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ABSTRACT

This Teacher Education and Mathematics (TEAM) module is designed to establish an atmosphere in the classroom in which students will feel free to share their thoughts, reactions, and experiences related to mathematics and mathematics teachers. It consists of an instructor's text and student materials. The instructor's text provides (1) specific directions for guiding the lessons and (2) commentary designed to help teachers build positive mathematics attitudes. The format is one of "facing pages" whereby the right-hand page provides step-by-step teaching directives and the left-hand page furnishes commentary that articulates a philosophy, provides explanations, and suggests psychological approaches. The "commentary and notes" page also allots space for the instructor's use and when no commentary applies, the entire page is alloted to "notes." Student materials include two sheets for use during instructional sessions--"Twelve Thoughts about Math," and "A Typical Day" as well as reading material ("Twelve Math Myths") for use after sessions. These materials are taken from the book "Mind Over Math" by Stanley Kogelman and Joseph Warren. (JN)

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SE 045 890

Demystifying Math

A Course to Reduce Math Anxiety and Sex-Role Stereotyping in Elementary Education

TEAM

TEACHER EDUCATION AND MATHEMATICS

Queens College of the City University of New York
Women's Educational Equity Act Program/U.S. Department of Education

TEAM

TEACHER EDUCATION AND MATHEMATICS

A Course to Reduce
Math Anxiety and Sex-Role Stereotyping
in Elementary Education

DEMYSTIFYING MATH

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TEAM

DEMYSTIFYING MATH

I

INSTRUCTOR'S TEXT

The Demystifying Math module was prepared for use as a first part of a course or in a workshop series teaching new attitudes towards mathematics. It consists of two parts; Instructor's Text and Student Materials. Both parts can be kept in a loose-leaf notebook.

The Instructor's Text provides specific directions for guiding the lessons and also, has commentary designed to help teachers build positive math attitudes. The format is one of "facing pages" whereby the right-hand page provides step by step teaching directives and the left-hand page furnishes commentary that articulates a philosphy, provides explanations, attitudinal interventions, instructional alternatives and suggests psychological approaches. The "Commentary and Notes" page also allots space for the instructor's use and when no commentary applies the entire page is alloted to "Notes."

Student Materials includes two sheets for use during instructional sessions--"Twelve Thoughts about Math" and, "A Typical Day" as well as reading material ("Twelve Math Myths") for use after sessions. These materials are taken from the book, Mind Over Math, by Stanley Kogelman and Joseph Warren. This module was developed in collaboration with Stanley Kogelman.

COMMENTARY AND NOTES

Although the Instructor's Text provides a thorough outline of classroom activities, it is not a script to be followed to the letter. Instead, the text should be thought of as a guide in terms of both direction and philosophy. Personal differences in teaching and learning styles are expected and, in fact, encouraged.

Recommended references are:

Kogelman, Stanley, and Joseph Warren. Mind Over Math. New York: Dial Press, 1978.

Tobias, Sheila. Overcoming Math Anxiety. New York: W. W. Norton, 1978.

It would be useful to put Mind Over Math on reserve at your library to facilitate students' access to this book. Chapters 1 and 4 are directly related to this module and have supplementary use after classroom discussion. (See page I-24.)

OVERVIEW

This attitudinal module is designed to establish an atmosphere in the classroom in which students will feel free to share their thoughts, reactions, and experiences related to math and math teachers. Ideally, this atmosphere will carry through the semester, so that students who have become assertive and involved in learning and questioning will remain so.

The point of view reflected here is that learning mathematics is more than just an intellectual pursuit. Psychological factors can and do play a major role in the learning process. Since math requires intense concentration, students are likely to experience emotional interference while trying to study it. In addition, the focused thinking style that math requires may be foreign to many students.

It is desirable to have the seats arranged in a circle to encourage participation and interaction.

The first 15 to 30 minutes should be devoted to acquainting the class members with you and with each other. Your remarks will serve as the model for the kind of comments you expect from the students.

COMMENTARY AND NOTES

Conducting this session requires you to change roles from teacher to facilitator to facilitator and listener. It is more important for you to explore and encourage student participation by questioning than it is to attempt to answer questions and offer explanations.

It can be encouraging and inspiring to students if you relate personal conflicts that have been resolved. It is reasonable to assume that students themselves experience many of the same conflicts, and it can be supportive for them to know that you are really not so different from them. This knowledge can enhance your potential as a role model because it encourages students to identify with you. The model you present may influence the students' future approach to <u>their</u> students when they become teachers.

How much you say depends on what is easy and comfortable for you. It's better to say a little in a relaxed and informal way than to say a lot stiffly or uncomfortably.

