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ABSTRACT

This workbook is designed as an easy-to-read, slower-paced mathematics text for students who have learning, reading, and language problems. Helping students fulfill mathematics requirements for graduation is a goal; the book can be used as the core or supplement to the mathematics curriculum in mainstreamed or special education classrooms, in mathematics laboratories, or as part of sheltered workshop and vocational training programs. Basic concepts and skills about decimals and percents, including computation with decimals and using proportions to solve percent problems, are presented. All 10 units begin with a brief discussion of how decimals or percents are used in real life and list a few key words. Lessons teach only one major concept per page. A comprehension exercise ends each lesson, usually with answers given on the page so students can quickly check their mastery. Most lessons are reinforced with at least one worksheet, available in the teacher's guide. The units are titled: Decimals Are Part of Our Lives, Comparing Decimals, Adding and Subtracting Decimals, Multiplying Decimals, Dividing Decimals, Percents, Ratios, Proportions, Solving Percent Problems, and Renaming Decimals and Percents. A short glossary is included. (MNS)

MATH IN ACTION

Rose Lock • Evelyn Morabe-Murphy

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MATH IN ACTION

DECIMALS and PERCENTS

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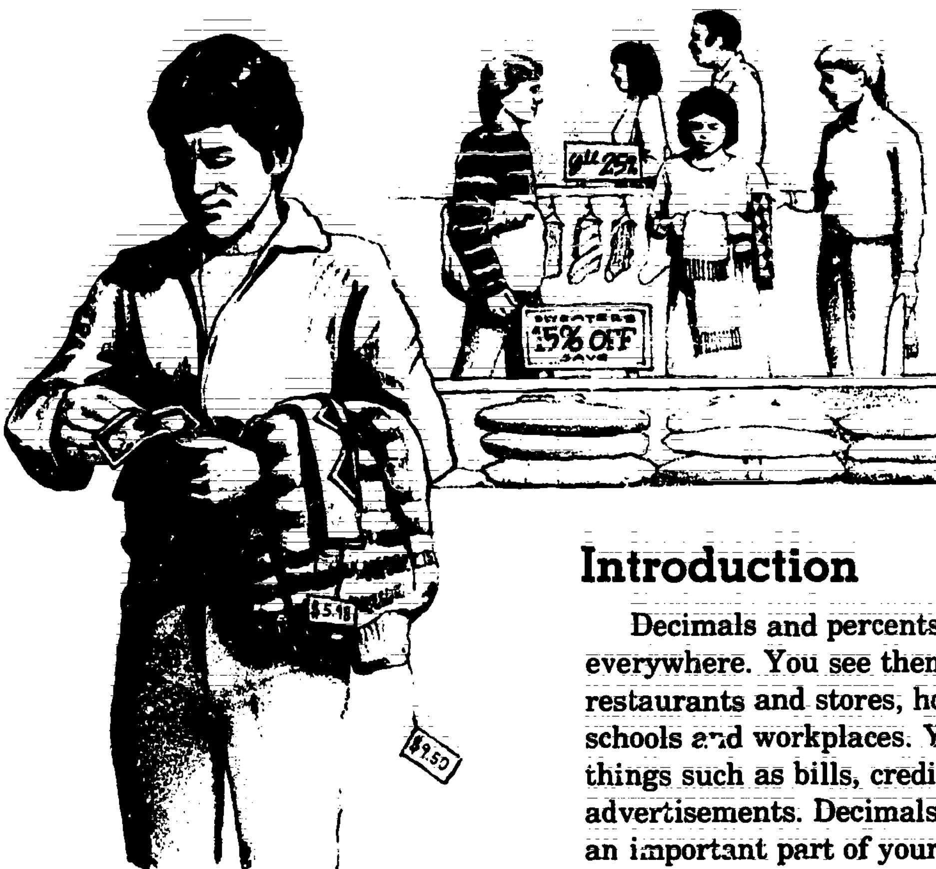


Introduction

Decimals and percents! You see them everywhere. You see them in places such as restaurants and stores, hospitals and banks, schools and workplaces. You see them on things such as bills, credit applications, and advertisements. Decimals and percents are an important part of your everyday life.

But exactly what are decimals?

And exactly what are percents?

How do we use them to solve everyday problems?

You'll learn answers to those questions in this book. You'll learn to understand decimals and percents. You'll learn how to read and write them correctly. And you'll learn how to solve problems that use decimals or percents.

Many of us have a hard time solving problems that use decimals or percents. For example:

a. *28 is what percent of 112?*
b. *What is 15% of $48.17?*
c. *$15.10 is 25% of what amount?*

Which of those problems are hard for you? Write them on a piece of paper, and put the paper away in a place where you can find it later. Then, when you finish this book, bring out that paper. See how easily you can solve those problems by using what you've learned!

Unit 1

Decimals Are Part of Our Lives

Imagine this:

You and some friends are driving to a place called River City. You need gas for your car. You see this sign at a gas station:

GAS
$1.09 per gallon

You stop at that gas station and fill your gas tank. You look at the gas pump to see how much you must pay. The pump shows you used 9.8 gallons. It also shows the cost of the gas: $10.68.

Now that your car is full of gas, you get on the freeway. You read a sign that tells you how far it is to River City:

River City
32.5 Miles

$1.09 per gallon; **9.8** gallons; **$10.68** total; **32.5** miles. Those are all a kind of number we use every day in real life. Those numbers are **decimals**.

We use decimals when we want to describe amounts of things. For example, we use decimals to talk about the money we spend, the gas we buy, and the miles we travel. What else might we use decimals to talk about?

Decimals are a special kind of number. In this unit, you'll learn why they are special. You'll learn how to read and write decimals. And you'll learn how to use decimals to show different kinds of amounts.

Math Words

Look up these words in the glossary at the back of the book. Find out what they mean.

digit **numeral** **place value**

Showing Parts of Whole Amounts

Fractions are parts of whole amounts.

You can show fractions by using a *fraction bar* (A fraction bar is the line between the denominator and the numerator.) Those kinds of fractions are called **common fractions**. $\frac{2}{10}$ is an example of a common fraction.

You can also show fractions by using a **decimal point**. Fractions that use a decimal point are called **decimal fractions** or just **decimals**. .25 is an example of a decimal fraction.

You've used common fractions to show:

- a part of a whole amount ($\frac{1}{2}$),
- a whole amount ($\frac{3}{1}$), and
- a whole amount plus a fraction ($1\frac{1}{2}$).

Decimals can also show those same kinds of amounts.

A Part of a Whole Amount

These decimals show part of an amount, or a fraction. Say them out loud like this: *point five*. Are the numbers written at the *left* or *right* of the decimal point?

.5 .33 .700

The numbers are written at the *right* of the decimal point. Numbers written that way show a part of a whole amount, or a fraction.

A Whole Amount

These decimals show whole amounts. Say them out loud like this: *nine point zero*. Are the whole amounts shown at the right or left of the decimal point?

9.0 141.00 22.000

Numbers written at the *left* of the decimal points show whole amounts. The zeros (0) at the right of the decimal point show there are no fractions. When you show whole amounts, you can write one or more zeros after the decimal point.

A Whole Amount and a Fraction

These decimals show a whole amount and a decimal fraction. Fractions like these are called **mixed decimals**. Say them out loud. On which side of the decimal point is the whole amount? On which side is the fraction?

3.5 1.51 118.90 1.033

> Remember:
>
> The decimal point separates whole numbers from fractions.
>
> - Numbers at the left of the decimal point show whole amounts.
> - Numbers at the right of the decimal point show fractions (parts of a whole amount).

Exercise

Answer these questions. Then check your answers.

1. What kind of amounts are shown at the left of the decimal point?
2. What kind of amounts are shown at the right of the decimal point?
3. Copy these numbers. Next to each number write: *decimal fraction, whole amount,* or *mixed decimal.*

a. 2.0	**e.** 25.000
b. 36.1	**f.** 1.355
c. .50	**g.** 228.25
d. .600	**h.** 35.08

Answers: 1. whole amount 2. decimal fractions 3. a. whole amount b. mixed decimal c. decimal fraction d. decimal fraction e. whole amount f. mixed decimal g. mixed decimal h. mixed decimal

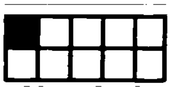
The Powers of Ten

All fractions have denominators. Their denominators show how many equal parts make up a whole amount. A whole amount can be divided into different numbers of equal parts. For example, **box A** is divided into five equal parts. Each part is $\frac{1}{5}$ of the whole amount. That means each part is *one of five parts*.

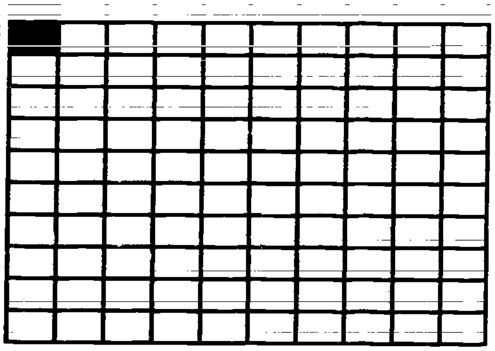
A  **Each part is $\frac{1}{5}$**

But decimals are a special kind of fraction. They have only one kind of denominator. That's because decimals divide whole amounts into only a certain number of equal parts. That number must be 10, or a ***power*** of 10. (That means 10×10, or $10 \times 10 \times 10$, or $10 \times 10 \times 10 \times 10$, and so on.) 100 and 1,000 are examples of numbers that are powers of ten.

Look at **box B**. It is divided into ten equal parts. Each part is 1 of 10 parts, or $\frac{1}{10}$. We'd write the decimal **.1** to show that each part is one of ten parts. .1 is the same amount as $\frac{1}{10}$.

B  **Each part is $\frac{1}{10}$ or .1**

Now look at **box C**. It is divided into 100 equal parts. Each part is 1 of 100 parts, or $\frac{1}{100}$. We'd write the decimal **.01** to show that each is 1 of 100 parts. .01 is the same amount as $\frac{1}{100}$.

C  **Each part is $\frac{1}{100}$ or .01**

Think of a dollar. It is a whole amount that can be divided into 100 pennies. That's 100 equal parts. How would you write each part as a decimal?

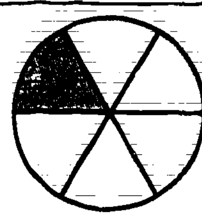
Right! **$.01** is the answer.

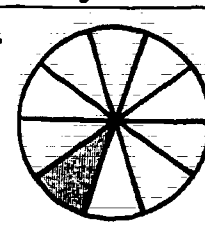
Exercise

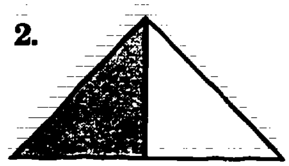
Look at the shapes below. Which shapes are divided into decimal fractions? Which shapes are divided into other fractions? Number a sheet of paper from 1 to 6. Then do this:

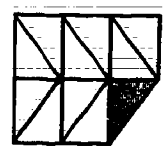
- Write the decimal **.1** for the shapes that have decimal fractions.
- Write a common fraction for the other shapes. (Example: $\frac{1}{5}$.) Be sure to use the correct denominators.

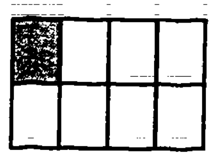
When you are finished, check your answers.

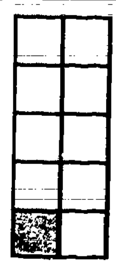
1. 

4. 

2. 

5. 

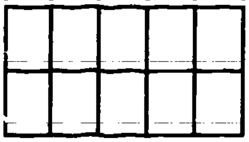
3. 

6. 

Answers: 1. $\frac{1}{6}$ 2. $\frac{1}{2}$ 3. $\frac{1}{8}$ 4. .1 5. $\frac{1}{11}$ 6. .1

Fractions or Decimals

Look at this box. It is divided into ten equal parts.



Each part is $\frac{1}{10}$, or a fraction. (From now on, when we say *fraction* we mean *common fraction.*) Each part is also .1, or a decimal. Here's how you'd count 9 of the parts as decimals:

.1 .2 .3 .4 .5 .6 .7 .8 .9

How would you count all ten parts as decimals?

All ten parts make a whole amount. That whole amount is equal to 1, a whole number. (Remember: whole amounts are written at the left of a decimal point.) So, 1.0 (or 1.00) is the decimal number that shows all ten parts. You'd count all ten parts as decimals like this:

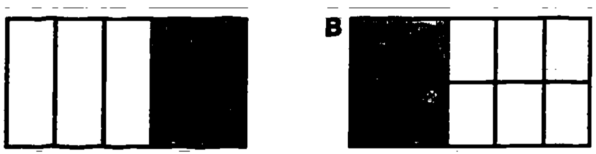
.1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 (or 1.00)

Fractions and decimals such as $\frac{1}{10}$ and .1 are **equivalents**. That means they equal the same amount. All decimals have equivalent fractions.

Compare These Boxes

You can see how decimals and fractions are equivalents. Box A is divided into fractions. Box B is divided into decimals. The shaded parts in both boxes are equal in amount. What fraction describes the shaded parts in box A? What decimal describes the shaded part in box B?

A B



Box A has $\frac{2}{5}$ parts. Box B has .4 parts.
So: $\frac{2}{5} = .4$

Exercise

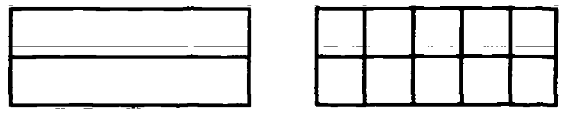
To find the answer, count the equivalent parts in each box.

Write the decimal equivalents of these fractions.

1. a. $\frac{1}{2} = .5$

b. $\frac{2}{2} =$ ■

Each part is $\frac{1}{2}$ — Each part is .1

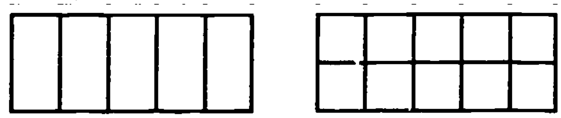


2. a. $\frac{3}{5} = .6$

b. $\frac{1}{5} =$ ■

c. $\frac{4}{5} =$ ■

Each part is $\frac{1}{5}$ — Each part is .1

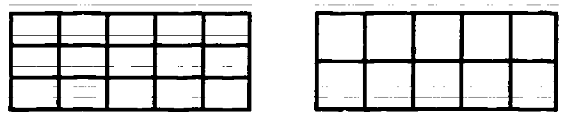


3. a. $\frac{6}{15} = .4$

b. $\frac{12}{15} =$ ■

c. $\frac{9}{15} =$ ■

Each part is $\frac{1}{15}$ — Each part is .1

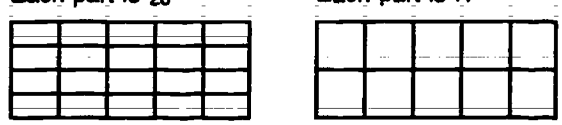


4. a. $\frac{4}{20} =$ ■

b. $\frac{8}{20} =$ ■

c. $\frac{20}{20} =$ ■

Each part is $\frac{1}{20}$ — Each part is .1



Answers: 1. a. $.5 = \frac{1}{2}$ **b.** $1.0\ (1.00) = \frac{2}{2}$
2. a. $.6 = \frac{3}{5}$ **b.** $.2 = \frac{1}{5}$ **c.** $.8 = \frac{4}{5}$
3. a. $.4 = \frac{6}{15}$ **b.** $.8 = \frac{12}{15}$ **c.** $.6 = \frac{9}{15}$
4. a. $.2 = \frac{4}{20}$ **b.** $.4 = \frac{8}{20}$ **c.** $1.0\ (1.00) = \frac{20}{20}$

Every Place Has a Value

You know that a decimal point separates whole numbers from decimal fractions. Whole numbers and decimal fractions are written with ***digits.*** (Digits are the numbers 1 to 9 and 0.)

Look at the digits in this number:

7246.135

What digits are in the whole number? What digits are in the decimal fraction?

A Place for Each Digit

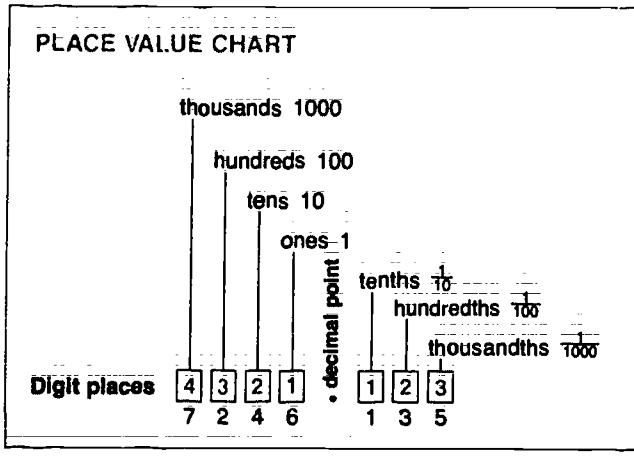
Every digit is in a certain ***place.*** For example, the digit 6 is *one place* left of the decimal point. The digit 4 is *two places*, the digit 2 is *three places*, and the digit 7 is *four places* left of it. How many places right of the decimal point is the digit 1? How many places right is the digit 3?

Each digit has a ***place value.*** That means each digit has a certain number value because of the place it is in.

Look at the place value chart. It shows the place value of each digit. Find the decimal point. Which side of the decimal point shows place values for the whole number? Which side shows place values for the decimal fraction?

Whole Number Places

The places at the left of the decimal point are for digits in whole numbers. Each place has a value that is a certain whole amount. For example, the first place next to the decimal point has the value of *ones.* The second place has the value of *tens.* What value does the third place have? What value does the fourth place have?



Decimal Places

Now look at the places that are right of the decimal point. Those places are for digits in decimal fractions. Each place has a value that is a fraction. For example, the first place has the value of *tenths.* The second place has the value of *hundredths.* What is the value of the third place?

Now look at these digits: 7246.135
What is the place value of each digit?

Answer:	Whole numbers	Decimals
	7 - thousands	1 - tenths
	2 - hundreds	3 - hundredths
	4 - tens	5 - thousandths
	6 - ones	

Exercise

Look at the digit that's underlined in each decimal number. What value does it have? Check your answers.

1. 3.$\underline{1}$8
2. 6.73$\underline{5}$
3. $\underline{4}$1.21
4. 19.0$\underline{7}$
5. $\underline{6}$34.21
6. .7$\underline{8}$3
7. 1$\underline{2}$.09
8. $\underline{2}$,890.13

Answers: 1. tenths 2. thousandths 3. tens 4. hundredths 5. hundreds 6. hundredths 7. ones 8. thousands

Reading Decimals

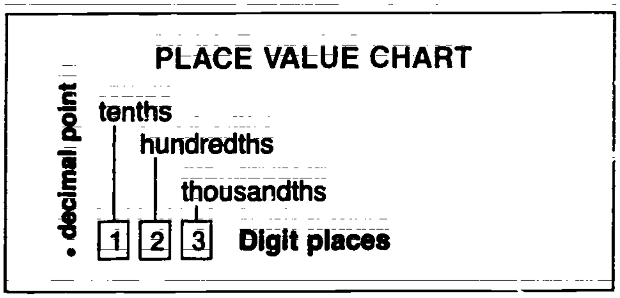
When you read a common fraction out loud, you say the ***numerator*** and the ***denominator.*** You read a decimal fraction in exactly the same way.

In a common fraction, the *numerator* is the number above the fraction bar. It shows how many equal parts are counted out of a whole amount. The *denominator* is the number under the fraction bar. It shows how many equal parts the whole amount is divided into.

In decimal fractions, the numerator is the ***numeral*** at the right of the decimal point. For example, the numerator for the decimal fraction .25 is the numeral 25. How can you tell what the denominator is?

You can tell the denominator of a decimal by counting the digits in the numerator. The place value of the *last* digit shows the denominator of the decimal.

Look at the place value chart. What is the value of the first place? Of the second place? Of the third place?



Decimals of Tenths

Look at this decimal: **.2**

There is one digit in the numerator. The digit is one place right of the decimal point. The value of that first decimal place is *tenths*. So, you'd read the decimal as *two tenths* (2 tenths). It sounds the same as saying the common fraction $\frac{2}{10}$ out loud.

Decimals of Hundredths

Look at this decimal: **.25**

There are two digits in the numerator. The last digit is two places right of the decimal point. What is the value of that second decimal place? How would you read the decimal?

The value of the second decimal place is *hundredths*. So you'd read .25 as *twenty-five hundredths* (25 hundredths)—just like the common fraction $\frac{25}{100}$.

Decimals of Thousandths

Now look at this decimal: **.251**

There are three digits in the numerator. The last digit is three places right of the decimal point. What is the value of the third decimal place? How would you read the decimal?

Answer: The value of the third decimal place is thousandths. You'd read the decimal .251 as two hundred fifty-one thousandths (251 thousandths or $\frac{251}{1000}$).

Exercise

With a partner, take turns reading these decimals out loud. Check your answers.

1. .1	**4.** .252
2. .8	**5.** .219
3. .17	**6.** .99

Answers: **1.** one tenth **2.** eight tenths **3.** 17 hundredths **4.** 252 thousandths **5.** 219 thousandths **6.** 99 hundredths

Writing Decimals

You learned that each place after a decimal point has the value of a certain fraction. A digit that fills the first decimal place has a value of tenths. A digit that fills the second decimal place has a value of hundredths. And a digit that fills the third decimal place has a value of thousandths.

Of course a decimal can have more than three digits after the decimal. (For example, you'd write six digits to show a millionth of an amount.) But the decimals you'll use the most will be tenths, hundredths, and thousandths.

Count Digits to be Sure

It's easy to make a mistake when you're writing decimals. You may mean to write the decimal that shows hundredths. But you write a decimal that shows tenths.

How can you make sure you are writing the correct decimal?

One way is to *count* the digits that fill the places behind the decimal point. Each decimal fraction has only a certain number of digits.

There's an easy way to remember the right number of digits: Look at these common fractions and their decimals. How many zeros are in the denominator? How many digits are after the decimal point?

$\frac{1}{10}$ .1 $\frac{21}{100}$ .21 $\frac{345}{1000}$ .345

Notice this: In each set, the number of digits in the decimal is the *same* as the number of zeros in the denominator.

How many digits should there be in these decimal fractions?

a. tenths.......... ■ digits
b. hundredths ..... ■ digits
c. thousandths .... ■ digits

Answers: a. tenths - 1 digit
b. hundredths - 2 digits
c. thousandths - 3 digits

Write It Down

Here are some steps that can help you write decimals correctly:

1. Write the decimal point *first*.
2. Then write the numerator after the decimal point. The place value of the last digit should show the denominator of the fraction.

Then Check It

To check the decimal you've written, follow these steps:

1. Write the amount as a common fraction.
2. Look at the denominator of the fraction. Count the zeros. Look at the decimal. Count the digits after the decimal point. There should be the same number of digits after the decimal point as there are zeros in the denominator.

Exercise

On a sheet of paper, write decimals for these amounts. Follow the steps above and check your decimals.

1. Four tenths
2. Forty-two hundredths
3. Eight tenths
4. One hundred sixty-eight thousandths
5. Thirty hundredths
6. Sixteen hundredths
7. Nine tenths
8. Thirteen hundredths
9. Three hundred thousandths
10. One tenth

Filling All the Places

Hundredths

Suppose you are writing the decimal for $\frac{3}{100}$. Its denominator is 100. You know you must fill two decimal places to show that denominator. That means you will write two digits after the decimal point.

But look at the numerator of $\frac{3}{100}$. How many digits are in the numerator?

Right! There is only *one* digit in the numerator—the digit 3. Here's how to fill the two decimal places when there is only one digit in the numerator: Write a zero *in front* of that digit.

1. **Write the decimal point.**
   Then write a zero in the first decimal place.

   tenths hundredths thousandths

   . 0 _ _

2. **In the second decimal place, write the digit of the numerator.**
   What is the place value of that digit?

   tenths hundredths thousandths

   . 0 3 _

   You read .03 as *three hundredths.*

Exercise

Write the decimals that show these amounts. Then check your answers.

