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ABSTRACT

The need for mathematics teachers to incorporate instruction in problem-solving skills into the curriculum is discussed. Results of a study of problem-solving instruction in the classrooms of nine seventh grade teachers are briefly noted. There were significant differences between the teachers in how they planned, organized, and delivered lessons, but there was little difference between regular lessons and problem-solving lessons. Moreover, students showed little improvement in problem solving. A complete description of the study is included. How the teachers taught is described in some detail. Implications are discussed in terms of organizational style, problem-solving content, and other teaching methods. Several figures and tables are supplied. (MNS)

A LOOK AT MATH TEACHERS AND PROBLEM SOLVING

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Problem Solving in Intermediate Mathematics Study

Secondary Science and Mathematics Improvement Program

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A LOOK AT MATH TEACHERS AND PROBLEM SOLVING

Math teachers are currently under considerable pressure to incorporate instruction in problem-solving skills into their curriculum. This focus on problem solving has been brought about by several national studies showing that our students are fairly proficient in computational skills, but that they have difficulty applying those skills in problem situations. As a result, problem solving has become one of the most popular topics in mathematics education.

Despite this new emphasis, it is unclear whether asking teachers to stress problem solving will affect the instruction students receive. Much of the research done on problem solving has looked at students and how they solve problems rather than at teachers and how they might promote problem-solving skills. Consequently, there are few precedents for math teachers who wish to implement problem-solving instruction.

As part of a larger program on improving secondary science and math instruction, the Problem Solving in Intermediate Mathematics Study (PSIM) recently examined problem-solving instruction in the classrooms of nine public school mathematics teachers. The purpose of the study was to take a close look at how teachers teach problem-solving skills. In order to do this, the study examined:

(1) teachers' views of problem solving and their plans for problem-solving instruction;

(2) teachers' organization and delivery of both regular and problem-solving lessons; and

(3) students' learning of problem-solving skills.

The study discovered that there were significant differences between the teachers in how they planned, organized, and delivered lessons. There was little change, however, between the teachers' instruction during regular lessons and their instruction during problem-solving lessons. Furthermore, students who participated in the problem-solving lessons showed little improvement in their problem-solving abilities.

There would appear to be a strong need for more understanding of the nature of problem-solving instruction if students' skills in this area are to be improved. The development of such knowledge, however, will have to take into account the realities and constraints of the classroom and the existing curriculum. If problem-solving instruction is to be successfully implemented, it must be responsive to the needs of teachers as well as students. It is hoped that the following description of teachers teaching problem solving will contribute to the long-range goal of improving students' problem-solving skills.

The PSIM study obtained information on the teachers' problem-solving instruction through a series of interviews and classroom observations. Teachers were asked to talk about their views on teaching mathematics, their perceptions of problem solving, their preparation for lessons, and the organization and operation of their classrooms. In addition, the teachers were observed during five days of regular math instruction and during a special six-day unit on problem solving.

About the Study

The Teachers and Their Setting. The nine participating teachers taught seventh-grade mathematics at four public schools. All of the participating teachers had secondary certification. Four had master's degrees in administration. One teacher had a math major in college; six teachers had math minors; two had no specialized math training.

The class sizes ranged from 16 to 34 students. The district in which the schools were located served 32,000 students from a medium-sized Western city. The community was made up primarily of low-middle- to middle-class residents, including a small proportion of minorities.

What the Teachers Were Asked to Do. The teachers attended a three-hour workshop and were given copies of a teacher workbook prepared as part of the PSIM study. The workbook contained an overview of the study, a three-page discussion of problem solving, a five-page discussion of four specific problem-solving skills on which the teachers were asked to focus, a list of possible activities, and 240 word problems. The activities and word problems were similar to problem-solving materials found in standard textbooks.

The teachers were given one month to plan a six-day unit on problem solving. They were encouraged to organize their lessons in any way they wanted. The teachers were free to add other skills to their lessons and to use whatever materials they wished.

