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ABSTRACT

This study considered the meaning that was given to knowing/doing mathematics in classrooms comprising the observational study conducted by the Wisconsin Center for Education Research during 1978-81. The study interprets the work of teachers and students, and considers what constitutes appropriate mathematical knowledge for children to learn. A field study was employed in two schools where revised topics of Developing Mathematical Processes (DMP) had been taught. Data were gathered from ten teachers using classroom observations and interviews. This study had access to interviews conducted after each topic had been taught. Results showed children learn not only subject matter, but also appropriate forms in which to cast knowledge. Teachers predominantly saw their role as managing efficient transfer of a body of subject matter to students, and children had limited opportunities to engage in creating and testing mathematical knowledge as a result. Teacher modification of DMP activities to meet student needs was predominantly adjustment of instructional procedures rather than content modification. Content was modified when teachers displayed a sense of ownership and control over what they were teaching. In those cases, children were helped to bring personal meaning to what had been learned. The management approach to instruction is related to a center-out model of curriculum, viewing teachers and students as consumers of knowledge. (Author/JM)

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Mathematical Knowledge and School Work: A Case Study of the Teaching of Developing Mathematical Processes

by Walter M. Stephens

June 1982

Wisconsin Center for Education Research
an institute for the study of diversity in schooling

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MATHEMATICAL KNOWLEDGE AND SCHOOL WORK: A CASE STUDY OF THE TEACHING OF DEVELOPING MATHEMATICAL PROCESSES

by

Walter Maxwell Stephens

Doctoral Dissertation From
The Program on Student Diversity and Classroom Processes

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ABSTRACT

This study asked what meaning has been given to knowing and doing mathematics in those classrooms which comprised the classroom observational study conducted by the Wisconsin Center for Education Research during 1978-81. This study interprets the work of teachers, the work of students, and what constitutes appropriate mathematical knowledge for children to learn.

A field study was employed in the two schools where the revised topics of Developing Mathematical Processes (DMP) had been taught. Data were gathered from ten teachers using classroom observations and interviews. This study also had access to interviews which had been conducted with teachers after each topic had been taught for the first time.

The notions of school work and mathematical knowledge were indispensible for showing that children in school learn not only the subject matter of mathematics, but through their work, they are also taught the appropriate forms in which to cast their knowledge. In the predominant pattern of teaching DMP, teachers saw their role as managing the efficient transfer of a body of subject matter to students. Whenever this management approach

to instruction prevailed, children had limited opportunties to engage in creating and testing mathematical knowledge.

Teachers did modify the activities of DMP to meet the perceived needs of students. However, the predominant pattern of change was one of adjustment to the procedures of instruction. Only rarely was the content of DMP modified directly to meet the needs of students or better to implement the mathematical goals of DMP. Only in these few instances of constructive adaptation did teachers display a sense of ownership of and control over what they were teaching. In those instances, children were also helped to bring personal meaning to what they had learned.

A management approach to instruction is related to a center-out model of curriculum development--and to a wider economic perspective--where teachers and children are regarded essentially as consumers of predefined knowledge. The study concludes with some recommendations for an alternate model of curriculum change.

Chapter 1

A WAY OF LOOKING AT MATHEMATICS TEACHING

Introduction

From the classroom observational study (Romberg, Small, & Carnahan, 1979), which was carried out by the Mathematics Work Group of the Wisconsin Center for Education Research, there is clear evidence that teachers have used the same curriculum materials in quite different ways (Romberg, Stephens, Buchanan, & Steinberg, in preparation). During 1978-81, those studies followed the implementation of an innovative mathematics program in two elementary schools in Madison.

Starting from the data gathered from the observational study, I asked whether these differences and variations in how teachers have implemented the program, Developing Mathematical Processes (DMP) (Romberg, Harvey, Moser, & Montgomery, 1974-76), could be explained by reference to the different orientations, beliefs, and values which teachers brought to the teaching and learning of mathematics.

The principal question of this study was: what meaning has been given to knowing and doing mathematics in those classrooms which have comprised the observational study? This question was broken down into three specific questions: first, how has mathematical knowledge been defined; second, how has the work of teachers been defined; and third, how has the work of students been defined?

In answering these questions, I asked first, what assumptions about the work of teachers, about the work of students, and about mathematical knowledge were made by those who developed DMP. Second, I asked how these orienting constructs have been defined by teachers themselves. Finally, I asked what discrepancies exist between the developers' perspective, and the perspectives of teachers on knowing and doing mathematics. In answering this latter questions, it was necessary to focus upon discrepancies between teachers' expressed definitions of these three orienting constructs and their actual practice.

My study was an extension of the longitudinal study conducted by the Mathematics Work Group of the Wisconsin Center for Education Research during the years 1979-81. That study had been designed to examine changes in children's learning of addition and subtraction skills monitored over a two and one-half year period. Ten curriculum units had been prepared for the longitudinal study, and each unit was written in the same pedagogical style as DMP with which teachers were familiar. The distribution of these ten curriculum units over the three grade levels is shown in the diagram below:

S1, S2, S3	S4, S5, S6, A1, A2	A3, A4
Grade One	Grade Two	Grade Three

Topics S1 to S6 introduced pupils to writing mathematical sentences and to solving those sentences using "basic" number facts. Topics A1 to A4 introduced pupils to the algorithms of addition and subtraction for solving mathematical sentences. The duration of the longitudinal study corresponded to Grades 1, 2, and 3, during which time the symbolic procedures for adding and subtracting are normally taught to children.

The observational data accumulated during that time dealt with content covered by teachers and teachers' and pupils' actions during the implementation of each of the ten curriculum units. The observational procedure was adapted from instruments developed for the study of instructional time by the staff of the Beginning Teacher Evaluation Study (BTES) (Fisher, Berliner, Filby, Marliave, Cahen, Dishaw, & Moore, 1978); by the Center's IGE Evaluation Work Group (Webb & Romberg, 1979); and from an earlier observational coding procedure developed by McLeod (1972) for his study of the effectiveness of DMP's inservice training program. For each child in the sample, the specific instructional activity which he/she was supposed to be working on and its structure were coded during each minute of instruction. Next, the student's actions were coded, according to whether he/she was working in a large group, a small group, or working independently; was engaged in discussion with the teacher, or with other students, and so on. Finally, the teacher's actions were coded, according to whether the teacher was giving verbal instructions to pupils, giving feedback, making structuring comments, etc. Thus, the observational procedure coded what was taught, teachers' actions and students' actions during each day of instruction on each of the ten topics in each of six classrooms.

An analysis of the classroom observational data (Romberg, Stephens, Buchanan, & Steinberg, in preparation) shows that there was considerable variation in the amount of time allocated to the coverage of content. For example, the number of minutes allocated to one of the topics, S5, varied from 525 minutes in one class to 945 minutes in another. Although such variations were important for showing differences among classrooms, there was more precise information on content coverages as well: the number of minutes spent on particular content areas is also accessible from classroom observational data. During instruction on Topic S5, for example, the number of minutes spent on verbal comparison problems (subtraction) varied from 134 minutes in one class to 247 minutes in another. It should be noted that the two classes with these extremes were not the same classes which differed most in total allocated time on Topic S5.

These variations were indicative of the phenomena to be investigated in my study. I show that the different conceptions held by teachers about their own work, about the work of children, and about appropriate mathematical knowledge for children to learn provide a framework for understanding in interpreting these variations.

The theoretical constructs of mathematical knowledge and school work are not to be thought of as conceptually independent variables such as one would adopt in a quantitative correlational study. Fundamental to this study is my belief that work and knowledge are interrelated; that what teachers and children do in mathematics classrooms, namely their work, defines the possibilities of their roles in creating and testing mathematical knowledge. I chose these orienting constructs in order to interpret variations in time allocation, the selection of content, and classroom behavior, because these notions of school work and mathematical knowledge had enabled me to interpret teachers' accounts of their own teaching which they had provided in interviews conducted during the classroom observational study. These constructs seemed to provide a tool for explaining why these accounts presented a different picture of the teaching of DMP than one would have expected if one had been guided by the suggestions and recommendations contained in the Teachers' Notes which accompanied the DMP materials. Furthermore, my own observations of those same teachers had convinced me that DMP was being implemented often in ways which reflected different beliefs, purposes, and values than had been assumed by the DMP authors.

At a more general level, I had expected to find variations in the way in which DMP was being taught for the following four reasons. First, although the decision to implement DMP was made by teachers and principals in consultation with the developers of DMP, it would have been naive to assume that these various parties all had a unified view of what mathematical knowledge should be taught and how it might best be learned. Second, the fact that mathematics was being taught to children in an elementary school would suggest that a teacher's pedagogical principles and practices, and the constraints of that particular learning setting would influence what mathematics children were taught and how it was taught to them. Third, one ought not have assumed that mathematical inquiry would fit comfortably into the time slots of the conventional classroom lesson. And fourth, one needed to recognize that schools have a wider social mandate than simply to teach mathematics, or any collection of subjects: through what they teach teachers, schools help to define and to legitimate what counts as work by teachers and students, and what kinds of knowledge are to be valued more highly than others.

At a specific level, I needed to attend to two features of the phenomena to be investigated. First, teachers were implementing a mathematics program which, unlike many textbooks, incorporated very explicit recommendations and suggestions for classroom instruction and how children might learn mathematics. These recommendations and suggestions were intended to challenge many traditional patterns of teaching mathematics in the elementary school. For example, small-group work and open-ended activities were proposed, and teachers were encouraged to assist children in developing their own strategies for solving addition and subtraction problems. Second, although the curriculum units were written in the general pedagogical style of

DMP with which teachers were familiar, the particular topics to be taught were new, reflecting a more careful analysis of the types of addition and subtraction which children are introduced to in the early grades of elementary school. These two features have implications for the principal questions of my study.

Romberg and Price (1981) argue that an innovative program is rarely assimilated into a school in the manner intended by the developers. Hence, the need to consider discrepancies between the developers' perspectives on the three orienting constructs and those of teachers. Sometimes, the response to an innovation is purely nominal, where new labels are adopted to cover unchanged practices. However, as Romberg and Price (1981) argue, there is a need to distinguish between different kinds of innovation even where teachers perceive themselves as actually implementing an innovation. Popkewitz, Tabachnick, and Wehlage (1982), for example, propose a threefold classification to characterize the actual responses by schools to the implementation of Individually Guided Education (Klausmeier, 1977). They describe schools which are constructive, technical, or illusory users of IGE.

Developing Mathematical Processes (DMP)

The authors of DMP saw their task as one of presenting an innovative program of instruction in elementary school mathematics which was (1) mathematically sound, (2) psychologically sound, and (3) pedagogically sound (Romberg, 1976).

DMP grew out of IGE. The structure of its materials, its topic-by-topic development, and its recommendations for classroom management made it readily adaptable to IGE settings. Its implementation was intended to fit into an Instructional Programming Model (IPM) which is a salient feature of IGE. This model assumed that instruction would be differentiated to provide for individual differences in general; and, specifically, that groups would be formed and reformed to work on different topics. The model also assumed that instruction will be differentiated within topics in order to reflect the special needs of sub-groups of students.

The mathematical perspective adopted by the developers of DMP is a measurement/modeling approach (Romberg, 1976). This perspective is based on the premise that the language of mathematics can be derived from applied problems. Through this approach, a child is helped to discover and apply mathematical language and processes through the use of one of the two attributes, length and numerousness. Children's perceptions of mathematics are developed through the processes of classifying, comparing, ordering (seriating), equalizing, and representing. Thus, children are encouraged to work through a variety of measurement activities, with each activity emphasizing a particular attribute, to ways of describing relationships between objects, then to ways of symbolically representing these attributes and the relationships between them, and finally, to representing mathematical sentences with real objects to validate their answers.

From a psychological perspective, the authors of DMP asked what attributes are initially familiar to children by means of which they could impose a structure on a set of objects by using those attributes. They used materials which were perceptually and intuitively meaningful to children by means of which they could be introduced easily to a symbolic language and to model an applied problem.

From its pedagogical perspective, DMP changed the teachers' role from one of provider of information and of correct procedures, to one of guide who leads children to mathematical knowledge. Children were steered away from dealing with problems which have only one correct answer, to problems which have many possible correct responses. They were encouraged to solve problems in different ways, and even to solve problems in their own way relying upon a mastery of techniques and skills developed earlier in DMP (cf. Romberg, 1977, p. 82).

The developers also expected that teachers would utilize a variety of settings in which children can learn mathematics, where teachers no longer alternate exclusively between whole group instruction and independent application. Not only did the developers recommend a variety of sizes of groups as an effective means of differentiating instruction, they also argued that children can, and do, learn mathematics by interacting with each other as they solve problems, discuss observations, and share discoveries among themselves (cf. Romberg, 1977, p. 86).

Embodied in these three perspectives--mathematical, psychological, and pedagogical--which the developers have brought to DMP are clear implications for the work of teachers, for the work of children, and for the kind of mathematical knowledge children should learn. These implications are examined more fully in the third chapter of this study.

Methodological Implications

The orienting constructs of this study--the work of teachers, the work of students, and how mathematical knowledge has been defined for children to learn--have provided a framework for understanding discrepancies between developers' perspectives and the perspectives and practice of teachers. They also provided a framework for interpreting variations of practice among teachers themselves. Knowledge of teachers' classroom practice was derived, in the first place from the classroom observational study, and from the Topic Interviews which were part of the classroom observational study. That study (Romberg, Small, & Carnahan, 1979) used time as a measure of the amount of learning done by students, and of the kind of activities in which teachers and students have been engaged, whereas this study presents a qualitative analysis of the phenomena using the three orienting constructs: the work of teachers, the work of students, and how mathematical knowledge has been defined.

In order to elicit the specific meanings which teachers and developers have given to these three constructs, I used a field-study approach. This approach included an analysis of DMP materials and of records of interviews conducted with teachers during the observational study. I also conducted a series of interviews with teachers and observed current teaching of DMP by selected teachers.

This study shows that variations and discrepancies between teachers' practice and developers' expectations, and variations of practice among teachers themselves--can be understood as reflecting the specific meanings which teachers have given to these orienting constructs. These qualitative interpretations of the quantitative data derived from the observational study and from observations of current practice comprise the findings of my proposed study. At one level, these findings are related to a range of issues and problems in the teaching of mathematics in elementary schools. However, at a more general level, I have related the findings of this study to some perenniel problems of epistemology and to contemporary social and ethical problems regard- the nature and definition of human work.

Related Studies

In this section, I refer to five studies which represent a cross-section of theoretical positions which guide those who use a field-study approach to the study of schooling: first, the IGE Evaluation Study Phase III by Popkewitz, Tabachnick, and Wehlage (1982); second, the study, "Teaching and Learning in English Primary Schools," by Berlak, Berlak, Bagenstos, and Mikel (1975); third, a paper by Smith and Sendelbach (1975), "Teacher Intentions for Science Instruction and Their Antecedents in Program Materials"; fourth, to Brandau's (1981) paper, "The Practice of an Elementary Mathematics Teacher--an Ethnographic Approach to the Study of Problem Solving"; and finally, Anyon's (1981) study, "Social Class and School Knowledge."

Each of these studies uses a field-study approach to investigate certain aspects of schooling. At one end, the IGE Evaluation Study Phase III deals with a group of schools which were judged from a national sample of IGE schools to be exemplary practitioners of IGE. At the other end, Brandau's study discusses the work of one teacher.

There is a common element to the first three studies. Each can be described as trying to penetrate the rhetoric of educational discourse in order to discern a deeper meaning of the events being studied. I use the term "rhetoric," in a nonpejorative sense, to connote commonly accepted ways of describing school events; and refer to those uses of the rhetoric of education which tend to attribute common practices and shared beliefs to schools and teachers.

Those schools which were studied in the IGE Evaluation Study Phase III can be clustered together under the description, "These schools are all exemplary users of Individually Guided Education." On other occasions, the rhetoric may refer to less specific elements of school philosophy and practice, when one speaks, for example, of "open" primary schools in the context of English education. Sometimes, one may use the rhetoric of education to describe a group of schools as all using a specific instructional program: "These schools all organize their mathematics program around DMP." These uses of educational rhetoric exemplify perfectly legitimate ways of referring to certain features in common to the schools being discussed. But one must be wary of assuming that these expressions which facilitate ease of reference and description betoken an identity of belief and practice. Beneath these common descriptions, there may exist a diversity of belief and practice, and a degree of complexity in each, which tends to be obscured by the uses of educational rhetoric.

Popkewitz, Tabachnick, and Wehlage (1982) sought to understand how IGE had been implemented in six schools which were considered to be model users of IGE. They had expected that there would be considerable similarity among these exemplary schools, but as their contact with each of the schools deepened over the course of one year they

> encountered fundamental variations in the conditions and social implications in the six schools studied, as well as different institutional responses to the technologies of IGE; to describe these variations and their significance . . . (they) created three categories: technical, constructive, and illusory schooling (Popkewitz, Tabachnick, & Wehlage, 1982, p. 6).

Guiding their observations of the schools and their interviews with teachers and other local educators was the question. What meaning has this school given to the philosophy and practice of Individually Guided Education? By means of the three categories--technical, constructive, and illusory schooling--the authors sought to give consistency to the ways in which IGE had been interpreted in each school. In particular, they used this threefold classification in order to describe how the adoption of IGE had affected the work of students, the work of teachers, and how knowledge was defined for children to learn.

The study be Berlak et al. (1975) can also be viewed as an attempt to penetrate the rhetoric of educational discourse, and to ascertain how teachers actually define their own work and that of students. In their study of several "informal" primary schools in England, Berlak et al. (1975) were confronted by a rhetoric of open education which had depicted these schools as places where children tended to make decisions about what should be learned, where children did not rely on extrinsic motivation from their teacher in carrying out their work, and where they tended also to set their own standards. As Berlak et al. (1975) noted, however, some contradictions were apparent among the various descriptions of open education in English primary schools. Accordingly, they "became increasingly certain that whatever accounted for (teachers') behavior, it was not the beliefs attributed to them by the Plowden report" (Berlak et al., 1975, p. 222).

In seeking to explore the meanings given by teachers to teaching an open classroom, Berlak et al. (1975) depicted teachers as drawn to both poles of a dilemma: whether, for example, the teacher makes decisions about what is to be learned or whether pupils make these decisions for themselves; whether the impetus or learning should come from the pupils or whether the teacher should intervene in order to initiate learning and to ensure that it is sustained by pupils. Their study presented 15 such dilemmas which were intended to represent the kind of decisions which teachers in open classrooms were required to address and resolve. The authors argued that the meanings given by teachers to teaching in an open classroom were best depicted by the patterns which showed how they resolved these dilemmas. Their study presented examples of the patterns displayed by teachers in resolving these tensions. They concluded that:

> The language of freedom, self-motivation, child-set standards, commonly used to characterize these schools, even with added qualifications, does not . . . capture the complexity of the actual schooling (Berlak, et al., 1975, pp. 239-240).

Smith and Sendelbach (1979) in their study, "Teacher Intentions for Science Instruction and Their Antecedents in Program Materials," focused on the extent to which the literal guidelines for the program were interpreted by teachers and incorporated into their plans for classroom instruction. Their attention was directed to a study of how teachers determined what was to be taught. Their study used interviews and stimulated recall through the use of videotapes in order to have teachers articulate the decisions made in planning lessons. Their study sought to determine the nature of the modifications made by the teachers to the original material; and, furthermore, to account for some of the differences between a teacher's intended approach and the rhetoric of the guidelines contained in the program materials. Their study depicts teachers as actively engaged in giving their own meaning to a program of instruction. Teachers are not depicted as making only minor alterations in pacing and small adjustments to the content. Smith and Sendelbach (1979) depict teachers as making significant changes in the content and presentation of instruction.

The fourth study, by Brandau (1981), looked at how a teacher's belief about mathematics and ideas for mathematics teaching are realized or compromised in the classroom. In an "ethnographic study" of one teacher, Brandau has sought to describe and account for those occasions when the teacher seemed to come closer to realizing her ideals, and those other situations, sometimes within the teacher's control, sometimes outside her control, when her ideals become compromised. Brandau's interest is uncovering the layers of meaning which the teacher has given to her own work.

As well as seeing the instructional process as the medium through which the teacher realized her ideals, or was led to compromise them, or experienced conflict as the process of realizing some goals seemed to compromise others, Brandau (1981) saw the teacher, on these occasions, as engaged in problem solving by trying out new strategies and new styles of teaching. This search for new strategies and style is given impetus, Brandau argues, by the "practical impediments in . . . mathematics instruction" (p. 5). Through an analysis of incidents which exemplify this search, Brandau has depicted the richness of the teachers' problem-solving strategies as she negotiates a complex course between conflict and possible compromise of her beliefs and ideals on the one hand, and their realization, on the other.

In considering these four studies, the term "field studies" can be applied as the element which links them to my proposed study. This term is able to link together the procedures employed by the investigators: their reliance upon interviews with teachers, and upon classroom observations which are frequently supplemented by an analysis of curriculum materials. Those who employ a field-study approach have chosen, on the other hand, not to employ an experimental approach in which it would have been necessary to specify in advance the variables to be manipulated, and to set up treatment and control groups. However, if one recognizes the term, "field-study," as part of the rhetoric of educational discourse, there are dangers of oversimplification in applying it to my own study and those which I have just discussed. In grouping together all these studies under the same heading, the danger is that one is apt to ignore important features which distinguish these related field studies from one another.

The most significant feature which distinguishes each of these studies from one another is its theoretical orientation. While each may be said to adopt a field-study approach to the study of schooling, how the particular events and actions being studied are interpreted is tied to the theoretical perspective which each investigator has adopted.

In their study of IGE schools, Popkewitz, Tabachnick, and Wehlage (1982) draw upon perspectives from the sociology of knowledge and political theory. Using insights from sociologists, such as Mannheim (1952) and Durkheim (1961), these investigators explore the interplay between the institutional and cultural context of schooling, and the influence of these factors on how the work of teachers and students is defined, and what constitutes desirable knowledge. From Mannheim (1952), for example, one can identify an assumption that teachers have only a limited capacity to stand back from the mental boundaries established by their objective positions in the social order; and also a belief that the language (or rhetoric) of schooling masks the divergent and often conflicting interests of groups in society. From Durkheim (1961), the authors present the notion of individual consciousness as an extension of a collective consciousness, through which individual perceptions are shaped by means of shared images and values. Elsewhere, Popkewitz (1981) explores conflicting interpretations of the mandate given to schools "to educate":

> Schooling also has, of course, a mandate to educate, but that mandate is ambiguous and is open to many interpretations. To 'prepare children for a democratic society', for example, can mean (a) to develop individual intellectual capacity so one can engage in rational discourse; (b) to develop an appreciation of and loyalty to given social institutions and practices; (c) to learn functional skills so one can be a productive member of a work-force. (p. 190)

Popkewitz, Tabachnick, and Wehlage (1982) have argued that the social and cultural context of each community which they studied is an important factor in interpreting the practices and priorities of a local school. They also argue that the pedagogical practices which are established in each school, and the professional or occupational context in which teaching is carried out are critical factors in determining the work of students, the work of teachers, and how knowledge is defined.

Thus, Popkewitz, Tabachnick, and Wehlage (1982) assert that schools are places where quite different institutional conditions, different styles of work, different conceptions of what constitutes desirable knowledge, and different professional ideologies are maintained. These influences are presented as powerful modifiers of philosophies and programs in ways one would not expect to occur if one attended only to the rhetoric in which schools described themselves or in which program materials were elaborated.

The studies by Berlak et al. (1975) and Brandau (1981), by contrast, pay less attention to social, political, and cultural factors beyond the classroom, and depict teachers as actively negotiating for themselves the meaning of classroom events. Both studies describe teachers' classroom behavior in terms of their beliefs and how children learn, and in terms of teachers' beliefs about their own role in children's learning. The theoretical perspective of these two studies is indebted to the theory of symbolic interactionism as articulated by Mead (1934) and Blumer (1969). Acknowledging his own indebtedness to

G. H. Mead, Blumer (1969) argues that

> human beings interpret or "define" each others' actions instead of merely reacting to each others' actions. Their "response" is not made directly to the actions of one another but instead is based on the meaning which they attach to such actions. Thus, human interactions is mediated by the use of symbols, by interpretation, or by ascertaining the meaning of one another's actions. (p. 79)

To some extent, one can link the study by Smith and Sendelbach (1979) to this theoretical perspective. However, I am less certain of the theoretical perspective which they have brought to their study.

To extend these differences among field studies even further, I include Anyon's (1981) study, "Social Class and School Knowledge," in which she employs a Marxist perspective to examine the content and methodology of teaching in five elementary schools. Anyon argues that, despite some similarities in curriculum topics and materials being used in the five schools, there were important differences in the content of instruction and in the methods of teaching; and that these differences reflect and are explained by different class stratifications in the schools themselves. She describes the schools as falling under four different categories: "working class," "middle class," "affluent professional," and "executive elite." According to Anyon, what is taught and how children are taught in these different schools reflects "class conflicts in educational knowledge and its distribution . . . in order (for example) the struggle to impose the knowledge of powerful groups on the working class and in student resistance to this class-based curriculum" (p. 38).

In discussing the teaching of mathematics in the working class schools, Anyon (1981) depicts students as spending a great deal of their time carrying out procedures, "the purposes of which were often unexplained, and which were seemingly unconnected to thought processes or decision making of their own" (p. 8). In the middle class school, she discerned more flexibility in regard to the procedures which children were expected to follow. There, the teacher tended to set out several alternative methods of solving a problem, and made efforts to ensure that children understood what they were doing. In the professional school, on the other hand, the teacher placed a greater emphasis on children's building up mathematical knowledge through discovery techniques or through direct experience; whereas, in the executive schools, these patterns of teaching were extended even further to include explicit problem solving, testing hypotheses about mathematical variables, and encouraging pupils to justify the reasonableness of their answers.

Like those who approach the study of schooling from a symbolic-interactionist perspective, Anyon asks what does it mean to teach and be taught in these schools; but, unlike them, she argues that in their interpretation of school events they seem to underestimate "the extent to which the negotiation of meaning in social situations takes place within a context of material and other givens where certain things are non-negotiable" (cf. Sharp & Green, 1975, p. 29). From a Marxist perspective, Anyon (1981) argues that:

> To grow up in the modern capitalist class is not only to enjoy travel, luxury, good schools, and financial wealth; it is also to have to maintain power in the face of others competing with you, within an irrational economic system that is increasingly difficult to predict, manage and control. (p. 38)

Anyon contends that schooling is so rooted in an economic/class structure that nothing short of a revolution can break the bonds in which individuals are enmeshed; and that the rhetoric of education necessarily reflects interests in the dominant class, and thereby defines and limits the way in which individuals perceive and interact within schools. On the other hand, Popkewitz, Tabachnick, and Wehlage (1982) take a stance which is less deterministic. They do not endorse Anyon's belief in the overriding influence of pervasive class stratification. Yet, they do believe, like her, that schooling is a social enterprise whose practices reflect the beliefs, values, and purposes embodied in conceptions of work and knowledge as these are lived out in the wider society. Likewise, they believe that schooling reflects tensions and contradictions implicit in those conceptions of work and knowledge. However, they do believe that individuals can break free, at least in part, from some of these socially imposed "givens" by identifying and reflecting upon the assumptions which underpin the pedagogical, occupational, and social contexts of schooling.

In employing the orienting constructs of school work and mathematical knowledge, I draw upon a social and political perspective similar to that adopted by Popkewitz, Tabachnick, and Wehlage (1982). They depict schools as places of work where teachers and students interact to establish, modify, and carry out social purposes. Schools are also places where conceptions of knowledge are developed and maintained (cf. Popkewitz, Tabachnick, & Wehlage, 1982, p. 11). However, Anyon's (1981) study remains significant for this study because of her fundamental assumption that knowledge is not just in people's minds, but emerges out of the productive social relationships which constitute school work for teachers and students.

Although the schools in which my study was conducted did not appear to differ on socio-economic criteria, as was the case with Anyon's (1981) study, conceptions of mathematical knowledge there appeared different from what the authors of DMP had intended, and these differences seemed to be embedded in different conceptions of what constituted appropriate work for teachers and students. My preliminary observations of classrooms where DMP was being taught did suggest that different conceptions of work and knowledge were present among such classrooms. If such differences in the curriculum-in-use were present among these classrooms, I would have to account for them in terms of my own theoretical framework.

The task of interpretation which I embark upon this study reflects similar interests to those of Brandau (1981) and which Berlak et al. (1975) brought to their study of English primary schools. There, it will be remembered they were investigating the extent to which the rhetoric of child-centred education was reflected in actual pedagogical practice. Berlak et al. (1975) posited a number of "tensions" or "dilemmas," to use their own words, which are embedded in the relationship between the public rhetoric of English elementary education and pedagogical practice. Berlak et al. (1975) conclude that, far from reflecting a uncritical adoption of the public rhetoric of child-centred education, teachers' actions can be interpreted as a "specific pattern of resolution of these tensions or conflicts which are manifest in their behavior" (p. 224).

Their interpretation of teachers' actions is based explicitly upon a symbolic interactionist framework which follows the work of Mead (1934) and Blumer (1969). This interpretative framework treats teachers' actions as components of classroom life, where the classroom itself is perceived as a miniature society in its own right, autonomous, not subject to profound change, and tending towards the harmonious resolution of conflict--as is implied by the metaphors of "dilemma," "tension," "conflict," and "resolution" which Berlak et al. (1975) employ. This same use of language is evident in the Brandau (1981) study.

Since the learner-centred approach of DMP is similar to the stated philosophy of the English elementary school, I could have adopted a similar approach to my analysis of teachers' actions. However in adopting a symbolic interactionist framework for the interpretation of teachers' actions, I would be treating as commonplace and as unambiguous those assumptions of work and knowledge implied by teachers' actions and embodied in the life classrooms being studied. As Masemann (1981) comments about such studies:

> the essential "givenness" of the researcher's assumptions concerning the nature of the educational task and its relation to wider social and political meanings can remain intact (p. 4).

She argues that an approach, such as adopted by Berlak et al. (1979) and Brandau (1981), is limited in its possibilities

> for (it) neither takes us out of the classroom and into the world of political or economic concerns, and (it) neither allows for the possibility of theory building (p. 4).

My own perspective is that pedagogical practices are social forms which mirror political and social relations and epistemological assumptions from the wider society. My aim is to show that more general conceptions of work and knowledge have become encapsulated in classroom practices. Therefore, in my analysis of lessons observed, and of accompanying classroom practices, I develop these conceptions of school work and mathematical knowledge by examining the role given to the individual pupil both as learner and as member of a classroom group, by relating these roles to a prevailing management approach to teaching, and by relating that pattern of teaching to its underlying assumptions about mathematical knowledge. Far from seeing pedagogical practices as expressions of the life of a micro-organic and autonomous classroom community which is able to resolve its own tensions and conflicts according to a pedagogical logic shaped by idiosyncratic or "commonsense" assumptions, I take as a starting point that pedagogical practices can be seen more fruitfully

> as a mediating link between the subjective consciousness of the individual and community lifestyles and cognitive styles (Popkewitz, in press).

Synopsis of the Study

In the following chapter, I discuss methodological and conceptual issues which attend the use of a field-study approach. The criticism, that the results of field studies are unable to be generalized, is addressed. Then, I describe the specific methodology of this study.

In the third chapter, DMP is related to the growth of Individually Guided Education, as a manifestation of the movement toward school reform in the 1970s, and to several other innovative programs in mathematics teaching which were developed at the same time. Implicit in each of these contexts are notions of school work and mathematical knowledge which were to become explicit in the implementation of DMP.

These theoretical constructs are applied in Chapter 4 to analysis of the Topic Interviews which were conducted by teachers by members of the Mathematics Work Group between 1979 and 1981. In the following chapter, those preliminary hypotheses which were derived from the Topic Interviews are extended by my own observations of classroom teaching and by discussions with teachers.

In the sixth chapter, I ask whether there has been a predominant pattern to the modifications effected by teachers in their implementation of DMP. There, a management-approach to instruction and accompanying patterns of technical changes in the procedures of instruction are depicted as the principal and pervasive features of the teaching of DMP.

In the concluding chapter, I argue that the beliefs, purposes, and values which have underpinned the teaching of DMP can be interpreted in terms of a management approach to instruction. I contend that this approach is shaped by an economic perspective within which children are seen as consumers of a fixed body of subject matter. This same economic perspective is also implicit in the way in which DMP has been implanted in schools. Having been designed and developed outside the schools in which it was to be used, DMP has been presented to teachers as a curriculum to be managed by them. Just as children are seen as consumers of a fixed body of subject matter, so teachers are seen essentially as consumers and managers of an externally produced curriculum.

Chapter 2

METHODOLOGY AND OVERVIEW

Before dealing with the specific questions which this study addresses, it is important to confront head-on, so to speak, a criticism which is often made of field studies as a class of educational research: that, while they may seek to address important questions about teaching and learning, it is not possible to generalize their findings which are no more than rich descriptions of singular events. To ask to what extent the findings of field studies are generalizable provides an opportunity for me to explore the relationship between one's theoretical perspective and the questions one seeks to address; as well as an opportunity to argue that the findings which flow from these questions can aspire to a degree of generality which the critics of field studies have been unwilling to recognize or concede. The following argument is an abridgment of a position which I have elaborated elsewhere (Stephens, 1982). The salient points of that position, I believe, should be stated at the outset of this study. Having discussed these conceptual issues relating to field studies, I present the methodology of this study and an overview of its findings.

