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ABSTRACT

The concept of three curriculums--intended, implemented, and attained--is central to this report. In the second international mathematics study, students were surveyed at age 13 (grade 8 in the United States) and in the terminal grade of secondary schooling; this analysis is limited to the first group. How each of the three curriculums was assessed is discussed. The intended and implemented content coverage is briefly discussed, with a table showing differences between the coverage for arithmetic, algebra, geometry, statistics, and measurement for 16 countries. Differences between student and teacher indices are shown in a table. The attained curriculum is discussed in detail, including appraisal by type of items. For most countries and for most mathematics topics, intention runs ahead of implementation. Teachers agreed with students on what had been taught in algebra and measurement, but on the other three topics there were a large number of items showing discrepant results. The appendix includes 23 data tables. (MNS)

Second International Mathematics Study

Perceptions of the Intended and Implemented Mathematics Curriculum

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The model of the three curriculums is central to this report. At the level of the education system there is the intended curriculum, containing the 'official' view of what is prescribed to be taught in mathematics in each country. At the second level, the level of the classroom, there is the implemented curriculum, the body of mathematical knowledge which is actually taught, or presumed taught, to students by teachers. Finally, the information and skills mastered by the students, as demonstrated in tests and questionnaires, makes up the attained or achieved mathematics curriculum.

The level of correspondence between the intended and the implemented curriculums might be termed the conformity of the educational system. In most countries, and for most mathematics topics, intention runs ahead of implementation. Such 'non-conformity' may occur for several reasons. The official curriculum developers may be over-optimistic about what teachers are reasonably able to cover in their courses, or the presence of differentiated or individualised instruction may imply different levels of coverage for students of differing abilities. Finally, teachers' perceptions of exactly what mathematics has already been taught to their students may be inaccurate. This phenomenon is particularly likely in countries where the tested population falls at the first year of post-primary school.

By and large, students agreed with their mathematics teachers on the extent of syllabus coverage. The correspondence was reasonably close in arithmetic, algebra and measurement, in most countries, but rather less so in geometry and statistics. In general, the student index of implemented content coverage was greater than the teacher index, and the reason for this was almost invariably a large proportion of students claiming that they had had the opportunity to learn the necessary mathematics 'in previous years'. Sometimes this exposure may have been in mathematics classes at an earlier stage, in primary rather than secondary school, sometimes in the course of other school subjects, special coaching lessons or mathematics competitions, and sometimes just in the process of informal learning outside the classroom environment altogether. Particular topics showing large discrepancies across several countries were square roots, indices, sequences, congruence, similarity and applications of Pythagoras' Theorem.

In a similar way, the degree of articulation between the implemented curriculum and the attained curriculum could be seen as a measure of the efficiency of an educational system. A system could not be described as 'efficient' if mathematical topics which students are given an 'opportunity-to-learn', because they are in the intended and the implemented curriculums and regarded as important, are not topics on which they can show high levels of achievement when tested. Conversely, students cannot be expected to perform well on material which they have not been taught. An index of 'efficiency', such as that developed in this report, is a necessary accompaniment to achievement scores when comparisons are being made between countries with different curricular emphases in mathematics.

Introduction

Underpinning the curriculum analysis of the Second Study of Mathematics conducted by the International Association for the Evaluation of Educational Achievement (IEA) is a simple, but powerful model, embodying the concept of the three curriculums. At the level of the educational system, there is the intended curriculum, the collective of intended outcomes, supplemented by course outlines, official syllabuses, examination prescriptions and textbooks which prescribes what it is intended should be taught in mathematics, in each country.

The second level deals with the classroom, where the content becomes translated into reality by the individual teacher, or implemented. Thus the implemented curriculum will reflect the personal preferences and biases of teachers, the coverage of text-books used, as well as the size and composition of the classes in which mathematics is taught. It is clear that the implemented curriculum need not bear a strong resemblance to the intended curriculum, and that it is the implemented curriculum which finally determines the student's opportunity to learn mathematics.

Finally, the attained curriculum is that assimilated by the student; the body of mathematical knowledge acquired and attitudes engendered in the process of studying the subject. Attempts can be made to measure this attained curriculum by tests and various types of questionnaire. The extent to which there is a strong correspondence between the three curriculums is an important concern of the IEA study, and some aspects of this correspondence, or lack of correspondence, relating in particular to the implemented curriculum, form the basis for the present investigation.

Target Populations

Two populations of students formed the focus of the Second IEA Mathematics Study, approximately paralleling those considered in the first study, carried out with more limited goals and terms of reference in 12 countries in 1964 (Husen, 1967). The target Population A in the second study consisted of all students in the grade in which the modal number of students had attained the age of 13.0 - 13.11 years by the middle of the school year. For those countries participating in this study, the national Population A's spanned four grade levels around the primary-postprimary divide, ranging from one year before the divide (e.g. in Hungary) to three years after it (e.g. in France).

Population B was defined internationally to consist of all students who were in the normally accepted terminal grade of the secondary education system in a participating country, and who were studying mathematics as a substantial part (approximately five hours per week) of their academic program. The analysis to be presented in this report is limited to the first of these populations, Population A.

Space considerations prevent a detailed consideration of the survey procedures and sampling methodology of the study. These have been fully documented, and will be reported in the three official volumes from the study. It may be sufficient to note here that large, representative samples of students and their teachers were drawn from each participating country, under the direction of their own IEA National Centers, following acceptance of a sampling plan by an international sampling committee.[1] Careful national and international validation of measuring instruments and data collection has been a feature of the study at every point, and there is good evidence to suggest that the instruments used reached an acceptable level of reliability. Centralised analysis of all data was carried out in Wellington, New Zealand for the cross-sectional study and Champaign, Illinois for the associated longitudinal study.

Intended Curriculum

At the time of the first IEA mathematics study, the curriculum reform movement in mathematics was only in its infancy. Since then, vigorous curriculum development has taken place in many countries, and the second study has as its central concern a detailed analysis of the various mathematics curriculums offered, and the settings in which they operate.

The curricular intentions of each country were obtained by examining syllabuses, curriculum guides and textbooks, and also through a series of Working Papers involving questionnaires completed by the IEA National Centers. From these sources, a comprehensive 'menu' of mathematics topics was drawn up, designed to include all those topics likely to be taught at the two levels in any IEA country. The Population A content outline originally contained 133 categories under five broad classifications: arithmetic, algebra, geometry, statistics and measurement.

Following the approach outlined by Wilson in Bloom et al (1981), a 'content by behaviors' grid was formed to determine the patterns of emphasis of subject matter appropriate for such a study. The vertical dimension of the grid consisted of the five topic areas listed above, whilst the horizontal dimension was divided into four levels of increasing cognitive complexity: computation, comprehension, application and analysis. Computation was defined to include the ability to recall specific facts, use mathematics terminology and carry out algorithms; comprehension implied the ability to recognise concepts, principles and rules, follow a line of mathematical reasoning

1. See Appendix Tables A2 to A5 for sample numbers.

or read and interpret a problem. The application behavioral level required the solving of routine problems, making comparisons, recognising patterns and analysing data, while the highest level of all, analysis, was defined to include the solving of non-routine problems, discovering relationships and formulating generalizations.

Following a lengthy process of international validation, described fully in the reports from the Study, weights were attached to the various cells in the grid, and an international pool of items selected to reflect the agreed-on emphasis. At a later stage in the study, prior to testing, IEA National Centers were asked to rate each item in the international pool on how acceptable or appropriate its subject matter would be in their own country.

They were advised that the key element in judging the appropriateness of an item should be the mathematical skill or knowledge contained in the item rather than its particular form.

The following rating scale was used:

2 - the item is highly appropriate to the national curriculum
1 - the item is acceptable
0 - the item is inappropriate (because its mathematical content is not in the curriculum).

After categories 1 and 2 had been combined, these ratings formed the basis for the variable termed the index of intended content coverage, the percentage of items in the test regarded as acceptable or highly appropriate in each country. The index may be regarded simply as providing an indication whether an item was seen to be in the official syllabus for a particular country, or not. It gives a general impression of the 'fit' of the international item pool to the curriculum of each country, and a vantage point from which to consider other aspects of the teaching process.

Implemented Curriculum

The teacher has well been called the 'gatekeeper' of the curriculum. Within the constraints of the education system in each country, it is the classroom teacher who eventually decides what aspects of the curriculum shall be taught, how much time shall be allotted to each topic, and how it shall be taught. For the international study of the implemented curriculum, therefore, a number of questionnaires were developed, to be completed by teachers. From one of these it was possible to obtain a measure which previous IEA studies had described as 'opportunity-to-learn'. In the present study this variable has been re-named as implemented content coverage. If the mathematical content required to answer a particular item has not been taught, either in the year of testing or in some prior year, it would be unreasonable to regard low scores on that item as an indicator that something is amiss. The students have simply not been given the 'opportunity-to-learn' it, in school, because it has not been covered. The curriculum has not been implemented as it was intended.

In order to quantify this measure, three separate questions were asked of each teacher about each item in the test:

During this school year, did you teach or review the mathematics needed to answer the item correctly?

__A No
__B Yes

If, in this school year, you did not teach or review the mathematics needed to answer this item correctly, was it because:

__A It had been taught prior to this school year
__B It will be taught later (this year or later)
__C It is not in the school curriculum at all
__D For other reasons.

From these questions, it was possible to determine whether the mathematics content for an item had been:

- taught or reviewed during the year
- not taught or reviewed, because it was assumed that it had been covered in a previous year
- not taught or reviewed.

The sum of the first two categories provided a 'teacher opportunity-to-learn' score, or teacher index of implemented content coverage, for each item, to be interpreted as the percentage of teachers judging that their class had had the opportunity to learn the necessary content and skills to tackle the item. When aggregated to the country level, after appropriate weighting, it provided a national indicator for each item, relating to the implemented curriculum, an index which could conveniently be compared with the appropriateness index, relating to the intended curriculum.