Although the study offered problem-solving materials to the teachers, it did not provide suggestions of problem-solving teaching methods. This was, in part, because knowledge about such methods is limited. A much larger reason, however, was that the researchers were most interested in learning the teachers' own approaches to problem-solving instruction. In this way, the study would reveal what in fact happened when teachers were asked to provide more problem-solving instruction.

The Problem-Solving Skills. The teachers were asked to concentrate on these four skills:

(1) identifying information necessary to solve a problem that is not in the problem statement;

(2) separating relevant from irrelevant information;

(3) identifying intermediate steps in a multiple-step problem; and

(4) representing the information in a problem through a table or diagram.

A sample of questions relating to these skills is contained in Figure 1.

How the Teachers Taught

The teachers' problem-solving instruction was examined in regard to their planning of lessons, their organization and delivery of lessons, and the effect of the instruction on students' learning. Selected comments from the teachers on each of these areas are contained in Table 1.

Teacher Planning. Although the time spent in planning varied quite a bit from teacher to teacher (the range was from 1 to 12 hours), the planning process of the teachers seemed fairly routine. The teachers themselves reported few differences in their planning for teaching problem solving as compared with their usual planning. Planning activity centered mainly on objectives and problem sets.

A major difference between teachers was whether their lesson plans were based on problem sets or on the problem-solving skills. Most of the teachers focused first on the objectives and then gathered problem sets to fit the objectives. Three teachers gathered problem sets and then worked to fit the problem-solving skills into the problem sets. Teachers who concentrated first on the problem sets provided less direct teaching of the four skills.

During planning for the problem-solving unit, six of the nine teachers added problem-solving "key words" to the list of problem-solving topics to be covered. Key words were words or terms (such as "in all") that would aid students in recognizing the mathematical operations required to solve a word problem. As noted earlier, the teachers were encouraged to add content or materials other than those provided by the study. The teachers felt that knowledge of key words was a useful problem-solving skill for the students to learn.

There was a significant exception to the otherwise routine planning for the problem-solving unit. One teacher believed that giving students a set of word problems in which each problem required the same mathematical operation was the same as giving them computational problems. Other than mixing types of problems, however, this teacher was at a loss for alternative instructional methods. This frustration resulted in the inability of this teacher to prepare a plan. The teacher ultimately decided to teach the problem-solving unit "intuitively" and "wing it."

Teacher Organization. The study looked at the ways in which the teachers organized the problem-solving lessons in terms of both time

and content. An examination of time allocations revealed how the teachers taught, while an examination of content showed what they taught.

Time. Records from classroom observations allowed each teacher's instruction to be divided into blocks or segments of time with a clear focus or goal. The major instructional time segments were (1) development of new ideas or concepts, (2) practice of new material, or (3) review, either of assignments to be checked or of previously introduced concepts. When information on these time segments was compiled and compared, the results showed dramatic differences in organization between teachers. There was very little change, however, between the teachers' organization for regular math instruction and their organization for problem-solving instruction. Each teacher seemed to have a characteristic and consistent pattern of time use that was unaffected by a change from regular math to problem-solving content.

Figure 2 illustrates the time spent on development, practice, and review by each of the teachers during regular math instruction. Figure 3 provides the same information for problem-solving instruction.

Although the amount of time allocated to development, practice, or review varied, all of the teachers placed a strong emphasis on a traditional approach of recitation and seatwork. This was true for both regular and problem-solving instruction. Students were given information by the teacher in a whole-class setting; this recitation period would include checking assignments and modeling new problems. Then students would individually practice the new problems in a seatwork setting.

Content. When the problem-solving lessons of the nine teachers were studied, it was clear that the teachers had emphasized the problem-solving skills requested of them. Figure 4 shows the breakdown of the overall class time by content.

Almost half of the class time (49%) was devoted specifically to the four designated skills, while 12 percent went toward key words, the problem-solving topic that had been added by the teachers. About 20 percent of the time was used for general practice of word problems that did not emphasize particular problem-solving skills. The rest of the time was spent on other kinds of math problems or on non-math activities.

