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AUTHOR Sealey, Jean
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ABSTRACT

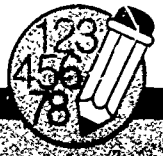
Some possible reasons why remediation is needed in mathematics instruction are presented. The need to diagnose the problem and then provide appropriate, effective remediation is discussed. Four guiding principles for diagnosis are presented, followed by six questions covering hypotheses about the source of the difficulty. Suggestions are then given for remedial work on errors with basic facts and on computational procedures. The game of "Multivision" is included. Finally, grouping for remediation is discussed. (MNS)

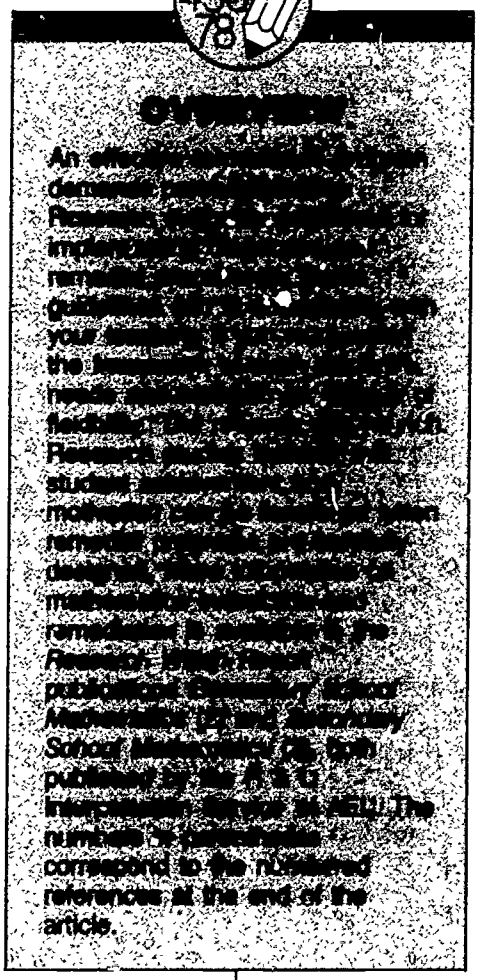
R & D INTERPRETATION SERVICE BULLETIN

MATHEMATICS

Remediation

Jean Sealey





QUESTION: *I am a sixth-grade mathematics teacher. About a third of my students seem to be at the second grade level in basic skills. How do I bring their skills up to an appropriate level in a way that will keep them motivated in the face of material they are seeing for the fourth or fifth year in a row?*

Of the basic skills areas, mathematics is the one we often associate with learning inhibited by emotional factors. We even use the term "math anxiety" to describe the fear many students experience when faced with learning mathematical concepts or doing even simple calculations. Perhaps we introduce concepts before children have reached the stage of development that allows them to understand the concept. Perhaps we do not afford children enough opportunity to manipulate concrete objects before moving to abstract symbols. Perhaps we emphasize mechanical steps of algorithms without developing a student's ability to think mathematically. Whatever the reason, many school children have difficulty with mathematics, a difficulty that, if not remedied, carries over into adulthood.

Remediation is an instructional event. We cannot rely on presenting the same material in the same way to remediate students' mathematical difficulties. Nor can we rely solely on organizational procedure. Grouping students is an organizational procedure that can be *part* of a remedial program, but grouping students by ability level does not constitute a remedial program in and of itself. Effective remediation in mathematics demands that we analyze the problem and seek alternative methods of instruction.

Because we use two words — diagnosis and remediation — to describe two steps in one process, we sometimes think of diagnosis as a separate event. Diagnosis is, however, an integral part of the remediation process. We must have an accurate assessment, or diagnosis, of the student's problem areas and strengths before we can even begin to prescribe remedies. Unless we intend to do something about the problem, that is remediate it, why would we bother to diagnosis it? In other words, effective remediation is not possible without an accurate diagnosis, and diagnosis without remediation serves no purpose. Before we suggest some specific remedial techniques, then, let's look at some of the guidelines and methods of diagnosis.

