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ABSTRACT

This project was designed to produce a program evaluation and needs assessment of the 1982 Alberta Elementary School Mathematics Program in terms of cognitive level comparisons between student responses and curricular demands. Assessment procedures consisted of 14 individual interviews and 8 paper-and-pencil tests based on children's responses to mathematical cognition tasks in one-one interviews. The same criteria embedded in student response assessments were applied to demands made by mathematics curriculum objectives, textbooks materials, classroom activities, and Alberta Education Assessment Test items. The project sampled curricular cognitive demands and student cognitive response levels from the six elementary school grades and across five mathematics topic strands (numeration, operations and properties, measurement, geometry, and graphing). Answers to various research questions varied by topic stand, grade level, and demand component. However, several general findings are noted. About three-quarters of all student responses were at the concrete operational level, with remaining answers at the preoperational level. About three-quarters of all demands were at the concrete operational level, with most of the remaining quarter at the early formal level. In general, cognitive demands made by the curriculum were found to correspond well to distributions of student cognitive responses in most topics and at most grade levels. (Author/JN)

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Assessing Cognitive Levels in Classrooms

(ACLIC)

Final Report

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Prepared Under Contract to

ALBERTA EDUCATION

September, 1984

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FOREWORD

". . . the so-called aptitudes of 'good' students in mathematics or physics, etc., consist above all in their being able to adapt to the type of instruction offered them, whereas students who are 'bad' in these fields, but successful in others, are actually able to master the problems they appear not to understand--on condition that they approach them by another route.

"What they do not understand are the 'lessons' and not the subject. Thus it may be--and we have verified it in many cases--that a student's incapacity in a particular subject is owing to a too-rapid passage from the qualitative structure of the problems (by simple logical reasoning but without the immediate introduction of numerical relations and metric laws) to the quantitative or mathematical formulation (in the sense of previously worked-out equations) normally employed by the physicist . . . even in mathematics many failures in school are owing to this excessively rapid passage from the qualitative (logical) to the quantitative (numerical)."

Piaget, J. (1972). A Structural Foundation for Tomorrow's Education. Prospects : Quarterly Review of Education, UNESCO, 2, Spring 1972.

ABSTRACT

The Assessing Cognitive Levels in Classrooms (ACLIC) project was designed to produce a program evaluation and needs assessment of the 1982 Alberta Elementary School Mathematics Program in terms of cognitive level comparisons between pupil responses and curricular demands. At the heart of the ACLIC Project is a question that has frequently been asked in the mathematical education literature, past and present: Is there a reasonable fit between the instructional demands implied by a mathematics curriculum and the response levels attained by the intended students?

In the ACLIC Project, cognitive assessment procedures were developed for the whole range of mathematics topics in the elementary school grades. These consisted of fourteen individual interviews and eight paper-and-pencil tests based on children's responses to mathematical cognition tasks in one-one interviews. The same criteria imbedded in the student response assessments were applied to the demands made by mathematics curriculum objectives, textbook materials, classroom activities, and Alberta Education Achievement Test items.

The project sampled curricular cognitive demands and student cognitive response levels from the six elementary school grades and across five mathematics topic strands: Numeration, Operations and Properties, Measurement, Geometry, and Graphing.

Students were selected from across the province to ensure a sample that was provincially representative, including an appropriate urban/rural balance. In all, 1767 interview task assessments were made of the responses from 360 Grade 1 to 3 students who were interviewed by 23 Teacher-Interviewers. Grade 3 to 6 student response levels were assessed by means of eight paper-and-pencil tests that were completed by 1677 students. The information collected was used to answer the following questions:

1) What levels of cognitive ability are demonstrated in mathematics topic contexts by Alberta students in each of Grades One through Six (ages six through eleven)?

2) What are the levels of cognitive demand made on students at each grade level by:

i) the curriculum objectives identified by the Elementary Mathematics Curriculum Guide, Alberta Education, 1982,

ii) the prescribed textual resources,

iii) teacher presentations, and

iv) teacher-made tests?

3) How well do the curricular demands (made by Curriculum Objectives, texts, teacher presentations, and tests) fit the distributions of student cognitive ability at each grade level in particular mathematics topic strands?

The answers to the three research questions varied a great deal with topic strand, grade level, and demand component and they really should be considered in specific contexts. However, to provide a brief overall summary, general answers follow.

1) About three-quarters of all student responses were at the Concrete Operational level (Early Concrete and Late Concrete levels together). The remaining quarter were primarily at the Preoperational level in the early grades and at the Early Formal and Formal Operational levels in the higher grades.

2) About three-quarters of all demands were at the Concrete Operational level (Early Concrete and Late Concrete combined). Of the remaining quarter, most occurred at the Early Formal level, with small percentages at the Preoperational and Formal Operational levels. The distributions of Curriculum Objective demands showed the greatest consistency across strands and the most consistent pattern of increasing demands as the grade level increased. The pattern of the demands of the Textbooks was found to be very similar to that of the Curriculum Objectives. The Classroom lessons observed made predominantly Concrete Operational (Early Concrete and Late Concrete combined) demands in the lower grades and predominantly Formal Operational (Early Formal and Formal combined) demands in the higher grades. The cognitive demands of the Achievement Test items were mainly Concrete Operational (Early Concrete

and Late Concrete combined) in Grade 3 and mainly Early Formal Operational in Grade 6. There were no Preoperational demands, and in only one instance were there Formal Operational demands, that being at Grade 5 in Operations.

3) In general, the cognitive demands made by the curriculum and its interpretations have been found to correspond reasonably well to the distributions of student cognitive responses in most topics and at most grade levels. In most areas there were some matches, some demand distributions significantly lower, and some demand distributions significantly higher than the corresponding student response distributions. The best overall fit between demand distributions and the student response distribution occurred at the Grade 4 level where 64% of the demand and response distributions matched. However, there were also some striking mismatches. For example, at Grade 5 in three of the four strands all of the demand distributions were significantly higher than the corresponding response distributions. In Grade 1 less than one-tenth of the demand distributions matched the corresponding response distributions. The one topic strand in which the demand/response pattern was particularly noteworthy was Measurement--all of the Measurement demand distributions were significantly higher than the pupil response distributions at every grade level.

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Dr. K. Lovell for the excellent summary of Piaget's theory that he shared more than ten years ago and which provided a concise and consistent set of guidelines for our cognitive assessments

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The Teacher-Interviewers and their students, principals and administrators, from many centres in Alberta, who played a central role in the project

The principals, teachers, and students in schools throughout the province who participated in the paper-and-pencil assessments, another essential component of the project

The Calgary teachers who volunteered to have their mathematics lessons and students observed to provide assessments of classroom cognitive demands

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The ACLIC Steering Committee for its helpful guidance, reactions and advice during the term of the project; specifically: G. Ditto, W. Eddy, W. Hathaway, P. McLaughlin, G. Popowich

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ASSESSING COGNITIVE LEVELS IN CLASSROOMS (ACLIC)

OUTLINE OF THE STUDY

Introduction

The Assessing Cognitive Levels in Classrooms (ACLIC) project was designed to produce a program evaluation and needs assessment of the 1982 Alberta Elementary School Mathematics Program in terms of comparisons between the levels of pupil cognitive response and the levels of curricular cognitive demand. As hypothesized, in some topics and at some grade levels, the distributions of cognitive demands made by the curriculum and its presentation correspond reasonably well to the distributions of student cognitive responses, but in some other cases there are striking mismatches. The mismatches are most often characterized by a lack of suitable material for the significant numbers of students who are still responding Preoperationally in mathematics, especially in the lower grades, but also by a lack of suitable material for the cognitively most able. The report which follows documents the theoretical and practical background of the Project, its research design, and the detailed findings that have led to these and other assertions. The recommendations made on the basis of the findings contain practical implications for the curriculum itself and for classroom interpretations of the curriculum.

Background

At the heart of the ACLIC Project is a question that has frequently been asked in the mathematical education literature, past and present: Is there a reasonable fit between the instructional demands implied by a mathematics curriculum and the response levels of which the intended students are capable? It has not been uncommon for teachers, from Kindergarten through university, to despair of ways to present mathematics "understandably" to the many who seem to find the subject more or less inscrutable. Although the demand/response question has very likely been asked as long as formal instruction in mathematics has been offered, research techniques have been developed only recently for addressing the issue and for obtaining reliable data from which answers could be formulated.

One research study that has addressed a similar demand/response question in the context of Grade Seven and Grade Eight mathematics was the Calgary Junior High School Mathematics Project (CJHMP). Considerable gaps were found between student cognitive response levels and curricular demands in fraction and ratio topics. It was also found that process-oriented teaching materials could narrow the demand/response gap and improve student achievement and attitudes in these topics.

In the ACLIC Project, cognitive assessment procedures were developed for the whole range of mathematics topics in the elementary school grades. These consisted of fourteen individual interviews and eight paper-and-pencil tests drawn from the intellectual development work and clinical interviews of Jean Piaget, the mathematical development research of Kevin Collis (University of Tasmania), the mathematical interview and test construction work of Kath Hart (Chelsea College, London), interviews conducted and reported by Robert Davis (University of Illinois, Urbana), and the Australian Council for Educational Research (ACER) Mathematics Profile Series paper-and-pencil cognitive assessment tests based on Collis' research findings.

Review of the Literature

How Children Learn

A basic concept in the work of Piaget and of Skemp is that of a schema, a cognitive structure which has reference to a class of similar action sequences from past experience (Harrison, 1969, p. 94; Flavell, 1963, pp. 52-53). A schema can be thought of as the structure common to all those acts that an individual considers to be equivalent. For example, if a child's experiences have led him to believe that putting three objects with four objects results in a group of seven objects, a basic schema has been constructed that will enable the child to understand that "3 + 4 = 7." Any problematic situation requiring behaviour which is already generally represented in the child's mind is handled by being assimilated to the schema. Learning that "3 + 4 = 7" is assimilated to the knowledge that 3 objects and 4 objects make 7 objects. Furthermore, a child with such an operational schema is in a position to understand that three hundred plus four hundred equals seven hundred without ever having to count out that many beads or matchsticks (Skemp, 1958, p. 70). However, when a child encounters a new situation in which none of the existing schemas appear to be appropriate, new relevant experiences are needed from which new or modified schemas can be built to accommodate to the new circumstances. As with life in general, an individual learns to adapt to a mathematical environment through the interplay of assimilation and accommodation. While it is clear that a child needs the relevant schemas to understand and to process situations that are obvious analogues or applications of structures built in the past, it is even clearer that the schemas must be well founded and understood if there is to be successful accommodation to new learning situations. Any teaching/learning design which can ensure that each individual will be able to build the prerequisite concepts, before or while tackling new learning tasks, promises to facilitate effective, enjoyable, useful, and transferable learning. Unfortunately, mathematics learning of this kind will inevitably be out of reach for those students encountering curricular demands that are usually or always significantly above the cognitive levels at which they are able to respond. The ACLIC Project was designed to provide assessments of pupil response levels, of curricular demand levels, and of the "goodness-of-fit" between responses and demands so that problems like the one just described might be minimized.

Piaget once stated that an individual's apparent failure to grasp the most basic concepts of elementary mathematics stems not from a lack of any special aptitude but rather from inadequate preparation, with its inevitable, concomitant emotional blocking. He went on to point out that the frequent failure of formal education can be traced to the tendency to begin with language, illustrations and narrated action rather than real, practical action. Preparation for mathematics education must begin with the encouragement of concrete manipulations that foster awareness of basic logical, numerical, and measurement relationships. This practical activity needs to be systematically developed and amplified throughout the primary grades, leading to basic physical and mechanical experiments by the time secondary education begins (Piaget, 1951, pp. 95-98). In a similar vein, Eleanor Duckworth, discussing implications from Piaget's work, has said:

> You cannot further understanding in a child simply by talking to him. Good pedagogy must involve presenting the child with situations in which he himself experiments, in the broadest sense of that term--trying things out to see what happens, manipulating things, manipulating symbols, posing questions, and seeking his own answers, reconciling what he finds at one time with what he finds at another, comparing his findings with those of other children. (Duckworth, 1964, p. 497)

A teacher who possesses a certain concept may find it difficult to realize that a pupil does not. Once a concept has been formed, it is so obvious to the possessor that it is perceived as part of the data in a situation in which the concept is appropriate. It is difficult to imagine perceiving the data in any other way. An added difficulty arises because pupils can learn by rote all the required procedures in the early stages of concept learning so that whether or not they are really developing the required schemas is not readily apparent. However, it is worth the effort to find out because, unless the pupil is provided with the appropriate kinds of repeated experience necessary to form the basic concepts and operations, verbal and blackboard teaching will lead to rote memorization. The concepts required for learning superordinate concepts will not be formed, and the student will be incapable of really understanding mathematics (Skemp, 1962, pp. 9, 10; 1960, p. 50). The ACLIC Project cognitive assessment materials were designed to help teachers become more aware of the concepts and cognitive processes that their pupils can or cannot use. Presumably, such information could help teachers to provide learning activities that are suited to the cognitive levels at which the students are able to respond.

A disheartening picture of the long-term results of teaching that is continuously geared well above pupil response levels has been provided by the research of Louis Johannot (1947). Johannot found that, in the great majority of the Genevan adolescents interviewed, abstract reasoning was absolutely artificial, merely pseudo-reasoning. Reasoning had been replaced by learned procedures and method types. The role that active, creative intelligence played was seen to be no more than a secondary one (Johannot, 1947, p. 113; Harrison, 1976 Translation, p. 101). More recently, small scale trials in Calgary of Johannot's interview tasks have given ample indication that the situation is little different across an ocean and thirty years later (e.g., Tyrell, 1976).

A concise and yet meaning-filled and useful contrast between the form of "reasoning" Johannot found and that which proponents of contemporary mathematics learning theory would prefer to see fostered, has been offered by Richard Skemp. He has dramatically contrasted instrumental understanding, characterized by using "rules without reasons," with relational understanding, "knowing both what to do and why." An instrumental approach to mathematics learning characteristically consists of (or stresses) the learning of rules for proceeding from given data to the correct answers to given questions (each sequent step is determined solely by the immediate conditions) On the other hand, a relational approach to mathematics learning engages the learner in building a conceptual structure (schema) from which a variety of strategies for solving any given problem can be produced (Skemp, 1978, pp. 9, 14). How to encourage relational learning is suggested by Skemp's description of the way in which "schemas" are constructed by being "built" and "tested" in three different "modes." In Mode 1, a person builds schemas from direct experience and tests them by experiment against expectations in actuality (the environment in which one's physical actions and activities take place (Skemp, 1979, p. 21)). In Mode 2, schemas are constructed through communication with others and tested through discussion with others. Mode 3 schema construction proceeds "from within" by formation of higher order concepts, by extrapolation, by the use of imagination and intuition (creativity), and by testing according to internal consistency with one's "inner reality" of existing knowledge and beliefs (Skemp, 1979, p. 163).

When mathematics teaching neglects the development of mathematical reasoning structures in favour of a steady diet of narrowly focused "explanations" and "exercises," it should come as no surprise that the development of relational thinking is found to be at a very low level. While mathematical strategies cannot be "given" to pupils, they can and must be built by the child through process-oriented experiences and discussions that are appropriate for the level of cognitive development the child has reached. It is important, then, to be able to assess the cognitive level at which a pupil is able to respond in a given mathematics context.

Assessment of Pupil Cognitive Response Levels

Interviews. The work of Piaget and his associates has provided a rich source of cognitive assessment tasks covering a wide range of contexts, including many relevant to school mathematics learning. A detailed search of the Piagetian and neo-Piagetian literature resulted in a collection of interview tasks designed to assess student understandings of the key concepts and skills contained in the 1982 Alberta Education Elementary Mathematics Curriculum Guide, Grades 1 to 3. Once suitable tasks were identified, they were edited, simplified, and adapted for use by the ACLIC Teacher-Interviewers. Capsule descriptions of all of the interview tasks used are given in the Major Sources of Data section of this report, under the heading "Cognitive Response Assessment: Interviews." Also, the "Interview Record" sheets for each of the ACLIC interviews are contained in Appendix 1. Since the details for each interview are available elsewhere in the report, only a Strand-Topic listing with reference citations is given here.

Strand	Interview Topic(s)	Reference Source(s)
Numeration	Equivalence; Pre-multiplication	Copeland, 1974a, pp. 137-139 Copeland, 1974b, pp. 26-27 Piaget, 1952, pp. 203-220
	Fractions	Piaget, Inhelder & Szeminska, 1960, pp. 302-335
	Serial, Ordinal, Cardinal Correspondence	Piaget, 1952, pp. 106-121
	Place Value	Brown, Hart, Kucheman, 1984, p. 6 Davis & McKnight, 1980
Operations	Addition Concept	Bye, 1975
	Additive Composition	Piaget, 1952, pp. 185-186
	Commutativity of Addition	Schroeder & Bye, 1984
	Pre-multiplication	Copeland, 1974a, pp. 137-139 Copeland, 1974b, pp. 26-27 Piaget, 1952, pp. 203-220
	Subtraction	Brown, Hart, Kucheman, 1984, p.6 Davis & McKnight, 1980
Measurement	Length	Copeland, 1974b, pp. 64, 65 Hart, 1981, pp. 11-12
	Time	Piaget, 1969, pp. 188-195
	Weight	Piaget & Inhelder, 1974, pp. 183-202

Strand	Interview Topic(s)	Reference Source(s)
Geometry & Graphing	Classification, Inclusion	Copeland, 1974b, pp. 39-40
	Graphing	Piaget, 1960, pp. 153-169
	Loci	Piaget, 1960, pp. 209-225
	Order (Linear, Circular)	Piaget, 1956, pp. 80-103

Paper-and-pencil Tests. While it was decided early in the ACLIC Project that the Grades 1, 2 and 3 cognitive response assessments could best be made with individual "Piagetian" interviews, paper-and-pencil cognitive assessments for the pupils in Grades 3 through 6 were considered feasible, largely on the basis of the successful development by Onslow (1976) of a multiple choice test for determining Piagetian developmental levels in the context of Ratio and Proportion and by Kieren (In Bye, Harrison & Brindley, 1980) of a short answer paper-and-pencil test for assessing cognitive response levels in the context of Fractions. These two tests were designed for pupils of Junior High School age, but many of the items could be adapted for the upper elementary grades. Previous successes in designing paper-and-pencil tests for assessing Piagetian levels of response at the upper elementary and secondary levels have been reported by Tisher (1971) and Shayer (Shayer, Kucheman & Wylam, 1976). In each of the cases cited, comparisons between the paper-and-pencil test ratings and those based on individual interviews established that there was a high level of agreement (of the order of 80%) between the two modes of assessment.

Following a lead in a personal communication from Professor Skemp, the ACLIC team reconsidered the work of Collis in Australia. A reference by Biggs & Collis (1982) to a paper-and-pencil test on Operations that had been constructed from Collis' cognitive interview findings led to obtaining from the Australian Council for Educational Research (ACER) the Operations test from the ACER Mathematics Profile Series (Cornish & Wines, 1978). Finding that the Operations test items were compatible with the ACLIC Interview tasks and with the Operations and Properties topics of the Alberta Elementary Mathematics Curriculum, the complete Mathematics Profile Series was obtained. The items used in the ACLIC assessments were selected according to difficulty, topic coverage, and cognitive response elicited. Four tests were constructed for Grade 3 & 4, and four for Grades 5 & 6, covering Number, Operations, Measurement and Geometry & Graphing.

Assessment of Curricular Cognitive Demands

A number of studies have assessed the cognitive demands of secondary science curriculum objectives and textbooks (e.g., Shayer, 1972; Lyon, 1979) but it is unknown if, outside of local studies, similar assessments have been conducted in the area of school mathematics.

The Calgary Junior High School Mathematics Project (CJHMP; Bye, Harrison & Brindley, 1980) addressed the question of the cognitive demands of two topic areas in the Alberta Junior High School Mathematics Curriculum Guide: rational numbers and ratios. Two sets of cognitive demand level criteria were generated by inference from Onslow's "Ratio and Proportion" test, Kieren's "Fractional Number Thinking Test," and related Piagetian research literature on the two topics. The CJHMP research team extracted descriptions of characteristic responses at each cognitive level from the Onslow and Kieren test items and from related research study findings. The expectations of each curriculum objective, each textbook explanation or exercise, each classroom expectation, and each achievement test item were matched (in terms of the least demanding interpretation) with the demand criteria that had been generated from the pupil response assessments. A similar approach was taken in generating the cognitive demand criteria for the ACLIC Project.

With assessment devices for rating the cognitive levels at which students are responding in the various mathematics topic areas and with sets of criteria for assessing the cognitive demands of the curriculum, it is possible to assess the "goodness-of-fit" between pupil responses and curriculum demands (See the Analysis of Data section of this report for a detailed description of the statistical procedures used.). Knowing how the expectations or demands of the curriculum compare with actual student responses should provide a sound basis on which to "tailor" instruction to the pupils for whom it is intended.

Teaching Tailored to Personal Schemas

In 1972, Jean Piaget gave an uncharacteristically direct set of recommendations to mathematics and science teachers regarding teaching methods and apparently "less able" students:

> . . . the so-called aptitudes of "good" students in mathematics or physics, etc., consist above all in their being able to adapt to the type of instruction offered them, whereas students who are "bad" in these fields, but successful in others, are actually able to master the problems they appear not to understand--on condition that they approach them by another route.
>
> What they do not understand are the "lessons" and not the subject. Thus it may be--and we have verified it in many cases--that a student's incapacity in a particular subject is owing to a too-rapid passage from the qualitative structure of the problems (by simple logical reasoning but without the immediate introduction of numerical relations and metric laws) to the quantitative or mathematical formulation (in the sense of previously worked-out equations) normally employed by the physicist . . . even in mathematics many failures in school are owing to this excessively rapid passage from the qualitative (logical) to the quantitative (numerical). (Piaget, 1972, p. 17)

He went on to recommend that teachers should not lecture, transmitting ready-made solutions, but should rather create situations which pose useful problems so that the child or adolescent can experiment to reconstruct basic principles (if not rediscover them). Furthermore, the teacher should be ready to provide counter-examples to help his students reflect on, and reconsider, hasty solutions. Such a teacher-organizer, acting as a mentor stimulating initiative and research, needs to know his own subject as well as being familiar with the nature of intellectual development (Piaget, 1972, p. 18).

In a similar vein, a British researcher has captured the essence of what is most important when instructional materials and methods are being chosen for fostering the development of relational thinking:

> If we are to foster intellectual development within the educational framework, then it is essential that the teacher critically examines the concepts underlying the subject matter he hopes to teach; he must also be aware of the approximate limitations set on understanding by the various stages of operational thinking . . . Piaget has shown that efficient learning cannot take place if new data is so far removed from the child's experience that accommodation cannot take place. There is every indication . . . that restructuring of thinking is more likely to be brought about by the child actively operating on his environment, rather than constantly thinking in a teacher directed situation. (Hughes, 1965, pp. 110-111)

A key Piagetian concept highlighted by Flavell (1963, p. 368), maintains that intellectual development is marked by a gradual transformation of overt actions into mental operations. Helping pupils to perform actions with decreasing support from external objects can be facilitated by having them act on physical objects that exemplify given concepts, patterns, or operations; then on pictorial or semi-symbolic representations; then on imagined objects and operations while keeping symbolic records, and so on, until the abstract concepts emerge in a form that can not only be used meaningfully but can also be reinterpreted, as required, in terms of previous levels of representation (rather than existing purely at a symbolic level).

Teaching can cooperate with and guide learning by taking into account student developmental levels in the topics at hand and choosing methodology accordingly, by creating situations that pose useful and challenging problems for students to investigate, by encouraging the search for patterns, and by arranging for students to encounter counter-examples while testing hypotheses and building abstractions, generalizations and formalizations.

Purpose and Problems Examined

The purpose of the ACLIC Project was to gather evidence to provide answers to three research questions:

1) What levels of cognitive response are demonstrated in mathematics topic contexts by Alberta students in each of Grades 1 through 6?

2) What are the levels of cognitive demand made on students at each grade level by:

i) the curriculum objectives identified by the Elementary Mathematics Curriculum Guide, Alberta Education, 1982,

ii) the prescribed textual resources,

iii) teacher presentations, and

iv) representative achievement tests?

3) How well do the distributions of curricular demands (made by the curriculum objectives, texts, teacher presentations, and tests) fit the distributions of student cognitive responses at each grade level in each mathematics topic strand?

The Scope of the Study

A Steering Committee consisting of representatives from the Calgary Board of Education's Mathematics Team, Alberta Education, and the Calgary Catholic Board of Education was established to assist in charting the course of the project. The Project Director was the Calgary Board of Education Mathematics Specialist and the ACLIC Research Team members were University of Calgary mathematics education instructors and a teacher pursuing graduate studies.

The project sampled curricular cognitive demands and student cognitive response levels from the six elementary school grades and across five mathematics topic strands: Numeration, Operations and Properties, Measurement, Geometry, and Graphing.

Students were selected from across the province to ensure a sample that was provincially representative, including an appropriate urban/rural balance. In all, 1767 interview task assessments were made of the responses from 360 Grade 1 to 3 students who were interviewed by 23 Teacher-Interviewers. Grade 3 to 6 student response levels were assessed by means of eight paper-and-pencil tests that were completed by 1677 students.

All 364 of the objectives stated in the 1982 Alberta Education Elementary Mathematics Curriculum Guide were rated as to level of cognitive demand by using the Cognitive Levels definitions stated on the following pages and the Cognitive Demand Level Criteria summarized in Appendix 2. The same cognitive demand criteria were used to rate all of the explanations, problems, and exercises in two of the authorized elementary school mathematics textbooks at each of the six grade levels (62 306 items were assessed altogether). At each grade level one of the texts assessed was from one of the two series most often chosen by Alberta schools, and the other was from one of the remaining authorized series. Using the same criteria, cognitive demand assessments were made of 36 classroom lessons in mathematics (A total of 1586 minutes of classroom lessons were observed.). All 95 of the items in Parts A and B of the 1982 Grade 3 and 1983 Grade 6 Alberta Mathematics Achievement Tests were rated for cognitive demand (Purely computational items were not included.).

Definitions of Terms

Achievement

In this study the term "achievement" is used to refer to learning that focuses on the mastery of facts, skills, concepts, or routine problem-solving strategies (Biggs and Collis, 1982). Achievement is associated with instrumental understanding, using rules without reasons (Skemp, 1978), and with the kind of surface learning that arises from committing to memory for quick recall various facts, names, definitions, rules, . . . (Bell & Shiu, 1981)

Cognition

In contrast with "achievement", the term "cognition" is used to refer to the capability of using skills, facts, or concepts in a way that demonstrates understanding of what has been learned, such as, for example, in solving a problem, carrying out a task, or making a judgement (Bell & Shiu, 1981). Cognition is associated with relational understanding, knowing both what to do and why (Skemp, 1978), and with the kind of deep learning characterized by the making of connections in memory, by the construction of a conceptual structure from which a variety of strategies for solving given problems can be produced, by the recognition of similar relations in new situations, and by the development of strategies for directing mental processes (Bell & Shiu, 1981; Skemp, 1978).

Cognitive Levels

The following Cognitive Levels criteria have been edited from a summary of "Piaget's Structure" (Lovell, K., personal communication, 1974). They were used to ensure consistency in the selection, development, and interpretation of the individual student interviews, the paper-and-pencil tests, and the cognitive demand criteria.

Preoperational (PO) thinking is characterized by intuitive and transductive thinking from one particular to another, thinking limited by the particular state of the situation considered, isolated centrings on one feature only, dealing only with one problem at a time, inability to relate one problem to another in the same situation, unsystematic, partial, fragmented, inconsistent thinking, and lack of reversibility of thought (inability to work back from an inconsistency).

Early Concrete Operational (EC) thinking is characterized by faulty inductive and deductive logic, generally unsuccessful attempts to consider or relate more than one feature of a situation, attempts at reversibility that end in confusion, incomplete or inconsistent attempts to classify facts, and uncertain judgements.

Late Concrete Operational (LC) thinking is characterized by inductive and deductive logic limited to concrete situations which involve visual or sensory data, successful classification of tangible data, successful systematic thinking and relating of two or more facts without extension or generalization from one concrete field to another, reversibility when concrete data are being operated with, a tendency to judge purely verbal problems and problem situations in terms of their content as specifically related to personal experience, and concentration on relating things visibly or tangibly present.

Early Formal Operational (EF) thinking is characterized by reasonably advanced and consistent inductive and deductive logic limited by the concrete elements in the situation, generally unsuccessful attempts at abstract and propositional thinking, and generally unsuccessful attempts to go outside of known data to form hypotheses. [The ACLIC Team found the term Concrete Generalization, which has been proposed by Collis (Biggs & Collis, 1982, p. 19), useful when characterizing and identifying Early Formal responses and expectations.]

Formal Operational (F) thinking is characterized by hypothetical and deductive thinking, consideration of data in terms of provisionally true or false propositions to be tested out in thought, logical thinking in symbolic and abstract form, recognition of the incompatibility of certain facts with an hypothesis, evidence of a preference to begin consideration of a situation with a theory rather than just the facts, and reasoning by implication at an abstract level.

Cognitive Demand Levels

The Cognitive Demand Levels used in this study have been generated using the Piagetian Cognitive Levels criteria described above. The Cognitive Demand Level criteria were derived, in the first instance, by assessing the cognitive expectations of each of the objectives in the Alberta Education Elementary Mathematics Curriculum in terms of the kind of student response required for successfully reaching the objective. These response expectations (demands) were rated as Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational according to the criteria in the Cognitive Levels definitions, the assessment criteria in the ACLIC student interviews, and the items in the ACLIC paper-and-pencil tests. Briefly, the Cognitive Demand Level criteria consist of specific subject matter examples of levels of pupil cognitive response expectations. A detailed listing of the Cognitive Demand Level criteria is included in Appendix 2.

Cognitive Response Levels

The ACLIC Project interview tasks and test items were chosen or designed to provide cognitive assessments that conform with the criteria contained in the Cognitive Levels definitions described above. In each cognitive assessment the student's responses were rated as Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational.

Concrete Generalization

Concrete generalization is a term used by Collis (Biggs & Collis, 1982, p. 19) to identify Early Formal responses that are transitional from Concrete to Formal. Such responses are characterized by the ability to generalize concrete patterns and to cope with formal structures, but only in concrete embodiments (Cornish and Wines, 1977, pp. 16, 17).

Operation

A cognitve "operation" is an interiorized, reversible action that is coordinated with other interiorized actions in the structure of a group containing certain laws that apply to the whole. (Battro, 1973, 121) "Operation . . . is nothing but an articulated intuition rendered mobile and entirely reversible, because it was emptied of its representative content and subsists as simple 'intention' . . ." (Piaget, 1970, 26; Battro, 1973, 122) "Operations are reversible because they contain everything possible, whereas reality is irreversible to the degree that it is drawing only a sample from among these possibilities." (Battro, 122)

Research Methodology

Essentially, the Project was conducted in two phases, each roughly one year in duration, as described in the following paragraphs.

Phase 1 (Year 1), The Developmental Phase

Criteria were established from the research literature for identifying student levels of cognitive response to specific elementary school mathematics tasks. Relevant sources were searched for interview tasks and paper-and-pencil tests or items that could be used to identify the levels of student cognitive response. The ACLIC interview tasks, which were used in Grades 1 to 3, were drawn largely from the research of Piaget and from interpretations of the original research conducted by his group at the Institute of Education, University of Geneva. Testing specialists were consulted regarding suitable cognitive assessment test items, sample sizes, and statistical analyses. For Grades 3 to 6, eight paper-and-pencil tests (Grade 3/4 and 5/6 versions of tests covering Number, Operations, Measurement, and Space concepts) were developed by drawing cognitive assessment items primarily from tests developed by the Australian Council for Educational Research (Cornish & Wines, 1978) and also from the ACLIC interviews, the Chelsea Diagnostic Mathematics Tests (Brown, Hart & Kucheman, 1984), and the work of Robert Davis (Davis & McKnight, 1980). The choice and development of the cognitive assessment instruments was guided by reference to the criteria contained in the definitions of the Cognitive Response Levels included in the Definitions of Terms section of this report.

The individual interviews and the paper-and-pencil tests were field tested in five Calgary schools that were not included in later parts of the Project. The information gained from the field testing proved invaluable in subsequent refinements of the assessment materials.

Twenty-four teacher-interviewers were identified by Alberta Education Regional Mathematics Consultants, Mathematics Supervisors of various school districts, and Superintendents. In late August and early October, 1984, the prospective Teacher-Interviewers participated in four full days of workshops conducted by ACLIC personnel to prepare them for conducting the interviews as well as enabling them to have a part in the interview refinement process. The ACLIC interview workshops included analyses of video-taped sample interviews covering each of the major mathematics topics at each grade level as well as discussions of the interview procedures, criteria for rating the cognitive levels of student responses (as guided by the interview procedure/record sheets, copies of which are included in Appendix 1), and interview "role-playing" using the complete set of manipulatives that had been assembled for conducting each interview.

One Teacher-Interviewer had to drop out before the actual interviewing began because of a shift to a new and especially demanding non-classroom assignment.

A detailed schedule for student interviews, paper-and-pencil testing, cognitive demands analyses, and printing timelines was drawn up for Phase 2.

Phase 2 (Year 2), The Assessment Phase

Part of the evidence for Research Question 1 was gathered in October, 1984, when each of the twenty-three Teacher-Interviewers conducted individual interviews with fifteen Grade 1 to 3 students on four to six topics in two or more sessions totalling approximately one hour. Two of the Teacher-Interviewers agreed to interview an additional seven and eight students, respectively, so that the originally planned total of 360 interviewed students could be accommodated. In all, 1767 interview procedure/record sheets were collected from 360 students in Grades 1 to 3. Among the materials assembled for the interviews were: counters, base ten blocks, dice, specially prepared game boards, beads, weighted plasticine balls, "pacers" that clicked once every second, stopwatches, flowers, attribute blocks, and a variety of simple cut-outs and manipulatives. The rest of the data for addressing Research Question 1) was collected in November, 1984 when 1677 Grades 3 to 6 students responded to one of eight paper-and-pencil cognitive assessment tests covering Number, Operations, Measurement, or Space (Geometry & Graphing) at either the Grade 3/4 or Grade 5/6 level.

Kolmogorov-Smirnov goodness-of-fit tests were used to assess the comparability of the distributions of cognitive level ratings of Grade 3 pupil responses to the interviews and to the paper-and-pencil tests.

For Research Question 2), the cognitive demands (expectations) of the curriculum were assessed by analyzing: a) the objectives in the five content strands of the 1982 Alberta Elementary Mathematics Curriculum Guide, b) the corresponding parts of authorized textbooks, c) observations from classroom mathematics lessons, and d) the items in Parts A and B of the Grade 3 and of the Grade 6 provincial Mathematics Achievement exams. The curricular demand analyses were based on cognitive demand criteria derived from the criteria used to rate student cognitive response levels (i.e., those imbedded in the student interviews and in the paper-and-pencil tests as well as in the Cognitive Response Level definitions listed under Definitions of Terms).

Appendix 2 contains documentation of the cognitive demand criteria that were used in the Project.

Evidence pertaining to Research Question 3 (How well do the distributions of curricular demands fit the distributions of student cognitive response at each grade level in particular mathematics topic strands?) was obtained using Kolmogorov-Smirnov goodness-of-fit tests to compare the observed student cognitive response distributions with the corresponding curricular demand distributions.

Major Sources of Data

The major sources of data in the ACLIC Project were: cognitive response assessment interviews, cognitive response assessment paper-and-pencil tests, and cognitive demand assessments of curriculum objectives, textbook materials, classroom observations, and Alberta Education Achievement Tests.

Cognitive Response Assessment: Interviews

Numeration Interviews

Equivalence and Pre-multiplication (For Grades 1 and 2). Asked to distribute (10) flowers, one-for-one, into (10) vases and then seeing the flowers made into a "spread out" bouquet, the child was asked to distribute another (10) flowers into the vases. After these flowers were formed into a "tightly packed" bouquet, the child's concept of lasting equivalence was assessed by asking: "Are there as many flowers here [spread out] as there [tightly packed] . . . or are there more in one of the bouquets?" The child's grasp of multiplicative composition was assessed by the question: "If we put all of the flowers in the vases with the same number in each vase, how many flowers will there be in each vase?" After a third set of (10) flowers was introduced, the question was repeated.

