DOCUMENT RESUME

ED 264 138 SE 046 303

TITLE	Problem Solving Challenge for Mathematics. 1985 Junior High School Curriculum.
INSTITUTION	Alberta Dept. of Education, Edmonton.
PUB DATE	85
NOTE	57p.
PUB TYPE	Guides - Classroom Use - Guides (For Teachers) (052)
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	Junior High Schools; Mathematics Education; *Mathematics Instruction; *Problem Sets; *Problem Solving; *Secondary School Mathematics; *Teaching Methods
IDENTIFIERS	*Alberta

ABSTRACT

This document is designed to assist teachers in helping students in further development of problem-solving skills. It consists of: a statement of purpose; an introduction (noting the place of problem-solving in junior high school mathematics curricula); a definition of problem-solving; a four-stage general framework for solving problems (which includes understanding the problem, developing a plan, carrying out the plan, and looking back); a list of strategies for each of the four stages; and ways to evaluate problem-solving. The major portion of the document consists of: (1) six sample problems which show how the strategies in the problem-solving model may be applied instructionally; (2) classroom problems, organized separately foı grades 7, 8, and 9; (3) computer problems, which also use the steps in the four-stage model; and (4) challenge problems related to number systems, ratio and proportion, measurement, geometry, and algebra. Answers to the problems are provided. A bibliography and (in appendices) a framework for multiple-choice tests and methods for evaluating problem-solving performance are included. (JN)

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1985 JUNIOR HIGH SCHOOL

Curriculum Alberta

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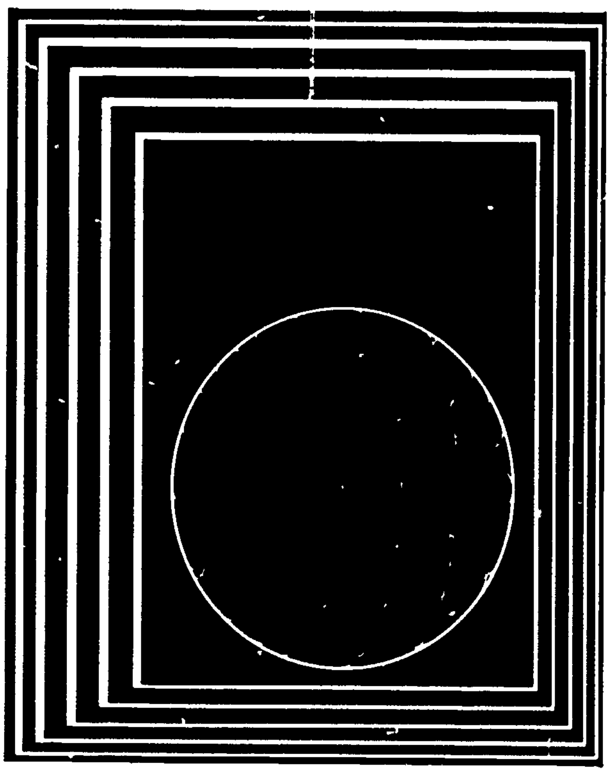

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PROBLEM SOLVING CHALLENGE FOR MATHEMATICS

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ACKNOWLEDGEMENTS

Alberta Education acknowledges with appreciation the contributions of the following members of the Problem Solving Monograph Committee operating under the direction of the Mathematics Curriculum Coordinating Committee and Curriculum Policies Committee and to Joan Worth for reviewing the manuscript.

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To Irene Sonnenberg
for her diligence in typing
this document.

To Dr. Joan Worth, Ron Commaert
for their assistance in reviewing
the document.

TABLE OF CONTENTS

PURPOSE

The monograph provides background information. ideas and sample activities for teachers in their efforts to assist students in further development of problem solving skills. The content provided in this monograph is a continuation of the problem solving model and related skills outlined in the elementary edition. LET PROBLEM SOLVING BE THE FOCUS FOR 1980's.

This monograph on problem solving should be directly helpful to you in answering five questions.

1. What is the place of problem solving in junior high mathematics?
2. What is a useful model for problem solving and what are the related skills and strategies?
3. How can one teach problem solving strategies to junior high school students?
4. How can problems be integrated with particular mathematical topics of the junior high curriculum?
5. How can one evaluate student performance with respect to problem solving?
6. What are some sources of problem solving activities?

INTRODUCTION

"Problem solving. the ability to reason and apply mathematics in problem situations is considered an integral part of the basic skills required for mathematical literacy. The ability to solve problems increases with importance in light of the rapidly changing demands of today's technological society. Mathematics plays an important role in developing within each student the problem solving skills that will serve throughout life."[1]

Thus. one of the major goals of junior high school mathematics is to provide for the continuing development of problem solving skills and strategies. Using the framework of Polya's model. as outlined in this monograph. will help provide students with ways for organizing their efforts. This framework is intended to help teachers and students view problem solving as a process that consists of several interrelated stages of action that lead toward the solution of a problem.

Problem solving must not be viewed as an isolated activity. Use of the suggested model and related skills should be incorporated into a teaching philosophy which makes problem solving an integral part of the mathematics program. Many of the problems illustrated in this monograph show how problem solving skills are learned and utilized in understanding mathematical concepts.

The teacher's role is to challenge students to think critically. to foster interesting ideas. and to serve as a facilitator in developing problem solving skills. The teacher must create a positive climate. conducive to problem solving. The teacher's own enthusiasm and capability to recognize willingness and perseverance on the part of the student are significant factors in the success in developing these strategies. The teacher must be supportive and encourage risk taking on the part of students. Teachers must encourage creative problem solving approaches and be willing to accept unconventional solutions.

Junior high school students should begin to assume more responsibility for their problem solving effort. expanding their tolerance for frustration and looking to themselves and to each other for interpretation and evaluation of results. At this level active hypothesis development. conjecture testing. generalizing.

inductive and deductive reasoning and independent learning should be stressed.

The problem solving methodology suggested in the monograph should help students feel a sense of involvement and satisfaction. as their input helps shape the mathematical experiences they encounter. Through this increased participation students should be better motivated to study additional topics. while increasing the number of opportunities to enjoy personal fulfillment.

[1] Elementary Mathematics Curriculum Guide 1982

WHAT IS A PROBLEM?

Some problems have solutions that are readily obvious to the students because of their previous knowledge and experience. For other problem types. the solution is not immediately evident and requires the testing and application of one or more strategies.

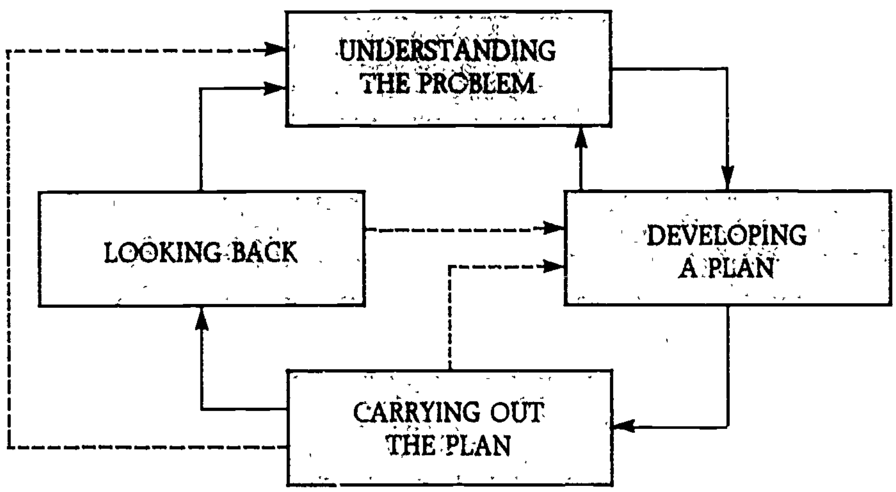
A problem involves a situation in which an individual or group is required to perform a task for which no immediate method of solution is evident.

A GENERAL FRAMEWORK FOR PROBLEM SOLVING

There is no SINGLE best strategy or method for either solving problems or teaching problem solving. However. within a general framework there are strategies that can be learned which will improve the problem solving abilities of both children and adults. Research does indicate that problem solving performance is enhanced when students are taught to use a VARIETY of strategies or heuristics. both general and specific.

Students require an overall framework for solving problems and a repertoire of strategies and skills that may be used within that structure. Several approaches and techniques have been proposed for problem solving. Polya's model provides the basis for the problem solving framework recommended in the junior high mathematics program of study.

The framework consists of four stages:



The framework should not be interpreted as a fixed sequence of stages and strategies that must be rigidly followed. Nor are the stages discrete or inseparable: their use will depend on the problem and on the individual student. Students may not always use all the stages. nor use them in a set order. The inherent flexibility of this framework allows it to be applied effectively to a wide variety of problem solving situations.

STRATEGIES FOR PROBLEM SOLVING

How does one teach problem solving strategies? Problem solving strategies are best learned by doing. Since there are many strategies and skills of problem solving it is best to look at the six sample problems as illustrative of different ways to teach problem solving strategies.

In teaching "Understanding the Problem" it is useful to teach students to read carefully. As suggested, one can have students underline important parts of a problem statement, rewrite it in short sentences, have students actually act out the problem, draw pictures of problems and write other problems that they think are like the given problems.

In teaching "Developing a Plan" and "Carrying Out the Plan", a junior high teacher should be aware of and help students be aware of the mathematical ideas which they can use in a problem.

In teaching "Looking Back", help students check the reasonableness of their answers, carefully identify the steps or computations they did, and write problems which are alike, easier, more difficult or different from the given problem.

A key to teaching problem solving is appropriate problem selection. Teachers should choose a problem that fits with the mathematics being taught and the background of their students.

On a general level, the following should be stressed:

1. Several strategies may be tried before a suitable one is found.
2. There may be more than one solution to a given problem.
3. Sometimes one must back away from a problem for a while in order to get a fresh look before proceeding.

1. [illegible]

The major purpose of this stage is to involve the students in thinking about the problem before attempting a solution. Initially, the teacher asks questions and suggests strategies that will focus attention on the information and conditions in the problem.

Strategies in this stage of the process include:

- Using concrete materials (manipulatives)
- Interpreting pictures, diagrams, charts and graphs
- Looking for patterns
- Identifying key words
- Simulating situations
- Drawing diagrams
- Restating the problem in your own words (internalize the problem)
- Asking relevant questions
- Identifying wanted, given and needed data
- Identifying extraneous information
- Considering alternative interpretations

2. DEVELOPING A PLAN (CHOOSING A STRATEGY)

This is the planning stage during which students consider and choose strategies for solving the problem. Students should be encouraged to choose alternative methods of solving problems. It is important that students consider and use strategies other than computation, and learn to accept these as legitimate problem solving strategies.

Strategies to be considered include:

- Acting or simulating situations
- Experimenting through use of manipulatives
- Collecting and organizing information (charts, graphs)
- Applying patterns
- Choosing and applying the appropriate operations
- Formulating an equation
- Guessing and checking - Identifying and applying relationships
- Making diagrams and models
- Using a simpler problem
- Using logic or reason
- Constructing flow charts
- Breaking problems into smaller parts
- Working backwards
- Developing a symbol or code system

This stage is very closely related to "developing a plan", but is listed separately in order to highlight the importance of the prior planning stage. Too often the whole focus of problem solving has been on this third stage, with emphasis on computing to get the right answer. In the general framework the "carrying out the plan" stage is merely doing what was planned in stage two.

Strategies in this stage of the process include:

- Acting it out
- Using manipulatives
- Collecting and organizing information (charts, graphs)
- Applying patterns
- Choosing and performing the appropriate operation
- Writing and solving a number sentence
- Guessing and checking
- Identifying and applying relationships
- Making diagrams and models
- Using a simpler problem
- Using logic or reason
- Executing flow charts
- Selecting appropriate symbols or notations
- Working backwards
- Accounting for all possibilities
- Recognizing limits and/or eliminating possibilities
- Looking at problems from varying points of view

This stage encourages the student to assess the effectiveness of the solution process. Students should learn to relate their answers to the question in the problem as one way of verifying that they have indeed solved the problem. Reflecting on the plans made and evaluating the strategies used assist students to become aware of the appropriateness of different strategies for a particular problem. This stage helps students to think through problems and to generalize the process for new situations.

Strategies in the stage of the process include:

- Stating an answer to the problem
- Restating the problem with the answer
- Checking the answer
- Determining the reasonableness of the answer
- Explaining the answer
- Reviewing the solution process
- Considering the possibility of other answers
- Looking for alternative ways to solve the problem
- Making and solving similar problems
- Generalizing solutions
- Documenting the process

EVALUATION OF PROBLEM SOLVING

Presently, reliable paper and pencil tests for measuring problem solving abilities are not available. Some standardized tests do include a measure of some problem solving skills. however. little evidence can be provided on how students "play the game" of problem solving. The evaluation of problem solving growth lies with the analysis procedures and the observation techniques of mathematics teachers.

The evaluation of problem solving performance is not synonymous with grading problem solving. Recognition and subsequent reward by a teacher for selecting appropriate strategies, willingness, and perseverance are key components of the evaluation component for problem solving.

Ideal evaluation of students' problem solving performance is best achieved by talking to the students individually while they are solving a problem. Since this is not possible in most cases, it is possible to obtain an accurate assessment of a student's problem solving performance through the combined use of two evaluation practices: (1) analysis of written work on a problem, and (2) observing and questioning students while they are working in a class situation.

1. Analysis of Written Work

It is important to understand how students are attempting problems and, in particular, which parts of the problem solving process are causing difficulty. Even though students get correct answers, the strategies should be analyzed to avoid the use of incorrect or inefficient procedures.

Figure 1 and Figure 2 illustrate two types of rating scales for evaluation. The scale in Figure 2 is specifically used when evaluating particular skills. The scale in Figure 1 is best used for evaluation of the complete four-step process.

FIGURE 1

A Point System for Analyzing Student Work

Understanding the Problem
- 0 completely misinterprets the problem
- 1 interprets part of the problem
- 2 complete understanding of the problem

Choosing and Implementing Strategies
- 0 no attempt
- 1 partly correct strategy
- 2 a strategy that should lead to a correct solution

Getting the Answer
- 0 no answer or completely inappropriate
- 1 copying error, computational error or partial answer
- 2 correct solution

FIGURE 2

Problem Solving Rating Scale

Indicators	Rating
Understanding the Problem	
–	
– Strategies (List strategies	1 2 3 4 5
– developed)	- - - - -
–	
Developing a Plan	
–	
– Strategies (List strategies	1 2 3 4 5
– developed)	- - - - -
–	
Carrying Out the Plan	
–	
– Strategies (List strategies	1 2 3 4 5
– developed)	- - - - -
–	
Looking Back	
–	
– Strategies (List strategies	1 2 3 4 5
– developed)	- - - - -
–	

Score ____________

2. Observing and Questioning Students

Several of the strategies for problem solving are assessed best by careful observation and questioning of students while they are involved in solving problems. Informal evaluative comments can be directed to students as they work, or observations can be recorded and shared later at parent or student conferences, for example.

A problem solving record form such as Figure 3 may be used for recording observations. A more structured observation schedule as in Figure 4 may be used. The important point is that observational data should be included in the evaluation of students' problem solving performance.*

*Charles Randall "Evaluation and Problem Solving" Arithmatic Teacher, January 1983 pp 6-7, 54.

