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ABSTRACT

This study hypothesized that subjects who display proportional responses on the Pouring Water Task have developed the ability to comprehend logical arguments of the form referred to as "reasoning to a contradiction," while subjects who display additive responses on the same task have not. To test this hypothesis, 100 additive and proportional high school subjects (mean age 16.4 years) were administered three versions of a four-card task requiring them to reason to a contradiction before, immediately after, and one month after verbal instruction in the use of the reasoning pattern. Results were generally as predicted as most of the additive subjects failed the immediate and delayed posttest problems (62 percent and 80 percent respectively) while most of the proportional subjects succeeded (80 percent and 71 percent respectively). Group differences were significant (p less than .001). Since science is an enterprise based upon the process of falsification of hypothesis, it seems clear that this fundamental property is lost on many students as indicated by the results. Clearly, if these students are to grasp this property, a far greater effort by teachers and curriculum developers will have to be made. (Author/JN)

The Four-Card Problem Resolved?

Proportional Reasoning and Reasoning to a Contradiction

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ABSTRACT

To test the hypothesis that subjects (Ss) who display proportional responses on the Pouring Water Task have developed the ability to comprehend logical arguments of the form referred to as "reasoning to a contradiction", while Ss who display additive responses on the same task have not, 100 additive and proportional high school Ss (mean age 16.4 years) were administered three versions of a four-card task requiring them to reason to a contradiction before, immediately after, and one month after verbal instruction in use of the reasoning pattern. Results were generally as predicted as most of the additive Ss failed the immediate and delayed posttest problems (62% and 80%, respectively) while most of the proportional Ss succeeded (80% and 71%, respectively). Group differences were significant ($p < .001$) in both cases.

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Introduction

Scientific hypothesis testing takes the general form of what logicians refer to as reductio ad absurdum or reasoning to a contradiction (e.g., Ambrose and Lazerowitz, 1948). Suppose for instance that an investigator tentatively answers the question, "How do salmon navigate upstream to spawn?" by advancing the hypothesis that they navigate using their sense of sight (p). To test this hypothesis he then blind folds some salmon and deduces that they will then fail to successfully navigate upstream (if p then q or $p \supset q$). Upon finding that the blindfolding has in no way impaired the salmons' ability to navigate ($p \cdot \bar{q}$) the investigator is, therefore, able to conclude that the salmon do not navigate by using their sense of sight ($\bar{p}$). The original hypothesis has been contradicted (i.e., reduced to the absurd) hence other hypotheses must be advanced and tested.

This same pattern of reasoning is required to solve the now classic Wason Four-Card Problem (Wason and Johnson-Laird, 1972). The Wason Four-Card Problem has generated a considerable amount of interest among psychologists and science educators primarily because of its obvious relationship to scientific reasoning and to aspects of Piaget's theory of formal operational thought (e.g., Bady, 1979; Braine, 1977; Ennis, 1975; Evans, 1980; Lawson, 1983a,1983b; Moshman and Thompson, 1981). In general terms the Wason Problem is of interest because, although it is written in a context familiar to all (i.e., card, letters and numbers), and appears to require only use of this presumably general and important pattern of reasoning, practically no one answers it correctly. As

Wason and Johnson-Laird (1972) point out, "Even some professional logicians have been known to err in an embarrassing fashion, and only the rare individual takes us by surprise and gets it right. It is impossible to predict who it will be." (p. 173). Does the considerable difficulty of the Wason problem imply that people are generally incapable of logical reasoning of this form? Certainly, it would appear so. Recent studies by Lawson (1983a,1983b) suggest that many adults fail to apply logical reasoning because they assimilate the problem to a biconditional interpretation and to schemes involving probabilistic considerations and the need to control variables. This difficulty could be anticipated as the problem requires the use of reasoning patterns presumably acquired through interaction in a probabilistic world with its shades of gray to the non-probabilistic black and white context of the four-card problem. But must we, therefore, conclude that the underlying logic of the four-card problem has not been acquired by presumably rational persons as Wason and Johnson-Laird (1972), Ennis (1975), and others have? I think not.

The purpose of the present study is to test the hypothesis that the underlying logic required for solution of the four-card problem has indeed been acquired by rational adolescents. Further, the hypothesis is advanced that the adolescents who have acquired this logical reasoning pattern are precisely those who, in the Piagetian tradition, have entered the formal operational stage of intellectual development. Contrary to the Wason, Johnson-Laird conclusion that it is impossible to predict who will solve the problem correctly, the present study seeks to show that it is indeed possible - under the appropriate circumstances.

Method

To test these hypotheses a sample of 100 adolescent (14.9 years to 19.1 years X age = 16.4 years) were first divided into two groups based upon responses to a pouring water task requiring the use of proportional reasoning (presumably an indication of formal stage reasoning) (cf. Lawson, Karplus and Adi, 1978). The two groups were those that utilized an incorrect additive strategy to solve the task (presumably the non-formal, or concrete operational adolescents) and those that utilized the correct proportions stracegy (presumably the formal operational adolescents). Both groups of adolescents were then individually administered a version of the four-card problem with feedback when errors where made so that all subjects were informed of the logically correct responses (i.e., (1) turn over the E card because an odd number on the other side could falsify the rule, (2) turn over the 7 card because a vowel could falsify the rule, (3) ignore the K card because no rule is given regarding consonants, and (4) ignore the 4 card because neither a vowel nor consonant on the other side would falsify the rule).

