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ABSTRACT

This report was part of the total scope of The Microcomputers and Middle School Problem Solving project. It employed representational models during problem solving in order to study: (1) observable patterns in self-directed use of representational models; (2) correlations between use of models and problem-solving ability; and (3) the motives of students for choosing to use representational models. Seventh-grade students were given 25 number theory problems in the context of some meaningful situation, as opposed to a mathematically abstract situation. A set of representational models was provided for voluntary use. The project covered the entire year and took place in three stages. A Cold Test, a Pretest, and a Posttest were administered to the students. Attitudes and other information were gathered from questionnaires given before and after the study. Each ability group used models a greater percentage of time on the Posttest than on the Cold Test, and the percentage of problem solving success increased from the Cold Test to the Posttest for each ability group. Above average and average ability males used representational models more on the Pretest and Posttest than above average and average ability females. The more concrete the model, the more it was used. Problems, including Cold Problem solving pretest and problem solving pretest and posttest, questionnaires, and computer menu are appended. (LMO)

Representational Models in Middle School Problem Solving

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Representational Models in Middle School Problem Solving

The use of representational models, such as counters, blocks, and pictures, while learning mathematical concepts and operations continues to be of interest to researchers (Khoury & Behr, 1982; Post 1980; Threadgill-Sowder & Juilfs, 1980) some of whom (Lesh, Landau, & Hamilton, 1983) have recently focused on the influence of these models during the problem-solving process. The trend in problem-solving research had been on finding traits and practices of successful problem solvers to determine what the teaching of problem solving should include (Lester, 1983). With this trend, however, came the realization that many of these traits might not be teachable. There have since been several (e.g., Hart, 1984; Lesh, Landau, & Hamilton, 1983; Schoenfeld, 1983a) who have been approaching problem-solving research through the "structure of cognitive processes and abilities" (Lester, 1983, p. 252). This shift away from researching competence toward researching performance holds promise for learning about the characteristics

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of the problem representations that average problem solvers find useful.

The notion of conceptual "model" (or "structure") is central to understanding the performances of students during the problem-solving process. Lesh et al.(1983) define a conceptual model as an adaptive structure consisting of within-concept systems, between-concept systems, systems of representations, and systems of modeling processes (p. 264). In other words, a conceptual model is the dynamics of using the knowledge of what something means; what it means in relation to what other things mean; the ways to represent these ideas or actions, like written symbols, spoken language, pictures, concrete models, graphs, etc.; and the processes needed to interface existing understandings, real situations, and representations.

The choice of representational model and the action on that model influence the level and quality of the understanding of underlying structures of the mathematical idea needed in the problem-solving process. This is supported by the recommendations for mathematics curriculum and instruction in the report of a National Council of Teachers of Mathematics (NCTM) conference on the impact of technology on school mathematics, (Corbitt, 1985), which states that the middle school mathematics curriculum should build on the recommendations for the K-4 program as well as consider the middle grades recommendations listed. The K-4 recommendations include "experience with physical manipulations and other concrete representations" (p. 245) with an emphasis on meaning and understanding. The middle grades recommendations include "emphasis on the development of 'number sense' . . . [and] nontraditional methods of problem solving" (pp. 245-246).

Schoenfeld (1983b) has expressed concern that the current "problem-solving movement" may take us to a "back-to-back-to-basics movement" if mathematical understandings are not developed in the process. Strengthening mathematical understandings during problem-solving activities is a function of an insightful teacher and the representational models available for use (Lesh, 1982; Lesh & Schultz, 1983; Schultz, 1984).

This study has taken the position that middle school problem solvers are in the process of refining conceptual understandings needed to solve problems. Since middle school children are still in transition from the concrete operational to the formal operational stage of cognitive development, representational models were used. This report on representational models is part of the total scope of The Microcomputers and Middle School Problem Solving (MMSPS) project (Grant No. SED-80-166561, Kantowski & Schultz). My interest in this collaborative effort centered on a search for (a) observable patterns in self-directed use of representational models, (b) correlations between use of models and problem-solving ability, and (c) the motives of students for choosing to use representational models. The Soviet teaching experiment, a nonexperimental method by Western standards, was used where seventh graders' performance on nonroutine number theory problems was studied, while representational models were available for voluntary use.