DIAGNOSIS

DIAGNOSIS INVOLVES MORE THAN scoring tests or assignments. It involves directly observing the student as he or she works and analyzing the student's errors. When working with students to diagnose mathematics difficulties, certain guiding principles are best kept in mind (1).

Be accepting. Remember that the student is usually aware of his difficulties. It is important that you establish an atmosphere of acceptance so that the student will feel free to respond.

Collect data. During diagnosis, you are collecting data, not evaluating and not teaching. An accurate diagnosis requires you to observe errors. If you correct incorrect responses or if you attempt to instruct, the student will be less likely to expose his areas of weakness.

Be thorough. Diagnosis is not a one-time session. It is an on-going process. Always be alert to the student's performance — during the instruction, as well as during actual diagnostic sessions.

Search for error patterns. A diagnosis cannot be made from a single error. Looking for error patterns is a problem-solving activity. Look for similar types of errors, look for exceptions to the patterns. Unless you correctly identify the student's errors, you cannot design or implement a successful remediation program.

REMEDIATION

WHEN YOU HAVE ANALYZED THE student's errors thoroughly, it is time to generate hypotheses about the source of the difficulty. Before beginning remediation, ask yourself these questions:

- Is this student lacking self-confidence? Encouragement and self confidence are aids to learning any subject.
- Has this student learned the prerequisite background skills? Students must know how to multiply before they can carry out long division problems.
- Does the student know all the steps in the procedure? Some of the shortcuts you take for granted in certain algorithms may have confused the student; it may be helpful to work through the problem step-by-step without omitting any of the details.
- Would it help to depict the problem graphically? For some students a picture or diagram may provide the concreteness they need to better understand the concept.
- Do I project a positive attitude? Convince the student with your words and actions that you believe in her ability to learn the material. The most effective remediation techniques and materials will not help a student who is convinced that she cannot learn.
- Have I discussed the difficulty with the student? Honesty, respect, and a desire to help must be communicated to the student when discussing the nature and extent of the difficulty. Choose a time that does not take him away from other activities. Protect the student's privacy by choosing an appropriate time for the discussion. Involve the student in the actual planning by explaining the procedures that you will use and by discussing your reasons and goals for remediation. Tell students what content areas will be covered.

How you design the remediation itself depends on the type of errors the student makes. There are two general types of error patterns: errors in basic facts and errors in computational procedures (4).

Basic Facts

Basic addition, subtraction, multiplication, and division fact errors occur when students haven't memorized these basic mathematical facts. Students need to practice facts to memorize them, but drills don't have to be repetitive and boring. *

- Let students drill each other.
- Present facts in new ways.
  9 + 3 is one less than 10 + 3
- Point out regularities:
  (a) $5 \times 2 = 2 \times 5$ (b) $a \times 1 = a$
- Allow students to use their fingers, toes, or to tap with a pencil to count.
- Vary paper and pencil drills with flashcards, oral drills, games, etc.
- Provide the opportunity for drill sessions regularly. Encourage all students to participate.

Computational Procedures

Errors in computational procedures occur when students write problems incorrectly, don't pay attention to operation signs, or don't know the steps of the procedure.

- First, make sure students know how to write the problem correctly.

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  63   .....align the numbers
x 21
  63
 126   .....shift the second product
1323   .....add columns
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MATHEMATICS REMEDIATION
Play a Game

Children satisfy their curiosity and need to interact with the environment by playing games. You can capitalize on this natural inclination to play games in your mathematics remediation program.

Many educational games are available commercially. Games that you make, however, allow you to specify the purpose and level of difficulty most appropriate for your students. We suggest that you create a few games. Or have your students create games and challenge them to see who can make the best math game.