Following the individual instruction period which lasted for approximately 5-10 minutes, all Ss were administered a logically isomorphic four-card problem within another context [i.e., Given the rule, "If a card has a triangle on one side then it has red dots on the other side", which cards, (triange, square, gre dots, red dots) should be turned over to test the rule?] Responses were scored correct only if both of the correct cards were selected (i.e., the triange and green dots cards) and logical reasons for these selections were stated. This testing was considered the immediate posttest. One month later during a delayed posttest all Ss were administered a third version of the

four-card problem (i.e., If a card has a fish on one side then it has stripes on the other side) and scored as correct or incorrect in the same manner.

Predictions

Based upon previous research and the argument that problems of the four card type are misleading due to their nonprobabilistic context, none of the Ss were predicted to correctly select the E and 7 cards on the initial testing. Based upon the hypothesis that concrete operational Ss (operationally defined as those who responded with 8 to the Pouring Water Task) have failed to acquire the mental structures necessary to assimilate short-term instruction on the logic of the four-card problem, they were predicted to fail the immediate and delayed posttest four-card problems. Some limited success due to rote application of instruction was viewed as possible on the immediate posttest but was not expected to be retained for the delayed posttest.

The formal operational Ss (operationally defined as those responding with 9 to the Pouring Water Task) were predicted to select the correct cards on the immediate and delayed posttests presumably because the mental structures which guide the logical reasoning involved in the problems have developed thus can allow assimilation and retention of the instruction.

Results

Upon initial testing only one S (a proportional S) selected the correct cards (i.e., p and $\bar{q}$). Table 1 shows the results of the immediate posttest. Of the 45 additive Ss only 17 Ss (28%) gave correct responses. Most incorrect responses consisted of selecting the q rather than the $\bar{q}$ card. However, none of the Ss failed to correctly select the p card. Of the 55 proportional Ss, 44

(80%) correctly selected only the p and $\bar{q}$ cards. Additive versus proportional group differences were highly significant ($X^2 = 22.2$, d.f. = 1, $p < .001$).

Table 2 shows the results of the delayed posttest. Of the 45 additive Ss only 9 (20%) gave correct responses. Again most of the mistakes consisted of selecting the q card. Of the 55 proportional Ss, 39 (71%) gave correct responses. Additive versus proportional group differences were again highly significant ($X^2 = 25.7$, d.f. = 1, $p < .001$). Note that for both groups of Ss a greater percentage of incorrect responses were given on the delayed posttest but that, as expected, the percentage of additive Ss who regressed was greater than the percentage of proportional Ss (18% more failures on the delayed posttest among additive Ss versus only 9% more among proportional Ss).

Discussion

Results were generally as predicted lending support to the studies' hypotheses. It would appear that most Ss identified as "formal operational" based upon proportional responses to the Pouring Water Task are able to assimilate and retain short-term verbal instruction on the logic of the four-card problem while most Ss identified as "concrete operational", based upon additive responses to the Pouring Water Task, are not. These results are interpreted to mean that the basic patterns of logical reasoning referred to as reasoning to a contradiction had "developed" in the formal Ss but not in the concrete Ss prior to instruction, thus allowing only the formal Ss to comprehend and retain the brief instruction. These results are not viewed as supportive of the view that advanced reasoning is always isomorphic with propositional logic (c.f. Inhelder and Piaget, 1958, p. 304) but rather that

some adolescents have developed mental structures that allow reasoning to be consistent with propositional logic under certain conditions. Likewise some adolescents have not developed those structures.

Implications for Science Instruction

These results indicate that some students are capable of comprehending verbal arguments of the form $p \supset q$, $p \therefore q$, $\bar{q} \therefore \overline{p \supset q}$ while others are not. Since science (as opposed to say religion) is an enterprise that is based upon the process of falsification of hypotheses in this manner, it seems clear that this fundamental property is lost on many students. Clearly if these students are to grasp this property a far greater effort by teachers and curriculum developers will have to be made.

Table 1

A Comparison of Immediate Posttest 4 Card Task Performance With Pouring Water Task Response

		Pouring Water Task Response	
		Additive	Proportional
4 Card Task Performance	Incorrect	28	11
	Correct	17	44

$\chi^2 = 22.2$, d.f. = 1, $p < .001$

Table 2

A Comparison of Delayed Posttest Task Performance With Pouring Water Task Response

		Pouring Water Task Response	
		Additive	Proportional
4 Card Task Performance	Incorrect	36	16
	Correct	9	39

$\chi^2 = 25.7$, d.f. = 1, $p<.001$

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