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AUTHOR Suydam, Marilyn N., Ed.; Kasten, Margaret L., Ed.
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ABSTRACT

This issue of the journal contains abstracts and critiques of 11 research reports. They concern: a case study of one child's strategies for mental addition; children's understanding of fractions; computation routines prescribed by schools; models in verbal problem solving; predicting eighth-grade algebra achievement; the effectiveness of microcomputers and flashcards for basic fact practice; minorities and mathematics achievement; order and equivalence of rational numbers; children's mathematical difficulties; sex differences in geometry proof writing; and mathematics achievement by American Indians and others. Research reports and articles listed in "Resources in Education" (RIE) and "Current Index to Journals in Education" (CIJE) for April-June 1985 are also listed. (MNS)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Fall 1985

Baroody, Arthur J. THE CASE OF FELICIA: A YOUNG CHILD'S STRATEGIES FOR REDUCING MEMORY DEMANDS DURING MENTAL ADDITION. Cognition and Instruction 1: 109-116; Winter 1984.

Abstract and comments prepared for I.M.E. by J. LARRY MARTIN, Missouri Southern State College.

1. Purpose

"The Case of Felicia" is not a study following typical research design. It is a report of results obtained through semi-structured clinical interviews spaced over a year's time with one child. A logically hypothesized, yet previously unobserved, mental addition strategy called "counting-all starting with the larger addend" is used by Felicia.

2. Rationale

Children use various counting strategies to find sums, the most basic being "counting-all." Using this strategy with concrete objects puts little load on memory. Children count out a number of objects for each addend, combine them physically or mentally, and count the union starting with one. If done mentally, the procedure places a much heavier load on working memory and variations on strategy are possible.

The least efficient strategy is "counting-all starting with the first addend" (CAF). The child stores in memory the two addends, counts to the first addend, then continues the count while simultaneously enumerating the second addend. For example, to find 3 + 4 the child must count 1, 2, 3, then continue the count while keeping track of the second addend. Thus the count is "1, 2, 3,; 4(1), 5(2), 6(3), 7(4)." This procedure requires a double count. For this example the double count would be for four steps, determined by the size of the second addend.

A modification which decreases memory demands is the "counting-on from first" (COF) strategy. Starting with the cardinal number of the first addend, the child continues the count while enumerating the second addend (e.g., 3 + 4: "3; 4(1), 5(2), 6(3), 7(4)). Note that the first addend is not enumerated. Once again, however, the double count has four steps and is determined by the size of the second addend.

Eventually, most children use another modification called the "counting-on from larger" (COL) strategy. Starting with the cardinal number of the larger addend, the child continues the count sequence as the smaller addend is enumerated (e.g., 3 + 4: 4; 5(1), 6(2), 7(3)). The double count is reduced to the size of the smaller addend, hence reducing working memory load.

Another logically possible, yet previously unobserved, mental addition strategy would be a "counting-all starting with the larger addend" (CAL) procedure. Just as COL reduces the size of the double count from the COF strategy, CAL would reduce the size of the double count requirement from the CAF strategy. This study reports the case of a child who used the CAL strategy.

3. Research Design and Procedures

The investigation used a semi-structured clinical interview technique for twelve interviews. Interviews with the subject began when she was 4 years and 9 months old and continued periodically for over a year. She was described as outgoing, verbal, and bilingual. Her parents are both educators.

The interviewer posed addition tasks or problems to the subject and then followed with a flexible questioning approach. All interviews were videotaped and then transcribed.

4. Findings

Over the course of the case study, the subject used the CAL strategy selectively. Her response to problems such as 2 + 4 and 3 + 5 was to count the larger addend first then continue the count with the smaller addend. By reversing the order of the addends, she reduced the size of the double count. Although she only occasionally used the strategy with concrete materials, she often used it with mental addition where it can be used to decrease the memory load. She also resorted to this strategy when she was trying especially hard to be right.

Accuracy using the mental counting-all process increased over the twelve interviews. Over the first five interviews, the subject was accurate 50% of the time on problems requiring a double count of one or two steps and about 10% of the time where double counts were three steps or greater. For the seven more recent interviews, accuracy rose to over 90% on problems requiring a double count of one or two steps and to about 67% for problems requiring a double count of three or more steps.

The subject was also found to use a modification of her CAL strategy on problems having a two-digit addend. Her modification appeared much like counting-on yet she actively rejected counting-on for basic problems. Her strategy seemed to be to start counting at a decade as a referent, continue to count until reaching the cardinality of the two-digit addend, and then to procede with the more difficult double count. For example, 32 + 3 might bring a response of "thirty (pause)--two, thirty-three, thirty-four, thirty-five." Observed errors such as 32 + 6 = 36 are not inconsistent with this postulated strategy, for counting in unfamiliar territory could cause the loss of the units in the two-digit addend.

5. Interpretations

This case study reveals a novel counting strategy for finding mental sums. The CAL strategy minimizes the mentally demanding double count by counting the larger addend first. Since counting-on strategies were resisted by the child, apparently she did not appreciate the fact that counting the larger addend is redundant to producing its cardinality. During the investigation, it was discovered that the subject varied her strategy according to the size of the addends. For problems having a double-digit addend, she chose to abbreviate her counting-all procedure to produce a counting-on like response. However, it was not true counting-on. It was rather a counting-all method based upon the "compositional structure" of our numeration system.

Abstractor's Comments

"The Case of Felicia" is an interesting one and well reported. The investigator is to be commended for his search for the previously unobserved counting strategy and his thoughtful interpretation of the interesting modification for larger addends. The study is intellectually satisfying because it fills a gap in our knowledge of children's counting strategies for finding sums. However, one is left unsatisfied as to what the educational significance of the findings are. I would prefer some statements concerning the educational significance of the findings. How should I modify teacher training? How should the classroom teacher alter her or his strategies? What impact should the results have on the elementary curriculum?

Behr, Merlyn J.; Wachsmuth, Ipke; and Post, Thomas R. CONSTRUCT A SUM: A MEASURE OF CHILDREN'S UNDERSTANDING OF FRACTION SIZE. Journal for Research in Mathematics Education 16: 120-131; March 1985.

Abstract and comments prepared for I.M.E. by THOMAS E. KIEREN, University of Alberta.

1. Purpose

The purposes of this study were to gain information about fifth-grade students' performances while constructing a rational number as a two-addend sum close to 1, and to gain insights into cognitive mechanisms used by successful children.

2. Rationale

Because knowledge of fraction size is important in performing computations and in problem solving with rational numbers, it is important though difficult to assess such knowledge in children. The research literature on whole numbers suggests that ability to give goal estimates of computation is related to a child's concept of number size. The previous research with respect to rational numbers is very sparse, with NAEP results indicating a rather low level of performance even for older children on an addition estimation task. Thus it is of interest to construct a research task which will allow for the study of elementary school children's performances in which they must realize the magnitude of rational numbers (fractions) and operate with such magnitudes.

3. Research Design and Procedures

This study was part of the Rational Number Project of 1982-83, a large set of investigations studying children's quantitative notions of rational numbers.

Subjects

Eight children each from teaching experiments in Minneapolis and DeKalb, Illinois, were the subjects in this study. They had received some 20 weeks of instruction on fractions at the time of the first interview and 27 weeks of instruction at the time of the second interview discussed here. This instruction had included some work on rounding numbers and considerable concrete and symbolic experience with adding rational numbers, but no formal instruction on strategies for estimating the sum of two numbers.

Task

Each child was interviewed twice while doing two versions of the task. Each child was given 6 cards each containing a numeral and asked to "put the numbers inside the boxes (on a form board showing a sum of 2 fractional forms) so that when you add them the number is as close to 1 as possible, but not equal to 1." To discourage actual adding a time limit of one minute was given. Version 1 used the numerals 1, 3, 4, 5, 6, 7 and Version 2 used 11, 3, 4, 5, 6, 7. To prevent construction of well-known "1 sums", e.g. 1/3 + 4/6 = 1 the constraint of close but not equal was added. It was thought that successful solution of the task required the following:

a) knowledge of the necessity of and capability of constructing fractions less than 1.

b) knowledge that each fraction is greater than zero and that the sum is greater than either addend.

c) ability to realize the relationships in size between a constructed fraction, 1, and a needed fraction.

Each child was given version 1 after 20 weeks of instruction and versions 1 and 2 after 27 weeks of instruction. To give a child a score, the ratio of the deviation from 1 and 1 was expressed as a percent. Each child was rated according to her or his average deviation.

4. Findings

The range of average deviation for the 16 children studied was 3 to 187 percent. Children were classified as high ($\leq$ 10%), middle (between 10 and 30 percent), and low ($\geq$ 30%), this done on a pragmatic basis.

Using children's explanations of their responses, their responses were categorized on the basis of the cognitive strategies used to perform the task, some of which produced responses of higher accuracy and level than others. These categories were correct reference point comparison, mental algorithm computation, incorrect reference point comparison (i.e., made incorrect comparisons to references such as 1, 1/2, etc.), incorrect mental computation algorithms, and gross quantitative estimates. The first two of these categories were associated with high performance (6 and 13% deviations) and the incorrect reference point comparison with moderate success (27% deviations). No children who ranked as low performers used correct reference point or mental computation strategies.

