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AUTHOR Balka, Don S.; Wiles, Clyde A.
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ABSTRACT

The 1985 Indiana Mathematics Contest included tests in Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive. Test-writing responsibilities were delegated to mathematics and mathematics education faculties at four state universities, with the mathematical content based on course objectives in the state guidelines. Fourteen sites across Indiana served as test centers, with 2672 students participating, an increase of 278 students from 1984. Outstanding Scholar awards were presented to 152 students. Analysis of student performance on selected items of each test, based on a 20 percent proportional stratified random sample, is then given. Known content, unknown content, and discriminators for each test are discussed. Students appeared to have a good command of fundamental algebraic procedures, but experienced difficulty in applying these procedures to situations involving equations and inequalities and in solving verbal problems in the first course, and with logarithmic and exponential functions, permutations and combinations, and analysis of functions in the second course. In Geometry, circles and their relationships was a discriminating content area, while finite mathematics topics played a definite role in determining who did well on the Comprehensive test. Overall, problem solving continues to be an area of concern. Appendices contain the four tests. (MNS)

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THE 1985 INDIANA STATE MATHEMATICS CONTEST: TESTS AND ANALYSIS OF STUDENT PERFORMANCE

Don S. Balka,
Saint Mary's College
Notre Dame, IN
Clyde A. Wiles,
Indiana University-Northwest
Gary, IN

INTRODUCTION

The 1985 Indiana State High School Mathematics Contest was held on April 27 at fourteen sites across the state. Funded by the Indiana Council of Teachers of Mathematics, and also by student registration fees, the contest has grown from 1700 participants in 1983 to almost 2700 participants this year.

Major goals of the contest remain the same:

1) To stimulate interest in the study of mathematics;
2) To recognize outstanding mathematics students;
3) To foster communication among mathematics students;
4) To facilitate communication within the mathematics education community in Indiana;
5) To promote appreciation of mathematical excellence;
6) To recognize outstanding mathematical achievement.

With increased interest in the contest and ongoing planning for future contests, the Director of the State High School Mathematics Contest has now become a member of the Executive Board of the Indiana Council of Teachers of Mathematics, with responsibilities delineated in the ICTM constitution. Additionally, the financial relationship between ICTM and the contest has been established.

The Contest Committee, composed of interested college and university mathematics educators, has remained intact since revival of the contest in the 1983. Ball State University, Muncie, continues to serve as the coordinating agency for printing, test distribution, test analysis, and financial disbursements.

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TEST CONSTRUCTION

Test writing responsibilities were delegated to mathematics and mathematics education faculties at four state universities: Ball State University, Algebra (First Course); Indiana University - Bloomington, Geometry; Purdue University - North Central, Algebra (Second Course); and Indiana University - Purdue University - Fort Wayne, Comprehensive. As in previous contests, the mathematical content of each test was based on course objectives listed in Guidelines for Mathematics Instruction in Indiana Schools (1977), published by the Indiana Department of Public Instruction. Items were solicited from classroom teachers throughout the state or were formulated by the test writers. Using standard test construction procedures, the four multiple-choice (90 minute time limit) tests were finalized with the following number of items:

Algebra (First Course)	48 items
Geometry	31 items
Algebra (Second Course)	60 items
Comprehensive	28 items

TEST ADMINISTRATION

Fourteen sites across the state served as test centers, with a total of 2672 students participating, an increase of 278 students from 1984. Each site coordinator was responsible for administering and grading tests, presenting certificates to all participants and "Scholar" awards to those students who correctly answered 75 per cent of the test questions, and arranging any local program for students.

All test results were sent to Ball State University for analysis and to determine students scoring in the top five per cent on each test. These students received a special certificate designating them as "Outstanding Scholars." Award cut-off scores for Algebra (First Course), Geometry, Algebra (Second Course), and Comprehensive Tests were, respectively, 40, 23, 46, and 24. Table 1 below provides information on student participation at each site for each test.

Table 1
1985 Indiana State Mathematics Contest
Student Participation

Site	Test			
	Algebra(1st)	Geometry	Algebra(2nd)	Comprehensive
Ball State	63	49	47	50
Butler	126	70	83	77
Franklin	43	41	42	18
Indiana State	111	78	50	20
I.S.U. Evansville	83	57	39	20
I.U. Bloomington	17	22	14	20
I.U. Richmond	25	29	33	21
I.U. Kokomo	45	58	41	30
I.U. Gary	65	48	53	40
I.U.P.U. Ft. Wayne	91	48	36	24
I.U. New Albany	47	46	40	48
Purdue	58	62	66	83
Purdue Westville	54	25	23	21
Saint Mary's	60	44	29	39
Totals	888	677	596	511

TEST ANALYSIS

The remainder of this article presents an analysis of student performance on selected items of each test, based on a 20 per cent proportional stratified random sample. Sample sizes for each test at each site are presented in Table 2. Complete copies of all four tests are included in the Appendix.

Table 2
Sample Sizes by Test Site and Test

Site	Test			
	Algebra(1st)	Geometry	Algebra(2nd)	Comprehensive
Ball State	12	10	9	10
Butler	25	14	17	15
Franklin	9	8	8	4
Indiana State	22	16	10	4
I.S.U. Evansville	17	11	8	4
I.U. Bloomington	4	4	3	4
I.U. Richmond	5	6	7	4
I.U. Kokomo	9	12	8	6
I.U. Gary	13	10	11	8
I.U.P.U. Ft. Wayne	18	10	7	5
I.U. New Albany	9	9	8	10
Purdue	12	12	13	17
Purdue Westville	11	5	5	4
Saint Mary's	12	9	6	8
Totals	178	136	120	103

The analysis, identical in format to that for the 1984 contest, focused on the following questions:

1. What do students know?
2. What don't students know?
3. What items make a difference in student performance?

Operationally, answers to each question were found, respectively, by:

1. An item difficulty of .70 or higher.
2. An item difficulty of .25 or less.
3. A discrimination index of .59 or higher.

Algebra (First Course)

The Algebra (First Course) Test consisted of 48 items, with a mean of 25.60 and a standard deviation of 8.74 for the sample of 178 students. The reliability of the test, using the KR-20 statistic, was .88. Scores for the sample ranged from 6 to 43. Outstanding Scholar Awards were presented to 40 of the 888 participating students.

KNOWN CONTENT. Analysis of the sample data produced 10 questions with item difficulties of .70 or greater, providing an indication of what students do know. Seven of the items (Items 1, 3, 6, 10, 26, 38, 39) fell into the Comprehension level of Bloom's Taxonomy of Cognitive Objectives, while the remaining three (Items 20, 27, 32) fell into the Application level. Item 38 had an item difficulty of .955, and was regarded as a statistically unacceptable test item. The content focus for most of the items was on simple procedures such as adding, subtracting, and multiplying algebraic expressions.

UNKNOWN CONTENT. Only 4 items (Items 4, 9, 14, 33) had item difficulties less than .25, indicating what students do not know. All of these items appear to be at Bloom's Comprehension level. With a negative discrimination index, Item 4 is a statistically unacceptable item. Responses to this item suggest that students have not been exposed to graphing procedures for determining whether a relation is a function, or, in fact, have not been exposed to algebraic equations with two second degree variables.

Item 9 was also a statistically unacceptable item. Two reasons seem apparent. First, a typographical error in the word "factorization" may have caused difficulty in understanding the problem. Second, the "magical" appearance of the expression $3x - 3x$ (= 0) in the factorization might have been appropriately identified as use of the Additive Inverse Property, and therefore, by process of elimination, the Identity Property would not be used (81 of 178 responded with alternative a).

Responses to Item 14 suggest that students either do not know the factorization of the sum of two cubes, or they cannot effectively deal with proportions using multiplication and division of algebraic expressions. Over one-third of the sample chose alternative b, the result after incorrectly factoring $x^3 + y^3$ as $(x + y)(x^2 + y^2)$ and then multiplying $4x(x^2 + y^2)$. The same number of students, however, chose alternative e. Students apparently did not attempt to eliminate responses by checking.

Simplification of a complex fraction is a "traditional" problem in algebra, and as many mathematics teachers can testify, a source of frustration for their students. On Item 33, only 19 of the 178 students in the sample correctly responded. Over one-fourth of the students chose alternative a, apparently "cancelling" the denominator $(x + 1)$ and the last terms $(1/2x)$.

DISCRIMINATORS. There were 9 items which most strongly discriminated the highest scoring students from the lowest scoring students. Five of these items fell in the Comprehension level (Items 12, 13, 35, 40, 43) and four fell in the Application level (Items 19, 20, 34, 47).

As a discriminator, Item 35 presents an interesting study. The performance objective including the idea of scientific notation is listed in the Guidelines for grades six, seven, and eight, however, less than one-half of the students in the sample correctly responded. The natural questions are evident: Is this notion being taught in Algebra, and has it been taught previously? If not, why not? With a demand by society for increased emphasis on science, for computer literacy and more formal computer training, knowledge of scientific notation is certainly necessary.

The 1985 analysis identifies an area for discrimination that also occurred in the 1984 Algebra Test analysis: solving equations and inequalties (Items 13, 19, 43). Although students are able to perform basic algebraic procedures, the

results indicate that they are unable to apply this knowledge in more complex problems. Yet, in comparing the responses to Items 13 and 18 involving inequalities, this argument would seem to be challenged. Of the 178 respondents, 121 (70%) correctly solved the inequality. One possible explanation is that the presence of a simple rational expression, x/3, in Item 13 was the point of concern. The analysis of the 1984 Algebra (First Course) Test identified rational expressions as a problem area.

Two of the discriminators (Items 34 and 47) were verbal problems, another source of difficulty for algebra students. Although Item 47 had a discrimination index of 0.77, almost 2/3 of the sample correctly answered the digit-place value problem. The results would appear to indicate once again that the lowest students are unable to apply their knowledge of basic algebraic procedures in problem solving situations.