1. $\frac{1}{100}$
2. $\frac{9}{10}$
3. $\frac{3}{10}$
4. $\frac{5}{100}$
5. $\frac{7}{100}$
6. $\frac{6}{10}$

Answers: 1. .01 2. .9 3. .3 4. .05 5. .07 6. .6

Thousandths

Now, suppose you are writing a decimal for $\frac{4}{1000}$. There is only one digit in the numerator. But you must fill three decimal places when the denominator is 1,000. To do that, write zeros in front of that digit.

1. **Write the decimal point.**
   Then in the first decimal place, write a zero.

   tenths hundredths thousandths

   . 0 _ _

2. **Write another zero in the second decimal place.**

   tenths hundredths thousandths

   . 0 0 _

3. **Then write the digit of the numerator.**

   tenths hundredths thousandths

   . 0 0 4

   You read .004 as *four thousandths.*

Now, suppose you are writing a decimal for $\frac{23}{1000}$. How would you write that decimal?

Answer: $\frac{23}{1000}$ = .023

Exercise

Write the decimals that show these amounts. Then check your answers.

1. $\frac{5}{100}$
2. $\frac{32}{1000}$
3. $\frac{18}{1000}$
4. $\frac{3}{1000}$
5. $\frac{7}{100}$
6. $\frac{99}{1000}$

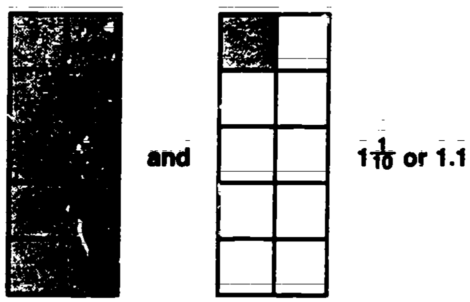
Answers: 1. .05 2. .032 3. .018 4. .003 5. .07 6. .099

Whole Amounts and Decimals

Reading Mixed Decimals

An amount made up of a whole number and a common fraction is called a **mixed number.** $1\frac{1}{10}$ is an example of a mixed number. To read $1\frac{1}{10}$ you'd say *one <u>and</u> one tenth.*

An amount made up of a whole number and a decimal fraction is called a **mixed decimal.** 1.1 is an example of a mixed decimal. To read it you'd say *one <u>and</u> one tenth.* Notice this: You say a mixed decimal exactly the way you say a mixed number.



The decimal point separates the whole number from the decimal fraction. The whole number is written at the left of the decimal point. The fraction is written at the right. You say *and* when you see the decimal point.

Practice reading these decimals out loud:

a. 2.5 (two <u>and</u> five tenths)
b. 7.03 (seven <u>and</u> three hundredths)
c. 6.25 (six <u>and</u> 25 hundredths)
d. 1.333 (one <u>and</u> 333 thousandths)
e. 3.010 (three <u>and</u> ten thousandths)

Writing Mixed Decimals

Here's how to write a mixed decimal for the mixed number $7\frac{9}{10}$. Write it in the order that you read it: *seven and nine tenths.*

1. **First, write the whole number.**
2. **Then write the decimal point.**
3. **Then write the decimal.**

Now, how do you think you'd write decimals for these amounts? (Think of the decimal places for 100 and 1,000. Then fill the empty places with zeros.)

a. $5\frac{1}{100}$ (five and one hundredth)
b. $3\frac{10}{1000}$ (three and ten thousandths)
c. $4\frac{1}{1000}$ (four and one thousandth)

Answers: a. 5.01 **b.** 3.010 **c.** 4.001

Exercise

Write mixed decimals for these amounts. Then check your answers.

1. one and two tenths
2. one and two hundredths
3. one and 20 hundredths
4. one and two thousandths
5. one and 20 thousandths
6. one and 200 thousandths
7. $2\frac{3}{10}$
8. $2\frac{3}{100}$
9. $2\frac{3}{1000}$
10. $2\frac{30}{100}$
11. $2\frac{30}{1000}$
12. $2\frac{300}{1000}$

Answers: 1. 1.2 **2.** 1.02 **3.** 1.20 **4.** 1.002 **5.** 1.020 **6.** 1.200 **7.** 2.3 **8.** 2.03 **9.** 2.003 **10.** 2.30 **11.** 2.030 **12.** 2.300

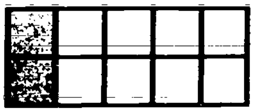
Unit Review

Check Yourself

1. Look at the shaded parts in each drawing. On note paper, write the decimal that stands for the shaded parts in each drawing. (Hint: Count the parts that are shaded.)

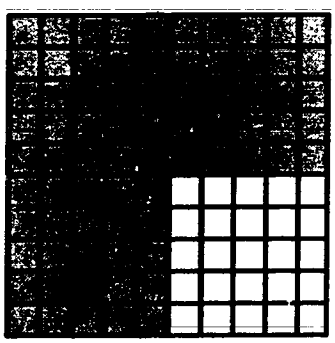
   .75 .9 .2 .40

   a.

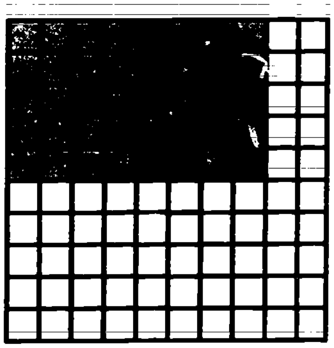


   b.

   c.



   d.



2. Look at the digits that are underlined in these numbers. What is the place value of each digit?

   a. 0.5
   b. 5.12
   c. 53.4
   d. 1.25
   e. 0.105
   f. .555

3. Write these amounts as decimals on notepaper.

   a. One and three tenths
   b. Thirty-five hundredths
   c. Seventeen thousandths
   d. Eleven and seventeen hundredths
   e. Five and eight thousandths

4. Choose the right word or words to complete each sentence below.

   decimals **place value**
   digit **power**
   numeral

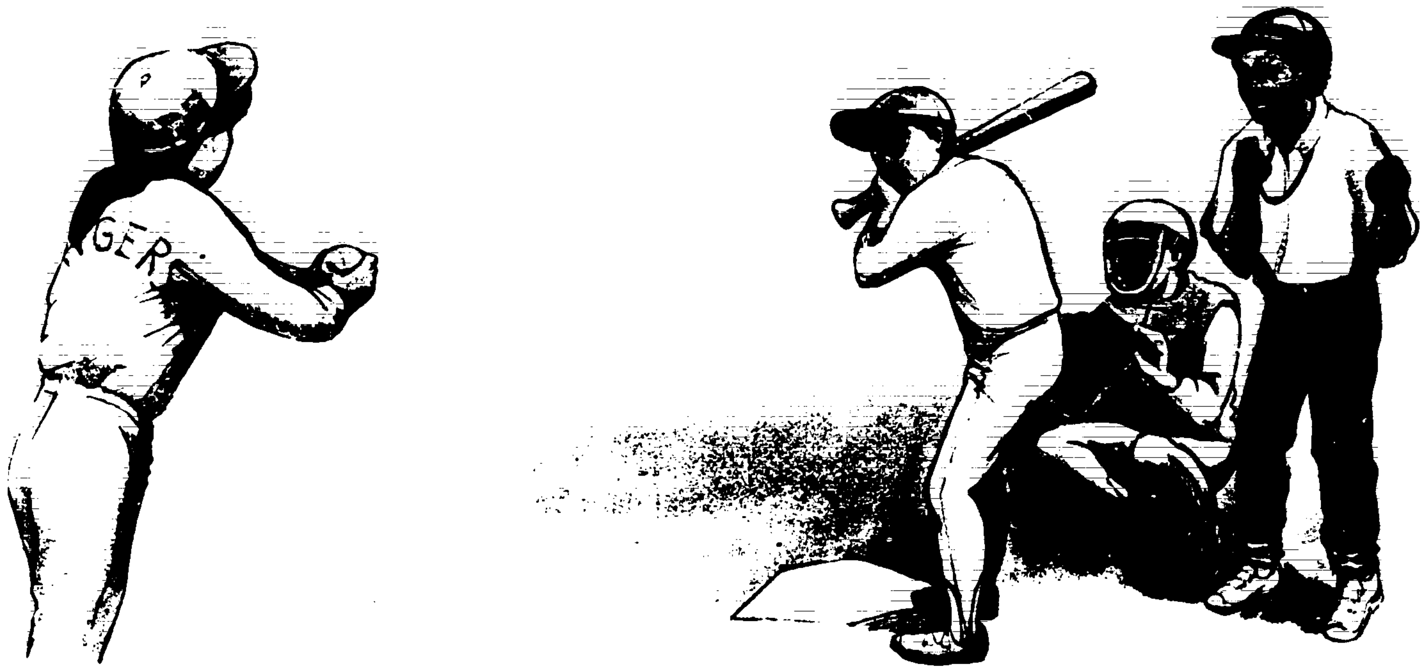
   a. Decimals show whole amounts that are divided into equal parts of 10 or a ______ of ten.
   b. ______ is another name for decimal fractions.
   c. The ______ of .2 is tenths.
   d. Every ______ in a decimal number has a certain value.
   e. The ______ at the right of the decimal point is the numerator of a decimal fraction.

Bonus Work

1. Gather things such as food labels, bills, receipts, and store ads that show decimal numbers. Then make a poster with those things.
2. Make flash cards of different decimal numbers. Practice reading the amounts out loud with your classmates.

Unit 2

Comparing Decimals



Let's say you are helping the coach of a softball team. She gives you this job to do: Put a list of batting averages in order. To do that, you must show who has the highest batting average, the next highest, and so on. Here is part of that list.

Player	Batting Average
Aquino	.312
Hasaad	.28
Running Deer	.257
Samuelson	.3
Williams	.275

Batting averages are given in decimals. To put them in order, you must compare the decimals with each other. But notice this: The decimals on the list don't all have the same denominator. One is in tenths, one is in hundredths, and the others are in thousandths. How can you compare those different decimals?

Before you can put those decimals in order, you must *rename* some of them. In other words, you must change the numerator and denominator without changing the value of the decimal.

You'll meet many math problems in real life like that. You must put decimals in order or decide which decimal is larger or smaller. The decimals won't always have the same denominator. And you'll have to rename some of the decimals.

In this unit, you'll learn how to rename decimals. You'll also learn how to compare decimals and put them in order.

> Math Words
>
> Look up these words in the glossary. What do they mean?
>
> **like decimals** **rename**
> **unlike decimals**

Like or Unlike?

Decimals That Are Alike

Common fractions that have the same denominator are alike. For example, $\frac{1}{8}$ and $\frac{5}{8}$ are common fractions that are like each other.

Decimal fractions can also be like each other. When do you think they are alike?

Right! Decimal fractions are alike when they have the same denominator. For example, the decimals are alike in each set below. What is the denominator in each set?

a.	.2	.5	.7	.8	.9
b.	.21	.03	.45	.70	
c.	.213	.003	.045	.700	

Answer: a. tenths **b.** hundredths **c.** thousandths

Notice this: Decimals that are alike have the *same number of decimal places.*

Decimals that don't have the same denominator and don't have the same number of decimal places are *unlike.* For example, these decimals are unlike:

.2 .04 .478

What is the denominator of each decimal? How many decimal places does each have?

Mixed Decimals That Are Alike

Now look at these mixed decimals:

2.5 2.05

Are they like or unlike each other?

To tell if mixed decimals are alike or unlike, *look at their decimal fractions.* Mixed decimals that are alike have the same denominator and the same number of decimal places.

2.5 and 2.05 are mixed decimals that are *not* alike. What is the denominator of each of those decimals? How many decimal places does each decimal have?

> Remember
>
> Decimals and mixed decimals that are alike have the same denominators. They also have the same number of decimal places.

Exercise

Look at these sets of decimals. Write *like* for the sets that show decimals that are like each other. Write *unlike* for the sets that show decimals that are not like each other. Then check your answers.

1.	.2	.04	.02
2.	.010	.348	.10
3.	.03	.30	.42
4.	.55	.5	.555
5.	.512	.007	.125
6.	.9	.5	.1
7.	4.52	57.49	257.01
8.	1.32	2.5	3.002
9.	3.5	3.52	3.552
10.	2.629	22.438	643.134
11.	.702	0.72	.072
12.	11.19	11.18	11.20

Answers: 1. unlike **2.** unlike **3.** like **4.** unlike **5.** like **6.** like **7.** like **8.** unlike **9.** unlike **10.** like **11.** unlike **12.** like

Renaming Decimals

Decimals must be alike when you compare them. If they are not alike, you must rename them so they all have the same number of decimal places—and the same denominator. When you rename a decimal, you change its numerator and denominator without changing its value.

Suppose you are comparing these unlike decimals:

.6 **.05** **.845**

Here's how to make those decimals alike:

1. Find the decimal with the most decimal places.

Count the places after the decimal point. Each decimal must have that number of decimal places.

tenths
hundredths
thousandths

. [1] [2] [3] decimal places

. 0 5
. 6
. 8 4 5

Which decimal has the most decimal places? How many places does it have?

2. Next, rename the other decimals so they have the same number of decimal places.

To do that, put zeros after the last digit of each decimal.

tenths
hundredths
thousandths

. [1] [2] [3] decimal places

. 0 5 0
. 6 0 0
. 8 4 5

How many decimal places does .6 have? How many zeros must you put after it?

How many decimal places does .05 have? How many zeros must you put after it?

Renaming Mixed Decimals

Sometimes you must rename mixed decimals such as:

2.005 **78.96** **548.9**

To do that, *look at only the decimal part* of the mixed decimal. Then follow the steps for renaming decimals.

1. Find the mixed decimal that has the most decimal places and count them.

tenths
hundredths
thousandths

Whole number places [ ] [ ] [ ] . [1] [2] [3] decimal places

2 . 0 0 5
7 8 . 9 6
5 4 8 . 9

2. Rename the other mixed decimals.

tenths
hundredths
thousandths

Whole number places [ ] [ ] [ ] . [1] [2] [3] decimal places

2 . 0 0 5
7 8 . 9 6 **0**
5 4 8 . 9 **0** **0**

Exercise

Copy these sets of decimals and mixed decimals. Then rename decimals to make all decimals in a set alike.

1. .5 .13   **2.** .265 .29   **3.** .7 .699   **4.** 2.8 2.95

5. .6 .60 .006   **6.** .3 .78 .125   **7.** 5.002 90.36 236.9   **8.** .2 3.6 15.35

Answers:

1. .50 .13   **2.** .265 .290   **3.** .700 .699   **4.** 2.80 2.95

5. .600 .600 .006   **6.** .300 .780 .125   **7.** 5.002 90.360 236.900   **8.** .20 3.60 15.35

Which Is Greater?

Picture this:

You are shopping for food. You see these prices: $.65 $.09

The prices are in decimals. What kind of decimal fractions are they: tenths, hundredths, or thousandths?

The decimals have two decimal places. So they are decimal fractions of hundredths. $.65 is the same as $\frac{65}{100}$. And $.09 is the same as $\frac{9}{100}$.

Which is greater: $.65 or $.09?

Look at the Numerators

When you compare common fractions that have the same denominators, you look at their numerators. Larger fractions have larger numerators. And smaller fractions have smaller numerators. For example, $\frac{3}{4}$ is bigger than $\frac{1}{4}$.

When you compare decimal fractions, you look at the numeral after the decimal point. (Remember: that numeral is the numerator of the decimal.) Larger decimals have higher numbers. Smaller decimals have lower numbers. So $.65 ($\frac{65}{100}$) is greater than $.09 ($\frac{9}{100}$).

$.65 > $.09

And $.09 ($\frac{9}{100}$) is smaller than $.65 ($\frac{65}{100}$).

$.09 < $.65

Now look at these two sets. In each set, which amount is greater? Which is smaller?

a. .8 .3

b. .876 .898

Answers: **a.** .8 > (is greater than) .3
.3 < (is smaller than) .8
b. .898 > (is greater than) .876
.876 < (is smaller than) .898

Remember
> means *is greater than*
< means *is smaller than*

Comparing Mixed Decimals

Let's say you're buying a can of tomato sauce. You compare the amount of sauce in two cans. One can has 6.2 ounces of sauce. And the other has 5.9 ounces of sauce. Which can has the greater amount?

When you compare mixed decimals, look first at the whole number. The larger mixed decimal has the higher whole number. The smaller mixed decimal has the smaller whole number. So 6.2 ($6\frac{2}{10}$) is greater than 5.9 ($5\frac{9}{10}$).

6.2 > 5.9

And 5.9 ($5\frac{9}{10}$) is smaller than 6.2 ($6\frac{2}{10}$).

5.9 < 6.2

Suppose you are comparing two cans that have these amounts:

10.4 ounces 10.9 ounces

The two amounts have the same whole number. To compare them, you must look at the decimal fraction. Which amount is greater? Which amount is smaller?

Answer: 10.9 > (is greater than) 10.4
10.4 < (is smaller than) 10.9

Exercise

Copy each set of decimal numbers. Write < or > between them to show if the first decimal number is greater or smaller than the second one. Then check your answers.

1. .18 ■ .21
2. .56 ■ .92
3. 2.44 ■ 3.25
4. 5.86 ■ 5.43
5. 1.09 ■ .90
6. 15.078 ■ 15.519
7. .0323 ■ .3076
8. 1.80 ■ 1.09

Answers: **1.** .18 < .21 **2.** .56 < .92
3. 2.44 < 3.25 **4.** 5.86 > 5.43 **5.** 1.09 > .90
6. 15.078 < 15.519 **7.** .0323 < .3076
8. 1.80 > 1.09

Putting Decimals in Order

These decimals and mixed decimals are not in order. Here's how to put them in order from largest to smallest.

.8 .9 .23 .07 1.09 1.9 2.1

1. First, rename the decimals that have the fewest decimal places.

Which decimals are they? What do you rename them as?

Answer: .8 becomes .80 .9 becomes .90
1.9 becomes 1.90 2.1 becomes 2.10

2. Next, compare the whole numbers.

Put them in order from highest to lowest. If some mixed decimals have the same whole number, put them in order according to their *decimals*.

.80 .90 .23 .07 **1.09** **1.90** **2.10**

How would you put the mixed decimals in order?

Answer: 2.10 > 1.90 > 1.09

3. Now finish putting the list in order.

Look at the decimals. Put the decimals in order from highest to lowest. How would you do that?

2.10 1.90 1.09 **.80** **.90** **.23** **.07**

Answer: 2.10 > 1.90 > 1.09 > .90 > .80 > .23 > .07

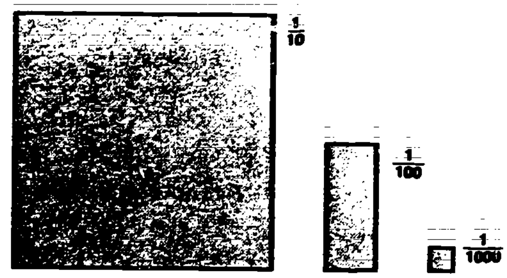
Exercise

1. Put these decimals and mixed decimals in order from highest to lowest.

a.	.5	1.02	3.9	.08
b.	.028	.75	3.293	.5
c.	1.05	3.0	1.26	.88

2. Copy the batting average on page 15. List them from highest to lowest.

Answers: 1. a. 3.90 > 1.02 > .50 > .08
b. 3.293 > .750 > .500 > .028
c. 3.00 > 1.26 > 1.05 > .88
2. .312 > .300 > .280 > .275 > .257



.1 > .01 > .001

Decimal Sizes

Sometimes decimals have the same numerators. For example: .1, .01, and .001 have the same numerators. But their denominators are different. Which of the three is the largest decimal? Which is the next largest? Which is the smallest?

.1 is the largest decimal. It stands for $\frac{1}{10}$ of a whole amount. .01 is the next largest. It stands for $\frac{1}{100}$ of a whole amount. And .001 is the smallest. It stands for $\frac{1}{1000}$ of a whole amount. You know that a tenth is much larger than a hundredth of a whole amount. And a hundredth is much larger than a thousandth.

If you *rename* those decimals, it is easier to see which is largest. So, rename them all to three decimal places. You'll see that:

.100 > .010 > .001

Exercise

Put the numbers in order from largest to smallest. Write > between each number.

1. .2 ■ .02 ■ .002
2. .009 ■ .09 ■ .9
3. 1.1 ■ 1.001 ■ 1.01
4. 5.006 ■ 5.06 ■ 5.6
5. 2.4 ■ 2.004 ■ 2.040
6. 4.70 ■ 4.07 ■ 4.007

Answers: 1. .2 > .02 > .002
2. .9 > .09 > .009 3. 1.1 > 1.01 > 1.001
4. 5.6 > 5.06 > 5.006 5. 2.4 > 2.040 > 2.004
6. 4.70 > 4.07 > 4.007

Unit Review

Check Yourself

1. Copy each set of decimals. Rename each set so that the decimals are alike.
   a. .7, .95
   b. .251, .17
   c. .006, .6
   d. 1.5, 1.32, 1.05
   e. 8.06, 8.2, 8.198
   f. 6.107, 6.23, 6.08
2. Copy each set of decimals. Write > or < between each set. (Remember to rename unlike decimals as like ones.)
   a. .25 ■ .2
   b. .09 ■ .009
   c. .3 ■ .299
   d. .573 ■ .65
   e. .1 ■ .01
   f. .075 ■ .7
3. Copy each set of decimals. Write > or < between each set. (Remember to rename unlike decimals as like ones.)
   a. 4.5 ■ 4.9
   b. 13.89 ■ 13.5
   c. 6.098 ■ 3.3
   d. 10.01 ■ 10.384
   e. 17.6898 ■ 18.9
   f. 103.48 ■ 103.11
4. On notepaper, arrange each set of decimals from highest to lowest.
   a. .6, .1, .5
   b. .18, .05, .40
   c. .136, .538, .095
   d. .007, .7, .07
   e. 8.4, 7.85, 6.1
   f. 3.12, 2.9, 5.02
   g. 2.365, 2.62, 2.4
   h. 7.03, 7.10, 7.353
5. Choose the right word or words to complete each sentence.
   like decimals
   rename
   unlike decimals
   a. .12 and .506 are _____.
   b. When you change an unlike decimal to a like decimal, you _____ it.
   c. .12 and .20 are _____.

Bonus Work

1. Write different decimal fractions and mixed decimals on small pieces of paper. You can make up the numbers or copy them from a math book. Mix up the pieces. Then ask your classmates to arrange the numbers in order from lowest to highest.
2. Compare the unit prices for the things listed below. Go to two different stores or get ads for two different stores. (Make sure you compare the same size of those things at both stores.) Write the unit price at each store. Then write > or < between the prices.

Things	Store 1		Store 2
a. can of tomato soup	_____	■	_____
b. 1 pound of oranges	_____	■	_____
c. bag of potato chips	_____	■	_____
d. bottle of shampoo	_____	■	_____

Unit 3

Adding and Subtracting Decimals

Let's say you have a checking account. You're figuring the *balance* of your account. The balance is the amount of money you have in the account.

Suppose your balance is $50.00.

On Tuesday you buy art paper, pencils, and pens. You write a check for $12.30 to pay for them.

On Thursday you buy a jacket. You write a check for $25.78 to pay for it.

And on Friday you get paid $36.75 for working part time. You deposit all of it in your checking account. What is the balance of your checking account now?

The balance of a checking account always changes. It changes when you write a check and take out money. It changes when you deposit money into the account.

To figure the balance of your account, you'd add and subtract decimals. You'd subtract a decimal amount each time you wrote a check. And you'd add a decimal amount each time you deposited money.

There are certain things you must do in order to add and subtract decimals correctly. You'll learn what they are in this unit.

Answer: The balance is $48.67

> Word Check
>
> Look up these words in the glossary. What do they mean?
>
> **decimal place** **line up**

Lining Up

Look at these two math problems.

$$\begin{array}{r} .2 \\ +\ 6.3 \\ \hline 8.3 \end{array} \qquad \begin{array}{r} 4.8 \\ -\ .2 \\ \hline .28 \end{array}$$

The answers are wrong. Why are they wrong?

When you add or subtract decimals, you must do this: *add or subtract digits that have the same place value*. For example, you add or subtract only tenths and tenths, hundredths and hundredths, and thousandths and thousandths.