5. Interpretations

There were vivid performance differences between high- and low-scoring children and corresponding vivid differences in strategies used. High scorers made accurate use of reference points, were flexible and used concepts of equivalence. Low scorers were poor at or made little use of such processes and knowledge. The middle-scoring students appeared to be in a transition state. The authors conclude that these results relate to others which seem to indicate that the issues of estimation, flexibility, quantitative concept of rational number, and translation between modes (concrete - pictorial - symbolic) are inter-related and demand careful attention from researchers.

A second set of interpretations regards the task itself. Since half of these grade five students were low scoring, it can be concluded that the task was difficult for students. For students who do not yet see a rational number, e.g., 3/4, as a conceptual unit, this task requires a large memory load. It further entails instances of complex tasks such as upper and lower bounds. To use the "construct a sum" task in instruction, the authors recommend a series of related background tasks be done first.

Abstractor's Comments

This report makes several valuable contributions to our knowledge of rational numbers. In the first instance the research task is itself very interesting. The authors see the task, because successful performance often involved correct reference estimation (as a way to "perceive" the global organization of the problem domain" (p. 129), as a task which identifies the progress from manipulative dependent to manipulative independent thought.

This "bridge" is important on two counts. The first is in terms of language use. To grow from an intuitive knowledge of fractional numbers (or any mathematics) to a more symbolic technical knowledge requires a move from an imminent use of language (language tied to or a metaphor for action/objects). Such independence from objects comes at a lower level through using language independently but analogously for object/actions or at a higher level, analytically. Because of the time constraint on this task it would seem that the high scorers were likely using language at this level.

The second element of the bridge is the passage from intuitive to technical/standard mathematical knowledge. Under the former a person likely integrates imagery, informal language, and thinking tools to accomplish tasks. Under the latter the person represents and reasons from more symbolic expressions. This research starts to present us with evidence of such a bridge and describes rather clearly the processes which mark a successful transition.

To study this transition problem (and the related information processing hypotheses noted by the authors), it would be interesting to study the middle and especially the low scorers on the construct-a-sum task. The two high-level performance process categories (reference point and mental algorithm use) are positive in nature and clearly described. The other three categories are less clear and, as suggested by the authors, students may have shown behaviours simply in response to being overwhelmed with the task. It seems that their performances could be more positively identified by following up a deviating response with an alternative question such as those given ("a fraction closer to 1 than 5/6, 7/8, etc."). The researcher (or teacher) could also make the problem more explicitly quantitative in language ("George and Willie are playing the just less than 1 game. Willie has played a 1/3 unit piece. Of these pieces ___, ___, ___, ___, which should George play to get just less than one whole unit?") This kind of question might explicitly provoke an analogy to action. Finally, of course, one could see how "concrete" one had to make the situation to allow for a correct response.

The authors of this paper have provided students of rational number thinking with insights into a child's attachment of quantity to rational number. They have provided us with a research and teaching tool which is suggestive of a path of research which will describe the transition from intuitive to technical knowledge of rational numbers.

Carraher, Terezina N. and Schliemann, Analucia D. COMPUTATION ROUTINES PRESCRIBED BY SCHOOLS: HELP OR HINDRANCE? Journal for Research in Mathematics Education 16: 37-44; January 1985.

Abstract and comments prepared for I.M.E. by DAVID F. ROBITAILLE, The University of British Columbia, Vancouver.

1. Purpose

The purpose of the study was to examine the algorithms used by students in arithmetic, and to investigate the nature of the errors they made in applying those procedures.

2. Rationale

Previous research indicates that the algorithmic procedures utilized by children "may not be so closely modelled after procedures learned in school." Children frequently develop their own algorithms and use them consistently. Some authors have suggested dividing algorithms into two general classes: those which deal with countable or manipulable objects, and those that, like most of the algorithms taught in school, involve only the manipulation of symbols. The authors advance the conjecture that the kinds of errors students make in applying algorithms should be related to the category of algorithm involved. For example, they hypothesize that algorithms in the first category would be unlikely to yield "senseless" results because students can immediately verify the reasonableness of the result obtained.

3. Research Design and Procedures

The study was carried out in Brazil, using 50 students aged 7-13 who were randomly selected from among the population of students in six schools, including both private and public, who had been identified as number conservers and who had successfully completed a counting task. The children were interviewed individually. Each was presented with the same set of seven addition and subtraction exercises to perform and asked to explain how they had arrived at each answer. The items involved one- and two-digit numbers, and some required either "carrying" or "borrowing".

4. Findings

Four strategies used by students to complete the exercises were identified. Two of these, counting and application of formal algorithms, accounted for the vast majority of cases. The other two, analyzing numbers into 10s and 1s and then working with the basic facts involved, and deducing results from previous exercises, were used infrequently. Counting strategies resulted in very few errors and most of the incorrect answers so obtained were within one unit of the correct response. Use of formal algorithms resulted in the highest percentage of incorrect answers, and a very large portion of those fell into the category of "senseless errors": 12 + 4 = 56.

5. Interpretations

Results of this study indicate that the twofold categorization of computational procedures used by students earlier is a useful one, and that the errors associated with these two approaches were fundamentally different in kind. The authors recommend that teachers encourage students to verify their answers by using a procedure different from the one used to obtain the answer. Their results also show, as other researchers have found, that students frequently do develop their own computational procedures.

Abstractor's Comments

The major finding of this study is that students frequently create, and use in a consistent manner, their own algorithms rather than the ones traditionally taught in school. This is an important finding, one that confirms similar results from other studies, and one that has important implications for mathematics education. Teachers, researchers, and curriculum developers should investigate the motives which prompt students to disregard formal algorithms, with a view to making the teaching of mathematics more meaningful to students.

This study was well-designed and well-executed; moreover, the paper provides a clear and succinct description of the project and its findings. The authors are to be commended for their efforts.

Fischbein, Efraim; Deri, Maria; Nello, Maria Sainati; and Marino, Maria Sciolis. THE ROLE OF IMPLICIT MODELS IN SOLVING VERBAL PROBLEMS IN MULTIPLICATION AND DIVISION. Journal for Research in Mathematics Education 16: 3-17; January 1985.

Abstract and comments prepared for I.M.E. by FRANK K. LESTER, JR. and PETER KLOOSTERMAN, Indiana University.

1. Purpose

The assumption behind this research was that students form "implicit models" of arithmetical operations and that these models influence the way students solve verbal problems. The purpose of the study was to assess the influence of implicit models in the solution of verbal problems involving multiplication or division.

2. Rationale

The authors justify this study with the premise that when children are given verbal problems with the same content but with different numerical data, the correctness of their solution strategies may vary. We should be able to test the assumption that a child's "implicit model" imposes constraints on his or her solution process by looking at the child's solution to problems where the size and decimal nature of the numbers rather than the content of the problems is varied. This study hypothesizes that children rely on one implicit model for multiplication and two models for division. The multiplication model was that of repeated addition, which has the constraint that multiplying by a fractional quantity is intuitively inappropriate. The first division model, partitive division, involves the division of an object or collection of objects into an equal number of sets. Such a model has the constraints that the dividend must be larger than the divisor and that the divisor must be a whole number. The second division model, quotative division, involves the question of how many times a given quantity is contained in a larger quantity. This model, which can be thought of as repeated subtraction when whole numbers are used, has only the constraint that the dividend must be larger than the divisor.

3. Research Design and Procedures

The sample consisted of students in grades 5 (10 or 11 years old; n = 228), 7 (12 or 13 years old; n = 202), and 9 (14 or 15 years old; n = 198) from 13 different schools in Pisa, Italy. The instrument used employed 42 one-step verbal problems. An example would be problem nubmer 1: "On the highway a car travels 2 km in 1 minute. If the speed of the car is constant, how far does it travel in 15 minutes?" (p. 9). Of the 42 items, 12 involved multiplication, 14 involved division, and the rest required addition or subtraction. Two 21-item tests were constructed. Each contained half of the items from each computational category. Subjects completed one of the two tests by indicating the arithmetic calcuation necessary to solve the problem. Subjects were not asked to find the actual solution. Only the multiplication and division items were used in the analyses of this study.

4. Findings

Tables showing each of the multiplication and division problems (translated into English) and results for the problems (% correct at each grade level as well as the most common error for each problem) were included. Discussion of the results focused on items of similar content where the size of the numbers had been varied. Following are tables which summarize the results for the 12 multiplication and 14 division problems used in the study.

Table 1

Summary of Responses to Multiplication Problems

Problem Number	Operation	% Correct Grade 5	% Correct Grade 7	% Correct Grade 9
1	2 x 15	84	96	98
2	1500 x 3	97	91	99
3	0.75 x 15	79	74	76
4	15 x 0.75	57	57	46
5	15 x 0.75	27	18	35
6	15000 x 0.75	53	57	52
7	15000 x 0.65	43	43	40
8	1.25 x 15	84	91	94
9	15 x 1.25	54	38	46
10	3.25 x 15	91	93	98
11	15 x 3.25	80	85	86
12	14 x 3.70	73	78	87

Table 2

Summary of Responses to Division Problems

Problem Number	Operation	Implicit Model	% Correct Grade 5	% Correct Grade 7	% Correct Grade 9
13	75 ÷ 5	partitive	89	93	99
14	96 ÷ 8	partitive	77	90	88
15	1500 ÷ 3	partitive	70	89	95
16	5 ÷ 15	partitive	20	24	41
17	5 ÷ 12	partitive	14	30	40
18	35000 ÷ 1400	quotative	68	79	94
19	280 ÷ 20	quotative	44	77	80
20	3.25 ÷ 5	partitive	73	71	84
21	0.75 ÷ 5	partitive	85	77	83
22	1.25 ÷ 5	partitive	66	74	70
23	900 ÷ 0.75	partitive	22	25	40
24	3 ÷ 0.15	quotative	22	38	55
25	10 ÷ 1.25	quotative	31	63	79
26	15 ÷ 3.25	quotative	41	62	90

5. Interpretations

Differences in results for problems 3 and 4 were attributed to the decimal operator in number 4. Similar results for problems 1 and 2 and for 4 and 6 suggest that size of the numbers alone does not influence the way the children solve the problem. The authors claim this finding supports the assumption that it is the decmial nature rather than the size of the numbers used which causes children difficulties. The large differences in performance between problems 8 and 9 was attributed to the assertion that a child's implicit model for multiplication does not allow for repeated addition of a fractional quantity. The fact that scores on problems 10 and 11 were high regardless of the order of the numbers multiplied was explained by saying that decimals considerably greater than one are big enough to override some feelings about the impropriety of multiplying by a decimal.

