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AUTHOR	Bernard, Y. F.
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ABSTRACT

This research is aimed at diagnosing the problem-solving skill of elementary school children and subsequently improving their competence in early mathematics. To reach this goal, a computer assisted instruction (CAI) program on open sentences, incorporating knowledge about students' problem solving strategies and misconceptions, was constructed. Two studies are reported, the first to gain more insight into the process of problem solving and the second to evaluate the functioning of the CAI program in the classroom. In the first study, 339 pupils in grades 2 and 3 were given a test, followed by interviews with 16 high performers and 16 low performers. Their answer patterns are discussed, and models of knowledge, the diagnostic program, and the remedial program are described. In the second study, the CAI program was evaluated with second-grade pupils in five schools. It was concluded that children can learn fundamental concepts relatively quickly with a computer program; one-third profited significantly. (MNS)

DEVELOPMENT OF A CAI-PROGRAM WITHIN THE FIELD OF EARLY MATHEMATICS

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Y.F. Barnard & J.A.C. Sandberg

University of Utrecht
Department of Educational Research

Y.F.Barnard
University of Utrecht
Department of Educational Research
Heidelberglaan 2
3584 CS Utrecht
tel: 31-30-534926

DEVELOPMENT OF A CAI-PROGRAM WITHIN THE FIELD OF EARLY MATHEMATICS

ABSTRACT

This paper presents research on the development of a CAI-program within the field of early mathematics. One of the problems children are presented with in the first grades of primary schools are socalled open sentences (3 + * = 7). These problems are often difficult for children because it is not immediately clear what operation brings about the solution. Research has been conducted aimed at clarifying the problem solving strategies children use in the domain of open sentences. Based on the analysis of problem solving strategies, computer models have been constructed describing knowledge and misconceptions underlying the solving strategies. Furthermore, we tried to develop a CAI-program that is able to diagnose the strategies being used and to remedy lacking knowledge and misconceptions. This program has been introduced in second grades of five elementary schools and has been used by children for some six weeks. The results indicate that one third of the children profit significantly from working with the program. These results are further discussed.

INTRODUCTION

One of the major aims of formal education is the development of problem solving skills. Psychological research has shown that processes underlying problem solving are often meaningful and therefore interpretable. Errors that students make during problem solving are not due to carelessness, but often just result from lack of general or domain-specific knowledge combined with existing misconceptions about the problem-domain.

It is practically impossible for teachers in most schools to attend to the individual needs of each student. Due to a lack of diagnostic instruments and research within this area it is difficult to diagnose a student's specific level of skill. Evaluation of students' learning processes takes place mostly on the performance-level (is the answer right or wrong) and not on the competence-level (in which way was the answer constructed). In general remedial teaching is aimed at getting correct answers instead of enhancing deeper knowledge and correcting misconceptions. Misconceptions can be very persistent, they are not easily detected because often students get correct answers, despite the fact that they do have misconceptions about the domain at hand.

The individual approach needed for diagnosing and remedial teaching at a deeper level of students' problem solving skill may be facilitated by using Computer Assisted Instruction (CAI). But the traditional CAI-programs share the disadvantages of classroom teaching: emphasis on observable behavior without considering cognitive processes. If CAI is to play a meaningful part in instruction based on a deeper understanding of students' difficulties, CAI-programs must possess the following characteristics:

a) sufficient knowledge of the subject matter and a large base of domain tasks and problems
b) knowledge of possible behavior of students, problem solving strategies, underlying knowledge and misconceptions. Based on the interaction with the student the system has to build a model of the student
c) knowledge about teaching strategies and tactics
d) knowledge about interaction-processes between a computer-program and students
e) metacognitive knowledge

It may be clear that only intelligent, integrated tutoring systems of sufficient complexity can encompass all these characteristics (Sleeman & Brown, 1982, Wielinga, 1985).

The research described in this paper is aimed at diagnosing the problem solving skill of elementary school children and subsequently improving subjects' competence within the domain of early mathematics. In order to reach this goal, a CAI-program has been constructed. Although a truly integrated, intelligent tutoring system was not feasible within the frame-work of the project, we tried to develop a program that has knowledge about students' problem solving-strategies and misconceptions.