You can expect that there will be a close relationship between past experiences and present attitudes. It is crucial that you resist the temptation to explain or to justify the attitudes of former teachers. An important message is being developed that should be underscored--<u>as teachers we can have an enormous impact on students</u>. Learners may remember what teachers say for a lifetime.

Always avoid words or looks that imply that a student is stupid. When people believe that they are "dumb," then failure is seen as proof of their inadequacy. ("If I'm dumb, then I'm hopeless and there is no sense working harder if I'm only going to fail anyway.")

BEGINNING THE PROGRAM

In introducing this session, make some comments about math anxiety. (See the Introduction to the Instructor's Handbook.) Consider including the following points:

1. Math anxiety is an intense emotional reaction to math that makes learning difficult, if not impossible.

2. Many people, perhaps a majority, dread having to deal with math, and they make major career decisions based on avoiding math. This action increasingly limits career choice, since almost all careers require some math. Even history and journalism programs now have a statistics requirement.

3. Math anxiety appears to affect women more than men, but that appearance may be due to the reluctance on the part of men to admit to difficulties. In any case, women seem to avoid math more than men do.

Introduce yourself with a brief description. You could include information about your academic background, development of interest in education, work background, career shifts, conflicting career goals, conflict of marriage and family considerations with career, and how those conflicts were resolved. Discussing current conflicts is not recommended.

In addition, give some of your math history. It is particularly helpful for students to know that there were points at which you experienced difficulty with math.

After your initial statement, ask the students to introduce themselves and say something about their reasons for taking the course and their feelings about math. Allow students to respond spontaneously or, if they are reluctant to respond, go around the circle, giving each student a turn to speak.

While a student is speaking, don't interrupt unless someone seems to be going on excessively. Don't hesitate to ask clarifying questions when a student has finished speaking. For example, if someone says that she or he has always hated math, ask if there are particularly memorable or unfavorable math teachers in that person's past. If a former teacher has done something that embarrassed or humiliated a student, don't hesitate to be empathic. Show the student that you understand how uncomfortable the situation was. As students relate their math histories, expect diverse responses, ranging from liking math and looking forward to teaching it to hating math and wishing it would go away.

Ask at intervals if others feel the same way that the speaker does. This is generally more helpful than responding directly to students' remarks, and such comments serve to facilitate group interaction. Encourage students to interact with one another by asking for similar experiences and attitudes. For example, if one participant says that she experiences a wall coming down when she is faced with math, ask what the wall feels like and then ask if anyone else experiences a similar barrier.

The following are additional questions you might ask to stimulate discussion:

1. Why are you taking this course?

2. What do you expect of the course?

3. What subjects do you most look forward to teaching?

4. Do you remember any elementary school teachers in connection with math?

5. What do you think mathematicians are like? What are the stereotypes of mathematicians?

6. If you felt you were good at math, how would that affect your career goals?

Assign students the task of writing a "math autobiography." You may limit the scope of the task by asking them to describe a limited number of incidents or recollections. Or you may ask them to write about how they use mathematics currently in their lives.

The object of the discussion is not to convince everyone that these statements represent myths, but rather to explore the ideas contained in the statements and to examine the consequences of believing in them. Indeed, you may find yourself in agreement with one or more of these statements.

In fact, using your fingers indicates that you do know how answers are arrived at and what they mean. As teachers, we all need to be flexible and allow for different ways of learning and solving problems. The child who uses his or her fingers apparently understands the concepts. Left alone, she or he will probably make a gradual transition away from counting on fingers, and may use them only when feeling anxious or under pressure.

MATH MYTHS*

Math myths are those statements about math that are part of our folklore. The goals of this discussion are for the students to:

1. Develop a realistic view of mathematics and how it is studied and used.

2. Understand the consequences of believing certain math myths.

3. Explore their own experiences with and attitudes toward math.

4. Begin to develop an attitude toward math (for example, to recognize that there is no "best way" to do a math problem) that can influence their future teaching so that positive attitudes are passed on to their pupils.

Allow 30 to 60 minutes for discussing myths about math and how those myths affect attitude and performance. Free interchange of ideas should be encouraged.

Begin by passing out "Twelve Thoughts about Math" (see Student Materials, II-3). Say something like:

> I'd like you all to take a look at the statements about math on the sheet that is being passed out. Please think about these statements and indicate which ones you agree with and which ones you disagree with. I'm not going to look at your responses. This is for your own reference. After you look this list over, we will discuss each of the statements.

Allow two or three minutes for the students to look over the sheet. Then begin the discussion by asking if anyone would like to comment on any of the statements. If there is no spontaneous response, ask how many students agreed with the statement, "It's bad to count on your fingers." Note that you can start with any statement you like and that it is not necessary to go in order. The order should be based on your own preference, as well as the trend of class discussion. For example, talking about intuition tends to lead naturally to a discussion of creativity in mathematics.

