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ABSTRACT

Multivariate methods were used to identify between-set factors relating the criterion set of eleven Wechsler Adult Intelligence Scale Revised subtest variables to the predictor set of demographic variables: age, race, sex, education, occupation, geographic region, and urban versus rural residence. Although factor analysis is usually used to identify within-set factors, results from this method were used in comparison to those from canonical correlation, or multivariate analysis. Canonical correlation analysis was used with applications of three different orthogonal (varimax criterion) rotation strategies: (1) simultaneous, symmetrical, and joint rotation of two sets of related factors; (2) an asymmetrical rotation of each set's correlations to its canonical variates; and (3) simultaneous, symmetrical rotation of a single composite set of factors. The results from the factor analysis, the symmetrical rotation, and the asymmetrical rotation were not as helpful as the third strategy for identifying and interpreting between set factors. References and tables are appended. (Author/GDC)

Comparison of Multivariate Methods

To Identify Between Set Factors

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Running head: MULTIVARIATE COMPARISONS

Abstract

Multivariate methods were used to identify between set factors re.ating the criterion set of 11 WAIS-R subtest variables . the predictor set of demographic variables. Although factor analysis is usually used to identify within set factors, results from this method were used in comparison to those from canonical correlation analysis in response to Thorndike's (1976) criticisms regarding rotation of canonical components. Canonical correlation analysis was used with applications of three orthogonal (varimax criterion) tation strategies to enhance interpretability. The nree rotation strategies were: (a) simultaneous, symmetrical, and joint rotation of two sets of related factors (Cliff & Krus, 1976), (b) an asymmetrical rotation of each set's correlations to its canon cal variates (Huba & Bentler, 1982), and (c) simu'taneous, symmetrical rotation of a single composite set of factors (Chastain & Joe, 1985). The results from the factor analysis, the symmetrical rotation, and the asymmetrical rotation were not as helpful as the Joe and Chastain procedure (Joe, Chastain, & Simpson, 1984) for identifying and interpreting between set factors. These results were intuitively and empirically sound and were produced by a major, extant, widely used statistical package (SAS, 1985).

Analyses that employ canonical variates as vehicles for interpretation have proven to be practically indispensable in multivariate techniques, especially with their inclusions in major statistical packages (Krus, Reynolds, & Krus, 1976). The interpretation of canonical variates has been readily grasped in multiple discriminant analysis and multivariate analysis of variance (Hall, 1969), but difficulties have been encountered in trying to interpret the canonical results from canonical correlation analysis (Thompson, 1980; Thompson, 1984). These difficulties seem unusual when viewed from the perspectives of Baggaley, Knapp, and Fornell (cited in Thompson, 1984). Baggaley noted that canonical correlation is the most general case of the general linear model. Knapp stated that virtually all parametric techniques are special cases of canonical correlation analysis. Fornell noted that multiple regression, analysis of variance (ANOVA), and MANOVA were special cases of canonical analysis. Harris (1975) noted both the generalization and the specialization of canonical analysis. Table 1 shows the relationship of canonical correlation analysis (CCA) to various methods. The appropriate canonical correlation analysis, according to the number and type of variables in each set, is analogous to its alias, alternate parametric technique. These generalizations of canonical correlation analysis makes it both powerful and complex.

Canonical correlation analysis (CCA) focuses on the correlations (linear association) between a linear combination of variables in one set (SI) and a linear combination of variables in another set (SI*). CCA determines a series of maximum linear relationships between these two sets of variables by resolving the two sets of variables into two sets of variates (CVI and CVI*). Thus, the canonical variates are linear composites of the original variables and are selected so that the paired variates are correlated maximally: (a) the first canonical correlation (CC1) is the maximum correlation between the frist pair of canonical variates (CVI1 and CVI*1), (b) the second canonical carrelation (CC2) is the largest possible correlation between the second pair of canonical variates (CVI2 and CVI*2), and (c) so on. Except for the paired variates, variates must be uncorrelated within and between sets (i.e., r=0 for CVI1 and CVI2; r=0 for CVI*1 and CVI*2; and r=0 for CVI1 and CVI*2).