This paper presents two studies, the first aimed at gaining more insight in the process of problem solving within the domain of early mathematics and the second aimed at the evaluation of the functioning of the CAI-program within the classroom.

RESEARCH

DOMAIN

The domain chosen is that of elementary mathematics, in particular 'open sentences' (e.g. 6 - * = 2). There are twelve different types of open sentences to be distinguished, depending on the placement of the equalizing-sign, the identity of the unknown, and the operation-sign.

	type		type
1)	3 + 6 = *	7)	* = 2 + 5
2)	9 - 2 = *	8)	* = 8 - 3
3)	3 + * = 5	9)	9 = * + 4
4)	5 - * = 2	10a)	5 = * - 2
5)	* + 6 = 8	10b)	2 = * - 5
6a)	* - 3 = 5	11)	7 = 3 + *
6b)	* - 5 = 3	12)	4 = 6 - *

table 1 : examples of all possible types of open sentences

Though the area seems to be quite simple in itself, difficulties may arise because from the surface features it is not immediately clear what operation needs to be performed. The literature on this subject shows that children of seven to nine years old have difficulties with these problems (Lindvall & Ibarra, 1980, De-Corte & Verschaffel, 1979, 1980).

STUDY 1: THE SOLVING OF OPEN SENTENCES

The research started with administering an open sentences test to 339 second and third grade children from six different elementary schools, each school using a different method in teaching mathematics. The majority of these methods does not teach open sentences directly. These types of problems are mostly used in practising number facts. None of the methods put a large emphasis on open sentences.

The test for the second graders (seven to eight years old) used numbers up to twelve, the test for the third graders (eight to nine years old) used numbers up to twenty. The reason for this diversion was that we were not interested in measuring knowledge of numberfacts or technical ability but in the way children solved the problems. Therefore, we wanted to avoid errors due to the use of incorrect numberfacts for the second graders. For the third graders, on the other hand, by using somewhat larger numbers, we hoped to avoid the children were having all the answers correct by knowing the problems by heart.

First the results of the test show that third graders do not perform any better than second graders. This clearly shows that practice does not pay. Secondly, errors were, in general, not due to lack of technical ability to add and subtract, most children performed well on the canonical types 1 and 2 (a + b = * and a - b = *).

grade	mean score	relative score (corrected for number of items)
2 (n: 183)	42.42 (i: 60)	0.77
3 (n: 156)	81.85 (i: 108)	0.76

i = number of items
n = number of pupils

table 2 : results of the open sentences test

INDIVIDUAL INTERVIEWS

General testing results like the ones above only tell us something about the products of a cognitive process, not about the process itself. To get more insight in this process, 16 high performers and 16 low performers have been selected. As high performers we selected children that had a high score (80% or more correct) on mostly all different problem-types. As low performers children who had difficulties with at least three problem-types were selected. Children in this category were required to solve the canonical types 1 and 2 correctly. The

children were selected in a way that garantueed that all types of problems were represented among the 'problematic' ones.
Each child has been interviewed individually and these interviews were audio-taped. The children were presented with a series of 15 problems, in which all types were represented. First the child was asked to read the problem aloud. Then he or she was asked to solve the problem and to explain the solution. After that the child was asked to produce a story problem fitting the structure of the open sentence and finally to reproduce the open sentence from memory. The protocols of the interviews were transcribed and subsequently analyzed.

PROTOCOL-ANALYSIS

Looking at the answer-patterns on the different types of problems, it seemed at first that the children behaved very inconsistently, answering one problem correctly and the next incorrectly. But taking a closer look at the children's way of talking about the solution, it became clear that their answer-pattern could be interpreted in terms of very systematic behaviour. For instance, some children explained their solutions in terms of the operation-sign: 'Always add when it says +'. Other children always read a problem spontaneously backwards when the problem 'started' with the unknown (e.g. $* - 6 = 4$) and subsequently solved it backwards (answer 2 in this case). Each child was found to use one overall strategy. Six of such overall strategies were identified.