A response indicating that a child considered one of the bouquets to have more than the other was rated as Preoperational in terms of concepts of equivalence and multiplication. Accurate use of one-one correspondence but inability to predict the number of flowers per vase was rated as Early Concrete Operational. Consistent resistance to perceptual distraction and maintenance of the equality of the numbers of flowers in the bunches (e.g. ". . . because each was in its own vase") was rated as at least Late Concrete Operational. Successful prediction of the number of flowers per vase was rated as being at least at the Late Concrete Operational level in terms of multiplicative composition. (Copeland, 1974a, 137-139; Copeland, 1974b, 26-27; Piaget, 1952, 203-220)

The record sheet for this interview task, which came to be called "FLOWERS," is included in Appendix 1.

Fractions (for Grades 1, 2, and 3). The child was shown a strip of adding machine tape, longer than wide, and was told: "This is quite a thin piece of toffee. The bears (two "bear" stickers mounted on two white cards) want to eat it all up. To be fair, each should get just the same amount. How shall we do it? . . . You can use these (pencil, scissors, "sticks," . . .)." Then: "How did you do it?" (or, "Tell me what you did.") When the child had divided the whole, the following question was posed: "Would these pieces taken together make up as much as the whole strip of toffee that we started with . . . or more . . . or less?" . . . "Tell me why." Then, a third bear was brought out and the problem reposed with a new strip of adding machine tape. Alternatively, the procedure could be started with three bears. As appropriate, the procedure was repeated for quarters, fifths, or sixths.

At the Preoperational level of response the child experiences real difficulty trying to divide the toffee into two equal parts, ending up with, for example, more than two parts, or approximately equal small portions with some of the toffee undivided, or all of the toffee shared unequally, or three portions (confusing the number of cuts with the number of parts). Early

Concrete Operational responses are marked by successful production of two equal parts but without realization that the original whole must necessarily equal the sum of its original parts. The ability to make 3 equal parts and to intuitively (not operationally) realize that there is conservation of the whole is rated Late Concrete Operational. Early Formal Operational responses characteristically involve handling the trichotomy (three equal parts) problem by means of an anticipatory schema (i.e., an a priori understanding of the relations between the fractions sought and the original whole) and operational conservation of the whole. At this level, division into fifths and sixths is also handled by means of an anticipatory schema. (Piaget, Inhelder & Szeminska, 1960, 302-335).

The record sheets for this task, which came to be called "TOFFEE," are included in Appendix 1.

Serial, Ordinal, Cardinal Correspondence (For Grade 1). In this interview (based on Piaget, 1952, 96-121), the child was presented with ten toy drums, graduated in size (the largest at least twice the size of the smallest) but in disarray, and ten "drumsticks" (wooden toothpicks, each cut to match the radius of the drum to which it belongs). Information about the child's notions of serial, ordinal, and cardinal correspondence was gathered from the responses to the following five questions and related procedures:

Question 1: "The drums are going to be in a parade. Arrange the drums and drumsticks so that each drum is with the right size of drumstick." [Interviewer Note: Discuss until it is clear that the child understands the principle of serial correspondence.]

Question 2: [Interviewer Note: Once the rows of drums and drumsticks have been arranged in correspondence with one another, in clear view of the child, move the drums closer to one another and the drumsticks further apart, but maintaining the distance between the rows of drums and drumsticks. Then, touching one of the drums . . .] "Which drumstick will go with this one?" [Repeat, choosing in order or at random according to how the child answers.]

Question 3: [Interviewer Note: After several repetitions of Question 2, reverse one of the series.] "Which drumstick will go with this one?" [As in Question 2]

Question 4: [One or both series disarranged] "Which drumstick belongs to this drum?"

Question 5: (Which enables determination of "the exact level of the child's understanding" (Piaget, 1952, 98)) [Interviewer Note: Mingle all of the elements of the two series. Pick out a drum, say number 5.] "Some of the drums are going in the next parade, but not all of them--only those that are bigger (or smaller) than this drum. Find the drumsticks belonging to the drums that are going in the parade and those belonging to the drums not going."

These five questions, posed separately to the child, were reduced to three more general problems for systematizing the results (in accordance with Piaget, 1952, 98):

1. constructing a serial correspondence or similarity (Q. 1)

2. determining a serial correspondence when it is no longer directly perceived (transition to ordinal correspondence) (Q. 2 and Q. 3)

3. reconstructing the ordinal correspondence when the intuitive series are destroyed (Q. 4 and Q. 5)

Regarding the child's ability to construct a serial correspondence (Q. 1), inability to make the drums and drumsticks correspond and inability to form the two individual series were rated as Preoperational responses. An Early Concrete Operational response features spontaneous construction of the correct series, following some trial and error, and successful solution of the problem of serial correspondence (especially by the method of double seriation) (Piaget, 1952, 102, 103). Late Concrete Operational responses are characterized by the child continually considering the set of relationships among all of the elements, at each new step looking for the biggest (smallest) of the remaining elements (no trial or error). Responses at this level frequently demonstrate an obvious ease of operation including establishing the correspondence immediately, without previously seriating the drums and drumsticks (Piaget, 1952, 106)

Regarding assessment of the child's ability to move from serial to ordinal correspondence (Q. 2 & Q. 3), a characteristic Preoperational response would have the child losing all notion of correspondence when one of the series is displaced, merely matching elements that happen to be opposite one another. Early Concrete Operational responses include attempts to find the correct correspondence "by empirical means or by counting" (Piaget, 1952, 106) hampered by constant confusion between the correct position and that of the preceding one. Solving the problem by coordinating estimates of the required position with that of the cardinal value of the sets in question (involving both qualitative serial and ordinal numerical correspondences) is rated a Late Concrete Operational response (Piaget, 1952, 106-114).

The child's attempts to reconstruct a cardinal correspondence (Q. 4 & Q. 5) are rated Preoperational when no correspondence is made, when the series are not reconstructed, or when "matching" elements are chosen at random. Early Concrete Operational responses include problem solution attempts that lack systematic re-seriation or cardination. Late Concrete Operational responses typically involve achieving reconstruction of the series by co-ordinating ordination and cardination (Piaget, 1952, 115-121).

The record sheets for this task, which was called "DRUMS," is included in Appendix 1.

Subtraction/Place Value (Brown, Hart & Kucheman, 1984; Davis & McKnight, 1980). This interview explores subtraction concepts as well as place value ideas and it is described under Operations and Properties Interviews, which follows.

Operations and Properties Interviews

Addition Concept (Invariance, Reversibility). (For Grade 1) The materials for this interview consisted of 11 toy cars (seven of one color, four of another) and a mat on which was drawn a "parking lot" with 11 parking spaces. The interviewer began parking the seven cars in the first seven stalls demonstrating one-to-one correspondence, and invited the child to continue parking the rest of the cars. Next all the cars were driven out of the parking lot. Then the cars were parked again, this time beginning with the ones of which there were four. After the four cars of one color and one of the remaining seven cars were parked, the interviewer asked, "If you finish parking all the cars, will all the stalls be full?" and, if yes, "Will there be any cars left over?" If both these questions were answered correctly, the child's response was rated as at least Early Concrete Operational. If the child was unable to construct the one-to-one correspondence between stalls and cars, or if he was unable or unwilling to predict whether the spaces would be full, or if he predicted incorrectly, the response was rated as Preoperational. This task was adapted from an unpublished working paper (Bye, 1975).

The record sheet for this interview, called the "PARKING" task, is included in Appendix 1. Note that the responses listed on the record sheet under Early Concrete Operational were actually classified as Preopertional, while those listed under Late Concrete Operational were actually classified as "at least Early Concrete Operational."

<u>Additive Composition</u> (Invariance, Sharing). (For Grades 1 and 2) The first task of this interview (additive composition and invariance) was conducted by the interviewer telling the following story while demonstrating with poker chips placed on a card with two halves marked "Yesterday" and "Today." There is a boy (girl) whose mother makes cookies he likes very much. Every morning he asks his mother for cookies to take to school. Yesterday his mother gave him four cookies to eat at recess. In the afternoon he asked for cookies again and mother gave him four more cookies to eat in the afternoon. Today he asked for cookies in the morning and his mother gave him four, and in the afternoon he asked for cookies and again his mother gave him four. Now, yesterday he ate four cookies in the morning and four more cookies in the afternoon. But today he was so busy at recess he only had time to eat one cookie. So he saved the rest of the cookies to eat in the afternoon (demonstrated by moving three cookies over next to the four for today's afternoon).

The interviewer then asked, "Did the boy have more cookies yesterday (pointing to all of yesterday's cookies) or more cookies today (pointing to all of today's cookies) or did he have the same on both days?"

The response of a child who did not know or was uncertain which day had more chips was rated as Preoperational.

A child who initially chose one day as having more than the other but who when pressed for justification suggested counting and discovered that both had eight was rated as Early Concrete Operational.

A child who knew that both days had the same number even without counting and who based his justification on the equality of the numbers or on a reversibility or identity argument was rated as Late Concrete Operational.

The second task of this interview (sharing) involved 24 chips put into two clearly unequal piles. The interviewer asked, "Suppose I put these cookies into two piles, these (the larger pile) for you and these (the smaller pile) for me. Would that be fair? What could you do with all these cookies to put them into two piles so that it would be fair?

The responses of children who judged the two unequal piles to be "fair" or who made the piles "equal" by estimation without counting or checking in any way were rated as Preoperational.

Children who equalized the piles by a series of trials and errors and counted or matched to verify success were rated as Early Concrete Operational.

Responses involving systematic sharing (e.g. putting all in one pile and removing two at a time) or using operations on numbers were classified as Late Concrete Operational. (Piaget, 1952, pp. 185-186)

The record sheet for this interview, called "COOKIES", is included in Appendix 1.

Commutativity of Addition (For Grades 1, 2, and 3). The materials for this interview consisted of two identical open-top boxes, about 10 cm x 15 cm x 2 cm, separated into two compartments by a wooden divider. Each box held 24 small blocks which could easily be moved from one side of the box to the other. With all the blocks on the one side of the divider in each box, the child was asked whether the two boxes contained the same number of blocks. If necessary, counting the blocks was suggested. In the child's view the interviewer moved six of the blocks to the right side of one of the boxes and explained that in the other box some blocks would be moved but that the child would not be able to watch. The interviewer then arranged the boxes so that they were mirror images of one another and placed them side by side concealing the six blocks on one side of one box.

The interviewer then asked "Can you tell me how many blocks are hidden under my hand?" and, if the answer sounded like a guess, asked questions such as: "Are you sure? Is there any way you could know for sure, instead of just thinking there might be that many?"

The responses of children who did not know or guessed or estimated were rated as Preoperational. Children who guessed six and supported this by reference to the six in the other box, but did not verify that the other sides of each box each had the same number (18), were rated as Early Concrete Operational. Responses in which the children determined that there were six by computation (e.g. 24 - 18 = N, 18 + N = 24, etc.), by counting on from 18 to 24, or by noting the symmetry of the six and 18 in one box and the 18 and six in the other were rated as Late Concrete Operational.

If it was necessary to verify child's method of solution, the interviewer took up one box and moved some blocks from one side to the other without the child seeing the moves, arranging the box with eight on one side and sixteen on the other. The box was then presented with the eight blocks covered. A child who noticed that there were fewer showing so there must be more hidden was said to be using compensation. If this idea was applied as an estimation, then the response was rated Early Concrete Operational. If the child argued that since there were two fewer showing (16 instead of 18), there must be two more hidden (10 rather than 8), the response was rated Late Concrete Operational.

The record sheet for this interview, called DIVIDED BOXES (Schroeder & Bye, 1984), is included in Appendix 1.

<u>Equivalence and Pre-multiplication</u> (Copeland 1974a, 137-139; Copeland, 1974b, 26-27; Piaget, 1952, 203-220). This interview covers concepts of numerical equivalence and foundational ideas for multiplication. It is described under <u>Numeration Interviews</u>.

<u>Subtraction/Place Value</u> (For Grades 1 to 3). This interview began with a game using base ten blocks to ensure that the child had some familiarity with structured units, tens, hundreds, . . . material. Third grade children were

then asked: 1) to show 365 with the blocks, 2) to read 699, 3) to write the successor of 699, and 4) to write the number 2 less than 300 [Tasks 3) and 4) were suggested by items in a Chelsea Diagnostic Mathematics Test (Brown, Hart, Kucheman, 1984, 6)]. Then, the following tasks were posed: 5) 527 - 332, 6) 702 - 25, and 7) 4 002 subtract 25 [6) and 7) were suggested by Davis and McKnight, 1980)] with the child being encouraged to describe what was done, and why, and to use the blocks, if needed. Second grade children were taken as far as task 6), but, the numbers 365, 699, 699, and 300 were replaced by 36, 69, 69, and 50, respectively, and 527 - 332 and 702 - 25 were replaced by 32 - 18 and 102 - 25. At the Grade 1 level only tasks 1) to 4) with the numbers 36, 39, 39, and 20 were used.

Correct responses to one or more of 1) to 3), only, were rated as Early Concrete Operational. Correct responses to 4) and/or 5), in addition to the preceding, were rated Late Concrete Operational. Correct responses to 6) or 7) (with some explanation of the procedure, with or without blocks) were rated Early Formal Operational.

The record sheets for these interview tasks, which were called "BLOCKS1," "BLOCKS2," and "BLOCKS3," respectively, are included in Appendix 1.

Measurement Interviews.

Length (For Grades 1 and 2). This interview, concerned with conservation of length and comparison of length, was conducted using two sets of task cards. The first set (cards A1 to A6) was adapted from an unpublished working paper (Bye, 1975); the second set was based on items constructed for the Concepts in Secondary Mathematics and Science (CSMS) project (Hart, 1981, pp.11-12).

Glued onto each of the cards in set A were two identical wooden sticks (about 1 mm x 5 mm x 100 mm). The sticks were placed in various orientations as is shown in the diagrams in Appendix 1. To begin the task, the interviewer gave the child several loose sticks identical to those on the cards and explained that the cards to follow had sticks like those glued onto them. As each card was presented the interviewer asked, "Are these two sticks both the same length or is one stick longer than the other? Which one is longer? Can you tell or show me how you know?"

The cards in set B each contained two lines drawn on one centimetre squared paper. On cards B1 to B4 the two lines were of different lengths but their endpoints were vertically aligned. The two lines on card B5 were the same length but their endpoints were not aligned. As each card was presented the same set of questions was asked as for set A.

As the interviews proceeded the interviewer made note not only of whether the answers were correct but also of the reasons given in response to the question, "How do you know?" Characteristic modes of thinking demonstrated by students in responding to the question are listed below. Each child's overall response for each of the two sets of tasks was rated as Preoperational, Early Concrete Operational, or Late Concrete Operational according to the levels of cognition predominating in the responses to each set.

When children based their judgements on perception, or focused on endpoints in isolation without taking into consideration the relation of the opposite endpoints (set A) or the shape of the line in between (set B), their responses were rated as Preoperational. Typically these children judged the sticks on card A2 and the lines on card B5 to be unequal because one end of one extends beyond the corresponding end of the other, while the lines on cards B1 to B4 were considered to be equal in length because the ends matched.

Responses of children who considered both ends of each line and attempted to coordinate relationships between corresponding endpoints were rated as Early Concrete Operational. For example, some such children "measured" using a stick or strip of paper to check the alignment of the ends rather than placing the measuring tool along the lines.

Children who correctly completed all or most of the tasks using measuring tools placed along the lengths of the objects to be compared and argued that since each object matched the same length on the measuring tool the objects themselves must match (transitive reasoning) were rated as Late Concrete Operational. Responses involving small errors using measuring tools and judgements that the sticks in set A differed very slightly in length were considered Late Concrete Operational as long as the logic was essentially as described above.

The record sheets for this interview, called LENGTH, are included in Appendix 1.

Time (For Grades 1, 2, and 3). The child was first asked to Count up to 15 along with the clicks produced by a "pacer" [one click per second] and to notice how far the hand of a stopwatch had moved by the end of the count [i.e., to the 15 seconds mark].

Then the stop-watch was masked and the instructions given to the child were: "Count up to 15 again, but this time twice as quickly . . . count two numbers to every click . . . . How far do you think that the [hidden] hand of the stop-watch went while you were counting faster? Why?" The following questions were also asked. "Does counting quickly take more time than counting slowly . . . or less time . . . or the same amount of time?" "Does the watch go more slowly at one time and more quickly at another . . . or does it always go the same?"

Responding that the stop-watch hand runs more or less rapidly according to the speed of work being timed (in this case, "counting to 15") was rated Preoperational. Inability to correlate the work done by oneself (the count of 15) with the steady motion of the stop-watch hand (appreciation of the conservation of velocity but inability to apply it to more than one moving body), refusal to make any predictions on the grounds that it is impossible to do so, inability to ascribe a unique unit of time or a common duration to motions having different velocities, or comparing the two counting speeds directly without reference to the watch was rated Early Concrete Operational. Late Concrete Operational Responses are characterized by predictions that the hand will stop before 15 (realizing that the speed of the watch is not affected by the speed of the work timed) accompanied by inaccurate guesses. Early Formal Operational responses include predictions of 7 1/2 (exact correlation between task duration and displacement of stop-watch hand) and approximately 7 1/2, with a logical explanation. (Piaget, 1969, 188-195).

The "TIME" interview record sheets are included in Appendix 1.

Weight (For Grades 1, 2, and 3). In Task One the child was asked to order by weight three plasticine "pebbles" whose weight could not be guessed from their volumes. A balance scale was available. Only two pebbles were allowed to be touched at the same time. In Task Two, three pebbles, the smallest being the heaviest, the largest the lightest, were given to the child to order from lightest to heaviest. In Task Three the child was first given six pebbles whose weights could not be determined by inspection alone and, then, three pebbles of identical volume but differing weights. Each time the child was asked to "arrange in order from lightest to heaviest by weighing two at a time." In Task Four the child was asked to arrange ten pebbles of the same volume but different weights in order from lightest to heaviest. In Task Five the child was asked to identify which of three identical matchboxes was the lightest and which the heaviest in each of three situations described as follows: 1) Box A is heavier than Box B and Box B is heavier than Box C, 2) Box A is heavier than Box B and Box C is lighter than Box B, and 3) Box B is lighter than Box A and heavier than Box C.

An Early Formal Operational response, indicating that the concept of weight seriation is fully developed, is characterized by correct orderings in all cases with only minor errors, if any. Correctly coordinating all of the weight relations in Tasks One and Two, correctly seriating in Task Four, but showing inability to coordinate the inverse relations in Task Three mark Late Concrete Operational responses. An Early Concrete Operational response is exemplified by an empirical approach to Task Four and failure to coordinate successive constructions. In Task Three it is established that A>B and C>D but that this does not tell anything about the other relations between A, B, C, and D. In a typical Preoperational response, Task One and Task Two are answered incorrectly because the pebbles are weighed one at a time and without any correlation. Both Preoperational and Early Concrete Operational responses are often based on size considerations, as when the child notes that "Pebble A is heaviest but it is lightest because it is smallest." The student responses were assigned cognitive level ratings on the basis of the overall patterns of response to the various subtasks contained in the Weight interview. (Piaget & Inhelder, 1974, 183-202).

The record sheet for the "WEIGHT" task is included in Appendix 1.

Geometry and Graphing Interviews

Classification, Inclusion (For Grades 1, 2, and 3). A set of geometric solids was placed on the table. The child was asked to sort them into two piles so that "things in each group are alike in some way." Then the child was asked "How are all the objects in this group alike?" The procedure was repeated but with the request that the objects be sorted in another way. Task Two was conducted with red and blue cardboard cutouts of squares and circles. After establishing that the child knew which were squares and which were circles, the interviewer asked two questions: 1) Are all the circles blue? and 2) Are all the blue ones circles? . . . Why? Grade 2 and 3 children were given an additional task consisting of a repetition of Task One with the solids replaced by large and small, single-thickness attribute shapes.

A typical Preoperational response shows inconsistency in naming an attribute common to all of the objects in a group and inability to consider an entire group of objects simultaneously in order to name a single common attibute. An Early Concrete Operational response is characterized by one or two different groupings of the objects but with an inability to identify the common attributes of each group. Late Concrete Operational responses clearly show evidence of flexible thinking in the classification tasks and in the identification of attributes in each subgroup formed. Late Concrete Operational responses include correct use of the concept of inclusion in Task Two. Interpreting the question "Are all of the circles some of the blues?" as "Are all the circles all the blues?" is rated as Early Concrete Operational. A Preoperational response is characterized by the child's inability to separate the circles as a class from the whole collection. "All" can only mean "the whole set of objects" at the Preoperational level of response. (Copeland, 1974b. 39-40).

The record sheets for the Classification, Inclusion task, which was called "SORTING," is included in Appendix 1.

<u>Graphing</u> (For Grade 3). For this interview, which was concerned with locating a point in two-dimensional space, the materials were sheets of plain white rectangular paper, a thirty-centimetre ruler (marked in centimetres only), an unmarked stick, strips of paper, lengths of string, pencils and markers. Two identical sheets of paper were placed at opposite corners of a table. On one of them a point, P, was marked in red about halfway between the centre of the rectangle and one of its corners. The child was asked to mark a point on the second sheet in the same position as P on the first, so that if the second sheet were placed on top of the first, the two points would coincide. The children were encouraged to use whichever of the measuring

tools they wished. After the first attempt the sheets were superimposed and the results evaluated. Children were then given a chance to try again if they wished.

Children's responses were rated as Preoperational it they placed their point by visual estimate and made no use of the materials provided and no attempt to measure.

Responses were rated as Early Concrete Operational if the point was located visually and measuring devices were used perceptually and inappropriately. Typical responses at this level involve measuring one distance only, either obliquely from one corner of the rectangle to the point or from one edge to the point.

When, through a process of trial and error, children discovered the need for two-dimensional measurement, their responses were rated as Late Concrete Operational.

When there was no trial-and-error behavior and the child immediately coordinated the two rectangular measurements, the response was rated as Early Formal Operational. (Piaget, 1960, pp. 153-169)

The record sheet for this interview is included in Appendix 1 under the name "DOT."

<u>Loci</u> (For Grades 2 and 3). The child watched as the interviewer marked two points on a blank piece of paper, saying "Let's imagine these are trees. Where can you stand to be the same distance from either tree?" The child would indicate the positions. Next, the child was asked to do the same for two series of trees lying on lines perpendicular to one another. In the third task, the child was asked to place beads to show where the trees might be planted in order to be "the same distance" or "just as far" from the dot.

A typical Early Formal Operational response demonstrates reasoning by recurrence, for example: After determining a few points in the series, the child concludes that all points in the circle or straight line must have the same property. Typical Late Concrete Operational responses show only an "inkling" of the "locus" and are simply an extension of the method used to place the first bead. There are occasional equidistance errors due to overemphasis on continuing in a chosen direction and to disregard for considerations of symmetry. Early Concrete Operational Responses are manifested by one or two solutions estimated perceptually, but fairly accurately, or by various responses produced apparently at random and irregularly. In Task Three no attempt is made to measure. A Preoperational Response is illustrated by random choice of points without regard for distance. (Piaget, 1960, 209-225).

The record sheet for the "LOCI" interview is included in Appendix 1.

Order (Linear, Circular) (For Grades 1 and 2). Shown a linear string of nine vari-coloured beads, the child was first asked to arrange a duplicate set of loose beads in the same order, then to arrange the loose beads in the reverse order, and, finally, to reproduce in linear order a string of twelve vari-coloured beads that were presented in a "figure 8" pattern.

Correct responses to all of the tasks were rated Late Concrete Operational, demonstrating an ability to order in both linear and circular arrangements, which requires the concept of reversibility. Correct responses to Tasks One and Two, only, were rated Early Concrete, indicating an ability to arrange in reverse order. An inability to coordinate a whole row of beads with a given linear ordering marks a Preoperational response. (Piaget, 1956, 80-103)

The record sheet for this interview, which came to be called "BEADS," is included in Appendix 1.

Cognitive Response Assessment: Paper-and-Pencil Tests

Eight different paper-and-pencil tests were developed: a Grade 3/4 and a Grade 5/6 test for each of Number, Operations, Measurement, and Space (Geometry and Graphing). Items were drawn, in the main, from tests developed by the Australian Council for Educational Research (ACER), the Mathematics Profile Series (MPS) and the ACER Mathematics (AM) Series. Other item sources were Brown, Hart, & Kucheman (1984) and Davis and McKnight (1980). All of the items were selected for assessing student cognitive response levels in accordance with the Cognitive Response Level criteria compiled by the ACLIC Team. The ACER items are based on cognition research by Collis (1972. 1975) and procedures have been established by ACER for connecting item performance with Piagetian levels. The other items were chosen according to the ACLIC criteria and were found to provide cognitive response assessments consistent with those of the ACER items.

Appropriate items from the Grade 3 and 6 Alberta Education Achievement Tests were placed on seven of the eight ACLIC paper and pencil tests so that the performance of the Grade 3 to 6 students in the sample could be compared with that of Grade 3 and 6 students across the province. A series of chi-square goodness-of-fit tests was conducted to determine whether the differences between the performances observed among students in the sample were significantly different from what would be predicted from the item statistics produced in the large-scale testing program. These tests are reported in detail in Appendix 3.

In comparing Grade 3 and 4 students actual performances on individual items with the results of the Grade 3 provincial test, 18 of the 30 differences were not statistically significant. The Grade 3 ACLIC students who were approximately half a year younger than the Grade 3 students who took the achievement test performed significantly lower in seven of the 15 comparisons, but the Grade 4 students who were approximately half a year older than the achievement test sample students scored significantly higher in three cases and significantly lower in two cases. The Grade 5 students who were about a year and a half younger than the Grade 6 achievement test sample students scored significantly lower in 18 of 21 comparisons. But the Grade 6 students who were about half a year younger than the students in the achievement test sample scored significantly lower in nine cases, no different in 11, and significantly higher in one case.

From these results it is concluded that the students in the ACLIC test sample are probably representative of students in the same grade across the province.

For each of the ACLIC paper-and-pencil tests, Table 1 lists the number of cognitive items, the mean item difficulty, the reliability, and the number of students who wrote the test. The reliabilities as measured by Cronbach's alpha ranged from 0.653 to 0.861, with a median of 0.717. Test reliabilities in this range are generally considered quite adequate in research of this sort (Nunnally, 1967).

The Number 3/4 test contains 26 items, four from ACER MPS, 13 from ACER AM, 3 from Brown, Hart, & Kucheman (1984), one from Davis & McKnight (1980), and 5 from the 1983 Alberta Education Grade [illegible] Mathematics Achievement Test.

Table 1

ACLIC Paper-and-pencil Test Cognitive Item Characteristics

ACLIC Test	Number of Items	Mean Item Difficulty	Reliability (Cronbach Alpha)	Number of Students
Number 3/4	21	0.539	0.653	209
Operations 3/4	20	0.718	0.861	229
Measurement 3/4	28	0.525	0.706	208
Space 3/4	25	0.389	0.661	202
Number 5/6	21	0.359	0.665	182
Operations 5/6	30	0.683	0.848	186
Measurement 5/6	30	0.464	0.728	251
Space 5/6	31	0.478	0.773	245

The Number 5/6 test contained 30 items, 21 from the ACER MPS and AM series and 9 from the 1983 Alberta Education Grade 6 Mathematics Achievement test. The number tests call for students to respond to items relating to representation of number, place value, expanded notation, rounding, decimal concepts, fractions, ratios, basic computation and comparison of numbers. The Number test items are similar to:

Which of the following gives 749 rounded to the nearest hundred?

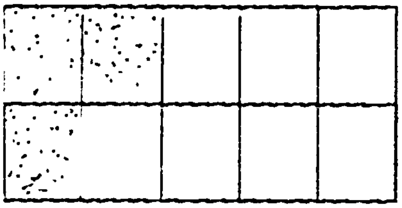
A 700 B 750 C 740 D 800

(Similar to ACER, 1980a, Unit I, p. 2, q. 3)

The number that is 2 less than 4000 is ______

(Similar to Brown, Hart & Kucheman, 1984, p.6)

What fraction of the figure is shaded?



A 0.4 B 0.3

C 0.7 D 0.6

(Similar to ACER, 1970a, p. 2, q. 7)

The Operations 3/4 test consists of 25 items, 20 of which came from the ACER MPS, and the other five were from the Alberta Education Grade 3 Mathematics Achievement Test. Operations 5/6 consists of 33 items, 30 of which were drawn from the ACER MPS and the balance from the 1983 Alberta Education Grade 6 Mathematics Achievement Test. The ACER items call for the student to demonstrate operational understanding of commutative, associative, distributive, inverse, and identity properties with numbers that are either single-digit or multi-digit. The only modification of the ACER Operations items for the purposes of the ACLIC study was the elimination of signed number distractors as these are outside of the Alberta Curriculum until Grade 6. The ACER Operations items are similar in format to:

2 + 5 = 5 + []

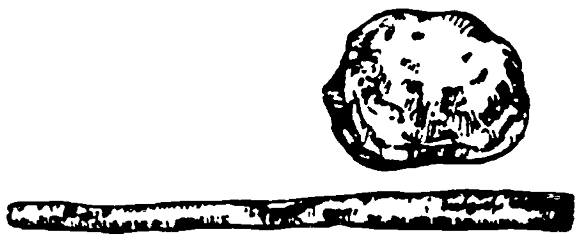
358 X 1 = []

(4 + 7) + 3 = [] + (7 + 3)

(Similar to ACER, 1977, pp. 1, 3, qs. 1, 24, 8)

The Measurement 3/4 test contained 28 items, all of which were drawn from the ACER MPS and AM series. The Measurement 5/6 test contained 35 items, 30 from the ACER MPS and AM series and 5 from the Alberta Education Grade 6 Mathematics Achievement Test. These tests call on students to demonstrate: conservation of length, area, volume, and mass; iteration of units of measurement; perceptions of time; the numerical representation of measurement; and the concept of rate. Sample items are:

"This lump of clay can be rolled into this shape.



"No clay was added and no clay was taken away. The long shape has

A less clay. B the same amount of clay. C more clay."

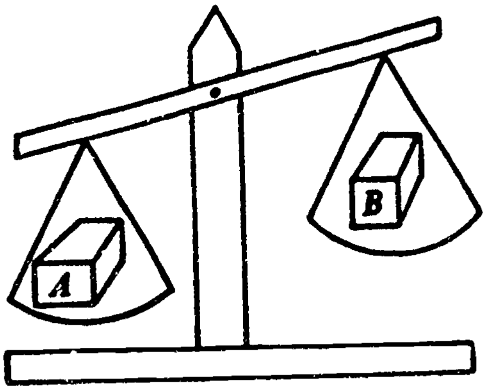
(ACER, 1971, p. 2, q. 3)

"Which box is heavier?

A box A

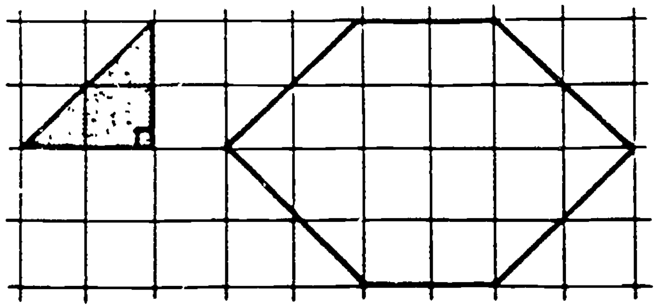
B box B

C There is no way of telling"



(ACER, 1970b, p. 2, q. 2)

"In the diagram the shaded triangle represents a unit which can be used to measure area.



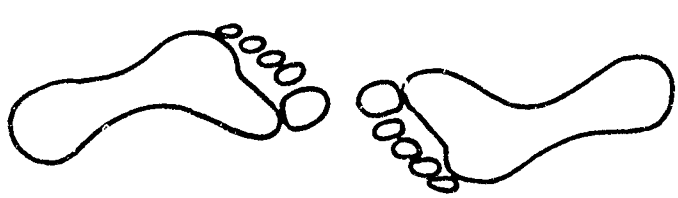
Which one of the following gives the number of such units in the hexagon?

A 5 B 6 C 8 D 12 "

(ACER, 1979, Unit II, p. 10, q. 18)

The Space 3/4 (Geometry and Graphing) test contains 30 items, 25 from the ACER MPS and 5 from the 1983 Alberta Education Grade 3 Mathematics Achievement Test. Space 5/6 contained 36 items, 32 from the ACER MPS and 4 from the 1983 Alberta Education Grade 6 Achievement Test. The Space tests call on students to demonstrate an understanding of order, reversibility, transformations, 3-dimensional visualization, location of objects in a coordinate plane, ability to handle multiple attributes of shapes, and knowledge of symmetry. The following items illustrate the types of questions included in the Space tests:

"These two foot prints were seen on the sand.



They were made by

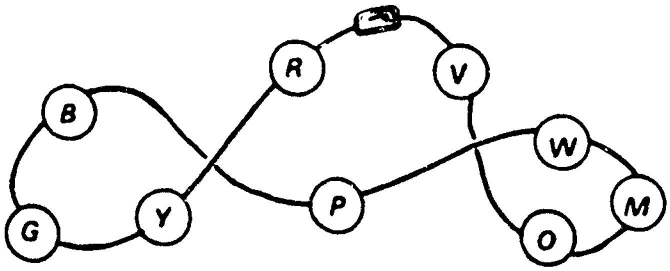
A a left foot and a right foot B two left feet C two right feet "

(ACER, 1978, Unit I, p. 4, q. 2)

"If the bead labelled V was threaded first, what was the order for threading the <u>rest</u> of the beads?

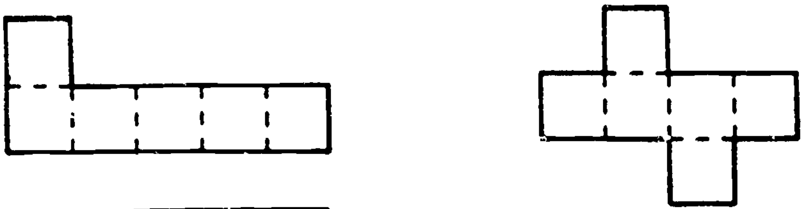
A W, M, O, P, Y, G, B, R　　B W, M, O, P, B, G, Y, R

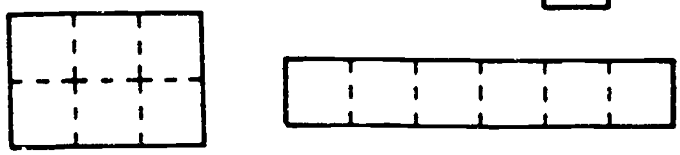
C O, M, W, P, Y, G, B, R　　D O, M, W, P, B, G, Y, R "



(Acer, 1978, Unit I, p. 4, q. 3)

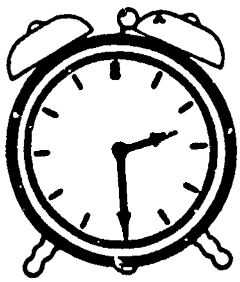
"Which of the following shapes can be folded on the dotted lines to from a cube?





(ACER, 1978, Unit I, p. 4, q. 4)

"This is the reflection of a clock in a mirror. It appears to be reading 'half past two.'



The time shown by the actual clock would be

A 2:30　　B 6:12　　C 9:30　　D 10:30 "

(ACER, 1978, Unit I, p. 9, q. 28)

Cognitive Demand Assessment: Curriculum Objectives

The ACLIC Team assessed the cognitive demand levels of all of the objectives stated in the 1982 Alberta Education Elementary Mathematics Curriculum Guide using the Cognitive Levels definitions stated previously and the Cognitive Demand Level Criteria summarized in Appendix 2. In Appendix 5 each of the curriculum objectives is listed along with one or more cognitive demand level ratings, depending on the number of subparts implied in the statement of the objective.

Cognitive Demand Assessment: Textbooks

The Cognitive Levels definitions and the Cognitive Demand Level Criteria (See Appendix 2.) were used to rate the cognitive demand levels of the explanations, problems, and exercises in two of the four authorized elementary school mathematics textbooks at each of the six grade levels (62 306 items were assessed altogether). Only those textbook items included in the Alberta curriculum were assessed. At each grade level at least one of the texts assessed was from one of the two series most often chosen by Alberta schools.

