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ABSTRACT

Most studies of children's solution processes on simple addition and subtraction word problems have used individual interviews or the analysis of error patterns on paper-and-pencil tests as the primary data-gathering techniques. The present paper reports an investigation in which the contribution of eye-movement data was explored for studying those aspects of the problem-solving process that are inaccessible with the methods mentioned above, especially the text-comprehension processes contributing to the construction of a problem representation, and the subject's decision-making processes in choosing a solution strategy. Eleven addition and subtraction word problems were administered individually to 9 high-ability and 11 low-ability first graders. For each problem, eye-movement data were collected while children read and solved the tasks; afterwards they were asked to explain verbally how they arrived at their answer. Besides the usual findings concerning problem difficulty, solution strategies, typical errors and solution times, an anlysis of the gaze durations and the sequences of fixations is presented. In addition to some interesting findings concerning solution processes, the study also showed that eye-movement registration can be used easily with young children, and is appropriate for collecting data on their cognitive processes. Five pages of references and seven data tables conclude the document. (Author/MNS)

EYE-MOVEMENT DATA AS ACCESS TO SOLUTION PROCESSES OF ELEMENTARY ADDITION AND SUBTRACTION PROBLEMS

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Abstract

Most empirical studies of children's solution processes on simple addition and subtraction word problems have used individual interviews or the analysis of error patterns on paper-and-pencil tests as the primary data-gathering techniques. The present paper reports an investigation in which the contribution of eye-movement data was explored for studying those aspects of the problem-solving process, that are inaccessible with the methods mentioned above, especially the text-comprehension processes contributing to the construction of a problem representation, and the subject's decision-making processes in choosing a solution strategy.

Eleven addition and subtraction word problems were administered individually to nine high-ability and eleven low-ability first graders. For each problem, eye-movement data were collected while children read and solved the tasks; afterwards they were asked to explain verbally how they arrived at their answer. Besides the usual findings concerning problem difficulty, solution strategies, typical errors, and solution times, two kinds of eye-movement results are presented, namely an analysis of the gaze durations for different parts or areas of the problem text (based on the total sample), and the sequences of fixations of those areas (based on the data of six children). In addition to some interesting findings concerning children's solution processes, the study also showed convincingly that eye-movement registration can be used easily with young children, and is very appropriate for collecting data on their cognitive processes.

1. Introduction

Elementary arithmetic word problems have already been submitted to a lot of research during the past decennia. This research was mainly focussed on the relationship between various task characteristics (e.g. the number of words in the problem, the mathematical structure of the problem, the presence of so-called key-words...) and subject variables (e.g. general reasoning ability, verbal and mathematical abilities) with problem-solving performance. However, a theoretical framework involving an integrated and detailed account of the distinct aspects of problem solution (problem difficulty, problem representation, solution strategies, typical errors...) for a variety of problem types was completely lacking (Verschaffel, 1984).

At the end of the seventies, the information-processing approach began to permeat research on word problem solving. A number of scholars agreed on the acute problems to be solved in this domain and on the main appropriate theoretical concepts and research methods to attack these problems. The aim was to build a detailed theory of the cognitive structures and internal processes underlying performance and of the development in these structures and processes over time (Romberg, 1982). From a methodological point of view, two main categories of investigations can be distinguished. Some researchers have collected empirical data concerning the level of difficulty of different types of word problems, the strategies children use to solve these poblems and the nature of their errors, using individual interviews as the main data-gathering technique (Carpenter & Moser, 1982; Lindvall & Ibarra, 1980). Other investigators were involved in building computer-implemented models constituting a well-founded and unified account of the internal processes and cognitive structures underlying children's performance on those verbal problems (Riley, Greeno & Heller, 1983; Briars & Larkin, 1984).

Since 1979 we have also been working on a research project in which an attempt is being made to acquire a better understanding of the development of young children's problem-solving skills and processes with respect to elementary arithmetic word problems (see e.g. De Corte & Verschaffel, 1985a and b, in press a, b and c; De Corte, Verschaffel & De Win, 1985). In our past investigations empirical data were collected mainly using individual interviews and the analysis of error patterns on paper-and-pencil tests.

Based on these investigations on the one hand, and the vast literature on (arithmetic) problem solving on the other, we have developed a hypothetical model of competent problem solving comprising four stages. First, a complex, goal-oriented text-processing activity occurs : starting from the verbal text, the pupil constructs a global, internal representation of the problem in terms of sets and set-relations. On the basis of that representation, the problem solver then selects an appropriate formal arithmetic operation or in informal counting strategy to find the unknown element in the problem representation. In the third stage, the selected action or operation is executed. Finally, verification actions are performed to check the correctness of the solution found in the preceding stage.

As stated above, the first stage of the solution process is perceived as a goal-oriented text-processing activity. The emerging problem representation is considered to be the result of a complex interaction of bottom-up and top-down analysis : that is, the processing of the verbal input as well as the activity of the subjects' cognitive schemata contribute to the construction of the problem representation. Two main categories of schemata are distinguished : (1) semantic schemata, representing the subjects' knowlege about increasing and decreasing, combining and comparing groups of objects (the Change, Combine and Compare schema respectively), and (2) the word problem schema, which involves knowledge of the structure of word problems, their role and intent in mathematics instruction, and the implicit rules and assumptions that need to be known when "playing the game of school word problems" (De Corte & Verschaffel, 1985a).

Our investigations have provided a rich set of specific findings concerning the appropriate as well as inappropriate information structures formed in representing word problems, and the variety and the development in the solution strategies used to solve the problem. However, until now, several other aspects of the problem-solving process, such as the text-comprehension variables and processes contributing to the construction of these information structures or the subject's decision-making processes while choosing a specific solution strategy, have received almost no attention neither in our and others' empirical work nor in the theoretical (computer) analyses (see also Carpenter, 1985; De Corte & Verschaffel, 1985b; Kintsch & Greeno, 1985). Undoubtedly, this is largely due to the above-mentioned research methods' inaccesibility to these aspects of the word problem-solving process (De Corte & Verschaffel, in press a and d).

Recently we have started to apply eye-movement registration as a new and complementary technique for generating and/or testing specific hypotheses concerning children's understanding and problem-solving processes on elementary arithmetic problems. Cumulating research evidence has shown that there exists a strong relationship between people's information-encoding and -manipulating processes in visual display situations on the one hand, and their eye movements on the other (Fisher, Monty & Senders, 1981; Just & Carpenter, 1980; Rayner, 1978). Therefore, one can hypothesize that eye-movement data will be helpful in unravelling certain aspects of arithmetic problem solving that are unaccessible with other techniques, such as paper-and-pencil tests and individual interviews.

In this contribution we present the design and some results of an exploratory eye-movement investigation undertaken during the last year. The goal of this study was twofold. First, we wanted to collect empirical data concerning the above-mentioned model of elementary arithmetic word problem solving, and, as such, to contribute to the construction of a theory on problem solving in general. Second, it was aimed to make a significant and original contribution to the methodology of research on children's arithmetic problem solving. Indeed, although eye-movement registration has a long history in experimental psychology in general and in cognitive research in particular - in 1859 Helmholtz published already on eye-movements -, we know of little or no research in which such data have been collected in research on young children's elementary arithmetic problem solving.

2. Method

In the present study a series of numerical and verbal problems were administered individually to a group of high-ability (H) and low-ability (L) first graders near the end of the school year. With respect to each problem we collected eye-movement data while the child read and solved the problem, together with retrospection data in response to the question how (s)he arrived at the answer. In this section we give a more detailed description of the subjects and the problems involved in this study, and of the procedures used to register and to analyze the eye movements.

2.1 Subjects

The subjects were 22 first graders from an elementary school in Leuven. It seemed impossible to differentiate them into a high-ability and a low-ability group on the basis of the scores on the teacher-made tests and the teacher's judgments; therefore the two subgroups were formed on the basis of their combined score on the numerical and verbal problems in the present study. Unfortunately, during the analysis of the eye-movements, data from two children, both belonging to the H-group, got lost. As a consequence, the H- and L-group consist of 9 and 11 children respectively.

The experiment took place in April. At that time, the children had received intensive training in writing and solving numerical addition and subtraction problems with numbers up to 20; they also had already been taught word problems involving these operations (see De Corte, Verschaffel, Janssens & Joillet (1985) for a description of the main characteristics of the mathematics program).

2.2 Materials

The materials used in this study consisted of 32 numerical and 11 verbal problems involving addition and subtraction with numbers up to 20. However, in the present paper we only deal with word problems (see Table 1).

