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ABSTRACT

Noting the growing interest in how writing activities might complement current teaching techniques and improve learning in mathematics, this paper presents a progress report on the use of writing assignments in freshman precalculus courses at a Minnesota university. The paper first presents a rationale for this teaching technique and the original goals of the project, and then provides examples of students' inability to write an effective definition of a reference angle (although they were able to compute correctly a reference angle problem), indicating a lack of verbal strategies. The paper points out that one of the most useful techniques in the process of working with the students was showing the class writing efforts on slides. It then provides another example of poor verbal strategies, noting how writing can improve requisite understanding of mathematical concepts. The paper concludes with some observations drawn from one year's efforts at using writing assignments in mathematics classes: the assignments (1) improve class participation, (2) broaden opportunities for conceptual growth even for better students in the course, (3) stimulate meaningful discussions of mathematics as a language and of strategies for learning it, (4) emphasize the kinds of writing given less attention in orthodox content area writing programs, and (5) force one to consider curriculum concerns. (HTH)

WRITING AND LEARNING COLLEGE MATHEMATICS

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Models for Excellence--
Writing and Learning Across the Curriculum

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Introduction

In most college curricula, few subjects of study are more broadly separated in the minds of teachers and students than mathematics and writing. As disciplines, mathematics and English have very few touching points, and many practitioners in each feel profound insecurities and anxieties about the other's subject matter. Furthermore, mathematics has traditionally been taught either by lecture or through programmed sequences of problems, or by some combination of the two, and this system has never found much use for the elaborateness of the discursive and even explorative form of writing activity. However, with the growth of the idea that writing can be an effective instrument for learning, and particularly as a result of cross-curriculum writing workshops, there is a growing interest in how writing activities might complement present teaching techniques and improve learning in mathematics. In fact, writing assignments may address in a unique way the main problems facing the mathematics teacher: helping students to read texts, teaching them how to learn a new concept and recognize when they have understood it, and encouraging students to learn in such a way that they retain the material for the next courses in the mathematics sequence.

This paper is a progress report on an effort to use writing assignments in freshman-level pre-calculus courses to improve the learning situation in those classes. The aims of using writing assignments in these mathematics classes were originally defined as follows:

1) to facilitate their learning and retention of basic concepts;
2) to give the students some experience in various types of writing relative to mathematics: namely, on the spot and overnight summaries, reaction journals, expressive writing relative to their feelings toward math and their problems in learning mathematics, problem-definition and description, and demonstrations of comprehension in tests, etc. [for examples, see appendix],
3) to provide the instructor with feedback concerning students' grasp of and facility with those concepts in a way that would facilitate our interaction as teacher and student.

The idea must be stressed that writing here is not being used to introduce students to writing in the style of journals, but as a tool for use in learning. In the process of this experimentation, students' control of grammar and mechanics was generally so closely linked with their ability to use logic, that grading on that basis never became an issue.

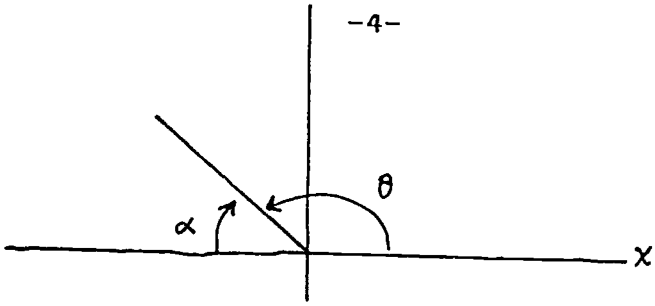
The present discussion concerns two particularly crucial issues that received special emphasis in this session--the importance of assignments that translate visual identification into verbal description, and of assignments that stimulate the growth of skill in explicitly algorithmic thinking and procedures.

Why use Writing in teaching mathematics?