Cognitive Demand Assessment: Classroom Observations

Assessment of the cognitive demands made in regular classroom lessons was the only item in the ACLIC proposal that was greeted with strong reservations when the idea was discussed with teachers. For a time, the Project Team considered dropping this part of the planned cognitive demand assessments, but decided to proceed when encouraged to do so by the Steering Committee and when the Project Director expressed confidence that thirty or more elementary school teachers from the Calgary Board of Education would be willing to volunteer to have one of their classes observed for the purposes of the project. The response to the circulated request for teachers was gratifying, indeed, and a total of thirty-six classroom observations were arranged. Thirty-four Calgary Board of Education classes and two Calgary Catholic Board classes were observed. With the exception of Measurement in Grades 4 and 6, classroom lessons were observed at each grade level in every topic strand. Of the four kinds of cognitive demand assessment conducted in the ACLIC Project, only the classroom observations cannot be claimed to be representative of teachers across the province. The teachers observed were self-selected through the very process of volunteering, and they came only from two large urban school systems. Given the circumstances under which the classroom observations were arranged, the lessons observed would very likely represent better than average approaches to the teaching of elementary school mathematics in the province of Alberta.

Given the considerable pressures on the Project Team to complete the other cognitive assessments, and in the interests of consistent observations, it was decided to ask Richard Holmes to be the ACLIC Classroom Observer. Although the classroom observations had to start before the cognitive demand assessments of the Elementary Mathematics Curriculum Objectives could be completed, with the attendant refinement of the ACLIC cognitive demand criteria, the Project Team had numerous detailed discussions that included the Classroom Observer and that treated in detail the general descriptions of the cognitive response levels (see Definitions of Terms), specific mathematics topic examples from the interviews and paper-and-pencil tests, and trial cognitive assessments of videotaped lessons. The emphasis was on assessing the cognitive level of the expectations of students in the lessons observed. The discussions sharpened the observation procedures and critera, leading to an observation form that was designed to facilitate concise minute-by-minute observations throughout a whole class period. A copy of the Observation Record Sheet is included in Appendix 3.

In the actual classroom observations, paced by minute-interval beeps from a slide-rule computer, the Classroom Observer made cognitive level ratings every minute of the major emphases in the teacher's presentations and in the activities being engaged in by the majority of the students. The Observer followed the teacher when there were three or more groups working in the class or when at least half of the class was with the teacher. Copies of the worksheets used in each class were collected for subsequent cognitive assessment and a note was made of any textbook or workbook pages assigned to be completed in class. When centres were used, the type of activity and materials at each centre were recorded for later assessment. Split grades were treated as two grades with records being made of the separate lessons, assignments, and other materials.

With one or two exceptions (in which a "special" lesson was apparently or allegedly presented), the lessons observed were those normally taught by the teacher as part of the regular program.

Cognitive Demand Assessment: Achievement Tests

Using the Cognitive Levels definitions and the Cognitive Demand Level Criteria (See Appendix 2), cognitive demand assessments were made of all 95 of the items in Parts A and B of the 1982 Grade 3 and 1983 Grade 6 Alberta Mathematics Achievement Tests. The purely computational items contained in the other parts of the Achievement Tests were not included.

Analysis of Data

The data for the first and second research questions investigated in the ACLIC Project were collected from student Interview and Paper-and-pencil cognitive response assessments and from cognitive demand assessments of the curriculum objectives, textbook materials, classroom observations, and Alberta Education achievement test items. The findings were tabulated and presented in the form of of percentage distributions of responses or demands found at the Preoperational, Early Concrete Operational, Late Concrete Operational, Early Formal Operational, or Formal Operational levels.

The third research question investigated by the ACLIC Project asked: "How well do the distributions of curricular cognitive demands fit the distributions of student cognitive responses at each grade level in each mathematics topic strand?" In the process of choosing an appropriate statistical analysis to provide answers for this question, an assumption was made that the observed distributions of student cognitive responses should be reflected, at least approximately, by the distributions of cognitive demands found in the curricular material the students are expected to learn. In one sense, Research Question 3 asks how well the curricular materials anticipate the cognitive levels at which the intended students normally operate, given a particular mathematics learning context. If, for example, many students are still operating at a Preoperational level in a given topic but they are expected to learn only from materials that require at least Concrete Operational thinking, a cognitive mismatch between students and curriculum is apparent. Similarly, a mismatch can occur when there is little or no material of suitable sophistication to challenge Early Formal or Formal Operational thinkers. To answer Research Question 3, two-sample Kolmogorov-Smirnov tests (Siegel, 1956, 127-136) were used to determine whether or not there were significant differences between the ordered distributions of student responses and the ordered distributions of the relevant curricular demands at each grade level and in each topic strand. The Kolmogorov-Smirnov procedure assesses the absolute differences between the cumulative proportions produced by the two frequency distributions being compared. It tests whether two independent samples have been drawn from populations with the same distribution (Siegel, 1956, 127). The maximum difference observed between two cumulative proportion distributions, the Kolmogorov-Smirnov "D," can be compared with a critical "D," for which the sampling distribution is known under the hypothesis of no significant differences between the distributions. For a given sample size, probabilities can be associated with the occurence of a difference as large as the observed K-S "D." Each Curricular Cognitive Demand frequency distribution was compared with that of the observed Pupil Cognitive Responses after the Demand frequencies had been scaled so that their totals matched the number of students assessed by interview or by paper-and-pencil test. The Null Hypothesis of no significant difference between two distributions was accepted if the probability of observing the calculated K-S D was greater than 0.05. For N=60 and p=0.05, the critical two-sample K-S D value is 0.248. When the observed K-S D exceeded this value, the null hypothesis was rejected, indicating that there was a significant difference between the two distributions. While the K-S D statistic itself is not affected by the sample size, N, the critical value of "D" is. For example, for N=100 and p=0.05, the critical two-sample K-S D is 0.192 (as compared with 0.248 when N=60).

FINDINGS OF THE STUDY

The presentation of the findings of the study is organized in the following way. First, the cognitive levels of student responses are reported by strand and by grade. Next, the levels of cognitive demands of objectives, textbooks, classroom presentations, and provincial achievement tests are detailed. Finally demands are compared with responses by strand and by grade.

Cognitive Levels of Students' Responses.

Cognitive levels of students' responses were observed in interviews (Grades 1 to 3) and in paper-and-pencil tests (Grades 3 to 6). Since these modes of assessment are so different, an important question is whether the two methods are consistent and yeild the same or similar distributions of students tc cognitve response levels. At the Grade 3 level where some students who were interviewed also took paper-and-pencil tests, this question is answered in the affirmative by means of a statistical test presented in Figures 21, 23, 25, and 27, and described in a later section.

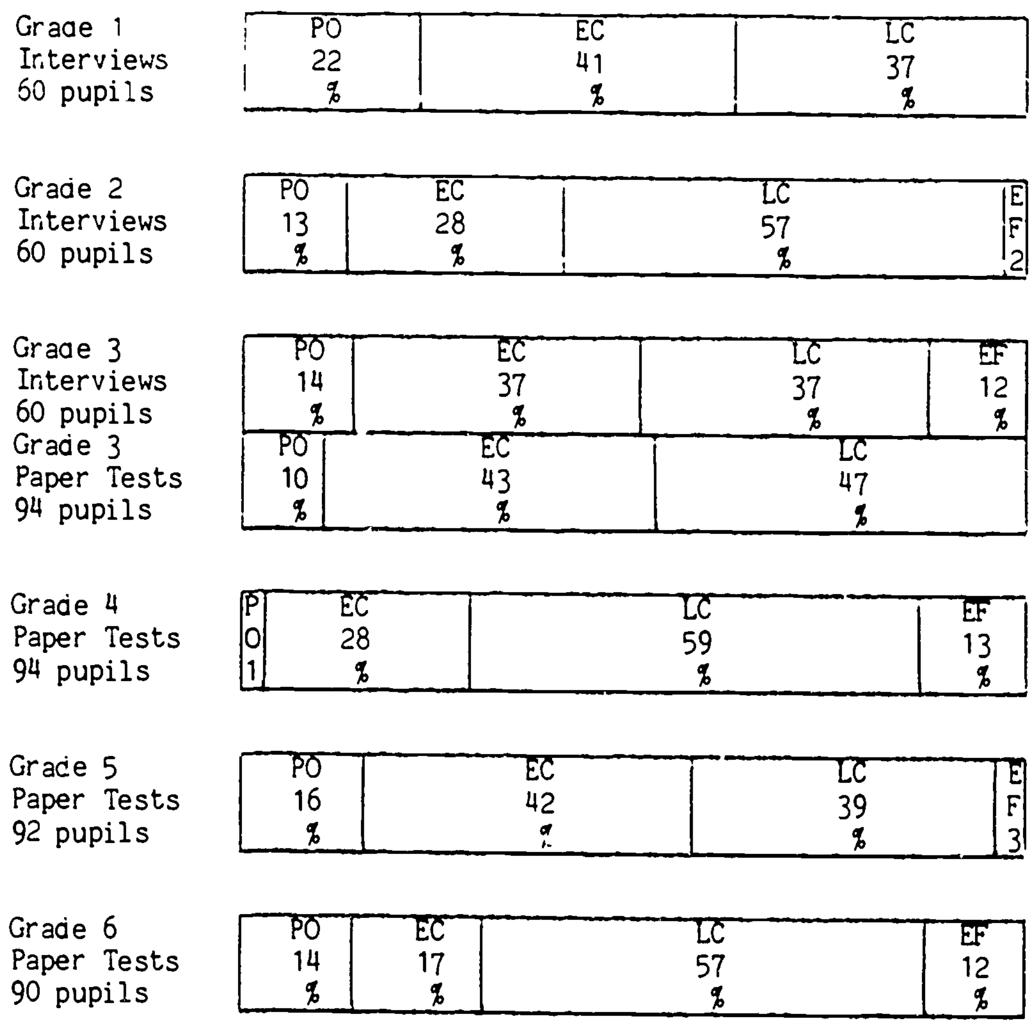
Numeration.

The findings regarding the cognitive levels of students' responses in the Numeration strand are presented in Figure 1.

The percentage of Preoperational responses ranged from a high of 22% in Grade 1 to a low of 1% in Grade 4. Interestingly there were more Preoperational responses in Grades 5 and 6 (16% and 14%, respectively) than in the two preceeding grades. One reason for this apparent anomoly may be that the tasks and test items differ for the different grades. The test for Grades 5 and 6 placed a heavy emphasis on fractions and decimals, topics at a high cognitive level. Students who performed Preoperationally in this area might not have done so in the context of whole numbers which received greater emphasis in the earlier grades.

It is interesting to note that at each grade three-quarters or more of

Figure 1: Cognitive Levels of Pupils' Responses, Numeratic



the responses were at concrete operational levels (Early Concrete and Late Concrete combined).

The percentage of Early Formal Operational responses increased from 0 in Grade 1 and 2% in Grade 2 to about 12% in Grades 3, 4, and 6. Again, the fact that the percentage does not rise steadily is probably due to differences in the mathematical contexts in which the assessments were made.

Operations.

Findings with respect to cognitive levels of students' responses in Operations are presented in Figure 2.

The percentage of Preoperational responses decreased steadily from 29% in Grade 1 and 13% in Grade 2 to about 4% in Grade 3 and nil thereafter. As in Numeration, a substantial majority of responses were Concrete Operational in Grades 1 to 5. But by Grade 6 the percentage at these levels was found to be only 34%, the Early Formal Operational and Formal Operational responses having risen to 66%.

Measurement.

The findings regarding the cognitive levels of students' responses in Measurement are presented in Figure 3.

In this strand about two-thirds of the responses of Grade 1 children and about half of the responses of Grade 2 children were Preoperational. The fraction drops to about a third in Grade 3 and even less in Grades 4, 5, and 6. Concrete Operational responses are in the majority from Grade 2 onward. The percentage of Early Formal Operational and Formal Operational responses increases from 0 in Grade 1 and 2% in Grade 2 to 20% in Grade 6.

Geometry and Graphing.

The findings concerning the cognitive levels of students' reponses in Geometry and Graphing are shown in Figure 4.

The percentage of responses that were Preoperational declined steadily

Figure 2: Cognitive Levels of Pupils' Responses, Operations

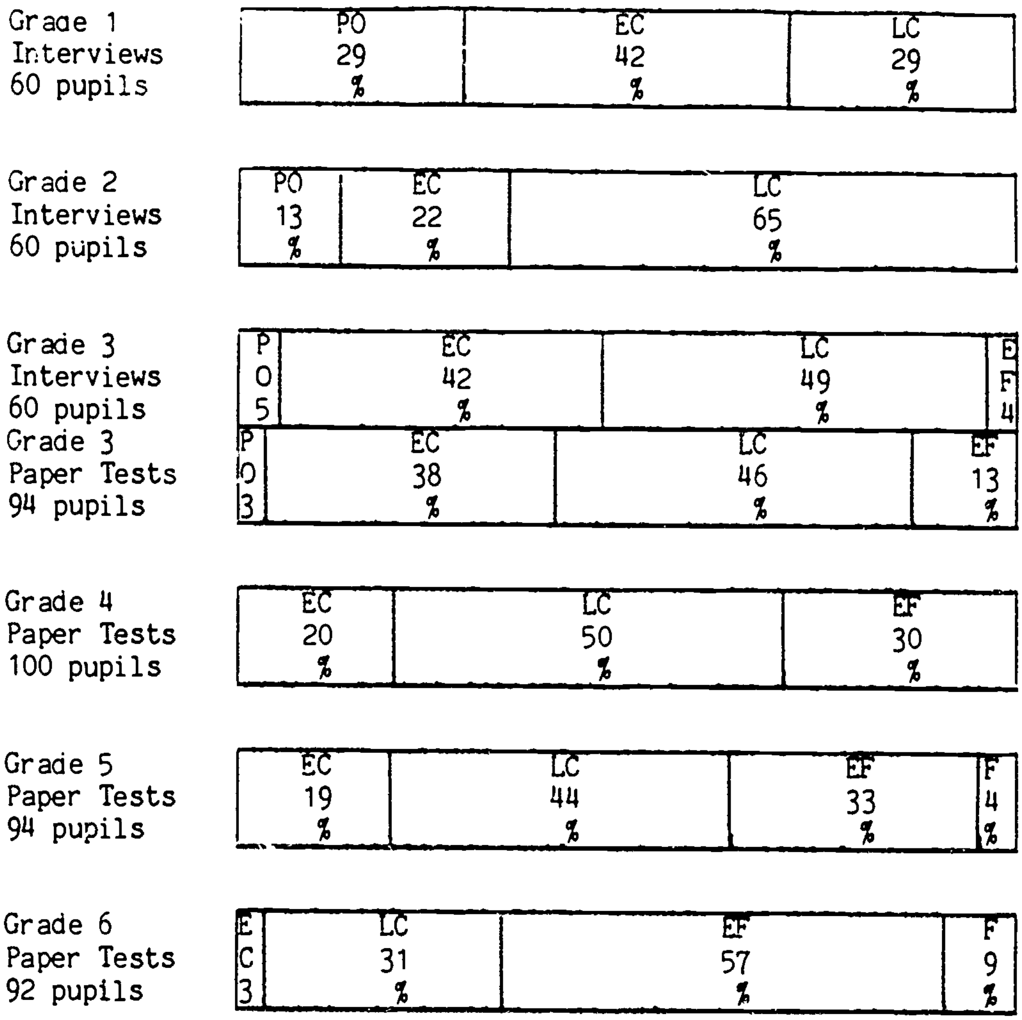


Figure 3: Cognitive Levels of Pupils' Responses, Measurement

	PO	EC	LC	EF	F
Grade 1 Interviews 60 pupils	64 %	21 %	15 %		
Grade 2 Interviews 60 pupils	49 %	27 %	22 %	EF 2	
Grade 3 Interviews 60 pupils	33 %	37 %	21 %	9 %	
Grade Paper Tests 92 pupils	27 %	53 %	18 %	EF 2	
Grade 4 Paper Tests 116 pupils	9 %	48 %	39 %	4 %	
Grade 5 Paper Tests 138 pupils	21 %	23 %	50 %	6 %	
Grade 6 Paper Tests 113 pupils	4 %	14 %	62 %	19 %	1 %

Figure 4: Cognitive Levels of Pupils' Responses, Geometry & Graphing

	PO	EC	LC	EF	F
Grade 1 Interviews 60 pupils	37 %	22 %	41 %		
Grade 2 Interviews 60 pupils	23 %	49 %	26 %	2	
Grade 3 Interviews 60 pupils	26 %	40 %	27 %	7 %	
Grade 3 Paper Tests 112 pupils	24 %	43 %	33 %		
Grade 4 Paper Tests 90 pupils	14 %	36 %	41 %	9 %	
Grade 5 Paper Tests 151 pupils	13 %	46 %	36 %	5 %	
Grade 6 Paper Tests 94 pupils	4 %	19 %	54 %	22 %	1 %

from 37% in Grade 1 to 4% in Grade 6, while the percentage of Formal Operational responses rose fairly evenly from 0 in Grade 1 and 2% in Grade 2 to 23% in Grade 6. About two-thirds or more of the responses were Concrete Operational in all six Grades.

An interesting observation arising from the analysis of the pupils' responses to the Classification-Inclusion tasks was that the additional two-dimensional task assigned to the Grade 2 and 3 children produced more correct responses than did the comparable task with the solids. This raises a question about which is more demanding cognitively in the concrete mode, working with three-dimensional or two-dimensional shapes.

Cognitive Levels of Students' Responses by Grades.

Figures 5 and 6 show the levels of students' responses for each grade. Generally the pattern which emerges is that the percentage of Preoperational responses decreases and the percentage of Early Formal and Formal Operational responses increases from Grade 1 to 6. At each grade level the clear majority of responses is at the Concrete Operational level (Early Concrete and Late Concrete combined).

Generalizing across grades, the Measurement strand stands out as the one with the largest proportions of Preoperational responses and the smallest proportions of Early Formal Operational responses. On the other hand, the Operations strand appears to have the greatest percentages of higher-level responses.

Curricular Cognitive Demands.

Cognitive Demands of Curriculum Objectives.

A detailed listing of the Grade 1 to 6 curriculum objectives along with their assessed cognitive demand levels is included in Appendix 5. In the sections which follow the curriculum objective demand level findings are summarized by strand.

Figure 5

Grade 1 Pupil Response Levels by Strands

Strand	PO	EC	LC
Numeration	22 %	41 %	37 %
Operations	29 %	42 %	29 %
Measurement	64 %	21 %	15 %
Geometry & Graphing	37 %	22 %	41 %

Grade 2 Pupil Response Levels by Strands

Strand	PO	EC	LC	EF
Numeration	13 %	28 %	57 %	2
Operations	13 %	22 %	65 %	
Measurement	49 %	27 %	22 %	2
Geometry & Graphing	23 %	49 %	26 %	2

Grade 3 Pupil Response Levels by Strands

Strand	PO	EC	LC	EF
Numeration	14 %	37 %	37 %	12 %
Operations	5	42 %	49 %	4
Measurement	33 %	37 %	21 %	9 %
Geometry & Graphing	26 %	40 %	27 %	7 %

Figure 6

Grade 4 Pupil Response Levels by Strands

Strand	PO	EC	LC	EF	F
Numeration	1	28 %	59 %	13 %	
Operations		20 %	50 %	30 %	
Measurement	9 %	48 %	39 %	4 %	
Geometry & Graphing	14 %	36 %	41 %	9 %	

Grade 5 Pupil Response Levels by Strands

Strand	PO	EC	LC	EF	F
Numeration	16 %	42 %	39 %	3	
Operations		19 %	44 %	33 %	4 %
Measurement	21 %	23 %	50 %	6 %	
Geometry & Graphing	13 %	46 %	36 %	5 %	

Grade 6 Pupil Response Levels by Strands

Strand	PO	EC	LC	EF	F
Numeration	14 %	17 %	57 %	12 %	
Operations		3	31 %	57 %	9 %
Measurement	4	14 %	62 %	19 %	1 %
Geometry & Graphing	4 %	19 %	54 %	22 %	1 %

Numeration. Figure 7 shows the cognitive demand levels of objectives in the Numeration strand. Demands at the Preoperational level are found only in Grade 1 (17%). Early Concrete Operational demands are included in each grade, steadily decreasing from 66% in Grade 1 to 14% in Grade 6. The percentage of Late Concrete Operational demands increases from 17% in Grade 1 to 55% in Grades 3 and 4, but decreases thereafter. Early Formal Operational demands are not found before Grade 4, but increase from 27% in Grade 4 to 54% in Grade 5 and 50% in Grade 6.

Operations. Cognitive demand levels of Operations objectives are shown in Figure 8. There are no objectives with Preoperational demand levels in this strand. The proportion of Early Concrete Operational demands decreases from more than two-thirds in Grades 1 and 2 to about a quarter in Grades 4, 5, and 6. As in the Numeration strand, the percentage of objectives at the Late Concrete Operational level increases then declines, from about 30% in Grades 1 and 2, to about 50% in Grades 3 and 4, to about 20% in Grades 5 and 6. Early Formal Operational demands first appear in Grade 3 (8%) and rise to nearly 50% in Grades 5 and 6. In Grade 6, 8% of the demands are at the Formal Operational level.

Measurement. Figure 9 displays the cognitive demand levels of Measurement objectives. Preoperational demands are found only in Grades 1 and 2 (26% and 13%, respectively). Objectives with demands at the Early Concrete Operational level are prominent in Grades 1 and 2 (42% and 29%, respectively), but constitute about 10% or less of the objectives in later grades. The percentage of Late Concrete Operational demands rises then declines as in the previous two strands. Early Formal Operational demands first appear in Grade 4 (7%), increasing to 26% in Grade 5 and 60% in Grade 6.

Geometry & Graphing. The cognitive demand levels of objectives in the Geometry and Graphing strands are displayed in Figure 10. Preoperational

Figure 7: Cognitive Demands of Numeration Objectives

Grade 1, 6 items	PO 17%	EC 66%	LC 17%
Grade 2, 9 items	EC 56%	LC 44%	
Grade 3, 11 items	EC 45%	LC 55%	
Grade 4, 11 items	EC 18%	LC 55%	EF 27%
Grade 5, 13 items	EC 15%	LC 31%	EF 54%
Grade 6, 14 items	EC 14%	LC 36%	EF 50%

Figure 8: Cognitive Demands of Operations Objectives

Grade 1, 6 items	EC 67%	LC 33%		
Grade 2, 9 items	EC 78%	LC 22%		
Grade 3, 26 items	EC 42%	LC 50%	EF 8%	
Grade 4, 23 items	EC 26%	LC 52%	EF 22%	
Grade 5, 25 items	EC 24%	LC 28%	EF 48%	
Grade 6, 25 items	EC 28%	LC 20%	EF 44%	F 8%

Figure 9: Cognitive Demands of Measurement Objectives

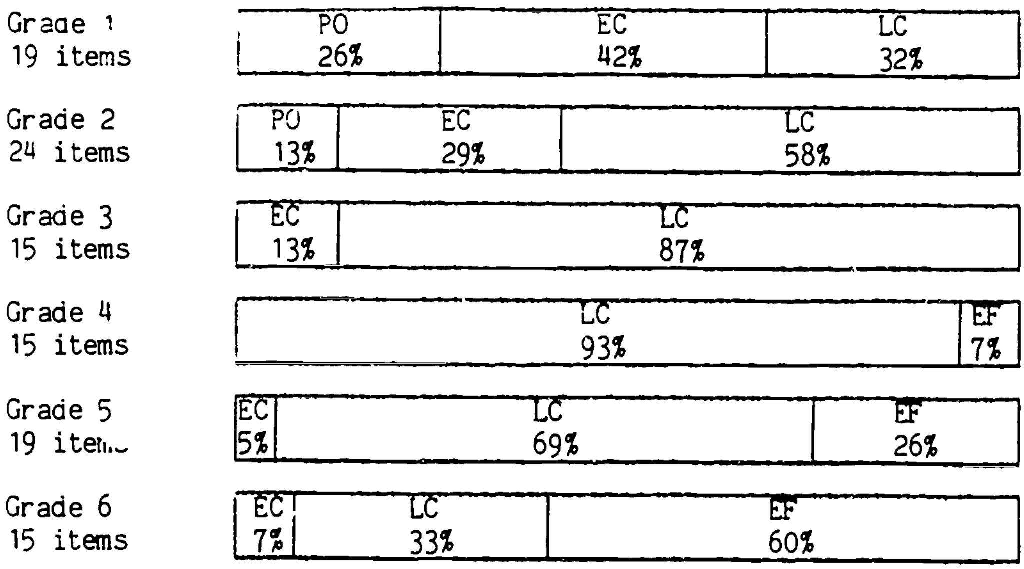
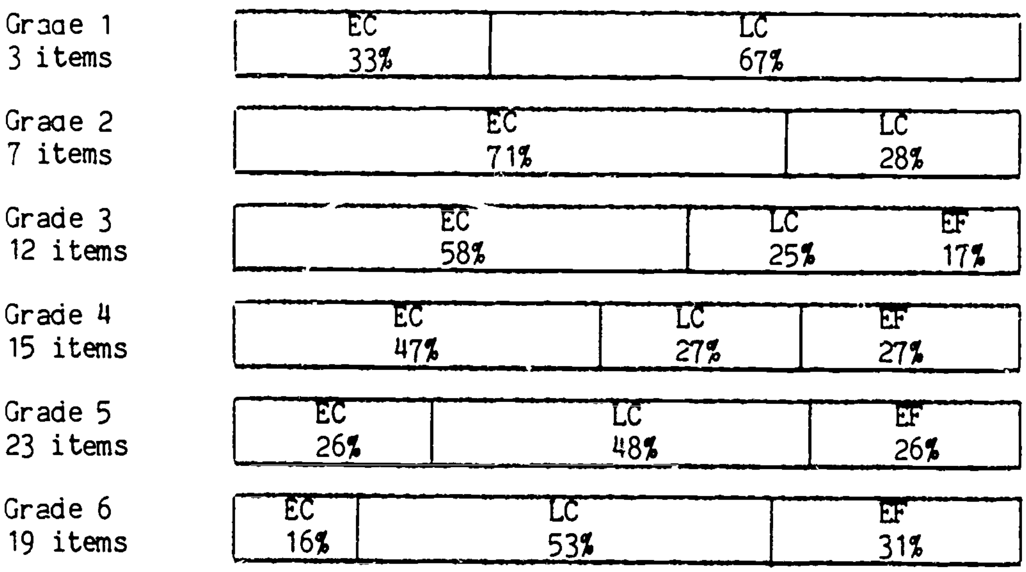


Figure 10: Cognitive Demands of Geometry & Graphing Objectives



demands occur only in Grade 1 (33%). The percentage of Early Concrete Operational demands declines steadily from 67% in Grade 1 to 16% in Grade 6, while the percentage of Late Concrete Operational demands rises from 0 in Grade 1 and 29% in Grade 2 to 53% in Grade 6. Early Formal Operational demands first appear in Grade 3 (17%) and increase steadily to 31% in Grade 6.

Cognitive Demands of Prescribed Textbooks.

As described previously, the Cognitive Levels definitions and the Cognitive Demand Level Criteria (see Appendix 5) were used to assess the cognitive demand levels of every curriculum-related explanation, problem, and exercise in two prescribed textbooks at each grade level.

Numeration. Figure 11 presents the levels of cognitive demands of Numeration items in prescribed textbooks. Demands at the Preoperational level occurred in Grade 1 (22%), but rarely (3% or less) in later grades. The percentage of textbook demands at the Early Concrete Operational level declines from 68% in Grade 1 to only 1% in Grade 6. The percentage of Late Concrete Operational demands rises from 10% in Grade 1 to a maximum of 55% in Grade 4, then falls to 15% by Grade 6. Early Formal Operational demands begin in Grade 4 at 23% and predominate by Grade 6, where they represent 84% of the items.

Operations. Cognitive demand levels of Operations items in textbooks are shown in Figure 12. There are no textbook items with Preoperational demand levels in this strand. The Early Concrete Operational demands range from 91% in Grade 1 to 4% in Grade 6. The percentage of Late Concrete Operational dem[illegible] increases from 9% in Grade 1 to a maximum of 56% in Grade 4 then decreases to 31% in Grade 6. Early Formal Operational demands first appear in Grade 3 (4%) and reach a level of 57% in Grade 6. A further 9% of the Grade 6 demands are at the Formal Operational level.

Measurement. Figure 13 shows the cognitive demand levels of

Figure 11: Cognitive Demands of Textbooks, Numeration

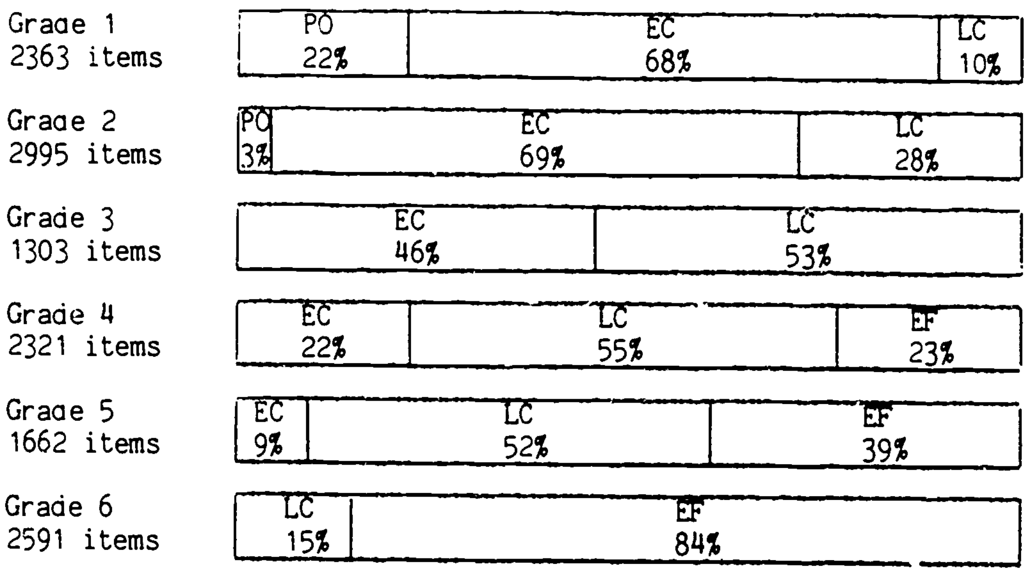
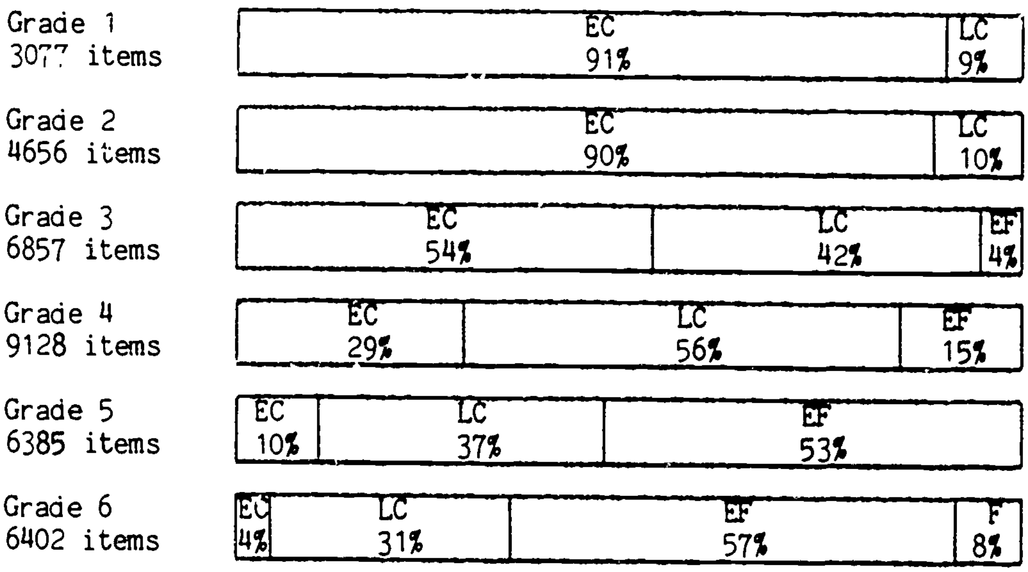


Figure 12: Cognitive Demands of Textbooks, Operations



Measurement items in textbooks. The percentage of Preoperational demands was 7% in Grade 1, but negligible in later grades. Early Concrete Operational demands declined from 81% in Grade 1 to 10% in Grade 6. Late Concrete Operational demands increased from 12% in Grade 1 to a peak of 75% in Grade 3, then declined to 29% in Grade 5 and 41% in Grade 6. The percentage of Early Formal Operational demands is 0 in Grades 1 to 3, 24% in Grade 4, 63% in Grade 5, and 49% in Grade 6.

Geometry & Graphing. The cognitive demand levels of textbook items on Geometry and Graphing are presented in Figure 14. The percentage of Preoperational demands was 49% in Grade 1 and 29% in Grade 2, but insignificant in the other grades. About half or slightly more of the demands at each grade level were at the Early Concrete Operational level. The percentage of textbook demands at the Late Concrete Operational level was 0 in Grade 1 and about 20% in all the other grades except Grade 5 where it was 45%. Early Formal Operational demands increased from 0 in Grades 1 and 2 and 4% in Grade 3 to a maximum of 22% in Grade 6.

Cognitive Demands of Classroom Observations.

A detailed tabulation of the classroom observations showing the numbers of minutes observed at each cognitive demand level for each grade and strand is included in Appendix 6. In the sections which follow, these findings are summarized by strand.

Numeration. A summary of the levels of cognitive demands of classroom observations in Numeration is presented in Figure 15. No Preoperational demands were observed, but Early Concrete Operational demands were found in Grade 1 (97%) and Grade 2 (38%). Late Concrete Operational demands comprised about half of the sample of demands in Grades 2, 3, and 5 and all of the small sample (8 minutes) of Numeration demands in Grade 3. The percentage of Early Formal Operational demands was insignificant in Grades 1 to 3 but rose

Figure 13: Cognitive Demands of Textbooks Measurement

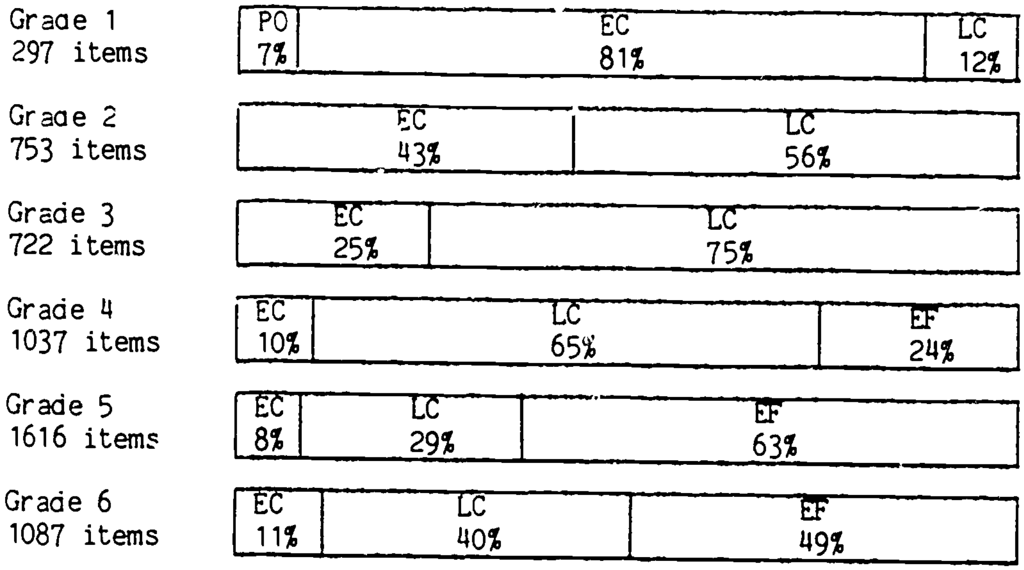
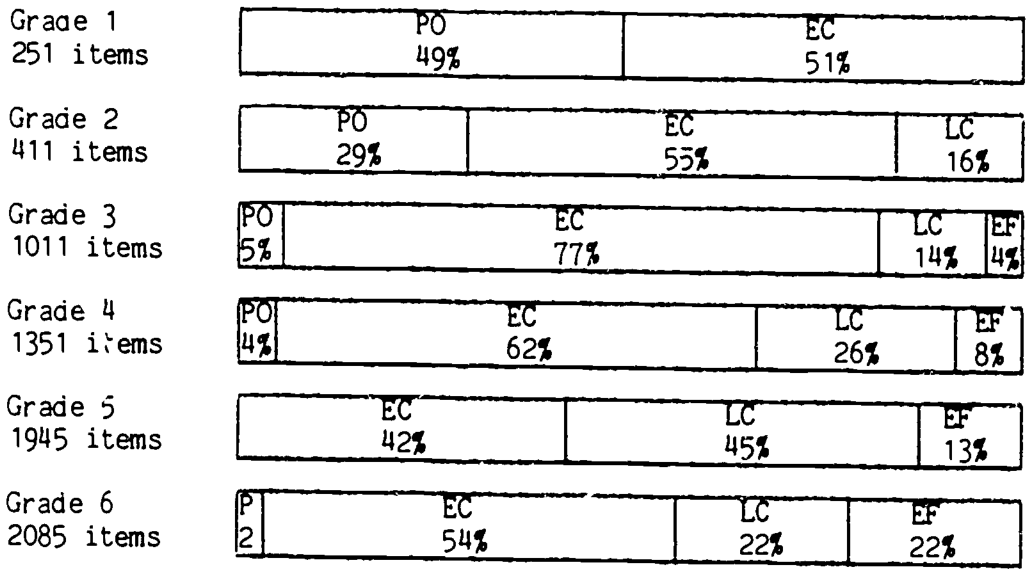


Figure 14: Cognitive Demands of Textbooks, Geometry & Graphing



steadily from 43% in Grade 4 to 91% in Grade 6.