Method

Subjects

All three seventh grade sections of a middle-class suburban Atlanta public school participated in the project, which became part of their respective classroom routines. However, data for just 34 of the 78 students were collected for the purposes of the study. These students were in a departmentalized setting in which they attended mathematics class for one hour each day in the same classroom with the same teacher. Eleven subjects were classified as above average, 13 as average, and 10 as below average in mathematics achievement according to their California Achievement Test results and their mathematics placement levels available at the beginning of the school year. Students were eager and enthusiastic about participating. And there was a great deal of parent support due to the microcomputer component in the study.

Materials and Instruments

Number theory problems. All but one of the 25 number theory problems presented were in the context of some meaningful or hypothetically meaningful situation, as opposed to a mathematically abstract situation. (See Appendix for complete listings of problems.) One of the meaningful/hypothetically meaningful problems was as follows: "E.T. was sitting in the closet eating Reese's Pieces. When he started he had 48 Pieces. When he stopped he had 26 Pieces which he put into piles. The first pile contained only orange Reese's Pieces. The second pile contained only brown Reese's Pieces. The third pile was yellow, and the fourth was green. He noticed he had 3 more orange than brown, 3 more brown than yellow, and 3 more yellow than green. How many Reese's Pieces were in each pile?" The mathematically abstract problem was: "The perimeter of a right triangle is 24 cm, and the area is 24 sq cm. What are the lengths of the sides of the triangle?"

All problems presented were in sets of five to nine problems per set, where each set was for instructional, computer, or testing purposes. Five were given during the Cold Problem-Solving Pretest (herein referred to as Cold Test) at the onset of the project; five were given during instruction on problem solving; five during the Problem-Solving Pretest (herein referred to as Pretest), which were the same as the Cold Test problems; nine were given as computer problems (however, technical difficulties eliminated one problem)--detailed discussion on computer problems is under microcomputer problem section; and six were given as the Problem-Solving Posttest (herein referred to as Posttest).

Each problem in a set corresponded to at least one other problem in the other sets according to mathematical structure. Table 1 presents the names of the problems organized according to their structure and purpose.

Representational models. A set of representational models (referred to as "provided materials" throughout the study) was provided for the subjects for voluntary use for each problem given. Table 2 provides descriptions and classifications of models made available for each problem. Models are classified according to whether they are directly meaningful, indirectly meaningful, or nonmeaningful, as well as concrete, pictorial, or symbolic.

The concrete-pictorial-symbolic classification has long been a useful reference to teachers and researchers. Post (1980) summarized the thinking of Piaget, Bruner, and Dienes on this organizational scheme. Note that concrete in the context of this study does not necessarily mean manipulative. The term manipulative is applied to a model on which some action is performed. The term "concrete manipulative," therefore, means a three-dimensional object on which some action is performed.

The directly meaningful, indirectly meaningful, and nonmeaningful classification is then applied across the concrete, pictorial, and symbolic types of models. That is, a concrete model can be directly meaningful, such as the Reese's Pieces were in the E.T. problem. In general, a directly meaningful model is

Table 1

Problems used in the Microcomputers and Middle School Problem-Solving Project

Lessons	Cold Problem-Solving Pretest and Problem-Solving Pretest	Computer	Problem-Solving Posttest
		Little Bird Quarks and Yippies	John's Boxes
Mark's Train	Douglas's Chairs	Goats and Ducks	Bikes and Trikes
E.T.	Steve's Group	Globs in a Goblet	Bottle Caps
Calvin's Family	Sam's Ride	Upright Triangle Lucinda's Children	Hiram's Ride
Rhonda's Pennies	Della Fever	Buttons Rows of Robots	Parking Cars
Andrea's Party	Susan's Party	Fueling Around	Fantastic Fifteen

Table 2

Representational models used in the Microcomputer and Middle School Problem-Solving Project