Here's how to make sure you are adding or subtracting the right digits: When you write the problem, *line up the decimal points*. Then *line up digits that have the same place value*. In other words, write each number so that the decimal points and digits line up in columns. For example:

$$\begin{array}{r|l} .& 2 \\ +\ 6 .& 3 \\ \hline 6 .& 8 \end{array} \qquad \begin{array}{r|l} 4 .& 8 \\ - \quad .& 2 \\ \hline 4 .& 6 \end{array}$$

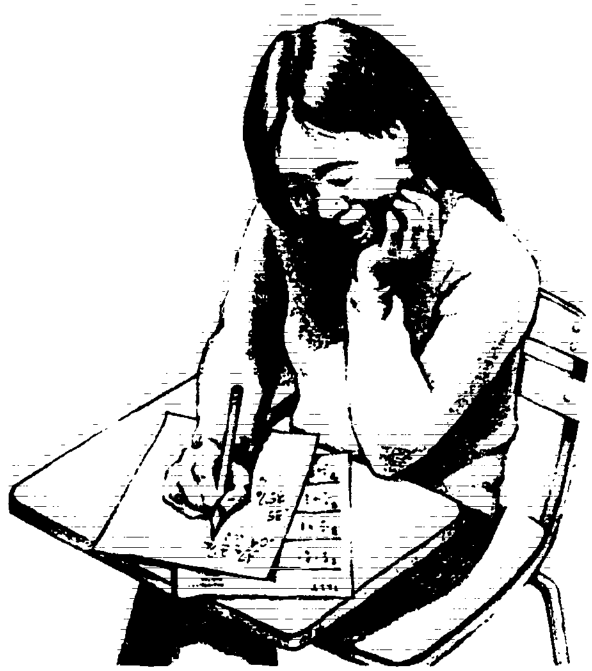
The answers are now correct.

How would you write these numbers so the decimal points and digits line up correctly?

a. 3.88 1.23 15.57

b. 105.33 12.89

Answers: **a.** 3.88 / 1.23 / 15.57 **b.** 105.33 / 12.89



A Rule to Remember

When you add or subtract decimals, line up their decimal points. Line up digits that have the same place value.

Exercise

On a sheet of paper, write these sets of numbers. Line up their decimal points and digits correctly. Then check your answers.

1. .2 .5 .9
2. 1.49 3.78 3.03
3. 24.892 10.586
4. .589 .321
5. 1.80 0.70
6. 425.38 12.16

Answers:
1. .2 / .5 / .9
2. 1.49 / 3.78 / 3.03
3. 24.892 / 10.586
4. .589 / .321
5. 1.80 / 0.70
6. 425.38 / 12.16

Renaming Unlike Decimals

Decimals that you add or subtract must be alike. In other words, all the decimals in a problem must have the same number of decimal places. If they don't, you must rename the decimals that have fewer decimal places. How do you rename decimals?

Right! You rename a decimal by putting zeros after the last digit. Here's how to make sure you rename decimals correctly in addition or subtraction problems.

Suppose you are adding these decimals.

.3 .16 .728

1. First, line up the numbers according to their decimal points.

Line up digits that have the same place value.

.3
.16
.728

2. Next, find the decimal with the most decimal places.

Then rename the other decimals so that they have the same number of decimal places.

.300
.160
.728

a. Which decimal has the most decimal places?

b. Which decimals do you rename? What do you rename them as?

Exercise

Write each set of decimals. First, line up the decimal points correctly. Then make all the decimals in the set alike by renaming. Check your answer when you are finished.

1. .6 .25
2. .675 .25
3. .4 .126
4. .245 .3 .12
5. .007 .352 .7
6. .9 .2 .1 .009
7. .24 .18 .02 .28 .2
8. .078 .21 .08 .02 .2 .509

Answers: a. .728 has the most decimal places.
b. You rename .3 and .16
.3 is renamed as .300
.16 is renamed as .160

Answers:
1. .60 .25
2. .675 .250
3. .400 .126
4. .245 .300 .120
5. .007 .352 .700
6. .900 .200 .100 .009
7. .24 .18 .02 .28 .20
8. .078 .210 .080 .020 .200 .509

Mixed Decimals and Whole Numbers

Sometimes you'll have to solve problems that have decimals, mixed decimals, and whole numbers. For example:

25.7 3.87 .559 35

The numbers are not alike. To make them alike, you must rename three of the numbers. Which numbers are they?

You must rename the mixed decimals and the whole number. Here's how you'd write a problem with those kinds of numbers:

1. First, give the whole number a decimal point.

Write it *behind* the whole number. Why would you write the decimal point behind and not in front of the whole number? (Hint: look at the place value chart on page 9.)

25.7 3.87 .559 35.

2. Next, line up the numbers according to their decimal points.

Line up digits that have the same place value. Where does the whole number go?

```
25.7
 3.87
  .559
35.
```

3. Find the decimal that has the most decimal places.

Rename the other numbers so that they are like that decimal. Notice how the whole number is renamed.

```
25.700
 3.870
  .559
35.000
```

Look at the list that has the renamed numbers.

a. Which decimal has the most decimal places? How many does it have?

b. Which mixed decimals are renamed? What do you rename them as?

c. How do you rename the whole number? What do you rename it as?

Answers:

a. .559 has the most decimal places. It has three.

b. 25.7 and 3.87 are the mixed decimals you rename. 25.7 is renamed as 25.700 and 3.87 is renamed as 3.870

c. The whole number is renamed by first writing a decimal point behind it, then writing three zeros after the decimal point. So 35 is renamed as 35.000

Exercise

Write each set of numbers. First, line up the numbers correctly. Then make all the numbers in the set alike by renaming. Check your answers when you are finished.

1. .6 2.25
2. 34.605 21
3. 6 .126 1.02
4. 8.245 20.3 12
5. 7 .352 .5 .15
6. 10.9 5.2 .1 .009 226

Answers:

1. .60
2.25
2. 34.605
21.000
3. 6.000
.126
1.020
4. 8.245
20.300
12.000
5. 7.000
.352
.500
.150
6. 10.900
5.200
.100
.009
226.000

Adding Decimals

Remember this rule when you add decimals: *Line up everything in straight columns!*

Line up decimal points exactly over each other. Line up digits that have the same place value. If you don't, your answer will be wrong!

Suppose you are solving this problem:

.1 + .23 + .456

To solve the problem correctly, follow these steps:

1. Line up everything in straight columns.

Line up decimal points and digits that have the same value. Make sure the columns are straight.

		10th	100th	1000th
	.	1		
	.	2	3	
+	.	4	5	6

Which digits have a place value of tenths?

Which digits have a place value of hundredths?

Which digit has a place value of thousandths?

2. Rename decimals that have fewer decimal places.

		10th	100th	1000th
	.	1	**0**	**0**
	.	2	3	**0**
+	.	4	5	6

How many decimal places should each decimal have?

Which decimals do you rename? What do you rename them as?

3. Add the decimals and solve the problem.

		10th	100th	1000th
	.	1	0	0
	.	2	3	0
+	.	4	5	6
	.	**7**	**8**	**6**

Look at the decimal places in the problem and the answer. Notice this:

- All decimal points line up.
- All digits with the same place value line up.

The answer is like the decimals in the problem. It has the *same number of decimal places.*

Exercise

Write these problems. Be sure to line up everything in straight columns. Solve the problems. Then check your answers.

1. .01 + .4 + .125
2. .006 + .9 + .02
3. .2 + .140 + .01 + .116
4. .5 + .122 + .101 + .123 + .001
5. .300 + 2 + .111 + .04 + 3
6. .1 + .4 + .2 + .06 + .009

Answers:

1. .010 + .400 + + .125 = .535
2. .006 + .900 + .020 = .926
3. .200 + .140 + .010 + .116 = .466
4. .500 + .122 + .101 + .123 + .001 = .847
5. .300 + 2.000 + .111 + .040 + 3.000 = 5.451
6. .100 + .400 + .200 + .060 + .009 = .769

Carrying

When you add decimals, you must often carry amounts. Sometimes you must carry amounts from the decimal fraction to the whole number. Here's how to solve those kinds of problems.

Example: 1.29 + 19.901

1. Line up everything in straight columns.

Line up the decimal points. Line up digits with the same place value.

		1	.	2	9	
+	1	9	.	9	0	1

2. Rename decimals with fewer decimal places.

```
   1.290
+ 19.901
```

3. Add the columns.

Starting with the right column, add the digits in each column. If a column adds up to a two-digit number, carry the first digit to the next column on the left.

```
 1 1
   1.2 9 0
+ 1 9.9 0 1
  2 1.1 9 1
```

4. Write the decimal point.

Your columns and decimal points should be in straight lines. Then you can *bring down* your new decimal point right under the others.

```
 1 1
   1.2 9 0
+ 1 9.9 0 1
  2 1.1 9 1
```

What would happen if your columns were out of line?

Right! You wouldn't know where to put your decimal point. Or you might add the wrong digits together.

5. Check your decimal point.

Look at the problem again. How many columns of digits are there *to the right* of the decimal point?

```
 1 1
   1.2 9 0
+ 1 9.9 0 1
  2 1.1 9 1
       ↑ ↑ ↑
```

There are three columns. Now look at your answer. How many columns are there to the right of your decimal point? The number of columns should be the same.

Exercise

Write these problems. Be sure to line up everything in straight columns. Solve the problems. Then check your answers.

1. 1.01 + .49
2. .06 + .94 + .1
3. .2 + .140 + .96 + .116
4. .5 + 9.4 + 21.101 + 81.129
5. 25.18 + .4 + 5.08 + .5
6. 1.321 + 2 + 43.7 + 5.099

Answers:

1. 1.01 + 1.49 = 2.50
2. .06 + .94 + .10 = 1.10
3. .200 + .140 + .960 + .116 = 1.416
4. .500 + 9.400 + 21.101 + 81.129 = 112.130
5. 25.18 + .40 + 5.08 + .50 = 31.16
6. 1.321 + 2.000 + 43.700 + 5.099 = 52.120

Subtracting Decimals

When you subtract decimals, you must also remember this rule: *Line up everything in straight columns!*

You will get a wrong answer if you don't line up the decimal points and digits of a problem. That's because you can only subtract digits from other digits that have the same place value.

To solve subtraction problems correctly, follow these steps.

Example: .297 − .16

1. Line up everything in straight columns.

Line up decimal points and digits that have the same value. Make sure the columns are straight.

		10th	100th	1000th
	.	2	9	7
−	.	1	6	

Which digits have a place value of tenths?

Which digits have a place value of hundredths?

Which digit has a place value of thousandths?

2. Rename one of the decimals.

		10th	100th	1000th
	.	2	9	7
−	.	1	6	0

How many decimal places should each decimal have?

Which decimal do you rename? What do you rename it as?

3. Subtract the decimals and solve the problem.

		10th	100th	1000th
	.	2	9	7
−	.	1	6	0
	.	1	3	7

How many decimal places does the answer have?

Exercise

Write these problems. Be sure to line up everything in straight columns. Solve the problems. Then check your answers.

1. .44 − .12
2. .406 − .2
3. .145 − .114
4. .522 − .001
5. .358 − .043
6. .647 − .536

Answers:

1. .44 − .12 = .32
2. .406 − .200 = .206
3. .145 − .114 = .031
4. .522 − .001 = .521
5. .358 − .043 = .315
6. .647 − .536 = .111

Borrowing

When you subtract whole numbers, you often *borrow* an amount. For example:

$$\begin{array}{r} {\scriptstyle 2\ 14} \\ \cancel{3}\,\cancel{4} \\ -\ 1\ 6 \\ \hline 1\ 8 \end{array}$$

You must also sometimes borrow when you subtract decimals. Here's how to solve those kinds of problems.

Example: 9.5 − 6.75

1. **Line up everything in straight columns.**
   Line up the decimal points. Line up digits with the same place value.

	9	.	5	
−	6	.	7	5

2. **Rename one of the decimals.**

$$\begin{array}{r} 9.50 \\ -\ 6.75 \\ \hline \end{array}$$

3. **Subtract.**
   Starting with the right column, subtract the bottom digit from the top digit. If the top digit is too small, borrow 1 from the digit that's next to it at left.

$$\begin{array}{r} {\scriptstyle 4\ 10} \\ 9\,.\,\cancel{5}\,\cancel{0} \\ -\ 6\,.\,7\ 5 \\ \hline 5 \end{array} \qquad \begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 8\ \cancel{4}\ 10} \\ \cancel{9}\,.\,\cancel{5}\,\cancel{0} \\ -\ 6\,.\,7\ 5 \\ \hline 2\ \ 7\ 5 \end{array}$$

4. **Write the decimal point.**
   Put the decimal point in the answer. Bring it down so that it's under the other decimal points.

$$\begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 8\ \cancel{4}\ 10} \\ \cancel{9}\,.\,\cancel{5}\,\cancel{0} \\ -\ 6\,.\,7\ 5 \\ \hline 2\,.\,7\ 5 \end{array}$$

5. **Check your decimal point.**
   Look at your answer. How many columns are there to the right of your decimal point?

$$\begin{array}{r} {\scriptstyle 14} \\ {\scriptstyle 8\ \cancel{4}\ 10} \\ \cancel{9}\,.\,\cancel{5}\,\cancel{0} \\ -\ 6\,.\,7\ 5 \\ \hline 2\,.\,7\ 5 \\ \uparrow\ \uparrow \end{array}$$

Exercise

Write these problems. Be sure to line up everything in straight columns. Solve the problems. Then check your answers.

1. .94 − .06
2. 1.01 − .49
3. .140 − .116
4. 21.10 − 8.129
5. 25.18 − 5.3
6. 42.7 − 6.099

Answers:

1. $\begin{array}{r} .94 \\ -\ .06 \\ \hline .88 \end{array}$ 2. $\begin{array}{r} 1.01 \\ -\ .49 \\ \hline .52 \end{array}$ 3. $\begin{array}{r} .140 \\ -\ .116 \\ \hline .024 \end{array}$

4. $\begin{array}{r} 21.100 \\ -\ 8.129 \\ \hline 12.971 \end{array}$ 5. $\begin{array}{r} 25.18 \\ -\ 5.30 \\ \hline 19.88 \end{array}$ 6. $\begin{array}{r} 42.700 \\ -\ 6.099 \\ \hline 36.601 \end{array}$

Unit Review

Check Yourself

Copy and solve the problems below.

1. Add these like decimals.

a. .621 + .453 + .009

b. 2.35 + 3.51 + 6.88

c.
$$\begin{array}{r} 23.04 \\ +\ \ 7.86 \\ \hline \end{array}$$

d.
$$\begin{array}{r} \$1.32 \\ .93 \\ +\ \ 8.04 \\ \hline \end{array}$$

2. Subtract these like decimals.

a. .85 − .62

b.
$$\begin{array}{r} 12.8 \\ -\ 10.8 \\ \hline \end{array}$$

c. $3.11 − .49

d.
$$\begin{array}{r} .981 \\ -\ .703 \\ \hline \end{array}$$

3. Add these unlike decimals.

a. 3.68 + .5 + 1.9

b. .671 + .95 + .8

c.
$$\begin{array}{r} .008 \\ +\ .23 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 4.30 \\ 1.6 \\ +\ 10.389 \\ \hline \end{array}$$

4. Subtract these unlike decimals.

a.
$$\begin{array}{r} 3.13 \\ -\ 1.782 \\ \hline \end{array}$$

b. .8 − .345

c.
$$\begin{array}{r} 32.898 \\ -\ 12.96 \\ \hline \end{array}$$

d. 1.2 − .069

5. Add or subtract the decimals from the whole numbers.

a.
$$\begin{array}{r} 16 \\ +\ \ 2.8 \\ \hline \end{array}$$

b.
$$\begin{array}{r} 8.23 \\ -\ 3 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 4 \\ -\ 1.62 \\ \hline \end{array}$$

d.
$$\begin{array}{r} 4 \\ 3.87 \\ +\ 9.2 \\ \hline \end{array}$$

6. Choose the right words to complete each sentence.

decimal places **line up**

a. ______ decimal points in addition problems.

b. Line up ______ when you add or subtract.

Bonus Work

1. How many addition problems can you make out of these numbers so that they add up to 20?

2.89 5.6 10.175 4.11 1.335

2. Record everything you spend in a day. Make a form like this to help you.

Date: ____________

How much money do you have to spend?

____

What You Spend For	**Amount**
____________	______
____________	______
____________	______

How much money do you have at the end of the day? ____________

Unit 4

Multiplying Decimals

Picture this:

You have a part-time job. You work different hours every week. Your pay is $4.15 an hour. This week you work 13.5 hours. How much do you earn?

After work, you get on a bus. You have a doctor's appointment. The doctor's office is 12 bus stops away. It takes the bus about 5.5 minutes to travel from bus stop to bus stop. How long will it take you to get to the doctor's office?

On your way home, you shop for food. You buy 3.2 pounds of meat. It costs $1.99 a pound. How much will you pay for the meat?

Each of those real life problems have to do with decimals. To solve those problems, you must multiply those decimals.

In the unit before this, you learned to add or subtract decimals. You learned that you must line up decimal points and digits in order to get a correct answer.

You don't have to line up decimal points and digits when you multiply decimals. But you do have to do other things to get a correct answer. You'll learn about them in this unit.

Math Words
Look up these words in the glossary. What do they mean?
factor **product**

Factors and Products

You'll be using two important words in this unit. The first word is ***factor.*** Factors are the numbers you multiply. The other word is ***product.*** A product is the answer you get when you multiply factors. For example:

$2 \times .30 = .60$

2 and .30 are factors and .60 is the product.

Exercise

Look at these problems and their answers. Which numbers are factors? Which are products?

1. $3.6 \times .2 = .72$

2. $25 \times .3 = 7.5$

3. $13.6 \times 12.3 = 167.28$

4.
$$\begin{array}{r} 3.98 \\ \times \quad .2 \\ \hline .796 \end{array}$$

5.
$$\begin{array}{r} 100 \\ \times \ .05 \\ \hline 5.00 \end{array}$$

6.
$$\begin{array}{r} 460 \\ \times .002 \\ \hline .920 \end{array}$$

Answers: **1.** factors: 3.6, .2 product: .72
2. factors: 25, .3 product: 7.5
3. factors: 13.6, 12.3 product: 167.28
4. factors: 3.98, .2 product: .796
5. factors: 100, .05 product: 5.00
6. factors: 460, .002 product: .920

Writing the Problem

When you write an addition or subtraction problem, you must line the numbers up correctly. If you are adding or subtracting decimals that are not alike, you must rename them so that they all have the same number of decimal places.

You don't do any of that when you *multiply* decimals. You don't rename factors. And you don't line up the factors according to decimal points and decimal places. Instead, you line them up in a way that makes it easy to multiply each number. Look at this multiplication problem. What lines up?

$$\begin{array}{r} 3.04 \\ \times \quad .2 \\ \hline .608 \end{array}$$

Right! The *last* digits in both factors line up.

Look at the factors and the product in the problem. Then answer these questions.

a. Do the decimal points line up?

b. How many decimal places are in each factor?

c. How many decimal places are in the product?

Answers:
a. The decimal points don't line up.
b. There are two decimal places in one factor and one decimal place in the other factor.
c. There are three decimal places in the product.

Where Does the Decimal Point Go?

Suppose you are multiplying this problem:

$$\begin{array}{r} 4.61 \\ \times \;\; .2 \\ \hline 922 \end{array}$$

Where would you put the decimal point in the product? To find out, do this:

1. First, count *all* the decimal places in each factor.

How many decimal places are there?

4.61	(2 decimal places)
× .2	(1 decimal place)
922	

2. Next, add up the decimal places.

Find the total of all decimal places in both factors. How many decimal places are there altogether?

4.61	(2 decimal places)
× .2	(+ 1 decimal place)
922	(3 decimal places)

3. Count decimal places.

Now, look at the product. Start from the last digit at the right and *count* decimal places. Count the same number of decimal places as there are in the total of the factors. Write the decimal point in front of those decimal places. In front of which number do you write the decimal point?

4 . 6 1	(2 decimal places)
× . 2	(1 decimal place)
. 9 2 2	(3 decimal places)
↑ ↑ ↑	
3 2 1	

Look at this problem. Practice counting decimal places. Where would you put the decimal point in the product?

2.112	(? decimal places)
× .13	(? decimal places)
27456	(? decimal places)

Answer: .27456

Exercise

Some of the products for these problems are wrong. Their decimal points are not in the right place. Copy all the problems and their products. But write the decimal points correctly in the products. Then check your answers.

1. 2.3 × .6 = 13.8
2. 7.38 × .3 = 2.214
3. 4.221 × .7 = 295.47
4. 7.3 × 5 = 3.65
5. 15.3 × .13 = 198.9
6. 23.01 × 0.2 = 4.602

Answers: 1. 1.38 2. 2.214 3. 2.9547 4. 36.5 5. 1.989 6. .4602

Getting the Correct Product

Follow these steps when you multiply numbers that have decimals. They will help you get the right answer.

Example: 2.5 × 1.31

1. Write the problem.

Write one of the factors on top. Write the other factor on the bottom. Be sure to line up the last digits of both factors.

```
    1 . 3 | 1
×       2 . 5
-------------
```

2. Multiply.

Multiply the factors the same way you multiply whole numbers. Pay no attention to the decimal points.

```
   1.31
×   2.5
-------
    655
   262
-------
   3275
```

3. Add up all the decimal places.

Count the decimal places in each factor. Add them up and find the total number of decimal places.

```
   1.31    (  2 decimal places)
×   2.5    (+ 1 decimal place)
-------    -------------------
   3275    (  3 decimal places)
```

4. Write the decimal point.

Starting from the last digit in the product, count from right to left. Count the same number of decimal places as there are in the total of the factors. Write a decimal point in front of those places.

```
    1 . 3 1    (2 decimal places)
×     2 . 5    (1 decimal place)
-----------
  3 . 2 7 5    (3 decimal places)
      ↑ ↑ ↑
      3 2 1
```

Exercise

Some of the products in these problems are wrong. Copy the problems and multiply them correctly. Follow the steps you just learned. Write the correct products. Then check your answers.

1. 0.6 × 1.23 = .831
2. 1.156 × .3 = 32.61
3. 3.5 × .5 = 1.75
4. 10.5 × 4.25 = 44.7
5. 1.3 × .2 = 62.2
6. .15 × 63 = 94.5
7. 10 × .73 = 6.30
8. 2.7 × .06 = .162
9. .44 × 12 = 82.5
10. 8.6 × .54 = 4.633

Answers: 1. .738 2. .3468 3. 1.75 4. 44.625 5. .26 6. 9.45 7. 7.30 8. .162 9. 5.28 10. 4.644

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Completing the Product

Sometimes when you multiply a problem you'll get an answer that does not have enough places. For example, look at this problem. How many total decimal places are in the factors? How many places are in the product?

$$\begin{array}{r} 1.14 \\ \times\ .03 \\ \hline 342 \end{array}$$

There is a total of four decimal places in the factors. So the answer must have four decimal places. But there are only three places in the product. You must write the product so that it has one more decimal place.

Here's how to do that: Write a zero *in front* of the product. Then write a decimal point in front of that zero.

$$\begin{array}{r} 1.14 \\ \times\ .03 \\ \hline .0342 \end{array}$$

Now look at this problem. Then answer the questions.

$$\begin{array}{r} .12 \\ \times\ .3 \\ \hline 36 \end{array}$$

a. How many total decimal places are in the factors?
b. How many places are in the product?
c. How many zeros should you write in the product?
d. What is the correct answer?

Remember:
When a product does not have enough places, do this:
- Write zeros in front of the product. Write as many zeros as you need decimal places.
- Write a decimal point in front of those zeros.

Exercise

Copy these problems. Write the decimal point correctly in each problem. Then check your answers.