Table 1

The word problems in Table 1 represent eight different types from the Riley et al. (1983) classification schema : Combine 1, Combine 2, Change 1, Change 3, Change 5, Compare 1, Compare 3 and Compare 5. For the Combine 2, the Change 3 and the Compare 3 problem, a variant was also inserted, in which the known sets were given in an inversed order. The hypothesis was that this factor, together with the semantic structure, would influence problem difficulty, especially in Change problems, in which inversing the order of presentation of the start and the change set results in an incongruence between the real temporal sequence of events, on the one hand, and the order in which they are described in the text, on the other (De Corte, Verschaffel & De Win, 1985; Nesher, 1982). Consequently, four addition and seven subtraction problems were included in this study. The number triples were : 4-7-11, 5-7-12, 3-8-11, 4-8-12, 5-8-13, 3-9-12, 4-9-13, 5-9-14. As it was practically impossible to randomize the order of the problems for each subject, three different sequences of the eleven word problems were administered.

Eye-movement data can be obtained in several ways. In the present study they were collected using a German apparatus, namely Debic 80. This system is based on the corneal reflection-pupil center principle, which can e summarized as follows (De Graef, Van Rensbergen & d'Ydewalle, 1985). The Debic 80 system luminates the eye with an infrared lightbeam; at the same time, the eye is observed by an infrared video camera (see Figure 1). With the help of this camera, the difference between the center of the pupil and the corneal reflection is registered. On the basis of this image, Debic 80 computes the vector between the center of the pupil and the corneal reflection. This vector corresponds to a certain angular rotation of the eye. Starting from this vector, the system is able to compute the point in the visual field the subject is looking at. This computation is done every 20 milliseconds and is represented in two different ways. First, the visual stimulus, together with the point the subject is looking at, are recorded on video; on the monitor the subject's point of regard is represented as the intersection of a vertical and a horizontal axis superimposed on the slide (see Figure 1). Second, the coo dinates of these subsequent intersections are stored on computer tape.

Figure 1

For each child the data collection took place separatedly for the numerical and the verbal problems. Half of the children were presented the numerical problems first; the others started with the verbal problems. Each session lasted 20-30 minutes. The child was seating in a chair with the interviewer standing by his side; the problems were presented on slides and projected on a screen (see Figure 2).

Figure 2

After the child had solved a problem, he was asked how he arrived at the answer. However, contrary to our past investigations in which verbal protocols were used as data (see e.g. De Corte & Verschaffel, in press a), no further probing questions were asked.

2.4 Analysis of the eye-movement data

The analysis of the computer data representing children's subsequent points of regard, was done using a program developed by De Graef et al. (1985). Before the program can be applied, one has to define the elements or the areas of the perceptual field one is interested in. Due to computer-power limitations, we could not construct separate areas for each word of the problem. Therefore, we made for each problem a grid consisting of five horizontal and six vertical zones. While the horizontal zones were always the same, the vertical ones differed for each problem taking into account the place of the numbers and particular words (such as "more", "altogether"). As an illustration, Figure 3 presents the grid for the Compare 5 problem. The coordinates of the angular points of the zones serve as input for the computer program.

Figure 3

Furthermore, for each solution process two time measures has to be introduced into the computer, namely the starting and end points, i.e. the exact moment of the presentation of the slide and of the subject's answer respectively. Once all these values are put into the computer, the data generated by the Debic 80 system can be transformed into a sequence of symbols referring to the distinct areas of the stimulus field, together with the number of measurements during which the subject's eye was in that area.

Of course, this enormous mass of data - there are fifty measurements per second and on the average a solution process takes 30 seconds ! - had to be further reduced before one can interprete them appropriately. In the present study this was done in two different ways. First, we computed for each solution process the total number of measurements and the percentage of the total solution time during which the subject was looking at each particular area; these gaze durations per zone were related to several task and subject variables using analysis of variance. Second, Debic's raw data were reduced to sequences of eye fixations on distinct parts of the problem text. The first analysis was automatized and is based on the eye-movement data of all twenty children; the second was done by hand, and therefore deals only with six subjects.

3. Results

In this section we discuss some main findings of both analyses of the eye-movement data. However, before we summarize the findings concerning the other aspects of performance collected during this study, namely problem difficulty, typical errors, and solution times.

3.1. Problem difficulty, typical errors, solution times

3.1.1 Problem difficulty

Table 2 shows the performances of the high-ability and the low-ability group on the eleven word problems. A child's answer was scored as correct when the correct answer was given, but, in this case, also when the solution differed only one or two units from the exact answer due to a so-called technical error (De Corte & Verschaffel, 1981, p. 766).

Table 2

Generally speaking, the results of the H-group are in accordance with those of prior research (Briars & Larkin, 1984; De Corte & Verschaffel, in press c; Nesher, Riley & Greeno, 1983; Morales, Shute & Pellegrino, 1984; Riley et al., 1983). The Change 1 and Combine 1 problems were the easiest : both were correclty solved by all children. The most difficult problems were the Change 5 and the Compare 5 problems, eliciting only one and two correct solutions respectively. All other problems were intermediate in difficulty level.

In the L-group, the Change 1 and the Combine 1 problems were also solved correctly by all children. All other problems elicited very little correct answers, except the two Compare 3 problems that were of intermediate difficulty; together with the easy Change 1 and Combine 1 problems, they constituted the four addition problems in our study; all other problems involve subtraction. Similar surprisingly "high" scores on Compare 3 problems by low achievers who had been exposed to mathematics instruction for some time, were found by Verschaffel (1984). There are several hypothetical explanations for this finding. The child may have interpreted the difficult Compare 3 problem in terms of a more familiar Change 1 or Combine 1 problem (see also Escarabajal, 1985). Or he may have applied the key-word strategy, in which his selection of the appropriate arithmetic operation is not based on a global analysis of the problem situation, but on the occurrence of an isolated key word in the problem text, i.e. the word "more" which is associated with addition (see also Nesher & Teubal, 1975). It is even possible that the child simply used the best known and most familiar arithmetic operation : adding the two given numers (see also Goodstein, Cawley, Gordon & Helfgott, 1971). In this respect, we mention that for the subtraction problems used in this study, each of these three inappropriate strategies necessary leads to an incorrect answer.

Besides the semantic structure, we also have examined the effect of another task variable on the difficulty level of the word problems, namely the order of the sentences in the problem, which was varied in the Combine 2, Change 3, and Compare 3 problem types (see Table 1). However, this variable did not seem to significantly influence problem difficulty. Similar results were obtained by Nesher (1982) and Boons (in preparation).

3.1.2 Typical errors

In view of a more detailed analysis of children's responses, their wrong answers were classified in one of the following categories :

(1) Wrong operation (WO), i.e. performing the wrong operation, such as adding the two given numbers in the problem instead of subtracting the smaller number from the larger one (or the inverse);

(2) given number error (GN), answering with one of the given numbers in the problem, either the smaller (SGN) or the larger (LGN) one;

(3) a miscellaneous category (MC), containing errors for which we had no ready explanation;

(4) no answer (NA).

Table 3 gives the distribution of the incorrect answers over these different categories for each problem separately.

Table 3

As said before, the Combine 1 and the Change 1 problem were correctly solved by all children. For the other two addition problems - the Compare 3 problem and its inversed variant - the most frequently occurring error was the larger given number (LGN); WO errors were almost completely lacking.

With respect to the subtraction problems, both Combine 2 problems were mostly answered with a WO error; only one LGN error was observed for each problem. The Change 3, the Compare 1 and the Compare 5 problems also elicited significantly more WO than GN errors, although the latter's frequency was somewhat higher than on the Combine 2 problems. Change 5 was the only problem type for which more GN than WO errors were found; however, a significant number of WO errors were elicited too.

These error data are certainly not in accordance with the predictions implemented in the Riley, Greeno & Heller (1983) and Briars & Larkin (1984) computer models, nor are they congruent with the results of other empirical investigations (Carpenter, Hiebert & Moser, 1981; De Corte & Verschaffel, 1985a; in press c; Lindvall & Tamburino, 1981). However, the written presentation of the problems, and the fact that no concrete materials were available to the child, may account for the present findings. Indeed, other studies of our Center suggest that these two context variables, together with group testing instead of individual interviewing, may lead to a drastic increase of the number of WO answers at the expense of other error categories such as LGN, SGN and NA (Boons, in preparation; De Corte, Verschaffel & De Win, 1985; Pauwels, in preparation).