A Question of Generalizability

Critics of field studies have argued that the results of field studies are not generalizable and are, at best, rich idiosyncratic descriptions of teachers' actions. The issue is made more complex when some of those who engage in field studies appear reluctant to claim that their findings have any general application, preferring to settle for more limited claims, that, for example, their findings are intended only to be intelligible to those who read them. Such disputation and uncertainty leads one to ask how educational researchers define generalizability and how they expect it to be demonstrated. These issues become more important in educational research as field studies are employed to understand curriculum problems.

In this section, I argue that there has been a tendency to limit claims of generalizability to cases where one could demonstrate an appropriate use of sampling theory. Those who accept this view have narrowly defined the notion of generalizability in terms of sampling theory, thus reducing the possibility of a science of education. These views are examined in the first part of this section. In the second part, I discuss the possibility of research producing generalizations which lead to theory. This view of generalization is identified with the interpretation of school events. In these uses of generalization in educational research, I argue that sampling theory is not a necessary condition.

At first glance, one may wonder why such questions should arise. Surely, it may be argued, field studies are intended to interpret, explain, and produce understanding of the actions being investigated; and in that sense, the investigators are concerned with making generalizable conclusions. Wolcott, for example, in his study Teachers Versus Technocrats (1977) sought to explain the effects of a system of planning and communication on teachers in one school district. To suggest that Wolcott's analysis of the impact of that innovation may not have any relevance beyond that school district would appear to do an injustice to his intentions.

The Received View

Critics of field studies, however, are unlikely to concede that the possible relevance of a field study to other sites is sufficient to justify a claim to generalizable results. It is one thing, they argue, to intend that one's field has a wider application and explanatory power beyond the sites which comprise it. It is an altogether different issue to achieve success in this enterprise. According to what may be called the conventional, or received view, one can generalize with assurance only by ensuring that the instances under investigation are truly representative of the population, and by carefully eliminating bias from the data one has chosen to study. The received view has been succinctly articulated by Julian Simon (1978).

> A good principle is that you should generalize from your data if you can reasonably regard them as a fair sample of the universe to which you want to generalize. Everything that we know about bias in samples, therefore, comes to bear on the problem. If the sample was randomly drawn from a universe, then you can infer that what is true of the sample is true of the universe. But when the sample is not randomly drawn from the universe, the generalization is certainly not automatic (p. 390).

In applying his view to an anthropological study such as Wolcott's, Simon can be interpreted as requiring Wolcott to establish that the discovered patterns could be expected to occur, with some measure of confidence, in other sites where similar innovations had been implemented. That seems quite a modest requirement to make of Wolcott; and one which would not force him to argue that exact "carbon copies" of those patterns are likely to occur in other sites, only that similar innovations could be expected to produce similar patterns.

However, among those who support the received view, there appears to be a more radical critique of the findings of field studies: that, in the absence of controls against bias in sampling, no generalizable conclusions can be drawn from a study in which a single group is investigated as it undergoes change. This line of criticism seems to question whether any valid conceptual knowledge can be obtained through a field study of one instance. Campbell and Stanley (1963), representative of the received view, once argued that field studies "have such a total absence of control as to be of almost no scientific value" (p. 176); at best, it may be said that field studies are based upon "a tedious collection of specific detail . . . and involve the error of misplaced precision" (p. 177). Campbell and Stanley (1963) conclude:

> How much more valuable the study would be if the one set of observations were reduced by half and the saved effort directed to the study in equal detail of an appropriate comparison instance (p. 177).

The received view of generalizability seems to employ a procedural definition of generalizability. According to such a definition, the results of a study can be said to be generalizable because the right sampling procedures have been employed. This sense of generalizability might be called "horizontal" generalizability because it implies that, if the same sampling procedures were performed on a population which has the same characteristics as the one being studied, or another sample from the same population, one would expect to have one's results replicated in the second study. This definition of generalizability is therefore related to the use of certain sampling procedures which enable one to attach a quantified estimate of likelihood to the repeatability of one's findings. The essential feature of this definition is that it rests on certain notions of probability which enable one to predict how the elements of a sample, and the characteristics they bear, relate to a general population having the same characteristics.

There are dangers in solidifying one sense of generalizability as it relates to educational research; and yet it seems that the notion of "horizontal" generalizability is frequently employed by those, for example, who present conclusions from correlational studies of teaching behavior. If "horizontal" generalizability is what these researchers imply when they use the term, then it is clear that the statistical and sampling procedures which are intended to produce repeatability of the results of a study are distinct from the procedures one would use in order to explain the actions and events being studied; that is, explaining why certain actions by teachers are more effective in helping students to learn. Gage (1978), for example, reports several studies of specific variables in teachers' behavior and their effect on pupils' achievement in reading and mathematics in the early grades of elementary school. One such study, by Brophy and Evertson (1974), reports that teachers' criticism of wrong answers by pupils is highly correlated with their adjusted achievement in reading. If Brophy and Evertson were to explain why teachers' criticism of wrong answers is more effective in certain contexts, than not criticizing their answers, they would need to appeal to other considerations than those they had used to ensure that their results were repeatable.

The argument about generalizability could indeed rest with this sense of "horizontal" generalizability if researchers limited themselves only to making claims about other samples of the same population or about samples of other populations which bear the same characteristics as the one studied. However, they do use generalizations to interpret what has been studied. That is, they look beyond the specific situation in ways which simplify that situation, and at the same time link features of that situation to more abstract and general considerations. This other sense of generalizability might be called "vertical."

The idea of a "vertical" generalization is the one which I wish to attach to the interpretation of school events. This sense of "vertical" generalization can be aspired to by field studies and by conventional empirical research as well. The contrast between "vertical" and "horizontal" generalizability can be illustrated by making a distinction between building interpretative theory and showing that these interpretations are likely to be useful in studying school events. For building interpretative theory, there may be value in selecting an instance which offers clearly delineated features and telling contrasts--as did Wolcott in his study Teachers Versus Technocrats (1977). However, once developed, these interpretations need to have some more general application; and an investigator will need to convince a skeptical audience that there are grounds for being confident that the features which gave rise to the interpretative theory are likely to be borne out across a range of groups or instances.

In the present study, I use the orienting constructs--the work of teachers, the work of students, and how mathematical knowledge is defined--in order to interpret variations in how mathematics has been taught in those classrooms which participated in the three-year longitudinal study. These orienting constructs are intended to lead to some interpretative generalizations within and across schools. Once these generalizations are reached, it is important to argue that they apply not only to the two schools being studied, but are likely to apply to other schools as well. Showing the reader how to relate the original descriptions and interpretations which were derived from a particular context to a wider social context of assumptions, beliefs, values, and purposes is a central issue of a field study. This problem confronted Cusick in his study, Inside High School (1973). There, Cusick shows an apparent reluctance about making generalizations of any sort. His reticence is typical of others who adopt a field-study approach to educational research. It is as though these investigators have been so intimidated by the notion of "horizontal" generalizability and its tie to sampling theory that they see themselves as disqualified from making generalizations altogether.

Cusick, in his study, Inside High School (1973), when confronted by the objection that his study, "dealing with a limited and perhaps unique sample, may be ungeneralizable," (p. 231) seems to back away from the issue of generalizability, and claims only to offer a description of a high school which is intelligible (my emphasis) even to those who have never participated. Not that Cusick refrains from drawing some general recommendations from his study. Cusick (1973) recommends, for example, that "teachers should be prepared to accept school as a place where conflict is inevitable," (p. 226) and that they should give primary importance to their instructional role.

My response to Cusick is that his study is intelligible only to the extent to which his readers can understand those beliefs and assumptions which underlie the practice of high school education in the United States. For, despite his disclaimer on the issue of generalizability, Cusick believes that the characteristics which define the social and learning environment of the students at Horatio Gates High School are probably (my emphasis) shared by most American high schools. These have been elaborated by Wehlage (1981) following Cusick's own listing:

1. Subject matter specialization for teachers and students,
2. Vertical organization of people, with students at the bottom,
3. A doctrine of adolescent inferiority, which denies students initiative and responsibility,
4. Downward communication flow from teachers to students,
5. Batch processing of students by a single teacher,
6. Routinization of activity for students and teachers,
7. Dependence on rules and regulations for students and teachers,

8. Future reward orientation for students, and
9. Supporting physical structure to facilitate the above (p. 221; cf. Cusick, 1973, pp. 208-209).

Whether each of the above claims is well founded, and whether every detail of Cusick's study is sound is not implied here. What is being argued is that the characteristics which Cusick presents as defining the social and learning environment of Horatio Gates High School already embody interpretations of school events. These characteristics are notable as much for introducing all kinds of nonobservable perceptions and understandings. Nor are they explicable as generalizations derived from evidence which can be described in everyday observable terms (cf. Ryan, 1970, p. 72). I argue that in his description of defining characteristics of the social and learning environment of an American high school, Cusick can be seen as using the kind of interpretation which I have called "vertical" generalization.

Cusick's descriptions, such as "vertical organization," "batch processing," and "routinization," interpret school events in terms of more general social categories. In using categories of description from industrial production and management theory, he is able to interpret school events as having important features in common with these enterprises. His argument can be thought of as a kind of "existence proof"; that is to say, there exist in high schools patterns of actions and events which are well accounted for, not in conventional educational terms, but in the language and discourse of industrial production and management theory. It is in this sense that one can depict Cusick's study as aspiring to a kind of "vertical" generalizability.

His claim that these characteristics are probably shared by most American high schools, although unproved, would, if true, ensure that his interpretations had a wide "horizontal" generalizability. In other words, "vertical" generalizations need a certain level of "horizontal" generalizability if they are to be treated as useful theoretical knowledge about schooling.

Using Cusick's study as an example, I argue that the findings of field studies are generalizable in the "vertical" sense, if the particular events and actions being studied can be interpreted, in terms of a rationally defensible and consistent theoretical framework, as instances of more general social ideas. The theoretical framework in terms of which these issues are cast provides "orienting categories" from which generalizations may be formed (cf. Tabachnick, 1981). Cusick, for example, draws his "orienting categories" from a social interactionist theory of behavior, especially from the works of Blumer (1969). While other field studies may adopt a somewhat different theoretical framework from that used by Cusick, field studies, in general, reflect a common interest in documenting and interpreting the intentional decisions of the parties engaged in the actions and events being studied, and to the intersubjective agreements shared among the participants.

Appraising Interpretative Generalizations

When conventional empirical studies claim some measure of "horizontal" generalizability for their results, these claims presuppose that certain sampling procedures have been used. While there are no procedural checks to ensure that "vertical" generalizations embody sound interpretations of the events being studied, these interpretations need to be checked with other facts about those events (cf. Homans, 1967); and, at the same time, critical appraisal needs to be directed at the rationality and consistency of the theoretical framework employed by the investigator.

The theoretical framework of a field study provides the "orienting categories" within which an investigator attempts to interpret events and actions. Tabachnick (1981) has argued that these orienting categories, unlike the operational categories of conventional empirical research, are "open to change and development as a result of encountering the action of being studied" (p. 84). That this should be so is not a sign of theoretical weakness, but rather a direct result of the purposes one brings to a field study. If one aims to look beyond the particular events and actions in ways which simplify them, and at the same time link them to more general categories of social behavior, then, even if one has a consistent theoretical framework of interpretation, one cannot determine in advance how that theoretical framework applies to the particular event being studied, or whether all aspects of the theory are equally relevant to those events. Furthermore, one might also be led to modify one's theoretical framework in the light of the events themselves. For these reasons, the orienting categories which an investigator brings to a field study apply in a different manner than the operational categories of conventional empirical studies.

<u>Work and Knowledge</u>

I have already set out the orienting constructs which will guide my proposed study: the work of teachers, the work of students, and how mathematical knowledge has been defined. These constructs have been employed by Popkewitz, Tabachnick, and Wehlage (1982) in their study of the implementation of Individually Guided Education. There, these constructs have their roots in the social and political theories of Durkheim (1961), Gouldner (1973), and Mannheim (1952). The theoretical implications of these constructs have been elaborated by Popkewitz and Wehlage (1977). They emphasize that in a field-based study the concepts of "work" and "knowledge" are not operationally defined terms. Their role in field-based inquiry is to orient the investigator's attention to three different levels of social action and the interrelationships among these levels. Work can therefore be viewed as:

(1) the activities in which people are engaged,
(2) the interactions created by those activities, and
(3) the sentiments that accompany the activities and interactions (Popkewitz & Wehlage, 1977, p. 70).

The notion of "work," understood in this broad sense, can be used to direct one's attention to the social interactions between teachers and pupils, and to assist one to clarify the assumptions and implications of school practices (cf. Popkewitz & Wehlage, 1977, p. 70). It can also be used to illuminate the relationship between the actions of teachers and pupils, and mathematical knowledge. By examining the assumptions, beliefs, values, and purposes embodied in the interactions between teachers and pupils, one is led to ask: what kinds of mathematical knowledge are presented to children in these transactions?

What is the source of this knowledge? Do children, for example, participate in the creation of mathematical knowledge? What are children to come to believe about the creation and testing of mathematical knowledge as a result of their work on DMP?

It is important to show that the interpretative generalizations which I was able to develop through use of these constructs are likely to apply to other schools than those which comprise the immediate focus of my study. My attention was, therefore, directed at those features which the two schools have in common with many other elementary schools, and by reason of which one is justified in arguing for the "horizontal" generalizability of one's findings: the prevalence of age-graded and self-contained classroom groups; the practice of the two schools to group children for mathematics instruction according to ability; the use of an externally developed mathematics program; and the tendency of teachers to focus upon issues of classroom management in the teaching of DMP. Having related my interpretative generalizations to these features of elementary education, then it is reasonable to suggest that the findings of this study are likely to apply to other schools with similar features where an innovative mathematics program has been introduced.

Synopsis

In this section, I have argued that the findings of field studies are generalizable. Most typically, these generalizations take the form of interpretations of the specific events and actions being studied, and are intended to link these events and actions to more general descriptions of social behavior. If these interpretative generalizations are to be useful, a reader needs to be convinced that similar interpretations are likely to apply to instances which can be identified as similar to the cases studied. That is a pressing question which needs to be addressed by all those who use field studies to generate theoretical knowledge about school events. However, in a particular study, whether the enterprise of building interpretative generalizations is successful will depend on the accuracy of one's observations and upon the rationality and consistency of the theoretical framework which one brings to that study; especially in the form of orienting categories derived from that framework. Where these are lacking, or not well developed, an investigator runs the risk of offering no more than a rich description of particular events and actions; and will have to head off the charge that the study involves an error of misplaced precision. But where a consistent and rationally defensible framework has been employed, investigators should not be intimidated by critics of field studies who define generalizability in terms of sampling theory. I have argued that there is another sense in which one can aim to develop generalizable findings, for which sampling theory is not a necessary condition. Generalizations in this "vertical" sense can be developed by conventional empirical studies and by field studies equally well.

Methodology

The methodology to be followed needed to display fit, some consonance, with the principal question of this proposed study: What does it mean to know and do mathematics in the classrooms which have participated in the longitudinal study? It needed also to display some consonance with the three orienting constructs which will be employed in addressing the above question: The work of teachers, the work of students, and how mathematical knowledge has been defined. A conventional experimental or quasi-experimental approach to these issues using multiple regression analyses or survey instruments did not seem appropriate to dealing with matters of action and practice at the day-to-day level. The descriptive data from the observational study have depicted knowing and doing mathematics in these classrooms in terms of a large number of conceptually independent and operationally defined categories relating to the behavior of teachers and pupils. One of the major tasks of the proposed study has been to interpret these pieces of seemingly independent information.

The issues which I proposed to investigate were closely linked to a complex network of teachers' perceptions, and classroom and school practices. Teasing apart the threads of this network is at the heart of field work and is reflected in its methodology (cf. Smith, 1981, p. 102).

How the developers have defined the work of teachers, the work of students, and appropriate mathematical knowledge could be ascertained, in part, from their expectations for classroom practice and content development which are stated in the curriculum materials; from their recommendations for allocated time implicit in the observational study; and from other documents which describe the aims and objectives of DMP and its relationship to IGE An investigation of these recommendations and expectations formed a part of my "field work" approach.

A field work approach was necessary to ascertain how teachers gave practical expression to the orienting constructs which guide this study; and to relate the definitions of school work and mathematical knowledge which were embedded in classroom practice to patterns of the institutional life of schools. A field work approach was therefore necessary for eliciting discrepancies between teachers' and developers' perspectives on the three orienting constructs, and also for eliciting discrepancies between teachers' perspectives as stated and their actual practice.

Schedule for Field Work

1981

October	preliminary meetings with school principals to discuss study analysis of transcripts of topic interviews (two months)
November	initial meetings with teachers to discuss purpose of study and to arrange individual meetings
December	interviews with ten teachers at Grades 1-3 (one month)

1982

January	classroom observations of all teachers, to be followed by a second interview

Analysis of Topic Interviews

Initially, I analyzed the transcripts of Topic Interviews (Stephens, Romberg, in preparation). The questions which comprised the Topic Interviews are given in Appendix A. The interviews themselves had been conducted by members of the Mathematics Work Group with individual teachers at the end of each of the ten topics covered during the observational study. These interviews were intended to ascertain the degree of importance which teachers gave to the topic, and also to elicit from teachers some specific comments about their own teaching: "Which activities in the topic do you feel most useful? Are there activities which you consider superfluous? Did you change/add any activities? Why?" At the start of the school year, teachers were asked to describe their approach to planning and instruction: "Do you plan day by day, activity by activity, topic by topic, or use some other scheme? On what basis do you decide whether an activity should be carried out through seatwork or be teacher directed?"

These interviews were important not only for the responses which teachers gave to the above questions, but also for the implicit characterizations given by teachers to the mathematical content of instruction, for how they perceived their role in the classroom, and for the ways in which they describe their students. Implicit in the transcripts, therefore, were interpretations which teachers had given to the orienting constructs which guided this study. An initial analysis of these transcripts led to some preliminary hypotheses which were tested in my subsequent interviews and observations.

Interviews and Observations

Following the analysis of transcripts, the next step was to interview all eleven teachers who were continuing to teach DMP at the same grade level as was the case during the observational study. One of the eleven said that she was unwilling to participate any further in this study. These interviews had three purposes. First, they provided an opportunity to refine and develop further the preliminary hypotheses and interpretations which were derived from the analysis of transcripts. Thus, they were intended to elicit from teachers their expectations for the work of children, what they consider to be appropriate mathematical knowledge for children to learn, and their perceptions of their own role. Second, the interviews were used to discuss with teachers those aspects of their selection of content, of their allocation of time, and of their classroom behavior and organization in those respects in which they have appeared to differ from the expectations and recommendations of the developers. A third purpose of these interviews was to gauge the extent to which teachers have modified their teaching of DMP since the time of the classroom observations and Topic Interviews. Some changes in teaching and in organization of content were to be expected, since teachers of Grade 1 were in the 1981/82 school year teaching DMP for a fourth time; those of Grade 2 for the third time; and those of Grade 3 for a second time. It was essential to describe and characterize the nature of such changes as have occurred. Any pattern in these changes would serve to refute a conjecture that the teachers had acted idiosyncratically. Furthermore, the changes themselves would reflect teachers' conceptions of work and knowledge.

Insights and interpretations derived from these interviews needed to be checked against teachers' actual practice. For that reason, I observed all available teachers as they taught DMP. It happened that, during the 1981/1982 school year, two teachers (Teacher H and Teacher J) who had taught DMP at Grade 3 level in School 2 were no longer teaching mathematics to their children. Teachers H and J were teaching Science and Social Studies respectively across all groups. As a result, eight teachers in the two schools were continuing to teach DMP at the same grade level as they had done at the time of the classroom observational study and the initial Topic Interviews. Two teachers (Teachers A and E) were teaching at Grade 1--one in each school. Three teachers (Teachers B, F, and G) were teaching at Grade 2--one in School 1 and two in School 2. Three teachers (Teachers C, D, and K) were teaching at Grade 3--two in School 1 and one in School 2. Teacher K was responsible for all mathematics teaching at Grade 3.

These observations took place in January and February 1982. Each teacher was observed at least once between the first and second interviews. One teacher, Teacher A, was observed three times. Three were observed twice--Teachers B, C, and F. Teacher G was observed twice in the classroom, and twice on videotapes of her lessons which were recorded in the previous year as part of the classroom observational study.

In arranging to visit classrooms it was not possible to request of teachers that they teach a specific topic. At each grade level, there was already at the time these observations were conducted a wide spread in content being covered among the classes. One exception occurred with Teachers C and D in School 1 who were still planning to keep close together in their teaching of DMP. Therefore, for all teachers being observed, I asked that no special arrangements be made to their normal planning, and that they continue to teach whatever they would have done had I not been coming.

While observing lessons, I kept a written record of what the teacher was doing, and endeavoured to keep a word-for-word account of what teachers said to the whole class. Likewise, I was able to keep a fairly accurate record of children's responses to teachers' questions. In the fifth chapter, I present an account of one lesson observed for each teacher.

It is clear that one cannot generalize about a teacher's implementation of DMP simply on the strength of one lesson observed. However, my observations were able to be related to the Topic Interviews (Stephens, Romberg, in preparation) and to my own initial interviews with all teachers. Moreover, these observations were followed by a second interview with all teachers. Thus, it was possible to see whether the beliefs, purposes, and values which teachers had expressed in interviews were consistent with their classroom practice. It was also possible to place their comments made to me and my observations of their teaching in the context of the Topic Interviews which had been recorded at least twelve months earlier. From these various perspectives it was possible to construct a picture of how teachers were implementing DMP and to discern differences and similarities among them.

It was necessary to follow up these observations with a second interview. Thus, these observations and my subsequent interviews with teachers provided a further means of ascertaining the extent to which teachers' stated intentions and expectations were lived out in their actual teaching.

Each stage of the field study was used to check and further refine interpretations developed at the previous stage. This process of development attended to similarities and differences both between and within the two schools. The process also relied upon comparison in another sense: my tentative interpretations were constantly being checked with other members of the Mathematics Work Group, including those who have acted as classroom observers during the observational study, and those who have acquired through their management of the longitudinal study extensive knowledge about and insight into the two schools and their teachers.

Conducting and Reporting Interviews

My first interviews with teachers, the transcripts of which are contained in Appendix B, followed the same pattern as the Topic Interviews (Stephens, Romberg, in preparation). Like the Topic Interviews, they contained questions about planning, instruction, and the topic currently being taught. The questions to be asked were submitted in advance to teachers. As a result, teachers often came to the interview with prepared notes on which they based their responses.

I did not follow the interview format mechanically. Before meeting with each teacher I reviewed their earlier Topic Interviews. If there were issues from the Topic Interviews which needed further clarification, for example, on the practice of grouping pupils by ability at that particular grade level, I sought answers to these issues in the course of the interview. At other times, I found it necessary to interpolate my own questions in order to clarify teachers' immediate responses. If it appeared that certain questions would be redundant because a teacher had already answered them indirectly, those questions were either omitted or passed over very quickly.

The principal difference between this first series of interviews and the earlier Topic Interviews was the inclusion of questions which were intended specifically to relate current teaching of DMP to that of previous years. In each case, I asked teachers to describe how their teaching had changed since they had started with DMP. Only in one instance had a teacher commenced to teach DMP in the previous year. All others had been teaching DMP for at least two years.

There were also two teachers in School 2 who were no longer teaching DMP in Grade 3, although they were continuing to teach other subjects at that grade level. In their case, I asked them to refer back to their teaching of DMP in the previous year. For all teachers, I decided to invite them to consider what advice they would give to a teacher who had not taught DMP before. My intention behind this question was to give teachers an opportunity to reflect upon their own experiences with DMP, and to place themselves in the position of one who was teaching DMP for the first time. Although this question was not submitted to teachers in advance, they had no difficulty responding to it.

The duration of each interview was between 30 and 45 minutes. Interviews were tape-recorded, and the transcripts are contained in the appendices of this study. In preparing the transcripts, I found it imperative to exercise editorial discretion. There were times when the conversational nature of the interviews gave rise to statements which tended to be fragmented and disjointed. As interviewer, I had no difficulty in following what teachers had said, but after sharing the unedited transcripts with members of the Mathematics Work Group and other colleagues, it was clear that without further editing the transcripts would have been very difficult to read for anyone who had no prior knowledge of DMP or of the schools involved in the study. This would have done a serious injustice to the ideas and reflections of the teachers who had participated in these interviews, and therefore justified some editorial license on my part.

However, in making adjustments to sentence length and unctuation, and in eliminating gross colloquialisms, I have endeavored to keep as close as possible to what teachers actually said, and to their own style of saying things.

The second series of interviews, the transcripts of which are contained in Appendix C, took place in January and February 1982, at least one month after my initial interviews with teachers. In the intervening time, I had been able to observe teachers as they taught DMP. This second interview usually took place in the same week--often on the same day--as these observations. As with the first interview, questions to be asked in the second interview were submitted in advance to teachers. They were therefore aware that I expected to seek further elucidation of issues arising out of the first interview, and that I would also use this occasion to discuss aspects of their own teaching which I had observed, for example, whether what I had observed had been typical of their teaching or more of an exception to their usual practice. It was not possible to list in advance questions of this sort. Nevertheless, teachers were fully apprised that this kind of question would be included in the second interview.

Within the second interview, I also presented teachers with a synopsis of data from the classroom observational study. This synopsis of classroom observational data comprises Appendix D. One of the problems in having teachers comprehend and discuss these data was that the information had been gathered as far back as 1979; nor were the teachers aware then of the categories under which data had been classified. Therefore, in presenting teachers with information about their teaching in a previous year, I found it helpful to present them with data gathered at the same time from other teachers at the same grade level. I then invited them to comment upon the data which related to their own teaching, and, in particular, to comment upon differences in time allocation between teachers. This synopsis of classroom observational data, presented information on actual time spent teaching S- and A-Topics at each year level. It comprised measures of total time spent on each Topic appropriate to that grade level, and showed total time spent as a percentage of the estimated time required to teach those Topics; time spent on activities where the objectives had been ranked as "very important" by members of the Mathematics Work Group; time spent on activities which emphasized the acquisition of new concepts; time spent on activities which contained applications of addition and subtraction; and time spent on activities where it had been anticipated that emphasis would be given to pupils' discussing their findings. In presenting these last four measures of time spent, I had ranked each class with other classes at the same year level in terms of time spent as a percentage of total time spent, and as percentage of estimated time. Teachers' responses to this part of the second interview are considered in the latter part of this study.

Another important objective of the second interview was to give teachers an opportunity to reflect in a more global fashion on their teaching of DMP than had been possible in my first interview--or in the Topic Interviews--where the questions were directed at the topic currently being taught. In the second interview, I hoped to elicit from teachers their perceptions of the mathematical skills and understanding which they hoped would be fostered as a result of that year's experience with DMP. Likewise, I hoped to have teachers reflect on those features of DMP which they saw as desirable, which they would like to see embodied in another mathematics program, or, on the other hand, which they would like to see avoided.

Teachers' responses to these questions helped to elucidate their conceptions of work and knowledge, and at the same time complement the impressions which I had formed of the underlying constructs of this study from my analysis of earlier interviews and of observations of classroom teaching.

Summary of Findings

I found that the beliefs, purposes, and values which teachers expressed in these interviews, and those which were implicit in their classroom practices, reflected more than the personal standpoints of the teachers themselves, but served rather to disclose an inheritance of beliefs, purposes, and values which define many features of institutional life in elementary schools.

From the Topic Interviews there emerged several preliminary hypotheses which were to be tested in subsequent interviews and observations. These were: the bestowal of a collective identity on the children being taught, a preference for teacher-directed and whole-group instruction, an assumption that children were dependent on the teacher to decide what was to be learned and how learning was to take place, the subordination of the mathematical content of DMP to concerns about and procedures for classroom management, and a tendency to treat the mathematical content of DMP as crystallized subject matter which was extrinsic to both pupils and teachers.

These preliminary findings reflected tensions which had been implicit in the development of DMP, especially when seen in its relationship to IGE. Intended as a curriculum program to be used in IGE schools, DMP had failed to identify and challenge the existing traditions of school work and mathematical knowledge which it was intended to change. For instance, DMP had been based upon a constructivist theory of learning in which children were to be active in developing mathematical knowledge under the guidance of their teacher, but persistence of behavioral objectives in DMP did offer scope for teachers to treat mathematical knowledge as subject matter to be conveyed to students in a predefined form. IGE had also emphasized the efficient management of instruction through flexible groupings of students and different patterns of "task" organization. It had depicted learning as a management process through which children were guided by teachers, and in which they were to be supported by appropriate learning environments. IGE had not attended to the social dimension of classroom learning in which were embedded assumptions about what constituted desirable knowledge and about how individuals were to relate to one another; nor did it attend to the fact that different patterns of task organization and classroom management profoundly affected the nature of school knowledge. These tensions and unresolved questions were carried into the development of DMP and became explicit in its implementation.

Classroom observations confirmed the existence of these tensions and unresolved questions in the teaching of DMP. Although teachers tended to follow the recommendations of the DMP authors in their use of manipulative materials, there was a clear trend to reassert a management perspective on instruction whenever more difficult or abstract concepts and skills were being taught. This was especially prevalent with groups of children who were described as "low" or "slow." Individual children tended to become submerged in a group process of learning where they followed instructions which were given to the group as a whole, and where they were expected to work in isolation from other students.

Pervasive features of this pattern of instruction included the elimination of small-group activities and a disinclination to allow children to work collaboratively, a more detailed prescription of "tasks" for children to perform, a closer supervision of children's performances on these tasks, the elimination or simplfication of activities which teachers considered to be "too difficult" or "too challenging" for children, an emphasis on teacher-led discussion, the establishing of rules of procedure for solving problems, and a preference to persist with a single method of solution even when children were experiencing difficulty with that method. The changes which did occur in most classes reflected a management approach to instruction. Rarely was the content of DMP modified directly or adapted to meet children's needs. However, the impact of a management approach to instruction was to alter the way mathematics was presented to children, and thus the content of DMP was changed.

Behind this approach to instruction are assumptions about mathematical knowledge and how children are to acquire it. What is to be learned is a body of concepts and skills, subject matter, which is crystallized and extrinsic. That subject matter is to be learned in the same way by all children. They are to demonstrate that they have learned what has been presented to them if they can reproduce and apply it according to predefined criteria.

Although this management approach to instruction, often accompanied by teacher-directed and whole-group learning, was most obvious in "low" or "slow" groups, it was present in a more subdued form in those groups where children were characterized as "brighter" or "sharper." There, children were given more responsibility for completing assigned work, and they were expected to have less difficulty in completing advanced work. Nevertheless, although there were changes in the pacing and supervision of instruction for the "brighter" groups, the content of instruction tended to remain static for all children. There was also an infrequent use of alternate or optional activities within these classes. Thus, the management of instruction had been adapted for these children who were less likely to present problems in learning.

A management approach to instruction had been effected by a series of technical changes to the activities recommended by the DMP authors. Fundamental to that pattern of teaching is a subordination of the mathematical goals of DMP to the requirements of smooth and efficient management of instruction. With the notable exception of one teacher, this pattern of teaching was evident across all classes.

In the concluding chapter of this study, I present a conception of mathematical inquiry as an intellectual craft which is supported and developed within a scholarly community. I relate that account of mathematical inquiry to that which was envisioned by the DMP authors, and ask how that vision of mathematical inquiry has been affecte by a management approach to the teaching of DMP. Its impact was evident in four important aspects of mathematical inquiry: abstracting, inventing, proving, and applying. Whenever a management approach to instruction was adopted, opportunities for children to engage in activities which embodied these processes were limited. Moreover, under this pattern of instruction, children's inventing, proving, and applying mathematics tended to be redefined in a narrower sense than had been intended by the DMP authors.

The beliefs, purposes, and values which underpin a management approach to instruction are at variance with the intentions of those who wrote DMP, and equally at variance with the beliefs, values, and purposes which support mathematical inquiry as I presented it. The conduct of mathematical inquiry requires that children exercise judgment and choice, that they are able to choose among alternate strategies, that they have a sense that they are able to create and test that mathematical knowledge. To put it briefly, mathematics needs to be presented to them as a human creation, marked by good guesses as well as bad, and by evolving standards of proof. By contrast, a management approach to instruction so emphasizes the performance of predefined tasks, the role of the teacher as manager of all significant instructional decisions, and the teaching of a fixed body of subject matter that opportunities for individual choice and judgment are severly curtailed, along with any sense that children are participants in a collaborative enterprise of building mathematical knowledge.

The fact that DMP has been implemented in ways in which its developers did not expect or intend cannot be explained by reference to the "mindlessness" of the developers--if only they had been more explicit in their directions; or by reference to the "mindlessness" of teachers--if only they had studied the materials more closely and understood their intent better. These are naive explanations which do an injustice to the DMP authors and to those teachers whom I observed. The fact that DMP has been implemented as I have described it is evidence of a pervasive and enduring set of beliefs, purposes, and values which define appropriate relationships between teachers and children, what constitutes desirable mathematical knowledge for children to acquire, how they are to acquire it, and having acquired it how they are to demonstrate competence in it. Behind a management approach to instruction are two assumptions: first, that children are consumers of mathematical knowledge which has taken the form of a fixed body of subject matter to be conveyed to them; and, second, that teachers are consumers or managers of an externally prepared curriculum package in which that subject matter has been incorporated. Neither in the case of children in what they learn, nor in the case of teachers in what they teach is there likely to be a lively sense of ownership of and control over their respective fields of work. This wider economic perspective serves as a reminder that patterns of school work and mathematical knowledge are not shaped by a logic which is peculiar to schools themselves; nor are they likely to be altered as simply as has been assumed by those who have produced innovative curricula.

