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ABSTRACT

The Mathematics 13-23-33 program consists of core and elective components, both mandatory, for three courses. The core represents the common set of minimum educational objectives prescribed for all students taking the program; the elective component allows for variety and flexibility in the choice of topics. Emphasis is placed on inductive, experimental approaches to the understanding and assimilation of mathematics content. Problem solving, applications, and statistics are strong facets of the program. Suggested time allocations are included. Objectives, comments, applications, and textbook references are listed for each course. Objectives and comments or activities for the electives are also included. Appendices contain the Alberta metrication policy, problem-solving strategies, and recommendations for school mathematics of the 1980s. (MNS)

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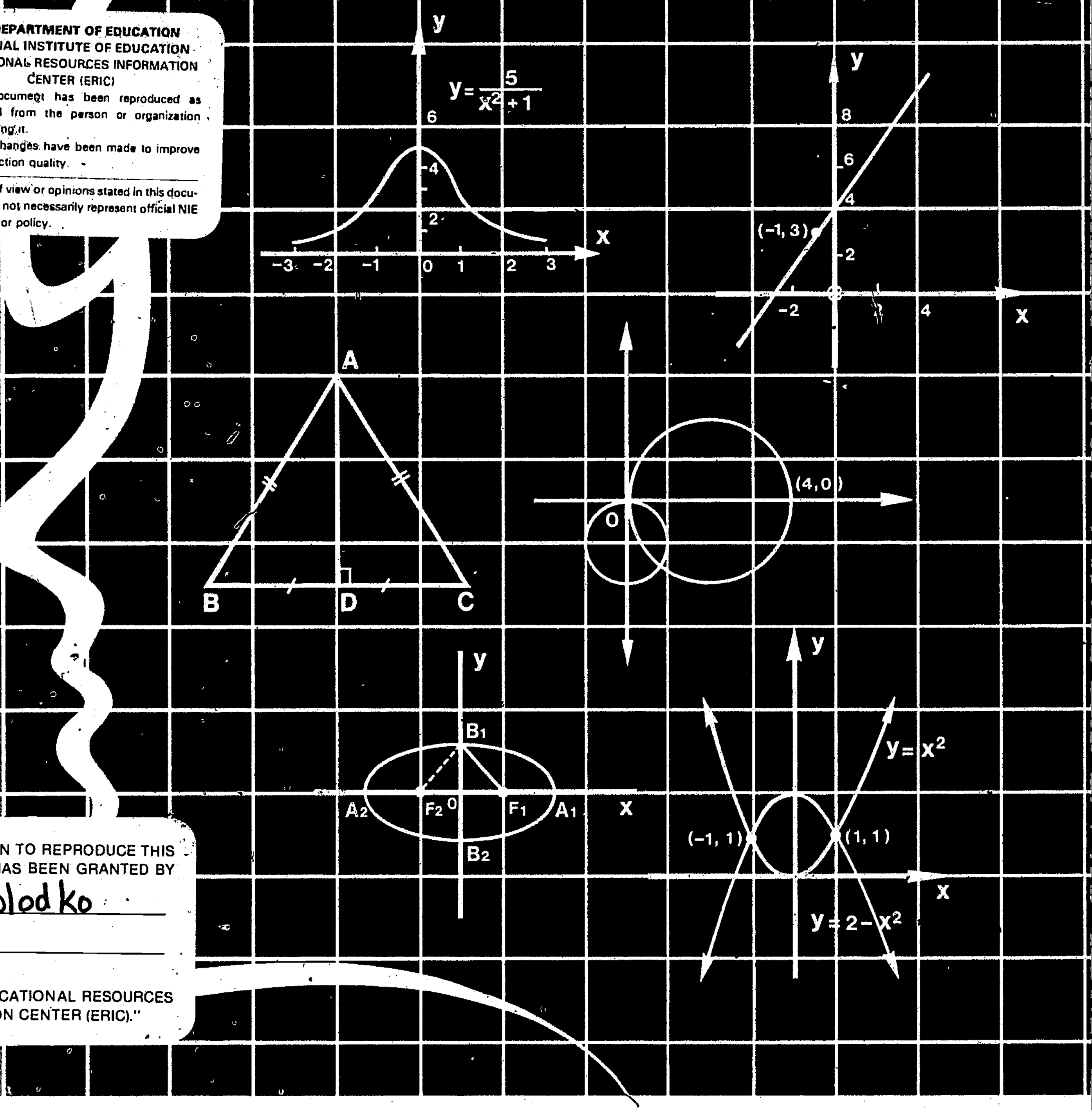

SE 046 042

MATHEMATICS 13-23-33

CURRICULUM GUIDE 1983

Curriculum

Alberta
EDUCATION

TABLE OF CONTENTS

NOTE: This publication is a service document. The advice and direction offered is suggestive except where it duplicates or paraphrases the contents of the Program of Studies. In these instances, the content is in the same distinctive manner as this notice so that the reader may readily identify all prescriptive statements or segments of the document.

ACKNOWLEDGEMENTS

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PROGRAM RATIONALE

The ability to quantify information and to perform mathematical operations is an important component of public education and one's ability to function effectively in today's society. Mathematics plays a significant role in developing mathematical skills and understandings needed by students as they progress through their school years and when later, as adults, they prepare themselves to enter the career field.

Mathematics as a formal study of mathematical concepts and relationships must be in balance with applied use of mathematics in everyday situations. It is in this regard that the development of problem solving becomes an important part of the schooling process as students learn to apply mathematical solutions to the diversity of problems encountered in daily living.

Mathematics programs must provide a strong knowledge base and at the same time assist in the development of mathematical reasoning. Pedagogically, the program should be based on a solid foundation of concrete experiences in the early school years, leading to progressively increasing levels of cognitive development. Programs should be flexible and take into account students' individual abilities and rates of learning. Topics traditionally covered in the past may become less important as new demands require greater use of microelectronic application in processing information. Mathematics as a part of the basic education of students at all levels must ensure continuity and yet be responsive to emerging influences that are characteristic of our changing times.

It is toward these ends that the high school mathematics program has been developed and organized. Three program streams (Mathematics 10-20-30;13-23-33) are available to students in recognition of the need to provide differentiated programs to accommodate varying mathematical abilities and the background required for specific career patterns. Each program route varies in the mathematics subject matter presented, the instructional emphasis, and time spent on concept development.

The program has been organized in a "core-elective" format with mandatory components of mathematical subject matter prescribed for all students. This provides a measure of commonality in the instruction and mathematical background of students taking a particular program route. Flexibility is provided in the elective component which allows for the selection of topics to meet student needs or as a means of addressing emerging applications or mathematics in new fields of human endeavor.

The influence of microelectronic technology on mathematics programming is only beginning to be felt. The integration and use of calculators and computers in the mathematics program is strongly encouraged in the program and in the instructional process. A number of elective topics and suggestions in the Comments and Applications section of this guide are designed to facilitate their use.

The senior high school mathematics program has also recognized the major recommendations of the National Council of Teachers of Mathematics. These recommendations have been developed to serve as "an agenda for action" for mathematics education in the 1980's. The emphases placed on problem solving, integration of microelectronic technology, applications, and the flexibility needed within a curriculum are incorporated in the overall design and content of the Alberta program.

PROGRAM STRUCTURE

Core Elective Format

The Mathematics 13-23-33 program consists of core and elective components. The core component of the program represents the common set of minimum educational objectives prescribed for all students taking the program. The elective component allows for variety and flexibility in the choice of topics to be covered within the guidelines outlined on page 7 of this document. The core and elective are both mandatory requirements of the senior high school mathematics program.

Common Core Content

Topics and objectives common to both the Mathematics 10-20-30 and 13-23-33 programs have been identified as content basic to each of the program streams (see chart, page 10). While the content statements (objectives) are identical, it is intended that the time and approach, instructionally, would vary for each program. Mathematics 10-20-30 should emphasize the theoretical development of mathematics concepts and relationships through deductive reasoning. Mathematics 13-23-33 places greater emphasis on inductive, experimental approaches to the understanding and assimilation of mathematics subject matter.

Independent Core Content

The independent core component serves to differentiate between the two program streams by identifying specific concepts and skills unique to each program. The distinction is further enhanced by way of the depth to which concepts are covered and the emphasis placed on abstract/theoretical concept development. Independent core topics covered at lower grade levels in Mathematics 10-20-30 are repeated, in some instances, in the Mathematics 13-23-33 program at higher grade levels. Again, the approach and depth should vary for each program stream.

Electives

Elective topics for Mathematics 13-23-33 have been provided for at the Grade 11 and 12 levels only. The Grade 10 course excludes the elective component to ensure sufficient time for developing a strong mathematics foundation for sequent courses. For further information regarding the electives, please refer to page 7 of this guide.

PROGRAM EMPHASIS

Problem Solving

A major emphasis in mathematics education has traditionally been placed on the ability of students to solve problems. The skills attained through experiences that are mathematics related and relevant to everyday situations assume greater significance and importance in preparing students to function in a changing world. Hence, a major emphasis of the senior high school mathematics program is the maintenance and further development of problem solving skills developed in earlier grades. A number of applied problem situations are suggested toward the achievement of this goal, in the applications column of the Objectives Statement sections beginning on page 15 (Mathematics 13 Core), page 29 (Mathematics 23 Core), and page 51 (Mathematics 33 Core).

Typically, the problems presented to students in the past required nothing more than a repetition of operations involving a series of computations and, in many cases, number transfer in stated formulas. The solution process was largely mechanical in nature once the steps needed to arrive at an answer were known (or stated) and subsequently practiced by the student. Problems necessitating the use of a variety of problem-solving techniques were seldom presented, and the need to reach "the right answer" was stressed as the most important outcome, rather than the development of skills and processes that could be internalized and applied to more than one type or set of problems. The senior high school mathematics program recognizes the importance of obtaining the "right answer" but also stresses the development of problem-solving skills which can be applied to almost any problem situation. Appendix B on page 82 outlines a four-stage model as a means of initiating and developing problem-solving skills.

Applications

Making the study of mathematics both meaningful and relevant to students is regarded as an important function of mathematics education at all levels. The senior high school mathematics program has made a serious attempt to do so by suggesting interesting and relevant situations where mathematical concepts and skills may be applied to real life situations. These suggestions appear in the <u>Comments</u> and <u>Applications</u> columns of the Objectives Statement sections referred to in paragraph one, above. Teachers are encouraged to draw problems from a variety of contexts as concepts are developed in the instructional process.

Applications, like problem solving, should be integrated into the overall program rather than be dealt with as an independent unit. Whenever possible, integration and coordination with other subject areas should be encouraged. When applications require extensive computations or data storage, the use of a calculator and microcomputer should be encouraged.

Statistics

For a number of years, statistics has played an increasing role in public information. The public is continually presented with statistical information, such as the consumer price index, baseball averages, weather forecasting, election polls and stock market indices, in addition to those being used extensively in research and industry.

In any enterprise where large amounts of data need to be handled and processed, and where predictions are to be made, statistical methods play a vital role. Statistical techniques are also used in designing telephone exchanges; optimally setting traffic lights; designing insurance policies; determining the reliability of a particular make and model of car; criminal detection, and market research, to mention only a few.

The introduction of statistics is intended to familiarize the student with the elementary descriptive measures that form the basis of any further work with large data sets. In most cases the data should be collected by the students, preferably different data for each student. The subsequent analysis then becomes more meaningful and hopefully provides not only statistical insight, but some new knowledge about our surroundings. In order to understand and intelligently discuss much of the information to which we are daily subjected, some knowledge of the terminology and underlying assumptions of statistics is necessary.

Statistics is an applied science. To motivate this subject, especially at the introductory level, one should concentrate on the experimental approach. This strand provides the opportunity for field trips to collect data. By displaying this data in various forms via graphs, it is possible to make some basic inferences about the data source, and it is important to make some inference in order to justify collection of data. It is equally important to ask what further information would be worth knowing, and to discuss how students might set about discovering it.

PROGRAM PLANNING

The topics and objectives for each course are not meant to be followed in any particular sequence and will be largely influenced by teacher preference and the order of presentation in the prescribed and recommended learning resources. Teachers are encouraged to develop long range plans and to consider the time allocations suggested for each topic.

THE ELECTIVE COMPONENT

The elective component is a mandatory part of the senior high school program at the Grade 11 and 12 levels. The elective component primarily offers an opportunity for students to spend time on interesting and useful areas of mathematics not necessarily contained in the core and independent core components.