A check on this measure was provided by the students themselves, who were asked to respond to each item by saying whether they had been taught the necessary mathematics 'this year', 'in previous years' or 'never'. The sum of the first two categories for this variable gave a 'student opportunity-to-learn' score, or student index of implemented content coverage, paralleling the teacher index, for each item. When aggregated to the national level, this index could be interpreted as the percentage of students in the country claiming that they had had a chance to learn the mathematics needed to do the item.

Several countries did not request this information from their students. Some were concerned about the extra load it would place upon them in answering the tests, and the consequent increase in time needed. Others were unsure about the likely validity of any answers which might be given. Nevertheless for the subset of countries for which both teacher and student indices were available, a comparison between them, and an examination of large discrepancies which occurred for particular items, provides some interesting insights into the teaching and learning process, and is a central concern of this report.

Attained Curriculum

In order to assess pupil outcomes, that is, the attained curriculum, test items, questionnaires and attitude scales were developed. The validation process is fully described in the official volumes from the Study. The achievement items were designed to measure the pupils' knowledge of areas of the mathematics curriculum which had been agreed as important by the National Centers and their committees in participating countries. Some countries ran a longitudinal study utilizing a pre-test/post-test design on student achievement. Other countries confined themselves to a cross-sectional study, using the post-test only. For the purposes of this study, the outcome measures (which could be regarded as indices of attained content coverage), are various sub-scores on the post-test measure, expressed as national percentages of students choosing the correct response on the constituent items. Only those 157 items common to both the cross-sectional and the longitudinal studies are included in this analysis. All are multi-choice items, and no corrections for guessing have been applied, as the current state of research on this topic suggests that standard formulae are probably not applicable.

A final index forms something of a bridge between the implemented and the attained curriculum. This has been termed the teacher estimate of student achievement, and was obtained by asking teachers what percentage of the students in the target class did they estimate would get each item correct, without guessing. By aggregating these item responses to a national level, after appropriate weighting, it is possible to see how accurate were teachers' perceptions both of what they had taught and how well they thought their classes would perform on each item. The index was also designed to detect whether teachers, in assessing the 'opportunity-to-learn' scores for their students, were able to distance themselves from the likely performance of the same students on the post-test. If the relationship between this index and the 'opportunity-to-learn' index were too close, it would cast some doubt on their validity. We would then be justified in calling into question the capacity of teachers to assess accurately whether or not their students had had the necessary exposure to particular topics to allow them to display mastery of these topics in tests and examinations. Analyses in selected countries have shown only moderate correlations between the two indices, and, with certain exceptions to be noted and discussed later, it would appear that teachers in most countries are able to assess rather accurately the probable test performance of their students. The validity of their 'opportunity-to-learn' assessments is, of course, the major focus of the present paper.

Relationships Between the Curriculums

One might anticipate a close relationship between the intended curriculum, the implemented curriculum, and the attained curriculum, in each country. The goals and aspirations of the mathematics educators who have the responsibility of shaping the curriculum are accurately perceived by teachers and translated into appropriate and effective instructional sequences in the classroom. Students in their turn, have an alert perception of the dimensions of the curriculum which they are studying, and this is reflected in the measured outcomes of their learning. In brief, topics which are in the syllabus, are being taught by teachers and mastered by pupils.

A lack of articulation between the intended and the implemented curriculums could then be interpreted as a lack of 'conformity' in the educational system, a signal that the intentions of the policy makers and curriculum developers are not being put into practice with a comprehensiveness that they would consider desirable. Some teachers, especially in decentralised school systems not constrained by the dictates of national examinations, may hold a contrary view. They would argue that the individual needs of students with differing goals and abilities cannot be met by a 'lock-step' approach, and that such 'non-conformity' is a desirable feature of an educational system. Good teachers should welcome the professional freedom to choose their own textbooks and plan their own curriculum emphases and flexible work sequences.

A lack of articulation between the implemented and the attained curriculum could, on the other hand, be interpreted as a lack of 'efficiency' in the system, if topics which students are given an 'opportunity-to-learn' (because they are in the intended and implemented curriculums and regarded as important) are not topics on which they can demonstrate high levels of achievement when tested. This could be attributed to poor teaching, inadequate facilities, or skimpy treatment of topics because of overfull timetables and competing curriculum offerings. It is also likely to be influenced by the varying emphases and preferences expressed by teachers for particular topics, and corresponding emphases and preferences shown by students in studying them. It may also be the result of inaccurate or highly unrealistic perceptions on the part of teachers of the cumulative body of mathematics to which students have been exposed, inside and outside the classroom.

The appendix to this report contains a number of detailed tables for those countries in the survey for which data were available for at least three of the five variables being considered. Tables A1 to A5 summarize scores for each variable in turn across subtests and countries; Tables A6 to A22 give data for each country in turn across subtests and variables. In each case the items upon which the tables are based are the pool of 157 common to both cross-sectional and longitudinal studies (46 arithmetic, 30 algebra, 39 geometry, 18 statistics, and 24 measurement).

What follows is a very brief summary of the main outlines of the section on the intended and implemented curriculums, to be found in Chapters 7 and 8 of the first volume of the IEA Second Study of Mathematics, provisionally entitled <u>The International Mathematics Curriculum</u>. This will set the scene for the more detailed analysis of discrepancies between student and teacher perceptions of content coverage given at the end of the report.

<u>Intended Content Coverage</u>

The item appropriateness ratings given in Table A1 form a convenient way of describing the content of the intended curriculum in each country, and as they are tied to the actual items in the international pool, they allow comparisons with the other indices relating to the implemented curriculum. They also give a general impression of how well the international item pool fitted the mathematics curriculum in each country. For example, the arithmetic items appeared to articulate well with the intended curriculum in every country, being a perfect fit in the U.S.A. and generally showing better than an 85 percent fit in the other countries. The measurement items also appeared to be generally within the intended curriculum in all countries except Belgium and France. Coverage of statistics was very low in these countries, also in Israel and Luxembourg, where it barely figured in the intended curriculum at all. The algebra items were generally regarded as appropriate in all countries, except for Canada (Ontario) and the U.S.A., where the Population A students were in primary rather than secondary school. Geometry showed a very diverse pattern, with enormous variation between countries. There is clearly no world consensus about what geometry should be taught at this level, and no country found more than about seven-eighths of the items appropriate; many indeed found no more than half the items fitted their official curriculum.

Implemented Content Coverage

The scores given in Tables A2 and A3 give a summary of the perceptions of teachers and students in each country as to which of the items in the international pool they regarded as having been 'covered' in their mathematics instruction. As far as the teachers were concerned, this required them to say whether they believed their students had had the 'opportunity-to-learn' the mathematics needed to do each item, either because they had taught or reviewed it themselves during the year, or because they believed that it would have been taught in a previous year. The figures in Table A2 take on more meaning when they are compared with those in Table A1, to see where major discrepancies lie between what was intended to be covered, and what teachers assumed or claimed had actually been covered. The differences between respective entries in these two tables are given in Table 1.

A few examples should make the method of interpretation clear. In Belgium (Fl.) the difference between the two indices for the Algebra subtest is given as 22 percent; reference to Tables A1 and A2 shows that 93 percent of the items were judged by the Belgian (Fl.) National Center as being appropriate for Population A, but teachers reported that, on average, only 71 percent of these items would have been covered prior to or during the Population A year in the country's classrooms, a difference of 22 percent. A difference in the reverse direction occurs in the Statistics subtest. The official view is that virtually none of the items would be appropriate (11 percent), but teachers assumed that their students would have had the 'opportunity-to-learn' as many as 38 percent of the items, a difference of -27 percent. For the other three subtests, the discrepancies are less than 20 percent; and the weighted total across all subtests is zero.

It should of course, be recognized that the two indices (intention and implementation) are not strictly comparable, since the first is obtained at the national level in each country, the second at the classroom (teacher) level and aggregated to the national level. But by focussing only on large discrepancies some cautious generalizations are in order. For some National Centers, notably Belgium (Fl.) and France, it appears that the appropriateness ratings may underestimate the index of intended coverage, since National Committees, in providing the ratings, judged some items inappropriate if the content had been prescribed to be taught earlier in the school system than the Population A level. This could account for some of the negative differences in Table 1. Most countries classified such items as appropriate, since they fell within the cumulative mathematics syllabus up to the time of testing.

Teachers, on the other hand, in rating the items in some subtests, notably Statistics, could be taking into account likely prior learning of mathematical concepts in other subjects of the curriculum (social

Table 1 Differences* between indices of intended and implemented content coverage for Population A, by major subtest and country

Country	Arithmetic %	Algebra %	Geometry %	Statistics %	Measure-ment %	Weighted Total %
Belgium (Fl.)	9	22	-6	-27	-16	0
Canada (B.C.)	9	2	-7	42	18	9
Canada (Ont.)	-2	-14	27	16	-6	4
England & Wales	19	30	33	19	18	24
Finland	10	8	30	33	23	19
France	-4	11	11	-23	-30	-4
Hungary	2	-1	-1	-2	3	0
Israel	25	-3	33	-47	41	16
Japan	9	10	36	25	5	17
Luxembourg	14	21	-12	-10	-3	4
Netherlands	8	7	11	29	5	11
New Zealand	26	34	28	40	30	30
Swaziland	2	-4	-16	6	-5	-4
Sweden	20	26	13	53	28	24
Thailand	7	-10	14	21	14	8
U.S.A.	16	-6	12	28	25	14

*These differences have been calculated by subtracting each weighted national teacher 'opportunity-to-learn' mean (TOTL) from the corresponding item appropriateness rating (APPR), as given in Tables A1 and A2.
In this and the following three tables, differences exceeding 10 percent might be regarded as interpretable. (See Statistical Note, p.30)

studies, woodwork, technical drawing), and this could also lead to negative differences. A number of graphical items in the tests, for example, could have been encountered by students at an earlier stage in other school settings outside the mathematics classroom.

But most of the differences in the table are positive, suggesting that, in general, intention runs ahead of implementation. In some countries, such as England and Wales, Finland, Israel, New Zealand, Sweden, and U.S.A., substantial differences run across all or almost all subtests; other entries such as Geometry for Ontario or Statistics for British Columbia show a substantial positive discrepancy in just one subtest.