Chapter 3

THE ORIGINS OF DMP

In order to depict the conceptions of work and knowledge implicit in DMP, it is necessary to place DMP in the context of a long and important tradition aimed at the reform of elementary education in the United States. DMP originated as part of the movement toward Individually Guided Education (IGE). These two movements need to be seen as part of a longer movement of curriculum and school reform which found expression in the United States in the 1960s and 1970s.

To view DMP, then, as just another mathematics program without linking its development with that of IGE would be to ignore a whole current of educational change which characterized the school reform movement of that time. However, in this chapter, I argue that DMP and IGE both failed to challenge existing traditions of school work and knowledge or even to understand what those traditions were, even though each aspired to radical reform -- of schooling, in the case of IGE, and of mathematics teaching and learning in the case of DMP.

In this chapter, I discuss, first, the development of IGE and how it sought to change elementary education; second, the development of DMP; and third the conceptions of work and knowledge incorporated into DMP.

Development of IGE

This section presents a sense of the history of the educational ferment which led to the development of IGE. In a retrospective study, Romberg (in preparation-b) argues that the genesis of IGE can be traced back to the 1950s when parts of the academic community were directing a concentrated barrage of criticism at the lack of intellectual training in U.S. public schools. This criticism was directed especially at the alleged fruits of the progressive education movement and those schools of education which had aligned themselves with a "life adjustment" curriculum.

Kliebard (1979) sees in this stream of criticism a persistent charge that educationists had turned their backs on the rich heritage of intellectual culture, which they had a duty to hold in trust, and had replaced it instead by "a kind of rehearsal for the efficient performance of predicted adult activities" (p. 281). Against this social efficiency movement in education, Bestor (1953), one persistent critic of progressive education, argued that schooling could not possibly prepare students for the diverse and unfathomable demands of a nebulous future; on the contrary, its role was properly construed as custodian and transmitter of the society's intellectual culture and heritage to the next generation. This persistent criticism of contemporary pedagogy, Romberg (in preparation-b) argues, helped to prepare the ground for the wave of national reaction which overtook the U.S. following the apparent demonstration of Soviet technological superiority in the launching and retrieval of Sputnik in 1957.

That technological "defeat" for the U.S., and the often shrill reactions of politicians and the press, gave credence to the allegation that public schools were "soft" on intellectual development, especially in the sciences and mathematics. This same movement of criticism of contemporary pedagogy embodied an implicit call for reform; and, as such, drew attention to those already established, but not well recognized, efforts in curriculum development. As far back as 1952, the University of Illinois Commitee and School Mathematics (UICSM), for example, had commenced to develop new curriculum materials "which emphasized the structure of the disciplines" (Romberg, in preparation-b). Programs such as this were among the first to benefit from federal support for research into and development of new curricula which followed the passing of the National Defense Education Act in 1959.

In this new wave of criticism of public schools, the academic community continued to have persuasive advocates such as Jacques Barzun (1961). But the chorus of criticism was not joined by "born-again" advocates of social efficiency such as Admiral Rickover (1959). Not only was Rickover advocating a return to hard discipline in academic subjects, but his rationale, unlike that of Bestor and Barzun, was that these disciplines were needed to ensure the survival and technological supremacy of American society.

At the same time, criticism of the public schools was mounting in other segments of society. Minority groups, notably the black community, pointed to evidence that their children were not performing well on standardized tests. They were demanding that the schools seek to remedy these disadvantages under which their children appeared to be placed.

Yet, at a time when criticism was being directed at public schools from several quarters, there continued in Post-World War II United States a buoyant confidence that the same technical expertise and procedures of rational management, which had produced a military victory and a revitalized economy, could be harnessed to redress these weaknesses in and to improve the quality of schooling itself.

Among those who saw themselves well qualified and equipped to bring knowledge and technical expertise to the reform of schooling were the psychologists. Psychologists, such as E. L. Thorndike (1922), had exercised a profound influence on educational practice in the 1920s, but psychology had grown in confidence and stature largely as a result of the development of training programs based on behaviorist principles during the Second World War. That which had achieved so much in the development of military training programs appeared to hold considerable promise in the reform of schooling. It is no coincidence, therefore, that an eminent educational psychologist, Herbert J. Klausmeier, was the founding father of IGE and nurtured the concept of Individually Guided Education from its beginnings in the 1960s.

The entrance of psychologists into the enterprise of improving schooling brought with it a mixture of new rhetoric and diverse intellectual traditions. Not only did the rhetoric of reform bring a new terminology to bear on the task of the school, as illustrated by the terms "cognitive learning," "individualized instruction," and "multi-unit school," but promising new directions to be investigated as well. Some of these directions pointed toward "aptitude-by-treatment interaction" (Cronbach, 1975), others pointed to the development of "hierarchies of learning" (Gagne, 1962), or of more intricate taxonomies of educational objectives (cf. Bloom, 1956).

In this movement for change, Romberg (in preparation-b) detects three recurring themes:

1. An emphasis of the cognitive processes which underlie the growth of concepts and skills,
2. The emergence of an engineering model of curriculum development, itself rooted in systems analysis and tied to a behaviorist conception of change,
3. A preoccupation with individual differences among students, particularly with regard to different aptitudes and rates of learning; a preoccupation which was influenced deeply by current psychological research and a long tradition of psychometric measurement.

Persuasive though these themes were as banners behind which those committed to reform were to unite, these themes grew out of and were supported by diverse theoretical positions. Different and often conflicting theories of learning were brought to bear on the task of school reform. On one side, the taxonomies of educational objectives advanced by Bloom (1956) and the learning hierarchies of Gagne (1962) were rooted in a behaviorist tradition; but from another perspective, children were seen as moving through distinct stages of cognitive development (cf. Piaget, 1926, 1955) and as needing an educational environment in step with their current stage of cognitive development, and, at the same time, assisting children to move into the next stage. These divergent theoretical perspectives were to be reflected in the curriculum innovations which accompanied the various reform movements such as IGE.

In one sense, it was clear what IGE was against. It was against the traditional pattern of the age-graded and self-contained classroom. Its proponents argued that the traditional classroom was an inefficient means of fostering "an environment in which the individual students learn at rates appropriate to each student and in a manner suitable to each student's learning style and other intellectual and personal characteristics" (Klausmeier, 1977, p. 7). IGE proposed that individual students with common learning needs should be grouped together, and their learning managed according to the principles and procedures of the IGE Instructional Planning Model.

IGE sought to challenge the belief that schools were achieving what they claimed to be doing: namely, providing a quality education for each child according to her/his level of ability and achievement. IGE believed that schooling was a goal-oriented activity; but real accomplishment may fall a long way short of what is implied by the rhetoric of schooling, and may be altogether different. To assist schools to accomplish the goal of a quality education for each child

IGE proposed a set of conditions which were intended to remedy the inefficiencies of the traditional classroom. These were the creation of "clearly defined roles and responsibilities, shared decision making, continuous pupil progress, personalized instruction, active learning, evaluation related to instructional objectives, involvement of parents and support from the community, and support by responsible education agencies" (Klausmeier, 1977, p. 7).

These conditions were to be effected by implementing the following steps which are constitutive of IGE:

1. the multiunit organization,
2. instructional planning for the individual student,
3. instructional programming and use of compatible curricular materials,
4. evaluation (of students) for educational decision making,
5. home-school-community relations,
6. facilitative environments, and
7. continuing research and development. (Klausmeier, Rossmiller, and Saily, 1977, pp. xvii-xviii.)

While these components are intended to foster the attainment of those conditions cited above, the focus of change has shifted to procedural and organizational issues. In effect, IGE had set out to bring greater efficiency to the predominant traditions of work and knowledge in the elementary school.

IGE made efficient management of instruction one of the central concerns of its pedagogy. Like the traditional classroom, IGE continued to use time as a principal instrument of control of instruction. Not only is the school day broken up into sequential time modules, at the end of which children are moved from one activity to another; but time also takes on a more subtle dimension in indicating "appropriate times" when certain pieces of schoolwork are to be taught. These appropriate times then become part of the folklore of schooling: for example, multiplication and division are traditionally reserved for initial teaching in the fourth grade; geometry is the traditional mathematical subject to be studied in year 10.

For IGE, as in the traditional classroom, knowledge continued to be equated with the artifacts of instruction. Those artifacts were derived from a body of subject matter. It was the task of instructional planning to ensure that this subject matter was transmitted to pupils in the most efficient manner possible.

IGE brought a management perspective to instruction within which existing traditions of school work and knowledge appeared commonplace. But as soon as one recognizes that pedagogy is a social process which links individuals to communal forms of knowledge and work, one sees that IGE, in challenging the procedures of traditional schooling, had merely substituted a different technology of instruction.

In the latter years of IGE, much research and in-service work with teachers was devoted to specifying the outcomes of IGE and the performance objectives for its implementation. (See IGE Implementor's

Manual, Evers, Fruth, Heffernan, Karges, and Krupa, 1975.) This supports my concern that the implementation of IGE often tended to be seen as a technical or procedural affair; and that, instead of keeping a sharp focus on the purposes of change and its challenge to entrenched conceptions of work and knowledge in the elementary school, IGE implementation became preoccupied with marking the extent to which schools had implemented certain performance objectives. This distinction between "technical" and "constructive" schooling has provided a sharp analytical tool for the IGE Evaluation Study Phase III (Popkewitz, Tabachnick, & Wehlage, 1982).

The progressive development and diffusion of IGE has been elaborated elsewhere by Klausmeier (1977). It is sufficent to say that by the end of 1973 the diffusion of IGE had been so extensive that a national Association for Individually Guided Education had been established. This initiative was taken by the IGE coord tors in twelve of the states together with the support of the Wisconsin R & D Center and the IGE Teacher Education Project at the University of Wisconsin. By the end of the 1974-75 school year, the number of schools implementing IGE was estimated by Klausmeier (1977) to have exceeded two thousand.

Therefore, it is within the context of this important national movement for the reform and improvement of elementary education that DMP must be situated. Its own development was a deliberate response by mathematics educators at the University of Wisconsin-Madison to produce curriculum materials in mathematics which were compatible with the principles and practice of IGE.

Development of DMP

In order to adapt instruction to the needs of individual students, the developers of IGE proposed a model of instructional programming which set down the requirements for successful teaching within IGE. This model emphasized the setting of instructional objectives, the planning and implementation of a program of study for each child within the overall program, and assessing students for the attainment of predefined objectives. It was not assumed that all students would engage in the same kind or number of activities; and although teachers were to use the same objectives to evaluate students' learning, they were expected to vary the amount of attention and guidance given to individuals. Variations were also expected in the use of printed and audio-visual materials by students; in their use of space and equipment; and in the amount of time spent by students in the various modes of learning suggested by the IGE model, for example, in one-to-one teaching, small group work led either by the teacher or a student, or individual seatwork (cf. Klausmeier, 1977, p. 1(

Romberg (in preparation-b) reviews the difficulties which confronted teachers as they tried to implement the IGE model of instruction using the curriculum materials then available. These materials, usually textbooks, often had no clearly specified objectives, their evaluation of students' learning was not objectively referenced, and the activities embodied in the materials either lacked the variety of approaches, which IGE had recommended, or the activities were simply unrelated to any objectives.

These problems could have been overcome by having teachers write their own materials, using them in the local school and possibly sharing them among a group of schools; or the task of curriculum development could have been undertaken by a group of mathematics educators, including practicing teachers, who had the time and the resources to do so. The latter option was adopted, and DMP became one of three major curriculum development projects undertaken by the Wisconsin Research and Development Center for Cognitive Learning (WRDCCL). The other two projects were the Wisconsin Design for Reading Skill Development (Otto and Askov, 1974) and the Pre-Reading Skills Program (Venezky, Pittleman, Kamm, and Leslie, 1974).

There was no dearth of modern mathematics materials on which to base DMP. But almost all reflected a modern-structural approach to mathematics. The authors of DMP did not believe that this kind of mathematics was appropriate for children in elementary school, nor did they see in the new textual materials the variety of activities which would be needed by a program intended to fit an IGE model of instruction. Many of the modern mathematics textbooks seemed to assume that, since the mathematical content had been recast in a modern structural form, the task of instruction was to convey or transmit this new content to pupils. Put in another way, the "new" mathematics, like that which it was intended to supplant, was still seen as something extrinsic, as something which needed to be passed on to pupils and learned by them.

DMP was designed on a modeling approach to mathematics, and measurement was to be the basis of mathematical modeling. DMP also made specific provision for teachers to group children in a variety of ways, and to adopt a flexible sequence of instruction for particular children or groups of children, through station work, alternate activities, and optional activities. It sought to avoid the pitfalls of both the traditional approach to the teaching of arithmetic and of the modern-structural approach to the teaching of mathematics. The authors of DMP saw their task as one of developing "a second generation" program of mathematics instruction, one which built upon the insights of the first wave of innovative programs which emerged during the 1960s, but at the same time avoiding the mistakes of that era.

Like the developers of innovative mathematics programs in the 1960s, the authors of DMP saw the traditional approach to the teaching of mathematics as being almost exclusively preoccupied with arithmetic. There, the content of instruction seemed to have been chosen on the basis of "pragmatic eclecticism," resulting in programs which were seldom more than lists of topics to be covered or competencies to be mastered. Kaufman and Steiner (1968) describe the predicament of traditional mathematics courses succinctly:

> All mathematics taught was understood more or less as a static body of knowledge, prefabricated in the one and only correct way to be handed down to the next generation. (p. 3)

Rarely was there any attempt to organize instruction around a unifying theme from mathematics itself. Often the emphasis was on "practical arithmetic," and a traditional course in arithmetic might have consisted of adding and subtracting sundry commodities, money, weights, and measures; multiplication and division, converting between various units of measurement; finding averages, areas, perimeters, and volumes; and so on. Little attempt had been made to develop a "feel for the whole."

The pedagogical approach to the teaching of elementary arithmetic had been deeply influenced by the work of associationist psychologists such as E. L. Thorndike (1922). Since they had identified the subject matter of arithmetic with the formation of stimulus-response bonds, strong support had been given to the use of drill and practice in the teaching of arithmetic.

Largely as a reaction to the eclecticism of the traditional approach and to its lack of structural unity, the new programs which accompanied the curriculum reform movement of the 1960s adopted a modern-structured approach to mathematics. These programs aimed to present mathematics in a more organized and logical form. Many of the developers of the new programs were themselves mathematicians with an interest in displaying the logical structure of mathematical systems. Their approach, as Romberg (in preparation-a) comments,

> is quite different from the traditional approach. Sets, functions, statistics and probability, and logical thinking are explicitly included in the modern structural approach. Moreover, even when the goals of the two programs are the same, the concepts and skills may differ.

For example, the goal of teaching arithmetic is common to traditional and to modern structural approaches, but the latter will stress relationships between arithmetical operations (subtraction being the inverse operation of addition, division as the operational inverse of multiplication and so on), as much as having children perform the four operations correctly. Both approaches emphasize the applications of mathematics: the first uses the relevance of an application to consumers or citizens as the criterion for inclusion in a mathematics program, whereas the second will use applications in order to introduce students to the concepts of mathematical modeling.

Criticisms of the modern structural approach have focused on the difficulty of dealing with the concepts of mathematical structure in ways which have meaning for young and non-expert students of mathematics. Fremont (1967) critizes the modern structural approach for its failure to attend to the problem that

> the student cannot possibly appreciate the role of unification if he has no comprehension of what is being unified. More than that, because of the fact that the student does not yet know of the need for or the importance of unification, he is being asked to accept the teacher's word for the fact that this is an important idea to study. (p. 175)

Three Other Mathematics Programs

The first generation of programs which comprised the modern structural approach to mathematics instruction tended to be preoccupied with the "reform" of content, and had neglected to attend to the psychological and pedagogical implications of these new programs. DMP was one of a second generation of innovative programs which aimed to bring new insights into psychology and pedagogy to bear on mathematics teaching. But it was not alone in this enterprise. Three other programs are worthy of presentation because they represent different strands among these "second generation" programs; and in particular show how different approaches to work and knowledge can give rise to quite different artifacts of curriculum development. The descriptions which follow utilize the rhetoric of the curriculum developers to highlight the points of contrast among the developers' perspectives; these descriptions are not intended to illustrate the actual implementation of the programs being discussed.

At the far end of the spectrum, one could place the Direct Instruction Model of Teaching which was sponsored by the University of Oregon. This model of instruction was oriented specifically toward the teaching of basic skills in arithmetic and reading. Its basis lay in the principles of operant conditioning and behavior analysis of B. F. Skinner (1953). However, the instructional model itself was developed from Becker's "work on the systematic use of reinforcement procedures in the classroom and Engelmann's work in the Bereiter and Engelmann (1966) preschool program" (Becker & Carnine, 1980, p. 432).

Taking its name as an acronym from Direct Instruction System for Teaching and Remediation, DISTAR Arithmetic (Engelmann and Carnine, 1969) takes a stand on individualization which is diametrically opposite to that of DMP. Arguing from a strict behaviorist position, the proponents of a Direct Instruction Model of Teaching claim that the requirements for an instructional program are determined by what is to be taught not by attention to who is being taught. In a program like DISTAR Arithmetic, individual differences between students are only significant in determining which group a child will belong to. Children are expected to enter the program at different levels, and according to the level of performance they will be placed in a group comprising five to ten children. The lowest performance groups will be smallest and the highest performance groups the largest. Groups are to be made as homogeneous as possible for each unit of instruction, but they are not intended to be static during the course of the year. Once children are grouped, they face the same sequence of tasks and questions. They will be taught by teachers using the same techniques, and they will be expected to achieve a predetermined level of mastery before being allowed to proceed to the next task.

DISTAR Arithmetic is a predetermined series of tasks or routines to be learned. The content of instruction is carefully analyzed in terms of behavioral objectives upon which instruction is then based. The quality of teaching is made uniform by ensuring that teachers follow a script which sets out in precise detail what they are to say and do. While the use of behavioral objectives is paramount, there is no attempt made to link together the content of instruction under unifying themes which aim to mirror the structure of mathematics. Thus, knowledge is defined as an artifact of instruction, and is quite extrinsic to both teacher and students. Not only is the work of teachers predetermined by the design of the program, the work of students is also predefined in terms of compliance with the program design.

Like the Direct Instruction Model in respect to its strong behaviorist base, but completely unlike it in respect to individualization is Individually Prescribed Instruction (IPI) Mathematics (Lipson, Kohut, and Thomas, 1967) developed by the Mathematics Curriculum Staff at the Learning Research and Development Center at the University of Pittsburgh.

The developers of IPI express with great confidence their reliance upon behaviorist psychology. For example, Jones (1976) proclaims almost smugly that

> Research on instrumental behavior and the effects of reinforcement has, as a matter of fact, led to the only major application of learning theory to educational technology: the development of programmed instructional techniques (p. 15)

The program materials present mathematics as a collection of routines to be learned and mastered before students proceed to the next task. In the elementary school, the content of mathematical instruction is heavily dominated by arithmetic stressing competence in the four basic operations including the use of algorithms. There is also a substantial section on geometry in the elementary program. Each page of the program materials clearly specifies the level, unit, and skill to be mastered by the students. There is no variation expected in how pupils approach each problem. Each problem is intended to contribute toward the learning of a predefined skill or concept, and as such the possibility of a multiplicity of correct responses is deliberately excluded in the design of the program.

Lindvall and Bolvin (1976) set down the principles of programmed instruction upon which IPI is based:

> The development of a program requires that the behaviors that lead to terminal behaviors are carefully analyzed and sequenced in a hierarchical order such that each behavior builds on the objective immediately below it in the sequence and is prerequisite to those that follow it. (p. 178)

The implementation of IPI requires an extensive battery of self-managed instructional materials for individual learners to use. Thus, IPI interprets individualization strictly as self-managed instruction: Individualization means working through the curriculum on an individual basis. The work of students is to follow the sequence of tasks allocated to them by the teacher; and to have the quality of their work appraised by the teacher. The work of the teacher is depicted as very similar to that of a factory manager: Teachers are to make use of a predetermined system for determining individual abilities and needs; they are to set attainable goals for students to reach; they are to provide appropriate conditions under which students can achieve those goals; and they are to monitor the operation of the program itself so that adjustments can be made where necessary to IPI's administrative procedures and instructional practices.

DMP, as I have already noted, broke with the strong behaviorist tradition embodied in DISTAR Arithmetic and IPI, but maintained a behavioral orientation in its organization. DMP attempted to integrate a constructivist theory of learning with an instructional process where the outcomes of learning were behaviorally referenced. These objectives were intended to identify what children were expected to learn; but the authors of DMP did not intend that these behavioral objectives would define all the mathematics that children would learn; nor was it expected that instruction should be determined by focusing narrowly upon these objectives. It should be noted that although the overriding goal of DMP was that children would become independent problem solvers, the DMP authors made no attempt to break down "problem solving" into a list of constituent behaviors to be taught. Indeed, they believed that this was an impossible and inappropriate task, and that any attempt to specify problem solving in terms of behavioral objectives would distort the nature of mathematical problem solving. Following a constructivist approach to learning, the authors of DMP believed that children's own insights into mathematics and their latent strategies could be capitalized upon by the teacher in assisting children to become independent and self-assured problem solvers. These insights and strategies would need to be articulated through dialogue and discussion, both between teacher and pupils and between pupils themselves.

Thus, DMP never interpreted individualization in the restricted sense of independent, self-managed instruction as did IPI; nor did it see tightly prescribed group instruction, along the lines of DISTAR Arithmetic, as providing a suitable environment for the development of mathematical insight and competence in problem solving. The authors of DMP fully expected that teachers would be faced with having to compromise between the needs and interests of the individual student and the welfare of the whole group; just as teachers would have to chart a difficult course between taking charge of instruction and leaving children free to explore mathematics on their own. The authors of DMP clearly saw teachers as responsible for children's mathematical development, but they hoped that teachers' pedagogical judgments would be tempered by a vision of mathematics in which children could make choices, apply guesswork, and upon which they could apply their own insights. Unlike the previous two programs, DMP sought to unify children's mathematical experiences by adopting a modeling approach through measurement.

Finally, and at the other end of the spectrum from the Direct Teaching Model, one could place the Comprehensive School Mathematics Program (CSMP) Elementary School Curriculum (Kaufman & Sterling, CEMREL, 1975). This program totally disavows the use of behavioral objectives, adopts a constructivist learning theory, and interprets individualization in terms of independent discovery of mathematical relationships by children.

This program made no a priori assumptions about what mathematics will be learned by children. However as Kaufmann and Steiner (1968) argue, the domain of elementary school mathematics

> will be characterized by certain levels of abstraction and certain degrees of rigor and complexity, but it will also be determined by categories such as "significant," "adequate," "liberating," "motivating," etc. . . . It includes the incorporation of new mathematical ideas at an elementary level and an elaboration of different approaches to one and the same mathematical topic, . . . the investigation of the concept of the spiral approach, etc. (p. 6)

As might be expected, the mathematical community was heavily involved in the choice of content and the design of the curriculum materials. When CSMP materials were produced by CEMREL, they had a major emphasis on the following areas of mathematical investigation:

strategies for logical thinking and classification,

relations among objects, especially upon numerical relations,

measurement and geometry,

probability, statistics and strategies for counting.

These were to be its several unifying themes. The instructional program of CSMP had three focal points. The primary point of focus was the nature of mathematical content. But the authors saw that there needed to be "vehicles" which carried the content and its applications directly to children, and which allowed the children to interact directly with mathematical material. CSMP tried to achieve this without introducing abstract mathematical language and formality, as had been the case with many of the first generation of innovative mathematical programs. CSMP also developed a "pedagogy of situations" where children would be encouraged to share their insights, suggestions and observations with the whole class. Thus, the classroom was depicted as a microcosm of the mathematical community; and the collective insight and mathematical experience of the class as a whole was considered at the arbitrator of the validity of pupils' individual contributions.

The individual student within CSMP, like the professional mathematician, was expected to work alone "at least 90% of the time . . . in small groups with a teacher available whenever needed" (Kaufman & Steiner, 1968, p. 14). The role of the teacher within CSMP was depicted as one of stimulating children's interest and insight into mathematics. There was to be ample provision for individualization through suggested activities and worksheets, once children are introduced to and have developed an interest in a particular mathematical idea.

While representing a radical departure from programs such as DISTAR Arithmetic and IPI, CSMP bears some resemblances to DMP. Both programs sought to present mathematics using certain unifying themes from the discipline itself.. Both programs hoped to assist students to become independent problem-solvers, and had adopted a constructivist theory of learning which accorded children the status of having their own mathematical insights to contribute to the process of learning. Unlike CSMP, DMP preferred to specify, in a minimal way, the desired outcomes of children's learning in terms of behavioral objectives, whereas CSMP refused to countenance behavioral objectives in any form. On paper, at least, it seemed that CSMP gave the teacher a role which was concerned more with assisting children and facilitating their learning, than was the case in DMP where it was expected that teachers would often make presentations to the whole class. Whether in practice CSMP would work out that way is not issue. In terms of its developers' intentions, it allows one to see DMP, as do the other two programs, in a wider context of reform in mathematics education, and of continuing debate about the work of teachers, the work of students, and what constitutes appropriate mathematical knowledge for children to learn.

Developing Mathematical Processes

How the authors of DMP interpreted the work of teachers, the work of students, and what was appropriate mathematical knowledge for children to learn can be ascertained from the pedagogical, psychological, and mathematical perspectives which the authors of DMP espoused. Those perspectives are now discussed in greater detail, and their implications considered for the underlying constructs of this study.

Pedagogical Perspective

Arguing that the child's point of view should be considered in developing a mathematics program for the elementary school, the authors of DMP sought to chart a middle course between responding solely to children's interests and needs--a course which they took to be unrealistic and impractical--and placing a dominant emphasis on content which had tended to characterize the modern structured approach. This latter approach had failed, they argued, to utilize relevant knowledge about children's cognitive development and about how children learn. The pedagogical approach of DMP presented the teacher's role as being broader than simply presenting or transmitting knowledge to students. The teacher's role was depicted also as one of questioning students about mathematical ideas, guiding their insights and strategies, observing their work, and summarizing their conclusions.

The pedagogical approach of DMP was to be linked to the philosophy of IGE and its basic premise that individualization should be reflected in the proper management of instruction. IGE was committed in general to the belief that, since children learn at different rates and in different ways, alternative approaches to learning concepts were called for. This emphasis on the individual learner w s not taken by the authors of DMP or IGE to imply that children should learn independently. Indeed, in the revised S- and A-Topics, many activities are introduced by the teacher to the whole class. Thereafter, children might engage in station work, a small group activity, or in individual seatwork. DMP saw interaction between the teacher and children, and between children themselves as crucial in helping children to organize and reflect upon their own ideas, and to articulate the strategies available to them. This focus on interaction was also seen as an important means of reducing competition among children, and of fostering a spirit of collaboration in the classroom.

However, the subordination of the individual student to the whole group was a feature of the pedagogy of DMP. The authors did recognize that their focus upon groups with common learning needs would present teachers with no alternative but to place the interests of the group first:

> we believe that you cannot take care of every child's individual needs, interests, and abilities every minute of the day. You have to stay sane. You have to make compromises and also look to what is best for the group. (Romberg, Harvey, Moser, and Montgomery, 1975, p. 72)

There are tensions in these implied definitions of the work of teachers and students. On one side, teachers are seen as responsible for setting instructional objectives and for ensuring that they are pursued by children. From another perspective, teachers are depicted as facilitating children's learning and as creating opportunities where children can pursue their own interests, make their own choices, and develop their own strategies to solve problems.

Thus, the pedagogical perspective of DMP, while trying to accommodate the child's point of view in the teaching and learning of mathematics, left unresolved the relationship of the individual child to the processes of group instruction. Given a deeply embedded practice of whole group instruction in the traditional elementary school, the authors of DMP should have confronted this issue more explicitly. As it was, their pedagogical advice could have been interpreted as suggestions for good classroom management within the <u>status quo</u>, rather than as an invitation to radical reform. As such the content of DMP was likely to remain subordinate to teachers' concerns for efficient classroom management.

Psychological Perspective

In developing a modeling approach to mathematics into a program of instruction, the authors of DMP committed themselves to a constructivist theory of learning. This view is that children are active agents in organizing their own learning, and thus can be assisted by a teacher able to elicit strategies available to them. The developers believed, therefore, that children should begin with concrete experiences and move to the abstract. They endorsed Lovell's (1972) dictum that, "it is abstractions from actions performed on objects and not the objects themselves that aid forward knowledge of mathematical ideas." For example, children in Grades K-2 are first made familiar with the basic attributes of length, weight, capacity, shape, and numerousness. At the same time, children are led through three phases of representing attributes: first, a physical phase, then a pictorial phase, and finally a symbolic phase. These three phases are related to the development of processes which represent and transform attributes. Attributes are described and classified; they are compared and ordered; sets of objects can be equalized, joined, or separated with regard to an attribute; at a later stage, children will be introduced to partitioning and grouping a set of objects.

However, along with Lovell (1972), the authors of DMP also believed that, parallel with this gradual process of abstraction from physical situations, children also need to be introduced to appropriate symbolization and the working of examples. Thus, the authors of DMP endorsed the use of drill and practice as important aids to increase speed and accuracy at a time when the child is already able to perform symbolic operations. But they did advise against the premature introduction of drill and practice.

Another feature of the psychological perspective of DMP was a belief that children should be motivated, not by the extrinsic attraction of rewards and competition, but by the intrinsic challenge presented to children when they are confronted by a problem situation. Problems were to be chosen and presented in a variety of formats and settings. Three criteria were to be used in the selection of problems which would be intrinsically motivating to children:

(a) problems come from the child's environment,

(b) the child sees them as problems to which she/he is able to find solutions,

(c) the problem tasks provide immediate and informative feedback. (cf. Romberg, in preparation-a)

However, motivation needs to be placed in a wider context than simply the selection of problems. Other aspects of motivation were drawn from IGE, where teachers were expected to motivate students by focusing their attention on desired objectives--although, in the case of DMP, these objectives were not expressed in the form of many behavioral objectives--by helping students to set and attain goals, by providing feedback and appropriate praise, and by establishing good patterns of work for children to imitate.

These prescriptions and recommendations are drawn from a psychological perspective on learning which filters out the social, epistemological, and ethical dimensions of school work and mathematical knowledge. The acquisition and application of mathematical knowledge cannot be viewed solely as a psychological process when mathematics becomes part of classroom work. Embedded in classroom instruction are assumptions about what knowledge it is desirable for children to learn, how they are to learn, and how they are to show competence in what they have learned. A psychological perspective, such as that adopted by the DMP authors, is neutral regarding the degree of responsibility and autonomy which children ought to exercise in the creation and testing of mathematical knowledge. The same perspective is neutral regarding the kind of mathematical knowledge as learned and the social relationships which should accompany that learning.

By casting children in the role of learners, IGE, as did DMP, seemed to assume that "wise adults can plan, organize, and make decisions about instruction based on information about differences (among children)" (Romberg, in preparation-b). In IGE and DMP, this paternalist view of the work of children found expression in the objective-referenced model of instruction. The attainment of these objectives and the sequencing of instruction for different children rests with the teacher.

The fact that teachers were assumed to be responsible for making decisions about children's learning is not a criticism of DMP. What is critical is that the authors' pedagogical and psychological perspectives, by leaving unchallenged existing traditions of school work and mathematical knowledge, allowed teachers to assimilate DMP into the status quo.

Mathematical Perspective

Furthermore, the emphasis on behavioral objectives, by identifying only the logical qualities of students' thought, seemed to divorce mathematical inquiry from the acquisition of personal knowledge through choice, judgment, responsibility, and control over what is learned. Mathematical inquiry thus tended to be portrayed as a set of crystallized logical forms to which students needed to be introduced and which they needed to assimilate and reproduce.

It might be said that the constructivist orientation of DMP was intended to provide an effective counterbalance to this distortion of mathematical knowledge by recognizing and encouraging children's active role in learning: in being helped to develop their own strategies for solving problems, in choosing between alternate strategies, and in explaining mathematical ideas in their own terms rather than employing predefined terms of the teacher. While committed in the use of behavioral objectives, the authors of DMP argued that they were using these objectives to identify what children were expected to learn, and that these behavioral objectives were not to be used as a basis of instruction. Theirs was a distinction between objective referenced instruction and objective based instruction.

However, the focus upon the logical qualities of mathematical thought and the procedures which children were to apply to mathematical inquiry, whether expressed in terms of behavioral objectives or not, presented a view of mathematical knowledge where a sense of the whole was likely to be absent, and where mathematical thought was divorced from asking "why?", searching, imagination, guesswork, tentativeness, and the use of proof and evidence, all of which have characterized mathematics as a human activity.