	example	a: * - 2 = 7		
		b: 6 = * + 2		
		c: 7 - * = 4		
strategy	answer	a	b	c
1. Add all. When the problem structure is not conform the canonical structure add the 2 givens from the problem statement		9	8	11
2. Interpret the operation-sign as a direct instruction to perform the stated operation on the 2 given numbers		?	8	3
3. Read and solve the problem from the right to the left when the equalizing sign is placed on the left		9	4	3
4. Read and solve the problem from the right to the left when the problem first states the unknown		?	4	3
5. Bridge the gap between the two given numbers. When the structure is not canonical, then the difference between the largest and the smallest number in the problem is determined		5	4	3
6. Expert		9	8	3

table 3 : strategies

note: ? = impasse

Although most problems are 'solvable' with one of the general strategies, sometimes a child gets in trouble. For example, using strategy 4 : * - 4 = 6, reading backwards the problem says 6 equals 4 minus something. But this is an impossible problem for children. So there is an impasse, which has to be overcome in order to come up with an answer. One may wonder why children do not often just say a problem is impossible. It seems that our education provides children at an early age with the notion of solvability: there is an answer and that answer has to be a number (Gelman & Gallistel, 1978). Repair theory (VanLehn, 1983) is exactly based on this notion of solvability, for it predicts the use of 'repairs' to overcome impasses, so there will be an answer to a problem anyhow.

In the interviews impasses were revealed:

- when children hesitated a long time before giving an answer
- when children gave several different answers and were not able to decide on one of them
- when occasionally a child would say that he or she could not work the problem or that the problem was impossible in itself

In two cases the children in this study could find themselves in an impasse: when they thought it neccessary to subtract a larger number from a smaller one or when the problem was actually an impossible one by the way they read it. In both cases the impasse arises because children have no notion of negative numbers.

Children have different means to overcome impasses at their disposal, for instance they try to subtract all the same and give zero as the answer. The following repairs have been found to be used by the children using strategies 2, 3, and 4:

- Add the two given numbers
- Subtract the smaller from the larger number
- Subtract till you cannot any further, answer 0
- Say the problem is impossible

MODELS OF KNOWLEDGE

How to explain the general strategies children have been found to be using. An explanation in terms of underlying knowledge and misconceptions seems to be adequate . By supposing such an implicit knowledge-base one is able to predict the answer-patterns that have been found. The literature on problem solving makes the assumption of such a knowledge-base plausible, for reasons of cognitive parsimony: integrated knowledge, however incorrect this may be, instead of isolated procedures for one and every type of problems. In other domains, such as thermo-dynamics and vertical subtraction, the behaviour of students is interpreted in terms of correct knowledge, incorrect knowledge and lacking knowledge as well. Impasses and 'repairs' to solve them are also general phenomena (Brown & VanLehn, 1982, VanLehn, 1983, Jansweyer, et al. 1986).

In order to acquire further insight in the knowledge structure underlying children's solutions, simulation-models have been constructed, implemented in PROLOG. At the one hand, these simulation-models form a theoretical support, for they do predict the different answer patterns correctly, on the other hand actually constructing and implementing the models gives a more accurate view on what (incorrect) knowledge structures there must be present.

The models are stated in terms of declarative and procedural knowledge. Declarative knowledge is defined as knowledge about the principles and facts of a domain. Procedural knowledge concerns rules that prescibe how declarative knowledge has to be used when a problem in the domain is to be solved. With both these kinds of knowledge a representation of the problem can be constructed that forms the basis of the final solution.

The models give explicit knowledge rules, this does not mean that pupils can formulate them explicitly, the rules are supposed to be an abstraction of deeper knowledge structures.

Table 5 illustrates the models.

Model	Declarative knowledge	Procedural knowledge
1.	canonical form numberfacts operators	Find the operation and the two numbers. If the form is canonical, perform the operation. If the form is not canonical, add the two numbers.