Operations. Figure 16 presents a summary of the levels of cognitive demand in Operations observed in classrooms. No demands at the Preoperational level were observed. Early Concrete Operational demands ranged from 97% in Grade 1 to 2% in Grade 5 and none in Grade 6. Late Concrete Operational demands rose from 23% in Grade 2 to a maximum of 43% in Grade 3 and declined to 28% in Grade 5 and 0 in Grade 6. Early Formal Operational demands first appeared in Grade 2 (17%) and peaked at 70% in Grade 5. The demands in Grade 6 were 22% Early Concrete Operational with the remaining 78% Formal Operational.

Measurement. Figure 17 summarizes the cognitive demands of classroom observations in Measurement. Unfortunately, no classroom observations were made of measurement activities in Grades 4 and 6. In Grade 1 76% of the demands were at the Early Concrete Operational level, only 1% were Late Concrete Operational, and 22% were Early Formal Operational. In Grades 2, 3, and 5 about three-quarters or more of the demands were Late Concrete Operational and the rest were Early Formal Operational.

Geometry & Graphing. The cognitive demands of classroom observations involving Geometry and Graphing are presented in Figure 18. The situation in Grades 1 to 3 is notable for its lack of Late Concrete Operational demands (6% in Grade 3 only). In Grades 1 and 3 about two-thirds of the demands were Early Concrete Operational and one-third were Early Formal Operational, while in Grade 2 these fractions were reversed. In Grade 4 the demands were split almost evenly between the Early Concrete Operational and Late Concrete Operational categories. Grade 5 has some Early Concrete Operational, about half Late Concrete Operational, and some Early Formal Operational demands. In Grade 6 the demands were almost entirely Late Concrete Operational, except for a few (11%) Early Formal Operational.

Figure 15: Cognitive Demands of Classroom Observations, Numeration

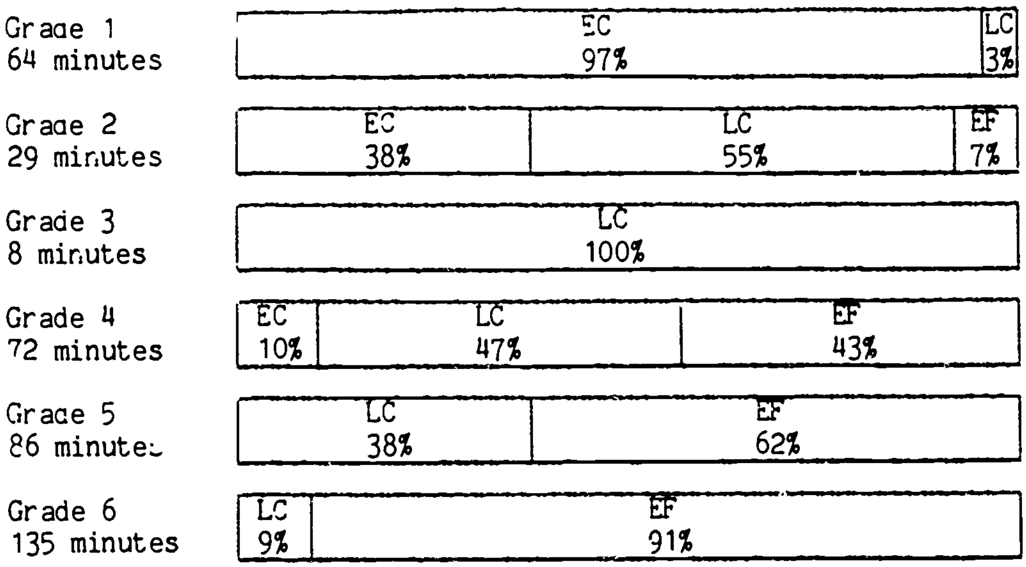


Figure 16: Cognitive Demands of Classroom Observations, Operations

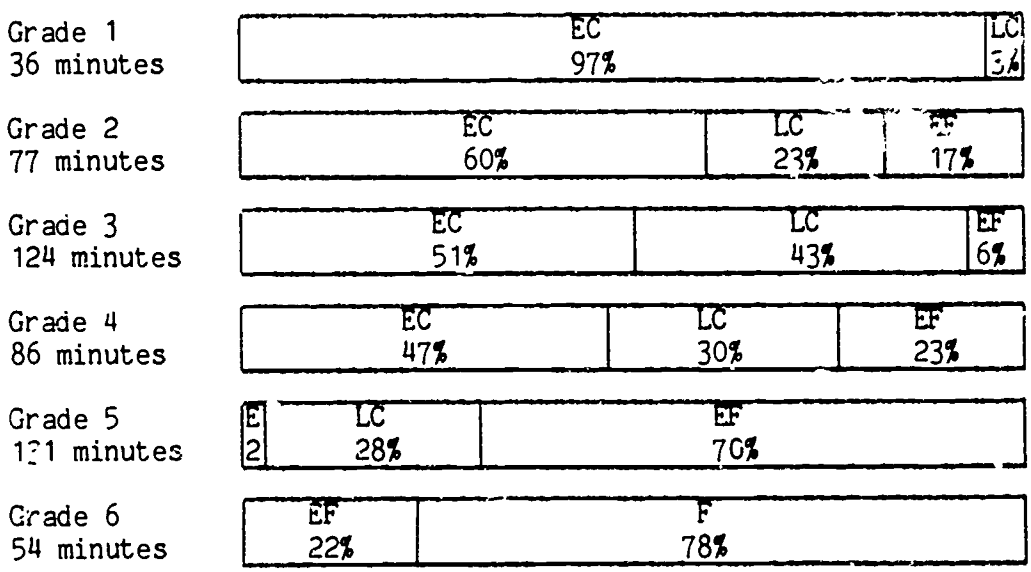


Figure 17: Cognitive Demands of Classrooms Observed, Measurement

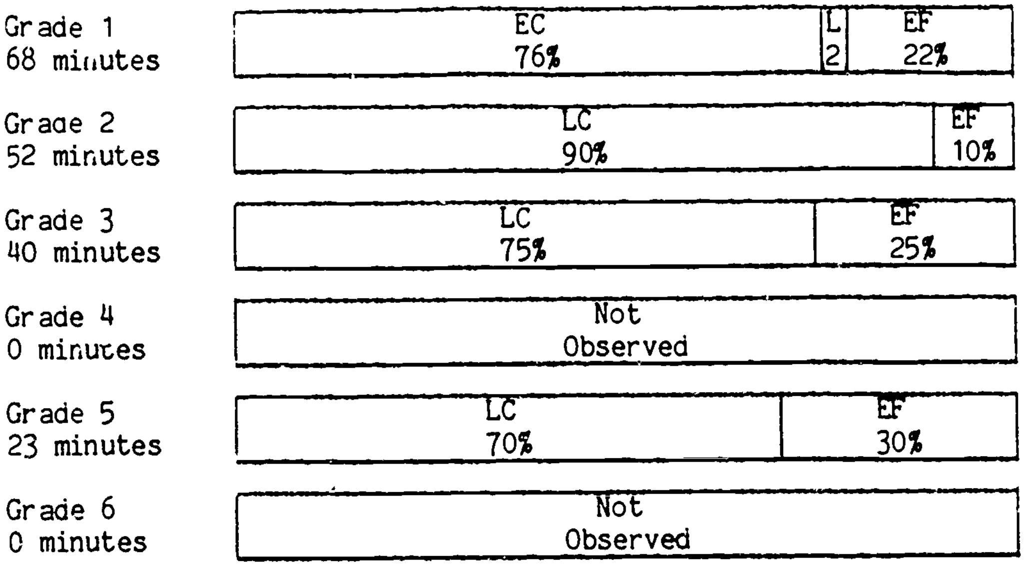
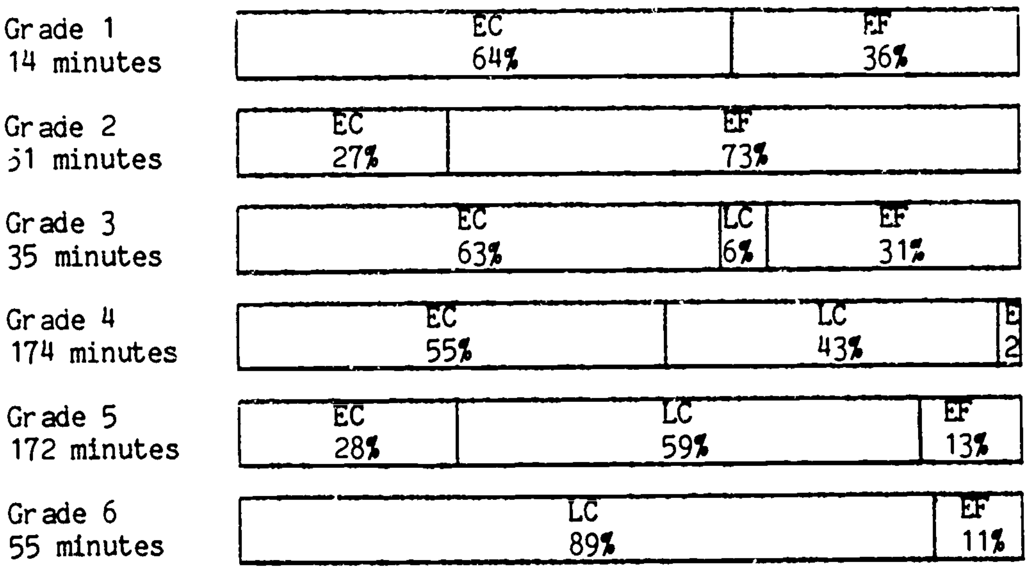


Figure 18: Cognitive Demands of Classrooms Observed, Geometry & Graphing



Cognitive Demands of Provincial Achievement Tests.

Appendix 7 contains a detailed listing of the topics and cognitive demands of items in Parts A and B of the Alberta Mathematics Achievement Tests for Grade 3 (June 1982) and Grade 6 (June 1983). The remaining Parts of these tests which contain computational exercises only were not assessed. The sections which follow summarize the findings.

Grade 3. Figure 21 shows the distributions of the cognitive demands of the 50 items of the Grade 3 Achievement Test by curriculum strand and for the test as a whole. Except for one item in Geometry and Graphing which had a demand at the Early Formal Operational level, all the item demands were either Early Concrete Operational or Late Concrete Operational. For the test as a whole the demands were about one-third Early and two-thirds Late Concrete Operational. The Numeration strand and Geometry and Graphing strand had demands about evenly divided between Early and Late Concrete Operational, while the Operations and Measurement strands were more heavily weighted with Late Concrete Operational demands (86% and 83%, respectively).

Grade 6. The cognitive demands of the 45 items on the Grade 6 Achievement Test are shown in Figure 22. One item in Numeration and one in Geometry had demands at the Early Concrete Operational level. Late Concrete Operational items made up 29% of the test as a whole, those being 11% of the Measurement items, 27% of the items in Numeration and in Operations, and 50% of those in Geometry and Graphing. Three-fifths of the items had demands at the Early Formal Operational level; this comprised about 90% of the Measurement items and about half of the items in the other strands. Three items in Operations (20% of the strand, 7% of the whole test) were at the Formal Operational level.

Figure 19: Cognitive Demands of Grade 3 Provincial Achievement Test

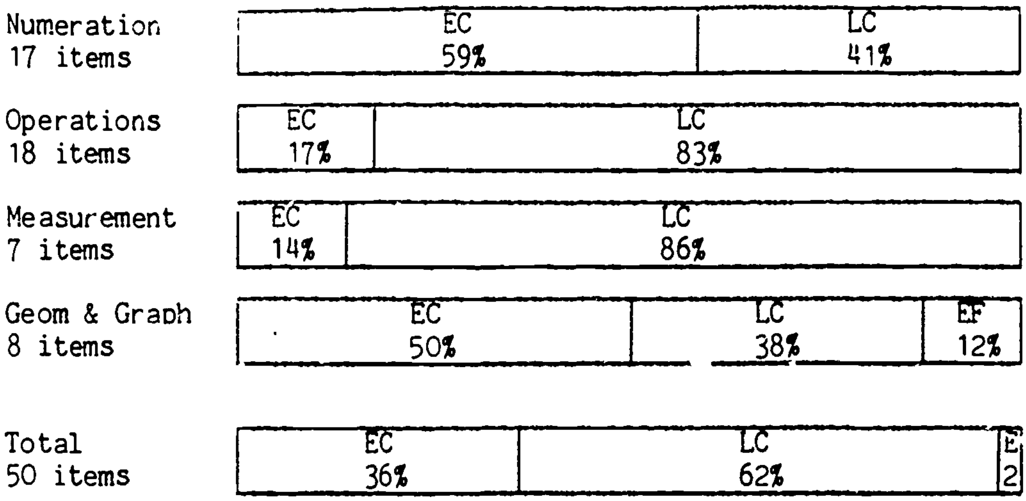
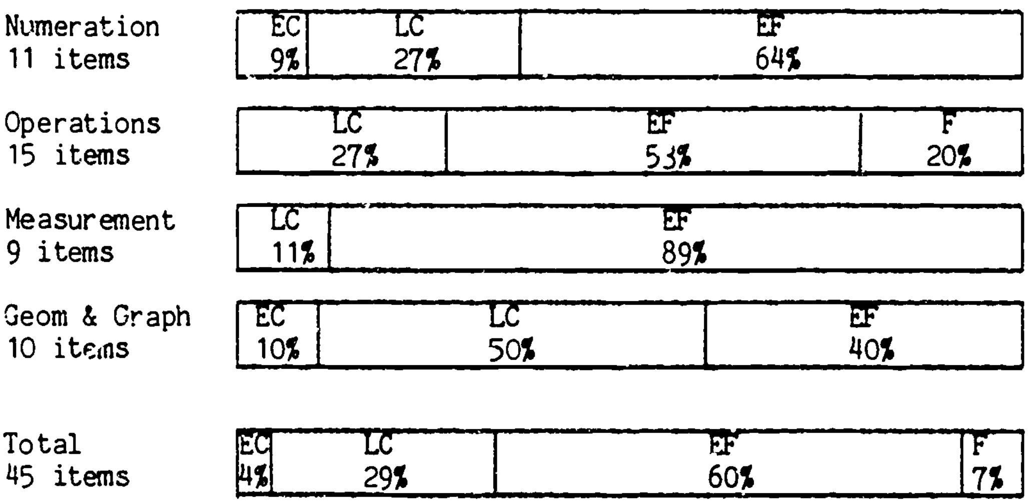


Figure 20: Cognitive Demands of Grade 6 Provincial Achievement Test



Cognitive Demands Compared with Student Cognitive Response Levels

Numeration

Numeration, Grades 1 to 3

The findings from the comparisons between curricular demand and pupil response distributions in Numeration strand topics in Division One are summarized in Figure 21.

At the Grade 1 level, the distribution of Numeration curriculum objectives matched the distribution of pupil responses reasonably well. However, the Grade 1 textbooks and classrooms proved to be short of Late Concrete Operational material (10% and 3%, respectively, whereas 37% of the Grade 1 students responded at this level in Numeration topics). Even though there were no Preoperational demands observed in the classroom (compared with 22% of the student responses at this level) this difference, alone, was not large enough to cause rejection of the Null Hypothesis.

The Grade 2 Numeration findings showed a mismatch only between textbook demands and pupil responses. While 59% of the Grade 2 students responded at the Late Concrete or Early Formal levels, only 28% of the textbook material made comparable demands. The overabundance of Early Concrete demands, as compared with student responses, would also have been sufficient to cause the null hypothesis to be rejected.

As shown in Figure 21, two assessments of pupil cognitive response level were used in Grade 3. Paper-and-pencil cognitive assessment tests covering "Number" concepts were administered to 115 Grade 3 pupils. Of these, 60 had previously been interviewed using number concept and numeration tasks. The distributions of cognitive ratings from the interview and paper-and-pencil assessments were not significantly different, as indicated by a K-S D of 0.117 which could occur by chance in more than 20% of frequency distributions drawn from the same population. The distributions of demands by the Grade 3

Figure 21

Curricular Demand and Pupil Response Contrasts: Numeration, Grades 1 to 3

	PO %	EC %	LC %	K-S D*	Probability	Decision
Gr.1 Interview Ratings 60 pupils	22	41	37			
Gr.1 Curric. Objectives 6 items	17	66	17	D=0.205	p=0.162	Accept
Gr.1 Textbooks (Numeration) 2363 items	22	68	10	D=0.268	p=0.028	Reject
Gr.1 Classroom Observations 64 minutes		97	3	D=0.340	p=0.003	Reject

	PO %	EC %	LC %	EF %	K-S D*	Probability	Decision
Gr.2 Interview Ratings 60 pupils	13	28	57	2			
Gr.2 Curric. Objectives 9 items		56	44		D=0.145	p>0.200	Accept
Gr.2 Textbooks (Numeration) 2995 items	3	69	28		D=0.307	p=0.007	Reject
Gr.2 Classroom Observations 29 minutes		38	55	7	D=0.129	p>0.200	Accept

	PO %	EC %	LC %	EF %	Interview K-S D*	Interview Prob.	Interview Dec'n	Paper K-S D*	Paper Prob.	Paper Dec'n
Gr.3 Interview Ratings 60 pupils	14	37	37	12						
Paper & Pencil Tests (ACER) 115 pupils	10	43	47		0.117	>0.200	Acc.			
Gr.3 Curric. Objectives 11 items		45	55		0.142	>0.200	Acc.	0.096	>0.200	Acc.
Gr.3 Textbooks (Numeration) 1303 items	1	46	53		0.130	>0.200	Acc.	0.084	>0.200	Acc.
Gr.3 Classroom Observations 8 minutes			100		0.508	<0.001	Rej.	0.530	<0.001	Rej.
Achievement Test (Gr.3) 17 items		59	41		0.142	>0.200	Acc.	0.096	>0.200	Acc.

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

Numeration curricular objectives, textbooks, and achievement test items all matched reasonably well with the student cognitive response distributions, whether assessed by interview or paper-and-pencil test. As for classroom demands in Grade 3, only 8 minutes were observed in which Numeration topics were treated. This lesson segment contained only Late Concrete demands, which resulted in a mismatch because of the lack of provision for those students responding at the Preoperational and Early Concrete levels. The brevity of the classroom observation should be taken into account in the interpretation of this particular finding.

Numeration, Grades 4 to 6

Figure 22 summarizes the Numeration findings for Grades 4 to 6. The cognitive response distributions are from paper-and-pencil assessments of number and numeration concepts and understandings.

The Grade 4 findings are much like those of Grade 3, with a mismatch only between the classroom demands and the pupil responses. Here the observation time is much longer (72 minutes), but the mismatch is due to an excess of Early Formal demands as compared with pupils responses at this level (43% versus 13%), which implies a shortage of demands below the Early Concrete level.

The Grade 5 and 6 findings show mismatches between demands and pupil responses for every comparison. In every case the large percentage of Early Formal demands as compared with the small percentage of responses at that level (3% and 12%) is enough to cause rejection of the null hypothesis. This finding can also be viewed as indicating that there are insufficient demands at or below the Late Concrete level. In the case of Grade 5 textbooks, it is the discrepancy between the demands at the lowest level rated, Early Concrete (9%), and the combined Preoperational and Early Concrete responses (58%) that is reflected in the observed K-S D.

Figure 22

Curricular Demand and Pupil Response Contrasts: Numeration, Grades 4 to 6

Source	PO	EC	LC	EF	K-S D*	Probability	Decision
Gr.4 P&P Test Ratings, 94 pupils	1%	28%	59%	13%			
Gr.4 Curric. Objectives, 11 items		18%	55%	27%	D=0.145	p>0.200	Accept
Gr.4 Textbooks (Numeration), 2321 items		22%	55%	23%	D=0.098	p>0.200	Accept
Gr.4 Classroom Observations, 72 minutes		10%	47%	43%	D=0.303	p<0.001	Reject
Gr.5 P&P Test Ratings, 92 pupils	16%	42%	39%	3%			
Gr.5 Curric. Objectives, 13 items		15%	31%	54%	D=0.506	p<0.001	Reject
Gr.5 Textbooks (Numeration), 1662 items		9%	52%	39%	D=0.485	p<0.001	Reject
Gr.5 Classroom Observations, 86 minutes			38%	62%	D=0.584	p<0.001	Reject
Gr.6 P&P Test Ratings, 90 pupils	14%	17%	57%	12%			
Gr.6 Curric. Objectives, 14 items		14%	36%	50%	D=0.378	p<0.001	Reject
Gr.6 Textbooks (Numeration), 2591 items		1	15%	84%	D=0.718	p<0.001	Reject
Gr.6 Classroom Observations, 135 minutes			9%	91%	D=0.789	p<0.001	Reject
Achievement Test (Gr.6), 11 items		9%	27%	64%	D=0.514	p<0.001	Reject

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (N = number of pupils).

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Overall, there is a reasonable match between curriculum objective demands and pupil responses in the Numeration topics from Grade 1 through Grade 4. A reasonable match between textbook demands and pupil responses was found in Grades 3 and 4 but not in the earlier or later grades. All of the demand distributions in Grades 5 and 6 are marked by a significant excess at the Early Formal level (with a corresponding shortfall of material posing demands at or below the Late Concrete level).

Operations

Operations, Grades 1 to 3

Figure 23 summarizes the comparisons made between curricular demands and pupil responses in the Operations strand from Grade 1 to Grade 3.

Each of the demand distributions in Grades 1 and 2 were found to differ significantly from the relevant pupil response distributions. In Grade 1, 29% of the students were responding Preoperationally but none of the demands addressed that level, resulting in a mismatch. In Grade 2 the mismatches resulted from a lack of material suited to the 65% responding at the Late Concrete level coupled with the complementary overabundance of material at or below the Early Concrete. The demands of the curriculum objectives, the textbooks, and the classroom in Grade 3 were distributed in a manner not significantly different from the pupil response demands. However, the Grade 3 Achievement Test items dealing with Operations were largely at the Late Concrete demand level (83% as compared with 53% and 59% of the pupil responses rated at the Late Concrete and Early Formal operational levels, combined), which implies insufficient items at or below the Early Concrete level.

Operations, Grades 4 to 6

A summary of the demand/response contrasts observed in the Operations strand from Grade 4 to 6 is presented in Figure 24.

Reasonable matches between the pupil responses and the demands of the

Figure 23

Curricular Demand and Pupil Response Contrasts: Operations, Grades 1 to 3

	PO	EC	LC	K-S D* Probability Decision
Gr.1 Interview Ratings 60 pupils	29 %	42 %	29 %	
Gr.1 Curric. Objectives 6 items		67 %	33 %	D=0.290 p=0.014 Reject
Gr.1 Textbooks (Operations) 3077 items		91 %	9 %	D=0.290 p=0.014 Reject
Gr.1 Classroom Observations 36 minutes		97 %	3	D=0.290 p=0.014 Reject

	PO	EC	LC	EF	K-S D* Probability Decision
Gr.2 Interview Ratings 60 pupils	13 %	22 %	65 %		
Gr.2 Curric. Objectives 9 items		78 %	22 %		D=0.430 p<0.001 Reject
Gr.2 Textbooks (Operations) 4656 items		90 %	10 %		D=0.552 p<0.001 Reject
Gr.2 Classroom Observations 77 minutes		60 %	23 %	17 %	D=0.249 p=0.049 Reject

	PO	EC	LC	EF	Inter-view	K-S D* Prob. Dec'n
Gr.3 Interview Ratings 60 pupils	5	42 %	49 %	4		
Paper & Pencil Tests (ACER) 94 pupils	3	38 %	46 %	13 %	Paper K-S D* Prob.	0.086 >0.200 Acc.
Gr.3 Curric. Objectives 26 items		42 %	50 %	8 %	0.051 >0.200 Acc.	0.052 >0.200 Acc.
Gr.3 Textbooks (Operations) 6857 items		54 %	42 %	4	0.125 >0.200 Acc.	0.065 >0.200 Acc.
Gr.3 Classroom Observations 124 minutes		51 %	43 %	6	0.093 >0.200 Acc.	0.050 >0.200 Acc.
Achievement Test (Gr.3) 18 items		17 %	83 %		0.248 0.006 Rej.	0.308 <0.007 Rej.

PO-Preoperational; EC-Early Concrete; LC-Late Concrete;
EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

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Figure 24

Curricular Demand and Pupil Response Contrasts: Operations, Grades 4 to 6

	EC	LC	EF	F	K-S D* / Probability / Decision
Gr.4 P&P Test Ratings 100 pupils	20 %	50 %	30 %		K-S D* Probability Decision
Gr.4 Curric. Objectives 23 items	26 %	52 %	22 %		D=0.083 p>0.200 Accept
Gr.4 Textbooks (Operations) 9128 items	29 %	56 %	15 %		D=0.154 p=0.180 Accept
Gr.4 Classroom Observations 86 minutes	47 %	30 %	23 %		D=0.265 p=0.002 Reject
Gr.5 P&P Test Ratings 94 pupils	19 %	44 %	33 %	4 %	K-S D* Probability Decision
Gr.5 Curric. Objectives 25 items	24 %	28 %	48 %		D=0.108 p>0.200 Accept
Gr.5 Textbooks (Operations) 6385 items	10 %	37 %	53 %		D=0.158 p=0.190 Accept
Gr.5 Classroom Observations 131 minutes	2	28 %	70 %		D=0.330 p<0.001 Reject
Gr.6 P&P Test Ratings 92 pupils	3	31 %	57 %	9 %	K-S D* Probability Decision
Gr.6 Curric. Objectives 25 items	28 %	20 %	44 %	8 %	D=0.247 p=0.008 Reject
Gr.6 Textbooks (Operations) 6402 items	4	31 %	57 %	8 %	D=0.018 p>0.200 Accept
Gr.6 Classroom Observations 54 minutes			22 %	78 %	D=0.691 p<0.001 Reject
Achievement Test (Gr.6) 15 items		27 %	53 %	20 %	D=0.113 p>0.200 Accept

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

curriculum objectives and of the textbooks were found in Grades 4 and 5. The same was found with the textbooks in Grade 6. This was not the case for the Grade 6 curriculum objectives nor for the classroom observations in all three Division Two grades. The Grade 6 curriculum objectives were 28% Early Concrete Operational but only 3% of the pupils responses were at this level. The largest discrepancies between pupil responses and classroom demands were as follows: Grade 4, 47% Early Concrete demand contrasted with 20% response at that level; Grade 5, 70% Early Formal demand versus 37% combined Early Formal and Formal pupil responses; and Grade 6, 78% Formal demands compared with only 9% of the pupil responses classified at that level. The distribution of the Grade 6 achievement test items dealing with operations was not significantly different from that of the pupil responses.

Measurement

Measurement, Grades 1 to 3

The summary of the demand/response contrasts presented in Figure 25 shows that all of the demand distributions were significantly different from the pupil response distributions.

In Grades 1 and 2, the mismatches between pupil responses and both the demands of the curriculm objectives and of the classroom can be attributed to the lack of provision for students operating Preoperationally in measurement contexts (64% and 49% of the Grade 1 and 2 responses, respectively, in contrast with only 1% to 26% of the demands occurring at the Preopertional level). This was also the case with the Grade 1 classroom observations (no Preoperational demands as compared with 64% of the pupil responses occurring at this level). An even greater discrepancy was found at the Grade 2 level where classroom demands were not less than Late Concrete but where 76% of the student responses were.

As for Measurement in Grade 3, the pupil response ratings from the

Figure 25

Curricular Demand and Pupil Response Contrasts: Measurement, Grades 1 to 3

Grade 1	PO	EC	LC	EF	K-S D* Probability Decision
Gr.1 Interview Ratings 60 pupils	64 %	21 %	15 %		
Gr.1 Curric. Objectives 19 items	26 %	42 %	32 %		D=0.378 p<0.001 Reject
Gr.1 Textbooks (Measurement) 297 items	7 %	81 %	12 %		D=0.574 p<0.001 Reject
Gr.1 Classroom Observations 68 minutes		76 %	2	22 %	D=0.641 p<0.001 Reject

Grade 2	PO	EC	LC	EF	K-S D* Probability Decision
Gr.2 Interview Ratings 60 pupils	49 %	27 %	22 %	2	
Gr.2 Curric. Objectives 24 items	13 %	29 %	58 %		D=0.369 p<0.001 Reject
Gr.2 Textbooks (Measurement) 753 items	1	43 %	56 %		D=0.484 p<0.001 Reject
Gr.2 Classroom Observations 52 minutes			90 %	10 %	D=0.759 p<0.001 Reject

Grade 3	PO	EC	LC	EF	Interview K-S D* Prob.	Paper K-S D* Prob. Dec'n
Gr.3 Interview Ratings 60 pupils	33 %	37 %	21 %	9 %	Inter-view	K-S D* Prob. Dec'n
Paper & Pencil Tests (ACER) 92 pupils	27 %	53 %	18 %	2	Paper K-S D* Prob.	0.103 >0.200 Acc.
Gr.3 Curric. Objectives 15 items		13 %	87 %		0.671 <0.001 Rej.	0.568 <0.001 Rej.
Gr.3 Textbooks (Measurement) 722 items		25 %	75 %		0.559 <0.001 Rej.	0.456 <0.001 Rej.
Gr.3 Classroom Observations 40 minutes			75 %	25 %	0.804 <0.001 Rej.	0.702 <0.001 Rej.
Achievement Test (Gr.3) 7 items		14 %	86 %		0.661 <0.001 Rej.	0.559 <0.001 Rej.

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

interviews and from the paper-and-pencil tests were in agreement (K-S D = 0.103; $p > 0.200$), but the demand/response mismatch pattern continued. The curriculum objective, textbook and achievement test demands were, respectively, 87%, 75% and 86% at the Late Concrete Level as compared with 30% and 20% of the student responses at that le[illegible] and above. All of the classroom demands were Late Concrete or Early Formal but 70 to 80% of the student responses were at or below the Early Concrete level.

Measurement, Grades 4 to 6

The situation in the Grades 4 to 6 Measurement strand was found to be much like that in Grades 1 to 3, as can be seen in Figure 26. In each of the three Division Two grades there are either very few or no demands at the Preoperational and Early Concrete levels, in contrast with 18% to 57% of the student responses occuring at these levels, or there is an overabundance of Early Formal demands (which implies a shortage of lower level demands).

Geometry and Graphing

Geometry and Graphing, Grades 1 to 3

Figure 27 summarizes the demand/response contrasts found in Grades 1 to 3 in topics in Geometry and Graphing.

There were significant mismatches between the Grade 1 pupil response distribution and the demand distributions of the curriculum objectives, textbooks, and classroom. This occurred because there were no demands at the Late Concrete operational level to correspond to 41% of the pupil responses reaching this level in the contexts of Geometry and Graphing.

No significant differences were found between the Grade 2 pupil response distribution and the corresponding curriculum objective and textbook demand distributions. However, a marked mismatch between the Grade 2 classroom demand distribution and that of the pupil responses was found. This mismatch is due to the large proportion of Early Formal demands (73%) and the very

Figure 26

Curricular Demand and Pupil Response Contrasts: Measurement, Grades 4 to 6

	PO	EC	LC	EF	F	K-S D* Probability Decision
Gr.4 P&P Test Ratings 116 pupils	9 %	48 %	39 %	4 %		
Gr.4 Curric. Objectives 15 items			93 %	7 %		D=0.569 p<0.001 Reject
Gr.4 Textbooks (Measurement) 1037 items		10 %	66 %	24 %		D=0.465 p<0.001 Reject
Gr.4 Classroom Observations	[Not Observed]					
Gr.5 P&P Test Ratings 138 pupils	21 %	23 %	50 %	6 %		
Gr.5 Curric. Objectives 19 items		5 %	69 %	26 %		D=0.389 p<0.001 Reject
Gr.5 Textbooks (Measurement) 1616 items		8 %	29 %	63 %		D=0.574 p<0.001 Reject
Gr.5 Classroom Observations 23 minutes			70 %	30 %		D=0.442 p<0.001 Reject
Gr.6 P&P Test Ratings 113 pupils	4	14 %	62 %	19 %	1 %	
Gr.6 Curric. Objectives 15 items		7 %	33 %	60 %		D=0.396 p<0.001 Reject
Gr.6 Textbooks (Measurement) 1087 items		11 %	40 %	49 %		D=0.284 p<0.001 Reject
Gr.6 Classroom Observations	[Not Observed]					
Achievement Test (Gr.6) 9 items			11 %	89 %		D=0.685 p<0.001 Reject

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (N = number of pupils).

Figure 27

Curricular Demand and Pupil Response Contrasts: Geometry & Graphing, Grades 1 to 3

	Distribution (%)	K-S D* Probability Decision
Gr.1 Interview Ratings 60 pupils	PO 37 % / EC 22 % / LC 41 %	
Gr.1 Curric. Objectives 3 items	PO 33 % / EC 67 %	D=0.413 p<0.001 Reject
Gr.1 Textbooks (Geometry) 251 items	PO 49 % / EC 51 %	D=0.413 p<0.001 Reject
Gr.1 Classroom Observations 14 minutes	EC 64 % / EF 36 %	D=0.367 p=0.001 Reject

	Distribution (%)	K-S D* Probability Decision
Gr.2 Interview Ratings 60 pupils	PO 23 % / EC 49 % / LC 26 % / EF 2	
Gr.2 Curric. Objectives 7 items	EC 71 % / LC 29 %	D=0.226 p=0.093 Accept
Gr.2 Textbooks (Geometry) 411 items	PO 29 % / EC 55 % / LC 16 %	D=0.129 p>0.200 Accept
Gr.2 Classroom Observations 51 minutes	EC 27 % / EF 73 %	D=0.706 p<0.001 Reject

	Distribution (%)	Interview	K-S D* Prob. Dec'n
Gr.3 Interview Ratings 60 pupils	PO 26 % / EC 40 % / LC 27 % / EF 7 %		
Paper & Pencil Tests (ACER) 112 pupils	PO 24 % / EC 43 % / LC 33 %	Paper K-S D* Prob.	0.072 >0.200 Acc.
Gr.3 Curric. Objectives 12 items	EC 58 % / LC 25 % / EF 17 %	0.241 0.004 Rej.	0.258 0.039 Rej.
Gr.3 Textbooks (Geometry) 1011 items	PO 5 % / EC 77 % / LC 14 % / EF 4 %	0.192 0.035 Rej.	0.209 0.147 Acc.
Gr.3 Classroom Observations 35 minutes	EC 63 % / LC 6 % / EF 31 %	0.314 <0.001 Rej.	0.258 0.039 Rej.
Achievement Test (Gr.3) 8 items	EC 50 % / LC 38 % / EF 12 %	0.241 0.004 Rej.	0.258 0.039 Rej.

PO-Preoperational; EC-Early Concrete; LC-Late Concrete; EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

small proportion of student responses at this level (2%). It is also attributable to insufficient demands at or below the Late Concrete operational level.

In Grade 3 significant mismatches were found in every response/demand contrast (with the exception that the Grade 3 test of goodness-of-fit between textbook demand and pupil response distributions indicated a reasonable match when the interview responses were used and a borderline mismatch when the paper-and-pencil responses were used). The curriculum objective, textbook, and achievement test demand/response mismatches arose from the sparsity of demands at the Preoperational level as contrasted with 26% and 24% of the pupil responses occurring at that level. The classroom observation demand/response mismatch can be attributed to a lack of demands at or below the Late Concrete operational level (where 93 to 100% of the pupil responses were found).