Item 12, an equation involving a radical, could be grouped in the equation and inequality area, however, additional procedures are necessary for solving this item. Approximately one-half of the sample correctly answered the item, while almost one-third incorrectly chose alternative c, failing to square both sides of the equation twice.

Even with 70 per cent of the sample correctly answering Item 20, it still discriminated the highest scorers from the lowest scorers. In viewing the responses, failure to follow through on directions would appear to be the major reason for error. Students added both sides of the triangle, but failed to subtract from the perimeter to find the third side.

In summarizing the 1985 Algebra (First Course) Test results, it is gratifying to note that students who took this test have a good command of fundamental algebraic procedures. They experience difficulty in applying these procedures to situations involving equations and inequalities, and in solving verbal problems.

Algebra (Second Course)

On the 60-item Algebra (Second Course) Test, the range of scores for the random sample of 120 students was from 12 to 60. The sample mean was 29.77, with a standard deviation of 10.02. The reliability of the test, using the KR-20 statistic, was .89. Outstanding Scholar Awards were presented to 32 students.

KNOWN CONTENT. An item analysis identified 12 questions having item difficulties of .70 or greater. Using Bloom's Taxonomy of Cognitive Objectives, three items (Items 2, 15, 39) were found to be at the Comprehension level, while the remaining items (Items 8, 11, 20, 21, 22, 39, 44, 45, 48, 51) were at the

Application level. A broad range of content was covered by the items, including integer arithmetic, linear and quadratic equations, systems of equations, and work jobs.

UNKNOWN CONTENT. Questions with item difficulties less than .25 were examined to determine what students did not know. There were 8 test items in this category, three at the Analysis level (Items 30, 31, 42) and the other five items at the Application level (Items 3, 14, 23, 33, 50).

These items identify essentially the same three content areas of concern as found in the analysis of the 1984 contest results:

1. Basic understanding of exponential and logarithmic functions;
2. Fundamental knowledge of permutations and combinations; and
3. Ability to analyze conditions relating to solutions of elementary algebraic equations or of algebraic functions.

Students were generally unsuccessful in solving logarithmic and exponential equations. They continue to have difficulty analyzing properties of functions and finding their zeros. The item analysis showed Item 3 to be the most difficult item on the test. Although 79 of 120 students in the sample were able to determine by some procedure that the function had a minimum, only nine students were able to find the correct value. Item 42 is a number theory problem requiring students to identify a pattern in the unit's digit of powers of three. In observing the mixed respones to this item, it is apparent that the problem solving ability of students is the difficulty.

DISCRIMINATORS. There were nine test items which most strongly discriminated between the highest scoring and lowest scoring students in the sample. These items were categorized as follows: Application level (Items 16, 28, 29, 41, 53, 54, 57, 59) and Analysis level (Item 4). The one area clearly identified by the discrimination indices was that of exponents and exponential functions. To a lesser extent, linear equations and inequalities also discriminated.

In summary, it appears that that students have a good grasp of many "traditional" Algebra (Second Course) topics and that they have improved in the area of polynomial algebra compared to 1984 results. Yet, three major problem areas remain: 1) logarithmic and exponential functions, 2) permutations and combinations, and 3) analysis of functions, equations, their roots, and the nature of their roots. With test results for three years (1983-85) showing students having difficulty in the areas of exponential and logarithmic functions, mathematics teachers need to be aware of the approaches textbooks use as new mathematics books are reviewed throughout Indiana.

GEOMETRY

The Geometry Test consisted of 31 items. Outstanding Scholar Awards were given to 54 students. The scores for the sample of 136 students ranged from 5 to 28 with a mean of 14.82 and a standard deviation of 5.09. The reliability, using the KR-20 statistic, was .75. These descriptive data reveal a test with good range and discrimination characteristics, and one that covered a broad spectrum of topics in the geometry curriculum. With the exception of one item, all test items were at the Comprehension through Analysis levels of Bloom's Cognitive Taxonomy.

KNOWN CONTENT. With item difficulties greater than .70, Items 1, 2, and 17 measured content that was well known by the students who took this test. Most students responded correctly to Item 1, an item requiring only knowledge of arc notation. Thus, it was the only item of the test with unsatisfactory discrimination characteristics. Item 2 is a Comprehension level item relating to supplementary angles that requires solving a linear equation in one unknown, and Item 17 is an Application level item involving knowledge of the Pythagorean relation. In sum, there is evidence that the students had mastered at the Comprehension/Application level content relating to common notation, the Pythagorean relation, and properties of simple polygons.

UNKNOWN CONTENT. The only item with a difficulty index less than .25 was Item 26. This Application level item concerns the surface area of a cylinder and how this surface area relates to changes in the radius. Incorrect answers by students suggest that the areas of the circular bases were neglected or that volume was confused with surface area.

Items 11, 12, 22, 25, and 30 had item difficulties between .25 and .35. While guess work is involved in deciding if the difficulties students had with these items stemmed from the cognitive level of the item or from the tenuousness with which the knowledge was held, the following generalizations are offered. Content weakenesses may be found in:

1. relationships among distance, surface area, and volume of common solids;
2. relationships among circles and polygons;
3. the general topic of locus; and
4. the meaning of distance on a curved surface.

At a somewhat more general level, the nature of proof and the practical use of theorems may also require attention if the geometry program of these students is to be improved.

DISCRIMINATORS. Items 4, 22, and 27 had discrimination indices greater than .5º. At the Application level, Item 4 involved similarity of nested right triangles and the Pythagorean relation. With the most popular wrong response being "none of the above," evidence suggests that students did not understand similarity in this context.

Item 22, at the Application/Analysis level, involved a generalization concerning intersection of the diagonals of parallelograms. Students tended to choose response E, seemingly confusing "can be proved" with "might be an example of." It was noted previously that this item was a particularly difficult item for the group. The fact that it also discriminated well suggests that it was known only by those who did very well overall.

Item 27, a Comprehension item, involves chords, radii, and the Pythagorean relation. Since other evidence shows that the Pythagorean relation was well known, the best conclusion seems to be that higher scoring students acquired more information about circles. One other item on the test involving circles and chords could have been solved by a "guess and check" strategy; thus, the evidence suggests that circles and their relationships is a discriminating content area, with students who did well also having a deeper understanding of similarity and proof.

COMPREHENSIVE

The Comprehensive Test consisted of 28 items. Outstanding Scholar Awards were presented to 26 students. The mean for the sample of 103 students was 14.56 with a standard deviaion of 4.65. Scores for the sample ranged from 6 to 26. The test reliability, using the KR-20 statistic, was .77.

KNOWN CONTENT. Eight items had difficulty indices of .70 or higher (Items 1, 2, 5, 7, 18, 20, 21, 23). All of these items are classified at the Knowledge level of Bloom's taxonomy. Item 1 involves knowledge of the period of a repeating decimal. Item 23 involves counting squares on a subset of a checkerboard. Items 5, 7, and 20 all involve sequences of numbers generated according to some linear equation rule. These items involve content that is frequently encountered within the junior high mathematics curriculum, and thus these routine problems were in fact routine. Item 2 involves knowledge of the sine function graph; Item 18 is a geometry exercise based on similar triangles and the "3-4-5" Pythagorean triple; and Item 21 required knowledge of slopes of parallel lines.

UNKNOWN CONTENT. Items 3, 9, and 14, all at the Application/Analysis level, had difficulty indices below .25, and represent content areas that students do not know well.

Perhaps the most interesting of these is Item 3. There are many opportunities for error, given the known confusion that exists about relationship among perimeter, surface area, and volume. However, the item had a very respectable discrimination factor, leading one to believe that students who have developed visual insight and the ability to keep track of computations did well on the test.

Item 9 deals with propositional logic where quantifiers are used. Unfortunately, one of the foils was at least as defensible as the given correct answer. With 25% of the sample choosing response A, it appears that this item may not represent untaught or unknown content.

Item 14 is a routine problem in combinatorics. Based upon the number of incorrect respones, it appears that the content was not learned. In light of today's emphasis on finite or discrete mathematics, this content area deserves more attention than it seems to have received.

DISCRIMINATORS. The Comprehensive Test contained four items that had very low discrimination factors. Items 5, 7, and 20 were too easy to provide discriminations among students, and Item 9 was too difficult; however, four items had discrimination indices greater than .59 (Items 3, 11, 15, 22).

Item 3 was discussed previously. Items 11, 15, and 22 all involve number theory content. Additionally, Item 15 involves multiple proportions, and Item 22, statistics and/or finite mathematics. All of these problems can be solved through algebraic analysis.

In summary, the test was of reasonable difficulty, and reliably differentiated among those who took it. Knowledge level items of appropriate content were well known. Students who did relatively well were distinguished by their success with problems at the higher cognitive level, and with an apparently broader range of content knowledge. The analysis provides evidence that finite mathematics topics played a definite role in determining who did well and who did not.

GENERAL SUMMARY

Analysis of test items for representative samples of students on each of the four tests provides insight into what our good students know and do not know. Results show that students generally know fundamental procedures in all areas. Problem solving ability, in a broad sense, is weak, with many students unable to perform well at higher cognitive levels. The position of finite or discrete mathematics in the high school mathematics curriculum needs special consideration. Students who did well on the Algebra (Second Course) or Comprehensive Tests were successful in this content area. Results on all tests also suggest that students lack global views of certain content areas, in addition to fundamental procedures, to better analyze the mathematics studied.

Overall, the results are encouraging. Yet, our encouragement must be tempered by the fact that these test results represent work of our best students. It is the continued hope of the State Mathematics Contest Committee that teachers use the tests as sources for problem solving activities across the high school mathematics curriculum, and the analysis to help identify areas which may require special attention.

