DOCUMENT RESUME

ED 269 267 SE 046 586

AUTHOR Cookson, Connie; Moser, James M.
TITLE Coordinated Study Individual Interview Procedures. Working Paper No. 290.
INSTITUTION Wisconsin Univ., Madison. Research and Development Center for Individualized Schooling.
SPONS AGENCY National Inst. of Education (ED), Washington, DC.
PUB DATE May 80
GRANT OB-NIE-G-80-0117
NOTE 37p.; For related documents, see SE 046 583-590.
PUB TYPE Reports - Research/Technical (143)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Addition; Educational Research; Elementary Education; *Elementary School Mathematics; Error Patterns; Interviews; Manipulative Materials; *Mathematic Instruction; Problem Solving; *Research Methodology; Student Behavior; *Subtraction
IDENTIFIERS *Mathematics Education Research

ABSTRACT

This paper describes the various procedures associated with the individual interviews that are part of the data gathering processes of the Coordinated Study being carried out by the Mathematics Work Group of the Wisconsin Research and Development Center for Individualized Schooling. The first major section describes the six basic verbal addition and subtraction problem types used in the study, how they were selected and how they are varied by substitution of verbal and numerical terms. The second section briefly characterizes the general interview procedures and verbal protocols. The final section gives definitions of the various student behaviors that can be expected in response to the presentation of the verbal problems. The behaviors are classified by models, correctness, strategies, and errors. (Author)

Working Paper 290

Coordinated Study Individual Interview Procedures

by

Connie Cookson and James M. Moser

Report from the Project on

Studies in Mathematics

Thomas A. Romberg and Thomas P. Carpenter

Faculty Associates

James M. Moser

Senior Scientist

Wisconsin Research and Development Center
for Individualized Schooling
The University of Wisconsin
Madison, Wisconsin

May 1980

Published by the Wisconsin Research and Development Center for Individualized Schooling. The project presented or reported herein was performed pursuant to a grant from the National Institute of Education, Department of Health, Education, and Welfare. However, the opinions expressed herein do not necessarily reflect the position or policy of the National Institute of Education, and no official endorsement by the National Institute of Education should be inferred.

Center Grant No. OB-NIE-G-80-0117

MISSION STATEMENT

The mission of the Wisconsin Research and Development Center is to improve the quality of education by addressing the full range of issues and problems related to individualized schooling. Teaching, learning, and the problems of individualization are given concurrent attention in the Center's efforts to discover processes and develop strategies and materials for use in the schools. The Center pursues its mission by

- conducting and synthesizing research to clarify the processes of school-age children's learning and development

- conducting and synthesizing research to clarify effective approaches to teaching students basic skills and concepts

- developing and demonstrating improved instructional strategies, processes, and materials for students, teachers, and school administrators

- providing assistance to educators which helps transfer the outcomes of research and development to improved practice in local schools and teacher education institutions

The Wisconsin Research and Development Center is supported with funds from the National Institute of Education and the University of Wisconsin.

WISCONSIN RESEARCH AND DEVELOPMENT
CENTER FOR INDIVIDUALIZED SCHOOLING

Table of Contents

List of Tables

List of Figures

Abstract

This paper describes the various procedures associated with the individual interviews that are part of the data gathering processes of the Coordinated Study being carried out by the Mathematics Work Group of the Wisconsin Research and Development Center for Individualized Schooling. The first major section describes the six basic verbal addition and subtraction problem types used in the study, how they were selected and how they are varied by substitution of verbal and numerical terms. The second section briefly characterizes the general interview procedures and verbal protocols. The final section gives definitions of the various student behaviors that can be expected in response to the presentation of the verbal problems. The behaviors are classified by models, correctness, strategies, and errors.

A major aim of mathematical instruction is to enable students to acquire concepts and skills requisite for solving problems of many types. A principle goal of mathematical education research is to understand how children acquire those concepts and skills and to understand how selected pedagogical and psychological factors are related to their acquisition. The Mathematics Work Group of the Wisconsin Research and Development Center for Individualized Schooling is presently conducting a program of research focused on a small set of those concepts and skills. Our interest lies in arithmetical learning, and in particular, in the acquisition of concepts and skills related to addition and subtraction of whole numbers.

The research program is attempting to relate pupil performance on selected arithmetic skills to pupil cognitive processes, instructional materials, and teachers' classroom behaviors. The interrelationship of these variables is depicted in Figure 1. Using this framework, we are proceeding to:

1. identify important addition and subtraction skills;

2. review past empirical data or collect new data on these skills;

3. re-examine these mathematical skills and hypothesize how they are related to underlying cognitive skills;

4. examine the instructional materials designed to teach these skills; and

5. conduct a series of empirical studies on the appropriateness of particular teacher classroom behaviors, the appropriateness of instructional materials, and the relationship of specific cognitive skills to