Large positive differences may come about for several reasons. The first which comes to mind is simply that there is more in the curriculum than is currently being taught in any given country; government officials responsible for the development of the mathematics curriculum are unduly optimistic about what teachers are reasonably able to cover during the regular course of a year's work (or should have covered in previous years). Under these circumstances, teachers may omit parts of the curriculum due to lack of time.

A second explanation may be that in countries where streaming or differentiated schooling is practised, classes of less able students may not cover as much mathematics as classes of average or above average ability. Different content may be specified for the different streams or programs, and only the higher ability streams may be expected to cover the full content as intended in official prescriptions.

Another situation likely to lead to large positive discrepancies can occur in countries where Population A falls at the first year of post-primary school. This occurs in British Columbia, Finland, Japan, New Zealand and Sweden. It is probable that teachers at this level will not be entirely familiar with content taught in different types of school at the primary level, and so will be unable to rate accurately whether their present students have covered it or not. This could well account, in part at least, for the number of large positive discrepancies in these countries.

The final observation above leads naturally on to a consideration of the second measure of the implemented curriculum, the student index of implemented content coverage, or student 'opportunity-to-learn' score. This was designed as a check on the teacher index, to throw light on such eventualities as those just mentioned, and provide an independent student perception of what mathematics had actually been taught. A number of National Centres did not collect this information, and some of those that did collect it were doubtful about its validity. No discussion of this index therefore occurs in The International Mathematics Curriculum, but it may not be out of place to discuss it here, provided due caution is exercised in its interpretation.

Table 2 presents a series of percentage differences between the student and teacher indices of implemented coverage, constructed in a similar way to those in Table 1. Differences are not as large, suggesting a greater consonance between teachers and their students, than between teachers and those who prescribe the curriculum. There appears to be good agreement in arithmetic and in measurement, except in Israel, and also in algebra. The presence of negative differences in this table suggests that there are some items in the tests which do not appear to be familiar to students but which are regarded by teachers as having been covered. This is especially so in Swaziland and may be either a reflection of over-optimism on the part of the teachers or lack of appreciation of exactly what they had been taught on the part of the students. Generally the differences are positive, and it is interesting to conjecture why this should be so.

Table 2 Differences* between student and teacher indices of implemented content coverage for Population A, by major subtest and country

Country	Arithmetic %	Algebra %	Geometry %	Statistics %	Measurement %	Weighted Total %
Canada (B.C.)	-3	-10	12	19	3	3
France	-8	-7	17	11	-11	0
Hungary	1	-2	2	-1	-3	0
Israel	20	3	31	31	24	21
Japan	3	7	30	15	-1	11
Luxembourg	-7	0	12	13	-4	2
New Zealand	8	4	14	12	9	9
Nigeria	3	3	4	6	5	4
Swaziland	-7	-14	-5	-12	-11	-9
Thailand	1	0	20	18	4	8

*These differences have been calculated by subtracting each weighted national teacher 'opportunity-to-learn' mean (TOTL) from the corresponding national student 'opportunity-to-learn' mean (SOTL), as given in Tables A2 and A3.

National Centers in some countries, such as Thailand, reported a general tendency on the part of students to over-rate all items. This tendency also showed up in Israel (very strongly), Japan, New Zealand, and Nigeria, but need not necessarily be interpreted as undue optimism or lack of realism on the part of the students. There may be an element of this, where students failed to detect hidden difficulties in an item because it had a superficially familiar appearance. Teachers may have been able to detect the hidden traps which, in their view, placed the item outside the syllabus.

But, perhaps a more likely reason why the student index of implemented coverage should be larger that the teacher index is that students were quite correctly identifying the item as belonging to a topic they had covered previously, but one about which their current classroom teacher may have lacked information. More detailed analysis, at the item level, and not presented here, shows that this is indeed the case. Almost invariably, the major contributing cause for large discrepancies was a high percentage of responses from students claiming they had been taught the mathematics necessary to do the item 'in previous years' (See p.5). Sometimes this may have been in mathematics classes at an earlier stage, in primary rather than secondary school, sometimes incidentally in the course of instruction in other subjects, sometimes outside the school during special coaching lessons, and perhaps sometimes just in the process of informal learning away from school altogether. These differences are explored at the item level in subsequent analyses, illustrated by reference to comments drawn from a number of countries. They clearly suggest that a treatment of content coverage, seen purely from a teacher perspective, may give only a partial picture of the cumulative body of mathematical knowledge to which students have been exposed.

Table 3 Differences* between teacher indices of implemented content coverage and student post-test scores for Population A, by major subtest and country

Country	Arithmetic %	Algebra %	Geometry %	Statistics %	Measurement %	Weighted Total %
Belgium (Fl.)	18	18	-11	-20	25	8
Canada (B.C.)	29	37	8	-9	26	20
Canada (Ont.)	33	29	7	5	35	23
England & Wales	31	23	9	4	33	21
Finland	30	25	-4	-7	17	14
France	28	31	5	-7	33	20
Hungary	34	41	33	25	35	34
Israel	17	36	5	-5	13	15
Japan	25	23	-6	5	26	15
Luxembourg	34	22	10	-5	32	21
Netherlands	22	22	14	-34	21	13
New Zealand	22	23	15	3	25	19
Nigeria	38	40	38	27	40	37
Swaziland	53	62	49	47	57	54
Sweden	27	18	-4	-9	19	12
Thailand	43	45	19	12	38	33
U.S.A.	33	27	6	15	34	23

*These differences have been calculated by subtracting each national mean post-test score (POST) from the corresponding weighted national teacher opportunity-to-learn mean (TOTL), as given in Tables A2 and A5

Attained Content Coverage

Table 3 shows an important set of comparisons at the subtest level, that between the teacher index of implemented content coverage and the index of attained content coverage (mean student scores on the post-test, expressed as percentages). The differences in the table might be regarded as a crude measure of the efficiency of the learning process in the classrooms of each country, and as such take on considerable significance. The raw data are contained in Appendix Tables A2 and A5. For the purposes of this table, it is assumed that the teacher index of the implemented curriculum is a more valid indicator that the student index, although for certain topics in some countries there is evidence that the student index may be preferable. However, the teacher index was supplied by more countries, and was generally taken to be better one to use in the official reports of the study.

The results in the table suggest that the countries fall into three reasonably well-separated groups. The first, containing Belgium (Fl.) Israel, Japan, Netherlands, Sweden and Finland, shows small differences in the weighted total column, not more than 15 percent. The second, with weighted total differences clustering tightly around 20 percent, contains Canada (B.C.), Canada (Ont.), England and Wales, France, Luxembourg, New Zealand and U.S.A. The third, with differences well in excess of 30 percent, contains Hungary, Nigeria, Swaziland and Thailand. An examination of the rank orders of entries in each subtest column shows a fair degree of consistency; Group 3 countries tend to show the largest discrepancies in each topic area, Group 1 countries the smallest.[2]

If we are to take the teacher's perception of what has been taught, or presumed taught, to be accurate, then this suggests that Group 1 countries are the most 'efficient' in producing test results, and Group 3 the least. This difference is an important statistic to consider alongside post-test scores in each country, for it would be unwarranted and unfair to pass judgement about student performance on the basis of post-test scores alone, without some accompanying idea of how much of the mathematics in this test had been taught, or presumed taught, by teachers.

The below-average post-test scores for Finland, England and Wales, Luxembourg, New Zealand, Sweden and U.S.A. (Table A5) can be viewed in a new light when it is realised that their students have not had the 'opportunity-to-learn' as much of the mathematics in the international item pool as those in other countries classified in Groups 1 and 2. When this is taken into account, their performances are rather more comparable. Or at least that is how it would appear if teacher perceptions of content coverage are to be given credence as possessing adequate validity.

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2. The cautions given in Table A2 with respect to Hungary, Israel and Swaziland results should be noted here, as they may affect these comparisons.

Statistics and geometry show up as two subtests in which a considerable amount of incidental learning takes place, almost entirely in Group 1 countries, and so the differences are smaller, even negative in a few countries. What may not be taught in the classroom is picked up in some other way. For the remaining three subtests it would seem as though much of the learning takes place in the classroom, and the measure of what mathematics is presumed taught and the measure of what is demonstrated as learned, differ more widely.

Another relationship worthy of at least passing attention is that between the prior teachers' estimate or assessment of what they believed their students would score on the post-test and the actual subsequent performance of the students. The definitions of these scores were given on pp. 5-6. The differences between these two sets of scores, aggregated with appropriate weighting to the national level, are given in Table 4. The raw data are contained in Appendix Tables A4 and A5. To illustrate: the difference of 11 for Nigeria shows a consistent tendency on the part of Nigerian teachers at the Population A level to over-estimate the performance of their students in the post-test. This is particularly pronounced in the measurement subtest,

Table 4 Differences* between teacher estimates of student achievement and student post-test scores for Population A, by major subtest and country

Country	Arithmetic %	Algebra %	Geometry %	Statistics %	Measure-ment %	Weighted Total %
Belgium (Fl.)	-10	-2	8	1	0	-1
Canada (B.C.)	-4	4	13	-7	2	2
Canada (Ont.)	2	14	14	0	3	7
England & Wales	-2	-3	-10	-16	1	-5
Finland	0	0	-15	-14	-2	-6
France	-5	1	14	2	-1	2
Hungary	3	4	1	-3	8	3
Israel	3	-2	-16	-25	-3	-7
Japan	-2	-1	-13	-11	4	-5
Luxembourg	4	2	-5	-6	8	1
Netherlands	-2	1	-3	-18	2	-3
New Zealand	-6	7	-2	-15	1	-2
Nigeria	9	14	13	4	15	11
Swaziland	11	14	8	6	12	10
Sweden	-6	-3	-9	-15	-11	-8
Thailand	6	10	7	2	-2	5
U.S.A.	-4	5	11	-11	6	2

*These differences have been calculated by subtracting each national mean post-test score (POST) from the corresponding weighted national mean teacher estimates of student achievement (TESA), as given in Tables A4 and A5.