The time allotment for the elective component should be approximately 15 hours at the grade 11 level and 20 hours at the grade 12 level. It is suggested that the 15 or 20 hours of electives be determined according to the plans of the individual teacher.

Structure of the Electives

The elective component of the program may be:

1) a content area not prescribed as a core or independent core topic

OR

2) a locally developed unit as determined by the teacher or school system

OR

3) an extension of the subject matter in any of the core or independent core topics to provide students with enrichment.

Guidelines

Teachers should keep the following guidelines in mind:

1) The topics are open-ended so that the interests and abilities of students may be taken into account

2) Student initiated projects may be considered as an elective

3) A teacher should make use of any appropriate resources

4) Electives should be included in the course throughout the year, wherever appropriate. These should not necessarily be taught during a 15 or 20 hour block. Where more than one elective is included, the time for each elective does not necessarily have to be the same.

5) The elective component of the program should be included in the evaluation of the students.

6) Prerequisite core material may be required before some electives are attempted. Particular elective topics have been recommended for the different courses. (see program chart page 10).

Suggested Elective Outlines

The elective component is designed to be an interesting and motivational aspect of the mathematics program. The outlines (on pages 41 and 63) for elective topics are intended to act as guidelines for individual teachers. Teachers may wish to follow the suggested outlines, or incorporate their own ideas for the elective component. The prescribed references provide useful material for many of the elective units.

Topics within the elective component of the course should be utilized to challenge the academically talented students. Such topics as Binomial Theorem and Topology, for example, may be used to provide a challenge to students of this calibre. Teachers may also extend core topics and concepts to a higher level of complexity to meet the needs of the stronger academic student.

Suggested Time Allocations

The time allocated to topics in the elective component of the course may vary according to the topic(s) chosen and instructional preferences of the teacher. A total of 15 or 20 hours may be devoted to cover one elective topic, or alternatively, two or more topics. The time allotted to elective topic(s) is at the discretion of the teacher.

Example 1: Linear Programming covering all 15 hours.
Probability covering all 20 hours.

Example 2: Linear Programming and Inequalities covering 15 hours.
Matrices and Vectors covering 20 hours.

Example 3: Absolute Value/Inequalities/Complex Numbers covering 15 hours.
History of Math/Induction/Matrices covering 20 hours.

Mathematics 13

TOPIC	NUMBER OF HOURS
Number Systems	12
Equations and Graphing	22
Presentation of Data and Descriptive Statistics	12
Variation	10
Exponents	15
Polynomials	22
Trigonometry	12
Geometry	20
TOTAL HOURS	125

Mathematics 23

TOPIC	NUMBER OF HOURS
Radicals	12
Polynomials	17
Co-ordinate Geometry	18
Presentation of Data and Descriptive Statistics	10
Systems of Equations	16
Geometry	15
Trigonometry	15
Variation	7
Electives	15
TOTAL HOURS	125

Mathematics 33

TOPIC	NUMBER OF HOURS
Relations and Functions	15
Trigonometry	24
Statistics	21
Quadratic Functions, Equations	30
Logarithms	15
Electives	20
TOTAL HOURS	125

MATHEMATICS PROGRAM OVERVIEW

ELECTIVE	INDEPENDENT CORE	CORE	INDEPENDENT CORE	ELECTIVE
Arrangements and Selections History of Math Math Induction Matrices Probability Topology Vectors Binomial Theorem	MATH 30 Sequences, Series and Limits Trigonometry Quadratic Relations (Conics) Polynomial Functions	Presentation of Data and Descriptive Statistics. Logarithms Trigonometry	MATH 33 Relations and Functions Quadratic Functions, Equations and Applications	Absolute Value Complex Numbers Consumer Mathematics History of Math Linear Programming Probability Topology Vectors
Absolute Value Complex Numbers Inequalities Linear Programming Math Art Transformational Geometry	MATH 20 Relations and Functions Polynomials Systems of Equations Quadratic Functions, Equations and Applications Radicals Geometry	Coordinate Geometry Systems of Equations Radicals Variation Trigonometry Geometry Presentation of Data Descriptive Statistics Polynomials	MATH 23 Radicals Geometry Polynomials	Area and Volume Consumer Math Inequalities Math Art Transformational Geometry
	MATH 10 Number Systems Exponents and Radicals Geometry (Deductive) Polynomials Equations and Graphing	Number Systems Variation Exponents Equations and Graphing Geometry Polynomials Trigonometry Presentation of Data and Descriptive Statistics	MATH 13 Geometry (Inductive)	

LEARNING RESOURCES

Mathematics 13

Prescribed Reference: MATHEMATICS FOR A MODERN WORLD: BOOK 2

E. G. Carli, J. C. Egsgard, C. Psica, J. J. Del Grande

Toronto: Gage Educational Publishing Ltd., 1975.

Prescribed Reference: APPLIED MATHEMATICS FOR TODAY: INTRODUCTION

D. Dottori, R. McVean, G. Knill, J. Seymour

Toronto: McGraw-Hill Ryerson Ltd., 1980.

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Prescribed Reference: APPLIED MATHEMATICS FOR TODAY: INTERMEDIATE

Book 1 Second Edition

D. Dottori, G. Knill, J. Seymour

McGraw-Hill Ryerson Ltd., 1976.

Recommended Reference: MATHEMATICS FOR A MODERN WORLD

Book 3 Second Edition

A. G. Burns, R. G. Pinkney, J. J. Del Grande

Gage Publishing Ltd., 1976.

Mathematics 33

Prescribed Reference: APPLIED MATHEMATICS FOR TODAY: SENIOR

Book 1 Second Edition

D. Dottori, G. Knill, J. Seymour

McGraw-Hill Ryerson Ltd., 1977

Recommended Reference: MATHEMATICS FOR A MODERN WORLD

Book 4

E. G. Carli, J. C. Egsgard, C. Psica, J. J. Del Grande

Gage Publishing Ltd., 1975.

Learning Resource Approvals

In terms of provincial policy, learning resources are those print, nonprint and electronic courseware materials used by teachers or students to facilitate teaching and learning.

PRESCRIBED LEARNING RESOURCES are those learning resources approved by the Minister as being most appropriate for meeting the majority of goals and objectives for courses, or substantial components of courses, outlined in provincial Programs of Study.

RECOMMENDED LEARNING RESOURCES are those learning resources approved by Alberta Education because they complement Prescribed Learning Resources by making an important contribution to the attainment of one or more of the major goals of courses outlined in the provincial Programs of Study.

SUPPLEMENTARY LEARNING RESOURCES are those additional learning resources identified by teachers, school boards or Alberta Education to support courses outlined in the provincial Programs of Study by reinforcing or enriching the learning experience.

PROGRAM OF STUDIES

GOALS OF THE SENIOR HIGH SCHOOL MATHEMATICS PROGRAM

Although the different courses of the senior high school mathematics program have different specific objectives, the goals of the senior high mathematics program are set forth in relation to three main expectations and needs: those of the individual, those of the discipline of mathematics and those of society at large. They are listed as follows:

Student Development

a) To develop in each student a positive attitude towards mathematics

b) To develop an appreciation of the contribution of mathematics to the progress of civilization.

c) To develop the ability to utilize mathematical concepts, skills and processes.

d) To develop the powers of logical analysis and inquiry.

e) To develop an ability to communicate mathematical ideas clearly and correctly to others.

Discipline of Mathematics

a) To provide an understanding that mathematics is a language using carefully defined terms and concise symbolic representations.

b) To provide an understanding of the concepts, skills and processes of mathematics.

c) To provide an understanding of the common unifying structure in mathematics.

d) To furnish a mode of reasoning and problem solving with a capability of using mathematics and mathematical reasoning in practical situations.

Societal Needs

a) To develop a mathematical competence in students in order to function as citizens in today's society.

b) To develop an appreciation of the importance and relevance of mathematics as part of the cultural heritage that assists people to utilize relationships that influence their environment.

c) To develop an appreciation of the role of mathematics in man's total environment.

MATHEMATICS 13
CORE

Mathematics 13

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
A. NUMBER SYSTEMS				
1. Identify numbers as natural, whole, integral and rational.	Many students entering this program should have some knowledge of the number systems. However, several concepts will require both practice and maintenance to develop previously taught skills.	When working with these basic number concepts practical problems should be used as much as possible. Basic problem-solving shows the need for accurate calculations and applications. Application problems can also motivate the students to grasp these skills fully and also provide motivation to study further the irrational numbers and the real numbers.	39-41	47-48.
2. Add, subtract, multiply and divide rational numbers.				2, 3, 51, 52
3. Convert a rational number from decimal form to fractional form $\frac{a}{b}$ and vice versa.				48-50
4. Apply percentage to consumer-related problems: **a) simple interest** **b) discounts and mark-ups** **c) commissions**	The calculator should be used to facilitate the exploration of numbers more fully.	Applications related to student interest such as stereo equipment and automobiles may be used.	173-177	53, 54, 339-351

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OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
B. EQUATIONS AND GRAPHING				
1. Maintain skills in solving first degree equations with rational coefficients.	It is not necessary to include fractional equations with variables in the denominator.		116-120	
2. Solve word problems whose solutions are based on first degree equations with rational coefficients.	Several practical problems exist in the sciences, business and technical fields that require the development of equations and their respective algebraic and graphical solutions. Problem solving approaches should be emphasized.		132-133	181-195
3. Identify and use the terms: quadrant, origin, axis, coordinate, ordered pair, abscissa and ordinate.		Latitude and Longitude; grid plans of cities and laying out of new townsites; telephone rates based on a grid system; grid organization of an oil field.	84-99 103	123-127
4. Recognize and graph ordered pairs.	Game approaches, such as "Battleship" and picture plotting, can be used to reinforce graphing concepts. Computer games often involve grids and the plotting of points.			
5. Interpret graphs of straight lines.	Students should be able to read the values of a variable from a graph. The references listed go beyond the study of linear equations. It will be necessary to choose questions carefully and supplement from other resources.	Several examples such as supply-cost graphs, Celsius-Fahrenheit graphs, distance-time graphs can be used.		137-139
6. Graph linear equations using: a) ordered pairs b) intercepts				

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
C. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS				
1. Organize data by: **a) collecting various types of data** **b) grouping data into classes** **c) determining the frequency of each class** **d) defining class width (interval), class boundaries, and class mark.**	As an extension to the graphing work previously done, students should be able to collect and organize data so that decisions, inferences, and predictions can be made. A knowledge of the basic concepts of statistics will stimulate problem solving. Grouping data into classes is used here to construct a histogram.	Statistical applications involving the students themselves are plentiful. Some examples are: their heights to the nearest centimetre, the lengths of their forearms, the sizes of their wrists in millimetres, etc.	21-33	25-40
2. Graph data using bar graphs, circle graphs, histograms and frequency polygons.				
3. Calculate the mean, median and mode for ungrouped (raw) data.				
4. Select the most suitable of the three types of averages for a given set of ungrouped (raw) data.				
5. Select a suitable sample from a given population.				

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
D. VARIATION				
1. Identify direct variation.	Review skills relating to graphing, the construction of tables and units of measurement.		146-163	202-210
2. Identify inverse variation.		Boyles law relating pressure (P) and volume (V) could be developed to illustrate inverse variation.		
3. Identify partial variation.		In police work, it is important to know that the length of the humerus is proportional to the height of the corpse. $H = 2.89 b + 70.6$ (Male) $H = 2.75 b + 71.4$ (Female) where H = height and b = length of humerus.		
4. Solve problems based on direct, inverse and partial variation.	Have the students develop practical problems that are familiar to them, such as wage earnings, consumer problems and science problems.			
5. Find the constant of proportionality for a given variation.	Variation problems involving variables other than the first degree are not recommended.			