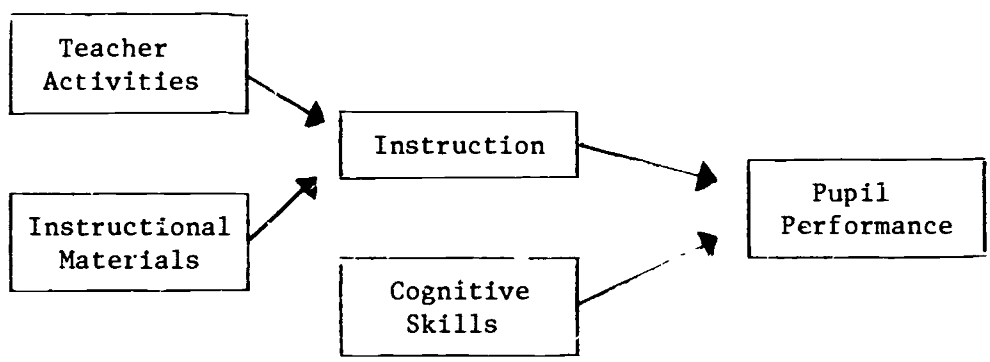

Figure 1. Factors influencing pupil performance.

mathematical skills.

The work of the Mathematics Work Group is built around the conceptual framework exemplified in Figure 1. The empirical and theoretical investigations generally involve two or more of the factors depicted, and have been organized into four major categories. These are a conceptual paper series, a set of short empirical studies, a major longitudinal study, and an invitational conference of scholars

This paper relates to the longitudinal study. Approximately 150 students in three separate schools have been identified as subjects for the study and are being followed for about three years. Pupil performance will be measured in several ways:

1. Individual interviews. At several times during each school year, individual children are administered a set of problem tasks dealing with addition and subtraction. The interviewer attempts to ascertain the children's solution strategy, correctness of answer, type of errors made, and modeling procedures.

2. Group administered paper-and-pencil tests. There are two separate categories of tests:

   a. Achievement monitoring. These tests measure pupil progress toward a set of performance objectives that are contained in the instructional materials. By means of matrix sampling procedures, estimates are made of group performance. Achievement monitoring tests related to arithmetic objectives are given shortly after the completion of the instructional units.

   b. Topic inventories. These are very short tests that measure pupil progress toward mastery of the objectives of a specific instructional unit, or topic. Every subject takes the same test, resulting in a measure of individual performance.

Instruction and classroom environment are assessed by direct classroom observation of teacher actions, pupil behaviors, and instructional materials. A trained observer is present each day the instructional units, or topics, dealing with arithmetic objectives are being used. Organizational and grouping measures are noted, along with indications of interactions between teacher and pupils, and among pupils. Measures of pupil engaged time are estimated by observing six target students.

The purpose of this paper is to describe in some detail the individual interview procedures. Results obtained from the set of interviews to be conducted over a period of three years are not contained in this paper. A series of Center working papers (e.g., Kouba & Moser, 1980) will document the results, with one paper planned for each separate interview. In the following major sections c this paper, descriptions of problem types used in the interviews, procedures and protocols, and pupil response categories will be presented.

Problem Types

Throughout all problem solving interviews, six basic addition and subtraction verbal problems have been utilized. This section shows how problems were analyzed and selected, how specific problem wording was determined, and how various number combinations were assigned to the problems.

Problem analysis. An initial concern of our research was to characterize basic problem types that provide different interpretations of addition and subtraction operations. The analyses of the verbal problems that serve as the basis for this research are presented in Moser (1979) and Carpenter and Moser (1979) in which a model characterizing 21 possible problems is described. From the 21 problem types, six basic problem types are included in the interview. The specific problems were selected because:

1. they are representative of problems commonly included in elementary mathematics texts,

2. they include the three basic but different types associated with subtraction.

3. they are problems that younger subjects are most likely to be able to solve,

4. they elicit different patterns of solution, as indicated by an earlier pilot study (Carpenter, Hiebert, and Moser, in press).

5. there should be more than one addition problem, and

6. exemplars of both action and static problems are to be included.

Examples of each of the problem types and the order they are given in the interviews are presented in Table 1.

Problem wording. The research design of the Coordinated Study calls for the same six basic problem types to be administered up to four times in a single interview as well as repeated in each of the separate interviews. It is necessary to change the wording of the problems so that the children will not remember or immediately recognize the problem situation. Consequently, an algorithm has been designed to maintain consistency as nearly as possible in terms of sentence length, syntax, and vocabulary difficulty. An algorithm for each problem type has been designed.

Consider the following specific example of Task 1, the Joining (addition)

Table 1

Representative Addition and Subtraction Problems

1. Joining (addition)	Wally had 3 pennies. His father gave him 5 more pennies. How many pennies did Wally have altogether?
2. Separating (subtraction)	Tim had 11 candies. He gave 7 candies to Martha. How many candies did Tim have left?
3. Part-Part-Whole missing addend (subtraction)	There are 6 children on the playground. 4 are boys and the rest are girls. How many girls are on the playground?
4. Part-Part-Whole (addition)	Sara has 6 sugar donuts. She also has 9 plain donuts. How many donuts does Sara have altogether?
5. Comparison (subtraction)	Joe has 3 balloons. His sister Connie has 5 balloons. How many more balloons does Connie have than Joe?
6. Joining missing addend (subtraction)	Kathy has 5 pencils. How many more pencils does she have to put with them so she has 7 pencils altogether?

problem:

Wally had 3 pennies.
His father gave him 5 more pennies.
How many pennies did Wally have altogether?