Geometry and Graphing, Grades 4 to 6

As summarized in Figure 28, the demand/response picture in Grades 4 to 6 is a mixed one.

All three Grade 4 Geometry and Graphing demand distributions were accepted as being not significantly different from the pupil response distribution.

In Grade 5, all three Geometry and Graphing distributions proved to be significantly different from that of the pupil responses. In each case, the percentage of demands at or below the Early Concrete level was substantially below 59%, the corresponding figure for pupil responses.

The Grade 6 distributions of the Graphing and Geometry demands made by the curriculum objectives and by the achievement test items were not significantly different from that of the pupil responses. As for the textbooks, there was an overabundance of demands at or below the Early

Figure 28

Curricular Demand and Pupil Response Contrasts:
Geometry & Graphing, Grades 4 to 6

	PO	EC	LC	EF	F	K-S D* Probability Decision
Gr.4 P&P Test Ratings 90 pupils	14 %	36 %	41 %	9 %		K-S D* Probability Decision
Gr.4 Curric. Objectives 15 items		47 %	27 %	27 %		D=0.178 p=0.117 Accept
Gr.4 Textbooks (Geom&Graph'g) 1351 items	4 %	62 %	26 %	8 %		D=0.157 p>0.200 Accept
Gr.4 Classroom Observations 174 minutes		55 %	43 %	2		D=0.144 p>0.200 Accept
Gr.5 P&P Test Ratings 151 pupils	13 %	46 %	36 %	5 %		K-S D* Probability Decision
Gr.5 Curric. Objectives 23 items		26 %	48 %	26 %		D=0.329 p<0.001 Reject
Gr.5 Textbooks (Geom&Graph'g) 1945 items		42 %	45 %	13 %		D=0.173 p=0.023 Reject
Gr.5 Classroom Observations 172 minutes		28 %	59 %	13 %		D=0.305 p<0.001 Reject
Gr.6 P&P Test Ratings 94 pupils	4 %	19 %	54 %	22 %	1 %	K-S D* Probability Decision
Gr.6 Curric. Objectives 19 items		16 %	53 %	31 %		D=0.082 p>0.200 Accept
Gr.6 Textbooks (Geom&Graph'g) 2085 items	2	54 %	22 %	22 %		D=0.331 p<0.001 Reject
Gr.6 Classroom Observations 55 minutes			89 %	11 %		D=0.234 p=0.013 Reject
Achievement Test (Gr.6) 10 items		10 %	50 %	40 %		D=0.166 p=0.152 Accept

PO-Preoperational; EC-Early Concrete; LC-Late Concrete;
EF-Early Formal; F-Formal

*Kolmogorov-Smirnov Goodness of Fit Test: Each Curricular Cognitive Demand frequency distribution was compared with that of the corresponding Pupil Cognitive Responses to Interview or Paper-and-Pencil assessments. Two distributions were considered not significantly different if the probability of observing the calculated K-S D was greater than 0.05 (with N = number of pupils).

Concrete level (56% demands contrasted with 23% responses), which corresponds to a lack of demands at or above the Late Concrete level. The Grade 6 classroom demands for Geometry and Graphing exhibited a significantly greater proportion of demands than responses at or below the Late Concrete level (89% contrasted with 77%).

DISCUSSION OF FINDINGS

Number of Matches and Mismatches between Demand and Response Distributions by Curriculum Strands

For discussion of general overall patterns the data in Figures 21 through 28 are summarized in Table 2. This table shows by strand the number of instances in which the curricular cognitive demand distributions were significantly higher (H) than, or lower (L) than, or not significantly different (NSD) from, the corresponding student response distributions. The data are reported for each grade and for all grades together.

Table 2 shows that in 55% of all of the demand/response comparisons the demand distributions were significantly higher than those of the student responses, in 32% of the comparisons the distributions were not significantly different, and in 13% of the comparisons the demand distributions were lower than the distributions of student responses. The demand distributions generally compare favourably with the response distributions in all strands except Measurement.

A trend is evident over the six grades in the Numeration strand. As shown in Table 2, all of the demand distributions match or are lower than the response distributions in Grades 1 and 2, but there is a gradual shift with increasing grade level until all demand distributions are significantly higher than those of the student responses in Grades 5 and 6. It appears that the program starts 'easy' and gets 'harder' (where 'easy' and 'harder' are assessed in terms of the relative numbers of demand distributions that are higher than, no different from, or lower than the corresponding response distributions). The number of cases in which the demands are higher are: 0, 0, 1, 1, 3, 4, for the six grades in order. This can be contrasted with the Operations strand where, using the same criteria, the program starts 'harder' (three out of three demand distributions higher), becomes 'easier' by Grade 4

Table 2

Numbers of Matches and Mismatches between Demand and Response Distributions by Curriculum Strand

	Numeration			Operations			Measurement			Geom/Graph			Totals			
Grade	H	NSD	L	H	NSD	L	H	NSD	L	H	NSD	L	H	NSD	L	Total
1	0	1	2	3	0	0	3	0	0	1	0	2	7 58%	1	4 33%	12
2	0	2	1	1	0	2	3	0	0	1	2	0	5 42%	4	3 25%	12
3	1	3	0	1	3	0	4	0	0	3	1	0	9 56%	7	0 0%	16
4	1	2	0	0	2	1	2	0	0	0	3	0	3 27%	7	1 9%	11
5	3	0	0	1	2	0	3	0	0	3	0	0	10 83%	2	0 0%	12
6	4	0	0	1	2	1	3	0	0	1	2	1	9 60%	4	2 13%	15
Total	9	8	3	7	9	4	18	0	0	9	8	3	43	25	10	78
% of Total	45%	40%	15%	35%	45%	20%	100%	0%	0%	45%	40%	15%	55%	32%	13%	100%

H: demand distribution significantly higher than response distribution

NSD: no significant difference between demand and response distributions

L: demand distribution significantly lower than response distribution (zero out of four demand distributions higher), and then increases in difficulty at Grades 5 and 6 (one out of three and one out of four demand distributions higher).

The best fit occurs in Operations, where 45% of the demand distributions matched the response distributions, 35% of the demand distributions were higher, and 20% were lower. Numeration and Geometry & Graphing each have 40% of the demand and response distrib ,ions matched, 45% of the demand distributions higher and 15% lower. In Measurement the situation is dramatically different: 100% of the demand distributions are higher than the response distributions. It may be that many aspects of measurement are intrinsically highly demanding. The frequent use of scales resembling number lines and inferences involving transitivity are two examples. Although number lines do not appear in the Numeration strand until Grade 4, the need to read scales arises in Grade 2 or earlier in the Measurement strand.

In Geometry & Graphing the number of matches and mismatches shows no particular pattern across the grades. The demand distributions for Grades 3 and 5 were predominantly higher than the response distributions, with 3 out of 4 and 3 out of 3 demand distributions higher than those of the responses. Grades 1, 2, 4 and 6 show a variety of match/mismatch configurations.

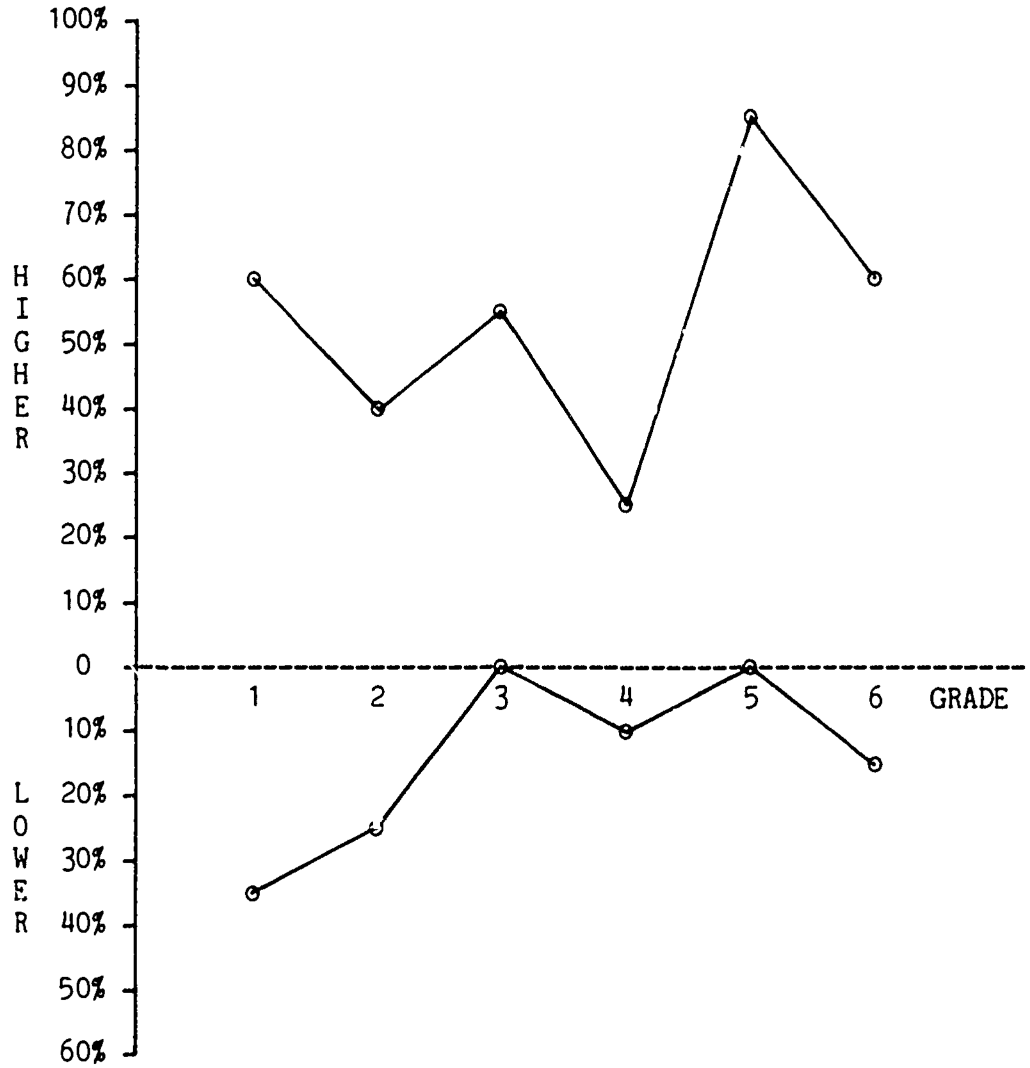
When looking at Table 2 by grades several features stand out. For example, the Grade 1 demand distributions in Operations are all higher than the response distributions. Many six- and-seven-year-olds respond Preoperationally in the topics of the Operations strand. However, it is difficult to state Operations demands in Preoperational terms. Hence, it is not unexpected that all three Grade 1 demand distributions would be higher than the corresponding response level distributions, and reference to Figure 23 confirms that this is, in fact, the case. Since these mismatches result from a lack of demands at the Preoperational level to match the 29% of the responses which were Preoperational, it may be that the only recourse is to give the Preoperational students manipulative materials and experiences in which arithmetic operations are embedded in order to provide rudimentary concrete activities to bridge the gap to the Early Concrete level.

Table 2 shows that in Grades 5 and 6 the demand distributions in the Numeration strand are all higher than the corresponding response distributions. While earlier grades deal mainly with whole numbers, the topics emphasized in Numeration in Grades 5 and 6 are "place values greater than hundred thousands and less than hundredths," "fractions (decimal and common)," and "ratios." The findings of the Calgary Junior High Mathematics Project indicated that these topics have an inherently high level of cognitive demand As Figure 22 shows, there is a heavy emphasis on Early Formal Operational demands in the Numeration strand in Grades 5 and 6, but there is a lack of pupil responses at this level. Figure 24 indicates that in the Operations strand in Grades 5 and 6 there are substantial numbers of demands at the Early Formal and Formal Operational levels, but in this strand there are sufficient student responses at these levels to match the curriculum and textbook demands. In order for the computations of the Operations strand to be carried out meaningfully, there must be parallel development of number concepts in the Numeration strand. It is interesting to note that the percentage of Early Formal and Formal Operational responses is so much smaller in the Numeration strand than in the Operations strand in these grades. This is an example of "broken front" development, in which students' cognitive abilities are more advanced in one strand than in another.

A graph of the total number of mismatches per grade yields the interesting profile shown in Figure 29. This graph shows that in four of the six grades the percentage of cases in which the demand distributions are higher than the response distributions is fairly consistently in the 40% to

Figure 29

Percentage of Demand Distributions Significantly Higher or Lower than Student Response Distributions, by Grade



60% range. The two exceptions to this statement are Grades 4 and 5. The Grade 5 program stands out as being the 'hardest' or the most challenging, cognitively, with 85% of the demand distributions higher, and none lower than the corresponding response distributions. Grade 4 appears to have the best fit between demand distributions and response distributions with only 27% of the demand distributions higher and 9% lower. The cases in which the demand distributions are lower than the response distributions occur mainly in Grades 1 and 2. The largest percentage of mismatches is found in Grade 1 where 58% of the demand distributions were higher, 33% were lower and only 9% matched the response distributions.

The findings from the comparisons for Grade 4 are of particular interest in view of the fact that during the revision of the curriculum for 1982 complaints from teachers suggested that the then current Grade 4 program was too demanding. Adjustments were made at that time in the whole curriculum, producing the favourable situation for Grade 4.

Number of Matches and Mismatches Between Demand and Response Distributions by Demand Areas

Table 3 has been produced from the data in Figures 21 through 28 in order to facilitate the discussion of patterns across the grades in each demand area (Curriculum Objectives, Textbooks, Classroom Observations, and Achievement Tests). This table records the number of instances in which the curricular cognitive demand distributions were higher (H) than, lower (L) than, or not significantly different (NSD) from the student response distributions (for each grade and for all grades together).

The Curriculum shows the best overall fit with 46% of the demand distributions matching, 42% higher, and 12% lower than the response distributions. The programs for Grades 3 and 4 display the best

Table 3

Numbers of Matches and Mismatches between Demand and Response Distributions by Demand Area

	Curriculum			Textbooks			Classroom			Achievement Tests			Totals			
Grade	H	NSD	L	H	NSD	L	H	NSD	L	H	NSD	L	H	NSD	L	Total
1	2	1	1	2	0	2	3	0	1	-	-	-	7	1	4	12
2	1	2	1	1	1	2	3	1	0	-	-	-	5	4	3	12
3	1	3	0	2	2	0	3	1	0	3	1	0	9	7	0	16
4	1	3	0	1	3	0	1	1	1	-	-	-	3	7	1	11
5	3	1	0	3	1	0	4	0	0	-	-	-	10	2	0	12
6	2	1	1	2	1	1	3	0	0	2	2	0	9	4	2	15
Total	10	11	3	11	8	5	17	3	2	5	3	0	43	25	10	78
% of Total	42%	46%	12%	46%	33%	21%	77%	14%	9%	63%	38%	0%	55%	32%	13%	100%

H: demand distribution significantly higher than response distribution

NSD: no significant difference between demand and response distributions

L: demand distribution significantly lower than response distribution demand/response fit, each with three matches and one case in which the demand distribution is higher. Grade 2 has two matches, one demand distribution higher, and one lower. Grades 1 and 6 have one match each, two demand distributions higher, and one lower. Grade 5, with only one match, has three demand distributions higher than the response distribution.

With respect to Textbooks, 33% of the demand and response distributions match, in 46% of the cases the demands are higher, and in 21% the demands are lower than the responses. Again, the best match is at Grade 4 where there are three matches and one demand distribution higher than the distribution of the responses. Grade 3 has two matches and two with the demands higher. Grades 2, 5, and 6 have only one match each with Grade 2 having one demand distribution higher and two lower, Grade 5 with three higher, and Grade 6 with two higher and one lower than the corresponding response distributions. Since only those portions of the textbooks that correspond to the Alberta curriculum were analyzed, it might be expected that the match here would tend to reflect the match or mismatch in the Curriculum area. Hence, it is not surprising that the Grades 2 through 4 textbook demand distributions match well with the response distributions and the Grades 1, 5, and 6 demand distributions match less well, since this is the case with the Curriculum.

The summary of comparisons between demand distributions and corresponding response distributions in the Classroom is particularly noteworthy. Since the demands recorded in the Classroom presumably reflect the demands of the Curriculum it is not surprising that the best fit between demand and response distributions occurs at Grade 4. The most critical areas were Grades 5 and 6 where all the demand distributions were higher than those of the corresponding responses. The Classroom demand distributions at Grades 1, 2 and 3 do not reflect the demand distributions of the Curriculum as well as one would expect. In Grade 1, three demand distributions were higher and one was lower.

In each of Grades 2 and 3, three demand distributions were higher and one macched. Overall, 77% of the observed demand distributions were higher than the corresponding distributions of student responses, only 14% were not significantly different, and 9% were lower. This is particularly interesting because the teachers that were observed had volunteered to participate and were generally considered to be above average teachers. Also, for the most part, they seemed aware of the kinds of learning activities that the observer would presumably consider appropriate.

The Achievement Test findings show that the demand levels were higher than student response levels in five of the eight comparisons. Only three matches occur. At Grade 3 there was a match in one strand (Numeration), while the demand distributions were higher than student responses in the other three strands. At Grade 6 there were two matches (Operations and Geometry & Graphing), but the demand distributions were higher than the response distributions in the other two strands.

It should be noted that the distributions of student responses were obtained late in the fall term, whereas the demands of the Achievement Tests reflect end-of-year expectations. Furthermore, the Achievement Test items included in the ACLIC assessments did not include any items from the computation sections which have lower cognitive demands (largely at the Late Concrete Operational level). Similarly, the demands of the Curriculum and Textbooks were assessed for the whole school term. The Classroom demands were assessed at the end of the school term. Moreover, the teacher sample was small, and the number of minutes observed in some strands was very small. These factors might well account for the lack of fit where the demands were higher than the responses. Any report of a cognitive mismatch should be viewed conservatively.

Even in the light of the preceding cautions, the fact remains that 77% of the Classroom demand distributions were significantly higher than the pupil response distributions, which might well lead to mechanical learning rather than the understanding, internalization and cognitive development that is preferred. However, the large mismatch might well be accommodated by supplementary learning activities designed to bridge the gap and promote understanding, meaning, and internalization of the concept under study.

An important theoretical question is: How closely should the demand distribution match the corresponding student response distribution? It could be asserted that the demand level should be higher than the corresponding response level in order to promote intellectual development. However, an action plan based on this assertion should include instruction geared to bridging the gap between the levels of students' responses and the levels of demands. Another important consideration is that even with a "perfect" match there are always higher level demands for all but the cognitively most advanced in a given grade.

In any case, the immediate practical question is how large must a discrepancy between the demand and response distributions be in order to be judged "too large." In the foregoing statistical comparisons, the critical K-S D value was roughly 0.25 so that two distributions were considered to be significantly different when the maximum difference between the cumulative percentage distributions for corresponding cognitive levels exceeded 25%. (A difference this large between two distributions drawn from the same population would be observed by chance only 5 times in a hundred.)

CONCLUSIONS

The Assessing Cognitive Levels in Classrooms Project has produced data analyses and interpretations that provide answers to the three research questions posed on page 12. In the following sections, the questions are restated along with general answers and then a number of specific conclusions are drawn.

> 1) What levels of cognitive response are demonstrated in mathematics topic contexts by Alberta students in each of Grades 1 through 6?

In a representative sample of students at each grade level, a wide range of cognitive levels of student response was found across the four topic strands. In general, about three-quarters of all student responses were at the Concrete Operational level (Early Concrete and Late Concrete levels together). The remaining quarter were primarily at the Preoperational level in the early grades and at the Early Formal and Formal Operational levels in the higher grades. (See Figures 1 to 6.)

The younger students responded more frequently at the Preoperational and Early Concrete Operational levels than did the older students. In Grades 1 and 2 a substantial proportion of responses were at the Preoperational level. The percentage of Preoperational responses dropped as grade level increased from Grade 3 to Grade 6. At the Grade 6 level only a very small percentage of the responses were rated Preoperational, and the percentage of student responses at the Early Formal Operational level was dramatically higher than in the earlier grades. None of the Grade 1 and very few of the Grade 2 students responded above the Late Concrete Operational level, but the percentages of Early Formal Operational responses generally increased with grade level. In Grades 5 and 6 a small number of responses were at the Formal

Operational level.

Figures 5 and 6 show that cognitive abilities develop on a "broken front." That is, at any grade level, the percentage of responses at a particular cognitive level is not consistent across the strands. For instance, at the Grade 2 level 13%, 13%, 49% and 23% responded at the Preoperational level in Numeration, Operations, Measurement and Geometry & Graphing, respectively. This indicates that a student may be responding the Preoperational level in some strands but at a higher level in others (Piaget referred to this phenomenon as one form of decalage.)

Also, the percentage of responses at the lower cognitive levels in a strand may not decrease steadily across the six grades as one would expect. For instance, although the percentage of responses at the Preoperational level in Numeration generally decreased from Grade 1 to Grade 6, as shown in Figure 1, it actually increased in Grade 5 as compared with earlier grades. The differences in the sophistication of the mathematical content in the strand at the various grade levels may account for this discrepancy. For example, it is possible that students who responded at a Concrete Operational level to a task involving whole numbers in Grade 2 or 3 might respond Preoperationally in Grade 6 in the context of fractions and decimals. (Piaget referred to this phenomenon as another form of decalage.)

2) What are the levels of cognitive demand made on students at each grade level by:

i) the curriculum objectives identified by the <u>Elementary Mathematics Curriculum Guide</u>, Alberta Education, 1982,

ii) the prescribed textual resources,

iii) teacher presentations, and

iv) representative achievement tests?

The predominant level of demand is the Concrete Operational. About three-quarters of all demands are at the Concrete Operational level (Early Concrete and Late Concrete combined). Of the remaining quarter, most occur at the Early Formal level, with small percentages at the Preoperational and

Formal Operational levels. (See Figures 7 to 20.)

As would be expected, the demand levels increased with increasing grade levels. The numbers of Preoperational demands ranged from a substantial number in some strands at Grade 1 to very few, if any, at Grade 6. The numbers of Early Formal Operational demands increased from very few at Grades 1 and 2 to a substantial number at Grade 6.

The distributions of Curriculum objective demands showed the greatest consistency across strands and the most consistent pattern of increasing demands as the grade level increased. Only in Grades 1 and 2 were there any Preoperational demands. Early and Late Concrete Operational demands predominate from Grades 1 to 4, but in Grades 5 and 6 Formal Operational (Early Formal and Formal combined) demands predominate, by a slight margin, in all strands except Geometry & Graphing. (See Figures 7 to 10.)

The pattern of the demands of the Textbooks is similar to that of the Curriculum objectives (Figures 11 to 14).

The Classroom lessons observed made predominantly Concrete Operational (Early Concrete and Late Concrete combined) demands in the lower grades and predominantly Formal Operational (Early Formal and Formal combined) demands in the higher grades. Except in Measurement, the demands usually included some Early Concrete demands. In Measurement there were Early Concrete Operational demands only in Grade 1; all other demands in the strand were Late Concrete or Early Formal. In Grades 1 and 2 in Geometry & Graphing there were Early Concrete Operational and Early Formal Operational demands but no Late Concrete Operational demands. In Grade 6 Operations there were only Early Formal and Formal Operational demands. (See Figures 15 to 18.)

Parts A and B of the 1982 Grade 3 and 1983 Grade 6 Alberta Education Achievement Tests were assessed. Their cognitive demands were mainly Concrete Operational (Early Concrete and Concrete combined) in Grade 3 and mainly Early

Formal Operational in Grade 6 (Figures 19 and 20). There were no Preoperational demands, and in only one instance were there Formal Operational demands, that being at Grade 6 in Operations. Overall, in Grade 3 the demands were 36% Early Concrete, 62% Late Concrete and 2% Formal. In Grade 6, 4% of the demands were Early Concrete, 29% were Late Concrete, 60% were Early Formal and 7% were Formal. It should be noted that the sections of the tests containing only computational items were not assessed. Also, because of the small number of test items in any one strand, a change in only one or two questions can make a significant change in the distribution.

> 3) How well do the distributions of curricular demands (made by the curriculum objectives, texts, teacher presentations, and tests) fit the distributions of student cognitive responses at each grade level in each mathematics topic strand?

In general, the answer to Question 3) is that the cognitive demands made by the curriculum and its interpretations have been found to correspond reasonably well to the distributions of student cognitive responses in most topics and at most grade levels. In most areas there were some matches, some demand distributions lower, and some demand distributions higher than the corresponding student response levels. The best overall fit between demand distributions and the student response distribution occurred at the Grade 4 level where 64% of the demand and response distributions matched. However, there were also some striking mismatches. For example, at Grade 5 in three of the four strands all of the demand distributions were higher than the corresponding response distributions, and 83% of all of the demand distributions in this grade were higher than the response distributions. In Grade 1 only 9% of the demand distributions matched the corresponding response distributions, but 58% of the demand distributions were higher while 33% were lower than the response distributions. (See Figures 21 to 29 and Tables 2 and 3.)

Three strands, Numeration, Operations, and Geometry & Graphing, had reasonably consistent patterns of matches and mismatches, with 40% to 45% matches, 35% to 45% of the demand distributions higher than, and 15% to 20% of the demand distributions lower than the response distributions (Table 2). The one strand that stands out is Measurement, where 100% of the demand distributions were significantly higher than the response distributions.

Two of the four demand areas (Curriculum Objectives and Textbooks) had very similar patterns of matches and mismatches across the grades (Table 3). In 46% and 33% of the demand/response distribution comparisons there were matches; 42% and 46% of the demand distributions were higher; and 12% and 21% of the demand distributions were lower than the response distributions, respectively.

The Achievement Test demand/response comparisons produced 38% matches, 63% of the demand distributions higher and none of the demand distributions lower than the response distributions. That the computation items with generally lower demand levels were not included in the analysis and that the Achievement Tests represent year-end expectations should be taken into account when considering these findings.

The distributions of demands observed in Classrooms matched the distributions of student responses less well than the other areas. In all, 77% of the Classroom demand distributions were significantly higher than the corresponding distributions of student responses, while 14% matched, and 9% were lower. The best fit occured in Grade 4 where one demand distribution was higher, one was lower, and one was not significantly different from the corresponding response distribution. In Grades 5 and 6 all the demand distributions were higher than response distributions, but the results in the other grades were not so extreme. In interpreting these findings it is important to note that the number of teachers involved was small, and the number of minutes observed in some grades and strands was very small.

In general, the mismatches that were found are most often characterized by a lack of suitable material for students still responding Preoperationally, especially in the lower grades. But in some instances a lack of suitable material for the cognitively most able was also found.

The statements in the section headed <u>Number of Matches and Mismatches Between Demand and Response Distributions by Curriculum Strands</u> lead to the following conclusions.

1. The demand distributions generally compare favourably with the response distributions in all strands except Measurement.

2. The distributions of student response levels in the early grades in the Numeration strand were generally higher than the distributions of demands, but in the later grades the demand levels were generally higher than the response levels.

3. In the Operations strand the distributions of cognitive demands generally matched or were lower than the distributions of responses (with the exception of Grade 1, as noted earlier).

4. The cognitive demands of the Measurement strand were at levels consistently exceeding the levels of response of the students.

5. The distributions of cognitive demands in Geometry & Graphing generally matched the distributions of responses with the exception of Grades 3 and 5 where the demand levels usually exceeded the response levels.

6. Overall, the total program provides the best fit between demands and responses at the Grade 4 level. Since there are more instances in Grade 5 than in any other grade in which the demand distributions are higher than the response distributions, it appears the Grade 5 is the most demanding.

The statements in the section titled <u>Number of Matches and Mismatches between Demand and Response Distributions by Demand Areas</u> lead to the following conclusions.

1. Across the six grades the patterns of matches between cognitive response distributions and cognitive demand distributions were very similar for Curriculum Objectives and Textbooks. The fit was best in Grade 4. In Grade 5 the cognitive demand distributions were predominantly higher than the distributions of cognitive responses. The Curriculm Objectives and Textbook demand

distributions were generally closer to the student response distributions than were the demand distributions of the Classroom and the Achievement Tests.

2. Generally, the distributions of cognitive demands observed in classrooms were higher than the distributions of cognitive responses of the students. It should be noted that the sample of teachers was limited, and the number of minutes observed in some grades and strands was very small.

3. In the two grades with Achievement Tests, Grades 3 and 6, the demand levels of the tests were generally higher than the response levels of the students. It should be noted that computation items, which have generally lower cognitive demand levels, were not included in the assessment.

RECOMMENDATIONS

The recommendations which follow are based on the conclusions presented above. Their main focus is on increasing the extent to which the distributions of curricular cognitive demands match the distributions of student cognitive responses.

1. It is recommended that the curriculum be reviewed and revised where necessary to include more objectives of lower demand levels in order to provide adequate small steps to bridge any gaps between the student cognitive response levels and curricular cognitive demand levels. This is particularly important in Grades 5 and 6 Numeration, in Grade 1 Operations, in all grades in Measurement, and in grades 3 and 5 in Geometry and Graphing.

2. It is recommended that the curriculum be reviewed and revised where necessary to provide more adequately for the cognitively most able. The following cases have been identified as ones in which the distributions of curricular demand are lower than the distributions of student responses: Numeration, Grades 1 and 2; Operations, Grades 2, 4, and 6; and Geometry & Graphing, Grades 1 and 6.

3. It is recommended that learning materials (e.g., prescribed textbooks) be supplemented to provide more learning experiences at the lower demand levels, especially the Preoperational and Early Concrete Operational levels.

4. It is recommended that suitable strategies such as learning activities with appropriate materials, instructional and questioning techniques, and suitable assignments and practice exercises, all promoting the development of cognitive structures, be provided in each classroom so that the teacher can better assist students to move from their demonstrated response levels to the higher levels being demanded by the curriculum.

5. It is recommended that the curriculum be reviewed considering both cognitive demands and achievement demands in order to more evenly distribute the challenges so that all grades have a consistent demand/response pattern, perhaps even approximating that of Grade 4.

6. It is recommended that activities be developed that are suitable for groups of students operating across a wide range of cognitive levels. Such activities should allow students to operate at their own levels and interact productively with other students at higher levels.

7. It is recommended that the "Notes and Comments" sections of the Curriculum guide be revised and expanded to provide more explicit and appropriate suggestions for developing students' cognitive abilities.

IMPLICATIONS FOR FURTHER RESEARCH

1. Although the ACLIC project was an extensive one, it would be valuable to investigate in greater depth selected critical topics in each strand. This would require more interviews or paper-and-pencil test items reflecting a more comprehensive profile of each topic selected.

2. A study might be conducted to explore whether teaching resources, materials, and inservice programs centred on cognitive structures can help students bridge gaps between their current cognitive response levels and the higher cognitive demand levels of the curriculum and its presentation.

3. A comprehensive diagnostic and remediation program could be undertaken in which interviews or paper-and-pencil items would be structured to cover the cognitive aspects of various concepts and skills. Based on the strengths and weaknesses exhibited by a student's responses, a remediation program could be designed and tested.

4. A study might be conducted to investigate the cognitive abilities of apparently high achieving students.

5. A study might be conducted to determine whether the needs of gifted and talented students are being met with respect to developing their cognitive abilities.

6. Fundamental theoretical questions that have been raised could be investigated further. For example, How closely should cognitive demand distributions match corresponding student response distributions? Can activities and teaching strategies be designed to close the gaps between student response levels and program demand levels? Will such activities result in higher student achievement?

7. A study might be conducted to determine whether students' performances change when their teacher is made aware of the contrasts between student cognitive response levels and program cognitive demand levels.

8. Each strand of the secondary mathematics program could be assessed in terms of distributions of curricular cognitive demands and student cognitive responses.

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Appendix 1

ACLIC Interview Record Sheets

REVISED

ACLIC INTERVIEW RECORD: FLOWERS (About 10 minutes)

NAME: ______ BOY/GIRL GRADE: ______

AGE: ______ BIRTHDAY: ______ SCHOOL: ______

INTERVIEWER: ______ DATE: ______ CASSETTE: ______ COUNTER: ______

(Copeland, 1974, 137 to 139; Copeland, 1974b, 26, 27; Piaget, 1952, 203 to 220)

MATERIALS: 10 large white flowers
10 vases [plastic parfait "glasses", arranged in a triangular "ten-pins" pattern]
10 red flowers
10 other flowers
10 chenille "stems"
2 containers for flowers [one a bottle with a tall, narrow neck; the other a short, wide basket with a florist's styrofoam arrangement block]

PROCEDURE:

Ask child to put a large white flower in each vase. Then remove the flowers and arrange them in the styrofoam block in the basket. Ask child to put a red flower in each vase. Then remove the red flowers and place them in the tall, narrow-necked container.

QUESTION 1: "Are there as many flowers here [red] as there [white]?" (or, "Is there the same number of red flowers as white flowers?") "How do you know?"

If the child says that there are more of one kind of flowers than the other: Interchange the red and white flowers in the containers so that the red ones are spread out and the white ones are bunched together. "And now . . ." (repeating QUESTION 1 if necessary). "Where were the flowers before?" . . . "Were they [the red ones] exactly right?" . . . "And what about those [the white ones]?"

QUESTION 2: "If we put all the flowers into the vases with the same number in each vase, how many would there be in each vase?"

Then introduce the third set of flowers and repeat the procedure for establishing that there are just enough of these flowers for the vases. Then repeat QUESTION 2.

Optional: Repeat QUESTION 2 with chenille "stems."

COGNITIVE LEVELS OF RESPONSE

EQUIVALENCE (QUESTION 1)

PREOPERATIONAL:

☐ Unable to regard equivalences between flowers and vases as lasting or to use this information to answer QUESTION 1

☐ One set of flowers seen as having a larger number than the other [perceptual distraction producing non-equivalence]

☐ OTHER PREOPERATIONAL RESPONSE/COMMENT:

EARLY CONCRETE OPERATIONAL:

☐ Though fooled by perception, child finds correct answer by trial and error or questioning, but can be fooled again (May make direct comparison between the sets of flowers without using the vases to compose the equivalences established.)

☐ Intuitive composition depending on perceptual content and not yet operational (1-1 correspondence without lasting equivalence)

☐ OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENT:

LATE CONCRETE OPERATIONAL:

☐ Child at operational level, no longer swayed by perception; the logic of the reversibility of composed equivalences overcomes any perceptual distraction (Piaget, 1952, 213) ("I count with the vases.")

☐ OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENT:

FLOWERS RECORD 3 of 3

MULTIPLICATION (QUESTION 2)

PREOPERATIONAL:

☐ Can make one-one correspondence of flowers to vases, but if one set of flowers is bunched and the other spread out, the latter is seen as having more flowers; cannot apply transitive property logic (If #BF = #V and #V = #WF, then #BF = #WF)

OTHER PREOPERATIONAL RESPONSE/COMMENT:

EARLY CONCRETE OPERATIONAL:

☐ Child gradually succeeds by repeating experiment or by responding to interviewer questions like: "How many blue flowers were in the vases?" and "How many white flowers were in the vases?"; still distracted by perceptual factors

☐ OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENT:

LATE CONCRETE OPERATIONAL:

☐ Immediate use of the logic of the transitive property, with the conclusion that since one red flower and one white flower went into each vase, there must be the same number of red flowers as white flowers

☐ Successful answer to the question: "If we put all the flowers in the vases with the same number in each vase, how many flowers would be in each vase?" and to the same question repeated after a third set of ten flowers is introduced

☐ OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENT:

REVISED

ACLIC INTERVIEW RECORD: TOFFEE (About 10 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______

AGE: ______ BIRTHDAY: ______ SCHOOL: ______

INTERVIEWER: ______ DATE: ______ CASSETTE: ______ COUNTER: ______

MATERIALS: A roll of adding machine tape
6 Bear Stickers mounted on cards
A pair of scissors
Pencil and/or marking pen
Plastic Stirsticks cut in half lengthwise and sideways (to make thin "sticks" that resist rolling)

METHOD: (Show the child a strip of adding machine tape, longer than wide.) "This is quite a thin piece of toffee (candy, cheese, . . .). The bears want to eat it all up. To be fair, each should get just the same amount. How shall we do it? . . . You can use these (pencil, scissors, "sticks," . . .)."

"How did you do it?" (or, "Tell me what you did.")