Keep in mind that games give students motivation, experience with numbers, and practice using skills in solving problems and applying mathematical skills. Games are enjoyable and rewarding to most children. Students are usually less threatened by a game than by paper-and-pencil activities thereby reducing anxiety.

Before making a game, you'll need to decide on the topic of instruction and the purpose of the game. For example, you may want a game that provides practice in basic facts. Next, design the game. Choose a format that is appropriate for the topic, for the purpose of instruction, and for the students' level of ability. Set the game in a context. For early elementary students, you might design a race track gameboard that requires students to answer single digit addition to advance around the track.

To increase the game's appeal and its effectiveness, remember to:

1. Use color to attract attention. Markers, cards, and game boards can be made from different colored construction paper, gift wrapping, or wallpaper.
2. Keep game boards simple. Extraneous material on the board distracts students from the learning objectives.
3. Use materials readily available in the classroom or at home. You will need construction paper, dice, flashcards, cardboard boxes, styrofoam, colored pens (or crayons). Use bottlecaps, wood scraps, rocks, Popsicle sticks, etc., for counters.
4. If possible, design a game board that can be used for practicing more than one skill simply by changing the problem cards.
5. Make answers readily available, with answer sheets, color codes, or answers on the backs of the game cards.

Sorry? Make that "Subtraction"

The game presented here is an adaptation of a popular commercial game, Sorry. It was selected as an example since you and your students may already be familiar with it, but don't be limited by this example. Your students may like the games you — or they — create even better.

MULTIVISION

The purpose of Multivision is to practice simple multiplication and division with one-digit divisors. You will need a game board, two markers per player, multiplication and division problem cards, chance cards, and one die.

Rules for Multivision

Two to four students can play. Each student rolls the die, and the player with the largest number goes first. Players take turns, proceeding clockwise around the board.

To leave "Start," players draw a multiplication or division card and give the correct answer to the problem on the card. If their answers are correct, players roll the die and move a token the number of spaces indicated on the face of the die. On subsequent turns, players repeat the same process, moving from where they landed on the previous play. When an incorrect response is given, the player draws a chance card and follows its direction to resume play.

The second token may not be moved from "Start" until the first token has been moved to "Home." To arrive at "Home," players must answer a multiplication or division card and roll the exact number of spaces needed to put their tokens on "Home." The winner is the first player to get both tokens to "Home."

Decide before play begins whether or not more than one player may occupy the same space. The rules may allow for opposing players to occupy the same space, or for one player to be "knocked back to 'Start'" by an opposing player already occupying the same space.

Problem Cards of Multivision

The multiplication and division cards may be made to fit the grade levels of the students and mathematical objectives of the teacher. The chance cards can be devised by students, but it is best that players are not penalized by losing a turn.

The multiplication and division cards can be self-checked by numbering the cards. Number multiplication cards in one color and division cards in another. Then make a checklist of answers for each operation, with the answer sheets coded to the operation cards. The person who gets multiplication card number 27, for example, can check the answer listed for number 27 on the multiplication check sheet.

Examples of problem cards and chance cards:

12 x 3

81 ÷ 3

9 x 6

Oops! Move back four spaces. Answer a division card to move forward again.

You can move ahead if you can answer one multiplication and one division card correctly.

Multivision appeared in the May 1982 issue of *Arithmetic Teacher* (1). The same article describes an adaptation of *Monopoly* for practicing addition and subtraction skills with fractions, along with recognizing basic fractional parts and matching a numeral with a pictorial representation of a fraction. Other game-related articles that have appeared in *Arithmetic Teacher* include those described below.