Problem	Model	Meaningfulness	Concreteness
Lesson Problems:			
Andrea's Party	Photocopied tickets	Indirect	Pictorial
Rhonda's Pennies	Pennies	Direct	Concrete
Calvin's Family	Number tiles	Indirect	Symbolic
Mark's Train	Cuisennaire Rods	Non-	Concrete
E.T.	Reese's Pieces	Direct	Concrete
Cold Problem-Solving Pretest and Problem-Solving Pretest:			
Susan's Party	Smiley face pictures	Indirect	Pictorial
Della Fever	Strips of squares	Non-	Pictorial
Sam's Ride	Number tiles	Indirect	Symbolic
Douglas's Chairs	Pictures of chairs	Indirect	Pictorial
Steve's Group	Pictures of running shoes	Indirect	Pictorial

(table continues)

Problem	Model	Meaningfulness	Concreteness
Problem-Solving Posttest			
John's Boxes	Sheet sectioned into six parts	Non-	Pictorial
Bikes and Trikes	Pictures of bicycles and tricycles	Indirect	Pictorial
Bottle Caps	Bottle caps	Direct	Concrete
Hiram's Ride	Number tiles	Indirect	Symbolic
Parking Cars	Play cars	Indirect	Concrete
Fantastic Fifteen	Blocks	Non-	Concrete
Computer Problems:			
Quarks and Yippies	Pictures of Yippies, plastic containers	Indirect	Pictorial
Lucinda's Children	Number tiles	Indirect	Symbolic
Rows of Robots	Blocks	Non-	Concrete
Fueling Around	Blocks	Non-	Concrete
Buttons	Bottle caps	Non-	Concrete
Upright Triangle	Paper strips	Non-	Pictorial
Globs in a Globlet	Cups and styrofoam pieces	Non-	Concrete
Goats and Ducks	Pictures of goats and ducks	Indirect	Pictorial
Little Bird	Pictures of birds, paper squares	Indirect	Pictorial

the actual object which a problem is about. Indirectly meaningful models are representations of real objects, such as the picture of bikes in the Bikes and Trikes problem. The significate (Flavell, 1963) of nonmeaningful models, on the other hand, is an altogether different thing from the model itself. For example, blocks were used to stand for people in the Fantastic Fifteen problem.

Questionnaires. A General Questionnaire was given before and after the study for the sake of obtaining background information on the subjects and, especially, their thoughts and feelings about problem solving, representational models, and microcomputers. Questions required responses to fit within a 4-point Likert-type scale, where 1 meant none, 2 meant very little, 3 meant pretty much, and 4 meant a lot. Responses to Items 10 and 11 are relevant to this report. A Post-Attitude Questionnaire using the same 4-point scale was administered at the close of the project, where items of concern to my investigation were Items 9 through 16. Items on this questionnaire sought reactions to the project itself. (See Appendix for relevant questions from each questionnaire.)

Microcomputer problems. Even though the microcomputer problem data are not the subject of this report, the following description of the microcomputer problem solving is presented below in order to offer the reader a more complete view of the study.

Nine problems were programmed on nine diskettes for student use in this project. Once a diskette was booted, a menu (see Appendix) requiring a Brite Pen to operate appeared. The student would first request to SEE PROBLEM, and then would proceed to use hint selections as he or she solved the problem. Each student was assigned a data diskette. All menu choices and responses were recorded for each problem worked on the data diskettes of the 34 subjects. The nonsubjects participating in the project were assigned to mock data diskettes on which no data were actually saved. Of particular importance to this report were the students' use of pictures, diagrams, and provided materials (representational models) which were routinely within reach in a labeled box on the computer stand. The provided materials were available independently of the software program and students had to remember to "let the computer know" they had used them by touching USE PROVIDED MATERIALS on the menu, and to mark their computer worksheet accordingly when they used the materials. The pictures and diagrams available through hint selection in the computer varied from static to animated formats. Many animated formats were under the control of the student. That is, the student could initiate or terminate a pictorial or symbolic demonstration of a problem. Students recorded all pencil-and-paper work on a problem worksheet prepared for each problem. More than one session with a problem was common for a student.

Procedures

Phase 1. The project took the entire 1982-1983 school year and occurred in three phases. On the first day of the first phase of the study we introduced ourselves (Hart [then Graduate Research Assistant], and myself), gave an orientation to the project, and administered the Pre-General Questionnaire to each seventh grade class.

This was followed the next day with the Cold Test, with no prior instruction from us about problem solving. Each student had envelopes of representational models available within reach for voluntary use. Specific models were designated for each problem. Students worked independently and had the entire hour to work the test.