In addition to the canonical correlations, there are two sets of canonical weights (function coefficients), two sets of intraset canonical loadings (structure coefficients), and two sets of interset canonical loadings (index coefficients). The index coefficients represent the correlations between the canonical variates and the variables of the other set (e.g., correlations between CVI and SI* or correlations between CVI* and SI). The average squared index coefficient is a redundancy coefficient (Rd) and is an index of the average proportion of variance in the variables in one set that is reproducible from the variables in the other set. The canonical function coeficients are analogous to regression b weights (raw canonical coefficients) or beta weights (standardized canonical coefficients) and can be used to compute canonical variate scores. Canonical structure coefficients are analogous to factor structure coefficients and a squared canonical structure coefficient represents the proportion of variance linearly shared by a variable with the variables' canonical composite. The use of canonical function structure coefficients should depend on the purpose(s) represented by the research question(s).

The purpose of this paper was to foster greater use of canonical correlation analysis in describing the relationships between two sets of variables. Just as rotation of factor loadings or principal components can be used to help interpret relationshps within a single set of variables, so also rotation of the canonical structure coefficients can be used to help interpret relationships between two sets of variables. The interpretability of multidimensional relationships was investigated by a comparison of multivariate methods to identify between set factors.

Method

Although factor analysis is usually used to determine within set variable structure, it was also used as a comparative method in this paper because of Thorndike's (1976) criticism of varimax rotation applied to canonical correlation. It was Thorndike's contentions that: (a) this application of rotation violates the fundamental logic of a canonical analysis to keep the two sets of variables separate, and (b): if the structure of the combined sets is of interest, then a traditional factor analysis should be performed. A factor analysis was performed with rotation of the resulting factors to Kaiser's (1958) varimax criterion to determine the validity of this criticism.

A canonical correlation analysis was performed with three rotation schemes. The factors were rotated to Kaiser's (1958) varimax criterion and the resulting solutions were compared to determine their utility, and, in some sense, the "best" method for identifying multidimensional relationships between two sets of variables.

Data Source and Variables

The 1981 Wechsler Adult Intelligence Scale-Revised (WAIS-R) standardization sample (N=1880) provided the necessary variables for investigation. A subset (N=1856) was chosen deleting 24 nonwhite adults who were classified as "other" (neither white nor black) since these were too few in number relative to whites (N=1664) and blacks (N=192). The total WAIS-R standardization sample represents national random stratified selections of subjects based on seven demographic variables. These variables were age, race, gender, education, occupation, region of residence, and urban versus rural residence. Further descriptions are provided elsewhere (Chastain & Reynolds, 1985; Wechsler, 1981).

Two sets of variables were formed. The first set consisted of the 11 WAIS-R subtests used to compute Full Scale IQ: Information, Digit Span, Vocabulary, Arithmetic Comprehension, Similarities, Picture Completion, Picture Arrangement, Block Design, Object Assembly, and Digit Symbol. These variables were measured in standard scores scaled to means of 10 and standard deviations of three. The second set of variables consisted of 11 demographic variables; the seven previously mentioned demographic variables and four additional variables of marital status, origin of birth, handedness, and birth order. Many of the 11 demographic variables have been studied and shown to be associated with IQ (Barona & Chastain, 1986; Barona, Reynolds, & Chastain, 1984).

Age and education were measured in years while birth order was measured on a scale of 1 to 9 (no one in this sample had more than 8 older siblings). Gender, race, urban-rural residence, handedness, and place of birth were dichotomously scored one or zero; a value of one was equivalent to male, white, urban residence, right handedness, and born in the U.S., respectively. Occupation, region of residence, and marital status were "dummy" coded. One category from each of these three variables was deleted from the analysis since this information was redundant and would preclude analysis. Occupation category number six, not in the labor force; region of residence category number three, the South region; and marital status category number four, widowed, were deleted from the analyses. The effect of dummy coding left 19 demographic indicators for the 11 original demographic variables.

Procedures

Three factor analyses were performed using a principal factor analysis procedure with varimax rotation applied to: (a) all WAIS-R variables of interest, (b) the 11 WAIS-R subtests, and (c) the 19 demographic indicators.

A canonical correlation analysis was performed on the two sets of variables. As noted by Van de Geer (1971), canonical correlation analysis as a generalized path analysis is analogous to a double factor analysis. Thus, three different rotation strategies were employed to determine an "optimal" rotation strategy.

The first rotation strategy followed a procedure originally recommended by Cliff and Krus (1976). The canonical coefficients for both sets of variables were simultaneously rotated to an orthogonal solution.