Rather than focus on what constitutes desirable mathematical knowledge for children to learn, the authors of DMP tend to elaborate their mathematical perspective from a pedagogical standpoint. This is illustrated by their approach to problem solving as a basic goal of DMP. In attaining this goal of problem solving, DMP does teach some broad strategies for writing and solving number sentences and story problems involving addition and subtraction. In the first round of published materials, an equalizing strategy was used as the principal means for solving problems (Romberg, Harvey, & McLeod, 1970). In the revised topics (S1-A4), a Part-Part-Whole analysis was adopted as the principal means of analyzing story problems prior to their solution.

However, the authors were aware that the solution of problems could easily become mechanical for children, and could indeed be taught in a mechanical fashion. From a pedagogical standpoint, the DMP authors believed that this problem could be avoided by requiring children to analyze the problem into what is known and what is not known, by writing and solving a number sentence which represented the problem, and then by determining whether the solution obtained is the correct solution of a problem sitation or number sentence. In validating their answers children might use estimates, reasoning, trial and error, or an alternative computation. Likewise, the authors advise teachers that

> problem solving cannot occur if the children do not understand the problem being posed. The situations must be meaningful to them. (Romberg, Harvey, Moser, and Montgomery, 1975, p. 48)

It would be a mistake to construe simply as a pedagogical or psychological issue the problem of how to make mathematical situations meaningful to children. How one resolves the issue depends on what a teacher considers to be appropriate mathematical knowledge for children to learn, and how one conceives of mathematics as a human activity.

Making a problem meaningful might be interpreted by one teacher as giving an explanation and expecting children to accept "as authoritative the teacher's system of thought and abstraction" (cf. Popkewitz & Wehlage, 1977, p. 8). In that case, patterns of mathematical thought and knowledge are being treated as extrinsic to children, and as needing to be adopted or accepted by them if they are to become successful problem-solvers. One can see a risk that the principal strategies of DMP--equalizing and the Part-Part-Whole classification--might be treated in this way. That is of being treated as ends in their own right rather than as bridges which enable teachers to assist children to develop confidence in a variety of strategies.

The authors of DMP clearly believed that they were helping to expound their mathematical perspective by making these pedagogical recommendations. In effect, they have assumed that teachers knew what constituted mathematical knowledge. That view, which was implicit in The Task Analysis for Developing Mathematical Processes (Romberg, Harvey, & McLeod, 1970), took mathematics to be a fixed body of subject matter, and thus the task of pedagogy was to present that subject matter in suitable sections for instruction. That view of mathematical knowledge was set out by Lovell (1972). For Lovell (1972), as for the DMP authors, mathematics is subject matter to which children are directed by the teacher:

> Since mathematics is a structured and interlocked set of relations expressed in symbols and governed by firm rules, the initiative and the direction of work must be the teacher's responsibility. (p. 177)

This vision of mathematical knowledge bears little resemblance to the history of mathematics as a human intellectual activity. From a historical and social perspective, mathematics occurs within a context of beliefs about what procedures are to be followed and what is to be countenanced as acceptable work. Moreover, this context of beliefs continues to develop as new mathematical insights are accepted within the mathematical community. These agreed conventions and intuitive understandings reflect a craft notion of mathematical inquiry. From within the mathematical community, standards of mathematical inquiry are practiced, maintained, and developed. These standards are certainly not the result of applying external rules. These important features of mathematical inquiry are lost when one treats mathematics as a set of logical relationships. What is lost is the process by which these relationships have been created and tested; their link with imagination, intuition, and an aesthetic sense.

It might be said that, in proclaiming problem-solving to be the basic theme of DMP, the authors were in sympathy with the above vision of mathematics. But, as I argued earlier, the authors showed a marked inclination to present problem-solving to teachers from a pedagogical perspective, and gave very slight attention to the mathematical implications of this principal goal of DMP. Thus, Romberg, Harvey, Moser, and Montgomery (1975) adivse that: "The children measure to solve a problem. They equalize to solve a problem. They order to solve a problem" (p. 48).

In the notes for teachers for Topic S3, one of the revised topics, the authors Kouba and Moser (1979) state:

> It is our feeling that with proper guidance and inst .ction the children can be made (sic) to see that addition and subtraction problems can be thought of in the context of part-part-whole and that the analysis hinges around deciding whether the two numbers in the problem represent the two parts or the whole and one of the parts. (p. 193)

With these comments, the authors of DMP have viewed mathematics as a set of crystallized logical forms to which children are to be introduced, and which they can be made to see.

Summary

From IGE, DMP inherited a behavioral orientation to the organization of instruction. Although IGE had challenged existing patterns of elementary schooling, it had merely proposed a different technology, thus leaving unquestioned existing traditions of school work and knowledge.

While the authors of DMP maintained a similar pattern of organization through, for example, the use of behaviorally referenced instruction, they sought to counterbalance the narrowing effects of behaviorism by adopting a constructivist approach to learning. They argued that their behavioral objectives were to serve only as a means of identifying what children had learned, and were not to define their learning in terms of the set of behaviorally prescribed tasks. Through a constructivist approach to learning the authors of DMP intended that children would be able to abstract from the actions which they performed on objects. By adopting a modeling approach through measurement, teachers were to help unify children's mathematical experiences.

However, although DMP through its constructivist approach to children's learning was intended to be a radical innovation in the teaching and learning of mathematics, its pedagogical approach did not appear to identify and challenge embedded practices of elementary schooling, such as the subordination of the content of instruction to the demands of classroom management. Likewise, DMP left unresolved the relationship between the needs of the individual child and the demands of whole-group instruction.

From a psychological perspective, the authors of DMP failed to realize that a constructivist theory of learning, when applied to a classroom setting, is powerless to answer questions about what constitutes desirable mathematical knowledge, how children are to acquire knowledge, and how they are to demonstrate competence in what they have learned. A constructivist theory of learning may be able to address some of the processes implied by these questions, but it is unable to provide a theoretical framework for classroom learning.

From a mathematical perspective, DMP tended to focus exclusively upon the logical relationships which are features of mathematical knowledge. In taking this approach, the authors seemed to give less attention to those intellectual dispositions--imagination, intuition, guesswork, tentativeness, the use of proof and evidence--which confer personal meaning upon mathematical knowledge. As a result, the acquisition of mathematical knowledge could be incorporated into a management approach to instruction where the teacher's task is to introduce children to a set of predetermined logical entities. Thus, the broader mathematical goal of DMP--of enabling children to participate in the creation and testing of mathematical knowledge--was likely to be incompletely realized.

Chapter 4

A PRELIMINARY ANALYSIS

Romberg (in preparation-b) has said that

> schooling is a collective experience. For the child, being in school means being in a crowd. For the teachers, it means always being responsible for a group of students. Thus, the problem of how a small number of adults can organize and manage a large number of children is the central organizational problem of schools.

These comments serve well to introduce the theme of this chapter which is to present a preliminary analysis of how teachers in the two DMP schools have construed their own work, the work of their students, and what constitutes appropriate mathematical knowledge for children to learn. My analysis is based upon the Topic Interviews (Stephens & Romberg, in preparation) recorded with 11 teachers in the two schools after they had taught each of the S- and A- Topics for the first time. These interviews were conducted by members of the Mathematics Work Group during the years 1979-1980. Their questions are included in Appendix A.

This analysis represents a preliminary charting of the landscape. It opens up questions which I pursue in interviews with teachers and observations of their teaching. It is necessarily preliminary for another reason: The Topic Interviews upon which my analysis is based were intended specifically to check with teachers how they had taught each of the S- and A-Topics, and were not intended to elucidate directly how teachers conceived of their own work, the work of their students, and of what constituted appropriate mathematical knowledge. However, these topic interviews are a rich source of inferences on each one of the three underlying constructs of this study. The discussion is also preliminary because teachers at the time of the Topic Interviews were still gaining acquaintance with DMP. At the time of my initial interviews with the same teachers in the latter part of 1981, some of them had taught the S-Topics for four years in their revised form, including a year when they were being used in pilot edition.

Reinterpreting the Topic Interviews

By linking the work of teachers and their conceptions of children's work and appropriate mathematical knowledge to a set of shared beliefs, purposes, and values, I am pointing to a social dimension of work and knowledge. This social dimension is not captured if one refers to what a teacher does, what pupils do, and what they learn solely in terms of a set of operationally defined and conceptually independent categories. Through their work, teachers enter into an inheritance of beliefs, purposes, and values in which the practice of teaching is framed in a particular society. To quote Popkewitz (1982a):

> Schooling is an historical enterprise in which each generation has to re-establish the significance and benevolence of its institutional arrangements. (p. 10)

Thus, the curriculum reform movement of the past 20 years needs to be seen as an important stage in this process of reappraisal and challenge of the traditions of work and knowledge in elementary and secondary schools. As I argued in the previous chapter, DMP has its roots in that movement. Likewise, the authors of DMP assumed that teachers, with suitable support and through the use of innovative materials, could modify and adapt conceptions of school work and mathematical knowledge.

However, a belief that teachers are capable of acting in a planned and rational way; capable, too, of deciding to adapt and modify their conditions of work, does not imply that their work is unattended by conditions of dependence: some dependence upon the resources available, upon other human beings, including those who have helped to shape the pattern of beliefs and values which give the practice of teaching its special character in a society. Furthermore, we cannot pretend that some of the "contemporary forms of social life are not potentially coercive. Our definitions of social conditions and our expectations are created by past and present patterns of action and belief" (Popkewitz, Tabachnick, & Wehlage, 1982, p. 9). From my reinterpretation of the Topic Interviews, I argue that constraints were imposed upon the kind of changes envisaged by the authors of DMP. These constraints were rooted in the set of beliefs, purposes, and values which defined school work and mathematical knowledge in the elementary school. These conceptions began to emerge from my analysis of the Topic Interviews.

Persistent Strands

From the Topic Interviews, a number of salient features of work and knowledge emerged. Conspicuous among these features were teachers' persistent ascriptions of a collective identity to the children they taught. A second feature of the Topic Interviews was a clear presumption on the part of most teachers in favor of whole-group and teacher-directed instruction. Not only was this presumption linked to teachers' sense of a collective group of pupils, it was also reinforced by a third feature of the Topic Interviews: that pupils in their work were dependent upon the teacher for direction; moreover, their work was usually defined solely by reference to what the teacher had specified. For their part, teachers were much concerned about the management of instruction and hence about the suitability of the activities recommended in the DMP. These concerns reflected a fourth strand in the Topic Interviews: the subordination of the mathematical content of DMP to concerns about the management of instruction and the rules by which instruction is to be conducted. By and large, teachers treated the mathematical content of DMP as extrinsic to pupils and as a body of subject matter to be conveyed to pupils. Furthermore, teachers seemed to have little sense of ownership over the mathematical content of DMP and thus treated it as beyond their own control to adapt and modify.

A collective indentity. The notion of schooling as a collective experience resonates throughout the Topic Interviews. In almost every case, one finds teachers referring to the collective experience of the classroom. The interviews are replete with such comments as:

"This year, I have the top group";

"(Mine are) an average group";

"My kids are not really that challenging";

"I have a lower group";

"My kids are not real comfortable with reading";

"For low kids, . . ."

What is more remarkable is that these characterizations were not made by teachers in response to a specific question as to how they would describe their classroom group. There was no question which was designed to elicit that kind of comment in the topic interviews. These comments arose quite unsolicited in teachers' responses to questions such as "Do you plan alone or with others?", "Do you find some of the activities unclear?", "Are there activities which you consider superfluous?". To say that these comments occurred in teachers' responses to questions which were not explicitly directed as having them characterize the group of children whom they were teaching is not to brand such comments as irrelevant. I interpret them, on one level, as serving to let the interviewer know the kind of considerations which the teacher believed would justify the decisions she/he had taken. Far from being irrelevant, these are the considerations used by teachers to justify their selection or omission of certain elements of curriculum content, or their decisions whether or not to implement certain classroom arrangements, such as working in small groups. For example,

> They (the DMP authors) could have left out all the words, and I could have just read the stories; and they could have left it very simple with the box and a spot for writing the sentence. Too much for my kids. (Teacher E)

> For a low kid, I think I'd leave out the patterns . . . . If something looks a little too difficult, or something that looks that it might not be a whole lot of value to them, I didn't use it. (Teacher K)

> I try to let the kids who can go ahead independently . . . . The slow group has very low reading ability so it's very hard for them to do anything that requires reading on their own. Maybe two kids can go ahead with the reading . . . . (Teacher F)

> The children I have (are) not the most advanced and so even the extra pages were extremely important for my kids for reinforcement. (Teacher B)

> All the kids in the top group understood that perfectly, it was just redundant. (Teacher J)

At the outset, one needs to interpret these comments as having been made by teachers in interviews conducted by members of the Mathematics Work Group. Sometimes the interviews were conducted by senior faculty members, at other times by full-time research assistants or by graduate assistants. These were the people who had devised DMP, or who, at least, would have been seen as coming into schools to do research on mathematics education. They would not have been seen as practicing teachers. Thus, at one level, one can see how teachers, being interviewed by "outsiders," might wish to claim some territory where their own expertise and perceptions cannot be challenged. An interviewer would be in no position to dispute a teacher in her/his description of "my group" or "my kids."

The practice of grouping children by ability for the purposes of learning mathematics is well established in both schools. Thus, the institutional practices of these schools not only conform to the belief that it is possible to group children according to mathematical ability, but the discourse of teachers is intended to show that the practice actually works. The potency of these consistent patterns of description is that they allow teachers to confer orderliness and rationality upon their actions, especially their decisions to adapt DMP. These descriptions enable teachers to display their own competence and responsiveness to the presumed needs of children. If, for example, parts of an activity are "too challenging" for "my group" it appears sensible to omit that activity altogether, or to reduce the challenge by going through the activity as a whole class exercise led by the teacher.

Teacher directed learning. Another important clue to the teachers' underlying conceptions of work is disclosed in the Topic Interviews when teachers refer to themselves as being responsible for making all significant instructional decisions. They see themselves as responsible for shepherding their students through DMP, directing, and telling children what to do, making decisions about what curriculum content will be presented or omitted, and preventing embarrassment and frustration should students come across material they could not handle. Some examples:

> The ones I felt were going to be too challenging for the kids so that they were going to be frustrated, I left out. (Teacher H)

> I liked the way it made them think, it was good for that, but I would never let them do it by themselves again. It was too frustrating for them. (Teacher D)

> The needed a lot of direction to follow all the steps . . . . They needed my direction. When I was able to direct them, I was really pleased because that is the first abstract thinking process they really had to do and were able to do it. They couldn't if I hadn't directed them. (Teacher B)

> If it looks like it's going to be too confusing for them, I do it with them . . . . I couldn't even imagine that all the kids could do that. We did them mostly together. (Teacher C)

The work of students is therefore directly affected by how teachers conceive of their role: children tend either to work as a whole group with their teacher directing them from the chalkboard or from the text-book, or else they tend to be engaged in independent seatwork at the direction of the teacher. Occasionally, children engage in structured activities by moving from one work station to another. But here again teachers usually direct pupils' movements from one station to another. There is in the Topic Interviews little evidence of children being encouraged to collaborate with one another. The above responses not only serve to create an impression of a collective group of pupils, but within the collective group there is a ssnse in which pupils are cut off from each other. They are not linked in any substantial way to one another. They are each linked through the process of group instruction to their teacher.

DMP, as remarked in the preceding chapter, saw teachers as responsible for directing and guiding children's learning. One cannot take exception to the fact that teachers express in the Topic Interviews a sense of being in charge and of being responsible for what their children are learning. However, my analysis of the Topic Interviews left unresolved the question of how much structuring and guidance was being employed. I feared that sometimes teachers had structured and guided children's work so extensively that children were left with no choices or judgments of their own to make, and that mathematical inquiry was reduced to following a set of well defined procedures and rules:

> . . . rather than let them try it by themselves, I did page eight with them and part of nine. (Teacher D)

> Having the teacher measure (the length around his/her neck and ankle, instead of pupils doing their own measurement), or you could even pick one child and use him/her as an example. (Either) would be a good idea, but I think it's hard enough to keep everybody on the same page, much less having everybody have a different number. (Teacher E)

> In the beginning a lot of the stuff was teacher directed because of the new concept . . . . The slow group has very low reading ability so it's very hard for them to do anything that requires reading on their own. (Teacher F)

Dependent pupils. Teachers' presentations of their own critical role in organizing children's learning and dispensing knowledge implies that they see pupils as dependent upon them, and that they need to define for pupils what will count as appropriate work and appropriate knowledge to be learned. Some measure of dependence upon teachers is only to be expected because children in these early grades of elementary school are tiny and vulnerable, but it is also in the nature of a collective classroom group that the children who comprise the group are dependent upon an adult to lead them--their teacher. Indeed, the conceptions of "pupil" and "student" imply dependence upon a teacher. This dependence is exhibited in teachers' references to "my group," "my kids," and "my class." It is also shown in the "needs" which are ascribed to the class group. These needs constitute a rationale for teacher-directed instruction. Their very existence seems to require the prompt and responsive attention of the teacher. Because these needs belong to the whole group, it is also likely that the interests of individual children will have to be subordinated to the needs of the class.

The most prevalent symbol of pupils' dependence on their teacher was the requirement adopted by almost all teachers that pupils needed to have their work corrected before being allowed to proceed to the next piece of assigned work.

> I had them go back and correct their errors . . . . I'm really a real stickler for (that) especially in this topic (S5) . . . because they were so careless. So I didn't even let them go on until they had every page "starred." (Teacher B)

While this is the most obvious sign of pupils' dependence upon their teacher, it is a practice which few could question. But at other times children are dependent upon their teacher to show them how too much independence can be a bad thing, as the following excerpt shows:

> These kids . . . are so cocky, they think they can do everything . . . . They really hit a snag on the last part of (this) topic. I . . . was going to give them all help on it. And they said, "Well, we know it. We don't need any help." And I said, "Then you go right ahead and do it by yourselves." Two of them did it . . . just like that. And the rest of them are struggling, so I let them struggle for a day and tomorrow I'm going to give them some help because they were so cocky they didn't need to listen before, now they know they need to. (Teacher J)

In the latter class, the children are depicted as being too independent for their own good: They needed to be taught a lesson that they ought to have relied upon their teacher's help and guidance in the first place. In the former case, children are dependent upon the teacher to check their work because they are careless when left to their own devices. However, in other cases, the teacher sees no alternative but to have the group follow her directions and her method of solution of mathematical problems at all times:

> I always had the feeling I was doing the work. Some days they really surprised me, but the majority of the time I had to put everything on the board and say: "Come on, this is what we're doing. Here's my answer put it down." They were that type of class. I'm really not sure if they really understood the underlying concepts at all. (Teacher E)

This is a more subtle form of dependence and one which has a direct bearing on what constitutes desirable mathematical knowledge for children to learn. No longer is the teacher seeing herself as a guide helping children to develop skills in problem solving. Teacher E sees herself as responsible for setting down the categories of analysis, and for leading students through the problems in which those categories would be applied. Her students are dependent upon her for this help because they are seen as deficient in mathematical knowledge and as being powerless to remedy this deficiency without explicit direction. In the course of the interview, Teacher E continued to refer to the problems which her students had experienced in Topic S3. This is the topic where children are introduced for the first time to the Part-Part-Whole method of analysis and solution of word problems involving addition and subtraction. The teacher agreed that story problems were important, but

> basically there was too much writing for my children. They couldn't read the stories anyway. (Teacher E)

From the outset, therefore, these children are deemed to be incapable of independent work in solving story problems. Although The Teacher's

Guide to Topic S3 recommended that children attempt to do certain problems on their own, Teacher E considered it necessary to change the activity:

> Mainly because of my low children . . . . I had to do more explanation beforehand and do it together. Where it says about the last five problems that your children do by themselves, I never did that. Maybe one. And then we could have twenty beforehand, before they had the one to do by themselves and they would get it all wrong. (Teacher E)

Therefore, the teacher decided that the word problems would be discussed and analyzed in front of the whole class, with the teacher taking a leading role in initiating the analysis and laying down the categories by which the analysis would proceed. Her pupils needed this kind of approach because, she claims, they were unable to deal with the wording of the story problems. Their task, by and large, was to be attentive to what the teacher said and did, and to follow a predefined pattern of analysis. These changes in procedure did effect a major change in how the mathematical content of DMP was presented to the children.

Teacher E appeared to be very despondent about her pupils' lack of success in solving story problems.

> A lot of the children as they got used to the stories would just look at the numbers real quick and write them in (into the Part-Part-Whole chart). They didn't wait for all the words to be read. (Interviewer) "So they don't get them in the right spot then?" Right. Sometimes by chance. (Teacher E)

Not only is it clear that there is a close relationship between teachers' conceptions of their own work and what they see as appropriate work for children; but where children are depicted as being limited in their ability to deal with mathematical problems, one gets the impression from the Topic Interviews that mathematical knowledge is treated as a fixed body of procedures. In this context, children's work seems to become so structured and organized that they are left only with predetermined tasks to perform.

Whether children are seen as capable of developing and articulating their own mathematical insights under the guidance of their teacher, or whether they are seen as dependent upon their teacher to present them with the rules and procedures to work through a fixed body of subject matter has profound implications for the kind of mathematics which they are taught, and how they are expected to display their competence in what they have learned. In the former case, it is likely that children will be encouraged to look upon mathematics as a body of knowledge which they can help to create. They are likely to be guided and assisted to devise alternative methods of analysis and solution, and to show that these alternative strategies are successful if their results can be validated. It was this vision of children's ability to create and test mathematical knowledge which the writers of DMP hoped would moderate any tendencies to treat children as empty vessels into which an unchanging body of mathematical knowledge was to be poured.

From my analysis of the Topic Interviews, I suspected that few teachers accepted this vision of children's ability to create and test mathematical knowledge. However, I confess that the Topic Interviews shed light only sparsely and indirectly upon the kind of mathematical knowledge which teachers consider appropriate for children to learn.

Where these inferences can be drawn, they are usually limited to those instances where teachers have experienced difficulty in communicating the underlying concepts of a particular topic to their children. Even though teachers were asked in the Topic Interviews to comment on how important they considered that topic to be, the paucity of their responses may serve to show that they were still coming to terms with the new material and a different mode of presentation. However, their reticence about the appropriateness of the mathematical content of DMP is more than compensated for by their many concerns about issues of management and control associated with the content of DMP.

Content subordinate to management. There was ample evidence that the content of DMP had become subordinate to the procedures of classroom control and of instructional management. This interpretation is illustrated by the following representative comments:

> I never have liked one group doing this (and another doing that). When Mrs. B. (a teacher's aide) is in the room, I might say, "These five children need a lot of extra help." And she might take them, and I might do another activity, but never alone. It's too hard. (Teacher E)

Later the same teacher added:

> You can do a thousand activities, and with this group, they'll go to their books and say, "I don't know what to do." They're a different type of group, and so I generally do all the activities together. (Teacher E).

Or there may be a dislike of a particular activity:

> The one (activity) I altered was "give me a hand." I would never tell my children to write on their hands ever . . . . I adjusted it to having numbers on different colored pieces of paper. (Teacher L)

The priority of management over mathematical content is further reinforced by teachers' responses to the question "Are there pupils for whom you plan especially?" Teachers tended to respond to this question from a management perspective: individual pupils who were in need of help were to be attended to within a context of whole group instruction. Teachers usually said that they planned to spend additional time with these students. One teacher said that, since her mathematics lesson was held in the period before morning recess, she usually asked those students who were having difficulty to stay behind after the lesson was finished. Other teachers said that they spent a great deal of time supervising the work of these particular students; as a Grade 1 teacher expressed it;

To see if something is happening or not happening. (Teacher A)

Some teachers also reported that they ensured that those children who were likely to have problems with their work were seated close to the teacher so that their work could be watched more attentively.

What is significant about these responses is that they make no reference to modifying the content of instruction in order to assist students who are experiencing difficulty in understanding the mathematical content of DMP. Individual difficulties are dealt with procedually within a context of whole group instruction. At other times when teachers anticipated that elements of mathematical content were likely to cause difficulty for most students, those elements were often omitted entirely. If some activities were considered "too challenging" for students to do on their own, teachers tended to take charge and "escorted" the class through the exercises in question.

These instances serve to draw attention to a major concern of teachers: that the conduct of the lesson should remain manageable by them at all times. For some teachers, this concern is exemplified in their wish to lead the class through problems, rather than having children work independently. For others, it is illustrated in a desire to keep children's work within established limits to that children are not creating too many different demands on their teacher at the one time. For others, it is illustrated in a dislike of small group work. But in each case the message tends to be the same: the mathematical content of DMP has become subordinated to the rules which govern classroom life. These rules are situated in a network of beliefs, values, and purposes which find expression in a model of teacher-directed instruction. There, the teacher is totally responsible for leading a group of students through a body of predefined content.

Extrinsic subject matter. Not only to teachers tend to see mathematics as a body of subject matter to which students need to be introduced, and to which they bring few insights of their own, but teachers themselves appeared in the Topic Interviews to view the mathematical content of DMP as extrinsic to their own intellectual habits of understanding, insight, and choice. It was something with which they needed to become familiar. It was also a domain of intellectual activity over which they saw themselves as having little control. Often when teachers referred to the mathematical content of DMP, they spoke in terms of "your math," "the system," "this stuff," or as one teacher commented:

> We have to get the kids thinking your math . . . . I want the kids saturated so they know. (Teacher A)

When asked what activities they considered unclear, teachers tended to respond in terms of the difficulties likely to be experienced by their students, such as reading difficulties in tackling word problems; or in terms of directions which appears unclear; only rarely did they respond in terms of the lack of clarity or appropriateness of the mathematical content. These references reinforced an impression that the mathematical content of DMP was something to be approached with some anxiety:

> I was a little worried that it was a little too abstract for them. (Teacher F)

> I guess I felt pressured getting it all in with my children--other kids, the teachers got done with theirs, their children, but I think with (my) load of children it's an awful lot to get done. (Teacher E)

For others, it was a question of staying with the material of the topic booklets:

> I stayed with the book . . . . We did some extra worksheets and things. (Teacher K)

> I enjoyed teaching it. I'm now getting into this math. Because, I mean, it was a new program. (Teacher C)

But there were signs of a possible change of approach among teachers who had used pilot editions of DMP materials in the previous year. One such teacher remarked.

> When I work with new material, I stay pretty close to what you want me to do. Because I have to find out what you're getting at . . . . This year, I'm doing it my way. I know

> your concepts and now I know mine, but I always feel I have to . . . think through with the person who wrote the manual before I can say, "Hey, I don't like that." (Teacher A)

There was further but limited evidence that some changes in the presentation of DMP were motivated by a better understanding of its mathematical objectives. One such instance was obtained in a Topic Interview with Teacher G. She was seeking a way to ensure that her pupils knew how to write the correct number sentence after solving a story problem using the Part-Part-Whole classification. Such a story problem might be: There are nine balls. Four disappear, How many are there now? When the children had recorded this information correctly in the Part-Part-Whole chart, it would appear as follows:

<table>
<tr><td colspan="2">9</td></tr>
<tr><td>4</td><td></td></tr>
</table>

But, as Teacher G commented, the children who knew that the answer was 5 would place that value in the chart which would then read:

<table>
<tr><td colspan="2">9</td></tr>
<tr><td>4</td><td>5</td></tr>
</table>

Too often, she noticed that pupils were writing a correct number sentence from the completed chart, but one which did not represent the transformation implied in the story problem. They might write a correct canonical sentence 4 + 5 = 9, which is consistent with the Part-Part-Whole chart, whereas the correct and appropriate canonical sentence would be 9 - 4 = 5. Teacher G realized that this was not what the authors of DMP had intended, although they may not have anticipated how easily children might be led into writing a correct but appropriate number sentence from a completed Part-Part-Whole chart.

> So I had to really be very careful and tell them that . . . . The only two numbers that go into the chart are the two numbers that are in the story problem. (Teacher G)

She then asked the children to leave a space for the missing number, and when they had found its values, they were to write it in using another color. Therefore,

> When they saw the chart they clearly saw the number that was a missing number was the one in red. (Teacher G)

Since the children were taught to write only canonical sentences, that is, those where the missing number appeared alone on the right of the "equals" sign, this strategy would clearly assist them to write a correct and appropriate number sentence.

This is one of the rare responses to the question, "Were there any activities that you found complex to teach?" where a teacher has looked more carefully at the mathematical objectives of DMP and has decided to change the content. It may be said that this represents only a minor change in the way which DMP has been presented. However, since the Topic Interviews established such a clear impression that the content of DMP had been subordinated to concerns of classroom management, I took this instance as a counterexample of that trend.

Teachers' adherence to the curriculum content of DMP and its accompanying vocabulary has been more consistent with what the developers intended than their adherence to some of the procedures recommended by the developers. These, as I have already said, are often modified in view of teachers' perceptions of their own role and their perceptions of what constitutes appropriate work for children. Those procedures and activities where the developes have recommended the use of manipulatives seem to have been most widely adopted. Those where the developers have recommended that students work in small groups, or carry out independent investigations, and thus move outside the direct supervision of their teacher, seem to have been the activities which were most frequently modified.

The Way Ahead

The Topic Interviews did confirm my suspicions, elaborated in the preceding chapter, that the implicit tensions in the philosophy of DMP would very quickly show themselves in its implemenation. Especially pertinent to these concerns was the relationship of the individual student to the classroom group. There were clear signs from the Topic Interviews that teachers had tended to focus upon the class group as the unit of instruction. Moreover, whole-group instruction appeared to be the predominant pattern, where the teacher was not simply responsible for children's learning, but totally in charge of all stages of instruction.

Hence, a major task of subsequent interviews and observations was to ascertain whether whole group or direct instruction was as prevalent as it appeared to be in the Topic Interviews. At the same time, it was necessary to ask what were the implications of these patterns of instruction for the work of teachers, the work of students, and how appropriate mathematical knowledge was defined for children to learn. One of the dangers in using a generic term such as "whole-group instruction" is that its use can blind an investigator to significant differences among classes where this pattern of instruction is discerned.

In my own interviews and observations, it was therefore essential to ask whether whole-group instruction produced similar definitions of work and knowledge for "the slow groups" as it did for "the sharper groups." From my analysis of the Topic Interviews, I was inclined to believe that children in the latter groups, while still expected to follow their teachers' directions, would be given more scope to work on their own, more responsibility for managing their own work, and greater opportunities to develop self-confidence in problem-solving. However, neither the Topic Interviews nor the Classroom Observational Study had been able to resolve this question. Admittedly, in the Topic Interviews, teachers had spoken of omitting activities which they believed would be "too frustrating" or "too challenging" for their children. But there was no way of knowing how these criteria were applied in day-to-day teaching, and whether they were applied differently for "slow kids" as opposed to "bright kids."

Furthermore, the paucity of references in the Topic Interviews to the mathematical content of DMP made it necessary to seek out further clues as to how mathematical knowledge was defined by what teachers did as well as by what they said. Not that the paucity of references to the mathematical content of DMP was an unilluminating feature of the Topic Interviews. It did suggest that mathematical content had become subordinated to issues of classroom management and control, thereby changing the kind of mathematics which children were taught. However, there were contrary indications in the same interviews when teachers spoke of helping children to solve problems and to become abstract thinkers. Were these aspirations to be dismissed as mere rhetoric? It was important to ascertain how these statements were reflected in actual practice. In this respect, I needed to look more closely at the social interactions between teachers and pupils: How were pupils to respond to their teachers' questions? What kind of help were they given in the course of a lesson? Were they dependent upon their teacher to tell them whether their answers were correct? How much direction did teachers give pupils before they commenced assigned work? What patterns of analysis were pupils to follow? Were these predefined by the teacher? Were pupils allowed to choose between alternative methods of analysis? Could they collaborate with other pupils in solving problems? These questions guided my observations and interviews with teachers.

Chapter 5

SOME LESSONS OBSERVED

My aim in this chapter is to illustrate how a culture of pedagogical beliefs and practices has affected the teaching of DMP in ways which the developers of DMP did not expect or intend. In illustrating this process of transformation, I leave behind the rhetoric of DMP and move into the classroom. If any of the developers of DMP had accompanied me in these observations, they would have recognized that DMP was being taught. The materials of DMP, the booklets, the manipulatives, and many of the recommended activities were all in evident use. But these observations confirmed an impression, already sensed from my analysis of the Topic Interviews, that DMP as taught reflected conceptions of school work and mathematical knowledge which were not aligned with those of its developers.

This chapter proceeds in two sections. First, I present descriptions of lessons which I observed. One lesson is presented for each of the eight teachers who were continuing to teach DMP in the 1981-1982 school year. After each description, I comment on the salient features of that lesson which have determined how mathematical knowledge and inquiry have been presented to children. Likewise, I refer to patterns of school work implicit in the lesson. Second, I interpret the actions of teachers in terms of more general patterns of action and belief which bear directly upon conceptions of school work and mathematical knowledge. These interpretations concern the work of the individual learner and the learner's relation to a group, differentiation between groups, and the prevalence of teacher-directed instruction.

Observations

The following descriptions report my observations of ten lessons by eight teachers. Each teacher was observed at least once, and from these observations I have selected one for each class group being taught. Where I had the opportunity to observe a teacher several times with the same class, I found a very consistent pattern of teaching across lessons. For that reason, my selection of one lesson from several available to report has been quite arbitrary. The observations which I report are presented by grade level.