1. $\begin{array}{r} .12 \\ \times\ .24 \\ \hline 288 \end{array}$

2. $\begin{array}{r} 7.9 \\ \times\ .008 \\ \hline 632 \end{array}$

3. $\begin{array}{r} .04 \\ \times\ .2 \\ \hline 8 \end{array}$

4. $\begin{array}{r} .214 \\ \times\ .21 \\ \hline 4494 \end{array}$

5. $\begin{array}{r} 1.62 \\ \times\ .03 \\ \hline 486 \end{array}$

6. $\begin{array}{r} .114 \\ \times\ .86 \\ \hline 9804 \end{array}$

Answers: a. three total decimal places **b.** two places in the product **c.** one zero **d.** .036

Answers: 1. .0288 **2.** .0632 **3.** .008 **4.** .04494 **5.** .0486 **6.** .09804

Rounding Decimals

Rounding Off

Suppose you are solving this real life problem: You are shopping for food. You buy vegetables that cost $.89 a pound. The vegetables weigh 1.25 pounds. How much do you pay for the vegetables?

	1.25	pounds
×	.89	cost per pound
	$1.1125	total cost

The answer to the problem is in *ten thousandths*. But when we do everyday shopping, we usually figure money in *hundredths*. To figure exactly how much to pay for the vegetables, you must round the answer to the nearest hundredth. (When we figure with money, we say we are rounding to the nearest cent or penny.)

The Nearest Hundredth

Here's how to round that answer to the nearest hundredth (cent or penny):

First find the digit in the thousandths place. Drop all digits after it.

$1.112

Now look at the digit in the thousandths decimal place. What digit is in that place?

If the digit in the thousandths place is less than 5, you'd *round off* the decimal. To do that, simply drop the digit that's in the thousandths place. The digit 2 is less than 5, so you'd round off the decimal like this:

$1.11

Exercise

Round off these amounts.

1. $1.0645
2. $ .993
3. $ .3912
4. $50.254

Answers: 1. $1.06 2. $.99 3. $.39 4. $50.25

Rounding Up

You now buy a fish that weighs 1.75 pounds. It costs $.98 a pound. How much do you pay for it?

	1.75	pounds
×	.98	cost per pound
	$1.715	total cost

Notice this: the digit in the thousandths decimal place is 5. When the digit in the thousandths place is 5 or more, you *round up* to the nearest hundredth (cent or penny). Here's how to do that:

- Round up the digit in the hundredths place to the next number.

  $1.725

- Drop the digit that's in the thousandths place.

  $1.72

How would you round these amounts?

a. $1.089
b. $ .999
c. $ .6777
d. $40.1388

Answers: a. $1.09 b. $1.00 c. $.68 d. $40.14

Exercise

Copy and solve these problems. Round your answers to the nearest hundredth.

1. $3.35 an hour × .5 hour
2. 1.5 pounds × $.69 a pound
3. $4.39 an hour × 1.25 hours
4. 12.72 gallons × $1.29 a gallon
5. 6.26 gallons × $1.09 a gallon
6. $1.75 per mile × 5.5 miles

Answers: 1. $1.68 2. $1.04 3. $5.49 4. $16.41 5. $6.82 6. $9.63

Unit Review

Check Yourself

1. Copy these multiplication problems. Write each decimal point in the correct place. You will need to add zeros to some products.

a.	4.5 × 3.1 1395	b.	1.03 × .045 4635	c.	18.02 × 1.49 268498
d.	3.5 × .02 70	e.	.678 × .01 678	f.	.75 × 24 1800

2. Copy and solve these multiplication problems.
   a. 2.981 × .3
   b. .135 × .04
   c. 2.5 × .16
   d. 44.3 × .007
   e. 22.6 × 12.18
   f. .134 × .124

3. Solve these word problems. Round answers to the nearest hundredth.
   a. Jamie fills the gas tank of her car with 8.7 gallons of gas. One gallon costs $1.139. What is the total price that Jamie pays?
   b. Kim works at the Lake Bakery. She works 18.75 hours this week. She earns $3.35 per hour. How much does she earn in all?
   c. The fruit stand sells bananas for $.39 a pound. Carl buys 3.5 pounds. How much does he pay?

4. Choose the right word to complete each sentence.

   factor **product** **round**

   a. You call the answer for numbers that you multiply a ____.
   b. To count decimal places in a ____, you start at its decimal point and count each place from *right* to *left*.
   c. ____ $.568 to $.57.

Bonus Work

1. Make a poster that shows the steps for one of these things:
   a. getting a total of decimal places from the factors of a multiplication problem.
   b. writing the decimal point in the product correctly.
   c. multiplying decimal numbers.
2. Make up a word problem about decimal numbers that you multiply. Solve the problem. Then give it to your classmates to solve.
3. How much would you earn if you worked these hours one week and if you were paid these wages? Round amounts to the nearest hundredth.

	Wage	Hours per Week
a.	$4.15	28.75 hours
b.	$5.37	41.5 hours
c.	$8.31	38.25 hours

Unit 5

Dividing Decimals

Picture this:

You have a job packing boxes. You pack 13 boxes in 60.25 minutes. How long does it take you to pack 1 box?

You want to buy a large bottle of soft drink. You can buy a 16-ounce bottle that costs $1.99, or you can buy a 22-ounce bottle that costs $2.15. Which bottle gives you more for your money?

You own a car. It takes 13.8 gallons of gas to fill up the gas tank. You can travel 260.75 miles on that tank of gas. How many miles can you travel on 1 gallon of gas?

Many real life problems are like the ones you just read. Those problems are usually about money, amounts that are measured, or time. What other real life problems can you think of that are about money, measurements, or time?

To find the answer to those problems, you must divide. The number you divide could be a decimal. The number you divide by could also be a decimal.

You must follow certain rules when you divide with decimals. If you don't, your answer won't be correct. You'll learn those rules in this unit.

Math Words

Look up these words in the glossary. What do they mean?

dividend **divisor**
quotient **remainder**

Division Words

In this unit, you'll use words that name parts of division problems. Here are three of those words:

divisor: A divisor is the number you divide by.

dividend: A dividend is the number you divide.

quotient: A quotient is the answer to a division problem.

Look at this math sentence:

$$1.5 \div 3 = .5$$

Which number is the divisor? Which number is the dividend? And which number is the quotient?

Right! 3 is the divisor. It is the number you'd divide by. 1.5 is the dividend. It is the number that will be divided. And .5 is the quotient. It is the answer you get when you divide 1.5 by 3.

Exercise

Look at these problems and their answers. Which numbers are divisors? Which are dividends? Which are quotients?

1. $6.4 \div .2 = 32$
2. $27 \div .3 = 90$
3. $167.28 \div 13.6 = 12.3$
4. $\begin{array}{r} 1.99 \\ 2\overline{)3.98} \end{array}$
5. $\begin{array}{r} 1.02 \\ 5\overline{)5.10} \end{array}$
6. $\begin{array}{r} 3.07 \\ .3\overline{).921} \end{array}$

Answers:

1. divisor: .2 dividend: 6.4 quotient: 32
2. divisor: .3 dividend: 27 quotient: 90
3. divisor: 13.6 dividend: 167.28 quotient: 12.3
4. divisor: 2 dividend: 3.98 quotient: 1.99
5. divisor: 5 dividend: 5.10 quotient: 1.02
6. divisor: .3 dividend: .921 quotient: 3.07

Copy the Problem Correctly

Look at this math sentence:

$$25.4 \div .5$$

Which number is the divisor? Which is the dividend?

When you're writing a division problem, it's easy to mix up the divisor and the dividend. It's also easy to copy a decimal wrong and put the decimal point in the wrong place. So always check the numbers after you write them. Make sure that:

- The number you *divide by* is in the divisor's place.

$$.5\overline{)}$$

- The number you *divide* is in the dividend's place.

$$\overline{)25.4}$$

- The numbers are copied correctly.

$$.5\overline{)25.4}$$

Exercise

Number a paper from 1 to 6. Then look at the math sentences. Look at the division problems under them. If the problem is written correctly, write *correct*. If the problem is not written correctly, write it correctly on your paper. Then check your work.

1. $18.3 \div .4$

   $.4\overline{)18.3}$

2. $9.02 \div 1.7$

   $9.02\overline{)1.7}$

3. $.87 \div .25$

   $2.5\overline{)8.7}$

4. $193.54 \div .5$

   $.5\overline{)1935.4}$

5. $.3873 \div .03$

   $.003\overline{).3783}$

6. $3.138 \div 3.13$

   $3.13\overline{)318.3}$

Answers:

1. correct
2. $1.7\overline{)9.02}$
3. $.25\overline{).87}$
4. $.5\overline{)193.54}$
5. $.03\overline{).3873}$
6. $3.13\overline{)3.138}$

Dividing by a Whole Number

Suppose you are dividing this problem. The divisor is a whole number.

$$1.55 \div 5$$

Follow these steps when dividing decimals by a whole number:

1. Write the problem.

Which number is the divisor? Which is the dividend?

$$5\overline{)1.55}$$

2. Write the decimal point for the quotient.

Find the decimal point in the dividend. Write another decimal point *directly* above it. That will be the decimal point in the quotient.

$$\begin{array}{r} . \\ 5\overline{)1.55} \end{array}$$

3. Divide the dividend.

Now divide the dividend the same way you divide whole numbers. As you divide each number, write the answer directly above the last digit in that number.

$$\begin{array}{r} .31 \\ 5\overline{)1.55} \end{array}$$

When you finish dividing, the decimal points and decimal places in both the quotient and the dividend should line up. Which numbers line up in the problem above?

A Rule to Remember

Here's the first rule to remember when you divide with decimals:

Rule 1: Line up decimal points and decimal places in the dividend and quotient.

Exercise

Write and solve these problems. Then check your answers. If you get a wrong answer, do the problem again.

1. .36 ÷ 3	6. 16.800 ÷ 8
2. .864 ÷ 2	7. 862.44 ÷ 2
3. .770 ÷ 7	8. 66.36 ÷ 12
4. 2.5 ÷ 5	9. 37.95 ÷ 15
5. 8.48 ÷ 4	10. 5.775 ÷ 25

Answers: 1. .12 2. .432 3. .110 4. .5 5. 2.12 6. 2.100 7. 431.22 8. 5.53 9. 2.53 10. .231

Filling All the Decimal Places

Look at the dividends and quotients in these problems. Notice how they line up.

$$\begin{array}{r} .324 \\ 2\overline{).648} \end{array} \qquad \begin{array}{r} 1.1 \\ 5\overline{)5.5} \end{array} \qquad \begin{array}{r} 3.01 \\ 3\overline{)9.03} \end{array}$$

In division problems, decimal points and decimal places should line up. Each decimal place should be filled by a digit.

But sometimes this happens with certain problems: There aren't enough digits to fill all the decimal places. Here's how to solve those problems:

1. Write the problem.

Example: .2 ÷ 5

$$5\overline{).2}$$

2. Write the decimal point for the quotient.

$$\begin{array}{r} . \\ 5\overline{).2} \end{array}$$

3. Rename the dividend if needed.

In the example, 2 cannot be divided by 5. You must rename the dividend to divide it. To do that, write a zero after the last digit in the dividend. Write only as many zeros as you need. How many zeros are needed in the example?

$$\begin{array}{r} . \\ 5\overline{).20} \end{array}$$

4. Divide the dividend.

As you divide a number, write the answer above the *last digit* of that number.

$$\begin{array}{r} .4 \\ 5\overline{).20} \end{array}$$

5. Fill empty decimal places.

The quotient has one filled decimal place and one empty decimal place. Write zero in the empty decimal place.

$$\begin{array}{r} .04 \\ 5\overline{).20} \end{array}$$

Now look at this problem: .3015 ÷ 15

$$\begin{array}{r} .201 \\ 15\overline{).3015} \\ \underline{30} \\ 1 \\ \underline{0} \\ 15 \\ \underline{15} \\ 00 \end{array}$$

a. How many filled decimal places are in the dividend?

b. How many filled decimal places should there be in the quotient?

c. What is the correct answer to the problem?

Answers:
a. There are four decimal places in the dividend.
b. There should be four decimal places in the quotient.
c. .0201

> **A Rule to Remember**
>
> Here's a second rule to remember when you divide with decimals:
>
> **Rule 2:** The quotient and dividend must have the same number of filled decimal places.

Exercise

Write and solve these problems. Then check your answers. If you get a wrong answer, do the problem again.

1. .36 ÷ 6	4. .12 ÷ 60
2. .5 ÷ 25	5. .3 ÷ 75
3. 6.3 ÷ 6	6. .60 ÷ 150

Answers: 1. .06 2. .02 3. 1.05 4. .002 5. .004 6. .004

Dividing with Decimals

Dividing a Decimal by a Decimal

Many problems will ask you to divide a decimal by a decimal. Before you can divide, you must rename the divisor as a whole number. To solve problems like that:

1. **Write a math sentence for the problem.**

   .3750 ÷ .50

2. **Rename the divisor as a whole number.**

   You rename a decimal as a whole number by moving its decimal point. You move it behind the last digit in the divisor. To show where the decimal point goes, put a mark behind the last digit.

   .3 7 5 0 ÷ .5 0.

3. **Rename the dividend.**

   When you move the decimal point in the divisor, you must also move the decimal point in the dividend. Count the decimal places that you move the decimal point in the divisor. Move the decimal point in the dividend the same number of places. Put a mark in the dividend to show where the decimal point goes.

   .3 7.5 0 ÷ .5 0.

   1 2   1 2

4. **Write the problem with the renamed divisor and dividend.**

   Now you can solve the problem.

```
          .7 5
   50. ) 37.5 0
         35 0
          2 5 0
          2 5 0
          0 0 0
```

Now suppose you solve this problem.

2.412 ÷ 1.2

a. What do you rename its divisor?

b. What do you rename its dividend?

c. What is the answer to the problem?

Answers: a. 12 b. 24.12 c. 2.01

Dividing a Whole Number

Here's how to divide a whole number by a decimal or mixed decimal.

1. **Write a math sentence.**

   Write the whole number with a decimal point after it.

   36. ÷ 1.20

2. **Rename the divisor as a whole number.**

   3 6. ÷ 1.2 0.

3. **Rename the dividend.**

   Count the number of place you move the decimal point in the divisor. In the dividend, write the same number of zeros after the decimal point. Move the decimal point behind the last zero. Put a mark there.

   3 6.0 0. ÷ 1.2 0.

   1 2   1 2

4. **Write the problem with the renamed divisor and dividend.**

   Now you can solve the problem. What's the answer?

   120.) 3600.

Answer: 30

> **A Rule to Remember**
>
> Here's a third rule to remember when you divide with decimals:
>
> **Rule 3:** If the divisor is a decimal or mixed decimal, rename it as a whole number.

Exercise

Rename these math sentences. Then write the problems and solve them.

1. .84 ÷ .2	**4.** .049 ÷ .7
2. 5.55 ÷ .5	**5.** 16 ÷ .04
3. .625 ÷ .025	**6.** 48 ÷ .08

Answers: 1. 4.2 2. 11.1 3. 25 4. .07 5. 400. 6. 600.

Getting a Problem to Divide Evenly

Look at this problem:

```
      .05
50 )2.83
    2 50
      33
```

Notice this: The divisor does not divide the dividend evenly. There is a *remainder*—a number left over. What number is the remainder?

In the problem, 50 does not divide 2.83 evenly. 33 is left over. But you can continue to divide until there are no remainders.

Here's how to solve those kinds of problems:

1. Rename the dividend.

Write a zero after the last digit in the dividend. Then divide as usual. Does the answer come out evenly?

```
      .056
50 )2.830
    2 50
      330
      300
       30
```

2. If you still get a remainder, rename the dividend again.

Sometimes writing one zero after the dividend is not enough. You still get a remainder. Add another zero and divide again. Keep doing that until you can finally divide the dividend evenly.

```
      .0566
50 )2.8300
    2 50
      330
      300
       300
       300
       000
```

Now solve these problems. Rename the dividend in each problem until you can divide it evenly.

1. 14)4.9   2. 4).27

Answers:

```
1.      .35       2.     .0675
   14 )4.90            4 ).2700
```

Exercise

Write these problems and solve them. Then check your answers.

1. .6 ÷ .25
2. 7.56 ÷ 21
3. 1.2 ÷ 20
4. 5.51 ÷ 38
5. 1.8 ÷ 16
6. .147 ÷ 1.75
7. 13.6 ÷ 32
8. .84 ÷ 1.12

Answers:

```
1.      2.4        2.       .36
   .25 ).60 0           21 )7.56
         50
         10 0
3.      .06        4.       .145
    20 )1.20            38 )5.510
5.      .1125      6.        .084
    16 )1.8000        1.75 ).14 700
7.      .425       8.        .75
    32 )13.600        1.12 ).84 00
                             78 4
                              5 60
```

Rounding Quotients

Suppose you divide a problem and this happens: You rename the dividend three times and you still get a remainder. The problems below are like that. (The three dots behind the quotients and dividends show that the problems can still be divided.)

1.
```
     .0333···
3 ) .1000···
     9
     10
      9
     10
      9
      1
```

2.
```
     .1285···
7 ) .9000···
     7
     20
     14
      60
      56
       40
       35
        5
```

You won't be able to divide those problems evenly, no matter how many times you rename the dividend. You'll always get a remainder.

When you solve those kinds of problems, do this: Round their answers. Round them to the nearest hundredth. Or round them to the nearest thousandth.

The Nearest Thousandth

You learned how to round a decimal to the nearest hundredth in Unit 4. Here's how to round a decimal to the nearest thousandth:

First find the digit in the ten thousandths place. Drop all digits after it.

.0033 .1285

If the digit in the ten thousandths place is less than 5, round it *off.*

$.0033 \Rightarrow .003$

If the digit in the ten thousandths place is 5 or more, drop it. Then round *up* the digit in the thousandths place.

$.1285 \rightarrow .129$

How would you round these numbers to the nearest thousandth?

a. 1.0645 **c.** .3912555
b. .9954 **d.** 5.038613

Answers: **a.** 1.065 **b.** .995 **c.** .391 **d.** 5.039

Exercise

1. Round these decimals to the nearest thousandth.
   a. 9.9994
   b. 426.0555
   c. .73612
2. Round these decimals to the nearest hundredth.
   a. 2.469
   b. 56.78411
   c. 303.876
3. Solve these problems. Round your answers to the nearest thousandth.
   a. 2.3 ÷ 3
   b. 2.6 ÷ 2.1
4. Solve these problems. Round your answers to the nearest hundredth.
   a. 3.1 ÷ .09
   b. 4 ÷ .6

Answers: **1. a.** 9.999 **b.** 426.056 **c.** .736
2. a. 2.47 **b.** 56.78 **c.** 303.88
3. a. .767 **b.** 1.238
4. a. 34.44 **b.** 6.67

Unit Review

Check Yourself

1. The quotients of these problems are wrong. Work each problem correctly. Then explain why each problem was wrong and what you did to correct it.

a. 8.32 ÷ 4

$$\begin{array}{r} .203 \\ 4\overline{).832} \end{array}$$

d. .0156 ÷ 3.9

$$\begin{array}{r} .4 \\ 39\overline{)015.6} \end{array}$$

b. 36 ÷ .12

$$\begin{array}{r} 3 \\ .12\overline{)36} \end{array}$$

e. 213.66 ÷ 3

$$\begin{array}{r} 712.2 \\ 3\overline{)2136.6} \end{array}$$

c. .025 ÷ .5

$$\begin{array}{r} .05 \\ 5\overline{)0.025} \end{array}$$

f. 6 ÷ .002

$$\begin{array}{r} 300 \\ 2\overline{)600} \end{array}$$

2. Copy and solve these problems on another sheet of paper. Round answers to the nearest hundredth.

Divide by these whole numbers.

a. 24.4 ÷ 4 **d.** 32 ÷ 14
b. .183 ÷ 5 **e.** \$2.18 ÷ 10
c. 6.2 ÷ 9 **f.** \$.75 ÷ 3

Divide by these decimals.

g. .08 ÷ .2 **j.** .007 ÷ .14
h. 12.992 ÷ .6 **k.** \$23.89 ÷ 2.5
i. 9.25 ÷ 3.7 **l.** \$115.35 ÷ 3.45

3. Read and solve these word problems.
   a. The Madison family is driving to Dallas, Texas. They travel 327.8 miles. Their car uses up 11 gallons of gas altogether. How many miles has their car traveled per gallon of gas?
   b. Larry drives 41.3 miles to work every day. It takes him .75 hour ($\frac{3}{4}$ hour) to drive that distance. How fast does he drive per hour? (Round your answer to the nearest hundredth.)
   c. Clara wants to get the most for her money when she buys a box of cereal. She could buy a 17.5-ounce box at \$2.85 or a 20-ounce box at \$2.99. Which box of cereal is a better buy? (Hint: Which costs less per ounce?)

4. Choose the correct word or words to complete each sentence below.

dividend **quotient**
divisor **remainder**

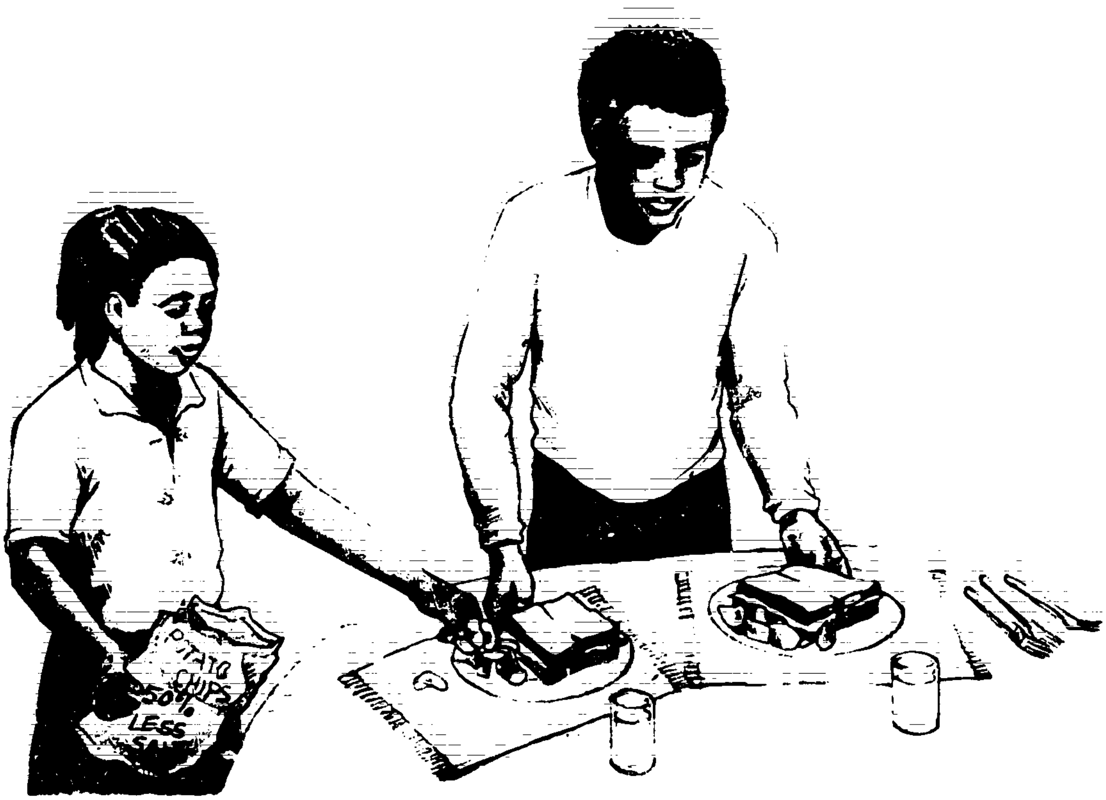
   a. When the ______ has a decimal, you must rename it as a whole number.
   b. The answer to a division problem is called a ______.
   c. When you move the decimal point in the divisor, you must also move the decimal point in the ______.
   d. When a number will not divide evenly, it leaves a ______.

Bonus Work

Go to a grocery store. Compare the prices and weights of different brands of the same kind of food. Then make a poster with your information. Use this title: ***Be a Wise Shopper.***

Unit 6

Percents



So far, you've shown parts of whole amounts in two ways: You've shown them as *common fractions*. And you've shown them as *decimals*.

Another way to show parts of whole amounts is with *percents*. A percent is simply another way to write a fraction.