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OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
E. EXPONENTS				
1. Utilize the following laws of exponents: **Where a, b, ε I; x, y $\in$ R; $x \neq 0$, $y \neq 0$** $x^a \cdot x^b = x^{a+b}$ $x^a + x^b = x^{a-b}$ $(x^a)^b = x^{ab}$ $(xy)^a = x^ay^a$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$ $x^0 = 1$ $x^{-a} = \frac{1}{x^a}$	The values of "a" and "b" should be specific integers while x and y should be monomials. Students should be able to recognize the parts of a power from an example. e.g., x^3: x^3 is the power x is the base 3 is the exponent Every attempt should be made to help students understand the reasons why these laws exist. Demonstrate that: $(2)^2 \cdot (2)^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = 2^5$ and then short cut to the law $(2)^2 \cdot (2)^3 = 2^{2+3} = 2^5$		58-63	11-22
2. Change a number from decimal form to scientific (standard) notation and vice versa.	This may be a good time to review metric units.			
3. Perform the operations of multiplication and division on numerals expressed in scientific (standard) notation.		Various science problems can be used to illustrate the use of large and small numbers in standard form.	14-19	

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OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
F. POLYNOMIALS				
1. Identify and use the following terms: **a) algebraic expression** **b) term** **c) factor** **d) monomial** **e) binomial** **f) trinomial** **g) polynomial** **h) coefficient** **i) degree**	Like and unlike terms should be clearly explained. Considerable time should be spent here on the language of mathematics. Order of operation should also be reviewed. Stress the difference between term and factor.		63-69 70-78	77-108
2. Evaluate a polynomial for given values of the variables.	Substitute values for the variables in some of the more common formulae used in science such as area, volume and measurement.			
3. Add and subtract polynomials.				
4. Multiply: **a) monomial x monomial** **b) monomial x binomial** **c) monomial x trinomial** **d) binomial x binomial**	Numerical examples $[(6 + 2)\ (6 - 3)]$ can be used to introduce algebraic multiplication of $(x + 2)\ (x - 3)$.			

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OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
5. **Write the expansions of** $(P + Q)^2$, $(P - Q)^2$ **and** $(P - Q) \cdot (P + Q)$ **and recognize them as general cases.**	Illustrations of $(p + q)^2$ can be presented $\begin{array}{c\|c\|c} & p & + q \\ \hline p & p^2 & pq \\ + & & \\ q & pq & q^2 \end{array}$	Show the biological implications in heredity such as a pea having a dominant wrinkled gene (W) and a recessive smooth gene (w) crossed with same (W + w). $(W + w)\ (W + w) = W^2 + 2Ww + w^2$ Dominant wrinkled pea (W^2) two heterogeneous wrinkled peas (2Ww) recessive smooth pea (w^2)	63-69 70-78	77-108
6. **Recognize and factor a polynomial with a common factor where the common factor is:** **a) a monomial** **b) a binomial**				
7. **Factor a trinomial of the form:** $ax^2 + bx + c$ **where** $a, b, c, \in I, a \neq 0$				
8. **Factor polynomials of the form:** $P^2 - Q^2$	Have students experiment inductively with questions of this type before teaching a specific method of factoring. RESTRICT P and Q to monomials.			

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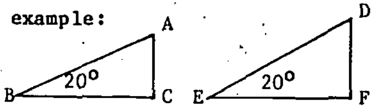
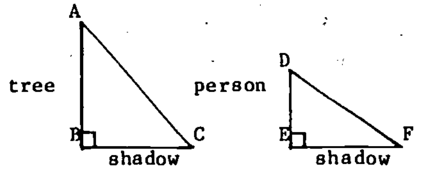
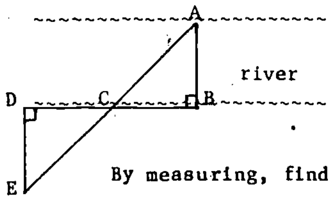
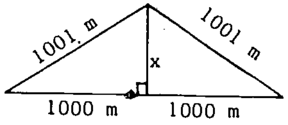
OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
9. Factor polynomials by using any combination of methods outlined in objectives 6 through 8.			63-79 70-78	77-108
10. Divide a polynomial by a: **a) monomial** **b) binomial**	Division of polynomials should be restricted to first degree binomials in one variable.			
G. TRIGONOMETRY				
1. Find the unknown sides in similar triangles.	It is worth introducing trigonometry by using similar triangles since this gives students practice in thinking in terms of ratios of sides rather than absolute measurement of the sides of a triangle. This concept is central to right triangle trigonometry: the ratio of a given pair of sides is always the same for a given angle in a right triangle regardless of the absolute measurement of the sides. example:  $\frac{AC}{AB} = \frac{DF}{DE} = \sin 20^\circ$	Problems involving right triangle trigonometry and similar triangles can be generated around the school. Some examples include: a) height of a flagpole by using shadows b) height of a building using trigonometry (measurement of angle by simple gravity protractor) c) distance across a river (real) or fictitious) using either trigonometry or similar triangles.	226-247	246-259
2. Apply similar triangles to practical problems.				
3. Define sine, cosine and tangent ratios for right angle triangles.				
4. Find the trigonometric ratios of the acute angles in a right triangle when the sides are given.				
5. Determine the trigonometric ratios of any given acute angle.				
6. Determine the measure of any acute angle given one of its trigonometric ratios.				

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
7. **Solve problems based on right triangles using trigonometric ratios.**	Outdoor measurement problems should be limited to simple right triangle problems. Instruments for distance and angle measurement can be made from very simple materials. Students should become familiar with trigonometric tables and how to use them but should be allowed to use the calculator.	INDIRECT MEASUREMENT USING SIMILAR TRIANGLES 1. Height of a tree  By measuring, find BC, EF, DE. Calculate AB $\Delta ABC \sim \Delta DEF$ Why? 2. Distance across a river pick a point or mark with a stake.  By measuring, find BC, DC, DE. Calculate AB. Point D, C and B are collinear. D and C are arbitrarily chosen. Choose point E so that E, C and A are collinear.	226-247	246-259

Mathematics 13

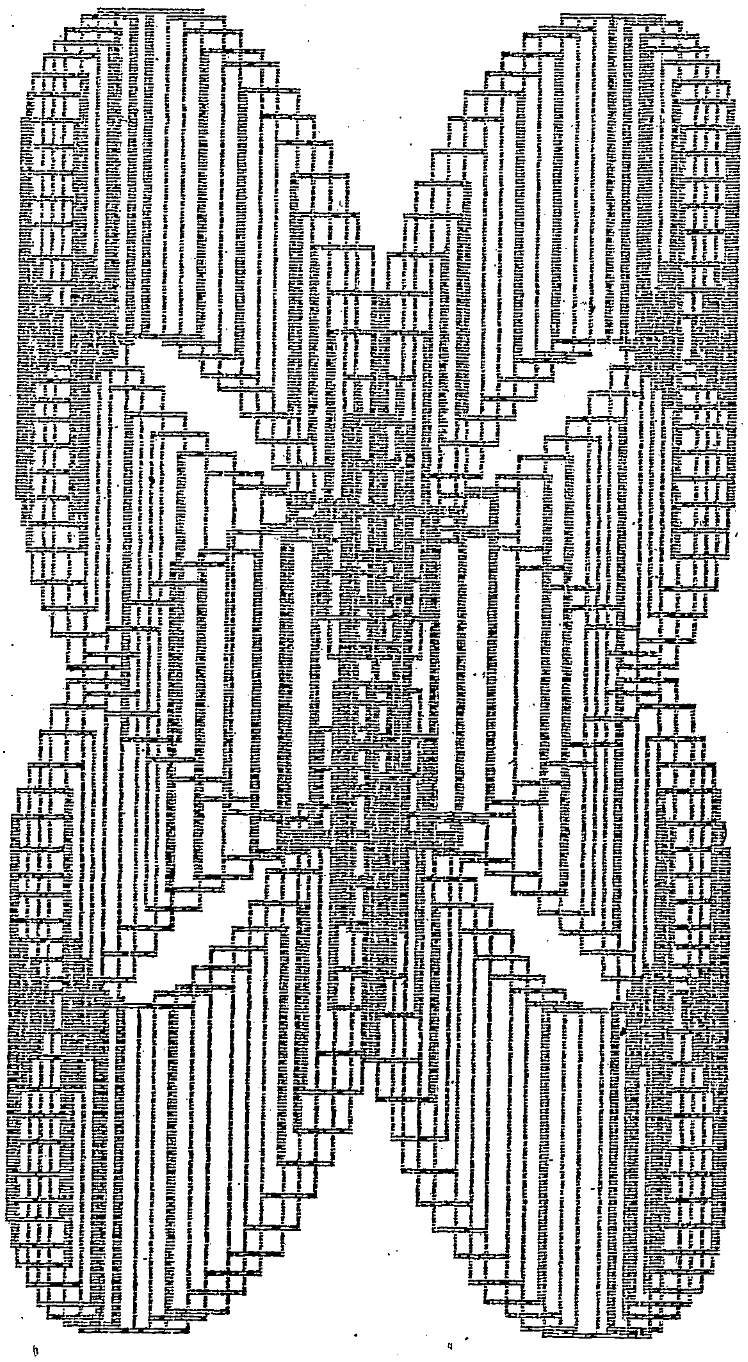
OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
H. GEOMETRY				
1. Recognize and use the following terms: vertex, side(ray), degree, straight angle, right angle, acute angle, obtuse angle, reflex angle, adjacent angles, complementary angles.	Using diagrams, the students should be able to describe each of these terms. To most students this is a review section.			
2. Recognize and use the following terms associated with triangles: equilateral, equiangular, isosceles, scalene and right triangles.	Where applicable, measurements could be assigned to different types of triangles and polygons so that perimeters and areas can be calculated.			
3. Use the Pythagoras theorem to solve right triangles and associated problems.		If a 2000 metre railway track expands 2 metres, how high will the track rise, assuming the ends are fixed? 		213-219
4. Recognize and use the following terms associated with polygons: quadrilateral, trapezoid, parallelelogram, rectangle, rhombus, square, regular polygon and diagonal.			199-230	

Mathematics 13

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
5. Recognize and use the following terms associated with parallel lines: transversal, corresponding angles, alternate angles and interior angles.	Students should be encouraged to find examples of parallel lines in the world around us.	Building construction: Doors, frames, windows, etc.	199-230	219-221
6. Recognize and use the following terms: congruency, similarity, perpendicular, bisector and perpendicular bisector.		Using parallel lines in the same plane (e.g., exercise book) have students draw transversals and discuss measurement of angles formed. Skew lines could be discussed.		
7. Measure an angle with a protractor.		Use the face of a clock with the minute hand on 12 and various positions of the hour hand. (Guess measurement first.)		
8. Construct an angle congruent to a given angle.	The mira may be used as an alternate to ruler and compass constructions.			
9. Construct the bisector of a given angle.				
10. Construct a perpendicular to a given line segment: **a) at a point on a segment** **b) through a point not on the same segment.**			248-249	
11. Construct the right bisector of a line segment.				

Mathematics 13

OBJECTIVES	COMMENTS	APPLICATIONS	AMT	MMW
12. Construct a line parallel to a given line.			248-249	219-221
13. Solve problems related to vertically opposite angles.			199-230	
14. State and apply the conditions for congruence of a triangles - SAS, SSS, ASA.				
15. State the conditions for similarity and solve related problems.				
16. State the conditions for parallelism and apply to solving related problems.				
17. State and apply area formulas for triangles, rectangles, squares, parallelograms and trapezoids.				
18. Solve problems involving numerical applications of the relationships and conditions described in objectives 13-17.				