The second rotation strategy was an asymmetric rotation of the canonical structure coefficients (Huba & Bentler, 1981). In an asymmetric rotation, each unrotated loading matrix is rotated separately. Huba, Palisoc, and Bentler (1982) have developed a program to perform both symmetric and asymmetric rotation to an orthogonal criterion similar to Kaiser's (1958) varimax (orthsim; Bentler, 1977). Also, the relative influence of one set of variables (e.g., the X predictor variables influence on the Y criterion variables) may be manipulated by an α parameter. This parameter in a symmetric rotation is $\alpha = .5$. That is, both sets of variables influence the other equally. The α parameter could be chosen to represent the proportion of variables in the smaller of the two sets of variables so that each variable has the same relative influence. In the present investigation, $\alpha = .5$ since there are 11 criterion variables and 11 demographic variables (represented by 19 indicators).

The third rotation strategy used a procedure that correlated all of the variables in both sets with linear composites (Joe, Chastain, & Simpson, 1984). The linear composites were determined by the addition of the corresponding canonical variates which were the optimally weighted composite variates from each set. Each of these summed linear composites was correlated with all of the original variables from both sets of variables to provide the canonical structure of the between sets association. This structure was subjected to an orthogonal rotation to the varimax criterion.

Results

Factor Analyses

Table 2 shows the intercorrelation matrix of all the variables from both sets of variables. This matrix was subjected to a maximum likelihood factor analysis. The resulting factors were then rotated to the varimax criterion. The test of HO: 7 were factors were sufficient vs. HA: more factors were needed was rejected by a chance probability of the observed $X^2 < .0001$. This was evidenced by eight eigenvalues greater than one. The first seven squared canonical correlations reported by this maximum likelihood procedure (convergence criterion satisfied with 12 iterations) were 1.00, 1.00, 1.00, 1.00, 0.95, 0.81, and 0.58. Only two factors could be identified as factors that were represented by variables from both sets ("between" set factors). The first factor identified a general factor in the sense that all 11 WAIS-R subtests had relatively high loadings on this factor (.48 to .92). Three demographic variables were related to this factor; education (.66), professional occupation (.31), and race (.29). The second factor identified a performance factor since WAIS-R subtests 7 through 11 define Performance IQ (-.33 to -.50). The demographic variables related to this factor were age (.86), and single marital status (-.49). The other factors in this solution were represented by only one or two variables from one of the original sets (e.g., factor 3 was represented by marital status, factor 4 was represented by region of residence, etc.).

The factor analysis of the 11 WAIS-R subtests, separately, revealed only two factors with eigenvalues above one. These factors were rotated to the varimax criterion. The two resulting factors were readily identified as verbal and performance factors. Factor one was defined by the first six WAIS-R subtests (these are used to compute Verbal IQ). Factor two was defined by the last five WAIS-R subtests (these are used to compute Performance IQ).

The factor analysis of the 11 demographic variables, separately, revealed 10 factors with eigenvalues over one. The orthogonal rotation results indicated relatively little multicollinearity since each factor was primarily defined by one variable. Only the first rotated factor accounted for over 10 percent of the variance (12 percent). The first rotated factor was the only factor defined by more than one variable: age (.77) and marital status (married, -.94; divorced, .80).

Canonical Correlation Analysis

The results of the canonical correlation analysis using the 11 WAIS-R subtests as the SI variables and the 19 demographic indicators (determined from the 11 demographic variables) as the SI* variables revealed 6 significant ($p < .05$) canonical correlations. These 6 canonical correlations with resulting F statistics and probabilities were: (1) .720 with $F = 16.429$ ($p = .0001$),

(2) .696 with F = 10.608 (p = .0001), (3) .397 with F = 3.956 (p = .0001), (" .228 with F = 2.216 (p = .0001), (5) .186 with F = 1.761 (p = .0001), and (6) .150 with F = 1.428 (p = .0064). The last 2 squared canonical correlations were only .03 and .02, respectively. This indicated that the first 3 canonical correlations might be the most meaningful. The squared canonical correlations for the first four canonical correlations were .52, .48, .16, and .05, respectively. There was a large decrease in R-square from the second to the third and from the third to the fourth squared canonical correlations. The multivariate test statistics yielded F (209, 17470) = 16.429, p = .0001, and Wilks' Lambda Λ = .18 with a total R-square between set var.a ion of .82.

Table 3 shows the results of the redundancy analysis. These results also seem to indicate that the first three canonical variates account for the most variance; .52, .48, .16 squared canonical correlations, respectively.