When the child has divided the whole, ask: "Would these pieces taken together (illustrating by a sweep of the hand) make up as much as the whole strip of toffee (candy, cheese, . . .) that we started with . . . or more . . . or less?" . . . "Tell me why."

Then, bring out a third bear and repose the problem with a new strip of adding machine tape. [Alternatively, START with three bears.]

As appropriate, proceed to quarters, fifths, or sixths . . . using the same procedure.

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

--real difficulty dividing toffee into two equal parts (halves), for example:

- [ ] more than two parts
- [ ] approximately equal small portions but rest of toffee undivided
- [ ] all of toffee shared but unequally
- [ ] three portions (confusing number of cuts with number of parts)

(Piaget, Inhelder, Szeminska, 1960, 303)

- [ ] OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL:

--problem of dichotomy solved (i.e., 2 equal parts)
--no realization that the original whole must necessarily equal the sum of its original parts

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL:

--trichotomy is solved (i.e., making 3 equal parts) and conservation of the whole is realized intuitively (but not operationally)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS):

--trichotomy handled by means of an anticipatory schema (i.e., an a priori understanding of the relations between fractions sought and the original whole); operational conservation of the whole
--division into fifths and sixths handled by means of an anticipatory schema

OTHER EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS) RESPONSE/COMMENTS:

ACLIC INTERVIEW RECORD: DRUMS (About 15 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______

AGE: ______ BIRTHDAY: ______________ SCHOOL: ______________

INTERVIEWER: ______________ DATE: ______ CASSETTE: ______ COUNTER: ______

MATERIALS: Ten drums, graduated in size, the largest at least twice the size of the smallest
Ten drumsticks [wooden toothpicks cut to length of drum radius]

QUESTION 1: "The drums are going to be in a parade. Arrange the drums and drumsticks so that each drum is with the right size of drumstick." (Discuss until it is clear that the child understands the principle of serial correspondence.)

QUESTION 2: (Once the rows of drums and drumsticks have been arranged in correspondence with one another, in clear view of the child, move the drums closer to one another and the drumsticks further apart, but maintaining the distance between the rows of drums and drumsticks.) [Touching one of the drums] "Which drumstick will go with this one?" (Repeat, choosing in order or at random according to the child's answers.)

QUESTION 3: (After several repetitions of QUESTION 2, reverse one of the series.) "Which drumstick will go with this one?" [As in QUESTION 2]

QUESTION 4: (One or both series disarranged) "Which drumstick belongs to this drum?"

QUESTION 5: [Which enables determination of "the exact level of the child's understanding" (Piaget, 1952, 98)] (Mingle all of the elements of the two series. Pick out a drum, say number 5.) "Some of the drums are going in the next parade, but not all of them--only those that are bigger (or smaller) than this drum. Find the drumsticks belonging to the drums that are going in the parade and those belonging to the drums not going."

[These five questions, which must be posed separately to the child, can be reduced to three more general problems for systematizing the results:

1. constructing a serial correspondence or similarity (Q. 1)

2. determining a serial correspondence when it is no longer directly perceived (transition to ordinal correspondence) (Q. 2 and Q. 3)

3. reconstructing the ordinal correspondence when the intuitive series are destroyed (Q. 4 and Q. 5) (Piaget, 1952, 98)]

CONSTRUCTION OF SERIAL CORRESPONDENCE (QUALITATIVE SIMILARITY) [Q. 1]

PREOPERATIONAL:

--child unable to make drums and drumsticks correspond and unable to form correctly isolated series (Piaget, 1952, 99)

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL:

--child capable of spontaneous construction of correct series following some trial and error

--successful solving of problem of serial correspondence (especially by the method of double seriation) (Piaget, 1952, 102, 103)

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL:

--child continually considers the set of relationships between all of the elements, at each new step looking for the biggest (smallest) of the remaining elements (no trial or error)

--ease of operation by immediate correspondence, without previously seriating drums and drumsticks (Piaget, 1952, 106)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

SERIAL CORRESPONDENCE TO ORDINAL CORRESPONDENCE [Q. 2 & Q. 3]

PREOPERATIONAL:

--child loses all notion of correspondence when one of the series is displaced, merely choosing elements opposite one another

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL:

--child tries to find the correct correspondence "by empirical means or by counting" (Piaget, 1952, 106) but constantly confuses the right position with that of the preceding term

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

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LATE CONCRETE OPERATIONAL:

--problem solved by coordinating estimate of required position with that of cardinal value of sets in question (involving both qualitative serial and ordinal numerical correspondences) (Piaget, 1952, 106-114)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

RECONSTRUCTION OF CARDINAL CORRESPONDENCE [Q. 4 & Q. 5]

PREOPERATIONAL:

--no correspondence, series not reconstructed, elements chosen at random

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL:

--attempt to solve problem lacks systematic re-seriation or cardination

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL:

--reconstruction achieved by co-ordination of ordination and cardination (Piaget, 1952, 115-121)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

REVISED

ACLIC INTERVIEW RECORD: BLOCKS1 (About 15 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______

AGE: ______ BIRTHDAY: ______________ SCHOOL: ______________

INTERVIEWER: __________ DATE: ______ CASSETTE: ______ COUNTER: ______

MATERIALS: 20 Flats, 20 Longs, 20 Units, 1 Die
BLOCKS1 Cards (as indicated below)
Pencil, eraser
Sheet of Bristol Board (or facsimile) on which to roll Die

PROCEDURES:

A. "Race to 50"
--"Have you used these [B, F, L, U]? What do you call . . . ?"
--Interviewer and child take turns rolling Die
--". . . soonest to 100."
--roll for "units" until enough to exchange for "long" (Use child's words.) "How many "units" in a "long"? (. . . "flat")
--after a "long" received, roll for "longs" until 100 reached or passed

[ ] Successful [ ] Unsuccessful

COMMENT:

B. "Show me |361| [printed on a card] using as few pieces of wood as you can."

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

C. "This meter counts the number of people that go in through a gate to a hockey game. . ."

| | | |3|1|9| [On card]

"How many have already gone through the gate?

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

"Tell me what the meter will show when one more person has gone through."

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least LATE CONCRETE OPERATIONAL.)

D. The number that is 2 less than 20 is ______. [On card]

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least LATE CONCRETE OPERATIONAL.)

[If not successful, use blocks.] (EARLY CONCRETE OPERATIONAL)

COMMENT:

[If appropriate, try BLOCKS2 Cards.]

REVISED

ACLIC INTERVIEW RECORD: BLOCKS2 (REV)

(About 15 minutes)

NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

MATERIALS: 4 Blocks, 20 Flats, 20 Longs, 20 Units, 1 Die
BLOCKS2 Cards (as indicated below)
Pencil, eraser
Sheet of Bristol Board (or facsimile) on which to roll Die

PROCEDURES:

A. "Race to 100"
--"Have you used these [B, F, L, U]? What do you call . . . ?"
--Interviewer and child take turns rolling Die
--". . . soonest to 100."
--roll for "units" until enough to exchange for "long" (Use child's words.) "How many "units" in a "long"? (. . . "flat")
--after a "long" received, roll for "longs" until 100 reached or passed

[ ] Successful [ ] Unsuccessful

COMMENT:

B. "Show me 1361 [printed on a card] using as few pieces of wood as you can."

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

C. "This meter counts the number of people that go in through a gate to a hockey game. . ."

| | | | 6 | 9 | [On card]

"How many have already gone through the gate?

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

"Show me what the meter will show when one more person has gone through.

| | | | | | | [On card]

Successful Unsuccessful

COMMENT:

(Successful response is at least LATE CONCRETE OPERATIONAL.)

D. The number that is 2 less than 50 is ______. [On card]

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least LATE CONCRETE OPERATIONAL.)

[If not successful, use blocks.] (EARLY CONCRETE OPERATIONAL)

COMMENT:

E. "Can you subtract?"

$$\begin{array}{r} 56 \\ \underline{-33} \end{array}$$ [On card]

[ ] Successful [ ] Unsuccessful

COMMENT (Please save child's work.):

(Successful response is at least LATE CONCRETE OPERATIONAL.)

$$\begin{array}{r} 32 \\ \underline{-18} \end{array}$$

"Tell me what you did [and why] when you were working that out."

[ ] Successful [ ] Unsuccessful

COMMENT (Please save child's work.):

[If unsuccessful, use F, L, U.] (CONCRETE OPERATIONAL)

[If appropriate, try: $\begin{array}{r} 102 \\ \underline{-25} \end{array}$]

REVISED

ACLIC INTERVIEW RECORD: BLOCKS3
(Grade 3, About 15 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______________

AGE: __________ BIRTHDAY: ______________ SCHOOL: ______________

INTERVIEWER: __________ DATE: __________ CASSETTE: __________ COUNTER: __________

MATERIALS: 4 Blocks, 20 Flats, 20 Longs, 20 Units, 1 Die
BLOCKS3 Cards (as indicated below)
Pencil, eraser
Sheet of Bristol Board (or facsimile) on which to roll Die

PROCEDURES:

A. "Race to 100"
--"Have you used these [B, F, L, U]? What do you call . . . ?"
--Interviewer and child take turns rolling Die
--". . . soonest to 100."
--roll for "units" until enough to exchange for "long" (Use child's words.) "How many "units" in a "long"? (. . . "flat")
--after a "long" received, roll for "longs" until 100 reached or passed

[ ] Successful [ ] Unsuccessful

COMMENT:

B. "Show me 1365 [printed on a card] using as few pieces of wood as you can."

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

C. "This meter counts the number of people that go in through a gate to a hockey game. . ."

| | |6|9|9| [On card]

"How many have already gone through the gate?" (Successful response is at least EARLY CONCRETE OPERATIONAL.)

[ ] Successful [ ] Unsuccessful

COMMENT:

(Successful response is at least EARLY CONCRETE OPERATIONAL.)

"Show me what the meter will show when one more person has gone through.

| | | | | | [On card]

Successful Unsuccessful

COMMENT (Please save or record child's response.):

(Successful response is at least LATE CONCRETE OPERATIONAL.)

D. The number that is 2 less than 300 is ______. [On card]

Successful Unsuccessful

COMMENT:

(Successful response is at least LATE CONCRETE OPERATIONAL.)

[If not successful, use blocks.] (EARLY CONCRETE OPERATIONAL)

E. "Can you subtract?"

```
 527
-332
```

[On card]

Successful Unsuccessful

COMMENT (Please save child's work.):

(Successful response is at least LATE CONCRETE OPERATIONAL.)

```
 702
- 25
```

"Tell me what you did, and why, when you were working that out."

Successful Unsuccessful

COMMENT (Please save child's work.):

(Successful response with logical explanation is at least EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS BEING FORMED).)

[If unsuccessful, use F, L, U.] (CONCRETE OPERATIONAL)

```
  4 002
-    25
```

"Tell me what you did [and why] when you were working that out."

( ) Successful ( ) Unsuccessful

COMMENT (Please save child's work.):

(Successful response with logical explanation is at least EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS BEING FORMED).)

[If unsuccessful, use B, F, L, U.] (CONCRETE OPERATIONAL)

ACLIC INTERVIEW RECORD -- PARKING (About 10 minutes)

NAME: ____________________ BOY/GIRL GRADE: ____________

AGE: __________ BIRTHDAY: __________ SCHOOL: ____________

INTERVIEWER: __________ DATE: __________ CASSETTE: __________ COUNTER: __________

MATERIALS: Eleven toy cars, 7 of one color, 4 of another.
Parking lot mat.

PROCEDURE: Interviewer and child together park seven cars then four cars. Child drives all the cars out of the parking lot. Child parks four cars then one of the seven.

QUESTION: If you finish parking all the cars, will all the stalls be full?
(If yes), Will there be any cars left over? If incorrect have the child continue parking the cars and ask the question again.

COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

☐ Child is unable to make one-to-one correspondence.

☐ Child is unable or unwilling to make any prediction.

☐ Child makes incorrect predictions and does not change them after completing the task.

☐ OTHER PREOPERATIONAL:

EARLY CONCRETE OPERATIONAL:

☐ Child makes incorrect prediction(s) but changes his mind as he completes the task.

☐ OTHER EARLY CONCRETE OPERATIONAL:

LATE CONCRETE OPERATIONAL:

☐ Child immediately predicts correctly.

☐ OTHER LATE CONCRETE OPERATIONAL:

ACLIC INTERVIEW RECORD--COOKIES

NAME: BOY/GIRL GRADE:

AGE: BIRTHDAY: SCHOOL:

INTERVIEWER: DATE: CASSETTE: COUNTER:

MATERIALS: Forty bingo chips.
Card (8 1/2 x 11) divided in half, halves marked "Yesterday" and "Today."

DIRECTIONS 1: . . . boy (girl) likes cookies that mother makes
. . . yesterday mother sent 4 cookies for morning recess and 4 for afternoon (Act out with chips on card, "Yesterday" side.)
. . . yesterday he (she) ate 4 cookies in the morning and 4 in the afternoon
. . . today, mother again sent 4 cookies for the morning and 4 for the afternoon (Act out on "Today" side.)
. . . but today he (she) was so busy he (she) only had time to eat 1 cookie in the morning and saved the rest of the cookies to eat in the afternoon (Move three cookies over next to the four for "Today's" afternoon.).

QUESTION 1: Did the boy (girl) have more cookies yesterday (pointing to all of yesterday's cookies) or more cookies today (pointing to all of today's cookies) or did he (she) have the same on both days? Tell me how you know.

QUESTION 1: COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

Child does not know or is not certain which day had more chips. Justification, if any, is based on perception.

OTHER PREOPERATIONAL:

EARLY CONCRETE OPERATIONAL:

Child initially chooses one day as having more than the other. When pressed for justification suggests counting and discovers they both have eight.

OTHER EARLY CONCRETE OPERATIONAL:

LATE CONCRETE OPERATIONAL:

Child knows that both days have the same number even without counting. Justifies on the basis of equality of number, on basis of reversibility (they can be rearranged as they were), or on the basis of identity (none were added or taken away, they are the same).

OTHER LATE CONCRETE OPERATIONAL:

DIRECTIONS 2: Take 24 chips and put them into two clearly unequal piles, one for the student, one for the interviewer.

QUESTION 2: Suppose I put these cookies into two piles, these (the larger pile) for you and these (the smaller pile) for me. Would that be fair? What could you do with all these cookies to put them into two piles so that it would be fair? Show me how you would do it.

QUESTION 2: COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

Child judges piles to be "fair" or child makes piles "equal" by estimation, without counting or checking in any way.

OTHER PREOPERATIONAL:

EARLY CONCRETE OPERATIONAL:

Child equalizes piles by a series of trials and errors, counts or matches to verify success.

OTHER EARLY CONCRETE OPERATIONAL:

LATE CONCRETE OPERATIONAL:

Child shares systematically by putting all in one pile and removing two (or an equal number for each person) at a time, or uses number facts to make two equal piles.

OTHER LATE CONCRETE OPERATIONAL:

ACLIC INTERVIEW RECORD: DIVIDED BOXES (About 5 minutes)

NAME: ________________ BOY/GIRL GRADE: ________

AGE: ________ BIRTHDAY: ________ SCHOOL: ________

INTERVIEWER: ________ DATE: ________ CASSETTE: ________ COUNTER: ________

MATERIALS: Two identical boxes, open at top but with a divider as shown (about 10 cm x 15 cm x 2 cm). Twenty-four small blocks in each box.

DIRECTIONS 1: With all the blocks on one side of the divider in each box, ask the child whether the two boxes are the same (whether they have the same number of blocks in them). If necessary, suggest counting the blocks. In child's view explain that you will move some of the blocks to one side of the divider; move six to the opposite side, leaving 18. Explain that in the other box you will also move some blocks, but that you do not want the child to see what you are doing. Arrange the box so that it is a mirror image of the first box. With your hand covering the six blocks on the one side of the second box, show the two boxes side by side.

QUESTION 1: Can you tell me how many blocks are hidden under my hand? If the answer sounds like a guess, ask questions such as: Are you sure? Is there any way you could know for sure, instead of just thinking there might be that many?

COGNITIVE LEVELS OF RESPONSE

PREOPERATIONAL:

Child does not know, or guesses, or estimates.

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL:

Child guesses six and supports this by reference to the six in other box, but does not verify that the other sides of each box each have the same number (18).

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL:

Child determines that there are six by computation (24 - 18 = N, 18 + N = 24, etc.), by counting on from 18 to 24, or by reference to the six and 18 in one box and the 18 and six in the other.

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENT:

DIRECTIONS 2: (May be used to verify child's method of solution.) Take up one box and explain that you are going to move some blocks from one side to the other without the child seeing the moves. Arrange the box with eight on one side and sixteen on the other. Present the box with the eight covered.

QUESTION 2: As before.

NOTE: A child who notices that there are fewer showing so there must be more hidden is using compensation. If this idea is applied as an estimation, then an EARLY CONCRETE OPERATIONAL response is indicated. If the child argues that since there are two fewer showing (16 instead of 18), there must be two more hidden (10 rather than 8), a LATE CONCRETE OPERATIONAL response is indicated.

To: Grade 1 Teachers, Group A

Re: Divided Boxes

This task with 24 blocks in each of the two boxes will probably be solved by older children through counting, but some children at the beginning of grade 1 may not be able to count 24 small objects reliably. So, the interviewer should present this task in such a way that it can be solved even without counting.

When first presenting the two boxes with all 24 counters on one side, make sure that the boxes are arranged identically (for example, four rows of six in each box). Then when presenting 6 and 18 in one box and 18 and a hidden number in the other, make sure that the two groups of 18 are identically arranged (for example, three rows of six). Finally, when presenting 8 and 16 and 16 and a hidden number, be sure to arrange the blocks so that the two boxes are mirror images of each other.

In assessing the cognitive level of the response, note that counting is not a requirement for the LATE CONCRETE OPERATIONAL level. A child who argues that there are "just as many" (even if he does not know how many) on the corresponding sides of the two boxes may be working at the LATE CONCRETE OPERATIONAL level.

REVISED

ACLIC INTERVIEW RECORD: LENGTH (About 5 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______________

AGE: ______________ BIRTHDAY: ______________ SCHOOL: ______________

INTERVIEWER: ______________ DATE: ______________ CASSETTE: ______________ COUNTER: ______________

MATERIALS: Two sets of task cards (A1 - A6 and B1 - B5)
Coffee stir sticks identical to those on cards A1 - A6
Strips of paper, pencils

PROCEDURE: Show the child the coffee stir sticks and explain that the cards (A1 - A6) you will show him (her) have sticks like them glued onto them. As you present each card ask "Are these two sticks both the same length or is one stick longer than the other? (Which one is longer?)" Also ask the child to tell or show how he knows.

Introduce cards B1 - B5 explaining that these cards each have two lines drawn on squared paper on them. Ask "Are these two lines the same length or is one longer than the other? Which one? How do you know?"

SUMMARY OF RESPONSES

The following abbreviations may be used to describe methods.

P - Perception - Child bases his answer on perception (e.g., "it looks longer.")

O - One end - Child notes that one end of one line extends beyond the other, without considering the relationship of the opposite ends.

S - Straighten - Child argues that if the curved or broken lines on cards B1, B2, and B4 were straightened they would be longer.

A - Align ends - Child attempts to compare corresponding ends of lines (especially on cards A1, A2, and B5).

M - Measure along - Child uses a tool carefully placed along the length of the object.

T - Transitivity - Child compares two different objects with a moveable tool and argues that if they both match the tool they must be equal.

I - Inaccurate - Child makes small errors using measuring tool and concludes that the sticks in card set A are slightly different in length.

Other Abbreviations/Comments:

LENGTH RECORD 2 OF 2

Card	Equal	Unequal	Method(s)							Comments
			P	O	S	A	M	T	I	
A1	+	−								
A2	+	−								
A3	+	−								
A4	+	−								
A5	+	−								
A6	+	−								
B1	−	+								
B2	−	+								
B3	−	+								
B4	−	+								
B5	+	−								

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

☐ Perception ☐ One end

Other evidence for PREOPERATIONAL level of response:

EARLY CONCRETE OPERATIONAL:

☐ Align ends

☐ B3 incorrect, others in B set correct

Other evidence for EARLY CONCRETE OPERATIONAL level of response:

LATE CONCRETE OPERATIONAL:

☐ Measure along ☐ Transitivity

Other evidence for LATE CONCRETE OPERATIONAL level of response:

REVISED

ACLIC INTERVIEW RECORD: TIME (About 10 minutes)

NAME: ______ BOY/GIRL GRADE: ______

AGE: ______ BIRTHDAY: ______ SCHOOL: ______

INTERVIEWER: ______ DATE: ______ CASSETTE: ______ COUNTER: ______

(Piaget, 1969. 188-195)

MATERIALS: Stop-watch (one complete rotation in 1/2 minute (30 seconds))
Metronome or facsimile (adjustable) [to produce 1 beat per second]
Opaque cap for stop-watch

QUESTIONS:

"Count up to 15 in time with the metronome, looking at the hand of the stop-watch." [Which advances from 0 to 15 seconds in the same time]

(Mask the stop-watch.) "Count up to 15 again, but this time twice as quickly . . . count two numbers to every beat. . . . How far do you think that the [hidden] hand of the stop-watch went while you were counting faster? Why?"

PREDICTION: Hand at ______

"Does counting quickly take more time than counting slowly . . . or less time . . . or the same amount of time?"

"Does the watch go more slowly at one time and more quickly at another . . . or does it always go the same?"

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL:

( ) --child thinks stop-watch hand runs more or less rapidly according to the speed of work whose duration is being timed (in this case, "counting to 15")
( ) --PREDICTION OFF (i.e., greater or less than 7 1/2)
( ) OTHER PREOPERATIONAL RESPONSE/COMMENTS:

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EARLY CONCRETE OPERATIONAL

--inability to correlate the work done by oneself (the count of 15) with the steady motion of the stopwatch hand (appreciation of the conservation of velocity but inability to apply it to more than one moving body)

--refusal to make any predictions on the grounds that it is impossible to do so

--inability to ascribe a unique unit of time or a common duration to motions having different velocities

--the two counting speeds compared directly without reference to time duration measured by watch

--PREDICTION OFF (i.e., greater than or less than 7 1/2)

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL:

--predicts that the hand will stop before 15 (realizing that the speed of the watch is not affected by the speed of the work timed), but guesses inaccurately

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS):

--PREDICTION: 7 1/2 (exact correlation between task duration and displacement of stopwatch hand) (Piaget, 1969, 194)

--PREDICTION: about 7 1/2, with logical explanation

OTHER EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS) RESPONSE/COMMENTS:

ACLIC INTERVIEW RECORD: WEIGHT (About 15 minutes)

NAME: ____________________ BOY/GIRL GRADE: ____________________

AGE: __________ BIRTHDAY: __________ SCHOOL: ____________________

INTERVIEWER: __________ DATE: __________ CASSETTE: __________ COUNTER: __________

MATERIALS: Three pebbles whose weight cannot be guessed from the volume
Beam balance
Two small empty boxes
Three balls of modelling clay, varying in size and with the smallest containing some lead shot, the second-smallest a pebble, and the largest of clay only
Three balls of modelling clay as in the preceding description but with the heaviest of medium size, and the lightest the largest
Six pebbles whose weight cannot be determined by inspection alone
Three pebbles of identical volume but different weight
Ten clay balls of the same volume but different weights
Three visually identical match boxes, one filled with sand, one filled with matches, and one empty

PROBLEM I

(Show child three pebbles whose weight cannot be guessed from their volumes. Say that the balance can be used to weigh the pebbles, or they can be weighed in the hand, whichever the child wishes.) "The rule for this game is: You must never touch more than two pebbles at a time." (Hand child two identical empty boxes for weighing the pebbles two at a time) "Put the heaviest pebble here (pointing to one side of table), put the lightest there (pointing to the other side), and the other in the middle."

COMMENTS:

PROBLEM II.

(Hand the child three balls of modelling clay, the smallest containing lead shot, second-smallest a pebble, largest only clay.) "These three balls do not have the weight they seem to have, so weigh them two at a time and try to put them in order from heaviest to lightest (or lightest to heaviest)."

(Repeat the preceding, but with the heaviest of medium size and the lightest the largest.)

COMMENTS:

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PROBLEM III.

(First, give child four to six pebbles whose weight differences cannot be seen by inspection alone, and then, three pebbles of identical volume but different weights. Each time, ask the child to: . . .) "Arrange these in order from lightest to heaviest (or, heaviest to lightest) by weighing them two at a time."

COMMENTS:

PROBLEM IV.

(Show the child ten clay balls of the same volume but different weights.) "Arrange these in order of increasing weight (or, from lightest to heaviest)." [No two-at-a-time restriction this time]

COMMENTS:

PROBLEM V.

(Hand the child three visually identical matchboxes, one of which is filled with sand, another with matches, and the third empty.) "These matchboxes are not the same weight. One is filled with sand, one with matches, and one is empty. Weigh them in your hand. Watch where I put the boxes." (Shuffle the boxes around and deliberately arrange them on the table in the form of a triangle.) "I'm going to ask you three questions and I'd like you to answer them by pointing to the right boxes, without opening or touching them.)

QUESTION V1. "This box (A) is heavier than that one (B), and that one (B) is heavier than this one (C). Which is the heaviest of the three boxes? And which is the lightest?" (Piaget and Inhelder, 1974, 185)

QUESTION V2. "This box (A) is heavier than that one (B), and this one (C) is lighter than that one (B). Which is the heaviest of the three? And which is the lightest?" (Piaget and Inhelder, 1974, 185)

QUESTION V3. "This box (B) is lighter than that one (A), and heavier than this one (C). Which is the heaviest and which is the lightest of the three?" (Piaget and Inhelder, 1974, 185)

COMMENTS:

COGNITIVE LEVEL OF RESPONSES

PREOPERATIONAL. Lack of composition.

--unable to solve Problems I and II because of weighing only two or three objects, often one at a time and without any correlation (Piaget and Inhelder, 1974, 185)

--OTHER PREOPERATIONAL RESPONSE(S):

EARLY CONCRETE OPERATIONAL. Empirical seriation.

--also unable to solve Problems I and II but because the relations are established by co-ordinating isolated pairs (e.g., A > B and A > C)

--in Problem III establishes that A > B and C > D, failing to appreciate that this tells nothing about the relationships between A, B and C, D

--attacks Problem IV (simple seriation) empirically but fails to co-ordinate successive constructions

--unable to solve Problem V (Piaget and Inhelder, 1974, 185)

--OTHER EARLY CONCRETE OPERATIONAL RESPONSE(S):

LATE CONCRETE OPERATIONAL. Operational seriation (concrete).

--correct series (A > B > C) constructed for Problems I and II by co-ordinating all the relations

--correct seriation for Problem IV

--unable to construct the logical system needed to solve Problem III because of a failure to co-ordinate the inverse relations (B < A and B > C, for example) in Question V3 (Piaget and Inhelder, 1974, 185)

--OTHER LATE CONCRETE OPERATIONAL RESPONSE(S):

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS)

--(advanced) operational seriation fully developed (Piaget and Inhelder, 1974, 185)

--OTHER EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS) RESPONSE(S):

REVISED

ACLIC INTERVIEW RECORD: SORTING (About 20 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______________

AGE: ________ BIRTHDAY: ______________ SCHOOL: ______________

INTERVIEWER: ________ DATE: ________ CASSETTE: ________ COUNTER: ________

MATERIALS:

GRADE 1

2 blank sheets of paper

A set of geometric solids including: a sphere, a large cube, a smaller cube, a cone, a long narrow cylinder, a shorter wide cylinder, a long rectangular prism, a pyramid, a triangular pyramid, a long triangular prism

Cardboard cutouts of three red squares, two blue squares and three blue circles.

GRADES 2 and 3

As for Grade 1, but with the addition of the following attribute blocks:

a large, thin, blue circle; a small, thin, yellow circle; small, thin, red square; small, thin, blue triangle; large, thin, blue rectangle; large, thin, red triangle.

PROCEDURE:

GRADE 1

CLASSIFICATION

A. Mix the objects up and place them on a table before the child. Ask the child to put the objects into 2 groups so that the things in each group are alike in some way. All of the objects must be used. Have him place each group of objects onto a sheet of paper. After the child has sorted them all, point to one of the groups and ask, "How are all the objects in this group alike?

GROUP ONE: ______________________________

____

[ ] CORRECT [ ] INCORRECT

GROUP TWO: ______________________________

____

[ ] CORRECT [ ] INCORRECT

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B. Mix the objects again and ask the child to group them into 2 groups, a different way. When he has finished sorting again ask, "How are all the objects in this group alike?" Then repeat the question for the second group.

GROUP ONE: ______________________________

____

( ) CORRECT ( ) INCORRECT

GROUP TWO: ______________________________

____

( ) CORRECT ( ) INCORRECT

INCLUSION:

Bring out the red and blue cardboard cutouts and arrange them in a line (e.g., blue square, red square, blue circle, red square, blue square, blue circle, red square, blue circle). Then ask: "What colour is this?" (pointing to one of the figures) "What shape is that?" And then . . .

"Are all the circles blue?"

( ) Successful ( ) Unsuccessful

COMMENTS:

"Are all the blue ones circles?" "Why?"

( ) Successful ( ) Unsuccessful

COMMENTS:

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SORTING RECORD 3 of 5

GRADES 2 and 3

As for Grade 1, but repeat the CLASSIFICATION procedure using the set of attribute blocks described under MATERIALS.

CLASSIFICATION (GEOMETRIC SHAPES), A.

GROUP ONE: ______________________________

____

[ ] CORRECT [ ] INCORRECT

GROUP TWO: ______________________________

____

[ ] CORRECT [ ] INCORRECT

CLASSIFICATION (GEOMETRIC SHAPES), B.

GROUP ONE: ______________________________

____

[ ] CORRECT [ ] INCORRECT

GROUP TWO: ______________________________

____

[ ] CORRECT [ ] INCORRECT

CLASSIFICATION (ATTRIBUTE BLOCKS), A.

GROUP ONE: ______________________________

____

[ ] CORRECT [ ] INCORRECT

GROUP TWO: ______________________________

____

[ ] CORRECT [ ] INCORRECT

CLASSIFICATION (ATTRIBUTE BLOCKS), B.

GROUP ONE: ______________________________

____

[ ] CORRECT [ ] INCORRECT

GROUP TWO: ______________________________

____

[ ] CORRECT [ ] INCORRECT

INCLUSION

"Are all the circles blue?"

( ) Successful ( ) Unsuccessful

COMMENTS:

"Are all the blue ones circles?" "Why?"

( ) Successful ( ) Unsuccessful

COMMENTS:

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

CLASSIFICATION

PREOPERATIONAL.

( ) Able to sort objects but inconsistent in naming an attribute common to all the objects in a group.
( ) Incapable of considering an entire group of objects simultaneously and of naming a single common attribute.

( ) OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL.

( ) Able to group the objects in only one or two ways. When the objects are mixed up again, unable to sort them into 2 different groups, on request.

( ) OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL.

( ) The child's thinking is flexible.

( ) OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

INCLUSION

PREOPERATIONAL.

Only knows what is "seen". ". . . cannot mentally separate the circles as a class from the whole series. 'All' can only mean . . . the whole of the graphic collection" (Copeland, 1974b, 39).

OTHER PREOPERATIONAL RESPONSE/COMMENTS:

EARLY CONCRETE OPERATIONAL

Successfully dissociates squares as a class from circles and red from blues, but not yet able to set up classes based on the logic of inclusion.

Response to "Are all the circles blue?": "No, because there are blue squares." Since the child does not yet have the logical structure required to answer the question "Are all of the circles some of the blues?" the question is interpreted "Are all the circles all the blues?"

OTHER EARLY CONCRETE OPERATIONAL RESPONSE/COMMENTS:

LATE CONCRETE OPERATIONAL.

Child can establish logical classes of "circles," "squares," and "blues," considering the entire heterogenous grouping "all" of the shapes, and the circles as "some" of the shapes which are blue. (Copeland, 1974b, 39, 40)

OTHER LATE CONCRETE OPERATIONAL RESPONSE/COMMENTS:

ACLIC INTERVIEW RECORD: DOT (About 15 minutes)

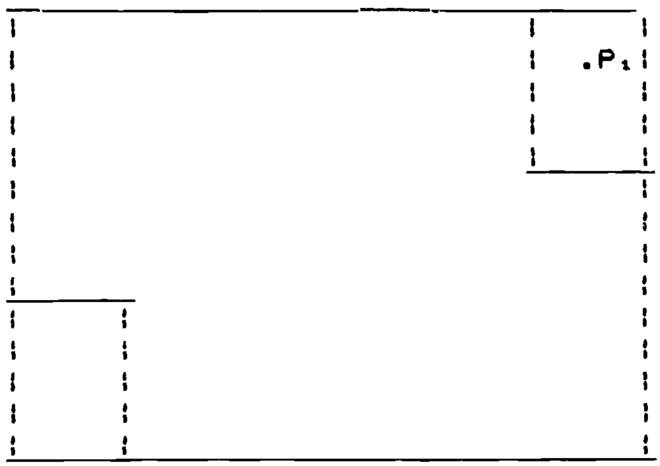
NAME: ____________________ BOY/GIRL GRADE: ____________

AGE: ________ BIRTHDAY: ____________ SCHOOL: ____________

INTERVIEWER: ____________ DATE: ________ CASSETTE: ________ COUNTER: ________

MATERIALS: sheets of plain white rectangular paper, a thirty centimetre ruler (marked in cm only), an unmarked stick, strips of paper, lengths of string.

PROCEDURES: Give the child two identical sheets of plain white rectangular paper, placing them at opposite corners of the table, as shown below. On one of the sheets mark a point, P_1, in red about halfway between the centre of the rectangle and its upper right-hand corner. Ask the child to mark a point on the second sheet in the same position as P_1 has on the first sheet . . . so that if the second sheet is placed on top of the first, the two points will be in the same place.



COGNITIVE LEVELS OF RESPONSE

PREOPERATIONAL:

☐ Children make no use whatever of the material provided. Instead of attempting to measure, they place their point by visual estimate.

☐ OTHER PREOPERATIONAL RESPONSE:

BEST COPY AVAILABLE

EARLY CONCRETE OPERATIONAL:

The point is located visually. Measuring devices are used perceptually and inappropriately.

Beginnings of measurement - however, measurement is one-dimensional. Oblique measurement is common from a corner of the rectangle.

OTHER EARLY CONCRETE OPERATIONAL RESPONSE:

LATE CONCRETE OPERATIONAL:

Empirical discovery of two-dimensional measurement. (Trial-and-error).

OTHER LATE CONCRETE OPERATIONAL RESPONSE:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS)

There is no trial-and- error behavior; the child immediately coordinates the two rectangular measurements.

OTHER EARLY FORMAL OPERATIONAL RESPONSE (CONCRETE GENERALIZATION):

REVISED

ACLIC INTERVIEW RECORD: LOCI (About 15 minutes)

NAME: ______________________ BOY/GIRL GRADE: ______________________

AGE: __________ BIRTHDAY: ______________________ SCHOOL: ______________________

INTERVIEWER: ______________ DATE: ______________ CASSETTE: ______________ COUNTER: ______________

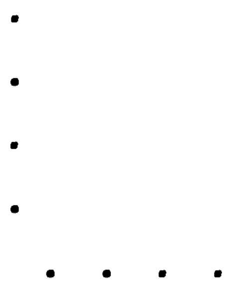
MATERIALS: a set of beads, ruler, pencil, sheets of paper, felt pen

PROCEDURE:

TASK 1: Begin with a blank sheet of paper. As the child watches, mark two points on the sheet, saying, "Let's imagine that this is a tree, and this is another tree." (The points should not be centred on the sheet.) "Where can you stand so as to be the same distance (or 'just as far') from either tree?" (Ask the child to indicate the position with a bead.) "Are there any other points?" After the questions have been answered in this form, remove bead(s) from paper and ask the child to draw all possible positions. If the child says that the beads are on a line, ask whether the points can touch, and how far the line can go.

TASK 2: The problem is extended to cover equidistance from several points A and A^1, B and B^1, etc., where points A, B, C and D lie in a straight line, and points A^1, B^1, C^1 and D^1 lie at corresponding distances in another line at right angles to it. (The locus of points equidistant from A and A^1, etc. is then the bisector of the angle.)

As the child watches, mark the points on the sheet, saying, "This is a row of trees and here is another row of trees. Where can you stand so as to be the same distance from (or 'just as far') from either row?"



TASK 3: A single dot is marked on a sheet of paper. Ask the child to show where a set of beads should be placed, or where a series of trees should stand, in order to be the same distance, or "just as far", from the dot.