MATHEMATICS 23
CORE

23

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
A. RADICALS			
1. Review the basic laws of exponents.	Students should be able to work with monomial bases containing literal and/or numerical factors. Exponents should be restricted to integers, e.g., $\left(x^3\right)^{-2}$, $\left(\frac{2}{3}\right)^3$		117-122
2. Identify each part of a radical expression.	Students should be able to identify each part of a radical expression from an example: $\sqrt{x^8}$: where 2 is the index, x^8 the radicand and $\sqrt{x^8}$ is the radical.		
3. Simplify radical expressions of form $^b\sqrt{x}$, b = 2,3	Change pure radicals to mixed radicals and vice versa. Illustrate with radicals of the form. $\sqrt[b]{x^m y^n}$, where b=2 or 3 and m and n are positive integers.		108-117

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
4. **Perform the four basic operations on radicals of the form** $\sqrt[b]{x}$, b = 2, 3	Addition and subtraction can be explained by using the distributive laws. For example: a) $2x + 3x = (2 + 3)x = 5x$ b) $2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$ Restrict the operations with cube roots to simple numerical radicands.	Radical equations can be applied to: a) determining braking distances of a car knowing its speed and type of road. Use: $s = \sqrt{30fd}$, where s = speed f = coefficient of friction and d = braking distance (skid marks) The formula can also be used to find speed by measuring the skid marks and thus determining whether the driver was speeding. b) duration of a hurricane or tornado. Use: $t = \sqrt[2]{\frac{d^3}{216}}$ or $t = \frac{d\sqrt{6d}}{18}$ in simplified form where t = time in hours and d = diameter of storm in km.	108-117
5. **Rationalize radical denominators that are monomials and binomials.**	Demonstrate the reason for dividing by a rational number. It is easier to write $\frac{\sqrt{10}}{2}$ as a decimal than the unrationalized form of $\frac{\sqrt{5}}{\sqrt{2}}$ which requires division by an approximate decimal. The index should be restricted to 2.		
6. **Solve radical equations containing one radical in one variable.**	Emphasize the isolation of the radical on one side of the equation before squaring.		
B. POLYNOMIALS			
1. **Maintain previous skills in algebraic operations.**	This should include both polynomials and rational expressions.		24-32
2. **Maintain previous skills of factoring.**			

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
3. Maintain previous skills of solving linear equations with one unknown.	This should include equations containing unsimplified polynomials.		32-36 42
4. Simplify rational expressions by factoring.	Review operations with rational numbers before proceeding to operations with rational expressions.		See AMT Senior 63-69
5. Perform the operation of multiplication and division with rational expressions.			
6. Perform the operations of addition and subtraction of rational expressions with: **a) the same denominator** **b) different denominators**			
C. COORDINATE GEOMETRY			
1. Maintain previous skills related to the following: quadrants, axes, origin, ordered pairs and intercepts.	The usefulness of mathematics increased greatly when mathematicians applied algebraic methods to geometry (in the 17th century) to form a subject called coordinate geometry.	Good references for this section are Math Is 4 and Holt Math 4. Many applications to this section exist in the trades-related fields.	

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
2. Determine the distance between two points.	Using two points on a graph, say P(1,2) and Q(5,4) derive the distance formula using the Pythagorean Theorem before generalizing the formula.	Determine length of rafters, support cables, etc.	66-70
3. Determine the coordinates of the midpoint of a line segment.	By taking two points on a graph, students can predict the midpoint before deriving the midpoint formula.		
4. Determine the slope of the line passing through two given points.	Since coordinate geometry is highly visual, students should be encouraged to do neat work so that they can better predict the slope of a line, state relationships between slopes of lines, graph lines from $Ax + By + C = 0$ or $y = mx + b$, and interpret graphs of straight lines.	If the slope of a roof is 1:4 and the span of the roof is 8 metres, calculate the rise of the roof and the length of the roof rafters.	46-57
5. State the relationship between slopes of **a) parallel lines** **b) perpendicular lines**		A staircase is to be built with a rise of 7 and a run of 12. Determine a method of using a carpenter square to cut the main support for risers and treads.	
6. Graph lines whose equations are in the form: $Ax + By + C = 0$ **or** $y = mx + b$, **using the slope and y-intercept.**			

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
7. Use the slope test to determine whether three points are collinear.			70-72
8. State the intercepts of a line by examining the graph.	To find the x-intercept the student must understand that along the x-axis, y=0. By putting y=0 in the equation one can get the x-intercept. Similarly, by putting x=0 one can get the y-intercept.		
9. Determine the intercepts of a line from its equation.			
10. Write an equation and draw the graph of: **a) a vertical line** **b) a horizontal line**			
11. Write the equation of a line and draw its graph given a slope and a point on the line.		NOTE: Many good examples of problems related to this section are found in various books. The references to be noted in the NCTM book, *A Sourcebook for Applications of School Mathematics*, (1980).	51,52

Mathematics 23

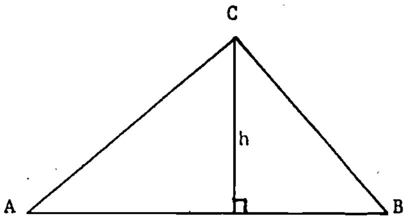
OBJECTIVES	COMMENTS	APPLICATIONS	AMT
12. Write the equation of a line passing through a given point and a) parallel to a given line b) perpendicular to a given line			51,52
13. Given two points: a) draw the graph of the line passing through them b) write the equation of the line passing through them			
D. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS			
1. Maintain previous skills of organizing ungrouped (raw) data.			322-331
2. Calculate the mean, median and mode of grouped data.			
3. Draw histograms and cumulative frequency histograms (ogives) for grouped data.	Draw histograms as a review of previous material. (See suggestions in Math 10 applications). Use these histograms to now construct a cumulative frequency histogram. Graphically, (see page 383 of Math Is/4) one can now determine any desired percentile. There are various ways to calculate percentile. Your prescribed reference indicates one approach.		
4. Graphically demonstrate how to obtain and interpret quartiles and percentiles.			332-335 338-340

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
E. SYSTEMS OF EQUATIONS			
1. Solve linear systems of equations both algebraically and graphically.	It would be worth introducing solutions by graphing before the algebraic solutions are taught. In the graphical solution, a student has a chance to understand how equivalent systems can be generated and used to arrive at a solution.	Problems involving two vehicles travelling at different speeds and leaving a fixed point at different times can be used. Using graphical or algebraic means, we can find out when the two vehicles meet.	81-93
2. Identify systems as having many solutions, one solution or no solution.	Systems can be described graphically showing: a) coincident lines (dependent system) b) parallel lines (inconsistent) c) intersecting lines (independent) Students should be able to determine the type of system by examining the slopes of the lines.		
3. Solve problems based on the solution of systems of equations.	Problems involving statements about numbers, supplementary and complementary angles, speeds of two vehicles, digits of numbers and investments can be used.		

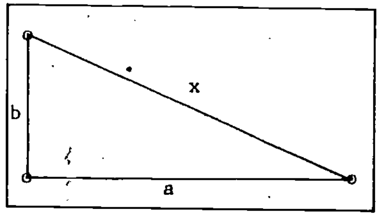
Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
F. GEOMETRY			
1. Define and illustrate the following terms related to the circle: radius, chord, interior, exterior, arc, semicircle, segment, sector, central angle, secant line and tangent line.	Have students draw: 1) the inscribed circle of a triangle 2) the circumscribed circle of a triangle	Discuss circular forms in nature, in the community, in art and in industry (e.g., wheels, gears, pulleys, etc.). One of the highlights of this section should be the experimental approach to geometry. Through an experimental approach, the students will become sensitized to the real world and many interesting problems will result. For example, why are the tops of all bottles circular? What property of the circle is being taken advantage of?	
2. Discover by experimentation the following circle relationships: a) A line through the centre of a circle and the midpoint of a chord is perpendicular to the chord. (Converses should also be considered.) b) The measure of an inscribed angle is half the measure of the central angle subtended by the same arc (or congruent arcs). c) Inscribed angles subtended by the same arc (or congruent arcs) are congruent. d) An angle inscribed in a semicircle is a right angle. e) A tangent line is perpendicular to the radius drawn to the point of contact. f) Tangent segments drawn to a circle from the same exterior points are congruent.			

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
g) **Angle between the tangent and the chord is equal to one-half the measure of the intercepted arc.**			
3. **Solve problems involving numerical applications of the relationships in Objective 2.**			
G. **TRIGONOMETRY**			
1. **Maintain previously developed skills in solving right triangles.**	This section is intended as a review of the trigonometry studied in Grade 10 Math. More complex problems can be introduced here.	a) To find the length of a tunnel through a mountain using an obtuse triangle.  Knowing h (height of mountain above AB) and the angles A and B, the length of AB can be calculated.	167-173

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
		b)  Three holes are to be drilled in a rectangular plate. Find the lengths of a and b, knowing the value of x and one of the acute angles.	
2. Identify cosecant, secant and cotangent as reciprocal ratios of sine, cosine and tangent.			158-159, 169
3. Determine the relative measures of the sides of: a) a 30 - 60 - 90 triangle b) a 45 - 45 - 90 triangle			159-161
4. Solve right triangles using any of the trigonometric ratios.			170-193
5. Solve problems involving right triangles including angles of elevation and depression.	Students should understand why angles of elevation and depression are equal.		

Mathematics 23

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
H. VARIATION			
1. Maintain previous skills of direct, inverse and partial variation.	Partial variation can be extended to include problems involving two equations in two unknowns.		18-21
2. Identify joint variation.			
3. Solve problems related to joint variation.			

MATHEMATICS 23
ELECTIVES

23

TRANSFORMATIONAL GEOMETRY

Euclid has been criticized for his use of the super-position argument to prove the familiar side-angle-side (SAS) congruence theorem. Simplified, the argument calls for one to pick up a triangle and attempt to fit it on another. While deficiences have sometimes been pointed out in Euclid's system of axioms, it may be noted that those seeking to evolve more logically satisfactory systems of axioms have generally attempted to by-pass the difficulty.

Although it may be necessary to justify every step of a proof one should not lose the intuitive appeal of Euclid's approach. Most geometry teachers probably explain congruence by placing one triangle over another. E.G., by cutting out cardboard triangles, by tracing one triangle over another, etc.

Basically then, we could consider the result of mapping a figure by some appropriate transformation.

References:

1. Bye, M., Griffiths, T. and Hanwell, A., *Holt Math 4*, Toronto: Holt, Rinehart and Winston, 1980, pages 265-303.

2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: An Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1977, pages 320-344.

3. Ebos, F., and Tuck, B., *Math is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979, pages 353-371.

4. *The Mathematics Teacher (NCTM): Transformation Geometry and the Artwork of M.C. Escher*, December 1976, pages 647-652.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To develop and describe the following different types of transformations:	Graph paper should be used whenever possible.
a) Translations	(For obvious reasons, these are also referred to as slides or glides.)
b) Reflections	A transparent plastic reflector called a MIRA would be an excellent tool in teaching reflections.
c) Rotations	The hands of a clock noting its centre is a very useful example.
d) Dilations	Most students know what similarity (same shape, different size) means. Use examples (photo enlarger, microscope, etc.), to explain dilations.

MATH ART

This unit is intended to give students an opportunity to apply in an interesting way some of the skills and concepts learned in Geometry (both Euclidean and Analytic). Once a student learns some of the simple principles of the artwork, he can create his own designs.

References.

1. *Art 'n Math,* Billings, Campbell and Schwandt (Action Math Associates, Inc.).
2. *Creating Escher - Type Drawings,* Rannucci and Teeters (Creative Publications).
3. *Graph Gallery,* Boyle, (Creative Publications).
4. *Paper Folding in the Classroom,* Johnson (NCTM Publications).
5. *Creative Constructions,* Seymour (Creative Publications)
6. *Line Designs,* Seymour (Creative Publications).
7. *Mathematics Teacher (NCTM)*
    - (a) Tangram Mathematics, February, 1977, pp. 143-146
    - (b) The Artist as Mathematician, April, 1977, pp. 298-308
    - (c) Filing (tesselations), March, 1978, pp. 199-202.
    - (d) Transformation Geometry and the Artwork of M.C. Escher, December, 1976, pp. 647-652.
    - (e) Creativity With Colors, March, 1976, pp. 215-218.
8. *Geometry,* Jacobs, (W. H. Freeman Publishing).

OBJECTIVES	COMMENTS/ACTIVITIES
1. Graphing pictures	
(a) Given a set or ordered pairs, locate these on a coordinate system and join them to form a picture.	

OBJECTIVES	COMMENTS/ACTIVITIES
1. Graphing Pictures (continued)	
(b) Given a set of linear equations, each with a restricted domain, draw the lines on a coordinate system to form a picture.	Student writes his name or suitable message in block letters on graph paper, then analyzes each letter to write the equations which represent them (restrict to straight lines).
2. Line Designs	
(a) Draw line designs within one angle.	Student creates his own pattern using basic construction techniques for angles. Alternate rectangles can be colored to form a pattern. The same approach can be used with colored thread and small nails on a sheet of plywood to create string designs.
3. Geometric Constructions	
(a) Perform basic constructions related to line segments, angles, perpendicular and parallel lines.	Student analyzes and recreates simple drawings from patterns.
(b) Draw a polygon within a circle using ruler and compass (e.g., hexagon, polygon, square).	Student creates and colors his own designs.
4. Tesselations	
(a) Draw a tesselation based on a rectangle, a triangle or other polygons.	Students find examples of tesselations; e.g., linoleum, floor tiles, wallpaper, brick wall, etc.
5. Escher Drawings	
(a) (An extension of tesselations). Draw a simple tesselation based on Escher's work and analyze how it is drawn.	Students create their own Escher - type drawings. Students analyze some of Escher's paintings to find geometric contradictions (e.g., "Belvoir").

