. $\frac{1}{3}$ and $\frac{4}{12}$
6. $\frac{2}{12}$ and $\frac{1}{6}$

Answers:

1. $\frac{8}{16}$ 2. $\frac{3}{12}$ 3. $\frac{4}{16}$ 4. $\frac{4}{20}$ 5. $\frac{4}{12}$ 6. $\frac{2}{12}$

Renaming Fractions to Higher Terms

Suppose you need to rename a fraction as an equivalent. And that equivalent must have higher terms. To do that, you must **raise the terms** of the fraction. In other words, you must change its numerator and denominator to higher terms.

Look at these equivalents:

$$\frac{1}{3} = \frac{4}{12}$$

$\frac{4}{12}$ is an equivalent you can get by raising the terms of $\frac{1}{3}$. What do you think you do to raise those terms?

You raise those terms by multiplying the numerator (1) and the denominator (3) by the *same whole number*. What whole number is it?

$$\frac{1}{3} = \frac{4}{12} \quad \text{or} \quad \frac{1}{3} = \frac{4}{12}$$

You raise $\frac{1}{3}$ to $\frac{4}{12}$ by multiplying its terms by 4. But you can raise the terms of a fraction by using any whole number—as long as you use the same number when you multiply.

A Rule to Remember

Here's a rule to remember when you must find an equivalent with higher terms:

Multiply the numerator of the fraction by a whole number; then multiply its denominator by the same whole number.

Exercise

A. Finish finding these equivalents with higher terms.

1. $\frac{2}{5} = \frac{4}{\ }$
2. $\frac{1}{8} = \frac{\ }{24}$
3. $\frac{4}{5} = \frac{\ }{\ }$
4. $\frac{2}{3} = \frac{\ }{\ }$
5. $\frac{1}{9} = \frac{\ }{\ }$
6. $\frac{3}{8} = \frac{\ }{\ }$

Answers: 1. $\frac{4}{10}$ 2. $\frac{3}{24}$ 3. $\frac{20}{25}$ 4. $\frac{8}{12}$ 5. $\frac{2}{18}$ 6. $\frac{18}{48}$

B. Find equivalents with higher terms for these fractions. (Use any whole number.)

1. $\frac{3}{4} = \frac{\ }{\ }$
2. $\frac{1}{6} = \frac{\ }{\ }$
3. $\frac{1}{4} = \frac{\ }{\ }$
4. $\frac{1}{2} = \frac{\ }{\ }$
5. $\frac{3}{5} = \frac{\ }{\ }$
6. $\frac{2}{7} = \frac{\ }{\ }$

Raising to the Next Highest Terms

Suppose you multiply the terms of a fraction by the whole numbers 2, 3, 4, 5, and so on. (Those are numbers that get higher by 1.) What happens to the terms each time you multiply them? Notice what happens to the terms of $\frac{2}{5}$:

$\frac{2}{5}$ $\frac{2 \times 2}{5 \times 2} = \frac{4}{10}$ $\frac{2 \times 3}{5 \times 3} = \frac{6}{15}$ $\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$ . . . (and so on)

With the fraction $\frac{2}{5}$, the term **2** gets higher as if you are counting by twos (2, 4, 6, 8 . . .). And the term **5** gets higher as if you are counting by fives (5, 10, 15, 20 . . .). If you keep counting, the terms will continue to get higher in those ways.

Fractions such as $\frac{2}{5}$, $\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$, and so on make up a group of equivalent fractions. Each fraction in the group is equal to the fraction with the lowest terms—$\frac{2}{5}$. And each fraction is raised to the next highest terms.

Suppose you are raising the fraction $\frac{1}{3}$ to its next highest terms. What would the terms be in this group of equivalent fractions?

$\frac{1}{3} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$ . . .

To check your answer, look at this table. It shows how $\frac{1}{3}$ changes to equivalent fractions with the next highest terms.

		×1	×2	×3	×4	×5	×6	×7	×8	×9	×10	
Numerator	1	1	2	3	4	5	6	7	8	9	10	. . .
Denominator	3	3	6	9	12	15	18	21	24	27	30	

Exercise

Copy each problem. Fill in the missing numbers. Then check your answers.

1. $\frac{2}{3} = \frac{4}{\blacksquare} = \frac{\blacksquare}{9} = \frac{8}{\blacksquare}$
2. $\frac{3}{4} = \frac{\blacksquare}{8} = \frac{9}{\blacksquare} = \frac{\blacksquare}{16}$
3. $\frac{1}{6} = \frac{2}{\blacksquare} = \frac{\blacksquare}{18} = \frac{4}{\blacksquare}$
4. $\frac{2}{5} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$
5. $\frac{1}{7} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$
6. $\frac{1}{8} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

Answers:
1. $\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$
2. $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$
3. $\frac{1}{6} = \frac{2}{12} = \frac{3}{18} = \frac{4}{24}$
4. $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$
5. $\frac{1}{7} = \frac{2}{14} = \frac{3}{21} = \frac{4}{28}$
6. $\frac{1}{8} = \frac{2}{16} = \frac{3}{24} = \frac{4}{32}$

Finding the Missing Term

Let's say you must change a fraction to a higher equivalent. You know what one of the higher terms is. But you must find the other. For example:

$$\frac{3}{4} = \frac{■}{20}$$

In that problem, the denominator of the equivalent fraction is *given*. But the numerator is missing. To solve that problem you'd do this:

- First, divide **4** (the denominator) into **20** (the given denominator).

$$\frac{3}{4} = \frac{■}{20}$$

Math Work

$$\begin{array}{r} 5 \\ 4\overline{)20} \end{array}$$

- Then multiply **3** (the numerator) by **5** (the answer you got).

$$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$$

Now, look at this problem: the numerator of the equivalent is given; the denominator is missing. You'd find the missing denominator the same way you'd find a missing numerator.

$$\frac{2}{3} = \frac{8}{■}$$

- First, divide **2** (the numerator) into **8** (the given numerator).

$$\frac{2}{3} = \frac{8}{■}$$

Math Work

$$\begin{array}{r} 4 \\ 2\overline{)8} \end{array}$$

- Then, multiply the **3** (the denominator) by the **4** (the answer you got).

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{■}$$

What's the answer?

Answer: $\frac{2}{3} = \frac{8}{12}$

Exercise

Find the missing terms for these equivalents. Then check your answers.

1. $\frac{2}{5} = \frac{■}{20}$
2. $\frac{3}{4} = \frac{■}{16}$
3. $\frac{4}{5} = \frac{■}{30}$
4. $\frac{1}{7} = \frac{■}{21}$
5. $\frac{4}{4} = \frac{12}{■}$
6. $\frac{5}{6} = \frac{10}{■}$
7. $\frac{2}{7} = \frac{14}{■}$
8. $\frac{3}{4} = \frac{18}{■}$

Answers: 1. $\frac{2}{5} = \frac{8}{20}$ 2. $\frac{3}{4} = \frac{12}{16}$ 3. $\frac{4}{5} = \frac{24}{30}$ 4. $\frac{1}{7} = \frac{3}{21}$ 5. $\frac{4}{4} = \frac{12}{12}$ 6. $\frac{5}{6} = \frac{10}{12}$ 7. $\frac{2}{7} = \frac{14}{49}$ 8. $\frac{3}{4} = \frac{18}{24}$

Reducing to the Lowest Terms

You'll often be told to *reduce a fraction to its lowest terms.*

When you **reduce** a fraction, you change its numerator and denominator to lower terms You do that by dividing both terms evenly by the *same* whole number—or **factor.**

Factors are numbers you multiply to get an amount. For example:

$2 \times 2 = 4 \qquad 2 \times 4 = 8$

2 and 2 are factors of 4. 2 and 4 are factors of 8.

Suppose 4 and 8 are terms of the fraction $\frac{4}{8}$. What happens when you divide both terms by the factor 2?

$$\frac{4}{8} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{2}{4}$$

You reduce $\frac{4}{8}$ to $\frac{2}{4}$ when you divide both terms by 2. Notice this: You can also reduce $\frac{2}{4}$ to lower terms by dividing its terms by 2.

$$\frac{4}{8} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{2}{4} \qquad \frac{2}{4} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{1}{2}$$

Can you reduce $\frac{1}{2}$ any further?

$\frac{1}{2}$ *cannot* be reduced any further. It is at its lowest terms.

$\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$ (lowest terms)

Exercise

1. Copy these fractions. Then reduce them to their lowest terms.

   a. $\frac{6}{12}$ b. $\frac{4}{14}$ c. $\frac{9}{12}$ d. $\frac{8}{10}$

   Answers: a. $\frac{1}{2}$ b. $\frac{2}{7}$ c. $\frac{3}{4}$ d. $\frac{4}{5}$

2. Write numerators to make these fractions with lowest terms.

   a. $\frac{■}{2}$ b. $\frac{■}{8}$ c. $\frac{■}{3}$ d. $\frac{■}{5}$

Using the Greatest Common Factor

You can quickly reduce a fraction to its lowest terms if you do this: Divide both terms by their *greatest common factor*. The greatest common factor is the greatest number you can evenly divide into *both* terms of a fraction.

Look at these examples. 1, 2, and 4 are common factors. Each can evenly divide the terms of $\frac{4}{8}$. But 4 is the greatest common factor. How many steps does it take to get the answer in example A? In example B?

Example A

$$\frac{4}{8} \begin{matrix} \div 1 = \\ \div 1 = \end{matrix} \frac{4}{8} \qquad \frac{4}{8} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{2}{4} \qquad \frac{2}{4} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{1}{2}$$

Example B

$$\frac{4}{8} \begin{matrix} \div 4 = \\ \div 4 = \end{matrix} \frac{1}{2}$$

Exercise

The terms of these fractions can be divided by these common factors: 1, 2, 3, 4, or 6. Write the greatest common factor for the terms in each fraction. Then reduce that fraction.

1. $\frac{6}{12} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{2}$

2. $\frac{6}{8} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{■}$

3. $\frac{2}{6} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{■}$

4. $\frac{8}{12} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{■}$

5. $\frac{9}{12} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{■}$

6. $\frac{6}{9} \begin{matrix} \div ■ = \\ \div ■ = \end{matrix} \frac{■}{■}$

Answers:

1. $\frac{6}{12} \begin{matrix} \div 6 = \\ \div 6 = \end{matrix} \frac{1}{2}$
2. $\frac{6}{8} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{3}{4}$
3. $\frac{2}{6} \begin{matrix} \div 2 = \\ \div 2 = \end{matrix} \frac{1}{3}$
4. $\frac{8}{12} \begin{matrix} \div 4 = \\ \div 4 = \end{matrix} \frac{2}{3}$
5. $\frac{9}{12} \begin{matrix} \div 3 = \\ \div 3 = \end{matrix} \frac{3}{4}$
6. $\frac{6}{9} \begin{matrix} \div 3 = \\ \div 3 = \end{matrix} \frac{2}{3}$

Finding Lowest Terms

Sometimes you can easily see how to reduce a fraction. But if you're not sure, you can make a table like this to help you.

Factor	÷1	÷2	÷3	÷4	÷5	÷6	÷7	÷8	÷9	÷10
Numerator										
Denominator										

Suppose you are reducing the fraction $\frac{12}{18}$. Here's how to use the table:

- First write the terms of the fraction under the factor 1. (That's because 12 ÷ 1 = 12 and 18 ÷ 1 = 18.)
- Then try to divide each factor into both terms. If it goes into a term exactly, write the answer. If it doesn't, write **No**.

	÷1	÷2	÷3	÷4	÷5	÷6	÷7	÷8	÷9	÷10
12	12	6	4	3	NO	2	NO	NO	NO	NO
18	18	9	6	NO	NO	3	NO	NO	2	NO

- Now look at the answers under the factors: Circle the ones that have both a numerator and a denominator. Those are the terms that have common factors. Look for the greatest common factor. The answer under the greatest common factor should be the lowest terms of the fraction. (If you think it isn't, make another table and reduce it further.)

	÷1	÷2	÷3	÷4	÷5	÷6	÷7	÷8	÷9	÷10
12	12	6	4	3	NO	2	NO	NO	NO	NO
18	18	9	6	NO	NO	3	NO	NO	2	NO

What is the greatest common factor?
What are the lowest terms of $\frac{12}{18}$?

Exercise

Use the table. Reduce these fractions to their lowest terms.

1. $\frac{9}{21} = \frac{\blacksquare}{\blacksquare}$ 2. $\frac{18}{27} = \frac{\blacksquare}{\blacksquare}$ 3. $\frac{12}{16} = \frac{\blacksquare}{\blacksquare}$ 4. $\frac{24}{28} = \frac{\blacksquare}{\blacksquare}$

Answers:

1. $\frac{3}{7}$ 2. $\frac{2}{3}$ 3. $\frac{3}{4}$ 4. $\frac{6}{7}$

The Simplest Form

When you solve fraction problems, you sometimes get answers like these. What's wrong with them?

$\frac{2}{4}$ $\frac{3}{2}$ $\frac{2}{2}$ $\frac{4}{1}$ $2\frac{3}{6}$

If you get answers like those, you are not finished solving the problems. Those numbers are higher equivalents of other numbers. You must rename them in their **simplest forms.** In other words, you must **simplify** them.

Each of those answers is simplified in a certain way. Look at them again. Do you know what their simplest forms are like?

- The simplest form of a proper fraction such as $\frac{2}{4}$ is an equivalent fraction with lowest terms.

  $\frac{2}{4} = \frac{1}{2}$ (equivalent with lowest terms)

- The simplest form of an improper fraction such as $\frac{3}{2}$ is a mixed number with lowest terms.

  $\frac{3}{2} = 1\frac{1}{2}$ (mixed number with lowest terms)

- The simplest form of an improper fraction such as $\frac{2}{2}$ is the whole number 1.

  $\frac{2}{2} = 1$ (whole number 1)

- The simplest form of an improper fraction such as $\frac{4}{1}$ is its equivalent whole number.

  $\frac{4}{1} = 4$ (equivalent whole number)

- The simplest form of a mixed number such as $2\frac{3}{6}$ is a mixed number with lowest terms.