This problem can be modified by noting its semantic structure and making appropriate changes as indicated by an algorithm. The symbols x, y and z stand for the numbers assigned to the problems (the following section describes how the numbers were actually assigned). The algorithm Task 1 is as follows:

1. Name had x noun_1.

   His/her noun_2 gave him/her y more noun_1.

   How many noun_1 did name have altogether?

   Substituting Joe, frogs, and friend in the appropriate positions does not in any way alter the problem class.

   Following are the algorithms designed for each of the remaining five problem types.

2. Separating (Subtraction)

   Name_1 had z noun_1.

   She/he gave y noun_1 to name_2.

   How many noun_1 did name_1 have left?

3. Part-Part-Whole, missing addend(Subtraction)

   There are z noun_1 $\text{prepositional phrase}_1$.

   y are noun_2 (or adjective_1) and the rest are noun_3 (or adjective_2).

   How many noun_3 (or adjective_2) are $\text{prepositional phrase}_1$?

4. Part-Part-Whole (Addition)

   Name has x adjective_1 noun_1.

   Name also has y adjective_2 noun_1.

   How many noun_1 does name_1 have altogether?

5. Comparison (Subtraction)

$\underline{\text{Name}}_1$ has y $\underline{\text{noun}}_1$.

His/her $\underline{\text{noun}}_2$ $\underline{\text{name}}_2$ has z $\underline{\text{noun}}_1$.

How many more $\underline{\text{no}}\ _1$ does $\underline{\text{name}}_2$ have than $\underline{\text{name}}_1$.

6. Joining, missing addend (Subtraction)

$\underline{\text{Name}}_1$ has y $\underline{\text{noun}}_1$.

How many more $\underline{\text{noun}}_1$ does he/she have to put with them so he/she has z $\underline{\text{noun}}_1$ altogether?

Number assignment. Within the problems, two of the three numbers from the number triple (x, y, z) defined by $x + y = z$, $x < y < z$ are given. In the two addition tasks, the smaller addend x appears first in the problem statement and the larger addend y appears second. In subtraction tasks 2 (Separating) and 3 (Part-Part-Whole, missing addend) the sum z appears first in the problem statement while the larger addend y appears second. The order of presentation of the numbers y and z is reversed in subtraction tasks 5 (Comparison) and 6 (Joining, missing addend) with y appearing first and z second.

For the set of interviews to be administered over the course of the study, several number domains were identified. Where feasible, number triples were selected from the domains according to the following guidelines:

1. Avoid "doubles" such as $x + x = z$ because it was hypothesized that children may operate differently with those combinations (cf. Groen & Parkman, 1972).

2. Avoid consecutive addends such as 6 + 7 = 13.

3. Avoid triples where the sum is 10 or any multiple of 10.

4. Avoid addends of 0 and 1.

Three number domains were selected. In the smaller number problems (referred to as the "b" problems), the additional guideline of $5 \leq z \leq 9$ was imposed. In the larger number problems (referred to as the "c" problems) the restriction on the sum was $11 \leq z \leq 15$. For the later interviews that began in January 1980, the domain of 2-digit numbers is included. In the 2-digit domain, two sub-domains were identified. In the first no regrouping ("borrowing" or "carrying") is required to determine a difference or sum when a computational algorithm is used. In the second numbers are utilized where regrouping is required. The no-regrouping set is called the "d" problem set while the regrouping set is referred to as the "e" problems. For the 2-digit problems, the sum z is restricted to numbers in the 20s and 30s. The actual number triples utilized in the study are listed in Table 2. The assignment of the number of triples to the six problem types is carried out using a six-by-six Latin square design. For each domain, this results in six sets of six problems which are then uniformly and randomly distributed across subjects.

Interview Procedures

Because a major interest of our research is the identification of problem solving processes used by young children, the individual interview technique was chosen as the most appropriate way to determine those processes. In this section the general procedures used in carrying out the individual interviews are described. It is well to remember that the set of interviews is spread out over a period of three years. While the procedures described in this section give a general characterization, some specific modifications do occur as the subjects mature and become more "test-wise." For example, questioning techniques do not have to be so elaborate once children know what

Table 2

Number Triples Used in Coordinated Study

		Two-digit numbers	
Smaller numbers "b"	larger numbers "c"	no regrouping "d"	regrouping "e"
2, 3, 5	3, 8, 11	12, 15, 27	12, 19, 31
2, 4, 6	4, 7, 11	12, 16, 28	13, 18, 31
2, 5, 7	5, 7, 12	11, 18, 29	14, 18, 32
2, 6, 8	4, 9, 13	13, 16, 29	16, 17, 33
3, 4, 7	6, 8, 14	14, 21, 35	15, 19, 34
3, 6, 9	6, 9, 15	14, 23, 37	17, 19, 36

types of probing questions are likely to be asked.