1 9 8 5

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

A L G E B R A T E S T

(FIRST COURSE)

THIS TEST WAS PREPARED BY THE SECONDARY MATHEMATICS EDUCATION COMMITTEE, DEPARTMENT OF MATHEMATICAL SCIENCES, BALL STATE UNIVERSITY.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school algebra, first course. For each of the 48 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

A. 8

B. 3

C. $\frac{1}{3}$

D. 4

E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the ninth grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)																
B	R	O	W	N		M	A	R	Y		A					

SEX
Ⓜ
●

GRADE
●
⑩
⑪
⑫

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. The product of $(2x - y)$ and $(4x + 3y)$ is

   a. $8x^2 + 2xy + 3y^2$

   b. $8x^2 + 2xy - 3y^2$

   c. $8x^2 + xy - 3y^2$

   d. $8x^2 - 3y^2$

   e. None of the above.

2. Find the value of $3[4 - 2^3 + 4 \div 2 \cdot 5]$.

   a. 24

   b. 20

   c. 18

   d. 0

   e. -30

3. If $\frac{1}{4}$ of a number is 6, then $\frac{1}{3}$ of the number is

   a. $\frac{1}{12}$

   b. $\frac{1}{8}$

   c. 8

   d. 12

   e. None of the above.

4. Which of the following relations is a function?

   a. $y^2 = 1 - x^2$

   b. $\frac{x^2}{a} + \frac{y^2}{b} = 1$

   c. $x = 3$

   d. All of the above.

   e. None of the above.

5. If $1 - \frac{4}{x} + \frac{4}{x^2} = 0$, then $\frac{2}{x}$ equals

   a. -1

   b. 1

   c. 2

   d. -1 or 2

   e. -1 or -2

6. If $x = -2$ and $y = \frac{1}{2}$, the value of $8y^2 - 4x + 1$ is

   a. 25

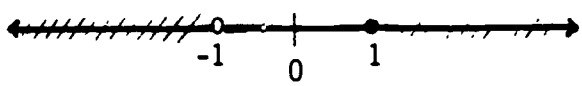
   b. 31

   c. 17

   d. 11

   e. -5

7. The set of points shaded in the graph to the right is denoted by



   a. $\{ x \mid x \geq 1 \} \cup \{ x \mid x < -1 \}$

   b. $\{ x \mid x \geq 1 \} \cap \{ x \mid x < -1 \}$

   c. $\{ x \mid x > 1 \} \cup \{ x \mid x \leq -1 \}$

   d. $\{ x \mid x > 1 \} \cap \{ x \mid x \leq -1 \}$

   e. None of the above.

8. If $x + y = 4a + 7$ and $x - y = 2a + 5$, then x equals

   a. $2a + 2$

   b. $2a + 5$

   c. $2a + 6$

   d. $3a + 6$

   e. None of the above.

9. One property not used in the fractorization to the right is the

$$
\begin{aligned}
& x^2 - 2x - 15 \\
&= x^2 - 2x - 15 + 3x - 3x \\
&= (x^2 - 5x) + (3x - 15) \\
&= x(x - 5) + 3(x - 5) \\
&= (x + 3)(x - 5)
\end{aligned}
$$

a. Identity Property

b. Commutative Property

c. Distributive Property

d. Associative Property

e. None of the above.

10. If $\frac{9}{N} - \frac{7}{N} = \frac{1}{6}$, then N = _____.

a. $\frac{1}{12}$

b. $\frac{1}{3}$

c. 6

d. 12

e. None of the above.

11. Consider the graphs of $2x + 4y = 3$ and $3x + 6y = 10$.

a. They intersect at $(\frac{1}{2}, \frac{1}{2})$.

b. They are parallel.

c. They do not intersect.

d. Both b and c are correct.

e. None of the above is correct.

12. If $\sqrt{x + 2} = 2$, then $(x + 2)^2$ equals

a. $\sqrt{2}$

b. 2

c. 4

d. 8

e. 16

13. The solution of the inequality $\frac{x}{3} + 2 \leq x$ is

a. $x \geq 1$

b. $x \geq 3$

c. $x \leq 3$

d. $x \geq \frac{3}{2}$

e. $x \geq 2$

14. What is the missing numerator? $\frac{4x}{x + y} = \frac{?}{x^3 + y^3}$

a. $4x^3 - 4x^2y + 4xy^2$

b. $4x^3 + 4xy^2$

c. $4x^3 + 4x^2 + 4xy^2$

d. $x^2 - xy + y^2$

e. None of the above.

15. The length of a rectangle is 4 inches more than twice the width. If the perimeter is 26 inches, the length is

a. 10 inches

b. $18\frac{2}{3}$ inches

c. 3 inches

d. 7 inches

e. 13 inches

16. The slope of a line perpendicular to $2x + 3y = 6$ is

a. 2

b. $-\frac{2}{3}$

c. $-\frac{3}{2}$

d. $\frac{3}{2}$

e. None of the above

17. Which of the following expressions is equivalent to $x(x - a) + a(x + a)$?

a. $(x + a)^2 (x - a)$

b. $(x + a)(x - a)^2$

c. $(x + a)^3$

d. $(x + a)^2$

e. $x^2 + a^2$

18. The solution of the inequality $-3(1 + 2x) \leq 15$ is

a. $x \leq -3$

b. $x \geq -\frac{18}{5}$

c. $x \geq -2$

d. $x \geq -3$

e. None of the above.

19. The solution for the equation $(x + 3)(2x - 4) + 3 = x^2 + (x + 8)(x - 1)$ is

a. $-\frac{4}{5}$

b. $-\frac{1}{5}$

c. $\frac{1}{2}$

d. $-\frac{17}{9}$

e. There is no solution.

20. The perimeter of a triangle is $6a + 5b - 2c$. If two sides are $a - b + 6c$ and $3a + 6b - 10c$, the third side is

a. $4a + 5b - 4c$

b. $10a + 10b - 6c$

c. $2a + 10b - 6c$

d. $2a - 6c$

e. $2a + 2c$

21. The two solutions of $2x^2 - x - 4 = 0$ are

a. both positive and equal

b. both negative and equal

c. opposite in sign

d. both positive but unequal

e. both negative but unequal.

22. The solution for the system of equations $\begin{cases} 3x - 5y = 14 \\ 2x - 7y = 2 \end{cases}$ has x-coordinate

a. $x = -8$

b. $x = -6$

c. $x = 2$

d. $x = 8$

e. $x = \frac{4}{3}$

23. $\sqrt{180}$ =

a. 90

b. 44.4

c. $6\sqrt{5}$

d. $90\sqrt{2}$

e. $2\sqrt{90}$

24. The solution set for $2x^2 - 8x + 5x - 20 = 0$ is

a. $\{-5, 4\}$

b. $\{-2, 4\}$

c. $\{\frac{5}{2}, 4\}$

d. $\{-\frac{5}{2}, 4\}$

e. None of the above.

25. The slope of the straight line passing through the points (1, 7) and (-3, -3) is

a. $-\frac{1}{2}$

b. $\frac{2}{5}$

c. -2

d. $\frac{5}{2}$

e. -1

26. $(x^2 - 3x + 2) - (3x^2 - 5x - 1) =$

a. $4x^2 - 8x + 1$

b. $2x^2 - 2x + 3$

c. $-2x^2 + 2x + 3$

d. $-2x^2 + 2x + 1$

e. None of the above

27. Chris has 15 coins. Some are nickels and the rest are quarters. If there is $1.95 in all, how many nickels are there?

a. 9

b. 7

c. 6

d. 3

e. None of the above.

28. The solution (x, y) for the system of equations $\begin{cases} \frac{1}{x} + \frac{1}{y} = 3 \\ \frac{1}{x} - \frac{1}{y} = -1 \end{cases}$ is

a. $(1, 2)$

b. $(1, \frac{1}{2})$

c. $(\frac{1}{2}, 1)$

d. $(1, -\frac{1}{2})$

e. None of the above.

29. The solution of the proportion $\frac{2.5}{8} = \frac{10.4}{x}$ is

a. $x = 31.28$

b. $x = 3.25$

c. $x = 33.28$

d. $x = 34.88$

e. None of the above.

30. For what value of c are the roots of the equation $x^2 + 6x + c = 0$ equal?

a. 6

b. 9

c. 12

d. 36

e. None of the above.

31. For the relation given by $y = 2x + 1$ having domain the set of positive integers less than 5, the range is

a. $\{y \mid y \text{ is an integer and } 1 < y < 11\}$

b. $\{y \mid y \text{ is an integer and } y < 11\}$

c. $\{y \mid y \text{ is an integer and } y > 1\}$

d. $\{y \mid y \text{ is an integer and } 1 < y < 8\}$

e. None of the above.

32. A number is 12 less than three times another number. If their sum is 108, what are the two numbers?

a. 29, 79

b. 30, 78

c. 48, 60

d. 43, 65

e. 44, 64

33. Simplify $\dfrac{\dfrac{x}{x+1} - \dfrac{1}{2x}}{\dfrac{1}{x+1} - \dfrac{1}{2x}}$

a. x

b. $2x + 1$

c. 1

d. $\dfrac{2x - 1}{x + 1}$

e. None of the above is correct.

34. Henry's motorboat can travel 12 mph in calm water. On a river, the boat could go 45 miles traveling with the current in the same amount of time that it went 27 miles against the current. Find the rate of the current.

a. 3 mph

b. 4 mph

c. 5 mph

d. 6 mph

e. None of the above.

35. Written in scientific notation .000965 would be

a. $.965 \times 10^{-4}$

b. 9.65×10^{4}

c. 965×10^{6}

d. 9.65×10^{-4}

e. None of the above.

36. If $a = -2$, $b = 3$, and $c = -1$, then the value of $\frac{b^2 - 3a}{3a - c} + |3a|$ is

a. -3

b. 3

c. 9

d. -15

e. None of the above.