A two-week instructional period followed in which students were given a series of lessons on number theory, heuristics, representational models, and the microcomputer. The Pretest, which had the same problems as the Cold Test, was given immediately after the two-week instructional period. This marked the end of the first phase of the project.

Phase 2. During the next eight months, students individually worked nine problems at one of the two microcomputer stations in their classroom. Their routine was to obtain their data diskette and current problem worksheet and problem diskette. They would then proceed to work the problem on the computer using the hint menu and provided materials when desired, recording their pencil-and-paper work on their worksheet.

Due to the number of students and having to participate in their regular mathematics class as well, they averaged one problem a month. The classroom teacher, who served as a Teacher Researcher (for which she was given a modest honorarium), assisted with the record keeping and logistics when one of the project staff was not present. Otherwise, a Computer Assistant (student selected by Teacher Researcher from each class), managed the flow of computer work in their respective classrooms and reported to the Teacher Researcher and project staff.

Phase 3. The third phase of the study consisted of the Posttest with models available for voluntary use, the Post-General Questionnaire, the Attitude Questionnaire, and individual interviews with selected students. This phase was completed within a week.

Results

Students coded each problem on each test indicating if they had used the "provided materials" available for that problem. We coded each solution attempt according to whether the student (a) abandoned the process, (b) reached an impasse, (c) reached a correct final result, (d) reached correct intermediate results, or (e) reached an incorrect final result (Lucas, 1979). To quantify this data we assigned a score of 1 if the student used materials and a score of 0 if the student did not use materials. Likewise, we assigned a score of 1 if the student reached a correct final result, .5 if the student reached a correct intermediate result, and 0 if the student reached an impasse or an incorrect final result.

Table 3 summarizes the use of representational models and problem-solving process scores according to test, ability groups, and sex. The representational models percentages indicate the proportion of problems for which students used provided materials in relation to all the problems attempted in a category. The process score percentages represent the proportion of average correctness of problems attempted on a test in relation to the best score possible. These descriptive data serve to detect patterns of self-directed use of representational models and the relationship between use of models and problem-solving success during testing.

Each ability group used provided materials a greater percentage of time on the Posttest than on the Cold Test, with the below average group showing the greatest increase in usage from 41.8% to 75%, whereas the average ability group increased from 33.8% to 60.6%, and the above average group from 44% to 55.6%. Likewise, each ability group used provided materials a greater percentage of time on the Posttest than on Pretest, however, with the below average group showing the least increase

Table 3

Percentages of use of representational models and problem-solving process scores according to test, ability, and sex

		Use of Representational Models			Problem-Solving Process Score		
		Sex			Sex		
Ability Group	Problem-Solving Test	Male (n_m)	Female (n_f)	Total (n_{m+f})	Male (n_m)	Female (n_f)	Total (n_{m+f})
		(2)	(8)	(10)	(2)	(8)	(10)
Above Average	Cold Test	20.0	40.0	44.0	35.0	20.0	23.0
	Pretest	20.0	15.0	16.0	75.0	33.8	42.0
	Posttest	83.3	47.6	55.6	91.7	72.6	76.9
		(5)	(8)	(13)	(5)	(8)	(13)
Average	Cold Test	44.0	27.5	33.8	8.0	26.3	19.2
	Pretest	32.0	20.0	24.6	30.0	28.8	29.2
	Posttest	75.0	52.4	60.6	83.3	54.8	65.2
		(7)	(4)	(11)	(7)	(.4)	(11)
Below Average	Cold Test	42.9	40.0	41.8	4.3	0.0	2.7
	Pretest	40.0	60.0	47.3	17.1	15.0	16.4
	Posttest	69.5	83.3	75.0	33.4	43.8	37.5

(table continues)

		Use of Representational Models			Problem-Solving Process Score		
		Sex			Sex		
Ability Group	Problem-Solving Test	Male (n_m)	Female (n_f)	Total (n_{m+f})	Male (n_m)	Female (n_f)	Total (n_{m+f})
		(14)	(20)	(34)	(14)	(20)	(34)
Total	Cold Test	40.0	35.0	37.0	10.0	18.5	15.0
	Pretest	34.3	26.0	29.4	30.0	28.0	28.5
	Posttest	63.6	59.0	63.9	50.7	53.0	59.4

in usage from 47.3% to 75%, the average ability group showing the next highest increase from 24.6% to 60.6%, and the above average group showing the greatest increase from 16% to 55.6%. When comparing the usage of provided materials by the ability groups from the Cold Test to the Pretest, it is found that there was a decrease in use by the average and above average groups, with the above average group showing the greatest decrease from 44% to 16%, the average group decreasing next from 33.8% to 24.6% percent, and the below average group showing a slight increase in use from 41.8% to 47.3%.

