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ABSTRACT

This is the second of four phases of reporting the findings of a study of school mathematics in 24 countries, including the United States. The report consists of five parts: (1) introduction; (2) eighth-grade findings; (3) twelfth-grade findings; (4) student attitudes toward mathematics; and (5) concluding remarks. Supporting documentation (including list of participating countries, sampling information and 41 figures) is included in appendices. A summary of findings is also presented under 20 headings: introductory; class types; class size; yearly hours of instruction; teacher characteristics; how mathematics teachers spent their time; role of textbook in eighth- and twelfth-grades; how student time was spent in class; homework; extent of calculator use in class; ways in which the calculator was used; content of the eighth-grade mathematics curriculum; content of the twelfth-grade mathematics curriculum; international studies in school achievement which require varying degrees of "compromises" in assessment procedures; how mathematics was taught in eighth-grade; student acheivement in eighth-grade mathematics; student achievement in twelfth-grade mathematics; sex differences in mathematics achievement; changes since the 1964 mathematics study; and student attitudes toward mathematics. (JN)

Second International Mathematics Study

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Summary Report for the United States

Contractor's Report

Second International Mathematics Study

Summary Report for the United States

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*Other members of the National Mathematics Committee and the National Technical Advisory Panel were involved in preparing and reviewing the reports from which this summary was drawn. For names see Appendix C.

**National Research Coordinator

May 1985

NCES 85-210

Foreword

This is the second of four phases of reporting the findings of a study of school mathematics in twenty-four countries,* including the United States. The data were collected in 1981-1982.

Phase I provided an abbreviated report, dealing only with the major findings in a summary fashion and providing limited comparative international data. The international data include various average scores across all countries taking part in the Study, as well as selected findings for those participants whose national reports had been released at the time of preparation of this report.

The Phase I report, released at the Allerton Conference, September 1984, has now been revised. THE PRESENT VERSION (JANUARY 1985) SUPERSEDES EARLIER VERSIONS OF THE U.S. REPORT.

In Phase III, a detailed report of U.S. findings will be released. This report includes information of item-level outcomes (achievement and attitudes) for students. A less technical report, designed for wide distribution, will also be available.

Phase IV consists of the full international reports of the various aspects of the Study, including data on individual, identified countries. The timetable is as follows:

Phase	Report	Release Date
I	Summary Report (Preliminary)	September 1984
II	Summary Report (Revised)	January 1985
III	Detailed U.S. Report	Spring 1985
IV	International Reports	Early 1986

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* Those countries are listed in Appendix A.

Summary of Findings

Finding	Page	Figure
1. Introductory		
*In 1981-1982, students and teachers in a national sample of about 500 classrooms in the U.S. joined in a study of school mathematics in two dozen countries.	1	
*Detailed information was gathered on what mathematics was in the curriculum, what mathematics was taught by teachers, how that mathematics was taught and what mathematics was learned by their students.	3	
*The Study was targeted at 13 year olds (eighth grade in the U.S.) and those studying college preparatory mathematics in the final year of secondary school (twelfth grade classes in the U.S.).	4	
2. Class Types		
*Four class types for the eighth grade were identified and analyzed separately: remedial, typical, enriched and algebra classes.	9	
*Two class types were identified and analyzed separately for the twelfth grade: precalculus and calculus.	51	
3. Class Size		
*Eighth grade mathematics classes in the U.S. had an average size of 26, somewhat lower than the nationwide class sizes for other school subjects. (Average for junior high school in the U.S. for social studies and science was 30 students). Male to female student ratio was 1 to 1.	9	
*Twelfth grade mathematics classes had an average size of 20. Enrollment by sex was 56 percent male to 44 percent female.	51	
4. Yearly Hours of Instruction		
*The average number of hours per year provided for eighth grade mathematics instruction was 145. This was comparable to the amount of time devoted to mathematics instruction at this grade level in the vast majority of countries in the Study.	9	1

	Page	Figure
*The average amount of twelfth grade mathematics instruction per year was about 150 hours per class. This was somewhat lower, on average, than the time devoted to college-preparatory mathematics instruction in many countries.	51	1
5. Teacher Characteristics		
*The teacher of a typical U.S. eighth grade mathematics class was experienced and well-trained, having 13 years of teaching experience, 9 or 10 semester courses in mathematics, 2 courses in mathematics teaching, and 4 courses in general education and pedagogy.	10	
*The teacher of a typical senior high school mathematics class had about 16 years of teaching experience with eight years at the senior level. This teacher had a median age of 40 years and had taken about 16 semester courses of mathematics.	52	
*Eighth grade teachers indicated that some of the more important factors in effective teaching were establishing and enforcing clear-cut rules for acceptable student behavior, making encouraging remarks to students as they work and reviewing tests with students shortly after they have been graded.	11	
*Twelfth grade teachers indicated that some of the more important factors in effective teaching were reviewing tests shortly after they have been graded and, for teachers of precalculus classes, clearing up problems from a previous lesson, establishing and enforcing clear-cut rules for behavior, and making encouraging remarks to students.	52	
*Teachers at both grade levels reported finding the classes easy to teach and finding most students attentive with relatively few of them especially fearful or anxious about mathematics.	11	
*All teachers for both grades, except teachers of eighth grade remedial classes, chose developing a systematic approach to solving problems as the most important goal in teaching mathematics. Knowing mathematical facts, principles and algorithms was also a highly rated goal by teachers for both grades.	53-55	

	Page	Figure
6. How Mathematics Teachers Spent Their Time		
*Much of eighth grade teachers' time in class was spent on presenting new content or reviewing old material. A relatively small but measureable proportion of time was spent on administrative tasks, including maintaining class order (discipline).	13	2
*Senior high teachers spent about 40 percent of their time in class presenting new material, 20 percent reviewing previous material, 30 percent of time supervising student work and 10 percent on administrative/management duties in class.	55	28
7. Role of Textbook (Eighth Grade and Twelfth Grade)		
*The textbook defined "boundaries" for mathematics taught by teachers. Limited use was made of resources beyond the textbook for either content or teaching methods.	14 56	3 29
8. How Student Time was Spent in Class (Eighth Grade and Twelfth Grade)		
*The majority of student time in mathematics class was spent listening to teacher presentations, doing seatwork or taking tests. Little time was spent in small group work.	14 56	4 30
9. Homework		
*Students in about 80 percent of eighth grade mathematics classes in the U.S. were reported to spend three hours per week or less on mathematics homework. The average was estimated by teachers to be about 2.3 hours per week (nearly 30 minutes per day).	14	5
*The typical twelfth grade mathematics student was estimated by teachers to do about 4 hours of homework per week.	56	5
10. Extent of Calculator Use in Class		
*Calculator usage in eighth grade mathematics class was low (data collected in 1981-1982). Only 1 class in 20 used calculators for two or more periods per week. One class in three was not allowed to use calculators. Another one-third of classes reported never using calculators.	15	6

	Page	Figure
*Calculator use was much higher in senior high school mathematics than in junior high school mathematics classes. About one-third of the senior classes used calculators in class two or more times a week. Another twenty percent of the classes never used, or were not allowed to use, the calculator in class.	56	6

11. Ways in Which the Calculator Was Used

	Page	Figure
*Calculators were used most commonly in eighth grade mathematics for checking answers to problems, for recreation or for doing projects. Little use was made of them in test-taking.	15	7
*Calculators were most commonly used in senior high mathematics for checking work or solving problems. About one-half of the classes used calculators on tests.	56	7

12. Content of the Eighth Grade Mathematics Curriculum

	Page	Figure
*In the typical eighth grade general mathematics class, about 50 percent of the cururiculum was arithmetic, 25 percent was algebra and the remainder was other topics such as geometry and measurement.	15	
*While students in the eighth grade algebra classes covered much of the standard material in a first year high school algebra course, they missed other content such as geometry, measurement and statistics.	15	
*Topics in geometry were taught internationally at the eighth grade, but not in many U.S. eighth grade classrooms.	15	
*The predominant characteristic of enriched eighth grade mathematics classes was an extended treatment of algebra and a modest treatment of geometry and measurement.	15	

13. Content of the Twelfth Grade Mathematics Curriculum

	Page	Figure
*Calculus and precalculus classes differed greatly in terms of subject matter covered and in terms of achievement.	57-59	
*Calculus classes were judged by teachers to have already covered much of the content taught to the precalculus classes in the twelfth grade year.	57-59	

	Page	Figure
19. Changes Since the 1964 Mathematics Study		
*Eighth grade classes showed a modest decline in end-of-year performance on the 36 items in common between the First International Mathematics Study (FIMS) and the present Second International Mathematics Study (SIMS). The declines were somewhat greater for more demanding comprehension and application items than they were for computation items.	44-46	21-22
*In the twelfth grade, comparison on the 20 items in common between FIMS and SIMS showed an overall modest increase in end-of-year performance, especially in elementary functions and calculus. Much, but not all, of this improvement was due to the high performance of the calculus classes. Much of the increase was seen in the more demanding comprehension questions and, for the calculus students, at the even more demanding application level.	70-74	34-35
20. Student Attitudes Towards Mathematics		
*Students held the view that the study of mathematics helps one to think logically and that the subject is a good one for creative people. Twelfth grade students in college preparatory mathematics classes were more positive in their view of mathematics than were eighth grade students.	79	20 33
*Overall, the students had a positive self-concept with respect to mathematics. Again, twelfth grade college preparatory students showed a more favorable view than eighth grade students.	81	
*The majority of students reported that they wanted to do well in mathematics, and that their parents wanted them to succeed in the subject.	81	
*Students perceived mathematics as a useful and important subject. The eighth grade students were undecided about whether they would like to have a job involving the use of mathematics, while two-thirds of the twelfth grade college preparatory mathematics students indicated they would like such a job.	82	

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Part I. INTRODUCTION

In 1981-82, students and teachers in about 500* mathematics classrooms across the United States joined their counterparts in some two dozen countries around the world in a comprehensive study of school mathematics. This study was designed to provide detailed information from each country about the content of the mathematics curriculum, how mathematics is taught and how much mathematics students learn. The information is intended to help policy analysts and mathematics educators in individual nations analyze their school programs and identify areas of strength and weakness, and to provide data which are useful to national officials as they plan for future directions in school mathematics in their own countries.

The Study was conducted by members of the International Association for the Evaluation of Educational Achievement, or IEA,** an international network of leading educational research institutions. The University of Illinois at Urbana-Champaign carried out the study in the United States.

In each country, a National Committee of specialists in mathematics education and testing was responsible for the Study.*** In addition, a National Technical Advisory Panel, which provided direction in technical matters related to the Study, was chaired by Edward Kifer of the University of Kentucky. Kenneth J. Travers of the University of Illinois at Urbana-Champaign was Project Director and Curtis McKnight of the University of Oklahoma served as National Research Coordinator.

The Study is the second to take place in a twenty-year period. In 1964, twelve countries****, including the United States, took part in a similar study. Hereafter, the present study will be called the Second International Mathematics Study, SIMS, or simply "the Study" for short. The 1964 study will be called the First International Mathematics Study or FIMS.

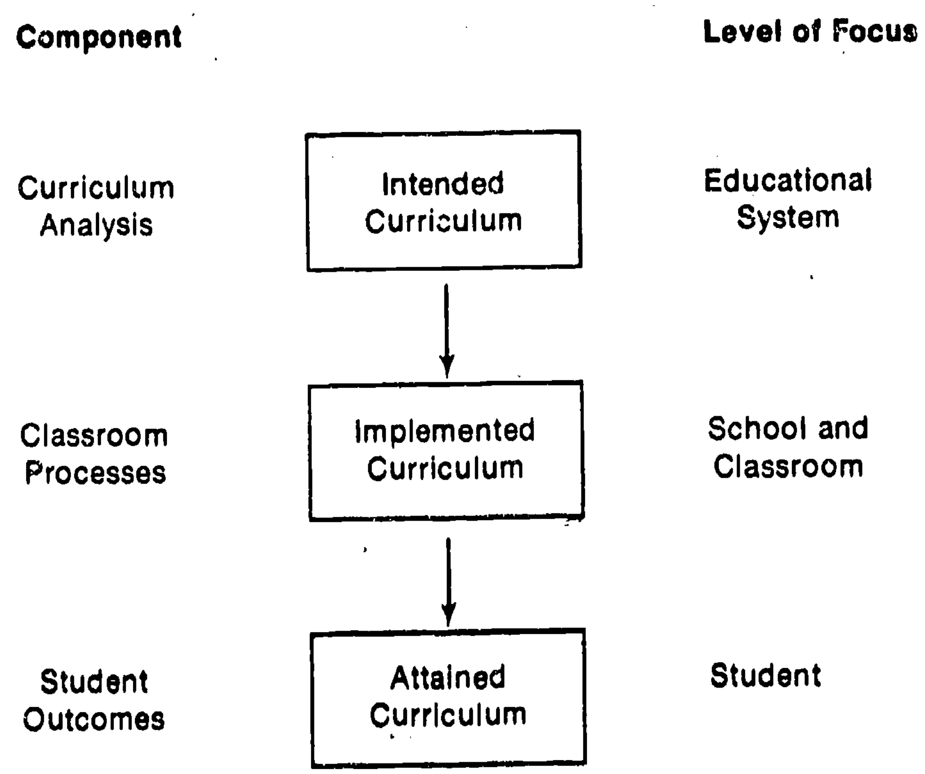
Each country provided its own costs for participating in these studies. In the United States, funding was provided by the National Institute of Education, the National Science Foundation and the National Center for Education Statistics. Further grants were provided by the William and Flora Hewlett Foundation and the Carnegie Corporation of New York to support the Allerton Conference to discuss the major findings of the Study. A dissemination grant was provided by the National Science Foundation in late 1984 to produce a more popularized version of the findings.

* For notes on numbers of classrooms, see Appendix D.
** Information on the IEA is in Appendix B.
*** Committee members are named in Appendix C.
**** The twelve countries are identified in Appendix A.

Components of the Second International Mathematics Study

The conceptual design of the Second International Mathematics Study has three components, as represented in the figure below:

THREE ASPECTS OF A CURRICULUM



Each of these components is the subject of an international report, scheduled for publication by Pergamon Press, London, England, in 1986.

The intended curriculum is reflected in curriculum guides, course outlines, syllabi and textbooks adopted at the educational system level. In most countries, nationally defined curricula emanate from a ministry of education or similar national body. In the U.S., such statements of intended goals or specifications of curricular content are developed in State Departments of Education or at the local school district level. Thus it is considerably more difficult to describe the intended curriculum for the U.S. than for almost any other of the participating countries.

The implemented curriculum focuses on the classroom level where the intended curriculum is taught by the teacher. Teachers may exercise their own judgments in translating curriculum guides or adopted textbooks into a program for their classes. Thus, their selection of topics or patterns of emphasis may not be consistent with those intended. To identify the implemented curriculum, a number of questionnaires were developed for completion by the individual classroom teacher. For example, teachers were asked whether or not they had provided instruction to the target class for each of the items on the achievement tests. They were also asked to provide detailed information on the number of class periods devoted to specific subtopics and which interpretations of selected concepts and processes they utilized. Such highly specific information on curriculum coverage coupled with similarly detailed information on instructional strategies permits a rather comprehensive characterization of what mathematics has been taught and how it was taught to the target populations.

The third component of the study addressed the attained curriculum - what students had learned as measured by the tests and questionnaires. Extensive achievement tests were designed to measure student knowledge and skill in areas of mathematics designated as important and appropriate for each population. The "fit" of these tests to the curriculum in individual countries varies substantially and is a limiting factor in international comparisons. Quite obviously, the tests contain items less appropriate in some countries than in others and may not always contain an adequate range of items to fully represent all curricula. This limitation restricts the nature of cross-national comparisons that can be drawn and argues against interpreting the study as an international contest. The student outcome measures also included a number of opinionnaires and attitude scales. These were intended to elicit student views as to the nature, importance, ease, and appeal of mathematics and selected mathematical processes.

This design, as implemented in the U.S., includes the following sources of data:

1. Questionnaires completed by school officials concerning school, teacher and mathematics program characteristics; organizational factors; and school and departmental policies affecting mathematics instruction.
2. Questionnaires completed by teachers to provide background information on experience, training, qualifications, beliefs and attitudes. Additional questionnaires to provide general information on instructional patterns (allocated time, ability of class, classroom organization and activities related to individualization of instruction, resources used; and goals and factors affecting instructional decisions), and beliefs about effective teaching. Additional questionnaires related to instruction on selected specific topics.
3. Ratings by teachers (opportunity to learn) of whether the content needed to respond to each item of the achievement tests had been taught that year, in prior years, or not at all, to their students.
4. Questionnaires completed by students providing background information (e.g., parents education and occupation), time spent on homework, and attitudes and beliefs related to mathematics.
5. Achievement tests completed by students at the beginning and at the end of the year.

Target Populations

The study included a survey of the mathematics curriculum provided for two groups of students, identified as target Population A and B.

Populations for the Study (During the 1981-1982 School Year)

1. All students enrolled in normal mathematics classes in the U.S. at Year 8 level (Population A).
2. All students enrolled in mathematics classes at Year 12 level in classes requiring as prerequisite two years of algebra and one year of geometry (Population B).

United States Sample

The United States sample in the Second International Mathematics Study was composed of students and teachers from approximately 500 classrooms in about 250 public and private schools across the United States. Students were tested by internationally developed mathematics achievement tests at the beginning of the 1981-82 academic year and again at the end of the school year. The testing also included an attitude questionnaire. These data were collected from approximately 7,000 eighth-grade students and 5,000 students enrolled in twelfth-grade mathematics classes. Technical details of the sample are in Appendix D.

While sampling errors of estimates have not been reported in this document, they are being prepared and will be shown in future more detailed reports of the Second International Mathematics Study. Because of the large sample sizes, the sampling errors tend to be small. For example, the standard error for estimates of item difficulties reported in Table 6, page 25, range between 1 and 2 percentage points. Interpretations of the findings of the study have taken into account the order of magnitude of standard errors.

Measurement of Mathematics Achievement. Survey estimates of student achievement, student attitudes, opportunity to learn, and classroom characteristics presented in this report are mostly percentages of respondents who gave a particular answer to questions. No attempt has been made to summarize the total student scores on mathematics tests for this report. Some statistics are based on responses to a single question while others are based on averages of correct responses to a number of questions or "items" in a topic area such as algebra or geometry.

Readers should be cautious about interpreting small comparative differences. All achievement items, or groups of items, result only in estimates of the level of knowledge of mathematics topics by students in the population. The choice of items that make up a mathematics topic might affect conclusions drawn from those figures. Tests of significance for sampling differences have not been conducted for all comparisons presented in this report and estimates of the reliability of the items have not been computed for each subtest.

Furthermore, the reader should be cautious about interpreting reported changes over time. Changes based on a large number of items are the most reliable. Changes that are derived from a small number of items have the greatest potential for chance differences. Also, the reader should note that some tables include fewer items than others for the same mathematics topic because it was necessary to restrict comparisons to identical items from earlier tests. Change scores in these tables would not have the same level of reliability as change scores based on a larger number of items. Future reports will contain a more detailed discussion of the reliability of responses to this survey.

One matter of concern in an international study of schooling is whether the other countries involved have comparable proportions of their students in the target populations defined. At Population A (eighth grade in the U.S.) virtually all students of that age are still involved in schooling in most countries. At Population B (twelfth grade in the U.S.) there is more variation. Table 1 presents, for several of the countries in the Study, the percentage of the age cohort that is in Population B, the percentage of the grade cohort in Population B and the percentage of the age cohort still in school.

Table 1
Proportion of Population B Students in Relevant Age Groups and Grade for Each Country: 1981

Country	Age Group (Years)	Pop B Percent of Age Group	Pop B Percent of Grade Group	Percent of Age Group in School
Belgium (FL)	17	9-10	25-30	65
British Columbia	17	30	38	82
England & Wales	17	6	35	17
Finland	18	15	38	59
Hungary	17	50	100	50
Israel	17	6	10	60
Japan	17	12	13	92
New Zealand	17	11	67	17
Ontario	18	19	55	33
Scotland	16	18	42	43
Sweden	18	12	50	24
U.S.A.	17	10-12	12-15	82

Notes: 1. Age group is estimated age at middle of school year.

2. While the fourth column represents the percent of the age cohort still in school, this does not imply that all these students are in the grade(s) from which the Population B sample is drawn. Thus the second column is not always a simple product of the third and fourth columns.

3. Data are obtained from national reports for each country. The ratio of high school graduates to population age 17 was 72 percent in the United States in 1981. U.S. data on enrollment were based on the school enrollment rates of persons 17 years old according to the October 1981 Current Population Survey. An additional 5 percent was enrolled in college or university.

It can be seen from Table 1 that the U.S. retains in school one of the largest proportions of the age cohort. In terms of the proportion of students enrolled in advanced college-preparatory mathematics, however, we are retaining a relatively small group -- about 10-12 percent (see the column in Table 1 headed "Population B percent of age group). Furthermore, a smaller proportion still of the U.S. Population B group is studying calculus (only about 2-3 percent), than most other countries in the Second International Study.

Part II. EIGHTH GRADE FINDINGS

Four Class Types in the Eighth Grade

Teachers were asked to characterize the main subject matter taught in the sampled classes as remedial, typical, or enriched. On the basis of course and textbook titles supplied by the teachers, a fourth category of classes, (first year) algebra, was identified, largely from those classes identified as enriched. For many of the analyses that follow, this information has been taken as identifying four different eighth grade class types. While this classification is based on teacher judgments (which were made without precise definitions available), the differences in student achievement levels by class type verify that this classification is at least roughly appropriate.

Of the 236 classes in the eighth grade sample, 155 were classified as typical, 24 as remedial, 26 as enriched and 31 as algebra classes. While these proportions may be representative, the actual numbers of all class types other than the typical classes are small enough to make generalizations for those class types tentative.

Class Size

Mathematics classes had a median size of 26 students and about 65 percent of the classes had 20-30 students. The median size of remedial mathematics classes was 21 students.

Class Periods of Mathematics Instruction

Mathematics was typically taught 5 periods per week. The median length of a class period was 50 minutes, with almost all classes having a length from 40 to 60 minutes. Over 75 percent of the classes had periods 45-55 minutes long.

Hours of Eighth Grade Mathematics Instruction Per Year

The median number of clock hours per year of mathematics instruction was 145. About 90 percent of the classes received between 115 and 180 hours of instruction. (See Figure 1.)

Teaching Assignments

The typical weekly teaching assignment for the eighth grade teacher included 25 or 30 class periods (71 percent of the teachers). Seventy percent of the teachers devoted either 20, 25 or 30 class periods to instruction in mathematics. Relatively few of the teachers had additional teaching responsibility in the area of science and a total of approximately 25 percent taught in some other area. Even fewer algebra teachers (13 percent) taught in another area. The predominant pattern in this sample was one of full-time mathematics instruction. A high percentage (50) of algebra class teachers reported themselves to be mathematics department chairs as did a fairly high percentage (30) of remedial class teachers.

Teacher Characteristics

The sample for the Study was drawn to be a representative sample of U.S. classrooms at each grade level and thus the teacher characteristics which follow should represent the teaching received by typical eighth grade U.S. students.

Gender. The ratio of female to male teachers for the national sample of eighth grade classes was almost 1:1. However, slightly more of the teachers of algebra classes were women (55 percent) and markedly more of the teachers of enriched classes were men (61 percent).

Age/Experience/Training. According to median data, the typical eighth grade mathematics class had a teacher who was experienced and well-trained. This teacher was about 37 years old with 13 years of teaching experience, 8 of which had been obtained in teaching eighth grade mathematics. This teacher's collegiate training has included 9 or 10 semester courses in mathematics, 2 courses in the teaching of mathematics, and 4 in general methods and pedagogy. Teachers for remedial classes were slightly younger (median age 35) and for enriched classes slightly older (38). Teachers of enriched classes tended to have more teaching experience generally (about 15 years) while teachers of remedial classes tended to have more eighth grade teaching experience (10 years). Oddly, teachers of both algebra and remedial classes tended to have taken slightly more mathematics while teachers of enriched classes had taken more work in general methods and pedagogy.

That the teaching corps was experienced and well trained is demonstrated by the age and experience of this sample: the national sample included relatively few younger teachers (only 15 percent below the age of 30, about the U.S. average), few inexperienced teachers (only 15 percent with less than 7 years experience), and few teachers with limited training in mathematics (less than 20 percent reported fewer than 5 semester courses and only 4 percent with fewer than 3 semester courses). In the area of mathematics methods and pedagogy, approximately one-third of the teachers had a single course, one-third had two courses, and one-third had three or more courses. There were no teachers in the sample who reported having had no collegiate training either in mathematics or in the teaching of mathematics.

Beliefs About Effective Teaching. The teachers were asked to rate a series of forty-one items concerning effective teaching. The teachers emphasized factors such as establishing and enforcing clear-cut rules for acceptable student behavior (rated very important by 67 percent of the teachers); making encouraging remarks to individuals as they work (62 percent); getting materials, equipment and space ready before class (55 percent); and reviewing tests with students shortly after they have been graded (55 percent). They tended to rate much lower specific techniques of instruction such as outlining and summarizing lessons, planning transitions, calling on non-volunteers, discussing feelings directly, etc.