OBJECTIVES	COMMENTS/ACTIVITIES
6. Paper Folding	
(a) Demonstrate the following concepts using paper folding: - angle bisector - bisection of a segment - line perpendicular to a given line - centre of a circle	
(b) Construct a parabola, hyperbola and ellipse using paper folding.	
7. Tangrams	
(a) Rearrange polygonal shapes to form geometric figures.	Using the 7 pieces of a tangram square, form rectangles, isosceles triangles, parallelograms and trapezoids.
(b) Illustrate basic properties of polygons (e.g., form an isosceles trapezoid and show the non-parallel sides are equal by moving the component parts around).	Use the tangram pieces to make up figures such as the letter "T", the number "1", a house, a cat, etc.

CONSUMER MATHEMATICS

It is becoming more and more important that students have some understanding of how mathematics is related to the consumer. A working knowledge of how the basic skills in mathematics are applied to consumer topics such as accessing money, spending money and the management of money is essential for a student to become a good citizen.

References:

1. *Mathematics in Life,* Gage Publishing Co.
2. *Mathematics Plus*, Houghton Mifflin
3. *Mathematics for the Real World,* Merrill/Bell and Howell Publishing Co.
4. *Mathematics for Daily Use,* Doubleday Publishing Co.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To develop an appreciation of how money is obtained. (a) Jobs - Descriptions - Applications - Wages	Identify different careers and discuss the wages paid for the related work. Identify what is needed for the accomplishment of different jobs. Discuss filling out applications and interviews.

OBJECTIVES	COMMENTS/ACTIVITIES
1. (Continued)	
(b) Payroll - Methods of payment - Deductions · Tax · Pension · Benefits	Have students discuss and analyze real or imaginary paychecks.
(c) Selling Products	Have students participate in such activities as a school store, concession or canteen.
(d) Record Keeping.	Discuss and practice the necessity for good record keeping.
2. To develop an understanding of wise and thrifty spending habits.	
(a) Buying	
- Comparative buying	Students can check out the cost of a shopping list of some everyday needs. Prices can be compared from store to store, by brand and by size. Prices could be prepared over a period of time to determine price changes and trends:
(b) Credit Buying - Types of credit - Types of payment - Interest rates - Installment buying - Sources of borrowed money	Students could determine the cost of buying such items as a car.
(c) Operation Costs	Students could determine the cost of operating a car for a period of a year. Things to take into account are: purchase price, licence, insurance, gasoline, service and maintenance, depreciation.

OBJECTIVES	COMMENTS/ACTIVITIES
2. (continued)	
(d) Bank Accounts	Students can study the various bank accounts probably best by an organized field trip to a local bank.
(e) Food, Shelter and Clothing	Generate activities with the students to have them role-play situations about the basic necessities of life.
3. Mathematics of Money Management	
(a) Insurance - Life - Property - Automobile	Have students study the various types of insurance programs in order to determine the advantages and disadvantages of each type.
(b) Home Ownership and Renting	Have students discuss the sale and rental of homes with the local real estate firms.
(c) Taxation	Make a study of income tax and fill out an income tax form. Discuss the local tax structures within a community.
(d) Investments	Have students make a study of the stock market.

INEQUALITIES

All or most of our past experience in mathematics has dealt with equalities. We can make use of this experience to solve inequalities since inequalities are solved in the same manner as equalities except when multiplying or dividing by a negative. In this case the direction of the inequality sign must be changed.

References:

1. Burns, A., Pinkney, R., and Del Grande, J. *Mathematics for a Modern World: Book 3;* Second Edition, Toronto, Gage Educational Publishing Ltd., 1976.

2. Ebos, F., and Tuck, B., *Math is 4,* Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Solve inequalities.	
2. Graph the solution sets of inequalities.	

AREA AND VOLUME

Observation tells us that geometric figures are common in everyday life. It is often necessary to find areas or volumes in order to solve everyday problems.

References:

1. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Applied Mathematics for Today: Introduction,* Toronto: McGraw-Hill Ryerson, Ltd., 1980.
2. Ebos, F., and Tuck, B., *Math is 4,* Don Mills: Thomas Nelson and Sons (Canada) Limited, 1970

OBJECTIVES	COMMENTS/ACTIVITIES
1. Determine the area (using the pertinent formula) for each of the following plane figures: triangles, squares, rectangles, parallelograms, trapezoids and circles.	Using different floor plans have students find the area of floors so that wall-to-wall carpet may be laid.
2. Determine the total area (or area of different regions) of various polyhedrons (prisms and pyramids), cylinders and cones.	Wall papering is popular. Have students find the quantity of wallpaper needed to cover walls in rooms of various shapes. Wallpaper (or adhesive covering material) is often used to decorate various household items like wastepaper baskets. Have students suggest different items and calculate the quantity of covering required.
3. Determine the volume of various polyhedrons (prisms and pyramids), cylinders, cones and spheres.	Using structural forms with various dimensions students could calculate the number of M^3 of concrete to be used. The capacity of swimming pools, silos, etc., of varying shapes and dimensions could be calculated.

NOTE: A review of (a) The theorem of Pythagoras and (b) π might be required.

MATHEMATICS 33
CORE

33

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
A. RELATIONS AND FUNCTIONS			
1. Define a relation and its domain and range, utilizing graphs and algebraic statements.	Relations may be defined by ordered pairs, graphs or open sentences. Family relations actually are examples which can be used to illustrate mathematics. Emphasis should be graphical. This would be an appropriate place to introduce the concept of absolute value as a way of expressing the domain and range. e.g., $x^2 + y^2 = 4$ has domain $-2 \leq x \leq 2$ which could be written $\lvert x \rvert \leq 2$. The domain and range should be determined from the graph of a function or relation. Finding domain and range algebraically need not be emphasized.		
2. Define the term function.	Experimental activities help motivate the students to learn this concept. A function may be defined as "a relation which has no 'fickle pickers'". The following example will illustrate this: R = { (2,3) (2,4) (3,5) } may be represented as: $2 \rightarrow 3$ using the matching notation, $\searrow 4$ 2 is a 'fickle picker' $3 \rightarrow 5$ because it picks more than one number of the range.		

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
2. continued	$S = \{(2,3)\ (4,5)\ (6,7)\}$ may be represented as: $2 \rightarrow 3$ there are no 'fickle $4 \rightarrow 5$ pickers', so this relation $6 \rightarrow 7$ is a function.		
3. Use functional notation f(x) for particular values of x.			104
4. Define and graph a linear function.			86
5. Show the relationship between the graphs of linear and quadratic functions and the roots of the corresponding equations.			72-73, 87
B. TRIGONOMETRY			
1. Maintain previously developed skills.	Review terms associated with rectangular coordinate systems: Review the Theorem of Pythagoras. Find coordinates of points on the x-axis and y-axis. Note: The relationship between degrees, minutes and decimal degrees should be discussed.		172-191 297 210-211

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
2. Solve problems related to right triangles including: **a) more than one triangle** **b) 3 dimensions**			182-191
3. Determine the value of a trigonometric ratio of any angle.	Angles should be restricted to positive angles less than or equal to 360 degrees. Related angles may be illustrated by geometric ideas such as congruent triangles or reflections. In actual practice students will find it easier to use the 3-step reduction of angles method. 1. Find the related angle. 2. Use the same function as the given one. 3. Determine the sign of the function.		216-224
4. Solve oblique triangles by using Law of Sines and/or Law of Cosines.	Law of Sines: Ambiguous problems in which two sides and an angle opposite one of them are given may have more than one solution depending on the relative measure of the sides. Better students may wish to pursue an analysis of this case in detail.		192-197 207-208
5. Apply the Law of Sines and Law of Cosines to practical problems.		In surveying, find inaccessible distances. In aerial navigation, determine which course to fly to counteract the effects of the wind.	198-199 208-209

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
6. **Derive and apply the following to practical problems.** **a) Quotient relations:** $\tan \theta = \frac{\sin \theta}{\cos \theta}, \cot \theta = \frac{\cos \theta}{\sin \theta}$ **b) Reciprocal relations:** $\csc \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}$ $\cot \theta = \frac{1}{\tan \theta}$ **c) Pythagorean relations:** $\sin^2 \theta + \cos^2 \theta = 1$ $1 + \tan^2 \theta = \sec^2 \theta$ $1 + \cot^2 \theta = \csc^2 \theta$	The intent is to introduce the student to the concept of an identity as a statement to be proved. These should be limited to the following types: a) conversion of both sides in terms of sin and cos b) squaring a binomial expression e.g., $(\cos A + \sin A)^2$ c) factoring expressions restricted to common factor or difference of squares.		233-238
7. **Solve simple trigonometric equations.**	Coverage should include equations of the linear and quadratic form. For example: 1. $3 \cos \theta + 1 = 2$ 2. $2 \sin^2 \theta + \sin \theta = 1$		

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
C. PRESENTATION OF DATA AND DESCRIPTIVE STATISTICS			
1. Maintain previously developed skills with ungrouped and grouped data.			25-40
2. Develop and apply standard deviation and z-scores.	z-score, $z = \frac{x - \bar{x}}{\sigma}$, may be introduced as a measure independent of units used. This allows easy use of tables involving areas under the normal curve.		41-44
3. Illustrate and develop the normal distribution.	1. Comparing the normal curve to histograms of progressively narrower intervals (or frequency polygons) shows how the smooth curve is obtained from actual data. 2. Relate measure of central tendency and dispersion to the normal curve.	Predictions, life expectancy, mortality tables.	
4. Introduce probability using an experimental approach.	Introduction to such terms as outcomes, sample space and event would be appropriate.		3-16 45-54

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
D. QUADRATIC FUNCTIONS, EQUATIONS AND APPLICATIONS			
1. Identify and express quadratic functions in the form $y = ax^2 + bx + c$ where $a, b, c \in R$, $a \neq 0$.			86-87
2. Identify and express quadratic equations in the form $ax^2 + bx + c = 0$ where $a, b, c \in R$, $a \neq 0$.			72-73
3. Graph a quadratic function using a table of values.	Students should become familiar with the graphs of quadratic functions of the forms: $y = ax^2$, $y = ax^2 + bx$, and $y = ax^2 + bx + c$.		88
4. Find the vertex, axis of symmetry, domain, range and maximum or minimum value of a quadratic function from its graph.	The importance of the vertex and axis of symmetry can be illustrated using appropriate problem solving applications.	A road passes under a railroad overpass in the form of a parabolic arch 5 metres high (at the apex) and 20 metres wide. The equation of the parabola is: $y = 5 - \frac{x^2}{20}$ Find the height of the tallest truck that can pass under the arch. Assume the truck is 3 metres wide, that it stays on its own side of the road, and that the centerline of the road passes directly under the apex of the arch.	88-94