2.	numberfacts operators repairs	Find the operation and the two numbers, perform the operation. If a greater number must be subtracted from a smaller: impasse. If impasse, select repair.
3.	normal form is: a +/- b = c numberfacts operators repairs	If the form is normal, follow model 6a or 6b. If the form is not normal (c = a +/- b), solve the problem from right to left. If a greater number must be subtracted from a smaller: impasse. If impasse, select repair.
4.	normal form does not begin with unknown numberfacts operators repairs	If the form is normal, follow model 6a or 6b. If the form is not normal, solve the problem from right to left. If a greater number must be subtracted from a smaller: impasse. If impasse, select repair.
5.	canonical form placement of the = sign is not essential part-whole relation numberfacts operators	If the form is canonical, find the operation and the two numbers, perform the operation. If the form is not normal, determine the greatest and the smallest number, determine the difference.
6a. (leads to correct answers on all types)	canonical form placement of the = sign is not essential part-whole relation numberfacts operators	If the form is canonical, find the operation and the two numbers, perform the operation. If the form is not normal, determine which of the elements is the whole and which are the parts. If the value of the whole is unknown, add the the parts. If one of the parts is unknown, subtract the other part from the whole.
6b. (expert strategy)	equality part-whole relation numberfacts operators	Determine which of the elements is the whole and which are the parts. If the value of the whole is unknown, add the the parts. If one of the parts is unknown, subtract the other part from the whole.

table 5 : illustration of the simulation-models

The description of the models shows that some of the strategies predominantly take superficial characteristics of problems into account, for example whether the problem says + or - (model 2). Other strategies are based on more fundamental knowledge, such as for instance knowledge of the part-whole relationship. Only model 6b is entirely based on knowledge of the mathematical principles underlying open sentences. That an expert does not pay much attention to superficial problem features was demonstrated in the interviews, where some of the experts failed to reproduce a problem literally. They did keep the relation between the different numbers in tact but not the syntactic structure, for instance reproducing * - 7 = 3 when the original problem was 3 = * - 7.

THE DIAGNOSTIC PROGRAM

Because each strategy gives rise to distinct answer-patterns, a diagnostic program could be constructed that is able to identify those patterns. In general, the diagnostic program is based on the principle: present no more problems than strictly nessecary for diagnosing a child's strategy. Therefore the program follows the following rules:

- First present those types with a maximum discriminatory power
- First identify those strategies that are based on a minimum of understanding or a maximum of understanding of the domain (strategy 1 and 6)
- Take into account the repairs that children might be using; different answer-patterns may lead to one and the same diagnosis due to the use of repairs.

The child sits behind the terminal and is shown a problem, for example 4 + * = 10. The child has to type his or her answer. The answer is placed on the dot in the problem-statement and the problem disappears to make room for the next one. The child is given opportunity to indicate that a problem cannot be solved (one of the repairs). One of the keys functions as 'cannot-key'. All in all, the child is presented with between twenty and forty open sentences, depending on the path through the program. The program registers all the answers. The diagnostic program does not offer any feedback, in order to avoid interference with the diagnostic process.

The first open sentences presented are of the canonical type. These types are used to check if a child can solve these problems correctly and can be considered to be 'technically competent'. For we are not interested in wrong answers due to counting errors or the use of incorrect number-facts. When the child makes too many mistakes on these problems the program is not suitable for this particular child. With the other types of problems wrong answers are not accounted as such, when the answer is one number below or above the expected answer. For instance, presenting 5 + * = 9, the answer 15 or 13 is viewed as resulting from a mistake in performing the adding procedure and is treated as if the answer is 14.

THE REMEDIAL PROGRAM

The remedial program closely follows the diagnostic categories, i.e. the six strategies. The remedial program works from the principle that the level of dialogue must be adjusted as much as possible to the pupil's specific level of knowledge and skill. The program is aimed at teaching knowledge about the central concepts of the domain, such as the part-whole relationship and equality. Thus, the program is aimed at enhancing deeper knowledge of the domain and not at teaching tricks for solving each seperate type of problem correctly.

principle	example from subprogram for strategy 2
1. Start remedying issues typical of the strategy being used	% 3 + * = 7 > 10
2. Try to evoke a conflict within the pupil by presenting counter examples (Socratic dialogue)	% 3 + 10 = * > 13 % 3 + 10 = 7 3 + 10 = 13 which is correct?
3. When no conflict arises, present the pupil with a concrete situation, in order to demonstrate the impossibility of some or other solution	% ____ _###_ ____ | 3 | |###| | 7 | 0---0 0-#-0 0---0 How many people got in at the busstop?
4. Never give the solution directly	% 3 + * = 7 >
5. Do not instruct pupils in algorithms connected to specific problems.	
6. Return to the diagnostic program as soon as the strategy specific issues have been settled satisfactorily	