(with teachers estimating a national mean of 45.2 percent, and students actually obtaining a national mean of 30.7 percent, a difference of approximately 15); it is least noticeable in the statistics subtest (with corresponding figures of 40.6 and 37.0, difference approximately 4). The presence of negative differences in many countries shows the opposite tendency, with teachers, nationally, under-estimating the likely performance of their students. A small negative difference would be expected, on average, because teachers were asked not to take guessing into account when making their estimates, and no allowance for guessing was built into the post-test scores. Teachers in Nigeria and Swaziland appeared to consistently over-estimate the performance of their students; to a lesser extent this is also true in Thailand and Ontario. The overall estimates in each of the other countries appear to be quite accurate, although there are major fluctuations in some of the subtests. Performance in geometry and statistics seems difficult to predict accurately, and it will be recalled that these two topic areas also seemed to generate the largest discrepancies between the two 'opportunity-to-learn' indices, discussed in Table 2. However, a comparison between teacher estimates of curricular coverage and teacher estimates of class achievement is rightly the topic of another study, and will not be elaborated on here.

Item Appraisal[3]

Table A23 in the Appendix presents the results of a more fine-grained analysis at the item level, similar to that contained in Table 2 for the five main topic areas; arithmetic, algebra, geometry, statistics and measurement. There were 73 items out of 157 which met the (arbitrary but convenient) criterion of showing a discrepancy of over 30 percentage points in at least one country between the teacher and student indices of implemented content coverage (21 out of 46 items in arithmetic, 6 out of 30 items in algebra, 27 out of 39 in geometry, 14 out of 18 in statistics, 5 out of 24 in measurement). In other words, there were 73 items where students differed quite widely from their teachers in their perceptions of whether they had had the opportunity-to-learn the mathematics necessary for success.

An example should aid in interpreting the entries in the table. The first entry refers to Item 7 in the core form of the longitudinal post-test (coded X7), which was re-positioned as Item 13 in the core form of the test used in the cross-sectional study. For this geometry item, three countries showed differences between student and teacher indices of implemented content coverage exceeding 30 percent, namely Canada (B.C.), New Zealand and Thailand. The differences of 45, 36 and 34 percent, respectively, are all positive, showing that in each of these countries the students took the more 'optimistic' view about their 'opportunity-to-learn' the mathematics needed to do this particular item.

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3. Thanks are due to the National Centers in Canada (B.C.), Hungary, Japan, New Zealand and Thailand for their help in carrying out this item appraisal.

The following section discusses a representative selection of those items showing large discrepancies in several countries. Comments on these items are appended from officers in the respective National Centers, giving their explanations for the discrepancies observed. To obtain reactions from all countries on all items would have been too large a task, but full statistics are included in Table A23 for completeness.

Some general characteristics of the discrepancy table can be noted. Israel, New Zealand and Japan show the largest number of positive discrepancies, confirming results already noted at the subtest level that, in these countries, teachers of Population A students may be unaware of what mathematical knowledge had been acquired at earlier levels, in school or outside the school, and so under-rate the exposure of their students to the mathematics needed to answer items in the test. Very few negative differences occur, reinforcing the observations already made at the subtest level that, in general, Population A students tend to err on the generous side in their perceptions of what they have been taught. This is least true in France, but even there, positive entries in Table A23 outweigh negative ones.

Arithmetic Items

Arithmetic and geometry are the subtests giving rise to the largest proportions of discrepant answers, across all countries in the survey. In arithmetic, however, the picture tends to be idiosyncratic. Different countries show discrepancies in different items; there are very few such items common to more than three countries, and most occur in just a single country.

Items A17 and D12,[4] involving the understanding of a sequence expressed in diagrammatic form, could be done by simple deduction, or drawing and counting rows, and do not necessarily involve mathematics which would have been specifically taught. This was the observation of the British Columbia National Center with regard to Item D12, and almost certainly it would apply to Item A17 as well. Students would have been familiar with diagrams of this type, but teachers may not have deliberately taught the actual mathematical skills needed to solve the problem. The fact that students said that they had had the 'opportunity-to-learn' the item does not necessarily mean they got it correct, of course, but simply that it looked familiar to them.

Items A30 and C33 deal with square roots, a topic not found in a number of official syllabuses at this level. Finland, Japan, New Zealand, Swaziland and Sweden reported these items as inappropriate for Population A students, and low intended coverage was indicated in Belgium (French and Flemish), France, Hungary and Ireland.

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4. In the following discussion, all items will be referred to by their longitudinal numbers. They are illustrated in the form in which they appeared in the New Zealand version of the post-test, but somewhat compacted and photo-reduced to save space.

A17

1st row	1	A	0
2nd row	1 - 1	B	1
3rd row	1 - 1 + 1	C	2
4th row	1 - 1 + 1 - 1	B	25
5th row	1 - 1 + 1 - 1 + 1	E	30

What is the sum of the 50th row?

D12 Matchsticks are arranged as follows:

If the pattern is continued, how many matchsticks are used in making the 10th figure?

A 30

B 33

C 36

D 39

E 42

C33 Since 4 x 9 = 36, $\sqrt{36}$ is equal to

A 4 x 9

B 4 x 3

C 2 x 9

D 2 x 3

E $\sqrt{2} \times \sqrt{3}$

A30 What is the square root of 12 x 75?

A 6.25

B 30

C 87

D 625

E 900

A24 If $10^2 \times 10^3 = 10^n$ then n is equal to

A 4

B 5

C 6

D 8

E 9

A33 0.00046 is equal to

A 46×16^{-3}

B 4.6×10^{-4}

C 0.46×10^{3}

D 4.6×10^{4}

E 46×10^{5}

The New Zealand situation is particularly interesting here, since examples of square roots are given in the two officially prescribed textbooks for the two years immediately prior to the Population A year, but they do not appear in the official, national syllabus until the year following. It appears that many primary and intermediate school teachers in New Zealand have followed the textbook and not the syllabus in teaching the topic. The teachers at Population A level, the first year of secondary schooling, do not appear to be aware of this prior learning, because they use a variety of different textbooks of their own choosing.

The presence of a square root button on cheap electronic calculators is another factor which should not be overlooked. These are commonly owned by students in New Zealand, and irrespective of whether the topic was in the syllabus or not, the way to get square roots from a calculator was likely to be well known by many Population A students. The concept itself may not have been grasped, of course, and so Item A30 may have looked more familiar to the students than Item C33. Results from Table A23 suggest that this was so, as the discrepancy between teacher and student estimates for Item A30 is 46 percent, for Item C33 only 32 percent.

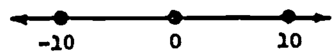
In Hungary, at the time of the Second IEA Mathematics Study, three mathematics curriculums were in use at the Population A level: the traditional curriculum, a 'transitory curriculum' and an experimental version of the 'new' curriculum. A little less than half the students would have been on the traditional 'classical' curriculum at the time of the survey. Square roots were included in the transitory and new experimental curriculums, but not in the traditional one. It is likely, then, that teachers accustomed to the traditional curriculum may not have had a clear idea of the extent of prior exposure of their students to the new material.

The Thai National Center noted that square roots were right at the end of the textbook in the Population A year, and the topic may not therefore have been covered by the time of the survey. The students may have misunderstood the problem as simply requiring them to find the product of the two numbers given, and there is evidence to suggest from an examination of their responses that this is just what they did.

Another topic which appeared to generate differing perceptions was that of indices. Items A24 and A33 (the latter classified as Algebra, but treated here for convenience) were again part of the transitory and experimental curriculums in Hungary, and were not generally taught in New Zealand. For these items, students may have had the opportunity to learn the elementary notions of 'standard form' in science classes, rather than in mathematics.

The Hungarian National Center noted that all of the topics generating differences (and there were only five for Hungary) were covered in the special mathematical clubs of the Bolyai Janos Mathematical Society. Since a large number of more able students participate in these activities, they may have answered 'yes' to the question on 'opportunity-to-learn', in spite of the fact that the topics were not mentioned in school. Hungary also made the suggestion that students scoring poorly on these items, and generally showing low performance in mathematics as a subject, may believe that they probably <u>had</u> been taught the mathematics necessary to do the items, and that failure was their own fault.

A35



Which of the following sequences of numbers is in the order in which they occur from left to right on the number line?

A $0, \frac{1}{2}, -1$

B $0, -1, \frac{1}{2}$

C $-1, -\frac{1}{2}, 0$

D $-1, 0, -\frac{1}{2}$

E $-\frac{1}{2}, -1, 0$

B5 For the table shown, a formula that relates m and n is

A $n = m$

B $n = 3m$

C $n = m^2 + 1$

D $n = -m^2 + 1$

E $n = 2m + 1$

m	-1	1	2	4
n	-1	3	5	9

C16 If $x = y = z = 1$, then $\frac{x - z}{x + y}$ is equal to

A -2

B -1

C 0

D $\frac{1}{2}$

E 1

<u>Algebra Items</u>

There are relatively few algebra items showing major country differences between student and teacher perceptions of 'opportunity-to-learn', and they fall into no simple groupings. Only three items (A35, B5 and C16) show differences exceeding 30 percent in more than one country. What is significant however, is that the differences are virtually all <u>negative</u>. Notably, A35 and C16 are items where the students are consistently under-rating their exposure to the topic, in comparison with their teachers. The items may be unusual in the way in which the information is presented, just a little 'off the beaten track', and therefore regarded as unfamiliar. The teachers on the other hand, may quite correctly regard the underlying principles as being within the syllabus, and consider that the mathematics necessary to do the items has been taught. More than that it is difficult to say. In general, student and teacher perceptions in algebra appear to correspond quite closely, worldwide.

A6 The triangles shown below are congruent. What is *x*?