There were class-to-class differences in time allocation to content. Three of the teachers, for example, spent about 65 percent of the class time on the four skills and about 16 percent of the time on key words, while another teacher spent more than 60 percent of the time on practicing word problems and 25 percent on the problem-solving skills.

Student Learning. In order to measure the impact of the problem-solving lessons on students' acquisition of the skills, students of the nine teachers were given a test on the four problem-solving skills

before and after the series of lessons. They were given a third test as well that measured their ability to solve word problems. Results from these three tests were compared against the results of the same tests given to a control group of three classes who received no instruction in problem solving. Table 2 shows the pretest and posttest results from all 12 clas es.

Overall, the students who received instruction in problem-solving skills showed about the same improvement as the control-group students who did not receive problem-solving instruction. The improvement in the mean class average of the group receiving instruction was only 0.6 of a point higher than that of the control group. This difference is not statistically significant.

Implications of the Study

The information obtained through this study indicates a need for better understanding of problem-solving instruction. The participating teachers were interested, experienced professionals who spent time planning their lessons and who organized and delivered math instruction using traditional teaching methods. An examination of the content of instruction during the special unit showed that much class time was devoted to problem-solving topics. Yet, the test results suggest that the problem-solving instruction had little effect on the level of the students' skills.

Some of the studies done on problem solving have led researchers to believe that this particular area of mathematics may be better taught using less traditional instructional methods. John Goodlad, commenting on a study of his own, said: "Many [mathematics teachers] want their students to be logical thinkers, to learn how to attack problems, and to think for themselves. Why, then, did so few mathematics teachers in our sample appear to get much beyond a relatively rote kind of teaching and textbook dependency not likely to develop powers of critical reasoning?"[1]

The teachers in the PSIM study placed much emphasis on showing and telling students how to do problems. This is considered an effective teaching method for developing computational skills, for which automatic responses and quick solutions are helpful. In problem solving, however, the emphasis is not speed but flexibility and adaptability.

Several explanations can be offered for why teachers may prefer traditional instructional methods when teaching problem solving. These have to do with the organizational styles of teachers, the need for better understanding of problem-solving content, and a general lack of knowledge concerning methods of teaching problem solving that are useful to teachers.

Organizational Style

The descriptions of regular and problem-solving instruction by the nine teachers in this study suggested that teachers tend to display individual organizational styles that they use consistently regardless of the topic of instruction. Organizational styles represent blueprints that aid teachers in the day-to-day operation of the classroom and in the maintenance of an orderly flow of events during instruction.

Classroom management and lesson planning are two areas in which the development of organizational styles is particularly useful to teachers. Only after teachers master their management tasks can they begin to consider other matters more closely related to instruction; teachers who are responsible for 25 to 35 students at a time must meet rigorous organizational challenges before they can begin to concentrate on student learning. An organizational style also simplifies lesson planning because, once teachers have established a routine for operating their classrooms, they need only to plan for events that change on a daily basis--mainly topics and assignments.

The development of organizational styles may be helpful, even necessary, for teachers. Problems may arise, however, if teachers never vary their styles. Rigid approaches could result in a sameness of instruction based on ease rather than on the requirements of the subject matter. The concept of an organizational style helps explain why the problem-solving instruction took on the same structure as the regular math instruction; the teachers of the study appeared to view problem solving as a new topic similar to other basic skills, and consequently, their instruction did not vary much.

Problem-Solving Content

The teachers in the study taught problem-solving skills in basically the same way they taught other math content, indicating that they did not perceive problem-solving content as significantly different. In addition, during planning and instruction, some teachers focused much more on solving problem sets than on developing problem-solving skills.

One problem-solving skill that the teachers did emphasize was what they called "key words." Many of the teachers felt that poor reading skills contributed to students' difficulty in problem solving, so the addition of key words may have been one way in which they tried to confront that problem. On the other hand, the use of key words represents a fairly concrete approach that may not be the best way to help students learn to analyze and solve problems.

It should be pointed out that one of the teachers did feel problem solving instruction needed to be different than regular math instruction, but this teacher was unable to come up with any alternative methods for teaching problem-solving content. Devising

means for teaching students how to be analytical or intuitive was a major obstacle.