Teacher A (Grade 1)

Teacher A's Grade 1 was preparing to do station work. Coins in various combinations had been attached to adhesive tape to paper cards. The cards were individually lettered and had been placed at locations throughout the classroom. The teacher introduced this activity as follows:

"Today you are going to be trains. Today you are going to stations."

She then handed out a worksheet to every student and continued:

"Put your name on this sheet at the top right-hand corner. Which hand is your right hand? Hold it high for me to see. Good . . . .

When you count the money on each card, write down the total and how many cents . . . .

You don't have to go to A, to B, to C, and so on.

You can start anywhere you like. But you must look carefully at the station name."

The stations were each identified on the worksheet by an appropriate letter. Beside each letter there was a space for the amount to be written in.

"Jason, don't forget the cents sign. Why? Because it could be six snowmen . . . . We have to use symbols.

We could write the word "cents." But that would be boring.

Patrick wrote "6¢" using such a small cents sign that I had to use my magnifying glass to read it. Be careful to write large, or else I will have to use another symbol."

The teacher wrote that symbol on the blackboard. She then asked the pupils what that symbol meant. They responded:

"Wrong."

The teacher then offered some further guidance for students:

"When you begin to count coins, what coin do you begin with . . . . The one that will buy the most. Count by dimes."

(On the cards, there was no coin larger in value than a dime.) Under the teacher's direction, the class started counting by 10s. Then she had them counting together by 5s.

"Could I count my pennies by 2s as a short cut?"

Then she led them into counting by 2s:

"2, 4, 6, 8, ...

Please work on your own today. Put all your books and papers in a row along the front of the room. Let's get started, trains."

Children proceeded around the stations. They were very busy counting coins and writing answers into their worksheets. The teacher moved about among the stations for the most part speaking to individual students and questioning them. Sometimes she would speak to the whole class:

"This is a good group. They are good listeners . . . .

Don't forget. There is an N over here by the tape recorder."

Some children needed to be helped in writing their numbers. A common problem was writing a figure back-to-front. Others carried with them a paper strip with the numbers written on it. Others referred to a number line posted on the wall. Some were able to write their numbers without such assistance. At one stage, the teacher interjected another request:

"Stop and make sure your name is on the sheet."

Later on, she asked:

"Boys and girls, where does the cents sign come - before or after the number?"

As the teacher checked students' work, she was enthusiastic in praising them for good work. Those who finished all the stations were given a color puzzle to complete on their own. The teacher brought the activity to a close by asking:

"Are we just about finished?"

Some students had not finished all the stations, but many had. The teacher collected cards from their various locations. Students seemed to know what to do next. They returned their pencils and crayons to where they had obtained them at the front of the room.

"I like the way Emily is putting her crayons back. Good thinking Emily . . . . The color sheet can be finished at home tonight. Come and join me around the rocking chair."

The remainder of the lesson was spent as a group answering many problems which were posed by the teacher in the following form.

"I gave the clerk a nickel to buy 3¢ of candy.
She gave the candy and how much money back?"

As students left the classroom for their lunch period, several who had not finished their cards were asked to remain behind.

They were required to complete their work under the direct supervision of the teacher.

Comments. On the surface, Teacher A's use of station work appears to be the antithesis of teacher-directed and whole-group instruction. Children are free to move around the stations in whatever order they choose. Only at the end of the lesson does some whole-group teaching take place. Some pupils who finished early were given a coloring activity to complete on their own.

However, on closer examination, there is a very high degree of structuring in the way in which each counting exercise is presented to the children. The coins to be counted were attached to a card by strips of transparent tape. They were also arranged in lines with coins of the same denomination being placed next to each other, usually on the one line or on two lines underneath each other. Clearly, this is a much more "tidy" arrangement of coins than children would encounter if they were given a handful of coins to count. Then, they would have to devise some strategy to order the coins before counting them. It was this important aspect of counting money which Teacher A had bypassed entirely. Moreover, Teacher A told the children before they commenced station work that in counting they should start with the coin of greatest value.

There is no doubt that children were required to apply their counting skills in this lesson. However, the structure, orderliness and support which Teacher A had introduced did simplify the nature of the application in such a way that features which are normally present were eliminated. This simplification would have made some sense if children were counting money for the first time, but that was not the case. The orderliness with which the coins were attached to cards was intended, in Teacher A's own words, to facilitate ease of storage, to avoid "confusion" if children were confronted with a pile of coins to count, and to ensure that they could cover a much larger number of stations within the lesson.

Teacher E (Grade 1)

Teacher E's Grade 1 was playing a game called "Chips In." Students were paired with one standing by a desk and the other seated with a blank piece of paper, ten chips on the desk and a container. Those standing were to act as the "watchers."

"Watchers, put your hands on your shoulders if your partner gets the sum right. Hands high in the air if you think the answer is wrong. I will give the answer after I have read the question and watched your hands.

Problem #1 8 - 3

Don't write the question just the answer . . . 5

If right, put a chip in the container.

Problem #2 6 + 3 . . . 9

Chip in if right, or chip out if wrong.

Problem #3 15 - 1 . . . 14

Detectives, you should be looking at your partner's paper.

Problem #4 10 - 8 . . . 2

Problem #5 7 + 4 . . . 11

Julie, is your partner done? (She hadn't moved her hands.)

Problem #6 8 + 5 . . . 13

Some children are not using their chips.

Problem . . .

Problem #10 16 + 0 . . . 16

James, are you watching your parner. You're there to see that he's honest."

Comments. On the surface, this part of a lesson appears to be a perfectly simple game in which pupils practice their recall of basic number facts. As such, it proceeded very smoothly. However, Teacher E spends considerable time ensuring that the correct procedures are being followed--that chips are being put into containers--and that the "watchers" are doing what they were told to do.

There was no evidence that any watchers consistently acted in that capacity. Most of the time they put their hands on their shoulders. Some seemed to go "off task" entirely. Indeed, Teacher E had made their role redundant when she herself gave out the answers. However, the effect of designating half the class as silent watchers was to enable Teacher E to have each "worker" in a desk on his/her own, rather than to have two children working side by side on the same problems. Beneath the surface impression of pupils engaging in a game are implicit values about how children are to work on their own, and how they are to follow instructions.

Teacher B (Grade 2)

As she handed back tests to her pupils. Teacher B announced to the whole class:

"All these people (five pupils) got 100% on the paper."

To the remainder, 13 pupils, papers were handed back, each with a comment addressed to the pupil. Then to the whole class, she said:

"When it says 'altogether,' what does that mean?"

Some students replied:

"Add on."

Those with mistakes were requested to correct their papers before the next lesson. Some pupils had missed out because, as the teacher said to me later,

"They had basic facts problems."

That day's task was then presented to the class. Pupils were asked to get their boards (a wooden board upon which they could rest their papers). They then gathered around the teacher ' o handed around duplicated copies of S5, page 20. The class was told that this was an important exercise because

"We got mixed up on Whole-and-Part last week."

Before commencing to do work on the sheet, the teacher presented two problems, similar to those on the sheet, for the class to discuss. The first was:

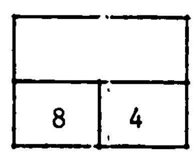
> "I had some mice. Eight ran away. I had four left. How many were there to start?"

She addressed the following questions to the class:

> "Is eight part or whole? Why?"

> "What is four?"

Several pupils voluntereed answers to those questions. They were invited to come to the blackboard and to fill in the numbers on a Part-Part-Whole chart which the teacher had drawn there. The completed chart was as follows:



The teacher's next question was:

> "What kind of a story will I write?"

A pupil responded:

> "Add on."

The teacher asked "Why?", and the pupil responded:

> "Because you know two parts."

Thus, the sentence 8 + 4 = 12 was written on the blackboard. The problem was then left. (There was no validation of the answer. In other words, there was no discussion of whether the answer, 12, fitted the original story problem.)

In the second problem presented to the class, a "part" was known as well as the "whole." The teacher asked what kind of a number sentence would she write. This required a subtraction sentence, "9 - 6." Pupils were able to answer this correctly. The problem was then left without validation. There may not have been time for that since the sheet of problems was to be done.

The first problem was read to the class by the teacher. The duplicated sheet required children to "Write a sentence. Use the chart to help. Then solve."

#1 20 

Some swim away.
14 are left.
How many swam away?

In introducting this problem, the teacher said:

"I want you to use your 'thinkers' for these problems. Circle '20.' Some swim away. Circle 'some.' Fourteen are left. circle '14.' Don't write a number sentence. Just fill in your chart."

The second and third problems on the sheet were read to the class:

#2 Some 

8 more come.
Now there are 11.
How many were there
to start with?

#3 15 

7 hopped away.
How many are left?

In the fourth problem, the teacher asked:

"What is 'some' in this problem?"

"Is 'some' all the caterpillars?"

#4 6 

Some more come.
Now there are 13.
How many came?

The pupils were slow to respond. So she added:

"No, because there were six to start with."

One student, Byron, said:

"Six is the whole."

The teacher then asked him

"Is that all the caterpillars? . . . Now there are 13"

(the latter sentence was read with emphasis).

Byron responded

"13 is a part"

The teacher countered:

"But, Byron, that would mean you have three parts."

From several other pupils, correct labels were offered for the numbers given, and the chart was completed to the teacher's satisfaction, and the missing number found.

In the fifth problem, the pupils had circled the word "some," and the numbers "8" and "4."

#5 Some 

8 jumped away.
4 are left.
How many were there to
start with?

The teacher then directed a question at Byron:

"Did all the squirrels jump away? Byron? Yes or No?"

Byron seemed confused:

"Yeah . . . no."

The teacher added:

"If it's 'no,' what is 8?

Eight jumped away."

She demonstrated with her arms how some squirrels might jump away, and added

"Did all the squirrels jump away?"

Then, as though to prove her point, she said:

"How can 4 be the whole and 8 the part?"

She had noticed that Byron had incorrectly placed these numbers in the Part-Part-Whole chart. The chart was then completed by students. Several needed to borrow an eraser.

For the rest of the lesson, Teacher B continued to lead the whole class through a Part-Part-Whole analysis of problems #6 to #12. There was no change in the pattern of instruction. Afterwards, Teacher B commented to me about the lesson, and said how disappointed she was that her children were unable to identify correctly the Part-Part-Whole structure of the problems on the worksheet:

"They were up the wall, today, especially Byron. Several children have learning disabilities. Mrs. Peters' group would not have had trouble with this sheet. (She was referring to the other grade-two group.)

But for the whole class, it's a matter of reading and comprehending the problem. Even with children reading it, they don't comprehend. For so many of the problems, I have to read it all through with them, or else they don't get it . . . . They have been off this topic (Part-Part-Whole) for a month, and they've forgotten."

She added an afterthought on the structure of the questions:

"You could leave out the last question (of the word problems). And if they labeled Part-Part-Whole correctly, they would get the answer anyway."

Comments. Teacher B's lesson is one of the clearest examples of teacher-directed and whole-group instruction. Even in the children's seating arrangements, Teacher B required that certain pupils, Byron, for example, sit close by her. The whole class was directed by Teacher B to tackle one problem at a time. The method of analysis was prescribed in advance: Teacher B's overriding objective was to have pupils identify and label the Part-Part-Whole structure of every story problem. Pupils were not required to write or solve a number sentence, although some appeared to do so on their own. Although it was clear that many children were finding difficulty with a Part-Part-Whole analysis, Teacher B considered it necessary to give more directed practice, and did not diverge from that pattern of teaching. Later, she said, that she had hoped to leave the children free to continue with some of the latter problems on their own. Therefore, her decision to continue with whole-group instruction can be seen as a procedural change.

The effect of the above procedural change was to solidify this one pattern of analysis, and to preclude the introduction of an alternative approach to the solution of story problems.

Teacher F (Grade 2)

Teacher F and her assistant had 12 pupils. The lesson commenced with the children gathered in front of their teacher. She was preparing to demonstrate a weighing activity taken from Topic S5, page 11. Instead of setting up weighing stations, as the DMP booklet had suggested, Teacher F demonstrated how to weigh various objects using cubes and washers as units. Individual children were asked to come to the scales and to count the number of cubes or washers needed to weigh various objects.

Teacher F represented on the chalkboard two different objects which she had weighed--a block of wood, called object #1, and a wooden cone, called object #2. Two students were asked to write next to each object the number of cubes each weighed. Teacher F then asked:

"Which weighed more? How many more did it weigh?"

The class agreed that object #1 was heavier than object #2. Teacher F continued:

"How much heavier was object #1 than object #2?"

She emphasized that these were different ways of asking the same question. (Object #1 weighed 12 cubes, object #2 weighed 5 cubes.) Some children called out:

"Five cubes."

Teacher F did not respond to their answer, but immediately asked:

"It's a take away."

She then handed out 12 cubes to one pupil whom she asked to check that there were in fact 12 cubes. Likewise, another pupil was given 5 cubes. Teacher F continued:

"Before I show you the answer, who can write the number sentence which shows the difference?"

She paused, and then counted the difference between the two sets of cubes. She wrote out the following number sentence:

$$12 - 5 = \square$$

A girl, Betsy, was asked to come out in order to complete this sentence. She correctly placed 7 in the box. Teacher F then told the pupils to open their Topic booklets at page 14. This page showed pictures of different objects. Alongside each object was a number of units indicating the weight of the object represented in the picture. Children were to circle which object weighs more, and then write a sentence which shows how to find "How much more?" Finally, they had to solve the sentence. Teacher F added some comments of her own:

"What if the book shows the picture of a shape?

See, the ball is 17 cubes big.

Sometimes, it (the book) says cubes, or washers,

or bricks. We don't have bricks.

You have to find the difference.

You have to write it as a take away."

Teacher F and her assistant then handed out cubes to each student.

"Start on page 14. Then go to page 15.

If you finish this page, I'll check it."

Teacher F and her assistant then moved among students who were working one to each desk. Children were able to use cubes to represent the subtraction. Most children used the cubes for some problems. As I moved about, I noticed that several children were writing a subtraction problem using the correct numbers, but placing the smaller number first. Teacher F noticed the same thing, and commented to the whole class:

"Can you take 12 from 3?"

Later, she added:

"Don't forget the equals sign.

What kind of problems are these?

They are subtraction problems--take aways."

The latter comments were appropriate because several pupils had represented the comparison problem by an addition, or they had placed the smaller number first. Several examples of children's work were:

$$3 - 12 = 8$$

$$2 + 9 = 11$$

$$0 + 6 = 0$$

The lesson concluded with children continuing to work on page 15.

Comments. Teacher F presents a further clear instance of teacher-directed and whole-group instruction, except for the latter part of the lesson when independent seatwork was assigned. Even then, however, individual errors were treated as issues to be brought to the attention of the whole group.

Teacher F said that she thought that her pupils would have a better chance of "following the weighing" if she did it, rather than have them "all spread out around the room." Furthermore, Teacher F told her pupils that they had to use a "take away" in order to find the difference in weights. The DMP authors had hoped that children would have the opportunity to weigh objects, and that they would be assisted to write an appropriate number sentence which expressed the difference between the two weights. In this lesson, children were simply told what kind of sentence was required--even so, some pupils had problems in writing a subtraction sentence in the correct form.

Teacher F justified her very explicit directions by referring to her children's inability "to work things out for themselves."

The effect of her directions is eliminate discussion of the relationship between a comparison or difference problem and the writing of a subtraction sentence. The focus of the exercise is then to ensure that children write the "take away" in an appropriate form, and that they obtain the right answer.

Teacher G (Grade 2)

Teacher G apologized at the start of the lesson that her children had been looking forward to spending today on timed tests. She hoped that I would not mind. She then addressed the class:

"Hands up if you know that kind of test you are supposed to take." Every student raised a hand. Teacher G then grouped students according to which test they were to take. Those taking the same tests were seated together. Teacher G added:

"Put your name on the test. One person from each group is to go and get the answer sheet for that test."

Fifteen different kinds of tests were available. Most students were working on four different tests. In order to start the children, Teacher G said:

"Put your pen on Row C. Now begin there."

After one minute, Teacher G called on children to stop writing. Children were asked to correct their neighbor's work. Answers were called out by the student who had collected the answer sheet. That student used the answer sheet to correct his/her neighbor's sheet. If children had reached the target for mastery--30 correct out of 42 items--they went to a wall chart and marked off that they had completed that particular test. Those who had successfully completed the test which they just taken went and obtained the next higher test from a table. Those who had not reached the required level for mastery obtained a fresh copy of the former test.

In order to prepare children for the next round of testing, Teacher G said:

> "Fold your arms and pretend that you are frozen. If your neighbor has their name on the test, raise your hand. This time, get ready to start at Row A. Begin."

After one minute, children stopped when Teacher G said to. She added:

> "With your four fingers point to the person who has the correct answers to your group."

Marking was completed as before. One student went to the wall chart and marked off his name. Several students told Teacher G that they were only two or three short of reaching the required number in order to "pass."

Teacher G was prepared to give children a third attempt. She saw that they were excited and sais:

> "We'll have a calming down exercise. Everyone, take a slow breath. Now put your pencil on Row B. Start when I get to 1--5...4...3...2...1."

One student was doing test #599 which was the highest level among

15 tests. Teacher G said that this student needed another two minutes beyond everyone else. The whole group watched the clock and remained silent until this student finished. Correcting was done as before. Several students went to the wall chart to record their successful completion of the test. Some of them showed their corrected copies to Teacher G. She merely looked at the total, but did no more. Several students said that they were within one or two of passing, and asked whether they could have one more try. Teacher G said:

> "If you would like to stay for one more test, you may do so during the break."

Half the class stayed in to do one more test. As students left the class, three told Teacher G that they had improved their scores so much during the lesson that they were expecting to reach 30 correct on the next occasion on which tests were given.

Comments. What could have been a highly regimented and threatening activity has been made into a cooperative and less stressful exercise by Teacher G. Students were responsible for checking their neighbor's work, for obtaining answer sheets, and for recording their own success on the wall chart. The class paused for an extra two minutes to allow one student to work on the most advanced level test.

Teacher G had 15 tests each graduated to be slightly more difficult than the one before it. She had feared her children might be too discouraged if they were expected to stay on one type of test on which a mixture of addition and subtraction problems occurred. Under her system, each student had already moved successfully through several tests, and was currently placed on a test which was only marginally more difficult than the one just completed. Therefore, she was able to attend to individaul differences and at the same time give each student confidence that reaching mastery on the current test was an attainable objective.

Although this particular activity was not part of DMP--it was the policy of the school district to give timed tests in "basic math facts"--this lesson afforded an impression of Teacher G's style of teaching. Her expectations were not uniform for the whole class. There was scope for individual students to exercise choice and judgment. There was a conscious attempt to adapt the mathematical content of her program to suit the needs of her children.

Teacher C (Grade 3)

Teacher C had spent the first half of the lesson taking her Grade 3 class through DMP Activity 41H, page 16. Students had been using chips to represent fractions of various quantities. Students and teacher had worked in unison with the teacher directing students what to do and engaging in a question-and-answer session as the students worked through questions under the teacher's direction.

She then said:

"Look at page 17. We are all finished with chips right now.

Let's read question #1 together."

Students then read the question. It depicted a collection of 14 crosses:

X X X X X X X

X X X X X X X

Students were asked to divide the set of X's into sevenths, and then to say how many X's were in 1/7, 2/7, and 5/7 of the set. The teacher asked the class:

"What does sevenths mean?"

One student, Jillian, offered as an answer:

"Circle seven. 7 + 7 = 14."

The teacher responded:

"Jillian, you're getting ahead of yourself."

She then circled the X's by twos on the blackboard, and said:

"That's exactly what we were told to do."

Children were then asked to complete their books as their teacher had done on the board. She advised:

"Your books should look just like the blackboard."

A student was then asked to read the first question. The teacher asked how many X's there were in 1/7 of the set. A student offered a correct response. She then said to the class:

"Put down two on that line. What does 2/7 mean?"

After a correct response had been offered by another student, the teacher said:

"Put down four on that line."

She then gave the class some drill questions on 3/7 and 5/7. She then turned to Jillian and asked:

"If you took away 7/7 , would you take away all of them?"

Jillian agreed. The class was then asked to look at question #2. This question presented students with a set of 20 squares arranged as follows:

X X X X X X X

X X X X X X X

X X X X X X

The class was asked to divide the set of squares into fifths. As the pupils were reaching for their pencils, the teacher asked:

"How many squares are there altogether? How many fives go into twenty? ..3? ..4?"

Students nodded approval at the second suggestion. The teacher then asked them to draw the squares into groups of four making sure that they got the groups close together. Teacher and students then went through the three questions relating to this problem.

In question #3, students were asked to divide a set of twenty circles into fourths. I followed the work of two students who were seated on the floor near me as they completed this question. Student A had divided the set of circles into groups of four. As the teacher saw him working, she commented:

"You have to get them into groups of five."

When the teacher moved on, Student B helped Student A to complete the first question on this problem:

"How many circles in 1/4 of the set?"

As the teacher worked through the other two questions on this problem, Student A wrote in the correct responses in his book as the teacher called out the correct numbers. The bell then concluded the lesson, somewhat unexpectedly. The teacher quickly collected students' worksheets for the problems which had just been completed. She said she would look over students' work and return the sheets to them on the next day. Chips were gathered up by the students. They were asked to take home a sheet of addition problems for homework.

Comments. Teacher C has presented a clear instance of teacher-directed and whole-group instruction. Her directions to pupils were very explicit. She gave specific attention to reading each problem to the whole group. Students were required to work on one problem at a time. It was expected that pupils would work on their own, but some "unofficial" collaboration between pupils was noted. Teacher C later admitted that she was aware that students did help one another from time to time, and that she preferred to "turn a blind eye" to these events.

Teacher C's concern to have the class group move in unison from one problem to another led her to announce answers before some children had an opportunity to reflect on the problem. However, the effect of her question-and-answer approach was more profound in the way it redefined the objective of Actibity 41H. There, it was hoped that children would be helped to discern the logical relationship between fractional parts and the number of equal groups. In problem #1, Jillian's incorrect interpretation of sevenths was sidestepped, and, instead, Teacher C demonstrated from the chalkboard that 1/7 of a set of fourteen comprised two elements. There was no discussion why this should be so. Likewise, in problem #2, Teacher C told the students that fifths meant five equal groups. The authors of DMP had intended that children be helped through discussion and individual investigation to see these relationships. Those opportunities were precluded by Teacher C's approach.

Teacher D (Grade 3)

Teacher D spent the first ten minutes of the lesson reviewing fractions. This was done as a preliminary exercise to DMP Activity 41H, p. 16. Children formed a circle around the teacher who handed out plastic counters to each child. As well as using counters to represent sets and fractional parts of sets, Teacher D also had children stand to represent fractional parts. For instance, she asked seven children to stand in front of the group, and then inquired whether it v possible to divide the set of seven into two halves. Children shook their heads to indicate that it was not possible. Teacher D continued:

"What would we need to do with the number we have?"

One pupil said that they would need one more to join the group, or one of the group to sit down. Teacher D said:

"OK. Let one sit down. How many are there in each half?"

Many hands went up, and Teacher D accepted the correct answer before asking:

"Who can divide the group into thirds?"

Peter came forward and separated the six students into three pairs. Teacher D then asked the following questions:

"How many in one third? How many in two thirds? How many in three thirds?"

After each question, Teacher D identified one student who gave the correct answer. Several similar problems were presented, but this time children had to use their counters to represent the fractional parts and to say how many counters belonged to each part.

Teacher D introduced the children to p. 16, problem #2. They were asked to count out ten chips in fron of them. Teacher D then asked them to divide the counters into two equal groups.

"How many in each half? BJ?"

BJ gave the correct answer. Teacher D continued:

"Can you divide them into thirds?"

She allowed a short time for children to try regrouping their counters before asking:

"Did you find three equal groups with none left over?"

Children nodded in disagreement. Teacher D then inquired:

"How many made an estimate to start with?"

About one quarter of the group raised their hands. Susan said that she had not bothered to rearrange the counters since three groups of three made nine.

After questions #3 and #4 were completed in a similar way, Teacher D requested that the children turn to p. 17 problem #1. She did not ask that the problem be read aloud.

"How many Xs are there? Make seven groups of two."

Immediately, she continued:

"How many in 1/7?"

After a short pause, she added:

"In one of those groups there are two. Write that in (in the space provided for the answer). How many in 2/7? That is two of those groups. Write in four. What about 5/7? That is ten. Does everyone understand where those numbers come from?

In question #2, students were presented with a set of 20 squares which were to be divided into fifths. Teacher D asked the children to try to make five groups using a pencil to group the squares on their worksheets. After allowing about one minute for children to work on their own, she then posed a string of questions in quick succession:

"Try one in each group. Will that make five groups? No. Try two in each group. Will that make five groups. No. Try three in each group . . . . Try four in each group. Is that OK?"

Several children nodded in approval. Teacher D continued:

"Write four in each group. Do you have five equal groups?"

Some children completed the task of circling off the squares into groups of four. Teacher D allowed only a short time for these pupils, before asking?

"How many in 2/5?"

Several children responded aloud:

"Eight."

Teacher D continued:

"How many in 3/5?"

One child gave the correct answer. Teacher D asked the next question:

"How many in 5/5?"

Several children said together:

"That's all the squares."

Teacher D then said:

"The rest of the pages will be fine for you to do alone."

Pupils went to their seats and commenced the assigned exercises. While they were working, Teacher D moved about the class looking at individual students' work. She had not warned the class in advance about problem #4 which required children to divide 16 arrows into sixths. Several children were already having difficulcy with this question, evidently believing that it was possible. After Teacher D had responded to the first request for help on this question, she then announced to the whole class:

#4 can't be done. Put a cross through that question.

The lesson concluded a few minutes after Teacher D had introduced the children to the problems on p. 18. She announced that they would continue with pages 18 and 19 next lesson.

Comments: The fact that Teachers C and D had taught the same lesson, allows one to make some comparison between the two teachers. Teacher C invested a great deal of time in ensuring that each problem was read aloud to the whole group. Also evident was the greater amount of prompting which she gave to her students. Teacher D, on the other hand, is much more confident that her pupils can read the problems on their own. She was also prepared to have pupils complete p. 17 after only two of the six problems on that page had been done as a class exercise. Later she admitted that she had not detected the "unsolvable" problem #4 before assigning it. Had she noticed it in advance, she said that she would have told the class that one of the problems could not be solved, and that they were expected to find it themselves.

In Teacher D's group, there appears to be less direction than was given by Teacher C. However, one needs to notice that, in dealing with problems on page 17, Teacher D has framed her questions to the class in such a way that the problem's solution is implicit in her questions. Problem #1, for example, asks that children divide a set of 14 Xs into sevenths. Teacher D told the children to make seven groups before asking how many Xs were in one seventh. In question #2, Teacher D directed the children to make five groups. Having been told to make five groups, all that remains is for children to enumerate how many squares there are in 2/5, 3/5, and

5/5. The relationship between fifths and five equal groups had been settled in the directions given to the class. That it had been settled in advance is all the more remarkable since the point of Activity 41H is to lead children to an understanding of this relationship between fractional parts and the number of equal groups.

Thus, while Teacher D appears to use fewer structuring comments than Teacher C, the effect of those comments which she does employ is to prevent children from investigating for themselves the crucial mathematical relationship which Activity 41H was intended to present. Teacher D's line of questioning attaches more importance to having children give correct answers to questions about the number of elements in certain fractional parts. The Activity had assumed that if children could give correct answers to these subsidiary questions they had discerned the prior relationships between fractional parts and the number of equal groups.

Teacher K (Grade 3)

On one afternoon I observed all three classes of the third grade of School 2 which were taught in 1981-82 by the one teacher. These classes are grouped by ability and are called Red, Blue, and White. Before joining the classes, I had forgotten to ask the teacher which classes corresponded to upper, middle, and lower ability groups. But that became clearer to me in the course of my observations.

The students in Red group were asked by their teacher as soon as they had taken their seats to turn to page 26 of Topic A3. The teacher asked:

"What is a Boggle?"

(A Boggle is an animal shape inside of which is written an addition or subtraction problem where the answer comes to 44. The students had to identify the true Boggles from those which had the same shape and appearance but whose number problem did not have an answer of 44.) The teacher continued:

"What does the Boggle have to equal if it is to be a true Boggle?"

Lower down on the page, students had to create their own Boggles by composing a number problem where the answer was 44.)

"What do they have to equal?"

A student answered "44." The teacher then went over the directions for completing page 26. A circle had to be drawn around the true Boggles.

"Then on the bottom, make seven Boggles of your own. Be careful to see that you create true Boggles on the bottom. Think about it, then write the problem."

The teacher then asked whether everything was clear to the students. Jamie asked:

"Do they want us to make up things which are not true Boggles?"

The teacher answered:

"No Jamie. Only true Boggles."

Another student, Sue, asked:

"Do we correct the answers if they are wrong?"

The teacher responded:

"No. You just don't circle those shapes."

Before setting the class to work, the teacher went over the directions for page 27. There another animal shape, called a Trixxle, was introduced.

"This page is done the same way as the page on Boggles."

Except, as the teacher pointed out, a true Trixxle was a shape where the answer to the addition or subtraction problem came to 41. He then added:

"I want you to be careful of writing this kind of addition:

$$\begin{array}{r} 42 \\ +\ 2 \\ \hline \\ \hline \end{array}$$ Why?

Richard volunteered to answer:

"That is not true."

The teacher agreed:

"You have to make sure the 2 is in the one's column . . . .
I want you to come up to my desk when finished. The
reward is a star for each page, but only when correct . . . .
I'll be walking around to help if you need it. Go to it."

Pupils brought their work to the teacher's table for correcting. Any mistakes were pointed out to them, and had to be corrected by the pupil and brought back to the teacher. The reward was two Smile stickers or stars if both pages were correct. No more than three students were allowed to be wa ting at the teacher's table at any one time.

The lesson with the Blue group commenced with the teacher returning time tests (timed tests of basic number facts) corrected from the previous day.

"Some of you are getting close to passing."

(That required a score of 100%.)

"Now that we are done with Topic A4, we will have the topic inventory today . . . . Prove to me that you did understand. Your work in class was good . . . . You don't have to finish the topic inventory in class. One part will be word problems. If you haven't done all the corrections (on the time test), I'd like you to take that home to do tonight."

The teacher then handed back the time tests, and proceeded to discuss the word problems on the topic inventory:

"Some of you will need Part-Part-Whole. Some of you will have to use (Part-Part-Whole) box. Do whatever it takes for you to get the problem right. I want to see the problem in compact form and the answer circled . . . . The "whole" goes in the top--that's the larger number. A lot of you use the box. That helps your confidence . . . ."

One student asked:

"Do we have to use the box if we don't want to?"

The teacher responded:

"No."

Joseph asked:

"Do we have to validate?"

The teacner replied:

"If you feel you need to, please do."

In introducing the second part of the topic inventory which dealt with addition and subtraction problems in compact form, the teacher advised students:

"Check the sign first. Some you will have to borrow. Some you won't . . . . I want quality work. I don't care how long it takes . . . . Take it away."

As soon as the students of White group had seated themselves, the teacher handed out folders with each student's name being read out. When the distribution of individual folders was complete, the teacher said:

"Let's have a productive day today. Let's roll."

Nothing more needed to be said and students opened their folders and commenced work. Most students were working from the DMP booklet 49-52. Soon six students were lining up at the teacher's table. Most of these were responding to a "See me" note which had been attached to completed assignments which had only just been returned. Some had questions to ask about the work which had been assigned that day. In the folders which had been handed out at the start of the lesson, each student had received individual instructions for the day's work. These directions told students which pages of the DMP booklet needed to be done that day. When worksheets were completed, they were placed in the "Done" side of the folder. At the end of the lesson, folders would be collected from students. It was therefore unnecessary for some pupils to speak with the teacher in the course of the lesson. If the assigned pages were completed before the lesson was over, students then spent time on additional activities which had been listed in the "Do" side of their folders.

Comments: One is struck by the contrasts among these three lessons. In the first, with Teacher K's "low" group, everything is spelled out in detail for the whole group. Individual questions are answered, but Teacher K intends that the answer apply to everyone. Admittedly, the types of questions prescribed allow for little variation in children's responses, but Teacher K accentuates the uniformity expected in children's work. In the second lesson, with Teacher K's "middle" group, there is some allowance made for individual differences: pupils are permitted to complete their tests at home, those who still need to may use the Part-Part-Whole chart to solve story problems. Nevertheless, a certain uniformity of response is imposed on the group: answers have to be circled, and problems have to be set out in a prescribed format. While Teacher K's directions embody more flexibility in this second group, they are still addressed to the whole group. In each class, the mathematical content--exercises on two pages of the DMP booklet in one case, and questions on a topic inventory in the other--tends to be treated as a fixed body of subject matter to be handled in an almost uniform manner by all pupils.

In the "top" group, where students are engaged on individualized assignments, far greater scope is allowed for pupils to manage their own work. Directions are given to individual students by written prescriptions attached to their folders. Students are expected to work through the assigned pages on their own, and when in difficulty to seek help from their teacher. They are not expected to seek assistance from other pupils. These more flexible arrangements do permit students to develop strategies of their own, although Teacher K does not guide them in that direction except to question those who seek help. Then, help is given on a one-to-one basis. But one needs to realize that this kind of guidance is applied selectively to those students who are consistently getting wrong answers, and to those who do not know what to do. Provided that children are getting right answers, they are not required to meet with their teacher. Their assignments for the next day are simply presented to them. Within this framework, individual choice of strategies and judgment in the use of those strategies necessarily occur, but Teacher K's procedures for managing this "top" group emphasize the completion of assigned work and getting correct answers.