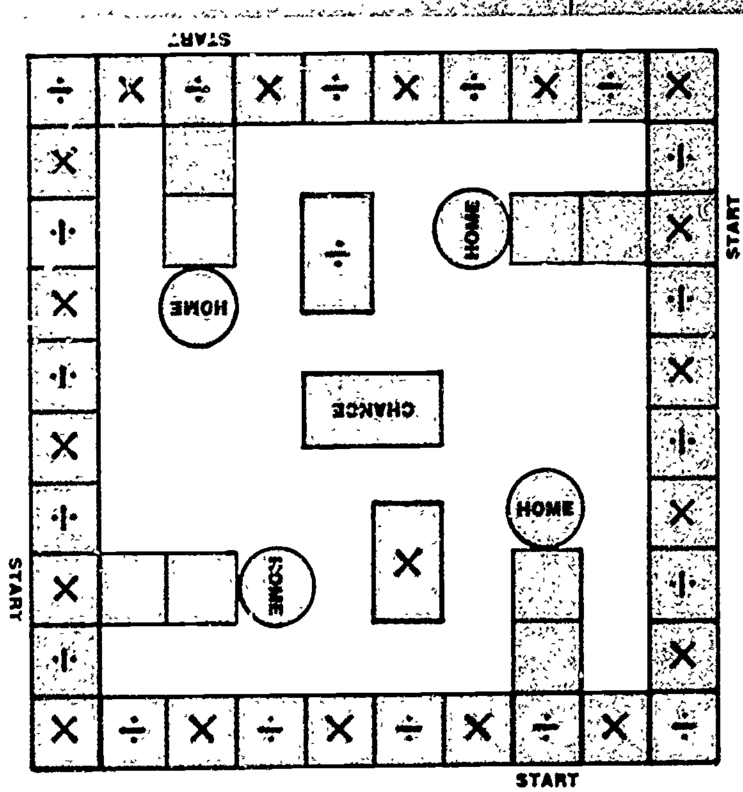
"Games People Play" (by Alan Barson, January 1982) defines formation games, chase games, and elimination games and gives examples of each type.

"Wrap It, Laminate It, and the Game Begins" (by Pauline Smith Weinstein, September 1980) gives instruction for making creative open boards with variations for reinforcing addition facts, the concept of place value, and telling time.

"Speed and Accuracy in Mathematics" (by Rudy B. Bende, January 1980) describes a game that simulates [illegible] The author suggests that it can be used at any grade level with [illegible]

[illegible]

1. [illegible] 29(9), 36-37.



Multivision Game Board

Players move around the board from their Start to Home base by answering multiplication and division problems.

- Draw attention to the operation sign.

Error: $\begin{array}{r} 22 \\ \times 15 \\ \hline 37 \end{array}$ Instead, write: $\begin{array}{r} 22 \\ \times 15 \\ \hline \end{array}$

Then work on computational procedures.

- Demonstrate the procedures step-by-step.

EXAMPLE: ONE-DIGIT MULTIPLIER

step 1: $\begin{array}{r} 28 \\ \times 3 \\ \hline 24 \\ 60 \\ \hline 80 \end{array}$ Multiply separately

$3 \times 8 = 24$
$3 \times 20 = 60$

step 2: $\begin{array}{r} 28 \\ \times 3 \\ \hline 84 \end{array}$ Combine steps

TWO-DIGIT MULTIPLIER

step 3: $\begin{array}{r} 28 \\ \times 13 \\ \hline 24 \\ 60 \\ 80 \\ 200 \\ \hline 364 \end{array}$ Multiply separately

$3 \times 8 = 24$
$3 \times 20 = 60$
$10 \times 8 = 80$
$10 \times 20 = 200$

step 4: $\begin{array}{r} 28 \\ \times 13 \\ \hline 84 \\ 280 \\ \hline 364 \end{array}$ Combine steps

step 5: Omit zero

- When a student makes an error such as $13\overline{)5239}$ $\begin{array}{r} 43 \\ 52 \\ \hline 39 \\ 39 \\ \hline \end{array}$

break the problem into steps.

$13\overline{)52}$ $\begin{array}{r} 4 \\ - \end{array}$ $13\overline{)3}$ $\begin{array}{r} 0 \end{array}$ $13\overline{)39}$ $\begin{array}{r} 3 \end{array}$

OR teach the student to estimate.