When division problems were considered, the very low scores for numbers 16 and 17 were attributed to students' reluctance to have a divisor which was larger than the dividend. This finding supported assumptions from both implicit models for division. Problems 18 and 19 involved quotative division and were more difficult for students than the corresponding partitive division problems (13 and 14). As these quotative problems involved larger numbers than the partitive problems, it was not possible to tell whether size of the numbers or change in the applicable implicit model was responsible for the variation in success rates. Students did better on problems 20 to 22 than they did on 16 and 17 despite the fact that both sets of problems violated the implicit models' requirements that divisor should be smaller than dividend. This was attributed to the students' desire not to violate the partitive model's rule that divisors must not be fractions. The low rate of correct response on number 23 was also attributed to this cause. Substantial variation in success across grade levels for problems 24 to 26 lead the authors to conclude that only the partitive model of division is intuitive. In contrast, the quotative model, necessary for problems 24 to 26, is learned through instruction.

In summarizing, the authors state: "Our findings support our belief that many of the difficulties children encounter when dealing with arithmetical concepts and operations can be explained in a similar fashion as arising from the conflict between formal algorithmic structures and related tacit, uncontrolled, primitive models" (pp. 14-15). The authors conjecture that primitive models result from natural human behavior in conjunction with the way mathematical concepts are taught in school.

Abstractor's Comments

In no field is the gulf between researcher and practitioner wider than in education. Thus, it is refreshing to read an educational research report that addresses a problem of so much potential interest to teachers: namely, the difficulty children have in solving word problems in arithmetic. Although the study is clearly not without flaw, it is soundly conceptualized and the data analysis, results, and conclusions are systematically reported. Furthermore, it attempts to resolve a particularly thorny question for researchers in this area: What sorts of mental representations, or models as the authors choose to call them, do children employ when they attempt to solve various kinds of word problems? Answers to questions of this type are extremely difficult to come by because researchers have no direct access to the cognitive processes one uses during the performance of a task. In fact, until very recently, human behavior researchers have avoided the study of covert behavior on the grounds that mental processes were an inaccessible "black box".

Rather than belaboring the various minor shortcomings (in the design, methodology, and interpretations) of this study, we have chosen to address certain inherent (perhaps) limitations of research of this sort. These limitations are concerned with: (1) the relationship between conceptual models and arithmetic procedures, (2) the design of research to test the validity of the hypothesized models, and (3) the research methodology employed.

Conceptual model - arithmetic procedures match. Both Cobb (1985) and Baroody (1985) have made the important point that the relationship between children's conceptual knowledge and the arithmetic procedures they use is often not clear. Indeed, it is possible that a child may have a particular conceptualization (model) of the meaning of an operation, say multiplication or division, but may at times not call upon that conceptualization when asked to identify or perform that operation. Baroody suggests two reasons for the lack of a match between conceptual model and procedures used: (1) the general human tendency towards parsimony in cognitive processing; and (2) the influence of other nonconceptual influences, such as beliefs, on an individual's decisions about procedures to employ. In the first case, it is common, if not natural, for an individual to exert as little mental effort as possible in solving a problem. This "drive" for cognitive efficiency may result in failure to access whatever cognitive model exists in the individual's mind. At the same time, an individual's beliefs about story problems may have a profound influence on how the information given in the story is processed. For example, Lester (1985) found that many third and fifth graders believe that any story problem can be solved by simply looking for "key words" (e.g., "in all" tells you to add; "left" tells you to subtract) and following a few other rules (e.g., if there are 3 numbers in a story you add; if there are 2 numbers in a story and one of them "divides the other evenly", you divide). It would seem that such beliefs override the conceptual models these children have for the operations. The concern then is that it is very difficult to know whether or not an individual calls upon available conceptual models simply by analyzing the individual's written responses.

The research design. A basic rule of research design is to be as careful as possible to develop research procedures that will provide data which will either support or deny the hypotheses under investigation. That is, the researcher must attempt to eliminate the possibility of alternative explanations. The research design employed in this study does not accomplish this goal. For example, the authors report that items 3, 4, and 5 involved the same numbers but "in Problem 3 the operator is a whole number, and in Problems 4 and 5 it is a decimal. At each grade, Problems 4 and 5 were more difficult than Problem 3, which supports our view that a decimal operator is a source of difficulty" (p. 8; our emphasis). However, it seems just as reasonable to say that the differences were due to the problem contexts (i.e., non-mathematical content). That is, the students might have been less familiar with problem situations involving "detergent" (item 5) than with "wheat" or "gypsum" (items 3 and 4 respectively), thereby making item 5 more difficult for them. (In fairness it should be noted that the authors did recognize this possiblity, but they failed to consider it in their interpretation of the results.) This sort of interpretation was made frequently with no concern about the dissimilarity in the contexts of items. Thus, as suggestive as the data may have been to support the original hypotheses, there is at least one other explanation. This is very unfortunate in view of the fact that it would not have been a particularly difficult matter to develop items with the same or very similar story contexts. This flaw in the design makes it difficult to draw conclusions with confidence.

Methodology employed. A third problem area has to do with limitations in the use of paper-and-pencil tests to gather information about the children's conceptualizations. Again, it is a fundamental principle of research that the data collected should bear directly on the nature of the phenomenon being studied. In order to be able to judge the extent to which various nonconceptual factors may have influenced the children's responses, the use of clinical interviews and "think aloud" procedures may have been more suitable.

Notwithstanding the fact that this study had some fundamental weaknesses, it is important to reiterate our belief that it is a timely, generally well-conceived piece of research which has addressed an important area of inquiry that could have long-term implications for instructional practice.

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Flexer, Barbara K. PREDICTING EIGHTH-GRADE ALGEBRA ACHIEVEMENT. Journal for Research in Mathematics Education 15: 352-360; November 1984.

Abstract and comments prepared for I.M.E by SIDNEY L. RACHLIN, University of Hawaii.

1. Purpose

The study was designed to investigate the relative value of students' grades in seventh-grade mathematics, scores on a test of general intelligence, mathematics achievement test scores, and algebra aptitude test scores for predicting success in eighth-grade algebra as measured by algebra grades and scores on a standardized first-year algebra test.

2. Rationale

Although a number of researchers have investigated the relative value of the selected cognitive variables for predicting success in a beginning algebra course, this work focused on courses taught in the ninth grade. "Very little is known about the factors that may be related to achievement among the academically talented students who are enrolled in algebra courses in the eighth grade" (p. 353).

3. Research Design and Procedures

The study is based on the statistical analysis of scores and grades for all 139 students selected for participation in an accelerated eighth-grade course at a suburban public school. Students were selected for the course on the basis of their performance in a seventh-grade "pre-algebra" course and their IQs. Predictor variables included the students' average percentage grades in seventh-grade mathematics; the students' IQs, as determined by the Lorge-Thorndike Intelligence Tests (1964); and students' prognosis for success in algebra, as measured by the Orleans-Hanna Algebra Prognosis Test (1968). Criterion variables included the students' average percentage grades for first-year algebra and students' achievement in algebra as measured by the Modern Algebra Test (1972). The means, standard deviations, and correlations of all variables for the students who completed the eighth-grade algebra course were examined. The data were further analyzed by means of two stepwise regression analyses--one for each criterion variable. Zero-order correlations between predictor and criterion variables for each sex were compared. Finally, a discriminant analysis was performed to determine the extent to which the same set of cognitive predictors could differentiate between those students who successfully completed eighth-grade algebra and those students who did not.

4. Findings

All predictor variables, except IQ, were significantly correlated with the algebra grade. All predictor variables, except performance in seventh-grade mathematics, were significantly correlated with students' scores on the algebra achievement test scores. The multiple correlation of all cognitive predictors with algebra grades was .56 and the multiple correlation of all cognitive predictors with algebra achievement test scores was .55. Discriminant analysis revealed that the six predictors differentiated between successful and unsuccessful students.

5. Interpretations

An algebra prognosis test was identified by the author as the best overall predictor of success in an eighth-grade algebra course. However, the author cautioned that use of an arbitrary cutoff score "stringent enough to prevent most of the potential failures is also likely to exclude a number of students who would succeed" (p. 358). The author suggests that alternative predictors for success in eighth-grade algebra should be explored.

