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ABSTRACT

This booklet describes basic counting principles and provides activities designed to develop basic skills in: (1) stable ordering; (2) one-to-one matching; (3) total number; (4) different objects; and (5) different order. These are extended to more advanced strategies aimed at building counting skills strong enough to provide a basis for adding, subtracting, and even multiplying and dividing. Strategies here include: (1) counting on; (2) counting backwards; (3) counting multiples; (4) combined counting; (5) near doubles; and (6) compensation. Included are a list of resources and a list of references. (JM)

STRATEGIES AND ACTIVITIES

COUNTING AND EARLY ARITHMETIC LEARNING

Learning activities and implications from recent cognitive research

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PREFACE

A great deal of research, much of it supported by the National Institute of Education through R&D Centers and individual grants, has been focused on early arithmetic learning. The purpose of this booklet is to draw implications from the results of research related to counting, and to show how counting can form a basis for learning arithmetic.

The material is aimed at teachers of children aged 4 to 8. The booklet presents a discussion of how children learn, together with specific examples of learning activities suggested by the research. Many different methods of presenting the activities are possible. The aim is not to propose a particular curriculum or teaching method. Instead, the goal is to provide clear, concrete information to teachers about effective ways to help children learn early arithmetic concepts.

While the material is intended for early childhood teachers, many parents will find the material readable and the activities easy to do with children at home. Also, the booklet may be useful to mathematics supervisors or university educators who present inservice workshops for teachers. Much of the material will also be relevant to special educators, who work with elementary school students.

The booklet was prepared by Dr. Gerald Kulm, National Institute of Education Senior Associate in Learning and Development. The material was reviewed by Dr. Patricia Campbell, Assistant Professor of Mathematics Education, University of Maryland; Mrs. Carolyn Johnson, Elementary Teacher, West Lafayette, Indiana; and Dr. Karen Schultz, Associate Professor of Mathematics Education, Georgia State University.

CONTENTS

COUNTING AND EARLY ARITHMETIC LEARNING

A striking result of recent research on arithmetic learning is the importance of counting. Young children use counting in natural, informal ways to solve familiar problems. Many preschool children invent their own counting strategies to solve problems that they are not expected to solve until second grade or later. Once in school, students often continue to use counting strategies, sometimes in spite of instructions by teachers not to do so.

Research has shown that early arithmetic teaching can build on children's counting skills. Instead of discouraging students from counting, preschool and first-grade work can develop and build on counting abilities, setting the stage for later, more formal symbolic work.

COUNTING PRINCIPLES

In order to build on counting, teachers must understand that counting is more than simply reciting a string of numbers in a rote fashion. A child must understand five principles before counting can be used effectively.

1. **STABLE ORDERING:** The child can count in a fixed sequence; "one, two, three, four," and so on. Note that if the child counts "one, two, three, four, five, seven, nine, ten," for example, skipping some numbers but always counting in the same order, s/he does understand this principle. Over time, the child can learn to fill in the missing numbers.

2. **ONE-TO-ONE MATCHING:** In counting objects, the child matches each number with one object and counts all of the objects.

3. **TOTAL NUMBER:** The child knows that the last number counted is the total number of objects.

4. **DIFFERENT OBJECTS:** The child knows that the objects to be counted need not all be of the same kind.

5. **DIFFERENT ORDER:** The child knows that the objects can be

counted in any order. For example, she can start counting with any of the objects being first.

ACTIVITIES TO DEVELOP COUNTING SKILLS

The first thing a teacher should do is determine how well a child can count. Then, a variety of activities can be used to correct, practice, or extend the child's counting ability. Most children count easily if the activity is meaningful. Active manipulation of familiar concrete objects can set the stage for developing a sense of number meaning through counting.

Here are some suggested activities for each of the five counting principles. These activities can be used to diagnose and to teach counting skills. Most of them can be modified and extended for different ages by using different objects or larger numbers.