We use percents a great deal in everyday life. Look around you. You can see and hear percents being used almost every place you go. For example, let's say:

- You're riding a bus to school. The person next to you is reading a newspaper. You read the headline:
  72% Vote in Election
- In your math class you get back a test you took. On the test you see:
  Your score—92%
- You buy a bag of chips for a snack. You read this on the package:
  50% less salt
- After school you go to a record store. You buy a record. The sales clerk tells you:
  "The tax is six and a half percent."
- When you get home, you listen to your new record. Somebody sings,
  "I love you one hundred percent!"

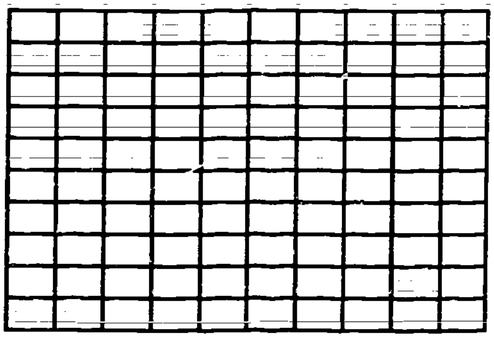
What are some other places where you might see or hear percents being used?

In this unit, you'll learn exactly what percents are. You'll learn what they mean, too.

Math Words
Look up these words in the glossary. What do they mean?
one hundred percent   **percent**

What Is a Percent?

Look at this shape. It is divided into 100 equal parts.



Each part is a fraction of all the parts. That fraction is *one hundredth* of all the parts.

Each part is also *one percent* of all the parts. A percent is another way to write a fraction.

But percents show only one kind of fraction. What kind of fraction do you think that is?

100 Equal Parts

Percents only show fractions of things that are divided into 100 *equal parts*. In fact the word *percent* means "part of one hundred." So, a percent is *one* part of one hundred parts ($\frac{1}{100}$). Two percent is *two* parts of one hundred parts ($\frac{2}{100}$). Three percent is *three* parts of one hundred parts ($\frac{3}{100}$), and so on.

Amounts that are divided into 100 equal parts can be shown in other ways besides percents. They can be shown as decimals. And they can be shown as common fractions. For example, you can show 1 part of 100 parts like this:

1 percent **or** .01 **or** $\frac{1}{100}$.

They are all the same. They all equal the same amount—one hundredth.

1 percent = .01 = $\frac{1}{100}$ (a hundredth)

What decimals and common fractions are the same as these percents?

a. 10 percent
b. 18 percent
c. 25 percent

Answer:
a. 10 percent = .10 = $\frac{10}{100}$ (10 hundredths)
b. 18 percent = .18 = $\frac{18}{100}$ (18 hundredths)
c. 25 percent = .25 = $\frac{25}{100}$ (25 hundredths)

> Remember:
> A percent is part of an amount that is divided into 100 equal parts. So 1 percent is the same as .01 or $\frac{1}{100}$.

Exercise

1. Write fractions that are the same as these percents.
   a. 4 percent **b.** 5 percent **c.** 20 percent **d.** 80 percent
   e. 9 percent **f.** 75 percent **g.** 66 percent **h.** 99 percent
2. Write decimals that are the same as these percents.
   a. 6 percent **b.** 7 percent **c.** 3 percent **d.** 30 percent
   e. 8 percent **f.** 28 percent **g.** 79 percent **h.** 50 percent
3. Write fractions and decimals that are the same as the percents in these sentences.
   a. She did 72 percent of her homework.
   b. 18 percent of the students are absent.
   c. 75 percent of the voters voted.

Answers:
1. a. $\frac{4}{100}$ **b.** $\frac{5}{100}$ **c.** $\frac{20}{100}$ **d.** $\frac{80}{100}$
e. $\frac{9}{100}$ **f.** $\frac{75}{100}$ **g.** $\frac{66}{100}$ **h.** $\frac{99}{100}$
2. a. .06 **b.** .07 **c.** .03 **d.** .30 **e.** .08 **f.** .28
g. .79 **h.** .50
3. a. 72 percent = $\frac{72}{100}$ = .72
b. 18 percent = $\frac{18}{100}$ = .18
c. 75 percent = $\frac{75}{100}$ = .75

Writing Percents

You learned that fractions are made up of numerators and denominators. Think of the fraction *one hundredth.* Which number is the numerator? Which number is the denominator?

When you write a common fraction, you show the numerator and denominator of *one hundredth* like this:

$$\frac{1}{100} \quad \frac{\text{numerator}}{\text{denominator}}$$

When you write the same fraction as a decimal, you use a decimal point and two decimal places to show the denominator. The numerator is the numeral in the decimal.

.01

How do you show the numerator and denominator of that fraction when you write it as a percent?

The Denominator Is 100

When you write a percent, the numeral you write is the numerator. The word *percent* tells you that the denominator is 100. You can also show the denominator by using the *percent sign: %.* For example:

$\frac{5}{100}$ is 5% or 5 *percent*

.05 is 5% or 5 *percent*

100 Pennies

Now think of a dollar. It can be divided into 100 equal parts. What are those parts?

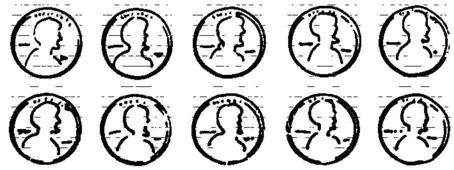
Right! the 100 equal parts of a dollar are pennies (or cents). What percent is one part of the dollar? What percent is 60 parts of the dollar?

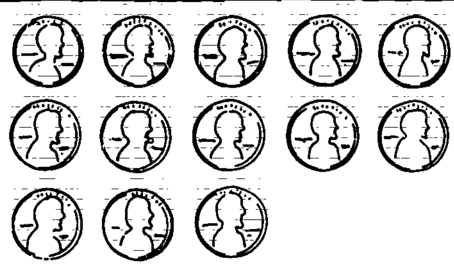
Each part of the dollar is 1%. So 60 parts are 60%.

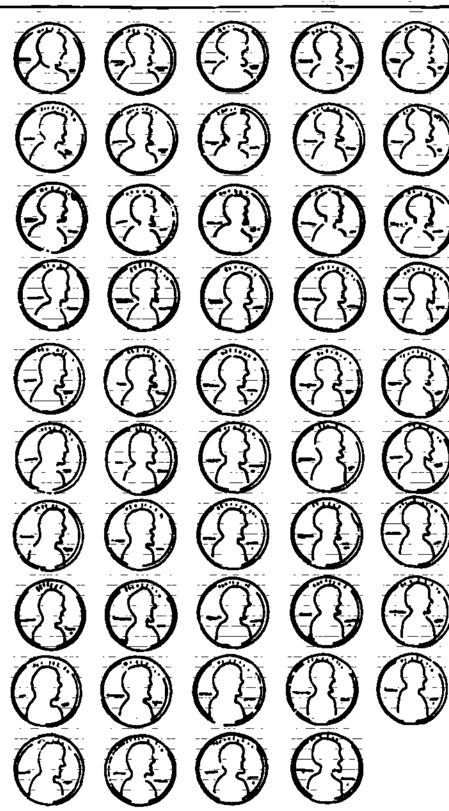
Exercise

Count the pennies in each picture. Then write the percent that shows what part of a dollar those pennies are. Use the percent sign.

1. 

2. 

3. 

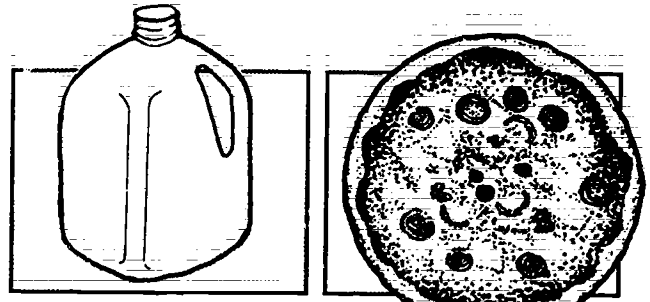
4. 

Answers: 1. 5% 2. 10% 3. 13% 4. 49%

A Hundred Percent

You've learned that percents show parts of whole amounts. And those amounts are divided into 100 equal parts. So 1% is one part of 100 parts. 20% is 20 parts of 100 parts. And 90% is 90 parts of 100 parts. What percent are all 100 parts?

Right! All 100 parts are 100%. All 100 parts are also one whole amount. So 100% stands for a whole amount.

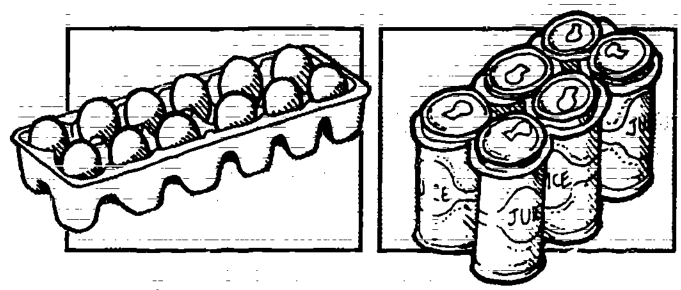


One whole thing = 100%

One Whole Thing

A whole amount can be *one whole thing*. For example, a gallon of milk is a whole amount. A whole pizza is also a whole amount. All of the pizza is 100% of one whole thing. And all of the milk is 100% of one whole thing.

What are other examples of things that are 100 percent of one whole thing?



One whole group = 100%

One Whole Group

A whole amount can be a *group* of things. For example, a carton of eggs is a whole amount. A six-pack of soft drinks is also a whole amount.

12 eggs in a carton are 100% of a whole group. And six cans of soft drink are 100% of another whole group.

Here's another example: A class of students can be a whole group. All the students that belong to a class make up 100% of a whole group. If 30 students belong to the class, 100% is 30. But if 12 students belong to the class, then 100% is 12.

How many students make up 100% of your class?

> **Remember:**
> 100% stands for a whole amount. That whole amount can be a whole group of things or one whole thing.

Exercise

What amounts make up 100% of these groups?

1. A high school basketball team playing on the court.
2. A baseball team playing on the field
3. A football team playing on the field
4. A class of two dozen people
5. A class of four dozen people

Answers:
1. 5 basketball players = 100%
2. 9 baseball players = 100%
3. 11 football players = 100%
4. 24 students = 100%
5. 48 people = 100%

How Many Hundredths?

Suppose you read a newspaper headline that tells you this: 75% of the people who voted in an election voted for a certain person. Can you figure out what percent did *not* vote for that person?

Remember: A whole amount is 100%. So, if 75% voted for the person, 25% did *not*.

100%	all voters	75%	for
− 75%	for	+ 25%	not for
25%	not for	100%	all voters

How many voters *actually* voted? You cannot tell. You only know the *percent* of voters. For example, 100% does not mean there were 100 voters. 100% stands for the total amount, whatever that total amount may be.

60 in All

Suppose you take a test for a driver's license. The test has 60 questions. If you answer all 60 correctly, what is your score?

Your score is 100%. If you answer 45 of those questions correctly, your score is 75%.

20 in All

Suppose you take a math test. The test has 20 questions. What is your score if you answer all 20 questions correctly?

Your score is also 100%. If you answer 15 questions correctly, your score is 75%.

They Are All 100%

60 correct questions is 100%. And 20 correct questions is 100%. 45 correct questions is 75%. And 15 correct questions is 75%. How can that be?

The two tests are made up of two different *whole amounts*. One whole amount is 60. The other whole amount is 20. When we use percents, we divide the different whole amounts the same way—into 100 equal parts.

To score a test of 60 questions, we divide the 60 into 100 equal parts. The 60 is now the same as *100 parts of 100 parts* or $\frac{100}{100}$. It is 100%, or the whole amount. And 35 questions is *75 parts of 100 parts*, or $\frac{75}{100}$. 35 is 75% of the whole amount.

To score a test of 20 questions, we divide the 20 into 100 equal parts. 20 is the same as *100 parts of 100 parts*. And 15 is the same as *75 parts of 100 parts*. What percent is 20 questions? How would you write that percent as a fraction? What percent is 15 questions? How would you write that percent as a fraction?

Answers: 20 is 100% or $\frac{100}{100}$
15 is 75% or $\frac{75}{100}$

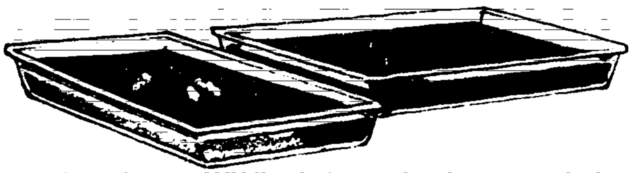
Exercise

1. Write percents and fractions for these numbers. The first is done.
   a. 10 questions = 50 parts of 100 parts
      10 questions = 50% = $\frac{50}{100}$
   b. 10 cups = 75 parts of 100 parts
      10 cups = ■% = $\frac{■}{■}$
   c. 25 miles = 15 parts of 100 parts
      25 miles = ■% = $\frac{■}{■}$
   d. 25 hours = 40 parts of 100 parts
      25 hours = ■% = $\frac{■}{■}$
2. Read and solve these word problems.
   a. Tom pays 75% of his loan. What percent is left to pay?
   b. Angel puts 15% of her paycheck into her saving account. What percent does she have left?

Answers: 1. a. 10 questions = 50% = $\frac{50}{100}$
b. 10 cups = 75% = $\frac{75}{100}$
c. 25 miles = 15% = $\frac{15}{100}$
d. 25 hours = 40% = $\frac{40}{100}$
2. a. 25% b. 85%

More Than 100%

Let's say you work in a deli. You are making a dessert. You must make enough to fill just one pan. But when you make the dessert, it fills *two pans*. You now have two times the amount of one pan.



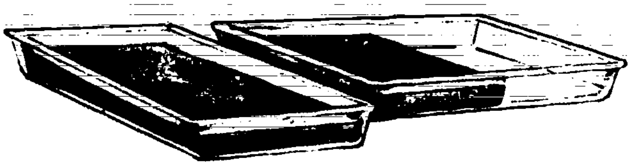
You have 100% of dessert when you fill one pan. You have 100% more dessert when you fill the other pan. What percent of dessert do you have altogether?

Right! You have *200%* of dessert.

When you work with percents, you sometimes need to show amounts that are more than one whole amount. You do that by adding the other amounts to the whole amount. For example:

$$100\% + 100\% = 200\%$$

Now suppose you make another batch of dessert. This time the dessert fills 1 pan. Then it fills half of another pan (50%). What percent of dessert do you have?



You have 150%. The pan that's filled is 100%. The pan that's half filled is 50%.

$$100\% + 50\% = 150\%$$

Exercise

Add these percents. Then check your answers.

1. 100% + 75%	4. 100% + 5% + 1%
2. 100% + 99%	5. 100% + 23% + 72%
3. 100% + 1%	6. 100% + 50% + 50%

Answers: 1. 175% 2. 199% 3. 101% 4. 106% 5. 195% 6. 200%

100% Plus a Fraction

Imagine that you are back in the deli, making another batch of dessert. This time you make enough to fill one pan and a tiny bit of another pan. That tiny bit is *less than one percent* of the pan. In fact, it is $\frac{1}{2}$ of a percent. What percent of dessert do you have altogether?

You have $100\frac{1}{2}\%$ of dessert. In other words, you have 100% (a whole amount) and $\frac{1}{2}\%$ (a fraction of one percent).

$$100\% + \tfrac{1}{2}\% = 100\tfrac{1}{2}\%$$

You'd read $100\frac{1}{2}\%$ as *one hundred and one-half percent.*

You can also show that amount with a decimal:

$$100\tfrac{1}{2}\% = 100.5\%$$

You'd read 100.5% as *one hundred and five-tenths percent.* Or *one hundred and point five percent.*

When you work with percents, you'll sometimes use amounts that are fractions of one percent. That one percent can be divided into any number of equal parts.

Suppose a percent is divided into three parts. What percent is one part? What percent is two parts?

Right! One part of that percent is $\frac{1}{3}\%$. And two parts are $\frac{2}{3}\%$.

Exercise

Add these percents.

1. $100\% + \frac{1}{5}\%$	3. 100% + .25%
2. $100\% + \frac{3}{4}\%$	4. 100% + .1% + .5%

Answers: 1. $100\frac{1}{5}\%$ 2. $100\frac{3}{4}\%$ 3. 100.25% 4. 100.6%

Unit Review

Check Yourself

1. What things make up one hundred percent?
   a. Name five *groups* of things that make up one hundred percent. (An example is one *carton* of eggs.)
   b. Name five *separate* things that make up 100 percent. (An example is one bicycle.)
2. How many hundredths do these percents show?

a. 18%	d. 78%
b. 42%	e. 11%
c. 87%	f. 39%

3. Read and solve these word problems.
   a. 85% of the students in Mt. Lander High School voted in the school election. What percent of students did not vote in the election?
   b. Jim pays for gasoline for the family car 35% of the time. What percent shows how often his parents pay for gas?
   c. Debbie gets her science test back. Her score shows she answered 82% of her test correctly. What percent of the test did she get wrong?
4. Choose the right word to complete each sentence.

   percent **one hundred percent**

   a. One _____ is a hundredth of a whole amount.
   b. _____ is all of 100 parts.

Bonus Work

1. Make a poster that shows how we use percents in real life. Look through newspapers and magazines for ads and stories that show percents. Cut them out and paste them on poster-sized paper.
2. Get a partner and play "What Makes Up 100%?" You'll need these coins (or write the amount of each coin on strips of paper): three quarters, five dimes, five nickels, and ten pennies. You'll also need a pencil, a sheet of paper, and a timer.

   Here's what you do:

   a. Set the timer for 10 minutes.
   b. Combine the amounts of any coins so that they add up to $1.00. That's 100%. Example: the amounts of two quarters and five dimes are 100%.
   c. Write the coins that you put together.
      Example: 2 quarters + 5 dimes
   d. Make as many combinations as you can before the time runs out.

Unit 7

Ratios

Imagine this:

Your favorite clothing store is having a one-day sale. All clothes are 50% off. If you buy a pair of pants on that day you will pay $16. If you buy that same pair of pants the next day, you will pay $32. The regular price is twice as much as the sale price. You'd save $1 out of $2.

Your favorite baseball team is off to a pretty good start. So far it has played 20 games. It has lost only 5 of those games. That means its win/loss record is 15 to 5—or 3 to 1.

You've just read two examples of using *ratios* in everyday life. We use ratios to compare a part of an amount with the whole amount. For example: We compare a sales price with the regular price.

We also use ratios to compare a part of an amount with other parts of that amount. For example: the win/loss record of a baseball team.

In this unit, you'll learn that percents are one kind of ratio. They are like the ratio in the first example. You'll also learn how to read and write those kinds of ratios. And you'll see how they are used in everyday life.

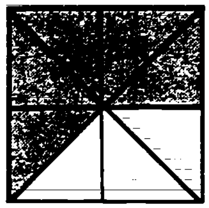
Math Words

Look up these words in the glossary. What do they mean?

ratio **reduce** **lowest terms**

Writing Ratios

Look at this shape. It is divided into 8 equal parts. Some of those parts are shaded. How many parts are shaded compared to the total parts?



To show how many shaded parts there are compared to the total parts, we can write a ratio like this:

shaded parts : total parts

5 : 8

When we write a ratio like that, we use a *colon* (:). The colon separates the number of parts from the number of total parts. Which number shows the parts: the number before or after the colon? Which shows the total parts: The number before or after the colon?

To read ratios that compare a part to its whole amount, we say the number of parts is "out of" the total number of parts. For example, read 5 : 8 like this: *five out of eight.*

Exercise

Read these ratios out loud. Which number shows the parts? Which number shows the total parts?

1. 6 : 12	**4.** 4 : 10
2. 3 : 10	**5.** 16 : 100
3. 9 : 20	**6.** 20 : 100

Answers:
1. 6 parts, 12 total parts
2. 3 parts, 10 total parts
3. 9 parts, 20 total parts
4. 4 parts, 10 total parts
5. 16 parts, 100 total parts
6. 20 parts, 100 total parts

Writing Ratios as Fractions

You learned to write ratios with a colon like this:

shaded parts : total parts

5 : 8

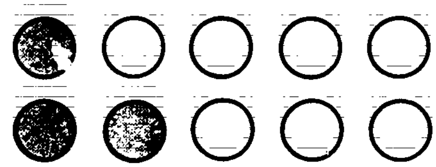
You can also write a ratio so that it looks like this:

$\frac{5}{8}$ **shaded parts / total parts**

Notice that the ratio now looks like a fraction. That's because a ratio *is* a fraction. A fraction shows how many parts there are out of a total of parts.

Which term of the fraction shows the part? Which term shows the total parts? Right. The numerator shows the part. The denominator shows the total parts.

Look at this group of shapes. How many shapes are there in all? How many shapes are shaded?



Right. There are ten shapes in all. Three out of the ten shapes are shaded.

a. Write a ratio that compares the shapes that are shaded to the total number of shapes. Write the ratio as a fraction.

b. Write a ratio that compares the shapes that are *not* shaded to the total number of shapes. Write that ratio as a fraction.

Answers: **a.** $\frac{3}{10}$ **shaded shapes / total shapes**

b. $\frac{7}{10}$ **shapes not shaded / total shapes**

Percents Are Ratios

When we use percents, we are using ratios. A percent describes a part out of a whole. For example, 25% is 25 parts out of 100 parts. You can easily see that a percent is a ratio when you rename it as a fraction. For example:

$$25\% = \frac{25}{100} \begin{array}{l}\textbf{parts}\\ \textbf{total parts}\end{array}$$

Here's how you'd write the ratio with a colon:

$$25\% = \frac{25}{100} \begin{array}{l}\textbf{parts}\\ \textbf{total parts}\end{array} = 25:100$$

You'd read that as *25 out of 100.*

Ratios for 100%

How would you write ratios for 100%? To write a fraction ratio for 100%, write 100 as the denominator and 100 as the numerator. To show that ratio with a colon, write the colon between the numerator and the denominator:

$$100\% = \frac{100}{100} = 100:100$$

You'd read that ratio as *100 out of 100.*

Ratios for More Than 100%

How would you write ratios for percents that are greater than 100%? You'd do the same thing: write 100 as the denominator. Write the part as the numerator. For example, here's how to write ratios for 250%:

$$250\% = \frac{250}{100} = 250:100$$

You'd read those ratios as *250 out of 100.*

Practice reading these ratios out loud. Remember: The numerator is the first number. The denominator is the second number.

a. $20\% = \frac{20}{100} = 20:100$
b. $55\% = \frac{55}{100} = 55:100$
c. $99\% = \frac{99}{100} = 99:100$
d. $100\% = \frac{100}{100} = 100:100$
e. $150\% = \frac{150}{100} = 150:100$

Exercise

1. Read these math sentences. Write ratios for the percents. First write them as fractions. Then write them as ratios with a colon.
   a. 26% girls in music class
   b. 44% people voted in the election
   c. 88% on the test
   d. 125% price increase
   e. 8% tax
   f. 15% tip
2. Write these ratios as percents. Next write them as fractions. Then write them as ratios with a colon.
   a. 28 out of 100 **d.** 90 out of 100
   b. 60 out of 100 **e.** 80 out of 100
   c. 40 out of 100 **f.** 75 out of 100

Answers:

1. **a.** $\frac{26}{100}$ 26 : 100 **d.** $\frac{125}{100}$ 125 : 100
   b. $\frac{44}{100}$ 44 : 100 **e.** $\frac{8}{100}$ 8 : 100
   c. $\frac{88}{100}$ 88 : 100 **f.** $\frac{15}{100}$ 15 : 100
2. **a.** 28% $\frac{28}{100}$ 28 : 100 **d.** 90% $\frac{90}{100}$ 90 : 100
   b. 60% $\frac{60}{100}$ 60 : 100 **e.** 80% $\frac{80}{100}$ 80 : 100
   c. 40% $\frac{40}{100}$ 40 : 100 **f.** 75% $\frac{75}{100}$ 75 : 100

Showing Ratios in Lowest Terms

At a school, several students were asked to answer a question on a survey. The question was: *Do you think the President is doing a good job?*

Out of all the students, 60% said *yes*. How do you show that ratio as a fraction? The answer is $\frac{3}{5}$.

On page 54, you learned to write fraction ratios for percents. You learned to write 100 as the denominator. So, 60% of all the students would seem to be written $\frac{60}{100}$. Why is the correct fraction ratio $\frac{3}{5}$ and not $\frac{60}{100}$?