Mode of presentation. The smaller number "b" problems and the larger number "c" problems are presented under two conditions. In the first mode for both domains, a set of manipulative materials is provided. A set of 20-30 2-cm cubes, about equally divided between blue and orange in color, is present on the table at which the child is seated. Prior to interviewing, the child is encouraged to handle the cubes and is told that they could be used to help solve the problem if desired. At no time is any child required to use the cubes; it is strictly voluntary. All subjects come from classrooms that encourage the use of concrete materials. Thus, the use of cubes is not an unfamiliar process to the children.

The second mode of presentation for these two number domains has no physical objects present. After the first set of six problems with cubes present is completed, the examiner asks the child to pick up all the cubes, put them into a container and then set them aside. The conditions with cubes present is called the "+" condition and without, the "-" condition. Since two different number domains are involved, this results in four actual presentations of six problems each, b+, b-, c+, and c-. When all four conditions are involved, the order of presentation of the problem sets is b+, b-, c+, and c-. These four conditions were assumed to present increasing levels of difficulty. Thus, the term "level" is applied to these problem conditions as well as to "d" and "e" problem sets.

When the 2-digit "d" and "e" problems are presented to the subjects, cubes are also present. About 50 cubes are used, equally divided between two colors. Both 2-cm solid cubes and Unifix cubes are employed, although at a given interview only one type is present. Paper and pencil is also made available to the child during the presentation of the 2-digit number problems. As with the cubes, the child is not required to use the paper and pencil.

Not all problem tasks are given to a child in a single sitting. Because of limited attention span, the general procedure is to administer no more than 12 tasks during one session. Thus, in the earlier interviews when only the "b" and "c" problems are used, the first day's portion consists of the b+ problems followed by the b- problems. On a subsequent day the c+ problems are given first with the c- problems following. When the "d" and "e" problem sets are presented they are both administered in a single day with the no regrouping problems presented first.

Assignment of problems. Six sets of problems are prepared for each problem level with the Latin square design providing a method for randomly assigning the six number combinations to the six problems. Each participating student is assigned a particular number combination set (deck) for each presentation category. No child is assigned, for a given problem level, the same deck number on any two consecutive interviews. An assignment sheet is prepared with each student's name and the problem deck assigned. Following is an example of an assignment sheet for an earlier interview with "b" and "c" problems.

	b+	b-	GO ON?	c+	c-
Sue Smith	1	4		3	5
Joe Blow	3	6		2	4
Tim Tashun	2	5		1	3

.

.

.

Termination procedures. The general goal for each interview is to present each child with b+, b- problems during the first interview session and c+, c- problems during the second session. So that children will not be subjected to unreasonable demands for their level of ability, these guidelines have been developed to terminate the interview of a child is producing no codable response:

1. If, during the b+ portion of the interview, the child fails to use any coherent or identifiable strategies while trying to solve three of the first four problems, the interview is terminated.

2. If the child solves two of the first four b+ problems, but is baffled by the last two problems, i.e., solves ust two of the six problems, the b portion of the interview is terminated. The interview is terminated not because 3 of the 4 problems are incorrect, but because the child is perplexed by the problems and employs highly inappropriate strategies.

3. If the child does reasonably well with the b+ problems, the b- problems are presented. The b- portion should be completed unless the child cannot solve three of the first four problems using the criteria cited above.

The decision of whether or not the child should go on to the c portion is made at the end of the b interview and recorded on the assignment sheet under the column headed "GO ON?" The guidelines for terminating the interview at the c+, c- level are the same as those given above.

Some children who rely heavily on physical modeling do well with b+

problems but fail to solve b- problems. In those few cases, a decision is made to branch only to c+ problems. It is inferred that the child cannot solve c- problems but that c+ problems might well be within his/her problem-solving abilities.

General interview protocol. A warm-up exercise precedes the first interview and helps the students get into the spirit of the interview.

Warm Up

1

We've asked you to come in because we're interested in finding out how boys and girls figure out answers to number stories or problems.

Here are some objects. I'm going to sort the objects into two piles. [BLUE SQUARE WILL BE CHILD' SHAPE. SORT 4-SIDED SHAPES INTO ONE GROUP AND 3-SIDED SHAPES INTO A SECOND GROUP.]

Would you put this [BLUE SQUARE] where you think it should go?

How did you decide to put it there?

The dialogue for the interview follows:

2

Now I'm going to read some stories with numbers. Each story has a question. For each question, I may ask you how you figured out your answer. To help me remember I will be writing down some things on this paper.

Here are some cubes to use to find the answers. [POUR OUT CUBES.] Would you straighten them out and get them ready to use? [BUSY YOURSELF SOME WAY.] Remeber, these are here to use to solve the problem.