  $2\frac{3}{6} = 2\frac{1}{3}$ (mixed number with lowest terms)

Exercise

1. How do you simplify a proper fraction such as $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{8}{12}$?
2. What is the simplest form of an improper fraction such as $\frac{3}{2}$ and $\frac{8}{5}$?
3. What is the simplest form of an improper fraction such as $\frac{2}{2}$ and $\frac{3}{3}$?
4. What is the simplest form of an improper fraction such as $\frac{4}{1}$ and $\frac{3}{1}$?
5. What is the simplest form of an improper fraction such as $2\frac{4}{6}$ and $1\frac{3}{9}$?
6. Which number in each group needs to be simplified? Write that number. Then tell what kind of form you'd simplify it to.

a. $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{4}$

b. $\frac{1}{2}$, $\frac{7}{8}$, $\frac{8}{3}$

c. $\frac{4}{5}$, $\frac{2}{2}$, $\frac{7}{8}$

d. $\frac{12}{13}$, $\frac{2}{5}$, $\frac{7}{1}$

e. $2\frac{1}{4}$, $3\frac{3}{12}$, $2\frac{3}{4}$

f. $6\frac{1}{3}$, $\frac{3}{8}$, $\frac{9}{9}$

Answers:

1. Reduce it to lowest terms.
2. A mixed number with lowest terms.
3. The whole number 1.
4. Its equivalent whole number.
5. A mixed number with lowest terms.
6. **a.** $\frac{2}{4}$. The simplest form is an equivalent with lowest terms. **b.** $\frac{8}{3}$. The simplest form is a mixed number with lowest terms. **c.** $\frac{2}{2}$. The simplest form is the whole number 1. **d.** $\frac{7}{1}$. The simplest form is the equivalent whole number. **e.** $3\frac{3}{12}$. The simplest form is a mixed number with lowest terms. **f.** $\frac{9}{9}$. The simplest form is the whole number 1.

Renaming Improper Fractions

To find the simplest form of an improper fraction, do this: *Divide its numerator by its denominator.*

$\frac{3}{2} = 1\frac{1}{2}$ $\quad \begin{array}{r} 1\frac{1}{2} \\ 2\overline{)3} \end{array}$

$\frac{2}{2} = 1$ $\quad \begin{array}{r} 1 \\ 2\overline{)2} \end{array}$

$\frac{2}{1} = 2$ $\quad \begin{array}{r} 2 \\ 1\overline{)2} \end{array}$

Suppose you must simplify an improper fraction like $\frac{12}{8}$. You can divide the numerator by the denominator, then reduce the answer:

$\frac{12}{8} = 1\frac{4}{8} = 1\frac{2}{4} = 1\frac{1}{2}$ $\quad \begin{array}{r} 1\frac{4}{8} \\ 8\overline{)12} \end{array}$

Or you can first reduce the fraction to lowest terms. Then divide that denominator by that number.

$\frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$ $\quad \begin{array}{r} 1\frac{1}{2} \\ 2\overline{)3} \end{array}$

Which way is quicker?

Exercise

Copy these improper fractions on note paper. Then rename them in their simplest forms. Show your math work on that paper.

1. $\frac{4}{2}$ = ■ **5.** $\frac{6}{4}$ = ■

2. $\frac{2}{2}$ = ■ **6.** $\frac{12}{9}$ = ■

3. $\frac{9}{1}$ = ■ **7.** $\frac{16}{6}$ = ■

4. $\frac{7}{5}$ = ■ **8.** $\frac{28}{8}$ = ■

Answers:

1. 2 **2.** 1 **3.** 9 **4.** $1\frac{2}{5}$ **5.** $1\frac{1}{2}$

6. $1\frac{1}{3}$ **7.** $2\frac{2}{3}$ **8.** $3\frac{1}{2}$

Renaming Mixed Numbers

Here's how to see if you have correctly renamed an improper fraction as a mixed number! Change the mixed number *back* to an improper fraction. The answer should be the improper fraction reduced to its lowest terms.

For example:

$\frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}$

To change the mixed number ($1\frac{2}{3}$) back to the improper fraction, do this:

1. First, write the problem like this. Notice that the *denominator is the same in both fractions.*

   $1\frac{2}{3} = \frac{\ }{3}$

2. Next, multiply the whole number (1) by the denominator in the mixed number (3).

   $1 \times 3 = 3$ $\qquad \underset{\times}{1}\frac{2}{3} = \frac{\blacksquare}{3}$

3. Then add the answer (3) and the numerator of the mixed number (2). Write the total at the top of the improper fraction.

   $1 \times 3 = 3$
   $3 + 2 = 5$ $\qquad \underset{\times}{1}\frac{\overset{+}{2}}{3} = \frac{5}{3}$

4. Notice this: The improper fraction is $\frac{10}{6}$ *reduced to its lowest terms.*

Exercise

Copy these mixed numbers. Then rename them as improper fractions. Show your work.

1. $1\frac{2}{5}$ = ☐ **2.** $1\frac{3}{4}$ = ☐ **3.** $1\frac{5}{8}$ = ☐

4. $2\frac{1}{3}$ = ☐ **5.** $3\frac{3}{4}$ = ☐ **6.** $4\frac{2}{5}$ = ☐

Answers:

1. $\frac{7}{5}$ **2.** $\frac{7}{4}$ **3.** $\frac{13}{8}$

4. $\frac{7}{3}$ **5.** $\frac{15}{4}$ **6.** $\frac{22}{5}$

Unit Review

Check Yourself

1. Copy each fraction. Then rename it in its simplest form.

   a. $\frac{2}{4}$
   b. $\frac{6}{8}$
   c. $\frac{3}{2}$
   d. $\frac{4}{1}$
   e. $2\frac{4}{8}$
   f. $\frac{2}{4}$
   g. $\frac{2}{2}$
   h. $\frac{40}{12}$
   i. $\frac{6}{1}$
   j. $4\frac{3}{6}$
   k. $\frac{7}{2}$
   l. $\frac{14}{24}$

2. Copy each set of fractions. Find the missing term in each set.

   a. $\frac{2}{3} = \frac{\blacksquare}{9}$
   b. $\frac{4}{5} = \frac{8}{\blacksquare}$
   c. $\frac{1}{2} = \frac{6}{\blacksquare}$
   d. $\frac{7}{8} = \frac{\blacksquare}{16}$
   e. $\frac{5}{6} = \frac{15}{\blacksquare}$
   f. $\frac{3}{4} = \frac{16}{\blacksquare}$
   g. $\frac{1}{5} = \frac{\blacksquare}{20}$
   h. $\frac{2}{7} = \frac{\blacksquare}{21}$

3. Name the terms of the fractions in these word problems. Then reduce the fractions to lowest terms.

   a. Ms. Wolf has 15 students in her math lab. 10 students are boys. What fraction of her class are boys?

   $\frac{\blacksquare}{\blacksquare}$ are boys = $\frac{\blacksquare}{\blacksquare}$ are boys

   b. Tang is 16 years old. He has lived in the U.S. for 8 years. What fraction of his life has Tang lived in the U.S.?

   $\frac{\blacksquare}{\blacksquare}$ years = $\frac{\blacksquare}{\blacksquare}$ years

4. Choose the right words to complete these sentences about fractions.

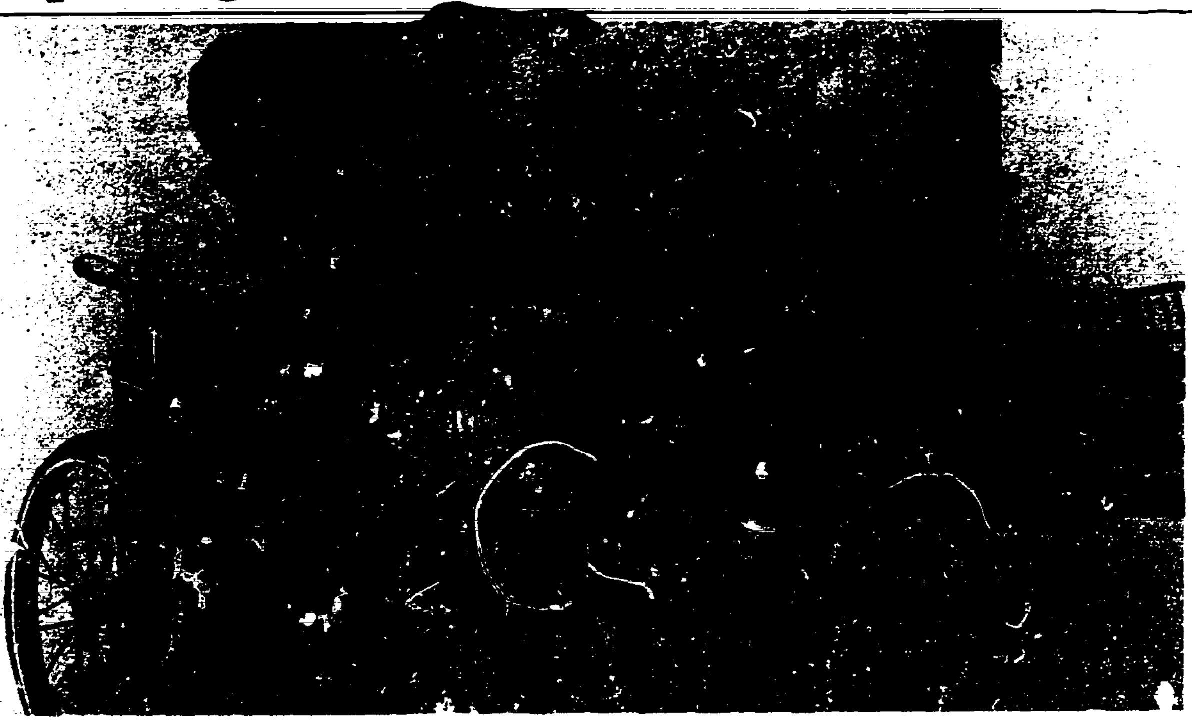
   factor
   raising
   reducing
   rename
   simplest form
   simplify

   a. When you change a fraction to an equivalent, you must ________ its terms so they are lower or higher.
   b. When you change the terms of a fraction so that the numbers are higher, you are ________ terms.
   c. When you change the terms of a fraction so that the numbers are lower, you are ________ terms.
   d. A number that you divide evenly into both terms of a fraction is called a ________.
   e. You ________ an improper fraction by renaming it as a mixed number.
   f. When you reduce a fraction to lowest terms, you are showing that fraction in its ________ ________.

Bonus Work

1. Make a poster that shows how to simplify different fractions.
2. Demonstrate to your class how to find the greatest common factor for one of the fractions: $\frac{32}{56}$, $\frac{56}{108}$, $\frac{91}{133}$.
3. Write some fractions that must be simplified. Ask your classmates to simplify them.

Comparing Fractions



Suppose someone hires you to straighten up his workshop. It's a mess! Tools are lying around everywhere! He tells you he wants you to put the tools away. He also tells you that some tools are missing. He wants to know which ones they are.

Then he tells you what to do: First sort all the wrenches and drill bits by size. Next list the tools that are missing by size. Then hang up the wrenches *in order of size* on a new pegboard—and leave space for the missing wrenches.

You see that the sizes of the wrenches and drill bits are marked in fractions. For example:

- Some drill-bit sizes are $\frac{1}{8}$″, $\frac{1}{4}$″, and $\frac{5}{16}$″. (″ stands for *inch.*)
- Some wrench sizes are $\frac{3}{8}$″, $\frac{1}{2}$″, and $\frac{9}{32}$″.

Your problem is this: How can you know which fraction size is larger when you compare fractions like $\frac{1}{4}$ and $\frac{5}{16}$? And how do you put fractions such as $\frac{3}{8}$, $\frac{1}{2}$, and $\frac{9}{32}$ in order by size?

Many jobs are like that. They ask you to work with fractions. You'll be able to do those jobs easily if you understand how to compare fractions.

In this unit, you will learn how to compare the sizes of fractions. You'll also learn how to put fractions in order.

You will see these math signs in some of the lessons: > and <.

> means *is more than.*
$5 > 3$ (5 is more than 3.)
< means *is less than.*
$3 < 5$ (3 is less than 5.)
Here's a way to remember the signs:
> *points* to the smaller amount: $5 > 3$.
< *opens* to the larger amount: $3 < 5$.

Math Words

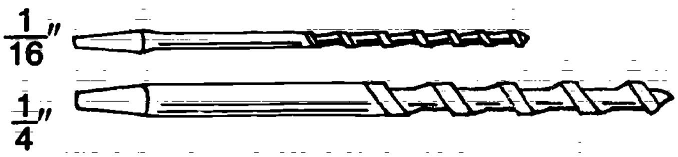
Look up these words in the glossary. What do they mean?

lowest common denominator
multiple

Different Denominators, Same Numerators

The first fractions you will compare have different denominators but the same numerators, such as $\frac{1}{4}$ and $\frac{1}{16}$. Those fractions can fool you. The ones with the larger denominators don't stand for the larger amounts.

For example, look at the two drill bits below. The fraction in front of each drill bit tells its size.



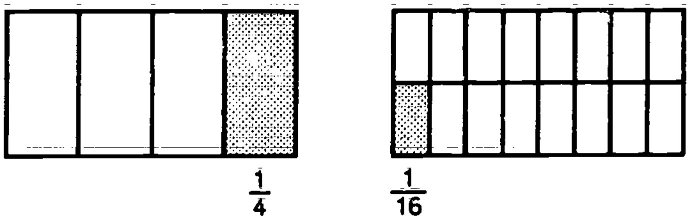
Which size drill bit is thinner: $\frac{1}{16}$" or $\frac{1}{4}$"? Which is thicker?

Right. The $\frac{1}{16}$" drill bit is thinner. The $\frac{1}{4}$" drill bit is thicker.

$\frac{1}{16} < \frac{1}{4}$ ($\frac{1}{16}$ is less than $\frac{1}{4}$.)

$\frac{1}{4} > \frac{1}{16}$ ($\frac{1}{4}$ is more than $\frac{1}{16}$.)

There's another way to show the same thing. Look at the boxes below.