To stimulate discussion about finger counting, ask how many of the students remember being forbidden to count on their fingers; then ask how many still do it (perhaps secretly). If they think it is wrong, ask

*Adapted from Chapter 2 of Mind Over Math, by Stanley Kogelman and Joseph Warren.

COMMENTARY AND NOTES

Stimulating memories of students' personal math history is a technique that is used a number of times in these modules. Students are encouraged to examine their attitudes and feelings and to reflect upon factors that fostered them.

Many of these points can be made during the discussion of each myth, and therefore a summary at the end of the session may not be necessary.

them why, and what they think the advantages are of not using fingers. Ask: "Are those advantages real or exaggerated?"

You can also discuss the consequences of the belief that it is bad to count on one's fingers--for example, feeling embarrassed and guilty or feeling that one can't be good at math if one has to use one's fingers. Perhaps the most important point is that attempts to stop finger counting don't work and eventually become counterproductive.

After most members of the class have had a chance to comment on a particular statement, give your own point of view. The material that follows can be used as the basis for your discussion of each myth. After you make your comments, ask if students disagree; then ask for further questions and comments. The discussion of math myths can lead to a discussion of school experiences with math. Tell students about your own experiences in learning math and encourage them to remember how they were taught math and how their attitudes developed. Depending on time, discuss some or all of the 12 math myths at length. If time runs short, make summary comments on myths you haven't discussed in detail.

CONSEQUENCES OF BELIEFS IN MATH MYTHS: HOW MYTHS LIMIT

Men are better in math than women:

- If you are a woman, you accept as "fact" that difficulty with math is part of your "being feminine."
- If you are a woman, you allow yourself to avoid math and thus severely limit your career options.
- If you are a man, you put women down and indicate that you don't really expect much of them when it comes to math or math-related activities.
- If you are a woman, you become dependent on men to do math and handle your finances for you.

Math is not creative:

- If you see yourself as a creative person, then you will not want to be involved with a "noncreative" subject like math.
- You will not allow yourself to use your imagination and creativity in solving math problems.

Math requires logic, not intuition:

- You don't allow yourself to use intuition, which is one of the best tools for getting started on math problems.

COMMENTARY AND NOTES

Encourage your students to respond to one another's insights and opinions. A peer opinion can have a powerful effect--more powerful than a professor's, in some cases.

- You have the false expectation that there is an automatic, logical solution to every problem. When the solution doesn't come to you, you conclude that you don't have the ability to do math.

- You put too much pressure on yourself, with the result that you don't relax and try to understand the problem.

You must always know how you got the answer:

- If you get an answer but can't explain it, you start to doubt yourself and think that you are wrong--even if you are right.

- You don't allow yourself time to sit down and discover how you figured out a problem.

There is one best way to do a math problem:

- You won't be able to develop your own method of solving a problem.

- Even when you do solve a problem, you will not feel that you succeeded, since your method was not the best one.

- You will think that everyone else has a better method and is therefore better in math than you are.

- You will hesitate to start a problem because you fear that you might do it the wrong way.

- You will try to remember how to do a problem rather than just doing it any way that you can.

It's always important to get the answer exactly right:

- You will be unable to allow yourself the flexibility and insight that are part of approximation.

- You will tend to focus on the negative rather than the positive. For example, if you make one careless mistake in a solution that is correct in terms of method, you will feel defeated. Instead, you should feel good about having understood the problem and about having made only one careless mistake.

It's bad to count on your fingers:

- If you use your fingers to count, you may feel childish or stupid.

- If someone sees you using your fingers, you may feel embarrassed. This potential for embarrassment can lead you to find private, secret ways to count that others can't see, such as using your hands behind your back or under a table or exerting slight pressure on your fingers that's imperceptible to others. Covert counting is often accompanied by guilt.

COMMENTARY AND NOTES

Encourage students to respond to one another. On some topics a peer opinion can be much more powerful than a professor's.

- You don't give yourself credit for what you know. Counting on your fingers suggests that you understand the process needed to solve the problem at hand.

- You can get caught up in trying to remember an answer and lose sight of the fact that there's more mathematical thinking involved when you count through a process than when you remember something you memorized in the past.

- You may deprive yourself of a simple, available problem-solving tool, thereby adding an unnecessary burden to a task.

Mathematicians do problems quickly, in their heads:

- Believing this leads you to try to do math problems faster than you are ready to do them. Speed comes from practice, and it is impossible to become quick at math (if that is your goal) without a gradual progression.

- If you keep thinking, "I don't do math fast enough," you will defeat yourself before you even start.