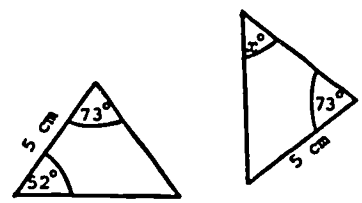
A 52

B 55

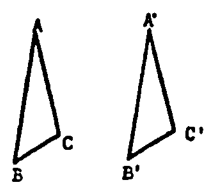
C 65

D 73

E 75



B14 Δ ABC and Δ A'B'C' are congruent and their corresponding sides are parallel. Δ ABC maps onto Δ A'B'C' by a



A reflection

B glide reflection

C rotation

D enlargement

E translation

C32 Triangle PQT can be rotated onto triangle SQR. The centre of rotation is

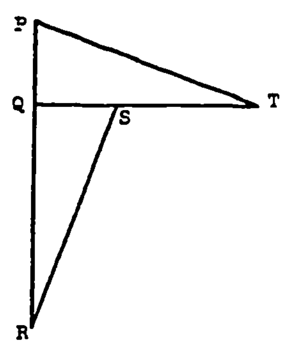
A Point P

B Point Q

C Point R

D Point S

E Point T



Geometry Items

A large number of geometry items show substantial discrepancies across many countries, and only a representative sample of them will be analysed here. . Three topics figure largely among these discrepant items: congruency, similarity and Pythagoras' theorem. Items A6, B14 and C32 deal with congruence (B14 and C32 in a transformation geometry context), and there were four other congruence items not illustrated here (Core21, C29, D16 and D22), showing large differences in several countries. Items Core40 (coded X40) and C9 deal with similar triangles.

The question to be raised here is how much do correct answers in geometry depend on intuition, 'common sense' or general knowledge, and how much do they depend upon instruction in mathematics. In New Zealand, intuitive ideas of congruence and similarity were included in the syllabus and in the official textbooks for the two years prior to the Population A year. But there is evidence to show that, at the time of the survey, secondary school teachers were not fully aware of this, and under-estimated the exposure of their pupils to geometrical concepts. They commonly begin to teach these topics anew within a transformational geometry framework in the Population A year, on the assumption that their students know little or nothing about them. A parallel situation prevails in British Columbia, where elementary school students are also presented with intuitive ideas of congruence and similarity, before the Population A year.

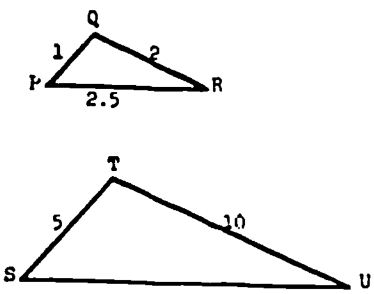
The Thai National Center observed for Item C32 that 'rotation' is a common everyday term, and remarked that students could probably solve the problem without understanding 'rotation' in the technical, transformational geometry sense. The teachers' responses, however, would largely be swayed by the fact that 'rotation' is not covered in the normal curriculum.

Results for Items B6 and D16 (not illustrated) further emphasise this tendency. It is reported that Thai teachers would regard 'line of symmetry' in Item B6 as not in the curriculum for Population A, but students, unfazed by the unfamiliar technical vocabulary, would attempt to solve the problem using intuitive ideas of congruence. Item D16 extends the concept of congruence to four-sided plane figures, and once again, students may have recognised the word 'congruence', and used their initiative in extending the 'same size and shape' schema into an unfamiliar situation. New Zealand teachers may have noted the term 'glide reflection' in one of the distractors for Item B14 and ruled the item as a whole out of the curriculum, whereas many students, seeing the general appearance of the item was familiar, and untroubled by such cues, would rate it as acceptable.

Similar triangles are covered right at the end of the official Grade 8 mathematics textbook in Thailand, and may not have been covered in all schools at the time of the IEA survey. Teachers would rate the items accordingly, but the students might not regard 'similar' in Items Core40 and C9 as a key word, and imagine that they were familiar with facts about triangles (Item C9) or perhaps thought they could solve Item Core40 by estimation or measurement.

Items B25 and C1 deal with Pythagorean triangles, a topic not in the intended curriculum in a number of countries at this level. The Hungarian National Center reported it as in the transitory and new experimental curriculums only, not in the traditional, and this could account for the discrepancy in their country, as it did for the arithmetic items previously mentioned. Pythagorean triangles of the (3,4,5) variety were given a very cursory treatment in the official syllabuses and textbooks for the last two years of primary school in New Zealand, but did not figure in the Population A year syllabus at all, at the time of the IEA survey. Teachers at this level were apparently unaware of the possible prior exposure of pupils to

X40



A 5

B 10

C 12.5

D 15

E 25

Triangles PQR and STU are similar. How long is $\overline{SU}$?

C9 If two triangles are SIMILAR, which of the following statements is TRUE?

A Their corresponding angles MUST be equal.

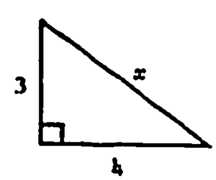
B Their corresponding sides MUST be equal.

C Their corresponding sides MUST be parallel.

D They MUST have the same area.

E They MUST have the same shape and size.

B25 Which of these is a correct statement for this triangle?



A. $x^2 = 3^2 + 4^2$

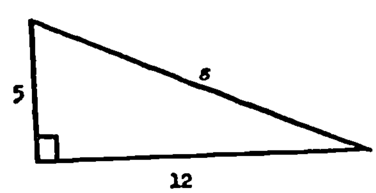
B $x^2 + 3^2 = 4^2$

C $x = 4^2 - 3^2$

D $x^2 = 4^2 - 3^2$

E $x = 4 + 3$

C1



What is the value of s?

A 7

B 13

C 15

D 17

E None of these

elementary applications of Pythagoras' theorem, and under-rated their familiarity with it. There is, too, at least the possibility that students may have come across the concept as the basis for generating right angles in woodwork or metalwork classes. On the other hand, in Item C1 (a 5,12,13 triangle and not a 3,4,5 one) they may simply have made a careful estimate, or used a little inspired guesswork. This last was the suggestion from the Thai National Center, which noted once again that the topic came near the end of the textbook used for Population A students, and may not have been covered by teache in time for the IEA survey.

X7

The diagram shows a cardboard cube which has been cut along some edges and folded out flat. If it is folded to again make the cube, which two corners will touch P?

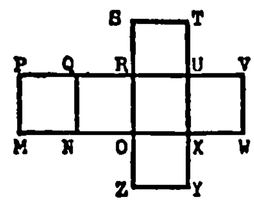
A corners Q and S

B corners T and Y

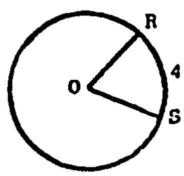
C corners W and Y

D corners T and V

E corners U and Y



X19



A 24

B 30

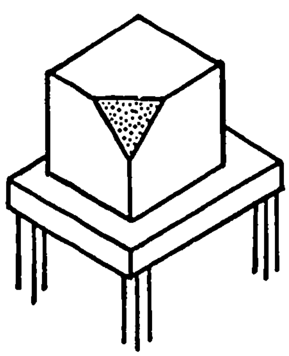
C 45

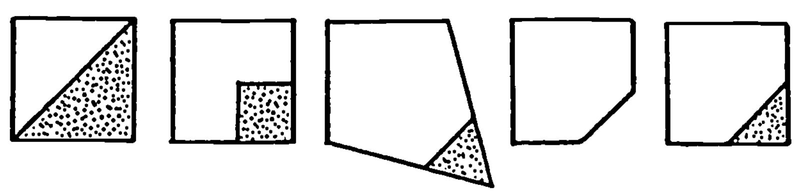
D 60

E 90

The length of the circumference of the circle with centre at O is 24 and the length of arc RS is 4. What is the size in degrees of the central angle ROS?

A15





A B C D E

The figure above shows a wooden cube with one corner cut off and shaded. Which of the following drawings shows how this cube would look when viewed from directly above it.

Three other geometry items deserve some mention here, Core7, Core19 and A15, as at least three countries showed discrepancies exceeding 30 percent between student and teacher perceptions of content coverage for these items. In Item Core7, students were being tested on their understanding of basic concepts of spatial relations, and National Centers in at least three countries observed that, although such an item would perhaps not be specifically taught by teachers in the classroom, nevertheless intuitive ideas of nets of 3-dimensional figures should have been introduced to students in elementary school. Item Core19 is another which teachers in many countries regarded as lying outside their syllabus, because 'length of arc of a circle' was not specifically included. But the intuitive mathematical underpinnings may well have been present with their students, in work with fractions, or pie charts or simply logical reasoning. Item A15 is another which tests basic spatial perceptions, and would be unlikely to find a place in the normal textbooks used by students at this level. The British Columbia National Center observed that basically no mathematics was needed, and the low rating on the part of teachers probably simply reflects this fact. The students said they knew enough to tackle the item, but the teachers did not regard it as mathematics in the formal sense.

C6 There are five black buttons and one red button in a jar. If you pull out one button at random, what is the probability that you will get the red button.

A 0

B $\frac{1}{6}$

C $\frac{1}{5}$

D $\frac{5}{6}$

E 1

Statistics Items

Although 14 out of 18 statistics items showed discrepancies of over 30 percent between teacher and student perceptions of implemented coverage in at least one country, these tended to be idiosyncratic. Very few items showed consistent discrepancies across a number of countries, and there were no topics within the subtest which stood out as providing more than their share. Only one is illustrated here, Item C6. This showed large positive differences across virtually every country listed in Table A23, and it is not difficult to see why. Elements of probability theory are not normally introduced into school syllabuses at this level, and this was the only item in the test on probability. But concepts of choice and chance are becoming increasingly common in the everyday life of students, in the games they play and the concepts they form, and they may well have felt that the material in the item was familiar, whether or not they were able to get it right. The item could almost be regarded as tapping general knowledge, and not formal mathematical skills.

X10	A solid plastic cube with edges 1 centimetre long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimetres long?	A	8 grams
		B	4 grams
		C	3 grams
		D	2 grams
		E	1 gram

Measurement Items

Only one item in the measurement subtest (Item Core10), showed differences exceeding 30 percent in more than one country. This would seem to be a topic where teacher and student perceptions of content coverage are very much in agreement, and such differences as do occur are idiosyncratic. The response from the British Columbia National Center suggests that in this item the students did not really perceive the difficulty of the problem, and notes that their response was rather too facile. This was confirmed by observations from the New Zealand National Center, with the comment that a large percentage of students made a serious conceptual error in choosing option D (2 grams) as the answer. The question clearly contains hidden traps, and the teachers' perceptions of whether or not items like this had been covered are likely to be the more accurate.