Interpretations

In my commentary on these lessons, I argue that whenever a pattern of teacher-directed and whole-group instruction was adopted the mathematical content of DMP has been altered in ways which its developers did not intend. From their point of view, teachers saw themselves as presenting mathematical knowledge to their pupils in the most efficient and direct way. However, under these conditions, mathematical inquiry was often cut short, and sometimes was preempted entirely.

Accompanying these instances of teacher-directed and whole-group instruction were patterns of work which were remarkably similar across all groups. These patterns of work define the role of the individual student and incorporate the work of individual students into a process of group learning. Beneath the surface similarities of whole-group instruction, one can detect different expectations about pupils' mathematical ability. For example, Teachers B, C, F, and K all saw themselves teaching "low" or "slow" groups where pupils needed to told what to do. Even though Teacher D saw her group as "sharper" and, therefore, as capable of completing an exercise on their own, her reliance upon teacher-directed instruction had the effect of reducing opportunities for her students to investigate mathematical relationships. Although Teacher A used work stations in her lesson, the activities already embodied such a high degree of structure that the money-counting exercises took on an artificial dimension. In these groups, mathematical inquiry was related to a fixed body of subject matter which students needed to answer.

A different conception of mathematical inquiry emerged in Teacher K's "top" group. A related conception of mathematical inquiry, and one closer to that intended by the DMP authors, emerged in Teacher G's group. In order to illustrate Teacher G's conception of mathematical inquriy, I have drawn upon observations of several lessons which I observed in her class.

This section uses the following headings under which more general interpretations are made of the lessons observed: individuals, classroom groups, teacher-directed instruction, and implications for mathematical knowledge.

Individuals

To work as an individual in each of the classrooms which I have described is to receive directions and to execute prescribed tasks. An individual pupil needs to be a good listener, to follow directions, and to have her/his work evaluated by the teacher.

Learning is also attending to one's own work even though everyone else may be doing the same task. Pupils are not only required to carry out directions which have been addressed to the whole group, it is also important for them to follow the procedures which have been expounded by the teacher. Nowhere is this more vividly shown than in the question-and-answer approach adopted by Teacher B in her lesson on Part-Part-Whole.

In all lessons observed, it was rare to see students assisting each other. On the few occasions when this did happen, it had not been planned by the teacher. In that sense, learning has become isolated individual task. Even though tasks were assigned in almost all cases to the group, each pupil seemed to know what was expected of her/him. Each was aware that the quality of her/his work would be judged by the teacher. The result of that judgment would either be praise or a request for correction.

Groups

In the lessons described above, individual learners are participants in a group experience which defines their role as workers. The rituals of group questioning, group assignments, and group identity--this the "fast group" or the "slow group"--help to create a "symbolic canopy" under which these pedagogical and instructional practices appear efficient, benevolent, rational, and beyond question (cf. Popkewitz, in press).

But teachers do differentiate between groups in what they define as work and knowledge. Teachers A, B, C, F, and K, for example, all describe their groups as "low" or "slow," where children need clear direction and guidance. Whatever individual differences exist within these groups tend to become merged into the group stereotype. Teachers were observed to spend considerable time explaining to pupils what had to be done. Often, they appeared to tell pupils what to do. Admittedly, these teachers were concerned to avoid frustrating their pupils, and so they did not assign work which they thought would be too challenging. However, their approach was one where the teacher often explained how to do the mathematics, rather than one where the teacher guided pupils to investigate relationships or to detect patterns on their own. When Teachers A, B, C, F, and K asked the their children what they had done, the teacher was seldom looking for an elaboration of the child's reasoning, but was determining whether the pupil's response was correct or not. By contrast, in Teacher K's "upper group," children were often asked to describe how they had tackled a problem, and their explanations were listened to attentively even though they may not have been successful in solving the problem.

In these "slow" or "low" groups, children are seen as needing to be checked upon by the use of active questioning from the teacher. Children are not seen as, nor are they expected to be, sources of mathematical insight. Within these same groups, there is an aversion by teachers to such risk-taking as might occur if children were left to their own devices to tackle more challenging or difficult work. On the contrary, the teachers of "faster," "brighter," or "sharper" groups are more at ease in letting children branch out on their own or to confront more challenging problems. Teacher K doesn't see the need for some very able children to seek advice in the course of their assignments. These children are trusted to sort things out for themselves. Teacher J said that she saw value in the likelihood of the "eager beavers," as she called them, experiencing some frustration. Teacher G encouraged pupils to attempt a higher level test even though she though she thought that there was little chance that they would pass on their first attempt. Nevertheless, they were encouraged to see value in that attempt. Teacher D said that in her "sharper" group she looks at the potential for frustration in an exercise in order to decide whether to do part of the exercise as a whole-class activity or to assign it totally for individual seatwork. This might appear to counter a general impression of greater risk taking with "sharper" or "faster" groups. However, when I observed Teacher D and Teacher C as they taught the same topic on fractions to their respective groups, Teacher D assigned her childern to do individual seatwork at the halfway point of the lesson. She clearly anticipated some questions from perplexed or "frustrated" students, but she was prepared to deal with such questions on a one-to-one basis. By contrast, Teacher C retained direction of the activity all throughout the lesson, and preferred to lead the pupils in lock-step fashion from one question to the next.

Although teachers treat groups differently according to the perceived ability of the group, these differences in no way weaken the power of this notion of group identity. If different treatments are justified, they are justified in terms of what the group is capable of doing. In this way, the notion of a group identity is able to legitimate different expectations of teachers towards the classes they teach. A pwoerful illustration of differentiation between groups is shown by Teacher K's expectations of work in the three groups being taught by that teacher.

In the "low" group, directions pertaining to every question were discussed in considerable detail with the whole class. Individual requests for clarification were accepted, but the answers were presented to the whole class. Children were given a very specific assignment--two pages of the DMP booklet--which they were expected to complete within that lesson.

For the "upper level" group, Teacher K had set a much brisker pace for students to follow, and, like the teachers in Anyon's (1981) "Executive Elite" School, Teacher K impressed upon students their responsibility for keeping up with their work, as it had been assigned to each one individually, and their responsibility also for seeking help from the teacher when they saw a need. Checking was not part of the daily routine in class. Teacher K's time was almost totally consumed in answering questions from individual students. If in correcting their work some problem was noted, Teacher K would write "See me." on the student's page. Students were responsible for following up these requests. But these instances were not widespread, as Teacher K said:

> There are kids there that rarely see me because they are so advanced . . . there are some really great kids in this group.

Teacher-directed Instruction

In the majority of the lessons observed, this pattern of teaching is the principal mode by whi, teachers define their own conceptions of work. It is the "symbolic canopy" of pedagogical practices under which teachers give meaning to conceptions of work and knowledge. Through a pattern of teacher-directed instruction, those conceptions of individuality and group membership, which I have already alluded to, are made to appear commonplace. By the same logic, the teacher is source of all significant instructional decisions in the classroom, is the one responsible for deciding what shall be learned and how learning will take place. It is a pedagogical process in which the focus of instruction is almost always on the whole group. This process of whole-group instruction is sustained by its own peculiar techniques and rules, such as questioning techniques through which teachers address questions to the whole class before an individual student is called upon to give an answer. Equally important in creating and sustaining a process of whole-group instruction is careful regulation of the pace and flow of instruction; the assignment of work to all members through the medium of the group; and the practice of giving directions and explanations to the class as a whole, but at the same time expecting that each student will listen to what is said and apply it to her/his own work.

To some extent, these features of teacher-directed instruction were present in all the lessons observed, including those not reported. Yet, the impact of teacher-directed instruction was not the only determining factor in how mathematics was presented to children. In those instances where teachers did tell children how to do the mathematics--what the relationships were which the children then had to apply, or what pattern of analysis they were required to use--it was not just the fact that the teacher was offering directions, but the content of directions which was crucial. The content of the directions assumed a view of mathematical knowledge as a fixed body of subject matter--concepts and skills--which needed to be conveyed to the children.

Teachers A, B, C, F, and K all saw themselves as teaching "low" groups, and clearly that perception affected the strategies which they employed. Teacher A may not have used whole-group instruction, but, like the others, she did impose a very high degree of structure and guidance upon children's station work--so much that important elements of counting money were either eliminated or greatly reduced. However, Teacher D, who described her group as the "sharper" group, provided very little opportunity for her children to investigate the relationships between fractional parts and the number of equal groups.

The common thread among Teachers A, B, C, D, F, and K is a conception which equates mathematical knowledge with learning a fixed body of subject matter. Even in Teacher K's "upper level" group, one can recognize that mathematical knowledge is still presented to pupils as a fixed body of content which they need to work through. Even though these pupils work at a much brisker pace than those in Teacher K's other two groups, even though the work prescribed for then is more demanding than that given to the others, Teacher K believes that all pupils should cover the same material if only at a different rate and at different times. But at least there is confidence that these pupils will be able to work on their own, with considerably less direct teaching than that experienced by the other groups taught by Teacher K. For these "upper level" students, mathematical knowledge has become identified with what they can work on for themselves, with what they can work out for themselves. That in itself is a significant difference between Teacher K's "low" group and the "upper level" group. In the latter group, there is much more scope for children to exercise choice and judgment, as well as an expectation that children there are capable of acquiring mathematical knowledge on their own. Like Teacher K, Teacher E allows her children more scope for independent work. In Topics S3 and S4, children are allowed to work independently on the DMP booklets. However, Teacher E expects the whole class to keep together on these exercises. Her emphasis on orderly management of the whole-group and on uniformity of responses were observed in several lessons. These features ensured that mathematics was presented to her children as a fixed body of rules and procedures, even though her children were given greater responsibility in applying these rules and procedures than were pupils in the "low" or "slow" groups.

The only clear exception to this perspective of mathematical knowledge was that of Teacher G. In all four observations of her lessons, I saw her giving students responsibility for their own work, presenting them with choices to make between alternate strategies for solving problems, and helping them to refine the strategies they already had. On these occasions, there were elements of teacher-directed instruction in Teacher G's approach, but the nature and content of her directions reflected a different conception of mathematical inquiry from that which was embedded in the other lessons which I observed--different conceptions of what mathematical knowledge children should acquire, how they should acquire it, and how they should demonstrate competence in what they had learned.

Mathematical Knowledge

In my review of the Topic Interviews, I claimed that teachers were viewing mathematical knowledge as extrinsic to themselves, as something which belonged to the writers of DMP and not as a body of knowledge over which they saw themselves as exercising control or choice. I conjectured then that this attitude towards mathematical knowledge might change as teachers became more familiar with the content and practices of DMP. However, in the lessons which I observed, with the exception of those of Teacher G and of Teacher K's "upper level," very little time is spent by teachers and students in

> explaining, concluding, informing, giving reasons, amassing evidence, demonstrating, defining, comparing (Buchmann, 1981, p. 19).

For the majority of teachers, mathematics continued to be viewed as something extrinsic and fixed to be passed on to pupils, and so instruction was identified with the transmission of a body of concepts and skills which bear little relationship to those features of personal knowledge to which Buchmann (1981) alludes.

It seemed that a pedagogy of teacher-directed instruction had allied itself with a picture of mathematical knowledge as a collection of crystallized forms which needed to be passed on to students. This conception of mathematical knowledge seemed to complement a picture of students as learners unable to construct mathematical knowledge even when guided by their teacher. Thus, with "deficient" learners, the body of concepts and skills to be learned can be treated as identical for all learners, and efficiency of transmission can become the overriding pedagogical consideration. Moreover, whole-group instruction is highly efficient for transmitting a fixed body of subject matter, provided pupils are perceived to be members of a relatively homogeneous group, and as deficient with respect to the same subject matter.

Implicit in the majority of lessons observed was a belief that children demonstrated their grasp of mathematical knowledge if they were able to replicate the procedures and patterns of analysis which had been taught to them. To allow pupils to choose between alternate strategies was seen as problematic and inefficient. Inefficient because there was a risk that children would confound alternate strategies. Problematic because teachers did not see the value in alternate approaches if they were able to teach children one procedure which produced correct answers. Thus, mathematical knowledge became identified with having a method for producing correct answers.

By contrast, the children in Teacher G's Grade 2 were expected, as were those in Teacher K's "upper level" Grade 3, to work at developing their own procedures. In one of Teacher G's lessons which I observed, children were expected to refine their own strategies for finding a "hidden number." They were to ask their teacher a series of questions of the form "Is it greater than . . .?" or "Is it less than . . .?" Very quickly, children progressed from asking random questions to adopting a more systematic approach. In another lesson, Teacher G asked her children to choose one of two methods for solving story problems, and, having tried both, to stay with the method which seemed to work "best for them." In these lessons, it was not assumed that mathematical knowledge is a fixed body of subject matter to be conveyed to pupils. On the contrary, the acquisition of mathematical knowledge was assumed to require active organization by each individual so that a coherent structure could emerge. These approaches assisted children to elicit those individual strategies of thinking and patterns of analysis which were available to them (cf. Lovell, 1972).

As with the pupils of Anyon's (1981) "affluent professional" school, these children were being taught that mathematical knowledge comes "from discovery and direct experience" (p. 18), and consists of concepts and skills which "are to be used to make sense (of situations) and (which) thus have personal value" (p. 23). Mathematics was not being presented as a predefined artifact of instruction. It needed to be given personal meaning by individual children.

Summary

The authors of DMP believed that there should be a match between pedagogical practice and their belief that children are able to participate in the creating and testing of mathematical knowledge. That goal was being attained by Teacher G. It was being attained to a lesser extent in Teacher K's "upper level" group. In other classes observed, a different perspective seemed to prevail. There, mathematical knowledge seemed to be equated with a fixed body of subject matter. This perspective, allied with a belief that children were deficient in that subject matter, found expression in patterns of teacher-directed instruction where pupils were often told how to do mathematics--what relationships to apply, what patterns of analysis to use. The pedagogical, social, and epistemological assumptions implicit in these patterns of instruction were not aligned with the perspectives of the DMP authors.

If the underlying assumptions of these predominant patterns of instruction are at such variance with beliefs, purposes, and values which were espoused by the DMP authors, I needed to ask how these patterns of instruction emerged and became consolidated. That is to be the principal question of the following chapter.

Chapter 6

THE PATTERN OF CHANGE

In this chapter, the changes and adaptations which have been introduced into DMP by teachers are discussed. This process of adaptation and change will be linked to the notions of "technical" and "constructive" change. My conclusion is that the predominant pattern of change can be described as "technical" rather than "constructive." In this way, I relate patterns of change to underlying conceptions of school work and mathematical knowledge.

The chapter has been divided into three sections. In the first, the distinction between technical and constructive change is addressed. Next, a synopsis of changes and adaptations which have taken place in the teaching of DMP is presented. These changes are presented at each grade level in the two schools being studied. Finally, I offer an analysis of these adaptations and changes in terms of conceptions of work and knowledge as embedded in classroom practices. Behind the predominant pattern of technical change is a conception of mathematical knowledge as a fixed body of subject matter needing to be learned by all pupils. This pattern of technical change can be viewed, therefore, as a series of adjustments to the procedures of instruction and classroom management. These adjustments are intended to facilitate the efficient transfer of a fixed body of mathematical knowledge in a classroom setting.

In presenting these synopses of change, I need to remind the reader that I am not evaluating the quality of teaching which I have observed and discussed with teachers. Effective teaching was taking place in the two schools being studied, but effectiveness did take on different connotations with different teachers, and often occurred in ways which were different from what the developers of DMP had intended.

Technical and Constructive Change

The metaphors "technical" and "constructive" were used originally by Popkewitz, Tabachnick, and Wehlage (1982) in their evaluation of six IGE schools. In their IGE Evaluation Study Phase III, they had used these terms to describe schools in terms of the predominant pattern of their implementation of IGE. In depicting "technical" schools, Popkewitz, Tabachnick, and Wehlage (1982) have argued that

> techniques have become the ends of school activity rather than a means of instruction, and technology provides an independent value system that gives definition to curriculum, classroom activity, and professional responsibility. (p. 61)

There, teachers sought for the most efficient ways of processing pupils, and the smooth management of instruction appeared to be more important than considering what was appropriate to teach, and how it could be taught most effectively. Knowledge in the technical schools was seen in terms of standardized units:

> All ideas and skills to be learned were presented in a discrete and sequenced form. (p. 87)

By contrast, in the "constructive" schools, the authors reported that:

> While teachers appeared to have adopted the main elements of the reform program, the technologies were used in ways which responded to the definitions and special requirements of the school. (p. 109)

There, the technologies of IGE were in evidence but they did not become priorities to be pursued in their own right, nor were they allowed to distract teachers from what they saw as their responsibility to decide what knowledge was to be taught, and how children were to learn.

In the following section, I apply the metaphors, technical and constructive, to changes and adaptations effected in DMP in the two schools being studied. I intend that these metaphors of change refer primarily to ways of thinking and styles of work. Thus, they should be taken as referring to the network of beliefs, purposes, and values which have given direction to the changes and adaptations described. Although the authors of the IGE Evaluation Study Phase III used the metaphors as descriptions of schools, their intent was to employ the expressions, "technical school" and "constructive school," as shorthand ways of referring to predominant patterns of belief, values, and action in those schools.

In this chapter, I intend to use the same metaphors, not to describe the two schools being studied, but to describe the patterns of change effected in the teaching of DMP.

Synopses of Change

The following accounts of change and adaptation in the teaching of DMP are based upon the interviews which I conducted with 10 teachers in the two schools. In presenting these synopses of change, I outline the principal respects in which adaptation and change have occurred at each grade level. The impact of these changes on conceptions of school work and mathematical knowledge is discussed in each instance. Finally, these changes are related in a more general way to patterns of technical and constructive change.

Grade 1

Even though some changes in content have taken place at this grade level in both schools, far more obvious are the changes in the nature of the learning activities which comprise DMP. These changes seem to reflect teachers' wishes to monitor and supervise children's work more closely than provided for in the DMP guidelines. Equally prominent among the rationales presented to justify these changes and adaptations are the special needs of the "low ability" or "slow" groups of students; and the belief that children need more structure and direction than allowed for in the DMP course materials.

Changes in content have been viewed by the teachers as supplementary to the central core of DMP activities. Each teacher at Grade 1 in the two schools has incorporated activities which introduce children to writing numbers, sequencing numbers from zero to 100; counting by 2's, 5's, and 10's, as well as additional activities on time and money. Tests which require students to recall basic addition and subtraction facts have also found their way into the two classes taught by Teacher A and Teacher E.

However, although both teachers see the content of the core as being expanded by the inclusion of additional subject matter, they have each adapted many of the activities into which the content of DMP has been incorporated.

Teacher A summed up her reasons for making these changes in the DMP activities in response to my question:

> So, many of the changes you have introduced in DMP activities are intended to break down those activities into more manageable bits for children?

"Yes, that would be the essence of any mathematics program" (Teacher A). Her children she says, need "more guidance, more support, more reinforcement, more constant checking to see if the concepts are there." This agenda for change can be illustrated by the following instance: Instead of having children pick out objects from around the room and then measure them--as one of the

DMP activities suggests--this teacher asks children to select an object and to measure its length or weight from a collection which she has prepared in advance. Each object in the collection is represented by a diagram on a worksheet which the children have. Having measured the length or weight of the object, children are required to record their measurement in the appropriate space on their worksheet.

In other measurement activities, where children are required to use paper clips, links, or straws to measure the length of an object, Teacher A finds that children are confused by the inexactness of measurements of objects randomly picked out from around the classroom. To counter this difficulty, the teacher exercises greater selection in the objects she requires to be measured:

> They (the children) want it to be closer to one or two. . . . This estimating is a hard concept for them. So the objects that I pick out and have them measure come out much closer than a random selection and that cuts down (on the confusion).
>
> I give them measurements that were fairly close, that would come to just about so many . . . Kids find that (i.e., the recommended activity) a little harder than your manual says it was.

Teacher A added:

> It's much better if I say, "Let's measure our desks," as opposed to "Let's measure anything in the room."

When I asked her later whether she was concerned primarily by the imprecision of the measurements which children would be confronted with if they have to pick out objects from around the classroom, she said:

> No, that is not it at all . . . I think that they might tumble over each other . . . With my kids I might have four over here saying, "What are we supposed to do?" I think the idea that you are going to give this activity and this youngster is going to go over here . . . and another youngster is going to go over there and find this. . . . That's not quite the way it is going to work out.

This teacher's desire to institute "more guidance, more support, more reinforcement, (and) more constant checking" had led her to modify many activities which the book recommends to be done by individuals or small groups.

> There are many activities that are too individualized. They rely too much on the individual. . . . Some of my children are too immature to go over and sift out this process. . . . Many times, I take them and we'll do it together. . . . It works out much more successfully.
>
> I think sometimes their desire to have the children make these discoveries on their own is overrated.

On the other hand, Teacher A recognized that these were some trade-offs in adopting a group approach to many of these DMP activities:

> It doesn't allow for that independent thinking and that independent type of reasoning, but I think that some of the youngsters would never do it anyway.

In observing this teacher's class, I found that the children were either doing station work or were grouped around their teacher for a traditional lesson. It seemed that she did not rely on small group activity at all. Later she clarified this observation by saying that there were three ways in which she grouped:

> We are all together. We go off as individuals or parts, and I supervise and move around giving help whenever it is needed; and then . . . the closest I get to a small group is that I have children who are having difficulty stay with me a few minutes. . . . On occasion, I pull youngsters out for a presentation. . . . But that is the exception rather than the rule.

Some predominant directions can be inferred from these changes to activities within DMP. For Teacher A, the mathematical content of these activities is clearly subordinate to her concerns for smooth management of instruction. As well as preferring to do things "together" with her class, Teacher A has also simplified the content of the activities. In justifying these moves, she claims that her children are too young and too immature to be left to do independent investigation. These overriding concerns for good classroom management have imposed a uniformity upon children's mathematical inquiry, and have curtailed opportunities where children have to confront ambiguity and so develop strategies for making ambiguous situations more amenable to mathematical investigation.

Similar features were evident in the implementation of DMP by the other Grade 1 teacher (Teacher E). The importance which this teacher attached to the ability of the group was more clearly stated, since in previous years Teacher E had taught the "lower" group. In reflecting upon this experience, she said:

> I always felt with this particular mathematics program that when you have a low group . . . they're really only imitating. I feel like they've just kind of echoing back everything I say, and that there is not always an understanding, especially when you get to story problems. . . . I feel like I'm just practically doing it for them and giving them the answers . . . and they're just writing it down because that's what I tell them. I don't know if there's always a real understanding with the very bottom group.

Her concern about the needs of the "low" group had direct implications for instruction:

> When you have a low group you always have to spend every possible minute on just the very basic things. You can't throw the extra activities in . . . I would just need the basic work.

However, even with this year's "bright" group, the teacher prefers to introduce every topic to the whole class. Those students who have an occasional problem can be more easily helped than in previous years.

> Because the majority of children can take care of themselves. And so they can just go on without too much direction, and so I'm free to handle the one or two who need extra help.

Even though there is greater confidence in the ability of the "bright" students to "take care of themselves," one should not infer that the teacher's concern for efficient and smooth organization is not obvious. Teacher E refers to her procedures for conducting station work:

> Sometimes I have a little bell (such) that when the bell rings then they move to the next station with their partner.

During station work, Teacher E also decides on the activities to be done by students:

> I can see what the activity is getting at and therefore I modify it.

Activities are therefore modified not just according to the teacher's perception of pupil ability, but also according to her preference to keep the group together and so to monitor its work more closely. This concern is illustrated by several changes she has effected this year with her "bright" group. In an activity where children were supposed to measure a partner's wrist and neck, Teacher E had them come and measure her neck and wrist,

> So that everybody has the same numbers up above so that they can see that they're staying together with the class.

In another activity where each child was to write down a number and to apply changes to that number, using addition and subtraction operations, according to directions given by Teacher E, the following change was made:

> I give them the number. The whole class has the same number. . . . It's easier to check and it's easy for the children to check and see if they are right.

In presenting story problems for the first time, the teacher prefers to read the problems to the class. Her advice to the students is:

> If you want to make sure what the story problem is saying to you, and what they want you to do, wait and I will read each one. And those who know how to do it will go ahead, and the other ones will wait for me and I will just read it.

Her preferences for directing instruction and for leading pupils through story problems become more pronounced when more difficult types of story problems are encountered:

> not your basic joining and separating. It really gets confusing. So what I do many times is read the story problem. I'll read it once and then I'll say it in my own words. I don't know if that's really the correct thing to do or not, but I don't always feel that the children understand what they are trying to do here.

Some other topics, by contrast, lend themselves to more individual work but there are limits set to how far the class can become spread out:

> This year I do a lot more where I just introduce something and they go ahead on their own. Some topics we all do individually where they go ahead. Sometimes when we do S3 or S4, they just go on on their own. (You would expect them to cover the same material but at a different pace?) Right . . . with our regrouping we can pretty much go through a unit without waiting for a few stragglers.

As in the other Grade 1, there was a similar focus on teaching the class as a group. This helped to promote more efficient management of instruction. When difficult concepts and processes are encountered, the class works as a group with the teacher taking the lead. Likewise, the teacher's role in monitoring students' learning is raised to major importance. It is essential to be able to check quickly and efficiently on the correctness of pupils' work. Teacher E argues that children need to know whether they are right or wrong without having to wait. But her emphasis on keeping the group together, on uniformity of responses, and on efficient checking of these uniform responses are consistent with her goal of having children cover the content prescribed in the DMP booklets. With this year's "bright" group, there are more opportunities for children to exercise independence than in previous years when Teacher E had the "slow" group. But Teacher E's concern for smooth management and for keeping the group together on the same task does limit these opportunities for independent inquiry among her "brighter" students.

Grade 2

Two quite different patterns of change have emerged at this level in School 2. For Teacher F of "the slower moving" group, there has been little time to do the extra things. Time spent on geometry has been cut back because Teacher F sees her children needing a firm background in addition and subtraction when they move into Grade 3 next year. At the same time, her teaching has become more direct and tasks simplified so that pupils can achieve some success. In the other group, Teacher G has developed many additional activities using ideas implicit in the DMP materials. These changes which she has introduced reflect her belief that children are participants in the creation and testing of mathematical knowledge.

By contrast, Teacher F spoke of the special difficulties which she was experiencing with her "low" group:

> However, I have regrouped within that group. . . . I have two or three that can work almost totally independently--read their own directions and go on from there. Probably six or seven more that work--that do part of it together--we do the first couple of ones on the page and they get the idea and then they can go on from there. Then others that I have to work almost one-to-one with.

Teacher F depicts her pupils as needing a lot of practice and whatever success they can from DMP. Their previous topic on solving number sentences had proved to be very difficult:

> The missing addends were really difficult for them, even with things to manipulate.

However, their current topic on grouping was described as

> a little more concrete to them at this point, and they are meeting with a lot of success which they really need.

When faced with what she saw as difficult activities, such as using the Part-Part-Whole chart, the remedy was plenty of directed practice:

> And so a lot of times we just practiced--we read the story and labeled the different things part-part-whole. And even transferring from the story where it was labeled correctly to the chart was a difficult problem for them. . . . With this group now . . . I find myself doing less reasoning out with them. . . . I don't like to do this a lot--but with the part-part-whole, I almost had to say--"Let's just learn this rule."

Even with the current activity with which the students were finding fewer problems, Teacher F did not think that they should do all the

> stations with the counting objects and regrouping. I'm not going to insist that they do all 12 stations but I would like them to do at least half of them to get the idea. . . .

Optional activities often are omitted, not because pupils have no need to do them, but because of the pressure of time to cover other topics which Teacher F sees as more crucial to pupils' survival in the next grade. Faced with these problems, Teacher F places less emphasis on reasoning and prefers to give her children a set of rules and procedures which they can follow, although she is not certain that they understand what the rules and procedures are about. Some activities have been omitted entirely, while others have been "trimmed back" to their bare bones. Teacher F has settled for a much reduced conception of mathematical knowledge for her pupils. Her hope is that by the end of second grade they will have mastered that body of subject matter--especially the addition and subtraction algorithms--which she sees as necessary for their "survival" in Grade 3.

In School 1, Teacher B also speaks of having a "slow" group. Although she reports that her children do engage in station work, Teacher B says that her preference is to work with the whole class as a group:

> I don't like them just to push pencils. So we do spend a lot of time together. Probably the greater part of the time in math, we are together.

In those activities where the pupils are likely to experience difficulty, Teacher B sees whole-class instruction as alleviating two problems. It can overcome the difficulties encountered by those who "do not have good facility in reading." It can also ensure that the process to be taught is executed in the same way by all children. Teacher B's comments on these techniques of group instruction serve to illuminate her teaching style in the lesson reported earlier:

> I have to remember that DMP is not a reading program. Some children who do not have good facility in reading, so they are held back . . . Many times, I read problems to my kids and that's why we label the "stuff" together.
>
> Number values are circled to attribute "whole" and "parts" by labeling them. . . . If they see certain key words like "part" or "some" they circle those too.
>
> Even if we don't say what they are. . . . If I read the problem to the group, they can label.

Likewise, with the topic of comparison sentences:

> We do it together a lot. We talk about each one. That way . . . I get a lot of feedback as far as how the whole group is understanding.

Later on, when Teacher B feels confident that the pupils have mastered the ideas being taught in this way, they will be expected to work on their own. However, at this stage she sees a need to monitor pupils' work very closely:

> They will do a lot of this on their own, and I will move around to see what they are doing and pick out children who are still having blatant problems. And with them individually I also find it very valuable

> when they hand in any work that it gets corrected by me, and they must correct everything before they move on to a new activity.

Because Teacher B sees many of her children needing "a lot more reinforcement," she therefore prefers to cover most or all of the optional activities.

> Because my kids really need it.

Indeed, her preference to cover many activities which otherwise might have been skipped is one of the major reasons offered by Teacher B to explain why she had spent more time on mathematics than any other Grade 2 class in the observational study:

> I'm never intimidated by spending more time. . . .
> I've always had the low group.

When Teacher B speaks of spending more time in doing mathematics, that time will be spent largely in whole-group and teacher-directed instruction, where Teacher B will set out the pattern of inquiry or method of analysis for children to follow. In that context, her close monitoring of children's work makes good sense. For Teacher B, mathematical inquiry is following a set of procedures which produce the correct answer. Her adaptations to the activities of DMP confirm a view of mathematics as a collection of crystallized forms which need to be passed on to pupils, and which they are required to reproduce.

Like Teacher F and Teacher B, Teacher G's management of children can be described as smooth and efficient. But her pattern of instruction is not based upon a conception of mathematics as a fixed body of subject matter to be mastered in the same way by all pupils. Therefore, her pedagogy bears a closer relationship to the underlying goals of DMP. In her class, the children are encouraged to take greater responsibility for correcting their own work, they are engaged frequently on tasks which are matched to their own interest and level of achievement; they are encouraged to adopt different strategies to the solution of problems; and these different strategies are presented to them in the expectation that they will choose the one which works best for them. Additional activities are included in Teacher G's Grade 2 program. Some of these are intended to support concepts and skills which children have learned at an earlier stage, whereas others are intended to build bridges with topics which will be encountered later.

To illustrate this pattern of change in Teacher G's class, I refer to the timed tests which were described in the previous chapter. These were not introduced simply as an end in their own right. Teacher G sees a close connection between children's ability to solve story problems and their familiarity with "basic" facts. She has also introduced children to number pairs, or as she calls them "10 relations." Each number is related to another number such that the pair of numbers adds up to 10. For example, if one number is 2, then the other is 8. So far, the children have explored only combinations of numbers which add up to 10. But Teacher G says: "Eventually, I might give them 17, and they will say 'Minus 7'." By helping children to become more familiar with patterns among numbers, Teacher G hopes that they will become more confident in solving story problems, and less tied to one method of solution, namely Part-Part-Whole. Already, she has introduced children to estimating as an alternate method for solving story problems. Before students use the Part-Part-Whole classification to analyze a story problem, Teacher G asks them:

> "Just for your estimate, do you think the answer is going to be more or less than one of the numbers? Put down an M or an L."

Teacher G continues:

> And now I'm getting to the point of saying, "You choose. Do you want to do the chart or do you want to use the estimate? Do the one that you think helps you the most." Many are (using the estimate). I'm hoping that most will feel comfortable with that.

These changes which have been effected by teacher G have focused on the need to help children acquire a sense of relatedness among the various concepts and skills embodied in DMP. Teacher G claims that some of these relations were not well developed in the DMP materials and that much of her own innovations have been to make these relations more explicit. This dynamic feature of mathematical knowledge is apparent to her pupils when they are asked to choose between strategies and to refine their own procedures. In

Teacher G's class, the management of instruction is more clearly subordinate to the mathematical goals which she has adopted. There, mathematics is not presented to children in terms of crystallized forms of knowledge, but as knowledge to which they need to bring understanding and personal meaning.