The unrotated standardized canonical structure coefficients are reproduced in Table 4. The first column of loadings revealed a general factor identified by all 11 WAIS-R subtests (loadings ranged from .57 to .90) and many of the demographic variables. The demographic variables were: (a) education (.80), (b) age (-.64), (c) birth order (-.27), (d) professional occupation (.37), (e) managerial occupation (.32), (f) race (.31), and (g) single marital status (.28).

The structural canonical coefficients corresponding to significant canonical correlations were rotated to the varimax criterion to assist in identifying variables that were related between sets from the canonical correlation analysis. Since the initial analysis showed a maximum of six dimensions accounting for the interrelationships between the 11 subtests and 19 demographic indicators, the structures for the six, five, four, three, and two dimensional solutions were each rotated separately. The purpose of this strategy was to identify solutions in which variables from both sets of variables loaded on each dimension. Because the three dimensional solution was deemed most satisfactory elsewhere (Chastain & Joe, 1985) and the other solutions did not aid interpretability, only the three dimensional rotated solutions will be presented for the rotation strategies in this paper.

Simultaneous Symmetric Rotations. Table 5 shows the rotated three dimensional solution. This solution has moved the loadings in the matrix closer to simple structure, but ease of interpretation was not enhanced.

Sixteen variables or indicators had loadings above .3 on the first rotated factor. The first rotated factor again represented a general intellignece factor related most highly to education (.94). The high loadings for education on factor one (.94), for age (.94) on factor two, and for sex (.85) on factor three do not seem either intuitively or empirically likely. The highest bivariate correlations between any one of the 11 subtests and demographic variables were: (a) education = .591, (b) age = -.510, and (c) sex = .161.

Asymmetric Rotations. Tables 6 and 7 contain the rotated factor patterns for the 11 subtests and 19 demographic indicators, respectively. Each set was rotated separately which allowed an asymmetric rather than symmetric rotation since the rotations were independent. The results for the 11 subtests were very close to the results of the factor analysis of the 11 subtests (c.f. pages 10-11). A two-dimensional solution seemed most appropriate for this set of variables indicative of the verbal and performance factors found previously.

This result indicates a problem with separate rotations. The number of within set factors may be different in the two sets (c.f. pages 10-11), but the transformation matrices used in canonical correlation analysis must be of the same rank. Asymmetric rotation may help in interpretation, but may also be conceptually unsound. Whether the rotation strategy is symmetric or asymmetric, the problem of separate rotations and the number of factors rotated has not been addressed previously.

The demographic variable loadings were very similar to those loadings when rotated simultaneously. Some of the loadings were: (a) education on factor one = .94, (b) age on factor two = .94, and (c) gender on factor three = .86. The other loadings were similar in magnitude and in direction to the symmetric rotation solution in Table 5. The most dramatic differences from the results in Table 5 and the results from the separate rotations were found among the subtests. There were no negative loadings in the separate rotation since all of the subtests were positively related to each other. The separate roatations did not help interpretability in any meaningful way.

Symmetric Rotation of Additive Composites. Table 8 contains the rotated loadings for each variable on the first three linear composites. The first rotated canonical factor was defined as a general intelligence factor with loadings on the 11 subtests ranging from .42 to .80. There were three demographic variables that had loadings of .35 or above on this general factor: education (.86), professional occupation (.37), and race (.35). The second rotated canonical factor was interpreted as an age-relatd performance factor since age (.-88), single marital status (.52), and all five performance subtests (.36 to .70) were related to this factor. The third rotated canonical factor was interpreted as a manual dexterity factor or a gender-specific factor. This factor was defined by the performance subtest of Block Design (.49), gender (.72), and skilled worker occupation (.38).

Discussion

There are distinct advantages to using canonical correlation analysis with rotation over traditional factor analysis procedures of the objective is to identify between set factors. The Joe and Chastain procedure may not have demonstrated a superiority over other rotation strategies, but their canonical factors seemed intuitively and empirically better than results from other canonical rotation strategies and factor analyses in this instance.

There were, however, many issues not addressed in this comparison. Carlson noted (in Thompson, 1984) that rotation destroys the biorthogonal property of the variates to be interpreted. Therefore, there is a need to assess: (a) the degree to which the canonicals are changed to quasi-canonicals, (b) the altered canonical transformation, (c) recomputed redundancies (Reynolds & Jackosfsky, 1981), (d) cross-validation of results, and (e) invariance estimates.