Both boxes are the same size. The box on the left has 4 equal parts. $\frac{1}{4}$ of that box is shaded.

The box on the right has 16 equal parts. $\frac{1}{16}$ of that box is shaded.

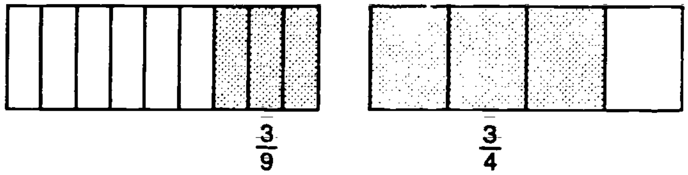
Which fraction amount is smaller: $\frac{1}{4}$ or $\frac{1}{16}$? Which is larger?

Right. $\frac{1}{16}$ is smaller. $\frac{1}{4}$ is larger.

$$\frac{1}{16} < \frac{1}{4}$$

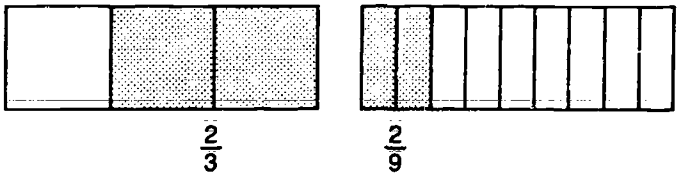
$$\frac{1}{4} > \frac{1}{16}$$

Look at the boxes below. Which is smaller: $\frac{3}{9}$ or $\frac{3}{4}$?



$\frac{3}{9} < \frac{3}{4}$ ($\frac{3}{9}$ is less than $\frac{3}{4}$.)

Which is larger: $\frac{2}{3}$ or $\frac{2}{9}$?



$\frac{2}{3} > \frac{2}{9}$ ($\frac{2}{3}$ is more than $\frac{2}{9}$.)

Remember:

When you compare fractions that have different denominators but the same numerators, remember this: A denominator tells how many equal parts there are in a whole thing or group. So:

- The *larger* the denominator, the *smaller* the parts.
- The *smaller* the denominator, the *larger* the parts.

Exercise

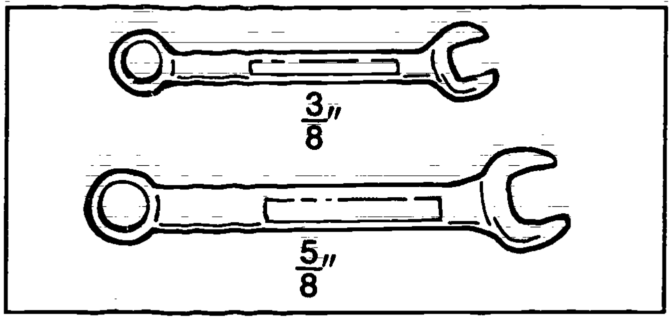
Copy each set of fractions. Write > or < between them to show if the first fraction is more or less than the second fraction. Then check your answers.

1. $\frac{2}{5}$ ■ $\frac{2}{3}$
2. $\frac{4}{7}$ ■ $\frac{4}{9}$
3. $\frac{7}{16}$ ■ $\frac{7}{12}$
4. $\frac{8}{15}$ ■ $\frac{8}{11}$
5. $\frac{5}{11}$ ■ $\frac{5}{16}$
6. $\frac{7}{16}$ ■ $\frac{7}{32}$

Answers: 1. $\frac{2}{5} < \frac{2}{3}$ 2. $\frac{4}{7} > \frac{4}{9}$ 3. $\frac{7}{16} < \frac{7}{12}$
4. $\frac{8}{15} < \frac{8}{11}$ 5. $\frac{5}{11} > \frac{5}{16}$ 6. $\frac{7}{16} > \frac{7}{32}$

Same Denominators, Different Numerators

Look at the fractions under the wrenches below. What do you notice about the denominators? What do you notice about the numerators?



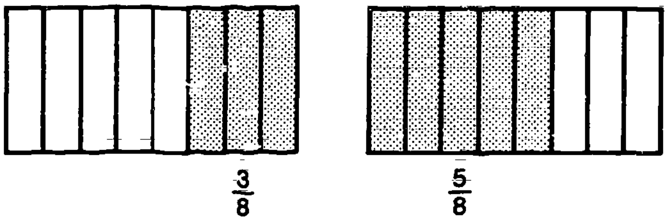
Right. The denominators are the same, but the numerators are different.

Which size wrench is smaller: $\frac{3}{8}''$ or $\frac{5}{8}''$? Which is larger?

$\frac{3}{8} < \frac{5}{8}$ ($\frac{3}{8}$ is less than $\frac{5}{8}$.)

$\frac{5}{8} > \frac{3}{8}$ ($\frac{5}{8}$ is more than $\frac{3}{8}$.)

Now look at the boxes below.



Both boxes are the same size. And each box has 8 equal parts. Each part is $\frac{1}{8}$ of the whole box.

$\frac{3}{8}$ of the box on the left is shaded.

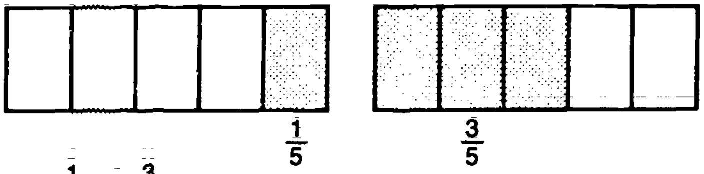
And $\frac{5}{8}$ of the box on the right is shaded.

Which fraction amount is smaller: $\frac{3}{8}$ or $\frac{5}{8}$? Which is larger?

$\frac{3}{8} < \frac{5}{8}$

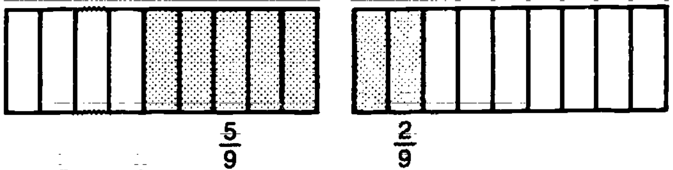
$\frac{5}{8} > \frac{3}{8}$

Look at these boxes. Which is smaller: $\frac{1}{5}$ or $\frac{3}{5}$?



$\frac{1}{5} < \frac{3}{5}$

Which is larger: $\frac{5}{9}$ or $\frac{2}{9}$?



$\frac{5}{9} > \frac{2}{9}$

Remember:

When you compare fractions that have the same denominators:

- The *larger* the numerator, the *larger* the amount.
- The *smaller* the numerator, the *smaller* the amount.

Exercise

Copy each set of fractions. Write > or < between them to show if the first fraction is more or less than the second fraction. Then check your answers.

1. $\frac{2}{3}$ ■ $\frac{1}{3}$
2. $\frac{3}{5}$ ■ $\frac{4}{5}$
3. $\frac{4}{7}$ ■ $\frac{6}{7}$
4. $\frac{3}{4}$ ■ $\frac{1}{4}$
5. $\frac{7}{8}$ ■ $\frac{5}{8}$
6. $\frac{5}{9}$ ■ $\frac{7}{9}$

Answers: 1. $\frac{2}{3} > \frac{1}{3}$ 2. $\frac{3}{5} < \frac{4}{5}$ 3. $\frac{4}{7} < \frac{6}{7}$
4. $\frac{3}{4} > \frac{1}{4}$ 5. $\frac{7}{8} > \frac{5}{8}$ 6. $\frac{5}{9} < \frac{7}{9}$

Different Denominators, Different Numerators

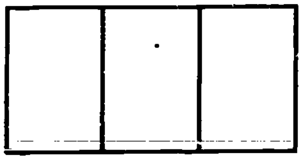
You know how to tell which of two fractions is larger when:

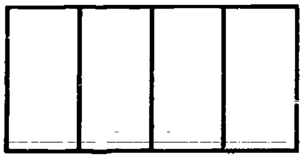
- the denominators are different but the numerators are the same: $\frac{3}{4} > \frac{3}{8}$;
- the denominators are the same but the numerators are different: $\frac{7}{12} > \frac{5}{12}$.

But what if both the denominators and the numerators are different? For example, look at the fractions below.

$\frac{2}{3}$ ■ $\frac{3}{4}$ different numerators / different denominators

Which fraction is smaller? Which fraction is larger? You can't tell, because you are comparing different parts: *thirds* and *fourths*.


thirds


fourths

Before you can compare fractions like $\frac{2}{3}$ and $\frac{3}{4}$, you have to rename one or both fractions. That means you must change them so both fractions have the same denominators.

The denominators of $\frac{2}{3}$ and $\frac{3}{4}$ can both be raised to 12. That number is the lowest denominator both terms can be raised to. So it is called their *lowest common denominator*.

Below are the two fractions raised to their **LCD.** (LCD stands for lowest common denominator.) They now have the same denominators, so you can compare them. Which is larger: $\frac{2}{3}$ or $\frac{3}{4}$?

$\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} = \frac{9}{12}$

$\frac{9}{12}$ has a larger numerator than $\frac{8}{12}$. So, $\frac{3}{4}$ is larger than $\frac{2}{3}$.

$\frac{3}{4} > \frac{2}{3}$ $\frac{2}{3} < \frac{3}{4}$

Review: How to Raise Terms

On page 29, you learned how to raise a fraction to a given denominator. You'll be raising fractions like that in the exercise below. Here's a quick review:

$\frac{2}{3} = \frac{}{12}$ (given denominator)

1. First, divide the denominator of the fraction into the given denominator. $3\overset{4}{\overline{)12}}$
2. Then, multiply the numerator of the fraction by the answer. $\frac{2}{3} \times 4 = \frac{8}{12}$ or $\frac{2}{3} = \frac{8}{12}$

Exercise

Raise these fractions to higher terms. Then write > or < to show which fractions are larger or smaller.

1. $\frac{1}{5} = \frac{■}{20}$ $\frac{1}{4} = \frac{■}{20}$ so: $\frac{1}{5}$ ■ $\frac{1}{4}$

2. $\frac{2}{3} = \frac{■}{21}$ $\frac{3}{7} = \frac{■}{21}$ so: $\frac{2}{3}$ ■ $\frac{3}{7}$

3. $\frac{1}{2} = \frac{■}{6}$ $\frac{2}{3} = \frac{■}{6}$ so: $\frac{1}{2}$ ■ $\frac{2}{3}$

4. $\frac{3}{4} = \frac{■}{12}$ $\frac{5}{6} = \frac{■}{12}$ so: $\frac{3}{4}$ ■ $\frac{5}{6}$

5. $\frac{4}{5} = \frac{■}{35}$ $\frac{2}{7} = \frac{■}{35}$ so: $\frac{4}{5}$ ■ $\frac{2}{7}$

6. $\frac{1}{8} = \frac{■}{72}$ $\frac{2}{9} = \frac{■}{72}$ so: $\frac{1}{8}$ ■ $\frac{2}{9}$

Answers:

1. $\frac{1}{5} = \frac{4}{20}$ $\frac{1}{4} = \frac{5}{20}$ so: $\frac{1}{5} < \frac{1}{4}$

2. $\frac{2}{3} = \frac{14}{21}$ $\frac{3}{7} = \frac{9}{21}$ so: $\frac{2}{3} > \frac{3}{7}$

3. $\frac{1}{2} = \frac{3}{6}$ $\frac{2}{3} = \frac{4}{6}$ so: $\frac{1}{2} < \frac{2}{3}$

4. $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$ so: $\frac{3}{4} < \frac{5}{6}$

5. $\frac{4}{5} = \frac{28}{35}$ $\frac{2}{7} = \frac{10}{35}$ so: $\frac{4}{5} > \frac{2}{7}$

6. $\frac{1}{8} = \frac{9}{72}$ $\frac{2}{9} = \frac{16}{72}$ so: $\frac{1}{8} < \frac{2}{9}$

Finding the LCD

Before you can compare fractions such as $\frac{1}{3}$ and $\frac{2}{5}$, you must do this: Find their lowest common denominator—their LCD. Here's how to find the LCD of two fractions:

1. Multiply the denominator of the first fraction by 1, 2, 3, and so on.
2. Multiply the denominator of the other fraction by 1, 2, 3, and so on.
3. Keep multiplying both denominators until you get a *multiple* that is the same for both denominators. (A multiple is the answer you get when you multiply.)

Example: Find the LCD of $\frac{1}{3}$ and $\frac{2}{5}$.

$\frac{1}{3} \times 1 = \frac{}{3}$	$\frac{2}{5} \times 1 = \frac{}{5}$
$\frac{1}{3} \times 2 = \frac{}{6}$	$\frac{2}{5} \times 2 = \frac{}{10}$
$\frac{1}{3} \times 3 = \frac{}{9}$	$\frac{2}{5} \times 3 = \frac{}{⑮}$
$\frac{1}{3} \times 4 = \frac{}{12}$	$\frac{2}{5} \times 4 = \frac{}{20}$
$\frac{1}{3} \times 5 = \frac{}{⑮}$	

When you multiply the denominator of $\frac{1}{3}$ by 5, the multiple is 15. When you multiply the denominator of $\frac{2}{5}$ by 3, the multiple is 15. So, LCD = 15.

Suppose you are comparing three or more fractions. You'd multiply each denominator until you find a multiple that's the same for all of them.

Example: Find the LCD of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.

You can make a table to help you easily find the LCD of fractions. First, draw a table to look like this:

Write ×1, ×2, ×3, and so on at the top of the table. Write the denominators of each fraction in front of the table.

		×1	×2	×3	×4	×5	×6	×7
$\frac{1}{2}$	2							
$\frac{2}{3}$	3							
$\frac{3}{4}$	4							

Now, multiply each denominator by each number at the top of the table. Write the multiples in the boxes under each number. Stop when you reach the same multiple for each denominator. What is the LCD of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$?

	×1	×2	×3	×4	×5	×6	×7
2	2	4	6	8	10	⑫	
3	3	6	9	⑫	15		
4	4	8	⑫	16	20		

LCD = ■ **Answer:** LCD = 12

Exercise

Copy each set of fractions. Find the lowest common denominator of each set. Then check your answers.