Despite the availability of information and materials related to problem solving, it seems that teachers need more support and better preparation for teaching problem-solving content. Increased understanding of problem-solving content, together with a heightened awareness that instructional methods can be changed to fit specific content, should perhaps be a focus of teacher training. Surely teachers are more inclined to use a variety of teaching methods when they feel comfortable with the subject matter.

Other Teaching Methods

As far as we could determine, the teachers in the study exhibited few teaching methods that were particular to problem-solving instruction. Without access to or knowledge of alternative methods for teaching problem-solving, the teachers appeared to use their normal methods.

In organizing problem-solving instruction, teachers may need at least initially to devote more time to planning in order to develop different instructional methods. Because the goals of learning problem solving are so different from those of learning other math skills, students may also require more class time to learn problem solving. Setting aside such time is problematic, of course, given current demands on both teachers and students.

Recognizing that planning new teaching methods every time the content changed would require more time than most teachers have available, it seems that what is necessary is the judicious use of well-planned, integrated lessons. Careful planning could allow, for example, opportunities to learn problem-solving skills in conjunction with other kinds of content.

Given the demands on teachers and the obvious constraints of managing classrooms, it is very important that any new instructional methods developed for problem solving be sensitive to the forces that guide teachers' daily planning and delivery of instruction. Studies have shown, for example, that when given alternative or innovative programs, teachers are likely to redefine the intended goals and practices into traditional terms in order to fit the realities of their classrooms. The developers of new teaching methods, therefore, must take the teachers perspective into account.

Summary

The PSIM study looked at the instruction of nine seventh-grade math teachers to see how they taught students problem-solving skills. The teachers' instruction for problem solving turned out to be very similar to their instruction for regular math skills. Tests showed

that the problem-solving instruction had very little impact on the students' problem-solving abilities.

This report suggests several explanations for these findings. First, teachers seem to have characteristic organizational styles that appear to be used consistently, regardless of the content of the lessons. Second, teachers may not perceive problem solving as a content area that requires different instructional strategies. And third, teachers may need further support and training in order to integrate alternative methods for teaching problem solving into their existing programs.

The strikingly different goal of problem-solving instruction--that is, the development of flexibility rather than automaticity in thinking--places greater pedagogical demands on teachers and requires alternative teaching methods. Because problem-solving content is more difficult for students than regular math content, teachers are under greater pressure to present problem-solving material in a way that produces the fewest difficulties for students and the fewest management problems for themselves. Although teachers, as usual, are in the front line of this conflict, the PSIM study suggested that they may benefit from some assistance. If the strong yet competing demands being made on teachers are to be met, teachers and researchers must work together to find more effective and efficient ways to help students learn problem-solving skills.

NOTE

1. Goodlad, John. (1984). A place called school. New York: McGraw-Hill Book Company, pp. 209-210.

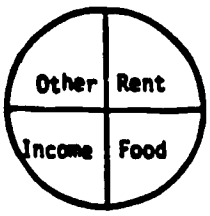
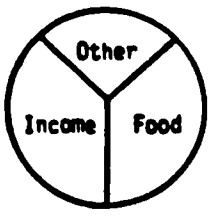
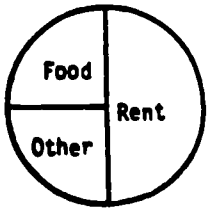
PROBLEM-SOLVING SKILLS	SAMPLE ITEMS
1. Identify information necessary to solve a problem that is not in the problem statement.	The members of a club decided to share the cost of a party. Each member brought $5. Food for the party cost $25. What fact could you use to figure out how much money the members have left after they pay for the food? a. There are 7 members in the club. b. Decorations for the party cost $10. c. The members invited 25 people to the party.
2. Separate relevant numerical information from irrelevant numerical information in the problem statement.	Circle the numbers you would use to solve this problem: A theater had 600 seats in 20 rows. Workers installed 7 new rows of seats with 35 seats in each row. How many new seats did the workers install?
3. Establish sub-goals; identify intermediate steps in a multiple-step problem.	In the morning, 8 inches of rain fell at a rate of 2 inches per hour. In the afternoon, it rained for 4 hours at a slower rate of 1 inch per hour. How many hours did the rainstorm last? What question do you need to answer before you can solve the problem? a. How many hours did it rain in the morning? b. How much rain fell in the afternoon? c. What was the average rate of rainfall?
4. Represent the information in a problem statement through a table or diagram.	A college student earns $500 per month. Out of this he pays $125 for food and twice that amount for rent. How much money does he have left for other expenses? Which picture shows the information in the problem? A.  B.  C. 