Grade 3

In School 2, all three mathematics groups are arranged by ability and are taught by the same teacher. The most significant change in 1981/1982 has been to allow the "top" group to proceed through DMP in independent assignments. Since these pupils completed Topic A4 in Grade 2, they are now engaged in working through the DMP booklets from #43 onwards. Under this new arrangement, the teacher's role, to use Teacher K's words, is to act

> basically as a resource person . . . if there is a certain group of kids that are approaching the same area at the same time, I'll small group with them . . . otherwise there are kids who rarely see me because they're so advanced.

The manner in which children in this "upper level" group work on individual assignments was depicted in the previous chapter. According to Teacher K, this new arrangement is working smoothly and efficiently, except that "sometimes I find myself teaching the same lesson twenty times." This difficulty occurs because students are now so spread out that they are seldom together enough to be treated to a small group presentation nor does the teacher assume that they will all find difficulty with the same activity.

Although this new arrangement is described as a system of individualized assignments, the content of instruction is not varied for individual students. Even where the DMP booklets list several alternate activities (e.g., 49E, F, G), Teacher K prefers to have all students do all activities. Where optional activities (e.g., 49I) are available, this teacher would like to see all students attempting this activity. In essence, therefore, students proceed through the same content at an individual pace as directed by the teacher. Given this uniformity of content to be covered by individual students, this change in instruction for the "upper level" group is confined to the technology and procedures of individual pacing. The process of allocating individual assignments is defended as a more efficient management of instruction for this group of children. It does not aim to provide different learning activities for individual students. I present this as a clear example of technical change, one where the primary focus is placed on the procedures and techniques of instruction rather than upon the content of instruction.

In the other two groups, children work together as a group most of the time, and the teacher usually presents an introduction to each activity. In the middle group, individual work is assigned after an initial introduction. For the lower group, independent work is confined only to carefully specified activities. This group, according to Teacher K, has "a lot of reading problems":

> If there is explaining to do, we go over that, or if there are activities, we go over that. Then, I usually assign a page or two. . . . I give it so that . . . the quicker kids are going to be done quite early and so that the average student is going to get done in that period of time.

For these "quicker" workers, there are numerous games provided which reinforce basic number facts and drill. In this lower group, where reading problems are vexing to the teacher, an acute problem arose when students encountered word problems:

> There is no way those kids can do that without me. . . . For the first couple of problems, I've been reading and having them pick out key words--what process we take to figure out the answer, eventually hoping that they will be (able to) handle (these problems). Right now, they can't though. . . . It would set them off on the wrong foot.

Apart from reading problems to the pupils and "having them analyze what I'm saying to them"--a procedure similar to that adopted by Teacher E in Grade 1--Teacher K has also prepared additional sheets in which word problems are presented to students in their most elementary and skeletal form. These sheets are intended to provide students with success in solving less complex word problems than those contained in the DMP topics, and so to avoid their developing bad attitudes toward this large component of Topics A3 and A4.

Thus, the students' perceived inability to do story problems on their own has motivated their teacher to change the way word problems are presented. The primary direction of this change is to simplify the semantic structure of the problems themselves, and to adopt a group approach to the reading and analysis of word problems, an approach where the teacher leads the class through a predetermined pattern of analysis. Once more, the direction of change has been to alter the techniques of presentation and instruction. Under this new arrangement, the work of pupils is more clearly specified and directed by the teacher. These new approaches are intended to foster greater success in the way pupils handle word problems. Once more, the path chosen is not one which seeks to address directly the content of instruction, in this case how students might penetrate the semantic structure of word problems, and so write and solve an appropriate number sentence. I present this as a further instance of technical change.

These patterns in the implementation of DMP in Grade 3 are not new to the 1981/1982 school year. Similar responses to those presented above were evident in the previous year. Then, Teacher J who had the "top" group was faced with the challenge of bright and fast-working students. However, she preferred not to proceed with a program of individual assignments:

> I generally tried to keep the class sort of together. The book is usually set up so that there are a lot of extra activities for children who can go ahead, and seeing they were all high, I felt . . . they

> really got more out of it if you would make a presentation of a new subject to a group rather than do it for each child, because . . . you don't do as good a job when you're hurried.

Those who finished early were assigned an activity which may not have needed as much stress in importance; or it may have been a "fun" topic which although "quite challenging I would let them do it by themselves."

There were particular problems created by these "fast workers" for the smooth management of instruction and the equitable treatment of all students. Teacher J comments:

> The children who are "eager beavers" keep on pushing you . . . and asking for your attention all the time, and some child who needs your help is not getting your help.

Her remedy was, in part, to work as a group, at least in the introductory phase of an activity. Toward the end of an activity, Teacher J used "challenging" activities to occupy these "eager beavers." She saw value in their coming up against problems which were demanding and challenging because she believed that they could handle the challenge, and because challenges were good for them. How unlike teachers of the "lower" groups who were careful to shield their children from potentially difficult experiences.

> I think it's OK to let the "eager beavers" go ahead, and, then if they get frustrated once in a while, that's good for them.

Thus, the content and activities of DMP can be used to keep bright students and "fast workers" occupied. To put it another way, the content and activities of DMP can themselves be used as instruments for the smooth and efficient management of group instruction, and as a means of dealing with students whose eagerness to push ahead is likely to interfere with the teacher's ability to deal with slower learners. This focus upon utilizing the content of DMP to support and maintain certain procedures of classroom management shows that the content of DMP, construed as a fixed body of subject matter to be covered, can itself be used to effect technical change in instruction.

Teacher H also taught Grade 3 in the previous year. Then, she reported that her children who comprised the "middle" group came into Grade 3 still struggling with Topic A3. She said that the "top" group had finished with Topic A4 by the end of Grade 2. So, Teacher H saw no sense in expecting her "middle" group to catch up to the "top" kids:

> They came to the school behind. . . . I didn't feel that need to catch up and be with those--the first and second groups.

During the year, she reported that they covered

> a great deal (of) basic addition and subtraction problems . . . They really needed a lot of reinforcement and additional practice, and that was good for them . . . in a more motivating way so that the drudgery of computation (was avoided).

These children also needed help with word problems. Their difficulties were depicted by Teacher H as reading problems at root, and not mathematical. Therefore,

> we read the problems together and I would stress different words like "altogether."

In dealing with word problems in this way, Teacher H took special pains to alert pupils to the presence of key words. She described her approach in the following way:

> I would say, "There's one word in the story problem which is going to give you a big clue and tell you what you should be doing--adding or subtracting--what clue word would that be?" . . . They would underline their key word. And that's how we did most of the story problems. . . . Towards the end of the year, . . . if I gave some additional story problems I'd try having them do it by themselves. But again, because some of these kids experience problems in reading a lot of the time, I'd just read the problem to them.

Here again is presented a change in the techniques of instruction rather than a change in the way content is analyzed and developed. In other aspects of DMP, Teacher H found that pupils were able to do number problems if she simplified them. Sometimes she decided that optional activities should be left out "because they would be too confusing for the children." At other times, she found that if pupils were given some "warm-up" activities to work through as a class group, they would then be in a better position to do assigned work on their own:

> They have to have a pre-exercise before a page where we might do a lot of board work together first. That

> made a real difference last year. Because they then got to the page and they could do that independently, usually by themselves.

These modifications also took the form of changes to the techniques and procedures of instruction. Rather than assign students to individual work after the teacher had presented a new idea, some of the same work was treated as a class activity on the blackboard. These pre-exercises did not embody a new approach to the problems which were later done independently. Basically, a fixed body of content has been simplified to assist pupils to move more confidently into independent work later in the lesson.

In School 1, Teachers C and D see value in their children working together as a group, although this concern is stronger for the group which is described as the low group. Teacher D, who has the "sharper group," says:

> I generally keep them with me. If they're going to be doing an activity that requires a small group, I'll just divide into groups of four or five, and make sure there's one child in there who would be able to lead and help the others. For the most part, they're with me.

Asked how did she determine whether an activity should be done by individual seatwork or class directed, Teacher D said:

> I look ahead to see if it's going to be just too difficult for them. Judging the frustration level. If you're going to have half the class coming back to you saying, "I don't get it," you might as well keep them all with you in the beginning.

However, with this sharper group, the teacher hopes that after an initial introduction children are able to proceed independently with work which has been assigned for the whole class to do. With some difficult topics, Teacher D introduces the activity and works through several problems with the group, then

> toward the end of the page you would have four or five problems that you would want them to do on their own, but you would say to them, "Anyone who wants to stay and make sure they've got it can stay with me."

But other activities, especially games and station work, are either streamlined or omitted because they entail practice which the teacher believes her children do not need.

With the slower group, Teacher C is less comfortable having the children working on their own:

> I would say the majority of times it is the class together. Sometimes there may be small groups that come up: maybe they just "blew" a page or they didn't understand the concept, and I would say, "Before you start the page, those who are not sure come back to the chair."
>
> The group likes to do things on the board. It's instant feedback.

Once again, this pattern of group work becomes more pronounced when the students encounter more challenging activities, such as word problems:

> We did most of the story problems, at least we read through (them) together and they solved them. We might have said, "What would you do, would you add on or take away, and why?" And when they would just put the plus (next to the problem) and they'd have to go back and do the problem themselves, but at least they had that guide.

Unlike the "sharper group" which can afford to skip various activities, Teacher C sees her group as needing

> more of the options and more of the other reinforcing. . . . I do have numerous sheets that I could use if they need to have extra work. I would make up with things like that. . . . it kind of goes according to their ability. . . . That is why at times we are behind them (the "upper" group).

Both teachers see mathematics as a fixed body of subject matter, and the teacher as presentor of that material. Teacher C, seeing herself as having a special responsibility for the "slower" students, takes a very direct role in the presentation of the concepts and skills to be learned. Her preference for a pattern of direct instruction comes through very clearly in her preceding comments. It is illustrated by her very direct style of teaching in the lesson which I observed and which she later described as a typical lesson. It is, perhaps, best summed up in her comments about the group she teaches:

> The group as a whole needs to do a lot of things together. I see that as soon as they are allowed to go to their seats. They just go on, "What are we supposed to be doing now?" . . . They get lost. A lot of the group gets lost on the reading material. . . . I like to do it as a group.

I asked Teacher C what she saw as her key role as a teacher in these large group presentations. She said:

> Getting it across to the group, and also working individually. Hopefully, the individual child will get it, with help, from the person next to them, or the teacher, or by questions, or just by grasping it.

Teacher C's concern about "getting it across" reflects her attention to the procedures of instruction. What is to be taught is a set of ideas and skills which she presents in a discrete and sequenced form. The means of instruction have assumed a priority in their own right, and there is comparatively little attention being given to the ends of mathematics instruction. As Popkewitz, Tabachnick, and Wehlage (1982) comment, "Ordering knowledge in this way enabled teachers to devote full attention to the procedures of implementation" (p. 87). Teacher C's approach to the teaching of DMP reflected similar concerns as did most other teachers in the two schools being studied. They were clearly shared by Teacher C's co-worker in Grade 3, Teacher D.

Although the other Grade 3 class is seen by Teacher D as the "sharper group," more capable of moving quickly through DMP, there is still a clear sense in which Teacher D sees herself as bearing the primary responsibility for introducing a new concept and thereafter for assigning a work program for the group. I asked her what she saw as central to her role as a teacher of mathematics. She replied:

> In this program? (Yes.) The central role would be the explanation of the new concept. From there you're organizing their learning. You're seeing which pages need to be done today, which need to be done as a group. But as a teacher where I'm most valuable is in the beginning, introducing the concept.

Thus, Teacher D sees her task as covering the content of the DMP booklets. As my observations showed, her "explanations" of the concepts which were being presented to children often left children with little to investigate on their own. Their task was to apply the concept as it had been presented, and to work through the exercises in the DMP booklets

Teachers' Conceptions of Mathematical Knowledge

I have argued that behind so many of the changes which teachers have introduced into the teaching of DMP is a conception of mathematics as a fixed body of subject matter to be conveyed to students. This conception has, I have argued, been implicit in the nature of the changes effected by teachers. This interpretation is supported by those questions in my interviews which asked them to describe what mathematical understanding they hoped to achieve through their teaching of DMP.

Teachers' responses to the question, "What mathematical understanding and skills do you hope to achieve with your children in the course of this year?", often presented answers in terms of a record of topics to be covered or skills to be mastered. Put in another way, they tended to report on "what it was time for children to learn" at that year's level. Popkewitz (1982b) describes this kind of response as reflecting a compendium of unexamined folklore of schooling. In this respect, Romberg (in preparation-c) argues that the distinction between mathematical knowledge and the record of that knowledge is crucial. For children, the acquisition of mathematical knowledge cannot be represented adequately by citing a list of topics to be learned, since that knowledge is acquired in a classroom instructional setting where the purposes and values embedded in that setting shape not only what is learned but also how children are to learn.

All too often, teachers' responses tended to filter out references to the social context in which mathematical skills and understanding were to develop. Thus, from a Grade 3 teacher:

> I would expect them to be able to add and subtract with accuracy and some speed. To have at least the understanding of multiplication and division, and to a certain extent some memorization of facts. . . . I would also expect them to be able to understand what a fraction is telling them. (Teacher D)

From a Grade 2 teacher:

> I hope that they will be able to solve open-ended sentences, and have the process pretty well under their belts. . . . I hope that they would be able to see expanded notation . . . and to write their own expanded notation for certain problems. . . . Also a pretty solid handle on the basic facts, plus or minus, through 20. (Teacher B)

From a Grade 1 teacher:

> If I had to narrow it down, . . . I would like them to know the Part-Part-Whole concept. I would like them to be able to identify from a story . . . what pertinent information they have regarding a "whole" and a "part." I would like them to know that, if they have a "whole" and a "part," it's a take away, if it's a "part" and a "part" it's an add on. I would like them to know that as well as they know their own name. (Teacher A)

But even though these responses may reflect an "unexamined folklore" of mathematics teaching, is it not possible for these responses to define the relationship between school work and mathematical knowledge at certain points in time? Could it not be said, for example, of children in Teacher A's Grade 1, that at some time of the year they were engaged in learning and applying the Part-Part-Whole classification to story problems? Is this not an adequate way of representing a connection between school work and mathematical knowledge?

This account is unsatisfactory because it treats what children were doing as an isolated intellectual exercise. The acquisition and application of mathematical knowledge is profoundly changed when mathematics becomes part of classroom work. Indeed, the notion of work as a social and ethical construct is necessary in order to portray effectively this social dimension of the acquisition and application of mathematical knowledge in the context of classroom instruction. Two questions then arise: What conceptions of work are involved in classroom instruction; and what conceptions of mathematical knowledge are embedded in these conceptions of work? With these questions now made explicit, one can see that behind teachers' responses there was an assumption that the content to be taught was independent of the procedures of instruction. Instruction had its own rules of management and task organization. In an important sense, these rules and procedures could be adapted to the teaching of any subject matter. Notions of work, as a social and personal activity related to the development of mathematical knowledge, have been filtered out of the above responses.

They have been filtered out because conceptions of what is appropriate work for teachers and children have already been presumed in a management perspective of instruction. Within that perspective, the technical changes effected by teachers appeared to be rational and normal.

However, in Teacher G's response to the same question, it was possible to discern quite different assumptions about school work and mathematical knowledge. Teacher G responded in terms of the mathematical objectives which she hoped to develop with her students:

> I hope . . . that students will be able to determine for themselves the correct process for solving story problems . . . that they will be able to decide whether to add or subtract. Maybe they will use the chart (Part-Part-Whole), maybe they won't. . . . But the basic one is being able to estimate what the answer should be, and knowing that the answer might be greater than the numbers given, or might be less than the numbers given. (Teacher G)

Thus, Teacher G's response conveys a sense that what constitutes appropriate work for children needs to reflect one's perspective of what it is to know and do mathematics. She implies that children are expected to exercise some independence of choice and judgment in determining the correct process for the solution of problems; and that there is no single and predetermined procedure which she expects them to replicate.

For other teachers, knowing and doing mathematics seemed to consist in acquiring a fixed set of concepts and skills which had been predefined for children to learn. This conception of knowing and doing mathematics cannot be separated from teachers' conceptions of school work which were rooted in a management perspective of instruction. Within this set of beliefs, purposes, and values, the teacher's role is to convey a fixed body of subject matter to children with little attention being given to how that knowledge was once created and tested. The majority of the changes which teachers have effected in the implementation of DMP can be interpreted within this management perspective as adjustments to the procedures of instruction. In that sense, they are called technical changes.

Technical Change

Through my interviews with teachers and observations of their lessons, as discussed in this and in the preceding chapter, several features have emerged which enable me to depict a predominant pattern of technical change in the implementation of DMP. These features are:

--the imposition of a pattern of whole-group instruction within which teachers have accentuated their supervisory and managerial role over children's learning, and within which teachers have preferred to interact directly with children through group processes and to reduce contact among children themselves;

--a strong tendency for teachers to treat the mathematical content of DMP as subordinate to their concerns for orderly classroom management and control;

--where these two features have led teachers to change the instructional activities of DMP, these changes have, in general, been confined to changes in the procedures by which the content of DMP has been presented, with no direct attempt to modify the nature of the content itself.

In particular, this predominant pattern in the implementation of DMP has been illustrated by:

--modification of instructional activities in order to make children's responses more uniform and to ensure that children were given more guidance, more reinforcement, and more constant checking;

--elimination of activities which were believed to be "too challenging" or "too demanding," or the transformation of such small-group or individual activities into whole-group instruction;

--introduction of fixed rules and procedures in teaching children to solve story problems, and to persist with a single method of analysis and solution even when children's difficulties with that method had made its continued use questionable;

--a disinclination to have children work collaboratively in order to overcome reading difficulties encountered by individual children; on the contrary the presence of reading difficulties was used as a rationale for whole-group presentations;

--a tendency on the part of teachers to preempt choices which might have been made by students, and to decide in advance what would be in students' best interests.

Within this pattern of technical change, there has been an almost exclusive focus upon the teacher as the one who presents instruction, and the one who develops concepts and skills for those being taught. Indeed, in their interviews, teachers have most frequently argued that this direct style of instruction was what their children really needed. Teachers preferred to adopt a whole-group approach to instruction because they hoped to monitor more closely what children were doing. Within this pattern of instruction, teachers could provide the kind of immediate feedback which they saw pupils as needing. When children had completed assigned seatwork, for example, they usually presented completed work to their teacher for correction during the lesson, or their folders containing completed work were collected at the end of the lesson. In many cases, children needed to make corrections to their mistakes before they were allowed to move on to the next activity.

Many of these features of teahnical change have been derived from a model of instruction which received theoretical endorsement in the principles of behavior analysis. Thus, a persistent pattern of technical change has expanded the behaviorally referenced objectives of DMP, and has supplanted its constructively oriented activities with a pattern of teaching more compatible with a behaviorist orientation to teaching.

If I am right in linking the management perspective of instruction, within which these technical changes derive meaning, to a behaviorist program of instructional design, then one would expect this orientation to have a powerful bearing upon teachers' conceptions of mathematical knowledge. Strong evidence of that orientation is shown in the way in which most teachers approach the solution of story problems. A behaviorist program of instructional design would require that the objectives of solving story problems, once having been specified unambiguously, then be articulated into teachable component groups (cf. Becker & Carnine, 1980, p. 452). Thus, one can understand more clearly why teachers, in giving special attention to a Part-Part-Whole analysis, have often taught it as an activity in its own right without linking it directly to the solution of word problems. It is seen as a component skill which children must master before they can solve word problems. This component skill can be taught separately from the other skills which, taken together, will utlimately enable children to solve word problems. Thus, teachers have tended to make explicit every step in the strategy, and to require pupils to perform each step in overt form: first, that children should identify correctly "whole" and "parts"; and only then should they apply the rule p + p = w or w - p = p. They have also encouraged pupils to follow, initially at least, the same steps in solving other similar problems (cf. Becker & Barnine, 1980, p. 452).

Since an overt response is encouraged at each stage of performing these component strategies, the teacher is well placed to

> precisely pinpoint the exact skills in a strategy that cause a learner's difficulty. (Becker & Carnine, 1980, p. 452).

Thus, pupils' difficulties have tended to be seen in the context of one component response. In the teaching of Part-Part-Whole, these difficulties have been presented as failing to identify correctly the "whole" and the "parts." Children's difficulties were seldom related to any wider context, such as needing to penetrate the variety of semantic forms in order to comprehend the rule or action or procedure embodied in the story problem. However, if pupils' difficulties are related to a restricted task-analysis of identifying "parts" and "whole," one remedy would be to give additional practice in completing Part-Part-Whole charts.

In such a context, it has been possible to treat a Part-Part-Whole analysis as an exercise in applying labels and in using a formula. The issue of whether these artifacts of instruction bear any resemblance to mathematical activity, as it would be construed by a mathematician, is not a valid question for a management approach to instruction. That approach enables one to handle whatever artifact of instruction one chooses. Mathematical kr wledge, therefore, is identical with whatever is to be learned. Like the teachers in the technical IGE schools, teachers who operate within this model have identified knowledge with the artifacts of instruction. This results in reducing learning to a sequence of objectives to be mastered, and thus creates a division between the work of students and conceptions of mathematical knowledge. Each artifact to be learned becomes separated from any disciplinary logic which might relate the activities of students to conceptions of mathematical craftsmanship (cf. Popkewitz, Tabachnick, & Wehlage, 1982, p. 88).

Constructive Change

By contrast, constructive change seeks to relate adaptations in the teaching of DMP to its broader mathematical goals of having children participate in the creation and testing of mathematical knowledge. Constructive change is able to dissociate these broader goals from a management approach to organization in which they often appear to be embedded.

In helping children to become competent and confident in the solution of mathematical problems, DMP intended the teacher's role to be one of enabling children to use a variety of strategies in order to penetrate the different semantic structures in which mathematical problems are presented. A pattern of constructive change will be identified by adaptations in the teaching of DMP which assist children to develop a sense of control and responsibility in the particular strategies which they use, and in their choice of strategies.

This approach to knowing and doing mathematics embodies a different view from that implicit in the patterns of technical change which I have described. Here is a view of mathematical inquiry as a kind of intellectual craftmanship, and a corresponding belief that this perspective should shape the way in which DMP is taught.

Within a pattern of constructive change there is a conscious attempt to develop a sense of coherence and congruence between one's conceptions of school work and what constitutes desirable mathematical knowledge for children to learn. This link is illustrated most clearly by the pattern of implementation adopted by Teacher G.

But, how does one explain the fact that Teacher G is an exception to the general pattern of a management-centered approach to instruction? Although it is easy to illustrate how Teacher G tended to provide the clearest instances of constructive implementation of DMP, I am unable to posit causes for these departures from the general pattern. It is clear that her own conception of what constituted desirable mathematical knowledge for children to learn was sufficiently different from that of other teachers. To say this is to offer a general description of the differences between Teacher G and other teachers. However, it is not an explanation.

The fact that Teacher G tended to be an exception to the general pattern of implementation of DMP is not a piece of contradictory evidence. It does not refute my finding that a management perspective to instruction was the pervasive feature of the way DMP was taught in the schools being studied. On the contrary, I take Teacher G's constructive implementation of DMP as showing that those features of schooling which give rise to a management approach to instruction are not totally coercive, despite the fact that they are pervasive. There is, so to speak, enough slippage within the institutional life of schools such that constructive implementation, while an exception, can still flourish.

On the other hand, it is a significant feature of the institutional life of schools that the constructive approach of Teacher G can remain relatively unknown among her co-workers and without effect on their teaching of DMP. In neither school did teachers appear to share experiences of teaching DMP; and when they did, their discussions focused upon management issues such as teachers' progress in covering the content of the various DMP topics; the inclusion of additional materials; the introduction of timed tests; and how students were to be grouped for mathematics, and whether any particular students should be moved from one ability group to another.

Within a management perspective on instruction there was an assumption that DMP was being implemented as intended. Of course, individual teachers saw the need to make adjustments to the procedures of instruction--selection of additional activities, omission of activities thought to be too difficult or too challenging, variations in pacing within one's own grade--but these adjustments were seen to arise because of the special needs of one's group. From my interviews with teachers, there is little evidence that the nature and extent of these technical or procedural changes were discussed with other teachers. At most, they were communicated to other teachers as one might pass on a piece of information so that others would know what one was doing. But there was no evidence that teachers discussed the impact of these changes on the teaching of DMP.

The prevalence of a management approach to instruction strongly suggested that teachers had a very limited sense of ownership of the "material" which they were teaching. Their role seemed to be one primarily of adapting the procedures of instruction to meet the perceived needs of their children. They themselves tended to display little control over the development and testing of mathematical ideas to which their children were to be introduced. By contrast, Teacher G did display a sense of competence in the development and testing of mathematical ideas which she presented to her children. While her constructive implementation of DMP shows that a management perspective is not totally pervasive, that same perspective seemed to ensure that Teacher G's constructive adaptations of DMP were unlikely to be known to or adopted by her colleagues.

Chapter 7

CONCEPTIONS OF MATHEMATICAL KNOWLEDGE AND SCHOOL WORK

Introduction

In a tentative conclusion to his study, Tasks and Social Relationships in Classrooms, Bossert (1979), having examined the effect of different patterns of task organization on patterns of social relationships in the classroom, suggests that how tasks are organized in classrooms must be pertinent to the moral or normative role of the school in the socialization of children. Absent, however, from Bossert's (1979) study is any attempt to explicate any logical connection between conceptions of work and knowledge and different patterns of "task organization." While it may seem commonplace for him to refer to classrooms as "places where teachers and pupils work" (p. 7), there is little recognition in his study that one's conception of work is embedded in a network of moral and social considerations.

While it is obvious that all learning is rooted in a social process (Berger & Luckmann, 1967), there is considerable disputation about the psychological and social conditions under which knowledge is developed. As Popkewitz (1982b) argues, notions of children's competency are inextricably related to a normative view of society in terms of which competency is located. Thus, a management perspective of instruction depicts children as learners or consumers of knowledge; as needing to be managed in their introduction to a body of knowledge extrinsically conceived; and as needing to be tested in order to ascertain whether they can reproduce or apply what they have received. The fact that items of knowledge have been cast in predefined terms; the fact that children have to forego their own preferences and choices regarding strategies in demonstrating what they have learned; the fact that they enter into competition, or are at least isolated from others as they learn, cannot be set aside as mere incidental features to the processes of teaching and learning. Those features define teaching and learning as social events by incorporating assumptions as to how children are to relate to each other and their teacher, and how the content of instruction is defined.

One is struck in reflecting upon Bossert's (1979) study, how the social and ethical dimensions of school work and knowledge have been filtered out by focusing instead on considerations of task organization and classroom management. It is an irony of Bossert's study of the sociology of classroom organization that he fails to consider the interplay between the social context, which defines the work of teachers and students, and the conceptions of school knowledge which are embedded in that context. In this study also, teachers have brought assumptions about their own work and about appropriate work of students to their teaching of DMP. With few exceptions, their approach to instruction has been from a management perspective. The beliefs, purposes, and values which underpin this perspective have been an impediment to the constructive implementation of DMP, and have served as a barrier behind which assumptions about school work and mathematical knowledge can remain unquestioned and unchanged.

In this chapter, I argue that knowing and doing mathematics needs to be related to the creation and testing of mathematical knowledge within the scholarly community. There, mathematical inquiry can be seen as an intellectual craft which is practiced and developed in a community whose function is to legitimate standards of acceptable work and what constitutes appropriate questions and standards of proof. This picture of mathematical inquiry is often ignored by those who present mathematics as a collection of logical entities, themselves beyond dispute, and mathematical inquiry as applying these logical artifacts and so generating fresh ones in accordance with fixed rules. This picture of mathematical inquiry is used to reflect upon those conceptions of knowing and doing mathematics which were espoused by the DMP authors, as well as those which were evident in the implementation of DMP.

I have already argued that a management perspective of instruction has been the principal feature of the implementation of DMP. That predominant pattern of implementation did shape the nature of mathematical inquiry in classrooms in ways which the authors of DMP had not intended. In this final chapter, I summarize the impact of that pattern of instruction on the nature of mathematical inquiry in the classrooms where DMP was implemented.

Finally, I claim that a management perspective of instruction can be explicated by relating its underlying conceptions of school work to the beliefs, purposes, and values which sustain the economic structures of society. Unlike Bossert (1979) who purports to discover law-like relationships between different patterns of task organization and social relationships in the classroom, I argue that those social relationships which are assumed by a management perspective on instruction reflect the social and ethical paradoxes of work in the economic order beyond classrooms. To ignore the connection between classroom relationships and this wider social context is to be blind to the genesis of those relationships and how they determine conceptions of school work. It is also to be overly simplistic in one's attempts to effect reformative change in schools.

Mathematical Knowledge

Polanyi (1958) posed the question whether mathematics was simply a collection of tautologies and necessary truths. It is evident, however, from the debates of the nineteenth century between Frege and other mathematicians about the foundations of arithmetic that we cannot tell in advance

> whether the axioms of arithmetic are consistent; and if they are not, any particular theorem of arithmetic may be talse. Therefore these theorems are not tautologies. They are and must always remain tentative. (Polanyi, 1958, p. 187).

Similar problems confront any attempt which defines mathematics as the collection of theorems which can be derived by logically correct steps from a set of axioms which are themselves mutually consistent. The very weakness of this attempt is that it fails to consider, or allows one the possibility of considering, how the axioms are selected in the first place. Second, the history of mathematics shows that it is possible for mathematical inquiry to take place using theorems which have not been at the time formalized in accordance with strict logical procedure; as an instance, one needs only to refer to the work of those who instituted the infinitesimal calculus. Third, from among the infinitely many possible combinations of theorems which can be deduced from a given set of axioms, many will be useless and trivial, and only a tiny fraction will be regarded by mathematicians as significant (cf. Poincare, 1929).

Polanyi (1958) continues this argument by reasserting the place of intuition in mathematics. He refers to its essential role in anticipating mathematical theorems, in teaching and remembering them, and in the way they achieve recognition and endorsement by the mathematical community. Hence, doing mathematics, far from being an isolated intellectual exercise, is itself tied to a social context of intellectual connoisseurship and debate leading to conceptual reform and a continuing search for elegance and beauty (cf. Polanyi, 1958, p. 189).

The recent history of mathematics, to illustrate Polanyi's argument, has been characterized by trenchant and often bitter debate over conceptual revisions, such as those regarding mathematical continuity and infinity which were proposed by Cantor (1845-1918), but blocked by the mathematician Kronecker, and finally published in 1874 only as a result of Dedekind's intervention. In the present century, Cantor's revisions have emerged into acceptance and importance as fundamental to the development of a theory of functions, and of topology and analysis. If Cantor's conceptual revisions could have been supported by rigorous logical argument at the time of their origin, those bitter debates among and periods of uncertainty for mathematicians would not have occurred. But conceptual revisions are never accepted on purely logical grounds alone. Their impact on existing mathematical theory has to be discovered, and their ability to support an alternative theoretical structure has to be judiciously scrutinized by mathematicians. The eventual resurgence and endorsement of Cantor's theories about the foundations of arithmetic are, as Polanyi (1958) argues, evidence of the conserving influence and mutual surveillance of the community of mathematicians,

> a community which can be kept coherent only by the passionate vigilance of universities, journals and

> meetings fostering these values and imposing the same respect for them on all mathematicians. (p. 192)

One cannot, therefore, hope to capture a sense of what it is to do mathematics simply by looking at individual mathematicians. A mathematician engages in mathematics as a member of a learned community, and the commitments of various groups within that community as to what constitutes acceptable mathematics create the context in which the individual mathematics works.

Because it is not possible to draw a line in advance around the possible forms of argument which may be used in mathematical proofs, and because mathematicians view argument as essential to mathematical inquiry, they ask what acceptable mathematics should be like, and what methods of proof should be countenanced. Thus, doing mathematics cannot be regarded as a mechanical performance, or as an activity in which individuals engage by following predetermined rules. In this light, mathematical inquiry can be interpreted as embodying the elements of a craft than as a technical discipline. The idea of mathematical inquiry being akin to a craft directs attention to the personal autonomy and responsibility which is exercised by mathematicians in the creation and testing of mathematical knowledge. It also directs attention to the relationship of mathematical inquiry to imagination, intuition, and aesthetics (cf. Popkewitz, 1977). That is not to say that mathematicians are free to anything they like. As in other crafts, there will be agreement, in a broad sense, about what procedures are to be followed and what is likely to be countenanced as acceptable work. These agreements arise from discourse within the mathematical community, and are not seen solely as impositions of external standards.

Members of the mathematical community have a shared way of "seeing" mathematical inquiry. Their mutual discourse reinforces preferred forms, a sense of appropriateness, of elegance, of acceptable conceptual structures (cf. King & Brownell, 1966). In this respect, Hagstrom (1965) has argued that social control in science and mathematics

> is exercised in an exchange system, a system wherein gifts of information are exchanged for recognition from scientific colleagues . . . By rewarding conformity, this exchange system reinforces commitment to the higher goals and norms of the scientific community, and it induces flexibility (my emphasis) with regard to specific goals and norms. (p. 52)

Not only does a scientific community promote and reinforce its own standards of what constitutes acceptable work, but, as Hagstrom (1965) suggests, a major characteristic of a mathematical/scientific community is the continued evolution of its standards. Not only does the range of acceptable methods vary, but in mathematics, especially, the standards of rigor have themselves been subject to continued modification and refinement, a point well illustrated by E. T. Bell (1945):

> How did the master analysts of the 18th century--the Bernoullies, Euler, Lagrange, Laplace--contrive to get consistently right results in by far the greater part of their work in both pure and applied mathematics?