1. Developing Stable Ordering

- ***Use rhymes, fingerplays, and songs*** "one, two, buckle my shoe; three, four, shut the door; . . ."

- ***Play simple board games in which the child counts while moving a game piece along.***

- ***Use oral counting***
  Count in unison
  Count off, one by one, with each child taking one step forward
  Count in groups: one child counts to 3, the next to 6, and so on

2. Developing One-to-One Matching Skills

- ***Physically move the object as it is counted*** Use a counting frame (make one with heavy cord and styrofoam balls). Slide a ball to the end as it is counted.

  Use a pegboard and golf tees; chips and egg cartons

- ***Touch each object as it is counted*** Use everyday objects to count: pencils, blocks, candy, books

  Count objects arranged in lines, arrays, circles, at random. Change the arrangement and count again.

- ***Count while watching objects being moved or touched one at a time***

by the teacher or a classmate

- *Count while looking or listening* Use everyday objects: windows, pictures, parked cars, ceiling tiles, books

  Count handclaps or rings of a bell

3. Developing the Total Number concept

- *Use counting to determine "how many"* Recognize the total number of a set; count to verify, if necessary (Glue beans to small cardboard squares). Count sets of discrete familiar objects. books, children, buttons. Repeat the last number counted. Conceal objects in your hand or a container, then take them out and count one at a time to find out how many are hidden.

4. Developing the concept of Different Objects

- *Compare the number of related objects* Count four shoes, count four socks. How many of each are there? Are there as many shoes as socks? Match them to check.

- *Compare the number of similar objects with mixed objects* Put 6 pencils in a container. Put 2 clips and 4 erasers in another container. How many objects are in each container? Are there the same number of objects in each container?

- *Build a group of a given size* Use a group of mixed objects to make a group of 7. Make a group of 5 with not two objects alike. Make a group of 6 with two like objects, and the others all different.

5. Developing the concept of Different Order

- *Use different starting points in counting objects* Place objects in a circle and have the child count them. Then point to one of the objects and say, "Now count again starting here." Repeat, starting with different objects.

  Use mixed objects, arranged at random. Point to one of them and say, "Start counting here, make that the 'one.' " Repeat.

- *Fix the number name of one object* Use a group of fixed objects. Point at one of them and say, "Make that the 'three'. Count the objects." Put objects in a line and point to the object on the end, and

say, "Make that the 'one'." Then move the object to the other end and have the child count.

BUILDING ON THE COUNTING SKILLS

Early work with adding, subtracting, and even multiplying and dividing can be built upon strong counting skills. Research shows that children do not learn simply by mastering a skill in isolation and storing it in memory. Material taught this way is soon forgotten and almost never applied to new situations. Students learn best when ideas are meaningful and connected to previous knowledge. For children in first and second grade, previous knowledge is that which they have learned informally, often outside of school.

Generally, children start by counting physical objects, often aided by touching and/or moving each object. Later, they are able to recite the counting words in a sequence to solve a problem. Intermediate stages may involve the use of fingers, taps of a pencil, nodding the head, or other actions to keep track of the numbers counted or remaining. Instead of discouraging these behaviors, teachers should recognize that they are a natural part of many children's development of number understanding.

As children begin to do arithmetic, they use counting approaches. Often, they "invent" counting strategies in order to find sums or differences which have not been memorized. There is evidence that children who do not invent these strategies can be taught how to use them. The advantage of this approach over rote memorization is that children learn the basic facts more meaningfully, and have something to fall back upon if they forget a fact.

Several invented strategies have been identified. Alert, experienced teachers have probably noticed their students use some of these strategies.

1. **COUNTING ON:** To find a sum, the child starts with one number, then "counts on" the other. For example, to find 5+3, the child counts: "five (pause), six, seven, eight." Many children automatically develop the idea that it is more efficient to start with the larger number.

   Counting on is also used for certain subtraction problems. For example, Maria has $5 and needs $8 to buy a record. How much more money does she need to save? The child again counts: "five (pause), six, seven, eight," keeping track of how many were counted on (three numbers), which is the answer.