Mathematics 33

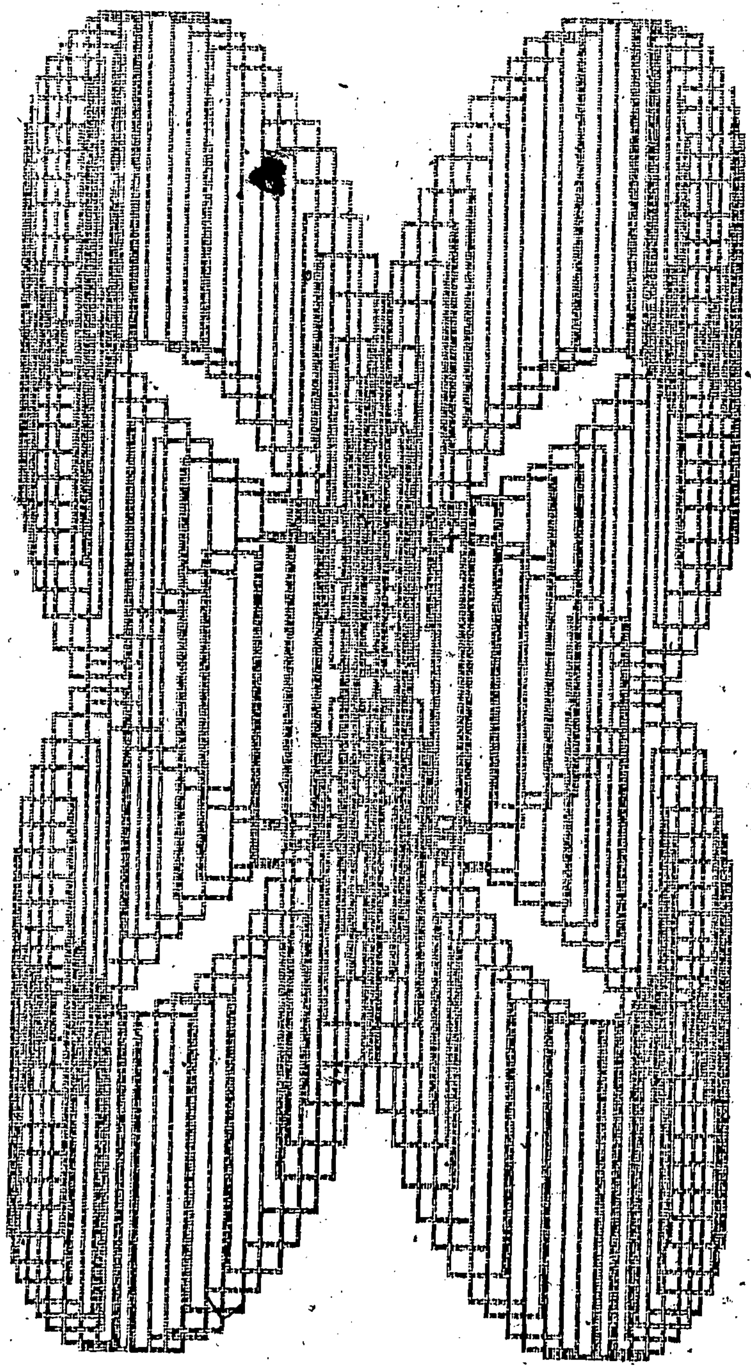
OBJECTIVES	COMMENTS	APPLICATIONS	AMT
5. Use the formula for vertex and axis of symmetry if the quadratic function is given in the form $y = ax^2 + bx + c$.	It is suggested that the formula be derived before it is used.		
6. State the relationship between the graph of a quadratic function and the roots of the corresponding equation.			92-95
7. Write quadratic equations in the form: $ax^2 + bx + c = 0$ and specify the value of a, b, and c.	Initial graphing of quadratics by a table of values acquaints students with the more important points of a quadratic and also develops an appreciation for graphing quadratics using these critical points.		
8. Use the method of completing the square of a quadratic function to find the vertex, axis of symmetry, range and maximum or minimum value. Draw the graph using this information.	The value of (a) should be restricted to the set of integers.		
9. Solve problems involving the maximum or minimum value of a quadratic function.	Problems involving maximum profit can be attempted with a strong Math 33 class, but should not be emphasized as a rule.	a. The Wishbones have 30 metres of fence with which to make a rectangular dog run. If they use a side of the house as one side of the run, what dimensions will give the maximum area?	96-100

<table>
<tr><th>OBJECTIVES</th><th>COMMENTS</th><th>APPLICATIONS</th><th>AMT</th></tr>
<tr><td>9. continued</td><td></td><td>b. Fast Freddie's Theatre charges $4.00 per ticket, and it has had a full house of 400 nightly. The manager estimates that the ticket sales would decrease by 50 for each $1.00 increase in the ticket cost. What is the most profitable price to charge? (A simple arithmetic solution using a table of values could be used to introduce the concept of maximum value).
<table>
<tr><td>N</td><td>400</td><td>350</td><td>300</td><td>250</td></tr>
<tr><td>C</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>I</td><td>1600</td><td>1750</td><td>1800</td><td>1750</td></tr>
<tr><td></td><td></td><td></td><td>↑
Max.</td><td></td></tr>
</table></td><td>96-100</td></tr>
<tr><td>10. Compute the real roots of a quadratic equation by:
a) factoring
b) using the quadratic formula or completing the square</td><td></td><td></td><td>74-79</td></tr>
<tr><td>11. Define and evaluate the discriminant of a quadratic equation.</td><td></td><td></td><td></td></tr>
<tr><td>12. State the nature of the roots by examining the discriminant.</td><td></td><td></td><td></td></tr>
</table>

OBJECTIVES	COMMENTS	APPLICATIONS	AMT
13. Solve problems whose solutions are based on quadratic equations.		a. A club bought a snowmobile for $720.00, planning to divide the expenses equally among the members. However, two members withdrew from the club, increasing each share by $4.00. How many members were in the club originally? b. How wide a strip must be sodded around a square court 10 metres on a side, so that one-half of the court is sodded. c. A 6 centimetre square is cut from each corner of a square piece of sheet metal. The sides are folded up to form an open box having a volume of 60 cc. What is the length of a side of the original square piece of sheet metal? d. Two students agree to share the work of cutting the grass on a large lot measuring 80 metres by 60 metres. One student starts by cutting a strip all of the way around the lot and continuing around in this way. How wide will this strip be when half the work is completed?	79-84
E. LOGARITHMS			
1. Maintain previous skills on exponents.			159-160

Mathematics 33

OBJECTIVES	COMMENTS	APPLICATIONS	AMT.
2. Identify and graph exponential functions.	This study of logarithms should emphasize the functions, their graphs and their use in applications.	Logarithms are frequently used to describe rates of growth, rates of decay, sound intensity, light intensity and earthquake intensity.	148-151
3. Convert equations from exponential form to logarithmic form and vice versa.		Sample Questions a. Loudness of sound is measured on a decibel scale according to the formula: $D = 10 \text{ Log } (L)$, where D is the number of decibels of sound and L is the loudness of the sound. How many times louder is the sound of 52 decibels than the sound of 37 decibels? b. A scientist predicts that 20 grams of a radioactive substance will decay in such a way that after (T) days the number of grams remaining (G) may be estimated according to the formula $G = 20X^{-T/15}$ Determine to one decimal place, the number of days required for the substance to reduce to 8 g. c. The length of time to double your money at 12% is found by solving $2 = (1.12)^n$ for n.	161-164
4. Solve logarithmic equations by converting to exponential form.	$\log_{16}x = \frac{-5}{4}$ $\log_x \frac{27}{8} = -3$ $\log_2 \frac{1}{16} = x$		
5. Define the inverse of an exponential function in logarithmic form.	Emphasize the relationship between exponential and logarithmic functions.		
6. Evaluate expressions and solve equations involving logarithmic form and exponential form.	Limit equations to base 10.		164-170
7. State and use the basic laws or properties of logarithms for: **a) products** **b) quotients** **c) powers** **d) roots**	Logarithms are no longer needed to do complex numerical calculations. However, familiarity with properties of logs is necessary to solve certain exponential equations. Students should be able to recognize that: a. $\log (2a) = \log 2 + \log a$ b. $\log \frac{4}{a} = \log 4 - \log a$ c. $\log a^3 = 3 \log a$, and vice versa.		
8. Use logarithms to solve practical problems.			



MATHEMATICS 33
ELECTIVES

33

ABSOLUTE VALUE

In some problems only the magnitude of a value is required. E.g., How much did the temperature increase from 9:00 a.m. to 3:00 p.m. if it were - 8°C at 9:00 a.m. and 4°C at 3:00 p.m.?

We would disregard the direction on a number line in order to solve the above. In other words, we would use the absolute value of the numbers.

References:

1. Ebos, F., and Tuck, B., *Math Is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.
2. Dottori, D., Knill, G., and Stewart, J., *Foundations of Mathematics for Tomorrow: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1978.
3. Carter, J., Clark, J., Porter, B., and Stouffer, M., *Mathematics Alive 3*, Toronto: Copp Clark Publishing, 1978.
4. Bye, M., and Elliott, H., *Math Probe 3*, Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Use the absolute value symbol correctly.	
2. Use absolute value to indicate the measure of the distance between the point and the origin on a number line.	
3. Use absolute value to tell the distance between any two points on the number line.	
4. Solve equations like $|2x|-|4 = 8$	
5. Find the solution sets of inequalities like $|3x| < 12$ and graph the solution set.	

COMPLEX NUMBERS

Complex numbers are a natural outgrowth of the study of quadratic equations. Development of a new number system through the introduction of a new symbol can be illustrated. For example, in expanding the system No to I, we merely introduce a new symbol: the minus sign. Similarly for the complex numbers, we introduce a new symbol, i, to represent $\sqrt{-1}$. This allows us to solve quadratic equations which have no solution in the real number system.

Although few practical applications of complex numbers exist at a level which high school students can understand, complex members are used extensively by electrical engineers. Several examples of this, at a very basic level, are included in reference 3.

References:

1. *Algebra Two and Trigonometry*, Vogeli et al, Silver-Burdett Company.

2. *Holt Algebra 2 With Trigonometry*, Nichols et al, Holt, Rinehart and Winston, 1974.

3. *Using Advanced Algebra*, Travers et al, Laidlaw Publishers, 1975.

4. *Algebra and Trigonometry: Structure and Method (Book 2)*, Dolciani et al, Houghton, Mifflin, 1977.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Demonstrate the need for a new number system beyond the real numbers in order to solve equations of the form $x^2 + 1 = 0$.	
2. (a) Perform simple arithmetic operations with complex numbers of the form $a = bi$. (b) Evaluate the conjugate and use it in division of complex numbers. (c) Solve for unknowns for equations in the form $2 + 3i = x + yi$.	

OBJECTIVES	COMMENTS/ACTIVITIES
3. Graph complex numbers on an Argand diagram, (rectangular, coordinate system) using a vector to represent the complex number.	
4. Evaluate the absolute value (modules) of a complex number and interpret it as the length of a vector on an Argand diagram.	
5. Solve quadratic equations which have non-real roots.	
6. Solve higher degree equations by special factoring methods (sum or differences of cubes, differences of squares).	
7. Find roots of a number algebraically. (e.g., $x^3 + 1 = 0$ can be used to find the 3 cube roots of -1).	
8. Find the roots of a number of graphical means, using an Argand diagram. (All roots represented by vectors with an equal length).	
9. Solve application problems of complex numbers (impedence of an electrical circuit).	

CONSUMER MATHEMATICS

It is becoming more and more important that students have some understanding of how mathematics is related to the consumer. A working knowledge of how the basic skills in mathematics are applied to consumer topics such as accessing money, spending money and the management of money is essential for a student to become a good citizen.

References:

1. *Mathematics in Life*, Gage Publishing Co.
2. *Mathematics Plus*, Houghton Mifflin
3. *Mathematics for the Real World*, Merrill/Bell and Howell Publishing Co.
4. *Mathematics for Daily Use*, Doubleday Publishing Co.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To develop an appreciation of how money is obtained.	Identify different careers and discuss the wages paid for the related work.
(a) Jobs	Identify what is needed for the accomplishment of different jobs.
- Descriptions - Applications - Wages	Discuss filling out applications and interviews.

OBJECTIVES	COMMENTS/ACTIVITIES
1. (Continued)	
(b) Payroll - Methods of payment - Deductions · Tax · Pension · Benefits	Have students discuss and analyze real or imaginary paychecks.
(c) Selling Products	Have students participate in such activities as a school store, concession or canteen.
(d) Record Keeping.	Discuss and practice the necessity for good record keeping.
2. To develop an understanding of wise and thrifty spending habits.	
(a) Buying	
- Comparative buying	Students can check out the cost of a shopping list of some everyday needs. Prices can be compared from store to store, by brand and by size. Prices could be prepared over a period of time to determine price changes and trends.
(b) Credit Buying - Types of credit - Types of payment - Interest rates - Installment buying - Sources of borrowed money	Students could determine the cost of buying such items as a car.
(c) Operation Costs	Students could determine the cost of operating a car for a period of a year. Things to take into account are: purchase price, licence, insurance, gasoline, service and maintenance, depreciation.

OBJECTIVES	COMMENTS/ACTIVITIES
2. (continued)	
(d) Bank Accounts	Students can study the various bank accounts probably best by an organized field trip to a local bank.
(e) Food, Shelter and Clothing	Generate activities with the students to have them role-play situations about the basic necessities of life.
3. Mathematics of Money Management	
(a) Insurance - Life - Property - Automobile	Have students study the various types of insurance programs in order to determine the advantages and disadvantages of each type.
(b) Home Ownership and Renting	Have students discuss the sale and rental of homes with the local real estate firms.
(c) Taxation	Make a study of income tax and fill out an income tax form. Discuss the local tax structures within a community.
(d) Investments	Have students make a study of the stock market.