> What these great mathematicians mistook for valid reasoning at the very beginning of the calculus is now universally regarded as unsound. (p. 153)

Nor does Bell (1945) have the last word. In taking issue with his last sentence, it is now known that during the 1970s, mathematical logicians, such as Robinson (1970) and Keisler (1971), found a way to make rigorous the intuitively attractive infinitesimal calculus which was developed by Newton and Leibniz and extended by those master analysts to whom Bell refers.

This dynamic and social character of mathematical inquiry is well illustrated in three of the six goals proposed for mathematics instruction by Buck (1965). These goals are intended to be more or less independent of specific courses of mathematics instruction, and are intended to "help to supply an answer to the person who asks: "Aside from its technological importance, what are the educational values of mathematics?":

> Goal 2: To convey the fact that mathematics is built upon intuitive understandings and agreed conventions, and that these are not eternally fixed.
>
> Goal 3: To demonstrate that mathematics is a human activity and that its history is marked by inventions, discoveries, guesses, both good and bad, and that the frontier of its growth is covered by interesting unanswered questions.
>
> Goal 4: To contrast "argument by authority" and "argument by evidence and proof"; to explain the difference between "not provéd" and "disproved," and between a constructive proof and a nonconstructive proof. (Buck, 1965, pp. 949-952)

These goals, as Romberg and Harvey (1969) note,

> reflect a belief that the primary value of mathematics is in its relationship to reality, that mathematics is an abstract but humanly created image of reality. From this perspective, it is our belief that these goals can only be attained through the human activity of creating mathematics. (p. 3)

Implications for DMP

How might this vision of mathematical inquiry as an intellectual craft be reflected in classrooms as Romberg and Harvey (1969) hoped it would? Children would need to sense that they were participating in the creation and testing of mathematical knowledge. They would need also, to some extent, to become their own authoriites in dealing with mathematical ideas. They would need to understand that abstracting, inventing, proving, and applying mathematics are activities which take place in a context of mutually agreed and developing standards as to what constitutes acceptable abstractions, proofs, inventions, and applications of mathematics.

How was this vision of mathematical inquiry as an intellectual craft reflected in DMP? In an earlier chapter, I pointed to two separate and contrasting strands in how the authors of DMP conceived of mathematical inquiry. On the one hand, there was a tendency to portray mathematical knowledge as a set of crystallized logical forms to which children were to be introduced by their teacher. On the other hand, the DMP authors recognized that children's mathematical investigations needed to advance beyond these predefined patterns of analysis to a point where children could try out their own strategies for solving number sentences and story problems (Romberg, Harvey, Moser, & Montgomery, 1975, p. 50).

In reflecting upon the implementation of DMP, it is, however, all too easy to point to these shortcomings in the way in which the course materials were presented to teachers, and so to argue that DMP would have been implemented differently than it was if only the authors had been more explicit and consistent in what they wanted to achieve. But to focus on these issues is to miss the mark entirely. Although DMP was intended to transform the teaching and learning of mathematics in the elementary school, its authors had failed to challenge the traditions of existing schooling, or even to understand what they were. The conceptions of work and knowledge which most teachers brought to the implementation of DMP were embedded in a management perspective of instruction where the focus of instruction was on the efficient transmission of a fixed body of subject matter to the children who comprised the classroom group.

Implications for Mathematical Knowledge

This predominant pattern of instruction led teachers to focus their attention on the management of the classroom group. It thus diverted their attention away from the processes used by individual students. Their attention was directed instead to ascertaining whether the outcomes of the work of individual students conformed to those patterns which had been prescribed for the whole group. Likewise, most teachers acted as though their students comprised a relatively homogeneous ability group. Especially in groups where children were depicted as comprising a "low" or "slow" group, they were taught as though they were deficient in mathematical knowledge and had nothing of their own to contribute. When these children were experiencing difficulty in grasping a new concept or skill, their difficulties were interpreted as a call for more intensive practice, or for the presentation of more simplified examples, rather than as a sign that an alternative approach to the content of DMP might be warranted. In all, the content of DMP was treated as crystallized, and as extrinsic to pupils; and was presented often as a series of tasks to which they needed to be introduced. If pupils were seen as comprising a "bright" group, they were expected to exercise more responsibility in completing their own work, but they were not expected to exercise choice or judgment beyond what was required by the text.

Thus, the implementation of DMP was assimilated into an existing network of beliefs, purposes, and values derived from a management perspective of instruction. Adapted by technical changes to conform to this predominant pattern of instruction, DMP was in fact assimilated as an ameliorative change into existing patterns of mathematical instruction. Ameliorative changes, as

Romberg and Price (1981) argue, are perceived to make some ongoing school practice more efficient, but they do not challenge existing conceptions of work and knowledge embedded in the culture of schools.

These ameliorative changes which accompanied the implementation of DMP were seen on the many occasions when teachers followed recommended activities, especially when children were to use blocks, counters, or other manipulative materials to represent the mathematical transformations of combining, separating, joining, and comparing objects. Likewise, counters and other visual devices were used as recommended in order to present addition and subtraction when regrouping, for example, between tens and units was required. In these instances, it was hoped that children would be helped to develop an understanding of the mathematical concepts embodied in these concrete situations.

However, a predominant management perspective tended to reassert itself when children were being introduced to concepts and skills which teachers saw as more demanding, or when teachers anticipated that their group would experience difficulty with a given activity, even though the same activity might not be thought difficult for a "brighter" group. That same perspective was also evident in those classes where I observed teachers who preferred to demonstrate the uses of manipulative materials for the whole class rather than have children attempt to use manipulatives themselves or in small groups. These observations were confined to classes which had been described as "slow" or "low."

Whenever a management approach to instruction emerged in the teaching of DMP, its presence was a sign that those elements of a given activity which were intended to provide a constructivist framework for children's learning had been abandoned or substantially modified. As a result, the content of the activity was treated as prescribed material to be mastered. Its presentation tended to become more clearly behaviorally referenced and prescriptive. Often the concept or skill to be taught was presented as a task separated from the mathematical context which gave it meaning. This was most clearly demonstrated in the tendency to treat the Part-Part-Whole analysis as a logical entity in its own right and thus divorced from the writing of a number sentence in order to solve a story problem. Under these circumstances it was difficult to describe children as creating mathematical knowledge.

Likewise, a management approach to instruction, which was frequently attended by patterns of teacher-directed and whole-group instruction, narrowed opportunities for children to test mathematical knowledge. These opportunities were usually confined to the validation of answers to addition and subtraction problems. Frequently children needed to present their answers to the teacher for checking. That process extended not only to ascertaining whether children had the correct answer, but to checking whether they had used the prescribed method.

In order to give more specific attention to the creation and testing of mathematical knowledge within the teaching of DMP, I refer to the definition which has been given by Romberg (in preparation-c) where he speaks of four related activities which are special to mathematical inquiry: abstracting, inventing, proving, and applying. I argue that a management approach to instruction has limited the opportunities for children to engage in these four activities; and, furthermore, that it has imposed a more restrictive and limited definition on those activities.

Abstracting. The fundamental process of DMP--describing, classifying, comparing, ordering, joining, separating, and grouping--are all instances of mathematical abstraction. Many of the activities of DMP have required children to use manipulative materials to represent and validate mathematical transformations in which these processes have been embodied. Teachers reported that, in general, they were comfortable with those activities which incorporated the use of manipulatives. Likewise, they said that they had implemented many measurement activities, including introductions to geometry and fractions, much as the DMP booklets had recommended. Although there were occasions when teachers did tend to limit the scope and variety of measurement activities, it could be argued that, even where a management approach to instruction has intruded into the teaching of DMP, abstracting remained a strong feature of the implementation of DMP.

Where that pattern was most evident, for example, in the teaching of Part-Part-Whole, children were still able to see that the abstract arithmetical operations of addition and subtraction applied with remarkable generality to a wide range of story problems. However, it should be noted that children were often taught to rely exclusively on the Part-Part-Whole analysis in order to abstract a mathematical transformation from the semantic structure of a story problem. This almost total reliance upon one method of analysis has limited children's experience in using a variety of approaches for penetrating the semantic structure of story problems. Nor have they been encouraged, with the exception of one teacher, to explore different approaches in analyzing the structure of story problems, where abstracting becomes intertwined with inventing, proving, and applying. It is clear, then, that a management approach to instruction did impose restrictions on the kinds of experiences in abstracting to which children were introduced.

It is possible to view abstracting from a psychological perspective as a process in which an individual engages. However, mathematical inquiry is not to be identified with a psychological process of abstracting. It is indeed a necessary condition of mathematical inquiry that one engages in abstracting. But what makes that inquiry mathematical are the kinds of abstractions--concepts and skills--and the norms and standards by which abstraction is regulated.

Likewise, one must avoid treating inventing, proving, and applying as psychological processes if one wishes to use these activities to identify mathematical inquiry. For that purpose, these activities need to be seen as performances which take place in the context of an intellectual discipline; and as mathematical performances these activities need to be seen in a context of agreed rules as to what constitutes a successful attempt at inventing, proving, and applying. In its relation to these three performances, a pervasive management approach to instruction not only presents a restricted range of experiences, as in the case of abstracting, but it tends to redefine what constitutes inventing, proving, and applying in radically different ways.

Inventing. Romberg (in preparation-c) defines inventing as creating a law or a relationship. In a similar spirit, the authors of DMP hoped

> that exposing children to a wide variety of problems will lead to a willingness to tackle new problems, confidence in their ability to handle new problems, and the ability to apply problem solving techniques. (Romberg, Harvey, Moser, & Montgomery, 1975, p. 50)

Romberg (in preparation-c) claims that inventing may be seen as "discovering relationships which lead to abstractions, theorems, models, and so forth, known to the mathematical community but not to the student." This sense of "inventing" appears especially relevant to school mathematics. However, the emphasis placed by most teachers on standardized and fixed procedures has as immediate impact on children's experience of inventing. This was most clearly illustrated in the teaching of Part-Part-Whole. When that one method of analyzing and solving story problems was presented as the only method which children could use, there was little inventiveness to be seen in pupils application of a predetermined pattern of analysis in order to establish a mathematical relationship between "parts" and "whole."

The authors of DMP had hoped that children, after becoming confident in a part-part-whole analysis, would indeed be helped to invent alternative approaches to the solution of story problems. However, within a management perspective, children were unlikely to be introduced to any other method of solution. Their work was to become competent users of that single pattern of analysis. Thus, a management approach to instruction has imposed such limiting prescriptions on what children learned, on how they were to learn, and how their learning was evaluated, that "inventing" in any valid sense was unlikely to occur.

Moreover, the activity of inventing cannot be divorced from having one's invention or discovery recognized as such. But in the predominant pattern of instruction, the individual child was often separated from discussion with other students and submerged in a group process, or required to carry out tasks specifically assigned to that student by the teacher. Thus, there was little opportunity for discussion with other students or opportunities for collaboration in devising new approaches to problem solving. Hence, variations in methods of problem solving were unlikely to be encouraged. Indeed, such variations as might have led to inventions would have been seen initially as deviations from a pattern of response which the class had been taught to follow. It was not surprising, therefore, that teachers reported that children did not use alternative strategies in solving problems; or when teachers reported that their children did use alternative strategies in solving word problems, they added that the children were usually wrong.

Proving. I did observe many instances of children proving relationships between numbers, but all too often these instances were confined to validating an answer to a number problem. Validation or checking was often performed using counters or blocks to represent a mathematical transformation. In Grade 3, children were encouraged to validate answers to subtraction problems, for example, by adding the answer and the number which had been subtracted (the subtrahend). But these instances of proving were only a tiny element of what the DMP authors aspired to when they urged that "Children need to be left alone at times with objects or pictures or pencil and paper to try their own methods" (Romberg, Harvey, Moser, & Montgomery, 1975, p. 50). Leaving children on their own, or guiding them to attempt their own methods do not constitute proving. But these activities are a seedbed upon which experiences in proving can be developed. Yet as my observations and interviews showed, these activities tended to be avoided by teachers, especially by those who saw their group as "low" or "slow." Only one teacher showed clear signs of developing a sense of proving when she asked her children to estimate whether the solution to a story problem was likely to be greater or less than the number given; to record their prediction; and then to prove whether their prediction was correct by using whatever method worked best for them. Here, a context of agreed rules has been established which allowed pupils to ascertain whether their estimate has been proved correct. Unlike the single predetermined pattern of analysis as used in other classrooms observed, this context of agreed rules did allow children to explore their own methods and to recognize when those methods were correct. In this way, children were able to develop a sense that they were their own authorities in dealing with mathematical ideas. Opportunities such as this, to develop a sense of personal responsibility and control over mathematical knowledge, were not observed in other classes.

Applying. The notion of applying is so pervasive throughout DMP that one might think that this feature of mathematical inquiry was least affected by a management approach to instruction. Did not children apply mathematical techniques to the solution of story problems? Did they not make use of measurement activities in order to represent and validate mathemtical transformations? Did they not apply mathematics to their investigations of space and shape?

Yet if one asks who did the applying and under what circumstances were these applications made, one can see how this predominant pattern of instruction could redefine the very idea of applying mathematics. One was forced to ask whether pupils were applying mathematics when they watched their teacher demonstrate applications for the whole group. One also had to ask what kind of applications were being made, when the range and variety of measurements was reduced in order to keep instruction more efficient and orderly, or when imprecision in children's measurements was deliberately precluded for the same reasons. The effect of this latter kind of technical change was not merely to make instruction more orderly and efficient, it also altered the nature of the applications which children performed. For example, by providing objects whose length could be measured exactly so that children were not confronted by imprecision in their measurements, the teacher had set aside two important elements of the process of measurement. These were the possibility of using a systematic procedure of measuring to the nearest unit, and of establishing that if more exact measurement is desired, smaller units need to be used (cf. Romberg, Harvey, Moser, & Montgomery, 1975, p. 34).

Mathematical applications are made in a physical world where there is variety, ambiguity, and imprecision. For that reason, one's mathematical models always embody some simplifications of the physical data, but those simplifications are warranted in order that one's model can be understood and can work. But to simplify one's data in advance in order to make one's teaching more efficient and orderly is to simplify data for the wrong reason and to confuse children about the nature of the physical data. If they are to apply mathematics, children need to know that the data are not always precise, and that one has to come up with acceptable procedures for handling imprecision, for example, by agreeing to measure to the nearest unit, or to accept as equally correct the two units which are on either side of the measurement.

In summary, I have described mathematical inquiry as an intellectual craft which takes place in a context of rules, some agreed upon and some still evolving, which are maintained and developed by a community of mathematicians. I have argued that this vision of mathematics was grasped by the authors of DMP, and that it was realized in some of the classrooms where DMP was implemented. However, whenever a management perspective of instruction emerged, mathematical knowledge became extrinsic to students and mathematical inquiry became crystallized: opportunities for children to participate in the development of agreed rules for mathematical inquiry were severely curtailed; their experiences of abstracting were often limited by the priorities and procedures which accompanied that perspective. Severe limitations also applied to the opportunities available to children for inventing, proving, and applying mathematical relationships. But, more importantly, children's conceptions of these mathematical activities were likely to be redefined by a management approach to instruction. To reinterpret Bossert (1979), the very patterns of task organization within a management approach to instruction have profoundly changed what it means to know and do mathematics.

Implications for School Work

Those patterns of task organization also define a notion of school work. How are these conceptions to be elucidated? As I said earlier, Bossert (1979) sees a connection between different patterns of task organization in classrooms and the moral or normative role of the school in the socialization of children. Regrettably, Bossert leaves this issue unexplored. It is imperative for me to consider those patterns of school work which are embedded in a management approach to instruction, and to ask whether they are merely the products of different patterns of task organization in the classroom, or whether these patterns of school work can be interpreted in the light of more general social conditions.

I argue that those patterns of school work which are embedded in that pattern of instruction reflect features of the ethical and social predicament of human work in the wider social and economic order. To fail to relate these forms of school work to more general conditions of human work is to be blind to the genesis of those forms of school work, and to fall into the simplisitic position of thinking that conceptions of school work are susceptible to change merely by introducing a new curriculum in which alternative conceptions of work are embodied. Because this naive optimism tended to be assumed by the developers of DMP, my study has had to confront the gap between the rehetoric and aspirations which were expressed in the curriculum materials and actual practices in the classrooms.

The following analysis looks at patterns of school work in a wider philosophical perspective than one usually associates with a discussion of issues in mathematics education. However, a widened perspective is necessary in order to explicate the conceptions of school work which are embedded in a management approach to instruction. Wihtin this wider framework, I consider some general problems of curriculum innovation which have attended the implementation of DMP, and, with the advantage of hindsight, I recommend the adoption of alternative approaches to assist schools in the improvement of mathematics teaching.

School Work and the Predicament of Human Work

To illustrate some features of the contemporary predicament of human labor and their relationship to these concepts of school work, I draw upon the encyclical letter, On Human Work (1981), by Pope John Paul II. This document embodies a statement of personalist critical theory: notably in its affirmation of the priority of the human person over the technical processes of production; in its appreciation of the fact that through work those who work enter into the heritage of workers of previous generations, and that through work human community is built; and in its attack on a false dichotomy between labor and capital.

These perspectives on human work allow me to bring a personalist critical theory to the analysis of school work. Some principles of that theory can be drawn from the document, On Human Work (1981). First, the human person is seen as "a subjective self capable of acting in a planned or rational way, capable of deciding about himself" (Section 6). Second, "since work in its subjective aspect is always a personal action, it follows that the whole person . . . participates in it whether it is manual or intellectual" (Section 24). Thus, in its subjective aspects, work is seen as participation in creativity activity; only the human person is able to work, other creatures perform tasks (cf. Section 25).

However, the fact that human work is inextricably tied to the social conditions of human existence leads to a twofold perspective:

> human life is built up every day from work, from work it derives its specific dignity, but at the same time . . . there are fresh fears and threats connected with this basic dimension of human existence. (Section 1).

Among these threats is the mechanization of work which supplants the human dimension of work, "taking away all personal satisfaction, and the incentive to creativity and responsibility" (Section 5). Another threat comes from the mechanization of work in its capacity to isolate workers from each other, and to fragment their efforts, thus submerging them into a group process of production and depriving them of personal authority over their work, and of a sense of community with other workers.

At the root of these problems and threats is an "economistic" perspective, namely

> that of considering human labor solely according to its economic purpose . . . thereby placing . . . the personal in a position of subordination to material reality. (Section 13)

The priority of economic structures over the social conditions of human existence is a theme reflected in contemporary analysis of curriculum innovation and school change. It is taken up by Apple (1979) and by House (1974). In The Politics of Educational Innovation (1974), House argues that educational innovation is shaped by an economic perspective which demands constant innovation and change within an unchallenged and unchanged network of social structures and relationships. Within this economic perspective, innovation is essentially ameliorative, and thus serves to maintain the status quo of a modern technocratic society. House (1974) continues:

> the basic fact about a technocratic structure is that the status quo is maintained by constant innovation . . . While it is easy to perceive the status quo in a traditional structure, the status quo of a modernistic society is not so easy to discern because constant innovation holds a strange fascination. But for society as a whole, there are changes in content, not of structure. Innovations allowable within that structure have very definite limits. (p. 256)

The effect of this economistic perspective is the conceptual separation of labor from capital, and in treating them as separate elements in production. Relating this economic perspective to one's ideas about teaching, one is able to elucidate assumptions about soical relationships and knowledge which underpin a management approach to instruction. There learners are separated from the content of instruction. Far from being producers of knowledge, learners are perceived as consumers of predefined subject matter Within this same perspective, the individual child is seen as a member of a group process in which knowledge is managed and transferred to learners. Knowledge is depicted as subject matter which is to be transferred to individuals by an efficiently managed group process. Mastery of that knowledge is specified in terms of predefined criteria of performance, and those criteria are to be applied uniformly to all members of the group.

This "economistic" perspective, with its focus on the child as learner and consumer of predefined subject matter, allows one to interpret instruction as a system of management which is directed toward the efficient transfer of predefined knowledge. Thus, technical change is able to be interpreted as adaptations within a system of instructional management. Such adaptations are deemed to be necessary for the more efficient transfer of knowledge. These adaptations are to be initiated by the teacher, who as leader and monitor of the system of instructional management, is responsible for making appropriate adjustments to administrative procedures and instructional practices. Within a management perspective of instruction, a new curriculum program like DMP can be adopted, but its adoption is merely a change in content, leaving unchallenged underlying notions of what is to count as appropriate work and knowledge. Having been assimilated within this perspective, DMP as implemented becomes an instance of ameliorative change.

Behind this economic perspective is the picture of a world comprised of separate individuals whose claims and interests fundamentally conflict. Through its moral and social arrangements, classroom management, therefore, provides a mode of regulating conflicting individual interests, and at the same time allows for the development of communal activities (cf. Gilligan, 1982). Thus, a management approach to instruction looks to impartial adherence to rules, and to conceptions of justice and fairness which presume that learners are isolated from each other, and that they are dependent upon the teacher. A counter argument that these arrangements are intended to foster children's own good has to be balanced by the fact that decisions about what children are to learn, how they are to learn, and the value of what they have learned are made for them by "wise" adults. Considerations of justice and fairness pervaded the paternalistic decisions which teachers made about whole-group instruction, in, for example, the need to keep the group together; in the unfairness, as a teacher expressed it, of having the "eager beavers" pressing for attention when someone who "really needed help" was not being attended to. These same considerations were reflected in the "needs" which teachers ascribed to the class group, and in the problems which teachers perceived in their management of instruction. These considerations were evident in the pressure felt by teachers to skip over certain topics so that students could spend more time on addition and subtraction which "they needed" to master before entering the following grade.

These considerations appear to give a human face to a management approach to instruction, but they do not disguise its underlying economistic assumptions about instruction. Its insistent emphasis on the individual learner and the management of school work provides little foundation for a sense of responsibility, connection and care between individuals, for a sense of personal satisfaction and the incentive to creativity and responsibility, and for a sense of intellectual inquiry as an essentially communal activity.

A personalist critical theory stands in direct contrast to an economistic perspective of work and its attendant social relationships. On the contrary, it affirms a normative conception of work as an activity with unites people. This perspective, as expounded in On Human Work (1981), helps to illuminate the social and communal context of mathematical inquiry:

> It is characteristic of work that it first and foremost unites people. In this consists its social power: the power to build community (Section 20) . . . through human work man enters into two inheritances: The inheritance of what is given to the whole of humanity in the resources of nature, and the inheritance of what others have already developed on the basis of those resources. (Section 13)

Applying this perspective to school work, one recognizes that through their work children enter into a heritage, not simply of accumulated mathematical facts or of subject matter, but of beliefs, purposes, and values which define the activity of mathematicians. It is this dimension of school work which is systematically obscured by a management approach to instruction, and is symptomatic of its economistic assumptions. By contrast, a personalist critical theory focuses on students as participants in the creation of mathematical knowledge. Knowledge is seen as requiring choice and decision; and, therefore, as undergoing dynamic change as individuals apply knowledge and communicate it to others.

To complete this analysis of the implementation of DMP, I apply the same principles of personalist critical theory to elucidate the work of teachers. Thus far, my interpretations have focused on the way in which DMP has been taught, but it is important to ask why a management approach to instruction, and its attendant patterns of technical change, have been so prevalent in the implementation of DMP.

Arguing from a personalist critical theory, I claim that the way in which DMP was introduced into schools has in fact cast teachers in the role of managers of an externally developed and prepackaged curriculum (cf. Dalin, 1978, p. 16). Just as I have argued that teachers have tended to present mathematical knowledge to children as crystallized logical forms, I argue that DMP was presented to teachers as a crystallized form of a mathematics curriculum, over which they had little sense of ownership and control.

If in the predominant pattern of teaching DMP, children had limited opportunities to sense that they were participants in the creation and testing of mathematical knowledge, so in the implementation of DMP in schools, teachers had limited opportunities to participate in the creation and testing of a mathematics curriculum. Just as I have criticized a management approach to instruction for failing to assist children to bring a sense of personal meaning and ownership to what they learn, so in the way in which DMP was established in schools there was little opportunity for teachers to bring to the development of a mathematics curriculum the same personal qualities--imagination, intuition, choice, judgment, and aesthetic sensibility. These deficiencies can be interpreted as the consequences of introducing DMP to teachers as a curriculum development over which they had little sense of ownership and control. DMP had, in effect, been presented to teachers as a complete mathematics curriculum which they were to implement and manage.

Looking at DMP as an instance of curriculum development, I claim that the process of implementation of DMP in schools reflected at another level the economic relationships which in the classroom treated students as consumers of predefined subject matter. Thus, teachers were cast in the role of consumers of a prearranged mathematics curriculum. Those same "economistic" relationships which prevented children from sensing that they were being linked by their work to an intellectual community whose beliefs, purposes, and values defined mathematical inquiry, also prevented teachers from seeing that they were being linked by their work to the work of others whose beliefs, purposes, and values defined curriculum development.

Using a perspective from personalist critical theory, I have interpreted a management approach to instruction, which had been the principal feature of the teaching of DMP, in terms of the assumptions about the work of teachers which were implicit in the creation and implementation of DMP. The beliefs, purposes, and values which have attended that process have served to limit teachers' sense of ownership of and control over what they were expected to teach. Thus, a management approach to instruction is not to be interpreted as simply the creation of teachers themselves. Having been cast in the role of managers of externally produced curriculum, teachers have become enmeshed in a network of beliefs, purposes, and values which define a "center-out" approach to curriculum development. Within that model there has been a conceptual separation between the work of teachers and the creation and testing of a mathematics curriculum. The separation of the work of teachers from the creation and testing of a mathematics curriculum has, I argue, been paralleled by the separation of the work of students and the creation and testing of mathematical knowledge in the teaching of DMP.

Beneath this parallelism are assumptions about work and knowledge which have been shaped by an economic perspective which treats teachers and students, in their respective roles, as consumers of predefined knowledge, over which neither has a clear sense of ownership or control. It is a strength of a personalist critical theory that it enables one to detect these similarities and continuities between the predominant patterns of work and knowledge which have accompanied the implementation and teaching of DMP.

Summary

This study set out to ask what meaning has been given to knowing and doing mathematics in those classrooms which comprised the classroom observational study (Romberg, Small, & Carnahan, 1977) conducted by the Wisconsin Center for Education Research during 1978-81. This study interprets the work of teachers, the work of students, and what constitutes appropriate mathematical knowledge for children to learn.

A field-study methodology was employed in two Madison schools where the revised S- and A-Topics of *Developing Mathematical Processes* (DMP) (Romberg, Harvey, Moser, & Montgomery, 1974-1976) had been taught. Data were gathered from ten teachers using classroom observations and interviews. This study also had access to interviews which had been conducted with teachers after each Topic had been taught for the first time.

DMP was intended to reshape conceptions of mathematical knowledge and school work. It sought to create a pedagogy in which children would be active in the creating and testing of mathematical knowledge. It saw mathematical inquiry as requiring exploration, investigation, choice, and judgment. It believed that children could be assisted by their teachers to approach mathematical inquiry in this spirit.

The developers of DMP did not, however, reckon with pervasive features of the culture of schools which would need to be challenged if DMP was to have the effect which they intended. By adopting a center-out approach to the development and implementation of DMP in schools, the authors of DMP tended to reinforce assmptions about the ownership of mathematical knowledge which had been embedded in that pattern of curriculum development.

Moreover, the authors of DMP tended to ignore the craft features of mathematical inquiry, and represented mathematical knowledge as a set of crystallized logical entities. This approach diverted attention away from the social and intellectual processes by which mathematical knowledge is created and tested.

As I investigated how children were taught DMP, the notions of school work and mathematical knowledge were indispensible for showing that children in school learn not only the subject-matter of mathematics, but through their work, they are also taught the appropriate forms in which to cast their knowledge. In the predominant pattern of teaching DMP, teachers saw their role as managing the efficient transfer of a body of subject matter to students. Whenever this management approach to instruction prevailed, children had limited opportunities to engage in creating and testing mathematical knowledge. Especially in groups which were regarded by teachers as "low" or "slow," children were treated as deficient in mathematical knowledge.

As effective managers of instruction, teachers did modify the activities of DMP to meet the perceived needs of students. However, the predominant pattern of change was one of adjustment to the procedures of instruction. Only rarely was the content of DMP modified directly to meet the needs of students or to better implement the mathematical goals of DMP. Only in these few instances of constructive adaptation of DMP did teachers display a sense of ownership of and control over what they were teaching. In those instances, children were also helped to bring personal meaning to what they had learned.

Limitations of This Study

This study has been deliberately restricted to an examination of the teaching of DMP in the two schools which participated in the classroom observational study (Romberg, Small, & Carnahan, 1979). The schools were each serving middle-class areas of Madison. The teachers involved in the study were all regarded as experienced and effective teachers of elementary grades. Likewise, students were usually described as highly motivated; and very few if any were typical of those students whom one might expect to find in a large urban, ethnically diverse, or low-income community.

Moreover, the study concerned the implementation of DMP which had been developed by the Mathematics Work Group at the Wisconsin R & D Center. DMP was not like a typical textbook with a heavy emphasis on pencil-and-paper mathematics. More importantly, DMP had been introduced to the schools being studied as a fully developed and self-contained mathematics program.

Therefore, the conclusions which I have drawn about the predominant pattern of implementation of DMP may not be applicable to those cases where a mathematics curriculum has been developed by the teachers themselves. The fact that DMP was produced for the schools to implement may well explain why teachers have tended to refer to "your math" and have treated the mathematical content of DMP as extrinsic to themselves and their students.

Directions for Further Research

Two obvious suggestions for further research can be made. The first, would be to study the teaching of the same mathematics program in schools which have been selected on the basis of known differences: differences, for example, in the social or economic background of students, experience of teachers, and so forth.

A further suggestion for continuing research would be to study the implementation of a mathematics program which had been developed within a school. In that case, it would be important to ascertain whether a management approach to instruction, together with accompanying patterns of technical change, was as much in evidence where teachers appeared to have a greater sense of ownership and control over the process of curriculum development. Under these conditions, one would look to see greater evidence of constructive change.

The reader may find in the conclusions of this study, and in the above recommendations for further research, a pessimistic outlook for curriculum innovation and reform in schools. It might be thought if a well-designed program like DMP did not successfully accomplish the expectations of its authors, what is there left to recommend?

Part of the problem lies in the fact that in mathematics education the community of researchers and curriculum developers is sharply separated from those who teach mathematics in elementary and secondary schools. Given the prevailing pattern of center-out innovation, it is rare for curriculum developers to see the consequences of their ideas in the classroom at a stage when the fruits of their observations can be incorporated into and so influence the development of the curriculum program. Likewise, as Kilpatrick (1981) remarks, "teachers, who have been trained to depend on experts for answers, have little impetus to correct those ideas and improve their own understanding" (p. 27).

These relationships between teachers and curriculum developers have not been challenged by my own recommendations for further research. I admit that there are serious flaws in the model of center-out implementation of curriculum innovation and reform; and this study has documented some of those limitations. But, having recognized those limitations, there is a danger of shifting to the other extreme, and of recommending that curriculum innovation and reform should be entrusted solely to the local school.

There are sound arguments for recommending that the local school should exercise greater responsibility and autonomy in curriculum innovation and reform. However, the espousal of school-based curriculum development assumes that curriculum change can be treated independently of institutionally embedded conceptions of school work and appropriate knowledge for children to learn. A school-based approach to curriculum development, while giving proper attention to the initiatives and perceptions of local teachers, provides no guidance in enabling teachers to challenge existing traditions, meanings, and power relations in which teaching and learning are embedded in schools.

Kilpatrick (1981) offers the most constructive recommendation of greater collaboration between practitioners and researchers. Admittedly, the interests of these two parties to curriculum change are not identical, but collaboration between them is necessary if teachers are to be assisted in "stepping out of the stream of daily classroom experience and stopping to reflect on it" (Kilpatrick, 1981, p. 27). Collaboration is also necessary if curriculum developers are to share with teachers a sense of ownership of and control over the curriculum.

A more promising approach to curriculum innovation and reform would be for curriculum developers to present local schools with several alternative frameworks for curriculum innovation. Romberg (in preparation-c) has proposed, for example, the idea of a "storyshell curriculum" in which the basic story line could be set out initially in episodic form and only later expanded to fit into a larger tale. Th. role of the curriculum developer would be to assist schools to choose the framework which best suited their needs; to identify impediments to implementation; to specify needs of local teachers for in-service support in developing the framework they have chosen; to prepare materials for teaching; and to establish appropriate forms for the systematic monitoring of the ensuing implementation of the curriculum. These approaches have been discussed more fully by Romberg and Price (1981). In li -of my own study, there is no alternative to a completely new approach to curriculum innovation and reform. Both the center-out approach and a totally school-based approach are likely to prove ineffective, while giving the appearance of change.

This study has pointed to issues of curriculum development and school reform beyond what is usually attempted in studies of mathematics education. I contend, however, that research into mathematical education is likely to be stultified unless mathematics educators confront these larger issues. How work and knowledge are defined for schools, and who is in control of the curriculum are questions which researchers cannot avoid by retreating to a supposedly quieter terrain.

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