The effect of the sequence of the sentences in the problem text on the kind of erroneous answers was also examined. For two problem types, namely Combine 2 and Compare 3, the error pattern was roughly the same on both variants. With respect to the Change 3 problem, there seemed to be an effect of that task variable : while the usual Change 3 problem elicited four LGN and no SGN errors, the GN errors on the inversed variant were equally devided between the LGN and the SGN. This can be explained as follows. An inappropriate representation of a Change 3 problem may lead erroneously to the identification of the second given number as the answer set (see also Fischer, 1979), but also of the second number in the real temporal order of the problem (Verschaffel, 1984). In a normal Change 3 problem both inappropriate strategies lead to a LGN; in its inversed Change 3 variant, however, they result in different error types, i.e. a SGN and a LGN error respectively.

3.1.3 Solution times

Table 4 gives an overview of the solution times of both the H- and the L-group for the eleven problems. These data were submitted to a series of analyses of variance. In each analysis ability (H- versus L-group) was used as an independent variable together with one of the following task variables : (1) semantic structure (Combine, Change and Compare problems); (2) problem difficulty (the Combine 1 and the Change 1 versus all other problem types); (3) operation (addition versus subtraction problems); (4) sentence order (normal versus inversed problems).

Table 4

The most remarkable finding was that the overall solution time of the H children was not significantly lower than in the L group. On the contrary, the children from the L group tended to answer even more quickly than the H children, although this difference was not significant.

With respect to the task variables, higher solution times were observed for the Compare than for the Change and the Combine problems; however, these differences were again not significant. On the other hand, significant differences were observed for two other task characteristics : the easy problems were solved more quickly than the difficult ones ($F(1, 18) = 24.77$, $p. < 0.01$), and addition problems were solved more quickly than subtraction problems ($F(1, 18) = 16.35$, $p. < 0.01$). Finally, normal problems elicited shorter solution times than inversed ones, but, again, the differences did not reached significance.

Although they were not significant, some interesting interactions between the above-mentioned task and subject variables were found too. With respect to the first three task variables, namely semantic structure, problem difficulty and operation, the differences between the distinct problem types were greater in the H- than for the L-group; with respect to sentence order the observed difference in solution time between the regular and the inversed problems was totally due to the L-group.

3.2 Total gaze durations per area

For all word problem solving processes we computed the total gaze duration in each area of the slide. Figure 4 presents two examples of the output of this first analysis. For each zone the total number of measurements (each 20 milliseconds) as well as the percentage of the total solution time, are mentioned. Moreover, totals are given for the three problem sentences (respectively A, B and C) and for the "empty" areas above, beneath, to the right and to the left of the problem (R);

the letter M **refers** to the amount of missing data, i.e. the number of measurement moments at which for some reason the point of regard was not or could not be registrered.

Figure 4

Here too an analysis of variance was performed to examine the effects and the interactions of the task and subject variables mentioned above on the partition of the total solution time over the main categories, namely A B, C, R and M.

A first remarkable finding is the relatively strong focus on the numbers in the problem. Frequently about 25% of the total solution time was spent in the two (small) number areas. Sometimes the child's eye was on the numbers for more than half of the total solution time. This may - at least partially - account for the fact that the average gaze durations for the first and the second sentence were always remarkably higher than for the question sentence, even in those problems where this latter sentence contained more words. However, non-numerical areas containing words such as "eerst" (in English "first") or "bijgekregen" ("got more") in Change problems, were sometimes looked at very long too (up to 20 % of the total solution time).

On the other hand, children regularly answered a problem even without casting a glance at important parts of the problem, such as the question sentence or the words "more than" in Compare problems. An example is shown in Figure 4b.

The analysis of variance revealed one interesting finding concerning the effect of ability as a subject variable on the partition of the gaze durations over the total solution time. More specifically, we found a significant overall difference ($F(4, 72) = 2.35, p < 0.05$) between the distributions of the gaze durations in the distinct main areas (A, B, C, R and M) for the H- and the L-group. This was mainly due to the fact that the H-children spend more time in the question areas, while the reverse was true for the empty zones above, below, to the right and to the left of the problem.

Although these total gaze durations per area yielded some interesting findings concerning young children's processes while reading and solving word problems, they often are rather difficult to interpret, and moreover, they do not provide sufficiently detailed information to answer a number of important, more specific questions. For example, the total gaze durations in the numbers areas do not tell us how many times nor at what moment during the solution process, these numbers are looked at by the child. Neither do the observed long gaze durations on particular words tell us whether they were due to encoding difficulties in the initial reading of the problem text or to repeated rereadings during the subsequent solution of the problem. Finally, total gaze durations per area do also not provide answers to questions such as : Do children really read word problems ? If so, do they read the problems sentence by sentence ? What happens after the problem is read ?

To shed light on these questions another kind of analysis is needed based on the temporal sequence of the fixations of the distinct areas of the problem.

3.3 Sequence of fixations

3.3.1 Procedure of data reduction

This second analysis consisted of two stages. First, graphical representations of the raw eye-movement data were made on millimeter paper. On the top of each sheet the pre-specified areas of the perceptual field for each problem were delimited, and each measurement point (i.e. the area looket at every 20 milliseconds) was represented vertically by a tract of one millimeter in the corresponding area. The consecutive positions of the resulting broken line correspond to the sequence of the areas the child's eye was in during the solution process; the length of the line indicates how long the eye was in that particular area. It is obvious that these graphical representations provide a very detailed but easily legible description of the child's eye-movements and -fixations during the whole problem-solving process. Because in the present study we had to make these drawings by hand, the data of only six children have been analyzed until now (three from each ability group).

The second stage in the analysis consisted in reducing the obtained raw and long diagrams by aggregating the measurement data in terms of the following categories:

(1) Sentence reading (S1, S2 or S3) : the child reads the first, the second or the third problem sentence. To be coded in one of these categories, the child's eye-movements must show the typical eye-movement pattern of reading behavior, i.e. subsequent fixations in the distinct zones of that particular sentence from left to right. In the present study a fixation was opera-

tionalized as gazing at the same area during at least 160 milliseconds (Van Lieshout, 1982; Young & Sheena, 1975). When in reading a sentence a single word (e.g. "apples") received little or no attention, this was also categorized as sentence reading. Moreover, if a sentence - after being read - was totally or partially reread before the child's eye went to another sentence, these corresponding data were simply added to the preceeding measurements for the same sentence.

(2) Number reading (N1, N2) : the child is looking at the first or the second given number in the problem. To be scored in one of these categories there had to be a fixation in the corresponding particular number area. However, when this was accompanied by fixations of the adjacent areas without resulting in typical reading behavior, that whole part of protocol was aggregated into one single number catagory.

(3) Word reading (W1, W2, W3) : one or more words in the first, the second or the third sentence respectively are viewed. A piece of an eye-movement diagram was scored in one of these categories, when it contained fixations that could not be conceived as sentence or number reading. For example, reading the words "How many" or "Pete and Ann" in the second and the third sentence of the Combine 2 problem, when this was not part of reading the second and the third sentence respectively.

(4) Rest category (R) : a part of a diagram lasting more than 50 measurements (= 1 second) without fixations. If such a piece was shorter, it was devided between the two adjacent categories.

(5) Missing (M) : a period of missing data. Small pieces of missing data were also equally divided between the two adjacent categories.

The results of this data reduction were again graphically represented, using whole boxes, half boxes and small lines referring to whole sentence, word and number reading respectively. The length of the boxes and the lines indicates the duration of that particular category; in this case every millimeter represents 200 milliseconds.

We will first exemplarily discuss six of these diagrams; however, these examples cannot be conceived as representative of the eye-movement patterns of all children who gave the same answer on the corresponding problem, nor of the same child's eye-movement patterns on similar problems. Afterwards some findings of our detailed analysis of these reduced eye-movement diagrams of the six children in the present example will be presented.

3.3.2. Examples of reduced eye-movement protocols

Figure 5 shows the eye-movement diagrams of Joelle on the Change 1 and the Change 3 problem. This girl from the L-group solved both problems very quickly : in 16 and 14 seconds respectively. While the former was solved correctly, the latter was answered with a WO error. Interestingly, the eye-movement patterns were very similar for both problems : first there was typical reading behavior, involving, however, only the first and the second sentence. Afterwards Joelle's eyes jumped immediately toward the two given numbers, suggesting that she was "doing something" with them. From her answers to all problems we know that Joelle each time added both numbers. For the Change 1 and the Change 3 problem this strategy yielded the correct answer and a WO error respectively.