Math requires a good memory:

- If you keep trying to memorize things you don't understand, you will become increasingly confused.

- Memory is one of the first things to go under tension, so memorizing can cause you to do much worse under pressure.

- You will hate math if you memorize a bunch of isolated facts that are meaningless to you. It will always seem like a foreign language you don't understand.

Math is done by working intensely until the problem is solved:

- Believing this keeps you from gaining the benefits and insights that come from taking breaks.

- If you keep working too long, you will just get frustrated, dislike math, waste your time, and then give up. There is a big difference between saying, "I can't do it" and saying, "I can't do it now."

Some people have a "math mind" and some don't:

- Believing that a "math mind" is biological rather than learned causes you to become discouraged when math doesn't come easily.

- You will give up without putting in the time, effort, and concentration it takes to learn math.

COMMENTARY AND NOTES

Have students read "Twelve Math Myths" from Mind Over Math as a follow-up to this discussion (see Student Materials, II-5).

- Each failure will become a confirmation of your belief that you don't have the right kind of mind for math.
- You will tend not to accept help from others because you believe that a "math mind" can't be developed.

<u>There is a magic key to doing math</u>:

- You won't use your own intuitive sense to solve problems.
- You will tend to rely on formulas, expecting them to provide the magic key, rather than on figuring things out.
- You will keep crying to remember what to do, when the real way to solve a problem is through your own imagination and insight.

COMMENTARY AND NOTES

You may want to write a "Typical Day" in which the activities are appropriate--or typical--for the group, using this one as a guide.

By allowing students to express their feelings about what they are going to do, you are reinforcing the notion that psychological factors play an important role in doing math.

You can expect that while the papers are being passed out, there will be considerable restlessness, and that several students will repeatedly ask what is to be done. For many students, just knowing that they are going to have to look at numbers makes them so tense that they simply cannot hear what is being said. Instead, they drift off into their own thoughts and anxieties. This is an important insight for students to gain and it can be underscored later in discussion.

Students' askin' ou to repeat instructions is a good sign, because it means that students have caught themselves drifting off and are trying to refocus. Throughout this discussion you can use the technique of asking if others have feelings similar to the ones just expressed. This will help underscore both the similarities and the differences in individual responses.

A TYPICAL DAY*

The main purpose of the "Typical Day" exercise is to demonstrate that people tend to make things needlessly difficult for themselves. This activity provides an excellent opportunity to see the effects of math anxiety on problem-solving behavior.

In addition, as a preliminary experience with numbers, the activity allows students to work with word problems in a nonacademic context.

Begin by asking the students how they would feel about looking at some material that has words and numbers. With the first mention of numbers, there is likely to be a visible rise of tension in the group, marked by restlessness, talking, and giggling. Comments will probably range from "I'd rather not" to "Let's get on with it." Allow five or ten minutes for this discussion, as needed.

Reassure students that the purpose of this activity is not to have them solve math problems, but rather to provide a way for them to gain insight into how their initial reactions and expectations interfere with their ability to do mathematics.

After the initial discussion, say:

> I am going to hand out a story about some of the things that happen on a typical day. Numbers are included in it, as well as words. I am not going to ask any specific questions, and I am not going to collect papers. I want you to look the material over and write down any thoughts, feelings, and/or reactions you have in relation to the story. This is your primary task. As you read through the story, I would also like you to make up a problem you can solve from any part of the story. Remember, however, that primarily we are going to discuss your reactions to the story.

Don't hesitate to repeat the instruction, "I'd like you to write down your reactions and make up an easy problem--one you can solve." Allow five or ten minutes for the students to look over "A Typical Day." When most of the students seem to be through, say, "I'd like to begin the discussion of your reactions to the material." Ask if anyone would like to start. If you can't get any volunteers, start to go around the circle, asking each person for her or his reaction to the story. The following questions may help to stimulate discussion:

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*Adapted from Chapter 4 of Mind Over Math, by Stanley Kogelman and Joseph Warren. Copyright © 1978. Used with permission of The Dial Press.

You can expect some students to be very impatient to get on with a discussion of the math. But it is better to delay that discussion until everyone has expressed his or her responses, since once the discussion of the math begins, people forget their initial responses. Moreover, at this point many students are still too anxious to absorb any explanations you give, even if they themselves have asked for the explanations.

The major points given here can be emphasized throughout the discussion. If a student mentions one of the points, bring it to everyone's attention by saying, "That's an important strategy," or "Thinking that way can really help you cope with tension." Ask if other students have tried the same technique. Summarize periodically, or say, "Yes, that's similar to what Jane said about . . ." The more often students hear these ideas, the more likely they will begin to use them and believe them.