Correspondingly, the percentage of problem-solving success (based on a process/outcome rating) increased from the Cold Test to the Posttest for each ability group with the above average group showing the greatest improvement from 23% to 76.9%, the average group from 19.2% to 65.2%, and the below average group from 2.7% to 37.5%. The percentage of correct answers from the Pretest to the Posttest also increased for each ability group with the above average group again showing the greatest increase from 42% to 76.9%, the average group from 29.2% to 65.2%, and the below average group showing the least increase from 16.4% to 37.5%. Finally, when comparing the percentages of success from the Cold Test to the Pretest, each group showed an increase with the above average group having a slightly greater increase from 23% to 42%, the average group from 19.2% to 29.2%, and the below average group showing the smallest increase from 2.7% to 16.4%.

Overall, males used representation models slightly more than females. Specifically, they used representational models 5% more of the time on the Cold Test, 8.3% more of the time on the Pretest, and 4.6% more of the time on the Posttest. However, overall they did not have greater problem-solving success than the females, except for a 2% advantage on the Pretest.

There is also a trend looking at ability groups and sex. Above average and average ability males used representational models more on the Pretest (10% and 12% more, respectively) and Posttest (35.7% and 22.6% more, respectively) than above average and average ability females. Moreover, above average males had greater problem-solving success than above average females on the Cold Test, the Pretest, and the Posttest (15%, 41.2%, and 18.8% more, respectively). Average males had greater problem-solving success than average females on the Pretest and Posttest (1.2% and 28.5% more, respectively). There is less evidence of any trend with the below average males and females, however females did use representational models 13.8% more than males on the Posttest and were 10.4% more successful with problem solving--a reverse trend from the above average and average ability group performance.

While observing subjects' working problems during each of the classes' testing periods, other patterns seemed to emerge. It appeared that there was a propensity toward some types of models than others. Pursuant to this observation was an analysis across types of models. Table 4 shows the average percentages of use of representational models and problem-solving success across concreteness and meaningfulness of models for all tests.

The 77% use of concrete models, 43.7% use of pictorial models, and 21.8% use of symbolic models shows the more concrete the model, the more it was used. Students had the greatest problem-solving success (61.1%) when concrete models were used. When looking at meaningfulness of the model, the average results show the more meaningful the model, the less it was used. However, when looking at meaningfulness across concrete collapsed with pictorial models, a different result appears. The uneven distribution of model types within and between cells, however, tempers any sweeping conclusions.

Discussion

It is not known whether the increases in correct answers from the Cold Test to Posttest and from the Pretest to Posttest were due to the corresponding increased use of provided materials, or not. Increased correct answers could be computer effect or interaction effect between computer and use of provided materials. More research would have to test this out. It is interesting to note, however, that the largest increase in use of provided materials by each group occurred from the Pretest to the Posttest, specifically, the below average group went up by 27.7

Table 4

Average percentages of use of representational models and problem-solving success across concreteness and meaningfulness of models for all tests

	Directly Meaningful		Indirectly Meaningful		Non-Meaningful		Average	
	Models	Success	Models	Success	Models	Success	Models	Success
Concrete	82.9	80.0	72.4	25.0	75.7	78.3	77.0	66.1
Pictorial	--	--	44.8	26.3	41.1	17.6	43.7	23.7
Symbolic	21.8	52.2	--	--	--	--	21.8	52.2
Average	37.1	59.1	48.3	46.6	49.8	32.8		

percentage points, the average group went up by 36 percentage points, and the above average group went up the most by 39.6 percentage points.