Figure 1

Four problem-solving skills with sample problems

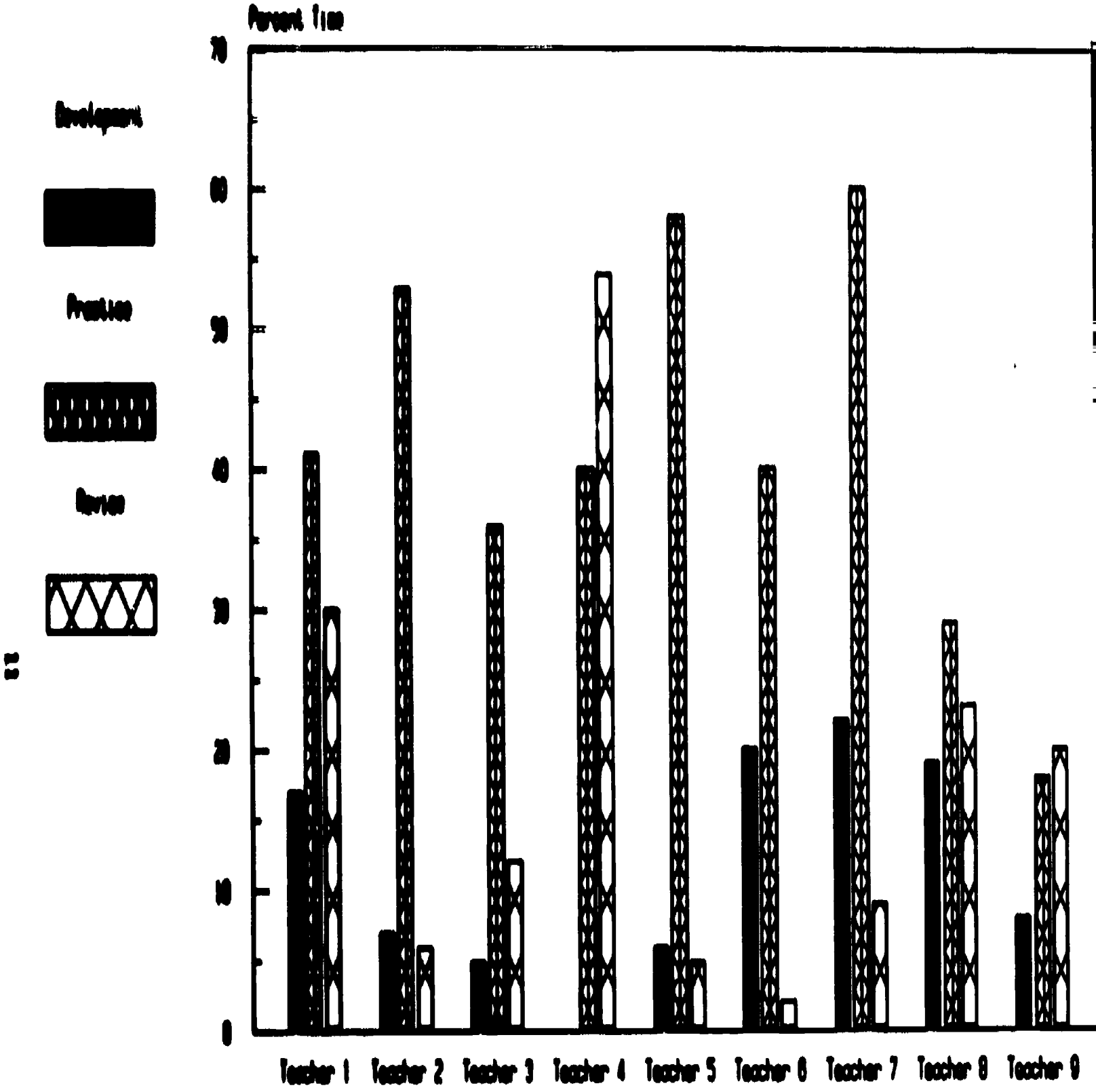


Figure 2. Differences between teachers in percent time spent in development, practice, and review segments during regular instruction