1. How many of you found yourselves drifting away when I began to give out the story?

2. What did you think of the story?

3. Was the story interesting or boring?

4. On what parts of the story did you write reactions?

5. On what parts of the story did you base your problem?

Students' reactions may include the following:

- I felt a wall coming down.
- I felt like I was being tested.
- I felt like everyone else was writing and I was the only one who didn't know what to do.
- It was boring having all those numbers.
- Who'd want to be so conscious of every last penny they spent!
- I felt more comfortable knowing I didn't have to hand it in.
- It was too easy.
- It reminds me of those awful word problems.

If students ask for an explanation of the math problems in "A Typical Day," say, "It is best for us to wait until after the discussion of your reactions to the material. Later I will be happy to discuss any questions you may have."

The major points to be underscored when discussing students' reactions are:

1. It is important to start with what you <u>can</u> do rather than with what is difficult for you.

2. When the numbers are difficult to deal with, change them to easier numbers that will give you insight into the problems.

3. Initial expectations and anxieties can prevent you from hearing instructions and from reading carefully and absorbing the necessary information.

4. Taking breaks helps. If you look away from the material for a few minutes, you may find that when you reread it, you'll notice information that you didn't see before.

COMMENTARY AND NOTES

The part of the story that usually gives students the most difficulty is the one about gas mileage. Many students blank out because they see decimals; others are concerned about the size of the tank.

Chapters 1 and 4 of Mind Over Math are directly related to this module. It may be fruitful for students to read these chapters after the classroom discussion, so that the reading can reinforce the points made during this discussion.

After the students have expressed their reactions to the material, ask if anyone remembers what the initial instructions were. Most students will remember that they were supposed to write down their reactions and make up a problem. Few will have heard that they were to make up a problem they could solve. Use this discussion to emphasize the fact that anxiety affects listening. This is something students can learn to watch for with their own pupils. When pupils are anxious, repeating the instructions does not help; it is necessary to reassure students and to let them calm down first so that they can listen.

When this discussion is complete, answer whatever questions students have about the different segments of "A Typical Day." Be alert to when students are tuning out and drifting away. Then ask them what caused this behavior, and how they felt.

At the end of the session, find out whether students have any further questions they would like to ask you.

You may assign students the task of writing a one- or two-page description of their own typical day that uses numbers. It can include a discussion of some problems students could have thought about, but avoided instead.

TEAM

DEMYSTIFYING MATH

II

STUDENT MATERIALS

Three Excerpts from Mind Over Math
by Stanley Kogelman and Joseph Warren

TWELVE THOUGHTS ABOUT MATH*

	Agree	Disagree
1. Men are better in math than women.		
2. Math is not creative.		
3. Math requires logic, not intuition.		
4. You must always know how you got the answer.		
5. There is one best way to do a math problem.		
6. It's always important to get the answer exactly right.		
7. It's bad to count on your fingers.		
8. Mathematicians do problems quickly, in their heads.		
9. Math requires a good memory.		
10. Math is done by working intensely until the problem is solved.		
11. Some people have a "math mind" and some don't.		
12. There is a magic key to doing math.		

*From Chapter 2 of Mind Over Math, by Stanley Kogelman and Joseph Warren. Copyright © 1978. Reprinted with permission of The Dial Press.

A TYPICAL DAY*

I really can't stand the cold. It must be around 20° right now. I wish it was summer and 85°.

I had some errands to run this morning before driving into Manhattan from Queens. I had run out of stamps so I went to the Post Office to buy two rolls of 15¢ stamps, one hundred to a roll, and a dispenser for 5¢. Then I went to the copy center to get ten copies of a recent newspaper article. The copies were 7¢ each for the first five and 6¢ for each additional copy.

I had to stop for gas and only needed 4.8 gallons, which cost $4.85. I realized that I had gotten pretty good mileage on my VW since I had gone 144 miles since my last fill-up.

Walter and I met for lunch at "La Garbage." Walter ordered a rare steak for $4.85 and coffee for 35¢. I had a chicken salad plate for $3.25 and coffee. The waiter put everything on one check, but we decided we would each pay for what we had ordered. We left about a 15% tip and went to a meeting.

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*From Chapter 4 of Mind Over Math, by Stanley Kogelman and Joseph Warren.

TWELVE MATH MYTHS*

Many of the most commonly held views of math are based on myths about the subject. These myths have resulted in false impressions about how math is done. They need to be dispelled.

ONE: MEN ARE BETTER IN MATH THAN WOMEN.

Research has failed to show any difference between men and women in mathematical ability. The perception of math as a masculine domain stems from other myths about the subject. Math is seen as the epitome of cool, impersonal logic--nonintuitive and abstract. This fits with the stereotypical image of men as cool, detached, and objective.