Abstractor's Comments

Hidden in the study's application of statistical tools are some interesting questions and some needed explorations and discussions. As the author points out, little research exists on the prediction of success for bright students taking algebra in eighth grade. Yet, the author provides very little in the way of background for this special population. For example, how were the students selected for a pre-algebra course in seventh grade? How many students enrolled in pre-algebra in seventh grade were not selected for algebra in eighth grade?

What is the definition for success in eighth grade algebra? Is a student who receives an F in eighth-grade algebra, but scores at the 50th percentile among students who take the course in ninth grade--successful? Although the author makes a strong case for the need for her study as a first step towards reducing the incidence of the trauma of failure experienced by previously high-achieving students, no mention is made of any affective measures. How do the attitudes of the 15 students who dropped the course differ from the 7 students who were required to repeat the course? How do these attitudes differ from the students who had pre-algebra in seventh grade, but were not permitted to take algebra in eighth grade? If a runner comes in last in a race, would it have been better for him to have not trained for the race ... to have not run the race at all?

In most cases, the author provided tables in sufficient detail to permit the reader to participate in the analysis of the data. This was not the case, however, with the author's analysis that the students who were required to repeat the course and the students who withdrew from the course "did not differ significantly on any of the predictor variables". A comparison of tables 1 and 4 shows that the mean IQs went down once the seven students who were required to repeat the course were removed from the sample. So, the mean IQs for the seven had to be greater than the average IQs of the 117 students who completed the algebra course successfully. Once these 7 IQs were averaged with the dropouts' IQs, a discriminant analysis revealed that the IQs of this combined group were significantly lower than the IQs for the successful students. This suggests that the IQs for the dropouts were also significantly lower than the IQs for the students who had to repeat the course. In addition to challenging the "preliminary analysis" performed by the author, this raises some questions about the weight placed on IQs in the selection process.

In her interpretation of the stu , the author recommends the use of an algebra prognosis test as the best overall predictor of success. As described by the author, the prognosis test used in the study is a composite of problems to solve, student-predicted mid-year algebra grades, and student-reported grades received most recently on their report cards in four major subjects. One predictor not used in the study was teacher-predicted mid-year and end-of-the-year algebra grades.

Throughout these remarks one common theme prevails. Although the study was conducted with a relatively small sample, no apparent effort was made to talk to the students or teachers involved. Such contact could have helped to provide a more meaningful and useful analysis.

One final, and perhaps more important, concern was raised by the study. <u>What is the "shelf life" of tests</u>? At what point should tests created for the curriculum applications of 1968-1972 no longer be used?

Fuson, Karen C. and Brinko, Kathleen T. THE COMPARATIVE EFFECTIVENESS OF MICROCOMPUTERS AND FLASH CARDS IN THE DRILL AND PRACTICE OF BASIC MATHEMATICS FACTS. Journal for Research in Mathematics Education 16: 225-232; May 1985.

Abstract and comments prepared for I.M.E. by THOMAS E. ROWAN, Montgomery County Public Schools, Rockville, Maryland.

1. Purpose

To examine whether microcomputer drill and practice of basic mathematics facts is more effective than flash cards when procedures for using the cards are designed to mimic the microcomputer actions.

2. Rationale

One of the uses of computers in mathematics education has been to provide drill and practice for learning the basic computational facts. This study considers whether the computer equipment is being put to good use in this role. If it is not, then perhaps other, more significant uses for microcomputers can be found.

3. Research Design and Procedures

The samples for this study consisted of 34 children who scored between 28 and 90 out of 100 on a timed subtraction pretest and 35 children who scored between 7 and 76 out of 81 on a timed division pretest. Two subgroups were formed from each of the subtraction and division groups. The subtraction subgroups had pretest means of 60.3 (SD = 18.2) and 59.8 (SD = 19.8). The division subgroups had pretest means of 41.4 (SD = 17.9) and 41.9 (SD = 16.3). One subgroup from each operation was randomly assigned to microcomputer drill and practice, the other to flash cards.

The experimental procedure used an Apple IIe microcomputer with a drill and practice program written especially for this experiment. The microcomputer was used only to give feedback on accuracy, to retest, and to record performance. It was not used to time speed. The flash card drill was designed to mimic the microcomputer by providing an accuracy check and a process which retested. Both groups of students (microcomputer and flash cards) used a stopwatch to check their speed. The students in the subtraction group tended to be accurate, but slow. The two subgroups on subtraction therefore concentrated on improving speed on all facts. The division group tended to be inaccurate, so the procedure was designed to improve accuracy. To accomplish this, the students in both the microcomputer and flash card subgroups were asked to select an individual file of 10 facts to be studied at a session. Any or all of these facts could be changed on any practice day. This allowed for practice tailored to the needs of the students.

The procedure followed by all children was the same. First, the children studied their file of facts twice. Next, the children took two self-administered practice tests timed with stopwatches. An important part of these practice tests, both with the flashcard group and with the microcomputer group, was immediate feedback and correction of errors. This occurred in the flashcard procedure by having the correct answers on the backs of the cards. Incorrectly answered facts were retested at the end of the session until they were correctly answered. Finally, the children kept logs of their progress.

On Friday of each week, the children took a progress test. After four weeks in the initial groups, the two conditions were switched so that the flash card group worked with microcomputers and vice versa.

A 2x5 (Practice Condition X Time) analysis of variance with repeated measures for the second factor was carried out for each group on the pretest and the first four weekly progress test scores. A separate analysis of variance test was carried out for the fifth and sixth weeks when the conditions were reversed.

4. Findings

Both groups (subtraction and division) showed significant increases in performance over time, but no differences were found between the subgroups which used microcomputers and those which used flash cards. Most children liked using the microcomputer better than the flash cards (63%), but 28% of the children preferred the flash cards. No relationship was found between previous experience, gains in test scores, and level of test scores or rank in group, and preference for the microcomputer.

5. Interpretations

The authors question the use of microcomputers for drill which requires only the retrieval of facts. The results of the study showed no achievement gains for students using it in this way. The microcomputer program was used in the design of the flash card procedure for the experiment, and such design activities might be an effective use of microcomputers. The author also noted that certain limitations were placed on the microcomputer program (e.g., the use of the stopwatch rather than the timing capability of the machine) in order to make the results more comparable.

It was suggested that future studies compare the microcomputer with traditional methods for drill on multistep procedures in mathematics. As in this study, the traditional procedure should be designed to mimic the actions of the microcomputer.

The authors call into question the use of microcomputers for drill in the simple retrieval of mathematical information. At the same time, they suggest that microcomputers might be used "mentally" as a means of extending noncomputer instructional methods.

Abstractor's Comments

From a research perspective, this study is very well designed and focuses on an issue of considerable current importance in mathematics education--the effective use of microcomputer technology. The fact that such great pains were taken to assure the research integrity of the study is one area for possible questioning. To what extent are microcomputers really being used in schools in the very limited drill-type activity which was a part of this study? If their use in this manner is not fairly extensive, then perhaps the results have only "laboratory" significance. It might have been better to choose a popular--if possible, one of the most popular commercial drill and practice programs--and to try to design the study around that.

The idea of using the microcomputer processes as a design tool for more effective noncomputer activities is an attractive one, but one wonders how well it would work with more complex procedures. It is certainly worth investigating.

Another issue which might interfere with the practicality of the results and recommendations of this study is that of human (teacher) time. How much extra time was required by the teacher in this study to run the noncomputer procedure? How much time would be required to run more complex noncomputer procedures? Will computer equipment become so readily available at some time in the future that use of noncomputer approaches will be unnecessary?

Matthews, Westina; Carpenter, Thomas P.; Lindquist, Mary Montgomery; and Silver, Edward A. THE THIRD NATIONAL ASSESSMENT: MINORITIES AND MATHEMATICS. Journal for Research in Mathematics Education 15: 165-171; March 1984.

Abstract and comments prepared for I.M.E. by B. ROSS TAYLOR, Minneapolis Public Schools.

1. Purpose

"A major objective of the National Assessment is to measure change in achievement over time." The objective of this article is to report on the information on minorities and mathematics that can be obtained from the results of the Third National Assessment in Mathematics.

2. Rationale

The results from the National Assessment are especially important because the size and composition of the National Assessment sample permit a description of the performance of black and Hispanic students within the national population. Information on other minority groups is not included because they were not sampled in sufficient numbers to provide reliable achievement data.

3. Research Design and Procedures

"The National Assessment assessed mathematics achievement in 1973, 1978, and 1982. Over 45,000 students participated in the most recent assessment. From 79% to 81% of the sample at each age were white, 12% to 14% black, 5% Hispanic, and 2% other minorities. The racial classification was based on the student's appearance and surname."

"Between 250 and 450 exercises covering a wide range of objectives were administered to the sample at each age group, with approximately 2000 students responding to each exercise. The exercises were classified according to four cognitive levels: (a) knowledge, (b) skill, (c) understanding, and (d) application."

The arti·le focuses on achievement of black and Hispanic students. In addition, selected results based on the achievemen· class of the student and the racial composition of the school were included because they provide an additional perspective. Achievement class is determined by a student's performance on the entire assessment booklet and reported by quartile. Racial composition is based on the information reported by the school principal.

