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ABSTRACT

A normally distributed data set of 1,000 values--ranging from 50 to 150, with a mean of 50 and a standard deviation of 20--was created in order to evaluate the bootstrap method of repeated random sampling. Nine bootstrap samples of N=10 and nine more bootstrap samples of N=25 were randomly selected. One thousand random samples were selected from each of the 18 bootstrap samples, and its mean and standard deviation were calculated. The cumulative means and standard deviations diverged from the parameter values as often, and to the same extent, as they converged toward them. It was also concluded that the bootstrap procedure was biased because it did not continue to approach the universe parameter as the number of iterations increased. The limit of convergence was not the universe parameter. Hence, the bootstrap hypothesis regarding point estimates of means and standard deviations was not supported. (Author/DDC)

An Evaluation of the Bootstrap Hypothesis Using Computer Simulation

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Abstract

A normally distributed universe of 1,000 values ranging from 50 to 150 with a mean of 50 and a standard deviation of 20 was created. Nine bootstrap samples of N=10 and nine more bootstrap samples of N=25 were randomly selected. One thousand random samples were selected from each of the 18 bootstrap samples and its mean and standard deviation were calculated. The cumulative means and standard deviations diverged from the parameter values as often and to the same extent as they converged toward them. Hence, the bootstrap hypothesis was not supported regarding point estimates of means and standard deviations.

Introduction

Diaconis and Efron (1983) argued that large sample results could be approached by repeated random sampling from a small sample thereby pulling oneself up by their bootstraps. An analogy with holograms is informative. It is possible to reconstruct an entire holographic image by cutting out any small section of the holographic plate and illunimating it. In short, properties of the whole are encoded in every part.

Diaconis and Efron (1983) demonstrated that the universe correlation between average Grade Point Average (GPA) and average Law School Admission Test (LSAT) scores for all 82 American Law Schools in 1973 could be approached by repeated random sampling from a bootstrap sample of 15 schools.

The purpose of the present study was to examine the extent to which point estimates for the mean and standard deviation converge toward their population values given repeated random sampling from a bootstrap sample.

Method

A data base of 1,000 normally distributed values was created by evaluating the equation for the normal curve using a mean of 50 and a standard deviation of 20 for abscissa values ranging from 50 to 150 in increments of 1. The calculated ordinate was rounded and served to indicate the number of times that the abscissa value appeared in the data base. A random sample of N = 10 data points was selected as the first bootstrap sample. A total of 1,000 random samples of N = 10 were obtained from this bootstrap sample and a mean and standard deviation was calculated for each sample. The cumulative mean and cumulative standard deviation was then calculated over the 1,000 values in blocks of 5 samples. This procedure varied the number of bootstrap calculations in the displayed result. The resulting 200 blocks were examined to determine if the initial estimates of the universe mean and standard deviation obtained from the original bootstrap sample converged toward the universe parameters for these statistics over the 1,000 replications of the bootstrap procedure.

The above procedure was replicated nine times using a different random bootstrap sample of N = 10 values. Then nine additional replications were carried out using nine different random bootstrap samples of N = 25. All calculations were implemented on a DEC-20/60 system using FORTRAN. The random sampling was governed by the IMSL routine labeled GGUD.

Results

Table 1 presents a summary of the results concerning the nine replications of the N = 10 bootstrap sample analysis. The data relate to the deviations between the sample value and the population parameter for the mean and standard deviation. For example, the mean of the first bootstrap sample was 1.44 units below the population mean; hence the entry of -1.4400. This starting position is labeled "From". The cumulative mean over 1,000 bootstrap sample means converged to a value that was 0.1816 greater than the population parameter; hence the entry of 0.1816 under the heading "To" indicating the terminus of the analysis. The standard deviation of the first bootstrap sample was 2.8113 larger than the universe parameter. The cumulative mean over 1,000 bootstrap sample standard deviations diverged to 5.7797 greater than the population parameter.

Table 1 indicates that the mean converged toward the universe parameter on replications 1, 2, 4, 5, and 8 but diverged from the universe parameter on replications 3, 6, 7, and 9. This is as even a split as is possible with 9 replications. However, the absolute degree of convergence exhibited over five replications averaged 2.5182 units whereas the absolute degree of divergence exhibited over four trials averaged but 0.9514 units.