HISTORY OF MATHEMATICS

The history of mathematics can provide many interesting discussions and lends itself to many interesting projects for the classroom. A look at historical topics is unlimited and is dependent only upon the creativity of the teacher.

Reference:

1. *Historical Topics for the Mathematics Classroom,* Thirty-first yearbook, National Council of Teachers of Mathematics, 1969.

NOTE: This yearbook also contains an excellent bibliography of the many books and articles available on historical topics in Mathematics.

OBJECTIVES	COMMENTS/ACTIVITIES
1. To acquire an appreciation for the historical development of mathematics from counting to modern day computers.	Discuss various types of early developments in mathematics. Many of these can be set in activity stations, as display areas or in written project form.
2. To humanize mathematics by looking at the lives of some of the mathematicians.	Investigate the lives of famous mathematicians by using different methods such as: reporting, writing essays, role-playing, discussions, films.
3. To interrelate mathematics with other subject areas such as science, music, social studies and art.	
4. To familiarize students with the use of various mathematical instruments which have been developed throughout the years.	Discuss, demonstrate the use of, or construct the basic mathematical instruments for:

OBJECTIVES

COMMENTS/ACTIVITIES

4. (Continued)

(a) Measurement
- sundial
- waterclock
- transit
- sextant
- angle mirror
- micrometer
- caliper
- trundle wheel

(b) Calculations
- slide rule
- logarithms
- calculators
- Napier's bones
- abacus
- computers

LINEAR PROGRAMMING

Linear programming involves the attempt to maximize results and minimize efforts to produce those results. Such attempts originated during World War II when the Allies attempted to maximize production and minimize costs. Manufacturing problems involving numbers of items and various other constraints, such as time required for production, can be analyzed in a systematic way using linear equations and inequalities.

The fundamental assumption of linear programming is: the maximum value of the parameter P, for the relation $P = Ax + By$, occurs at one of the vertices of the polygonal region determined by the various constraints in the problem. The maximum value of P can be found by examining these vertices.

References:

1. Ebos and Tuck, *Math is 4*, Thomas Nelson and Sons, 1979.
2. Travers, Dalton et al, *Using Advanced Algebra*, Doubleday, 1977.
3. Wigle, Jenning and Dowling, *Mathematical Pursuits Three*, Macmillan of Canada, 1977.
4. Dotton, Knill and Seymour, *Applied Mathematics for Today*, McGraw-Hill Ryerson, 1976.
5. Hanwell, Bye and Griffiths, *Holt Mathematics 4 (Second Edition)*, Holt, Rinehart and Winston, 1980.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Define the following terms: parameter, region, constraint, maximum point.	
2. Draw the graph of a linear equation in the form $Ax + By = C$.	
3. Draw the graph of a region defined by an inequality.	

OBJECTIVES	COMMENTS/ACTIVITIES
4. Determine a region bounded by several inequalities.	
5. Find the intersection of 2 linear equations by graphical or algebraic means.	
6. Find the maximum value of a perimeter P defined by P = Ax + By for a given set of constraints.	
7. Determine the value of a parameter P at each of the vertices of the polygon which results from graphing all the constraint conditions on x and y.	
8. Apply linear programming to solving business-oriented problems.	

PROBABILITY

There are numerous books which cover this topic, usually together with some statistics. In a short elective component, it is important to just get the feel of calculating some interesting probabilities. If one can experimentally verify or disprove ones' results, so much the better.

OBJECTIVES

The following concepts should be taught as a preamble:

1. The idea of an event E as the outcome of some experiment or trial.
2. The meaning of $P(E)$, the probability of this event occurring. Also the fact that probabilities are numbers between 0 and 1.
3. The concept of independence of events.
4. The meaning of mutually exclusive events.
5. If one has two events A *and* B, then: $P(A \text{ or } B) = P(A) = P(B) - P(A \text{ and } B)$.

COMMENTS/ACTIVITIES

If the above five ideas can be mastered, then one can follow with some simple combinatorial results before some interesting examples can be tried. The student should learn about:

a) Factorial notation

b) that $\binom{n}{k} = \dfrac{n!}{k!\,(n-k)!}$ represents the number of ways of arranging k objects among n locations.

c) the hypergeometric distribution.

One has a box containing N_1 black marbles and N_2 white marbles for a total of $N = N_1 + N_2$ marbles. If one now takes n marbles from the box (at random without replacement) then the probability one obtains n_1 black and n_2 white marbles is $\binom{N_1}{n_1} \binom{n_2}{n_2} \Big/ \binom{N}{n}$

e.g., if the box contains 5 black and 3 white marbles, then in drawing 2 marbles the probability one obtains one of each colour is:

$$\binom{5}{1} \binom{3}{1} \binom{8}{2} = 15/28$$

OBJECTIVES	COMMENTS/ACTIVITIES
continued	With these minimal tools, one can now tackle many interesting problems. For example, when dealt a 5-card hand from a standard deck, following are the poker hands one can obtain: a) nothing b) one pair c) 2 pairs d) 3 of a kind (a triple) e) full house (one triple, one pair) f) four of a kind g) flush (all in one suit) h) straight (5 cards in sequence) i) straight flush (5 cards in sequence in one suit) j) royal flush (highest straight-flush)

If one considers these 10 events as mutually exclusive, one can calculate the probability of each occurring and hence order, via probabilities, the sequence of winning events.

There is a multitude of variations on this one theme alone, namely what happens if you have a wild card? Does this change the order of winning events? All these can be answered, and if in doubt, deal a few hands to check your answer.

Reference:

1. Most of these notations can be found in any respectable text. One in which the card hands are discussed and the probabilities calculated is: *Basic Probability and Applications*, by M. Nosal., Published by W. B. Saunders.

TOPOLOGY

Reference:

1. *Excursions into Mathematics*, by Beck, Bleicher and Crowe. Published by Worth.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Euler's formula.	Pages 3 - 11.
2. The number of regular polyhedra.	Pages 12 - 16.
3. Tesselation of the plane: Use Euler's formula to show that triangles, hexagons and squares are the only regular polygons one can use to tile a floor.	
4. A European "football" is made of pieces of leather in the shape of regular pentagons and regular hexagons. These are sewed together so that each pentagon is surrounded by hexagons and each hexagon is surrounded (alternately) by three pentagons and three hexagons. Determine the number of pentagons and hexagons of such a football.	
5. Deltahedra: This section deals with non-regular polyhedra, all of whose faces are triangles. It also illustrates methods of constructing these polyhedra out of cardboard.	NOTE: The objectives listed are all fully illustrated in the text with numerous examples that can actually be constructed.
6. If time permits, there is a section of polyhedra without diagonals using Euler's formula.	

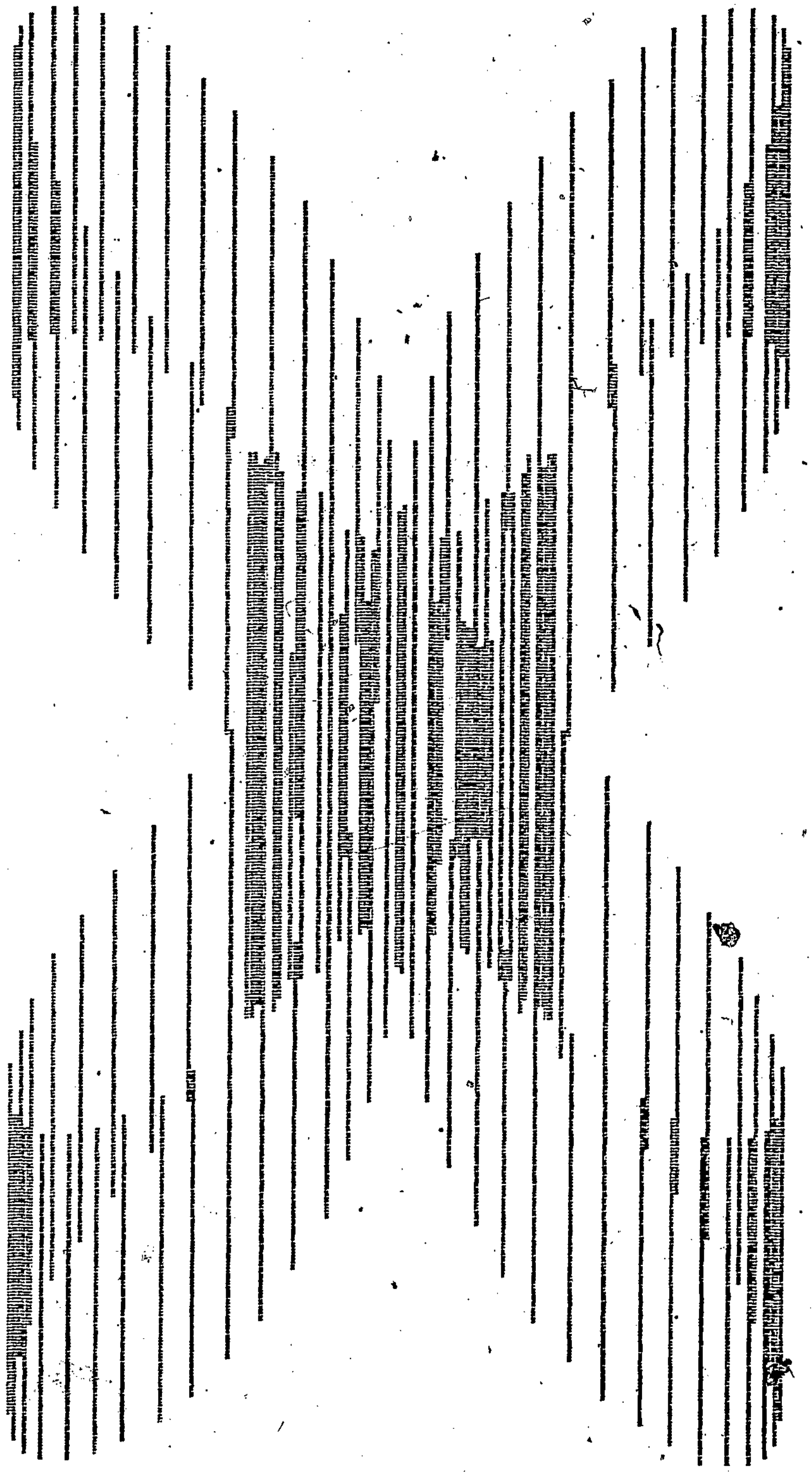
VECTORS

Vectors provide a useful tool for analyzing problems that deal with trips, forces, velocities, accelerations and displacement. A vector is a quantity that requires both a magnitude and direction to describe it.

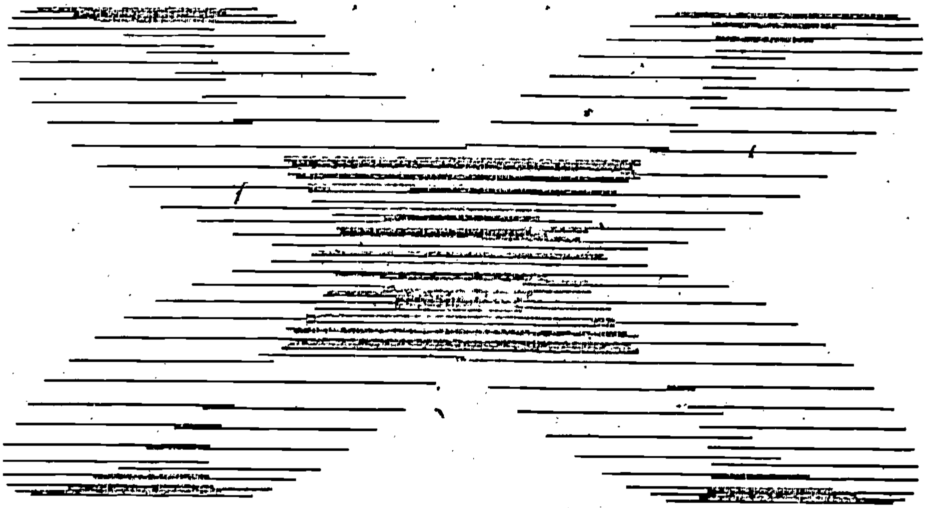
References:

1. Bye, M. and Elliott, H., *Math Probe 3*, Toronto: Holt, Rinehart and Winston of Canada, Ltd., 1973.
2. Burns, A.G. Pinkney, R.G., and Del Grande, J.J., *Mathematics for a Modern World: Book 3*, Toronto, Gage Educational Publishing Ltd., 1976.
3. Dottori, D., Knill, G. and Seymour, J., *Applied Mathematics for Today: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1976.
4. Dottori, D., McVean, R., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Introduction*, Toronto: McGraw-Hill Ryerson Ltd., 1974. Nichole, *Modern Intermediate Algebra*.
5. Dottori, D., Knill, G., and Seymour, J., *Foundations of Mathematics for Tomorrow: Intermediate*, Toronto: McGraw-Hill Ryerson Ltd., 1978.
6. Ebos, F., and Tuck, B., *Math is 4*, Don Mills: Thomas Nelson and Sons (Canada) Limited, 1979.
7. Hilton, A., *Vectors*, London: Macdonald Educational Colour Units, 1976.