4. Findings

"Between 1978 and 1982, important changes have occured in the performance of minorities on the National Assessment. Three major findings are the following:

- Although the mean performance level for black and Hispanic students continued to be below the national mean, 13-year-old black and Hispanic students made substantial gains in performance (6.5 percentage points) between the two assessments. Moreover, the gains made by black and Hispanic students were usually substantially larger than those made by their white counterparts. In general, the largest gains were on exercises assessing the lower cognitive levels of knowledge and skills.
- Students in schools with a heavy minority enrollment tended to perform below the national level, but they made signif nt performance gains between the two assessments.
- The performance of 17-year-olds on the third assessment appears to be directly related to the amount of mathematics they had studied. For each additional course taken there was a substantial increase in the performance of black and white students. Across higher level mathematics courses, the enrollment of black students was well below that of white students.

5. Interpretations

More black and Hispanic students are learning mathematics. Increases in achievement are greatest at the knowledge and skill levels. The results give additional support to the belief that schools are using compensatory resources to advantage.

Improvement in achievement by black and Hispanic students is important, but not sufficient. The higher-level cognitive processes of understanding and application must be addressed. Minorities must continue to be encouraged to enroll in advanced mathematics classes.

We can be encouraged by the increases in mathematics achievement of minority students. These increases should "serve as a challenge to educators as they work to realize the potential for improving the learning of mathematics by minority students."

Abstractor's Comments

The purpose of assessment is to define problems rather than to produce solutions. This article uses data from the Third National Assessment in Mathematics to help further define the problem of the achievement gap between black and Hispanic students on the one hand and white students on the other. In my opinion, the reduction of the achievement gaps between various ethnic groups is the most important problem that we face in mathematics education today.

Future National Assessments in Mathematics should focus more directly on gathering information about minorities and mathematics. The samples of American Indian students and Asian students should be made sufficiently large that reliable information about their achievement could be obtained. The data on all groups should be collected in a way that could provide information about the relationship between socioeconomic and ethnic factors.

The repots of the gains made by black and Hispanic students indicate that there is cause for optimism. In the past dozen years we have seen a strong focus by portions of the mathematics education research community on sex-equity issues. Over the same period, achievement and participation in mathematics by females have increased significantly. In the next dozen years, similar research efforts are needed on race-equity issues. We have every reason to believe that such efforts will be accompanied by substantial increases in achievement and participation by black, Hispanic, and American Indian students.

The new research efforts should seek to determine the factors that influence achievement of minority students. Special attention should be paid to the higher cognitive levels of understanding and application.

In summary, the Third National Assessment provides "good news" and "bad news." The "bad news" is that there are achievement gaps and the "good news" is that the gaps are decreasing. Research can help to identify the factors that contribute to the "good news"; then educators can use that information to eliminate the "bad news."

Post, Thomas R.; Wachsmuth, Ipke; Lesh, Richard; and Behr, Merlyn J. ORDER AND EQUIVALENCE OF RATIONAL NUMBERS: A COGNITIVE ANALYSIS. Journal for Research in Mathematics Education 16: 18-36; January 1985.

Abstract and comments prepared for I.M.E. by HAROLD W. MICK, Virginia Polytechnic Institute and State University, Blacksburg.

1. Purpose

This study is one of a series of reports from the Rational Number Project, a multiuniversity research effort funded by the National Science Foundation from 1979 through 1983. The purpose of this study was to identify patterns in the strategies used by two fourth-grade students as they performed tasks related to understanding the order and equivalence of rational numbers during an 18-week teaching experiment. The authors hypothesized that the identified patterns would fit into three characteristics of thought related to the children's successful performance of tasks on order and equivalence: (a) thought flexibility in coordinating between-mode translations, (b) thought flexibility for within-mode transformations, and (c) reasoning that becomes increasingly independent of specific concrete embodiments.

2. Rationale

The instructional materials reflected cognitive psychological principles as suggested by Piaget (1960), Bruner (1966), Dienes (1967), and Gagné and White (1978). Of particular interest was the role of physical models in facilitating the acquisition and use of mathematical concepts as the learner's understanding progressed from concrete to abstract. The authors assumed that the children's initial understanding of a fraction, m/n, is not derived from the natural numbers m and n but from embodiments (e.g., a picture of an object partitioned into n equal pieces with m of them shaded). The authors' analysis indicated that the development of children's rational number understanding appeared to be related to the following three ordered characteristics of thinking:

a. Thought Flexibility in Coordinating Translations. To understand the symbol, m/n, children need to know of the existing agreements identifying the symbol with associated pictorial and manipulative displays. Taken together, information about these agreements is adequate to make assignments in either direction between symbol and embodiment. The children's personal understanding of fraction depends on the extent to which they can carry out these directional translations.

b. Thought Flexibility for Transformation Within a Mode of Representations. Transformations affect change on states of knowledge within either symbol or embodiment modes of representation. For example, a child might transform 4/6 to 2/3 within the symbolic mode by dividing both the numerator and denominator by 2, and write 4/6 = 2/3. Or the child might transform a chip embodiment of 4/6 to a chip embodiment of 2/3.

c. Progressive Independence of Thought from Embodiments. Eventually, the children's manipulations with physical objects and pictures become internalized as mental constructs or operations. This permits the child to predict and plan displays in a coordinated way. For example, in comparing 5/6 with 2/3, a child might choose the unit to be six chips (or a multiple of 6) by mentally manipulating various sets of chips. Children who have developed to this point appear to use embodiments as a confirmation of a prejudgment based on a mental manipulation.

3. Research Design and Procedures

The authors presented data from two of the 12 fourth graders to analyze and exemplify the three major characteristics of thinking hypothesized. Passages from interview transcriptions and observer's notes were selected that illustrated the thinking strategies the two children employed. The two children, Bob and Jane (not their real names), had different levels of achievement before the teaching experiment began. Bob was clearly the more able in mathematics and in general school achievement. Bob's ability in mathematics was in the high range; Jane's ability in mathematics was in the low to middle range.

4. Findings

1. Thought Flexibility in Coordinating Translations. There were big differences between Bob's and Jane's understanding of the part-whole interpretation of fraction:

EXCERPT 1. (Interview following 2 days of instruction)

Interviewer: (Reads aloud) "One-third and one-fourth." Are they equal or is one less?

Bob: One third is less than one fourth because three is smaller than four.

(Interviewer alludes to a manipulative aid used in instruction.)

Bob: (Recalling that 1/4 was blue and 1/3 brown) One fourth is less than one third because four blues cover the whole, so they are smaller (than thirds, which require only three).

EXCERPT 3. (Interview after 7 days of instruction)

Interviewer: One fifth and one ninth, which is less?

Jane: One fifth is less ... because five is less than nine.

(Interviewer directs Jane to use colored parts.)

Jane: (Covers one circular unit with orange (1/5) parts and another with white (1/9) parts) It takes 9 white and 5 orange.

Interviewer: (Draws attention to colored parts) Which is less, one fifth or one ninth?

Jane: One fifth, because it takes five to cover this, and it takes nine to cover this (points to the circular units).

Interviewer asks about the size of the colored parts.)

Jane: One orange is bigger than one white. One fifth is less than one ninth.

2. Thought Flexibility for Transformations Within a Mode of Representation. Differences between Bob's and Jane's flexibility for transformations within an embodiment mode were made clear by their responses to a paper-folding display showing 2/3 = 4/6. Bob, when asked about 2/3 and 4/6 in the context of the paper-folding display, said they were equal and explained, "You don't have any more ... you have more parts but not more space...." Jane, when asked the same question, gave no response; she seemed to be confused by the simultaneous presence in the paper-folding display of 3 parts with 2 shaded and 6 parts with 4 shaded.

3. Progressive Independence of Thought from Embodiments. Again there were very noticeable differences in Bob's and Jane's independence of thought from embodiments.

EXCERPT 13. (Interview after 40 days of instruction)

Interviewer: (Writes "3/4 = 9/□") Find the number which goes in the box so the fractions are equal.

Bob: (Writes "12")...Three goes into nine three times, and four goes into twelve three times.

EXCERPT 16. (Final interview after 18 weeks of instruction; no pictures or manipulative aids were present)

Interviewer: (Writes "3/5 6/10") Three fifths and six tenths -- are they equal, or is one less?

Jane: Six tenths is greater ... If you have ten pieces, six covered, and five pieces with only three covered.

5. Interpretations

The authors hypothesized that thought flexibility in coordinating translations between the representational systems of fraction embodiments and mathematical symbols for fractions is a prerequisite to more abstract embodiment-independent thought. The data for Bob's progression supported the authors' hypothesized order of thought characteristics. However, Jane's progression never reached embodiment-independent thought, though Jane did seem to follow the hypothesized sequence as far as she progressed. The authors concluded that thought flexibility for transformations within the embodiment mode of representation seemed to facilitate thought flexibility for transformations within the mathematical symbol mode of representation. Consequently, children who have difficulty with transformations on embodiments almost surely will have difficulty making meaningful transformations on mathematical symbols.

Abstractor's Comments

The authors place their three characteristics of thought within the representational scheme of a commutative diagram:

$$\begin{array}{ccc} a/b & \longleftrightarrow & c/d \\ \Big\updownarrow & \checkmark & \Big\updownarrow \\ (a)/b & \longleftrightarrow & (c)/d \end{array}$$

It seems to me that they have the beginnings of a promising scheme that fits nicely into the information-processing context described by Resnick and Ford (1981, Chap. 9):

> For the first time, psychology has a language and a body of experimental methods that is simultaneously addressing both the skills involved in performance and the nature of the comprehension underlying that performance. (p. 197)

The scheme is not only worthy of further extension and refinement, but it has the potential to synthesize much of the research related to children's understanding of rational number. My following remarks are presented in the spirit that they might assist in a small way toward this anticipated development.