Another limitation of this paper concerns the work of Huba and Bentler (1981) and the ORSIM2 program developed by Huba, Palisoc, & Bentler (1982). This paper, by necessity, addressed only one aspect of their work. The ORSIM2 program has the advantages of greater flexibility and power by applying four rotations for each problem: (a) the simplicity criterion is maximized for the two sets by proportional weighting (to the number of variables in each set), (b) the simplicity criterion is maximized for the first set only, (c) the simplicity criterion is maximized for the second set only, and (d) each set is rotated separately with different simplicity criteria and an asymmetic adjustment of the canonical correlations. The drawbacks to using ORIM2 are: (a) user must compile the FORTRAN program onto system, (b) user needs to input two matrices of canonical correlation loadings (or weights) and a vector of canonical correlations, and (c) user must have a high level of expertise to specify parameters (e.g., α) and to interpret results.

The canonical correlation results for this paper were computed using the SAS (1985) procedures CANCORR and FACTOR. These procedures allowed separate or simultaneous rotations of raw canonical coefficients, standardized canonical coefficients (scoring coefficients), canonical structure coefficients, and the created linear composites. The FACTOR procedure in SAS can rotate the input matrices with many rotation strategies (such as varimax, quartimax, promax, etc.) and can provide plots (skree, factor by factor, etc.). Beyond the obvious advantages connected with using heavily supported software (i.e., algorithms, ease of use, programmability, documentation, etc.), the Joe and Chastain procedure is easier to grasp conceptually because it involves the rotation of a single set of factors rather than the joint rotation of two sets of related factors.

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Table 1

Canonical Correlation As General Linear Model

Variables(SI) Set One(Y)		Variables(SI*) Set Two(X)		C C A
Number	Type	Number	Type	Aliases
1	random	1	random	Pearson r
1	random	≥ 2	random	Multiple R
≥ 2	random	≥ 2	random	Multivariate Mult. R
1	random	≥ 1	dummy	ANOVA
≥ 2	random	≥ 1	dummy	MANOVA
1	random	≥ 2	both	ANCOVA
≥ 2	random	≥ 2	both	MANCOVA
≥ 1	dummy	≥ 1	random	DISCRIMINANT

Table 2

Lower Diagonal Intercorrelation Matrix

	VARIABLE	1	2	3	4	5	6	7	8	9	10
1.	SS1	1.000									
2.	SS2	0.466	1.000								
3.	SS3	0.817	0.519	1.000							
4.	SS4	0.628	0.563	0.642	1.000						
5.	SS5	0.691	0.457	0.754	0.582	1.000					
6.	SS6	0.658	0.468	0.700	0.565	0.669	1.000				
7.	SS7	0.499	0.401	0.515	0.478	0.501	0.572	1.000			
8.	SS8	0.470	0.407	0.473	0.455	0.468	0.531	0.590	1.000		
9.	SS9	0.467	0.445	0.467	0.550	0.447	0.540	0.603	0.553	1.000	
10.	SS10	0.384	0.357	0.386	0.429	0.386	0.471	0.590	0.488	0.691	1.000
11.	SS11	0.352	0.420	0.367	0.407	0.374	0.478	0.507	0.505	0.555	0.468
12.	CED	0.572	0.430	0.591	0.513	0.545	0.541	0.413	0.402	0.427	0.342
13.	CAGE	0.061	-0.121	0.082	-0.019	0.028	-0.164	-0.320	-0.360	-0.359	-0.333
14.	BIRTH	-0.185	-0.079	-0.184	-0.145	-0.130	-0.177	-0.146	-0.130	-0.158	-0.119
15.	RSEX	0.135	-0.018	0.022	0.161	0.043	0.002	0.076	[illegible]	0.120	0.049
16.	RRURAL	0.073	0.074	0.083	0.042	0.024	0.039	0.030	0.020	-0.008	-0.014
17.	OCC1	0.252	0.175	0.252	0.192	0.225	0.217	0.185	0.193	0.197	0.149
18.	OCC2	0.108	0.173	0.122	0.176	0.143	0.152	0.133	0.131	0.146	0.107
19.	OCC3	-0.026	-0.027	-0.062	0.003	-0.032	-0.018	0.071	0.082	0.124	0.097
20.	OCC4	-0.215	-0.139	-0.251	-0.174	-0.213	-0.168	-0.071	-0.043	-0.131	-0.082
21.	OCC5	-0.128	-0.104	-0.145	-0.102	-0.131	-0.105	-0.042	-0.067	-0.051	-0.048
22.	RRACE	0.232	0.150	0.262	0.245	0.239	0.205	0.195	0.135	0.268	0.196
23.	REG1	0.065	0.095	0.048	0.078	0.024	0.017	0.011	0.053	0.039	-0.026
24.	REG2	-0.070	-0.022	-0.051	-0.017	-0.050	-0.035	-0.019	-0.042	0.037	0.010
25.	REG4	0.052	-0.001	0.063	0.023	0.062	0.057	0.099	0.041	0.082	0.099
26.	RHAND	-0.009	0.012	0.001	-0.008	-0.002	-0.032	-0.026	-0.044	-0.029	-0.013
27.	USA	0.021	0.024	0.024	0.030	0.027	0.071	0.090	0.107	0.069	0.105
28.	STATUS1	-0.111	0.006	-0.146	-0.069	-0.124	0.032	0.119	0.129	0.161	0.134
29.	STATUS2	0.172	0.051	0.186	0.145	0.184	0.065	0.017	0.006	-0.003	0.005
30.	STATUS3	-0.032	-0.029	-0.001	-0.033	-0.026	-0.037	-0.034	-0.034	-0.057	-0.079