Perceptions of the Sampled Class. The teachers sampled indicated a higher liking for teaching mathematics generally than for teaching the specific class sampled for the Study. They reported typically about 20 students as attentive in class, about 4 as inattentive but not behavior problems, and 1 as a behavior problem (these were the medians of the distributions). Teachers of remedial classes, unsurprisingly, reported fewer attentive students and more behavioral problems. The median number of students reported by teachers as being anxious or fearful about mathematics was 1-3 and 90 percent of the teachers reported 6 or fewer. For remedial classes, the number of fearful students was consistently reported as higher.

Goals in Teaching Mathematics. Teachers were asked to rate the relative emphasis that should be given to each of nine objectives in mathematics instruction. Table 2 gives the results for all class types indicating the percentage of teachers who rated each goal as relatively more and relatively less importance.

Table 2
Relative Importance of Goals in Teaching Mathematics
as Rated by Eighth Grade Mathematics Teachers: U.S., 1981-82
(Percent of Teachers; All Class Types Pooled)

Goal	Relatively More Important	Relatively Less Important
Develop a systematic approach to solving problems	63	5
Develop an awareness of the importance of mathematics in everyday life	61	6
Perform computations with speed and accuracy	58	7
Know mathematical facts, principles and algorithms	55	7
Become interested in mathematics	45	8
Develop an attitude of inquiry	39	5
Understand the logical structure of mathematics	30	15
Develop an awareness of the importance of mathematics in the basic and applied sciences	20	20
Understand the nature of proof	12	64

The highest rated goals were developing a systematic approach to solving problems and developing an awareness of the importance of mathematics in everyday life (rated as relatively more important by 63 and 61 percent of the teachers, respectively). These were followed closely by performing computations with speed and accuracy (58 percent) and knowing mathematical facts, principles and algorithms (55 percent).

The results differed for some of the class types separately. Teachers of remedial classes reported that their most important goal was developing an awareness of the importance of mathematics to everyday life (84 percent) followed by the goals of knowing facts, principles and algorithms (64 percent), of performing computations with speed and accuracy (60 percent) and of becoming interested in mathematics (60 percent). Problem solving (48 percent) was in fifth place rather than first.

Teachers of algebra classes reported that their most important goal was problem solving (69 percent) followed by the goals of developing an attitude of inquiry (48 percent), of understanding the logical structure of mathematics (48 percent) and (a tie) of becoming interested in mathematics (41 percent) and of performing computations with speed and accuracy (41 percent).

How Mathematics Teachers Spent Their Time

Teachers were invited to estimate the number of minutes they devoted in a typical week to certain activities related to the target class. Figure 2 presents a summary of their responses.

There was substantial variation both in total time commitment and in the way that time was used. Making a profile of the typical teacher from this data, he or she can be described as having spent from one to two hours per week outside of class planning and preparing for the class and another hour to two hours grading student papers. Routine administrative duties such as taking attendance, making announcements, and setting up equipment required less than one-half hour per week and maintaining class order and disciplining students consumed five minutes or less of an average class period. The typical teacher spent from one to two hours of class time each week explaining content new to the students and about half as much time reviewing old material.

Teachers of remedial classes spent somewhat more time than did teachers of enriched and algebra classes in outside preparation and planning and considerably more time grading student papers and tests. They spent somewhat less time explaining new content and more time reviewing old content. They spent marginally more time in routine administration but almost twice as much time in establishing and maintaining order (although it still amounted to only a few minutes per day according to teacher reports).

Of course, since the target class was only one of five taught by the typical teacher, these time estimates must be multiplied by the number of classes taught (or some factor of that) to profile the entire teacher work week - and these activities reflect only a portion of teacher responsibilities.

Use of Instructional Resources

The student textbook was clearly the most consistently used resource in teaching. (See Figure 3.) Only a few teachers identified it as other than a primary or secondary source. Other locally or commercially produced materials including textbooks, workbooks, and worksheets were reported as used more often as a secondary source by the majority of teachers. Other materials seldom appeared as a primary source and were rarely or never used by 75-80 percent of the teachers. This was true even in areas such as geometry and measurement in which such materials might be considered most helpful.

How Mathematics Students Spent Their Time in Class

Teachers also estimated the average time spent by students in the target class during a typical week on selected class activities. (See Figure 4.)

The majority of student time was devoted to seatwork, blackboard work, or listening to lectures or explanations. A median of 40 minutes, about one class period, per week was spent taking tests and quizzes. Very little time in mathematics class was spent in individual or small group work. This differed little from class type to class type.

Homework

The teachers estimated that students in their classes typically spent 2.3 hours per week outside of class on assigned homework. About 80 percent of the classes were estimated as spending three hours or less per week on homework. (See Figure 5.)

Use of Calculators

Usage of calculators in mathematics classes is reported in Figure 6. At the time of data collection (1981-1982 school year) reported usage was low. Only four percent of the eighth grade teachers reported calculator use for two or more periods per week. About two-thirds of the eighth grade teachers reported calculators were never used or were not allowed in mathematics class.

Varieties of ways in which calculators were used or encouraged to be used are shown in Figure 7. The most common uses of the calculator for eighth grade were for recreational activities (puzzles, games, etc.), checking exercises and for projects. Only six percent of the classes reported using calculators when taking tests.

What Mathematics Did Teachers Plan to Teach?

The teachers reported the approximate number of class periods they expected to have spent by the end of the year on each of a selected list of topics. Table 3 presents this information. The results indicate that there was great variation expected between class types in the content of eighth grade mathematics taught. Generally speaking, topics in arithmetic predominated, with a median of 30 class periods (about one-sixth of the total eighth grade program) devoted to common and decimal fractions. Relatively little time was anticipated to be given to probability and statistics (about 4 periods per year, on average), with 90 percent of the eighth grade classes receiving 10 periods or less on this topic.

While summary data suggest adequate coverage of each topic, a detailed look reveals this coverage to have been fragmented into a number of small pieces. The international classroom process data indicate less fragmentation in some other countries. A frequent U.S. pattern of instruction was that of a large proportion of teachers devoting only a single lesson to a topic. The result was a "low intensity" coverage of many topics in the eighth grade curriculum.

Table 3
Median Anticipated Number of Periods for Selected Topics
For Eighth Grade Classes by Class Type: U.S., 1981-82

Topic	All Classes	Remedial	Typical	Enriched	Algebra
	Median Number of Periods				
Common Fractions	15	20	20	10	2
Decimal Fractions	15	20	15	10	1
Ratio and Proportion	10	10	10	6	4
Percentage	15	15	15	10	2
Measurement	10	10	12	10	1
Geometry	15	10	15	15	1
Formulae/Equations	20	2	20	26	50
Integers	10	10	15	10	8
Probability and Statistics	4	0	5	5	0
Number of Classes	236	24	155	26	31

Algebra Classes. At least fifty percent of those classes identified as eighth grade algebra classes gave very limited attention to non-algebraic topics. Other data available in the study suggest that of those teachers who did treat other topics in other content areas, many concentrated their efforts on quasi-algebraic aspects of the area. For example, they taught the topic of coordinates in geometry and taught verbal problems from other content areas that could be solved algebraically.

Typical and Enriched Classes. Distinctions between typical and enriched classes in items taught were not as sharp as might have been anticipated. The somewhat more restricted time committed to arithmetic topics by enriched classes and their more extended commitment to formulae and equations may represent an underestimate of the distinction between them and algebra classes. Other data suggest that distinctions in the topics of measurement and geometry not readily apparent here may represent differences in the content covered rather than only in the amount of time committed.

Remedial Classes. The curriculum for remedial classes was clearly characterized by a heavy concentration on common and decimal fractions and on percent. However, at least a few of the classes identified as remedial by the teacher must have covered a rather comprehensive general mathematics curriculum. Other data in the Study indicate that at least 50 percent of the remedial teachers offered little or no treatment of the algebraic topics or probability and statistics and 25 percent were similarly restricted in their view of geometry, measurement and ratio and proportion.

Teacher Coverage (Opportunity-to-Learn)

The data in Table 3 are an indication of teachers' planned coverage on a somewhat detailed list of topics. It is also useful to gain information on what mathematical content on the international test was reported actually taught by teachers.

The teacher coverage of the various content areas (opportunity-to-learn) for each of the four class types is given in Table 4. This opportunity-to-learn (OTL) measure was obtained by asking the teachers to respond to the following question for each item on the international test:

During the school year, did you teach or review the mathematics needed to answer this item correctly?

__Yes
__No

If, in this school year, you did <u>not</u> teach or review the mathematics needed to answer this item correctly, was it because:

__It had been taught prior to the school year
__It will be taught later (this year or later)
__It is not in the school curriculum
__For other reasons.

It should be noted that at various places in this report two different measures of OTL are used: "taught this year" and "taught up to and including this year."

Table 4
Teachers' Estimates of Mathematics Content Taught During Eighth Grade
Needed to Answer Cognitive Test Items: U.S., 1981-82
(Average Percent Across Items)

Content Area Number of Items	All Classes	Class Type: Remedial	Typical	Enriched	Algebra
Arithmetic (62 items)	75	76	80	84	42
Algebra (32 items)	66	37	64	80	86
Geometry (42 items)	39	25	41	54	24
Measurement (26 items)	58	53	64	75	20
Statistics (18 items)	51	48	58	59	15
Overall (180 items)	60	51	64	72	40
Number of Classes	287	29	189	31	38

Note: 1. Data include ratings from 51 teachers in the sample from whom complete data were not collected.

2. This table refers to content taught only during the eighth grade, not up to and including eighth grade.

The data in Table 4 are grouped according to the content area of the international test and, therefore, do not correspond exactly to the categories used in Table 3. To the extent that they do correspond, however, the data in Table 4 show a pattern similar to that in Table 3. Notice, for example, the high coverage of algebra topics by the algebra classes (also rather high for the enriched and the typical classes); the high coverage of measurement by the enriched classes (these classes also covered more of the geometry than the other class types); and the high arithmetic coverage by all but the algebra classes.

Table 4 also provides rough indicators of possible "ceilings" on student performance in certain topics. For example, since the remedial classes had only 25 percent of the geometry taught to them, it is unrealistic to expect high achievement on the geometry subtest. It should be kept in mind, however, that these data deal only with content coverage during the eighth grade, and do not include estimates of coverage for prior years. This latter information is provided, for example, in the leftmost column of Table 6 (page 24).

Mathematics Not in the Eighth Grade Curricula

An international study provides the opportunity to identify topics taught elsewhere but which are not in the U.S. eighth grade curriculum. A selection of such "non-curriculum areas" (based on topics taught in other countries but not as extensively in the U.S.) is shown in Table 5. For example, topics in transformational geometry, taught in some countries, were reported taught by only 12 percent of U.S. eighth grade teachers. (However, 25 percent of the enriched classes were taught this topic.) Content dealing with patterns in arithmetic was reported taught to only 18 percent of the classes.

Table 5
Average Percent of Test Items Taught from Topics Taught or Reviewed <u>the Least</u> in U.S. Eighth Grade Classes: 1981-82

Topic (Number of Items)	All Classes	Class Type: Remedial	Typical	Enriched	Algebra
Arithmetic					
Patterns (2 items)	18	23	18	27	6
Geometry					
Transformations (5 items)	12	6	12	25	4
Vectors (3 items)	1	6	1	1	1
Spatial Visualization (3 items)	15	16	16	27	3
Central Angle Measure (1 item)	20	14	24	29	0
Locus (1 item)	6	7	6	10	3
Probability/Statistics					
Circle Graphs (1 item)	23	17	25	35	6
Probability (2 items)	28	13	32	37	6

How Mathematics Was Taught

Information regarding how mathematics was taught in eighth grade was collected through detailed classroom process questionnaires completed by teachers in each of the following five content areas: Common and Decimal Fractions; Ratio, Proportion and Percent; Measurement; Geometry; and Algebra (Integers, Formulas and Equations). These questionnaires were completed by teachers immediately after they had taught the topic to their target class.

Note: In the discussion of this section, data are reported only for classes classified by their teachers as "typical" and not for remedial, enriched or algebra classes.

Fractions and Decimals. The teaching of fractions and decimals by the eighth grade teachers tended to be symbolic and formal rather than intuitive or concrete. Most teachers presented fractions as decimals or as quotients (See Figure 8). They similarly taught decimals as another way of writing fractions and as an extension of the place value system (See Figure 9).

The opinion section of the questionnaire indicated a formal approach to teaching decimals and fractions with a strong emphasis on computation proficiency. Most teachers believed that computation with fractions should be taught, that computational skill indicates understanding, and that drill should be continued until students are proficient. Many disagreed that problem solving, applications, estimating, and checking the reasonableness of answers should be emphasized more than computation.

Ratio, Proportion and Percent. As with fractions, a formal approach interpreting one concept by reference to another dominated the teaching of ratio, proportion, and percent. In Figure 10 we see that ratio was most often presented as a fraction and as a comparison. Percent, too, was interpreted as a fraction, and the interpretation of proportion most emphasized was equivalent ratios. Teachers emphasized solving proportions using cross products and solving proportional word problems by setting up a proportional equation. Setting up and solving the appropriate proportion was the preferred method for solving all three types of percent problems.

Integers and Equations. Integers were usually introduced through the number line and physical situations. However, when presenting the operations, teachers relied instead on giving rules. (See Figure 11).

Three-quarters of the teachers emphasized solving linear equations by performing the same operations on both sides. Exploratory or intuitive methods such as trial and error or arithmetic reasoning were rarely used. The teaching of algebra, like arithmetic, appears to have been rule-oriented and focused on symbol manipulation.

Formulas. The prevailing technique used for teaching formulas in general was to present a formula and explain the meaning of the terms. Questions on the teaching of specific formulas confirm this approach. For example, of eight approaches to presenting the formula for the area of a parallelogram, most teachers reported emphasizing presenting and demonstrating the formula (See Figure 12). The same was true for presenting the formula of a rectangular prism, the Pythagorean Theorem, and the number pi. Intuitive approaches using rectangular grids, paper cutting, measurement, or pictorial diagrams were not ignored but they were not emphasized in a majority of the "typical" classes. By contrast, the enriched classes tended to make greater use of these exploratory activities.

Geometry. Introducing topics in geometry also focused on a statement of definition and properties rather than on informal explorations and the use of students' intuitions. For instance, most teachers reported either teaching the angle sum theorem by presenting and demonstrating it or by having students sum the measures of the angles of triangles. Paper folding and tearing off corners of paper triangles were not emphasized. The most popular approach to congruent and similar triangles was merely to state the definitions and properties. (See Figure 13.) Measurement, construction and, for similarity, examples from the environment were also used but not emphasized. The same "show and tell" approach was emphasized for parallel lines.

Teachers opinions about the teaching of geometry were notably at odds with their reported practices. They affirmed that an intuitive approach is most meaningful, that concrete models and aids should be used and that activities to improve spatial ability should be included. But in reality the most emphasized approach was a statement of definitions; the only aids extensively used by the majority of the teachers were the ruler and protractor; and spatial relations were taught in only 39 percent of the reporting classes.

Summary. Overall, the approach to the teaching of eighth grade mathematics would appear to have been predominantly formal with an emphasis on rules, formulas, and computational skills as opposed to being informal, intuitive and exploratory. While this approach was emphasized there was evidence of the use of multiple approaches and exploratory activities with some topics and a belief by teachers that problem solving and intuitive approaches should receive more emphasis.

Some effort has been spent in getting quantitative indices of the relative emphasis of factors such as instruction using symbolic representations of content vs. instruction using more perceptual representations. Although perhaps surprising, findings thus far indicate that instruction tended to be more symbolic with remedial classes than with the other types of classes and tended to be more symbolic when reviewing content than when covering new subject matter. It also appears that measurement was taught almost exclusively by emphasizing symbolic representations rather than perceptual ones (but this was true in many countries).

Reasons for Emphasis or Non-Use. In addition to asking teachers which of a variety of approaches to a topic they emphasized, used or did not use, the classroom process questionnaires asked teachers to select from a list, the reason(s) why they chose to emphasize or chose not to use a particular approach.

The reasons given for choosing to emphasize an approach vary with the topic. However, the most frequently selected reason across all content topics was that the approach was "well known to me." This was followed by "easy for students to understand", "useful in math in subsequent grades," and "emphasized in students' text." The most frequently cited reason for not using a particular approach in 8 of the 11 topics was the fact that it "was not emphasized in students' text." This was followed by "never considered it."

It would seem that the factors influencing teaching decisions were driven by the student text and by teacher familiarity with the approach. This is in line with earlier indications that the text dominated content decisions.

What Mathematics Was Learned in the Eighth Grade

Students were tested in the following five content areas: arithmetic, algebra, geometry, measurement, and statistics. The same items were taken by the students at the beginning and end of the 1981-1982 school year, thus providing a measure of how much of these mathematics topics was learned in the school year.

The data in Table 6 reveal the fit of the achievement tests to the curriculum actually implemented in the U.S. according to teacher perceptions. See Figures 14 to 18 also.

Averaging across the entire set of 180 items, only 70 percent of the teachers reported that the relevant mathematics had been taught by the end of eighth grade. One would expect that in an international study the cognitive tests would be out of phase to some degree with the curriculum of each participating country. The limited level of coverage in U.S. schools reported in Tables 4 and 6 represents a severe constraint on interpretation of U.S. performance in the international context. This is most obviously the case for the area of geometry where, on average, less than one-half of the items (44 percent) had been taught by the end of the eighth grade.

Table 6
Average Percent of Items on the International Test Reported Taught and Learned in Eighth Grade Mathematics: U.S. and 19 Other Countries, 1981-82

	U.S.			
Topic (Number of items)	Opportunity to Learn (1)	Mean Pretest Score	Mean Posttest Score	Mean Posttest Score for 20 Countries (2)
Arithmetic (62 items)	87	42	51	51
Algebra (32 items)	69	32	43	43
Geometry (42 items)	44	31	38	41
Measurement (26 items)	70	35	42	51
Statistics (18 items)	73	53	57	55

(1) Opportunity-to-learn by the end of eighth grade--that is, up to and including eighth grade.
(2) The international means are based on a restricted set of 157 items in common between the international test and the U.S. National version of the test. The number of items by topic on the 157 item international test were: Arithmetic, 46; Algebra, 30; Geometry, 39; Measurement, 24; and Statistics, 18. In all cases the U.S. results differ less than 1 percent from those in the Table above when restricted to the set of 157 items. The countries included, in addition to the United States, were: Belgium (Flemish); Belgium (French); Canada (British Columbia); Canada (Ontario); England and Wales; Finland; France; Hong Kong; Hungary; Israel; Japan; Luxembourg; Netherlands; New Zealand; Nigeria; Scotland; Swaziland; Sweden; Thailand.
(3) Posttest data are based on 280 classes in the U.S.

Student Achievement in Eighth Grade Mathematics

The average achievement of students in the U.S. in arithmetic, algebra, and probability and statistics was at the average level of performance for all countries. Achievement was above the international mean for computational arithmetic (ability to calculate) but below the international mean for arithmetic items involving comprehension and the ability to solve problems.

Achievement in geometry for the U.S. was low, exceeded by three-fourths of the other countries. Within geometry, however, knowledge of transformational geometry, which is not a part of the curriculum of many U.S. schools, was at the international average. Achievement in measurement was very low compared to other countries. While the international test used metric units for all items involving units of measure, this is not sufficient to explain completely the low level of performance since many items involved measurement concepts in a way that did not require knowledge of metric system measures.

Achievement in Arithmetic. The arithmetic sub-test contained 62 items. Marginal curriculum coverage was reported for one item presenting a problem involving a least common multiple and for two items involving number patterns. With these exceptions, the items on the arithmetic sub-test could be considered part of the curriculum for most eighth grade classes or to have been covered in previous years. (Teachers of algebra frequently assumed the more fundamental skills as prerequisites.) Table 7 presents the results for achievement in arithmetic.

Table 7
Eighth Grade Achievement in Arithmetic: U.S., 1981-82

	Mean Percent Correct		
Topic (Number of Items)	Pre-test	Post-test	Difference
Whole Numbers (14)	51	57	+ 6
Common Fractions (12)	46	56	+10
Decimal Fractions (13)	41	50	+ 9
Ratio and Proportion (9)	37	44	+ 7
Percent (8)	33	45	+12
Powers (3)	42	54	+12
Square Root (3)	21	41	+20
All Arithmetic Items (62)	42	51	+ 9

For each of the topics identified in Table 7, posttest scores exceed pretest scores. However, in absolute terms posttest performance seemed surprisingly low, below 60 percent in all topic areas and just above the 50 percent level for the overall arithmetic sub-test. Note also that the overall change between the pretest and posttest means was just about 9 percent. (See Figure 14.) For some items the pretests, posttests or both are below 20 percent which is what would be expected if students were merely choosing answers at random.

Item-level Data: Arithmetic Here are two sample items from arithmetic along with some key information about performance on those items. Item 003 is a computation item on the addition of two fractions with unlike denominators. Just over half (57 percent) of the students in the average eighth grade classroom answered this correctly at the end of the year. This compares with 71 percent of U.S. eighth graders who did so in 1964 and 63 percent of students internationally (an average based on 20 countries) who did so in 1981-82. Item 108 involves division of one decimal number by another and, given the possible choices, focuses on the correct placement of the decimal point in the answer. End of year performance in the U.S. was 59 percent in 1982 compared with 66 percent in 1964 and also compared with 39 percent internationally in 1981-82. Teachers indicated that all U.S. eighth graders had been exposed to the content needed for the two items by the end of the eighth grade year.

Item 003

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1982	41	57
1964		71
1982 International Score		63

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	90	100

$\frac{2}{5} + \frac{3}{8}$ is equal to

A $\frac{5}{13}$

B $\frac{5}{40}$

C $\frac{6}{40}$

D $\frac{16}{15}$

*E $\frac{31}{40}$

Item 108

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	50	59
1964		66
1982 International Score		39

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	89	100

$.004\overline{)24.56}$

In the division above, the correct answer is

A 0.614

B 6.14

C 61.4

D 614

*E 6140

Achievement in Algebra. There were sharp differences between the National Committee's a priori judgments of what was likely to be a part of the eighth grade mathematics curriculum and the teacher coverage (OTL) data for a number of items on the algebra sub-test. This suggests the extent to which the U.S. curriculum lacks a uniform definition in algebra. For example, four items involving negative integers rated by the National Committee as not likely to be covered by many U.S. classes were reported by teachers as taught or reviewed in over 90 percent of the classes. In contrast, several items considered likely to be covered by most classes were reported as covered by less than 45 percent of the classes. Teachers and Committee members were in agreement that formal representation of the intersection and union of sets and simplifying a rational expression was not taught in most eighth grade classes. In general, the more formal concepts in algebra tended to be reported as covered by fewer than 70 percent of the classes.

Table 8 summarizes the achievement in algebra and Figure 15 portrays these data in a graph. Given that less than 70 percent of the items were reported as having been covered by the target classes prior to or in eighth grade, the limited achievement levels in this area may not be too surprising.

It should also be observed that for the eighth grade algebra classes, the range of items on the international test was not broad enough to portray the depth of coverage of their courses, which covered the same content as first year high school algebra courses. (e.g., content on rational exponents, factoring, the quadratic formula, rational expression and others.)

Table 8
Eighth Grade Achievement in Algebra: U.S., 1981-82

Topic (Number of items)	Mean Percent Correct Pretest	Posttest	Difference
Integers (6)	35	50	+15
Formulas/Expressions (14)	29	42	+13
Equations/Inequalities (9)	38	46	+ 8
Exponents/Sets (3)	20	25	+ 5
All Algebra Items (32)	32	43	+11

Notice that change between pretest and posttest was greatest for the topics of integers and formulas and expressions, which other analysis has shown were the algebraic topics covered most extensively by the sampled classes.

Item-level Data: Algebra Here are three sample items from algebra. Item 012 involves multiplication of two integers (signed numbers). Note that performance was below average internationally. Item 115 deals with substituting a negative number into an algebraic expression and then multiplying two negative numbers to find the value of the expression. Notice that performance on this item was lower than that for Item 012 which involves the same computation but less interpretation. Item 149 requires the forming of an algebraic expression to represent a quantity defined by a verbal expression. Less than one-half of the U.S. teachers reported teaching the content of this item by the end of the eighth grade. Performance on this item is rather low, and some 16 percentage points below the 1964 score. This drop is against the trend seen in the basic algebra items that were common to the First and Second International Mathematics Studies.

Item 012

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	21	55
1982 International Score		62

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	92	93

(-2) × (-3) is equal to

A -6

B -5

C -1

D 5

*E 6

Item 115

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	14	38
1964		23
1982 International Score		42

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	84	84

If $x = -3$, the value of $-3x$ is

A -9

B -6

C -1

D 1

*E 9

Item 149

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	43	54
1964		70
1982 International Score		62

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	45	46

A shopkeeper has x kg of tea in stock. He sells 15 kg and then receives a new lot weighing $2y$ kg. What weight of tea does he now have?

A $x - 15 - 2y$

B $x + 15 + 2y$

*C $x - 15 + 2y$

D $x + 15 - 2y$

E None of these

<u>Achievement in Geometry.</u> Teacher OTL reports indicated that curriculum coverage of items on the geometry subtest was low in the U.S., averaging only 44 percent across the set of Second Study test items. Most teachers seemed to have felt that both informal and formal transformational geometry were outside the U.S. curriculum. Teacher reports also indicated an average of only 22 percent of the classes had been exposed to items on spatial visualization.