Some students said in an interview that after working on the computer problems with all those hints, they realized they could probably do the problems (on the Posttest) if they had some help. The only "help" available during the test was the supply of "provided materials," which many reached for. It is also interesting to note that the average and above average groups used provided materials the least during the Pretest after formal instruction on how to use them. Perhaps the instruction had less an impact on them than a personal realization that was expressed by a couple students above. The experience of feeling the need for some "help" may have been more influential on their decision to use materials than being told by a teacher that they needed it.

Many students and teachers are under the impression that manipulatives are only for primary grade students or low-ability students. The results of this study suggest that, though the models were found to be useful for the below average group on the Posttest, they appeared to be more useful for the average and above average groups on that test.

In response to the Pre-General Questionnaire given before any instruction occurred, the students on the average indicated a strong dislike (1.44 on a scale of 4, with 4 meaning "a lot") for such things as counters or blocks to help solve a problem. One boy from the above average group, however, made the point that "They aren't available." Two boys--one from the above average, the other from the average group--referred to such materials as "babyish." They much preferred to use just pencil and paper as indicated by a score of 3.2. After the study, the Post-General Questionnaire results showed a score of 3.48 for an even stronger preference for pencil and paper only and a score of 1.57 for a slight increased preference for manipulative materials. One girl from the above average group even said, it's "easier" to just use pencil and paper. Something, however, motivated these students to have a change of heart in actual practice--a change of heart they would not admit. Again, not being required to use the materials was a factor. By using the materials voluntarily, not because of any assignment to do so, they may have internalized their use and not thought of them as some external thing forced on them.

Responses to the Post-Attitude Questionnaire administered at the end of the project offer more insight. When asked if they had to solve problems without the microcomputer, would they use provided materials more often, their score of 2.2, though not overwhelming, shows an inclination toward change. Comments were that they are "helpful," "useful," and they "help me a little." Similar to a comment heard in an interview with a student, one child wrote, "As hard as the problems were, I think the materials were not needed. I think that with the computer alone, we are able to solve the problem." The insinuation here is that if there were no computer to help solve the problems, then the provided materials might be useful.

When asked if by participating in this project, they realized that using provided materials can be an important activity in solving problems, their average score was 2.0. Even though one child said, "I think it is an important activity for other people but not for me," others commented with "I learned that sometimes you need a diagram or object to help you," Yes even though I didn't use them that much, I still knew they were there," and ". . . provided materials can be useful." Finally, when asked if they would like to use such materials in their regular mathematics class, they responded with an overall average score of 1.8. As one child sadly put it, "I wouldn't have time." Others wouldn't have anything to do with them. There is the feeling that there is a dislike for things like manipulatives when required to use them. These "hard line" feelings have developed out of years of experience and a one-year project could not possibly break down the bias.

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Appendix

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I. Problems

II. Questionnaires

Lesson Problems

Andrea's Party

1. Andrea is planning a party, She will give tickets to the football game as prizes. There will be four prizes: (1) the fourth prize winner will receive one ticket to the game; (2) the third prize winner will receive two tickets to the game; (3) the second prize winner will receive three tickets to the game; and (4) the first prize winner will receive four tickets to the game. "Well," said Andrea, "that means I will need 10 tickets in all. I wonder how many people could get prizes if I had 66 tickets?" Can you tell Andrea how many people would receive prizes?

Rhonda's Pennies

2. Rhonda has been saving her pennies to purchase some new baseball cards for her collection. She can't decide if she wants all gold (which cost the most), all silver, or all blue cards (which are least expensive). If she buys 5 blue cards she has 5 pennies left over. If she buys 4 silver cards she has 2 pennies left over. If she buys 3 gold cards she has no pennies left. How many pennies did Rhonda start out with?

Calvin's Family

3. Calvin and his younger brother and twin sisters live at 990 Candler Street. "Isn't it funny," said Calvin, "there are four children in our family, and the product of our ages is the same as our house number!" Can you find the ages of Calvin and his brother and sisters?

Mark's Train

4. Ms. Aaron assigned the mathematics class to make a train of Cuisinaire Rods 36 units long using only 2 different colors of rods. Mark made a train using purple rods (4 units long) and dark green rods (6 units long). He used 7 rods to make his train. How many purple and how many dark green rods did Mark use?

E.T.