1. I have some difficulty accepting the authors' position that children's initial understanding of a fraction, m/n, is derived from embodiments alone rather than from the natural numbers m and n. More likely, children's initial experiences with fractions occur outside school in natural situations like the family dinner table, where a child might request a half glass of milk. Or in a more mathematical context, fractions can result from a division of natural numbers, like $3 \div 4$, where sharing three candy bars among four children leads to both the part-whole, 3/4ths, and the commutative operator, $3/4 = 3(1/4(1)) = 1/4(3(1))$.

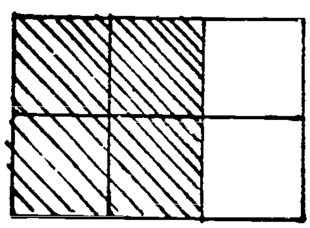
2. The authors seem not to distinguish between rational number and the fractions that lead to rational number abstraction. Hilton (1983) states the situation this way:

> Fractions always start life as parts of wholes. At this stage they are certainly not numbers, they are things -- 'half a coke', 'three quarters of the pie'. Moreover, they are, of necessity, proper parts. At a certain stage we pass from the things themselves to <u>amounts</u>, or <u>measures</u> of things. At this stage, we are entitled to say that $1/2 = 2/4$, and to introduce fractions greater than 1. (p. 38)

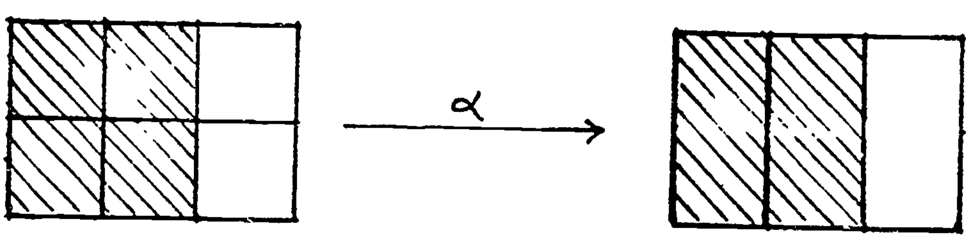
3. Looking more closely at translations between modes of representations, the authors' bidirectional "map" really is quite different in its application in the two directions. On the one hand children assign the appropriate symbol to a given embodiment situation. But on the other hand, children must create an embodiment situation of their choice; and whether they fold paper or sketch a picture, they unconsciously use the fraction as an operator on some unit to create the part-whole. This creative act uses the operator interpretation, which is subtle and often ignored in research literature and curriculum development. In this context, the fraction m/n is simply a composition of the whole number operations of division and multiplication -- both

well known to the children. It is my feeling that Jane could have benefited by spending more time creating part-wholes in order to abstract the rational number concept. Indeed, without the rational number concept she was groping with subsequent activities, as her interview excerpts indicated.

4. To explain why 4/6 = 2/3 or to solve the open sentence 4/6 = m/3, the commutative diagram scheme implies that the transformation 4/6 ---> 2/3 can be accomplished with the composition 4/6 ---> (4)/6 $\xrightarrow{\alpha}$ (2)/3 ---> 2/3, where α is an appropriate transformation within the representational system of fraction embodiments. This idea of structure preserving translations linking representational systems is a powerful one (see Hofstadter, 1980, Chap. 2). But in my judgment, care must be taken in using this powerful tool, particularly in the children's beginning experiences. For example, I would not use the single paper-folding display



to show 2/3 = 4/6. For example, Jane did not understand this representation at all. Indeed, the figure represents 4/6, not 2/3; to show that (4)/6 and (2)/3 cover equal areas, α needs to be applied to (4)/6 to get (2)/3. This act of repartitioning is somewhat difficult but necessary, and gives two part-whole situations to compare:



I believe Jane would have better understood the question about 2/3 and 4/6 within this context. Hilton (1983) comments about the similar situation of showing 1/2 = 2/4:

> Most texts, seeking to show that 1/2 = 2/4, demonstrate this by exhibiting the same portion of the same region. But what it (1/2 = 2/4) asserts is the equality of certain portions of equal regions -- it is, in other words, a statement about amounts and not about things. Try convincing a hamster that 5/5 of a hamster is the same as one hamster! The difficulty that all these texts have in explaining equality of fractions is that none is explicit about how a fraction becomes a number. (p. 38).

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Abstract and comments prepared for I.M.E. by STANLEY H. ERLWANGER, Concordia University, Montreal, Quebec.

1. Purpose

(a) To study the informal and formal mathematical knowledge of children with mathematical difficulties (referred to as MD); i.e., children of normal intelligence who are low achievers in school mathematics.

(b) To determine if the mathematical performance, concepts, and skills of MD children differ from those of children of the same age or younger who have an adequate achievement in mathematics.

The study tested three hypotheses:

H1. MD children are not seriously deficient in knowledge of basic mathematical concepts and non-algorithmic procedures.

H2. Difficulty with base ten concepts may underline a good deal of MD children's poor performance in arithmetic.

H3. MD children often experience difficulty with written arithmetic because they employ systematic strategies leading to errors.

The study also explored two areas concerning MD children's (a) proficiency in addition number fact knowledge and (b) problem-solving ability using the commutative and reciprocal principles, and handling word problems.

2. Research Design and Procedures

(a) Subjects: Three groups of pupils: 27 MD fourth graders (MD), 27 normal fourth graders (FG), and 27 normal third graders (TG). The MD and FG groups were selected from 677 pupils in ten schools using their CAT and Iowa Mathematics Achievement test scores. The TG group was randomly selected.

(b) Tasks: There were 14 tasks from five areas: (I) Informal Concepts and Calculational Skills (tasks 1-4), (II) Base Ten concepts and Enumeration Skills (tasks 5-9), (III) Error Strategies and Other Calculational Procedures (tasks 10-11), (IV) Knowledge of Number Facts (task 12) and (V) Problem Solving (tasks 13-14).

(c) Task Administration: The children were interviewed individually in three sessions for a total time of 50-70 minutes over a two-month period. A standardized method of interviewing and scoring was used. Some tasks were scored for accuracy (right/wrong) and others were categorized according to the type of error, strategy, or explanation given by the children.

3. Findings

Three types of results are reported comparing the MD group with the TG and FG groups: t-test, z-squared statistic, and strategies employed. Individual results are given for each of the 14 tasks followed by a general summary of the overall result.

Individual Results

I. Informal Concepts and Calculational Skills (Tasks 1-4).

Task 1 (which is more): The child was asked which of two large numbers is more (e.g., 9000 and 3200).

Task 2 (which is closer to): The child was asked which of two numbers on an imaginary number line was closer to a target number between them.

The results on tasks 1 and 2 show no significant difference among the three groups of children.

Task 3 (mental addition): The child was asked to solve addition problems mentally without using paper and pencil (e.g., 12 + 7, 220 + 110).

The results show MD children scored lower than FG but the same as TG.

'The strategies used were: counting addends together by ones, regrouping, and using the written addition algorithm as a mental strategy in which the numbers are operated upon as digits. The results for the three groups also shown: (i) a similar proportional trend of using strategies and (ii) use of an adaptive set of strategies for different sized sums; e.g., a counting strategy for sums less than 100 and a mental algorithm for sums over 50. Only the MD children departed from this pattern, with 47% of them using other inappropriate ways of finding sums over 100.

Task 4 (estimation): The child was read an addition problem and an appropriate answer (e.g., 91 + 24 = 50). The child was asked if the answer was close to or far away from the actual answer.

The results show no significant differences.

II. Base Ten Concepts and Related Enumeration (Tasks 5-9)

Task 5 (enumeration by tens): The child was asked how many dots were on a card. The dots were arranged in horizontal, alternating rows of ten red and ten blue.

The results show no significant differences.

Task 6 (counting large numbers): The child was asked how much money was in each of four piles (e.g., $430 shown as 4 x $100 and 3 x $10).

The t-test shows MD children perform significantly lower than FG but about the same as TG.

Task 7 (multiples or large numbers): The child was asked: How many X's are in Y (e.g., how many 10's are in 100).

The MD group performed significantly lower than FC but at the same level as TG.

Task 8 (large written numbers): The child was shown a pair of large numbers such as 799999 and 811111 and asked to point at the larger number.

The results show no significant differences.

Task 9 (representation of place value): The child was asked to read a written number (e.g., 25), to produce the same number of poker chips, and then to divide the pile of chips into quantities representing the value of each digit in the number.

The results show no significant difference.

III. Error Strategies and other Calculational Procedures (Tasks 10-11)

Task 10 (accuracy and bugs in written addition and subtraction): The child was asked to write down and solve addition and subtraction problems stated verbally, with two involving no alignment or re-grouping difficulties and four others involving each of these difficulties.

The results show: (a) the three groups performed with moderate to high accuracy. The MD performed lower than FG but the same as TG. (b) Six error strategies were found: (i) writing numbers as they sound (e.g., one hundred one as 1001), (ii) misalignment, (iii) wrong operation, (iv) addition bug, (v) subtraction bug and (vi) simple calculation.

Task 11 (monitoring errors): The child was asked to identify common errors given three correct and six incorrect addition problems written in vertical form.

The results show: (a) the MD group identified significantly less incorrect problems than FG, but were similar to TG; (b) the MD and TG groups were not as careful as FG in attending to details of written arithmetic.