Given the reports of such limited curriculum coverage in geometry, it might be unreasonable to expect high achievement levels. Table 9 and Figure 16 portray these data.

Table 9
Eighth Grade Achievement in Geometry: U.S., 1981-82

Topic (Number of items)	Mean Percent Correct Pretest	Posttest	Difference
Figures & Properties (15)	30	38	+ 8
Congruence/Similarity (8)	31	40	+ 9
Spatial Visualization (3)	47	52	+ 5
Pythagorean Relation (3)	25	33	+ 8
Coordinates (5)	24	34	+10
Transformations (8)	32	36	+ 4
All Geometry Items (42)	31	38	+ 7

Figures and their properties and congruence and similarity were the geometric topics most widely covered prior to or during the eighth grade year according to detailed analyses not reported here. Achievement for these topics was relatively high, as high as all other topics except spatial visualization.

On the surface, it seems contradictory that the highest achievement level would be attained in the subtopic of spatial visualization which was reported as not having been in the curriculum for over 65 percent of the classes. However, the three items in this area are highly visual and intuitive and may measure abilities to some extent independent of formal instruction. This idea seems supported by the low level of change on these items.

OTL data indicate that transformational geometry is not a standard part of eighth grade mathematics in the U.S., (covered in about 12 percent of classes) even though it is taught internationally (according to the OTL data for other countries). However, student achievement on this topic was at the international average. It appears that some aspects of transformational geometry are accessible to U.S. students without benefit of classroom instruction on the topic. They are somewhat intuitive, like the spatial visualization items. Again, change was relatively low.

Perhaps most disappointing in these data is the low level of achievement for the more basic items dealing with the classification and properties of geometric figures.

Item-level Data: Geometry Here are two sample items from the geometry sub-test. Item 122 requires the knowledge that the sum of the measures of the three angles of a triangle is 180 degrees. While considerable change took place (an average gain of 17 percent during the year), the final level of achievement is still just over 50 percent. Item 026 is an application (problem solving) item concerning similar triangles. End of year performance on this item was 41 percent for U.S. students compared to 56 percent in 1964 and 49 percent internationally (in 1982).

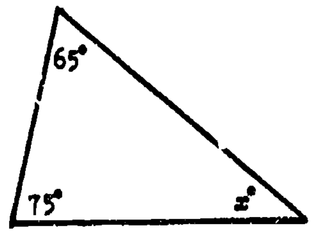
Item 122

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	35	52
1982 International Score		68

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	71	82



x is equal to

A 75

B 70

C 65

D 60

* E 40

Item 026

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	30	41
1964		56
1982 International Score		49

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	56	62

On level ground, a boy 5 units tall casts a shadow 3 units long. At the same time a nearby telephone pole 45 units high casts a shadow the length of which, in the same units, is

A 24

* B 27

C 30

D 60

E 75

• Achievement in Statistics. While only a single item could clearly be classified as dealing with probability and it was covered in less than 40 percent of the classes for the U.S., a number of items related to descriptive statistics were more widely covered.

Although Table 10 records the highest performance levels of any of the major topic area subtests, it also reflects the lowest pre- to posttest change. The teacher OTL data give evidence that this was a part of the curriculum in most eighth grade classes. Figure 17 portrays this information. However, for the particular items included, the achievement data indicate that it was an area of little growth compared with other mathematics topics. For only two of the eighteen items did the posttest-pretest difference reach 10 percent. Figure 17 portrays this information.

Table 10
Eighth Grade Achievement in Statistics: U.S., 1981-82

Topic (Number of items)	Mean Percent Correct Pre-test	Post-test	Difference
Averages (3)	64	66	+ 2
Tables and Graphs (13)	50	55	+ 5
Other (2)	53	59	+ 6
All Statistics Items (18)	53	57	+ 4

Item-level Data: Statistics Two sample items are presented for the topic of statistics. Item 067 is a simple problem solving item involving computing two averages and comparing them. Performance was high (72 percent correct at year's end) but was less than in 1964 and actually decreased over the course of the year. Item 160 involves the ability to answer questions using a line graph. Performance was again modest (59 percent) at year end but this represented a slight improvement over the 1964 performance.

Item 067

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	76	72
1964		84
1982 International Score		75

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	73	95

Joe had three test scores of 78, 76 and 74, while Mary had scores of 72, 82 and 74. How did Joe's average compare with Mary's?

A Joe's was 1 point higher.

B Joe's was 1 point lower.

*C Both averages were the same.

D Joe's was 2 points higher.

E Joe's was 2 points lower.

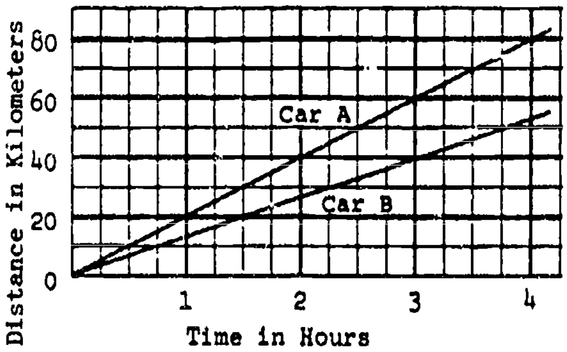
Item 160

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	47	59
1964		53
1982 International Score		52

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	50	68



Three hours after starting, car A is how many kilometers ahead of car B ?

A 2 B 10 C 15

*D 20 E 25

Achievement in Measurement. Measurement received one of the higher average OTL ratings for the U.S. The performance levels summarized in Table 11 are low, especially in view of this evidence of substantial curricular emphasis. Figure 18 shows these data as well.

The performance of U.S. students on some of the items clearly was limited by lack of familiarity with and use of metric units. For example, one item on the international test required the student to estimate the weight (mass) of a man in kilograms. Internationally, 88 percent of the students obtained the right answer, but only 35 percent of the U.S. students were able to respond correctly.

However, the performance by U.S. students on items not requiring knowledge of the metric system (not reported in detail here) provides enough evidence of a lack of understanding of the fundamental concepts of measurement to raise questions about how effectively measurement is taught in U.S. schools. It often has been suggested in the past that measurement should be a characterizing feature of the U.S. middle or junior high school mathematics curriculum and that metric measurement, in particular, be prominent if not dominant in that program. The data from this Study indicate that the goal of teaching measurement generally, and metric measure in particular, is not making progress in U.S. schools.

Table 11
Eighth Grade Achievement in Measurement: U.S., 1981-82

Topic (Number of items)	Mean Percent Correct Pre-test	Post-test	Difference
Standard Units (6)	43	46	+ 3
Linear Scales (3)	40	48	+ 8
Estimation (5)	52	56	+ 4
Area (8)	20	31	+11
Volume (4)	26	35	+ 9
All Measurement Items (26)	35	42	+ 7

Item-level Data: Measurement Here are four sample items from the measurement subtest. Item 038 involves reading a "ruler" (linear scale) but does not require knowledge of any system of units (such as the metric system). Notice that performance was quite low, both in 1982 and in 1964. Item 133 is a problem solving item that requires knowledge of how many meters are in a kilometer. Notice that the U.S. performance level is even lower than that for Item 038. Item 103 involves primarily an understanding of the additive (in this case, subtractive) property of area but, since metric units are indicated, concern about the metric system might affect performance. Notice that the performance is again somewhat low. Item 168 is one of several on the international test on the volume of a rectangular solid. While about one-half of the students were able to do the straight forward calculation to find volume given the three dimensions, performance dropped off dramatically (to well below that expected by random guessing) for this item (168) which requires deeper understanding of the concept of volume.

Item 038

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	38	47
1964		44
1982 International Score		40

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	54	86

On the above scale the reading indicated by the arrow is between

A 51 and 52

B 57 and 58

C 60 and 62

D 62 and 64

*E 64 and 66

Item 133

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	33	37
1982 International Score		60

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	58	80

How many pieces of pipe, each 20 meters long, would be required to construct a pipeline 1 kilometer in length?

A 5

*B 50

C 500

D 5000

E 50,000

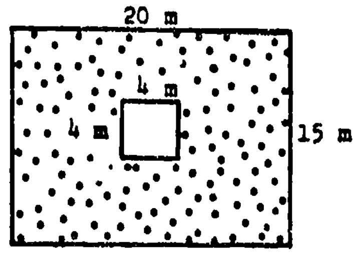
Item 103

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	30	44
1982 International Score		63

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	68	78



A square is removed from the rectangle as shown. What is the area of the remaining part?

A 316 m^2

B 300 m^2

*C 284 m^2

D 80 m^2

E 16 m^2

Item 168

Percent Correct

Year	Beginning 8th Grade	End 8th Grade
1981-82	9	10
1982 International Score		14

OTL

Year	During 8th Grade	Through 8th Grade
1981-82	45	54

A solid plastic cube with edges 1 centimeter long weighs 1 gram. How much will a solid cube of the same plastic weigh if each edge is 2 centimeters long?

*A 8 grams

B 4 grams

C 3 grams

D 2 grams

E 1 gram

Achievement by Class Type. Because the U.S. curriculum for eighth grade mathematics is characterized by variation in class type, in this section we present summaries of achievement by these class types.

In Table 12, pre- and posttest scores are reported for each class type for the five major content areas.

Table 12, Part 1
Eighth Grade Mean Achievement of Each Content Area by Class Type: U.S., 1981-82

Mean Percent by Class Type, Pre- and Posttest

Content Area	Remedial			Typical		
(Number of Items)	Pre	Post	Difference	Pre	Post	Difference
Arithmetic (62)	25	32	+ 7	38	47	+ 9
Algebra (32)	20	24	+ 4	28	38	+10
Geometry (42)	22	26	+ 4	28	35	+ 7
Measurement (26)	23	27	+ 4	32	39	+ 7
Statistics (18)	34	39	+ 5	50	55	+ 5
All Items (180)	24	29	+ 5	34	42	+ 8

(Table 12 continued on next page.)

Table 12, Part 2
Eighth Grade Mean Achievement of Each Content Area by Class Type:
U.S., 1981-82

Mean Percent by Class Type, Pre- and Posttest

Content Area	Enriched			Algebra		
(Number of Items)	Pre	Post	Difference	Pre	Post	Difference
Arithmetic (62)	54	64	+10	67	72	+ 5
Algebra (32)	40	57	+17	56	70	+14
Geometry (42)	39	50	+11	47	53	+ 6
Measurement (26)	43	52	+ 9	53	57	+ 4
Statistics (18)	65	68	+ 3	74	75	+ 1
All Items (180)	48	58	+10	59	65	+ 6

Figure 19 offers a graphical portrayal of some of these data for the four class types in just one area, the cluster of algebra items.

A number of observations can be made from the data in Table 12. A close relationship between class type and ability level is suggested by the consistent trend in pretest means. For remedial, typical, and enriched classes (but not in comparing enriched to algebra classes), the posttest mean of one classification never reaches the pretest mean of the next classification. Notice also that pre- to posttest differences were greater for the enriched than for the algebra classes for all topics - including algebra. This may reflect the fact that the tests fit the enriched classes best and the algebra classes less well. It also suggests that the range of test items in algebra may not be broad enough to capture the performance of classes devoting the major portion of the year to study in this area. The performance of students in remedial classes was near what would be expected if students were guessing randomly (twenty percent) on the pretest in four of the five content areas.

Sex Differences in Achievement in the Eighth Grade.

No strong differences were found in the mathematics achievement of eighth grade boys and girls. Boys and girls had nearly identical (within 2 percent) scores on each topic and changes from pre- to posttests were not much different. No differences between boys and girls were found on overall test scores in achievement. Analyzing the data to compare boys and girls on computational items and on comprehension and problem-solving items separately also showed no striking differences.

Changes in Eighth Grade Achievement Between 1964 and 1982

Forty items from the First International Mathematics Study were included in the test for the Second International Mathematics Study. Four of these items were either changed from open-ended to multiple choice items or the original multiple choice item was modified in some substantial way.

Thirty-six items from the First Study were included on the Second Study with either no changes or very minor editorial changes. Table 13 presents the results for these items grouped into the topics used previously for eighth grade results. The number of items in each topic cluster is given in parentheses after the topic name.

Table 13
Eighth Grade Posttest Achievement by Topic on Common Items for 1964 and 1982: U.S.

Topic (Number of Items)	1964-65	1981-82	Change
Arithmetic (14)	55	49	- 6
Algebra (10)	40	41	+ 1
Geometry (5)	40	34	- 6
Statistics (5)	57	54	- 3
Measurement (2)	35	37	+ 2
Overall (36)	48	45	- 3

The data indicate a modest decline in end-of-year performance from the eighth grade sample tested in 1964 to that tested in 1982. Figure 21 portrays these data. The magnitude of the negative change in arithmetic and geometry should be noted. It is also worth commenting that the U.S. position on these thirty-six items in the 1964 study was slightly below the international mean for 1964. Overall, achievement on these repeated items does not offer much encouragement except that we seem to have held our own in the areas of algebra and measurement.

It is of interest also to consider the results from the two studies in terms of different cognitive levels. Items were classified into four cognitive levels based on presumed difficulties of task. Level I was computation, Level II comprehension, and Levels III and IV were application and analysis, respectively. The number of items in each cluster is given in parentheses after the label. Table 14 presents achievement results in terms of these cognitive levels.

Table 14
Eighth Grade Posttest Achievement by Cognitive Level on Common Items for 1964 and 1982: U.S.

Cognitive Level (Number)of Items)	1964-65	1981-82	Change
Computation (13)	50	48	- 2
Comprehension (11)	50	46	- 4
Application and Analysis (12)	44	40	- 4

Figure 22 shows the 1964 to 1982 increase or decrease for each anchor item with the items grouped by cognitive level.

Notice that at each level there is a decline from 1964 to 1982. There is not only an overall drop across the board in comparing the results of the two studies, but performance seemed to decline a bit more at the higher cognitive levels.

Where Does the U.S. Stand Internationally? (Eighth Grade)

Figures 23 to 27 show the rank ordering of the twenty countries taking part in the Population A Study for the topics of arithmetic, algebra, geometry, and statistics and measurement. The following observations are made about these "staircase" graphs:

1. For most topics, the U.S. is close to the middle of the "staircase" -- that is close to the international average in achievement.

2. For most topics, the countries close to the U.S. in the graphs are rather similar in achievement. That is, there is little difference in the height of the "stairs" for the center cluster of countries.

3. In all topics Japan is highest in achievement. The average scores in Japan were about 10-20 percentage points higher than in the U.S.

4. The only countries identified are those whose national reports are available at this time.

Table 15 offers a comparison of Population A achievement pretest and posttest for the U.S. (means for the usual five topic areas) and the pooled international mean.

Table 15
Eighth Grade Achievement Means: U.S. and International, 1981-82
(Mean Percent Correct)

Topic	United States Pre	United States Post	United States Change	Twenty Country* Posttest Mean
Arithmetic	42	51	+9	51
Algebra	32	43	+11	43
Geometry	31	38	+7	41
Statistics	53	57	+4	55
Measurement	35	42	+7	51

* The countries included in addition to the United States, are: Belgium (Flemish); Belgium (French); Canada (British Columbia); Canada (Ontario); England and Wales; Finland; France; Hong Kong; Hungary; Israel; Japan; Luxembourg; Netherlands; New Zealand; Nigeria; Scotland; Swaziland; Sweden; Thailand.

Notice that the U.S. performs at or below the international mean in all areas except statistics and markedly below in measurement (possibly due to less familiarity with the metric system which was used consistently in the Second Study tests).

Some concern has been expressed about using means for comparison when it is likely that high achieving countries such as Japan may be outliers which raise the mean disproportionately. In answer to this and to get a better feel for where the U.S. stands internationally, Table 16 offers median and quartile data for the international distribution to compare to the U.S. posttest mean.

Table 16
Eighth Grade Posttest Achievement Comparisons: U.S. and International, 1981-82

	United States	International		
Topic	Posttest Mean	25th Percentile	Median	75th Percentile
Arithmetic	51	45	51	57
Algebra	43	39	43	50
Geometry	38	38	43	45
Statistics	57	52	57	60
Measurement	42	47	51	58

Notice that the U.S. students score at the international median in arithmetic, algebra and statistics. They score at the 25th percentile internationally in geometry and well below the 25th percentile in measurement. By any reasonable standard there is considerable cause for alarm in these comparisons.

Part III. TWELFTH GRADE FINDINGS

Two Class Types in the Twelfth Grade

To simplify the wide diversity of instructional patterns at the twelfth grade level, the 237 classrooms in the United States sample were classified into precalculus classes and calculus classes. The precalculus classes, 191 in number, were those that focused their efforts on teaching trigonometry, "college" algebra, and analytic geometry as indicated by their responses to specific classroom processes instruments. These classes might best be characterized as "senior mathematics," "elementary functions," or one of a group of other titles given to twelfth grade level courses taught to precalculus students. Some of these classes contained as much as a semester of troductory topics in calculus, but fell far short of what would be considered a semester course in the topic at the university level. The calculus classes, 46 in number, were those that were taught a full year course in the calculus, following the outline provided by the College Entrance Examination Board Advanced Placement syllabi for the topic. This was determined by an examination of the textbooks indicated and consideration of the classroom process questionnaire for calculus.

The decisions in making these classifications are supported by the different patterns of exposure to content and the different levels of achievement reported later. The data from the teacher questionnaires also show that teachers confirm the differences in the two groups. These, and other data presented later, make a strong case for the use of the two categories for analyzing and describing the curricula, instruction, and achievement in the U.S. twelfth grade classrooms participating in the Study.

Class Size

The student enrollment in the classes studied was composed of 56 percent males and 44 percent females. This same proportion was found in the precalculus and the calculus classes. The class sizes in both cases tended to be near 20, with the calculus classes having a median size of 17 and the precalculus classes a median size of 21.

Hours of Mathematics Instruction Per Year

The data on time allocated to mathematics instruction were very consistent, with classes in both areas reporting a median period length of 51 minutes. There was a slight difference in the total clock hours of instruction over the year between the two groups, with the precalculus classes receiving a median of 153 clock hours of instruction and the calculus classes 150 clock hours. (See Figure 1.)

Teaching Assignments

The teaching loads at the twelfth grade level varied more than at the eighth grade level. The most typical load was teaching 5 periods per day. However, the precalculus teachers ranged from 1 to 7 periods per day and the calculus teachers from 1 to 6 periods per day. Very few teachers in either category taught anything outside of mathematics. When a subject was taught outside of mathematics, it was most often in some aspect of science.

A large proportion of the calculus teachers also serves as department chairs in mathematics (44 percent compared with about 35 percent of precalculus teachers).

Teacher Characteristics

Gender. The teachers participating in the study were almost evenly divided between male (52 percent) and female (48 percent) teachers. However, some differences were noted in their assignments in the two types of curricula. In the precalculus classes, the ratio of male to female teachers was nearly 1 to 1. In the calculus classes there were significantly more male teachers than female teachers.

Age/Experience/Training. The median responses on teachers' backgrounds showed the typical teacher in the twelfth grade sample to be 40 years of age, with almost 16 years of teaching experience. Eight of these years had involved the teaching of twelfth grade mathematics. About ten percent of the teachers were between 55 and 65 years of age.

The teachers' responses to questions concerning their collegiate education showed a median of 16 semester courses in mathematics, 3 semester courses in the teaching of mathematics, and 6 semester courses in general methods and pedagogy. About 8 percent of the teachers reported little to no preparation in the areas of the teaching of mathematics and generic methods preparation.

Beliefs About Effective Teaching. Like the eighth grade teachers, the twelfth grade teachers were asked to respond to a set of items dealing with possible components of effective teaching. Here there were differences in the level of endorsement given statements by teachers in the two class types. Five of the maxims were selected as among the highest in importance by at least 50 percent of the precalculus teachers, while only one such was selected by at least 50 percent of the calculus teachers. The one maxim selected as important by both groups indicated that one should review tests with students shortly after they have been graded. This suggestion was endorsed by 65 percent of the precalculus teachers and 63 percent of the calculus instructors.

Other statements selected as important by precalculus teachers dealt with making encouraging remarks to students (53 percent), establishing and enforcing clear-cut rules of classroom behavior (54 percent), clearing up problems from a previous lesson (60 percent), and avoiding criticizing a student in front of a class (51 percent). These responses were also among the highest rated by calculus teachers, but their response levels indicate a more conservative approach to endorsing general maxims for effective teaching.

Perceptions of the Sampled Class. As at the eighth grade level, the teachers indicated that their sampled classes were perhaps a bit harder to teach than were their normal twelfth grade classes. However, 78 percent of the precalculus and 73 percent of the calculus teachers reported their target classes were either very easy or fairly easy to teach. Both class types reported that about 85 percent of their students were attentive in class and not behavior problems. The remaining students were viewed as being not attentive, but not discipline problems.

When asked to give the number of students in the sampled class which were especially fearful or anxious about mathematics, 78 percent of the precalculus teachers and 95 percent of the calculus teachers reported three or fewer students. Only precalculus classes ever reported more than six students as being especially fearful or anxious about mathematics.

Goals in Teaching Mathematics. Teachers were asked to rate the relative emphasis given to a set of nine goals characterizing objectives for the teaching of mathematics. Table 17 presents the results of these ratings.

Table 17
Relative Importance of Goals in Teaching Mathematics
as Rated by Twelfth Grade Precalculus and Calculus Teachers:
U.S., 1981-82
(Percents of Teachers)

	Precalculus		Calculus	
Goal	Relatively More Impt.	Relatively Less Impt.	Relatively More Impt.	Relatively Less Impt.
Develop a systematic approach to solving problems	65	5	65	0
Know mathematical facts, principles, and algorithms	42	9	54	7
Understand the logical structure of mathematics	41	11	37	11
Develop an attitude of inquiry	40	6	44	4
Develop an awareness of the importance of mathematics in everyday life	36	19	24	33
Become interested in mathematics	34	14	37	22
Develop an awareness of the importance of mathematics in the basic and applied sciences	32	15	48	9
Perform computations with speed and accuracy	24	23	13	30
Understand the nature of proof	12	38	15	41

The precalculus teachers chose to give relatively more emphasis in their target classes to developing a systematic approach to problem solving; to knowing mathematical facts, principles, and algorithms; and to understanding the logical structure of mathematics.

The calculus teachers indicated they had given relatively more emphasis to developing a systematic approach to problem solving; to knowing mathematical facts, principles, and algorithms; to developing an awareness of the importance of mathematics in the basic and applied sciences; and to developing an attitude of inquiry.

Both of the groups, as did the eighth grade teachers, gave their lowest emphasis rating to the goal of understanding the nature of proof. Only 12 percent of the precalculus teachers and 15 percent of the calculus teachers gave it relatively more emphasis than other objectives, while 38 and 41 percent of the teachers, respectively, noted that it received relatively less emphasis than the other goals with their target classes. Note, however, that relative importance of the goals was being sought rather than some sort of absolute importance.

How Mathematics Teachers Spent Their Time

The teachers were asked to estimate the number of minutes per week devoted to generic teaching tasks related to the teaching of their target class. The typical (median) precalculus teacher spent about 2 hours in preparing instruction, 1.5 hours in grading papers, 2 hours in explaining new content to the class, 1 hour in reviewing content taught previously, and about 25 minutes in dealing with classroom administrative and management details. The median calculus teacher followed much the same pattern, except the time allocations show an additional half-hour of preparation, only 50 minutes for reviewing previously taught material, and 15 minutes on classroom administrative and management tasks. As these responses deal only with a teacher's sampled class, they need to be multiplied by the number of classes taught per day (usually 5 or 6), or at least by the number of "preparations", to get an estimate of the total load placed on a classroom teacher's time.

Overall, the reported use of allocated in-class instructional time consisted of about 40 percent on developing new material, 20 percent on reviewing previously taught material, 10 percent on administrative and management tasks, and 30 percent on supervising student work in the classroom. (See Figure 28.)

Use of Instructional Resources

In examining instructional resources used, the textbook stood out as the most commonly and consistently used resource. The next most common resource was self-written tests (about 95 percent), followed by self-written materials (about 60 percent) and workbooks and supplementary texts (about 10 percent). (See Figure 29.)

How Mathematics Students Spent Their Time

Like the eighth grade students, the students in the sampled classes at the twelfth grade level spent the major portion of their class time listening to teacher presentations (130 minutes per week on the average). Doing seatwork (60 minutes per week on the average) and taking tests (about 45 minutes per week on the average) accounted for other large blocks of time. (See Figure 30.) While, on average, little time was spent on small group work, there was a great range in time devoted to this use of time across the sampled classrooms. Some classes spent as much as 80 minutes per week on small group activities while others spent virtually none.

Homework

Homework expectations differed markedly between the precalculus and the calculus classes. (See Figure 5.) In precalculus classes, teachers reported that the student in a typical class was expected to do about 4 hours of homework per week, with students in the middle fifty percent of the classes expected to do between 3 and 5 hours of homework per week. In calculus classes, teachers reported that the students were expected to complete 5 hours of homework per week, with the middle fifty percent of classes having from 4 to 6 hours of homework per week.

Use of Calculators

The use of calculators was more prevalent at the senior high school level than at the eighth grade level. About 33 percent of the classes used them 2 or more periods a week. (See Figure 6.) Another 28 percent of the classes used them 1 period a week or less. Teachers of eleven percent of the classes reported that their students did not use calculators, 9 percent indicated that calculators were not allowed in their classes, and about 20 percent of the teachers failed to provide information.