Table 2 (Continued)

	VARIABLE	11	12	13	14	15	16	17	18	19	20
11.	SS11	1.000									
12.	CED	0.451	1.000								
13.	CAGE	-0.510	-0.161	1.000							
14.	BIRTH	-0.158	-0.200	0.109	1.000						
15.	RSEX	-0.137	0.010	-0.001	-0.024	1.000					
16.	RRURAL	0.001	0.081	0.030	-0.023	-0.057	1.000				
17.	OCC1	0.222	0.316	-0.132	-0.067	0.043	0.040	1.000			
18.	OCC2	0.230	0.168	-0.145	-0.067	-0.007	0.046	-0.185	1.000		
19.	OCC3	0.011	-0.045	-0.148	-0.045	0.196	-0.068	-0.124	-0.189	1.000	
20.	OCC4	-0.062	-0.211	-0.140	0.116	0.093	-0.053	-0.182	-0.276	-0.186	1.000
21.	OCC5	-0.056	-0.108	-0.081	-0.003	0.107	0.009	-0.067	-0.103	-0.069	-0.101
22.	RRACE	0.136	0.163	0.044	-0.102	0.011	-0.080	0.061	0.056	0.071	-0.116
23.	REG1	0.048	0.069	0.046	-0.042	0.023	0.085	0.004	0.027	-0.015	-0.029
24.	REG2	0.056	-0.046	-0.014	0.002	-0.040	0.004	-0.008	0.002	0.007	-0.008
25.	REG4	-0.026	0.019	-0.015	0.028	0.014	0.078	-0.013	0.011	0.012	-0.003
26.	RHAND	0.019	-0.024	0.066	-0.011	-0.032	-0.056	0.001	-0.020	0.017	-0.029
27.	USA	0.122	0.109	-0.137	0.006	0.019	-0.045	0.031	0.054	0.041	0.006
28.	STATUS1	0.257	0.013	-0.627	-0.063	0.044	0.012	0.079	0.089	0.116	0.083
29.	STATUS2	-0.075	0.123	0.336	0.002	0.095	-0.057	-0.015	-0.026	-0.038	-0.071
30.	STATUS3	-0.061	-0.056	0.075	0.030	-0.054	0.049	-0.031	-0.014	-0.028	0.050

	VARIABLE	21	22	23	24	25	26	27	28	29	30
21.	OCC5	1.000									
22.	RRACE	-0.086	1.000								
23.	REG1	0.022	0.007	1.000							
24.	REG2	-0.031	0.099	-0.344	1.000						
25.	REG4	0.008	0.103	-0.268	-0.280	1.000					
26.	RHAND	0.010	0.042	[illegible]	-0.021	-0.030	1.000				
27.	USA	-0.002	-0.045	-0.046	-0.013	-0.027	-0.020	1.000			
28.	STATUS1	0.076	-0.063	0.008	0.014	-0.020	-0.063	0.082	1.000		
29.	STATUS2	-0.053	0.106	0.045	-0.004	-0.000	0.045	-0.049	-0.726	1.000	
30.	STATUS3	0.001	-0.074	-0.085	0.024	0.041	0.009	-0.012	-0.213	-0.270	1.000