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FIGURE 3

Recorded Anecdotal Observations

Wayne Gretzky

Nov. 4 Neat notebook — Outlines strategies well — Develops own problems

Jan. 15 Very willing to try problems and assist others

June 6 Learned all strategies to date — Helps others well

Dave Hunter

Dec. 4 Tries hard but has trouble selecting a strategy

Feb. 5 Knows how to complete tables

April 6 Continues to forget to check work

May 7 Getting better at suggesting solution strategies

FIGURE 4

Problem Solving Observation Checklist

Student: Dave Semenko

Date: November 14

	Frequently	Sometimes	Never
1. Selects appropriate solution strategies	___	___	___
2. Accurately implements solution strategies	___	___	___
3. Tries different solution strategies when stuck	___	___	___
4. Approaches problems in a systematic manner	___	___	___
5. Shows a willingness to try problems	___	___	___
6. Demonstrates self-confidence	___	___	___

PROBLEMS GRADES 7 TO 9

1. WHAT IS A PROBLEM?

A problem involves a situation in which one individual or group is required to perform a task and for which no immediate method of solution is evident. Problems that are of the traditional textbook variety are usually single task or computation oriented. Problems of this kind generally involve single calculations (i.e. the addition, subtraction, multiplication or division of the perceived values given in the problem). These "number crunching" problems are usually obvious to the student and are uni-dimensional in the skills development. Non-routine problems require multi-step processes and the testing and application of two or more strategies before the solution process is complete. A balance in the variety of problems present is necessary.

Good mathematical problems should include some of the following characteristics:[2]

(1) they involve mathematics in some way.
(2) they are of interest to the student.
(3) they require the student to interpret and modify the solution process if necessary.
(4) they allow for several methods of solution.
(5) they allow the student to feel that he or she wants to and can solve the problem.

There are many strategies and skills which may be brought to bear on any particular problem. The following six problems are illustrative of different approaches which may be taken to develop a comprehensive repertoire of problem solving skills.

[2] NCTM 35th Year Book

INTRODUCTION

Six problems have been selected to illustrate how the strategies in the problem solving model may be applied instructionally. The strategies vary from each of the problems given and are presented with a number of related questions and activities that will assist the teacher in working a problem through the four problem solving steps. Teachers are encouraged to use the problems as a springboard for introducing problem solving activities in their classroom.

2. SAMPLE PROBLEMS

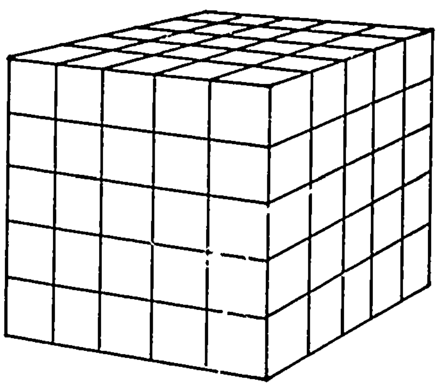
A wooden cube that measures ten centimetres along each edge is painted red. The painted cube is then cut into two-centimetre cubes. How many of the two-centimetre cubes do not have red paint on any face?

GRADE: Seven

STRAND: Geometry

UNDERSTANDING THE PROBLEM

- **Drawing a Diagram**



Initially the students should discuss the problem to demonstrate understanding of what is required. Students should draw the cube and draw the 2-cm cubes.

QUESTIONS:

Where are the cubes with three faces painted red?

Where are the cubes with two faces painted red?

Where are the cubes with one face painted red?

Where are the cubes with no face painted red?

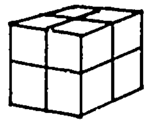
DEVELOPING A PLAN

- **To Develop a Pattern**

3 faces painted – found at the corners

There are 8 corners in a cube. There are 8 cubes with 3 faces painted red.

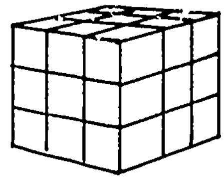
8 cubes



3 faces red – corners	8
2 faces red – edges	12
1 face red – face	6
0 faces red	27 - 26 = 1

One layer removed from each face
1 x 1 x 1 = 1

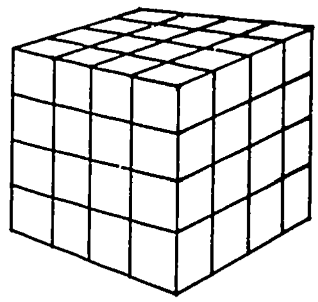
27 cubes



3 faces red – corners	= 8
2 faces red – edges 2 x 12	= 24
1 face red – faces 4 x 6	= 24
0 faces red	56

One layer removed
2 x 2 x 2 = 8

64 cubes



CARRYING OUT THE PLAN

- **Applying Patterns Developed to Solve the Problem**

Total number of cubes = 125

3 faces red – corners	= 8
2 faces red – edges 3 x 12	= 36
1 face red – faces 9 x 6	= 54

0 faces red – one layer removed
3 x 3 x 3 = 27

LOOKING BACK

- **Determining the Reasonableness of Answer**
- **Alternate Method**

There are 27 unpainted cubes.

Reasonableness of answer:

3 red faces	8
2 red faces	36
1 red face	54
0 red faces	27
Total =	125

There are 5 x 5 x 5 = 125 cubes.

All cubes have been accounted for.

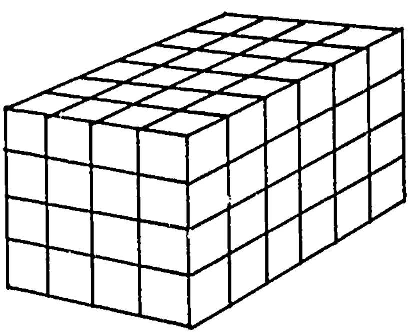
Note:
An alternate method of solving this problem would be to have the student remove outer layers using manipulatives.

EXTENSION:

If this rectangular solid is painted red and cut into cubes, how many cubes will have 3 faces painted red, 2 faces painted red, 1 face painted red, and 0 faces painted red?

ANSWER:

3 – 8
2 – 32
1 – 40
0 – 16



PROBLEM TWO

Janice went to a store, spent half of her money, and then spent $10 more. She went to a second store, spent half of her remaining money, and then spent $10 more. Then she had no money left. How much money did she have in the beginning wnen she went to the first store?

GRADE: Eight

STRAND: Number Systems

UNDERSTANDING THE PROBLEM

- **Acting It Out**
- **Simulating Situations**

Suppose you had $80 to start with.
At the first store:

you spent ½ (80) + 10	= $50
∴ you have 80 - 50	= $30 left

At the second store:

you spent ½ (30) + 10	= $25
∴ you have 30 - 25	= $5 left

DEVELOPING A PLAN

- **To Work Backwards**

CARRYING OUT THE PLAN

- **Working Backwards**

Janice had $10 before her last purchase in the second store. This is half of the money she had when she entered the second store, so she had $20 when she entered. In the first store she had $10 more than this, or $30, before she made her final purchase. But $30 is half of the money she had when she entered the first store, so she had $60 when she entered.

LOOKING BACK

- Stating an Answer to the Problem
- Restating the Problem with the Answer

She had $60 when she entered the first store.

She had $60.
She spent ½ (60) + 10 = $40 at the first store.
She had $20 left.
She spent ½ (20) + 10 at the second store.
She had 20 - 20 = 0 money left.

EXTENSION:

1. Susan delivers papers every day except Sunday. Every Saturday she collects $24.00. If each paper costs 25¢, how many customers does she have?

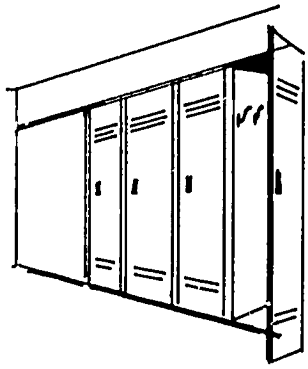
   ANSWER:
   16 customers

2. Three brothers – Jim, Mike and Eric – after finishing a meal in a restaurant, ordered a bowl of stewed prunes. While waiting for the prunes to be served, all three fell asleep. After a while, Jim woke up and found the prunes on the table. He ate his equal share and went back to sleep. Then Mike awoke, ate what he thought was his equal share of the remaining prunes, and went back to sleep. A little while later, all three brothers woke up and discovered that 8 prunes were left in the bowl. How many prunes were in the bowl originally?

   ANSWER:
   18 prunes

PROBLEM THREE

A school is built with only one long narrow hallway, but it is a very long hallway. There are 1000 lockers on one side of this hallway. On the first day of school, each student must leave the office and go down this long hallway to get out of the school. As the first student leaves she closes every locker door on her way out. The second student to leave the office opens every second locker door on his way out. The third student changes the state of every third door on her way out, that is, if a door that she is going to change is open she closes it and if it is closed she opens it. The fourth student changes the state of every fourth door, and so on, until the thousandth student leaves, changing the state of the thousandth door. The principal leaves the office and notices that some of the doors are open and some of them are closed. How many locker doors are closed and which ones are they?



GRADES: 7, 8, & 9

STRAND: Various

UNDERSTANDING THE PROBLEM

- Restating the Problem in Your Own Words
- Drawing Diagrams

Suggestions:
- Restatement of the problem in words with which each student is comfortable is necessary in order to make the problem personal for each student. This personal feeling toward the problem is very helpful and motivating especially for further use of the problem.
- A pictorial diagram will assist the student in understanding what is being done in the problem.

DEVELOPING A PLAN

- To Act or Simulate Situations

Suggestions:
- Students may suggest a model illustrating 1000 lockers and 1000 students to try working through the problem.
- Students may try a smaller manageable problem of say 20 lockers.
- Students may set this problem up in chart form.
- Students could experiment outside the classroom in the hallway.
- Students may wish to set up a concrete manipulative model. This would be very beneficial for those students still in the concrete stage of learning.

CARRYING OUT THE PLAN

- Making a Chart to Help Solve a Problem

Suggestions:

- It is important that the students realize that making a chart is helpful in solving this problem. It gives a record that can be read easily of what is happening as students pass down the hall opening and closing locker doors. One possibility for such a chart is shown below.

Locker Number

Student	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
2		O		O		O		O		O		O		O		O		O		O
3			O			C			O			C			O			C		
4				C				C				O				C				C
5					O					C					C					O
6						O						C						O		
7							O							C						
8								O								O				
9									C											C
10										O										
11											O									
12												O								
13													O							
14														O						
15															O					
16																C				
17																	O			
18																		O		
19																			O	
20																				O

Certain patterns begin to emerge as the chart is being developed. If it is done as a class project some students get excited about predicting the next door to be closed before it is entered on the chart.

LOOKING BACK

- Reviewing the Solution Process

Suggestions:

- It is a good idea to discuss some of the steps that the students have been using in solving the problems. Looking back over what they have done can help to reinforce some of the skills that may be useful in solving other problems. The careful restatement of the problem. the list of possible ways of solving the problem. the value of recording information in an organized fashion thus allowing us to see new patterns that may be useful. are all important steps that have been used here. Asking questions about why our answer is like it is gives us new and useful information. Although the answer to our original question is important it should be clear that the ensuing discussion and thinking are of at least as much value.

ANSWER:

Lockers closed: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625, 676, 729, 784, 841, 900, 961.

EXTENSION:

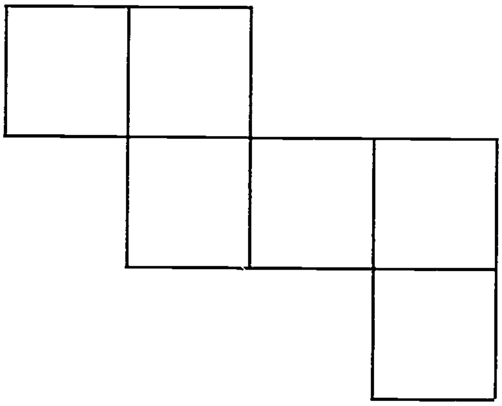
If 1000 students change 1000 lockers using the same procedure. how many times will locker 432 be changed?

ANSWER:

20 times

PROBLEM FOUR

This figure consists of six congruent squares and it has a total area of 294 cm^2. Find the perimeter of the figure.



UNDERSTANDING THE PROBLEM

- **Identifying Wanted, Given and Needed Information**
- **Interpreting Pictures**
- **Using Manipulatives**

Suggestions:
- The teacher may ask a series of questions about shapes, equivalent sides and meaning of area and perimeter.
- Discussion should include information on what is wanted, what is needed and what is given.

DEVELOPING A PLAN

- **To Identify and Apply Relationships**

Suggestions:
- Develop a plan whereby the student realizes that the whole is made up of smaller and equal parts
- Further indicate that to find the measure of one side of a small square we need to find the area of one small square.

CARRYING OUT THE PLAN

- **Identifying and Applying Relationships**

Suggestions:
- Once the area of one square is found proceed to find the measure of one side of one square.
- Count how many such sides there are to find the perimeter of the figure.

1. $294 \div 6$ or $\frac{294}{6} = 49\ cm^2$

2. $A = s^2$
   $s^2 = 49\ cm^2$
   $\therefore s = 7\ cm$

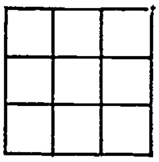
3. $P = 14s$
   $P = 14 \times 7$
   $P = 98\ cm$

LOOKING BACK

- **Explaining the Answer**

Suggestions:
- Have students explain that the perimeter was found by dividing a complex figure into 6 equal squares. Proceed to find the area of one of these 6 squares and further obtain the measure of one side of one square. Multiply that number by the number of equal sides that make up the perimeter of the figure.

EXTENSION:
- Give other examples whereby students have to divide a complex figure into simpler ones, to find the required answer.



1. Find the perimeter of the large square if the area of two of the smallest squares is 72 cm^2.
   ANSWER: 72 cm
2. Find the area of a square which has a perimeter of 28 cm.
   ANSWER: 49 cm^2
3. Find the dimensions of a square whose area is numerically equal to four times the perimeter.
   ANSWER: 16 units

A "break-dancing" floor is in the shape of a regular octagon. The segment from the centre of the octagon to a vertex is 16 m. Find the measure of the apothem of a regular octagon inscribed in a circle with a 16 m radius and a side of 12 m.

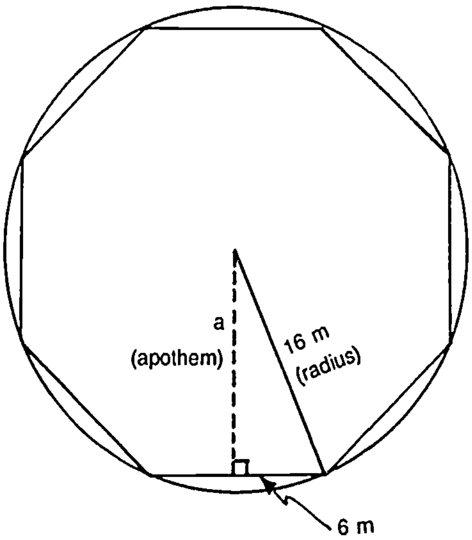
GRADE: Nine

STRAND: Geometry

UNDERSTANDING THE PROBLEM

- Identifying Key Words
- Drawing Diagrams
- Interpreting the Diagram

Key Words. regular octagon. apothem



DEVELOPING A PLAN

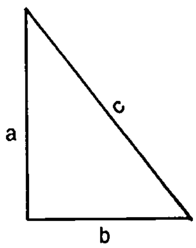
- To Choose an Appropriate Formula

- The apothem bisects (cuts in half) any side of a regular polygon. therefore. the triangle formed has a base of 6 m.
- The apothem drops perpendicular to the side. therefore: a right triangle is formed with the radius forming the hypotenuse.
- The formula is $c^2 = a^2 + b^2$ (Pythagoras)

CARRYING OUT THE PLAN

- Using a Formula

1. $c^2 = a^2 + b^2$
   $16^2 = a^2 + b^2$
   $256 = a^2 + 36$
   $a^2 = 256 - 36$
   $a^2 = 220$
   $a = \sqrt{220}$
   $a = 14.832$ m



LOOKING BACK

- Stating an Answer to the Problem

Answer:
The apothem of this regular octagon would measure approximately 15 m.

EXTENSION:

Calculate the area of the dance floor.