Table 1 also indicates that the standard deviation converged toward the universe parameter on replications 2, 4, 7, and 9 but diverged from the universe parameter on replications 1, 3, 5, 6, and 8. This is as even a split as is possible with 9 replications. The absolute degree of convergence exhibited over four replications averaged 1.7766 units which is nearly the same as the average absolute divergence of 1.1470 units displayed over five replications.

Averaging over the nine replications in Table 1 shows a slight trend toward convergence for both the mean and standard deviation.

Table 2 presents a summary of the results concerning the nine replications of the N = 25 bootstrap analysis. The data indicate that the mean converged toward the universe parameter on replications 2 and 8 but diverged from the universe parameter on replications 1, 3, 4, 5, 6, 7, and 9. The absolute degree of convergence averaged 1.8793 units over two replications. The absolute degree of divergence averaged 0.9246 units over seven replications.

Table 2 also indicates that the standard deviation converged toward the universe parameter on replications 3, 4, 7, 8, and 9 but diverged from the universe parameter on replications 1, 2, 5, and 6. The absolute degree of convergence averaged 0.8990 units over five replications which is nearly the same as the average absolute divergence of 0.8352 units displayed over four replications.

Averaging over the nine replications in Table 2 shows a trend toward divergence for both the mean and standard deviation.

Discussion

Two major conclusions emerged from the present bootstrap computer simulation. The first conclusion is that the bootstrap procedure gives rise to divergence with about the same frequency and to about the same extent as it gives rise to convergence with respect to the universe parameter. Diaconis and Efron (1983) admitted that "There are always a few samples for which the bootstrap does not work, and one cannot know in advance which they are" (p. 122). The present results indicate that this is an overly optimistic view of the bootstrap procedure regarding point estimation of the mean and standard deviation. Convergence toward the universe parameter occurs no more frequently than would be predicted by chance.

The second conclusion is that the bootstrap procedure is biased in that it does not continue to approach the universe parameter as the number of iterations increases. Said otherwise, the limit of convergence is not the universe parameter.

The overall conclusion is that small samples do not contain all of the information of a large sample in the way that a small piece of a holographic plate contains all of the information of the complete holographic plate. This conclusion is presently restricted to point estimations for means and standard deviations.

Reference

Diaconis, P., & Efron, B. (1983). Computer-intensive methods in statistics. Scientific American, 248, 116-130.

Table 1

Deviations of Means and Standard Deviations From Universe Parameters in Nine Bootstrap Samples of N = 10 Taken From A Universe of N = 1,000 Normally Distributed Values Ranging From 50 to 150 with a Mean of 100 and a Standard Deviation of 20.

	Mean		SD	
Replication	From	To	From	To
1	- 1.4400	0.1816	2.8113	5.7797
2	- 5.2200	- 3.8828	- 4.4130	- 0.6756
3	- 1.0400	- 2.0277	0.1724	- 1.2573
4	-11.5600	- 7.2983	- 3.7857	- 2.9335
5	11.1800	7.2789	0.7011	0.9241
6	- 0.9400	1.1450	- 4.4331	- 4.9152
7	- 2.6400	- 3.0890	-12.5286	-11.0903
8	- 9.1800	- 7.3472	- 8.6361	- 9.6127
9	- 6.4400	- 8.6040	- 1.5164	- 0.4381
Ave	- 3.0311	- 2.6271	- 3.5142	- 2.6910
SD	6.5186	5.0277	4.8034	5.2378

Table 2

Deviations of Means and Standard Deviations From Universe Parameters in Nine Bootstrap Samples of N = 25 Taken From A Universe of N = 1,000 Normally Distributed Values Ranging From 50 to 150 with a Mean of 100 and a Standard Deviation of 20.

	Mean		SD	
Replication	From	To	From	To
1	3.0960	4.1028	2.2087	2.2135
2	4.0400	3.6183	- 1.2126	- 2.2841
3	- 0.6960	- 0.8010	- 1.6144	- 0.0312
4	0.1120	1.1697	- 0.4389	- 0.3454
5	5.9120	6.5688	0.9809	2.2891
6	2.7520	2.9735	- 2.4949	- 3.4510
7	1.0720	3.1987	- 3.9119	- 3.0255
8	- 4.4960	- 1.1591	- 3.4843	- 2.2753
9	- 6.0080	- 7.3056	0.7827	0.0596
Ave	0.6427	1.3740	3.9159	4.0602
SD	- 1.0205	- 0.7744	2.0922	2.1308