In the twelfth grade, the most commonly reported instructional uses of calculators in the classroom were checking work, doing homework, and solving mathematics problems in class. (See Figure 7.) In contrast to the eighth graders, who were rarely allowed to use calculators on tests, about 50 percent of the twelfth graders were allowed to use calculators on examinations.

What Mathematics was Taught

Teachers were asked to report whether the content needed to respond to the items of the international test had been taught to their classes. These Opportunity-to-Learn (OTL) data are reported in Table 18.

The results show that the content taught to the precalculus and the calculus groups was different in distinctive ways. The greatest difference between the 191 precalculus classes and 46 calculus classes was, as expected, in elementary functions and calculus, which included differential and integral calculus. In number systems and algebra, for example, the precalculus classes received higher coverage than did the calculus classes. However, as Table 18 also shows, teachers judged that the calculus classes had been taught much of this material in preceeding years. For elementary functions and calculus, relatively little of the material was reported as being taught to the calculus classes prior to twelfth grade, but the content relating to 83 percent of the items was taught during the year.

Table 18, Part 1
Percentage of Cognitive Test Items Taught to Twelfth Grade Students:
U.S., 1981-82
(Average Percent Across Items)

Content Area (N items)	Precalculus (Number of classes, 191)			Calculus (Number of classes, 46)		
	Taught Before	Taught This Year	Never Taught	Taught Before	Taught This Year	Never Taught
Sets & Relations (7 items)	31	50	19	50	40	10
Number Systems (17 items)	39	42	19	75	14	11
Algebra (26 items)	34	52	14	53	41	6
Geometry (26 items)	21	40	39	41	26	33
Elementary Functions & Calculus (46 items)	8	37	55	9	83	8
Probability and Statistics (7 items)	29	14	57	50	6	44
Finite Mathematics (4 items)	29	21	50	62	8	30

(Table 18 continued on next page.)

Table 18, Part 2
Percentage of Cognitive Test Items Taught to Twelfth Grade Students:
U.S., 1981-82
(Average Percent Across Items)

	All Classes (Number of classes, 237)		
Content Area (N items)	Taught Before	Taught This Year	Never Taught
Sets & Relations (7 items)	35	48	17
Number Systems (17 items)	46	37	17
Algebra (26 items)	38	50	12
Geometry (26 items)	25	37	38
Elementary Functions & Calculus (46 items)	8	46	46
Probability and Statistics (7 items)	33	12	55
Finite Mathematics (4 items)	35	18	47

Mathematics Not in the Twelfth Grade Curricula

The development of the cognitive tests for the Second Study strove to provide items which were of current curricular interest to the countries participating. This process worked from a logical grid of curricular areas and topics common to most of the countries taking part. However, it was impossible to write an examination that fit each of a large number of different curricular programs perfectly. As a result there were items which some classes of students in the U.S. and in each other country had little or no opportunity to study.

In the present case, this method affected twelfth grade classes in the United States more often in the areas of geometry, probability and statistics, and finite mathematics than in the other areas shown in Table 18. In addition, the lack of opportunity to learn also affected the students in the precalculus classes in the elementary functions and calculus subtest. An overall comparison measure of this opportunity to learn showed the United States mean percentage of items actually taught (twelfth grade or before) to be about 65 percent overall and the international mean percentage for items actually taught to be 73 percent.

The test content items falling outside the United States curriculum generally involved topics in linear algebra (transformations and matrix algebra), parametric equations, combinatorics, and, for the precalculus classes, advanced topics in differentiation and integration.

Student Achievement in Twelfth Grade Mathematics

Table 19 contains achievement results for the twelfth grade by class types.

Table 19, Part 1
Twelfth Grade Achievement by Subtopic: U.S. and International, 1981-82
(Mean Percent Correct)

Content Area (N items)	United States: Precalculus (Number of classes, 191) OTL	Pre	Post	Difference	United States: Calculus (Number of classes, 46) OTL	Pre	Post	Difference	International* Posttest
Sets & Relations (7)	81	48	54	+6	90	66	64	-2	62
Number Systems (17)	81	33	38	+5	89	43	48	+5	50
Algebra (26)	87	35	40	+5	94	53	57	+4	57
Geometry (26)	60	24	30	+6	67	35	38	+3	42
Elem. Functions/ Calculus (46)	45	18	25	+7	92	26	49	+23	44
Probability & Statistics (7)	43	36	39	+3	57	48	48	0	50
Finite Mathematics (4)	50	24	29	+5	71	36	38	+2	44**

* The countries included, in addition to the U.S., are: Belgium (Flemish); Belgium (French); Canada (British Columbia); Canada (Ontario); England and Wales; Finland; Hong Kong; Hungary; Israel; Japan; New Zealand; Scotland; Sweden; Thailand.
**Estimated median score

(Table 19 continued on next page.)

Table 19, Part 2
Twelfth Grade Achievement by Subtopic: U.S. and International, 1981-82
(Mean Percent Correct)

Content Area (N items)	United States: All Classes (Number of classes, 237)				International* Posttest
	OTL	Pre	Post	Difference	
Sets & Relations (7)	83	52	56	+ 4	62
Number Systems (17)	83	35	40	+ 5	50
Algebra (26)	88	38	43	+ 5	57
Geometry (26)	62	26	31	+ 5	42
Elementary Functions/ Calculus (46)	54	19	29	+10	44
Probability & Statistics (7)	45	39	40	+ 1	50
Finite Mathematics (4)	53	27	31	+ 4	44**

Figures 31 and 32 portray in detail the results for two topic areas, algebra and analysis (elementary functions and calculus).

The data in Table 19 show the differences that existed between the students in the precalculus and calculus classes. Even in the instances where the calculus classes had little exposure to the content, their performance was still higher.

We note furthermore that in every content area (sets and relations, number systems, algebra, etc.) the end of the year average achievement of the precalculus classes was less (and in many cases considerably less) than the beginning of the year achievement of the calculus students.

It is important to observe that the great majority of U.S. senior high school students in fourth and fifth year mathematics classes (that is, those in precalculus classes) had an average performance level that was at or below that of the lower 25 percent of the countries. The end-of-year performance of the students in the calculus classes was at or near the international means for the various content areas, with the exception of geometry. Here, U.S. performance was below the international average. That is to say, the achievement of the highest scoring U.S. mathematics classes (2-3 percent of twelfth grade students) was at best comparable to the performance of the overall Population B classes in other countries. This was in spite of the fact that the U.S. calculus students represent a smaller fraction of the age cohort than is represented in the Population B of most of the other countries in the Second Study.

Item-level Data: Sets, Properties and Number Systems Here are data for eight items, chosen to illustrate key aspects of the twelfth grade findings. The first two items belong to the topic areas of Sets, Properties and Number Systems. The OTL data indicate that the vast majority of the classes were judged by teachers to have been exposed to the mathematical content of these items by the end of twelfth grade. On Item 001, dealing with operations on sets, 56 percent of the students were successful, a considerable improvement over the score of 21 percent obtained on this item in 1964. Much of this positive change is surely due to more widespread inclusion of this topic in the current mathematics curriculum than twenty years ago. Item 002 deals with removing nested parentheses and the operation of subtraction. The performance levels of 73 percent for the calculus classes and 58 percent for the precalculus classes are lower than expected of students in the twelfth grade. U.S. scores on both of these items are considerably below the international medians.

Item 001

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	54	52
Calculus	73	70
Overall	57	56
1964		21
1982 International Score		64

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	25	90
Calculus	2	95

If X and Y are sets, then $(X \cup Y) \cap (X \cap Y)$ is equal to

A X

B Y

C $X \cup Y$

*D $X \cap Y$

E $(X \cup Y) \cup (X \cap Y)$

Item 002

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	50	58
Calculus	62	73
Overall	52	60
1982 International Score		81

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	39	100
Calculus	28	100

For all rational numbers a, b, c, and d,

$a - (b + (c - d))$ is equal to

A $a - b + c - d$

*B $a - b - c + d$

C $a - b - c - d$

D $a - b + c + d$

E None of these

Item-level Data: Algebra and Analytic Geometry Item 006, dealing with zeroes of a function, saw gains of nearly twenty percentage points in both the precalculus and calculus classes during the year. These gains, consistent with the high reported teacher coverage of this content (74 percent in the calculus classes and 80 percent in the precalculus classes) indicate that this topic receives relatively major emphasis in both class types during twelfth grade. Notice that while the calculus classes perform at nearly the international median, the precalculus classes are well below the international median.

In Item 054, the order relationship between two functions is to be evaluated. Performance in 1982 is only marginally improved over that of 1964 and is well below the international score for 1982.

Item-level Data: Trigonometry and Probability Item 039, for the precalculus classes, had a beginning of the year score less than that expected from random guessing. Indeed, the distribution of wrong responses shows that about 1/3 of the students chose distractor E. End-of-year performance on this trigonometry item, which is less than 50 percent even for the calculus classes, is well below the international median of 64 percent. A very weak grasp of basic concepts is indicated.

Item 030 deals with probability. For this item, a slight preference among the wrong answers was shown for response E. This information, combined with the OTL data, suggests that many students are not familiar with how to deal with probability items extending beyond simple events. The only slightly improved performance on this item since 1964 indicates, further, that this topic has not received much increased emphasis since twenty years ago. Interestingly, international performance on this item is about ten points higher than in the U.S.

Item-level Data: Integral Calculus Performance on items 029 and 044 reflects understanding of the integral as measuring the area under a curve and the integral's relation to the graph of a curve. Very high OTL for the calculus classes is reported for both items. Item 029 showed especially large growth for the calculus classes, but even by the end of the year was below the international median. Item 044 was much more difficult for the U.S. classes and internationally The distribution of the wrong responses to the latter item shows that about 40 percent of the calculus students selected C as the correct answer. This results from confusing the maximum of the function over the interval with the maximum value of the integral. About 20 percent of the students incorrectly selected response E, failing to take into account the fact that the integral is decreasing in value over the interval from 6 to 10.

Item 006

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	26	44
Calculus	53	72
Overall	31	49
1982 International Score		73

OTL

Year	During 12th Grade	Through 12th Grade
Precalculus	80	98
Calculus	74	100

The curve defined by $y = 3x(x - 2)(2x + 1)$ intersects the x-axis only at the points

A $(-2,0)$ and $(\frac{1}{2},0)$

B $(2,0)$ and $(-\frac{1}{2},0)$

C $(3,0)$ and $(-2,0)$ and $(\frac{1}{2},0)$

D $(3,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

*E $(0,0)$ and $(2,0)$ and $(-\frac{1}{2},0)$

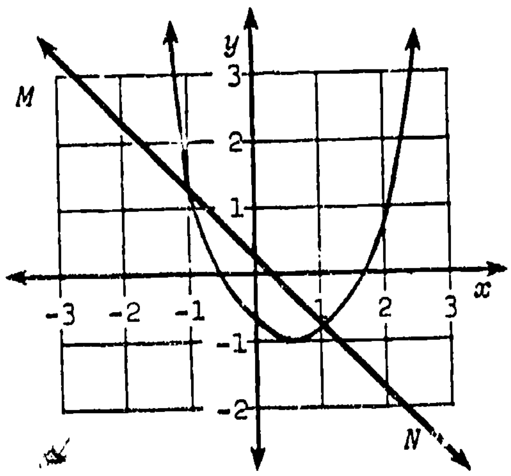
Item 054

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	22	31
Calculus	30	44
Overall	24	33
1964		29
1982 International Score		58

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	61	85
Calculus	61	98



For what values of x does the function represented by the straight line $\overleftrightarrow{MN}$ exceed the quadratic function?

*A $-1 < x < 1$

B $x < -1$ or $x > 1$

C $-\frac{3}{4} < x < 1\frac{1}{4}$

D $x > 0$

E $x > y$

Item 039

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	10	32
Calculus	38	44
Overall	16	34
1982 International Score		64

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	80	96
Calculus	64	100

If $\cos\theta = \frac{1}{2}$, then $\cos 2\theta$ is equal to

*A $-\frac{1}{2}$

B $\frac{1}{2}$

C $-\frac{\sqrt{3}}{2}$

D $\frac{\sqrt{3}}{2}$

E 1

Item 030

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	46	48
Calculus	63	56
Overall	50	49
1964		45
1982 International Score		58

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	23	40
Calculus	9	60

A set of 24 cards is numbered with the positive integers from 1 to 24. If the cards are shuffled and if only one is selected at random, what is the probability that the number on the card is divisible by 4 or 6?

A $\frac{1}{6}$

B $\frac{5}{24}$

C $\frac{1}{4}$

*D $\frac{1}{3}$

E $\frac{5}{12}$

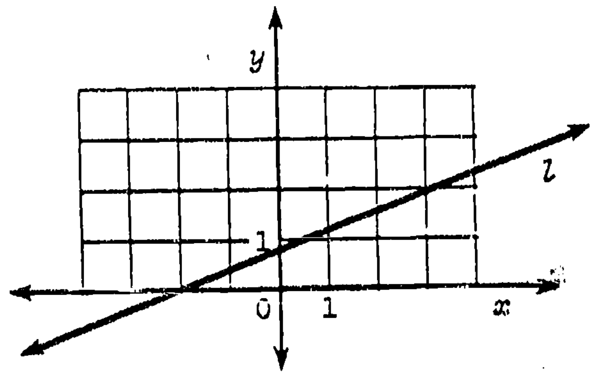
Item 029

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	18	26
Calculus	22	53
Overall	19	31
1982 International Score		59

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	20	21
Calculus	100	100



The line l in the figure is the graph of $y = f(x)$.

$\int_{-2}^{3} f(x)\,dx$ is equal to

A 3 B 4 C 4.5

*D 5 E 5.5

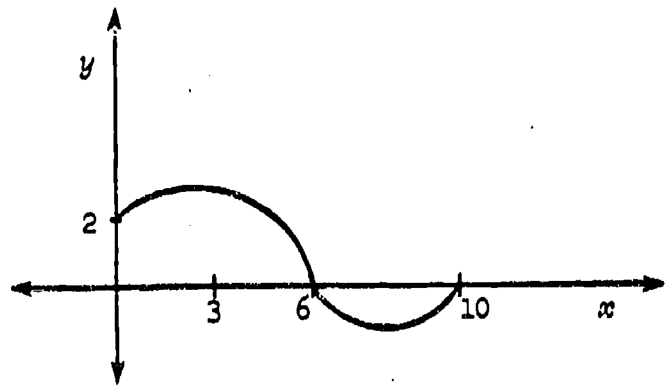
Item 044

Percent Correct

Year	Beginning 12th Grade	End 12th Grade
1981-82		
Precalculus	10	12
Calculus	7	26
Overall	9	15
1982 International Score		28

OTL

Year	During 12th Grade	Through 12th Grade
1981-82		
Precalculus	20	21
Calculus	95	98



The graph of the function f is shown above for $0 \leq x \leq 10$.

$\int_{0}^{a} f(x)\,dx$ attains its greatest value when a is equal to

A 0 B 2 C 3

*D 6 E 10

Sex Differences in Achievement in the Twelfth Grade

The data in Table 20 show the performance of the male and female students taking part in SIMS at the twelfth grade level. The data used in this analysis were the unweighted student scores by item for each sex. Unlike the findings for the eighth grade level, the data here illustrate a pattern of higher performance on the cognitive items for the male students involved in the study. In particular, the results show that the pretest means (except for Sets and Relations) were higher for the males than for the females and all the posttest means for the males exceeded those for the females. Furthermore, change scores for the males usually equaled or exceeded (often only marginally) those for the females on each of the topic areas considered. The size and consistency of these differences may be taken as suggesting one or more sex-related variables at work.

Table 20
Twelfth Grade Mathematics Achievement by Gender: U.S., 1981-82
(Unweighted Student-Level Data)

	(Number of Students, Female) 1999			(Number of Students, Male) 2559		
Topic (N items)	Pre	Post	Difference	Pre	Post	Difference
Sets/Relations (7)	53	55	+2	50	57	+7
Number Systems (7)	33	38	+5	35	41	+6
Algebra (26)	37	42	+5	39	45	+6
Geometry (26)	24	29	+5	27	32	+5
Elementary Functions and Calculus (46)	18	28	+10	20	31	+11
Probability and Statistics (7)	35	35	0	41	44	+3

Note:
Female enrollment by class type was calculus, [illegible] and precalculus, 1679.
Male enrollment by class type was calculus, 420 and precalculus, 2139.

Changes in Twelfth Grade Achievement Between 1964 and 1982

Twenty of the 136 twelfth grade cognitive test items were included in both the First International Mathematics Study and the Second International Mathematics Study with no significant changes. Another six items could potentially be selected for comparisons but were substantially altered and were not chosen for comparisons here. Table 21 presents the results for twenty items clustered by the subtest topics for twelfth grade. The number of items in each cluster is given in parentheses after the name of the cluster.

Table 21
Twelfth Grade Posttest Achievement by Topic on Common Items for 1964 and 1982: U.S.

Topics (Number of Items)	1964	1982	Change
Sets and Relations (1)	21	56	+35
Number Systems (1)	45	59	+14
Algebra (3)	44	45	+ 1
Geometry (4)	36	37	+ 1
Elementary Functions/ Calculus (10)	25	31	+ 6
Probability and Statistics (1)	45	49	+ 4
Overall (20)	32	38	+ 6

These data are portrayed in Figure 34.

The data show a slight overall increase in student performance from 1964 to 1982. The only topic in which significant change was found was that of elementary functions and calculus. Here there was a gain of six percent on a ten item subtest. It is possible, however, that this gain reflects the fact that greater proportions of students took calculus in 1982 than in 1964. (The numbers of students taking the Advanced Placement Calculus examination has been reported by the College Entrance Examination Board as going from 8,000 in 1964 to 32,000 in 1982.) There is also the possibility that the 1982 Population was more selective in college preparatory mathematics than was the corresponding Population in 1964. At any rate, the comparative data suggest that for the subject matter represented by the items in Table 21, the twelfth grade college preparatory mathematics students in the U.S. have not lost ground in the eighteen years between the First and Second International Mathematics Studies.

The data in Tables 21-26 are based on a posttest sample of 252 classes (208 precalculus and 44 calculus). Of the precalculus classes, 17 did not have complete Second Study data.

In order to provide an indication of how much of the gain was provided by the calculus classes in the twelfth grade sample, Table 22 presents the Second Study posttest results separated for those in calculus classes and those in precalculus classes. The First Study made no such distinction so only one set of scores can be reported from it and these scores serve as the common basis for computing change scores for both types of SIMS classes.

Table 22
Twelfth Grade Posttest Achievement for Common Items by Class Type: U.S., 1964 and 1982

	Precalculus (Number of Classes, 208)			Calculus (Number of Classes, 44)	
Topics (Number of Items)	1964	1982	Change	1982	Change
Sets & Relations (1)	21	52	+31	70	+49
Number Systems (1)	45	56	+11	72	+27
Algebra (3)	44	43	- 1	59	+15
Geometry (4)	36	36	0	45	+ 9
Elementary Functions/ Calculus (10)	25	27	+ 2	50	+25
Probability and Statistics (1)	45	48	+ 3	56	+11
Overall (20)	32	35	+ 3	53	+21

These data suggest that much of the overall change in Table 21 can be attributed to the 44 calculus classes. However, even the precalculus students showed a slightly higher score. The gain of 2 percentage points in elementary functions and calculus may indicate that even our less prepared precalculus students have made some advances in that topic area. The lack of change of the precalculus student in algebra and geometry is cause for concern.

As in the eighth grade, it is of interest whether the changes took place primarily in less cognitively demanding tasks of computation or in relatively more cognitively demanding tasks of comprehension, analysis and application. Table 23 presents the data from the twenty anchor items clustered in the four cognitive levels described earlier.

Table 23
Twelfth Grade Postest Achievement by Cognitive Level on Common Items for 1964 and 1982: U.S.

Level (Number of Items)	1964	1982	Change
Computation (6)	45	50	+ 5
Comprehension (7)	26	38	+12
Application and Analysis (7)	26	27	+ 1
Overall (20)	32	38	+ 6

Figure 35 portrays the increase or decrease of each common item with the items arranged by cognitive level.

The change of 12 percentage points in comprehension appears to be noteworthy given that there were seven items in that cluster. However, six of the seven items are from the area of elementary functions and calculus which has already been seen to elicit a significantly higher level of performance by the calculus classes than the precalculus classes. In order to get some feel for how much of the increase is due to the calculus classes, Table 24 breaks the three cognitive-level cluster results down by class type.

Table 24
Twelfth Grade Achievement on Common Items by Cognitive Level and Class Type: U.S., 1964 and 1982

	Precalculus (Number of Classes, 208)			Calculus (Number of Classes, 44)	
Level (Number of Items)	1964	1982	Change	1982	Change
Computation (6)	45	48	+ 3	59	+14
Comprehension (7)	26	33	+ 7	61	+35
Application and Analysis (7)	26	25	- 1	39	+13
Overall (20)	32	35	+ 3	53	+21

The data in Table 24 show a rather stable performance for precalculus students in computation and application and a sizeable gain in comprehension. The pattern of change is more dramatic and consistent in all areas for the calculus students but it is somewhat misleading to compare 1982 calculus students with a general group of students enrolled in the fourth year of secondary school mathematics in 1964 which included some calculus classes but also many others.

In spite of all necessary qualifying remarks, the pattern that emerges from Tables 21 through 24 is one of stability and mild gains, especially in analysis and in comprehension. Our best have held their own or become somewhat better in the last twenty years.

Where Does the U.S. Stand Internationally? (Twelfth Grade)

Figures 36 to 41 show the rank ordering of the countries taking part in the Population B study for the topics of sets and relations, number systems, algebra, geometry, probability and statistics, and elementary functions and calculus. The following observations can be made about the "staircase" graphs:

1. In some topics the U.S. calculus classes are among the upper half of the countries and are always higher than the U.S. precalculus classes.

2. In most topics the U.S. precalculus classes are among the bottom four or five countries.

3. The overall results, combining the two class types, do not differ markedly from the results for precalculus classes.

Table 25 presents the mean posttest data for the U.S. overall, for U.S. precalculus and calculus classes separately and the means for the international posttest.

Table 25
Twelfth Grade Posttest Achievement Means: U.S. and International, 1981-82
(Mean Percent Correct)

Topic (N items)	United States Precalc.	United States Calculus	United States Total	Fifteen Country* Posttest Mean
Sets and Relations (7)	54	64	56	62
Number Systems (17)	38	48	40	50
Algebra (26)	40	57	43	57
Geometry (26)	30	38	31	42
Elementary Functions/ Calculus (46)	25	49	29	44
Probability and Statistics (7)	39	48	40	50
Finite Mathematics (4)	29	38	31	44**

* The countries included, in addition to the U.S., are: Belgium (Flemish); Belgium (French); Canada (British Columbia); Canada (Ontario); England and Wales; Finland; Hong Kong; Hungary; Israel; Japan; New Zealand; Scotland; Sweden; Thailand.

** Estimated median score

Table 25 indicates that the total U.S. sample performs at a level markedly lower than the mean level of mathematics in countries participating in SIMS. The calculus classes in the U.S. sample performed at or near the mean level of the same comparison group internationally.

Table 26 presents a comparison of the U.S. posttest mean with international quartile and median data for all subtests except finite mathematics (for which the data were not available).

Table 26
Mean Achievement for U.S. Classes, Medians and Quartiles for 15 Countries: Twelfth Grade, 1981-82
(Percent Correct)

Topic (N items)	U.S. Means Precalc.	U.S. Means Calculus	U.S. Means Total	International 25th Percentile	International Median	International 75th Percentile
Sets & Relations (7)	54	64	56	51	61	72
Number Systems (17)	38	48	40	40	47	59
Algebra (26)	40	57	43	47	57	66
Geometry (26)	30	38	31	33	42	49
Elementary Functions/Calculus (46)	25	49	29	28	46	55
Probability/ Statistics (7)	39	48	40	38	46	64

The U.S. precalculus classes in most cases were close to the 25th percentile. The U.S. calculus classes were in most areas above the median but in no topic were they at the 75th percentile.

Part IV. STUDENT ATTITUDES TOWARDS MATHEMATICS

Attitudes towards mathematics were assessed in five areas. The same instruments were used for the eighth and twelfth grades, except for minor changes in the mathematical content of certain items to reflect the curriculum at each grade level. However, it must be kept in mind here, as well as in other parts of this report, that the two populations tested were quite different in their composition. At the eighth grade, virtually all young people of that age were in school and were taking mathematics (as was the case in the vast majority of countries taking part in the international study). At the twelfth grade, however, only a relatively small proportion of the students (about 12-15 percent) were taking college preparatory mathematics for the fourth year. (Indeed, some 20-25 percent of young people in the U.S. were no longer in school at twelfth grade in 1981-82.) Hence, the twelfth grade students in this study were a special group in that they had remained in school, maintained sufficient scholarship to be in a college preparatory program of studies, and had successfully completed <u>at least</u> three years of college preparatory high school mathematics.