IDENTIFICATION OF COGNITIVE LEVEL OF RESPONSE

PREOPERATIONAL.

Task 1. The child indicates a point at random without regard for distances involved.

Task 2.

Task 3.

COMMENTS:

EARLY CONCRETE OPERATIONAL.

Task 1: The child finds one solution, the midpoint estimated perceptually, but fairly accurately, or a few other points nearby.

Task 2: The child only considers two of the points or produces irregular and random intervals between various points.

Task 3: The child arranges the beads either in a row or else in an irregular ring around the point without any attempt to measure (without discovering for each point the point which is symmetrical to it in relation to the centre).

COMMENTS:

LATE CONCRETE OPERATIONAL.

Task 1. The child shows an inkling of the "locus" but this is achieved by extending the method used in
Task 2. placing the first (central bead) and placing one bead behind another in a continuous line
Task 3. following the same direction. There are occasional errors in equidistance and these due to over emphasis on continuing in a chosen direction, to the neglect of a careful return to the p .nt of departure, i.e., no thought is given to symmetry.

COMMENTS:

EARLY FORMAL OPERATIONAL (CONCRETE GENERALIZATIONS).

Task 1. The most important achievement at this stage is reasoning by recurrence. The child determines a few
Task 2. points in the series and immediately concludes that all points on the circle or straight line must
Task 3. have the same property.

COMMENTS: **BEST COPY AVAILABLE**

ACLIC INTERVIEW RECORD: BEADS (About 15 minutes)

Linear and Circular Order (Piaget, 1956, 80-103)

NAME: ______________________ BOY/GIRL GRADE: __________

AGE: ________ BIRTHDAY: ____________ SCHOOL: __________

INTERVIEWER: __________ DATE: ________ CASSETTE: ________ COUNTER: ________

MATERIALS: a string of nine vari-coloured beads arranged in a circle
a string of nine vari-coloured beads arranged in a simple linear order
a string of twelve vari-coloured beads arranged in a figure 8 pattern
lengths of string, loose vari-coloured beads

PROCEDURES:

TASK 1: Transposition of circular into simple linear order making the linear order correspond to beads arranged in a circular loop. "What would the necklace look like if it were in a straight line?"

☐ Successful ☐ Unsuccessful

COMMENTS:

TASK 2: Establishment of reverse order. Show the child a set of beads arranged in a row. Ask the child to arrange his own row of beads in the reverse order to that shown. (Can you start from the other end?)

☐ Successful ☐ Unsuccessful

COMMENTS:

TASK 3: Transposition of a figure 8 pattern into linear order. Reproducing a string of beads arranged in a figure 8 pattern, in simple linear order.

☐ Successful ☐ Unsuccessful

COMMENTS:

COGNITIVE LEVEL OF RESPONSES

PREOPERATIONAL.

Unable to make another row of beads in the same order. May arrange 2 beads in order correctly but unable to coordinate the whole sequence of beads into a given simple linear order.

OTHER PREOPERATIONAL RESPONSE:

EARLY CONCRETE OPERATIONAL.

Unable to transpose the circular order to a linear order and unable to make a row in reverse order. Toward the end of the EARLY CONCRETE OPERATIONAL Period a child may be able to reverse the order, but it is a trial-and-error process (Often loses track after centre of row, making the last half a copy of the model instead of its reverse).

O HER EARLY CONCRETE OPERATIONAL RESPONSE:

LATE CONCRETE OPERATIONAL.

Solves the problems quickly and with ease. Can reverse the order and correctly consider the intertwining relationship that exists in the figure 8 form.

OTHER LATE CONCRETE OPERATIONAL RESPONSE:

Appendix 2

Cognitive Demand Level Criteria

ACLIC Cognitive Demand Criteria, Numeration

Pre-operational (PO)

Counts to 10 but unable to conserve number because of perceptual distractions or inability to use transitive logic with numbers
Constructs a serial correspondence between two sets of ten objects graded in size in a regular manner
Experiences difficulty dividing a region into two equal parts (halves)

Early Concrete Operational (EC)

Constructs, represents, and identifies equivalent sets but can be fooled by perceptual distractors
Constructs and recognizes simple patterns using concrete objects or their images
Conserves number
Recognizes simple properties and relations in concrete materials
Uses one-way classification, forms simple hierarchies, and orders objects or their images using one major attribute
Can establish a serial correspondence between two sets of objects that are graded in size but may be confused when either series is reversed
Does not use cardination systematically
Able to perform composition of numbers intuitively with dependence on perceptual content and with lack of reversibility
Can construct a two- or three-digit number with base-ten blocks and can read the associated numerals
Can interpret simple fractions only in concrete and specific cases
Can divide a region (or set) into two equal parts but does not realize that the whole is necessarily the sum of its parts

Late Concrete Operational (LC)

Classifies data successfully, consistently
Exhibits reversibile thinking with concrete data
Recognizes relationships between things visibly or tangibly present
Uses the concepts of number conservation, transitivity, and reversibility in concrete contexts
Uses the logic of classes, differences, and relationships in direct or vicarious experiences
Generalizes the special properties of 0 and 1
Is aware of reversibility in the classification and seriation of objects or their images taking into account two major attributes
Seriates two sets of size-graded objects, or numbers, and co-ordinates the relevant ordinal and cardinal numbers
Uses an operational concept of numerical equivalence which is no longer subverted by perception
Consistently identifies the number "2 less than" any given one-, two-, or three-digit number
Makes three equal parts intuitively, grasping the concept of conservation of the whole
Rounds to the nearest hundred

Identifies one-half, one-third, one-quarter, one-fifth, or one-tenth of a shaded figure

Early Formal Operational (EF)

Sees x as a "rain check," as a generalized number, but with the idea that sooner or later it must be translated to a particular number
Generalizations from concrete experience are made but are only usable within the context of that concrete experience
Generalizes concrete patterns
Copes with formal structures only in concrete embodiments
Handles abstract ratio and rate concepts provided that they are related to concrete situations
Subdivides a given whole into thirds, fourths, fifths, or sixths by means of an anticipatory schema (i.e., an *a priori* understanding of the relations between the fractions sought and the original whole); operational conservation of the whole is understood
Systematically handles and/or explains three-digit subtractions with regrouping over a zero digit
Finds missing terms in ratios involving multiples greater than four, or terms that are two-digit or larger
Locates points on a number line to the nearest hundredth

Formal Operational (F)

Uses purely abstract thought; is able to combine novel results beyond personal experience
Hypothesizes about possible conclusions from theory, designing experiments to test those hypotheses
Handles formal structures in which the elements are abstract, including manipulations with large numbers where successive steps necessitate delay of arithmetic closure
Considers familiar rules and relationships as part of reality, capable of being operated on to produce all of the logical transformations
Is able to select appropriate transformations for particular relationships (as in real problem solving) implying a systematic overview, whether this is based on mathematical conventions or arises out of physical reality

ACLIC Cognitive Demand Criteria, Operations

Pre-operational (PO)

Handles "commutativity" and "multiplication by 1" intuitively and perceptually
[Counts from 1 to 10]

Early Concrete Operational (EC)

Resists premature closure (as in 213 + 342 = 342 + [])
Knows and uses Basic Facts of arithmetic operations (+, -, x, ÷)
Performs addition, subtraction, multiplication, and division on multidigit numbers without regrouping
Handles three addends without regrouping
Demonstrates understanding of qualitative compensation with numbers but quantitative accuracy inconsistent
Generalizes one-step patterns from the concrete (e.g., commutative and associative properties of addition)

Late Concrete Operational (LC)

Uses two or more steps or operations (e.g., x distributed over +)
Generalizes procedures from the concrete
Multiplies by 0
Performs addition, subtraction, multiplication, and division on multidigit numbers with regrouping (multidigit by one-digit division)
Uses multiple addends with regrouping (i. e., sums > 18)
Employs accurate quantitative compensation with numbers
Successfully relates two or more facts systematically
Interprets simple story problems

Early Formal Operational (EF)

Handles multidigit subtraction with borrowing across zero
Explains and uses multidigit computational algorithms successfully
Computes with multidigit multipliers containing zeros
Divides with quotients containing zeros
Multiplies by 10, 100, . . .
Uses "primitive placeholders" (e. g., "x as a raincheck")
Constructs abstract, propositional thinking from concrete elements

Formal Operational (F)

Successfully operates on relatively large numbers with systematic persistence over successive steps (e.g., (800 ÷ 25) ÷ 10 = [] ÷ (25 ÷ 10)
Successfully uses formal structures with abstract elements (e.g., f - e = [] - f)

ACLIC Cognitive Demand Criteria, Measurement

Pre-operational (PO)

Judges length by perception only, with endpoints considered in isolation
Is unable to apply the concept of equal distances between points
Believes that time passes more or less rapidly according to the speed of work being timed
Cannot identify heaviest, middle, and lightest weights given three objects and a balance; may compare weights two at a time but does not attempt to coordinate two comparisons

Early Concrete Operational (EC)

Deals with measurement of length or width but is unable to coordinate the two to make judgements about area
Uses concepts of measurement meaningfully only in concrete and specific cases
Recognizes simple properties and relations through practical manipulations of materials
Uses one-way classification, forms simple hierarchies, and orders on the basis of a major attribute
Cannot ascribe a unique unit of time or a common duration to motions at different velocities
Can estimate perceptually, and fairly accurately, the midpoint between two given points as well as a few other points equidistant from the given points
Can use beginning notions of one-dimensional measurement
Seriates weights empirically but is unable to use transitive inference
Conserves area and compares areas by counting units (visual iteration of area units)
Estimates length using metric units
Chooses an appropriate unit of length with which to measure an object
Conserves length and uses multiple steps in comparing lengths
Conserves volume (capacity) using numerical data, overcoming perceptual interference and recognizing that volume is independent of shape
Conserves quantity
Uses transitive inference to compare capacities

Late Concrete Operational (LC)

Uses inductive and deductive logic but only in concrete situations
Classifies data
Thinks systematically in concrete situations, relating two or more facts but not making extensions or generalizations
Thinks reversibly with concrete data
Successfully uses measurement systems in one dimension
Two- and three-dimensional measurement is handled only in concrete situations
Recognizes area and speed informally but not in terms of products and ratios of component dimensions
Uses reversibility of classification and seriation involving two major attributes of objects
Uses measuring tools placed along objects to be compared and uses transitivity

to make inferences about their lengths

Predicts that a task done more quickly will take less time but estimates (guesses) inaccurately, even when it is possible to calculate time elapsed from given information

Discovers two-dimensional measurement empirically (Trial-and-error)

Seriates weights operationally

Identifies the heaviest, middle and lightest of three given objects by co-ordinating all the relations

Orders as many as 10 objects from lightest to heaviest

Is unable to coordinate inverse relations (e.g., heavier and lighter than)

Makes a series of length comparisons using the transitive property

Compares areas by transformation of visual units and counting (unit iteration and coordination of multiple conditions)

Early Formal Operational (EF)

Uses fairly advanced inductive and deductive logic at a relatively abstract level but with dependence on the concrete elements in the situation

Attempts abstract and propositional thinking, but with limited success

Goes outside known data to form hypotheses, but with limited success

Predicts accurately the time taken by a task done twice as quickly as a referent task, having assembled the necessary data

Seriates weight and uses inverse relations

Mentally visualizes the integration of a number of units of measure (e.g., area, volume, . . .)

Conserves area and compares by counting units, where units are not visible, by mental iteration of units

Transforms non-square units of area

Visualizes the number of cubic units in an irregular shape

Formal Operational (F)

Uses hypothetical and deductive thinking

Uses data in terms of propositions to be tested out in thought

Thinks logically in symbolic and abstract form

Can begin with theory rather than with evidence

Reasons by implication at an abstract level

Uses formal structures with abstract elements

Uses proportionality and reciprocity fully

Considers all combinations of factors or relations in a theoretical or closed system

ACLIC Cognitive Demand Criteria: Geometry and Graphing

Pre-operational (PO)

Uses intuitive and transductive thinking
Thinking is related to the situation at hand
Focuses on only one feature at a time
Has an egocentric view of the world
Sorts objects but is inconsistent in naming common attribute(s) of each set
Locates a point on a sheet of paper similar to the model shown by using visual estimates only

Early Concrete Operational (EC)

Attempts inductive and deductive logic with limited success
Attempts to use reversibility, unsuccessfully
Identifies simple properties and relations in concrete objects only
Uses one-way classification, forms simple hierarchies, and orders on the basis of one major attribute
Dissociates squares from circles and reds from blues, for example, but is unable to handle class inclusion
Classifies (sorts) objects in one or two ways but is unable to dichotomize using negation
Can replicate the order of a set of objects but is unable to make the series in reverse or circular order
Relying completely on perception, finds only one point equidistant from two given points, the midpoint; chooses other points at random
When locating points equidistant from a given point, locations are chosen without measuring and in a row or an irregular ring
Locates a point on a sheet of paper similar to the model shown by using visual estimation or inappropriate measuring procedures, usually in one dimension

Late Concrete Operational (LC)

Uses inductive and deductive logic in concrete situations
Classifies data
Relates two or more facts without generalizing
Coordinates and uses several relevant attributes (e.g., length and width)
Uses conservation and transitivity of length
Generalizes symmetrical properties but not beyond a specific case
Uses compensatory manipulations
Concepts of one-dimensional space are well established but two- and three-dimensional ideas are limited to the concrete
Uses reversibility of classification and seriation, taking into account two major attributes of the objects being considered
Uses logical classes, e.g., "circles," "squares," and "blues," and interprets "all" and "some" appropriately
Classifies objects in several ways
Systematically reverses the order of objects in a row, a circle, or an intertwined arrangement
Locates a number of points equidistant from two points or from one point,

without using symmetry
Locates a point on a sheet similar to the model shown, using trial-and-error two-dimensional measurements

Early Formal Operational (EF)

Uses relatively more advanced inductive and deductive logic
Attempts abstract and propositional thinking, with limited success
Uses abstract ratio and rate concepts but only in the context of concrete situations
Uses graphical relations, including non-linear ones, and spatial transformations but with limited success unless they are related to concrete situations
Handles loci by reasoning by recurrence
Immediately locates a point on a sheet of paper similar to the model shown by using coordinated rectangular measurements

Formal Operational (F)

Uses hypothetical and deductive logic
Reasons by implication at an abstract level
Selects appropriate transformations for particular relationships
Uses abstract and proportional reasoning
Uses abstract ratio, rate, graphical, and spatial relations

Appendix 3:

Students' Performance on Items from Provincial Achievement Tests Included in ACLIC Paper and Pencil Tests.

Table A3.1: Students' Performance on Items from Grade 3 Provincial Achievement Test Included in ACLIC Grade 3/4 Tests.

ACLIC Grade 3/4 Test	Item Number ACLIC Test	Item Number Ach. Test	Item Difficulty Ach. Gr 3	Item Difficulty ACLIC Gr 3	Item Difficulty ACLIC Gr 4	Significance of Difference* Gr 3	Significance of Difference* Gr 4
Number	1	3	0.70	0.63	0.72	ns	ns
	2	10	0.57	0.57	0.69	ns	+
N=115 (Gr3)	3	15	0.54	0.62	0.52	ns	ns
N=94 (Gr4)	4	14	0.48	0.37	0.46	-	ns
	5	17	0.44	0.41	0.38	ns	ns
Operations	21	33	0.54	0.57	0.71	ns	+
	22	34	0.77	0.39	0.81	-	ns
N=94 (Gr3)	23	35	0.86	0.39	0.91	-	ns
N=100 (Gr4)	24	36	0.89	0.60	0.87	-	ns
	25	37	0.91	0.79	0.89	-	ns
Geom/Graph	26	20	0.48	0.43	0.54	ns	ns
	27	21	0.44	0.44	0.56	ns	+
N=112 (Gr3)	28	23	0.80	0.76	0.77	ns	ns
N= 90 (Gr4)	29	25	0.62	0.25	0.31	-	-
	30	24	0.66	0.37	0.28	-	-

* n s = Chi-squared goodness of fit test with df=1 indicated that the observed performance on the item was not significantly different (p=0.05) from that predicted by the p-value of the item for the provincial sample.

+ = ACLIC sample performed significantly better ($p<0.05$) than predicted.

- = ACLIC sample performed significantly worse ($p<0.05$) than predicted.

Table A3.2: Students' Performance on Items from Grade 6 Provincial Achievement Test Included in ACLIC Grade 5/6 Tests.

ACLIC Grade 5/6 Test	Item Number ACLIC Test	Item Number Ach. Test	Item Difficulty Ach. Gr 6	Item Difficulty ACLIC Gr 5	Item Difficulty ACLIC Gr 6	Significance of Difference* Gr 5	Significance of Difference* Gr 6
Number	1	1	0.67	0.57	0.72	-	ns
	2	2	0.53	0.61	0.52	ns	ns
N=92 (Gr5)	3	3	0.54	0.58	0.34	ns	-
N=90 (Gr6)	4	4	0.53	0.37	0.49	-	ns
	5	7	0.59	0.32	0.49	-	ns
	6	10	0.59	0.41	0.62	-	ns
	7	11	0.59	0.37	0.50	-	ns
	8	12	0.87	0.85	0.80	ns	-
	15	8	0.35	0.17	0.14	-	-
Operations	31	19	0.70	0.53	0.75	-	ns
N=94 (Gr5)	32	22	0.63	0.43	0.68	-	ns
N=92 (Gr6)	33	26	0.54	0.33	0.58	-	+
Measurement	31	44	0.59	0.29	0.34	-	-
	32	45	0.67	0.31	0.51	-	-
N=139 (Gr5)	33	30	0.60	0.48	0.65	-	ns
N=113 (Gr6)	34	29	0.44	0.24	0.33	-	-
	35	32	0.65	0.37	0.62	-	ns
Geom/Graph	32	44	0.59	0.13	0.22	-	-
	33	36	0.63	0.42	0.62	-	ns
N=151 (Gr5)	34	37	0.55	0.23	0.43	-	-
N= 94 (Gr6)	35	40	0.61	0.34	0.41	-	-

* n s = Chi-squared goodness of fit test with df=1 indicated that the observed performance on the item was not significantly different ($p=0.05$) from that predicted by the p-value of the item for the provincial sample.

\+ = ACLIC sample performed significantly better ($p<0.05$) than predicted.

\- = ACLIC sample performed significantly worse ($p<0.05$) than predicted.

Appendix 4:

Classroom Observation Form

Date ____________ School ______________________________________ Grade ________ Number: Status:

Topic/Objective ____

TIME	MATERIALS							TEACHER														COMMENTS
								Demand				Expository				Exploratory				Math	Non-	
	Text	WkB	B Bd	Chal	Conc	Pict	Swab	P	C	G	F	P	C	G	F	P	C	G	F	Mgmt	Math	
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Time	Text	WkBk	BkBd	Oral	Conc	Pict	Symb	Demand P	C	G	F	Expository P	C	G	F	Exploratory P	C	G	F	Math Mgmt	Non Math	Comments
25																						
26																						
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Appendix 5:

Cognitive Demands of Curriculum Objectives

Appendix 5: Cognitive Demands of Curriculum Objectives

	Grade 1 Numeration Strand Objectives	PO	EC	LC	EF	F
1.1.1	Matches members of two sets and determines equivalent and non-equivalent sets.		1			
1.1.2	Describes number relationships: more, fewer, greater than, less than, and equal to (no symbols).		1			
1.1.3	Associates a numeral with equivalent sets (0-10).	1				
1.1.4	Orders numbers 0-10.			1		
1.1.5	Reads a. writes numerals (0-100).		1			
1.1.6	Identifies the number of 10's and the number of 1's in any two-digit number.		1			
	Grade 1 Numeration Strand Total	1 17%	4 66%	1 17%	0 0%	0 0%
	Grade 2 Numeration Strand Objectives	PO	EC	LC	EF	F
2.1.1	Identifies the cardinal number associated with a set of objects.		1			
2.1.2	Orders numbers and recognizes "betweenness" (0-100).			1		
2.1.3	Reads and writes numerals (0-999).		1			
2.1.4	Names and uses ordinals first to tenth.			1		
2.1.5	Identifies the number of 100's, 10's, and 1's in a given three-digit numeral.		1			
2.1.6	Identifies multiples by counting by 5's, 10's, and 100's.		2	1		
2.1.7	Identifies, represents, and writes proper fractions (ha.ves, thirds, and quarters) in a concrete and pictorial setting.			1		
	Grade 2 Numeration Strand Total	0 0%	5 56%	4 44%	0 0%	0 0%

	Grade 3 Numeration Strand Objectives	PO	EC	LC	EF	F
3.1.1	Orders and determines "betweenness" of whole numbers (0-1 000) and understands symbols >, <, and = to show relationships.			1		
3.1.2	Reads and writes numerals (0-9 999).		1			
3.1.3	Identifies multiples by counting by 2's, 5's, 10's, 25's, and 100's (0-1 000).		2	3		
3.1.4	Identifies the number of 1 000's, 100's, 10's, and 1's in a number.		1			
3.1.5	Writes numbers in expanded notation (0-1 000) and vice versa.		1			
3.1.6	Identifies, writes, and compares proper fractions from concrete and pictorial representation (halves, thirds, quarters, fifths, and tenths).			1		
3.1.7	Reads and writes decimals from concrete and pictorial situations (tenths only).			1		
	Grade 3 Numeration Strand Total	0 0%	5 45%	6 55%	0 0%	0 0%

	Grade 4 Numeration Strand Objectives	PO	EC	LC	EF	F
4.1.1	Rounds whole numbers (limit: to the nearest thousand).			1		
4.1.2	Writes whole numbers in expanded notation and vice versa.			1		
4.1.3	Identifies and names place value of digits (0.01 - 99 999).		1	2		
4.1.4	Identifies, reads, and writes a fraction to represent a point on a number line, a part of a region, or a part of a set (emphasis on halves, thirds, quarters, fifths, and tenths).			1		
4.1.5	Identifies equivalent fractions.				1	
4.1.6	Reads, writes, and orders whole numbers and decimals (0.01 - 99 999).			1	1	
4.1.7	Regroups tenths and hundredths.				1	
	Grade 4 Numeration Strand Total	0 0%	2 18%	6 55%	3 27%	0 0%

	Grade 5 Numeration Strand Objectives	PO	EC	LC	EF	F
5.1.1	Identifies and names place value of digits (0.001 - 999 999).		1	2		
5.1.2	Rounds whole numbers (limit: to the nearest ten thousand).			1		
5.1.3	Rounds numbers to tenths and hundredths.				1	
5.1.4	Expresses and generates proportional ratios.				1	
5.1.5	Solves for the missing numeral in proportional ratios without using cross -products.				1	
5.1.6	Expresses tenths, hundredths, and thousandths as fractions or decimals.				1	
5.1.7	Generates equivalent fractions for halves, quarters, fifths, tenths, and hundredths.				1	
5.1.8	Regroups tenths, hundredths, and thousandths.				1	
5.1.9	Reads, writes, and orders whole numbers and decimals (0.001 - 999 999)		1	1	1	
	Grade 5 Numeration Strand Total	0 0%	2 15%	4 31%	7 54%	0 0%
	Grade 6 Numeration Strand Objectives	PO	EC	LC	EF	F
6.1.1	Identifies and names place value to billions (0.0001 - 1 000 000 000).		1	2		
6.1.2	Writes decimal numerals using expanded notation.				1	
6.1.3	Rounds numbers (0.0001 to 999 999 999).			1	1	
6.1.4	Identifies and uses proportional ratios.				1	
6.1.5	Expresses halves, quarters, and fifths as fractions or decimals.				1	
6.1.6	Expresses fractions and decimals as percents and vice versa.				1	

6.1.7	Identifies and orders integers.			1	1	
6.1.8	Reads, writes, and orders whole numbers and decimals (0.0001 - 1 000 000 000).		1	1	1	
	Grade 6 Numeration Strand Total	0 0%	2 14%	5 36%	7 50%	0 0%
		PO	EC	LC	EF	F

	Grade 1 Operations and Properties Objectives	PO	EC	LC	EF	F
1.2.1	Understands the process of addition and subtraction.		2			
1.2.2	Symbolizes addition and subtraction situations.			2		
1.2.3	Demonstrates mastery of basic facts involving sums and minuends through 9.		2			
	Grade 1 Operations and Properties Strand Total	0 0%	4 67%	2 33%	0 0%	0 0%
	Grade 2 Operations and Properties Objectives	**PO**	**EC**	**LC**	**EF**	**F**
2.2.1	Symbolizes addition and subtraction situations.			2		
2.2.2	Understands the basis of the commutative property for addition.		1			
2.2.3	Understands the processes of multiplication and division.		2			
2.2.4	Demonstrates mastery of basic facts involving sums and minuends to 18.		2			
2.2.5	Adds and subtracts to 99 without regrouping.		2			
	Grade 2 Operations and Properties Strand Total	0 0%	7 78%	2 22%	0 0%	0 0%
	Grade 3 Operations and Properties Objectives	**PO**	**EC**	**LC**	**EF**	**F**
3.2.1	Identifies addition, subtraction, multiplication, and division situations.			4		
3.2.2	Adds and subtracts two- or three-digit numbers with and without regrouping.		2	2	1	
3.2.3	Symbolizes multiplication and division situations.			2		
3.2.4	Understands the commutative property of addition and of multiplication.		2			
3.2.5	Identifies related sentences for addition, subtraction, multiplication, and division.			4		

3.2.6	Understands the unique effect of 0 and 1 in addition and multiplication.		3	1		
3.2.7	Demonstrates mastery of basic facts involving sums and minuends to 18 and products and dividends to 45.		4			
3.2.8	Multiplies whole numbers by 10 and 100.				1	
	Grade 3 Operations and Properties Strand Total	0 0%	11 42%	13 50%	2 8%	0 0%
	Grade 4 Operations and Properties Objectives	PO	EC	LC	EF	F
4.2.1	Adds and subtracts numbers using standard and expanded notation.		2	2	1	
4.2.2	Multiplies whole numbers by one- and two-digit whole numbers.			1	1	
4.2.3	Writes related sentences for addition, subtraction, multiplication, and division.			4		
4.2.4	Understands the associative property of addition and of multiplication.			2		
4.2.5	Demonstrates mastery of basic facts for sums and minuends to 18 and products and dividends through 81.		4			
4.2.6	Divides one- and two-digit whole numbers by a one-digit divisor (with and without remainders). Estimates quotients.			1	2	
4.2.7	Multiplies whole numbers by 10, 100, and 1000.				1	
4.2.8	Adds and subtracts decimals to hundredths.			2		
	Grade 4 Operations and Properties Strand Total	0 0	6 26%	12 52%	5 22%	0 0%

	Grade 5 Operations and Properties Objectives	PO	EC	LC	EF	F
5.2.1	Adds and subtracts whole numbers. Estimates sums and differences.		2	2	3	
5.2.2	Demonstrates mastery of basic facts.		4			
5.2.3	Multiplies whole numbers using one-, two-, and three-digit multipliers. Estimates products.			1	2	
5.2.4	Divides whole numbers using one- and two-digit divisors (with and without remainders). Estimates quotients.			1	3	
5.2.5	Multiplies and divides whole numbers and decimals by 10, 100, and 1000.				2	
5.2.6	Adds, subtracts, and multiplies decimals (sums, differences, and products to thousandths).			2	1	
5.2.7	Divides decimals by one-digit whole numbers.			1	1	
	Grade 5 Operations and Properties Strand Total	0 0%	6 24%	7 28%	12 48%	0 0%
	Grade 6 Operations and Properties Objectives	PO	EC	LC	EF	F
6.2.1	Adds and subtracts whole numbers and decimals. Estimates sums and differences.		2	2	3	
6.2.2	Demonstrates mastery of basic facts.		4			
6.2.3	Multiplies whole numbers and decimals using one-, two-, and three-digit multipliers. Estimates products.			1	3	
6.2.4	Divides whole numbers and decimals using one-, two-, and three-digit when number divisors.			1	2	
6.2.5	Divides whole numbers and decimals using one decimal place divisors.					1
6.2.6	Checks multiplication by division and division by multiplication.				2	
6.2.7	Mentally computes simple addition, subtraction, multiplication, and division.		1	1		

6.2.8	Calculates averages and percentages.				1	1
	Grade 6 Operations and Properties Strand Total	0 0% PO	7 28% EC	5 20% LC	11 44% EF	2 8% F

	Grade One Measurement Objectives	PO	EC	LC	EF	F
1.3.1	Tells time to the hour.		1			
1.3.2	Recites and orders the days of the week.	1	1			
1.3.3	Compares two or more objects as shorter, longer, thinner, thicker, heavier, and lighter than.		2	1		
1.3.4	Estimates,and measures using non-standard units of length, capacity, and mass.		1	5		
1.3.5	Identifies instruments for measuring length, capacity, mass, time, and temperature.	3	2			
1.3.6	Recognizes pennies, nickels, dimes, and quarters and states the value of each.	1	1			
	Grade 1 Measurement Strand Total	5 26%	8 42%	6 32%	0 0%	0 0%

	Grade Two Measurement Objectives	PO	EC	LC	EF	F
2.3.1	Tells time to the hour, half hour, and quarter hour.		1	2		
2.3.2	Writes the hour, half hour, and quarter hour using standard notation.		1	2		
2.3.3	Reads dates on the calendar.			1		
2.3.4	Recites months of the year in order.	1	1			
2.3.5	Reads the Celsius thermometer to five-degree intervals.			1		
2.3.6	Estimates and uses standard units of length, capacity, and mass with correct symbols -- m, cm, L, kg.		1	5		
2.3.7	Identifies appropriate measuring instruments for a given task.	2	3			
2.3.8	Counts a collection of coins up to 25¢.			1		

		PO	EC	LC	EF	F
2.3.9	Gives equivalent value of coins up to 25¢.			1		
2.3.10	Makes purchases up to 25¢.			1		
	Grade 2 Measurement Strand Total	3 13%	7 29%	14 58%	0 0%	0 0%
	Grade Three Measurement Objectives	PO	EC	LC	EF	F
3.3.1	Tells and writes the time to the nearest hour, half hour, quarter hour, and five-minute intervals.		1	3		
3.3.2	Orders months of the year.		1			
3.3.3	Reads the Celsius thermometer to one degree intervals and uses the symbol(°C).			1		
3.3.4	Counts collections of coins up to $1.00.			1		
3.3.5	Makes purchases and change up to $1.00.			1		
3.3.6	Extends estimation and measurement to include the use of the standard units kilometre and decimetre with symbols km and dm.			2		
3.3.7	Uses standard measuring instruments (metre stick, litre container, mass scales, calendar, Celsius thermometer).			5		
	Grade 3 Measurement Strand Total	0 0%	2 13%	13 87%	0 0%	0 0%
	Grade Four Measurement Objectives	PO	EC	LC	EF	F
4.3.1	Reads and writes time to minutes.			1		
4.3.2	Reads Celsius thermometer, and determines reasonableness of readings to given situations.			2		
4.3.3	Extends estimation and measurement to include the use of the standard units of millimetre, millilitre, and gram with symbols mm, mL, and g.			6		

4.3.4	Uses appropriate standard measuring units for length, capacity, and mass.			3		
4.3.5	Uses money (coins and bills) for purchasing and making change.			2		
4.3.6	Expresses linear measure to the nearest tenth and hundredth of a metre.				1	
	Grade 4 Measurement Strand Total	0 0%	0 0%	14 93%	1 7%	0 0%
	Grade Five Measurement Objectives	PO	EC	LC	EF	F
5.3.1	Reads and writes time to seconds.			1		
5.3.2	Reads the 24-hour clock.			1		
5.3.3	Extends estimations and measurements including tonne and its symbol t.			2		
5.3.4	Reads distances according to a scale.			1	1	
5.3.5	Draws 2-dimensional figures to scale using grid paper.			1	1	
5.3.6	Uses appropriate standard measuring units for length, capacity, and mass.			3		
5.3.7	Understands the system of metric prefixes including the use of symbols: kilo- (k), hecto- (h), deca- (da), basic unit, deci- (d), centi- (c), and milli- (m).		1	1		
5.3.8	Expresses linear measures in expanded form.			1		
5.3.9	Expresses equivalent linear measures.				1	
5.3.10	Finds perimeter of polygons without using formulas.			1		
5.3.11	Finds area of polygons without using formulas.			1	1	
5.3.12	Finds volume of rectangular solids without using formulas.				1	
	Grade 5 Measurement Strand Total	0 0%	1 5%	13 68%	5 26%	0 0%

	Grade Six Measurement Objectives	PO	EC	LC	EF	F
6.3.1	Finds perimeter of polygons with and without formulas.			1	1	
6.3.2	Finds area of triangles and rectangles using formulas.				2	
6.3.3	Finds volume of rectangular solids using formulas.				1	
6.3.4	Reads and determines distances according to a scale.			1	1	
6.3.5	Draws diagrams according to a scale.			1	1	
6.3.6	Reads the 24-hour clock and writes corresponding time notation.			1		
6.3.7	Understands and uses the system of metric prefixes including the use of symbols: kilo-, hecto-, deca-. basic unit, deci-, centi-, milli-.		1	1	1	
6.3.8	Expresses equivalent measures within units of length, capacity, mass, and time with symbols.				1	
6.3.9	Measures angles.				1	
	Grade 6 Measurement Strand Total	0 0%	1 7%	5 33%	9 60%	0 0%

	Grade 1 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
1.4.1	Classifies 3-dimensional objects according to various attributes.		1			
1.4.2	Recognizes and names circle, square, triangle, and rectangle.	1				
1.5.1	Collects data from the immediate environment to construct graphs using pictures or objects and discusses the results.		1			
	Grade 1 Space Total	1 33%	2 67%	0 0%	0 0%	0 0%
	Grade 2 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
2.4.1	Classifies 3-dimensional objects in relation to corners, edges, and faces.		1			
2.4.2	Classifies 2-dimensional figures in relation to boundaries, corners, and faces.		1			
2.4.3	Develops and continues patterns using 3-dimensional objects and 2-dimensional figures.		1	1		
2.4.4	Demonstrates symmetry through folding and cutting.		1			
2.5.1	Constructs and interprets pictographs and simple bar graphs using data collected from immediate environment.		1	1		
	Grade 2 Space Total	0 0%	5 71%	2 29%	0 0%	0 0%
	Grade 3 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
3.4.1	Classifies and identifies 3-dimensional objects and 2-dimensional figures.		1			
3.4.2	Constructs simple 3-dimensional objects.		1			
3.4.3	Constructs simple 2-dimensional figures.		1			

3.4.4	Identifies symmetric figures and draws lines of symmetry on 2-dimensional figures.		1			
3.5.1	Identifies the axes.		1			
3.5.2	Collects data and constructs pictographs and simple bar graphs.		1	1	1	
3.5.3	Interprets pictographs and simple bar graphs.		1	1	1	
3.5.4	Locates position of an object on a grid.			1		
	Grade 3 Space Total	0 0%	7 58%	3 25%	2 17%	0 0%
	Grade 4 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
4.4.1	Identifies properties of 3-dimensional objects and 2-dimensional figures.			2		
4.4.2	Constructs 3-dimensional objects and 2-dimensional figures.		2			
4.4.3	Determines whether or not a 2-dimensional figure is symmetric. Draws axes of symmetry.		1			
4.4.4	Translates (slides) and reflects (flips) concrete objects.		2			
4.5.1	Constructs pictographs and bar graphs.		1	1	1	
4.5.2	Interprets pictographs and bar graphs.		1	1	1	
4.5.3	Writes coordinates as ordered pairs.				1	
4.5.4	Graphs ordered pairs.				1	
	Grade 4 Space Total	0 0%	7 47%	4 27%	4 27%	0 0%

	Grade 5 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
5.4.1	Constructs and draws 2-dimensional figures.		1	1		
5.4.2	Distinguishes 2-dimensional figures as similar, congruent, or neither.			1		
5.4.3	Identifies and draws translations (slides), reflections (flips), and rotations (turns) of 2-dimensional figures.		2	4		
5.4.4	Tests congruency of polygons using translations, reflections, and rotations.			1		
5.4.5	Names corresponding sides and vertices of congruent polygons.			1		
5.4.6	Identifies and names line segments, lines, rays, and angles.		1			
5.5.1	Constructs pictographs, bar, and line graphs.		1	1	1	
5.5.2	Interprets and solves problems using pictographs, bar, line, and circle graphs.		1	2	2	
5.5.3	Reads and writes coordinates from a graph.				1	
5.5.4	Graphs ordered pairs.				1	
5.5.5	Generates ordered pairs from a given relationship.				1	
	Grade 5 Space Total	0 0%	6 26%	11 48%	6 26%	0 0%
	Grade 6 Space Objectives (Geometry and Graphing Strands)	PO	EC	LC	EF	F
6.4.1	Constructs and draws prisms, pyramids, cones, and cylinders.		1	1		
6.4.2	Draws and identifies radius, diameter, and circumference.		1			
6.4.3	Translates, rotates, reflects, and enlarges 2-dimensional figures.			4		