1. $\frac{2}{3}, \frac{5}{6}$ LCD = ■
2. $\frac{1}{3}, \frac{4}{9}$ LCD = ■
3. $\frac{4}{5}, \frac{7}{10}$ LCD = ■
4. $\frac{1}{2}, \frac{2}{3}$ LCD = ■
5. $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$ LCD = ■
6. $\frac{1}{4}, \frac{5}{6}, \frac{1}{8}$ LCD = ■

Answers: 1. 6 2. 9 3. 10
4. 6 5. 12 6. 24

Which Fraction Is Larger? Which Is Smaller?

You now know how to:

- find the lowest common denominator of two or more fractions;
- raise fractions to a given denominator; and
- compare fractions that have the same denominator.

You can now compare fractions such as $\frac{2}{3}$ and $\frac{4}{5}$. To figure out which fraction is larger, or which is smaller, follow these steps:

Step 1: Find the lowest common denominator.

LCD = 15

Math Work

x1	x2	x3	x4	x5	x6
3	6	9	12	$\textcircled{15}$	
5	10	$\textcircled{15}$	20		

$\frac{2}{3} = \frac{\ }{15}$

$\frac{4}{5} = \frac{\ }{15}$

Step 2: Raise the fractions to that lowest common denominator.

Math Work

$\frac{2}{3} = \frac{10}{15}$ $\quad \begin{array}{r}5\\3\overline{)15}\end{array}$ $\quad 2 \times 5 = 10$

$\frac{4}{5} = \frac{12}{15}$ $\quad \begin{array}{r}3\\5\overline{)15}\end{array}$ $\quad 4 \times 3 = 12$

Step 3: Show which fraction is larger.

Look at the numerators of the raised fractions. Compare the fractions.

Math Work

$\frac{2}{3} < \frac{4}{5}$ $\quad \frac{2}{3} = \frac{10}{15}$ $\quad \frac{4}{5} = \frac{12}{15}$

$\frac{10}{15} < \frac{12}{15}$ $\quad$ so: $\frac{2}{3} < \frac{4}{5}$

Exercise

Copy and compare each set of fractions. Show all your math work.

1. $\frac{2}{3}$ ■ $\frac{5}{6}$
2. $\frac{1}{3}$ ■ $\frac{4}{9}$
3. $\frac{4}{5}$ ■ $\frac{7}{10}$
4. $\frac{1}{4}$ ■ $\frac{2}{3}$
5. $\frac{1}{5}$ ■ $\frac{2}{3}$
6. $\frac{5}{6}$ ■ $\frac{3}{4}$

Answers:

1. $\frac{2}{3} < \frac{5}{6}$ 2. $\frac{1}{3} < \frac{4}{9}$ 3. $\frac{4}{5} > \frac{7}{10}$

4. $\frac{1}{4} < \frac{2}{3}$ 5. $\frac{1}{5} < \frac{2}{3}$ 6. $\frac{5}{6} > \frac{3}{4}$

Putting Fractions in Order by Size

Different Denominators, Same Numerators

Suppose you see this math problem:
Write these fractions in order
from smallest to largest:

$\frac{1}{16} \quad \frac{1}{4} \quad \frac{1}{8}$

To solve that problem, you must find which fraction is smallest in size, which is next in size, and which is largest. Here's what to do:

Step 1: Decide what kind of fractions you are comparing.

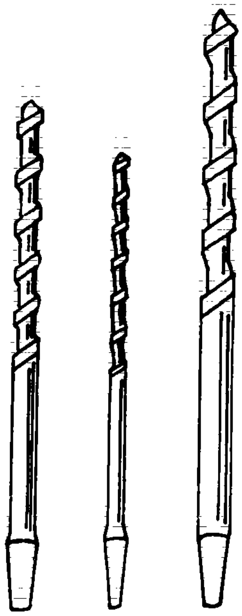
Look at $\frac{1}{16}$, $\frac{1}{4}$, and $\frac{1}{8}$. Which of these do they have?

- Different denominators but the same numerators.
- The same denominators but different numerators.
- Different denominators and different numerators.

Step 2: Compare the fractions to find the smallest.

$\frac{1}{16}$, $\frac{1}{4}$, and $\frac{1}{8}$ have different denominators, but the same numerators. You learned this: When the numerators are the same, the *largest* denominator stands for the *smallest* parts. Which fraction has the largest denominator: $\frac{1}{16}$, $\frac{1}{4}$, or $\frac{1}{8}$?

$\frac{1}{16}$ has the largest denominator. So $\frac{1}{16}$ is the smallest amount. The drill bits help you see that.



Step 3: Put the fractions in order.

If $\frac{1}{16}$ is the smallest fraction, which is the largest? How would you put $\frac{1}{16}$, $\frac{1}{4}$, and $\frac{1}{8}$ in order from smallest to largest?

$\frac{1}{\blacksquare} < \frac{1}{\blacksquare} < \frac{1}{\blacksquare}$

Answer: $\frac{1}{16} < \frac{1}{8} < \frac{1}{4}$

Exercise

Copy each set of fractions. Put them in order from smallest to largest. Then check your answers.

1. $\frac{2}{9}, \frac{2}{5}, \frac{2}{7}$
3. $\frac{4}{15}, \frac{4}{17}, \frac{4}{11}$
2. $\frac{1}{12}, \frac{1}{10}, \frac{1}{13}$
4. $\frac{3}{4}, \frac{3}{16}, \frac{3}{8}$

Answers:
1. $\frac{2}{9} < \frac{2}{7} < \frac{2}{5}$ 2. $\frac{1}{13} < \frac{1}{12} < \frac{1}{10}$
3. $\frac{4}{17} < \frac{4}{15} < \frac{4}{11}$ 4. $\frac{3}{16} < \frac{3}{8} < \frac{3}{4}$

Same Denominators, Different Numerators

Suppose you had to put this set of fractions in order from smallest to largest:

$\frac{1}{8}$ $\frac{5}{8}$ $\frac{3}{8}$

Here is what to do:

Step 1: Decide what kind of fractions they are.

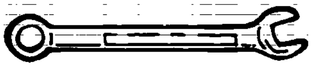
Look at the fractions $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{3}{8}$. What do you notice about them?

Right. They all have the same denominators but different numerators.

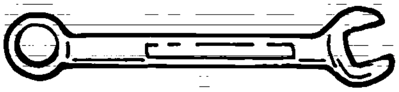
Step 2: Compare the fractions to find the smallest.

$\frac{1}{8}$, $\frac{5}{8}$, and $\frac{3}{8}$ have the same denominators. So they all stand for the same thing: 8 parts or 8 *eighths*. The numerators tell how many eighths make up certain amounts. Which is the smallest amount: 1 *eighth*, 5 *eighths*, or 3 *eighths*?

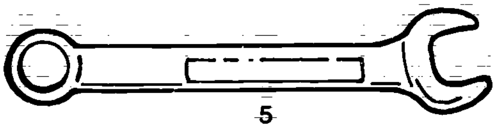
Right. 1 *eighth* is the smallest amount. So, $\frac{1}{8}$ is the smallest fraction. The wrenches help you see that.



$\frac{1}{8}$



$\frac{3}{8}$



$\frac{5}{8}$

Step 3: Put the fractions in order.

Now that you know the smallest fraction, you can put all the fractions in order. What is the order, starting with the smallest fraction?

$$\frac{\blacksquare}{8} < \frac{\blacksquare}{8} < \frac{\blacksquare}{8}$$

Answer: $\frac{1}{8} < \frac{3}{8} < \frac{5}{8}$

Exercise

Copy each set of fractions. Put them in order from smallest to largest. Then check your answers.

1. $\frac{2}{5}$, $\frac{1}{5}$, $\frac{4}{5}$
2. $\frac{6}{11}$, $\frac{4}{11}$, $\frac{3}{11}$
3. $\frac{8}{19}$, $\frac{1}{19}$, $\frac{5}{19}$
4. $\frac{5}{7}$, $\frac{6}{7}$, $\frac{2}{7}$

Answers:

1. $\frac{1}{5} < \frac{2}{5} < \frac{4}{5}$
2. $\frac{3}{11} < \frac{4}{11} < \frac{6}{11}$
3. $\frac{1}{19} < \frac{5}{19} < \frac{8}{19}$
4. $\frac{2}{7} < \frac{5}{7} < \frac{6}{7}$

Different Denominators, Different Numerators

Suppose you had to put these fractions in order from smallest to largest:

$\frac{3}{4}, \frac{5}{8}, \frac{1}{2}$

Here is what to do:

Step 1: Decide what kind of fractions you are comparing.

What do you notice about the denominators and the numerators of $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{2}$?

Right. The denominators and the numerators are different. You must rename the fractions so that they all have the same denominators.

Step 2: Find the lowest common denominator.

How do you find the LCD of all the fractions?

LCD = 8

$\frac{3}{4} = \frac{\ }{8}$

$\frac{5}{8} = \frac{\ }{8}$

$\frac{1}{2} = \frac{\ }{8}$

		x1	x2	x3	x4
$\frac{3}{4}$	4	4	8	12	16
$\frac{5}{8}$	8	8	16	24	32
$\frac{1}{2}$	2	2	4	6	8

Step 3: Rename the fractions.

What must you do to rename fractions?

$\overset{2}{4\overline{)8}}$ 3×2=6 $\frac{3}{4} = \frac{6}{8}$

$\overset{1}{8\overline{)8}}$ 5×1=5 $\frac{5}{8} = \frac{5}{8}$

$\overset{4}{2\overline{)8}}$ 1×4=4 $\frac{1}{2} = \frac{4}{8}$

Step 4: Put the fractions in order.

What must you do to compare the fractions?

$\frac{4}{8} < \frac{5}{8} < \frac{6}{8}$ so: $\frac{1}{2} < \frac{5}{8} < \frac{3}{4}$

Exercise

Copy each set of fractions. Put them in order from smallest to largest. Show your math work.

1. $\frac{3}{7}, \frac{3}{4}, \frac{3}{9}$
2. $\frac{5}{9}, \frac{5}{8}, \frac{5}{6}$
3. $\frac{3}{5}, \frac{1}{5}, \frac{4}{5}$
4. $\frac{4}{9}, \frac{8}{9}, \frac{5}{9}$
5. $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}$
6. $\frac{5}{6}, \frac{7}{8}, \frac{3}{4}$

Answers:

1. $\frac{3}{9} < \frac{3}{7} < \frac{3}{4}$ 2. $\frac{5}{9} < \frac{5}{8} < \frac{5}{6}$

3. $\frac{1}{5} < \frac{3}{5} < \frac{4}{5}$ 4. $\frac{4}{9} < \frac{5}{9} < \frac{8}{9}$

5. $\frac{1}{3} < \frac{1}{2} < \frac{3}{4}$ 6. $\frac{3}{4} < \frac{5}{6} < \frac{7}{8}$

Unit Review

Check Yourself

1. Copy each set of fractions. Write the correct sign to compare the fractions in each set. (> means *more than.* < means *less than.*)

   a. $\frac{2}{5}$ ■ $\frac{2}{3}$
   b. $\frac{5}{7}$ ■ $\frac{5}{9}$
   c. $\frac{3}{16}$ ■ $\frac{3}{8}$
   d. $\frac{4}{5}$ ■ $\frac{2}{5}$
   e. $\frac{6}{7}$ ■ $\frac{3}{7}$
   f. $\frac{5}{16}$ ■ $\frac{7}{16}$

2. Copy each set of fractions. Find the lowest common denominator of each set. Rename both fractions. Then compare the fractions by writing > or < between them.

   a. $\frac{2}{3}$ ■ $\frac{3}{5}$
   b. $\frac{3}{4}$ ■ $\frac{5}{6}$
   c. $\frac{2}{5}$ ■ $\frac{1}{3}$

3. Copy each set of fractions. Then arrange each set in order from smallest to largest. Show your work for sets **g.** and **h.**

   a. $\frac{3}{15}, \frac{3}{5}, \frac{3}{20}$
   b. $\frac{5}{16}, \frac{5}{9}, \frac{5}{6}$
   c. $\frac{7}{12}, \frac{7}{8}, \frac{7}{16}$
   d. $\frac{4}{9}, \frac{1}{9}, \frac{5}{9}$
   e. $\frac{2}{5}, \frac{4}{5}, \frac{3}{5}$
   f. $\frac{13}{16}, \frac{11}{16}, \frac{9}{16}$
   g. $\frac{2}{5}, \frac{1}{4}, \frac{3}{10}$
   h. $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}$

4. Choose the right word to complete each sentence.

   common **larger** **smaller**

   a. When the denominators are different but the numerators are the same ($\frac{3}{7}$, $\frac{3}{5}$), the fraction with the ________ denominator stands for the larger amount.
   b. When the denominators are the same but the numerators are different ($\frac{3}{8}$, $\frac{7}{8}$), the fraction with the ________ numerator stands for the larger amount.
   c. To compare fractions that have different denominators and different numerators, first find the lowest ________ denominator.

Bonus Work

1. Write different fractions on small pieces of paper. You can make up the fractions, copy them from a math book, or copy down the sizes of wrenches or drill bits. Mix up the pieces. Then ask your classmates to arrange the fractions in order from smallest to largest.
2. Get a set of wrenches. Trace their shapes on paper and mark the size of each wrench on the shapes. Cut out the shapes. Use them to make a poster comparing fractions. Use the signs for more than (>) and less than (<).

Unit 6

Multiplying Fractions

Suppose you want to make a pizza that is just big enough for yourself. But your recipe is for a pizza that's big enough for *two* people. What can you do?

You can make just half of the amount. To find half of an amount, you'd multiply it by $\frac{1}{2}$.

Here is part of a recipe for the sauce that goes on the pizza. Notice that almost all the amounts of ingredients are shown in fractions.