ANSWER:
712 m^2

How many dancers could be on the dance floor at once?

(Answers, of course, vary)

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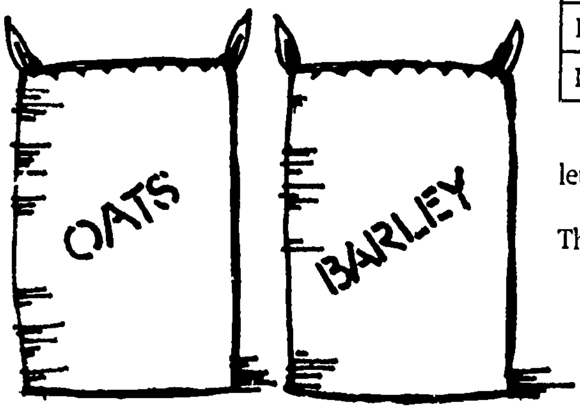
[illegible]

A hog farmer wishes to mix feed for his hogs. He wishes to mix 100 sacks of oats worth \$1.50 a sack with 50 sacks of barley worth \$2.50 a sack. What will be the value in cents per sack of the mixture?

GRADE: Nine

STRAND: Algebra (Mixture Problem)



UNDERSTANDING THE PROBLEM

- Restating the Problem in Your Own Words (internalize the problem).
- Asking Relevant Questions
- Identifying Wanted, Given and Needed Information

Sample Questions:

1. What types of feed are involved?
2. What is the cost/sack of each feed?
3. How many sacks will there be in the mixture?
4. How many sacks will there be of the mixture?
5. Are you looking for cost or for number of sacks?

DEVELOPING A PLAN

- To Collect and Organize Information (charts, graphs)
- To Formulate an Equation

	PRICE/SACK	# OF SACKS	TOTAL VALUE
OATS	\$1.50 or 150¢	100	150(100)
BARLEY	\$2.50 or 250¢	50	250(50)
MIXTURE	x	150	150x

let x = price/sack of mixture in cents

The Equation: $40(100) + 100(50) = 150x$

CARRYING OUT THE PLAN

- Solve the equation

$150(100) + 250(50) = 150x$
$15\ 000 + 12\ 500 = 150x$
$150x = 27\ 500$
$x = \$1.83$

LOOKING BACK

ANSWER:
It will cost \$1.83 for every sack of the mixture.

- Stating an answer to the problem
- Determining the reasonableness of the answer
- Making and solving similar problems

- Does the solution seem reasonable? Why or why not?

3. CLASSROOM PROBLEMS

The following problems have been developed for the teacher's use in the classroom. They are based on various strands outlined in the Program of Studies. Some suggested skills have been identified and strategies for solving these problems have been worked out. The material has been laid out so that it can be easily photocopied.

GRADE 7

PROBLEM 7.1

RATIO AND PROPORTION

A class of 20 students averages 66% on an examination; another class of 30 students averaged 56%. Find the average for all students.

UNDERSTAND
Identifying key words

LOOK BACK
Stating an answer to the problem. Determining the reasonableness of the answer

PLAN
To choose and apply the appropriate operation

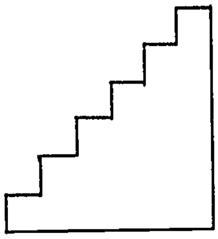
DO
Choosing and performing the appropriate operation

ANSWER:
60%

PROBLEM 7.2

GEOMETRY

Find the area of the figure below. The line segments for the "step" formation meet at right angles and are of 1 cm length.



UNDERSTAND
Identifying wanted, given and needed data

LOOK BACK
Stating an answer to the problem

PLAN
To partition

DO
Using a simpler problem carry out the partition and solve each part

ANSWER:
21 cm^2

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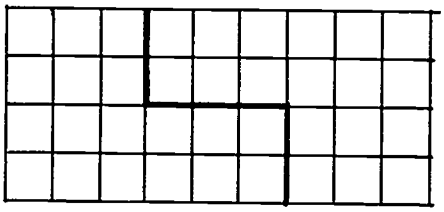
PROBLEM 7.3

GEOMETRY

Using a piece of graph paper. show how to cut the rectangle into two pieces that will fit together to fom a square.



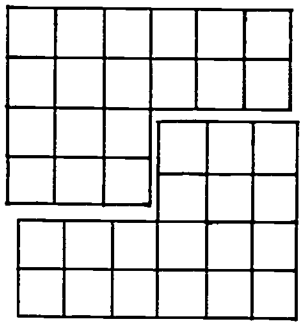
UNDERSTAND
Drawing diagrams

LOOK BACK
Reviewing the solution process

PLAN
To experiment through use of manipulatives

DO
Using manipulatives

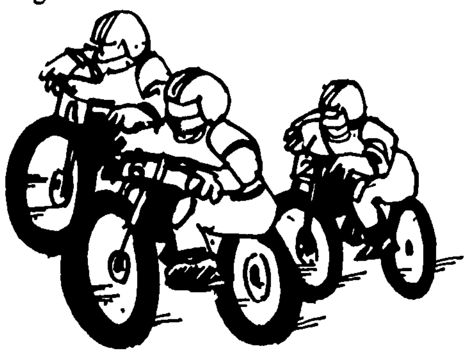
ANSWER:



PROBLEM 7.4

NUMBER SYSTEMS – RATIONALS

A mini-bike can go 38 km on one litre of gasoline. Its tank holds 5.8 L. How many kilometres can the bike go on 0.6 of a tank of gas?



UNDERSTAND
Restating the problem in your own words

LOOK BACK
Stating an answer to the problem
Determining the reasonableness of the answer

PLAN
To choose and apply the appropriate operation

DO
Choosing and performing the appropriate operation

ANSWER:
132.24 km

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PROBLEM 7.5

NUMBER SYSTEMS

Jane spent half of her money on a junior high dance ticket. One half of the remaining money was used for bus fare. She arrived home with $0.75. What amount of money did she have originally?

ANSWER:
$3.00

UNDERSTAND
Restating the problem in your own words

PLAN
To work backwards

DO
Working backwards

LOOK BACK
Stating an answer to the problem
Explaining the answer

PROBLEM 7.6

MEASUREMENT

The fencing that encloses a square lot is attached to upright posts placed 5 m apart. If twenty posts are necessary to fence the lot. what is its area?

ANSWER:
625 m^2

UNDERSTAND
Identifying wanted, given and needed data

PLAN
To make diagram and models
Explaining the answer

DO
Making diagram and models
Choosing and performing appropriate operation

LOOK BACK
Stating an answer to the problem
To choose and apply appropriate operation

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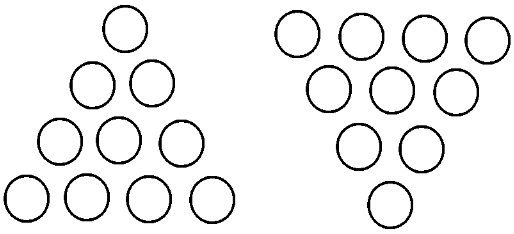
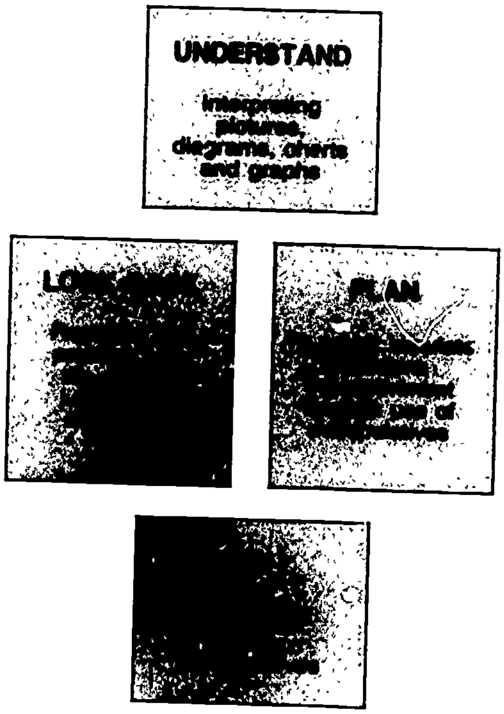
PROBLEM 7.7

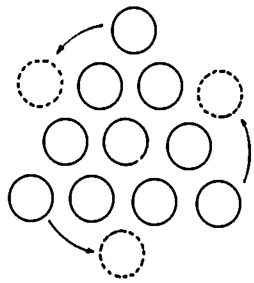
GEOMETRY

Move three coins in the figure on the left to make it like the figure on the right.





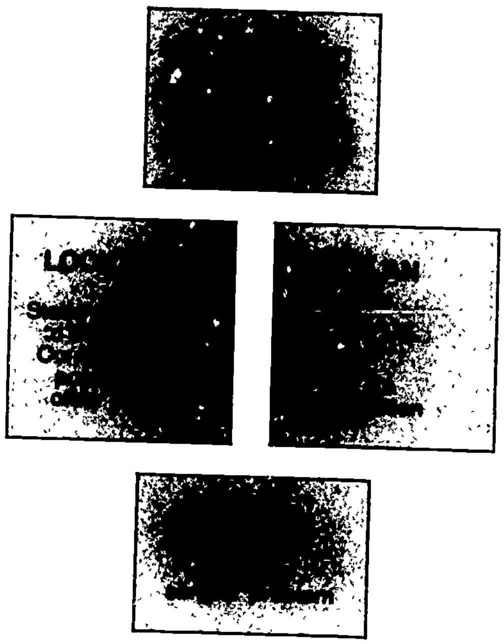
ANSWER:



PROBLEM 7.8

NUMBER SYSTEM

How many whole numbers less than 150 and greater than 0 do not contain the digit 3?



ANSWER:
116

PROBLEM 7.9

NUMBER SYSTEM

Julius Caesar wrote the Roman Numerals I, II, III, IV, and V in a special order from left to right. He wrote I before III but after IV. He wrote II after IV but before I. He wrote V after II but before III. If V was not the third numeral, in what order did he write these five numerals from left to right?

UNDERSTAND
Identifying key words

LOOK BACK
Stating an answer to the problem
Reviewing the solution process

PLAN
To experiment using manipulatives

DO
Using manipulatives

ANSWER:
IV, II, I, V, III

PROBLEM 7.10

NUMBER SYSTEM

If the counting numbers are arranged in four columns, as shown, under which letter will the number 101 appear?

A	B	C	D
1	2	3	4
8	7	6	5
9	10	11	12
"	"	14	13

UNDERSTAND
Asking relevant questions

LOOK BACK
Stating an answer to the problem
Generalizing solution

PLAN
To apply patterns

DO
Applying patterns

ANSWER:
D

PROBLEM 7.11

MEASUREMENT

You know that the perimeter of a certain rectangle measures 22 cm. If its length and width each measure a whole number in centimetres, how many different areas (in square centimetres) are possible for this rectangle?

UNDERSTAND
Drawing diagrams

LOOK BACK
Stating an answer to the problem
Determining the reasonableness of the answer

PLAN
To collect and organize information

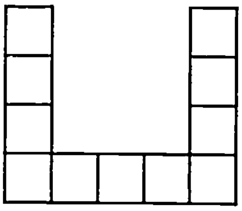
DO
Collecting and organizing information

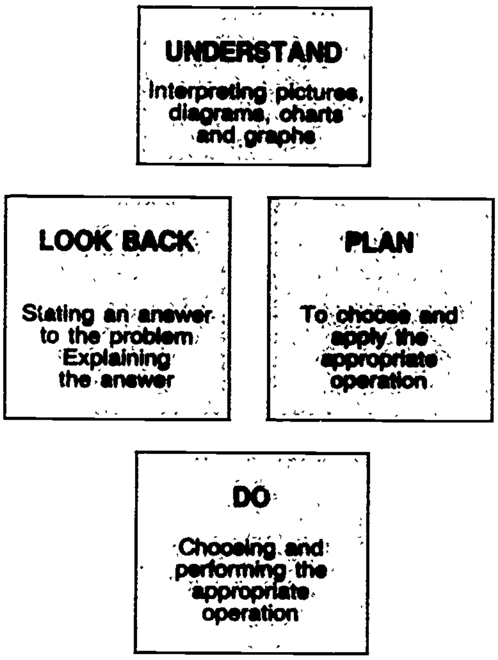
ANSWER:
5

PROBLEM 7.12

MEASUREMENT

The figure below is formed by eleven squares of the same size. If the area of the figure is 176 cm^2, what is its perimeter?



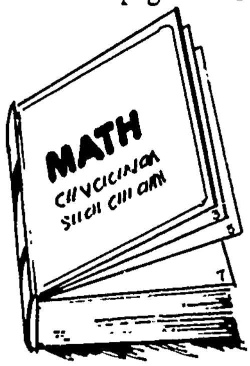


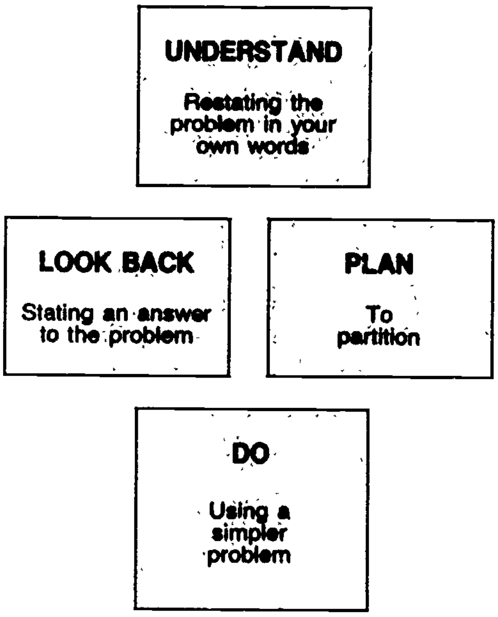
ANSWER:
96 cm

PROBLEM 7.13

NUMBER SYSTEM – WHOLE NUMBERS

A certain book has 500 pages numbered 1, 2, 3, and so on. How many times does the digit 1 appear in the page numbers?





ANSWER:
200

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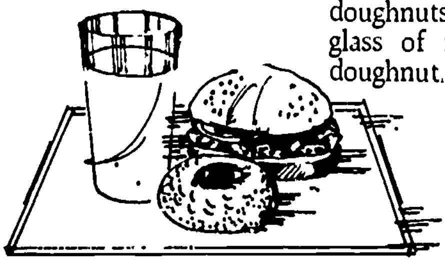
PROBLEM 7.14

NUMBER SYSTEM

Three friends ate lunch together. One spent $3.60 for a glass of milk, two hamburgers, and a doughnut. The second bought two glasses of milk, one hamburger, and two doughnuts for $3.00. If the third bought one glass of milk, one hamburger and one doughnut, how much did he pay?



UNDERSTAND
Identifying the key words

LOOK BACK
Stating an answer to the problem
Determining the reasonableness of the answer

PLAN
To formulate an equation

DO
Writing and solving a number system

ANSWER:
$2.20

PROBLEM 7.15

RATIO AND PROPORTION

In an economy move at a factory, all the workers had their salaries cut by 10%. They were all very unhappy about this and threatened to go on strike. At this point, the factory president stepped in and agreed to a 10% raise on their new salary. "Everyone should be happy again", he said with a smile. "Things are back the way they were to start with." Were they? Why?