[WHEN FINISHED WITH THE B+ QUESTIONS, COLLECT CUBES. SELECT THE PROPER BRANCH TO MOVE TO.]

Now I'm going to read some more stories. Again, I may ask you how you figured out your answer.

The verbal problems are read to the child and reread as often as necessary so that remembering the given numbers or relationships is not a factor, General questioning techniques. The goal of the interview is to identify the process(es) a child uses to solve a particular problem and then to accurately and reliably record appropriate information on a specially designed coding sheet. Whenever a child's behavior gives clear indication of the process(es) used, no questions are required. However, if the interviewer is unsure of what the child is doing, some probing questions are called for. No rigorous and systematized protocol is established. Rather, an interactive routine that varies with the problem type, the child involved, and the behavior exhibited by the child is suggested. Nevertheless, some general guidelines are followed.

The initial follow-up question is something akin to "How did you get that answer?" or "How did you decide that ____ is the answer?" Even if the interviewer strongly suspects that a child has counted, the question, "Did you count?" is not asked because the suggestion of counting might encourage a child to explain a solution process in that manner even if counting were not used. If a child volunteers that counting was used, then the appropriate questions are, "Did you count forward or backward?" and "What number did you start counting with?" When a child does not suggest counting, a question to use is, "Were you thinking of any numbers to yourself?" and if so, "What numbers?" or "How did they help you get the answer?" If prolonged questioning or probing seems to confuse or frustrate a child, it is broken off and the interviewer proceeds to the next problem.

Another report (Martin & Moser, 1980) presents much greater detail on specific questioning associated with particular problems and suspected strategies. That report is designed for persons who actually wish to carry out individual interviews as described in this paper.

Pupil Response Categories

This section characterizes the various types of student behaviors exhibited in the solution of individually presented verbal addition and subtraction problems. These behaviors are both hypothetical and actual, most having been observed either in pilot studies (Carpenter, et al., 1979; Carpenter, 1980) or in the Coordinated Study (Kouba & Moser, 1980). Student responses fall into four major categories: model, correctness, strategy, and error. Each will be completely described in the sections that follow.

Model. The term "model" can be thought of as a synonym for mode or method of representation. Once a problem situation is given, a child may opt to represent the numbers of the problem and/or the action or relationship described in the problem situation. The following categories are ways that children may carry out the representation. For a given problem, it is entirely possible that a child could choose more than one model.

Cubes. There are two major ways that cubes can be used. First, they are set out to represent the actual sets described in the problem situation. For example, if the Joining problem, "Wally has 3 pennies. His father gave him 6 more pennies. How many pennies did Wally have altogether?" is given, the child counts out a set of three cubes, then counts out a set of six cubes, which may be put aside in a separate pile or be adjoined to the set of three in a one-by-one basis, to represent the additional six pennies. Action performed on the cubes, if any, can be considered the child's interpretation and representation of the action or relationship between the sets described in the problem.

The second use of cubes occurs when a child is employing an advanced counting strategy (to be described in a subsequent section). When a child enters a sequence of counting numbers and counts forward or backward, the number of counting words spoken must be kept track of. Cubes are used as a tracking or memory device. For example, if a child begins a forward counting sequence at "eight" and goes on the "13," a set of five cubes might be set out one by one. Those five cubes do not represent a set of five objects as given in the original problem, but rather as counters to stand for the number words, "nine, ten, eleven, twelve, thirteen."

Fingers. Fingers are used in much the same way as the cubes described above, either to represent sets described in the problem, or to track numbers in a counting sequence.

Tallies. When paper and pencil are provided, the child can make tally marks on the paper, using them in the same way as fingers or cubes. The difference is that tally marks are not movable and joinable as are fingers or cubes. Where tally marks are used in a subtraction problem, a child may represent a separating action by crossing out tally marks already made.

Pictures. This quasi-symbolic representation might occur when paper and pencil is provided. Pictures would be used in much the same way as tally marks. However, the pictures made would be more graphic than tally marks. For example, if the problem situation dealt with apples, the child might draw circles with stems to represent the apples.

Numbers. Given paper and pencil, the child might write down one or more numbers. Usually, this would be used by the child to serve as a memory device. Since no operational symbol is involved, the expectation is that the actual solution process would be carried out by other means.

Number sentences. Once formal mathematics instruction has taken place, the child might choose to represent the numbers and the actions and relationships by means of a standard number sentence, or some variation that makes sense to the child. The sentence could be horizontal or vertical. When written in a vertical fashion, the operation symbol may well be left out by the child.

Organizing box. In the Coordinated Study, the children receive specific instruction on how to analyze problem situations. The general theme of this analytic procedure is to attempt to think of the problem entities and relationships in terms of a part-part-whole relationship. To help the student organize data, a graphic device known as the organizing box is used. It is shown at the right. A child could choose to use the box, writing in the particular portions the numbers of the problem thought to be the part(s) and the whole.

whole	
part	part

Other. Some children become very ingenious at representing numbers, particularly larger ones (when more than 10 fingers are needed), in problem conditions when cubes and paper and pencil are not provided. They visually or tactilely use other objects present in the interviewing room. Examples might be numerals on a clock face, lines in the surface of a table, square tiles on a floor, portions of a lighting or heating fixture, or buttons on clothing.