2. **COUNTING BACKWARDS:** Although this skill is difficult for many children, some are able to use it well enough to subtract. For example, to

find 9-3, the child counts: "nine (pause), eight, seven, six." This strategy should not be forced, and it is best applied when the number being subtracted is small.

3. **COUNTING MULTIPLES:** Many children learn on their own to count by "twos," "fives," or "tens." Later, they learn to count by other multiples, or even by hundreds or thousands. This skill provides a link to understanding multiplication and division. To multiply 4x2, count: "two, four, six, eight." To divide 15 by 5, count: "five, ten, fifteen."

4. **COMBINED COUNTING:** Counting on and counting backward can be combined to do fairly sophisticated mental arithmetic or estimation. For example, to add 53+22, "22 is 2 more than 20, so count: 53 (pause), 63, 73; now count on 2: 74, 75." To add 47+28, "28 is 2 less than 30 so count on: "47 (pause), 57, 67, 77; now count back 2: 76, 75.

5. **NEAR DOUBLES:** Most children learn the "doubles" addition facts first (1+1=2, 2+2=4, 3+3=6, etc). Many other facts can be derived as "near doubles" by counting on or backwards from a double. For example, to find 3+5, the child remembers that 3+3=6, then counts on 2 more to get 8.

6. **COMPENSATION:** This strategy is also based on doubles. To find 4+6, for example, the child remembers that 5+5=10, then reasons that 4 is 1 less and 6 is one more than 5, so the sum is 10.

The last two strategies make use of addition facts that have already been learned, and illustrate how the later facts can be learned with meaning. These strategies are useful as children learn facts with larger numbers which are difficult to count. Research has shown that children rely less and less on counting as they remember more facts. Strategies which link counting to facts already learned provide a way to extend the child's learning in a meaningful way.

ACTIVITIES TO DEVELOP COUNTING STRATEGIES

Many children's games and home activities involve counting. A rich preschool environment can help children develop a firm understanding of numbers, through counting. Later counting activities, such as those suggested here, can provide a critical link between informal and formal arithmetic skills. Often, children see school arithmetic as a completely separate activity from the number ideas they have learned informally. Counting activities can help them see connections between intuition and formal arithmetic.

1. Developing Counting On skills

- *Use familiar settings as story problems* You have 7 pennies and mother gives you 3 more pennies. How many pennies do you have now? Let children use pennies; later do the problem without objects.

- *Use objects to represent story items* Let blocks, or paper clips represent animals or cars, for example, in a story. Cover the objects with a box and do the problem.

- *Use oral counting* Play "Count to ten." Announce a starting number, then ask a student to count on, up to 10. For older students, use larger numbers, and count on by tens or hundreds.

2. Developing Counting Backward skills

- *Use familiar story problems* You have 7 candies and you eat two. How many candies are left? Use candies and count as they are removed: "seven, (pause), six, five."

- *Use objects* Start with some number of objects and announce how many will be removed; count down as objects are removed, one at a time.

- *Use oral counting* Missile launch count-down. Count the floors as the elevator goes down. Count with the clock as a timed game ends.

3. Developing Counting Multiples skills

- *Use natural pairs or arrays of objects* Count pairs of feet, eyes, ears by twos. Count window panes by threes, pennies by fives, days by sevens.

- *Use concrete objects* Group 12 objects by two, then count by twos. Repeat counting by threes; fours. Make "bundles" of 3s, 5s, 10s, with sticks or straws and rubber bands, then count.

  Play "Candy Factory." Produce pieces; make bars of 5 pieces; boxes of 10 bars. Count the day's output.

- *Use physical and oral activity* Walk along and say "one" softly with the left foot, stamp the right foot and say "two" aloud; repeat with odds softly, evens aloud.

4. Developing Advanced Counting skills

The following activities are appropriate for advanced students in second or third grade, or for all students fourth grade or above.