Five scales were used:

1. Mathematics as a Process
   How mathematics as a field of study is viewed

2. Mathematics and Myself
   Students' view of themselves as learners of mathematics

3. Mathematics and Society
   Students' view of the usefulness and importance of mathematics to society

4. Mathematics and Gender
   The extent to which mathematics is viewed as a male domain.

5. Calculators and Computers
   The extent to which calculators are viewed positively in the context of learning mathematics and the extent to which computers are positively viewed in a broader societal context.

The results for scales 1 through 4 are given in Table 27. Items were rated on a Likert scale from a low of 1 to a high of 5 with 3 as undecided. These data are shown in Figure 20 for the eighth grade and Figure 33 for the twelfth grade. Corresponding data for the calculators and computer scale were not available. However, the mean for this scale was 3.3 for the eighth grade and 3.7 for the twelfth grade.

Table 27
Summary of Attitude Scale Results by Grade: U.S., 1981-82

Scale	Eighth Grade		Twelfth Grade	
	Percent High Ratings	Percent Low Ratings	Percent High Ratings	Percent Low Ratings
Mathematics as a Process	35	19	48	16
Mathematics and Myself	58	15	72	12
Mathematics and Society	62	13	67	9
Mathematics and Gender	64	11	67	13

Mathcmatics as a Process

Students indicated that mathematics helped them think logically, was a good subject for creative people, and was a discipline where new discoveries are being made and where problems can be solved by different ways and by trial and error. In nearly every case, the twelfth grade students were slightly more process oriented than the eighth grade students (and the teachers more so than the students).

Although students at both levels tended to view mathematics as a set of rules to follow and memorize, the twelfth grade students were less so inclined. This may be a statement about the effectiveness of four years of study of high school mathematics in giving the twelfth grade students a better view of mathematics as a process. This contrasts with the limiting effect that years of arithmetic drill may have on eighth grade students' views of mathematics.

Mathematics and Myself

The twelfth grade students had a much more positive view and showed a more favorable response on all but one of the 19 items. Differences were greatest on an item, "I am not so good at mathematics," and another, "Mathematics is harder for me than for most people." On the high end of the scale there was a strong indication of satisfaction with success in problem solving and, for the twelfth grade group, an absence of fear of taking mathematics.

Home Support for Mathematics

The vast majority of the students claimed that they wanted to do well in mathematics and that their parents also wanted them to do well. At the eighth grade, 89 percent of the students agreed or strongly agreed with the item, "My parents want me to do very well in mathematics", while at the twelfth grade, 95 percent agreement was found.

Mathematics and Society

Both eighth and twelfth grade students perceived mathematics as useful and important. The twelfth grade overall had a slightly more positive view with 67 percent giving high ratings compared to 62 percent for the eighth grade. On most items the two groups tended to hold similar views. An exception was in relation to work. Most of the eighth grade (78 percent) agreed that it is important to know mathematics in order to get a job but were rather undecided about whether they would like to work at a job that let them use mathematics. In contrast, two-thirds of the twelfth grade said they would like such a job. Overall the two groups agreed on the extent to which mathematics is important for jobs, but the twelfth grade believed more strongly that mathematics is useful in everyday life.

Mathematics and Gender

The items in this scale were included to determine the extent to which mathematics was viewed by students as a male domain. A positive attitude was one that did not sex-role stereotype mathematics as a domain more appropriate for males than for females. Both groups held a strongly positive view of women and mathematics. Of all the scales, the most positive attitude was obtained on the Mathematics and Gender scale.

Similarities ceased when the responses were separated within each group by sex. The females believed much more strongly than the men that mathematics was as much for them as for their male peers. Not surprisingly, this belief was strongest for the twelfth grade women. The results found here reflect the findings of other research in the United States on sex-related differences in mathematics.

Calculators and Computers

This scale assessed students' attitudes toward calculators and computers. Both grades showed a positive attitude about the use of calculators in learning mathematics and about the role of computers in society. The twelfth grade students were somewhat more favorably disposed to the new technology than the eighth grade students. Both groups also showed a higher mean for the four computer items than for the four calculator items, with the twelfth grade students having the most favorable attitude toward each. However, it should be noted that all four calculator items dealt with the use of calculators in the context of learning mathematics while most of the computer items addressed the general use of computers in society. This contrasting focus probably accounts for the differences in results for the two sets of items.

Part V. CONCLUDING REMARKS

An international study of education achievement provides the unique opportunity to "step outside" and get a broader picture of teaching and learning in one's own country. The Second International Mathematics Study provides such a perspective.

It has been pointed out, for example, that our "typical" eighth grade mathematics program is dominated by arithmetic. But in spite of the emphasis placed on this subject matter, achievement was only at the international average for the twenty countries participating in Population A testing. Within the eighth grade there was dramatic differentiation of mathematics courses offered, extending from algebra and enriched mathematics for the more able students to remedial arithmetic for the less able. And while the algebra classes covered much of the content of first year high school algebra, they omitted other topics, such as geometry, measurement and probability.

Judging from data from various parts of the Second Study (not all presented here) it can be stated the eighth grade curriculum was typically a "low intensity" presentation. That is to say, many topics were dealt with only briefly--for perhaps a period or two. As a result, insufficient provision may have been made for developing a solid conceptual base upon which subsequent mathematics is to be learned. This impression of the curriculum was in marked contrast to the more "intense" approach to the study of mathematics found in some other countries, most notably in Japan.

For twelfth grade college preparatory mathematics, two main programs were found. Precalculus classes studied typical senior high school topics such as trigonometry and analytic geometry. Calculus classes followed the Advanced Placement Calculus syllabus. The calculus classes, representing our highest achieving mathematics students, performed at a level (especially in analysis) which compares favorably with international averages. However, the students in the standard twelfth grade college preparatory classes (called "precalculus" here) achieved at levels which were exceeded, on average, by about 75 percent of the countries in the Second Study who tested a Population B sample.

The twelfth grade program was built upon a foundation that is by most international standards highly compartmentalized. In the U.S., high school mathematics typically consists of one year of algebra, one year of geometry, another year of algebra, and then more advanced topics in the fourth year, such as analytic geometry, trigonometry, or calculus. In most countries of the world, a more integrated approach to mathematics is taken, in which the subject is presented in a more cohesive and unified fashion. It is plausible that the "fragmentation" and "low intensity" found in many of our mathematics programs could be allayed by a more integrated approach to the high school mathematics curriculum.

The Study has provided abundant data on characteristics of teachers (typically experienced and well qualified), use of class time (mostly spent in large group instruction) and instructional resources (the textbook typically defines the boundaries for the content which is taught).

There is a part of the Study yet to be reported upon in detail--how mathematics is taught in school. It is expected that as analysis of the data referred to in the present report continues, and as this additional information on classroom processes is considered, further important characterizations of the stat of mathematics education will be made possible.

Appendix A

LIST OF PARTICIPATING COUNTRIES

Australia*	Israel*
Belgium (Flemish)	Ivory Coast
Belgium (French)*	Japan*
Canada (British Columbia)	Luxembourg
Canada (Ontario)	Netherlands*
Chile	New Zealand
England* and Wales	Nigeria
Finland*	Scotland*
France*	Swaziland
Hong Kong	Sweden*
Hungary	Thailand
Ireland	United States*

*These countries also participated in the First International Mathematics Study in 1964. The Federal Republic of Germany took part in the First Study only.

Appendix B

Background on the International Association for the Evaluation of Educational Achievement

The International Association for the Evaluation of Educational Achievement (IEA), is an international, non-profit-making scientific association incorporated in Belgium for the principal purposes of: (a) undertaking educational research on an international scale; (b) promoting research aimed at examining educational problems in order to provide facts which can help in the ultimate improvement of educational systems; and (c) providing the means whereby research centers in the various member countries of IEA can undertake co-operative projects. The current chairman of the IEA Council is T. Neville Postlethwaite of the University of Hamburg, Federal Republic of Germany. Richard Wolf, Teachers College, Columbia University, is IEA Assembly Member for the United States.

The Mathematics Project Council, responsible for the Second Mathematics Study, is chaired by Roy W. Phillipps of the New Zealand Department of Education. Robert Garden, also of the New Zealand Department of Education, is International Project Co-ordinator for the Study. Kenneth J. Travers is Chairman of the International Mathematics Committee (IMC) which designed the Second International Mathematics Study and developed the international instruments. Other members of the IMC are: Sven Hilding, Sweden; Edward Kifer, United States; Gerard Pollock, Scotland; Tamas Varga, Hungary; and James Wilson, United States. A. I. Weinzweig, United States, is consulting mathematician and Richard Wolfe, Canada, is consulting psychometrician to the IMC.

Appendix C

National Committees

National Mathematics Committee

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Floyd Downs, Hillsdale High School, San Mateo, California
Edward Kifer, University of Kentucky
Curtis C. McKnight, University of Oklahoma (National Research Coordinator)
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A. I. Weinzweig, University of Illinois at Chicago
James Wilson, University of Georgia
Richard Wolf, Teachers College, Columbia University

National Technical Advisory Panel

Edward Kifer, University of Kentucky (Chairman)
Leigh Burstein, University of California-Los Angeles
Robert Linn, University of Illinois at Urbana-Champaign
William Schmidt, Michigan State University
Jack Schwille, Michigan State University
Richard Wolf, Teachers College, Columbia University
Richard Wolfe, Ontario Institute for Studies in Education

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David Robitaille, University of British Columbia
Leslie Steffe, University of Georgia
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Appendix D

Sampling Information

The U.S. was one of few countries conducting a full longitudinal study at each population level. The objective of the sample design was to produce a self-weighting sample for each population. A national sample of school districts was selected with probability proportional to size (PPS) and invited to participate in the study. Cooperating districts provided lists of schools and a sample of schools containing the appropriate grade levels was again selected with probability proportional to size. From class lists provided by the cooperating schools, individual target classes were randomly selected within each grade-level stratum.

I. Populations for the Study - 1981-1982 School Year

1. Population A: All students enrolled in normal mathematics classes in the U.S. at Year 8.

2. Population B: All students enrolled in mathematics classes at Year 12 (only classes requiring as prerequisites two years of algebra and one year of geometry).

II. Public School Sample

1. Stages and Methods of Selection:

Stage	Unit	Method*	Sampling Frame
1	Districts	PPS	NCES Public School Districts 1979-80 (tape)
2	Schools	PPS	School lists provided by districts
3	Classes	SRS	Class lists provided by schools
4	Students	All	---

*PPS means probability of selection proportional to size of district or school. SRS means systematic random sample.

2. Stratification of Public School Districts

a. Before sample selection in Stage 1, districts were first stratified according to type into three strata:

Stratum	District type	Number of School Districts	Estimated Eligible Student Population
1	8th grade only	3,528	227,089
2	12th grade only	692	254,219
3	8th and 12th grades	11,062	6,092,456

b. 1,451 public school districts with neither 8th or 12th grades were deleted from the sampling frame.

c. Within each of the three strata, districts were ordered by:

i. SMSA code
ii. State (alphabetically)
iii. Within state by county (alphabetically)

3. Selection of Public School Districts

a. Each stratum's sample allocation was proportional to its percentage of the total eligible student population.

b. Districts were selected PPS

c. Measures of size (M.O.S.) for PPS sampling were computed as follows:

Strata

1 and 2

$$\text{M.O.S.} = \frac{\text{Total district enrollment}}{\text{Number grades in district}}$$

Stratum 3

$$\text{M.O.S.} = \frac{\text{Total district enrollment}}{\text{Number grades in district} \times .5}$$

d. Probability of selecting a district:

$$P_{dist.} = \frac{\text{District M.O.S.}}{\text{Cumulated M.O.S.'s for the stratum}}$$

e.

Pop	Total Districts Selected	Not Approp	Net Sample	Cooperated	Coop Rate
A	194	9	185	93	50.3%
B	199	5	194	93	47.9%

4. Selection of schools

a. Frame: list supplied by school district

b. Objective was to select 2 schools per district in strata 1 and 2; and 4 schools per district (two 8th and two 12th) in stratum 3.

c. In districts where there was not the objective number of schools, all available schools were taken and quasi districts were established to meet the requirement.

d. M.O.S. for PPS selection of schools was total school enrollment.

e. When enrollment was not supplied, schools were chosen with equal probability.

f. Probability of selecting a school:

$$P_{school} = \frac{\text{School enrollment}}{\text{Total district enrollment}}$$

g.

Pop	Total Schools Selected	Not Approp	Net Sample	Cooperated	Coop Rate
A	185	1	184	126	68.5%
B	185	4	181	135	74.6%

5. Selection of Classes

   a. Frame: list supplied by school

   b. Ineligible classes were eliminated and two classes per school were randomly selected

   c.

Pop	Total Classes Selected	Not Approp	Net Sample	Cooperated	Coop Rate
A	287	-	287	234	81.5%
B	270	-	270	227	84.1%

III. Private Sample

1. Stages and Methods of Selection:

Stage	Unit	Method	Sampling Frame
1	Schools	PPS	NCES Non-public Elementary/Seconardy survey for 1976-77, 1977-78, and 1978-79 tape
2	Classes	SRS	Class lists provided by schools
3	Students	All	---

2. Stratification of Private Schools

Stratum	School Type	Eligible Students
4	8th grade only	319,790
5	8th and 12th grades	181,198
6	12th grade only	214,170

Sample allocation to a stratum was proportional to the stratum's percentage of the total eligible student population.

3. Selection of Schools

   a. School selected PPS

   b. M.O.S. = number of 8th or 12 grade students

   c. Probability of selecting a school in a stratum:

$$P_{school} = \frac{\text{8th or 12th grade enrollment}}{\text{eligible students in stratum}}$$

$$P_{school\ Str\ 5} = \frac{\text{8th and 12th grade enrollment}}{\text{eligible students in stratum}}$$

   d.

Pop	Total Schools Selected	Not Approp	Net Sample	Cooperated	Coop Rate
A	84	2	82	31	37.8%
B	38	3	35	15	42.9%

4. Selection of classes

   a. Classes were selected as in the public schools (see Section II, part 5).

   b.

Pop	Total Classes Selected	Not Approp	Net Sample	Cooperated	Coop Rate
A	51	-	51	46	90.2%
B	33	-	33	25	75.8%

Note on Cooperation Rate: As noted in the report, extensive data were gathered from schools during the Study year. The entire set of cognitive items was given to the students at the beginning and again at the end of the year. During the year, teachers responded to detailed questionnaires on how the subject matter was dealt with (six questionnaires for eighth grade and five questionnaires for twelfth grade). At the end of the year, teachers completed a background questionnaire as well as the opportunity-to-learn (OTL) instrument. The OTL questionnaire required the teacher to indicate, for each of the items in the pool (180 for eighth grade and 136 for twelfth grade) whether or not the mathematics on which the item was based had been taught to the target class. The SIMS instrumentation was very demanding of time and effort of those participating. This factor undoubtedly contributed to the relatively low participation on rate.

Numbers of Classes: Certain variations in numbers of classes are to be found in this report. These deviations are due to the following factors: (i) For analyses dealing with mathematics taught and learned during the school year, all classes with complete data, including classroom process data, were included. (ii) For international (end of year) comparisons, classrooms participating in the posttest (but with incomplete classroom process data) were included.

It should also be noted that in SIMS classrooms were randomly sampled, not teachers. Thus, while it would not be quite accurate to say that a representative sample of teacher has been drawn, it is more correct to say that we have portrayed the teaching received by a representative sample of students.

Appendix E

FIGURES

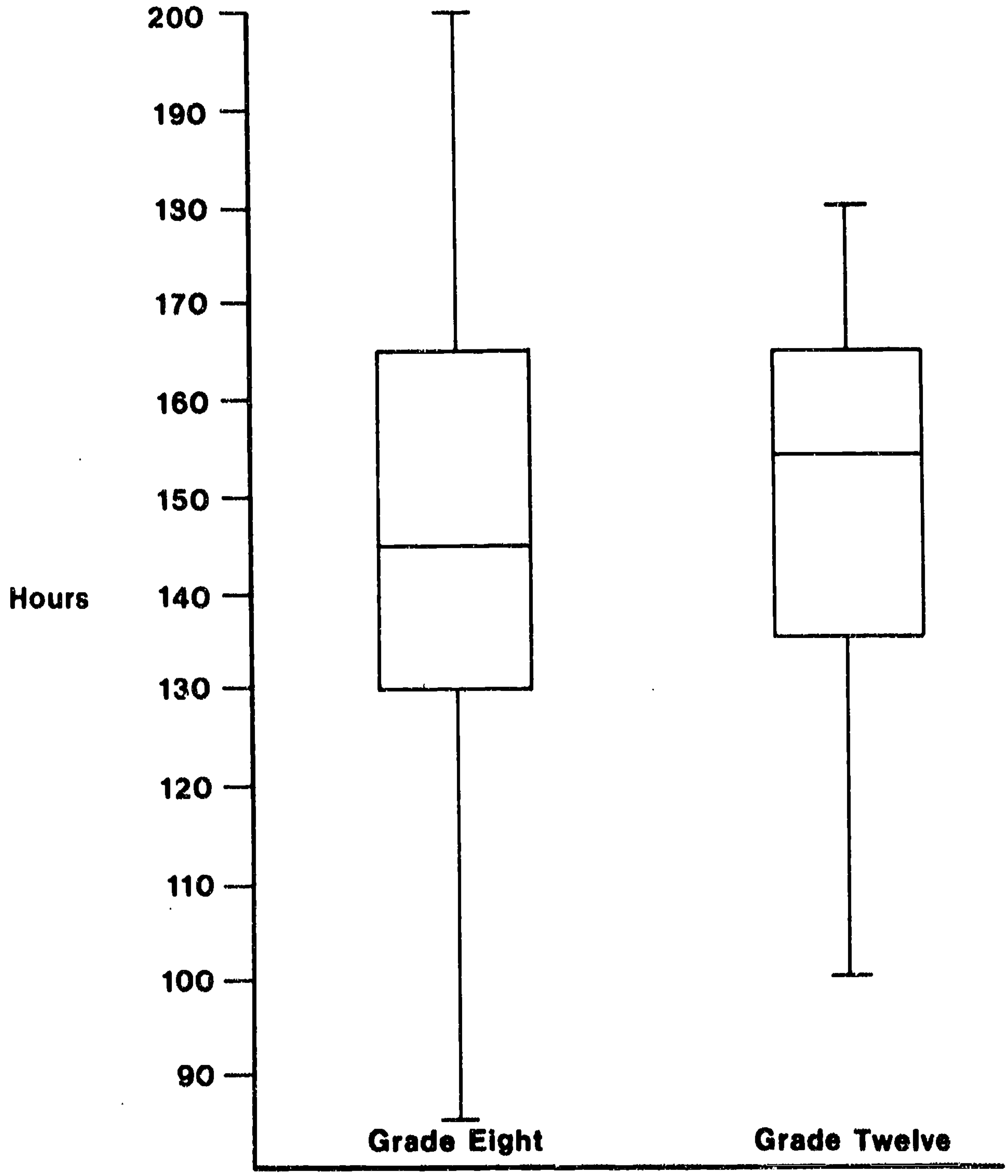


Figure 1. Hours of Mathematics Instruction Per Year Received by Mathematics Classes in U.S. Schools

Note: Box includes middle 50 percent of classes. Middle line across box is median: 145 hours for grade eight and 153 hours for grade twelve. Lines protruding from end of box ("whiskers") show range of middle 95% of classes.

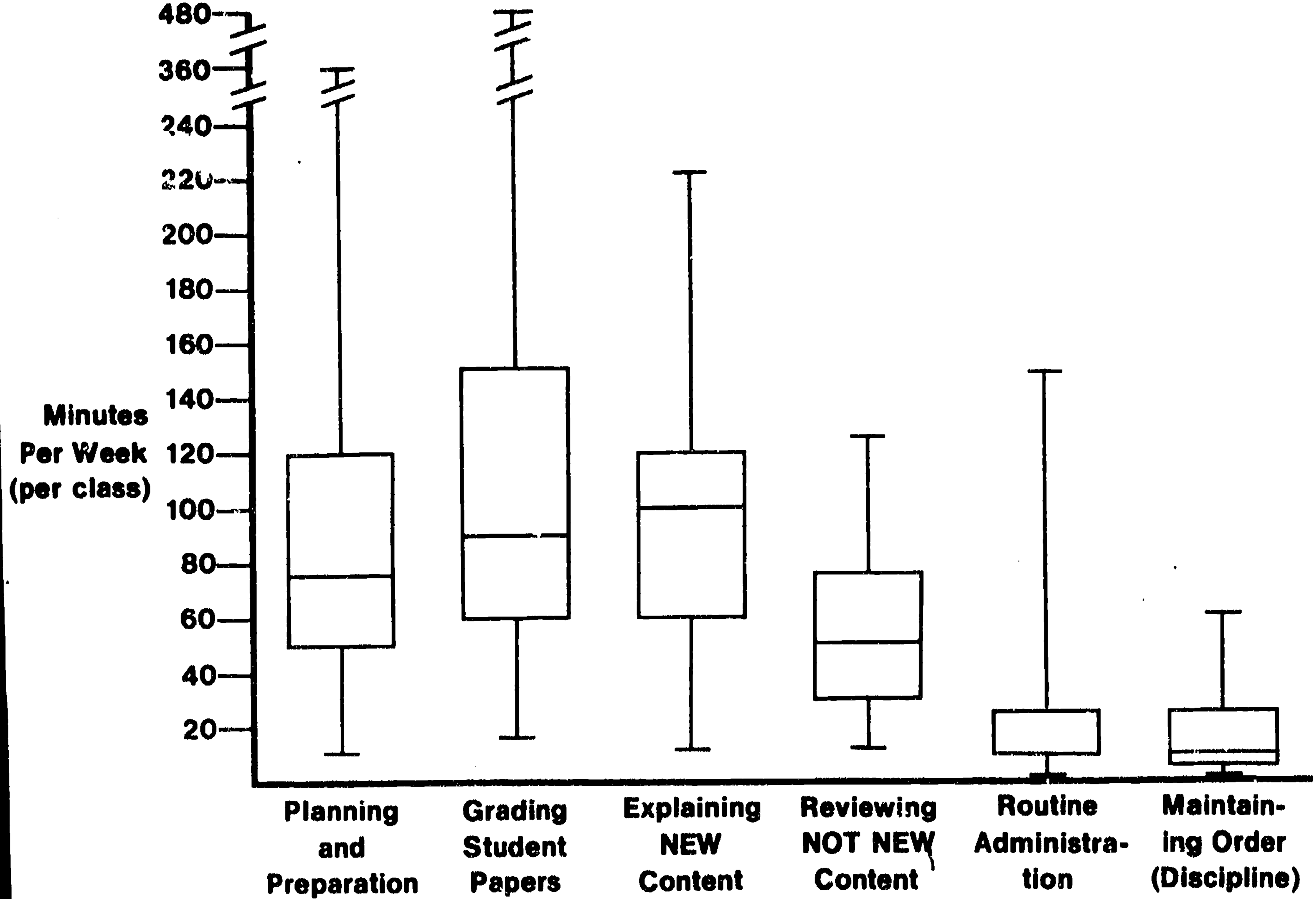


Figure 2. How Teachers' Time was Spent on Selected Teaching Activities for Eighth Grade Mathematics Classes

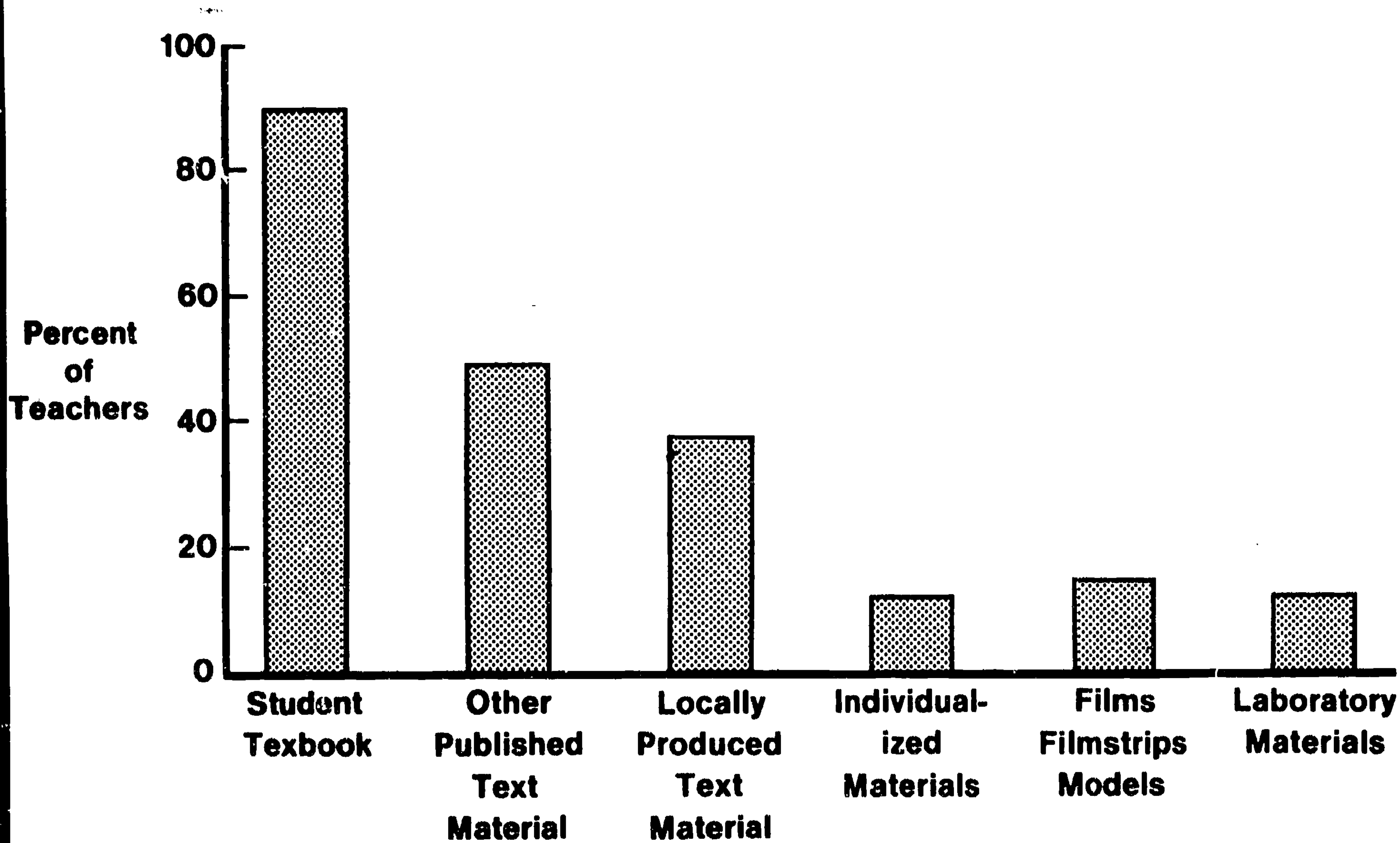


Figure 3 Percent of Teachers Using Various Instructional Resources in Eighth Grade Mathematics Classes