No action. It is quite possible that a child will use no visible means of representing a problem situation. A presumption is made that mental imagery might be brought into play. Another presumption is that a child is operating at a higher level of abstraction and is using recall of memorized information, or is performing some mental manipulation of number facts or number properties to derive an answer.

Correctness of Response

If the child responds with a numerical answer, then that response can be determined to be either correct or incorrect. It is quite possible for a child to end up with a seemingly correct answer that has been reached by an incorrect process. For example, when using a direct modeling of the problem with cubes, a child may miscount or misrepresent one set. Then in a subsequent counting a second miscount error occurs, offsetting the first error. In such instances the child is considered to have produced an incorrect response. A third possible type of response is one in which the child does not give any numerical answer, simply stating to the effect that he or she cannot do the problem at all or doesn't know the answer.

Strategy

The general sequence of problem solving steps expected of the young children in the longitudinal study is first to select the mode of representation - physical, symbolic, or mental - and then to carry out some sort of operation on the representation. The action can also be physical, symbolic, or mental. These actions upon representations are referred to as strategies. For the purposes of this paper, strategies will be classified as being appropriate only for addition, only for subtraction, or appropriate for either addition or subtraction.

Addition Strategies

Counting all. This involves construction of two separate sets, one for each addend that is counted, "one, two, . . . etc." and then counting the union of the two model sets, "one, two, . . . etc." The union ɔe made all at once after the second addend is modeled or it can be formed incrementally as a child adjoins cubes one at a time to the first set while modeling the second.

Thought not a frequent occurrence, counting all can be executed without the use of models. If so, the child counts either silently or aloud "one, two, . . . etc."

Subitizing. When a child models the two sets corresponding to the two addends, it is sometimes possible to immediately perceive the numerosity of the union set without having to individually count each member of that set. This subitizing behavior can be expected to occur only when the size of the union set is quite small, such as five or six in number, or when fingers are used as the modeling device.

Counting on from first. This strategy is generally used without models to represent the sets given in the problem. Rather, a sequence of counting words is employed. In this strategy the child enters a counting sequence at a point corresponding to the first addend read in the problem. In the problems in this study, the first addend read in the problem is always the smaller number. Counting is forward, and the number of spoken counting words (either silent or aloud) corresponds to the second addend. The final word spoken is then given as the answer. Knowing when to stop -- that is, when the required number of counting words have been recited -- is determined by internal tracking mechanism or by some external device such as cubes, fingers, or tally marks.

If counting on is used when sets are modeled, the behavior involves the child realizing that once a set has been correctly counted out, it does not have to be counted again. Thus, a child may look at the smaller modeled set and say, for example, "three" and then continue counting on, "four, five, six, . . . etc." while touching the objects in the other model set.

Counting on from larger. This is identical to the preceding "counting on" strategy with the exception that the counting sequence is entered at a point corresponding to the larger addend in the problem, which is always read second.

Subtraction Strategies

Separating. This involves modeling the given larger set, then taking away or crossing out element of that model set (usually one at a time) until a set equal to the smaller number has been separated. Counting the remainder set yields the answer. Counting of all three sets -- the original, the set taken away, and the remainder -- is effected "one, two, . . . etc." except where the remainder set is small enough that the child can determine its size by subitizing.

On rare occasions the child uses this strategy differently so that the remainder set is equal to the smaller given number in the problem. In this case, counting the number of objects that were taken away from the original model set yields the answer.

Adding on. This strategy begins with modeling the given smaller set. Then that set is incremented one at a time until the new model is equal in size to the larger number. Counting the number of model objects adjoined to the original smaller set yields the answer.

Matching. In this strategy, two sets, each counted out, "one, two, . . . etc." are used to model the two sets given in the problem. These two model sets are then physically or visually placed in a one to one correspondence and the excess of the larger set over the smaller set is determined as the answer. This determination is made by counting or subitizing. Matching is used almost exclusively on the Comparison problem.

Counting down. This strategy uses a counting sequence which is entered at a point corresponding to the larger given number (minuend) in the problem. Counting is backward. In most instances the counting sequence ends when the number of counting words spoken corresponds to the smaller given number (subtrahend) in the problem. The last spoken word is the answer. On rare occasions the counting sequence ends when the number word spoken corresponds to the smaller given number. Then the number of counting words spoken is the answer. In either case, the number of counting sequence words spoken is kept track of by an internal or external tracking mechanism (cubes, fingers, tally marks).

Counting up from given. This strategy is characterized by entry into a counting sequence at a point corresponding to the smaller number (subtrahend) given in the problem. Counting is forward. The counting sequence ends when the number corresponding to the larger given number (minuend) is spoken. The number of counting words spoken is the answer. Again, the number of words spoken is kept track of by some tracking mechanism.