UNDERSTAND
Identifying wanted, given, and needed data

LOOK BACK
Stating an answer to the problem
Restating the problem with the answer

PLAN
To choose and apply the appropriate operations

DO
Choosing and performing the appropriate operation

ANSWER:
answer may vary

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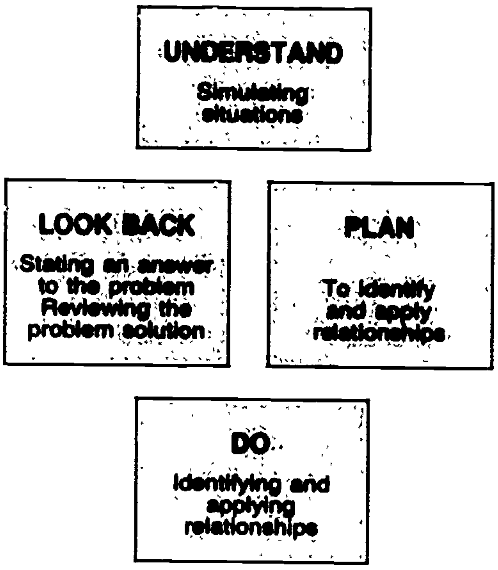
PROBLEM 7.16

WHOLE NUMBERS

Ten books of 100 pages each are arranged in order on a shelf. A bookworm starts on page 1 of the first book and eats through the 100th page of the last book. How many pages has it eaten through? Exclude covers and assume that one page equals one sheet.

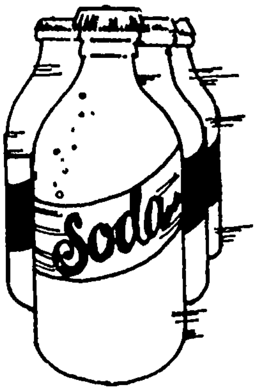


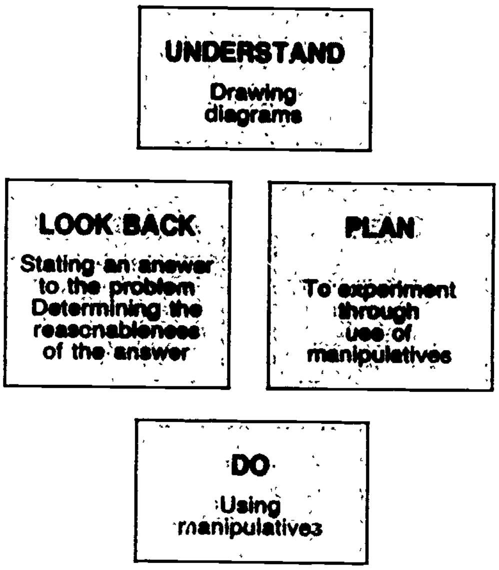
ANSWER:
802

PROBLEM 7.17

GEOMETRY

Eighteen bottles are placed in a 4 x 6 drink case. Every row and column contains an even number of bottles. Draw a diagram showing the arrangements of drink bottles in the case.





ANSWER:

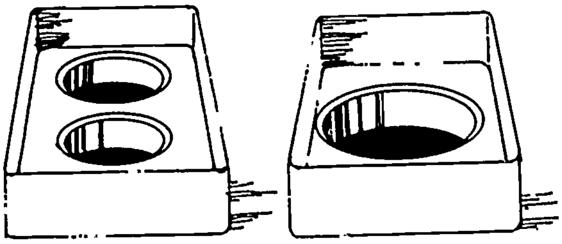
X	X	X			X
X	X			X	X
X	X		X		X
X	X	X	X	X	X

GRADE 8

PROBLEM 8.1

MEASUREMENT

Will two 5 cm drains give greater, less or the same drainage as a 10 cm drain?



ANSWER:
less

UNDERSTAND
Drawing a diagram

PLAN
To identify relationships

DO
Identifying and applying the relationships

LOOK BACK
Stating an answer to the problem
Looking for alternate solutions

PROBLEM 8.2

WHOLE NUMBERS

A bag of marbles can be divided into equal shares among 2, 3, 4, 5, or 6 friends. What is the least number of marbles that the bag could contain?

ANSWER:
60

UNDERSTAND
Restating the problem in your own words

PLAN
To use logic or reason

DO
Use logic or reason

LOOK BACK
Stating an answer to the problem
Determining the reasonableness of the answer

PROBLEM 8.3

WHOLE NUMBERS

In the addition below, each letter represents a different digit. What are the values of H, E, and A?

```
  H E
  H E
  H E
+ H E
-----
  A H
```

ANSWER:
H = 2, E = 3, A = 9

UNDERSTAND
Looking for patterns

PLAN
To identify and apply relationships

DO
Identifying and applying relationships

LOOK BACK
Stating an answer to the problem
Determining the reasonableness of the answer

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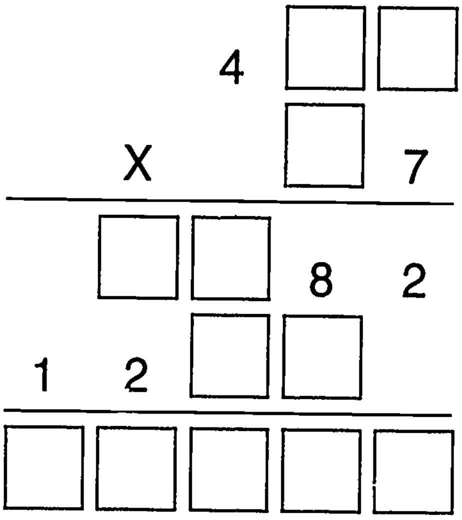
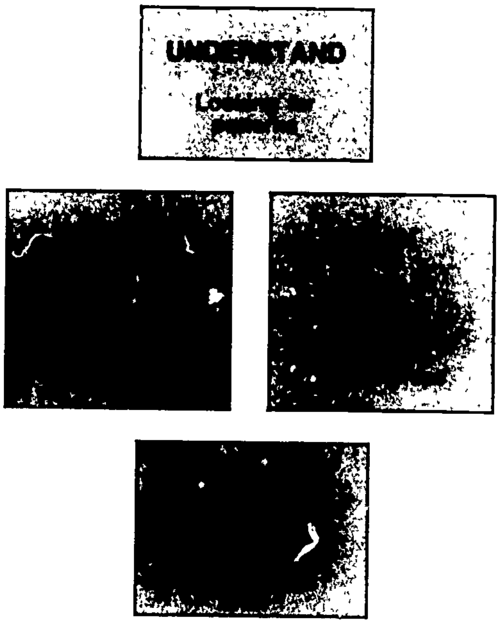
PROBLEM 8.4

WHOLE NUMBERS

In the multiplication below, each box represents a missing digit. What is the product?



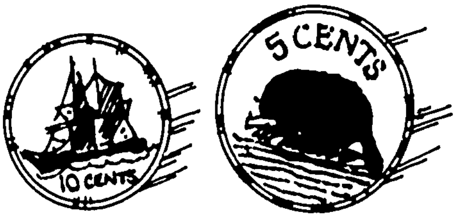


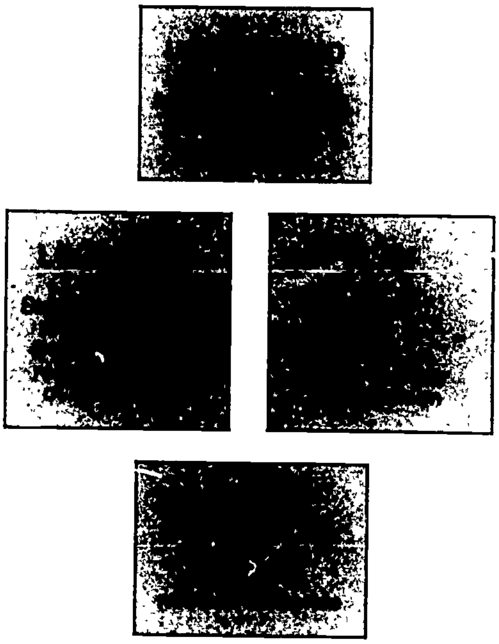
ANSWER:
15 762

PROBLEM 8.5

ALGEBRA

A dollar was changed for 16 coins consisting of just nickels and dimes. How many coins of each kind were in the change?



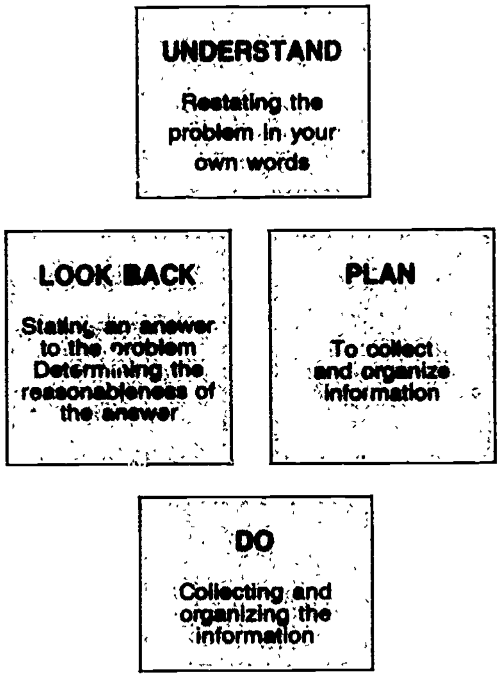


ANSWER:
4 dimes, 12 nickels

PROBLEM 8.6

WHOLE NUMBERS

I have exactly ten coins whose total value is $1. If three of the coins are quarters. what are the remaining coins and how many of each are there?

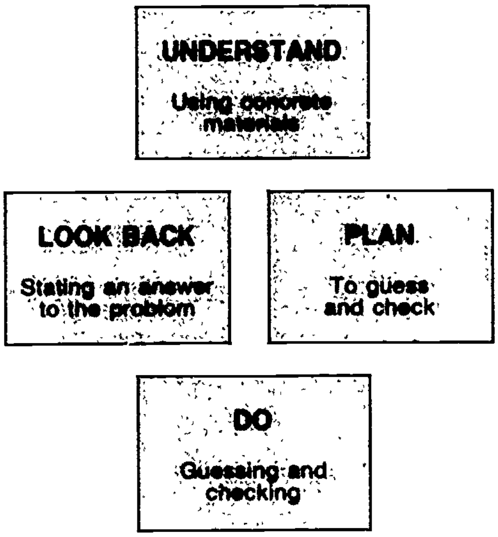


ANSWER:
5 pennies, 2 dimes

PROBLEM 8.7

WHOLE NUMBERS

When I open my mathematics book. there are two pages that face me. If the product of the two page numbers is 1806. what are the two page numbers?

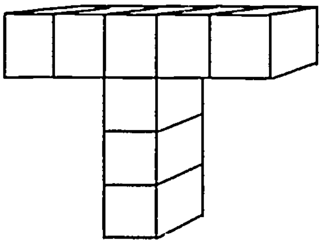


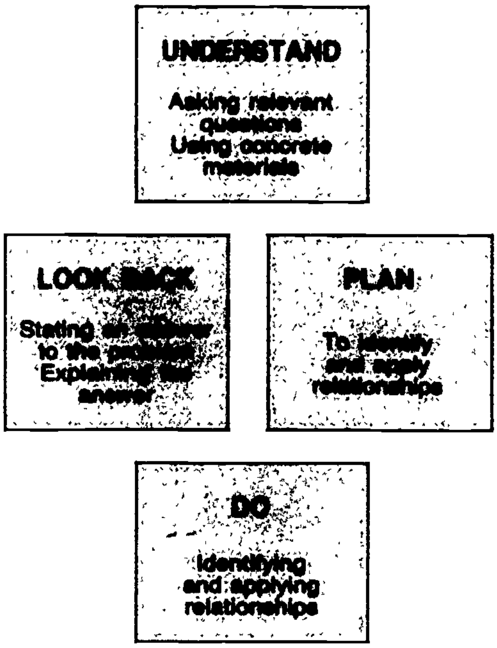
ANSWER:
42, 43

PROBLEM 8.8

GEOMETRY

Eight one-centimetre cubes are put together to form the T-shaped figure shown below. The complete outside of the T-shaped figure is painted red, and the one-centimetre cubes are then separated. How many of the cubes have exactly four red faces?



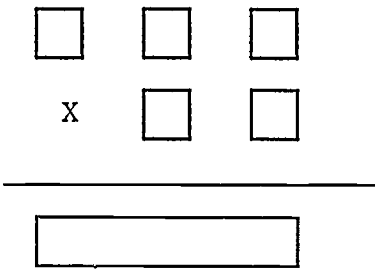


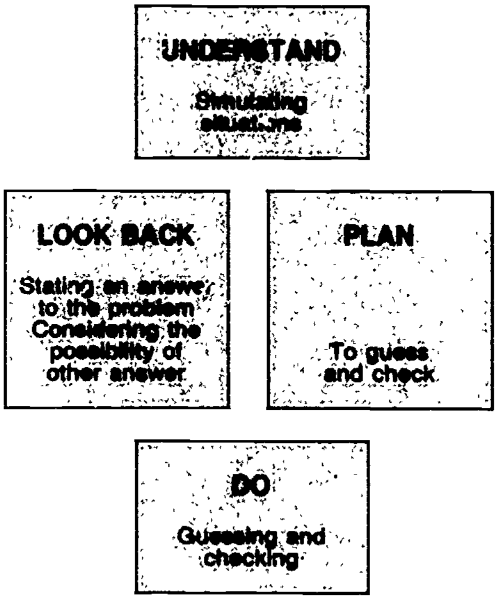
ANSWER:
4

PROBLEM 8.9

WHOLE NUMBERS

Arrange the numbers 2, 3, 4, 5, 6 in the boxes to get the largest possible product. Use each number once.



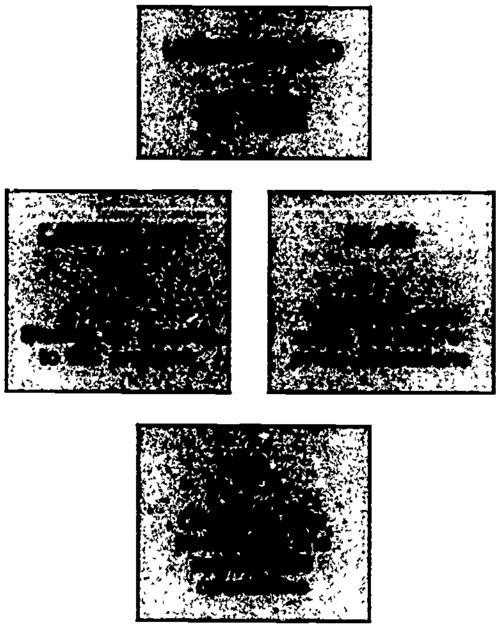


ANSWER:
542 x 63

PROBLEM 8.10

RATIO AND PROPORTION

Five boys wrote a mathematics test. The average mark was 68. If the marks of the four boys were 75, 62, 84, and 53, what was the mark of the fifth boy?

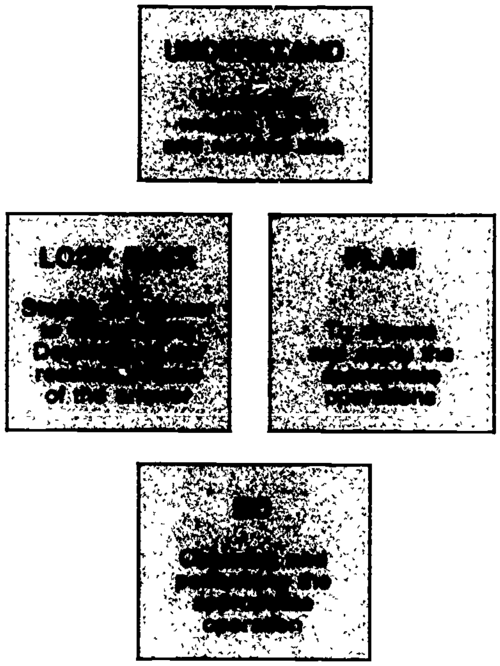


ANSWER:
66

PROBLEM 8.11

RATIONALS

In a hall, 1/3 of the people present are men, 1/4 of those present are women, and the rest are children. If there are 1152 people in the hall, how many chidren are there?