37. Which of the following relations is <u>not</u> a function?

a. $y = 2x + 1$

b. $y = x^2$

c. $x^2 + y^2 = 1$

d. $y = x^3$

e. None of the above.

38. Factor $x^2 + 10x + 21$

a. $x(x + 10)$

b. $x = 3$

c. $x = 7$

d. $(x + 5)(x + 2)$

e. None of the above.

39. If the sum of the three numbers is 45 and one of the numbers is r, the sum of the other two is

a. $45 + r$

b. $45 + 2r$

c. $45 - r$

d. $45 - 2r$

e. None of the above.

40. If $y = 2x$ and $z = 2y$ then $x + y + z$ equals

a. x

b. $3x$

c. $5x$

d. $7x$

e. $9x$

41. The distance between the two points (-1, 3) and (-4, 7) is

a. $\sqrt{13}$

b. 5

c. 25

d. $\sqrt{41}$

e. None of the above.

42. The width of a rectangular box is x feet, its length is $2x$ feet, and its height is $x-1$ feet. Find its volume in terms of x.

a. $2x^2 + x^3$ cu. ft.

b. $2x^3 + 2x^2$ cu. ft.

c. $2x^2 - 2x$ cu. ft.

d. $2x^3 - 2x^2$ cu. ft.

e. None of the above.

43. If $ax + b = 5$ and $a \neq 0$, then $x =$

a. $\frac{b + 5}{a}$

b. $\frac{5 - b}{a}$

c. $\frac{b - 5}{a}$

d. $5 - b$

e. None of the above.

44. Jenny is 2 years less than twice as old as Tom. In 5 years, Jenny's age will be $1\frac{1}{2}$ times Tom's age. How old is Jenny now?

a. 16

b. 6

c. 9

d. 10

e. [illegible]

45. Simplify $(x^2y)^3(-2x^3y^4)^2(-xy^2)$

a. $-2(xy)^{13}$

b. $(x^2y^2)^3(-2x^3y^4)^2$

c. $-(x^2y)^3(-3x^3y^4)$

d. $(-2x^2x^3xyy^4y^2)^5$

e. Cannot be simplified.

46. If $x^3 - 2x^2 - 3$ is multiplied by $2x^2 - 5$, the coefficient of x^2 in the product is

a. -16

b. -4

c. 0

d. 4

e. 16

47. The units digit of a two-digit number is 5 more than the tens digit and the number itself is three times as great as the sum of its digits. What is the number?

a. 38

b. -27

c. 27

d. 72

e. None of the above

48. Simplify $\frac{x - 9}{\sqrt{x} + 3}$

a. $\sqrt{x} - 3$

b. $\sqrt{x} - \frac{9}{\sqrt{x}} - 3$

c. $\frac{x^2 - 81}{x\sqrt{x} + 3x + 9\sqrt{x} + 27}$

d. $\frac{x - 9}{\sqrt{x}} + \frac{x - 9}{3}$

e. $\sqrt{x} - 6$

1985

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

ALGEBRA TEST

(SECOND COURSE)

THIS TEST WAS PREPARED BY THE MATHEMATICS
EDUCATION FACULTY AT PURDUE UNIVERSITY-NORTH CENTRAL.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in the second course of high school algebra. For each of the 60 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

A. 8

R. 3

C. $\frac{1}{3}$

D. 4

E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the eleventh grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX
Ⓜ
●

GRADE
⑨
⑩
●
⑫

DO NOT TURN PAGE UNTIL YOU ARE TOLD TO DO SO.

1. Which of the following is an instance of the distributive principle?

   a) $(x - y)(x + y) = (x - y)x + (x - y)y$
   b) $7 + x + 5 = 12 + x$
   c) $8 + x = x + 8$
   d) $(x + y)(x - y) = (x - y)(x + y)$
   e) None of these

2. Simplify: $-5 - [-3 - 2(-4)]$

   a) -10 b) -6 c) 4 d) -4 e) None of these

3. The function defined by $f(x) = 2x^2 - x + 3$ has a

   a) maximum value of 3
   b) minimum value of 23/8
   c) minimum value of 13/4
   d) minimum value of 3
   e) None of these

4. The graph of $f(x) = \left(\frac{1}{2}\right)^x$

   a) increases from left to right
   b) decreases from left to right
   c) increases, then decreases from left to right
   d) decreases, then increases from left to right
   e) None of these

5. The statement $|y - 2| < 3$ is equivalent to:

   a) $-1 > y > 5$ b) $-1 < y < 5$ c) $y > 5$

   d) the disjunction, $y < -1$ or $y > 5$ e) None of these

6. The roots of the quadratic equation $2x^2 + 4x - 3 = 0$ can be described as

   a) complex conjugates b) real and irrational c) real and rational

   d) integers e) None of these

7. $i^{65} - i^{64} =$

   a) $1 - i$ b) $i - 1$ c) i d) 0 e) None of these

8. The solutions of $2x(x - 2) = 3(4 - x)$ are

   a) both positive b) both negative c) both integers

   d) one positive and one negative e) None of these

9. An equation of a line through the point $(-2,-3)$ and perpendicular to the line through $(2,4)$ and $(7,3)$ is:

   a) $5x - y + 7 = 0$ b) $5x - y - 7 = 0$ c) $x + 5y + 17 = 0$

   d) $x - 5y + 17 = 0$ e) None of these

10. The graph of $f(x) = x^2 - 6x + 12$ will have

   a) No real zeros b) 1 real zero c) 2 real zeros

   d) more than 2 real zeros e) None of these

11. Simplify: $\sqrt{32} + \sqrt{98} - \sqrt{72} + \sqrt{18}$

a) $20\sqrt{2}$ b) $2\sqrt{14}$ c) $8\sqrt{2}$ d) $2\sqrt{2}$ e) None of these

12. Find the first term of an arithmetic sequence for which the eleventh term is 82 and the common difference is 7.

a) 5 b) 12 c) 157 d) 159 e) None of these

13. If $2x^{-1/3} = 3$, $x =$

a) $-2/9$ b) $-27/8$ c) $8/27$ d) $-9/2$ e) None of these

14. If $\log_{10}2 = a$ and $\log_{10}3 = b$, then $\log_{10}6\frac{2}{3} =$

a) $ab + \frac{a}{b}$ b) $(a + b) + (a - b)$ c) $\frac{1 + a}{b}$

d) $1 + a - b$ e) None of these

15. The equation $4 - (3 - (2 - (1 - x))) = 2$ has how many solutions?

a) None b) Infinite c) 1 d) 2 e) None of these

16. $\sqrt{2} \cdot \sqrt[3]{2} =$

a) $\sqrt[3]{4}$ b) $\sqrt[6]{4}$ c) $\sqrt[6]{32}$ d) 2 e) None of these

17. The solution of the system
$$\begin{aligned} y - z &= 1 \\ 3x - 2y + z &= 0 \\ 2x + 2y - z &= 5 \end{aligned}$$ includes

a) $x = 2$ b) $x = -2$ c) $y = 2$ d) $y = -2$ e) None of these

18. $a^3 - 8 =$

a) $(a - 2)(a^2 + 4a + 4)$ b) $(a + 2)(a^2 - 2a + 4)$

c) $(a - 2)^3$ d) $(a - 2)(a^2 + 2a + 4)$

e) None of these

19. If y varies directly as x and inversely as the square of z, and $y = 4/25$ when $x = 3$ and $z = 5$, what is y when $x = 9$ and $z = 2$?

a) $\frac{6}{5}$ b) 3 c) $\frac{9}{4}$ d) $\frac{9}{2}$ e) None of these

20. If $x - 2y = 7$ and $x + 2y = 5$, then $x =$

a) $\frac{1}{2}$ b) 0 c) 5 d) 6 e) None of these

21. The solution set of $x^2 - 3x = 40$ is

a) {8.-5} b) {8,5} c) {-8,5} d) {-8,-5} e) None of these

22. The distance between the points (-4,3) and (-2,2) is

a) $\sqrt{37}$ b) 3 c) $\sqrt{5}$ d) $\sqrt{61}$ e) None of these

23. A jar contains 10 balls, five of them blue and five of them red. If a person reaches into the jar and removes two balls, what is the probability that they are both blue?

a) 1/5 b) 5/18 c) 1/4 d) 1/2 e) None of these

24. If the graph of $ax + 3y = c$ contains the origin and the point (-6,-10) then,

a) neither a nor c can be determined
b) $a = 0$ and $c = 0$
c) $a = c$
d) $a = -5$ and $c = 0$
e) None of these

25. The expression $2x + 5 - \frac{2}{x - 2}$ is equivalent to

a) $\frac{2x + 3}{x - 2}$ b) $\frac{2x^2 + x - 12}{x - 2}$ c) $\frac{2x + 5}{x}$

d) $\frac{2x^2 - 12}{x - 2}$ e) None of these

26. What value of k will make $49x^2 + 14xy + k$ a perfect square trinomial?

a) 7 b) 2y c) $49y^2$ d) y^2 e) None of these

27. Evaluate: $2^{-1} + (2^2 + 2^{-3})^0 + \frac{1}{2^{-1}} + (-2)^0$

a) 3/2 b) 5/2 c) 7/2 d) 9/2 e) None of these

28. An equation of the line passing through points (3,-2) and (-2,3) is:

a) $y = 2x - 1$ b) $y = x - 1$ c) $y = -\frac{1}{2}x + 1$

d) $y = \frac{1}{2}x - 1$ e) None of these

29. Perform the indicated operation and express in a + bi form

$$[(3-4i)(2+i)] \div 2i$$

a) 0-5i b) -5+3i c) $-\frac{5}{2} - 3i$ d) $-\frac{5}{2} - 5i$ e) None of these

30. The greatest integer function, $y = [\![x]\!]$ is defined by:
For any real number x, y is the greatest integer less than or equal to x.
Which statement is true?