Pizza Sauce

$\frac{2}{3}$	cup tomato sauce
$\frac{1}{2}$	teaspoon sugar
$\frac{1}{2}$	teaspoon oregano
$\frac{1}{4}$	teaspoon pepper
1	clove garlic
$2\frac{1}{2}$	tablespoons onion

To find half of those amounts, you'd multiply each amount by $\frac{1}{2}$:

$\frac{1}{2} \times \frac{2}{3}$ cup tomato sauce

$\frac{1}{2} \times \frac{1}{2}$ teaspoon sugar

and so on.

Suppose you want to make *more* pizzas than the recipe is for? You want to make two of each amount. To do that you'd multiply each fraction by 2:

$2 \times \frac{2}{3}$ cup tomato sauce

$2 \times \frac{1}{2}$ teaspoon sugar

and so on.

Making a recipe is just one example of multiplying with fractions in real life. What are some other examples?

Math Words

Look up these words in the glossary. What do they mean?

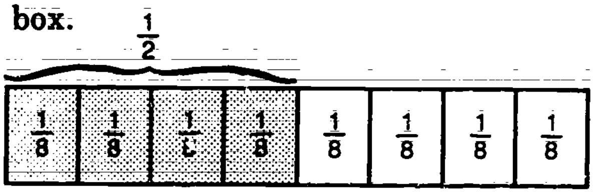
simplify before multiplying

Multiplying with Fractions

When you multiply an amount by a fraction, you do this: You find how much that fraction is *of* that amount.

Look at this box. It is divided into eight $\frac{1}{8}$ parts: $\frac{8}{8}$. The eight parts equal one whole box: $\frac{8}{8} = 1$.

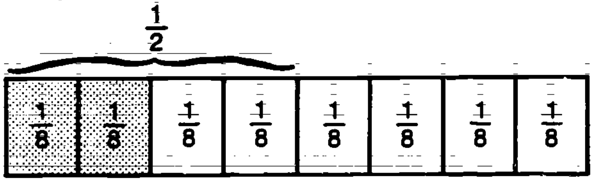
Suppose you find $\frac{1}{2}$ of the whole box. You are finding $\frac{1}{2}$ of 1 or $\frac{1}{2} \times 1$. Count the number of $\frac{1}{8}$ parts in $\frac{1}{2}$ of the box.



You should get four $\frac{1}{8}$ boxes—that's $\frac{4}{8}$ of the whole box. Now reduce $\frac{4}{8}$ to its simplest form: $\frac{4}{8} = \frac{1}{2}$. So:

$$\frac{1}{2} \times 1 = \frac{1}{2}$$

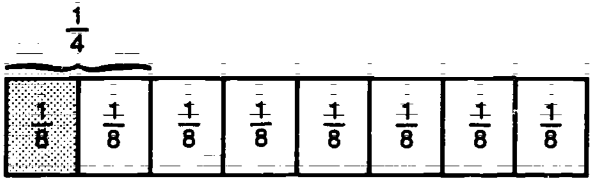
Suppose you find $\frac{1}{2}$ of half the box. You are finding $\frac{1}{2}$ of $\frac{1}{2}$ or $\frac{1}{2} \times \frac{1}{2}$. How many $\frac{1}{8}$ parts are in $\frac{1}{2}$ of half the box?



You should count two $\frac{1}{8}$ parts—$\frac{2}{8}$ of the whole box. What is $\frac{2}{8}$ reduced to its simplest form? Right! $\frac{2}{8} = \frac{1}{4}$. So:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Let's look at just $\frac{1}{4}$ of the whole box. If you find half of that $\frac{1}{4}$ part, you are finding $\frac{1}{2}$ of $\frac{1}{4}$, or $\frac{1}{2} \times \frac{1}{4}$. What's the answer?



$$\frac{1}{2} \times \frac{1}{4} = \frac{\blacksquare}{\blacksquare}$$

Answer: $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

Remember:

Look at the *amount* you are multiplying in each of these problems. Then look at the answer. Is the answer always larger or smaller than the amount?

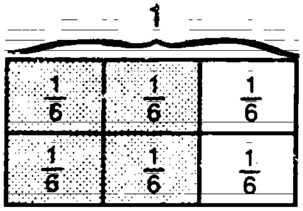
$\frac{1}{2} \times 1 = \frac{1}{2}$ $\quad \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\quad \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$

Remember this: When you multiply an amount by a fraction, *your answer is smaller than the amount.*

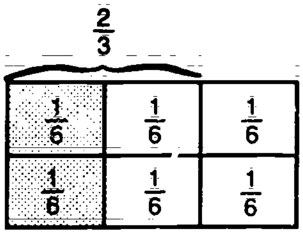
Exercise

These boxes are divided into 6 equal parts. Look at the boxes and answer the questions. Reduce your answers to the simplest forms.

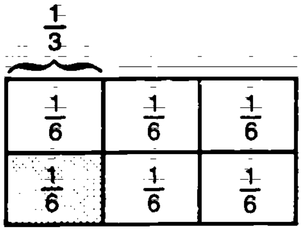
1. What is $\frac{2}{3}$ of 1? $\frac{2}{3} \times 1 = \frac{\blacksquare}{\blacksquare}$



2. What is $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{2} \times \frac{2}{3} = \frac{\blacksquare}{\blacksquare}$



3. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{2} \times \frac{1}{3} = \frac{\blacksquare}{\blacksquare}$



Answers: 1. $\frac{2}{3}$ 2. $\frac{1}{3}$ 3. $\frac{1}{6}$

Multiplying a Fraction by a Fraction

Multiply the Terms

Look at the fractions in this problem. Look closely at the answer. What do you think you must do to get that answer?

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Notice this: If you multiply the denominators of the fractions, you'll get the denominator of the answer.

$$\frac{1}{2} \times \frac{1}{3} = \frac{■}{6}$$

If you multiply the numerators of the fractions, you'll get the numerator of the answer.

$$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

That's how to multiply fractions: Multiply the denominators of those fractions; then multiply the numerators of those fractions.

Suppose you change the order of the fractions you are multiplying. Will you get a different answer? Solve these problems and find out.

$$\frac{2}{5} \times \frac{1}{3} = \frac{■}{■} \qquad \frac{1}{3} \times \frac{2}{5} = \frac{■}{■}$$

Answer: You can multiply fractions in any order. So, your answer to both problems should be the same: $\frac{2}{15}$.

Exercise

Copy and solve these problems. Then check your answers.

1. $\frac{2}{5} \times \frac{1}{5}$
2. $\frac{2}{3} \times \frac{2}{3}$
3. $\frac{4}{5} \times \frac{1}{3}$
4. $\frac{3}{4} \times \frac{1}{2}$
5. $\frac{6}{7} \times \frac{2}{3}$
6. $\frac{2}{9} \times \frac{2}{5}$

Answers: 1. $\frac{2}{25}$ 2. $\frac{4}{9}$ 3. $\frac{4}{15}$ 4. $\frac{3}{8}$ 5. $\frac{12}{21}$ 6. $\frac{4}{45}$

Reduce the Answer

Suppose you multiply these fractions. What's the answer?

$$\frac{1}{2} \times \frac{2}{3} = \frac{■}{■}$$

Did you get $\frac{1}{3}$? You should. Here's why: When you multiply the denominators, you get 6.

$$\frac{1}{2} \times \frac{2}{3} = \frac{■}{6}$$

When you multiply the numerators, you get 2.

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$

You learned that fraction answers must be in their simplest form. $\frac{2}{6}$ is not the simplest form of that fraction. You must reduce it.

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3} \text{ answer}$$

Remember:

To multiply a fraction by another fraction:

1. Multiply one denominator by the other denominator.
2. Multiply one numerator by the other numerator.
3. If you need to, reduce the answer to its lowest terms.

Exercise

Copy and solve these multiplication problems. If you need to, reduce answers to their lowest terms. Then check your answers.

1. $\frac{1}{2} \times \frac{2}{5}$
2. $\frac{3}{4} \times \frac{1}{3}$
3. $\frac{1}{5} \times \frac{2}{3}$
4. $\frac{2}{3} \times \frac{3}{4}$
5. $\frac{2}{5} \times \frac{1}{2}$
6. $\frac{5}{6} \times \frac{2}{3}$

Answers: 1. $\frac{1}{5}$ 2. $\frac{1}{4}$ 3. $\frac{2}{15}$ 4. $\frac{1}{2}$ 5. $\frac{1}{5}$ 6. $\frac{5}{9}$

Multiplying a Fraction by a Whole or Mixed Number

Suppose you want to double a recipe for pizza sauce. The recipe calls for $\frac{2}{3}$ cup of tomato sauce. You need two times that amount.

$$2 \times \frac{2}{3} = \frac{■}{■}$$

You must multiply a whole number by a fraction to find the answer. But you learned that you can only multiply numerators and denominators. A whole number does not have a denominator and a numerator. So, you cannot solve the problem until you change the whole number to an improper fraction. How do you do that? (If you're not sure, look on page 20.)

You can change the whole number 2 to an improper fraction by renaming it as $\frac{2}{1}$. Now you can multiply the fractions:

$$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3}$$

Notice that you get an improper fraction. You must simplify that fraction. (If you're not sure how to simplify improper fractions, look on page 33.) What's the answer?

$$\frac{2}{1} \times \frac{2}{3} = \frac{4}{3} = ■\frac{■}{■}$$

Answer: The answer is $1\frac{1}{3}$.

Now, suppose you want to increase the recipe $2\frac{1}{2}$ times. You'll need $2\frac{1}{2}$ times the $\frac{2}{3}$ cup of tomato sauce. $2\frac{1}{2}$ is a mixed number.

$$2\frac{1}{2} \times \frac{2}{3} = \frac{■}{■}$$

You must change the mixed number to an improper fraction before you can multiply it by a fraction. How do you change it? (If you're not sure, look on page 33.)

To change $2\frac{1}{2}$ to an improper fraction, you rename it as $\frac{5}{2}$.

$$\frac{5}{2} \times \frac{2}{3} = \frac{10}{6}$$

Next, reduce $\frac{10}{6}$:

$$\frac{5}{2} \times \frac{2}{3} = \frac{10}{6} = \frac{5}{3}$$

Last, rename $\frac{5}{3}$ as a mixed number. What's the answer?

$$\frac{5}{2} \times \frac{2}{3} = \frac{10}{6} = \frac{5}{3} = ■\frac{■}{■}$$

Answer: The answer is $1\frac{2}{3}$.

Remember:

When you multiply a fraction by a whole or mixed number:

1. Change the whole or mixed number to an improper fraction.
2. Reduce the answer if you can.
3. Rename an improper fraction answer as a whole or mixed number.

Exercise

Copy and solve these multiplication problems. Reduce answers if you can. Rename improper fractions. Then check your answers.

1. $2 \times \frac{2}{5}$
2. $5 \times \frac{1}{8}$
3. $4 \times \frac{2}{9}$
4. $4 \times \frac{1}{8}$
5. $2\frac{3}{4} \times \frac{2}{3}$
6. $3\frac{3}{5} \times \frac{3}{4}$

Answers: 1. $\frac{4}{5}$ 2. $\frac{5}{8}$ 3. $\frac{8}{9}$
4. $\frac{1}{2}$ 5. $1\frac{5}{6}$ 6. $2\frac{7}{10}$

Simplify Before You Multiply

Get a piece of paper and solve this problem. Count the steps it takes to get the final answer.

$$\frac{2}{3} \times \frac{5}{8} = \frac{\blacksquare}{\blacksquare}$$

You probably solved the problem in two steps, like this:

$$\frac{2}{3} \times \frac{5}{8} = \frac{10}{24} \begin{matrix} \div 2 \\ \div 2 \end{matrix} = \frac{5}{12}$$

You can save steps if you simplify a problem before you multiply it. Here's how: Look at the *numerator* in one fraction. Look at the *denominator* across it in the *other* fraction. See if you can divide those terms by *the same number*.

Look at the problem again. Which numerator and denominator can be divided by the same number?

$$\frac{2}{3} \times \frac{5}{8} = \frac{\blacksquare}{\blacksquare}$$

Right! The numerator 2 and the denominator 8 can be divided by the same number: 2. To simplify that problem, do this:

- Divide the numerator and denominator by the same number.

$$\overset{1}{2\overline{)2}} \qquad \overset{4}{2\overline{)8}}$$

- Draw a line through those terms and write in the new terms.

$$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{4}{\cancel{8}}} = \frac{\blacksquare}{\blacksquare}$$

Now, multiply the denominators, using the new term. Then multiply the numerators, using the new term.

$$\frac{\overset{1}{\cancel{2}}}{3} \times \frac{5}{\underset{4}{\cancel{8}}} = \frac{5}{12} \qquad \begin{matrix} 1 \times 5 = 5 \\ 3 \times 4 = 12 \end{matrix}$$

Sometimes you can simplify all the terms in a problem. Here's an example:

$$\frac{4}{9} \times \frac{3}{8} = \frac{\blacksquare}{\blacksquare}$$

- First, simplify the terms across one way:

$$\frac{\overset{1}{\cancel{4}}}{9} \times \frac{3}{\underset{2}{\cancel{8}}} = \frac{\blacksquare}{\blacksquare} \qquad \overset{1}{4\overline{)4}} \qquad \overset{2}{4\overline{)8}}$$

- Then, simplify the terms across the other way:

$$\frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} = \frac{\blacksquare}{\blacksquare} \qquad \overset{1}{3\overline{)3}} \qquad \overset{3}{3\overline{)9}}$$

Now, solve the problem. What's the answer?

Answer: The answer is $\frac{1}{6}$

Remember:

To multiply a fraction by another fraction in the shortest way, do this:

1. Simplify as many terms as you can.
2. Cross out old terms and write in new terms.
3. Multiply the numerators using the new terms. Then multiply the denominators using the new terms.
4. Reduce the answer if you need to.

Exercise

Copy and solve these multiplication problems. Simplify first. See if the answers can be reduced more. Then check your answers.