Table 3

Canonical Redundancy Analysis

Standardized Variance Explained

By Own Canonical Variables

	WAIS-R Subtests			Demographic Variables	
CV	Proportion	Cummulative Proportion	Canonical R-Square	Proportion	Cummulative Proportion
1	.46	.46	.52	.09	.09
2	.17	.63	.48	.08	.17
3	.06	.69	.16	.06	.23
4	.05	.74	.05	.05	.28
5	.05	.79	.03	.05	.33
6	.03	.82	.02	.06	.39
7	.04	.86	.02	.05	.44
8	.03	.89	.01	.04	.48
9	.04	.93	.01	.05	.53
10	.04	.97	.00	.05	.58
11	.03	1.00	.00	.05	.63

Table 4

Unrotated Canonical Structure Coefficients

Original Variable	Canonical Variates CVI1	CVI2	CVI3	CVI4	CVI5	CVI6
SS1	58 *	69 *	12	08	-22	-22
SS2	57 *	27	-12	30	15	19
SS3	60 *	71 *	-13	-20	-10	-09
SS4	60 *	53 *	25	29	16	14
SS5	59 *	59 *	-05	-08	-13	41 *
SS6	72 *	35 *	-06	-14	-26	-06
SS7	71 *	05	29	-12	-26	03
SS8	72 *	-02	24	18	-42 *	04
SS9	78 *	05	48 *	-24	27	00
SS10	65 *	-04	31	-36 *	-19	29
SS11	90 *	-22	-24	10	13	-04
	CVI*1	CVI*2	CVI*3	CVI*4	CVI*5	CVI*6
EDUCATION	80 *	49 *	-12	09	-14	-03
AGE	-64 *	72 *	-15	-01	11	09
BIRTH	-27	-12	-03	16	08	55 *
GENDER	-05	21	87 *	31	-07	-09
RURAL	03	11	-09	12	-10	-36 *
OCC1	37 *	17	-06	01	-08	-21
OCC2	32	01	-13	33	32	29
OCC3	06	-14	46 *	-12	-10	10
OCC4	-17	-34	07	30	-32	-03
OCC5	-12	-17	12	-03	07	-13
RACE	31	26	25	-42 *	39 *	17
REG1	08	06	01	46 *	27	-27
REG2	03	-12	-03	-14	54 *	-01
REG4	04	06	23	-49 *	-26	17
HAND	04	-03	08	02	-13	-12
USA	17	-11	-01	-05	-32	05
STATUS1	28	-51 *	09	01	11	-43 *
STATUS2	-02	39 *	03	-02	-12	41 *
STATUS3	-09	04	-07	-02	04	-15

NOTE: All loadings multiplied by 100, * denotes ≥ 35

Table 5

Rotated Symmetric Solution

Original Variable	Canonical Variates CVI1	CVI2	CVI3	
SS1	86 *	19	21	INFORMATION
SS2	63 *	-15	-04	DIGIT SPAN
SS3	92 *	17	-03	VOCABULARY
SS4	76 *	07	34 *	ARITHMETIC
SS5	83 *	10	04	COMPREHENSION
SS6	78 *	-18	05	SIMILARITIES
SS7	55 *	-37 *	39 *	PICTURE COMPLETION
SS8	51 *	-44 *	34	PICTURE ARRANGEMENT
SS9	57 *	-40 *	59 *	BLOCK DESIGN
SS10	44 *	-75 *	-11	OBJECT ASSEMBLY
SS11	59 *	-11	00	DIGIT SYMBOL
	CVI*1	CVI*2	CVI*3	
CED	94 *	-11	00	EDUCATION
CAGE	-03	94 *	-24	AGE
BIRTH	-27	07	-07	BIRTH ORDER
RSEX	-01	27	85 *	GENDER
RRURAL	10	06	-09	URBAN VS RURAL RES.
OCC1	40 *	-10	00	PROFESSIONAL
OCC2	27	-20	-08	MANAGERS
OCC3	-10	-11	46 *	SKILLED
OCC4	-35 *	-15	04	SEMI-SKILLED
OCC5	-22	-05	10	UNSKILLED
RRACE	37 *	03	29	RACIAL GROUP
REG1	10	00	02	NORTHEAST
REG2	-05	-11	-02	NORTH CENTRAL
REG4	04	04	23	WEST
RHAND	00	-04	08	RIGHT HANDEDNESS
USA	06	-19	01	U.S. BIRTHPLACE
STATUS1	-11	-57 *	12	SINGLE
STATUS2	22	32	03	MARRIED
STATUS3	-04	08	-08	DIVORCED