% = what appears on the screen
> = where the child gives an answer

table 6 : principles of the remedial program, with examples

In this example the central issue is the part-whole relation. For children, using strategy 2, do not realize that open sentences are an instance of the part-whole relation. In order to induce a cognitive conflict the child is presented with a counter example. The canonical form, which will be solved correctly, is meant to form a contrast with the incorrectly solved problem. The part-whole relation is never explicitly stated but has to be inferred. The program can also make use of more concrete ways of presenting a problem by showing animations. In the example, described in table 6, an animation of a moving bus with passengers, who get on and off, the bus is shown. Other animations are formed by balances and boxes with marbles. Several problem types, problematic in relation to the strategy being used, (for strategy 2 this means type 3,5, and 6, see table 1) are handled. When the child has shown to be able to solve at least 4 problems of each type correctly in succession the child is considered to have reached sufficient mastery of the central issue at hand. Next the child is send to the diagnostic program anew. For mastery of a new concept does not imply that a state of expertise has been reached, so a new diagnostic cycle is needed to find out whether the child needs more remedial teaching or not.

The principles 4 and 5, stated in table 6, are important in avoiding the teaching of tricks instead of mathematical concepts. In the next section the actual introduction of the CAI-program in the classroom will be considered.

STUDY 2: INTRODUCTION OF THE PROGRAM IN THE CLASSROOM

The CAI-program has been introduced in the second grade (children aged seven or eight years old) of five elementary schools. In order to evaluate the effect of the CAI-program an open sentences test was administered to the entire class before the children started to work with the program and simularly afterwards. Several weeks later the same test was administered again in order to be able to detect any lasting effect. The second grades of two other schools functioned as control groups. Their pupils only attented the regular school-program, without extra training in solving open sentences. The program operated for some six weeks in the schools.

In order to be able to use the program in elementary schools, the program had to be implemented in Simon's BASIC on a Commodore-64, for this is the only micro-computer commonly used in schools. This implementation demanded the inventivity of the programmer. Most of the children had never worked with a micro-computer before. So before starting to work with the program the children had to be made familiar with the computer. To this purpose the children were presented with an instructional program that illustrated the functioning of the keyboard and provided ample opportunity to practise the keys.

Each child worked on his or her own with the CAI-program. After half an hour at most the program was interrupted, when at least a integrated part of the program had been finished. Data on what the pupil had been doing in the program up to the moment of interruption were stored, so it was possible to continue where he or she had left off at some other time. When the pupil was diagnosed as an expert, he or she definitely left the program.

RESULTS

The actual implementation of the program within the classroom hold some problems of its own. Teachers were not inclined to let children work too often and for too long with the program. Rather than letting a child finish the program succesfully they took care that most children took their turn. So most children did not get longer than one hour and a half of remedial teaching. In the final analysis four groups of subjects had to be distinguished:

1. The control-group as defined above, supplemented with those children that were meant to work with the program but did not do so.
2. The children who did work with the program but did not reach the final diagnosis of expert.
3. The children who did work with the program and were eventually diagnosed as experts.
4. The children who were initially diagnosed by the program to be experts and therefore did not take part in the remedial division of the program.

Table 7 presents a summary of the results.

	1 control-group n = 76	2 non-experts n = 90	3 experts after remedy n = 41	4 experts n = 31
pretest	38.26	34.31	34.07	42.23
posttest 1	38.67	37.53	41.02	43.61
posttest 2	41.71	39.78	41.29	42.23

n = number of cases that actually took both the pretest and the two posttests

number of items is 48 per test

table 7: mean scores on the different tests

significant difference ($p < 0.05$)

on pretest between: group 3 and 4

on posttest 1 (corrected for scores on pretest) between: group 1 and 3; group 2 and 3

Group 3 performed significantly better on the first posttest than group 1. This is not the result of being experts anyhow, for they do perform significantly different from group 4 on the pretest. And furthermore, they do not perform any better on the pretest than the children who failed to become an expert. For these children the program seems to have worked well. The children of group 2 did not profit from the program.