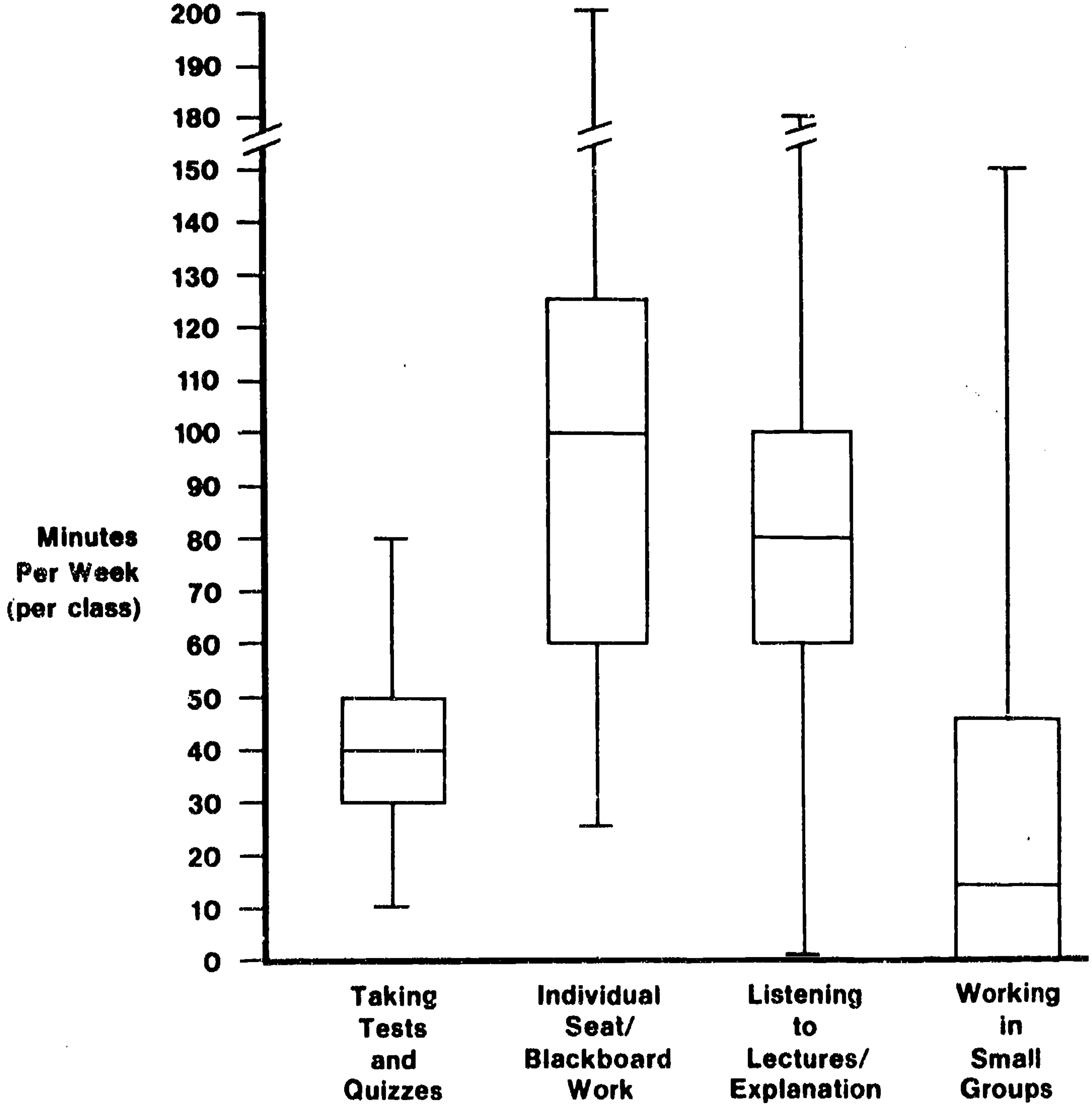


Figure 4 Amount of Student Time Spent on Selected Activities in Eighth Grade Mathematics Classes

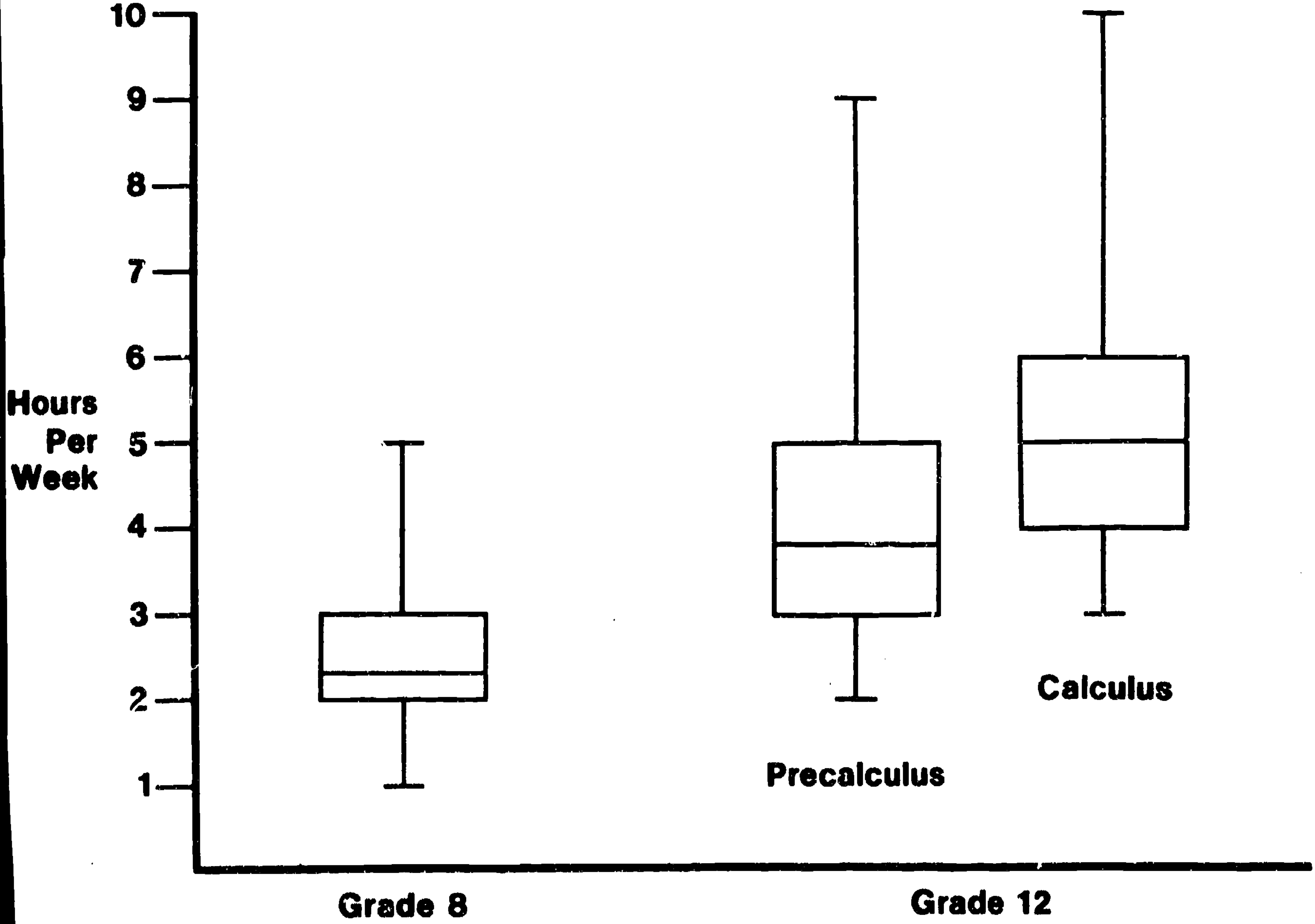


Figure 5 Hours of Homework Per Week Assigned in Mathematics Class

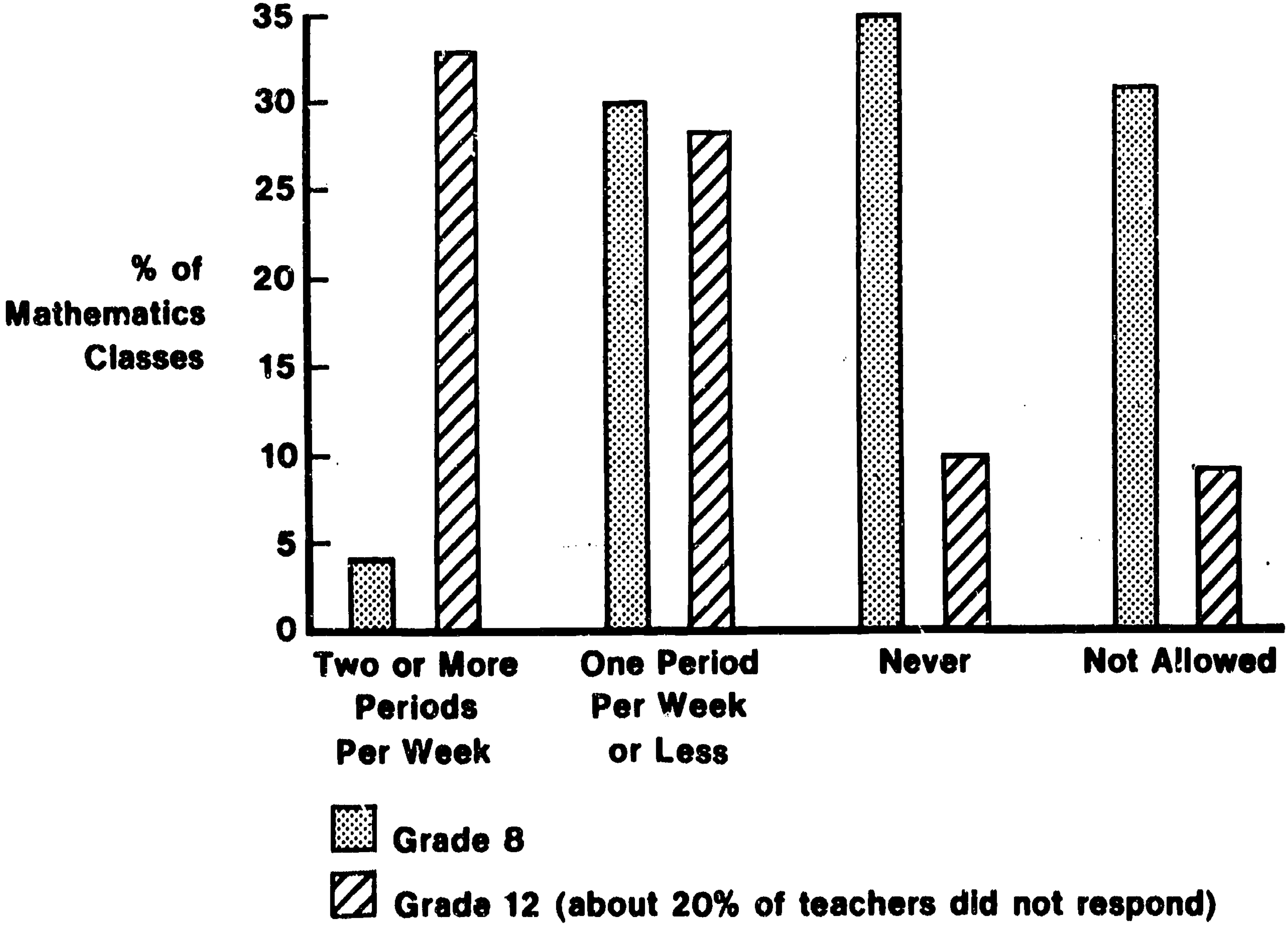


Figure 6 Frequency of Calculator Use in Eighth and Twelfth Grade Mathematics Classes

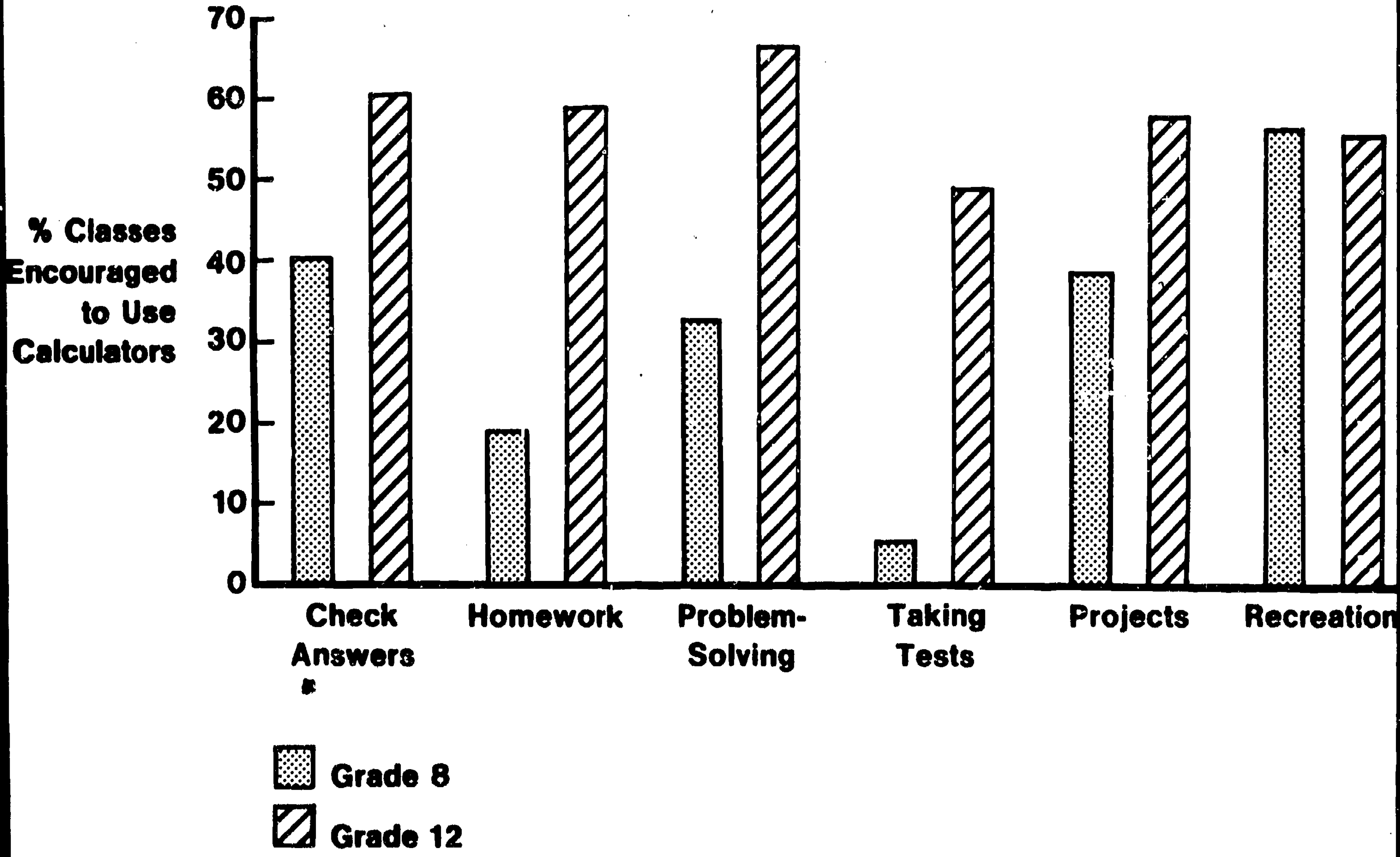


Figure 7 Kinds of Calculator Use in Eighth and Twelfth Grade Mathematics Classes

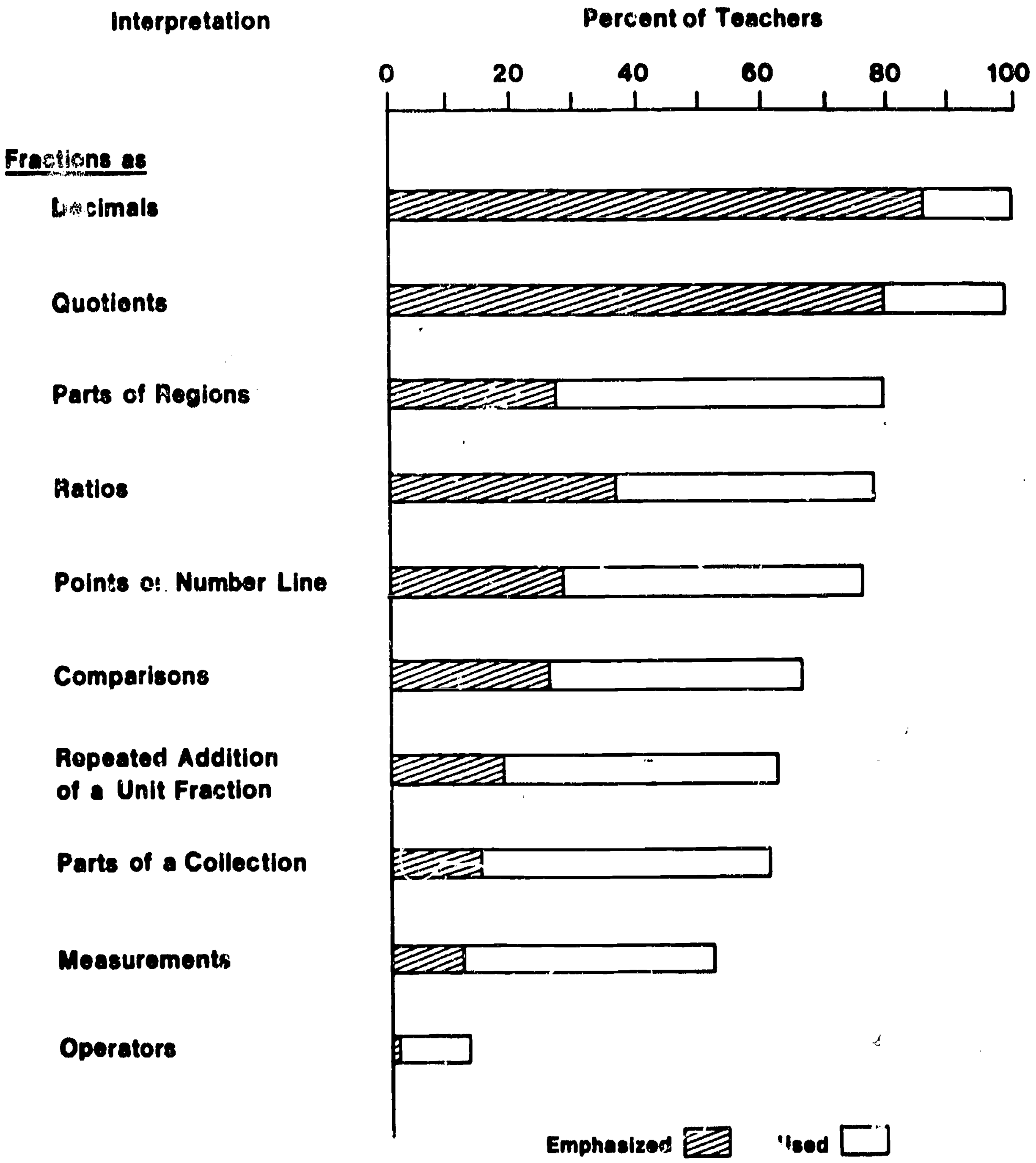


Figure 8 Interpretations of FRACTIONS from the Classroom Process Questionaires Used by Eighth Grade Teachers in Arithmetic Instruction

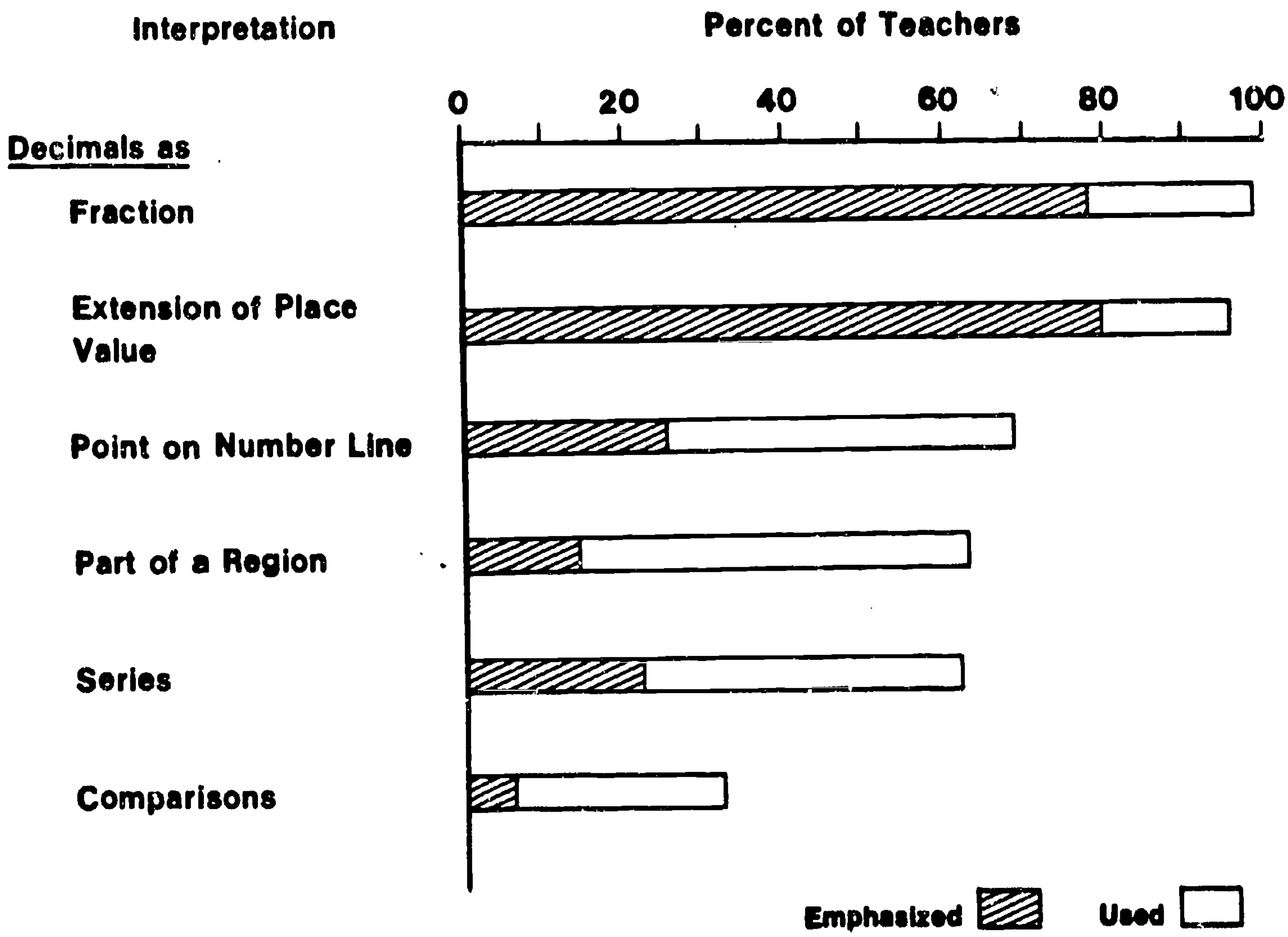


Figure 9 Interpretations of DECIMALS from the Classroom Process Questionnaires Emphasized or Used by Eighth Grade Teachers in Arithmetic Instruction