Coaching

Another element in explaining differences not encountered in the responses from other countries was mentioned by the Japanese National Center. Table A23 shows a consistent tendency for very large positive differences to occur between Japanese students and teachers in their perceptions of the implemented curriculum. These differences are by far the largest in the table, and occur in all subtests, but particularly in geometry. The cause appears to be the practice of large-scale tuition in mathematics outside the school in coaching classes of one form or another, a reflection of the intense pressure to achieve which is characteristic of the Japanese school system. Students attending such classes commonly cover more than the officially prescribed school curriculum. This phenomenon has already been noted in Hungary, with more able students attending the special mathematical clubs of the Bolyai Janos Mathematical Society. Informal mathematics clubs for students exist in other countries beside Japan and Hungary, but are generally seen as having a recreational and enthusiasm-generating role, and not a strict teaching one. The Japanese experience may be unique, and almost certainly accounts for the large discrepancies noted in many entries in Table A23.

Summary

1. The model of the three curriculums is central to this report. At the level of the education system there is the intended curriculum, containing the 'official' view of what is prescribed to be taught in mathematics in each country. At the second level, the level of the classroom, there is the implemented curriculum, the body of mathematical knowledge which is actually taught, or presumed taught, to students by teachers. Finally, the information and skills mastered by the students, as demonstrated in tests and questionnaires, makes up the attained or achieved mathematics curriculum.

2. It is obvious that the implemented curriculum need not bear a strong resemblance to the intended curriculum, and it is the implemented curriculum, not the intended one, which finally determines the students' opportunity to learn mathematics. The level of correspondence between the two curriculums might be termed the conformity of the educational system. The simple index of conformity used in this report demonstrates that in most countries, and for most mathematics topics, intention runs ahead of implementation. Large positive differences in all or almost all subtests showed up in England and Wales, Finland, Israel, New Zealand, Sweden and U.S.A. Such a lack of 'conformity' may occur for several reasons. The official curriculum developers may be over-optimistic about what teachers are reasonably able to cover in their courses. Or the presence of differentiated instruction, either in separate schools or in streamed classes, may mean that low ability students are not expected to cover as much mathematics as the official prescription suggests. Finally, teachers' perceptions of exactly what mathematics has already been taught to their students may be inaccurate. This phenomenon is particularly likely in countries where the tested population falls at the first year of post-primary school. This occurs in British Columbia, Finland, Japan, New Zealand and Sweden. Under these circumstances, teachers may not be entirely familiar with the mathematics taught at the primary stage, and so under-estimate both the exposure of students to particular topics and their likely performance on test items related to them.

Some teachers would argue that 'non-conformity' of this type in an educational system may not necessarily be a bad thing. This argument is likely to be met more often in decentralised school systems unconstrained by national, external examinations. Such teachers are likely to value the professional freedom to choose their own textbooks, and to plan flexible work programs to suit the individual needs of students of differing mathematical abilities.

3. An independent check on the 'conformity' of educational systems was provided, in a limited number of countries, by the students themselves, who rated their familiarity with the mathematics needed to do each item in a similar (but not identical) way to that of their teachers. By and large, the students agreed with their mathematics teachers on the extent of syllabus coverage. The correspondence was close in arithmetic, algebra and measurement, in most countries, but rather less so in geometry and statistics. In general, the student index of implemented content coverage was greater than the teacher index, and the reason for large positive discrepancies was almost invariably the large proportion of students claiming that they had had the opportunity to learn the necessary mathematics 'in previous years'. Sometimes this may have been in mathematics classes at an earlier stage, in primary rather than secondary school, sometimes in the course of other school subjects, special coaching lessons or mathematics competitions, and sometimes just in the process of informal learning outside the classroom environment altogether.

4. It is patently clear that there can also be a lack of correspondence between what teachers believe they have taught and what their students can demonstrate they have learnt. The degree of articulation between the implemented curriculum and the attained curriculum could be seen as a measure of the efficiency of an educational system. Such a system could not be described as 'efficient' if mathematical topics which students are given an 'opportunity-to-learn' (because they are in the intended and the implemented curriculums and regarded as important) are not topics on which they can show high levels of achievement when tested. Conversely, one ought not to criticise the performance of a system in which students are unable to perform well on topics which they have not been taught. The index of 'efficiency' used in this report is thus an important indicator to consider along with achievement scores, if unfair comparisons are not to be made between countries with different curricular emphases in mathematics.

5. Results from a detailed appraisal of particular items showed that the perceptions by teachers and students of the extent of the implemented curriculum were reasonably consonant in most topics in the algebra and measurement subtests, in every country. In other words, teachers agreed with students over what mathematics had been taught. In the other three subtests, there were a large number of items showing discrepant results. Arithmetic showed an idiosyncratic picture, with different countries showing discrepancies in different topics. These included square roots, indices and sequences, with teachers generally being more conservative in their perceptions of what material they believed their students would be familiar with. Geometry provided a large number of discrepant items, particularly in the topics of congruence, similarity and Pythagoras' Theorem. This almost certainly reflects a lack of knowledge on the part of teachers of work which had been covered previously in other schools, at the primary level, or of material picked up informally in other contexts outside the mathematics classroom.

References

Wilson, James W. "Evaluation of learning in secondary school mathematics" in: B.S. Bloom, J.T. Hastings, G.F. Madaus, Handbook of Formative and Summative Evaluation of Student Learning. McGraw-Hill, New York, 1971.

Husen T. International Study of Achievement in Mathematics (Vols. 1 and 2). Stockholm: Almqvist and Wiksell, and New York: John Wiley, 1967.

APPENDIX TABLES

The tables upon which this report is based are drawn from the datasets of the IEA, Second International Mathematics Study. The data in this report were processed at the International Data Processing Center, Wellington, New Zealand. Copies of the data tapes may be obtained from them or from the U.S. Department of Education, Center for Statistics. The survey procedures and sampling methods for this study will be reported in three official volumes of the Second IEA International Study of Mathematics. Their titles are: Volume I. Curriculum Analysis; Volume II. Data Collection and Analysis of the Cross-sectional Study; and Volume III. Analysis of the Longitudinal Study and Classroom Processes. These volumes will be published as they are completed by the IEA committees.

Statistical Note

Tables A1 to A5 which follow, plus Table A23, form the main source of data for this investigation. Tables A6 to A22 were derived from Tables A1 to A5 by rearranging the data by country, and rounding all percentages. The standard errors in Tables A2 to A5, along with the sample sizes listed, will give some idea of the precision with which the various percentage means can be estimated. Table A1, which was the fruit of the combined judgement of members of the IEA National Committee in each participating country, does not present statistics based on random samples, and so includes no standard error columns (S.E.) or sample N's.

As the four textual tables upon which the discussion largely focusses consist of columns of differences between percentage means, aggregated with appropriate weightings to the national level, some estimate of the size of difference which might be interpretable is perhaps desirable. This is not a straightforward matter. Some of the differences (those between means relating to subtests within a country) are derived from correlated variables, and the usual calculation for the significance of a difference between un-correlated means will over-estimate the percentage difference necessary for a statistically significant result. On the other hand, the samples in the survey were not simple random ones, but complex, multi-stage samples, subject to various stratum weightings to ensure the final statistics were nationally representative. This introduces the possibility of large 'design effects', specifically when the statistics relate to students who have been selected by intact class groups in differentiated systems where streaming, either overt or covert, is in operation. This factor will act in the opposite direction, and increase the size of difference necessary before statistical significance can be claimed. Finally, it is necessary to distinguish between statistical significance and educational significance. With large random samples, very small differences may become statistically significant, but for all practical purposes these differences may be negligible as far as educational importance is concerned.

A simple, and relatively conservative approach is suggested in interpreting the statistics shown in these tables. It is based on the observation that if two un-correlated means have standard errors of 1.8, the usual 't-test' suggests that a difference of about 5 points may be considered statistically significant at the 0.05 level ($t>1.96$). In fact, of all the standard errors in Tables A2 to A5, 262 out of 305 (86%) do not exceed 1.8 percent;

40 out of the 43 larger differences are in Tables A2 and A4, with their smaller samples, comprising teachers rather than students, and not subject to inflating design effects. If the figure of 5 percentage points is multiplied by a factor of 2, to make a very rough allowance for design effects resulting from the sampling strategies applied in the various countries, we arrive at a convenient (and slightly arbitrary) figure of 10 percentage points as a minimum which might be regarded as significant and worthy of interpretation. The present study will only concern itself with large differences, substantially greater than 10 percent.

Accuracy of the data

The statistics reported for the countries participating in the Second International Mathematics Study are based on sample surveys conducted within each participating country. A sufficient number of schools was chosen in each country to provide a national estimate of cognitive student achievement and other characteristics which might be compared with those of other countries. The statistics reported in this report contain national summaries of several topics (such as achievement levels on many mathematics items as completed by students or coverage levels as completed by teachers). Comparisons between items within a country are made with the same sample and are, therefore, more likely to be reliable than comparisons between countries.

Errors of sampling, data processing, loss of items in transmission of data from one country to the international processing center, and reporting errors may have occurred. Also, cultural biases may affect the meaning of items. The research centers participating in the study carefully evaluated each test item during meetings of representatives from each country reduce the possibility of bias through misinterpretation of its intent.

Each national center was provided with a sampling manual of detailed procedures. The manual suggested stratification by geographic region, systematic ordering of schools within strata, random selection of schools and of one or two intact classes within selected schools. It also recommended replacement of non responding schools from a parallel sample. These systematic procedures were followed in every country. However, some variations in their actual implementation occurred. A detailed report of the sampling procedures for each population in each country is available upon request from the Center for Statistics. This report shows that most countries followed strict rules for scientific sampling of classrooms for the study.

For the United States response rates at two stages of sampling were below 70 percent resulting in a total response rate of about

40 percent of the intended sample. All other countries, with the exception of England, were able to achieve a response rate of greater than 90 percent. In England, the response rate was about 71 percent. The reliability of national estimates for the United States when compared with other countries should be treated with caution.

A comparison of the characteristics of students in the achieved sample for the United States with those of the total student population suggests that the achieved sample may have included students with higher social and economic backgrounds than the national distributions would warrant. Thus, cognitive achievement scores for the United States are more likely to be biased upward than downward in this study.