a) The greatest integer function is one-to-one
b) $[\![2x]\!] = 2[\![x]\!]$
c) If $a < b$, then $[\![a]\!] < [\![b]\!]$
d) $[\![-3.5]\!] = -3$
e) None of these

31. Consider the absolute value function, $f(x) = |x|$. Which statement is not true?

a) If $x < 0$, $f(x) = -x$ b) $f(x) = f(-x)$ c) $f(x+y) = f(x)+f(y)$
d) $f(0) = 0$ e) None of these

32. The inverse of the function $f(x) = 2x - 1$ is

a) $f(x) = \frac{1}{2}x + 1$ b) $f(x) = 2x - 2$ c) $f(x) = \frac{1}{2}x - 2$
d) $f(x) = \frac{x+1}{2}$ e) None of these

33. Simplify: $\frac{x^{-1}}{y^{-1} + z^{-1}}$

a) $\frac{y+z}{x}$ b) $\frac{yz}{x}$ c) $\frac{xyz}{y+z}$ d) $\frac{yz}{x(y+z)}$ e) None of these

34. Evaluate $\begin{vmatrix} 1 & -2 & 3 \\ 0 & -1 & 5 \\ 6 & 4 & -3 \end{vmatrix}$

a) 25 b) 61 c) -55 d) -59 e) None of these

35. Factor: $x^2 - 2x + 1 - y^2$

a) $(x - 1 - y)^2$ b) $(x - 2x + y)(x - y)$ c) $(x + 1 - y)^2$
d) $(x - 1 + y)(x - 1 - y)$ e) None of these

36. If the domain of the function defined by $g(x) = (x + 2)^2 - 5$ is all real numbers, then the range is

a) $\{y : y \geqslant -5\}$ b) $\{y : y \geqslant 2\}$ c) $\{y : y \geqslant -2\}$

d) $\{y : y \geqslant -1\}$ e) None of these

37. If $x = \log_4 [\log_2 (\log_3 81)]$, $x =$

a) 1 b) 1/2 c) 2 d) -1 e) None of these

38. Which set of numbers is not closed under subtraction?

a) Rational Numbers b) Complex Numbers c) Integers

d) Natural Numbers e) None of these

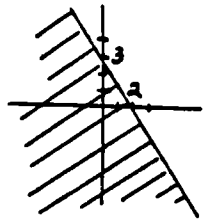
39. What is the slope of the line whose equation is $3y + 6x = 1$?

a) -3 b) 1/2 c) 1 d) 2 e) None of these

40. The solution for $\sqrt{x + 1} - \sqrt{x - 1} = 1$ is

a) 0 b) 1 c) 5/4 d) 1/4 e) None of these

41. Which inequality defines the shaded portion of the plane?



a) $3x + 2y - 6 \geqslant 0$ b) $2x + 3y - 6 \leqslant 0$ c) $3x + 2y - 6 \leqslant 0$

d) $2x + 3y - 6 \geqslant 0$ e) None of these

42. The units' digit of $(43,123)^{491}$ is:

a) 9 b) 7 c) 1 d) 3 e) None of these

43. Point $M(-2,-\frac{1}{2})$ is the midpoint of $\overline{AB}$. A is (3,-4). Then point B is

a) $(\frac{1}{2}, -\frac{9}{4})$ b) (-1,-3) c) (1,3) d) (-7,3) e) None of these

44. The product $(2x - 5y)^2 =$

a) $4x^2 + 25y^2$ b) $4x^2 - 10xy + 25y^2$ c) $4x^2 - 20xy + 25y^2$

d) $4x^2 - 25y^2$ e) None of these

45. $3(2 - 3a) - (5 - a) =$

a) $1 - 10a$ b) $1 - 8a$ c) $1 - 4a$ d) $11 - 8a$ e) None of these

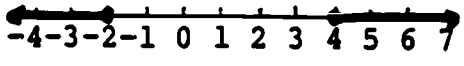
46. Which relation does not represent a function?

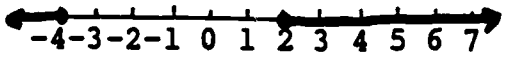
a) $\{(3,4),(2,7),(4,9),(7,4)\}$ b) $\{(x,y) \mid y = |x|\}$

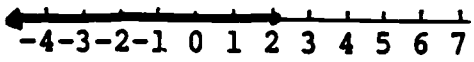
c) $\{(x,y) \mid y = 3x - 2\}$ d) $\{(x,y) \mid x = |y|\}$

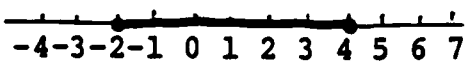
e) None of these

47. Which of the following is the graph of the solution set of $x^2 - 2x - 8 \leq 0$?

a) 

b) 

c) 

d) 

e) None of these

48. A bicycle wheel with a 26-inch diameter takes 10 revolutions to go a certain distance. How many revolutions will a wheel with a 20-inch diameter make in covering the same distance?

a) 11 b) 12 c) 13 d) 14 e) None of these

49. The graph of $x^2 + y^2 = 4 - 4x + 2y$ is a circle with

a) center (-2,1) and radius 3 b) center (-2,1) and radius 2

c) center (2,1) and radius 3 d) center (-2,-1) and radius 3

e) None of these

50. Evaluate $10 + 5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \ldots$

a) 19 b) 20 c) 21 d) infinite e) None of these

51. Machine A can do a job in 4 hours, while machine B can do the same job in 6 hours. Working together, how long does it take the two machines to do the job?

a) 5 hours b) 2.4 hours c) 15 minutes

d) 10 hours e) None of these

52. Which of the following equations has roots $1 + \sqrt{2}$ and $1 - \sqrt{2}$?

a) $x^2 - 2x + 5 = 0$ b) $x^2 - 5x + 2 = 0$ c) $x^2 - 2x - 1 = 0$

d) $x^2 - x + 2 = 0$ e) None of these

53. Solve for m: $B = \frac{dm}{d + m}$

a) $m = \frac{dB}{d - B}$ b) $m = \frac{dB}{d + B}$ c) $m = dB - d + B$

d) $m = dB - d - B$ e) None of these

54. If $f(x) = 2x^2 - 3x + 4$ and $g(x) = x + 5$, what is $g[f(-3)]$?

a) 0 b) 6 c) 18 d) 36 e) None of these

55. In how many ways can 5 different math books be arranged on a shelf?

a) 5 b) 15 c) 24 d) 32 e) None of these

56. If $\log 2 \doteq 0.3010$, what is $\log \sqrt{2}$, correct to four decimal places?

a) 0.0301 b) 0.6020 c) 0.1505 d) -0.3010 e) None of these

57. If $4^{x+2} = 2^{3x-1}$, $x =$

a) 5 b) 3 c) 3/2 d) 4 e) None of these

58. The number of points of intersection of the graphs: $y = x^2 + 2$, $4y = x^2 + y^2$ is

a) 0 b) 1 c) 2 d) 4 e) None of these

59. $\left(\frac{m^{5p}}{m^{5p-3}}\right)^{5p} =$

a) m^{20p^2-4p+3} b) $\frac{m}{15p}$ c) m^{15p} d) m^{8p} e) None of these

60. $\left(\frac{3}{4}\right)^{331} \cdot \left(-1\frac{1}{3}\right)^{328}$

a) $\frac{27}{64}$ b) $\frac{3}{4}$ c) $-2\frac{10}{27}$ d) $-\frac{27}{64}$ e) None of these

1985

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

GEOMETRY TEST

THIS TEST WAS PREPARED BY THE STAFF OF THE MATHEMATICS EDUCATION DEVELOPMENT CENTER, INDIANA UNIVERSITY-BLOOMINGTON.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school geometry. For each of the 31 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 30^0 = 90^0$, then x equals

   A. 120^0

   B. 3^0

   C. -60^0

   D. 60^0

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "60^0", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the tenth grade, would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

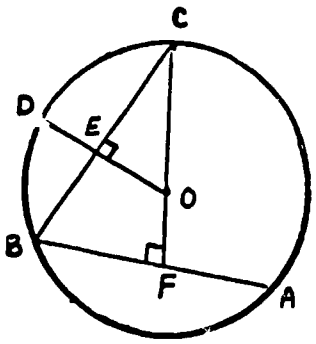
NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX
M
●

GRADE
9
●
11
12

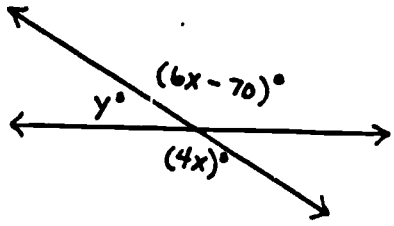
DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. O is the center of the circle shown and $\overset{\frown}{BC} = 120^\circ$. If AB = BC, what is the measure of $\overset{\frown}{AB}$?



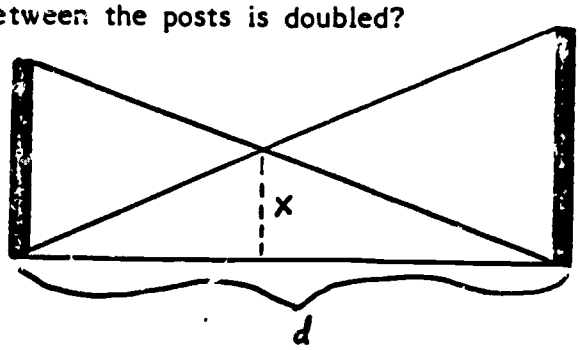
A. 60°
B. 120°
C. 180°
D. 240°
E. None of the above.

2. Two lines intersect to form the angles shown. Find the value of y.



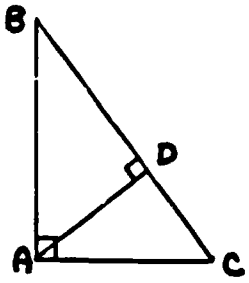
A. 55°
B. 145°
C. 35°
D. 40°
E. 140°

3. Two posts are set upright and a rope is stretched from the top of each post to the foot of the other post. The ropes cross at height x above the ground. How is the height affected when the distance d between the posts is doubled?