ANSWER:
480

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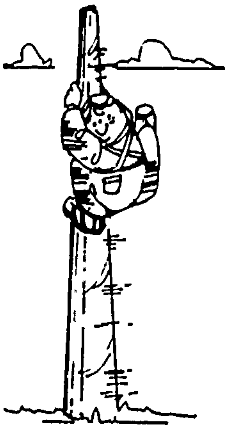
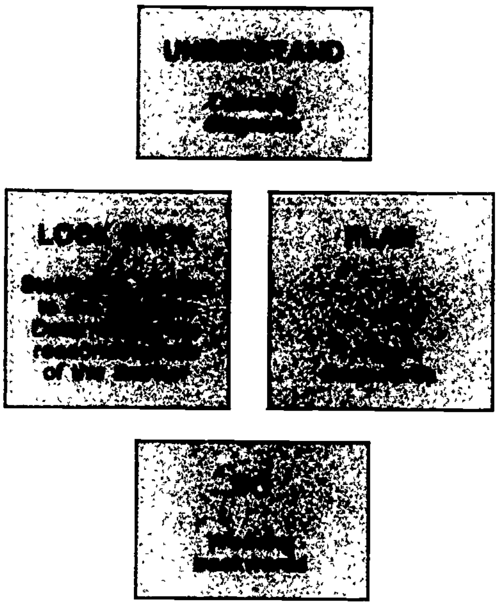
PROBLEM 8.12

RATIONALS

A boy attempts to climb a 10 m pole. At every attempt he climbs 1 m and slips back 1/2 m After how many attempts will he have reached the top?





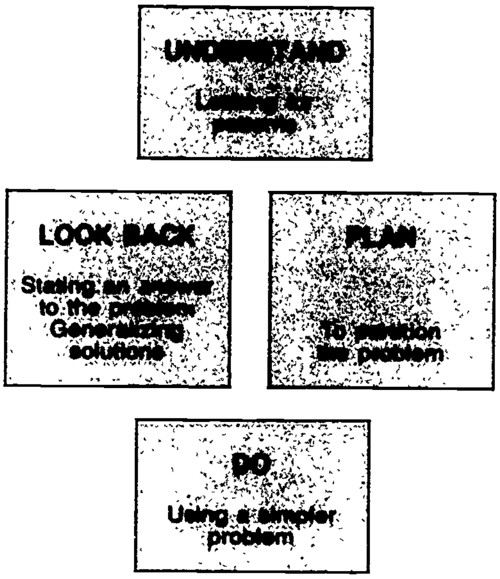
ANSWER:
19

PROBLEM 8.13

RATIONALS

Find the product:

$(1 - \frac{1}{2})(1 - \frac{1}{3})(1 - \frac{1}{4}) \ldots (1 - \frac{1}{98})(1 - \frac{1}{99})(1 - \frac{1}{100})$



ANSWER:
$\frac{1}{100}$

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PROBLEM 8.14

WHOLE NUMBERS

The numerals 333, 7777, and 88 all contain repeated, single digits. How many numerals between 11 and 999 999 contain repeated, single digits.?

UNDERSTAND

Looking for patterns
Identifying key words

LOOK BACK

Stating an answer to the problem
Explaining the answer

PLAN

To partition the problem

DO

Using a simpler problem

ANSWER:
43

PROBLEM 8.15

RATIO AND PROPORTION

On planet Crypton, ten percent of the population have two computers each, half of the remainder have none, and all the others have one computer each. How many computers are there on planet Crypton if there are 40000 inhabitants?

UNDERSTAND

Restating the problem in your own words

LOOK BACK

Stating an answer to the problem
Explaining the answer

PLAN

To choose and apply the appropriate operations

DO

Choosing and performing the appropriate operation

ANSWER:
26 000

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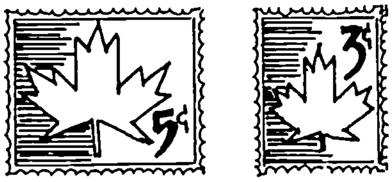
PROBLEM 9.1

GRADE 9

WHOLE NUMBERS

I have 4 three-cent stamps and 3 five-cent stamps. Using one or more of these stamps, how many different amounts of postage can I make?



UNDERSTAND
Identifying wanted, given and needed data

LOOK BACK
Stating an answer to the problem

PLAN
To [illegible] and [illegible] [illegible] Reviewing the solution process

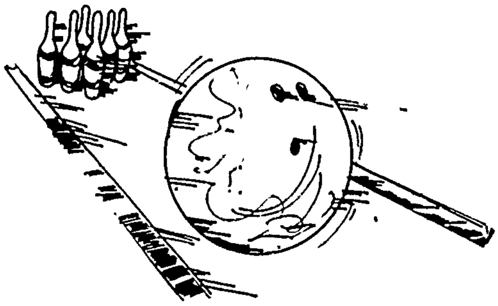
DO
Collecting and organizing the information

ANSWER:
19

PROBLEM 9.2

RATIO AND PROPORTION

In three bowling games, Alice scored 139, 143, and 144. What score will she need in a fourth game in order to have an average score of 145 for all four games?



UNDERSTAND
Identifying key words
Identifying wanted, given and needed data

LOOK BACK
Stating an answer to the problem

PLAN
To choose and apply the appropriate operations

DO
Choosing and performing the appropriate operations

ANSWER:
154

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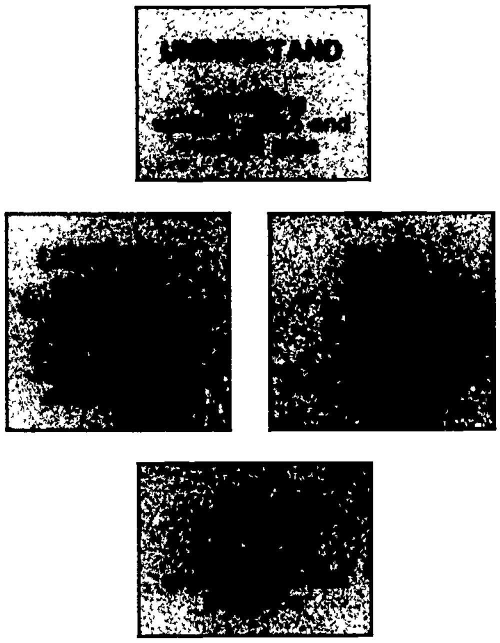
PROBLEM 9.3

WHOLE NUMBERS

During one school year. Nancy was given 25¢ for each math test she passed and was fined 50¢ for each math test she failed. By the end of the school year. Nancy passed 7 times as many math tests as she failed and she had a total of $3.75 How may tests did she fail?



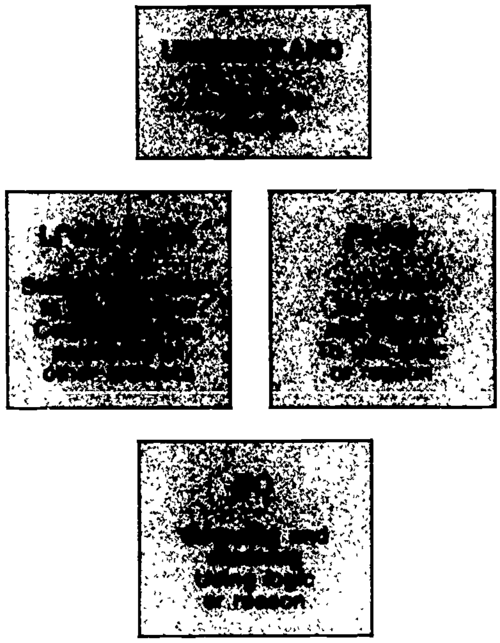
ANSWER:
3

PROBLEM 9.4

WHOLE NUMBERS

In the multiplication below. each letter represents a different digit. If A is not zero. what are the values of A. B. C. and D?

```
  A B C
x     C
-------
  D B C
```

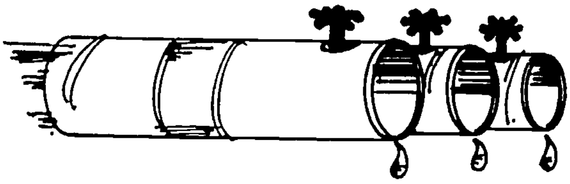


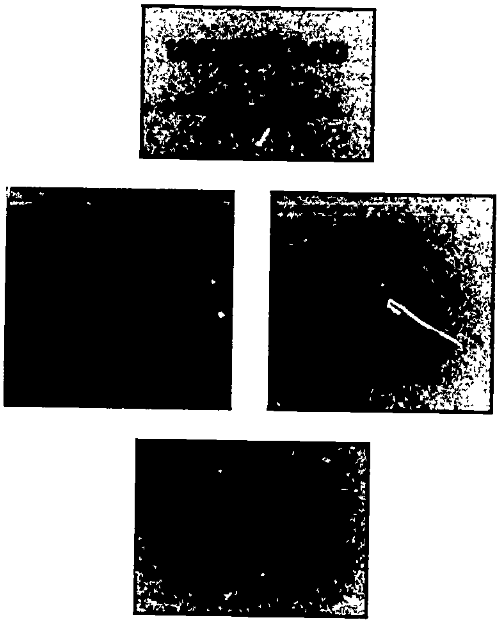
ANSWER:
125 x 5 = 625. 175 x 5 = 875

PROBLEM 9.5

RATIONALS

Three water pipes are used to fill a swimming pool. The first pipe alone takes 8 hours to fill the pool. the second pipe alone takes 12 hours to fill the pool. and the third pipe alone takes 24 hours to fill the pool. If all three pipes are opened at the same time. how long will it take to fill the pool?



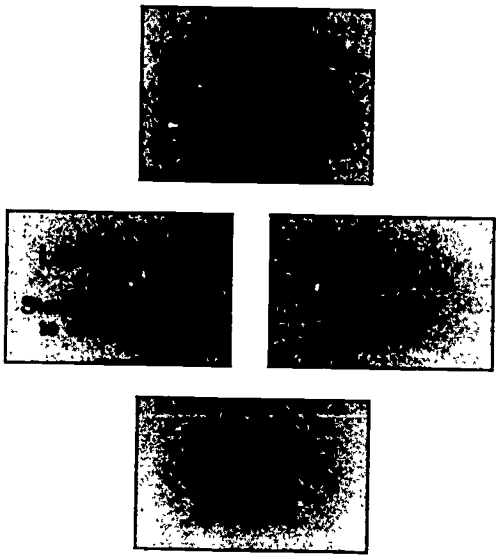


ANSWER:
4 hours

PROBLEM 9.6

WHOLE NUMBERS

The last digit of the product 3 x 3 is 9. the last digit of the product 3 x 3 x 3 is 7. and the last digit of the product 3 x 3 x 3 x 3 is 1. What is the last digit of the product when thirty-five 3's are multiplied?



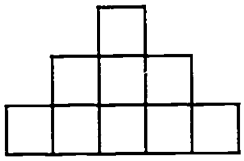
ANSWER:
7

PROBLEM 9.7

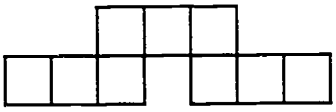
MEASUREMENT

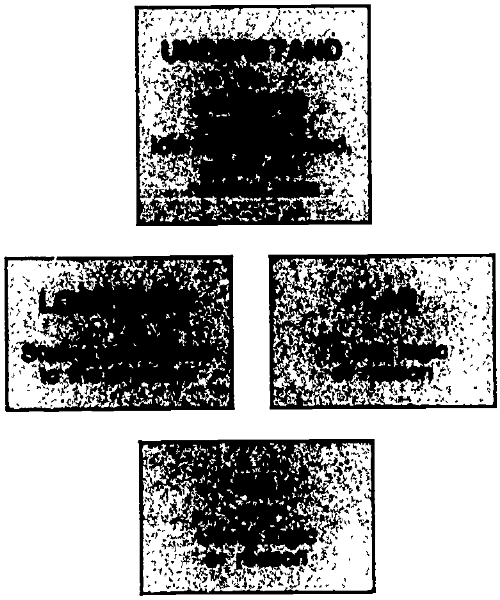
Figures A and B below are made up of congruent squares. If the perimeter of Figure A is 48 cm. what is the perimeter of Figure B?

A.



B.



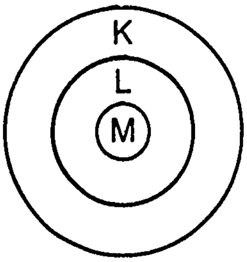


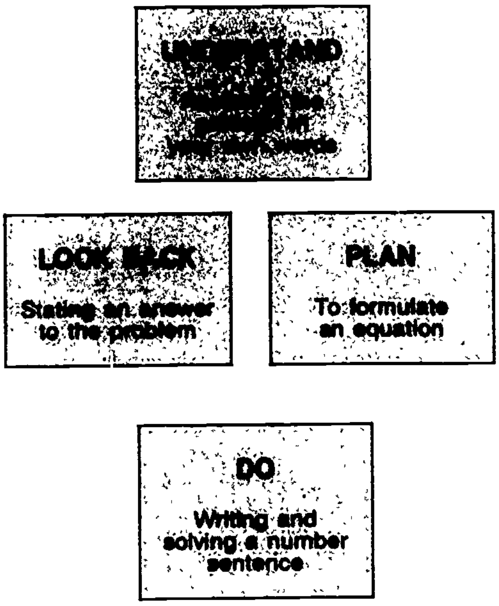
ANSWER:
60 cm

PROBLEM 9.8

WHOLE NUMBERS

Suppose K. L. and M represent the number of points assigned to the three target regions shown below. The sum of K and L is 11. the sum of L and M is 19. and the sum of K and M is 16. How many points are assigned to M?



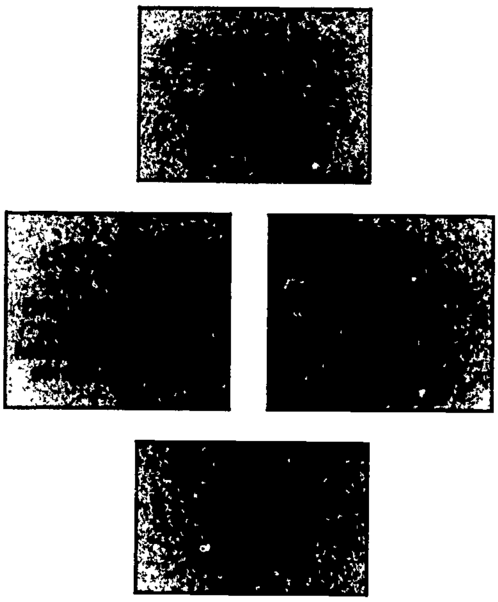


ANSWER:
12

PROBLEM 9.9

MEASUREMENT

There are eight baseballs. all exactly alike in size and appearance. but one is heavier than any of the other seven which are all the same weight With a balance scale. how can the heaviest baseball be positively determined with only two weighings?

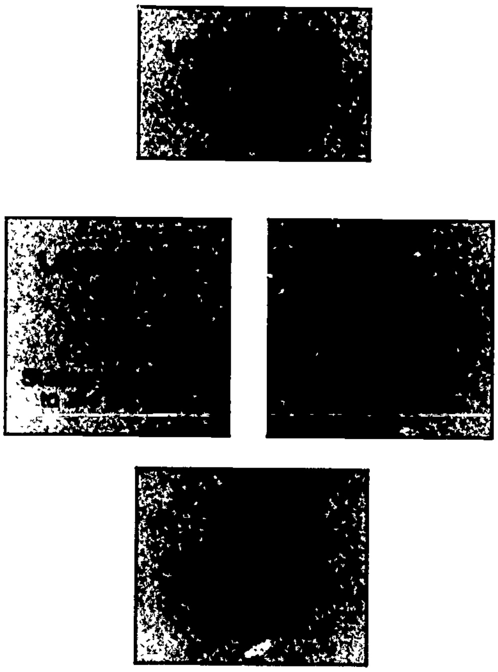


ANSWER:
student explanation may vary

PROBLEM 9.10

GEOMETRY

How many degrees does the minute hand of a clock pass through between 9:30 a.m. and 10:17 a.m.?