1. $\frac{1}{3} \times \frac{6}{7}$
2. $\frac{5}{6} \times \frac{2}{3}$
3. $\frac{1}{4} \times \frac{4}{5}$
4. $\frac{3}{8} \times \frac{4}{9}$
5. $\frac{2}{3} \times \frac{3}{4}$
6. $\frac{5}{9} \times \frac{3}{10}$

Answers:

1. $\frac{2}{7}$ 2. $\frac{5}{9}$ 3. $\frac{1}{5}$ 4. $\frac{1}{6}$ 5. $\frac{1}{2}$ 6. $\frac{1}{6}$

Unit Review

Check Yourself

1. Rename the improper fractions as whole or mixed numbers. Rename the whole and mixed numbers as improper fractions.

   a. $\frac{8}{4}$
   b. $\frac{12}{5}$
   c. $3\frac{2}{3}$
   d. 2

2. Multiply these fractions. Simplify the fractions first if you can. Check your answers to see if you can reduce them more.

   a. $\frac{1}{3} \times \frac{3}{5}$
   b. $\frac{1}{6} \times \frac{2}{3}$
   c. $\frac{2}{7} \times \frac{1}{4}$
   d. $\frac{6}{7} \times \frac{1}{10}$
   e. $\frac{5}{6} \times \frac{3}{10}$
   f. $\frac{3}{16} \times \frac{4}{9}$

3. Multiply these whole numbers and fractions. Reduce answers to lowest terms. Rename improper fractions.

   a. $3 \times \frac{1}{4}$
   b. $2 \times \frac{2}{9}$
   c. $4 \times \frac{3}{5}$
   d. $\frac{3}{8} \times 5$
   e. $\frac{4}{5} \times 6$
   f. $\frac{3}{9} \times 8$

4. Choose the right word to complete each sentence below.

   denominators **fraction**
   divide **simplify**

   a. To multiply fractions, first multiply the ________. Then multiply the numerators.
   b. To change an improper fraction to a whole or mixed number, ________ the numerator by the denominator.
   c. Before you multiply a whole number and a fraction, change the whole number to a ________.
   d. Before you multiply fractions, see if you can ________ the terms.

Bonus Work

1. Find some recipes in cookbooks, newspapers, or magazines. Copy the ingredients with fraction amounts. Double those amounts. Then halve them.
2. Look through newspapers. Cut out sale ads that offer "$\frac{1}{2}$ off," or "$\frac{1}{3}$ off," and so on. Make a poster with the ads.
3. Make up word problems about stores that have sales. Give the regular price. Then ask classmates to find how much the store is taking off. For example:

   ShoeMarket is having a "$\frac{1}{2}$ off" sale. How much will the store take off for shoes that usually cost $28?

Suppose you and a friend are walking to a party. You look at a map to see how to get there. You see that you can take Elm Street for $\frac{1}{4}$ of a mile, then take King Avenue for $\frac{5}{8}$ of a mile.

You also see another way: You can take Valdez Avenue for $\frac{3}{4}$ of a mile, then take Maple Street for $\frac{1}{8}$ of a mile.

How do you decide which way is shorter?

Right. You *add* the fractions.

Elm Street and King Avenue

$\frac{1}{4}$ mile + $\frac{5}{8}$ mile = $\frac{7}{8}$ mile

Valdez Avenue and Maple Street

$\frac{3}{4}$ mile + $\frac{1}{8}$ mile = $\frac{7}{8}$ mile

Both ways are $\frac{7}{8}$ of a mile long! So you can take either way.

That is an example of how you might have to add fractions in real life. What's an example of how you might have to *subtract* fractions in real life?

Math Words

Look up these words in the glossary. What do they mean?

like fractions
unlike fractions

Adding Like Fractions

Suppose you work in a candy store. A customer buys $\frac{1}{8}$ of a pound of vanilla fudge and $\frac{5}{8}$ of a pound of chocolate fudge. How much fudge does the customer buy altogether?

To find out, you add $\frac{1}{8}$ and $\frac{5}{8}$. You can write the problem across or down.

$$\frac{1}{8} + \frac{5}{8} = \frac{\blacksquare}{\blacksquare} \quad or \quad \begin{array}{r} \frac{1}{8} \\ +\frac{5}{8} \\ \hline \frac{\blacksquare}{\blacksquare} \end{array}$$

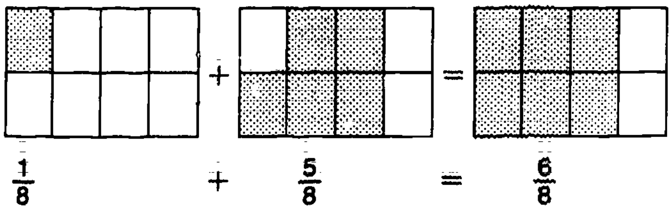
Notice that both fractions have the same denominator: 8. Fractions that have the same denominator are called **like fractions.** Like fractions stand for the same things. In this case, the things are $\frac{1}{8}$ parts, or *eighths*.

To add like fractions, add their numerators. *Do not add their denominators!* Instead, keep the same denominator for the answer.

$$\frac{1}{8} + \frac{5}{8} = \frac{6}{8} \quad or \quad \begin{array}{r} \frac{1}{8} \\ +\frac{5}{8} \\ \hline \frac{6}{8} \end{array}$$

Why do you think you keep the same denominator for the answer?

You keep the same denominator because you are adding the same things: *eighths*.



Now reduce the answer $\frac{6}{8}$ to its lowest terms.

$$\frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4} \quad or \quad \begin{array}{r} \frac{1}{8} \\ +\frac{5}{8} \\ \hline \frac{6}{8} = \frac{3}{4} \end{array}$$

The customer buys $\frac{3}{4}$ of a pound of fudge altogether.

Remember:

To add like fractions (fractions with the same denominator), do this:

1. Add the numerators.
2. Keep the same denominator.
3. Reduce the answer if you can.

Exercise

Copy and solve these addition problems. Reduce the answers if you can. Then check your answers.

1. $\frac{1}{3} + \frac{1}{3}$ **2.** $\frac{1}{4} + \frac{2}{4}$ **3.** $\frac{3}{5} + \frac{1}{5}$

4. $\begin{array}{r} \frac{2}{9} \\ +\frac{5}{9} \\ \hline \end{array}$ **5.** $\begin{array}{r} \frac{3}{10} \\ +\frac{5}{10} \\ \hline \end{array}$ **6.** $\begin{array}{r} \frac{5}{12} \\ +\frac{4}{12} \\ \hline \end{array}$

Answers: 1. $\frac{2}{3}$ 2. $\frac{3}{4}$ 3. $\frac{4}{5}$ 4. $\frac{7}{9}$ 5. $\frac{4}{5}$ 6. $\frac{3}{4}$

Subtracting Like Fractions

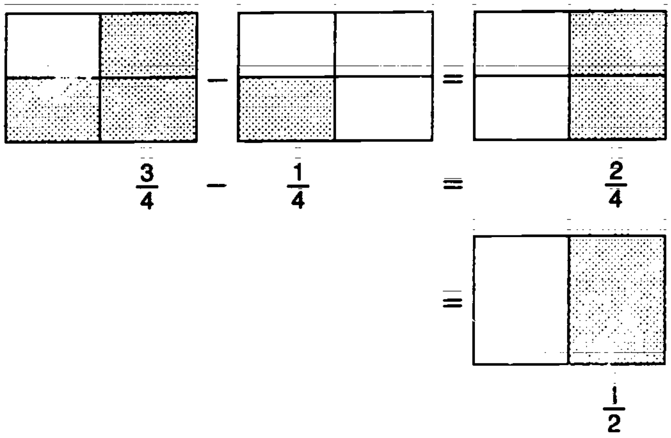
Suppose you work in a deli. You slice $\frac{3}{4}$ of a pound of Swiss cheese. A customer comes in and buys $\frac{1}{4}$ pound of the sliced cheese. How much sliced cheese is left?

To find the answer, you subtract the smaller fraction from the larger fraction. You can write the problem across or down.

$$\frac{3}{4} - \frac{1}{4} = \frac{\blacksquare}{\blacksquare} \quad or \quad \begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{4} \\ \hline \frac{\blacksquare}{\blacksquare} \end{array}$$

Notice that the two fractions are *like* fractions. That is, both fractions have the same denominator: 4.

To subtract the fractions, just subtract the numerators, and keep the same denominator. Reduce the answer if you can.



$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2} \quad or \quad \begin{array}{r} \frac{3}{4} \\ -\ \frac{1}{4} \\ \hline \frac{2}{4} = \frac{1}{2} \end{array}$$

Remember:

To subtract fractions with the same denominator, do this:

1. Subtract the smaller numerator from the larger numerator.
2. Keep the same denominator.
3. Reduce the answer if you can.

Exercise

Copy and solve these subtraction problems. Reduce the answers if you can. Then check your answers.

1. $\frac{7}{8} - \frac{2}{8}$ **2.** $\frac{4}{5} - \frac{3}{5}$ **3.** $\frac{6}{7} - \frac{1}{7}$

4. $\begin{array}{r} \frac{8}{10} \\ -\ \frac{4}{10} \\ \hline \end{array}$ **5.** $\begin{array}{r} \frac{5}{9} \\ -\ \frac{2}{9} \\ \hline \end{array}$ **6.** $\begin{array}{r} \frac{9}{12} \\ -\ \frac{6}{12} \\ \hline \end{array}$

Answers: **1.** $\frac{5}{8}$ **2.** $\frac{1}{5}$ **3.** $\frac{5}{7}$ **4.** $\frac{2}{5}$ **5.** $\frac{1}{3}$ **6.** $\frac{1}{4}$

Adding Unlike Fractions

Look at this problem. What do you notice about the denominators of the two fractions?

$$\begin{array}{r}\frac{5}{9}\\ +\frac{1}{3}\\ \hline \frac{\blacksquare}{\blacksquare}\end{array} \quad \text{or} \quad \frac{5}{9}+\frac{1}{3}=\frac{\blacksquare}{\blacksquare}$$

$\frac{5}{9}$ and $\frac{1}{3}$ are **unlike fractions.** They have different denominators. You must change the fractions so they both have the *same* denominator.

Here's how to add unlike fractions:

Step 1: First find the lowest common denominator of the fractions you are adding.

		×1	×2	×3
$\frac{1}{3}$	3	3	6	⑨
$\frac{5}{9}$	9	⑨	18	

LCD = 9

Step 2: Rename fractions.

Rename $\frac{1}{3}$ by raising it to the lowest common denominator—9. Why do you *not* have to rename $\frac{5}{9}$?

$$\begin{array}{r}\frac{5}{9} \longrightarrow \frac{5}{9}\\ +\frac{1\times 3}{3\times 3}=\frac{3}{9}\\ \hline \frac{\blacksquare}{\blacksquare}\end{array} \qquad \begin{array}{r}3\\ 3\overline{)9}\end{array} \qquad 3\times 1=3$$

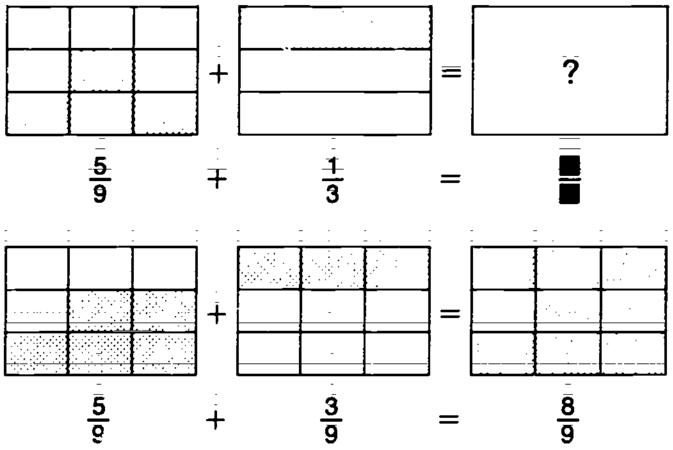
Step 3: Add the fractions.

Reduce the answer if you can. Rename an improper fraction.

$$\begin{array}{r}\frac{5}{9} \longrightarrow \frac{5}{9}\\ +\frac{1\times 3}{3\times 3}=\frac{3}{9}\\ \hline \frac{8}{9}\end{array} \quad \text{or} \quad \begin{array}{l}\frac{5}{9}+\frac{1}{3}=\\ \frac{5}{9}+\frac{3}{9}=\frac{8}{9}\end{array}$$

Can you reduce $\frac{8}{9}$?

Answer: $\frac{8}{9}$ cannot be reduced.



> **Remember:**
>
> To add unlike fractions (fractions with different denominators), do this:
>
> 1. Find the LCD of both fractions.
> 2. Rename fractions that have a different denominator.
> 3. Add the fractions.
> 4. Reduce the answer if you can.
> 5. Rename (simplify) an improper fraction.

Exercise

Copy and solve these addition problems. Reduce the answers if you can. Rename improper fractions. Then check your answers.

1. $\begin{array}{r}\frac{1}{3}\\ +\frac{2}{9}\\ \hline\end{array}$
2. $\begin{array}{r}\frac{1}{6}\\ +\frac{1}{4}\\ \hline\end{array}$
3. $\begin{array}{r}\frac{3}{10}\\ +\frac{1}{2}\\ \hline\end{array}$
4. $\frac{3}{4}+\frac{1}{2}$
5. $\frac{3}{4}+\frac{1}{3}$
6. $\frac{4}{5}+\frac{2}{3}$

Answers: 1. $\frac{5}{9}$ 2. $\frac{5}{12}$ 3. $\frac{4}{5}$ 4. $1\frac{1}{4}$ 5. $1\frac{1}{12}$ 6. $1\frac{7}{15}$

Subtracting Unlike Fractions

You know that you can only add fractions that have the same denominator. That's also true when you subtract fractions: They must have the same denominator.

Suppose you must subtract unlike fractions, such as these. Here's what to do:

$$\begin{array}{r} \frac{3}{4} \\ -\frac{1}{3} \\ \hline \frac{■}{■} \end{array} \quad \textit{or} \quad \frac{3}{4} - \frac{1}{3} = \frac{■}{■}$$

Step 1: First find the lowest common denominator of both fractions.