The results on the second posttest show no significant differences between the groups. But this is not due to a relapse of the children who became experts working with the program but to a marked progression within the other groups. Especially the control group has a very high score on all tests. It is not clear why the control group has a higher score in general than the other groups, but about thirty children from this group did not take all three tests and were therefore excluded from the sample. This might have resulted in an artificially high mean score for this group.

Approximately one third of the children that have been working with the program is finally diagnosed as expert. This is a fair result in itself for it means that children really may learn something from the program. On the other hand two third of the group who has been working with the program does not reach the expert-stage. Partially this may be due to 'flaws' in the computer program. On the basis of our findings and the commentary provided by the teachers the program will be corrected. On the other hand, a lot of children only worked for too small a period of time with the program. Table 8 presents the time spend on working with the remedial program, not including the time it took to diagnose the strategies being used. Diagnosing did not take very much time.

	mean time (in minutes)	std dev	median	range
group 2	75.86	43.04	69.83	8 - 208
group 3	47.78	46.26	30.25	7 - 228

table 8: time worked with the remedial program

The period of time it took children from group 3 to become experts is fairly short in our opinion, and even the longest period is still quite acceptable (228 minutes). It is plausible that when children from group 2 had gotten a chance to work with the program for some extra time, there would have been more experts in the end.

In conclusion it is clear that in principle children are able to learn fundamental, in this case mathematical, concepts in a relatively fast way with the aid of a computer program.

DISCUSSION

In the result section, the general outcomes of the evaluation of the CAI-program have been briefly discussed. There is a lot more to be said about the specific results, such as:

- the progression of the eventual experts on the different types of problems
- the different paths the children took through the program
- the relation between different diagnoses in succession

Because this paper focuses on the development of a CAI-program such matters as mentioned above will be addressed somewhere else.

The remedial program presented here has been based on one particular teaching strategy, namely 'Socratic Dialogue'. This method has advantages and drawbacks.

Stevens and Collins (1982) point out that children working with this method do eventually gain a lot of fundamental understanding, but this often takes a very long time. As our results show, the eventual experts did not need very much time on the average to become experts. On the other hand, the group of children that failed to reach the expert stage perhaps needed much more time than provided.

The remedial program tries to evoke a cognitive conflict within the pupil. However, a child will possibly need to possess a lot of metacognitive knowledge if he or she is to become aware of such a conflict. And it is not evident that children aged 7 or 8 years old do possess sufficient metacognitive knowledge (Brown, & Campione, 1981).

Furthermore, it might be that different groups of children profit more from one remedial strategy than another. Especially 'weak' performers are supposed to profit more from a remedial strategy that provides a very well structured learning environment. Therefore, research aimed at comparising different remedial strategies will be started in the near future.

Although the research on the CAI-program needs to be extended, the first results, presented here, indicate that this line of research could be a promising approach. The approach could be characterized as follows:

- detection and analysis of problem solving strategies in the domain chosen
- construction of models of knowledge and misconceptions underlying these strategies
- construction of a program that is able to diagnose these strategies
- construction of a remedial teaching program that is aimed at clarifying the central issues relevant in relation to the different problem solving strategies as described by the models

This approach is clearly much more limited than the one presented in the introduction section for the development of intelligent, integrated tutoring systems. For the present development of truly intelligent programs that could be introduced at large within Dutch schools is not feasible. On the one hand research on this kind of programs is still in an experimental stage and on the other the development of such programs is enormously time consuming. The approach proposed by the present authors is meant to contribute to the development of CAI-programs that are based on an understanding of problem solving processes in a domain and are ready to be used in the classroom in the near future.

CAI-programs, that are able to diagnose the knowledge and misconceptions of children and that are able to remedy on a conceptual level, can contribute to the individualization of education. Such programs would be powerful tools for teachers, for they can hardly be expected to be able themselves to assess each pupil's actual knowledge of a domain and his or her problem solving strategies, let alone taking these into account all the time.

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