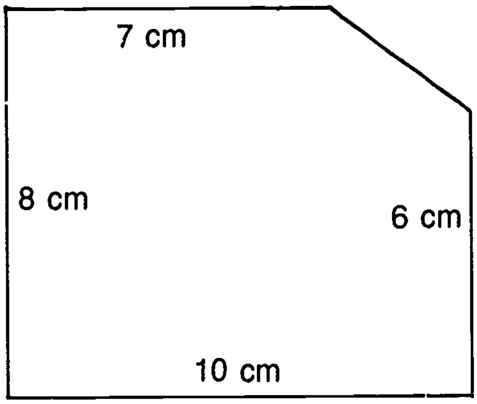


ANSWER:
282 degrees

PROBLEM 9.11

MEASUREMENT

Find the area:



ANSWER:
77 cm^2

UNDERSTAND
Interpreting pictures, diagrams charts and graphs

PLAN
To partition

DO
Using a simpler problem

LOOK BACK
Stating an answer to the problem

PROBLEM 9.12

MEASUREMENT

A cow is tethered by a rope 50 m long. The rope is fastened to a hood which is located 10 m from the centre on the longer side of the barn The barn measures 60 m by 30 m Over how much ground can the cow graze?

ANSWER:
4712 m^2

UNDERSTAND
Drawing diagrams

PLAN
To formulate the equation

DO
Writing and solving a number sentence

LOOK BACK
Stating an answer to the problem
Explaining the answer
Making and solving similar problems

ERIC

PROBLEM 9.13

RATIO AND PROPORTION

A train one kilometre long is travelling at a steady speed of 30 km/h. It enters a tunnel one kilometre long at 1:00 p.m. At what time does the rear of the train emerge from the tunnel?

UNDERSTAND
Drawing diagrams

LOOK BACK
Stating an answer to the problem. Reviewing the solution process

PLAN
To formulate an equation

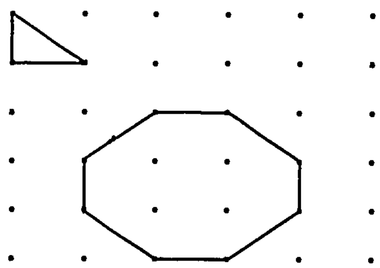
DO
Writing and solving a number sentence

ANSWER:
1:04

PROBLEM 9.14

GEOMETRY AND MEASUREMENT

The area of the triangle is 2 cm^2. What is the area of the octagon?



UNDERSTAND
Interpreting pictures, diagrams, charts and graphs

LOOK BACK
Stating an answer to the problem

PLAN
To partition

DO
Using a simpler problem

ANSWER:
28 cm^2

COMPUTER PROBLEMS

The following problems have been selected to illustrate how the strategies in the problem solving model may be applied for computer use. The strategies vary from each of the problems given and are presented with a number of related questions and activities that will assist the teacher in working a problem through the four problem solving steps. Teachers are encouraged to use the problems as a springboard for introducing problem solving activities on the computer.

COMPUTER PROBLEMS

1. Here are examples of twin primes.
   3,5; 11, 13.
   They are primes which differ by two. Write a computer procedure in BASIC (or LOGO) which will print out all the pairs of twin primes between 3 and 500.

2. Flying Start Consulting Ltd. will pay Mr. J Neide a $100 fee on day 1 and give him a raise of $1000 per day for each day thereafter. Double or Nothing Corporation will pay him $0.01 on day 1 and double his wage each day thereafter. Write a computer procedure which will list each day's wage, the total to date and will stop if the Double or Nothing total is larger than Flying Start before 100 days.
3. Here is a LOGO Procedure

```
TO INSPIRAL : ANGLE : SIDE : INC
IF : SIDE 1 THEN STOP
FD : SIDE
RT : ANGLE
INSPIRAL : ANGLE : SIDE : INC
```

   a. Find the value for : ANGLE which generates a 5 pointed star.
   b. Find a way of telling when the value of : ANGLE will generate a star shaped figure (or any number of sides).

4. Think of a young tree as a branch with one leaf. The next year it sprouts 2 branches in place of the leaf. The next year it sprouts 2 branches in place of each leaf.
   a. Write a LOGO procedure to grow such trees.
   b. How many leaves are there in five years?

(Richards Access Network, 1984)

CHALLENGE PROBLEMS

By definition a problem must be a challenge for the person solving it. But since as a teacher one is trying to provide mathematical problem solving experience for all, some students in classes may find the problems offered "easy". Other students may simply be very interested in mathematical problems as a task. For other students a teacher may want to provide the opportunity for creative mathematical expression. The purpose of this set of challenge problems is to provide teachers with a resource to use with students with relatively high levels of mathematical knowledge or interest.

Like any set of problems, these vary greatly. However, they can be characterized as having solutions which require good command of the related mathematical ideas, ability to rearrange information cleverly, several levels of thinking, complexity of thinking or the ability to generate and evaluate numerous alternatives. Because of the nature of this document and space requirements, most of the problems offered are given in verbal form. There are numerous physical puzzles (e.g. Rubic's cube, Chinese rings), geometric and topological problems (e.g. 4 coloured map problem), application problems (solving a local traffic flow problem, applying trigonometric ideas) and such things as mazes and chess problems which are not included. A teacher is encouraged to use these other types of challenge problems with students as well as those given here.

In solving problems students should be encouraged to use calculators or computers where appropriate. Some problems require extensive or complex calculations and calculators are a useful aid. Others require generation and organization of extensive data, generation of a number graphic representations. Computers are helpful in these circumstances. Further, doing a mathematical task using technology may be a problem in applied mathematics. A student may need to figure how a calculator does some computations or represent a problem solution as a computer algorithm or procedure.

There are many ways in which challenge problems can be used. Some are given below:

1. Post a problem of the week. Post individual or group solutions the next week. Continue this for the whole school year.
2. During problem solving lessons give differentiated assignments, by having some students work on challenge problems.
3. Use challenge problems as part of the curriculum for a mathematics option or as an activity for a mathematics club.
4. Sponsor local mathematics competitions. One could have "house league" mathematics teams. Teams could compete as teams or groups of individuals at noon hour mathematics matches. Challenge problems are useful for such contests.
5. Some problems admit extensive or elegant written (or drawn) solutions. Such problems could appear as part of a school paper. The paper could also have a "mathematics" page on which solutions were published.
6. Many students like to participate in mathematics contests sponsored locally, provincially and nationally. Challenge problems offer a good source of "practice" for such contests.

However these problems are used it is hoped that they will be an aid to improving the mathematical problem solving experiences of many students.

NUMBER SYSTEMS

1. With sufficient supply of 1¢, 2¢, 4¢, and 8¢ stamps, the number of different selections of stamps to make a postage total of 8¢ would be?

ANSWER:
10

2. John has just purchased an ACME paper cutter that will cut a stack of up to 500 sheets of paper in one operation. If no piece of paper is ever folded, what is the minimum number of operations needed to get 1983 pieces of paper, starting from one sheet?

ANSWER:
12

3. A freight train 500 m long passes through a 2000 m tunnel. If 60 seconds elapse from the time when the last car enters the tunnel to the time when the engine emerges from the other end, what is the speed of the train?

ANSWER:
25 m/s

4. "I want to call my uncle", said Mary "What is his new number?" "It is easy to remember", replied Bill. "The exchange is the same and only the last two digits are identical, and multiplied together they make the second. All four digits total 20." What is this four-figure number?

ANSWER:
5933

5. This "time statement" is also true as an addition sum for which each letter stands for a different digit.

```
  N I N E
  L E S S
    T W O
---------
S E V E N
```

What are the values of the letters?

ANSWER:

```
  8085
  7511
   362
 -----
 15958
```

6. The owner hired 3 watchmen to guard his orchard, but a thief still got in and stole some apples. On the way out, the thief met each watchman, one at a time. To each he gave half of the apples he had then, and 2 more besides. He escaped

with one apple. How many did he steal originally?

ANSWER:
36

7. Diophantus was a Greek Mathematician. When he died his epitaph read:

Diophantus

Passed 1/6 of his life in childhood, 1/12 in youth, and 1/7 as a bachelor. His son, was born 5 years after his marriage but died 4 years before 1/2 of his father's age.

How old was Diophantus?

ANSWER:
84

8. A used car dealer complains to his friend that today has been a bad day. He tells his friend he has sold two cars for $750 each. One of the sales yielded him a profit of 25%. On the other one he took a loss of 25%. "What are you worrying about?" asked his friend. "You had no loss whatsoever." "On the contrary, a substantial one", answers the car dealer. Who was right?

ANSWER:
$100 loss

9. 5 5 5 1 1 1 9 9 9
Above are three sets of digits: 3 fives, 3 ones, and 3 nines. These make a total of nine digits. The object is to cross out six of the digits and leave three so that when added together you have a sum of 20. How can this be done?

ANSWER:
2 – one, 1 – nine

10. The toll for an automobile crossing a certain bridge is 50¢. The machines in the "exact change" lanes accept any combination of coins that total exactly 50¢, but they do not accept pennies or half dollars. In how many different ways can the driver pay the toll?

ANSWER:
10

11. The houses on Main Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 7?

ANSWER:
24

12. A class of 64 students counted off by 1's beginning with number 1. Each student who counted an even number stood up. Then the students who were still seated counted by 1's again. Each student who counted an even number this time also stood up. After the fourth counting was completed, how many students remained seated?

ANSWER:
4

13. The six-digit number A42 73B is divisible by 72 without a remainder. Find the values of A and B.

ANSWER:
A=5, B=6

14. If a kindergarten teacher seats the class with 4 children on each bench, there will be 3 children who will not have a place. However, if 5 children are seated on each bench, there will be 2 empty places. What is the least number of children the class could have?

ANSWER:
23

15. What is the last digit in your answer for $4^{10\,000}$?

ANSWER:
6

16. What is the smallest number divisible by: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10?

ANSWER:
2 520

17. Margie is a blonde, Rose Mary a redhead, and Shirley is a brunette. They are married to Alex, Frank, and John, but
(a) Shirley does not like John

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(b) Rose Mary is married to John's brother
(c) Alex is married to Rose Mary's sister

Who is married to whom? (assume that married people like each other!)

ANSWER:
John-Margie, Frank-Rose Mary, Alex- Shirley

18. Place brackets (parentheses) to make a true statement.

$8 \div 4 + 2 \div 2 + 9 \div 3 + 3 \div 3 - 4 = 0$

ANSWER:
$(8 \div 4 + 2) \div 2 + (9 \div 3 + 3) \div 3 - 4 = 0$

19. I have two digits, one odd, one even. My remainders are equal when I am divided by 6 or by 8. My digits reversed make me smaller than I am. I have brothers and sisters, but I am the smallest I am more than half a hundred. Who am I?

ANSWER:
52

20. The product of two whole numbers is 1 000 000, but neither number contains a zero. What are the numbers?

ANSWER:
64 x 15 625

21. Each of the following sets of red and black checkers is to be arranged into piles. Each pile may contain only red or only black checkers. All piles, both red and black, must contain the same number of checkers. What is the greatest number of checkers that each pile can have?

a. 18 red, 30 black — **ANSWER:** 6

b. 84 red, 56 black — **ANSWER:** 28

c. 12 red, 60 black — **ANSWER:** 12

d. 21 red, 10 black — **ANSWER:** 1

22. Suppose that a printer is using an old-style printing press and needs one piece of type for each digit in the page numbers of a book. How many pieces of type will the printer need to number pages from 1 through 250?

ANSWER:
642

23. If a counting number ends in zeros, the zeros are called terminal zeros. For example 520 000 has four terminal zeros while 502 000 has just three terminal zeros. How many terminal zeros will 1 x 2 x 3 x 4 . . . x 20 have when written in standard form?

ANSWER:
4

24. A man worked 10 days. The first day he was paid $100. Each day thereafter he was paid 1/2 of what he received the day before. What was his total wage?

ANSWER:
$199.80

25. There are fewer than 6 dozen eggs in a basket. If I count them 2 at a time, there is 1 left over. If I count them 3 at a time, there are none left over. If I count them 4, 5 or 6 at a time, there are 3 left over. How many are there?

ANSWER:
63

26. What number divided by 2, 3, 4, 5, or 6 has a remainder of 1, but when divided by 7 has no remainder?

ANSWER:
301

27. A leak in the roof allows 2 drops through the first day, 4 drops through the second day, 8 drops the third, etc. When will the 500th drop fall through?

ANSWER:
8th day

28. The number of a past year is divided by 2 and the result turned upside down and divided by 3. then left right-side up and divided by 2. Then the digits in the result are reversed to make 13. What is the past year?

ANSWER:
1962

29. If six people were in a room and each one shook hands with every other person. how many handshakes were there?

ANSWER:
15

30. An amoeba is placed in a jar at 1:00 p.m. It reproduces by doubling every twenty minutes. How many amoebas will be in the jar as of 5:00 p.m. that day?

ANSWER:
4096

31. On December 1. Bob began saving 10¢ a day. Eight days later. Steven began saving 15¢ a day. On what date would they have the same amount in savings?

ANSWER:
December 24

32. A blacksmith said he would give a special rate for new horseshoes. He would charge 1¢ for the first nail. 2¢ for the second nail. 4¢ for the third. 8¢ for the fourth. etc. If each shoe takes eight nails. what is the cost of two new shoes for a horse?

ANSWER:
$655.35

33. The average weight of 4 football players is 90 kg. The weights of 3 of the players are 82 kg. 92 kg. and 108 kg. What is the weight of the fourth player?

ANSWER:
78 kg

34. Suppose you snap your fingers once after one minute has elapsed. Now. you wait for 2 minutes and snap your fingers again. and once again after 4 minutes. once again after 8 minutes. etc.. etc.. ... How many times will you have snapped your fingers at the end of 30 days?

ANSWER:
16

35. How many ways can a committee of two be selected from 5 people?

ANSWER:
10

36. There are 24 players on a baseball squad. 10 of the players can pitch. 6 can play first base. and 4 of the players can do both. How many players can neither pitch nor play first base?

ANSWER:
12

37. If you buy a goat for $20. sell it for $40. buy it again for $60. and sell it again for $80. how much money have you gained or lost on the combined deals?

ANSWER:
Gained $40

38. Rearranging the digits of the number 579 produces different numbers. What is the sum of all such numbers. including 579?

ANSWER:
4662

39. A ferryboat. when filled can carry 6 Pintos and 7 Toyotas or 8 Pintos and 4 Toyotas. If the ferryboat carries Toyotas only. then what is the maximum number that it can carry?

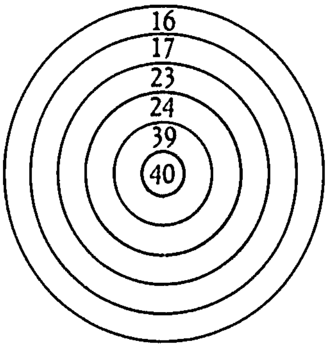
ANSWER:
16 Toyotas

40. The integer closest to $\frac{15}{2} - \frac{3}{5} \times \frac{11}{6}$ is

a. 3
b. 5
c. 6
d. 7
e. 9

ANSWER:
6

41. The rings on a target have values 16, 17, 23, 24, 39, 40.



How many arrows does it take to get exactly 100 points?

ANSWER:
16 + 16 + 17 + 17 + 17 + 17

42. A two-volume set of books stands on a bookshelf in the right order – volume 1 and volume 2. The pages of each together are 9 cm thick: the cover's are each 0.75 cm thick. A bookworm starts on the title page of volume 1 and eats through to the last page of volume 2. How far did it travel?