Present day mathematics teaching fails to address such translation problems in common syllabi and materials. Most mathematics courses describe procedures and then provide many practice problems which require an understanding of those procedures to carry out. The student will, in many if not most cases, have to explain the procedure to him/herself and evaluate that explanation and refine it in order to learn the procedure, but rarely, if ever, is there much opportunity for student-teacher interaction at that crucial stage of learning. As a result, mathematics seems to be a foreign language in an unfriendly country into which the students are thrust as tourists.

What is a Reference Angle?

This was a writing question, a definition, given on a trigonometry test. A reasonably accurate answer is, "An acute angle, α, which is formed by the x-axis and the terminal side of a given angle, θ."



From that reasonably clear statement, students slide very far indeed, despite the fact that they were all able to compute the reference angle in the next question. When having to represent the concept verbally, however, often extraordinary kinds of confusion get in the way. Some describe in terms of an example:

"An angle like if you have 400 degrees on the unit circle your reference angle would be 40 degrees."

or:

"A reference angle is 30°, 215°, 60°."

Others couldn't combine the minimal necessary terms and conditions:

"An angle with its terminal side on the x-axis."

or,

"An angle that is closest to the x-axis."

"A complementary angle which refers to the x-axis."

"The acute angle with the same terminal side."

"An angle that is acute. We use to simplify."(followed by a visual example)

Then there are answers that show the writers reaching for terms they don't control:

"The non-negative angle less than 360° that is equivalent to the initial angle."

"The angle in the first quadrant that has the same positive trignometric values as the angle ti at is given."

"An angle for which measurements of less than 90 degrees are made in relationship to the x-axis only."

"The angle that has to measure up supplementary with the angle given that tells which quadrant it is in."

"Another angle that is the same as your first angle but in simpler terms."

"The angle that is smaller than the original that has an equivalent trig function."

"An angle whose functions are the same within the first quadrant; may be + or -."

The last two approach trying to define by function rather than form, as does the following effort:

"An acute angle that is coterminal to another angle which is used to determine the trigonometric functions of the angle because they are the same as the acute angle."

One finds with such writing questions that relatively few answers are entirely wrong: the problem is learning verbal strategies of sufficiency. One need only imagine math students with their understanding reflected in such imprecise statements to see the importance of working with the students on their own verbal representations, by reacting, encouraging individual or group revising, etc. Such verbal imprecision indicates tourism in the subject at a pretty abject level. One of the most useful techniques in the process of working with the students was showing the class efforts on slides. Most students were amazed at the defitions the others produced, but very often required a lot of persuading to see the incompleteness of their own attempts.

How to Compute the Determinant of a Matrix

One assignment in a pre-business course was to answer this question for an educated lay-person. A matrix, for the uninitiated, is a grid of rows and columns of numbers, called elements. The elements of the matrix are subject to various kinds of computations, one of which is to produce, through a series of sums of products of elements, the "determinant of the matrix". What this question brought to the surface was that even when students have the ability to carry out the algorithmic, that is, formulaic directions, they have considerable difficulty articulating the explicit steps. Mathematicians know that facility in computation often precedes a complete undestanding of a problem: the eighteenth century grand mathematician Euler said this when he said that his pencil was faster than he was.

But algorithmic thinking must be considered the backbone of mathematics. And whether a student will go on to a higher course requiring proofs, or a computer programming course, or will simply ever develop the ability to read a textbook or succeed with a "word problem" or learn from a lecture or ask a question, the ability to talk about a process is a crucial skill that needs to be encouraged and developed. Unfortunately, as the short answer question has replaced the essay in humanities courses, the education of the math student in the verbalization of this process is neglected until the math teacher is faced with "hopeless" students.

In fact, none of the students who were asked this question were able to lay out the procedure with the degree of explicitness that would have allowed them to program it on the computer. The first response below is basically correct, but its vagueness indicates unsureness on the part of the student, probably as to the process, but certainly as to what an algorithmic explanation is.

1) evaluate for the row with the most zeroes

2) put in + and - signs--start with +,- across the top row.

3) evaluate for the first term: find the determinate of the smaller matrix.

4) evaluate for the next. Find the determinate of the next matrix.

5) write equation out.