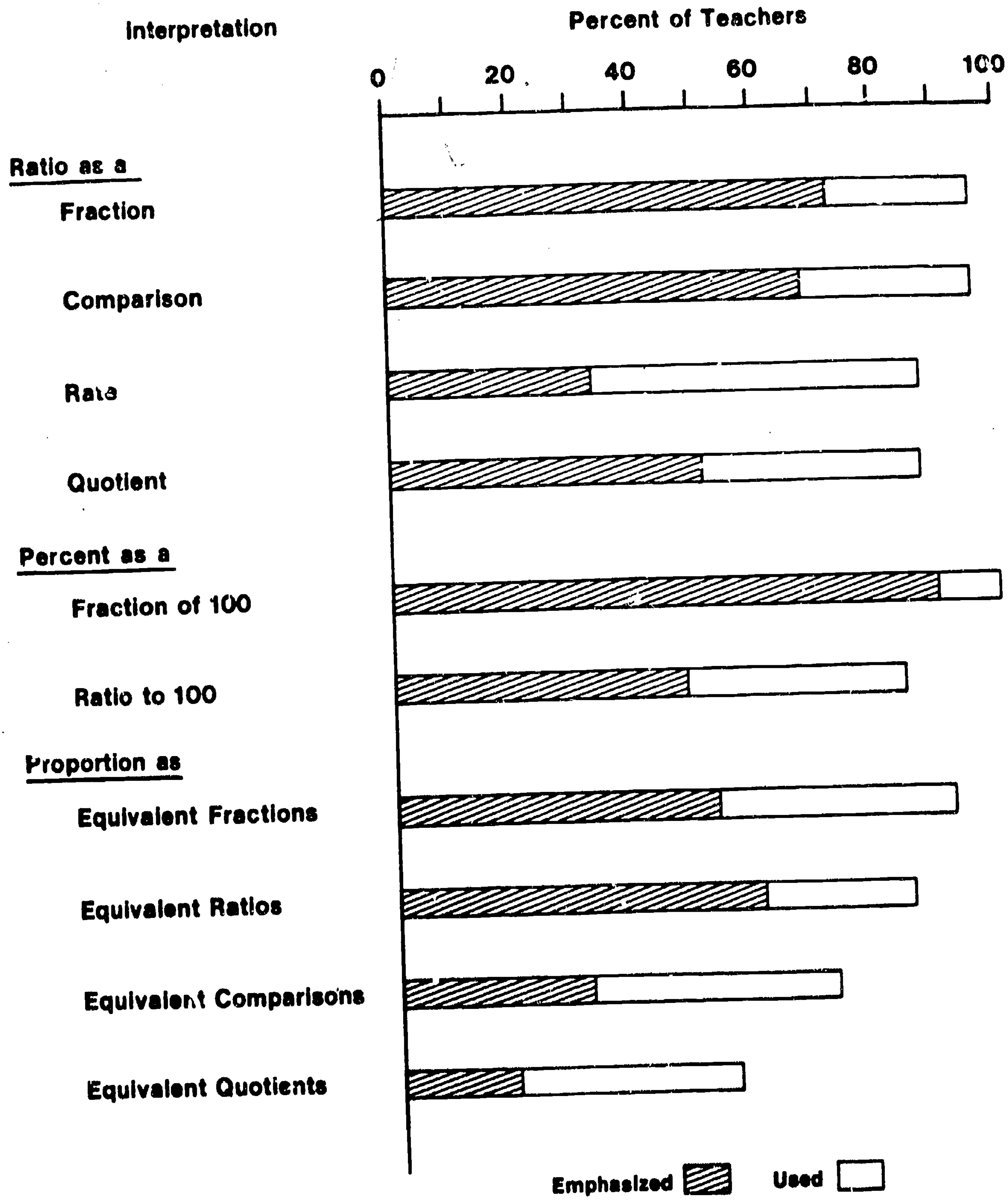


Figure 10 Interpretations of RATIO, PERCENT, AND PROPORTION from the Classroom Process Questionaires Emphasized or Used by Eighth Grade Teachers in Arithmetic Instruction

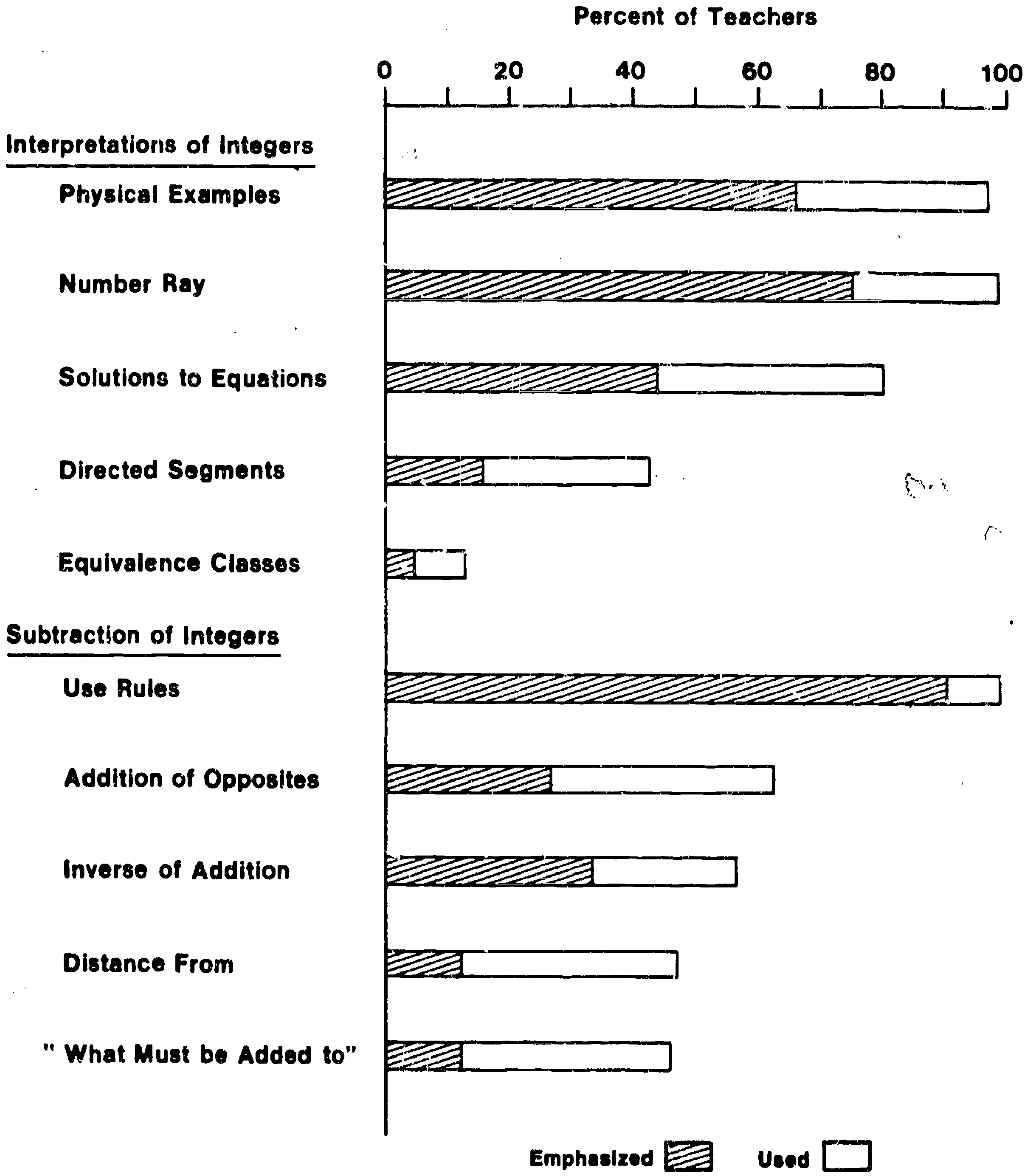


Figure 11 Methods of Teaching INTEGERS from the Classroom Process Questionaires Emphasized or Used by Eighth Grade Teachers in Algebra Instruction

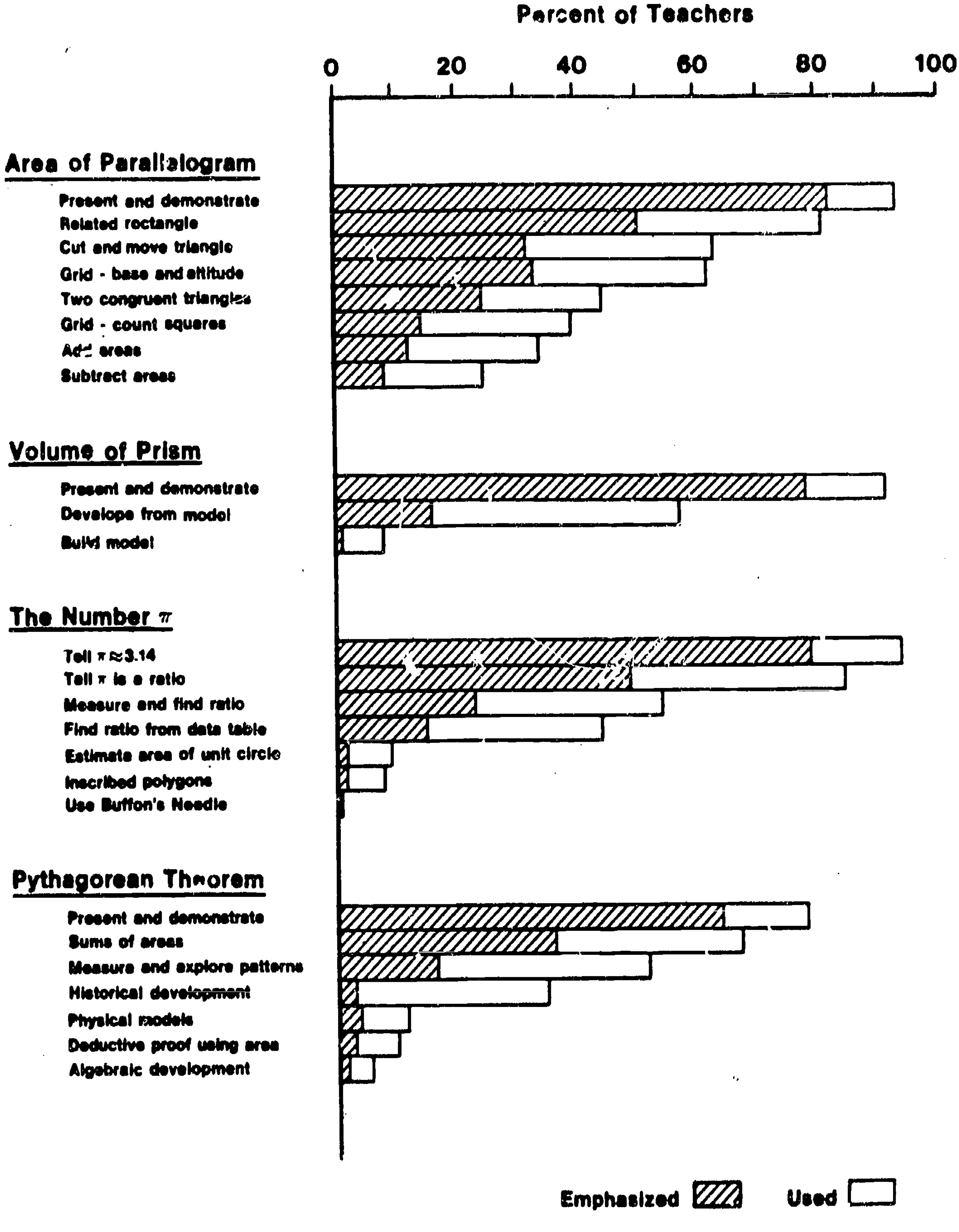


Figure 12 Methods from the Classroom Process Questionaires Emphasized or Used by Eighth Grade Teachers to Teach SELECTED FORMULAS

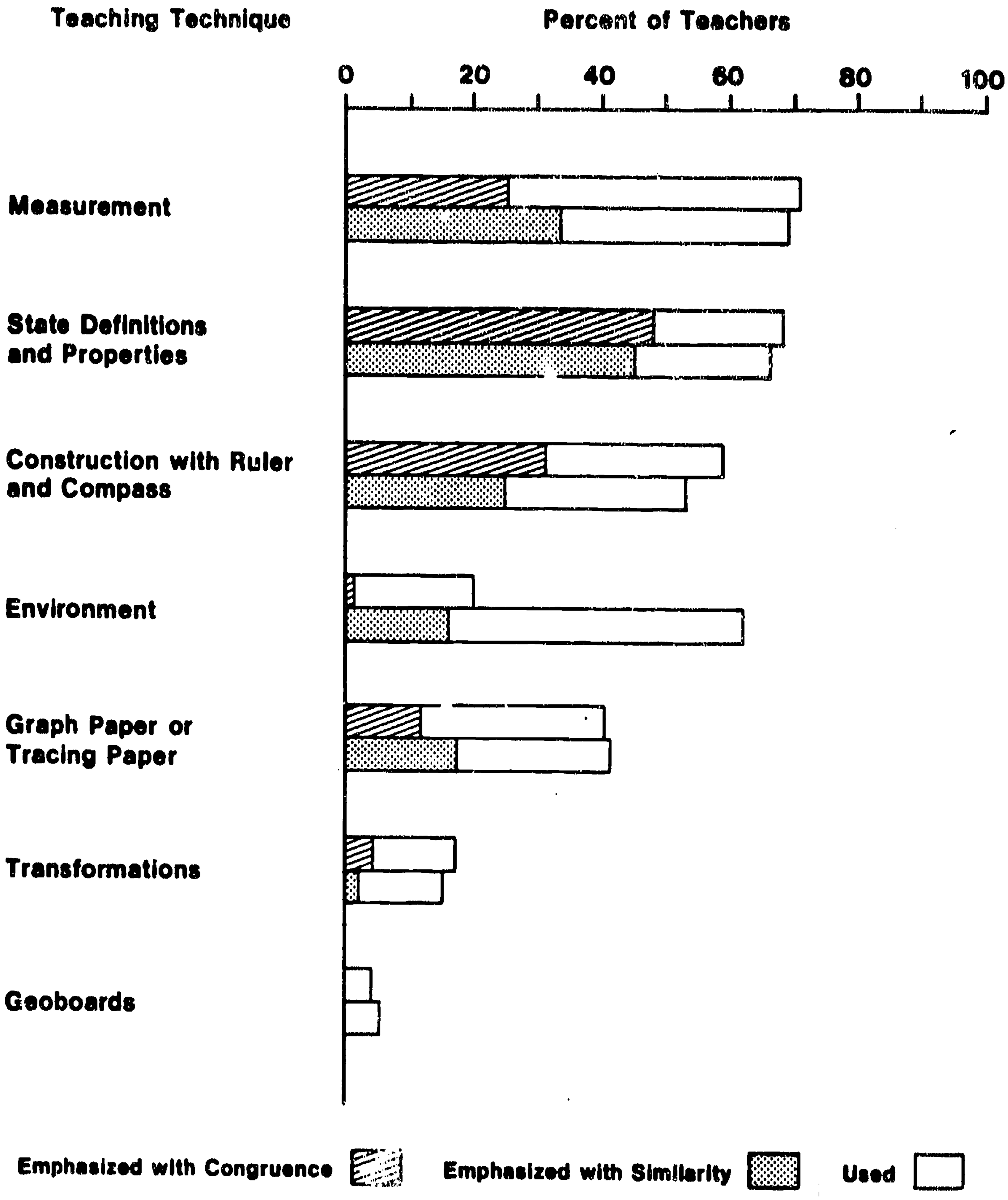


Figure 13 Techniques from the Classroom Process Questionaires for Teaching CONGRUENT AND SIMILAR TRIANGLES Emphasized or Used by Eighth Grade Teachers

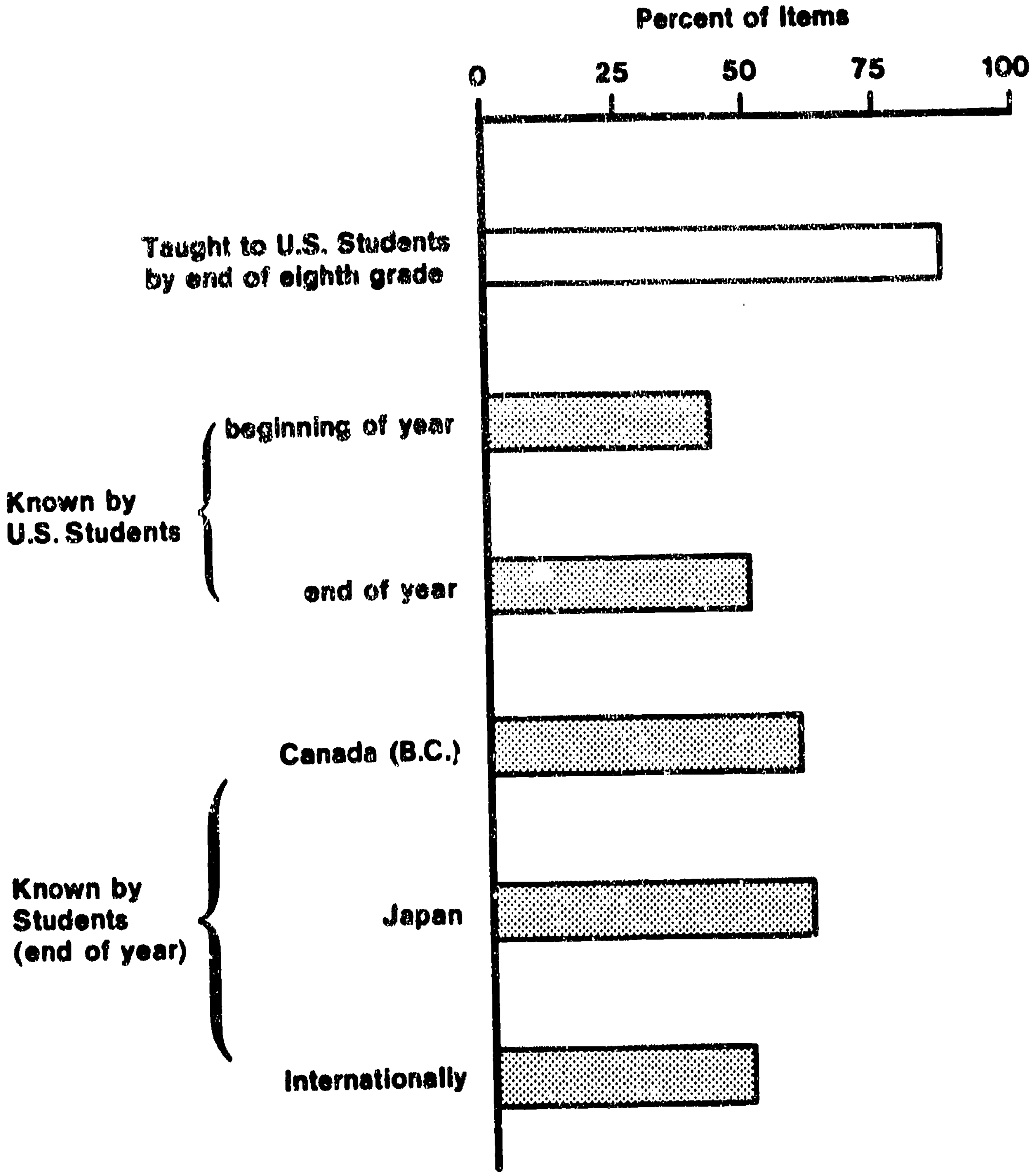


Figure 14 Average Percent of ARITHMETIC Items on International Test Taught and Learned (Eighth Grade)

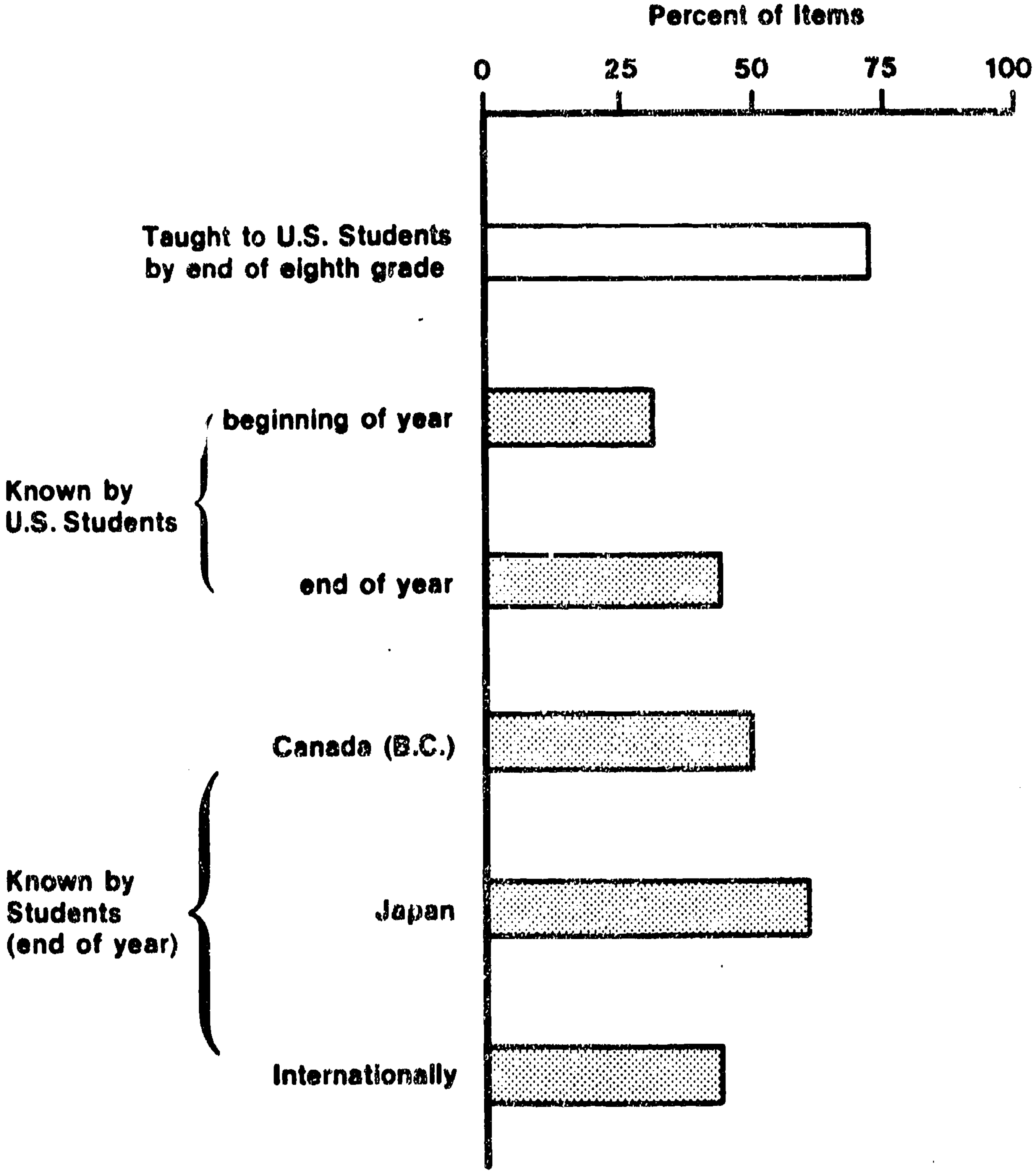


Figure 15 Average Percent of ALGEBRA Items on International Test Taught and Learned (Eighth Grade)

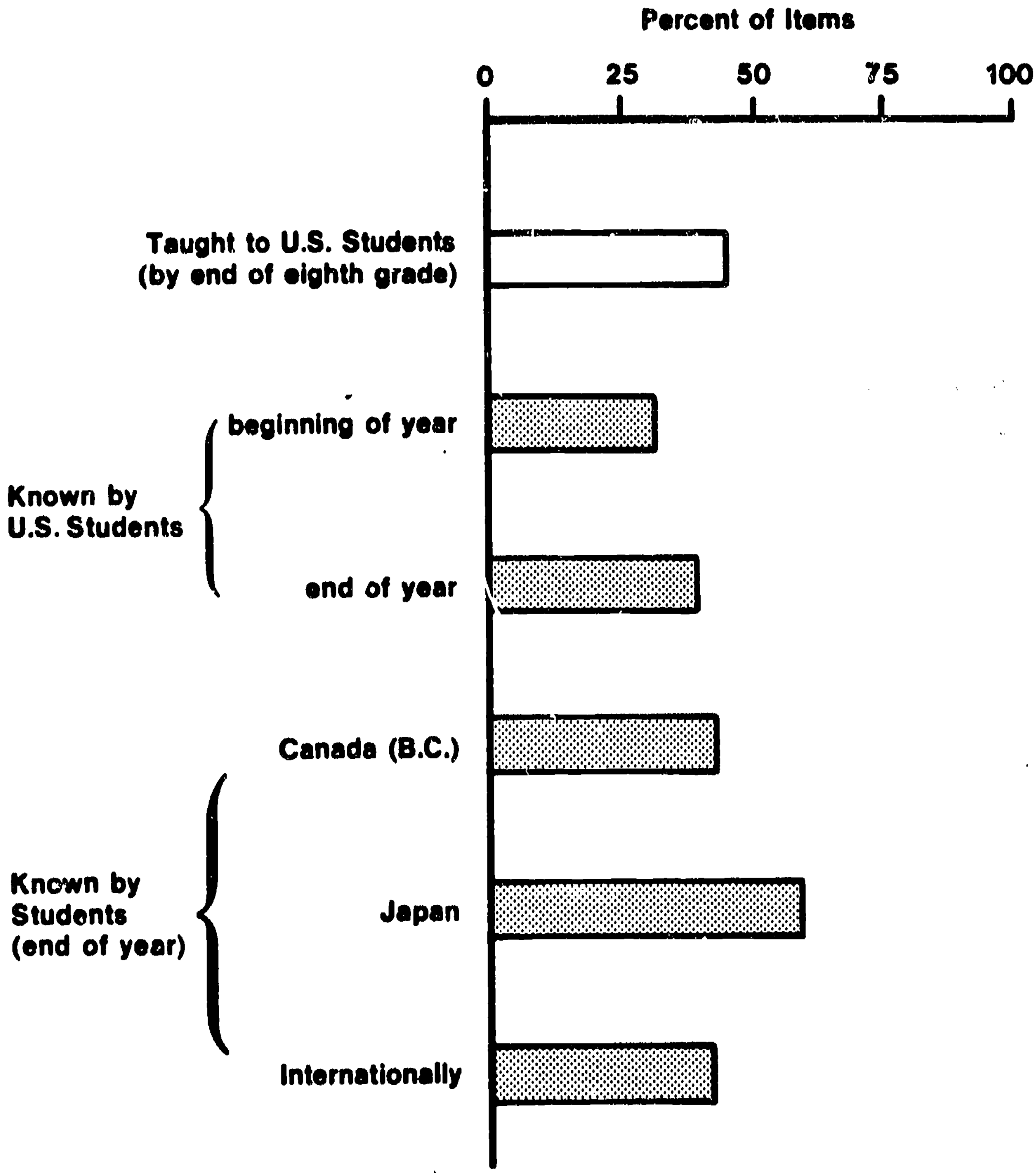


Figure 16 Average Percent of GEOMETRY Items on International Test Taught and Learned (Eighth Grade)