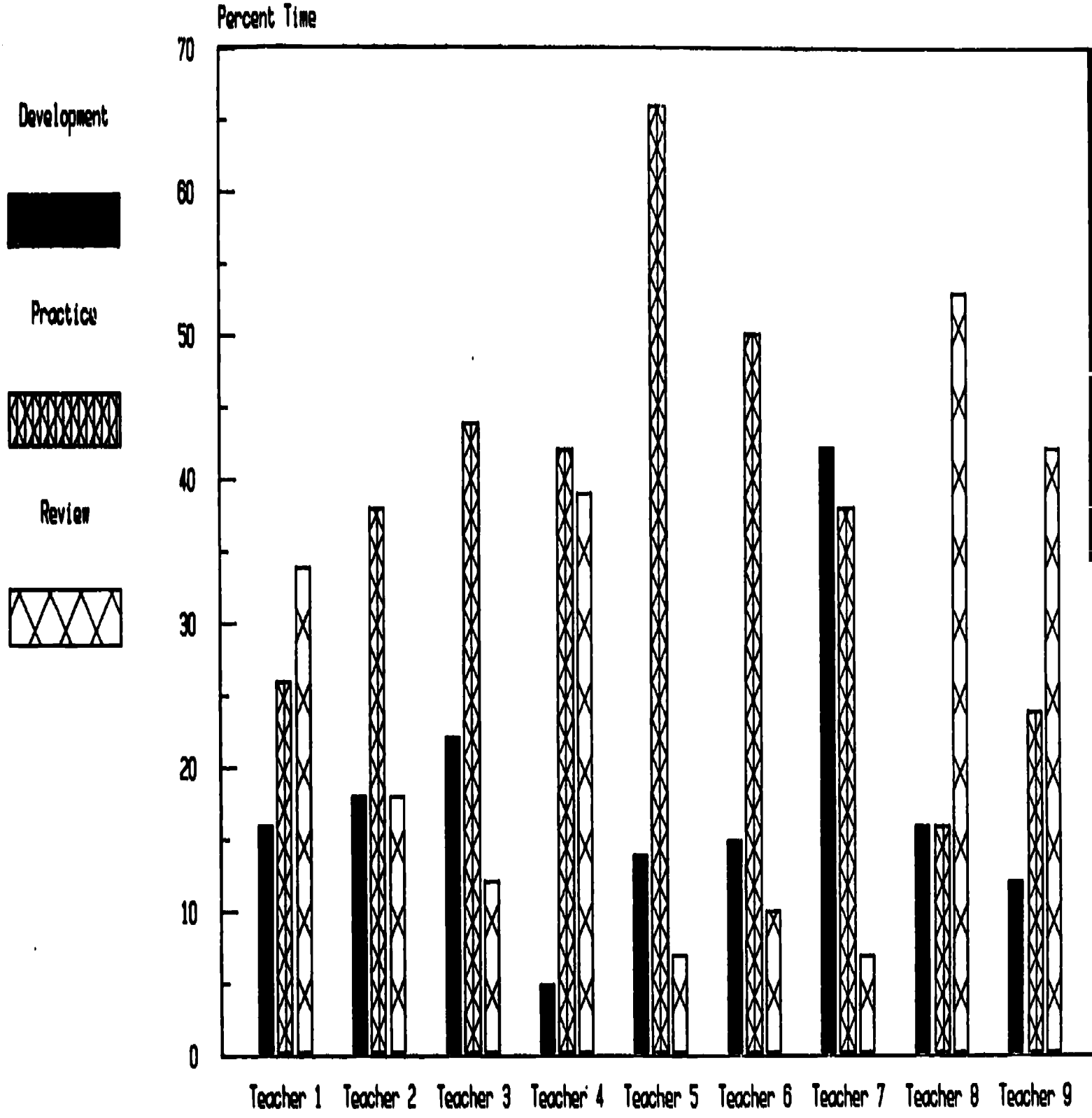


Figure 3. Differences between teachers in percent time spent in development, practice, and review segments during problem-solving instruction