Right! $\frac{3}{5}$ is the answer you get when you *reduce* $\frac{60}{100}$ to its lowest terms. When you write a ratio as a fraction, you can reduce it to its lowest terms.

You reduce a fraction by dividing its terms by a same number. For example:

$$\frac{60 \div 10}{100 \div 10} = \frac{6}{10}$$

You keep dividing the terms by a same number until you can't reduce them any more.

$$\frac{60 \div 10}{100 \div 10} = \frac{6 \div 2}{10 \div 2} = \frac{3}{5}$$

Let's go back to the survey. If 60% of all students said *yes*, then 40% said *no*. What fraction ratio of students said *no?* Reduce your answer to its lowest terms.

Answer: $\frac{2}{5}$ of the students said no.

$\frac{40}{100} = \frac{4}{10} = \frac{2}{5}$

Exercise

1. Finish finding the correct ratios for these percents. Reduce them to their lowest terms.
   a. $25\% = \frac{25}{100} = \frac{5}{20} =$
   b. $35\% = \frac{35}{100} =$
   c. $28\% = \frac{28}{100} = \frac{14}{50} =$
   d. $90\% = \frac{90}{100} = \frac{45}{50} =$
2. What is the ratio for each percent? Write the ratio as a fraction, then reduce the fraction to its lowest terms.
   a. 80% d. 16%
   b. 75% e. 58%
   c. 30% f. 45%
3. Write these ratios as fractions. Then reduce them to their lowest terms.
   a. 8 out of 10 d. 16 out of 48
   b. 75 out of 100 e. 18 out of 58
   c. 4 out of 6 f. 25 out of 100

Answers:

1. a. $\frac{1}{4}$ b. $\frac{7}{20}$ c. $\frac{7}{25}$ d. $\frac{9}{10}$
2. a. $80\% = \frac{80}{100} = \frac{4}{5}$ d. $16\% = \frac{16}{100} = \frac{4}{25}$
   b. $75\% = \frac{75}{100} = \frac{3}{4}$ e. $58\% = \frac{58}{100} = \frac{29}{50}$
   c. $30\% = \frac{30}{100} = \frac{3}{10}$ f. $45\% = \frac{45}{100} = \frac{9}{20}$
3. a. $\frac{8}{10} = \frac{4}{5}$ d. $\frac{16}{48} = \frac{1}{3}$
   b. $\frac{75}{100} = \frac{3}{4}$ e. $\frac{18}{58} = \frac{9}{29}$
   c. $\frac{4}{6} = \frac{2}{3}$ f. $\frac{25}{100} = \frac{1}{4}$

Finding Ratios in Word Problems

Sometimes a problem will ask you to find more than one ratio. For example:

> Tony takes a math test. He answers 75% of the questions correctly. What is the ratio of correct answers to all the answers? What is the ratio of wrong answers to all the answers?

Follow these steps to help you solve these kinds of problems:

1. Look for the sentences that tell you what ratios to find.

In this problem you are finding *two* ratios. One is the ratio of correct answers out of all answers. What is the other ratio?

2. Find the first ratio.

The problem tells you that 75% of the answers are correct. You know that a percent is a ratio. So, rename that percent as a fraction.

$$\frac{75}{100}$$

3. Reduce to lowest terms.

$$\frac{75 \div 5}{100 \div 5} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$$

The first ratio is this: 3 out of 4 answers are correct.

4. Find the other percent.

Now you must find the other ratio—the ratio of wrong answers to all answers. To do that, you must first figure out what percent of answers are wrong. The problem tells you that 75% of the answers are correct. How would you find the percent of answers that are wrong?

100%	**all answers**
− 75%	**correct answers**
25%	**wrong answers**

5. Find the second ratio.

Rename the percent as a fraction.

$$\frac{25}{100}$$

6. Reduce to lowest terms.

$$\frac{25 \div 5}{100 \div 5} = \frac{5 \div 5}{20 \div 5} = \frac{1}{4}$$

The other ratio is this: 1 out of 4 answers is wrong.

Exercise

Read and solve these word problems. Write fraction ratios. Then check your answers.

1. 44% of the seniors at a high school plan to go to a college. The rest plan to get full-time jobs. Out of all seniors, what is the ratio of those who plan to go to a college? What is the ratio of those who plan to work full time?
2. Kris is an All-City basketball player. She makes 60% of all baskets that she shoots in a game. What is the ratio of baskets she makes? Out of all the baskets she shoots, what is the ratio of baskets she misses?
3. Lin puts 30% of his pay into a savings account. He uses the rest to pay bills. What is the ratio of money he saves out of his pay? What is the ratio of money he spends out of his pay?

Answers:

1. $\frac{11}{25}$ plan to go to college; $\frac{14}{25}$ plan to work.
2. She makes $\frac{3}{5}$; she misses $\frac{2}{5}$
3. Lin saves $\frac{3}{10}$ of his pay; he spends $\frac{7}{10}$

Unit Review

Check Yourself

1. Explain which numbers show a part and which show a whole amount in these ratios.
   a. 2 : 3 b. $\frac{5}{11}$
2. Copy these fraction ratios. Reduce them to lowest terms.
   a. $\frac{8}{12}$ d. $\frac{12}{100}$
   b. $\frac{42}{84}$ e. $\frac{49}{81}$
   c. $\frac{88}{100}$ f. $\frac{66}{100}$
3. Find ratios for these amounts. Write fraction ratios and reduce them to lowest terms.
   a. 46 elderly people out of 121 church members
   b. four black teenagers out of 98 teenagers
   c. four out of seven Southeast Asian countries
   d. 33 pages out of 100 pages
   e. $72 out of $100
   f. 81 children out of 1,800 people
4. Read and solve these word problems. Write fraction ratios and reduce them to lowest terms.
   a. A drawing class has 28 students. 19 students use blue pens to do a project. 5 students use black pens. What ratio of students use blue pens?
   b. Tara gets an allowance every week. She spends 75% of it on food. What part of her total allowance is for food?
   c. The Cortez brothers are making a snack mix for their hike. 20% of their mix is raisins. 80% is nuts. What's the ratio of raisins to the whole snack mix?
5. Choose the right word to complete each sentence below.
   reduce lowest terms ratios
   a. We can use ______ to compare a part with its whole amount.
   b. You ______ a ratio to its lowest terms by dividing both terms by a same whole number.
   c. The ______ ______ for the ratio $\frac{8}{10}$ is $\frac{4}{5}$.

Bonus Work

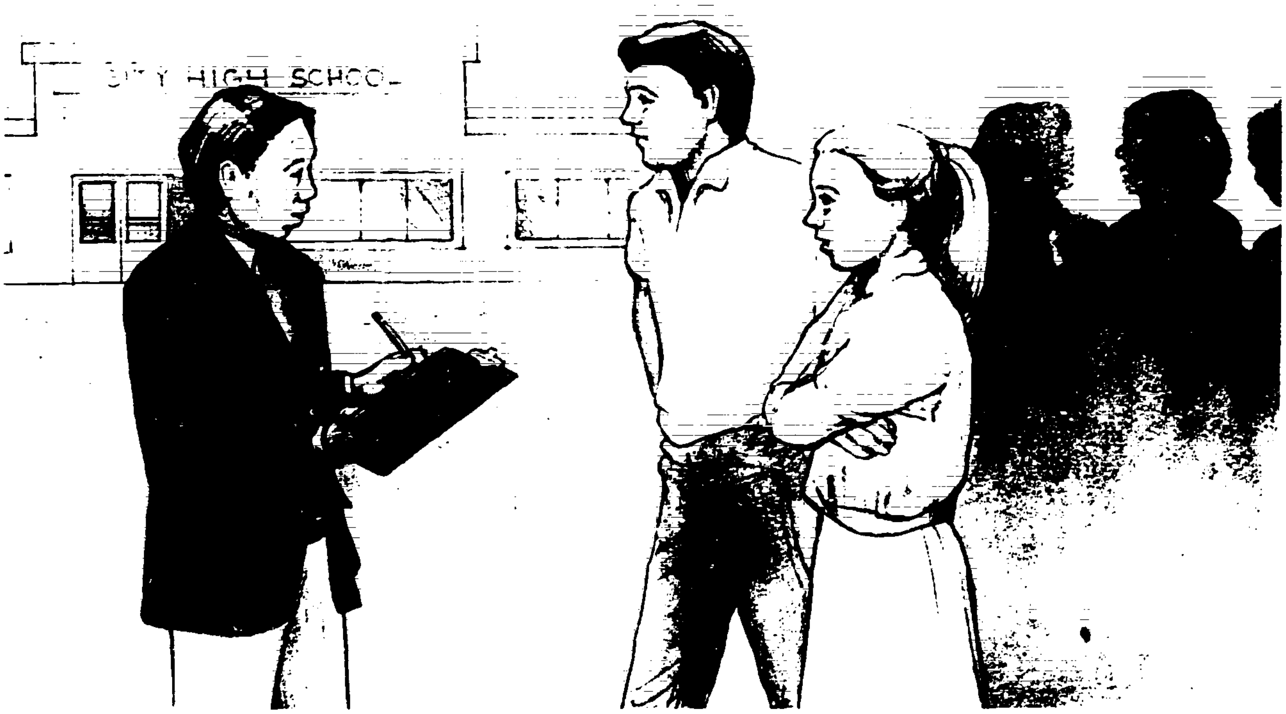
1. Use this chart to figure out how many hours out of 24 hours (a whole day) you spend on these activities. Then make a fraction ratio for each activity, showing how many hours out of 24 hours you spend. Reduce ratios to lowest terms. An example is shown.

Activity	Number of Hours	Ratio
Sleeping	8	$\frac{8}{24} = \frac{1}{3}$
Eating		
Walking		
Studying		
Watching TV		

2. How many people out of the total people in your class have these qualities? Find out. Then write ratios for each.
   a. How many are female? male?
   b. How many are right-handed? left-handed?
   c. How many say blue is their favorite color? green? red? yellow?

Unit 8

Proportions



Suppose you read this story in a newspaper:

200 graduating seniors from a high school were asked what they planned to do after they graduated.

Most students planned to go to college. Others planned to get a job or join a military service. Some didn't know what they would do. This chart shows how they answered.

	Number of Seniors	Percent of Seniors
college	110	55%
work	46	23%
military	12	6%
not sure	32	16%
Total	**200**	**100%**

You can figure out fraction ratios by looking at the chart. For example, if 23% of the students plan to work, then the ratio of those students to the whole group is $\frac{23}{100}$ or 23 out of 100.

But you can also write another kind of ratio for that same group of students. That ratio is $\frac{46}{200}$. It shows the *actual number* of students who planned to work, out of the *actual number* of students who were asked. In other words: 46 students out of a total of 200 students plan to work.

$\frac{23}{100}$ and $\frac{46}{200}$ are ***equivalent ratios.*** You can use equivalent ratios to solve problems about percents. But first you must learn how to make *proportions*.

Math Words

Look up these words in the glossary. What do they mean?

cross product **equivalent ratio**
proportion

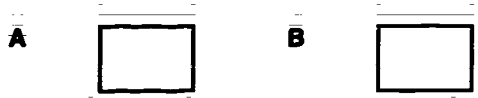
Equivalent Ratios

In Unit 7, you learned to reduce a ratio to its lowest terms. For example:

$$\frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$$

You do not change the value of the ratio when you reduce it.

Look at these two shapes. Shape A is the same size as shape B. They are *equivalents*.

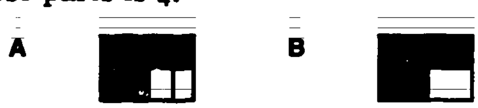


Let's divide shape A into 8 equal parts. The ratio of those parts is $\frac{8}{8}$. Let's divide shape B into 4 equal parts. The ratio of those parts is $\frac{4}{4}$.

Why is $\frac{8}{8}$ the same as $\frac{4}{4}$?

($\frac{8}{8}$ = 8 out of 8 parts) ($\frac{4}{4}$ = 4 out of 4 parts)

Now, suppose 6 parts out of 8 parts in shape A are shaded. The ratio of those parts is $\frac{6}{8}$. And suppose 3 parts out of 4 parts in shape B are shaded. The ratio of those parts is $\frac{3}{4}$.



($\frac{6}{8}$ = 6 out of 8 parts) ($\frac{3}{4}$ = 3 out of 4 parts)

Look at the size of all the shaded parts in shape A. Compare it with the size of all the shaded parts in shape B.

Why is the ratio $\frac{6}{8}$ equivalent to the ratio $\frac{3}{4}$?

Exercise

Write equivalents for these ratios. Be sure to reduce them to their lowest terms.

1. $\frac{25}{75}$ 2. $\frac{36}{81}$ 3. $\frac{68}{344}$ 4. $\frac{125}{600}$

Answers: 1. $\frac{1}{3}$ 2. $\frac{4}{9}$ 3. $\frac{17}{86}$ 4. $\frac{5}{24}$

Writing and Reading Proportions

You can use equivalent ratios to solve certain problems about percents. But to do that, you must first write the equivalent ratios as a *proportion*. You do that by writing the equivalent ratios with an equal sign.

For example, you know that $\frac{6}{8}$ and $\frac{3}{4}$ are equivalent ratios. Here's how to write them as a proportion.

$$\frac{6}{8} = \frac{3}{4}$$

You can read that proportion as: *6 out of 8 is the same as 3 out of 4.*

Now look at these equivalent ratios:

5 out of 10   1 out of 2

a. How would you write those ratios as a proportion?
b. How would you read that proportion?

Answers:
a. $\frac{5}{10} = \frac{1}{2}$
b. 5 out of 10 is the same as 1 out of 2

Exercise

Write a proportion for each set of equivalent ratios.

1. 15 out of 20 pounds is the same as 3 out of 4 pounds.
2. 18 girls out of 45 students is the same as 2 girls out of 5 students.
3. \$20 out of \$100 is the same as \$1 out of \$5.
4. 12 feet out of 38 feet is the same as 6 feet out of 19 feet.

Answers: 1. $\frac{15}{20} = \frac{3}{4}$ 2. $\frac{18}{45} = \frac{2}{5}$ 3. $\frac{20}{100} = \frac{1}{5}$ 4. $\frac{12}{38} = \frac{6}{19}$

Check by Multiplying

Look at this proportion:

$$\frac{2}{5} = \frac{40}{100}$$

Is the proportion true? Is 2 out of 5 really the same as 40 out of 100? Let's find out. Follow these steps:

1. Multiply across one way.

Multiply the numerator of the *first* ratio with the denominator of the *second* ratio. The answer is called a *cross product*. What is this cross product?

$\frac{2}{5} \quad \frac{40}{100}$ 2 x 100 = 200

2. Multiply across the other way.

Now multiply the denominator of the *first* ratio with the numerator of the *second* ratio. What is the cross product?

$\frac{2}{5} \quad \frac{40}{100}$ 5 x 40 = 200

3. Compare the two cross products.

Are the two cross products the same? If they are, the proportion is true.

$\frac{2}{5} \quad \frac{40}{100}$ 2 x 100 = 200

and 5 x 40 = 200

so 200 = 200

> **Remember:**
> If the cross products are the same, the proportion is true.

Exercise

See if these proportions are true or false. Find their cross products.

1. $\frac{2}{4} = \frac{50}{100}$ 2. $\frac{5}{10} = \frac{50}{100}$ 3. $\frac{1}{15} = \frac{20}{100}$

Answers: 1. true: 200 = 200
2. true: 500 = 500
3. false

Check by Reducing

Here's another way to see if a proportion is true. Reduce *both ratios to their lowest terms*. For example:

$$\frac{28}{56} = \frac{50}{100}$$

1. Reduce the first ratio.

$$\frac{\overset{1}{\cancel{28}}}{\underset{2}{\cancel{56}}} = \frac{50}{100}$$

$$\frac{28 \div 2}{56 \div 2} = \frac{14 \div 2}{28 \div 2} = \frac{7}{14 \div 7} \quad \frac{7}{\ } = \frac{1}{2}$$

2. Reduce the second ratio.

$$\frac{\overset{1}{\cancel{28}}}{\underset{2}{\cancel{56}}} = \frac{\overset{1}{\cancel{50}}}{\underset{2}{\cancel{100}}}$$

$$\frac{50 \div 2}{100 \div 2} = \frac{25 \div 5}{50 \div 5} = \frac{5 \div 5}{10 \div 5} = \frac{1}{2}$$

3. Compare the two reduced ratios.

Are the two reduced ratios the same? If they are, the proportion is true.

$$\frac{\overset{1}{\cancel{28}}}{\underset{2}{\cancel{56}}} = \frac{\overset{1}{\cancel{50}}}{\underset{2}{\cancel{100}}}$$

$$\frac{28}{56} = \frac{1}{2} \quad \frac{50}{100} = \frac{1}{2}$$

> **Remember:**
> If the reduced ratios are the same, the proportion is true.

Exercise

Find out if these proportions are true or false. Reduce the ratios to lowest terms.

1. $\frac{1}{20} = \frac{3}{100}$ 2. [illegible] $= \frac{7}{100}$ 3. $\frac{2}{8} = \frac{25}{100}$

Answers: 1. false 2. false 3. true: $\frac{1}{4} = \frac{1}{4}$

Using Proportions with Percents

Suppose you take a math test that has 30 problems. You get your test back. It shows two scores: $\frac{21}{30}$ and 70%. One score is a ratio that tells you how many of the 30 problems you actually got correct. Which score is that?

The other score tells you what percent of all the problems you got correct. How would you write that as a fraction ratio?

$\frac{21}{30}$ is the ratio that shows the actual number of correct problems out of all the problems. $\frac{70}{100}$ is the ratio that shows what part out of the total amount is correct. Both ratios equal the same amount. So both ratios are equivalents.

Writing a Proportion with Ratios

Here's how you'd write those equivalent ratios as a proportion:

$$\frac{\text{problems correct}}{\text{all problems}}\ \frac{21}{30} = \frac{70}{100}\ \frac{\text{parts}}{\text{total parts}}$$

You'd read that proportion like this: *21 out of 30 is the same as 70 out of 100.* In other words, 21 correct problems out of 30 is the same as 70% correct answers.

Now read this word problem.

> You have \$84. You spend \$21 on groceries. That's 25% of your money. Write a proportion that shows how much of the money you spend and what part of the whole amount it is.

a. What ratio shows the money you actually spend out of the money you actually have?

b. What ratio shows the part you spend out of the whole amount?

c. How would you show those ratios as a proportion?

Answers: a. $\frac{\$21}{\$84}$ b. $\frac{25}{100}$ c. $\frac{21}{84} = \frac{25}{100}$
(21 out of 84 is the same as 25 out of 100)

Exercise

Write a proportion for each problem. To see if the proportion is true, find the cross products or reduce both ratios.

1. You have \$18. You spend \$9. That's 50% of all the money.

$$\frac{\text{money you spend}}{\text{all the money}}\ \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}\ \frac{\text{part}}{\text{total amount}}$$

2. Jim saves 25% of his paycheck. His paycheck is for \$216. He puts \$54 of it into his savings account.

$$\frac{\text{savings}}{\text{paycheck}}\ \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}\ \frac{\text{part}}{\text{total amount}}$$

3. Bernie's Shoe Store is having a sale. All shoes are 50% off. A pair of shoes that costs \$48 now costs \$24 on sale.

$$\frac{\text{money you spend}}{\text{all the money}}\ \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}\ \frac{\text{part}}{\text{total amount}}$$

4. Jerry weighs 168 pounds. He loses 42 pounds. That's 25% of his total weight.

$$\frac{\text{weight he loses}}{\text{total weight}}\ \frac{\blacksquare}{\blacksquare} = \frac{\blacksquare}{\blacksquare}\ \frac{\text{part}}{\text{total amount}}$$

Answers: 1. $\frac{9}{18} = \frac{50}{100}$ 2. $\frac{54}{216} = \frac{25}{100}$ 3. $\frac{24}{48} = \frac{50}{100}$ 4. $\frac{42}{168} = \frac{25}{100}$

Dropping Zeros

Sometimes the two terms in a ratio will have zeros. For example:

$\frac{800}{4000}$

You'll find it easier to multiply or divide those terms if you *drop the zeros* in the ratio. When you drop zeros you quickly reduce the ratio to a smaller equivalent. Here's what to do:

1. Make sure both terms have zeros.

Do both terms of the ratio have zeros? If so, you can drop zeros from that ratio.

2. Start with the term that has the fewest zeros.

Find the term that has the *fewest* zeros. Draw lines through those zeros.

$\frac{8\cancel{00}}{4000}$

3. Count the same number of zeros in the other term.

Next, draw a line through the same number of zeros *in the other term.*

$\frac{800}{40\cancel{00}}$

4. Drop the zeros that have lines.

Now drop all the zeros that have lines through them. The reduced ratio is 8 out of 40. That ratio is equivalent to 800 out of 4000. They both equal the same amount.

$\frac{8\cancel{00}}{40\cancel{00}} = \frac{8}{40}$

Exercise

Drop the zeros in these ratios. Then check your answers.

1. $\frac{20}{3600}$ 2. $\frac{10}{100}$ 3. $\frac{250}{1000}$

4. $\frac{300}{400}$ 5. $\frac{900}{10{,}000}$ 6. $\frac{750}{95{,}000}$

Answers: 1. $\frac{2}{360}$ 2. $\frac{1}{10}$ 3. $\frac{25}{100}$ 4. $\frac{3}{4}$ 5. $\frac{9}{100}$ 6. $\frac{75}{9500}$

Zeros in a Proportion

Suppose you want to make sure this proportion is true.

$\frac{800}{4000} = \frac{22}{110}$

Follow these steps:

1. Look for zeros in the first ratio.

Both terms have zeros. So you can drop zeros from the ratio.

$\frac{8\cancel{00}}{40\cancel{00}} = \frac{22}{110}$

2. Look for zeros in the other ratio.

Only one term has a zero. So you can't drop zeros from the ratio.

$\frac{8\cancel{00}}{40\cancel{00}} = \frac{22}{110}$

Now see if the proportion is true.

3. Cross multiply:

$8 \times 110 = \blacksquare$ and $40 \times 22 = \blacksquare$

Or reduce:

$\frac{8}{40} = \frac{22}{110}$

$\frac{8}{40}\ \frac{8 = \blacksquare}{8 = \blacksquare}$ and $\frac{22}{110}\ \frac{2 = 11}{2 = 55}\ \frac{11 = \blacksquare}{11 = \blacksquare}$

a. Find the cross products. What are they?

b. Reduce both ratios to lowest terms. What are the reduced ratios?

Answers: **a.** 880 = 880 **b.** $\frac{1}{5} = \frac{1}{5}$

Exercise

Drop zeros wherever you can. Then see if these proportions are true.