Table A1 Population A item appropriateness ratings, by major subtest and country

Country	Arithmetic %	Algebra %	Geometry %	Statistics %	Measurement %
Belgium (Fl.)	84.8	93.3	25.6	11.1	66.7
Belgium (Fr.)	84.8	93.3	25.6	11.1	66.7
Canada (B.C.)	95.7	86.7	43.6	94.4	95.8
Canada (Ont.)	84.8	56.7	76.9	77.8	79.2
England & Wales	97.8	93.3	87.2	83.3	100.0
Finland	84.8	76.7	69.2	83.3	91.3
France	82.6	96.7	53.8	27.8	62.5
Hong Kong	91.3	80.0	69.2	83.3	91.7
Hungary	93.5	90.0	84.6	83.3	100.0
Israel	91.3	76.7	74.4	0.0	100.0
Japan	93.5	93.3	87.2	100.0	100.0
Luxembourg	93.5	73.3	23.1	22.2	79.2
Netherlands	89.1	80.0	76.9	61.1	87.5
New Zealand	93.5	96.7	87.2	100.0	100.0
Scotland	100.0	90.0	89.7	100.0	100.0
Swaziland	87.0	83.3	64.1	88.9	87.5
Sweden	87.0	75.9	48.7	100.0	95.8
Thailand	93.5	73.3	71.8	77.8	100.0
U.S.A.	100.0	63.3	56.4	100.0	100.0

Table A2 Population A teacher 'opportunity-to-learn' scores, by major subtest and country

Country	Arithmetic Mean %	Arithmetic S.E.	Algebra Mean %	Algebra S.E.	Geometry Mean %	Geometry S.E.	Statistics Mean %	Statistics S.E.	Measurement Mean %	Measurement S.E.	Median N
Belgium (Fl.)	75.6	1.4	70.9	2.6	31.3	1.5	38.4	2.3	83.1	1.4	138
Canada (B.C.)	86.9	1.2	84.6	1.6	50.6	2.6	52.0	3.4	77.6	2.7	78
Canada (Ont.)	87.0	1.0	71.0	2.0	50.3	2.3	61.5	2.5	85.4	1.6	159
England & Wales	79.1	0.9	63.1	1.6	53.8	1.4	64.0	1.8	81.9	1.1	379
Finland	75.1	0.7	68.5	1.2	38.8	1.5	50.5	1.6	68.7	1.3	199
France	86.1	0.5	86.1	0.9	43.3	1.1	50.7	1.8	92.0	0.5	333
Hungary	91.1	1.2	90.9	0.9	85.9	0.8	85.5	1.6	97.3	1.1	63
Israel	66.6	-	79.6	-	41.0	-	47.2	-	59.2	-	136
Japan	84.9	0.3	83.4	0.5	51.3	1.1	75.4	1.0	94.7	0.5	209
Luxembourg	79.4	0.9	52.8	2.1	35.0	1.2	32.3	1.2	81.7	1.0	84
Netherlands	81.3	0.9	73.1	1.5	66.3	1.3	31.7	2.1	82.5	1.2	224
New Zealand	67.4	1.4	62.7	1.7	59.3	1.4	59.9	2.6	70.4	2.1	169
Nigeria	78.7	-	72.5	-	64.5	-	63.7	-	71.0	-	30
Swaziland	84.9	-	87.3	-	79.7	-	82.8	-	92.3	-	24
Sweden	67.5	1.3	49.8	1.9	35.3	1.4	47.4	2.2	67.4	1.6	177
Thailand	86.1	1.0	83.0	1.1	57.9	2.5	57.1	3.4	86.3	1.4	90
U.S.A.	84.3	1.0	68.9	2.0	44.2	2.1	72.2	1.9	75.2	1.9	269

The final column contains the median number of teachers responding to the 'opportunity-to-learn' question on one or more post-test items, taken over the five test forms. This number is substantially the same as the number of teacher responses upon which the subtest scores in this table are based, with three exceptions; Hungary, Israel and Swaziland. In these three countries, and more particularly in Swaziland, response rates varied widely from item to item, and due caution should be exercised in interpreting results based upon these figures. In each case, the assumption has been made that the replies from non-responding teachers would have been distributed across categories in the same way as those of respondents. Because of this possible lack of validity, or small sample size, no standard errors have been given for Israel, Nigeria or Swaziland.

able A3 Population A student 'opportunity-to-learn' scores, by major subtest and country

try	Arithmetic		Algebra		Geometry		Statistics		Measurement		Median
	Mean %	S.E.	Mean %	S.E.	Mean %	S.E.	Mean %	S.E.	Mean %	S.E.	N
da (B.C.)	83.7	0.7	74.6	1.1	62.8	1.3	71.1	1.1	80.8	1.0	91
ce	78.3	0.4	79.0	0.6	59.8	0.6	61.5	0.6	81.5	0.5	193
ary	92.0	0.6	89.3	0.7	87.4	0.6	84.3	1.0	94.0	0.5	70
el	86.1	1.0	82.8	1.4	72.3	1.5	78.5	1.5	83.4	1.2	99
n	88.0	0.2	90.2	0.3	81.1	0.3	90.1	0.3	93.3	0.2	210
nbourg	72.3	1.0	52.9	2.1	47.3	1.2	44.8	1.9	77.7	0.9	113
Zealand	75.8	0.7	66.8	1.0	73.7	0.7	72.1	0.8	79.4	0.7	104
ria	81.2	1.0	75.8	1.3	68.8	1.4	70.1	1.5	75.5	1.3	48
land	82.6	-	78.3	-	77.7	-	80.8	-	83.5	-	-
iland	77.9	1.5	73.7	1.5	75.0	1.4	71.2	2.0	81.4	1.4	25
land	86.8	0.4	83.4	0.5	77.8	0.6	75.5	0.8	89.8	0.4	100

student 'opportunity-to-learn' scores were aggregated to school/class level, in order to calculate
weighted national estimates given in this table. The final column contains the median number of
ols/classes from which data were available in each country, taken over the five test forms. No entry
iven for Scotland, since intact classes were not sampled in that country.

e A4 Population A teacher estimates of student achievement, by major subtest and country

ry	Arithmetic Mean %	Arithmetic S.E.	Algebra Mean %	Algebra S.E.	Geometry Mean %	Geometry S.E.	Statistics Mean %	Statistics S.E.	Measurement Mean %	Measurement S.E.	N
um (Fl.)	47.6	1.1	50.6	1.3	50.3	1.3	58.9	1.2	58.0	1.1	264
a (B.C.)	53.7	1.9	52.3	2.3	55.0	2.2	54.2	1.8	53.8	2.2	87
a (Ont.)	56.0	1.8	55.9	1.7	57.2	2.1	56.7	1.4	53.7	1.8	107
nd & Wales	46.1	0.6	37.2	0.5	34.9	0.6	44.6	0.5	49.7	0.5	416
nd	45.4	1.3	43.9	1.1	28.6	1.0	44.1	1.1	49.4	1.3	204
e	52.8	1.5	55.7	1.9	52.2	1.9	59.6	1.5	58.2	1.3	185
ry	59.6	2.0	54.4	2.0	53.9	2.4	57.2	2.0	70.5	1.5	70
l	53.3	1.4	41.8	0.9	19.8	1.0	27.4	1.1	43.1	1.2	153
	58.2	1.5	59.0	1.4	44.5	1.6	59.5	1.1	72.2	1.0	212
bourg	49.4	1.6	33.3	1.1	20.6	1.5	30.9	1.4	58.2	1.4	107
rlands	56.9	0.8	52.3	1.1	48.6	1.2	47.6	0.9	63.8	0.9	235
ealand	39.5	1.7	46.1	1.8	42.4	1.8	42.0	1.3	46.3	1.6	97
ia	50.2	1.6	46.1	2.5	39.1	1.8	40.6	1.4	45.2	1.6	45
land	43.5	3.2	39.4	2.4	39.0	3.8	42.2	3.0	46.9	2.4	25
n	34.3	1.6	29.3	1.4	30.0	1.4	41.8	1.1	37.4	1.6	186
and	48.6	1.6	48.1	1.4	46.0	1.9	47.1	1.6	46.0	1.9	99
.	47.0	1.5	46.8	1.4	48.8	1.6	46.8	1.0	46.3	1.3	151

ble A5 Population A post-test scores, by major subtest and country

ntry	Arithmetic Mean %	Arithmetic S.E.	Algebra Mean %	Algebra S.E.	Geometry Mean %	Geometry S.E.	Statistics Mean %	Statistics S.E.	Measurement Mean %	Measurement S.E.	Total N
gium (Fl.)	58.0	1.4	52.9	1.7	42.5	1.1	58.2	1.5	58.2	1.3	3073
gium (Fr.)	57.0	1.8	49.1	2.0	42.8	1.5	52.0	1.7	56.8	1.5	2025
ada (B.C.)	58.0	1.3	47.9	1.4	42.3	1.2	61.3	1.3	51.9	1.3	2168
ada (Ont.)	54.5	0.9	42.0	0.8	43.2	0.8	57.0	0.9	50.8	0.9	4666
land & Wales	48.2	1.3	40.1	1.3	44.8	1.2	60.2	1.1	48.6	1.2	2612
land	45.5	1.0	43.6	0.9	43.2	0.8	57.6	1.0	51.3	0.9	4382
nce	57.7	0.5	55.0	0.8	38.0	0.5	57.4	0.6	59.5	0.4	8317
g Kong	55.1	1.5	43.2	1.2	42.5	1.0	55.9	1.4	52.6	1.4	5495
gary	56.8	1.5	50.4	1.6	53.4	1.4	60.4	1.3	62.1	1.2	1754
ael	49.9	1.5	44.0	1.6	35.9	1.3	51.9	1.5	46.4	1.3	3524
an	60.3	0.4	60.3	0.5	57.6	0.4	70.9	0.4	68.6	0.4	8091
embourg	45.4	1.3	31.2	1.7	25.3	0.8	37.3	1.3	50.1	1.1	2038
herlands	59.3	1.1	51.3	1.2	52.0	1.0	65.9	0.9	61.9	1.0	5418
Zealand	45.6	1.2	39.4	1.1	44.8	1.0	57.3	1.1	45.1	1.1	5176
eria	40.8	1.1	32.4	0.7	26.2	0.7	37.0	1.0	30.7	0.9	1414
tland	50.2	0.5	42.9	0.7	45.5	0.6	59.3	0.5	48.4	0.7	1320
ziland	32.3	1.4	25.1	1.5	31.1	1.3	36.0	1.7	35.2	1.3	817
den	40.6	0.9	32.3	0.8	39.4	0.8	56.3	1.1	48.7	1.0	3451
iland	43.1	1.3	37.7	1.0	39.3	0.9	45.3	1.0	48.3	1.1	3824
.A.	51.4	1.2	42.1	1.2	37.8	0.9	57.7	1.1	40.8	0.9	6648

final column contains the total numbers of students in each country attempting the core form of test. In Swaziland and Sweden each student took, in addition, two of the four rotated forms B, C or D; scores on the items contained in these forms are thus available from approximately -half this number of students. In all the other countries listed, each student attempted only one ated form, and item scores are thus available from approximately one-quarter of the Total N given. items in each subtest were distributed across the five forms of the test.