5. E.T. was sitting in the closet eating Reeses Pieces. When he started he had 48 pieces. When he stopped he had 26 pieces which he put into piles. The first pile contained only orange Reeses Pieces. The second pile contained brown Reeses Pieces. The third pile had yellow Reeses Pieces and the fourth had green Reeses Pieces. He noticed he had 3 more orange than brown, 3 more brown than yellow, and 3 more yellow than green. How many Reeses Pieces are in each pile?

Cold Problem-Solving Pretest
and
Problem-Solving Pretest

Susan's Party

1. Susan had a party for a new friend. There were eight people at the party (including Susan). Susan asked each person to shake hands with every person. How many handshakes were there?

Della Fever

2. Della Fever, disc jockey at Station WRTD, is shopping for carrying cases for her tapes. She has the choice of cases that hold 7, 8, 9, or 10 tapes each. If she bought 7-tape cases she would have 1 tape left over. If she bought 8-tape cases she would have 2 left over. If she bought 9-tape cases she would have 5 left over, and if she bought 10-tape cases she would have none left over. How many tapes does Della have?

Sam's Ride

3. Sam rides bus #252 to school. One day he noticed that if he multiplied his age by the age of his little sister (who wasn't even in school), and multiplied that result by his year in school, the product was his bus number. How old are Sam and his little sister and what is Sam's year in school?

Douglas's Chairs

4. At one table in the school cafeteria there is room for 20 people to sit. Some of the chairs have 3 legs and some have 4 legs. Douglas counted the legs of the chairs. He found that there were 71 legs in all. How many chairs had 4 legs and how many chairs had 3 legs?

Steve's Group

5. There are 3 groups (A, B, and C) of runners classified by time. Steve is in Group A. Group B has 10 more runners than Steve's group. Group C has 10 more runners than Group B. There are 69 runners in all. How many runners are in Steve's group?

Problem-Solving Posttest

John's Boxes

1. John had six boxes labeled one through six. In each box was a different present for each of his six children. John put the boxes in a row and began counting from left to right and back to the left, "one, two, three, four, five, six, seven, eight . . ." and so forth but he never counted the same box twice in a row. One of his children asked what he was doing. John said he was trying to find the box where he would land with the number 1001 because that box had the best present. John said whoever finds that box first can have the present in it. Susan said, "I know the answer right now." Can you figure out her answer?

Bikes and Trikes

2. Karen was standing on the corner counting bicycles and tricycles as they passed by. Her brother Vic asked her how many of each she had seen. Karen said, "I won't tell you how many of each I have seen, but I will tell you I've seen 22 wheels and 9 people." Can you figure out how many of each she had seen?

Bottle Caps

3. Daisy, Lynn and Steve were saving bottle caps for a contest. Altogether, they had saved 30 bottle caps, but Steve had 3 more than Lynn, and Lynn had 3 more than Daisy. How many bottle caps does each person have?

Hiram's Ride

4. One day while Hiram was riding his bike thinking about summer vacation, he realized that there were 52 school days left. "That's funny," he thought, "if I add my age to the number of miles I've biked, I get my lucky number 17. And if I multiply these same two numbers, I get 52 again." How many miles did Hiram bike and how old is he?

Computer Problems

Quarks and Yippies

1. Zeppo Quarkowski (you can call him Quark), must capture 500 Yippies for a new game and stuff each into one of six containers marked A, B, C, D, E, and F. Quark starts stuffing from left to right and then right to left and he never stuffs the same container twice in a row. Into which container will he stuff the 500th Yippie?

Rows of Robots

2. Jules has some robots. One day he decided to line them up in rows. When he placed them in two rows, he had one robot left over. When he placed them in three rows, again he had one left over. The same thing happened when he placed them in rows of four, five, or six. When he placed them in rows of seven, there were no robots left over. How many robots did Jules have?

Fueling Around

3. Qwrrl, pilot of the spaceship U.F.O. has 36 fuel containers that he needs to stack. However, he must stack them so that each layer has one few container than the layer below it. How many layers high will his stack be?

Lucinda's Children

4. Lucinda is 26 years old. She lives at 540 Littlewood Way. Her sister Betty has three children. One day Lucinda noticed that the sum of the ages of her sister's children is equal to her age (25) and the product of their ages is equal to her house number (540). How old are Betty's children?