OBJECTIVES	COMMENTS/ACTIVITIES
1. Understand those phenomena which possess magnitude and direction.	Describe five situations in which vectors are used.
2. Add and subtract vectors.	Create and solve problems using vectors.
3. Multiply a vector by a scalar.	
4. Solve problems using vectors.	



APPENDICES



APPENDIX A

Alberta EDUCATION

METRICATION POLICY

METRICATION POLICY

It is the policy of Alberta Education that:

1. SI units become the principal system of measurement in the curriculum of the schools in the province;
2. the change to the use of SI units in schools be such that the instructional programs are predominantly metric by June, 1978;
3. changes in the curriculum of individual schools proceed in concert with corresponding changes in industry and commerce;
4. selected consultative and in-service resources be made available to teachers for professional preparation in the integration of SI units in their instructional programs;
5. conversion of the schools to the metric system (SI) be carried out on a pre-planned basis with a gradual replacement and/or modification of measurement-sensitive resources;
6. conversion costs will generally be borne by the responsibility centre incurring them;
7. conversion will be accomplished by means of existing administrative structures, and there will be only a minimum number of short-term special purpose assignments associated with the change to SI.

EXPLICATION OF THE METRICATION POLICY

1 *. . . that SI units become the principal system of measurement in the curriculum of the schools in the province;*

1.1 As part of a world-wide movement to standardize commonly used elements of trade, Canada has committed itself to the use of SI units by the early 1980's. It is already evident that many sections of our society which use measures will be predominantly metric before 1980.

It has become imperative that the school curriculum prepare students to cope with metric measures in all facets of their life. In addition many students will require a detailed knowledge of more specilized units used in specific career fields.

1.2 Some imperial units will be in use for some time; therefore as teachers introduce SI they are expected to retain selected reference to imperial units. This teaching of specific imperial units should be related only to those that are relevant to student needs and should be kept to a minimum. Mathematical conversions from one system to the other are to be avoided wherever possible.

1.3 Resource materials for the classroom should use SI units as the principal measuring system with only such references to the imperial or "old" metric systems as are unavoidable.

2 *. . . that the change to the use of SI units in schools be such that the instructional programs are predominantly metric by June, 1978.*

2.1 In a predominantly metric program the student, depending upon age and ability, is familiar with the appropriate metric units of length, area, volume, mass and temperature, and uses these units in school for making and expressing measurements in all areas of the curriculum.

2.2 The date (June, 1978) is a general target by which it is expected that curriculum guides and programs of studies and the various acts and regulations will have been changed to reflect the new measurement language. Most schools will have begun the transition of their classroom programs by June, 1978; indeed many schools will have completed the change before this date.

2.3 It is entirely possible that some of the high school technical and vocational programs will have a need to continue teaching the use of imperial units. It may take a specific technological area some considerable time to make the change, especially one that uses machinery with a long working life.

2.4 The correct usage of the SI units will be directed by the **Metric Practice Guide** (CSA Z234.1 - 1973) or the **Metric Style Guide** (Council of Ministers of Education, Canada). A copy of the **Metric Style Guide** has been made available to each teacher (in English or French).

3 *. . . that changes in the curriculum of individual schools proceed in concert with corresponding changes in industry and commerce.*

3.1 The Canadian Metric Commission under the federal Ministry of Industry, Trade and Commerce has been coordinating the change to metrics since 1970. Timelines and metrication dates have been established for many sectors of the economy. Educators should keep themselves informed as to progress in both the technical areas in which they may be teaching and the activities occurring in the community.

3.2 The Department of Education will attempt to keep schools informed regarding progress in the use of metric units.

4 *. . . that selected consultative and inservice resources be made available to teachers for professional preparation in the integration of SI units in their instructional programs.*

4.1 Achieving metric conversion by the target date is dependent upon many factors, not the least of which is the whole-hearted cooperation by educators in carrying out their assigned roles. In preparing teachers and administrators to better cope with the changes, the Department of Education will provide consultative and inservice resources through the Regional Offices in Grande Prairie, Edmonton, Red Deer, Calgary and Lethbridge.

4.2 The staff of a school should plan co-operatively to bring a unified approach to the teaching and use of measurement. Metrication goes beyond the formal content of the curriculum. The changeover with its many implications will affect everyone associated with the process of education.

4.3 Since measurement is an activity related process, it follows that individuals — teachers as well as students — learn the metric units while measuring. That is, some degree of active involvement in measuring or in using the units that one is learning.

4.4 Very few people will have to know the entire metric system. That is, when presenting SI, one should only attempt to deal with that part of the system which is necessary for the task at hand.

5 *. . . that conversion of school programs to SI be carried out on a pre-planned basis with a gradual replacement and/or modification of measurement-sensitive resources.*

5.1 Most measurement-sensitive devices will have to be replaced over the next few years. Items ranging from inexpensive rulers to costly metal lathes will have to be either replaced or modified. Exactly when and how much modification versus replacement occurs depends upon the economics. For a machine that is near the end of its useful life, any modifications will have to be minimal. For a nearly new machine a scale replacement or recalibration may be the best course of action. In any event, common sense must prevail.

5.2 Any new acquisitions of machines or tools should be those with metric capabilities. In spite of the best planning and careful budgeting, there is bound to be some frustration as suppliers fail to deliver as promised and as people do not respond as predicted.

6 *. . . that conversion costs will generally be borne by the responsibility centre incurring them.*

6.1 The term "responsibility centre" is meant to refer to those parts of the administrative structure which are responsible for budgeting and purchasing services, materials, and equipment for schools.

6.2 By placing responsibility for costs as close as possible to the actual use of goods or services, it is hoped that there will be a greater accountability in the changing over to metrics.

6.3 Implicit in this policy is the idea that those exercising responsibility will do so with restraint.

7 *. . . that conversion will be accomplished by means of existing administrative structures, and there will only be a minimum number of short-term special-purpose assignments associated with the change to SI.*

7.1 In other words, as far as is possible, there will be only temporary positions associated with metrication. In addition, administrative costs are to be kept as low as possible, with the additional work load being handled on a contract or short-term assignment basis. In this way, it is hoped that the money spent on metrication will have maximum effect on the classroom.

APPENDIX B

Strategies of Problem Solving

In the teaching/learning of problem solving, an instructional approach should be used which helps students learn and choose procedures for solving problems. These procedures are easy to state and recognize, but they are often quite elusive when teaching. Difficulty frequently exists when teaching problem solving because, unlike the teaching of computational skills or concepts, there is no specific content involved. In problem solving, an individually acquired set of processes is brought to bear on a situation that confronts the individual.

There are generally four procedures (steps) which appear inherent in problem solving. These procedures, their descriptions and associated strategies have been compiled and adapted from a variety of sources and authors (George Polya, J.F. LeBlanc, Ohio Department of Education, Math Resource Project, 1980, NCTM Yearbook) and are listed below:

STEPS IN PROBLEM SOLVING

1) UNDERSTAND THE PROBLEM

What is the problem? What are you trying to find? What is happening? What are you asked to do?

Suggested Strategies:

- Paraphrase the problem or question (Restate the problem in your own words to internalize what the problem entails.)
- Identify wanted, given and needed information (Helps students focus on what is yet to be determined from problem statement as well as listing information so that they may better be able to discover a relationship between what is known and what is required.)
- Make a drawing (May help to depict the information of a problem, especially situations involving geometric ideas.)
- Act it out (Helps to picture how the problem actions occur and how they are related thereby giving a better understanding of the problem.)
- Check for hidden assumptions (What precisely does the problem say or not say? Are you assuming something that may ~~not~~ be implied? Beware of mistaken inferences.)

2) DEVISE A PLAN TO SOLVE THE PROBLEM

What operations should you use? What do you need to do to solve the problem? How can you obtain more information or data to seek the solution?

Suggested Strategies:

- Solve a simpler (or similar) problem (Momentarily set aside the original problem to work on a simpler or similar case. Hopefully the relationship of the simpler problem will point to the solution for the original problem.)

- Construct a table (Organizing data in tabular form makes it easier to establish patterns and to identify information which is missing.)

- Look for a pattern or trend (Does a pattern continue or exist? In connection with the use of a table, graph, etc., patterns or trends may be more apparent.)

- Solve part of the problem (Sometimes a series of actions each dependent upon the preceding one, is required to reach a solution. Similarly it may be that certain initial actions will either produce a solution or uncover additional information to simplify the task of solving the problem.)

- Make a graph or numberline (May help organize information in such a way that it makes the relationship between given information and desired solution more apparent.)

- Make a diagram or model (When using the model strategy attempt to select objects or actions to model those from the actual problem that represents the situation accurately and enables you to relate the simplified problem to the actual problem. May be used in connection with, or in place of other similar strategies, i.e., acting out the problem.)

- Guess and check (Guessing for a solution should not be associated with aimless casting about for an answer. The key element to this strategy is the "and check" when the problem solver checks his guesses against the problem conditions to determine how to improve his guess. This process is repeated until the answer appears reasonable. An advantage of this "guess and check" strategy is that it gets the individual involved in finding a solution by establishing a starting point from which he can progress. Used constructively with a table or graph this strategy may be a valuable tool.)

- Work backwards (Frequently, problems are posed in which the final conditions of an action are given and a condition is asked for which occurred earlier or which caused the final outcome. Under these circumstances working backwards may be valuable.)

- Change your point of view (Some problems require a different point of view to be taken. Often one tends to have a "mind set" or certain perspective of the problem which creates a difficulty in discovering a solution. Frequently, if the first plan adopted is not successful, the tendency is to return to the same point of view and adopt a new plan. This may be productive, but might also result in continuous failure to obtain a solution. Attempt to discard previous notions of the problem and try to redefine the problem in a completely different way.)

- Write an open sentence or equation (Often in conjunction with other strategies - using a table, diagram, etc., one selects appropriate notation and attempts to represent a relationship between given and sought information in an open sentence.)

3. CARRY OUT THE PLAN

For some students the strategy/strategies selected may not lend itself/themselves to a solution. If the plan does not work, the problem solver should revise the plan, review step 1, and/or try another plan or combination of plans from step 2.

4. LOOK BACK AT THE STEPS TAKEN (Consolidating Gains)

Is the result reasonable and correct? Is there another method of solution? Is there another solution? Is obtaining the answer the end of the problem?

- Generalize (Obtaining an answer is not necessarily the end of a problem. Re-examination of the problem, the result and the way it was obtained will frequently generate insights far more significant than the answer to the specific situation. It may enable the student to solve whole classes of similar and even more difficult problems.)

- Check the solution (The very length of a problem or the fact that symbolic notation is used may tend to make one lose sight of the original problem. Does the answer appear reasonable, does it satisfy all the problem requirements?)

- Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem solving is to study the processes that lead to solutions to problems. Once a solution is discovered, search the problem for further insights and unsuspected ideas and relationships.)

- Find another solution (Students tend to approach many problem situations with the expectation of only one correct solution. In many practical, daily life situations there may be many answers that are correct and acceptable.)

- Study the solution process (Studying the process of solution makes the activity of problem solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.

It must be noted that the four steps of the above model are not necessarily discreet. For example, one may move without notice into Step 2 while attempting to generate more information to understand the problem better.

If the 4-step model is used, the key is to select an appropriate strategy or strategies to help answer the questions suggested by each step. The strategies listed, and those devised by students will hopefully alter the problem information, organize it, expand it, and make it more easily understood. Strategies then may be thought of as the tools of problem solving and the 4-step model, the blueprint.

Recommendations for School Mathematics of the 1980s

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

The National Council of Teachers of Mathematics recommends that -

1. problem solving be the focus of school mathematics in the 1980's;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.