Figure 5

In Figure 6 one can see the eye-movement diagrams of Lieven, a high-ability child, on the Change 3 and the Compare 1 problem. Lieven's responses to these problems were a CA and NA respectively. Contrary to Joelle, he read all three sentences. Moreover, he spend most of his time reading the question sentence. There are also some interesting differences between the two diagrams in Figure 6. First, there is a great difference in the time Lieven needed to read the Change 3 and the Compare 1 problems, i.e. 44 versus 21 seconds respectively. Second, after reading the Change 3 problem, Lieven's eyes were switching between the two given numbers, suggesting that he was performing an arithmetic operation with them. On the contrary, in the protocol of the Compare 1 problem, such a stage does not occur. This is in accordance with Lieven's reactions to both problems. While the Change 3 problem was solved correctly, he said that he could not answer the Compare 1 problem. When the interviewer asked him why not, Lieven answered "I do not understand the last sentence".

Figure 6

Let us finally take a look at the eye-movement diagrams of Bert, a H-ability boy, on the Change 3 and the Compare 3 problem (see Figure 7). While Bert solved the former correctly, the latter was answered with the LGN. Bert's eye-movements on the Change 3 problem indicate that he first read the whole problem sentence by sentence and then jumped from one number to the other. However, these jumps are interrupted by rereading the question. Comparing Bert's protocol with that of Lieven on the same problem shows that while the latter spent 90 % of his time reading the problem and only 10% executing the arithmetic operation, almost the reverse is true for Bert. Bert's second protocol is a typical example in which a lot of rereading occurs. After reading the whole problem text, the last two sentences were read again; next the whole problem was reread, followed by a long fixation at the second number, which also was given as the answer.

Figure 7

3.3.3 Findings concerning the sequence of fixations in elementary word problem solution

Is the problem completely read ?

We first examine whether the eleven word problems were completely read by the six children involved in the second analysis.

Table 5

Table 5 shows in 15 of the 66 cases - almost 25 % - the problem was not fully read; each time the child neglected to look at the question. Eleven of these incomplete readings were coming from one child, namely Joelle (see Figure 5); the other four cases were produced by two other children, Bert and Femke. Joelle answered all eleven problems with the sum of the two given numbers, which yielded four correct and seven WO answers. Femke answered the Change 1 and the Compare 1 with a correct answer and a WO error respectively. Bert solved both Compare problems, of which he did not read the question, correctly. Theoretically, incomplete readings can be explained in two totally different ways.

On the one side, one could argue that incomplete reading reflects a superficial solution strategy, consisting mainly in looking for the two given numbers in the problem text and performing a rote computation; this computation may be based on the presence of a key word in the problem text (Nesher & Katriel, 1976) or it can simply be the operation that is best known and most familiar to the child

(Goodstein et al., 1971). By means of these "economical" strategies, children can obtain the correct answer for a number traditional elementary arithmetic word problems. As said before, in this study always adding the two given numbers yielded four correct answers.

On the other side, being able to answer a word problem before it is completely read, could be considered as an indication of expertise in that particular task domain. According to current theoretical models on mathematics word problem solving in general (Hinsley, Hayes & Simon, 1977; Mayer, 1982) and on elementary addition and subtraction word problem solving in particular (Briars & Larkin, 1984; De Corte & Verschaffel, 1985a; Riley, Greeno & Heller, 1983) the competent problem solver has organized knowledge of the essential components and basic relations underlying the main types of problems, called problem schemata. Understanding a word problem is then conceived as the activation of the appropriate schema and the mapping of the verbal problem statements onto that schema involving the correct assignment of the given and unknown quantities in the problem to its slots. Once a schema has been activated and its slots have been partially instantiated, further reading even might become a waste of time. For example, applying the Change schema most adults would already be able to answer a Change 1 problem after hearing the first two sentences (e.g. "Pete had 6 apples; Ann gave him 4 more apples"). However, although such problem schemata are assumed in the above-mentioned theoretical models, we know of little direct empirical evidence supporting their availability in beginning elementary school children (Verschaffel, 1984). Moreover, even for subjects who master these schemata and actively use them while reading and solving word problems, it still is useful to check their assumptions by reading the rest of the problem text. For example, a problem starting with the sentences "Pete has 3 apples; Ann has 7 apples", cannot only be followed by "How many apples do they have altogether ?", but also by "How many apples does Pete have more than Ann ?".

In summary our eye-movement data show that sometimes children do not read the whole problem before answering it. Based on the analysis of the gaze-durations (see Section 3.2), we expect the observed frequency in Table 5 to be representative for all twenty pupils in this study. It was argued that, theoretically, very superficial as well as deep-level processing strategies may account for incomplete reading. It is not possible to decide between these two alternative interpretations merely on the basis of a child's eye-movement protocol for a particular word problem. However, the regularity in Joelle's eye-movement protocols on the distinct problems together with her error and solution time data, point toward the first hypothetical explanation. For the other cases (Bert and Femke) further probing questions ("Why did you add both numbers ?"; "Can you retell the story") and the presentation of more problems would have been necessary to exclude one of the two alternative interpretations.

What happens during the first reading of the problem text ?

Word problems can be read in different ways. On the one hand, one can read systematically the subsequent sentences without any forward or backward jumps. On the other hand, certain passages (sentences, words, numbers) can be reread once, twice, or even more before arriving at the end of the first reading.

Table 6 gives an overview of the children's reading behavior from the presentation of the word problems till the end of their first reading. Cells with a dash refer to those cases in which the problem was read sentence by sentence without any interruptions. The symbols S1, S2, S3, W1, W2, W3, N1 and N2 indicate the elements being reread at least once during the first reading. For example, "S1,N2" means that the child had reread the first sentence and jumped backward (or forward) to the second given number before finishing the first reading of the problem.

Table 6

Table 6 reveals that the initial stage of the problem-solving process consisted mostly in reading the subsequent sentences in the problem without any interruption. In 21 cases, however, one or more numbers, words, or even whole sentences were reread before the child arrived at the end of the first reading. The data suggest some interesting hypotheses that need, of course, verification by analyzing the data of the other children in our study.

First, there seems to be a relationship between problem difficulty and children's initial reading behavior. For example, the two easiest problems - Combine 1 and Change 1 - were read very smoothly; the most difficult ones - Change 5 and Compare 5 - on the other hand, elicited a lot of rereading. Consequently, the average time needed to read the problem for the first time was also significantly larger for the difficult problems : 11 and 12 seconds for the Combine 1 and the Change 1 problem versus 18 and 19 seconds for the Change 5 and the

Compare 5 problem respectively.

Second, the eye-movement data also suggest a relationship between children's problem-solving ability and their initial reading pattern. We found not only that most rereadings were coming from the children of the H-group, but there was also a qualitative difference between both groups. While the rereading of the L-children consisted almost exclusively in jumping back and forth to the numbers in the problem, the children from the H-group frequently reviewed words and even whole sentences too.

What happens after the initial reading of the problem ?

Using the same symbols as in Table 6, Table 7 gives an overview of what happened during the rest of the problem-solving process, i.e. between the end of the first reading and the answer. A cell with a dash refers to a solution process that ended immediately after the first reading of the problem with or without giving the answer. "N1,N2,W2,S3" indicates that, after initially reading and before answering the problem, the child jumped on both given numbers, fixated a non-numerical part of the second sentence, and reread the question sentence (although not necessary in this order).

Table 7

As Table 7 shows, in the great majority of the solution processes both numbers were fixated. In fact, children frequently jumped two, three or even more times from one number to the other. Interestingly, in more than half of the processes words and sentences were reviewed too, indicating that semantic processing was not yet finished. Moreover, a closer look at Table 7 also reveals that the above-mentioned relationship between problem difficulty and children's reading behavior keeps holding after the first reading of the problem. For example, the two easiest problems - Change 1 and Combine 1 - elicited again less fixations on words and whole sentences than the more difficult ones. Taken together both findings suggest that, especially when children are confronted with a real problem, i.e. a task for which they do not have a ready-made answer, the solution process does not occur as a linear sequence of sharply distinguished stages, namely a representational and a computational stage. On the contrary, both aspects seem to alternate and interact in real problem solving.

The relationship between ability level and eye-movement patterns discussed above with respect to initial reading seems also to apply here. Indeed, after the first reading of the problem, non-numerical aspects (words, sentences) were reviewed more frequently by the children of the H-group than by the L-group children. Relating this to the hypothesis specified in the preceding paragraph, it seems plausible to assume that high-ability children have a more extensive representational stage consisting of semantic processing than their low-ability counterparts.

4. Discussion

During the last few years a large number of investigations has been done on elementary arithmetic word problem solving, using error analysis, individual interviews, and computer simulation as the main research techniques. While these studies have provided a large number of interesting theoretical constructs and empirical findings concerning the development of young children's skills and processes with respect to that verbal problems, a lot of questions remain, especially relating to the variables and processes involved in the construction of the problem representation and to those involved in the selection of the appropriate arithmetic operation. In this paper an exploratory investigation was presented in which a new technique, namely eye-movement registration, was used to unravel young children's processes while reading and solving elementary arithmetic word problems.