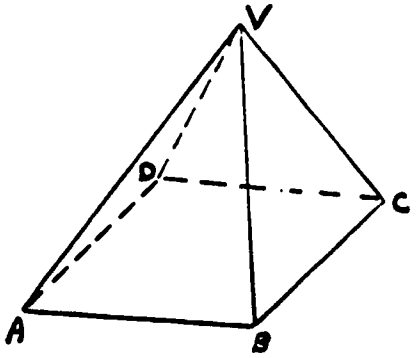
A. x is doubled.
B. x is squared.
C. x is halved.
D. x stays the same.
E. There is too little information to determine what happens to x.

4. Angles BAC and BDA are right angles. If AC = 12 units and AB = 16 units, what is the length of $\overline{AD}$?



A. 4.8 units
B. 9.6 units
C. 19.2 units
D. $16\sqrt{2}$ units
E. None of the above.

5. A regular square pyramid, V - ABCD, has a lateral edge, $\overline{AV}$, equal to a side of the base, $\overline{AB}$. Find the measure of $\angle VAC$.



A. 30°
B. 45°
C. 60°
D. 90°
E. None of the above.

6. The measure of each of the interior angles of a polygon is 160°. What is the sum of the measures of all of these angles?

A. 360°
B. 144°
C. 1600°
D. 2880°
E. None of the above.

7. Find the area of a triangle whose vertices are (5,8), (-6,-1), and (5,-4).

A. 6 sq. units
B. $6\sqrt{130}$ sq. units
C. $6\sqrt{202}$ sq. units
D. 132 sq. units
E. 66 sq. units

8. Find the perimeter of a square whose diagonal is $\sqrt{10}$.

A. 20 units
B. $4\sqrt{5}$ units
C. 5 units
D. $\sqrt{5}$ units
E. None of the above.

9. Find the measure of the angle formed by the hands of a clock at 4:45.

A. 127.5°
B. 145°
C. 135°
D. 120°
E. None of the above.

10. A parallelogram and triangle have equal areas. The base and the height of the parallelogram are 12 inches and 9 inches respectively. If the base of the triangle is 36 inches, what is the height of the triangle?

A. 1.5 inches
B. 3 inches
C. 6 inches
D. 9 inches
E. None of the above.

11. In general, for which type of polygon is it not possible to inscribe a circle?

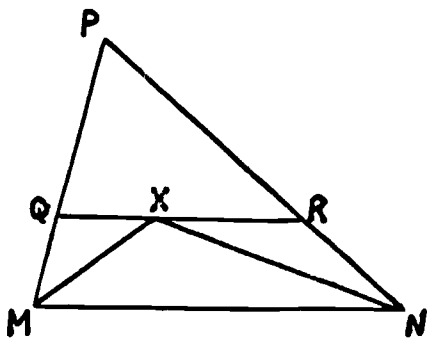
A. right triangle
B. obtuse triangle
C. rectangle
D. rhombus
E. All are always possible.

12. In a plane, what is the locus of points equidistant from the sides of a square?

A. A square
B. A pair of squares
C. A circle
D. A point
E. A straight line

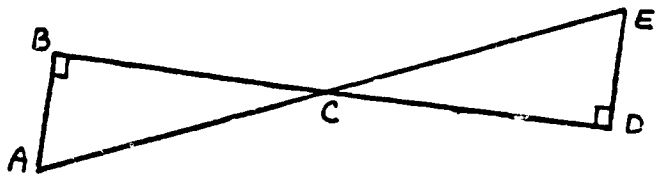
13. In ΔMPN, MP = 10 units, PN = 17 units, MN = 15 units. $\overline{MX}$ bisects $\angle M$. $\overline{NX}$ bisects $\angle N$. $\overline{QR} \parallel \overline{MN}$. Find the perimeter of ΔPQR.

A. 23.5 units
B. 25 units
C. 27 units
D. 42 units
E. None of the above.



14. $\angle B$ and $\angle D$ are right angles and AC = EC. Which method would be used to prove that triangles ABC and EDC are congruent ?

A. side-side-side
B. side-angle-side
C. angle-side-angle
D. side-side-angle
E. hypotenuse-leg



15. Given : "All goojums are jumgoos."
Which statement expresses a conclusion that follows logically from the given statement ?

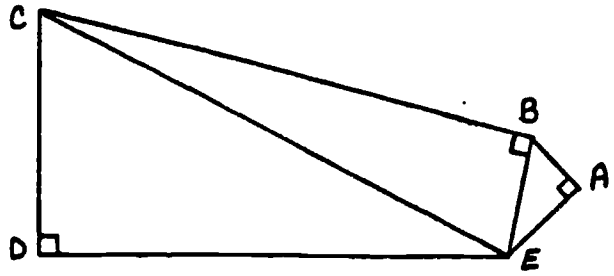
A. All jumgoos are goojums.
B. If a joom is not a jumgoo, then it is not a goojum.
C. If a joom is not a goojum, then it is not a jumgoo.
D. If a joom is a jumgoo, then it is a goojum.
E. None of the above.

16. The base of a right prism is a regular hexagon with side 4 cm. The height of the prism is 6 cm. What is the volume of the prism ?

A. $144\sqrt{3}$ cm^3
B. $72\sqrt{3}$ cm^3
C. $48\sqrt{3}$ cm^3
D. $288\sqrt{3}$ cm^3
E. None of the above.

17. $\angle CDE$, $\angle CBE$ and $\angle BAE$ are right angles. BA = 3cm., AE = 4cm., DE = 11cm., and DC = 7cm. What is the lenght of $\overline{BC}$?

A. 145 cm.
B. 170 cm.
C $\sqrt{195}$ cm.
D. $5\sqrt{1155}$ cm.
E. None of the above.



18. In right triangle ABC shown in the diagram. $\overline{DE} \perp \overline{BC}$. Also, AB = 3 units, AC = 4 units, BC = 5 units and DE = 1 unit. What is the length of $\overline{EC}$?

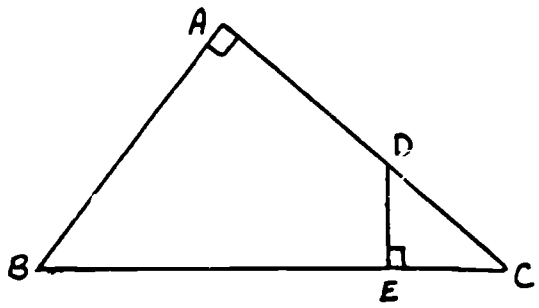
A. $\frac{4}{5}$ units

B. $\frac{3}{4}$ units

C. $\frac{5}{3}$ units

D. $\frac{4}{3}$ units

E. None of the above.



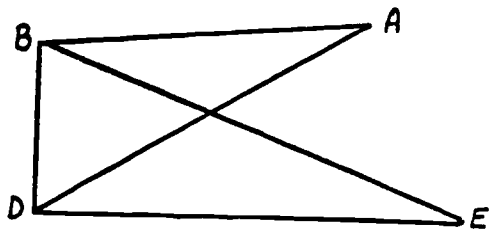
19. What is the minimum amount of information that is necessary to find the measure of <EBD as shown in the diagram ?

I. m(<ABE) = 25°

II. m(<BAD) = 30°

III. m(<BDA) = 75°

IV. m(<BED) = 20°



A. I

B. I and II

C. I and IV

D. I, II, and III

E. I, II, and IV

20. Suppose ΔPQR has the property that m<P = 2(m<Q). Also, point $S \in \overleftrightarrow{PR}$ but is not between P and R. If m<QRS = 120°, what type of triangle is ΔPQR ?

A. isosceles

B. right

C. obtuse

D. scalene

E. Not enough information is given.

21. If two planes perpendicular to the same line, L, are intersected by a third plane, then you know :

A. Line L must be perpendicular to the third plane.
B. Line L must be parallel to the third plane.
C. The lines of intersection are parallel.
D. The lines of intersection are perpendicular.
E. None of the above need necessarily apply.

22. A quadrilateral has bisecting diagonals. Using only this information, you can prove that it is a :

I. parallelogram
II. rhombus
III. rectangle
IV. square

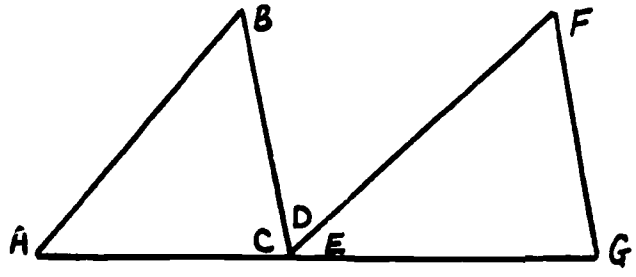
A. I
B. I and II
C. I and III
D. I, II, and IV
E. All four possibilities

23. The supplement of an angle is 7 times the complement. What is the measure of the supplement ?

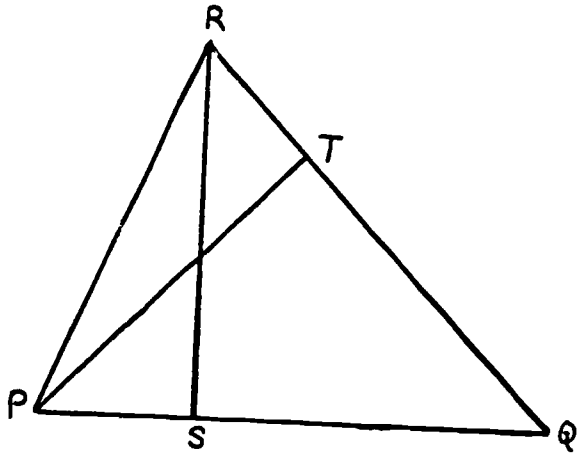
A. 15°
B. 75°
C. 90°
D. 105°
E. None of the above.