Globs in a Globlet

5. One hundred globs (striped pieces of gelatin that have grown old and hard) were divided up and placed in five globlets which everyone knows are the only containers that globs can be stored in without making a mess. But each globlet does not contain the same number of globs. In fact the second globlet has two more than the first, and so on down the line. How many globs are there in the first globlet?

Parking Cars

5. Paulette, the parking lot attendant, was parking cars in a very small parking lot. She found that if she parked cars in 3 equal rows, she had 1 left over; if she parked them in 4 equal rows, she had 1 left over; and, again, if she parked them in 6 equal rows, she had 1 left over. What is the smallest number of cars she could have parked?

Fantastic Fifteen

6. The Fantastic Fifteen was a circus group of acrobats. One stunt they performed was standing on each other's shoulders so that each row of acrobats was one less than the row below it. How many rows of acrobats did this stunt have?

Upright Triangle

6. The perimeter of a right triangle is 24 cm and the area of the right triangle is 24 sq cm. What are the lengths of the sides of the triangle?

Goats and Ducks

7. One day, Susie and Bill went to the farm. In one pen they saw some goats and ducks. As they left the farm, Susie remarked that in that pen there were 44 feet and 30 eyes. Bill asked how many goats there were, but Susie told him to figure it out for himself. How many goats were there?

Buttons

8. Jill had a button collection. One day she decided to put them into equal piles. She found that if she used 4 piles, she had 1 extra button. If she used 12 piles, she had 5 extra buttons, but if she used 10 piles, she was short 3 buttons. There were fewer than 100 buttons, but exactly how many buttons were there?

Little Bird

9. The local gym has five small trampolines lined up in a row. Little Bird's favorite aerobic exercise is jummping from one trampoline to another. His goal is to be able to work up to jumping trampolines for 20 minutes. If he always starts on the first trampoline and can jump 45 trampolines in one minute, on which trampoline will he land after 10 minutes? after 20 minutes?

GENERAL QUESTIONNAIRE

(Items Relevant to Report)

Please circle the most appropriate choice for the following.

1 = None 2 = Very little 3 = Pretty much 4 = A lot

	None	Very Little	Pretty Much	A Lot
•				
•				
•				
10. When I am solving a mathematics problem I just work it with pencil and paper while I think.	1	2	3	4

Comment--

11. When I am solving a mathematics problem I like to use materials like counters or blocks to solve it.	1	2	3	4

Comment--

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•
•

ATTITUDE QUESTIONNAIRE

(Items relevant to report)

For each of the following questions, please circle the number that best corresponds to your response about what is asked. The numbers stand for the following responses:.

1 = None 2 = Very little 3 = Pretty much 4 = A lot

	None	Very Little	Pretty Much	A Lot
• • •				
9. I used the provided materials to solve many of the problems. Comment--	1	2	3	4
10. If I had to solve problems without the microcomputer, I think that I would use the provided materials more often. Comment--	1	2	3	4
11. The pictures and diagrams provided were a help to me in solving the problems. Comment--	1	2	3	4

12. If I were solving problems without the microcomputer, I would draw my own pictures and diagrams more often. 1 2 3 4

Comment--

13. Participating in this project made me realize that using provided materials can be an important activity in solving problems. 1 2 3 4

Comment--

14. Participating in this project made me realize that drawing diagrams can be an important activity in solving problems. 1 2 3 4

Comment--

15. Now that I've spent some time solving problems using the computer I think that I am able to solve problems better even without the computer. 1 2 3 4

Comment--

16. I would like to use materials like those provided for the project in my regular mathematics classes when we solve problems. 1 2 3 4

Comment--

Menu

ACTION WORDS	OBJECT WORDS	
☐ DEFINE	☐ ANSWER	☐ CALCULATOR
☐ DRAW	☐ DIAGRAM	☐ COMPUTER PROGRAM
☐ MAKE	☐ FACTORS	☐ PENCIL AND PAPER
☐ SEE	☐ HELP	☐ PROVIDED MATERIALS
☐ USE	☐ PICTURE	☐ SIMILAR PROBLEM
☐ QUIT	☐ PROBLEM	☐ WORD / FORMULA
	☐ TABLE	

ERASE

☐ YES

☐ NO

SEE