		×1	×2	×3	×4
$\frac{3}{4}$	4	4	8	12	16
$\frac{1}{3}$	3	3	6	9	12

LCD = 12

Step 2: Rename the fractions so they both have the same denominator (the LCD).

$$\begin{array}{r} \frac{3 \times 3}{4 \times 3} = \frac{■}{12} \\ -\frac{1 \times 4}{3 \times 4} = \frac{■}{12} \\ \hline \frac{■}{■} \end{array}$$

$4\overline{)12}$ 3   3 × 3 = 9

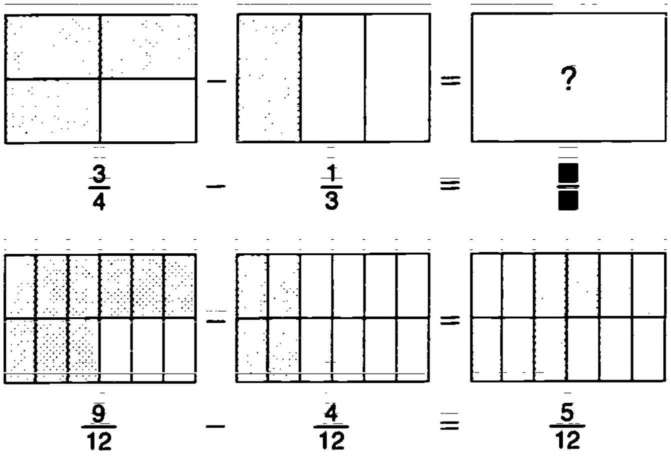
$3\overline{)12}$ 4   4 × 1 = 4

Step 3: Subtract the smaller fraction from the larger fraction. Reduce the answer if you can.

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ -\frac{1}{3} = \frac{4}{12} \\ \hline \frac{■}{■} \end{array} \quad \textit{or} \quad \frac{3}{4} - \frac{1}{3} = \quad \frac{9}{12} - \frac{4}{12} = \frac{■}{■}$$

What's the answer?

Answer: The answer is $\frac{5}{12}$.



$\frac{3}{4} - \frac{1}{3} = \frac{■}{■}$

$\frac{9}{12} - \frac{4}{12} = \frac{5}{12}$

Remember:

To subtract unlike fractions (fractions with different denominators):

1. Find the LCD of both fractions.
2. Rename fractions so both have the same denominator (the LCD).
3. Subtract the fractions.
4. Reduce the answer if you can.

Exercise

Copy and solve these subtraction problems. Reduce the answers if you can. Then check your answers.

1. $\begin{array}{r} \frac{5}{6} \\ -\frac{1}{3} \\ \hline \end{array}$ **2.** $\begin{array}{r} \frac{2}{3} \\ -\frac{4}{9} \\ \hline \end{array}$ **3.** $\begin{array}{r} \frac{5}{6} \\ -\frac{5}{12} \\ \hline \end{array}$

4. $\frac{2}{5} - \frac{1}{4}$ **5.** $\frac{3}{4} - \frac{1}{6}$ **6.** $\frac{5}{8} - \frac{2}{5}$

Answers: 1. $\frac{1}{2}$ 2. $\frac{2}{9}$ 3. $\frac{5}{12}$ 4. $\frac{3}{20}$ 5. $\frac{7}{12}$ 6. $\frac{9}{40}$

Subtracting from a Whole Number

You get a job painting a fence. You work 4 hours. You take $\frac{1}{2}$ hour of that time for lunch. You don't get paid for that $\frac{1}{2}$ hour. How much time do you get paid for?

To solve that problem, you subtract $\frac{1}{2}$ from the whole number 4. Here's how to solve that kind of problem:

$$\begin{array}{r} 4 \\ -\frac{1}{2} \\ \hline \blacksquare \end{array} \quad or \quad 4 - \frac{1}{2} = ?$$

Step 1: Change the whole number to a fraction.

$$\begin{array}{rcl} 4 & = & \frac{4}{1} \\ -\frac{1}{2} & = & \frac{1}{2} \\ \hline \blacksquare & & \blacksquare \end{array} \quad or \quad 4 - \frac{1}{2} = \frac{4}{1} - \frac{1}{2} = ?$$

Step 2: Find the LCD of both fractions.

You can easily see that the LCD is the same as the denominator in $\frac{1}{2}$.

		×1	×2
$\frac{4}{1}$	1	1	②
$\frac{1}{2}$	2	②	

Step 3: Rename the equivalent fraction.

$$\begin{array}{rcl} \frac{4}{1} & = & \frac{\blacksquare}{2} \\ -\frac{1}{2} & = & \frac{\blacksquare}{2} \\ \hline \end{array} \qquad \begin{array}{l} 1\overset{2}{\overline{)2}} \quad 2 \times 4 = 8 \\ 2\overset{1}{\overline{)2}} \quad 1 \times 1 = 1 \end{array}$$

Step 4: Subtract the fractions.

Reduce if you can. Then rename an improper fraction. What's the answer?

$$\begin{array}{rcl} \frac{4}{1} & = & \frac{8}{2} \\ -\frac{1}{2} & = & \frac{1}{2} \\ \hline & & \blacksquare \end{array} \quad or \quad 4 - \frac{1}{2} = \frac{4}{1} - \frac{1}{2} = \frac{8}{2} - \frac{1}{2} = \blacksquare$$

Answer: The answer is $3\frac{1}{2}$.

Subtracting from 1

Suppose you must solve this problem:

$$\begin{array}{rl} 1 & \text{hour} \\ -\frac{2}{3} & \text{hour} \\ \hline \blacksquare & \text{hour} \end{array} \quad or \quad 1 \text{ hour} - \frac{2}{3} \text{ hour} = ?$$

You can easily rename the whole number 1 as a fraction. That fraction should have the same denominator as $\frac{2}{3}$. What is that fraction?

$$1 = \frac{\blacksquare}{\blacksquare}$$

Right! $1 = \frac{3}{3}$. Now solve the problem. What's the answer?

$$\begin{array}{rcl} 1 & = & \frac{3}{3} \\ -\frac{2}{3} & = & \frac{2}{3} \\ \hline & & \frac{\blacksquare}{\blacksquare} \end{array} \quad or \quad 1 - \frac{2}{3} = \frac{3}{3} - \frac{2}{3} = \frac{\blacksquare}{\blacksquare}$$

Answer: The answer is $\frac{1}{3}$.

Exercise

Copy and solve these problems. Reduce the answers if you can. Rename improper fractions. Then check your answers.

1. $\begin{array}{r} 3 \\ -\frac{1}{2} \\ \hline \end{array}$ 2. $\begin{array}{r} 5 \\ -\frac{1}{4} \\ \hline \end{array}$ 3. $\begin{array}{r} 1 \\ -\frac{2}{3} \\ \hline \end{array}$ 4. $\begin{array}{r} 1 \\ -\frac{1}{4} \\ \hline \end{array}$

5. $8 - \frac{6}{8}$ 6. $10 - \frac{5}{6}$ 7. $1 - \frac{3}{6}$ 8. $1 - \frac{2}{8}$

Answers: 1. $2\frac{1}{2}$ 2. $4\frac{3}{4}$ 3. $\frac{1}{3}$ 4. $\frac{3}{4}$
5. $7\frac{1}{4}$ 6. $9\frac{1}{6}$ 7. $\frac{1}{2}$ 8. $\frac{3}{4}$

Subtracting from Mixed Numbers

Imagine this: You're making book shelves. You buy $3\frac{1}{2}$ boxes of nails. You use $\frac{2}{3}$ of a box. How many boxes do you have left?

To figure that out, you'd subtract $\frac{2}{3}$ from $3\frac{1}{2}$. In other words, you'd subtract a fraction from a mixed number.

$$\begin{array}{r} 3\frac{1}{2} \\ -\ \frac{2}{3} \\ \hline \blacksquare \end{array} \quad or \quad 3\frac{1}{2} - \frac{2}{3} = ?$$

You learned one way to subtract a fraction from a whole number: You rename the whole number as an improper fraction.

You can do the same thing when you subtract a fraction from a mixed number: You can rename the mixed number as an improper fraction. (If you're not sure how to do that, look at page 33.)

Here's how to solve a problem when you rename the mixed number:

Step 1: Rename the mixed number as an improper fraction.

$$\begin{array}{rcl} 3\frac{1}{2} & = & \frac{7}{2} \\ -\ \frac{2}{3} & = & \frac{2}{3} \\ \hline \blacksquare & & \blacksquare \end{array} \quad or \quad \begin{array}{l} 3\frac{1}{2} - \frac{2}{3} = \\ \frac{7}{2} - \frac{2}{3} = ? \end{array}$$

Step 2: Find the LCD of both fractions.

		×1	×2	×3
$\frac{1}{2}$	2	2	4	6
$\frac{2}{3}$	3	3	6	

LCD = 6

Step 3: Rename the fractions to the lowest common denominator.

What are the renamed fractions?

$$\begin{array}{rcccl} 3\frac{1}{2} & = & \frac{7}{2} & = & \frac{\blacksquare}{6} \\ -\ \frac{2}{3} & = & \frac{2}{3} & = & \frac{\blacksquare}{6} \\ \hline \end{array}$$

$2\overline{)6}$ with 3 above; $3 \times 7 = ▨$

$3\overline{)6}$ with 2 above; $2 \times 2 = ▨$

Step 4: Subtract the fractions.

What's the answer?

$$\begin{array}{rcccl} 3\frac{1}{2} & = & \frac{7}{2} & = & \frac{21}{6} \\ -\ \frac{2}{3} & = & \frac{2}{3} & = & \frac{4}{6} \\ \hline \end{array} \quad or \quad \begin{array}{l} 3\frac{1}{2} - \frac{2}{3} = \\ \frac{7}{2} - \frac{2}{3} = \\ \frac{21}{6} - \frac{4}{6} = \frac{\blacksquare}{\blacksquare} \end{array}$$

Answer: The answer is $\frac{17}{6} = 2\frac{5}{6}$.

Exercise

Copy and solve these subtraction problems. Show your math work. Reduce answers if you can. Rename improper fractions. Then check your answers.

1. $\begin{array}{r} 3\frac{1}{3} \\ -\ \frac{5}{6} \\ \hline \end{array}$ 2. $\begin{array}{r} 1\frac{1}{4} \\ -\ \frac{1}{3} \\ \hline \end{array}$ 3. $\begin{array}{r} 4\frac{1}{4} \\ -\ \frac{7}{8} \\ \hline \end{array}$

4. $\begin{array}{r} 2\frac{1}{5} \\ -\ \frac{9}{10} \\ \hline \end{array}$ 5. $\begin{array}{r} 1\frac{1}{3} \\ -\ \frac{2}{5} \\ \hline \end{array}$ 6. $\begin{array}{r} 9\frac{1}{8} \\ -\ \frac{3}{4} \\ \hline \end{array}$

Answers: 1. $2\frac{1}{2}$ 2. $\frac{11}{12}$ 3. $3\frac{3}{8}$ 4. $1\frac{3}{10}$ 5. $\frac{14}{15}$ 6. $8\frac{3}{8}$

Unit Review

Check Yourself

1. Copy and solve the problems below. Reduce answers if you can. Rename improper fractions.

Add these like fractions.

a. $\frac{2}{5} + \frac{1}{5}$

c. $\frac{2}{6} + \frac{3}{6}$

b. $\frac{3}{4} + \frac{2}{4}$

d. $\frac{3}{9} + \frac{3}{9}$

Subtract these like fractions.

e. $\frac{3}{5} - \frac{2}{5}$

g. $\frac{7}{9} - \frac{3}{9}$

f. $\frac{8}{10} - \frac{6}{10}$

h. $\frac{7}{12} - \frac{3}{12}$

Add these unlike fractions.

i. $\frac{1}{2} + \frac{1}{6}$

j. $\frac{2}{3} + \frac{5}{6}$

k. $\frac{1}{4} + \frac{2}{3}$

l. $\frac{2}{5} + \frac{1}{2}$

Subtract these unlike fractions.

m. $\frac{3}{4} - \frac{1}{8}$

n. $\frac{2}{3} - \frac{1}{6}$

o. $\frac{3}{5} - \frac{1}{2}$

p. $\frac{3}{4} - \frac{3}{5}$

Subtract the fractions from the whole or mixed numbers.

q. $1 - \frac{1}{3}$

r. $1 - \frac{2}{5}$

s. $3 - \frac{5}{9}$

t. $7 - \frac{3}{6}$

2. Choose the right word to complete each sentence.

denominators **numerators** **unlike**
like **rename**

a. ________ fractions have denominators that are the same.

b. ________ fractions have denominators that are different.

c. You add or subtract the ________ of fractions.

d. You do *not* add or subtract the ________ of fractions.

e. You ________ whole or mixed numbers when you subtract fractions from them.

Bonus Work

Solve these word problems.

1. There are 20 students in a math class. 3 students drive to school ($\frac{3}{20}$). 4 others ride bikes ($\frac{4}{20}$). 5 take a bus ($\frac{5}{20}$). The rest walk. What fraction of the class rides to school? (Hint: Add the fractions. Reduce your answer.)
2. $\frac{3}{8}$ of a cake is left after dinner. You get hungry later on and eat another $\frac{1}{8}$ of the cake. Now how much is left? (Hint: Subtract the fractions and reduce the answer.)

Dividing Fractions

Imagine this: You earn money by making and selling things. Right now, you are making birdhouses and doghouses.

You have $\frac{3}{4}$ of a quart of paint. You need $\frac{1}{8}$ of a quart of paint for each birdhouse. How many birdhouses can you paint?

To answer that question, you divide $\frac{3}{4}$ by $\frac{1}{8}$.

$$\frac{3}{4} \div \frac{1}{8} = 6$$

You can paint 6 birdhouses.

Suppose you paint 2 doghouses instead. How much paint can you use on each doghouse?

To answer that question, you divide $\frac{3}{4}$ by 2.

$$\frac{3}{4} \div 2 = \frac{3}{8}$$

You can use $\frac{3}{8}$ of a quart of paint on each doghouse.

You also earn money painting rooms. You have 2 gallons of paint. It takes $\frac{1}{5}$ of a gallon to paint one wall. How many walls can you paint?

You divide to find the answer.

$$2 \div \frac{1}{5} = 10$$

You can paint 10 walls.