ANSWER:
1.50 cm

RATIO AND PROPORTION

1. In a class of size x. the ratio of male students to female students is 4:3. One-seventh of the class are left-handed. and these are divided equally between the sexes. There are 15 right-handed females in the class. What is the size of the class?

ANSWER:
42

2. Michael has 2 containers. A and B. Container B holds twice as much as container A. A is 1/2 filled and B is 1/3 filled with syrup. The rest of each container he fills with water. He then pours the content of both containers A and B into a third container. What fraction of the total contents is water?

ANSWER:
$^{11}/_{18}$

3. When a farmer died. he wrote in his will the instructions on how to divide his 17 cows among his 3 sons as follows:
Tom gets 1/2 or 8 1/2 cows: Dick gets 1/3 or 5 2/3 cows: and Harry. the youngest gets 1/9 or 1 8/9 cows. This was not satisfactory at all. for none of the boys wanted part of a dead cow. Yet each wanted his full share. How can this be done?

ANSWER:
Borrow a cow
$\frac{1}{2}(18) = 9$; $\frac{1}{3}(18) = 6$; $\frac{1}{9}(18) = 2$

4. A man with fewer than 100 fish said. "If I had half as many more as I now have. and two fish and a half. I would have 100." How many fish did he have?

ANSWER:
65

5. A man obtained 1/8 of a dollar from one person. 1/6 from another. 1/5 from another. and 2/15 from another. How much did he get from all?

ANSWER:
$^{5}/_{8}$

6. If it takes one minute to make each cut. how long will it take to cut a 10 m pole into ten equal pieces?

ANSWER:
9 minutes

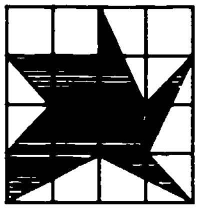
7. A man 180 cm tall casts a 45 cm shadow. If a telephone pole casts a 300 cm shadow. then the height of the pole is __________ cm.

ANSWER:
1200 cm

8. Brown had 9 pizzas and Jane had 6. They split them three ways evenly with Robinson who paid them $15. What is Brown's share of this money?

ANSWER:
$12.00

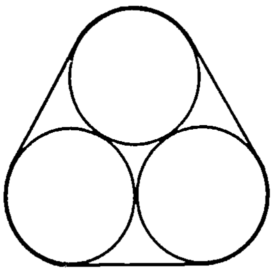
MEASUREMENT



1. Each of the squares in the 4 x 4 grid measures 1 cm x 1 cm. What is the area of the shaded part?

ANSWER:
7 cm²

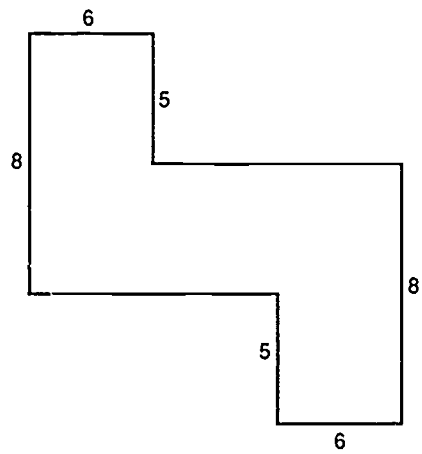
2.



Three pieces of steel rod, each of circular cross-section with radius 10 cm, are bound together by a band, as illustrated. What is the length of the band?

ANSWER:
122.8 cm

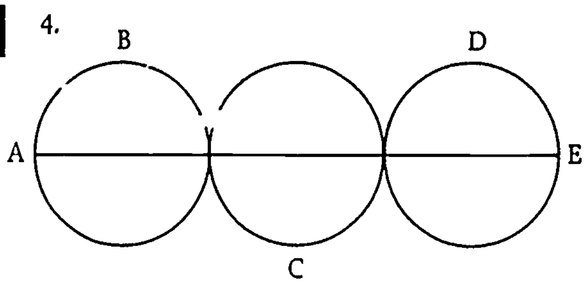
3.



If the area of this figure is 108 square units, then what is its perimeter?

ANSWER:
58

4.



The three touching circles shown are identical. Line segment AE = 42 cm long. How long is the curved path ABCDE?

ANSWER:
66 cm

5. Bill, John, Joe and Henry have to catch the six o'clock bus.
   a. Bill's watch is ten minutes fast, but he thinks it is five minutes slow.
   b. John's watch is ten minutes slow, but he thinks it is ten minutes fast.
   c. Joe's watch is five minutes slow but he thinks it is ten minutes fast.
   d. Henry's watch is five minutes fast but he is under the impression it is ten minutes slow.

   If each leave to catch the bus so he will just make it, if his time is what he thinks it is, who misses the bus?

ANSWER:
John, Joe

6. Pretend that blocks of wood that are either 6 dm or 7 dm long can be used as train cars and hooked together to make longer trains. Which of the following train-lengths cannot be made by hooking together either 6 dm cars, 7 dm cars or a combination of both?
   29 dm, 30 dm, 31 dm, 32 dm, 33 dm

ANSWER:
no 29

7. A faucet drips at the rate of 1 drop every 5 s. One drop of water is about 0.08 mL. It is estimated that about 30 000 homes have a leaky faucet.
   a. Calculate the total amount of water wasted in all the homes in a year

ANSWER:
15 137 280 L

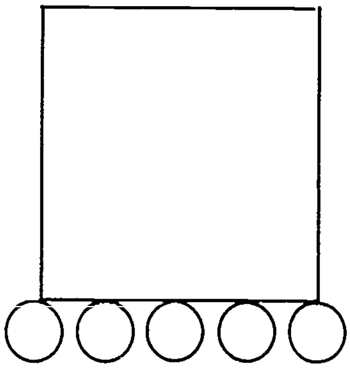
b. A container has a circular base. 1.1 m in diameter. What would be the height of the container needed to hold the water in (a)?

ANSWER:
16 m

8. Nine coins look exactly alike. but you know that one of them is counterfeit and weighs slightly less than the others. The only equipment that you have available is a balance scale. How can you find out which is the counterfeit coin by making just two weighings on this scale?

ANSWER:
Divide the coins into three equal groups. Weigh two of the groups. The counterfeit coin will be in the group that weighs the least. Set aside one coin from this group and weigh the other two. Continue weighing until you isolate the lightest (counterfeit) coin.

9 The figure below shows a crate being moved along the ground by rolling it on cylinders. If the circumference of each cylinder is 75 cm. how far does the crate move for each complete turn of the cylinders?



ANSWER:
1.5 m

10. A brick weighs 600 g plus 1/2 of its total weight. What is the total weight of the brick?

ANSWER:
1200 g

11. How many pieces of glass a half metre square fit into a frame a square metre?

ANSWER:
4

12. What is the difference between 6 dozen dozen and half a dozen dozen?

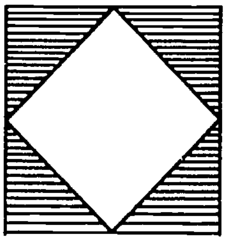
ANSWER:
792

13. Linda bought some eggs. She gave 1/2 her eggs and 1/2 an egg to her mom. Then she gave 1/2 her remaining eggs and 1/2 an egg to her aunt. Then she gave 1/2 of her eggs and 1/2 an egg to her sister. This left her with 1/4 dozen eggs. How many had she bought?

ANSWER:
31 eggs

14. A man had a house in which one of the windows was 1 metre square. He boarded up one half of this window but. to his surprise. found that he still had a square window that was 1 metre across and 1 metre from top to bottom. Draw a diagram that will show how this is done.

ANSWER:



15. Assume that you have a 3 L measure and a 5 L measure. What you want to do is to measure out exactly 4 L of water. How can this be done?

ANSWER:
3 –– 5
3 –– 5 (full) 1 L spare
3 L + 1 L = 4 L

16. A rectangular block has a length of 12 mm. a width of 10 mm. and a depth of 8 mm. What is its volume in cubic centimetres?

ANSWER:
0.96 cm²

17. Joe has some spheres. all of which weigh the same. He also has some cubes all of which weigh the same. He discovered that 4 spheres and 3 cubes weigh 37 g and that 3 spheres and 4 cubes

ERIC

weigh 33 g. What would one sphere and one cube together weigh?

ANSWER:
10 g

18. A rectangular box has volume 15 cm^3. If the length, width and height of the box are doubled, then what is the resulting volume?

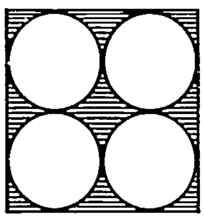
ANSWER:
120 cm^3

GEOMETRY

1. Two circles of radius 1, intersect in such a way that the two points of intersection and the centres of the circles are the vertices of a square. What is the area of the region common to both circles?

ANSWER:
$\frac{\pi}{2} - 1$

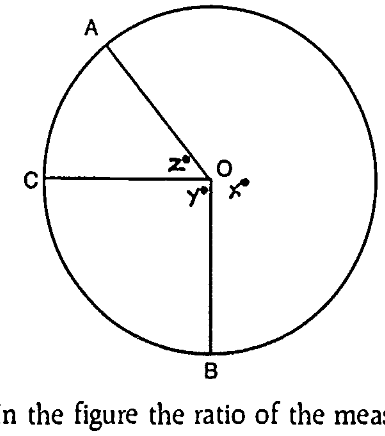
2.



The figure shows four circles of equal size within a square. Given that the radius of each of the circles is "a" centimetres, what is the total area of the shaded portion of the figure?

ANSWER:
$4a^2 (4-\pi)$

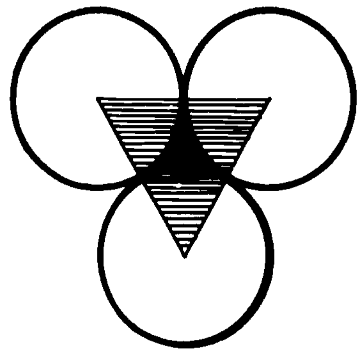
3.



In the figure the ratio of the measures of the angles shown is given by x:y:z = 9:4:2. What is the measure of $\angle$AOC?

ANSWER:
48 degrees

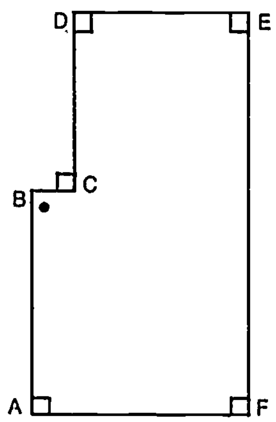
4.



In the diagram, each circle has a radius of 1, and the circles are externally tangent as shown. Find the area of the shaded region.

ANSWER:
$\sqrt{3} - \frac{\pi}{2}$

5.



ABCDEF represents a hole in a mini-golf course.

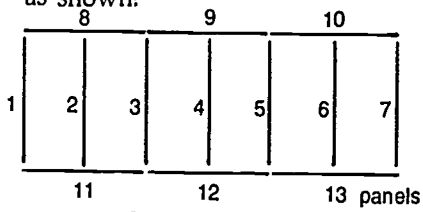
AB = 4 m
BC = 1 m
CD = 4 m
DE = 4 m

If the ball is at B and the cup is placed at D, describe a path for a hole-in-one that has a length less than 20 metres, and calculate its exact length.

ANSWER:
15 m

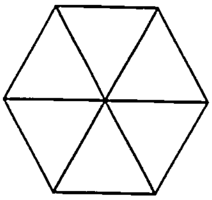
ERIC

6. At a 4H Club Show. 6 pig pens were made with 13 panels. all of equal length as shown:



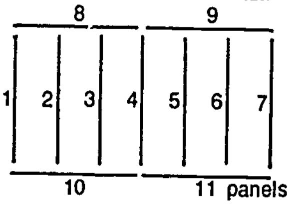
a. One of the panels were broken when a truck backed into it. The club members arranged the 12 panels so that they could still form 6 pens of equal size and same shape. How did they do it?

ANSWER:



b. Later. the manager needed one of the panels in another part of the show. The club members then made 6 pens of equal size and of the same shape with 11 panels. How did they do it?

ANSWER:

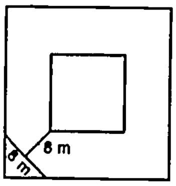


7. Fifty-six people sign up for a singles tennis toumament in which one lost match eliminates a player. To declare a champion what is the number of matches which must be played?

ANSWER:
55

8. Mr. Jackson had a garden in the shape of a square. Each year some of his melons from the garden were stolen. Since he depended on the produce for a living. he had a ditch 10 m wide and 10 m deep dug around the patch. He had the ditch filled with water. However. after only a night or two some of the melons were gone again and the thieves left behind two 8 m planks which they had used to gain access to the patch and to get out again. How did they do this?

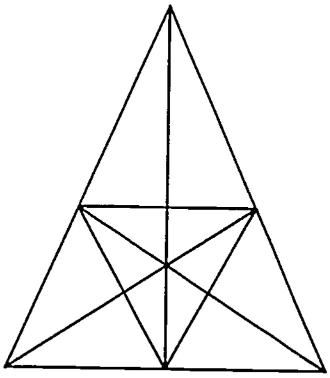
ANSWER:



9. Mr. Brook wants to build a container with a volume of 64 cm^3. He can build it either in the shape of a cube or of a rectangular prism. The rectangular prism would have to be 8 cm high and 4 cm wide. Which would have the smallest surface area. and by how much?

ANSWER:
cube, 16 m^2

10. How many triangles can you count?

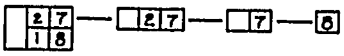


ANSWER:
47

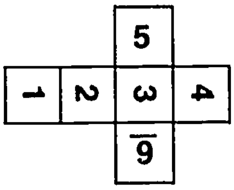
11. Describe how to fold this "map" so that numbered sections lie on top of one another in order from 1 to 8.

3	4	2	7
6	5	1	8

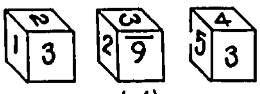
ANSWER:



12. The first figure below shows the network for a certain cube. One of the figures below the network is a drawing of this number cube. Which one is it?

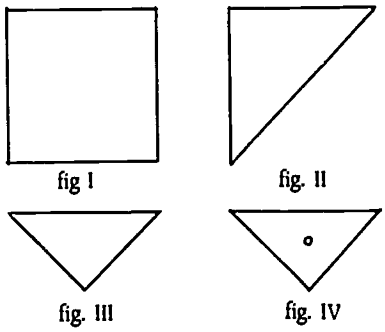


ANSWER:



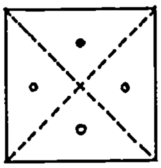
(✓)

13. A square piece of paper was folded as drawn in Figure I. II. and III. Figure IV shows where a small hole was punched through the paper.

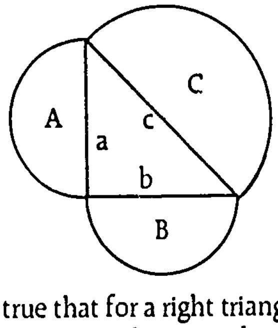


If the paper was completely unfolded which shows how it would look?

ANSWER:



14. Pythagoras showed that for a right triangle $a^2 + b^2 = c^2$



Is it true that for a right triangle the area of the semicircles are related by Area A + Area B = Area C?

ANSWER:
Yes

15. A number of girls are standing in a circle. They are evenly spaced and are numbered beginning with 1. Number 5 is opposite Number 16. How many girls are there in the circle?

ANSWER:
22 girls

16. A snail starts at the bottom of a well 16 m deep and crawls up 4 m each day. Each night. however. the poor thing slips back 3 m. How long will it take the snail to reach the top of the well?