24. In the figure shown, it would be possible to find the measure of <G if you knew the measure of :

A. Angles C and F
B. Angles A, B and E
C. Angles A, B, D and F
D. Angles A, B, C and D
E. None of the above.



25. In ΔPQR, $\overline{PT}$ and $\overline{RS}$ are altitudes. PR = 13 units, PS = 5 units and $m\angle Q = 45^{\circ}$. The length of $\overline{PT}$ is :



A. $\frac{17\sqrt{3}}{2}$ units

B. $\frac{17\sqrt{2}}{2}$ units

C. 12 units

D. $12\sqrt{2}$ units

E. None of the above.

26. A right circular cylinder has a radius of 1 inch and a height of 1 inch If the radius is doubled and the height is left unchanged, what happens to the surface area ?

A. It is doubled.

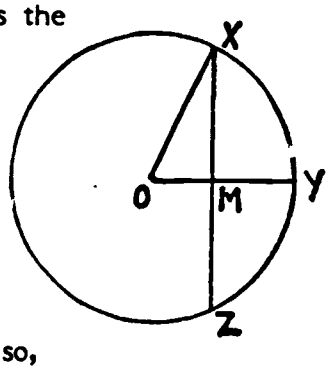
B. It is tripled.

C. It is quadrupled.

D. It is squared.

E. It increases by π sq. units.

27. $\overline{XZ} \perp \overline{OY}$. If OM = 8 units and MY = 9 units, what is the length of $\overline{XZ}$?



A. 15 units

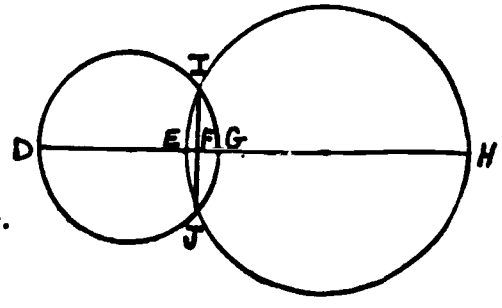
B. $6\sqrt{2}$ units

C. $12\sqrt{6}$ units

D. 30 units

E. None of the above.

28. In the diagram, two circles intersect at I and J. Also, DE = 24 cm., EG = 14 cm., and GH = 32 cm. What is the measure of $\overline{EF}$?



A. 12 cm.

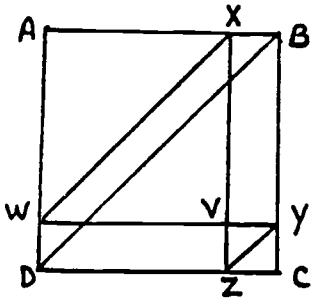
B. 10.5 cm.

C. 7 cm.

D. $\frac{28}{3}$ cm.

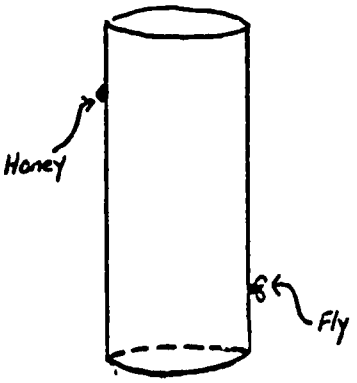
E. None of the above.

29. ABCD and AXVW are squares. BD = 15 mm. and YZ = 4 mm. What is the length of $\overline{WX}$?



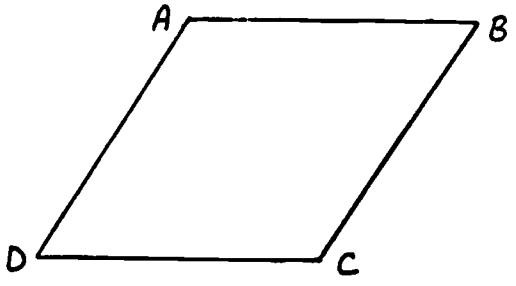
A. $2\sqrt{5}$ mm.
B. 11 mm.
C. $\frac{11\sqrt{2}}{2}$ mm.
D. 24 mm.
E. None of the above.

30. A drinking glass has the shape of a right circular cylinder. Its height is 4 inches and its circumference is 6 inches. On the outside of the glass 1 inch from the top of the glass is a drop of honey. On the outside, but on the opposite side is a fly. The fly is 1 inch from the bottom of the glass. What is the minimum distance the fly must walk in order to reach the honey ?



A. 13 inches
B. $2\sqrt{10}$ inches
C. $2\sqrt{13}$ inches
D. 5 inches
E. None of the above.

31. Quadrilateral ABCD is a rhombus. The length of AB is 5 units and the length of one of its diagonals is 6 units. The area of the rhombus is:



A. 36 sq. units
B. 40 sq. units
C. 48 sq. units
D. 60 sq. units
E. None of the above.

1985

STATE HIGH SCHOOL MATHEMATICS CONTEST

SPONSORED BY THE INDIANA COUNCIL OF TEACHERS OF MATHEMATICS

COMPREHENSIVE TEST

THIS TEST WAS PREPARED BY MEMBERS OF THE
MATHEMATICS FACULTY AT IUPU - FORT WAYNE.

DIRECTIONS FOR TEST:

DO NOT open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each of the 28 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by filling in the circle in the correct place on the separate answer sheet provided. A sample follows:

1. If $x + 2 = 6$, then x equals

   A. 8

   B. 3

   C. $\frac{1}{3}$

   D. 4

   E. none of these

	A	B	C	D	E
1.	①	②	③	●	⑤

The correct answer for the sample is "4", which is answer D; therefore, you should answer this question by filling in the circle D as indicated above.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any question. If you are unable to work any particular problem, it is to your advantage to guess at the answer rather than leave it blank. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin work. When you have finished one page, go on to the next page. The working time for the entire test is 90 minutes.

DIRECTIONS FOR ANSWER SHEET:

Fill in your name in the blanks provided. Above your name write the name of your school and the city where it is located, including zip code.

Along the side of your name indicate your sex and grade by filling in the circle provided. A sample follows:

SAMPLE: Mary A. Brown, who goes to Western High School in Muncie and is in the twelfth grade. would write across the top and fill in along the side.

WESTERN HIGH SCHOOL - MUNCIE, INDIANA 47306

NAME (Last, First, M.I.)														
B	R	O	W	N		M	A	R	Y		A			

SEX
Ⓜ
●

GRADE
⑨
⑩
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●

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO.

1. If a/b represents a lowest terms fraction whose decimal equivalent is repeating (a and b are positive integers), then the number of digits in the repeating cycle is:

   A. a - 1

   B. b - 1

   C. a

   D. b

   E. Not enough information

2. The function whose graph is twice the distance from the x-axis as $y = \sin x$ is:

   A. $y = \sin 2x$

   B. $y = 2 \sin x$

   C. $y = 2 + \sin x$

   D. $y = \sin(x + 2)$

   E. Both A & B

3. In the adjoining figure, $\overline{PQ}$ is a diagonal of the cube. If $\overline{PQ}$ has length a, determine the surface area of the cube.

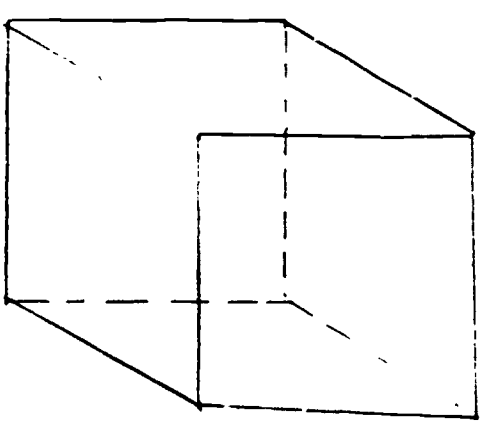
   A. $2a^2$

   B. $2\sqrt{2}\ a^2$

   C. $2\sqrt{3}\ a^2$

   D. $3\sqrt{3}\ a^2$

   E. $6a^2$



4. Find the smallest angle A in the first quadrant for which

$$16\cos^4 A - 16\cos^2 A + 3 = 0.$$

A. 30°

B. 60°

C. 45°

D. 0°

E. 90°

5. What is the next term in the sequence 5, 12, 26, 54, ...

A. 106

B. 134

C. 110

D. 80

E. 92

6. A running track has the shape of a square with a semicircle at each end. If it is a 300 meter track, what is the width of the square?

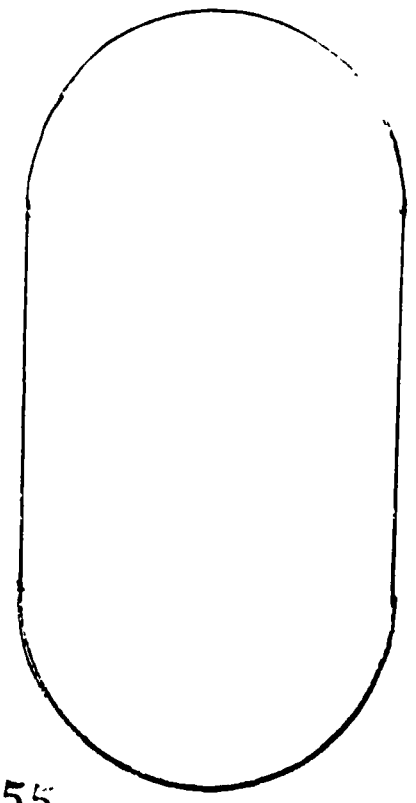
A. $\frac{300}{1 + \pi}$

B. $\frac{10}{\sqrt{\pi}}$

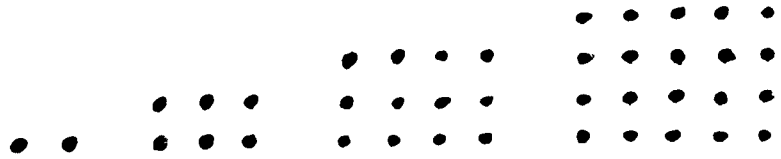
C. $\frac{10\sqrt{3}}{2 + \pi}$

D. $\frac{300}{2 + \pi}$

E. None of the above.



7. The following geometric array suggests a sequence of numbers.



What are the next two numbers in the sequence?