6.4.4	Identifies and tests congruency using translations (slides), reflections (flips), and rotations (turns).			1		
6.4.5	Names corresponding sides, vertices, and angles of congruent polygons.			1		
6.4.6	Identifies and names intersecting lines, parallel lines, perpendicular lines, and angles.		1			
6.5.1	Constructs pictographs, bar, and line graphs.			1	2	
6.5.2	Interprets and solves problems using pictographs, bar, line, and circle graphs.			2	2	
6.5.3	Locates points in all four quadrants.				1	
6.5.4	Generates and graphs ordered pairs from a given relationship (no negative numbers).				1	
	Grade 6 Space Total	0 0%	3 16%	10 53%	6 31%	0 0%
		PO	EC	LC	EF	F

Appendix 6:

Cognitive Demands Observed in Classrooms

Table A6.1: Cognitive Demands of Classrooms Observed, Grade 1

Class	Strand	PO	EC	LC	EF	F	Total
1	Geom/Graph	0	9	0	5	0	14
1	Measurement	0	11	1	2	0	14
2	Numeration	0	34	0	1	0	35
3	Measurement	0	18	0	11	0	29
4	Operations	0	35	1	0	0	36
5	Numeration	0	28	0	1	0	29
6	Measurement	0	23	0	2	0	25
All	Numeration	0 0%	62 97%	0 0%	2 3%	0 0%	64 100%
All	Operations	0 0%	35 97%	1 3%	0 0%	0 0%	36 100%
All	Measurement	0 0%	52 76%	1 1%	15 22%	0 0%	68 100%
All	Geom/Graph	0 0%	9 64%	0 0%	5 36%	0 0%	14 100%

Table A6.2: Cognitive Demands of Classrooms Observed, Grade 2

Class	Strand	PO	EC	LC	EF	F	Total
1	Operations	0	17	0	0	0	17
1	Geom/Graph	0	11	0	2	0	13
2	Measurement	0	0	24	0	0	24
2	Geom/Graph	0	3	0	35	0	38
3	Measurement	0	0	23	5	0	28
4	Numeration	0	5	7	0	0	12
4	Operations	0	18	0	0	0	18
5	Numeration	0	6	9	2	0	17
5	Operations	0	0	8	8	0	16
6	Operations	0	11	10	5	0	26
All	Numeration	0 0%	11 38%	16 55%	2 7%	0 0%	29 100%
All	Operations	0 0%	46 60%	18 23%	13 17%	0 0%	77 100%
All	Measurement	0 0%	0 0%	47 90%	5 10%	0 0%	52 100%
All	Geom/Graph	0 0%	14 27%	0 0%	37 73%	0 0%	51 100%

Table A6.3: Cognitive Demands of Classrooms Observed, Grade 3

Class	Strand	PO	EC	LC	EF	F	Total
1	Numeration	0	0	5	0	0	5
1	Operations	0	27	5	0	0	32
2	Measurement	0	0	30	10	0	40
3	Operations	0	4	42	2	0	48
4	Geom/Graph	0	22	2	11	0	35
4	Operations	0	10	0	0	0	10
4	Numeration	0	0	3	0	0	3
5	Operations	0	22	6	6	0	34
All	Numeration	0 0%	0 0%	8 100%	0 0%	0 0%	8 100%
All	Operations	0 0%	63 51%	53 43%	8 6%	0 0%	124 100%
All	Measurement	0 0%	0 0%	30 75%	10 25%	0 0%	40 100%
All	Geom/Graph	0 0%	22 63%	2 6%	11 31%	0 0%	35 100%

Table A6.4: Cognitive Demands of Classrooms Observed, Grade 4

Class	Strand	PO	EC	LC	EF	F	Total
1	Geom/Graph	0	20	4	2	0	26
2	Geom/Graph	0	13	36	1	0	50
3	Numeration	0	1	19	5	0	25
4	Geom/Graph	0	38	11	0	0	49
5	Geom/Graph	0	24	25	0	0	49
6	Numeration	0	6	15	26	0	47
7	Operations	0	11	0	20	0	31
8	Operations	0	29	26	0	0	55
All	Numeration	0 0%	7 10%	34 47%	31 43%	0 0%	72 100%
All	Operations	0 0%	40 47%	26 30%	20 23%	0 0%	86 100%
All	Measurement	-- --	-- --	-- --	-- --	-- --	-- --
All	Geom/Graph	0 0%	95 55%	76 44%	3 1%	0 0%	174 100%

Table A6.5: Cognitive Demands of Classrooms Observed, Grade 5

Class	Strand	PO	EC	LC	EF	F	Total
1	Operations	0	2	15	13	0	30
2	Geom/Graph	0	18	25	3	0	46
3	Geom/Graph	0	0	18	20	0	38
4	Numeration	0	0	28	17	0	45
5	Measurement	0	0	16	7	0	23
6	Geom/Graph	0	8	32	0	0	40
7	Operations	0	0	0	46	0	46
8	Numeration	0	0	5	36	0	41
9	Operations	0	0	22	33	0	55
10	Geom/Graph	0	23	25	0	0	48
All	Numeration	0 0%	0 0%	33 38%	53 62%	0 0%	86 100%
All	Operations	0 0%	2 2%	37 28%	92 70%	0 0%	131 100%
All	Measurement	0 0%	0 0%	16 70%	7 30%	0 0%	23 100%
All	Geom/Graph	0 0%	49 29%	100 58%	23 13%	0 0%	172 100%

Table A6.6: Cognitive Demands of Classrooms Observed, Grade 6

Class	Strand	PO	EC	LC	EF	F	Total
1	Numeration	0	0	1	25	0	26
2	Geom/Graph	0	0	49	6	0	55
3	Numeration	0	0	6	29	0	35
4	Numeration	0	0	5	35	0	40
5	Numeration	0	0	0	34	0	34
6	Operations	0	0	0	12	42	54
All	Numeration	0 0%	0 0%	12 9%	123 91%	0 0%	135 100%
All	Operations	0 0%	0 0%	0 0%	12 22%	42 78%	54 100%
All	Measurement	-- --	-- --	-- --	-- --	-- --	-- --
All	Geom/Graph	0 0%	0 0%	49 89%	6 11%	0 0%	55 100%

Appendix 7:

Cognitive Demands of Provincial Achievement Test Items,

Grades 3 and 6

Table A7.1: Cognitive Demands of Items in Alberta Grade 3 Achievement Test (1983).

Item No.	Cognitive Level	Curriculum Objective*	Facility (p-value)	Item No.	Cognitive Level	Curriculum Objective*	Facility (p-value)
1	LC	3.1.1	.786	26	LC	3.2.1	[illegible]
2	LC	3.1.1	.668	27	LC	3.2.2	.896
3	LC	3.1.1	.702	28	LC	3.2.2	.754
4	EC	3.1.3	.892	29	EC	3.2.2	.896
5	LC	3.1.3	.713	30	LC	3.2.2	.454
6	EC	3.1.2	.853	31	LC	3.2.3	.749
7	EC	3.1.2	.938	32	LC	3.2.5	.867
8	EC	3.2.1	.716	33	EC	3.2.4	.534
9	EC	3.1.4	.780	34	EC	3.2.6	.767
10	EC	3.1.5	.573	35	LC	3.2.5	.858
11	EC	3.1.5	.700	36	LC	3.2.5	.888
12	EC	3.1.5	.851	37	LC	3.2.5	.907
13	EC	3.1.5	.526	38	LC	3.2.8	.799
14	EC	3.1.5	.478	39	LC	3.2.1	.866
15	LC	3.1.7	.514	40	LC	3.2.1	.467
16	LC	3.1.6	.882	41	LC	3.2.1	.734
17	LC	3.1.7	.437	42	LC	3.2.1	.623
18	EC	3.4.1	.904	43	LC	3.2.1	.683
19	EC	3.4.1	.565	44	LC	3.3.1	.833
20	EC	3.4.2	.479	45	EC	3.3.2	.935
21	EF	3.4.2	.443	46	LC	3.3.4	.883
22	LC	3.5.2	.333	47	LC	3.3.5	.805
23	EC	3.5.2	.803	48	LC	3.3.3	.793
24	LC	3.5.4	.664	49	LC	3.3.6	.746
25	LC	3.5.4	.617	50	LC	3.3.7	.934

* Numbering scheme corresponds to that of Appendix 5, which lists each objective in full.

Table A7.2: Cognitive Demands of Items in Alberta Grade 6 Achievement Test (1983).

Item No.	Cognitive Level	Curriculum Objective*	Facility (p-value)	Item No.	Cognitive Level	Curriculum Objective*	Facility (p-value)
1	LC	6.1.1	.670	26	EF	6.2.3	.536
2	EC	6.1.1	.526	27	EF	6.3.1	.588
3	EF	6.1.2	.537	28	EF	6.3.2	.497
4	LC	6.1.3	.530	29	EF	6.3.1	.437
5	EF	6.1.4	.633	30	EF	6.3.3	.604
6	EF	6.1.4	.638	31	EF	6.3.3	.621
7	LC	6.1.5	.586	32	EF	6.3.5	.651
8	EF	6.1.6	.353	33	LC	6.3.6	.572
9	EF	6.1.7	.409	34	EF	6.3.8	.619
10	EF	6.1.8	.588	35	EF	6.3.8	.731
11	EF	6.1.8	.587	36	LC	6.4.1	.631
12	LC	6.2.1	.865	37	LC	6.4.3	.548
13	LC	5.2.1	.792	38	LC	6.4.3	.615
14	LC	6.2.1	.673	39	EC	6.4.6	.514
15	EF	6.2.3	.825	40	LC	6.4.3	.610
16	EF	5.2.4	.790	41	LC	6.5.2	.878
17	EF	5.2.3	.668	42	EF	6.5.2	.261
18	EF	6.2.4	.598	43	EF	6.5.2	.555
19	LC	6.2.4	.700	44	EF	6.5.3	.594
20	F	6.2.5	.434	45	EF	6.5.3	.671
21	F	6.2.5	.741				
22	EF	6.2.6	.632				
23	EF	5.2.5	.596				
24	EF	6.2.8	.677				
25	F	6.2.8	.677				

* Numbering scheme corresponds to that of Appendix 5, which lists each objective in full.

Appendix 8

ACLIC Paper-and-pencil Test Item Characteristics and

Source Ref^rences

APPENDIX 8

ACLIC Paper-and-pencil Test Characteristics

and Source References

Number 3/4

ACLIC ITEM #	SOURCE	MATHEMATICAL TOPIC	ACLIC P value (%) Gr 3	Gr 4	BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
1	5 3	Betweenness	63	72			70
2	5 10	Expanded Notation	57	69			57
3	5 15	Decimal	62	52			54
4	5 14	Expanded Notation	37	46			48
5	5 17	Fraction	41	38			44
6	2,AM1,19	Seriation	80	82	45*	LC	
7	2,AM1,20	Bridging - 100's	66	72	45*	LC	
8	2,AM1,21	Sequence	83	81	48*	LC	
9	2,AM1,22	Bridging - 10's	50	67	42*	LC	
10	2,AM2,2	Place Value	7[illegible]	81	35*	EC	
11	2,AM2,7	Comparing Numbers	90	84	45*	LC	
12	2,AM2,21	Comparing -2 Attributes	77	89	45*	LC	
13	2,AM2,26	Comparing Numbers	52	65	45*	LC	
14	2,AM2,27	Multiplying by 10	57	60	50*	EF	
15	2,AM4I,28	P.V. Multiple Step	24	32	55*	EF	
16	2,AM6I,1	Fractions	34	35	45*	LC	
17	2,AM6I,2	Fractions	60	64	45*	LC	
18	2,AM7I,7	Fractions	77	87	45*	LC	
19	3	Subtracting Across 0	48	48	47*	LC	
20	4	Subtracting Across 0	20	82	55*	EF	
21	1,UI,3	Rounding	19	59	45.0	LC	
22	1,UII,1	Fractions	38	51	45.5	LC	
23	1,UII,3	Estimating - x	13	12	47.5	LC	
24	1,UII,11	Problem Solving Fractions	47	52	47.9	LC	
25	3	Betweenness Fractions	06	07	48*	LC	
26	3	Bridging -100's	20	39	45*	LC	

1. ACER MPS Number Test
2. ACER AM 1,2. 4I, 6I, 7I
3. SESM - Brown, Hart & Kucheman
4. Davis & McKnight
5. Alberta Education Grade 3 Achievement Test

* ACLIC Calculated

APPENDIX 8

ACLIC Paper-and pencil Test Item Characteristics

and Source References

Number 5/6

ACLIC ITEM #	SOURCE	MATHEMATICAL CONTEXT	ACLIC P value (%) Gr 5	Gr 6	BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
1	3,1	Place Value-Decimals	57	72			67
2	3,2	Place Value-Decimals	61	52			53
3	3,3	Expanded Notation	58	34			54
4	3,4	Rounding	37	49			53
5	3,7	Representation-Fractions	32	49			59
6	3,10	Writing Numerals	41	62			59
7	3,11	Reading Numerals	37	50			59
8	3,12	Subtracting Across 0	85	80			87
9	1,UI,3	Rounding	63	72	45.0	LC	
10	1,UII,1	Fractions-Representation	67	74	45.5	LC	
11	1,UII,3	Estimating-Multiplication	50	67	47.8	LC	
12	1,UI,5	Problem-Solving Fractions	39	43	47.9	LC	
13	2,AM7,7	Fractions	18	29	52*	EF	
14	1,UI,14	Proportions	46	36	50.1	EF	
15	3,8	Percent	17	14			35
16	1,AM6,3	Equivalent Fractions	46	42	55*	EF	
17	4	Bridging - 100's	66	73	45*	LC	
18	4	Betweenness Fractions	21	26	48*	LC	
19	2,AM4,28	P.V.-Multiple Step	43	57	55*	EF	
20	1,UI,22	Proportion-Fractions	21	19	53.5	EF	
21	1,UII,6	Division-Decimals	26	37	50.1	EF	
22	1,UII,2	Fraction-Proportion	33	26	52.9	EF	
23	1,UII,9	Compensatory Manipulation	35	59	49.0	LC	
24	1,UIII,5	Compensatory Manipulation	23	43	52.3	EF	
25	1,UI,23	Decimals, +,-	13	22	54.0	EF	
26	1,UI,25	Proportions	10	23	54.5	EF	
27	1,UIII,14	Proportion -	26	36	57.2	EF	
28	1,UII,16	Fractions-Number Line	13	17	54.5	EF	
29	1,UI,24	P.V. Multiply by 10	12	24	54.2	EF	
30	1,UI,16	Fraction of a Set	11	10	50.6	EF	

1. ACER MPS Number Test
2. ACER AM
3. Alberta Education grade 6 Achievement Test
4. SESM, Brown, Hart & Kucheman

* ACLIC Calculated

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Operations 3/4

ACLIC ITEM #	SOURCE		CURRICULUM OBJECTIVE	ACLIC P value (%)		BRYTES	COG-NITIVE LEVEL	Alberta EDUCATION P value (%)
				Gr 3	Gr 4			
1	1		Commutative +	52	75	25.9	PO	
2	1		Commutative x	62	84	29.6	PO	
3	1		Identity-Division	71	68	35.0	EC	
4	1		Identity Property of 1	81	97	27.1	PO	
5	1		Associative x	50	59	37.8	EC	
6	1		Identities-Subtraction	51	59	36.9	EC	
7	1		Additive Identity	88	81	40.3	LC	
8	1		Associative +	49	57	41.4	LC	
9	1		Addition of 1	85	82	38.7	EC	
10	1		Multiplication 0	59	89	42.3	LC	
11	1		Commutative +	61	75	34.3	EC	
12	1		Commutative x	66	79	34.4	EC	
13	1		Premature Closure	80	83	37.2	EC	
14	1		Identity Property of 1	63	88	39.0	EC	
15	1		Associative x	67	78	36.4	EC	
16	1		- not Commutative	63	75	40.2	LC	
17	1		Additive Identity	78	86	40.3	LC	
18	1		Associative +	61	76	41.7	LC	
19	1		Add 1	77	81	41.4	LC	
20	1		Multiplication -0	52	81	48.1	LC	
21	2	33	Commutative +	57	71			48
22	2	34	Identity x	39	81			44
23	2	35	Missing Multiplier	39	91			80
24	2	36	Missing Addend	60	87			62
25	2	37	Missing Subtrahend	79	89			66

1. ACER MPS OPERATIONS
2. Alberta Education Grade 3 Achievement Test

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Operations 5/6

ACLIC ITEM #	SOURCE		MATHEMATICAL CONTEXT	ACLIC P value (%) Gr 5	Gr 6	BRYTES	COGNITIVE LEVEL	ALBERTA EDUCATION P value (%)
1	1		Commutativity +	83	88	25.9	PO	
2	1		Commutativity x	85	93	29.6	PO	
3	1		÷ not Commutative	69	90	35.0	EC	
4	1		Identity property of 1	95	97	27.0	PO	
5	1		Associativity x	69	88	37.8	EC	
6	1		- not Commutative	61	83	36.9	EC	
7	1		Additive Identity - 0	87	96	40.3	LC	
8	1		Associative +	57	83	41.4	LC	
9	1		Addition of 1	77	83	38.7	PO	
10	1		x Property of 0	81	85	42.3	LC	
11	1		Inverses +,-	89	97	44.5	LC	
12	1		Inverses +,÷	77	90	48.0	LC	
13	1		Compensatory Manipulations	43	63	46.9	LC	
14	1		Compensatory Manipulations	37	39	52.4	EF	
15	1		Compensatory Manipulations	28	28	50.7	EF	
16	1		Compensatory Manipulations	17	24	50.4	EF	
17	1		Distribution Property	44	79	56.7	EF	
18	1		÷ not Associative	23	15	53.2	EF	
19	1		Subtraction - not Associative	13	12	59.0	EF	
20	1		x Distribution over ÷	13	09	62.7	F	
21	1		Commutative +	69	88	34.3	EC	
22	1		Commutative x	70	92	34.4	EC	
23	1		÷ not Commutative	78	97	37.2	EC	
24	1		Identity Property of 1	86	95	39.0	EC	
25	1		x Associative	67	93	36.4	EC	
26	1		- not Commutative	70	91	40.2	LC	
27	1		+ Identity -0	89	95	40.3	LC	
28	1		+ Associativity	57	89	41.7	LC	
29	1		Addition of 1	76	77	41.4	LC	
30	1		x	61	71	48.1	LC	
31	2	19	Problem Solving	53	75			70
32	2	22	Division Checking	43	68			63
33	2	26	Problem Solving	33	58			54

1. ACER MPS Operations Test
2. Alberta Education Grade 6 Achievement Test

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Measurement Test 3/4

ACLIC ITEM #	SOURCE	MATHEMATICAL CONTEXT	ACLIC P values (%)		BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
			Gr 3	Gr 4			
1	2,11,1	Mass-Conservation	87	91	35*	EC	
2	2	Mass-Weighing	89	97	30*	EC	
3	2,12,13	Capacity-Conservation	63	69	42.1	LC	
4	2,10,3	Area-Iteration	53	57	39.4	EC	
5	2,11,10	Mass	66	75	33*	EC	
6	2,8,6	Length-Conservation	30	34	43.5	LC	
7	2,9,7	Length-Metric	40	63	37.6	EC	
8	2,9,8	Length-Estimation	83	89	35.6	EC	
9	2,9,9	Scale Drawing	57	62	45.2	LC	
10	2,8,10	Length-Conservation	84	78	42.2	LC	
11	2,13,7	Time-Duration	54	53	40*	LC	
12	2,11,12	Mass-Transitivity	59	56	45*	LC	
13	2,13,13	Seriation-Time	52	81	40*	LC	
14	2,11,20	Mass-Weighing	24	24	45*	LC	
15	2,11,23	Mass-Conservation	16	27	37*	EC	
16	2,11,16	Multiple Attributes	43	47	50*	EF	
17	2,13,22	Time-Duration	26	45	40*	LC	
18	2,10,5	Area-Ratio	46	57	38.3	EC	
19	2,9, 13	Length-Measurement	52	67	39.8	EC	
20	1,UII,30	Time-Clock	20	17	60.6	F	
21	2,10,9	Area-Conservation	09	11	52.6	EF	
22	1,UI,7	Area-Iteration	24	44	48.7	LC	
23	2,10,15	Area-	16	23	49.8	LC	
24	2,12,2	Capacity-Conservation	68	80	38.3	EC	
25	2,12,3	Quantity-Conservation	63	80	33.1	EC	
26	2,12,6	Capacity-Conservation	62	69	37.7	EC	
27	2,12,12	Capacity-Conservation	47	53	33.1	EC	
28	1,UII,25	Volume-Iteration	12	18	56.0	EF	

1. ACER MPS Measurement
2. ACER AM, 8, 9, 10, 11, 12, 13

* ACLIC Calculated

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Measurement 5/6

ACLIC ITEM #	SOURCE	CURRICULUM OBJECTIVE	ACLIC P values (%)		BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
			Gr 5	Gr 6			
1	1,UI,2	Metric Unit Size	79	89	43.9	LC	
2	1,UII,3	Scale Drawing	28	27	51.0	EF	
3	1,UI,11	Perimeter	54	54	51.5	EF	
4	1,UI,7	Area-Iteration	56	66	48.7	LC	
5	2,10,5	Area-Conservation	72	85	38.3	EC	
6	2,10,10	Predicting Relationship	78	78	43.9	LC	
7	2,10,11	Predicting Relationship	31	33	50.2	EF	
8	1,UII,16	Ratio-Iteration	09	14	53.7	EF	
9	1,UII,25	Volume-Iteration	14	28	56.0	EF	
10	1,UII,23	Area-Space Perception	32	28	55.3	EF	
11	1,UII,13	Area-Comparison	22	18	53.7	EF	
12	2,11,12	Mass-Transitivity	56	65	45*	LC	
13	2,11,16	Multiple Attributes	57	50	50*	EF	
14	1,UI,30	Using Scales	14	25	57.5	EF	
15	1,UI,4	Scale Drawings	58	72	45.2	LC	
16	1,UI,12	Graphs-Interpretation	57	79	47.8	LC	
17	1,UI,13	Graphs-Interpretation	35	42	50.8	EF	
18	2,12,13	Capacity-Conservation	45	65	42.1	LC	
19	2,12,22	Volume by Parts	35	35	45.8	LC	
20	2,12,23	Volume by Parts	35	35	49.9	LC	
21	2,12,26	Volume, Iteration	38	55	43.8	LC	
22	1,UI,10	Scale Drawings	42	55	50.2	EF	
23	1,UII,2	Angle-Estimation	40	73	49.2	LC	
24	1,UI,17	Speed-Proportionality	55	66	56.0	EF	
25	1,UI,18	Speed-Proportionality	42	49	55.2	EF	
26	1,UI,29	Multiple Attributes	19	37	56.8	EF	
27	1,UI,22	Perimeter	24	46	53.5	EF	
28	1,UII,18	Area-Iteration	22	41	54.0	EF	
29	2,13,22	Time-Duration	59	81	40*	LC	
30	2,12,7	Volume-Transitivity	44	71	40.0	LC	
31	3 44	Graphing	29	34			59
32	3 45	Speed-Graphs	31	51			67
33	3 30	Volume-Iteration	48	65			60
34	3 29	Area-	24	33			44
35	3 32	Scale	37	62			65

1. ACER MPS
2. ACER AM 10, 11, 12, 13
3. Alberta Education Grade 6 Achievement Test

* ACLIC Calculated

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Space 3/4

ACLIC ITEM #	SOURCE	MATHEMATICAL CONTEXT	ACLIC P value (%)		BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
			Gr 3	Gr 4			
1	1,UI,2	Spatial Orientation	58	83	41.8	LC	
2	1,UI,3	Ordering	62	64	42.5	LC	
3	1,UI,4	Space Orientation	50	78	44.0	LC	
4	1,UI,5	Shape Iteration	17	34	44.1	LC	
5	6	R-L Orientation	47	53	44.7	LC	
6	7	Graphing Ordered Pair	58	68	44.9	LC	
7	9	Properties-Shapes	60	60	45.9	LC	
8	15	Reflection-Detail	58	68	47.6	LC	
9	17	Spatial Orientation	15	23	48.5	LC	
10	19	Form Perception	45	38	49.1	LC	
11	22	Form Perception	23	32	49.0	LC	
12	26	Multiple Steps	20	29	49.8	LC	
13	UII,2	Volume-Iteration	33	49	44.8	LC	
14	3	Rotation	33	54	45.9	LC	
15	6	Rotation	12	09	46.5	LC	
16	7	Properties of Shape	49	47	48.7	LC	
17	10	Seriation	58	57	46.1	LC	
18	11	Transformations	40	78	50.2	EF	
19	12	Conservation-Length	26	34	50.3	EF	
20	24	3-D Space Perception	17	27	53.2	EF	
21	31	Space Perception	02	01	57.3	EF	
22	UIII,2	Rotation	20	33	48.4	LC	
23	3	Volume Iteration	24	32	48.5	LC	
24	6	Comparison-Area	29	40	50.1	EF	
25	24	Symmetry	26	24	59.9	EF	
26	2, 20	3-D space Perception	43	54			48
27	21	Volume-Iteration	44	56			44
28	23	Graphing-Interpretation	76	77			80
29	25	Coordinates-Ordered Pairs	25	31			62
30	24	Coordinates-Ordered Pairs	37	28			66

1. ACER MPS SPACE
2. Alberta Education Grade 3 Acehievement Test

APPENDIX 8

ACLIC Paper-and-pencil Test Item Characteristics

and Source References

Space 5/6

ACLIC ITEM #	SOURCE	MATHEMATICAL CONTEXT	ACLIC P value (%) GR 5	Gr 6	BRYTES	COG-NITIVE LEVEL	ALBERTA EDUCATION P value (%)
1	1,UI, 1	Spatial Orientation 3-D	63	80	39.9	EC	
2	2	Spatial Orientation 3-D	68	79	41.8	LC	
3	3	Order	56	77	42.5	LC	
4	4	Spatial Orientation 3-D	63	82	44.0	LC	
5	5	Shape-Iteration	36	49	44.1	LC	
6	6	Spatial-Orientation	64	72	44.6	LC	
7	7	Graphing-Ordered Pairs	70	85	44.9	LC	
8	8	Transformations	23	45	45.0	LC	
9	9	Properties of 2-D Shapes	60	79	45.7	LC	
10	10	Reflection-Detail	68	79	46.2	LC	
11	11	Space Perception 3-D	29	44	46.4	LC	
12	12	Space Orientation 2-D	29	44	46.5	LC	
13	13	Rotation-Orientation	77	79	46.9	LC	
14	14	Inverses - 2 Attributes	40	50	47.0	LC	
15	15	Reflection-Detail	61	83	47.6	LC	
16	16	Symmetry-Inverses	31	45	48.4	LC	
17	1,UI,17	Spatial Orientation	28	43	48.5	LC	
18	18	Spatial Orientation	42	63	48.6	LC	
19	19	Form Perception	42	44	49.1	LC	
20	1,UII,10	Order-	68	77	46.1	LC	
21	1,UI, 21	Spatial Orientation	31	36	49.3	LC	
22	1,UI, 22	Form Perception	30	45	49.0	LC	
23	1,UII,31	Space Perception	04	07	51.8	EF	
24	12	Conservation-Length	36	53	50.3	EF	
25	1,UI, 26	Multiple Steps	26	32	49.8	LC	
26	27	Space Perception 2-D	29	50	49.8	LC	
27	28	Reflections	28	45	50.0	EF	
28	29	Iteration 3-D	28	26	50.2	EF	
29	30	Spatial Orientation	29	28	51.2	EF	
30	31	Interrelations	50	64	51.8	EF	
31	1,UI, 32	Coordinates-Order Pairs	23	47	51.7	EF	
32	2 44	Graphing-Ordered Pairs	13	22			59
33	2 36	3-D Space Perception	42	62			63
34	2 37	Transformations	23	43			55
35	2 40	Translation	34	41			61
36	1,UI, 24	Comparison-Angles	28	53	49.4	LC	

1. ACER MPS
2. Alberta Education grade 6 Achievement Test

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APPENDIX 8

Raw Scores Corresponding to Cognitive Levels of Response

ACLIC Paper-and-Pencil Tests

ACLIC Test	Items*	PV	EC	LC	EF	F
Number 3/4	20	0-6	7-10	11-16	17-20	-
Number 5/6	21	0-5	6	7-12	13-18	19-21
Operations 3/4	20	0-4	5-12	13-18	19-20	-
Operations 5/6	30	0-2	3-10	11-20	21-27	28-30
Measurement 3/4	28	0-10	11-16	17-22	23-27	28
Measurement 5/6	30	0-8	9-11	12-19	20-26	27-30
Space 3/4	25	0-6	7-10	11-16	17-22	23-25
Space 5/6	31	0-8	9-13	14-21	22-28	29-31

* Number of items on cognitive scale; does not include Alberta Achievement Test items

Appendix 9:

Data Sources for Cognitive Levels of Response

Table A9.1: Data Sources for Cognitive Levels of Response, Numeration

Grade 1 Interviews	PO	EC	LC	EF	F	Total
BLOCKS 1	13	31	16	0	0	60
DRUMS (Correspondence 1)	14	17	27	0	0	58
(Correspondence 2)	16	25	17	0	0	58
(Ordination)	7	31	20	0	0	58
FLOWERS (Equivalence)	15	17	30	0	0	62
Interview Totals	65 22%	121 41%	110 37%	0 0%	0 0%	296 100%
Grade 2 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BLOCKS 2	0	8	50	1	0	59
FLOWERS (Equivalence)	5	13	41	0	0	59
TOFFEE	18	29	10	3	0	59
Interview Totals	23 13%	50 28%	101 57%	4 2%	0 0%	178 100%
Grade 3 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BLOCKS 3	0	27	28	5	0	60
TOFFEE	17	17	17	9	0	60
Interview Totals	17 14%	44 37%	45 37%	14 12%	0 0%	120 100%
Grade 3 Paper & Pencil Test	11 10%	50 43%	54 47%	0 0%	0 0%	115 100%
Grade 4 Paper & Pencil Test	1 1%	26 28%	55 59%	12 13%	0 0%	94 100%
Grade 5 Paper & Pencil Test	15 16%	38 42%	36 39%	3 3%	0 0%	92 100%
Grade 6 Paper & Pencil Test	13 14%	15 17%	51 57%	11 12%	0 0%	90 100%

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Table A9.2: Data Sources for Cognitive Levels of Response, Operations

Grade 1 Interviews	PO	EC	LC	EF	F	Total
BLOCKS 1	13	31	16	0	0	60
COOKIES (Invariance)	23	12	25	0	0	60
(Sharing)	16	19	25	0	0	60
DIVIDED BOXES	27	30	3	0	0	60
FLOWERS (Multiplication)	15	12	35	0	0	62
PARKING	11	49	0	0	0	60
Interview Totals	105 29%	153 42%	104 29%	0 0%	0 0%	362 100%
Grade 2 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BLOCKS 2	0	8	50	1	0	59
COOKIES (Invariance)	14	12	33	0	0	59
(Sharing)	1	17	41	0	0	59
DIVIDED BOXES	11	17	31	0	0	59
FLOWERS (Multiplication)	11	11	37	0	0	59
Interview Totals	37 13%	65 22%	192 65%	1 0%	0 0%	295 100%
Grade 3 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BLOCKS 3	0	27	28	5	0	60
DIVIDED BOXES	6	24	30	0	0	60
Interview Totals	6 5%	51 42%	58 49%	5 4%	0 0%	120 100%
Grade 3 Paper & Pencil Test	3 3%	36 38%	43 46%	12 13%	0 0%	94 100%
Grade 4 Paper & Pencil Test	0 0%	20 20%	50 50%	30 30%	0 0%	100 100%
Grade 5 Paper & Pencil Test	0 0%	18 19%	41 44%	31 33%	4 4%	94 100%
Grade 6 Paper & Pencil Test	0 0%	3 3%	28 31%	53 57%	8 9%	92 100%

Table 49.3: Data Sources for Cognitive Levels of Response, Measurement

Grade 1 Interviews	PO	EC	LC	EF	F	Total
LENGTH (Task Set A)	33	13	13	0	0	59
(Task Set B)	38	17	4	0	0	59
TIME	43	7	10	0	0	60
WEIGHT	36	13	7	0	0	56
Interview Totals	150 64%	50 21%	34 15%	0 0%	0 0%	234 100%
Grade 2 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
LENGTH (Task Set A)	25	24	9	0	0	58
(Task Set B)	30	9	19	0	0	58
TIME	32	16	13	1	0	62
WEIGHT	30	14	12	3	0	59
Interview Totals	117 49%	63 27%	53 22%	4 2%	0 0%	237 100%
Grade 3 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
TIME	22	14	17	7	0	60
WEIGHT	19	32	9	4	0	64
Interview Totals	41 33%	46 37%	26 21%	11 9%	0 0%	124 100%
Grade 3 Paper & Pencil Test	25 27%	49 53%	16 18%	2 2%	0 0%	92 100%
Grade 4 Paper & Pencil Test	11 9%	55 48%	45 39%	5 4%	0 0%	116 100%
Grade 5 Paper & Pencil Test	29 21%	32 23%	69 50%	8 6%	0 0%	138 100%
Grade 6 Paper & Pencil Test	4 4%	16 14%	70 62%	22 19%	1 1%	113 100%

Table A9.4: Data Sources for Cognitive Levels of Response, Geometry & Graphing

Grade 1 Interviews	PO	EC	LC	EF	F	Total
BEADS	8	12	10	0	0	30
SORTING (Classification)	45	12	3	0	0	60
(Inclusion)	2	9	49	0	0	60
Interview Totals	55	33	62	0	0	150
	37%	22%	41%	0%	0%	100%
Grade 2 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
BEADS	4	18	35	0	0	57
LOCI (Task 1)	21	33	4	1	0	59
(Task 2)	14	39	3	3	0	59
(Task 3)	10	44	2	3	0	59
SORTING (Classification)	30	27	5	0	0	62
(Inclusion)	2	14	46	0	0	62
Interview Totals	81	175	95	7	0	358
	23%	49%	26%	2%	0%	100%
Grade 3 Interviews	**PO**	**EC**	**LC**	**EF**	**F**	**Total**
DOT	13	29	12	6	0	60
LOCI (Task 1)	26	21	9	4	0	60
(Task 2)	19	27	9	5	0	60
(Task 3)	12	31	6	11	0	60
SORTING (Classification)	21	25	14	0	0	60
(Inclusion)	2	13	45	0	0	60
Interview Totals	93	146	95	26	0	360
	26%	40%	27%	7%	0%	100%
Grade 3 Paper & Pencil Test	27	48	37	0	0	112
	24%	43%	33%	0%	0%	100%
Grade 4 Paper & Pencil Test	13	32	37	8	0	90
	14%	36%	41%	9%	0%	100%
Grade 5 Paper & Pencil Test	19	70	55	7	0	151
	13%	46%	36%	5%	0%	100%
Grade 6 Paper & Pencil Test	4	18	50	21	1	94
	4%	19%	54%	22%	1%	100%

Appendix 10: Project Personnel

Project Director: Lois C. Marchand

Project Team Members:
- Marshall P. Bye
- Bruce Harrison (Project Coordinator)
- Richard A. Holmes
- Evelyn Sawicki (Graduate Student)
- Thomas L. Schroeder

Steering Committee Members:
- George Ditto (Chairman)
- Wes Eddy
- Warren Hathaway
- Pat McLaughlin
- Garry Popowich

Teacher-Interviewers:

Joan Adams
Nela Garcellano
Betty Miller
Lucille Kroeker
Barb Jonsson
Joanne Haines
Deborah Lawson
Arlie Fischback
Esther Shuffler
Joan Pagnucco
Dorothy MacInnis
Diane Congdon
Caroline Jones
Susan Buchynski
Lorraine McAuley
Fay Carswell
Susan Burgoyne
Evelyn Sawicki
Jean Crowder
Barbara Karbashewski
Elaine Gilchrist
Susan Lent
Patricia McKeage