Common Strategies

Use of number fact. In the smaller and larger number domains, the child can produce the answer by reason of recall of some previously internalized number fact. This strategy is marked by an almost instantaneous correct response. If queried by an interviewer as to how the answer was known so rapidly, the child responds with words to the effect, "I just know it." Because many children memorize triples of numbers (e.g., 4, 5, and 9 go together) the so-called fact for a subtraction task may well be the related addition fact. No differentiation is made in terms of ascertaining whether the known fact is an addition or subtraction fact.

Heuristic. In this instance an answer is determined by mental manipulation of some known number fact or relationship. Often the known facts that are manipulated are doubles facts (e.g., 6 + 6 = 12) or combinations that relate to 10 or a multiple of 10 (e.g., 7 + 3 = 10). This strategy can also be used in connection with the counting sequence strategies -- counting on, counting down and counting up from given. In these cases the number to be counted on (as in addition) or to be counted down (as in subtraction) can be decomposed by a heuristic in order to hasten the counting. This often happens in connection with a multiple of 10. For example, in solving an addition problem involving the addends of 14 and 17, the child might begin counting, "18, 19, 20" and then jump immediatelty to "30" followed by the answer of "31." In this example, the addend of 14 has been decomposed into 3 + 10 + 1 as the child was counting on. A reverse process of composition can also occur. As an example, suppose the missing addend problem involving the addend of 16 and the sum of 29 is posed. A child might begin counting up from the given number 16 by saying, "17, 18, 19, 20, and 9 more makes 29. Let's see, 4 and 9 is 13 so the answer is 13." In this example the answer of 13 is composed out of 4 and 9.

Algorithm. This strategy can occur only within the domain of 2-digit number problems. Essentially the child performs the standard algorithm for either adding or subtracting. Notation is in the vertical form. The ones place is operated on first and regrouping considerations come into play as necessary. Then the tens place is computed. Although many children will determine the sum or difference in the ones place by using a known number fact, this strategy does allow a child to use counting or heuristics to determine the sum or difference.

A variation known as an additive algorithm may occur in connection with a subtraction problem. For example, if the problem is a missing addend one, a child might write something akin to the following $\begin{array}{r}16\\ \underline{+}\\ 29\end{array}$ and proceed in an additive manner by asking him/herself, "Six plus how many is nine?" and after determining the response of "three" write the numeral 3 in the appropriate position below the numeral 6 of the upper number 16. In a similar fashion the number 1 is determined to be the appropriate one to place in front of the numeral 3 to give the final correct answer of 13.

Guess. Although this strategy is generally inappropriate, this problem solving process does represent the way in which some children attempt to determine an answer. A guess is simply that - volunteering some number which may or may not be close to the actual answer. Ordinarily, the child responds quickly without much evidence of thought or internal processing.

Given number. Again, this type of response is inappropriate. In this behavior, the child simply volunteers one of the two numbers contained in the problem as being the solution to the problem. Such a response is generally taken as an indicator that the child did not understand the problem posed.

Wrong operation. This behavior occurs when a child performs the operation of addition on the two given numbers when subtraction should have been used, or vice versa. Although clearly an inappropriate response, it does represent the child's way of attempting to solve the problem.

Error

The last three strategies described - guess, given number, and wrong operation - can be considered errors in the sense that their use invariably leads to the wrong solution. However, they are the child's way of attempting to solve the given problem. Another type of error is seen when a child chooses an appropriate strategy and still produces an incorrect answer. The error occurs in the incorrect execution of an appropriate strategy. This section characterizes this second type of error.

Miscount. This error occurs when a child fails to count correctly, either when using direct models such as cubes, fingers, or tally marks or when using the more sophisticated use of counting sequences, such as counting on, counting down, or counting up from given. When model sets are used, the miscount occurs when a child counts one or more elements of the model set twice, or fails to count one or more of those elements. When counting sequences are used, the miscount occurs if one of the numbers in the sequence is omitted, or most commonly, when the entry number corresponding to a given number of the problem is counted as one of the uttered words. In this latter case, the answer given is one shy of the actual number that should be recited as the answer.

Forgets data. In solving the problem, the child may forget some part of the given information (usually one of the given numbers) and carry out an operation or process on an incorrect number.

Wrong sentence. When representing the problem with a symbolic sentence, the child writes an inappropriate sentence. Ordinarily this will be evidenced by writing an addition sentence for a subtraction problem or vice versa.

Representational error. When symbolically representing a problem, a child may write a sentence, either correct or incorrect. However, in writing the 2-digit numbers, the child misrepresents one or both of the given numbers by transposing the digits of a number. For example, the symbol for "18" is written as 81.

Basic fact error. In the recollection of a basic fact, the child produces the wrong answer, such as, "I know that five plus eight is 14." This error can also occur in connection with the use of the computational algorithm where the child needs to recall a basic fact in order to determine the digit to put in the ones place of the answer.