The main findings can be summarized as follows. On the one hand, the data provide empirical evidence in favor of the hypothesis that semantic processing of the problem text is a crucial component of skilled word problem solving (De Corte & Verschaffel, 1985a; Kintsch & Greeno, 1985; Lindvall & Ibarra, 1980; Nesher, 1982; Riley, Greeno & Heller, 1983). For example, it was found that the high-ability children spent more time at reading and solving the problems - especially the complex ones - than the low-ability children, and that this was mainly due to the fact that they looked more and longer at the non-numerical elements in the problem text. On the other hand, our data also support the frequently-heard statement that failures and errors on word problems are due to inattentively reading of the whole problem; in fact, children sometimes answered without even casting a glance at crucial parts of the problem text. In this respect, on can ask to what extent such superficial solution strategies are a direct *result of the monotonous and tedious character* of the word problems children are confronted with (De Corte & Verschaffel, 1986). Finally, our eye-movement data force us to question the sequential and linear character of our theoretical model of competent problem solving, especially with respect to more complex problem types. Taking into account the exploratory character of this study, we advise against hasty generalization. The previous findings must certainly be verified in further research. However, it seems to us that the present results are promising with respect to the possibility of eye-movement registration for getting a more detailed picture of the processes intervening in the construction of a problem representation and in the selection of an appropriate operation, based on this representation.

A main goal of this investigation was precisely to explore the usefulness and the limitations of eye-movement data as access to solution processes of elementary arithmetic word problems. In this respect, the application of the technique with young children posed no special problems. First, the calibration data (which constitute an index of the reliability of the subject's eye-movement data, and are computed by the Debic system before the actual measurement phase) were, generally speaking, as good as in experiments with adults. Second, the percentage of missing data was mostly restricted to about 10-15 %; eye-movement measurements with more than 20 % missing data were rare. Third, the children were not at all disturbed by the presence of the eye-movement equipment nor by the unusual character of the testing situation. Undoubtedly, these positive results are largely due to several features of the Debic 80 system, compared to other methods for eye-movement registration : the child's head must not be fixed, nor are there attachments to the head; the infrared lightbeam is not perceived by the subject... Moreover, the interviewer spent much time at familiarizing the children with the situation by showing them the main components of the system and briefly explaining its functioning.

However, during the gathering and the analysis of the data, we also experienced some difficulties, that will be dealt with in our future work. First, eye-movement research is enormously time-consuming. However, this problem will become less and less important, due to the investigator's increasing familiarity with the eye-movement equipment on the one side, and to the further automatization of the data-reduction procedures on the other. Second, a specific problem of the present study was that no separate areas were constructed for each word on the slides; this made within-sentence analyses somewhat problematic. Although this technical problem can be solved, there will still remain a crucial issue, namely the size of the peripheral zone that can be viewed by the subject when his eye is on a particular point in the visual field; in other words, can a subject encode a word or a number without his eye's being in that particular area on the slide ? Third, because this investigation aimed mainly at exploring the usefulness of eye-movement data, relatively little attention was paid to the verbal data-gathering after the child had solved a problem. In line with our broad-spectrum principle concerning research methodology (De Corte, 1984), the concurrent application of both techniques will be pursued more systematically in our future research. Finally, taking into account the tremendous amount of data that eye-movement registration produces, future studies will be focused on more specific questions.

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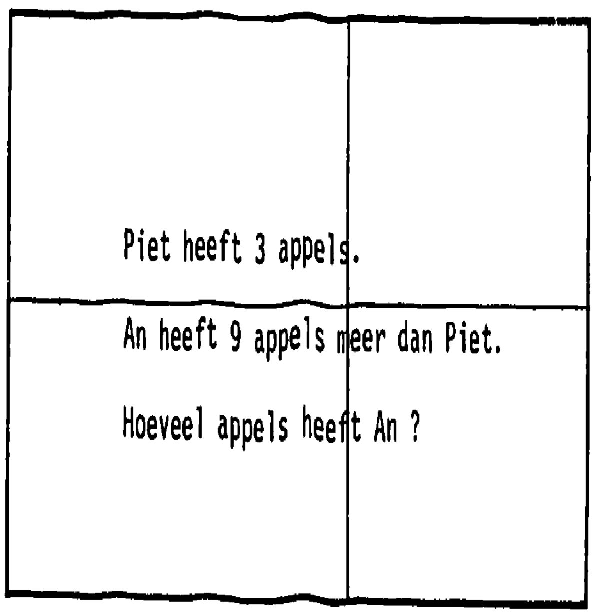
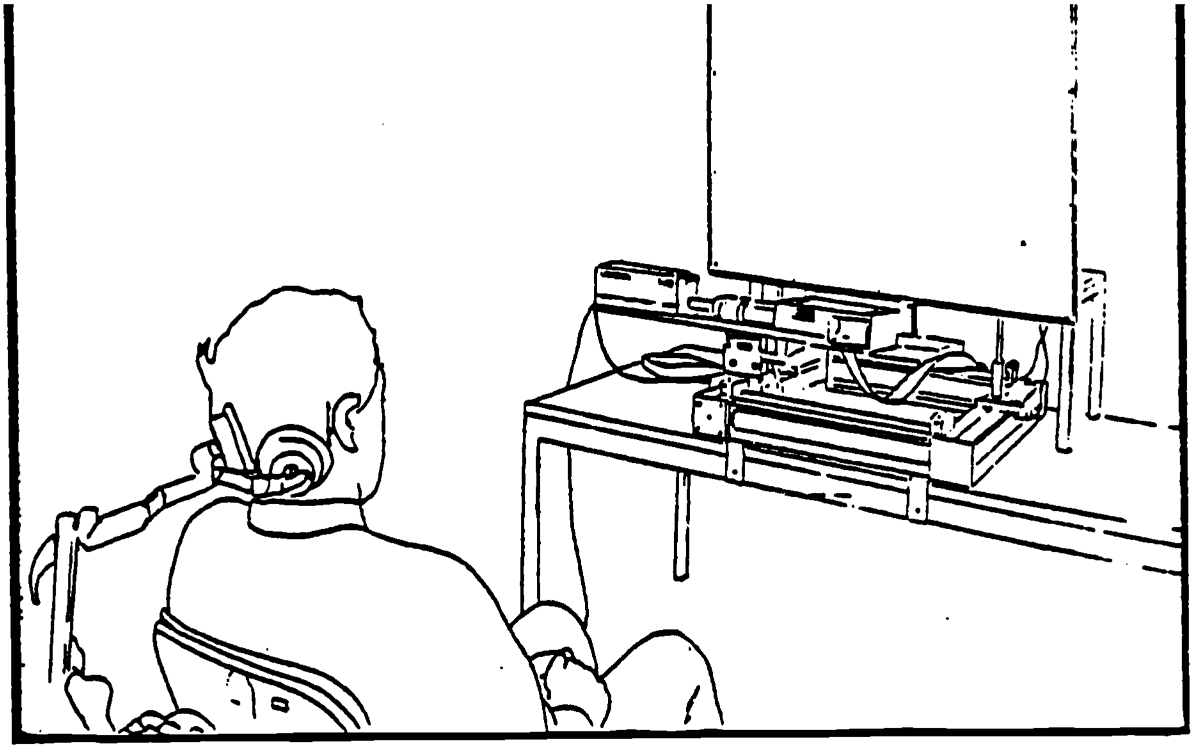


Figure 1 The intersection of the axes indicates the point where the subject is looking at