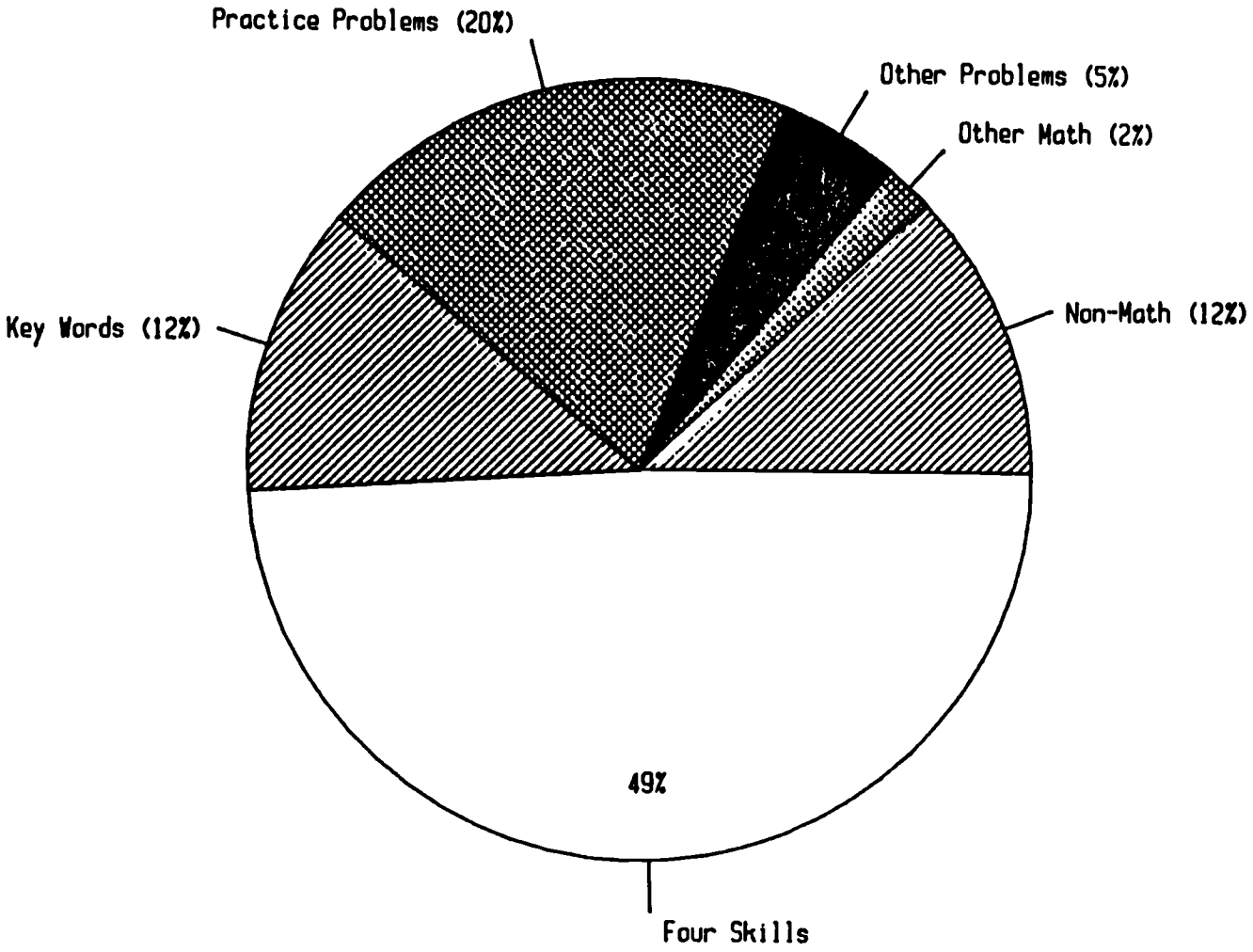


Figure 4. Topics of instruction for total class time

Table 1

Teachers' Comments on Problem Solving

Teaching Problem Solving
Teacher: "I'm still not convinced that I really know how to present [problem solving]. . . . You can talk and you can give them ideas, but then they don't have the patience after they miss a problem or enough intellectual curiosity . . . to say, 'Oh, I see what I did wrong'. . . . They just want to get the answer."
Teacher: "No, I don't think I'm going to change the way that I teach. . . . I'm just going to give them these word problems from day to day and see how they can handle them."
Teacher: "It's easier discipline-wise when you have a diverse group to sit everybody down at the same table or at their desk and do the same page at the same time in the same way. It's easy to check, it's quiet, it's orderly."
Teacher: "I would like to be able to do more of the latter, to have them understand how to do it alone, but I probably--because it's easier and quicker--show them how it's done."
Teacher: "A lot of times it just helps if you do a problem on the board. . . . Unless you do it on the board and show it to them step by step they won't get it. And then once they've seen it done, they can usually do it on their own."
Teacher: "I would like to think that if I show them three or four times and then turn them loose, they have good examples to follow. What I try to focus on is trying to have them look at examples and be smart enough to go back and put in the new numbers."
Planning for Problem Solving
Teacher: "The first thing I did was I got everything together that I could find. You know, you have to have the materials to work with, so I got everything I could find that dealt with word problems."
Teacher: "I got all the materials together, then I realized that I had a lot of stuff. . . . I looked at your objectives and pulled all the materials that I had that related to those types of objectives. . . . So at that point I had to decide how many problems did I want to have. Was it going to be two pages or three pages?"