A. 20, 30

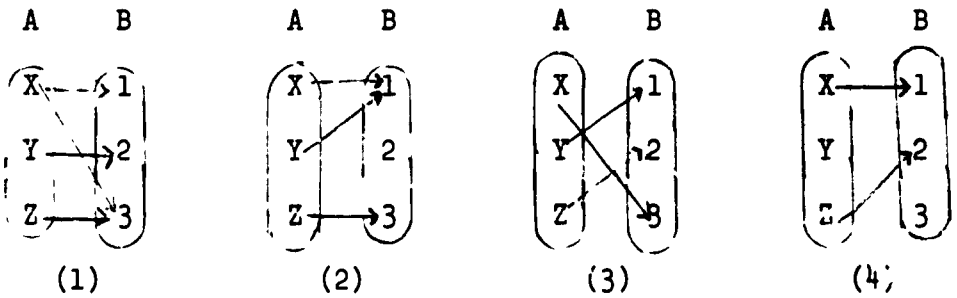
B. 30, 42

C. 42, 56

D. 56, 72

E. 72 90

8. Which of the following define functions from set A to set B?



A. (1) & (3)

B. (2) & (3)

C. (1) & (2)

D. (2) & (4)

E. None of the above.

9. The negation of "Some women are strong." is

   A. No woman is strong.

   B. All women are weak.

   C. Some women are weak.

   D. No woman is weak.

   E. All women are strong.

10. If $\log_{10} 2 = a$ and $\log_{10} 3 = b$, then $\log_5 12$ equals:

   A. $\frac{a + b}{1 + a}$

   B. $\frac{2a + b}{1 + a}$

   C. $\frac{a + 2b}{1 + a}$

   D. $\frac{2a + b}{1 - a}$

   E. $\frac{a + 2b}{1 - a}$

11. If N is an integral square, determine the next consecutive integral square in terms of N.

   A. $N^2 + 1$

   B. $(N + 1)^2$

   C. $N + 2\sqrt{N} + 1$

   D. $N^2 + 2\sqrt{N} + 1$

   E. None of the above.

12. If in a single purchase you are offered 3 successive discounts of 20 percent, 10 percent, and 5 percent in any order you wish, what order should you choose?

   A. 20%, 10%, 5%

   B. 20%, 5%, 10%

   C. 10%, 20%, 5%

   D. The order is irrelevent.

   E. Not enough information.

13. The expression $\frac{1^{4y-1}}{5^{-1} + 3^{-1}}$ is equal to:

A. $\frac{4y - 1}{8}$

B. 8

C. $\frac{15}{2}$

D. $\frac{15}{8}$

E. $\frac{8}{4y - 1}$

14. How many subcommittees composed of 4 Republicans and 3 Democrats can be formed from a committee consisting of 8 Republicans and 5 Democrats?

A. 1700

B. 1740

C. 80

D. 33,600

E. 700

15. Two numbers are such that their difference, their sum, and their product are to one another as 1 : 7 : 24. The product of the two numbers is:

A. 6

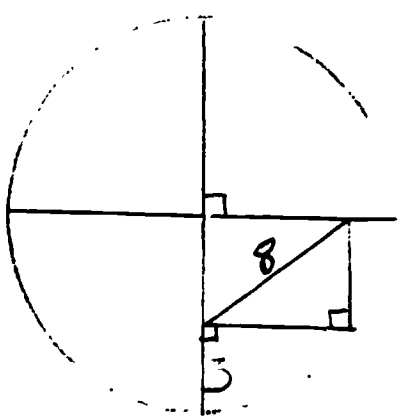
B. 12

C. 24

D. 48

E. 96

16. Determine the radius of the given circle.



A. 8

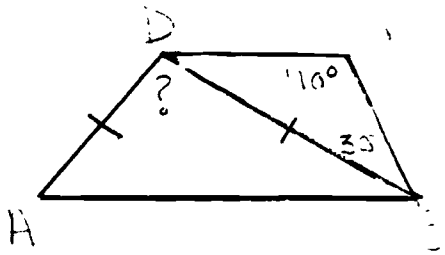
B. $4\sqrt{2}$

C. 5

D. 4

E. $4\sqrt{3}$

17. In trapezoid ABCD, sides $\overline{AB}$ and $\overline{CD}$ are parallel and diagonal $\overline{BD}$ and side $\overline{AD}$ have equal length. If $m\angle DCB = 110°$, and $m\angle CBD = 30°$, find $m\angle ADB$.



A. 80°

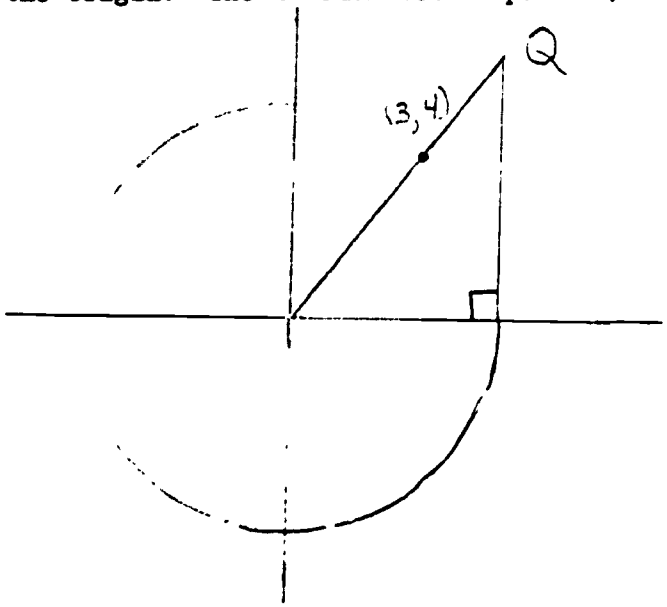
B. 90°

C. 100°

D. 110°

E. 120°

18. C is a circle with center at the origin. The coordinates of point Q are:



A. $(\frac{20}{3}, \frac{20}{3})$

B. $(5, \frac{4}{5})$

C. $(5, \frac{20}{3})$

D. (4,5)

E (6,8)

19. How many four-digit numbers have the same digits as 1990?

A. 24

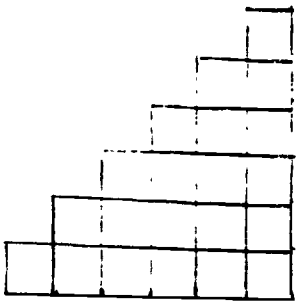
B. 12

C. 9

D. 6

E. 10

20. Molly is building a staircase out of blocks in the pattern shown. How many blocks will it take to build a staircase such that the last stair is 26 blocks high?



A. 351

B. 325

C. 676

D. 338

E. 26

21. If the graphs of $3x - 2y = 5$ and $6x - ay = 7$ do not intersect, then the value of a is :

A. 2

B. -2

C. $\frac{3}{2}$

D. 4

E. -4

22. Given the set of n numbers, $n > 1$, of which one is $1 - \frac{1}{n}$, and all the others are 1. The arithmetic mean of the n numbers is:

A. 1

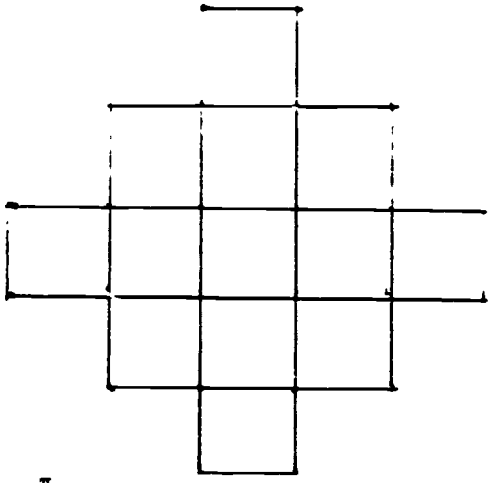
B. $n - \frac{1}{n}$

C. $n - \frac{1}{n^2}$

D. $1 - \frac{1}{n^2}$

E. $1 - \frac{1}{n} - \frac{1}{n^2}$

23. How many different squares are in the following figure?



A. 14

B. 9

C. 18

D. 16

E. 12

24. If $\sin \alpha = \frac{-5}{13}$ for $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, then $\sin 2\alpha$

A. $\frac{-10}{13}$

B. $\frac{-120}{13}$

C. $\frac{120}{169}$

D. $\frac{-120}{169}$

E. $\frac{-120}{130}$

25. The largest x coordinate of any point on the circle with center C(2,3) tangent to the line $y = x - 3$ is

A. $2\sqrt{2}$

B. 4

C. $4\sqrt{2}$

D. $2(\sqrt{2} + 1)$

E. 8

26. It takes a person ninety seconds to walk up an inoperative escalator. If the person stands still on the escalator when it is moving the trip takes sixty seconds. How long will the trip take if the person walks up the moving escalator?

A. 36

B. 75

C. 45

D. 48

E. Not enough information.

27. If an icosahedron is numbered like a die, with opposite faces summing to a constant what would the constant be?

A. 21

B. 7

C. 13

D. 12

E. Not enough information.

28. The sides of a trapezoid are one, two, three, and four centimeters long. What is its area?

A. 8

B. $\frac{6\sqrt{2}}{5}$

C. $\frac{10\sqrt{3}}{5}$

D. $\frac{10\sqrt{2}}{3}$

E. $\frac{3\sqrt{2}}{10}$