1. Find cross products.

   a. $\frac{10}{20} = \frac{50}{100}$ **b.** $\frac{40}{200} = \frac{14}{70}$

2. Reduce ratios to their lowest terms.

   a. $\frac{60}{100} = \frac{30}{50}$ **b.** $\frac{32}{80} = \frac{40}{100}$

Answers:
1. **a.** 10 = 10 **b.** 280 = 280
2. **a.** $\frac{3}{5} = \frac{3}{5}$ **b.** $\frac{2}{5} = \frac{2}{5}$

Unit Review

Check Yourself

1. Write a proportion for each of these equivalent ratios.
   a. 2 out of 4 is the same as 3 out of 6
   b. 8 out of 12 is the same as 2 out of 3
   c. 25 out of 30 is the same as 5 out of 6
   d. 22 out of 56 is the same as 11 out of 28
   e. 8 out of 9 is the same as 32 out of 36

2. Find out if these proportions are correct. Find the cross products or reduce ratios. Then write yes or no.
   a. $\frac{3}{5} = \frac{9}{15}$ d. $\frac{2}{100} = \frac{6}{300}$
   b. $\frac{7}{9} = \frac{10}{18}$ e. $\frac{4}{5} = \frac{18}{25}$
   c. $\frac{5}{6} = \frac{25}{30}$ f. $\frac{8}{100} = \frac{2}{25}$

3. Write a proportion for each of these equivalent ratios. Check the proportions by cross multiplying or reducing ratios.
   a. $\frac{5}{10}$ is the same as 50%
   b. $\frac{1}{2}$ is the same as 50%
   c. $\frac{3}{4}$ is the same as 75%
   d. $\frac{15}{20}$ is the same as 75%
   e. $\frac{25}{100}$ is the same as 25%
   f. $\frac{1}{4}$ is the same as 25%

4. Reduce these ratios to lowest terms. Drop zeros whenever you can.
   a. $\frac{300}{1200} = \frac{20}{80}$ d. $\frac{8}{56} = \frac{7}{49}$
   b. $\frac{40}{50} = \frac{360}{450}$ e. $\frac{160}{360} = \frac{12}{27}$
   c. $\frac{9}{27} = \frac{54}{162}$ f. $\frac{260}{520} = \frac{330}{660}$

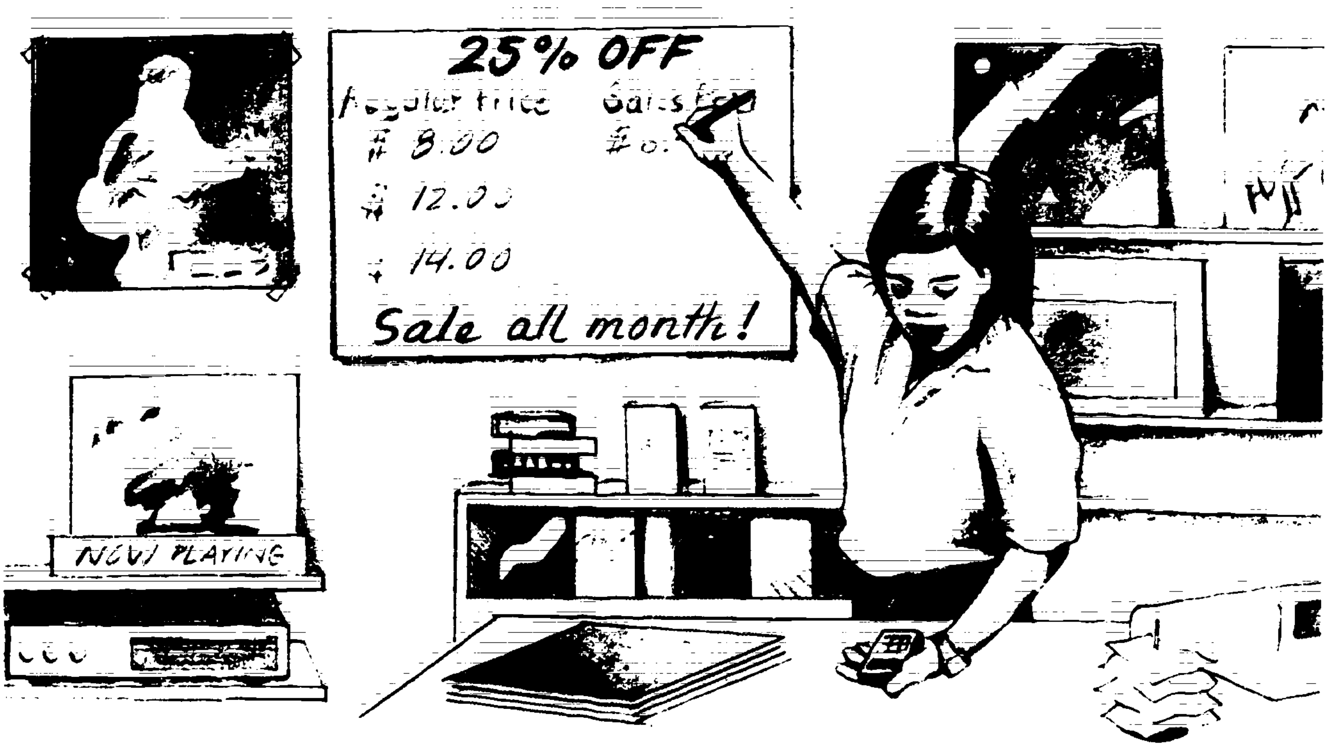
5. Choose the right word or words to complete each sentence.
   cross product **equivalent ratios**
   proportion
   a. A ______ shows two ratios that are equal.
   b. You get a ______ ______ when you multiply the numerator of one ratio with the denominator of the other ratio in a proportion.
   c. $\frac{1}{2}$ and $\frac{6}{12}$ compare the same part out of a whole. They are ______ ______.

Bonus Work

1. Look at the chart on page 58. Write a proportion for each answer on that chart. Make sure your proportion is correct by comparing the cross products.
2. Make a poster that shows how to check a proportion by comparing its cross products or reducing its terms.

Unit 9

Solving Percent Problems



Imagine this:

You're a salesclerk in a record store. Tomorrow the store is having a sale. Everything will be 25% off. Your boss tells you to figure the sale prices for records that cost $8, $12, and $14.

To figure the sale prices, you must first find 25% of the regular prices. Do you know how to find a percent of an amount?

Now suppose the sale is on. A customer wants to buy a record. The regular price for the record is $8. But the tag on the record has a sale price of $7. The customer says, "This price is *not* 25% off." How would you show her that it is or isn't?

Now imagine that the sale is over. Your boss wants you to mark the records back to their regular prices. You can't remember the regular price of some of the records. Those records have a sale price of $10.50. How would you figure the regular price?

These problems are all about percents. They are the kinds of problems you'll be solving in everyday life. You can solve all three kinds of problems easily by using a *proportion*.

In Unit 8, you learned to understand and write proportions. In this unit, you'll use what you've learned. You'll solve three different kinds of percent problems by using just one kind of proportion.

Math Words

Look up these words in the glossary. What do they mean?

known term **unknown term**

Four Terms

In the last unit, you learned to write proportions with fraction ratios. Fractions are made up of *terms*. What are terms?

Terms are the *numerator* and the *denominator* of the fraction. How many terms does a proportion have?

Right! a proportion has *four* terms. Proportions that have percents are always made up of these kinds of terms:

1. A denominator that shows an *actual total*.

   actual total $\frac{\blacksquare}{48} = \frac{\blacksquare}{\blacksquare}$

2. A numerator that shows the *actual parts* of the actual total.

   actual parts / actual total $\frac{36}{48} = \frac{\blacksquare}{\blacksquare}$

3. A denominator that stands for the *whole amount* when it is divided into percents (100 equal parts).

   actual parts / actual total $\frac{36}{48} = \frac{\blacksquare}{100}$ **whole amount**

4. A numerator that shows *parts of 100 percent* (the whole amount).

   actual parts / actual total $\frac{36}{48} = \frac{75}{100}$ **parts of percent** / whole amount

Now look at this proportion. Which ratio is equivalent to a percent? How can you tell?

$$\frac{20}{100} = \frac{8}{40}$$

Right! The first ratio has a denominator of 100. That tells you it is equivalent to a percent. Now, see if you can figure out which numbers show:

a. the parts of the percent

b. the actual total

c. the actual parts

Answers: a. 20% b. 40 c. 8

Known and Unknown Terms

A percent problem will ask you to find a *total*, a *part* of a total, or a *percent*. Notice this: They are all parts of a proportion. You can use a proportion to solve the problem.

The percent problem will tell you three terms of the proportion. Those are called **known terms**. To solve the problem, you must find the fourth term—the **unknown term**. This is an example of such a problem:

32 is ___% of 48?

Here's how to write the proportion:

1. **First write the ratio that has two known terms.**

   actual part / **actual total** $\frac{32}{48} = \frac{\blacksquare}{\blacksquare}$

2. **Then write a ratio with the other known term.**

   The percent sign tells you this: the other known term is the denominator 100.

   actual part / actual total $\frac{32}{48} = \frac{\blacksquare}{100}$ **whole amount**

3. **Write X for the unknown term.**

   actual part / actual total $\frac{32}{48} = \frac{X}{100}$ **part of percent** / whole amount

Now follow the steps above. Write a proportion for this problem:

15 is 50% of ___?

Answer: **part of percent** / **whole amount** $\frac{50}{100} = \frac{15}{X}$ **actual part** / **actual total**

Exercise

Finish writing these proportions.

1. 3 is ___% of 4? $\frac{3}{4} = \frac{\blacksquare}{100}$
2. 8 is 25% of ___? $\frac{\blacksquare}{100} = \frac{8}{\blacksquare}$
3. 50% is ___ of 180? $\frac{\blacksquare}{\blacksquare}$ $\frac{\blacksquare}{180}$

Answers: 1. $\frac{3}{4} = \frac{X}{100}$ 2. $\frac{25}{100} = \frac{8}{X}$ 3. $\frac{50}{100} = \frac{X}{180}$

Finding the Unknown Term

Suppose you're solving this problem:

$60 is 40% of ___?

The problem gives you three terms. The percent tells you the numerator (40) and denominator (100) of one ratio. And the amount ($60) is an actual part of an actual total. To solve the problem, you must find the *actual total*. Here's the proportion you'd write:

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{40}{100} = \frac{60}{X} \quad \frac{\textbf{actual part}}{\textbf{actual total}}$$

Follow these steps to find any unknown term in a proportion.

1. Reduce the terms in the first ratio to their lowest terms.

You'll find it easier to reduce if you first drop zeros.

Drop zeros:

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{4\cancel{0}}{10\cancel{0}} = \frac{60}{X} \quad \frac{\text{actual part}}{\text{actual total}}$$

Now Reduce:

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{2}{\cancel{40}}}{\underset{5}{\cancel{100}}} = \frac{60}{X} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$\frac{4 \div 2}{10 \div 2} = \frac{2}{5}$$

2. Find the cross product of two known terms.

$$\frac{\text{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{2}{\cancel{40}}}{\underset{5}{\cancel{100}}} = \frac{60}{X} \quad \frac{\textbf{actual part}}{\text{actual total}}$$

$$5 \times 60 = 300$$

3. Divide the cross product by the third known term.

$$\frac{\textbf{part of percent}}{\text{whole amount}} \quad \frac{\overset{2}{\cancel{40}}}{\underset{5}{\cancel{100}}} = \frac{60}{X} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$2\overline{)300} = 150$$

4. Replace X with the answer.

You now have the answer to the unknown term. Replace the **X** in the proportion with the answer.

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{2}{\cancel{40}}}{\underset{5}{\cancel{100}}} = \frac{60}{\underset{150}{\cancel{X}}} \quad \frac{\text{actual part}}{\textbf{actual total}}$$

5. Check your answer.

Check your answer by finding out if your proportion is true.

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{2}{\cancel{40}}}{\underset{5}{\cancel{100}}} = \frac{60}{\underset{150}{X}} \quad \frac{\textbf{actual part}}{\textbf{actual total}}$$

Compare cross products:

$$2 \times 150 = 300 \qquad 5 \times 60 = 300$$

Or reduce both ratios to lowest terms.

$$\frac{2}{5} = \frac{2}{5} \text{ and } \frac{60 \div 30}{150 \div 30} = \frac{2}{5}$$

Exercise

Find the unknown terms. Then check your answers by comparing cross products or reducing.

1. $\frac{1}{4} = \frac{X}{100}$
2. $\frac{90}{100} = \frac{27}{X}$
3. $\frac{12}{60} = \frac{X}{100}$
4. $\frac{10}{100} = \frac{2}{X}$
5. $\frac{45}{100} = \frac{X}{20}$
6. $\frac{60}{100} = \frac{X}{50}$

Finding a Part of the Total

Sometimes a problem will ask you questions such as:

What is 60% of $40?

Notice this: The problem gives you three terms. It gives you the *whole amount* in percents (100), the *part of the percent* (60) and the *actual total* ($40). To solve the problem, you must find an *actual part* of the actual total.

These steps show how to solve percent problems like that:

1. **Set up a proportion.**

First, write a ratio that is equivalent to the percent.

Then write a ratio of the actual total and the unknown term.

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{60}{100} = \frac{X}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

2. **Reduce the ratio with known terms.**

Drop zeros in the first ratio. Then reduce the ratio to lowest terms.

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}} = \frac{X}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

3. **Find the cross product of two known terms.**

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}} = \frac{X}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$3 \times 40 = 120$$

4. **Divide the cross product by the third known term.**

What is the answer?

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}} = \frac{X}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$5\overline{)120}^{\,24}$$

5. **Replace the X with the answer.**

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}} = \frac{\overset{24}{\cancel{X}}}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

6. **Check your answer.**

You can check your answer by finding out if the proportion is true. Is the proportion true?

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{3}{\cancel{60}}}{\underset{5}{\cancel{100}}} = \frac{\overset{24}{\cancel{X}}}{40} \quad \frac{\text{actual part}}{\text{actual total}}$$

Compare cross products:

$5 \times 24 =$ ■ and $3 \times 40 =$ ■

Or reduce both ratios to lowest terms.

$\frac{3}{5} = \frac{■}{■}$ and $\frac{24}{40}$ $\frac{8}{8} = \frac{■}{■}$

Answer:

$24 \times 5 = 120$ $3 \times 40 = 120$

$\frac{3}{5} = \frac{3}{5}$ Therefore, the proportion is true.

Exercise

Solve these problems by using a proportion. Then check to see if your proportions are true.

1. Find 25% of 16 gallons of paint.
2. What is 70% of $50?
3. Find 48% of 200 students.
4. Find 90% of 80 votes.
5. What is 75% of 124 miles?
6. What is 80% of $25?

Finding the Percent

This problem asks you to find a *percent.* The problem gives you three terms. One term is the *actual total* ($200). A second term is the *actual part* ($120). What is the third term?

> Kim gets a check for $200. She uses $120 of it to pay a bill. *What percent of $200 is $120?*

The third term is the denominator of the percent—the *whole amount.* The word *percent* in the problem is the clue that tells you that.

These steps show how to solve problems that ask you to find percents:

1. Set up a proportion.

First, write a ratio with the known terms. Then write a ratio of the whole amount and the unknown term.

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{120}{200} = \frac{X}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

2. If you can, reduce the ratio that has the known terms.

Drop zeros in the first ratio. Then reduce the ratio to lowest terms.

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{\overset{3}{12\cancel{0}}}{\underset{5}{20\cancel{0}}} = \frac{X}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

$$\frac{12 \div 4 = 3}{20 \div 4 = 5}$$

3. Find the cross product of the two known terms.

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{\overset{3}{12\cancel{0}}}{\underset{5}{20\cancel{0}}} = \frac{X}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

$$3 \times 100 = 300$$

4. Divide the cross product by the third known term.

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{\overset{3}{12\cancel{0}}}{\underset{5}{20\cancel{0}}} = \frac{X}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

$$5\overline{)300}\ \ 60$$

5. Replace the X with the answer.

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{\overset{3}{12\cancel{0}}}{\underset{5}{20\cancel{0}}} = \frac{\overset{60}{\cancel{X}}}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

6. Check your answer.

Check your answer by finding out if the proportion is true. Is the proportion true?

$$\frac{\text{actual part}}{\text{actual total}}\ \frac{\overset{3}{12\cancel{0}}}{\underset{5}{20\cancel{0}}} = \frac{\overset{60}{\cancel{X}}}{100}\ \frac{\text{part of percent}}{\text{whole amount}}$$

Compare cross products:

$3 \times 100 = ■$ and $5 \times 60 = ■$

Or reduce both ratios to lowest terms:

$\frac{3}{5} = \frac{■}{■}$ and $\frac{60}{100}\ \frac{20}{20} = \frac{■}{■}$

Answer:
$3 \times 100 = 300 \quad 5 \times 60 = 300$
$\frac{3}{5} = \frac{3}{5}$ Therefore, the proportion is true.

Exercise

Solve these problems by using a proportion. Then check to see if your proportions are true.

1. 14 miles is what percent of 56 miles?
2. What percent of 50 gallons is 8 gallons?
3. $24 is what percent of $80?
4. What percent of 90 people is 45 children?
5. 44 days is what percent of 110 days?
6. What percent of 2,500 votes is 2,000 votes?

Finding the Total Amount

Sometimes a problem asks you to find an *actual total amount*. This problem is an example. What three terms does the problem give you?

> Joe is shooting baskets. He makes 20 of them. That's 80% of all the baskets he shoots. *How many baskets did he shoot altogether?*

One term is the *actual number* of baskets Joe makes (20). The percent sign tells you another term—the *whole amount* in percents (100). And the number of the percent tells you the third term—the *part of the percent* (80).

These steps show how to solve problems that ask you to find an actual total.

1. Set up a proportion.

First, write a ratio with the known terms. What ratio is it?

Then write a ratio of the whole amount and the unknown term. Is the unknown term a denominator or a numerator?

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{80}{100} = \frac{20}{X} \quad \frac{\textbf{actual part}}{\textbf{actual total}}$$

2. Reduce the ratio that has the known terms.

Drop zeros. Then reduce the ratio to lowest terms.

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{4}{\cancel{80}}}{\underset{5}{\cancel{100}}} = \frac{20}{X} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$\frac{8}{10} \begin{matrix} \div 2 \\ \div 2 \end{matrix} = \frac{4}{5}$$

3. Find the cross product of the two known terms.

$$\frac{\text{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{4}{\cancel{80}}}{\underset{5}{\cancel{100}}} = \frac{20}{X} \quad \frac{\textbf{actual part}}{\text{actual total}}$$

$$5 \times 20 = 100$$

4. Divide the cross product by the third known term.

What is the answer?

$$\frac{\textbf{part of percent}}{\text{whole amount}} \quad \frac{\overset{4}{\cancel{80}}}{\underset{5}{\cancel{100}}} = \frac{20}{X} \quad \frac{\text{actual part}}{\text{actual total}}$$

$$4\overline{)100}\ \ 25$$

5. Replace the X with the answer.

$$\frac{\text{part of percent}}{\text{whole amount}} \quad \frac{\overset{4}{\cancel{80}}}{\underset{5}{\cancel{100}}} = \frac{20}{\underset{25}{\cancel{X}}} \quad \frac{\text{actual part}}{\textbf{actual total}}$$

6. Check your answer.

Check your answer by finding out if the proportion is true. Is the proportion true?

$$\frac{\textbf{part of percent}}{\textbf{whole amount}} \quad \frac{\overset{4}{\cancel{80}}}{\underset{5}{\cancel{100}}} = \frac{20}{\underset{25}{\cancel{X}}} \quad \frac{\textbf{actual part}}{\textbf{actual total}}$$

Compare cross products:

$4 \times 25 = \blacksquare \qquad 5 \times 20 = \blacksquare$

Or reduce both ratios to lowest terms.

$\frac{4}{5} = \frac{\blacksquare}{\blacksquare}$ and $\frac{20}{25} \begin{matrix} \div 5 \\ \div 5 \end{matrix} = \frac{\blacksquare}{\blacksquare}$

Answer:
$4 \times 25 = 100 \quad 5 \times 20 = 100$
$\frac{4}{5} = \frac{4}{5}$ Therefore, the proportion is true.

Exercise

Solve these problems by using a proportion. Then check to see if your proportions are true.

1. 10% of how much is $8?
2. 150 miles is 30% of how many miles?
3. 80% of how many pounds is 64 pounds?
4. $45 is 5% of how much money?
5. 186 people is 62% of what total amount?
6. 12% of what amount is $72?

Unit Review

Check Yourself

1. Read and solve these percent problems by using a proportion. *Find a part of a total amount.*
   a. What is 30% of $70?
   b. What is 90% of 40 gallons?
   c. Find 25% of 124 miles.
   d. 30 people took a test for a cashier's job. 80% passed the test. How many people passed?

   Find what percent a part is of the total amount.
   e. What percent of 16 feet is 12 feet?
   f. What percent of $120 is $96?
   g. What percent of 12 inches is 6 inches?
   h. Tony and Marc play eight sets of tennis. Tony wins six sets. What percent of the sets does Tony win?

   Find the total amount.
   i. 85% of what whole amount is $17?
   j. 10% of what whole amount is 21 persons?
   k. 45% of what total is 18 points?
   l. The science class is selling magazines to raise money for class field trips. Every student must sell $12 worth of magazines. That's 4% of the total amount. How much is the total amount?

2. Choose the right words to complete each sentence.
   known term **unknown term**
   a. When you're solving for an amount, you'd write an X for the ______ ______ in the proportion.
   b. When you write a proportion for a percent problem, it always has three ______ ______s.

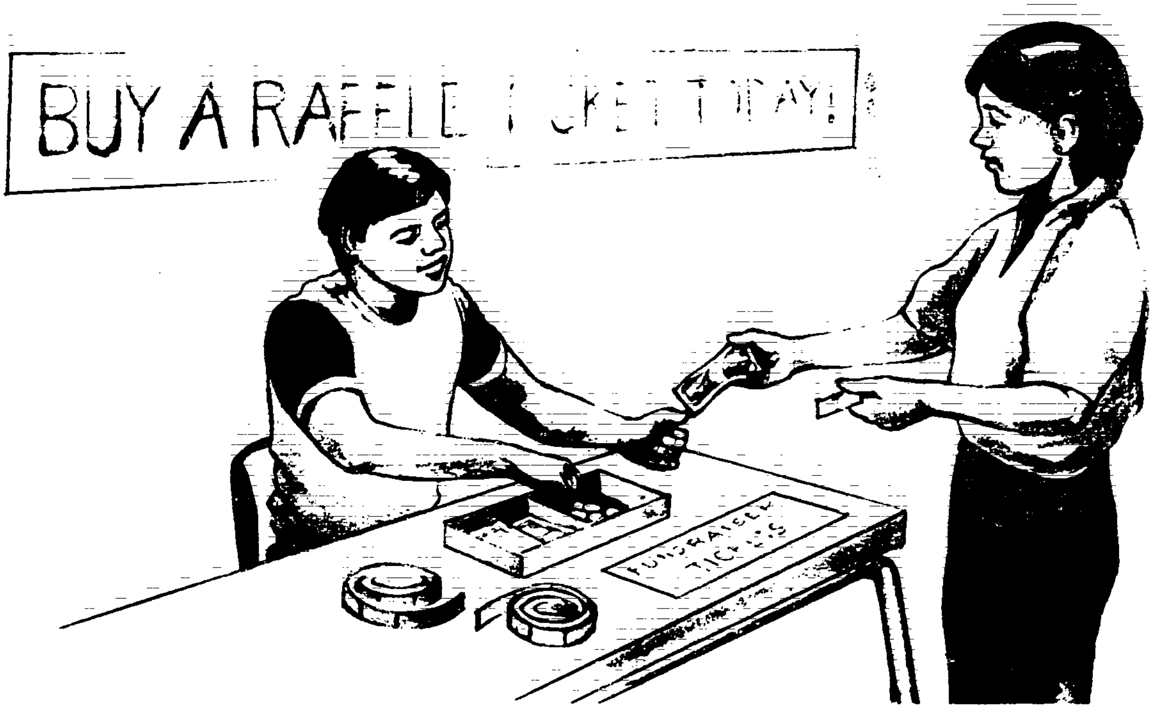
Bonus Work

1. Look through newspapers for different sale ads. Cut one ad out. Make up three percent problems using the amounts in the ad. Then give the ad and the problems to your classmates to solve.
2. Review how to find each of these things. Choose one of them. Demonstrate to your class how to find that thing.
   - a discount
   - a tip to a waiter or waitress
   - interest on a savings account
   - a sales tax
   - a tax such as social security from a paycheck
3. These students took a math test that had 40 problems. Find out what percent of the test each student got correct.

Student	Problems Correct	Percent Correct
Paul	30	
Jose	36	
Meiling	38	
Hans	34	
Rose	40	

Unit 10

Renaming Decimals and Percents



Imagine this:

Your club is having a fund raiser. You're selling tickets. They cost a *half dollar* (or $\frac{1}{2}$ of a dollar). Someone buys a ticket from you. She gives you a dollar bill. You give her *50 cents* back (or 50% of 100 cents). You must keep a list of all the money you take in. So you write .50 (or *50 hundredths*) on that list.

$\frac{1}{2}$ is a common fraction. 50% is a percent. And .50 is a decimal. They are three different ways to show the same thing. They are three different kinds of fractions.

You can rename each kind of fraction as the other two kinds of fractions. In other words, you can rename fractions as equivalent decimals and percents. You can rename decimals as equivalent fractions and percents. And you can rename percents as equivalent fractions and decimals. You'll learn how to do that in this unit.