NOTE: All loadings multiplied by 100, * denotes $\geq$ 35

Table 6

Separate Rotation For WAIS-R Subtests

Original Variable	Canonical Variates CVI1	CVI2	CVI3	
SS1	85 *	32	03	INFORMATION
SS2	49 *	23	35 *	DIGIT SPAN
SS3	90 *	15	21	VCCABULARY
SS4	70 *	46 *	01	ARITHMETIC
SS5	79 *	23	19	COMPREHENSION
SS6	61 *	36 *	37 *	SIMILARITIES
SS7	31	67 *	22	PICTURE COMPLETION
SS8	25	65 *	29	PICTURE ARRANGEMENT
SS9	31	85 *	14	BLOCK DESIGN
SS10	20	66 *	21	OBJECT ASSEMBLY
SS11	19	47 *	81 *	DIGIT SYMBOL

NOTE: All loadings multiplied by 100, * denotes $\geq$ 35

Table 7

Separate Rotation For Demographic Variables

Original Variable	Canonical Variates CVI*1	CVI*2	CVI*3	
CED	94 *	-02	01	EDUCATION
CAGE	-13	94 *	-21	AGE
BIRTH	-28	05	-07	BIRTH ORDER
RSEX	-04	24	86 *	GENDER
RRURAL	09	07	-08	URBAN VS RURAL RES.
OCC1	41 *	-06	00	PROFESSIONAL
OCC2	29	-16	-09	MANAGERS
OCC3	-09	-13	45 *	SKILLED
OCC4	-35 *	-19	04	SEMI-SKILLED
OCC5	-21	-08	10	UNSKILLED
RRACE	37 *	03	29	RACIAL GROUP
REG1	10	01	02	NORTHEAST
REG2	-04	-12	-03	NORTH CENTRAL
REG4	03	04	23	WEST
RHAND	00	-04	08	RIGHT HANDEDNESS
USA	08	-19	00	U.S. BIRTHPLACE
STATUS1	-05	-58 *	10	SINGLE
STATUS2	19	34	04	MARRIED
STATUS3	-05	08	-08	DIVORCED

NOTE: All loadings multiplied by 100, * denotes $\geq$ 35

Table 8

Rotated Loadings On Additive Components

Original Variable	Canonical Variates VW1	VW2	VW3	
SS1	80 *	-16	18	INFORMATION
SS2	58 *	15	-03	DIGIT SPAN
SS3	85 *	-15	-02	VOCABULARY
SS4	71 *	-05	28	ARITHMETIC
SS5	77 *	-08	04	COMPREHENSION
SS6	72 *	17	04	SIMILARITIES
SS7	51 *	36 *	32	PICTURE COMPLETION
SS8	48 *	42 *	28	PICTURE ARRANGEMENT
SS9	54 *	40 *	49 *	BLOCK DESIGN
SS10	42 *	39 *	32	OBJECT ASSEMBLY
SS11	54 *	70 *	-10	DIGIT SYMBOL
CED	86 *	11	00	EDUCATION
CAGE	-03	-88 *	-18	AGE
BIRTH	-25	-07	-06	BIRTH ORDER
RSEX	01	-22	72 *	GENDER
RRURAL	09	-06	-07	URBAN VS RURAL RES.
OCC1	37 *	09	00	PROFESSIONAL
OCC2	24	18	-07	MANAGERS
OCC3	-06	11	38 *	SKILLED
OCC4	-33	14	03	SEMI-SKILLED
OCC5	-20	05	08	UNSKILLED
RRACE	35 *	-02	24	RACIAL GROUP
REG1	09	00	01	NORTHEAST
REG2	-05	10	-02	NORTH CENTRAL
REG4	04	-03	20	WEST
RHAND	00	-04	-07	RIGHT HANDEDNESS
USA	05	18	00	U.S. BIRTHPLACE
STATUS1	-10	52 *	09	SINGLE
STATUS2	21	-29	03	MARRIED
STATUS3	-04	-08	-06	DIVORCED

NOTE: All loadings multiplied by 100, * denotes $\geq$ 35