Teacher: "I just took it from the point of view that this is what we want to do. We want to cover these four things and we want to emphasize understanding different types of problems."
Teacher: "I have left the last two days kind of open. . . . If it's not going like I want to then this will give me a little bit of time in there to adjust and get some material."

Table 1 (continued)

Learning Problem-Solving Skills
Teacher: "I can look at [math problems] and say 'Okay, I know that that means to multiply,' but then how do you tell the kids to recognize that two weeks later on a test?"
Teacher: "When a kid doesn't understand it, and I know intuitively that you divide . . . I don't really know how to explain it to them or how I know that that's what you do. . . . If they don't understand it intuitively, then I don't know how to teach it to them."
Teacher: "I find middle school kids become very restless, and their attention span is somewhat short. So if you go into a whole lot of analytical things--I mean a good example is when you teach them to multiply fractions. You could spend three days discussing why a reciprocal. They're much happier if you just say, 'Turn it over and multiply'."

Table 2

Average Student Scores on Problem-Solving Tests

Class	No. of Students	Skill Pretest	Skill Posttest	Change in Skill	Test of Word Problems
Problem Solving					
1	30	26.1	28.8	2.7	14.2
2	28	25.9	27.3	1.4	13.1
3	28	28.8	30.8	2.0	14.6
4	24	29.7	31.2	1.5	15.0
5	19	23.5	26.0	2.5	11.4
6	22	32.0	33.2	1.2	15.0
7	23	22.8	24.3	1.5	12.4
8	21	23.7	25.5	1.8	11.7
9	15	20.9	24.7	3.8	11.9
Group Mean		25.9	28.0	2.1	13.2
Comparison					
10	19	26.2	28.5	2.3	12.3
11	23	21.2	23.4	2.2	13.0
12	29	25.6	26.3	0.7	12.1
Group Mean		24.3	25.8	1.5	12.3

Note. The maximum score is 40 for the test of problem-solving skills; it is 18 for the test of word problems.