You'll need to divide with fractions often in real life. When you divide a fraction into an amount, you find out how many times that fraction goes into that amount. The answer can tell you if you have enough of that amount.

What's another example in real life where you divide a fraction into an amount?

Math Words

Look up these words in the glossary. What do they mean?

divisor **reciprocal**

Finding the Reciprocal

When you divide a fraction problem, you must first do something with the **divisor.** What is a divisor?

A divisor is the number you divide another number *by.* In a problem, it is the number that's written *after* the division sign (÷). Which number is the divisor in this problem?

$\frac{3}{4} \div \frac{1}{8} = ?$

One way to find the divisor is to say the problem like this: "$\frac{3}{4}$ *divided by* $\frac{1}{8}$ equals what?" The words *divided by* tell you that $\frac{1}{8}$ is the divisor.

Here's what you must first do when you divide a fraction problem: Find the **reciprocal** of the divisor. A reciprocal is an "upside-down" fraction. It is a fraction whose terms have been switched. For example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$. And the reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$. Here are other examples:

Fraction		**Reciprocal**
$\frac{2}{3}$	. . .	$\frac{3}{2}$
$\frac{1}{2}$	. . .	$\frac{2}{1}$
$\frac{5}{1}$	. . .	$\frac{1}{5}$
$\frac{6}{5}$	. . .	$\frac{5}{6}$

To find the reciprocal of a fraction, you simply do this: Write the fraction so its numerator is now its denominator and its denominator is now its numerator.

Look at this problem again. What is the reciprocal of its divisor?

$\frac{3}{4} \div \frac{1}{8} = ?$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.

Answer: The reciprocal is $\frac{8}{1}$.

Remember:

When you divide fraction problems remember that:

- the divisor is the number *after* the division sign;
- to find the reciprocal of a fraction, turn it "upside-down" (switch its numerator and denominator).

Exercise

Copy these problems. Draw a circle around each divisor. Then find the reciprocal of that divisor.

1. $\frac{1}{2} \div \frac{3}{4}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
2. $\frac{4}{5} \div \frac{1}{2}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
3. $2\frac{1}{2} \div \frac{1}{3}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
4. $\frac{3}{5} \div \frac{4}{3}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
5. $\frac{2}{7} \div \frac{3}{7}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
6. $\frac{14}{3} \div \frac{9}{7}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
7. $\frac{4}{12} \div \frac{24}{5}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.
8. $\frac{9}{1} \div \frac{9}{4}$ The reciprocal is $\frac{\blacksquare}{\blacksquare}$.

Answers: 1. $\frac{4}{3}$ 2. $\frac{2}{1}$ 3. $\frac{3}{1}$ 4. $\frac{3}{4}$
5. $\frac{7}{3}$ 6. $\frac{7}{9}$ 7. $\frac{5}{24}$ 8. $\frac{4}{9}$

Dividing a Fraction by a Fraction

Suppose you are solving this problem:

$$\frac{3}{4} \div \frac{1}{8} = ?$$

Here's what to do:

Step 1: Find the reciprocal of the divisor.

Circle the divisor. Then write the divisor's reciprocal under it.

$$\frac{3}{4} \div \underset{\frac{8}{1}}{\left(\frac{1}{8}\right)} = ?$$

Step 2: Rewrite the division problem as a multiplication problem.

First change the division sign (÷) to a multiplication sign (×). Then replace the divisor with its reciprocal.

$$\frac{3}{4} \div \underset{\frac{8}{1}}{\left(\frac{1}{8}\right)} = \frac{3}{4} \times \frac{8}{1}$$

Step 3: Simplify the problem if you can.

$$\frac{3}{4} \div \underset{\frac{8}{1}}{\left(\frac{1}{8}\right)} = \frac{3}{\cancelto{1}{4}} \times \frac{\cancelto{2}{8}}{1}$$

Step 4: Multiply the fractions.

Multiply the denominators. Multiply the numerators. Reduce your answer if you can. Rename an improper fraction.

$$\frac{3}{4} \div \underset{\frac{8}{1}}{\left(\frac{1}{8}\right)} = \frac{3}{\cancelto{1}{4}} \times \frac{\cancelto{2}{8}}{1} = \frac{6}{1} = 6$$

Now solve this problem. Copy the problem. Fill in the missing sign and numbers. Then check your work.

$$\frac{3}{8} \div \underset{\frac{\blacksquare}{\blacksquare}}{\frac{1}{4}} = \frac{3}{\underset{\blacksquare}{8}} \blacksquare \frac{\overset{\blacksquare}{\blacksquare}}{\blacksquare} = \frac{3}{2} = 1\frac{\blacksquare}{\blacksquare}$$

Answer:

$$\frac{3}{8} \div \underset{\frac{4}{1}}{\left(\frac{1}{4}\right)} = \frac{3}{\cancelto{2}{8}} \times \frac{\cancelto{1}{4}}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Remember:

To divide a fraction by a fraction, do this:

1. Find the reciprocal of the divisor.
2. Rewrite the division problem as a multiplication problem. Simplify if you can.
3. Multiply the denominators. Multiply the numerators.
4. Reduce the answer if you can. Rename an improper fraction.

Exercise

Copy and solve these division problems. Simplify if you can. Reduce answers. Rename improper fractions.

1. $\frac{2}{3} \div \frac{3}{5}$
2. $\frac{1}{6} \div \frac{3}{8}$
3. $\frac{1}{3} \div \frac{1}{5}$
4. $\frac{5}{8} \div \frac{1}{4}$
5. $\frac{2}{3} \div \frac{1}{9}$
6. $\frac{3}{5} \div \frac{9}{10}$

Answers: 1. $1\frac{1}{9}$ 2. $\frac{4}{9}$ 3. $1\frac{2}{3}$
4. $2\frac{1}{2}$ 5. 6 6. $\frac{2}{3}$

Dividing with Fractions and Whole or Mixed Numbers

Here's how to solve division problems that have whole or mixed numbers and fractions. See if you can answer the questions about each step.

Step 1: Rename the whole number.

$4 \div \frac{3}{4}$

a. How do you change a whole number to a fraction? (See pages 19 and 20.)

$4 \div \frac{3}{4} = \frac{4}{1} \div \frac{3}{4}$

Rename the mixed number.

$\frac{4}{9} \div 2\frac{2}{3}$

b. How do you change a mixed number to a fraction? (See page 33.)

$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \frac{8}{3}$

Step 2: Find the reciprocal of the divisor.

a. How do you find the reciprocal of the divisor in this problem?

$4 \div \frac{3}{4} = \frac{4}{1} \div \underset{\frac{4}{3}}{\enclose{circle}{\frac{3}{4}}}$

b. How do you find the reciprocal of the divisor in this problem?

$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \underset{\frac{3}{8}}{\enclose{circle}{\frac{8}{3}}}$

Step 3: Rewrite the division problem as a multiplication problem.

a. How would you rewrite this problem?

$4 \div \frac{3}{4} = \frac{4}{1} \div \underset{\frac{4}{3}}{\enclose{circle}{\frac{3}{4}}} = \frac{\blacksquare}{\blacksquare} \blacksquare \frac{\blacksquare}{\blacksquare}$

b. How would you rewrite this problem?

$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \underset{\frac{3}{8}}{\enclose{circle}{\frac{8}{3}}} = \frac{\blacksquare}{\blacksquare} \blacksquare \frac{\blacksquare}{\blacksquare}$

Step 4: Simplify the problem if you can.

a. Can you simplify this problem?

$4 \div \frac{3}{4} = \frac{4}{1} \div \underset{\frac{4}{3}}{\enclose{circle}{\frac{3}{4}}} = \frac{4}{1} \times \frac{4}{3}$

b. How would you simplify this problem?

$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \underset{\frac{3}{8}}{\enclose{circle}{\frac{8}{3}}} = \frac{4}{9} \times \frac{3}{8}$

Step 5: Multiply the fractions and find the answer.

a. Copy the problem and solve it. What's the answer?

$4 \div \frac{3}{4} = \frac{\blacksquare}{\blacksquare} \div \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} \blacksquare \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} = \blacksquare\frac{\blacksquare}{\blacksquare}$

b. Copy this problem and solve it. What's the answer?

$\frac{4}{9} \div 2\frac{2}{3} = \frac{\blacksquare}{\blacksquare} \div \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare} \blacksquare \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}$

Answer:

$4 \div \frac{3}{4} = \frac{4}{1} \div \overset{\frac{4}{3}}{\enclose{circle}{\frac{3}{4}}} = \frac{4}{1} \times \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3}$

Answer:

$\frac{4}{9} \div 2\frac{2}{3} = \frac{4}{9} \div \overset{\frac{3}{8}}{\enclose{circle}{\frac{8}{3}}} = \frac{\overset{1}{\cancel{4}}}{\underset{3}{\cancel{9}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{2}{\cancel{8}}} = \frac{1}{6}$

Unit Review

Check Yourself

1. Find the reciprocal of each fraction and whole number.

a. $\frac{3}{4}$

b. $\frac{2}{3}$

c. $\frac{3}{8}$

d. 2

e. 7

f. 5

2. Rewrite these division problems as multiplication problems, using reciprocals.

a. $\frac{2}{5} \div \frac{7}{8}$

b. $\frac{7}{9} \div \frac{3}{7}$

c. $\frac{3}{5} \div \frac{5}{9}$

d. $\frac{4}{7} \div 3$

e. $\frac{3}{4} \div 6$

f. $\frac{5}{6} \div 4$

3. Copy and solve these division problems. Simplify before multiplying if you can. Reduce answers. Rename improper fractions.

Divide these fractions.

a. $\frac{2}{7} \div \frac{1}{3}$

b. $\frac{1}{3} \div \frac{2}{5}$

c. $\frac{5}{8} \div \frac{2}{3}$

d. $\frac{2}{5} \div \frac{3}{4}$

Divide these fractions and whole or mixed numbers.

e. $\frac{5}{8} \div 3$

f. $\frac{3}{5} \div 4\frac{1}{5}$

g. $4 \div \frac{4}{5}$

h. $2\frac{1}{2} \div \frac{1}{4}$

4. Choose the right word to complete each sentence.

divisor **mixed** **upside**
fraction **multiply**

a. The fraction or whole number that follows the division sign is called the ________.

b. To find the reciprocal of a fraction, turn the fraction ________ down.

c. To divide with a whole number and a fraction, first change the whole number to a ________.

d. After you rewrite the division problem with a new sign and a reciprocal, ________ to find the answer.

e. Change answers that are improper fractions to whole or ____ numbers.

Bonus Work

1. Make up some fraction division problems. Then ask your classmates to find the answers.
2. Make a poster showing how to do these three kinds of division problems:
   - dividing a fraction by a fraction;
   - dividing a fraction by a whole number;
   - dividing a whole number by a fraction.

Congratulations!

You now understand about simple fractions. You know how to:

- show different kinds of fractions;
- name fractions;
- find equivalents;
- add, subtract, multiply, and divide simple fractions.

Glossary

a mount How much there is; the sum or total.

com mon The same.

com mon de nom i na tor A denominator that is the same in two or more fractions. $\frac{1}{4}$ and $\frac{3}{4}$ have a common denominator.

de nom i na tor The bottom number of a fraction. 4 is the denominator of $\frac{1}{4}$.

di vi sor The number you divide by in a division problem. The divisor always comes *after* the division sign. In this problem, $8 \div 2$, the divisor is 2.

e qual Having the same amount; having the same size and shape.

e quiv a lents Numbers that equal the same amount.

fac tors The numbers you multiply in a multiplication problem. In the problem 2×3, the factors are 2 and 3.

frac tion A number that stands for an equal part of a whole thing or whole group. Some fractions are $\frac{1}{8}$, $\frac{1}{4}$, and $\frac{1}{2}$.

im prop er frac tion A fraction whose numerator is the same as or larger than its denominator. $\frac{3}{3}$ and $\frac{6}{4}$ are improper fractions.

like frac tions Fractions with the same denominator. $\frac{1}{5}$ and $\frac{3}{5}$ are like fractions.

low est com mon de nom i na tor The lowest number that is a multiple of two or more denominators. 12 is the lowest common denominator of $\frac{2}{3}$ and $\frac{1}{4}$.

low est terms Numbers in a fraction that can't be reduced any further. The fraction $\frac{1}{2}$ has lowest terms.

mixed num ber A number that is made up of a whole number and a fraction. $3\frac{1}{5}$ is a mixed number.

mul ti ple The answer you get when you multiply one whole number by another whole number. Both numbers must be larger than 0. 12 is a multiple of 4 because $4 \times 3 = 12$.

name To say what the numbers are in a fraction, whole number, or mixed number.

nu mer a tor The top number of a fraction. 2 is the numerator of $\frac{2}{3}$.

pro per fraction A fraction whose denominator is larger than its numerator. $\frac{2}{5}$ is a proper fraction.

rais ing terms Renaming a fraction as an equivalent with higher terms. When you rename $\frac{1}{2}$ as $\frac{2}{4}$, you are raising terms.

re cip ro cal The fraction you get when you turn a fraction upside down. The reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$.

re duc ing terms Renaming a fraction as an equivalent with lower terms. When you rename $\frac{2}{4}$ as $\frac{1}{2}$, you are reducing terms.

rename To change the terms of a fraction; to change one kind of number to another kind of number.

sim plest form The lowest terms of a fraction, whole number, or mixed number.

sim pli fy To rename a fraction, whole number, or mixed number as its simplest form.

sim pli fy be fore mul ti ply ing To divide the numerator of one fraction and the denominator of another fraction by the same number. You do this before multiplying the two fractions to save having to reduce the answer.

terms The numbers in the numerator and denominator of a fraction.

un e qual Having a different amount; having a different size and shape.

un like frac tions Fractions with different denominators. $\frac{2}{3}$ and $\frac{3}{4}$ are unlike fractions.

whole num ber A number that stands for whole things or groups. 1, 2, and 3 are whole numbers.

Janus Books
a division of
Janus Book Publishers, Inc.
2501 Industrial Parkway West
Hayward, CA 94545
(415) 887-7070

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ISBN 0-8810[illegible]-044-3