Why do you think you'd need to rename decimals, percents, or fractions as equivalents?

One reason is this: Sometimes it's easier to find the answer to a problem if you rename it. For example, look at these two math problems. They are alike. Each asks you to subtract the same amount. Which is easier to subtract?

$\frac{3}{4} - \frac{1}{2}$ or $\begin{array}{r} .75 \\ -\ .50 \\ \hline \end{array}$

Math Word

Look up this word in the glossary. What does it mean?

simplify

Percents to Fractions

You already know how to rename percents as equivalent fractions. You know that the percent sign (%) stands for the denominator. And that the numeral of the percent stands for the numerator. In fact, you rename percents as equivalent fractions whenever you write a proportion that has a percent.

Follow these steps when you rename percents as equivalent fractions:

Problem: $50\% = \frac{■}{■}$

1. Write 100 as the denominator.

Remember: The percent sign stands for 100 equal parts. So the denominator is always 100.

$$50\% = \frac{■}{100}$$

2. Write the numeral of the percent as the numerator of the fraction.

Remember: The numeral of a percent is the same as the numerator of a fraction.

$$50\% = \frac{50}{100}$$

3. Simplify the fraction if you can.

When you show the equivalent fraction of a percent, always show it in its simplest form. So, reduce the fraction to its lowest terms if you can.

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$\frac{50 \div 5}{100 \div 5} = \frac{10 \div 10}{20 \div 10} = \frac{1}{2}$$

$$\text{so: } \frac{50}{100} = \frac{1}{2}$$

$$\text{and: } 50\% = \frac{1}{2}$$

Now, follow the steps. Rename this percent as a fraction. Then simplify the fraction.

$80\% = \frac{■}{■}$

a. What is the denominator before you simplify?

b. What is the numerator before you simplify?

c. What is the simplified equivalent fraction?

Answers: a. 100 **b.** 80 **c.** $\frac{4}{5}$

Exercise

Copy these percents. Then rename them to equivalent fractions. Reduce fractions to their simplest terms. Check your answers.

1. 85%	6. 1%
2. 60%	7. 35%
3. 12%	8. 25%
4. 48%	9. 3%
5. 10%	10. 75%

Answers: 1. $\frac{17}{20}$ **2.** $\frac{3}{5}$ **3.** $\frac{3}{25}$ **4.** $\frac{12}{25}$ **5.** $\frac{1}{10}$ **6.** $\frac{1}{100}$ **7.** $\frac{7}{20}$ **8.** $\frac{1}{4}$ **9.** $\frac{3}{100}$ **10.** $\frac{3}{4}$

Decimals to Fractions

Percents are a special kind of fraction. They only have denominators of 100. Decimals are also a special kind of fraction. What kind of denominators do decimals have?

Right! Decimals only have denominators of 10, 100, 1000, and other numbers that are powers of 10. How can you tell which denominator a decimal has?

Right again! You tell which denominator a decimal has by counting its decimal places.

Here's how to rename decimals as equivalent fractions:

Problem: $.04 = \frac{■}{■}$

1. Count decimal places and write the denominator.

First, write the digit 1 in the denominator.

$$.04 = \frac{■}{1}$$

Then count decimal places. Start at the decimal point and count to the right. Let each decimal place stand for one zero. How many decimal places are in the decimal?

$$\underset{1}{.0}\ \underset{2}{4}$$

Write the same number of zeros in the denominator behind the digit.

$$.04 = \frac{■}{100}$$

2. Write the numeral of the decimal as the numerator of the fraction.

Sometimes a decimal will have zeros that fill empty decimal places. The zero in **.04** is an example. When you write those kinds of decimals as fractions, drop zeros that fill empty decimal places.

$$.04 = \frac{04}{100} = \frac{4}{100}$$

3. Simplify the fraction if you can.

When you show the equivalent fraction of a decimal, always show its simplest form. So, reduce the fraction to its lowest terms.

$$.04 = \frac{4}{100} = \frac{1}{25}$$

$$\frac{4 \div 2}{100 \div 2} = \frac{2 \div 2}{50 \div 2} = \frac{1}{25}$$

so: $\frac{4}{100} = \frac{1}{25}$

and: $.04 = \frac{1}{25}$

Now, follow the steps above. Rename these decimals as fractions.

1. $.5 = \frac{■}{■}$
   a. What is the denominator before you simplify?
   b. What is the numerator before you simplify?
   c. What is the simplified equivalent fraction?

Answers: a. 10 b. 5 c. $\frac{1}{2}$

2. $.003 = \frac{■}{■}$
   a. What is the denominator?
   b. What is the numerator?
   c. Can you simplify the fraction?
   d. What is the equivalent fraction?

Answers: a. 1000 b. 3 c. no d. $\frac{3}{1000}$

Exercise

Rename these decimals as equivalent fractions. If you can, reduce the fractions to their lowest terms.

1. .9	6. .28
2. .32	7. .112
3. .60	8. .26
4. .825	9. .08
5. .009	10. .8

Answers: 1. $\frac{9}{10}$ 2. $\frac{8}{25}$ 3. $\frac{3}{5}$ 4. $\frac{33}{40}$ 5. $\frac{9}{1000}$ 6. $\frac{7}{25}$ 7. $\frac{14}{125}$ 8. $\frac{13}{50}$ 9. $\frac{2}{25}$ 10. $\frac{4}{5}$

Decimals to Percents

Using a Proportion

You know that a decimal can be renamed as a fraction. It can also be renamed as a percent. You can use a proportion to rename a decimal as a percent. Here's how.

Problem: $.5 = \_\_\%$

1. Rename the decimal as a fraction.

$$.5 = \frac{5}{10}$$

2. Set up a proportion.

For the first ratio: Write the equivalent fraction of the decimal.

$$\frac{5}{10} = \frac{■}{■}$$

For the second ratio: Write an equivalent fraction of the percent.

$$\frac{5}{10} = \frac{x}{100}$$

3. Find the unknown term.

First find the cross product of two known terms. What are those terms?

$\frac{5}{10} = \frac{x}{100}$ $\quad 5 \times 100 = 500$

Then divide the cross product by the third known term. What is that term?

$\frac{5}{10} = \frac{x}{100}$ $\quad 10\overline{)500}$ = 100

4. Rename the fraction as a percent.

The equivalent fraction has a denominator of 100. So, you can name it directly as a percent.

$$.5 = \frac{5}{10}$$

$$\text{and: } \frac{5}{10} = \frac{50}{100}$$

$$\text{and: } \frac{50}{100} = 50\%$$

$$\text{so: } .5 = 50\%$$

Exercise

Rename these decimals as percents. Use a proportion to find your answers.

1. .50
2. .05
3. .003
4. .125

Answers: 1. 50% 2. 5% 3. .3% 4. 12.5%

Using a Short Cut

Here's a quicker way to rename decimals as equivalent percents.

Problems:
a. $.5 = \_\_\%$ b. $.015 = \_\_\%$

1. Count two decimal places.

Start from the decimal point and count to the right. Count *two* decimal places.

a. .5 ■ b. .0 1 5
1 2 1 2

2. Fill any empty decimal place.

If a decimal has only one digit, the decimal place behind that digit will be empty. Write a zero in it.

a. .5 0 b. .0 1 5

3. Move the decimal point.

Move the decimal point so it is behind the second decimal place.

a. 5 0. b. 0 1. 5
1 2 1 2

4. Write a percent sign behind the last digit.

You can drop the decimal point if it is behind the last digit. You can also drop zeros that have no value.

a. 5 0 % b. 01. 5 % = 1.5%

Now, follow the steps above and rename this decimal: $.027 = \_\_\%$

a. Where do you move the decimal point?
b. Where do you write the percent sign?
c. What is the equivalent percent?

Answer: a. behind 2 b. behind 7 c. 2.7%

Exercise

Rename these decimals as equivalent percents. Then check your answers.

1. .65
2. .4
3. .019
4. .72
5. 1.45
6. .125

Answers: 1. 65% 2. 40% 3. 1.9% 4. 72% 5. 145% 6. 12.5%

Percents to Decimals

Finding by Dividing

You know how to rename a decimal and a percent as a fraction. And you know how to rename a decimal as a percent. Now you'll learn how to rename a percent as a decimal. Here's one way to do that:

Problem: 8% = . ?

1. Rename the percent as a fraction.

$8\% = \frac{8}{100} = .\_\_$

2. Divide the numerator by the denominator.

Notice this: 8 is smaller than 100. What do you do when you divide a small number by a large one?

$8\% = \frac{8}{100} = .08$ $\quad 100\overline{)8.00}$ = .08

So: 8% = .08

Now, rename this percent as an equivalent decimal.

150%

a. Rename the percent as a fraction. What is the equivalent fraction?
b. What is the denominator of the fraction?
c. What is the numerator?
d. To find the equivalent decimal, divide the numerator by the denominator. Keep dividing until your answer comes out evenly. What is the answer?

Answers: a. $\frac{150}{100}$ b. 100 c. 150 d. 1.5

Exercise

Rename these percents as decimals. To find the answer, divide the numerator by the denominator.

1. 50%	4. 25%	7. 185%
2. 75%	5. 2%	8. 125%
3. 93%	6. 5%	

Answers: 1. .5 2. .75 3. .93 4. .25 5. .02 6. .05 7. 1.85 8. 1.25

Another Short Cut

Here's a quicker way to rename percents as equivalent decimals.

Problems: a. 5% = . ? b. 11.50% = . ?

1. Count two decimal places.

Start from the decimal point. If the percent has no decimal point, start from the percent sign. Count to the *left*. Count two decimal places.

a. ■5 % b. 11. 50
2 places 2 places

2. Move the decimal point.

Move the decimal point so it is in front of the two decimal places.

a. .■5 b. .11 5 0
2 places 2 places

3. Fill any empty decimal place.

Drop the percent sign. Fill any empty decimal place. How do you fill it?

a. .0 5 So: 5% = .05

You can also drop zeros that have no value.

b. .1150 So: 11.50% = .115

Now, follow the steps above and rename this decimal:

12.50% = ?

a. Where do you move the decimal point?
b. What is the equivalent decimal?

Exercise

Rename these percents as equivalent decimals. Then check your answers.

1. 5%	4. 6.18%	7. .01%
2. 12%	5. 125%	8. 19.85%
3. 2.8%	6. 15%	9. 90%

Answers: 1. .05 2. .12 3. .028 4. .0618 5. 1.25 6. .15 7. .0001 8. .1985 9. .9

Renaming Fractions as Decimals and Percents

Powers of Ten

You now know this: You can rename a percent as an equivalent decimal and an equivalent common fraction. You can also rename common fractions as decimals and percents.

Suppose you are renaming these fractions:

$\frac{1}{10}$ $\frac{3}{100}$ $\frac{14}{1000}$

Notice this: The denominators are all *powers of 10*. What other kind of fraction has denominators that are powers of 10?

Here's how to rename fractions with denominators that are powers of ten.

1. Count zeros in the denominator.

First, rename the fraction as a decimal. The zeros in the fraction's denominator tell you how many decimal places the decimal will have. Let each zero stand for one decimal place. How many decimal places will decimals of tenths have? How many places will decimals of hundredths have? How many places will decimals of thousandths have?

$\frac{1}{10}$	$\frac{3}{100}$	$\frac{14}{1000}$
↑	↑↑	↑↑↑
1	12	123

2. Write the numerator.

Write a decimal point. Then write the numerator of the fraction after the decimal point. The last digit of the numerator should fill the last decimal place.

$\frac{1}{10} = .1$	$\frac{3}{100} = .■3$	$\frac{14}{1000} = .■14$
↑	↑↑	↑↑↑
1	12	123

Fill empty decimal places with zeros.

$\frac{1}{10} = .1$	$\frac{3}{100} = .03$	$\frac{14}{1000} = .014$
↑	↑↑	↑↑↑
1	12	123

3. Decide if the answer should be a decimal or a percent.

The answer you get in the second step is a decimal. If you need to find the equivalent percent, simply do this: Rename the decimal as a percent. How do you rename decimals as percents?

$\frac{1}{10} = .1■ = 1■. = 10\%$

2 places

$\frac{3}{100} = .03 = 03. = 3\%$

2 places

$\frac{14}{1000} = .014 = 01.4 = 1.4\%$

2 places

Now, follow the steps above. Rename this fraction as a decimal and then as a percent.

$\frac{3}{1000} = .\underline{?} = \underline{\quad}\%$

a. How many decimal places will the decimal have?

b. What is the equivalent decimal?

c. What is the equivalent percent?

Answers: a. three decimal places b. .003 c. .3%

Exercise

Rename these fractions first as decimals, then as percents.

1. $\frac{25}{100} = .\underline{?} = \underline{\quad}\%$
2. $\frac{50}{100} = .\underline{?} = \underline{\quad}\%$
3. $\frac{75}{100} = .\underline{?} = \underline{\quad}\%$
4. $\frac{5}{10} = .\underline{?} = \underline{\quad}\%$
5. $\frac{9}{10} = .\underline{?} = \underline{\quad}\%$
6. $\frac{2}{10} = .\underline{?} = \underline{\quad}\%$
7. $\frac{8}{10} = .\underline{?} = \underline{\quad}\%$
8. $\frac{2}{1000} = .\underline{?} = \underline{\quad}\%$
9. $\frac{52}{1000} = .\underline{?} = \underline{\quad}\%$
10. $\frac{123}{1000} = .\underline{?} = \underline{\quad}\%$

Answers are in the Teacher's Guide.

Fractions with Other Denominators

Suppose you're renaming a fraction whose denominator is *not* a power of ten. Here's how to use a proportion to rename it.

Problem: $\frac{3}{5}$ = ___% = ?

1. Set up a proportion.

For the first ratio: Write the fraction.

$\frac{3}{5} = \frac{\blacksquare}{\blacksquare}$

For the second ratio: Write the equivalent fraction of a percent.

$\frac{3}{5} = \frac{x}{100}$

2. Find the unknown term.

First find the cross product of two known terms. What are those terms?

$\frac{3}{5} = \frac{x}{100}$ $\quad 3 \times 100 = 300$

Then divide the cross product by the third known term. What is that term?

$\frac{3}{5} = \frac{x}{100}$ $\quad 5\overline{)300}$ = 60

3. Rename the equivalent fraction as a percent.

You now know both terms of the equivalent fraction. You can rename it as a percent. Write the numerator with a percent sign.

$\frac{3}{5} = \frac{\cancel{x}\,60}{100} = 60\%$ So: $\frac{3}{5}$ = **60%**

4. Rename the percent as a decimal if you wish.

You can now find the equivalent decimal of the fraction. Simply rename the percent as a decimal. How do you rename percents as decimals?

$\frac{3}{5}$ = 60% = 60. = $\underbrace{60.}_{\text{2 places}}$ = **.60**

So: $\frac{3}{5}$ = 60% = .60

Renaming Fractions by Dividing

Here's another way to rename a fraction: Divide its numerator by its denominator. The answer you get will be a decimal. You can then rename that decimal as a percent.

Problem: $\frac{3}{4}$ = ? = ___%

1. Divide the numerator by the denominator.

$4\overline{)3.00}$ = .75

2. Rename the decimal as a percent.

.75 = $\underbrace{75.}_{\text{2 places}}$ = 75% So: $\frac{3}{4}$ = .75 = 75%

Sometimes when you rename a fraction, it won't divide evenly: For example, when you rename $\frac{1}{3}$ or $\frac{6}{7}$ as a decimal or percent.

$3\overline{)1.000\ldots}$ = .333...
9
10
9
10
9
1

$7\overline{)6.0000\ldots}$ = .8571...
56
40
35
50
49
10
7
3

Round the decimal and percent of those fractions to the nearest hundredth:

$\frac{1}{3}$ = about .33 = about 33%

$\frac{6}{7}$ = about .86 = about 86%

Exercise

Use a proportion. Rename the fractions first as decimals and then as percents.

1. $\frac{2}{5}$ **2.** $\frac{2}{3}$ **3.** $\frac{1}{4}$ **4.** $\frac{1}{2}$

Rename these fractions first as decimals and then as percents. Divide their numerators by their denominators.

5. $\frac{1}{8}$ **6.** $\frac{3}{5}$ **7.** $\frac{3}{20}$ **8.** $\frac{5}{7}$

Renaming Mixed Amounts

Changing to Mixed Numbers

Sometimes you must rename decimals or percents that are greater than a whole amount. You must rename them as mixed numbers.

1.75 and 150% are examples of those kinds of amounts. To rename those amounts, do this: *Change the fraction only.*

Decimals to Mixed Numbers

Problem: Rename 1.75 as a mixed number.

1. **First, write the whole number.**

$$1.75 = 1\frac{■}{■}$$

2. **Then rename the decimal fraction as a common fraction.**

$$1.75 = 1\frac{75}{100}$$

3. **Reduce the fraction to its lowest terms.**

$$1.75 = 1\frac{\cancel{75}^{\,3}}{\cancel{100}_{\,4}}$$

Percents to Mixed Numbers

Problem: Rename 150% as a mixed number.

1. **Rename the percent as a decimal:**

$$150\% = 150. = 1.\underbrace{50}_{\text{2 places}}$$

2. **Write the whole number:**

$$150 = 1.50 = 1\frac{■}{■}$$

3. **Rename the decimal fraction as a common fraction.**

$$1.50 = 1\frac{50}{100}$$

4. **Next, reduce the fraction to its lowest terms.** What's the answer?

$$150\% = 1.50 = 1\frac{50}{100} = 1\frac{■}{■}$$

Answer: $150\% = 1\frac{1}{2}$

Changing to Decimals and Percents

Suppose you must rename a mixed number as a decimal or percent. First, write the whole number. Next, rename the fraction as a decimal. You can then rename the decimal as a percent if you wish.

Problem: $1\frac{1}{5}$ = ._?_ = __%

1. **Write the whole number.**
   Write the whole number with a decimal point.

$$1\frac{1}{5} = 1.$$

2. **Rename the fraction as a decimal.**

$$1\frac{1}{5} = 1.2 \qquad 5\overline{)1.0}^{\;.2}$$

3. **Rename the decimal as a percent, if you wish.**

$$1\frac{1}{5} = 1.2 = \mathbf{120\%}$$

$$1.2 = 1.2■. = 120. = 120\%$$

The fraction in the mixed number below will not divide evenly. How would you rename the mixed number as a decimal? As a percent?

$1\frac{1}{3}$ = ._?_ = __%

Answer: $1\frac{1}{3}$ = about 1.33 = about 133%

Exercise

1. Rename these as mixed numbers.
   a. 12.3 **b.** 45.12 **c.** 125% **d.** 285%
2. Rename these as decimals and percents.
   a. $3\frac{4}{5}$ = ._?_ = __%
   b. $5\frac{1}{3}$ = ._?_ = __%
   c. $9\frac{4}{8}$ = ._?_ = __%
   d. $3\frac{3}{10}$ = ._?_ = __%

Answers are in the Teacher's Guide.

Unit Review

Check Yourself

1. Rename these amounts as equivalent decimals.

a. $\frac{3}{4}$	e. 16%
b. $\frac{5}{8}$	f. 4%
c. $\frac{1}{5}$	g. 12%
d. $\frac{2}{3}$	h. .62%

2. Rename these amounts as equivalent fractions. Show the fractions in their simplest terms.

a. .05	e. 50%
b. .625	f. 88%
c. .8	g. 15%
d. 1.5	h. 4%

3. Rename these amounts as equivalent percents.

a. .03	e. $\frac{9}{10}$
b. .6	f. $\frac{12}{25}$
c. .81	g. $\frac{1}{4}$
d. .045	h. $\frac{7}{8}$

4. Rename these decimals and percents as mixed numbers.

a. 2.5	e. 105%
b. 3.75	f. 330%
c. 8.1	g. 212%
d. 4.62	h. 186%

5. Rename these mixed numbers as decimals and percents.

a. $2\frac{3}{5}$	d. $4\frac{3}{4}$
b. $5\frac{1}{4}$	e. $7\frac{1}{2}$
c. $3\frac{5}{8}$	f. $6\frac{3}{10}$

6. Choose the right word to complete each sentence.

equivalents	**powers of 10**
numeral	**simplify**

a. When you change a percent to a fraction, put the ______ in the numerator's place.
b. When you ______ a fraction, you show its lowest terms.
c. Decimal fractions all have denominators that are ______.
d. A fraction, a decimal, and a percent that show the same amount are called ______.

Bonus Work

1. Complete this table.

	Fraction	*Decimal*	*Percent*
a.	$\frac{1}{5}$		
b.		.5	
c.	$\frac{7}{10}$		
d.			45%
e.		.16	
f.			8%
g.	$\frac{2}{3}$		
h.		.64	
i.			100%

Glossary

cross pro duct The answer you get when you multiply the numerator of the first ratio with the denominator of the second ratio in a proportion. In this proportion, 4 and 6 are ***cross products.***

$\frac{1}{2} = \frac{3}{6}$ $\quad 1 \times 4 = 4$

$\quad 2 \times 3 = 6$

dec i mal A fraction that has a denominator that is 10 or a power of 10. For example, .2, .87, and .315 are decimals.

dec i mal pla ces Places at the right of the decimal point. The number of decimal places shows the denominators of decimal fractions.

de nom i na tor The total number of equal parts a whole amount is divided into.

di git The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. Digits are used to show amounts. For example, in the number 12, (twelve), **1** (one) and **2** (two) are ***digits.***

di vi dend The number you *divide* in a division problem. In this problem, 6 is the ***dividend:***

$\frac{2}{6}$ or $6 \div 2$

di vi sor The number you *divide by* in division problem. In this problem, 2 is the ***divisor:***

$2\overline{)6}$ or $6 \div 2$

e quiv a lent ra tios Two ratios that are equal. For example, $\frac{3}{4} = \frac{75}{100}$.

fac tor One of the numbers you multiply in a multiplication problem. In this problem, 2 and 3 are ***factors:***

$2 \times 3 = 6$

known terms The terms that are given in a proportion. For example, the ***known terms*** in this proportion are 16, 48, and 100:

$\frac{16}{48} = \frac{x}{100}$

like decimals Decimals that have the same number of decimal places. For example, .02 and .96 are ***like decimals.***

line up To write numbers in a column with certain parts of each number right above each other. For example, decimals can be lined up by their decimal points.

low est terms The numerator and denominator of a fraction that cannot be reduced. $\frac{1}{2}$ is a fraction at its lowest terms.

nu mer al A number.

nu mer a tor The number of equal parts that are counted out of a total of equal parts. 2 is the ***numerator*** of .2 and $\frac{2}{10}$.

one hun dred per cent A whole amount.

per cent A part of 100 equal parts. For example, 38 percent (38%) is $\frac{38}{100}$ or 38 parts out of 100 parts.

place The exact spot or place a digit is shown in a number. For example, in the number .24, the digit 4 is two ***places*** right of the decimal point.

place val ue The amount a place stands for. The place can be right or left of a decimal point. For example, in the decimal .42, the first *place* right of the decimal point stands for tenths. So 4 has a ***place value*** of tenths.

pow er The number of times a number is multiplied by itself. A power of 10 may be 10 × 10 (100), 10 × 10 × 10 (1,000), and so on.

pro duct The answer to a multiplication problem.

pro por tion A way of showing two ratios that equal each other. For example:

$\frac{6}{9} = \frac{2}{3}$

quo tient The answer to a division problem.

ra ti o A way of comparing two numbers. $\frac{1}{2}$, 1 out of 2, and 1 : 2 are all ways of showing the same ratio.

re duce To make smaller.

re main der The amount left over when a problem does not divide evenly.

re name To change a number to another number without changing its value. For example, .5 can be renamed as .50, and $\frac{1}{2}$ can be renamed as $\frac{2}{4}$.

round To shorten a decimal to a certain decimal place. For instance, you can round .876 to .88—the hundredths place.

sim pli fy Reduce a fraction to its lowest terms.

un known term The term in a proportion that is missing. For example, the numerator of the second ratio is the unknown term in this proportion.

$\frac{1}{2} = \frac{x}{4}$

un like dec i mals Decimals that do not have the same number of decimal places. For example, .3 and .33 are ***unlike decimals.***

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