Men are reluctant to admit they have problems so they express difficulty with math by saying, "I could do it if I tried. I just never worked hard enough." Women are often too ready to admit inadequacy and say, "I just can't do math. It's an ability I just don't have." Both may be expressing the same fears or anxieties about math. Russell Baker, a New York Times columnist, wrote humorously about his experiences with math (September 25, 1977):

> It is barking up the wrong tree for women to conclude that mathematics is more terrifying to them than to men . . . I first started fearing math near the end of the seventh grade . . . Many desperate years later I was one of that vast multitude of men who emerge from college without a doctorate in mathematics and have been scarred for life by inability to do our income-tax returns or verify the addition on restaurant checks . . . If men get more doctorates than women it is surely not because mathematics is "male-oriented," but because males, with their powerful instinct toward machismo, are ashamed to admit that when it comes to pi they are chicken.

Women mathematicians are expected to be less feminine than other women. This is not true. In fact, a 1960 study at UCLA by Phillip Lambert, which measured femininity of interest patterns in terms of current cultural stereotypes, showed that women math majors "were not only equal to nonmathematics majors in femininity, but significantly more feminine."

If more people saw math as an intuitive, creative human endeavor, then women and men might be expected to have equal ability and interest in mathematics.

*From Mind Over Math, by Stanley Kogelman and Joseph Warren. Copyright © 1978. Reprinted with permission of The Dial Press. This excerpt is from Chapter Two of the book.

TWO: MATH REQUIRES LOGIC, NOT INTUITION.

Few people are aware that intuition is the cornerstone of doing math and solving problems. The final product of mathematical work is completely different in appearance from the process by which the result was obtained. Einstein wrote:

> To these elementary laws (of physics) there leads no logical path, but only intuition supported by being sympathetically in touch with experience.

Newton, who invented the calculus, was unable to give the logical mathematical basis for his ideas. It took two hundred years and tremendous mathematical advances before his work could be "proved" correct.

Mathematicians always think intuitively first. The logical presentation of results follows, and may require far more work than coming up with the solution. The conception of math as strictly logical conflicts with the definition of intuition--"the act or faculty of knowing without the use of rational processes."

Math books may give the impression that math requires an especially "logical mind" because they are so ordered, precise, and logical--and because they are often impossible to follow. Even when a person can follow all the steps, there is a tendency to think, "I see what they did, but I could never have done it myself. I don't have a mathematical mind."

Math teachers usually spend considerable time preparing their lectures and working on problems. When problems are presented in class, the solutions have been revised to the point where they have become brief and logical. The scrap paper that represents the teacher's process of arriving at the solution is thrown away. Students are left with the impression that this carefully refined solution was arrived at by a magical process or divine inspiration.

In a painting you don't see the sketches, planning, and struggle that produced the finished work. Final mathematical form is the result of a process of using intuition, making false starts, and struggling until the solution is "seen." It is no more possible to sit down and solve a problem you have never seen before than it is possible to produce a work of art at one sitting by starting at the top of the canvas and working directly to the bottom.

In our society, intuition tends to be undervalued while logic and reason have the aura of religious truths. We all tend to react emotionally and intuitively first and then explain what we feel by means of logical argument. It is extremely difficult to convince anyone of anything through the use of logic alone. Logic can be used to support a point of view but in the final analysis the listener must be persuaded that the concept "feels" right.

Mathematical intuition refers to ideas about the answer or method of solution to a math problem. These ideas may not have evolved logically but rather seem to just pop into your head:

Michele: I asked my bank manager to explain compound interest but two elements of his explanation went absolutely over my head. When I got home, I decided to pretend it was I who had to explain it to someone else. When I reached the first point that I had been unable to understand, I hesitated for a moment. Suddenly it became crystal clear. This must be the intuition we discussed yesterday.

The second puzzlement was harder to unravel. I started playing with the numbers and found relationships I had not seen before. Again, in a flash, all became clear.

II-7

Everyone has mathematical intuition; they just have not learned to use or trust it. It is amazing how often the first idea you come up with turns out to be correct.

THREE: YOU MUST ALWAYS KNOW HOW YOU GOT THE ANSWER.

Getting the answer to a problem and knowing how the answer was derived are independent processes. One involves intuition, the other logic. If you are consistently right, then you know how to do the problem. There is no need to explain it.

Mathematicians are not always aware of the process by which they do math. Lightning calculators are able to do extensive computations in their heads with amazing speed and accuracy but rarely know how they get their answers.

What may appear to be guessing is really mathematical intuition at work. Demanding an explanation inhibits the use of this intuition and robs one of the pleasures of getting right answers!