IV. Knowledge of Number Facts (Task 12)

The child was asked to give quick answers to addition facts such as 2 - 5 and 6 - 3 without counting or calculating.

The results show MD children knew fewer facts than TG and FG.

V. Problem Solving Skills (Tasks 13 and 14)

Task 13 (use of principles): The child was asked to solve a pair of problems on commutativity (e.g., 12 - 7 and 7 - 12; 35 - 14 and 14 - 35) and a pair on reciprocity (e.g., 11 - 8 and 19 - 8; 9 - 12 and 27 - 2).

The results show no significant differences.

Task 14 (story problems): Each child was given nine story problems involving simple addition/subtraction, addition with several addends, complex subtraction, addition/subtraction with irrelevant information, multiplication, and division. The interviewer read the problem. The child was told he or she could think mentally, use paper and pencil or use fingers to solve the problem. After solving the problem, the child was to identify the strategy.

Ignoring simple calculational errors, the results show:

(a) most of the children approached the addition problems and simple subtraction problems correctly;

(b) MD performed significantly poorer than TG and FG. Less than half of the MD solved the complex subtraction, subtraction with irrelevant information, and multiplication problems.

Overall results

1. There are no significant differences between MD and either FG or TG on 7 of the 14 tasks (1, 2, 4, 5, 8, 9, and 13).
2. MD perform more poorly than FG and are similar to TG on 5 tasks (3, 6, 7, 10, and 11).
3. MD perform more poorly than both FG and TG on two tasks only (12 and 14).

4. Interpretations

The authors conclude as follows regarding the three hypotheses:

(a) Hypothesis H1 that MD children are not deficient in basic concepts and non-algorithmic procedures is supported by:
   - (i) no significant differences among the groups on tasks 1, 2, and 4.
   - (ii) MD show proficiency in tasks 1 and 4.
   - (iii) MD performed more poorly on task 3 than FG.
   - (iv) the proportional use of different addition strategies is similar among the three groups.

(b) The results for hypothesis H2 that difficulty with base ten concepts underlies mathematics difficulties are complex. They show that MD children (i) do not seem to differ from the other groups in elementary concepts of base ten notation, (ii) seem to possess basic concepts and skills but cannot use them on large numbers, and (iii) experience difficulty dealing with large numbers. The results do not support the hypothesis that inaccurate calculation stems from inadequate understanding of the base ten system.

(c) Hypothesis H3 that MD children's errors frequently result from common bugs is supported by (i) task 10 showing MD employ common bugs and their errors are not generated by unusual bugs, and (ii) task 11 showing MD are aware of common sources of bugs like TG but do not attend as carefully as do FG children to details of complex problems.

The authors conclude that one of the most severe difficulties of MD children is their poor knowledge of addition number facts which is significantly lower than for TG. They also observe that on problem solving, MD children do not differ from their peers in their use of insightful solutions.

The general conclusion by the authors is that their results suggest MD children of the type investigated display "normal, if immature and inefficient mathematical knowledge" (p. 242). They suggest the difficulties of MD "result from mundane factors such as immaturities of mathematical knowledge (e.g., bugs characteristic of younger children), inattention, poor execution of adequate strategies (e.g., mental addition), or lack of facility in dealing with large numbers" (p. 243). Hence remedial efforts should take a direct approach to correct these problems. The authors observe that the "one exception to 'essential cognitive normality' seems to involve MD children's poor knowledge of number facts ...(and)... this is a difficulty which is not yet adequately understood" (p. 243).

Abstractor's Comments

It is a pleasure to read a well-planned study that is written in a clear, concise, straightforward manner. The rationale underlying the hypotheses is well-presented. There is sufficient information for the reader about the procedures for selecting the three groups of children, for the construction of the 14 tasks, and for the method of presenting each task and recording the children's responses. The research design and statistical methods of analysis are appropriate for the type of comparisons made among the three groups of children. Finally, the results are reported for each of the 14 tasks as well as on subsets of tasks for the three hypotheses. In this way it is fairly easy to see how the authors arrive at their conclusions.

The study raises two significant issues about MD children. First, it shows such children are normal and their errors in arithmetic tend to be procedural in nature. The authors conclude from this that MD children perform poorly in arithmetic because of "mundane" factors and not cognitive deficiency or development lag. Do procedural errors in written arithmetic explain most of the errors by MD children? Do such errors persist over long periods of time and can they eventually lead to difficulties with algebra? Where do conceptual errors fit in? It seems to me that the tasks in the study focus more on the procedural aspects of written arithmetic than on conceptual aspects such as the various concepts underlying arithmetic operations. Thus the author's recommendation that remediation should take a direct approach to correcting errors has to be accepted with caution. If anything, this study suggests the need for more research concerned with the cognitive analysis of children's errors in arithmetic.

Secondly, the study draws attention once again to the surprising fact that MD children have difficulties with number facts. The study highlights in a sense the deviation here of MD children from their peers. Again this seems to be an area that is long overdue for some in-depth research.

As I read and reread the report I found myself somewhat concerned that the study was too extensive in its scope. It would have been quite sufficient to have investigated the three hypotheses alone. I wonder if it was really useful to include problem solving, even in an exploratory way. It is not clear how the nine types of story problems were selected and how they are consistent with research on problem solving at this level. I felt uncomfortable with the choice of story problems and the way in which they were presented to the children.

I am also at a loss as to why the authors chose to investigate addition number facts in the way they did, considering they knew "clinicians and teachers report that MD children have particular difficulty in remembering facts,.." (p.220). It might have been more useful at this stage to compare the responses of the three groups of children on (i) number facts on the four operations, (ii) number facts presented under different time requirements, (iii) number facts in which the order of the number pairs are reversed, and so on.

It is suggested that other researchers might be interested in comparing the results of this study with those on attitudes towards mathematics among children in grades 3 and 4.

Senk, Sharon and Usiskin, Zalman. GEOMETRY PROOF WRITING: A NEW VIEW OF SEX DIFFERENCES IN MATHEMATICS ABILITY. American Journal of Education 20: 187-201; February 1983.

Abstract and comments prepared for I.M.E. by JANE D. GAWRONSKI, Walnut Valley Unified School District, Walnut, California.

1. Purpose

This article reported on a large-scale study to determine ability of male and female students in senior high school classes to write geometry proofs. In addition, an explanation is proposed for inconsistent patterns of sex differences that characterize recent studies.

2. Rationale

In general, studies have indicated no systematic sex differences in performance in young children, but that by early adolescence boys begin to surpass girls on many mathematical tasks. By the end of high school this gap between males and females is both statistically and educationally significant. However, recent studies show declines in differences or no differences at all. The largest and most consistent sex differences have been on high-level cognitive tasks and particularly among higher-ability students. These differences in performance have been attributed to sex differences in tests of spatial ability.

With these reported differences, it might be expected that significant sex differences in performing geometry proofs would exist.

3. Research Design and Procedures

The Cognitive Development and Achievement in Secondary School Geometry (CDASSG) sample included 2,699 students in 99 geometry classes from 13 public high schools in five states. The subsample for the study reported here included all students in the geometry classes who had studied proof writing and whose teachers gave permission for the testing, a total of 1,520 students in 74 classes. The mean age was 16 years, 2 months.

During the first week of school, students were given a 25-minute test for entering knowledge of geometry terminology and facts. In the last month of the school year, students took the 40-minute Comprehensive Assessment Program (CAP) (1980) standardized geometry achievement test and one of three forms of a 35-minute proof test devised by CDASSG project personnel.

4. Findings

Findings were reported for 1,364 students, 690 males and 674 females, who took both a proof test and the entering geometry (EG) test. Students ranged from 7th to 12th graders with 63% in the tenth grade.

Differences between means for the three forms of the proof tests were significant indicating non-equivalence. Data are reported separately by form.

Raw mean scores on the proof tests were higher for males on two forms and for females on one, but none of the differences was statistically significant. The mean number of proofs correct was higher for males on all three forms, but never significantly.

Mean scores for girls were significantly lower than mean scores for boys on the EG test. When the proof total scores were adjusted using ANCOVA for entering geometry knowledge, adjusted mean proof total scores for females were higher than for males on all forms and significantly higher on one form. When the mean number of proofs correct were similarly adjusted, the results favored females on all three forms, but not significantly.

Although girls entered the high school geometry course with generally less geometry knowledge, by the end of the year there was no consistent difference between the sexes on proof-writing performance.

On the CAP test, administered at the end of the school year, boys' unadjusted means were significantly higher than girls' unadjusted means. Yet when the CAP scores were adjusted by ANCOVA for scores on the EG test, adjusted means for girls and boys were nearly identical.

When differences in entering geometry knowledge were taken into account, girls and boys learned both geometry problems and proof writing equally well.

A review of three subsets of high-achieving students was also made to determine sex differences. The first subset were the students who had perfect or near perfect scores on the form of the proof tests they completed. There were 37 females and 34 males in this subset. A second subset were students who were in grades 7 or 8 during the study and thus accelerated. Among this subset of 12 girls and 7 boys, no significant differences by sex were found between the means on either the total proof score or the number of proofs correct, adjusted or unadjusted. The third subset were those who scored in the top 3% nationwide as determined by the CAP norms. This subset consisted of 89 students, 31 female and 58 males in grades 7-10. This is consistent with other studies that found significantly more males than females score at the higher levels on a multiple choice test of standard content. However, proof-writing performance for this third subset indicated no sex-related differences.