Estimate: 5239 is about 5200

$$13\overline{)5200}\ \begin{array}{r} 400 \end{array}$$

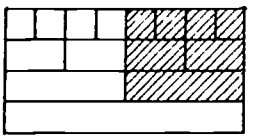
Is 43 a reasonable answer?

- Use pictures and diagrams to demonstrate

Multiplication $\begin{array}{r} 36 \\ \times 24 \\ \hline 24 \\ 120 \\ 120 \\ 600 \\ \hline 864 \end{array}$

	30	6
20	600	120
4	120	24

Equivalent Fractions
1/2 = 2/4 = 4/8



- Use alternative procedures

$736 = 600 + 120 + 16$
$-459 = 400 + 50 + 9$
$200 + 70 + 7 = 277$

- Use hand-held calculators.

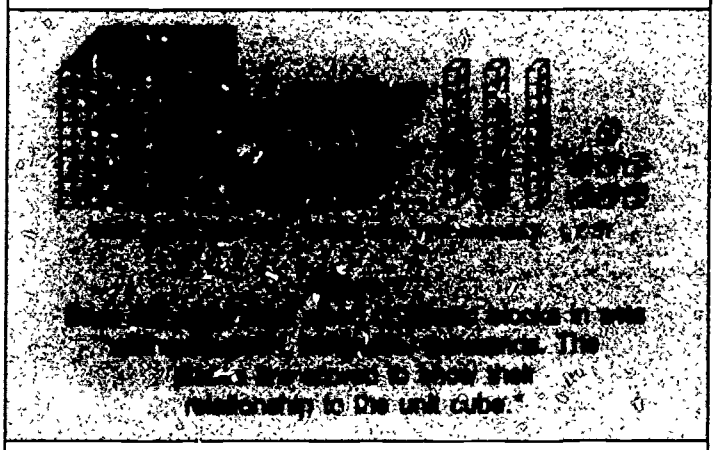
Group for Remediation—Carefully

Current research suggests that careful grouping of students for remedial mathematics can be part of an effective program. Explain the grouping procedure to the students and discuss the goals of the group. We suggest grouping students according to their developmental level and the skills they need. When grouping for remediation, keep these suggestions in mind:

Bolster self confidence by providing positive, immediate praise and reinforcement. Provide a progress chart for each student to publicly recognize achievement. Such a chart is also an aid in planning instruction.

Create flexible, temporary groups. The number of groups and students assigned to each should vary with lesson content and developmental level of students.

Have appropriate materials on hand. Let students choose the materials they want to work with. Games, base-ten blocks, Cuisenaire Rods (colored rods of graduated lengths), and other manipulatives provide the bridge from the concrete to the abstract that students need. Our insert offers suggestions about the use of games in remediation.



Encourage discussion within the group about discoveries each student has made. Provide opportunities for sharing and discussion by the class.

Form temporary student teams to practice basic facts and computational skills. To ensure effective practice, include on each team students who are

competent in the area being practiced, as well as students who need remediation.

SUMMARY

N EFFECTIVE REMEDIATION PROGRAM IN mathematics begins with a thorough analysis of a student's errors. The diagnosis and the child's developmental level provide the basis for remediation design. Mathematics remediation requires creative presentations of basic facts and computational procedures. Remember, too, that we can add concreteness to our demonstrations by using pictures and graphs. The challenge to teachers is to fit the student's difficulty to the appropriate remedial technique. Research has shown that this effort will be rewarded with gains in mathematics achievement. We have given you just a few of the many possibilities for remedial techniques with the hope that you will seek more information from other sources, such as those listed, and devise your own games and materials to vary your mathematics remediation.

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*Notes examples in this bulletin are from Driscoll (2) or adapted from Kulm (4) Figure 1 reprinted with permission (5).

SUGGESTED READINGS

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AEL
APPALACHIA EDUCATIONAL LABORATORY
Post Office Box 1348
Charleston, West Virginia 25325