Joan: I could get the right answer in school, but the proof had to be part of the answer. You had to know how you arrived at it. That was impossible for me so I felt that if I got the right answer, it was just luck. Somehow I had arrived at it but, since I didn't know how, I had to assume a little bird told me.

Proofs of mathematical results have not always been valued. Historically, the Arabic school of mathematics emphasized getting answers while the Greek school put proofs on an equal footing with results or answers.

Examining the process by which answers are obtained is useful in those instances where you find that consistently wrong answers keep appearing in the solution of a particular type of problem. In this case, you are concerned about where you are going wrong so you can correct yourself.

Teachers have to know methods so they can transmit them to others. Yet each person eventually comes up with his or her own methods in math, just as everyone develops a unique handwriting--even though everyone was taught the same way. This is the creative part of doing math.

FOUR: MATH IS NOT CREATIVE.

Creativity is as central to mathematics as it is to art, literature, and music. Mathematical theorems may seem to be "just thought up," but they actually represent the end result of the same creative process that leads to a painting or symphony.

The act of creation involves diametrical opposites--working intensely and relaxing--the frustration of failure and elation of discovery--the disappointment at realizing you've been on the wrong track and the satisfaction of seeing all the pieces fit together. It requires imagination, intellect, intuition, and an aesthetic feeling about the rightness of things.

Solving a math problem necessitates overcoming a difficulty in an imaginative way. Sometimes this is done intuitively without conscious awareness of the creative process. Then you know you have the solution but can't explain how you got it. When you do know that you have solved a problem, you are directly in touch with the originality in your work.

Creativity can be seen in all aspects of solving math problems. It can even be seen in the different ways people do arithmetic and in the variety of schemes people invent to count on their fingers.

FIVE: THERE IS A BEST WAY TO DO A MATH PROBL.

A math problem may be solved by a variety of methods which express individuality and originality--but there is no best way. New and interesting techniques for doing all levels of mathematics, from arithmetic to calculus, have been discovered by students.

Teachers and textbooks may give the impression that they are offering the best way to do a particular type of problem. But two teachers rarely explain the same thing in the same way and the presentation in a textbook is just an expression of the author's way of doing mathematics. Even multiplication and division are done in different ways in different countries.

When several people work on the same problem, each may feel that the other's method is better:

Gloria: I did the comparative shopping problem by long division.

Barbara: Oh, you did it the right way. I just estimated it.

Gloria: Your way seems easier, you got the answer without bothering to divide.

Barbara: But I couldn't get the exact answer.

The way math is done is very individual and personal and the best method is the one with which you feel most comfortable.

SIX: IT'S ALWAYS IMPORTANT TO GET THE ANSWER EXACTLY RIGHT.

The ability to obtain approximate answers is often more important than getting exact answers. When a complex mathematical problem is "solved" on the computer, the answer that is obtained is approximate. The "exact" solution often is unknown and cannot be determined. But it need not be determined because the approximation is good enough for all practical purposes.

Joan: After being married for ten years, I just found out that when my husband totals a check in a restaurant, he's not doing it penny by penny. He's sort of rounding it off and making sure that it's approximately seventeen dollars, or whatever.

I thought you had to make sure the whole thing was exactly right. That made it much more intimidating. When I found out he was just sort of knocking it off, I got kind of mad because all these years I had been attributing a higher level of computational ability to his performance in restaurants.

Feelings about the importance of the exact answer often are a reversion to early school years when arithmetic was taught as a skill. There, the emphasis was entirely on the answer. This led to the feeling that you were "good" when you got the right answer and "bad" when you did not.

Just because a problem has an exact answer does not mean there is always great value in finding it. It depends on the circumstances. A waiter must give you the correct check, but you only need to see if it is "about" right, because it doesn't really matter if it is off by a few cents. You can even use your intuition. When a

check is wrong, it tends to "feel" wrong and that feeling can be used as a clue that it is worth checking the addition.

SEVEN: IT'S BAD TO COUNT ON YOUR FINGERS.

There is nothing wrong with counting on fingers as an aid to doing arithmetic. In fact, most people find this to be very helpful.

Finger counting is often prohibited by parents and teachers and made fun of by peers. Those who use their fingers feel as if they are "cheating" or that they "shouldn't have to":

Marge: If you count on your fingers in public, it shows that you can't do it in your head. Then everybody knows you are a dolt.

To avoid embarrassment, people do it surreptitiously--behind their backs, under the table, or by tapping on their legs or even on their nose--and feel guilty about it. The use of flash cards and speed drills for multiplication and addition reinforces the idea that finger counting is "bad" and that math should be done by rote.

Counting on fingers actually indicates an understanding of arithmetic--more understanding than if everything were memorized. The abacus is really a sophisticated finger-counting machine that provides a fast and accurate aid to doing arithmetic. The Chinese use the abacus freely and feel no guilt about doing so.