No consistent pattern of statistically significant differences favoring either sex on any form of the proof tests was found in the sample as a whole or in the subsets examined.

5. Interpretations

The explanation suggested is that "when test items cover material that is taught and learned almost exclusively in the classroom, no pattern of sex differences tend to be found." This is consistent with other studies of routine tasks and this study shows that it holds for a high-level cognitive task, geometry proof writing.

Studies regarding the mathematical ability of talented boys and girls have assumed that the SAT-M items are unfamiliar to both sexes. However, unfamiliarity could easily be affected by experiences outside the mathematics classroom. Studies have shown that these informal experiences appear to be different for the sexes. In this regard, geometry proof is a unique topic, since it is unlikely to be encountered even by the most interested student outside of geometry classes.

A proposal is made that mathematical ability not be defined by tests for which out-of-class experiences can play an important role but rather "that mathematical ability be defined as the extent to which students learn routine or complex tasks involving topics that are not encountered even by interested students outside the classroom."

Abstractor's Comments

The extensive investigation reported here is a particularly interesting one. Numerous studies and hypotheses concerning the existence or non-existence of sex differences in mathematical ability have appeared in the literature. These studies have appeared in the popular press as well as the professional literature and occasionally have generated concern as well as controversy. This study builds on the knowledge base in the area by the acknowledgment of work previously done and provides an explanation for the results that is consistent with results from previous studies. Data are also provided to support the findings of the study.

Since this study is a school-based study, it has immediate relevance for mathematics teachers and supervisors. Additional study concerning the nature of the geometry courses, textbooks used, and classroom curriculum in geometry will also be of interest.

Witthuhn, Jan. PATTERNS OF STUDENT PERFORMANCE ON MATHEMATICS STRANDS FOR AMERICAN INDIANS AND OTHERS. Journal of Experimental Education 53: 58-63; Fall 1984.

Abstract and comments prepared for I.M.E. by MARGARIETE MONTAGUE WHEELER, Northern Illinois University.

1. Purpose

The abstract identified the purpose of the study to be an "investigation of patterns of performance among elementary aged students on...the mathematics curriculum of a large urban school district." Within the report, two questions further clarified the purpose of the research:

(a) What differences exist in the patterns of student performance on mathematics strands by ethnic group, sex, or socioeconomic class?

(b) Does the pattern of student performance differ for American Indian students from that of other populations of students?

2. Rationale

The researcher cites government publications and conference reports published within the past ten years to establish that the American Indian student is less likely to be enrolled in mathematics classes and more likely to exhibit special needs early during elementary school. Contributory causes have been attributed to low levels of expectation by school personnel and parents and to prevalence of anxiety toward mathematics even by those students aspiring to careers requiring mathematical competency. Inter-ethnic studies that consider achievement differences between American Indian students and other ethnic populations are few in number and lacking in specific reference to the various strands of mathematics typically found in school programs.

3. Research Design and Procedures

The sample for this study consisted of all kindergarten, first-, second-, and fourth-grade students from Minneapolis Public Schools. For each of the five ethnic groups identified (Asian, Black, Hispanic, Indian, White) the difference between the percentage represented by a particular ethnic group in the district population as a whole (40,197 students) and the percentage represented in the tested population (10,225 students) was three percentage points or less. About the same numbers of students were tested at each grade level and about the same number of boys as girls. The proportion of students in the upper SES to those in the lower SES was approximately two to one.

Students were administered a locally-developed criterion-referenced mathematics test. The test, eventually to be used for promotion decisions, had between 50 and 65 items depending on grade level. The kindergarten test covered only numeration, whereas the fourth-grade test had ten strands (numeration, whole number arithmetic (4), rational number concepts and computation, geometry, measurement, and problem solving). The first- and second-grade tests had fewer strands, but ones common to the fourth-grade test. Reliability measures on revised versions of the four instruments were calculated using both the Cronbach's alpha reliability coefficient and the Spearman-Brown split-half reliability coefficient. The lowest reliability coefficient among the eight reported was 0.862; the highest, 0.955.

Analysis of variance and several multiple classification analyses were used to determine the difference in total test score and strand score with respect to demographic variables and to determine the proportion of test score variability accounted for by these variables.

4. Findings

The researcher emphasized the following conclusions:

(a) Significant differences in the total test scores of students are related to the ethnic group and the socioeconomic classes of the students.

(b) Indian and Black students demonstrate strength on the geometry strand of the mathematics curriculum and difficulty on the numeration strand.

Additional results reported relative to the various analyses included the following:

(a) Asian students tend to be overrepresented in open programs and underrepresented in contemporary programs.

(b) Being Black is related to being from the lower SES, scoring poorly on mathematics tests, being highly mobile, and being overrepresented in fundamental programs and underrepresented in open programs.

(c) Hispanic students tend to be overrepresented in fundamental programs.

(d) Being Indian is related to being from the lower socioeconomic class, scoring poorly on mathematics tests and being over-represented in open and free programs and underrepresented in continuous progress programs.

(e) White students are generally from the higher socioeconomic class, are less mobile, score well on the mathematics tests, and are underrepresented in fundamental programs.

(f) At every grade level for every mathematical strand tested, discrepancies between Asians and Indians, Blacks and Whites, and Indians and Whites were statistically significant ($p = .05$).

(g) In each of the four grade levels, the independent variables, taken together, explain less than 20 percent of the variance in total test scores among students.

(h) Differences in both total and subtest scores by gender are not statistically significant at any grade level.

5. Interpretations

Based upon the results of the several analyses, the researcher concludes that classroom teachers should be aware that the demographic variables of ethnicity, gender, and socioeconomic class account for an increasing proportion of the variability in mathematics performance.

Recommendations for further research concerned examination of patterns of mathematical performance among older students, effects of mobility and school program type on mathematics achievement, distribution of student responses to test item distractors, and interaction between mode of instruction and ethnicity.

Abstractor's Comments

A status study of the patterns of student performance on mathematical topics among various ethnic groups was needed and continues to be needed. Influences on the learning and the participation of minorities in mathematics education is a complex intertwining of many variables. This study did little to further clarify the variables or to suggest interrelationships among the variables already partially understood.

Many of the problems with the "Findings" portion of the report may have actually been problems with the quality of the written report. To answer the question, "Are the patterns of performance on strands of the mathematics curriculum different for students from different ethnic groups?", the author tabulated data concerning Asian/White, Black/White, Hispanic/White and Indian/White discrepancies. The title of the paper leads the reader to expect, at the very least, tabulation of the results for the Indian/Asian, Indian/Black, and Indian/Hispanic contrasts. Indeed the author found it appropriate to globally reference Indian/Asian discrepancies and to quantitatively reference Indian/Black discrepancies in the text but not in the appropriate table. Fully half of the summary concerned "most" of the discrepancies; which mathematical strands summed to "most" is unknown.

A second instance where much more information, especially of the data included and excluded from the tables, would have been beneficial concerns the question, "Are there significant differences in strand scores by ethnic group?". Appropriately, the accompanying table presented scores in an ethnic group by mathematics strand matrix. It was not possible to identify from the table how column or row scores led to conclusions that "Indian and black students demonstrate strength on the geometry strand" or that "Indian and black students, ...have special difficulty with numeration."

A third instance where the data analysis was deficient concerned the question, "What is the relative importance of the independent variables as they relate to student performance?" The researcher identified three variables (membership in the black and Indian ethnic groups and lower socioeconomic class) which explained a "statistically significant portion of the variance in total student test scores of students at all grade levels studied". "Attendance" and the "fundamental" type of school program also should have been identified. For example, attendance ranked first, second, second, and third across the four grade levels in relative importance as related to student performance, whereas being Indian ranked seventh, fifth, fifth, and fourth. Obviously attendance was relatively more important than being Indian. Nevertheless the author fails to discuss attendance as a finding or to recommend future research regarding this variable.

Substantial editing is needed. Four instances of unnecessary ambiguity should suffice.

(a) Four of the five tables contain data with specific reference to ethnic group and grade level. While the total number of students in each grade level is found in the text, the ethnic distribution of students for a particular grade is never given.

(b) In Table 1 the reliability measures are shown. Two columns are headed *r*. The column referencing the Cronbach alpha reliability cannot be distinguished from that referencing the Spearman-Brown reliability.

(c) The relative importance of the independent variables can be inferred from Table 3. The total variability in test scores accounted for is given at each grade level for clusters of seven or eight ranked variables. It would have benefitted the reader to know not only that 13% to 19% of the total variability was accounted for but also the variance attributed to each variable in the cluster.

(d) On page 60, the author concluded that "differences (in total test or subtest scores) by gender are not statistically significant at any grade level." On page 61, the author concluded "with the exception of numeration at the first grade level, subtraction at the second grade level, and numeration, addition and subtraction at the fourth grade level, strand score differences by gender were not statistically significant". A possible contradiction is never resolved. The exceptions are not discussed further.

At times the observations and suggestions offered by the author do not appear to be related to the purposes of the report nor to the data collected. In particular, the call for "greater use of manipulatives and other hands-on experiences" and the observation that a Gagné-type analysis of terminal capabilities might increase mastery of numeration concepts by Indian and Black children was not anticipated. At other times, the absence of information was surprising. The absence of a discussion of the dimensions of the analysis of variance and of any reference to main effects is unusual.

The title and the purpose of this study on first examination appeared promising. Promise and reality did not intersect. This abstractor felt misled.

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