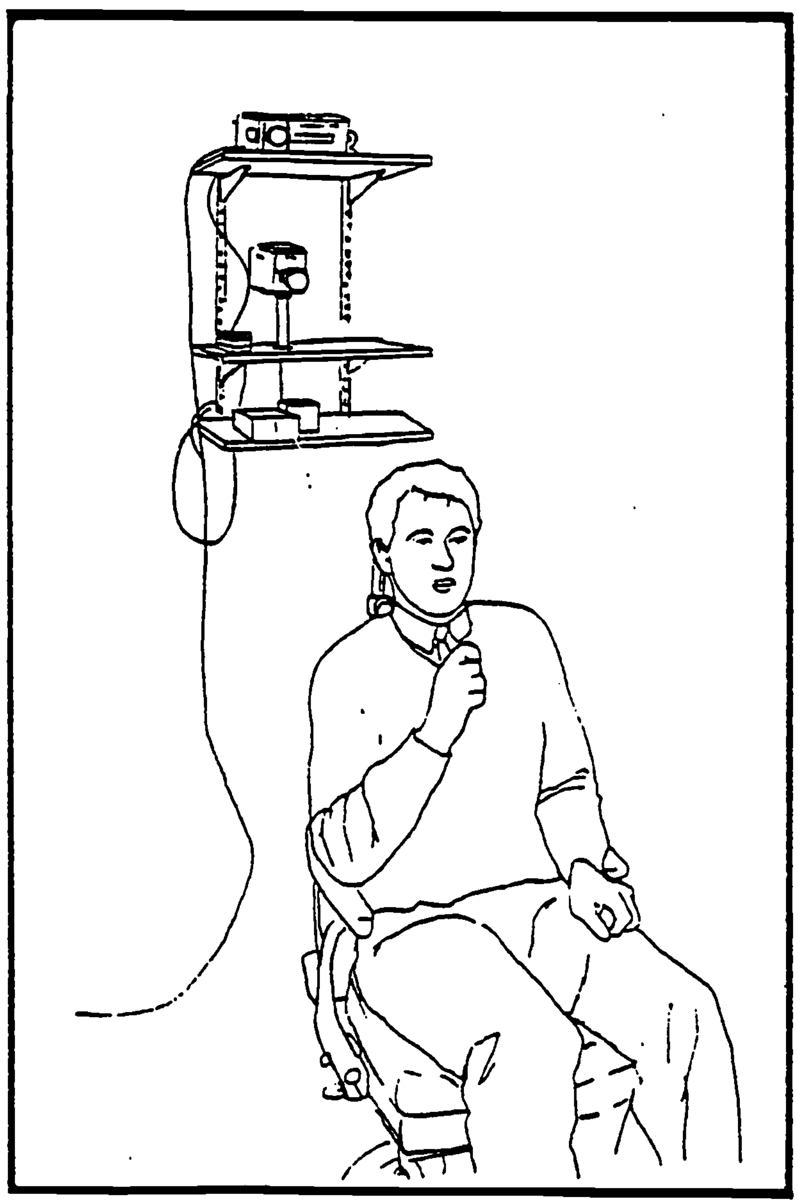


Figure 2 Graphic representation of the testing situaticn

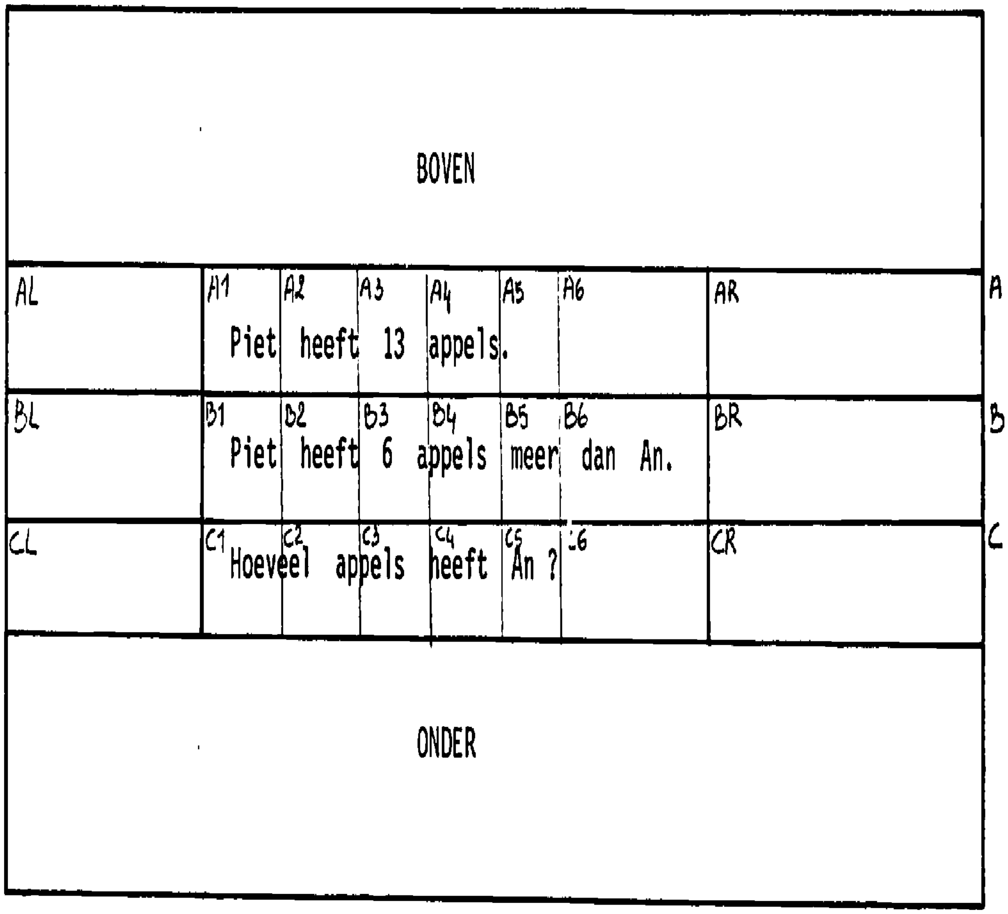


Figure 3 Areas distinguished for the Compare 5 problem

SUBJECT SARAR3 DIA 5 RANG 5

NTOT 3696 SEC 73,92

PIET HEEFT 13 APPELS.
PIET HEEFT 6 APPELS MEER DAN AN,
HOEVEEL APPELS HEEFT AN ?

CODE	A1	A2	A3	A4	A5	A6	A.			
SOM	55	85	436	75	0	0	651			
PCT	1,5	2,3	11,8	2,0	0,0	0,0	17,6			
CODE	B1	B2	B3	B4	B5	B6	B.			
SOM	40	193	551	238	187	397	1606			
PCT	1,1	5,2	14,9	6,4	5,1	10,7	43,5			
CODE	C1	C2	C3	C4	C5	C6	C.			
SOM	27	33	248	103	345	20	776			
PCT	0,7	0,9	6,7	2,8	9,3	0,5	21,0			
CODE	AL	AR	BL	BR	CL	CR	BOVEN	ONDER	R.	M
SOM	0	0	0	0	0	0	4	131	135	528
PCT	0,0	0,0	0,0	0,0	0,0	0,0	0,1	3,5	3,6	14,3

SUBJECT PIETR DIA 5 RANG 11

NTOT 2198 SEC 43,96

PIET HEEFT 13 APPELS.
PIET HEEFT 6 APPELS MEER DAN AN,
HOEVEEL APPELS HEEFT AN ?

CODE	A1	A2	A3	A4	A5	A6	A.			
SOM	11	187	752	105	2	0	1057			
PCT	0,5	8,5	34,2	4,8	0,1	0,0	48,1			
CODE	B1	B2	B3	B4	B5	B6	B.			
SOM	22	262	520	33	8	11	856			
PCT	1,0	11,9	23,7	1,5	0,4	0,5	38,9			
CODE	C1	C2	C3	C4	C5	C6	C.			
SOM	0	9	16	0	0	0	25			
PCT	0,0	0,4	0,7	0,0	0,0	0,0	1,1			
CODE	AL	AR	BL	BR	CL	CR	BOVEN	ONDER	R.	M
SOM	0	0	0	0	0	0	28	2	30	230
PCT	0,0	0,0	0,0	0,0	0,0	0,0	1,3	0,1	1,4	10,5