6) equals=

A motive for asking the question was the hope that the students could reconstruct the precision of their text in their own words (and perhaps even be inspired to complain less about it). However, this response is simply a set of partial directions, possibly useful as a reminder after the procedure has already been learned, but certainly inadequate as an explanation on its own, with the cryptic statements, #5 and #6, and the recursiveness of the definition in the other steps. One may have difficulty believing that a student can perform the steps without being able to explain them, but this is commonly the case, and makes obvious the need for the instructor to work on helping the student get beyond performance to understanding, and to see the need for doing so.

Some other student responses were worse. Imagine a non-mathematical layman trying to make sense of this attempt, from a conscientious but overwhelmed student, who profoundly disliked mathematics, although this course was required for her major.

1) --determinates are denoted by $|A|$, and A determinees what determinate.

2) --do not use the whole expansion, use cofactors: which is just telling us what sigh we give it. A posotive or negetive, by (-1) times its minor. . .

For a student to assume that this could be an adequate explanation indicates a habitual inability to grasp the notion adequacy in explanation, and the widespread existence of such inability should make clear the need for writing exercises of this sort as a basic part of any mathematics program.

Conclusion

There are some more general but tentative conclusions we have drawn from one year's effort with writing assignments that are worth sharing.

1) They improve class participation,

2) They create substantial hurdles for even the good students and thus broaden opportunities for conceptual growth even for the better students in a course,

3) They stimulate meaningful discussions of mathematics as a language and of strategies for learning it. They break down the fear-barriers of math as a "special", sacred language isolated from the "real" world.

4) One is lead to emphasize kinds of writing given less emphasis in orthodox writing-across-the-curriculum projects. Research projects and journal assignments have

been unwieldy in the ten-week college-course framework. The most useful assignments have been those that get at the students' grasp of concepts on the spot in class and overnight, and those that anticipate material to be covered the next day. Having the students write to other specific audiences than the teacher forces them to write more explicitly and to grasp the need for explicitness.

5) Above all, they force one to consider curriculum implications, particularly the way the difficulties with writing in mathematics illuminate a basic problem in the design of the curriculum for teaching mathematics in college where departmental controls and tight schedules provide little room for the kind of discursive trial-and-error approach to learning that writing assignments can make available in the mathematics class.

APPENDIX

TYPES OF PROJECTS WITH EXAMPLES

1.) Expressive Writing
--What is my math background, how do I feel about my experiences with math? What are my career goals? What is this course likely to be about?

2.) Summaries: to see where the class stands, an on-the-spot summary, or an overnight summary or an anticipatory summary
--Right now, what is your notion of a "radian" and why do we use them?
--What's wrong with this logic? (example follows in a reducing fractions problem)
--In your own words, explain how to factor a second degree polynomial.
--Tomorrow we will talk about the absolute value. In your own words, give a mathematical definition that does not use the word "positive".

3.) Journals (This was not a successful assignment)

4.) Visual Image Translation
--Discuss this graph, so that I would recognize it from the discussion alone, or over the telephone.

5.) Synopsize/organize tactics to use when solving a problem that help you.
--What is the difference between combinations and permutations and how do you tell them apart?
--When do you use various factoring procedures? What clues help you?
--Give a procedure for solving a distance-time-rate problem.

6.) Set forth a definition
--What would be a good definition of "degree of an angle"?

7.) Experiment with technical style
--Rewrite page 12 of the textbook, as you would like it to have read.

8.) Set up a series of steps--algorithms.

--Describe a guaranteed procedure for successfully computing square roots.

9.) Communicate your thoughts to a specific audience.
--Write to a boss of a company you are applying to. He doesn't like math, doesn't know any, but you--a student of Math XXX have the power to solve problem #31, which he needs the solution to. Be brief--he's impatient--but clear, and don't "wing it".

10.) Invent a problem
--You live in a family with 3 sisters and two brothers. Invent 2 combinations and 2 permutations problems. Example: In how many ways can we sit at the table? Provide answers, and explain how you recognized whether the problem was a combination or permutations one.