Table A6 **National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Belgium (Fl.)**

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	85	76	-	48	58
Algebra	30	93	71	-	51	53
Geometry	39	26	31	-	50	43
Statistics	18	11	38	-	59	58
Measurement	24	67	83	-	58	58
Total	157	61	61	-	52	53

NOTE : The following key applies to tables A6 to A22

APPR Item appropriateness index
TOTL Teacher 'opportunity-to-learn' index
SOTL Student 'opportunity-to-learn' index
TESA Teacher estimate of student achievement
POST Post-test score

See pp. 2-6 for definitions

Table A7 **National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Canada (B.C.)**

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	96	87	84	54	58
Algebra	30	87	85	75	52	48
Geometry	39	44	51	63	55	42
Statistics	18	94	52	71	54	61
Measurement	24	96	78	81	54	52
Total	157	81	72	75	54	52

Table A8 **National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Canada (Ont.)**

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	85	87	-	56	55
Algebra	30	57	71	-	56	42
Geometry	39	77	50	-	57	43
Statistics	18	78	62	-	57	57
Measurement	24	79	85	-	54	51
Total	157	76	72	-	56	49

Table A9 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : England and Wales

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	98	79	-	46	48
Algebra	30	93	63	-	37	40
Geometry	39	87	54	-	35	45
Statistics	18	83	64	-	45	60
Measurement	24	100	82	-	50	49
Total	157	93	68	-	42	47

Table A10 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Finland

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	85	75	-	45	46
Algebra	30	77	69	-	44	44
Geometry	39	69	39	-	29	43
Statistics	18	83	51	-	44	58
Measurement	24	91	69	-	49	51
Total	157	80	61	-	41	47

Table A11 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : France

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	83	86	78	53	58
Algebra	30	97	86	79	56	55
Geometry	39	54	43	60	52	38
Statistics	18	28	51	62	60	57
Measurement	24	63	92	81	58	60
Total	157	69	72	72	55	53

Table A12 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Hungary

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	94	91	92	60	57
Algebra	30	90	91	89	54	50
Geometry	39	85	86	87	54	53
Statistics	18	83	85	84	57	60
Measurement	24	100	97	94	71	62
Total	157	91	90	90	59	56

Table A13 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Israel

	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	91	66	86	53	50
Algebra	30	77	80	83	42	44
Geometry	39	74	41	72	20	36
Statistics	18	0	47	79	27	52
Measurement	24	100	59	83	43	46
Total	157	75	59	81	38	45

Table A14 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Japan

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	94	85	88	58	60
Algebra	30	93	83	90	59	60
Geometry	39	87	51	81	45	58
Statistics	18	100	75	90	60	71
Measurement	24	100	95	93	72	69
Total	157	94	77	88	57	62

Table A15 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Luxembourg

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	94	79	72	49	45
Algebra	30	73	53	53	33	31
Geometry	39	23	35	47	21	25
Statistics	18	22	32	45	31	37
Measurement	24	79	82	78	58	50
Total	157	62	58	60	38	37

Table A16 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Netherlands

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	89	81	-	57	59
Algebra	30	80	73	-	52	51
Geometry	39	77	66	-	49	52
Statistics	18	61	32	-	48	66
Measurement	24	88	83	-	64	62
Total	157	81	70	-	54	57

Table A17 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : New Zealand

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	94	67	76	40	46
Algebra	30	97	63	67	46	39
Geometry	39	87	59	74	42	45
Statistics	18	100	60	72	42	57
Measurement	24	100	70	79	46	45
Total	157	94	64	74	[illegible]	46

Table A18 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Nigeria

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	–	79	81	50	41
Algebra	30	–	73	76	46	32
Geometry	39	–	64	69	39	26
Statistics	18	–	64	70	41	37
Measurement	24	–	71	76	45	31
Total	157	–	71	75	45	34

Table A19 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Scotland

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	100	-	83	-	50
Algebra	30	90	-	78	-	43
Geometry	39	90	-	78	-	46
Statistics	18	100	-	81	-	59
Measurement	24	100	-	83	-	48
Total	157	96	-	81	-	48

Table A20 National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Swaziland

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	87	85	78	44	32
Algebra	30	83	87	74	39	25
Geometry	39	64	80	75	39	31
Statistics	18	89	83	71	42	36
Measurement	24	88	92	81	47	35
Total	157	81	85	76	42	31

Table A21 **National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : Thailand**

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	94	86	87	49	43
Algebra	30	73	83	83	48	38
Geometry	39	72	58	78	46	39
Statistics	18	78	57	76	47	45
Measurement	24	100	86	90	46	48
Total	157	84	75	83	47	42

Table A22 **National mean scores on selected variables related to the intended, implemented and attained curriculums, by topic subtest : U.S.A.**

Subtest	Number of items	INTENDED CURRICULUM	IMPLEMENTED CURRICULUM			ATTAINED CURRICULUM
		APPR %	TOTL %	SOTL %	TESA %	POST %
Arithmetic	46	100	84	-	47	51
Algebra	30	63	69	-	47	42
Geometry	39	56	44	-	49	38
Statistics	18	100	72	-	47	58
Measurement	24	100	75	-	46	41
Total	157	82	68	-	47	45

Table A23 **Differences* between student and teacher indices of implemented content coverage for Population A, by item and country**

ITEM LONG	ITEM XSEC	SUBT	COUNTRY CODE CBC	FRA	HUN	ISR	JAP	LUX	NZE	NIG	SWA	THA
X7	x13	GE	45					36			34	
X10	d17	ME	30			32			33			
X19	x32	GE	33			34			43			66
X21	c25	GE		32		42	63					
X33	x26	AR							37			
X36	x33	AR		-31								
X37	a11	AR		-31							-32	
X38	a16	AR		-32								
X39	x38	GE										37
X40	d29	GE	42	75		58	80	42	54			40
A2	x5	GE	32			36	32					
A6	x6	GE		30		37	68		36			
A8	b20	GE									38	
A10	a13	GE					46				-31	
A12	b18	ST					35					
A15	d1	GE	48			57		37	35	32		
A16	d26	AR	-34									
A17	b8	AR				45	48	30				
A19	a24	ST							36			
A22	x21	ST				35						
A24	a17	AR			35		30		33			
A27	x9	GE					37					
A30	x34	AR							46			39
A32	c31	AR							34			
A33	b29	AL			46				[illegible]		35	
A35	x1	AL	-34	-39					[illegible]			
B3	c14	ST				30						
B4	c10	ST				36						
B5	a2	AL					35				-30	36
B6	b15	GE				37						32
B8	a34	GE				33	38					
B10	d15	AR									-32	
B11	a6	GE									35	
B13	a4	GE				31	34					
B14	b14	GE				37	45		38			
B15	d22	AR					30					
B17	b30	ME								33		
B19	d34	ST	30									
B25	b3	GE			42		46					
B28	x24	ME		-41								
C1	x2	GE		68	50		58		60	30	62	47
C2	x30	ME							-30			
C6	d11	ST	56	42		49	86	39	41		33	48
C7	a19	ST	40			48						
C9	c7	GE	31	69		64	70	43	45			30
C10	d33	ST							35			
C16	c15	AL	-34							-31	-34	

	ITEM						COUNTRY CODE					
LONG	XSEC	SUBT	CBC	FRA	HUN	ISR	JAP	LUX	NZE	NIG	SWA	THA
C18	a20	AR							-34			
C19	d16	AR				33						
C20	d18	ME				30						
C21	c22	AR							42			
C22	d3	AR		-31								
C23	x28	GE					77					
C27	b19	ST	31									
C28	b9	AR				32						
C29	x36	GE		33		31	57	37				
C30	x27	ST										47
C32	d32	GE	40	57		56	43	34				38
C33	b22	AR			34				32			
D3	b12	AR							37			
D5	b34	AL									-56	
D8	b31	ST					34					
D11	d23	ST						31				
D12	x31	AR	36				47	34				
D13	a27	AR				34						
D15	x19	GE					44					32
D16	b25	GE					51	36				
D18	d27	GE				34						
D21	x15	ST		30				31				
D22	d24	GE				53	42					
D26	c34	AL					59					
D29	a18	GE					45					
D33	x35	AR						-37				

*These differences have been calculated by subtracting the (unweighted) national teacher 'opportunity-to-learn' mean (TOTL) from the corresponding national student 'opportunity-to-learn' mean (SOTL), for each item; only items where the absolute value of this difference reached 30 percentage points in at least one country have been included.

KEY:
- LONG Longitudinal test item number (X signifies Core Test)
- XSEC Cross-sectional test item number (x signifies Core Test)
- SUBT Subtest: AR (Arithmetic), AL (Algebra), GE (Geometry), ST (Statistics), ME (Measurement)

- CBC British Columbia
- FRA France
- HUN Hungary
- ISR Israel
- JAP Japan
- LUX Luxembourg
- NZE New Zealand
- NIG Nigeria
- SWA Swaziland
- THA Thailand