Figure 4 Two examples of the first analysis of the eye-movement data

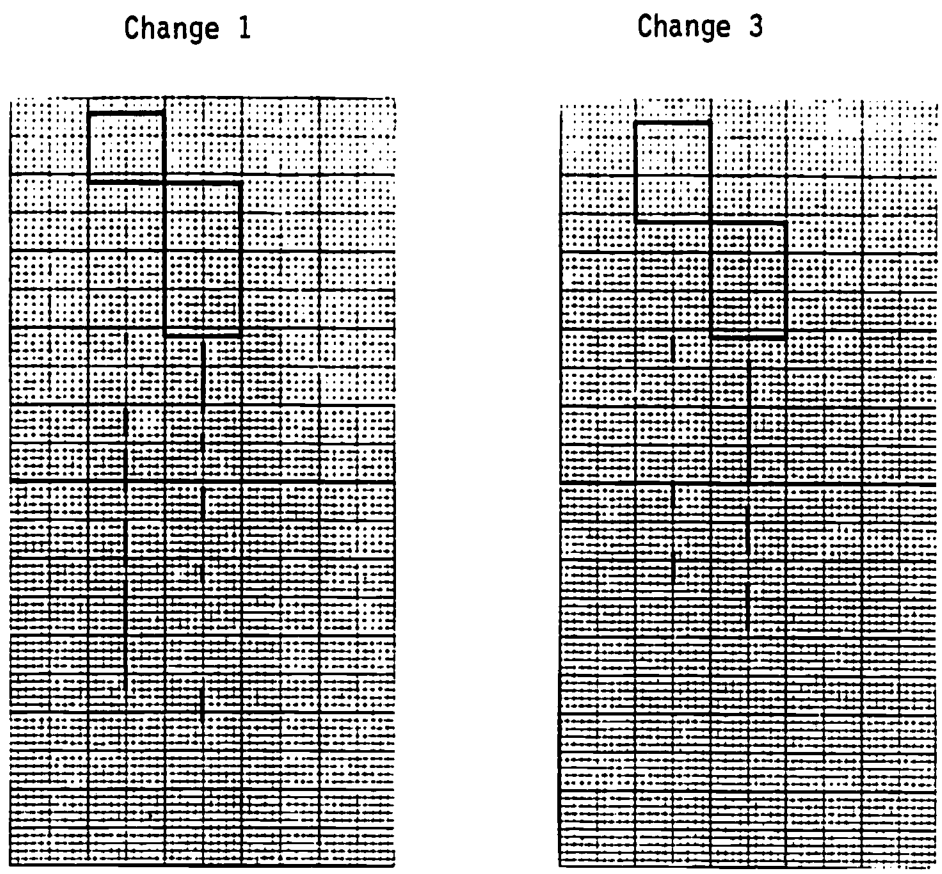


Figure 5 Joëlle's reduced eye-movement protocols for the Change 1 and the Change 3 problem

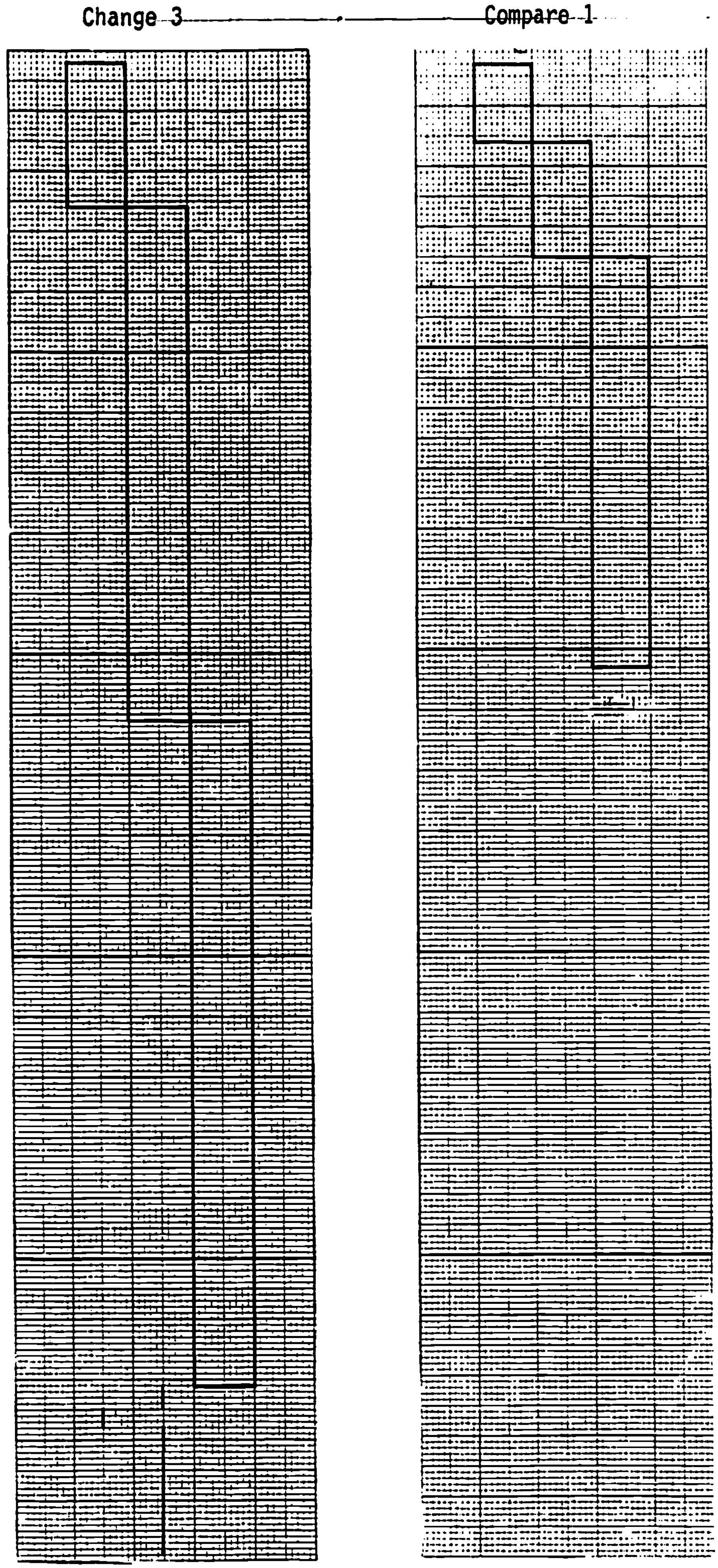


Figure 6 Lieven's reduced eye-movement protocols for the Change 3 and the Compare 1 problem

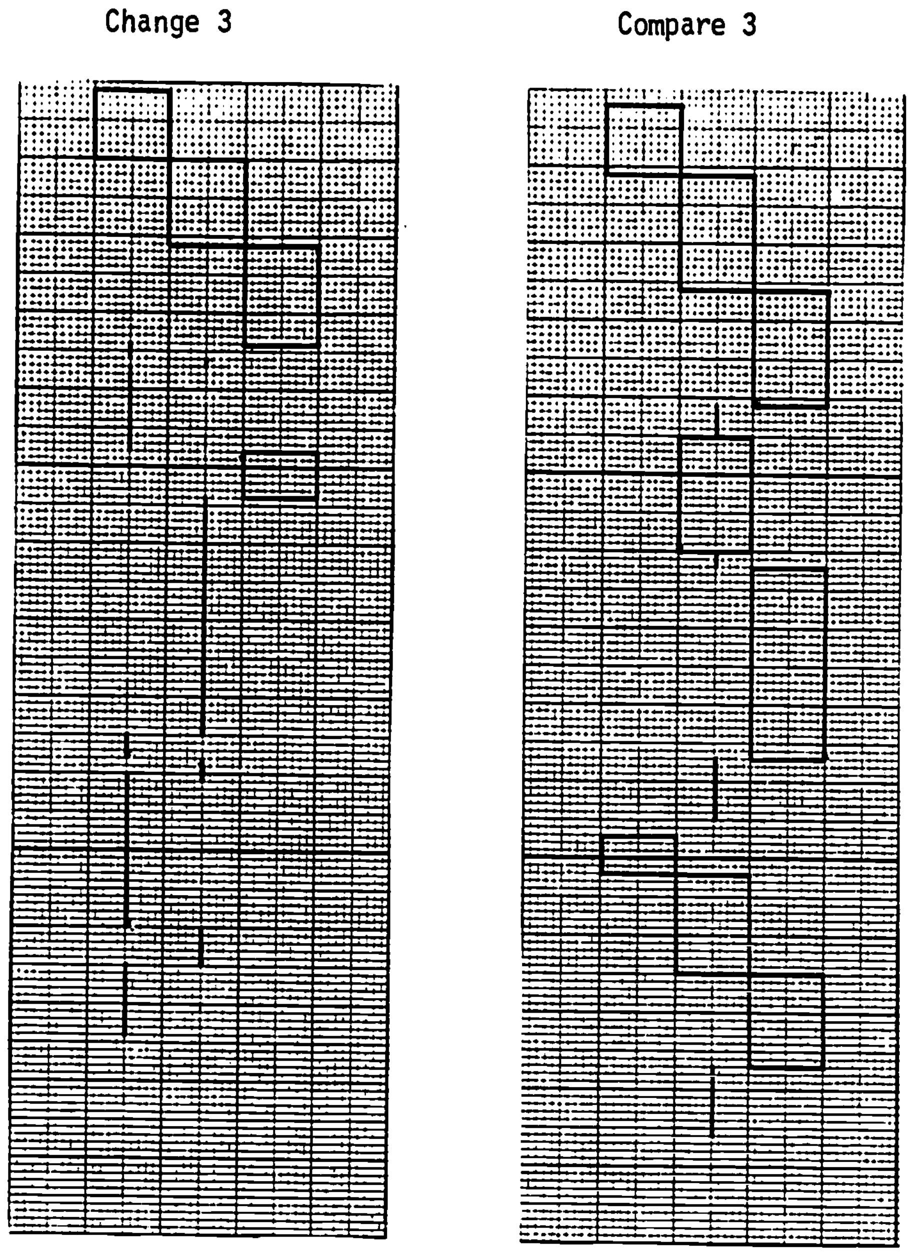


Figure 7 Bert's reduced eye-movement protocols for the inversed Change 3 and the Compare 3 problem