Examination of many of the finger schemes people have invented for themselves has shown them to be clever, imaginative and efficient. There is no reason to prohibit their use.

EIGHT: MATHEMATICIANS DO PROBLEMS QUICKLY, IN THEIR HEADS.

Solving new problems or learning new material is always difficult and time consuming. The only problems mathematicians do quickly are those they have done before. But they are often expected to be able to add long columns of figures quickly, solve complex problems with hardly any thought, and have all formulas at their fingertips. Mathematicians don't have these abilities and don't expect that they should.

Time-limited tests, flash cards, and arithmetic drills have all added to the impression that mathematical competence and speed are the same thing.

> Annie: When I have to pause for a second to consider and don't know the answer just like that, I feel like I'm stupid. I get angry at myself for going through too many steps. There is a little voice that says to me, "God, you're slow!"

Speed is not a measure of ability. It is the result of experience and practice. A concert pianist makes the performance of a difficult program look easy because he or she has studied and practiced for years. But mastery of a new piece still requires work. Mathematicians do not expect to solve new problems quickly without the use of pencil and paper. The time it takes

depends upon the number of similar problems they have solved, how confident they feel, how long it takes to get intuitive ideas--and a little luck.

When you expect to be able to solve a problem quickly and find that you can't, you tend to get discouraged and give up. There is no way to predict how long it will take to overcome the difficulty inherent in a problem. Once solved it may look "easy," but that does not mean that it was an easy problem or that it could have been done quickly.

NINE: MATH REQUIRES A GOOD MEMORY.

Doing math is like speaking a foreign language. When you are fluent you don't think about vocabulary or grammar. It is part of your thoughts and feelings. Knowing math means that concepts make sense to you and rules and formulas seem natural. This kind of knowledge cannot be gained through rote memorization.

The emphasis that is placed on memorization in doing multiplication and addition gives the impression that math requires a good memory. Later, it is natural to try to memorize procedures and formulas in algebra. But it is difficult to remember things you don't understand. As problems become more complex, memorization becomes increasingly difficult, if not impossible.

New concepts always take time to learn and it is easy to get frustrated and say to yourself, "I don't understand it and never will. I'd better memorize the rules." But not understanding a concept on the first attempt does not mean that a later attempt will fail. Everyone who has learned math has experienced a feeling of not being able to understand. It simply takes time and it really helps to try more than once.

TEN: MATH IS DONE BY WORKING INTENSELY UNTIL THE PROBLEM IS SOLVED.

Solving problems requires both resting and working intensely. Going away from a problem and later returning to it allows your mind time to assimilate ideas and develop new ones. Often, upon coming back to a problem a new insight is experienced which unlocks the solution.

It is generally felt that the way to learn is to make up your mind to do it, sit down, and then work hard. This may be true for adding a column of figures but it is not true for learning new concepts or solving math problems.

Resting is different from giving up. If you have worked intensely on a problem and think, "It's hopeless. I'll never get it," then you will leave the problem and completely stop thinking about it. If you think, "I can't get it now," then you will continue to think about it both consciously and unconsciously. Mathematicians--like writers and artists--often experience flashes of illumination during the periods of rest that alternate with working intensely. Henri Poincaré (1854-1912), one of the foremost mathematicians of all time, wrote:

> For fifteen days I strove to prove [a theorem]. I was then very ignorant; every day I seated myself at my work table, stayed an hour or two, tried a great number of combinations and reached no results. One evening, contrary to my custom, I drank black coffee and could not sleep. [During the night] ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination. By next morning I had established the [theorem], I had only to write out the results, which took but a few hours.

ELEVEN: SOME PEOPLE HAVE A "MATH MIND" AND SOME DON'T.

Do people really have different kinds of brains? The expectation is that those who have math minds grasp math quickly, easily and naturally. Concepts and ideas are mastered effortlessly. Problems are solved with barely a moment's hesitation. Correct answers just pop into their heads. If this is what you think, then when you look at a problem and don't know what to do immediately, you attribute it to being stupid or to not having a math mind. You get discouraged and say, "Why bother?" But the math mind is a myth.

Belief in myths about how math is done leads to a complete lack of self-confidence. But it is self-confidence that is one of the most important determining factors in mathematical performance.

We have yet to encounter anyone who could not attain his or her goals in math once the emotional blocks were removed.

PART ONE
WHAT IS MATH?

TWELVE: THERE IS A MAGIC KEY TO DOING MATH.

There is no formula, rule, or general guideline which will suddenly unlock the mysteries of math. But it is helpful to see math for what it is and for what it is not. If there is a key to doing math, it is in overcoming anxiety about the subject and in using the same skills you use to do everything else.