ANSWER:
13 days

17. Two vertical poles. 10 m high and 15 m high. stand 12 m apart. Find the distance. in metres. between the tops of the two poles?

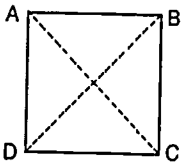
ANSWER:
13 m

18. Jack ran from Bixley to Quixley and then back to Bixley. Jill ran from Quixley to Bixley and back to Quixley. They ran at constant but different speeds along the same road. When they met the first time they were 500 m from Bixley. When they met the second time. they were 300 m from Quixley. How far was it from Bixley to Quixley?

ANSWER:
1200 m

ALGEBRA

1. In the figure. ABCD is a square having sides of length x cm and diagonals of length y cm. If $y^2 = (3x + 1)(x - 5) + 54$. what is the value of x. in centimetres?



ANSWER:
7

2. Find both values of x which satisfy: $x + 1/x = 5 + 1/5$

ANSWER:
5, $1/5$

3. A circular track is 1 000 m in circumference. Cyclist A races around the track at the rate of 700 m/min . cyclist B races at the rate of 800 m/min. and cyclist C races at the rate of 900 m/min. If the three cyclists start from the same position at the same time and cycle in the same direction. what is the least number of minutes it must take before all three are together again?

ANSWER:
10 minutes

4. The members of an Olympiad team contributed a total of $1.69 for refreshments for their weekly practice session. Each member contributed the same amount and paid for her or his share with exactly five coins. How many nickels were contributed by all the members together?

ANSWER:
26 nickels

5. In a math contest of 10 problems. 5 points were scored for each correct answer and 2 points were deducted from the score for each incorrect answer. If Steve worked all 10 problems and scored 29 points. how many correct answers did he have?

ANSWER:
7 correct

6. How many persons will it take to sort 400 boxes of stamps in 400 min. if 4 persons can sort 4 boxes in 4 minutes?

ANSWER:
4

7. Maple syrup sap is 3% pure maple syrup and 97% water. How much water needs to be evaporated from 500 L of maple sap to make a 30% maple syrup solution?

ANSWER:
450 L

8. Looking at the playground. I saw boys and dogs. Counting heads. I got 22. Counting legs I got 68. How many boys and how many dogs were there?

ANSWER:
10 boys, 12 dogs

9. Name 6 consecutive multiples of 5 which. when added together. make a sum between 340 and 350.

ANSWER:
45, 50, 55, 60, 65, 70

10. Two boys each had a different number of cars. Tim said. "If you give me 5. I'll have as many as you." Bill said. "If you give me 5. I'll have twice as many as you". How many did each have?

ANSWER:
25, 35

11. Mary had 20 coins. When she counted them she found she had the same value as if she had all nickels. but she had only 1 nickel. What coins did she have?

ANSWER:
1 nickel, 3 dimes, 15 pennies,
1 half-dollar

12. The sum of squares of two numbers is four less than the sum of one hundred plus half a hundred. What are the two numbers?

ANSWER:
5, 11

13. The H-B shop is having a sale on pencils
4 short pencils for 10¢
2 medium pencils for 10¢
1 long pencil for 10¢
What 20 pencils can Nancy buy for $1.00?

ANSWER:
12 s, 2 m, 6 l
8 s, 8 m, 4 l

14. A farmer buys 100 live animals for $100. How many of each does he buy if chickens are 10¢ each, pigs are $2 each, sheep are $3 each and cows are $50 each?

ANSWER:
1 cow, 4 sheep, 15 pigs, 80 chicks
OR 14 sheep, 26 pigs, 60 chicks

15. A man borrowed $3 500 and a year later paid back the loan plus interest with a cheque for $4 200. Find the annual rate of interest, in percent paid for the loan.

ANSWER:
20%

16. Each one of a group of ladies bought one item at a swap meet. All of the items sold for the same price. There was no tax. The total paid by the ladies was $2.03. If each item cost more than 10¢, how many women were there?

ANSWER:
7 women, 29¢

17. A donkey and a horse were carrying bales of hay. If the horse gave the donkey one bale, they would have the same amount. If the donkey gave the horse one of his bales the horse would have twice as many as the donkey. How many bales was each carrying?

ANSWER:
d-5, h-7

18. A man gave 4¢ each to some children. Had he given them 7¢ each, it would have taken 36¢ more. How many children were there?

ANSWER:
12

19. A tank of water with the plug removed empties at a uniform rate in 15 min. With the plug in, it fills at a uniform rate in 12 min. How long (in minutes) will it take to fill if the plug is removed and the tap is turned on.

ANSWER:
60 minutes

20. A frog ate 104 bugs in 4 days. Each day he ate 10 more than on the previous day. How many did he eat each day?

ANSWER:
11, 21, 31, 41

21. To encourage Jack to work his math problems correctly, his dad said he would pay him 10¢ for each correct answer and fine him 5¢ for each incorrect answer. If he received a dime after doing 25 problems, how many did Jack get right?

ANSWER:
9 correct 16 wrong

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A FRAMEWORK FOR MULTIPLE-CHOICE TESTS

Charles, Lester & O'Daffer, 1984

PART 1

THINKING PROCESSES	PROBLEM TYPES		
	1-STEP	MULTIPLE-STEP	PROCESS
1. Understand the question			
2. Understand the conditions and variables			
3. Select needed data			
4. Select appropriate subgoals and an appropriate solution strategy			
5. Correctly implement the solution strategy and attain subgoals			
6. Give an answer in terms of the data given.			
7. Evaluate the reasonableness of the answer			

PART 2

	PROBLEM TYPES		
	1-STEP	MULTIPLE-STEP	PROCESS
Get the correct answer			

EVALUATING PROBLEM SOLVING PERFORMANCE

BEYOND A CHECK
FOR THE CORRECT ANSWER

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Paper prepared for the annual meeting of The National Council of Teachers of Mathematics, San Antonio, Texas, April, 1985.

Assumptions Related to Evaluating Problem Solving Performance

1. Performance is influenced by attitudes and beliefs.
2. Constraints of the evaluation situation influence performance.
3. Individual interviews may be the most valid method to assess thinking processes.
4. The ability to get more correct answers is a desirable goal.
5. The purpose of evaluating is to make instructional decisions.

Realities of Evaluation in the Classroom

1. "Teachers don't make decisions at the .05 confidence level"?
2. Time does not permit student interviews to be the primary assessment technique.
3. Teachers have the opportunity for repeated observations of students.
4. Many teachers want to or are required to give grades.

Methods of Evaluating Problem Solving Performance

1. Individual student interviews
2. Multiple-choice tests
3. Open-ended tests for the correct answer
4. Holistic evaluation methods

IOWA PROBLEM SOLVING TEST

Get to Know the Problem

- You threw a baseball 5 meters farther than Tom did. You want to know how far your throw went. You could solve the problem if you knew:

  a) Tom's throw was 5 meters shorter than yours
  b) A meter is a little more than a yard
  c) A baseball is 8 inches around
  d) Tom's throw was 34 meters

Look Back

- In baseball it is 90 ft. from home plate to first base. To find how many yards it is from home plate to first base. divide 90 by 3 and the answer is 30 yards. Which problem below can be solved using exactly the same steps?

  a) Three identical baseball gloves cost $ 90 together. How much does one glove cost?
  b) A baseball costs $ 3. how much do 90 baseballs cost?
  c) There were 90 baseballs in a large box. The coach put in 3 more. How many are now in the box.
  d) There were 90 baseballs in a large box. The coach took out 3. How many are left in the box?

Lane County Mathematics Project

- Which two numbers continue the pattern?

  4. 5. 7. 10. 14. ____. ____

  a) 18. 23 b) 19. 25 c) 18. 22 d) 19.24

- Jim and Joe each have the same amount of money. Then Jim gives Joe 5¢. Jim now has half as much as Joe. How much did each boy have to begin with?

  a) 5¢ b) 10¢ c) 15¢ d) 20¢

CALIFORNIA ASSESSMENT PROGRAM

Problem Formulation

- Which problem is suggested by the problem below?

  a) What is the diameter of a 6-inch circle?
  b) What is the area of the floor?
  c) How much will it cost for 8 items?
  d) How many halves are there in 6 apples?

Interpretation

- A new school building has 40 classrooms. The school ordered 28 desks for each classroom. 1200 desks were delivered. Which of these is true?

  a) The correct number of desks was delivered
  b) Too many desks were delivered
  c) Not enough desks were delivered
  d) The school needed 200 more desks

ILLINOIS INVENTORY OF EDUCATIONAL PROGRESS

Understand the question in the problem

- Which statement is another way of asking what you are trying to find out in this problem?

  Problem: Jack and Denise divided the construction paper evenly among the 24 children in the room. Altogether they gave out 144 pieces of paper. How many pieces of paper did each child receive?

  a) How many pieces of construction paper did Jack and Denise give out altogether?
  b) How many pieces of construction paper was each of the 24 children given?
  c) How many children received the same number of pieces of construction paper?
  d) How many pieces of construction paper did Jack and Denise receive altogether?

Select appropriate subgoals to be obtained and an appropriate solution strategy

- Which is an appropriate first step in solving the following problem?

- **Problem:** Box seats cost $8 each and balcony seats costs $5 each. A person ordered 3 box seats and 6 balcony seats. What was the total cost for the tickets?

  a) Find the total number of seats
  b) Find the total cost for the balcony seats
  c) Find the total cost for the tickets
  d) Find the total number of tickets and the total costs of the tickets

Evaluate the reasonableness of the answer

- Which statement best tells why the answer given is NOT reasonable?

  Problem: Steve. Mike. and Holly took turns driving home from camp. Holly drove 80 km more than Mike. Mike drove 3 times as far as Steve. Steve drove 50 km. How long was the total drive?

  ANSWER: 130 km.

  a) Mike drove 150 km himself
  b) Steve. Mike. and Holly all took turns driving
  c) Steve drove 50 km himself
  d) Because you want to find the total distance

- Eight people signed up for the tennis tournament. How many matches were played in the tournament?

Possible Solutions:	Number of People	Number of Matches
A B C D E F G H	2	1
B C E E F G H	3	3
C D E F G H	4	6
D E F G H	5	10
E F G H	6	15
F G H	7	21
G H	8	28
H		

7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 28

There would be 28 matches played in the tournament.

Tennis tournament – Solution 1

Tennis tournament – Solution 2

HOLISTIC EVALUATION

Meaning of Holistic

"Holistic" is an adjective that comes from the noun "holism". Holism is a philosophy or theory that whole entities (like a student's solution to a problem) have an existence greater than the mere sum of their parts.

Types of Holistic Evaluation Techniques

1. analytic scales
2. dichotomous scales
3. general impression marking
4. primary trait scoring
5. focused holistic scoring

Analytic Scales

An analytic scale is a list of prominant features or characteristics of a solution with a numerical weighting attached to each feature Here is an example (Charles & Lester. 1982)

Understanding the Problem

0 – Completely misinterprets the problem
1 – Misinterprets part of the problem
2 – Complete understanding of the problem

Solving the Problem:

0 – No attempt or a totally inappropriate plan
1 – Partly correct procedure based on part of the problem interpreted correctly

2 – A plan that could lead to a correct solution with no arithmetic errors

Answering the Problem:

0 – No answer or wrong answer based on an inappropriate plan
1 – Copying error: computational error: partial answer for problem with multiple answers: answer labelled incorrectly
2 – Correct answer

Focused Holistic Scoring

"Focused holistic scoring" is a "holistic" method because it focuses on the total solu tion. and it is "focused" because it evaluates perfomance in tems of well-defined criteria. Typically. criteria are established for score points from 0 to 4. Following is an example (Charles. 1985).

FOCUSED HOLISTIC SCORING FOR PROCESS PROBLEMS

0 POINTS – Not Scoreable

These papers have any of the following characteristics:
- These are blank papers.
- The data in the problem may be simply recopied. but nothing is done with the data.
- These are papers which have an incorrect answer and no other work shown on the paper.

1 POINT – Unacceptable

These papers have any of the following characteristics:
- There is a start toward finding the solution. beyond just copying data from the problem.
- An inappropriate strategy is started. but not carried out. Also. if an inappropriate strategy is started and abandoned. there is no evidence the students tumed to another strategy. It appears that the student tried one approach that did not work and then "gave up".
- There is some work on the paper. but there is no evidence of any strategies. There is no logical organization to the work. The work looks as if random guesses were tried.
- The student just perfomed one or more computations trying to arrive at an answer that seemed reasonable when multiple computations were not called for in the solution.
- The student tried unsuccessfully to reach a subgoal.

2 POINTS – Poor

These papers have any of the following characteristics:
- There is some evidence of trying to use a strategy to find a solution.
- Infomation from the problem is used properly in the solution attempt. This would suggest students appeared to have understood something about the problem.
- The student used a strategy but a totally inappropriate one.
- An appropriate strategy was used. but it was not carried out far enough to help the student to find the solution (e.g.. the first 2 entries in an organized list).
- The student successfully reached a subgoal.
- Appropriate strategies were selected and implemented. but there is no answer to the problem.
- These papers may be ones with the correct answer. but the solution attempt is not systematic. The student's work appears random. The answer is correct. but there is no logical approach reflected by the student's work.

3 POINTS – Good

These papers have any of the following characteristics:
- The student has implemented a solution strategy that could have led to the correct solution. but the student misunderstood part of the problem or ignored a condition in the problem.
- There may be some evidence of temporarily pursuing an inappropriate strategy. but the student eventually used an appropriate one.
- All subgoals have been reached but the student incorrectly answered the problem for no apparent reason.
- Appropriate solution strategies were properly applied to arrive at an answer. but the answer was not given in terms of the data in the problem (e.g.. incorrect units were given). Also. this error appears to be one of misunderstanding not a careless error.
- A correct numerical answer with no units

is given and there is some evidence the student may not have understood what entity or entities the answer represented.
- The correct answer is given, and there is some evidence that appropriate solution strategies were selected. However, the implementation of the strategies is not clear.

4 POINTS — Excellent

These papers have any of the following characteristics:
- The student fully understood all of the information in the problem.
- The student selected an appropriate solution strategy or strategies.
- The student may have made an error in carrying-out the solution strategy. However, these errors do not reflect misunderstanding of either the problem or lack of knowledge how to implement the strategy, but rather they might include a copying error or a computational error.
- If appropriate, these papers might show evidence of an attempt to check student's own work.
- The answer is correct and was found through the use of appropriate solution strategies but the answer was not labelled correctly (i.e., given in terms of the data given in the problem) and occurred because of a careless error, not because of a misunderstanding.
- Appropriate strategies were selected and implemented. The correct answer was given in terms of the data in the problem.

SOME GUIDELINES FOR EVALUATING PROBLEM SOLVING PERFORMANCE AND ATTITUDES

1. Evaluation is not synonymous with grading. All teachers should have a plan for evaluating problem solving performance and attitudes.
2. Evaluate thinking processes as well as the correct answer.
3. Use observations of students' work.
4. Match evaluation to instructional content and emphases.
5. Assess attitudes and beliefs as well as performance.
6. Interview students individually if possible.
7. Every student does not have to be evaluated in every problem solving experience.
8. Inform students of your evaluation plan.

GOALS FOR THE TEACHING OF PROBLEM SOLVING (Charles et. al., 1985)

1. Improve students' willingness to try problems and improve their perseverance when solving problems.
2. Improve students' self-concepts with respect to their abilities to solve problems.
3. Make students aware of problem-solving strategies.
4. Make students aware of the value of approaching problems in a systematic manner.
5. Make students aware that many problems can be solved in more than one way.
6. Improve students' abilities to select appropriate solution strategies.
7. Improve students' abilities to implement solution strategies accurately.
8. Improve students' abilities to monitor and evaluate their thinking while solving problems.
9. Improve students' abilities to get more correct answers.

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