- ***Use oral counting by tens, hundreds, thousands*** Start at 30 and count by tens to 100. Start with 70 and count by hundreds, then tens to 500. Count backwards from 100 to 30, by tens.

- ***Play estimation games*** Give a starting and a "target" number, say 5 and 23. Count by 2s, 3s, etc. Who can hit the target?

  Give a problem and a range for its solution, say 38+27 and the range 60 to 70. Will the answer be in the range? Check with a calculator.

- ***Give a set of counting directions*** "Start with 6, count by 3s past 20, count back by 5s past 10. What is the answer?"

5. Developing Near Doubles skills

- ***Use sets of objects*** Show two sets of 3, and ask how many altogether. Now add 1 object to one of the sets. How many altogether. Add 1 more. Ask the child to count: "6 (pause), 7, 8."

- ***Use pictures*** Draw a set of 4 and a set of 5. Ask the child to draw lines connecting pairs of items in each set. How many are paired? How many altogether?

- ***Practice doubles and near doubles at the same time.*** 3+3, 3+4, 5+3, 4+3, 3+5.

6. Developing Compensation skills

- ***Use sets of objects*** Start with two sets of 4 objects each. How many altogether? Take 1 from one set and add it to the other. Now how many altogether? State the new fact (3+5=8).

  Start with a set of 10 objects. Separate into sets of 5. Then take 1 from one set and add it to the other. State the addition fact (4+6=10).

- ***Practice "families" of facts at the same time*** 3+7, 4+6, 5+5, 6+4, 7+3.

- ***In grades 3 or 4, use large numbers*** How much is 100+100? How much is 99+101? 99+102?

Many of the early mathematical concepts that children learn before starting school are natural, culture-free ideas that seem to develop in a variety of environments. Research across cultures, ethnic groups, and socio-economic settings indicates that early number concepts are present and competently used informally by virtually all children.

Early school experience which provides activities with counting, comparing, and estimating can significantly enrich the child's natural development. As children continue in school, their formal arithmetic learning should build upon and extend these informal skills and concepts in meaningful ways. Counting is a key to making this linkage between intuitive, invented skills and formal, symbolic learning.

RESOURCES

Children's Arithmetic: The Learning Process by Herbert Ginsburg gives insights about how children learn mathematics. Specific teaching suggestions are made, placing a major emphasis on counting as a basis for arithmetic. Published by D. Van Nostrand Company, 1977.

I Can Count the Petals of a Flower, by John and Stacey Wahi, is a full-color picture book for teaching children how to count, through beautiful photographs of flowers. Available from the National Council of Teachers of Mathematics.

Mathematics Learning in Early Childhood is a resource book for teaching mathematics to children aged 3-8. Teaching procedures and hundreds of activities emphasize problem solving and relating mathematics to the real world of the child. Available from the National Council of Teachers of Mathematics.

REFERENCES

Carpenter, T. P. & Moser, J. M. (1983). *The acquisition of addition and subtraction concepts.* In R. Lesh & M. Landau (Eds.), *Acquisition of mathematical concepts and processes.* New York: Academic Press.

Carpenter, T. P., Moser, J. M. & Romberg, T. A. (1982). *Addition and subtraction: A cognitive perspective.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Fuson, K. C., Richards, J. & Briars, D. J. (1982). The acquisition and elaboration of the number word sequence. In C. J. Brainerd (Ed.), *Children's logical and mathematical cognition.* New York: Springer-Verlag.

Gelman, R. & Gallistrel, C. R. (1978). *The child's understanding of number.* Cambridge: Harvard University Press.

Ginsburg, H. P. (1983). *The development of mathematical thinking.* New York: Academic Press.

Resnick, L. B. (1983). A developmental theory of number understanding. In H. P. Ginsburg (Ed.), *The development of mathematical thinking.* New York: Academic Press.

Steffe, L. P., von Glaserfeld, E., Richards, J. & Cobb, P. (1984). *Children's counting types: Philosophy, theory, and application.* New York: Praeger Press.