Table 1 Overview of the problems used in the story

Type(1)	Problem	Schema	Direction	Unknown	Operation
Combine 1	Pete has 5 apples : Ann has 8 apples: how many apples do Pete and Ann have altogether ?	Combine	-	Superset	Addition
Combine 2	Pete has 3 apples : Pete and Ann have 11 apples altogether; how many apples does Ann have ?	Combine	-	Subset	Subtraction
Combine 2*	Pete and Ann have 11 apples altogether; Pete has 4 apples; how many apples does Ann have ?	Combine	-	Subset	Subtraction
Change 1	Pete had 4 apples; Ann gave Pete 8 more apples; how many apples does Pete have now ?	Change	Increase	Result set	Addition
Change 3	First Pete had 5 apples; now Pete has 12 apples; how many apples did Pete get more ?	Change	Increase	Change set	Subtraction
Change 3*	Now Pete has 11 apples; first Pete had 4 apples; how many apples did Pete get more ?	Change	Increase	Change set	Subtraction
Change 5	Pete got 5 more apples; now Pete has 14 apples; how many apples did Pete have in the beginning ?	Change	Increase	Start set	Subtraction
Compare 1	Pete has 5 apples; Ann has 14 apples; how many apples does Ann have more than Pete ?	Compare	More	Difference set	Subtraction
Compare 3	Pete has 3 apples; Ann has 9 apples more than Pete; how many apples does Ann have ?	Compare	More	Compared set	Addition
Compare 3*	Ann has 4 apples more than Pete; Pete has 9 apples; how many apples does Ann have ?	Compare	More	Compared set	Addition
Compare 5	Pete has 13 apples; Pete has 6 apples more than Ann; how many apples does Ann have ?	Compare	More	Reference set	Subtraction

(1) The names refer to the categories distinguished by Riley, Greeno & Heller (1983); in the problems with an asterisk, the two given sets are presented in the inverse order.

Table 2 Number of correct answers in the high-ability and the low-ability groups on each problem

Type	H group (n=9)	L group (n=11)	Total (n=20)
Combine 1	9	11	20
Combine 2	6	3	9
Combine 2*	5	2	7
Change 1	9	11	20
Change 3	5	1	6
Change 3*	4	0	4
Change 5	1	1	2
Compare 1	4	0	3
Compare 3	4	6	10
Compare 3*	6	7	13
Compare 5	2	2	5

Table 3 Distribution of the incorrect answers in the high-ability and the low-ability groups on each problem

	H groups (n=9)						L groups (n=11)						Total (n=20)					
	CA	WO	SGN	LGN	NA	M	CA	WO	SGN	LGN	NA	M	CA	WO	SGN	LGN	NA	M
Combine 1	9	0	0	0	0	0	11	0	0	0	0	0	20	0	0	0	0	0
Combine 2	5	2	0	0	2	0	2	7	0	1	1	0	7	9	0	1	3	0
Combine 2*	6	2	0	0	1	0	3	6	0	1	1	0	9	8	0	1	2	0
Change 1	9	0	0	0	0	0	11	0	0	0	0	0	20	0	0	0	0	0
Change 3	5	2	0	2	0	0	1	5	0	3	2	0	6	7	0	5	2	0
Change 3*	4	3	1	1	0	0	0	8	1	1	1	0	4	11	2	2	1	0
Change 5	1	1	6	1	0	0	1	5	3	0	2	0	2	6	9	1	2	0
Compare 1	4	3	0	0	1	1	0	6	0	3	2	0	4	9	0	3	3	1
Compare 3	4	1	0	3	1	0	6	0	0	4	1	0	10	1	0	7	2	0
Compare 3*	6	0	3	0	0	0	7	0	2	0	2	0	13	0	5	0	2	0
Compare 5	2	2	1	0	3	1	2	4	0	2	2	1	4	6	1	2	5	2

CA = correct answer; WO = wrong operation; LGN = larger given number; SGN = smaller given number;
NA = no answer; M = miscallaneous category.

Table 4 Average solution times (in seconds) of the high-ability and the low-ability groups for each problem

	H group (n=9)	L group (n=11)	Total (n=20)
Combine 1	19	20	20
Combine 2	34	32	33
Combine 2*	34	43	39
Change 1	30	23	26
Change 3	33	27	30
Change 3*	39	42	41
Change 5	38	29	33
Compare 1	33	40	37
Compare 3	38	28	33
Compare 3*	22	28	25
Compare 5	52	33	42

Table 5 General overview of the six children's reading behavior on each problem

	L group			H group		
	Niki	Femke	Joëlle	Hans	Bert	Lieven
Combine 1	+	+	-	+	+	+
Combine 2	+	+	-	+	+	+
Combine 2*	+	+	-	+	+	+
Change 1	+	-	-	+	+	+
Change 3	+	+	-	+	+	+
Change 3*	+	+	-	+	+	+
Change 5	+	+	-	+	+	+
Compare 1	+	-	-	+	-	+
Compare 3	+	+	-	+	+	+
Compare 3*	+	+	-	+	-	+
Compare 5	+	+	-	+	+	+

+ = complete reading of the problem
- = incomplete reading of the problem

Table 6 Detailed overview of the six children's eye-movement patterns during the first reading of each problem

	L group			H group		
	Niki	Femke	Joëlle	Hans	Bert	Lieven
Combine 1	-	-	-	-	-	-
Combine 2	-	-	-	N1, N2, W1	N1, W1, W2	-
Combine 2*	-	N1, N2	-	-	N1	-
Change 1	-	-	-	-	W3, S1, S2	-
Change 3	-	-	-	N1	-	-
Change 3*	-	N1, N2	N1, N2	N1	-	-
Change 5	N2, W1, W2	N1	-	N1, N2, W1, S1, S2	-	-
Compare 1	-	-	-	N1	-	-
Compare 3	N1	N2, W2	N1, N2	N1, N2, S1, S2	-	-
Compare 3*	-	-	-	N1	-	-
Compare 5	-	N1, N2	-	-	S1, S2	N1, W2, S1

- = no rereading
N1, N2 = rereading of the first and the second given number respectively
W1, W2, W3 = rereading of one or more words in the first, the second and the third sentence respectively
S1, S2, S3 = rereading of the first, the second and the third sentence respectively

Table 7 Detailed overview of the six children's eye-movement patterns on each problem after it has been read for the first time

	L group			H group		
	Niki	Femke	Joëlle	Hans	Bert	Lieven
Combine 1	N1, N2, W3	N1, N2	N1, N2	N1, N2	N1, N2	N1, N2
Combine 2	W1	N1, N2, W1, W3	N2	N1, N2, W1, W2	N1, N2, W1	N1, N2, W3
Combine 2*	N1, N2, W2	N1, N2, S1, S2, S3	N1, N2, W2	N1, N2, W2, W3	N1, N2, S3	N1, W2
Change 1	N1, N2, W3	N1, N2	N1, N2	N1, N2	N1, N2	N1, N2, W3
Change 3	N1	N1, N2	N1, N2	N1, N2	N1, N2	N2, W2, S1, S2
Change 3*	N1, N2	N1, N2, S3	N1, N2	N1, N2, W1, S2, S3	N1, N2, S3	N1, N2
Change 5	N1, N2	N1, W1	N1, N2	N1, N2, W1	N1, N2, W2, S1, S2, S3	N1, N2, W1, W3, S1, S2
Compare 1	N1, N2, W2, W3, S1, S3	N1, N2	N1, N2	-	-	-
Compare 3	N1, N2	N1, N2, W3	N1, N2	N1, N2, W2, W3, S1, S3	N1, N2, S1, S2, S3	N1, N2, W1, W2, W3, S1, S2, S3
Compare 3*	N1, W1	N1, N2, W1, W3, S1, S2	N1, N2	N1, N2, W1, W2, W3	N1, N2, S1, S2	N1, N2, W3
Compare 5	N1, N2, W2	N1, N2, W2	N1, N2	N1, W3	N1, W1, W2	N1, N2, W2, W3 S2

- = no rereading
N1, N2 = looking at the first and the second given number respectively
W1, W2, W3 = rereading of one or more words in the first, the second and the third sentence respectively
S1, S2, S3 = rereading of the first, the second and the third sentence respectively