Computational error. This error is a misapplication of a 2-digit computational algorithm or the correct application of an incorrect or "buggy" (cf. Brown & Burton, 1978) algorithm. This is best described by several examples. For addition suppose the child writes $\begin{array}{r}17\\ +14\\ \hline 211\end{array}$. Presumably, the 7 and 4 were added to produce the 11 and then the 1 and 1 were added to get the 2. The defect in this algorithm is the failure to regroup and to consider the place value of the digits written in the answer. Another familiar "buggy" algorithm for subtraction occurs in $\begin{array}{r}36\\ -19\\ \hline 23\end{array}$ where the child follows rule of "take the smaller from the larger" and subtracts 6 from 9 and 1 from 3.

Wrong analysis. If a child opts to use the part-part-whole organizing box (described earlier in the section Models), he or she may err by putting the given numbers in the wrong position in the box which, if followed to its logical conclusion, leads to the use of the wrong operation.

Conclusion

This paper has set forth the basic conditions involved in the individual interviews that are part of the basic research into children's learning of addition and subtraction that is being carried out by the Mathematics Work Group of the Wisconsin Research and Development Center. The paper does not set forth any results from the interviews, which are being conducted three times a year during the school years 1978-79, 1979-80, and 1980-81. Those results are presented in reports prepared for each interview. While giving the definitions and descriptions of student problem solving behaviors as well as some of the general protocols, this paper should not be considered as a manual for conducting a similar interview. A separate report (Martin & Moser, 1980) exists for that purpose.

References

Brown, J. & Burton, R. Diagnostic models for procedural buts in basic mathematical skills. Cognitive Science, 1978, 2, 155-192.

Carpenter, T. & Moser, J. An investigation of the learning of addition and subtraction. (Theoretical Paper No. 79) Madison: Wisconsin Research and Development Center for Individualized Schooling, November 1979.

Carpenter, T., Hiebert, J. & Moser, J. First-grade children's initial solution processes for simple addition and subtraction problems. Journal for Research in Mathematics Education, in press.

Carpenter, T.P. The effect of initial instruction on first-grade children's solution processes for basic addition and subtraction problems. Paper presented at the annual meeting of the American Educational Research Association, Boston, April 1980.

Groen, G. & Parkman, J. A chronometric analysis of simple addition. Psychological Review, 1972, 79, 329-343.

Kouba V. & Moser, J. Results from the first individual interview (October 1978), coordinated study #1. (Working Paper No. 281) Madison: Wisconsin Research and Development Center for Individualized Schooling, 1980.

Martin, C. & Moser, J. Interview training procedures (Working Paper) Madison: Wisconsin Research and Development Center for Individualized Schooling, 1980.

Moser, J. Young children's representations of addition and subtraction problems. (Theoretical Paper No. 74) Madison: Wisconsin Research and Development Center for Individualized Schooling, November 1979.

Center Planning and Policy Committee

Richard A. Rossmiller
Center Director

Dale D. Johnson
Area Chairperson
Studies in Language:
Reading and Communication

Marvin J. Fruth
Area Chairperson
Studies in Implementation
of Individualized Schooling

Penelope L. Peterson
Area Chairperson
Studies of Instructional Programming
for the Individual Student

James M. Lipham
Area Chairperson
Studies of Administration and
Organization for Instruction

Thomas A. Romberg
Area Chairperson
Studies in Mathematics and Evaluation
of Practices in Individualized Schooling

Associated Faculty

Vernon L. Allen
Professor
Psychology

B. Dean Bowles
Professor
Educational Administration

Thomas P. Carpenter
Associate Professor
Curriculum and Instruction

W. Patrick Dickson
Assistant Professor
Child and Family Studies

Lloyd E. Frohreich
Associate Professor
Educational Administration

Marvin J. Fruth
Professor
Educational Administration

Dale D. Johnson
Professor
Curriculum and Instruction

Herbert J. Klausmeier
V.A.C. Henmon Professor
Educational Psychology

Joel R. Levin
Professor
Educational Psychology

James M. Lipham
Professor
Educational Administration

Dominic W. Massaro
Professor
Psychology

Donald M. McIsaac
Professor
Educational Administration

Wayne Otto
Professor
Curriculum and Instruction

Penelope L. Peterson
Assistant Professor
Educational Psychology

Thomas S. Popkewitz
Professor
Curriculum and Instruction

Gary G. Price
Assistant Professor
Curriculum and Instruction

W. Charles Read
Professor
English and Linguistics

Thomas A. Romberg
Professor
Curriculum and Instruction

Richard A. Rossmiller
Professor
Educational Administration

Peter A. Schreiber
Associate Professor
English and Linguistics

B. Robert Tabachnick
Professor
Curriculum and Instruction

Gary G. Wehlage
Professor
Curriculum and Instruction

Louise Cherry Wilkinson
Associate Professor
Educational Psychology

Steven R. Yussen
Professor
Educational Psychology

11/79