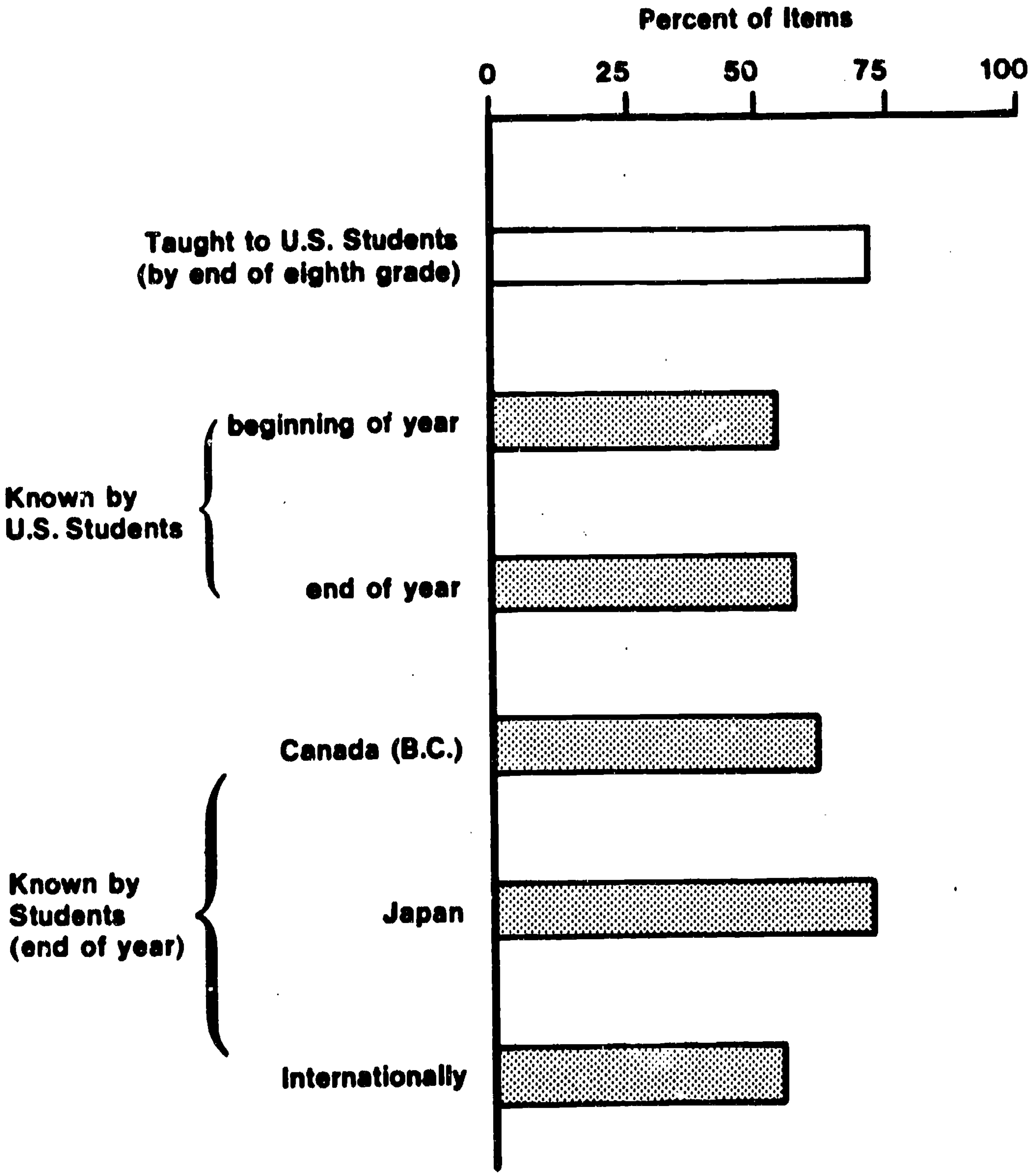


Figure 17 Average Percent of STATISTICS Items on International Test Taught and Learned (Eighth Grade)

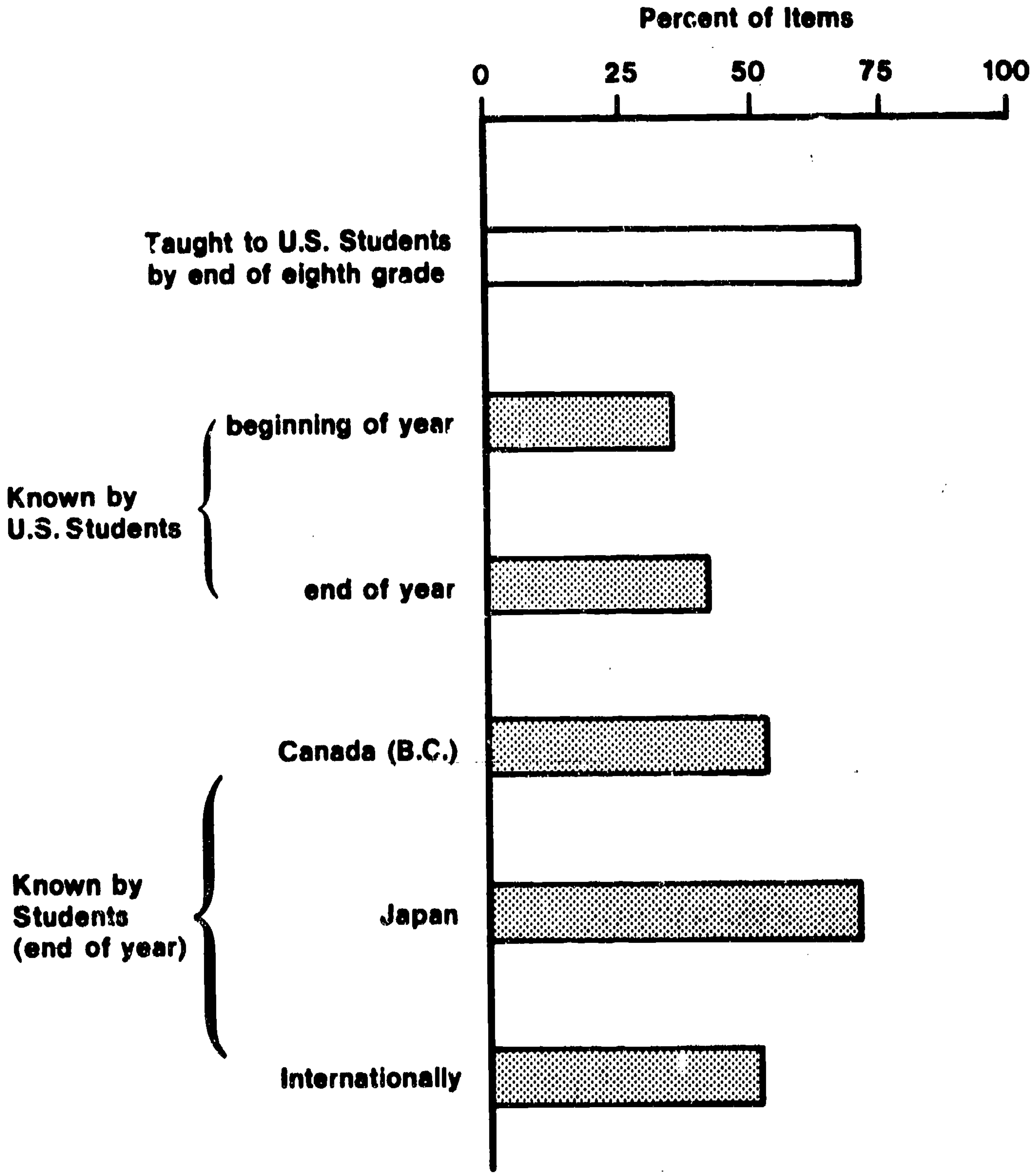


Figure 18 Average Percent of MEASUREMENT Items on International Test Taught and Learned (Eighth Grade)

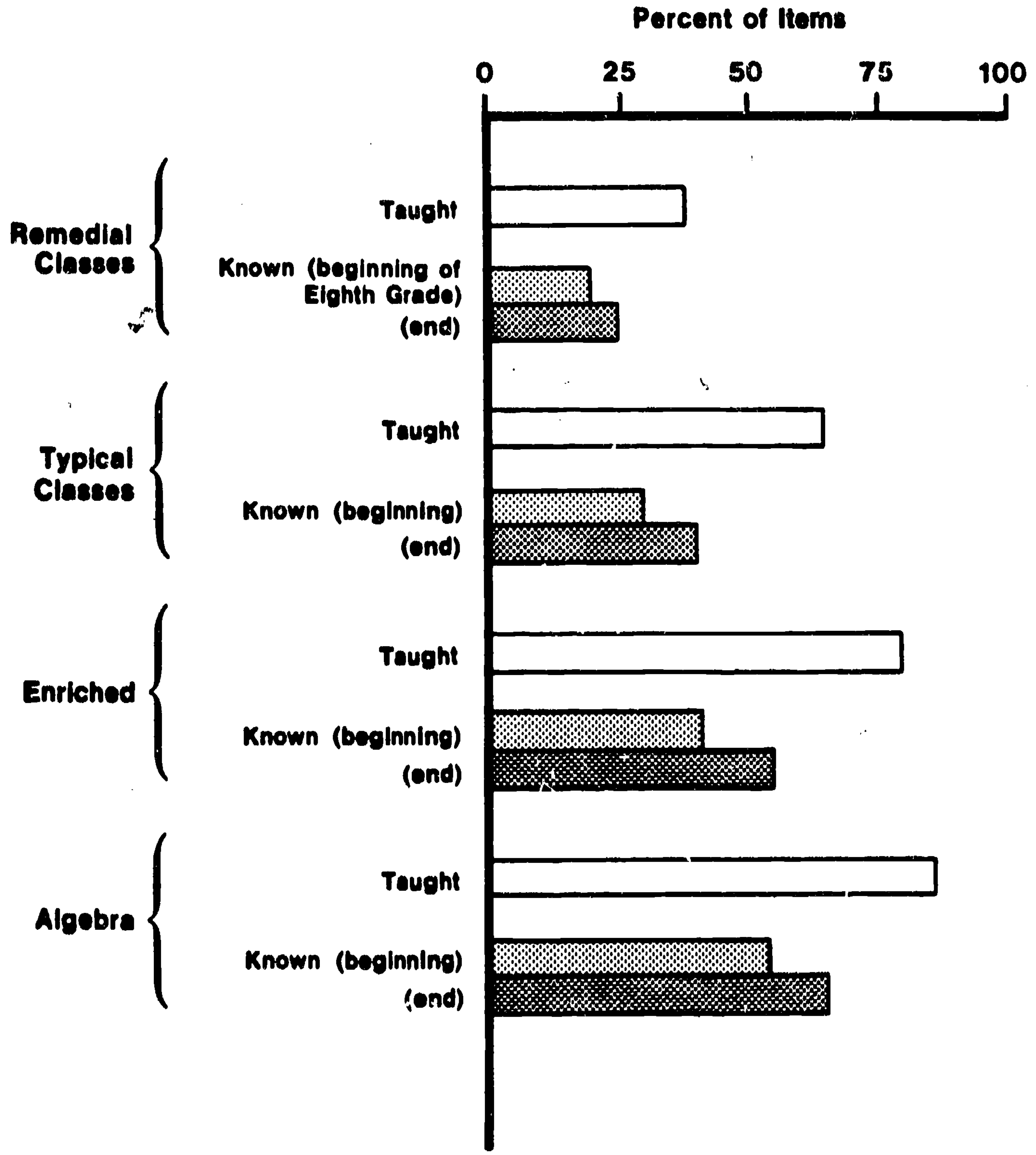


Figure 19 ALGEBRA Achievement of Eighth Grade Students by Class Type

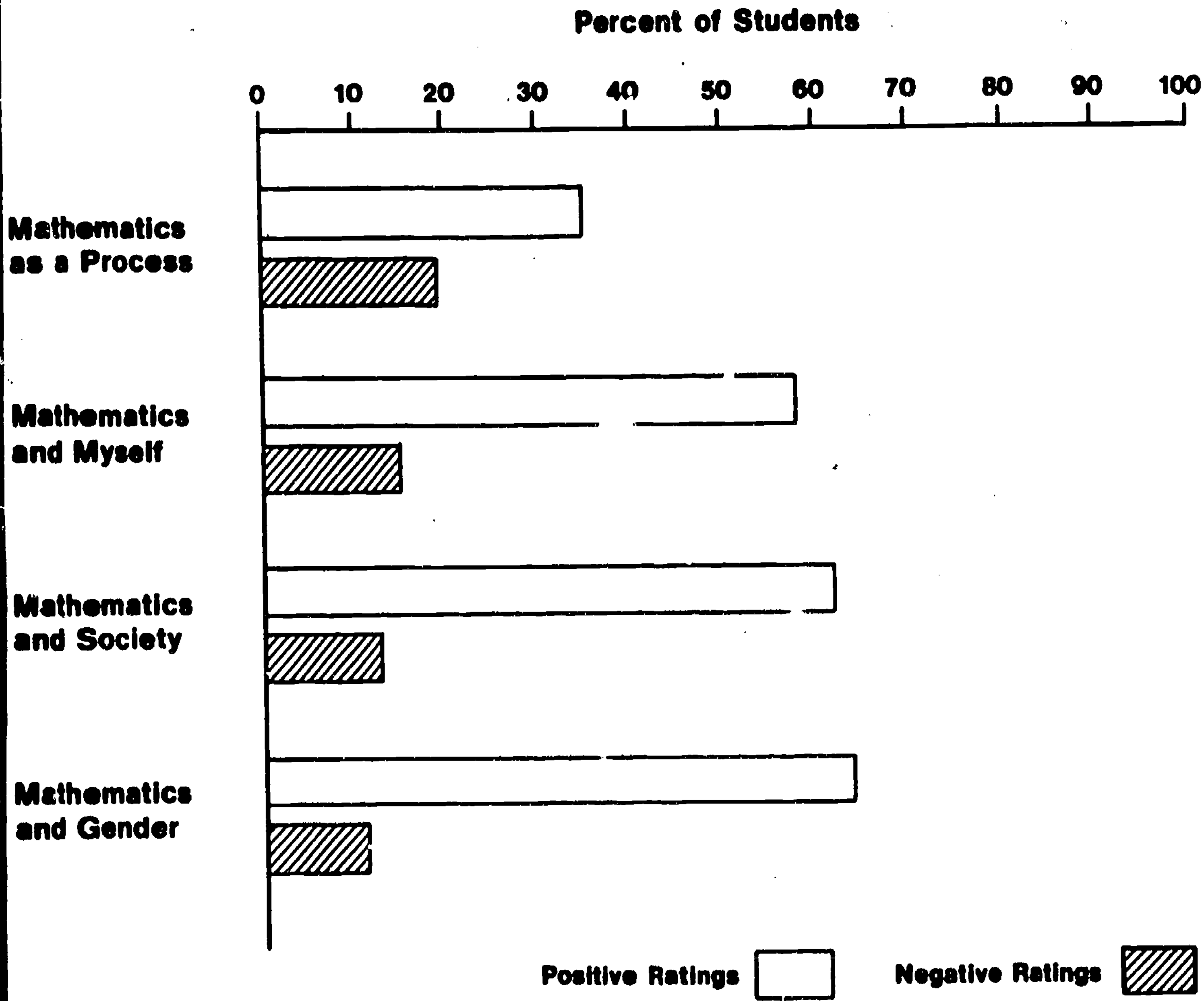


Figure 20 Percent Positive and Negative Ratings of Students for Attitude Scales in Eighth Grade Mathematics: U.S., 1981-82

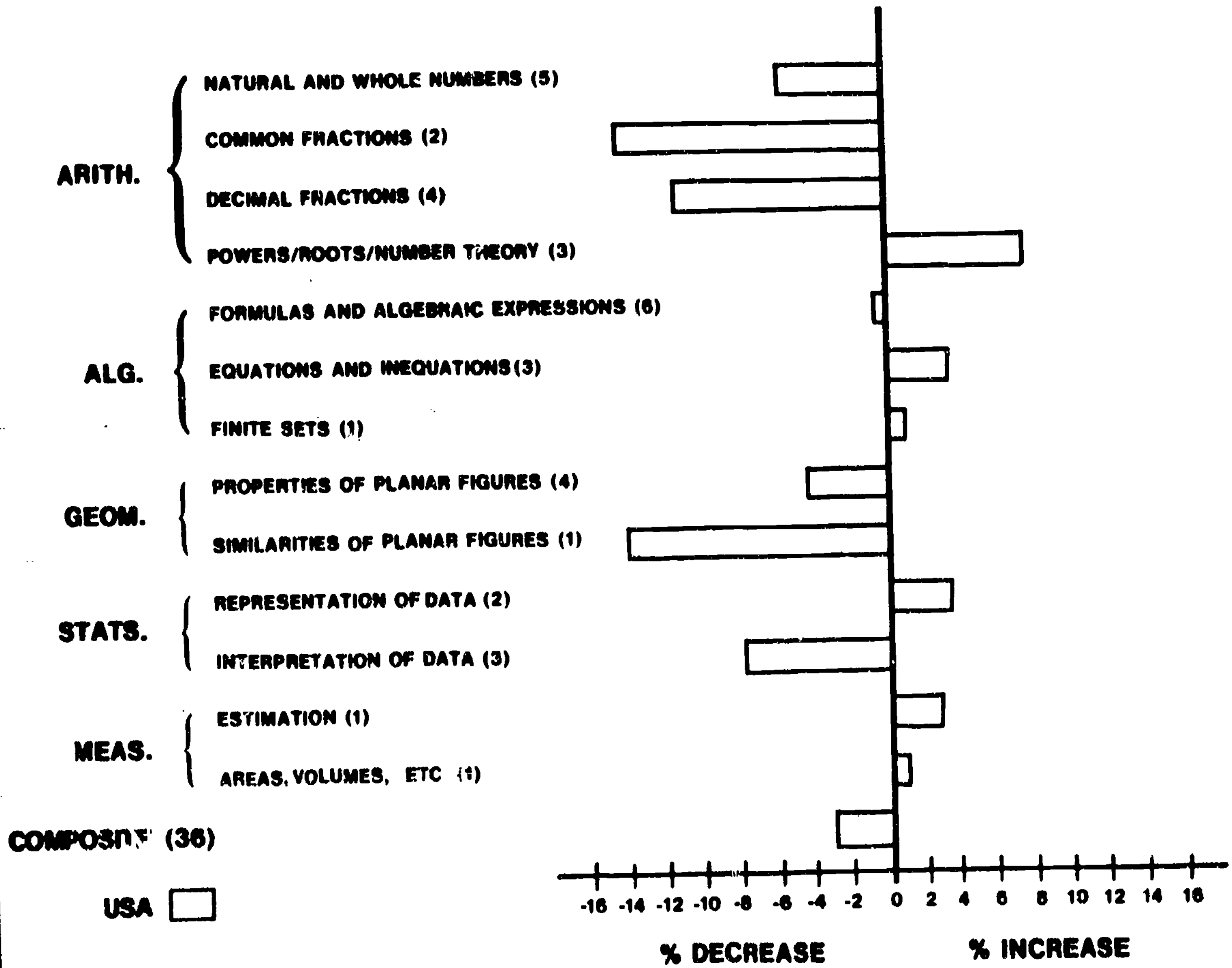


FIGURE 21 EIGHTH GRADE - CHANGES IN MEAN SCORES FROM 1964 TO 1982

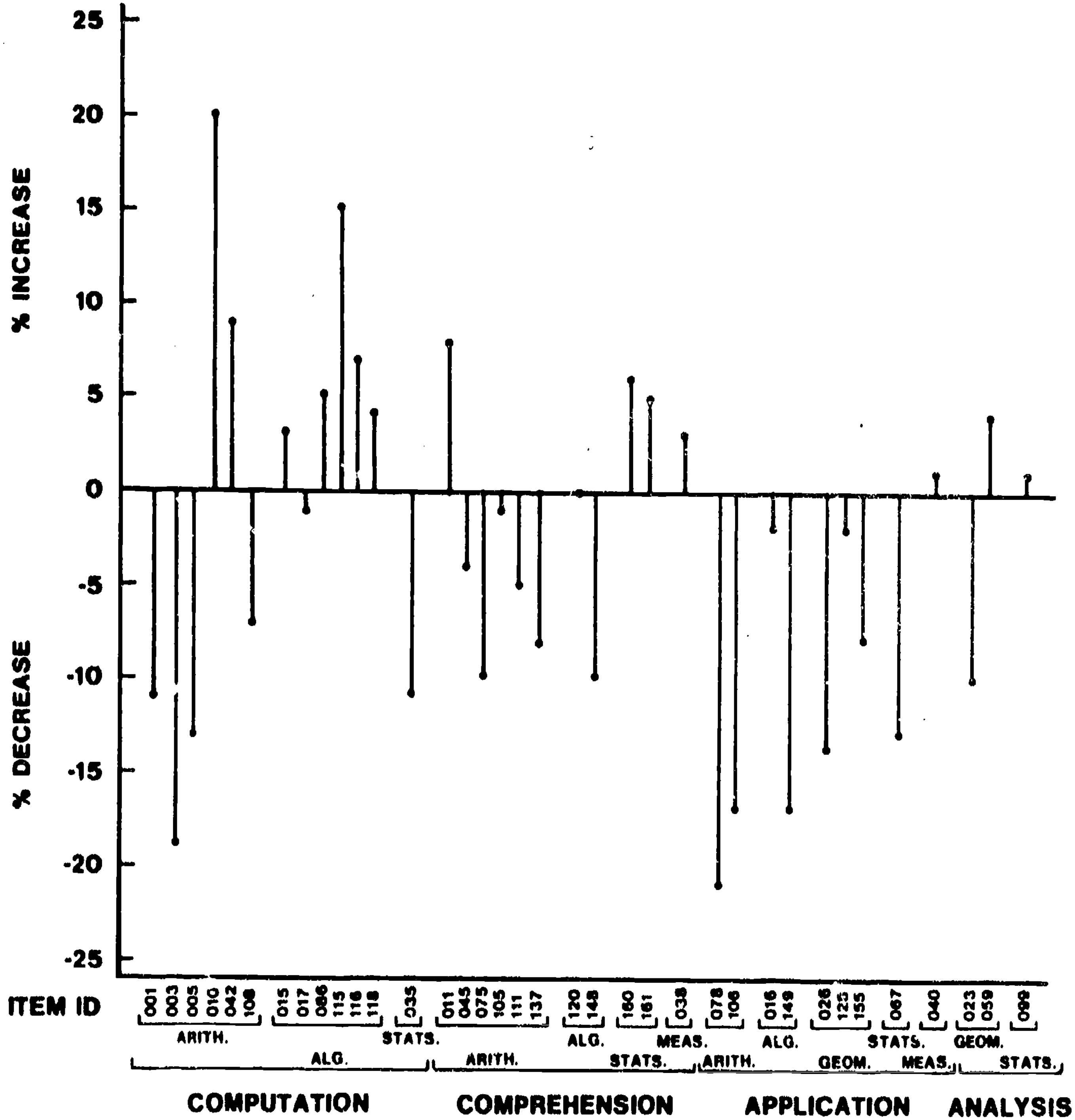


FIGURE 22 EIGHTH GRADE
STUDENT ACHIEVEMENT BY COGNITIVE LEVEL
CHANGE ON 36 ITEMS FROM 1964 TO 1982

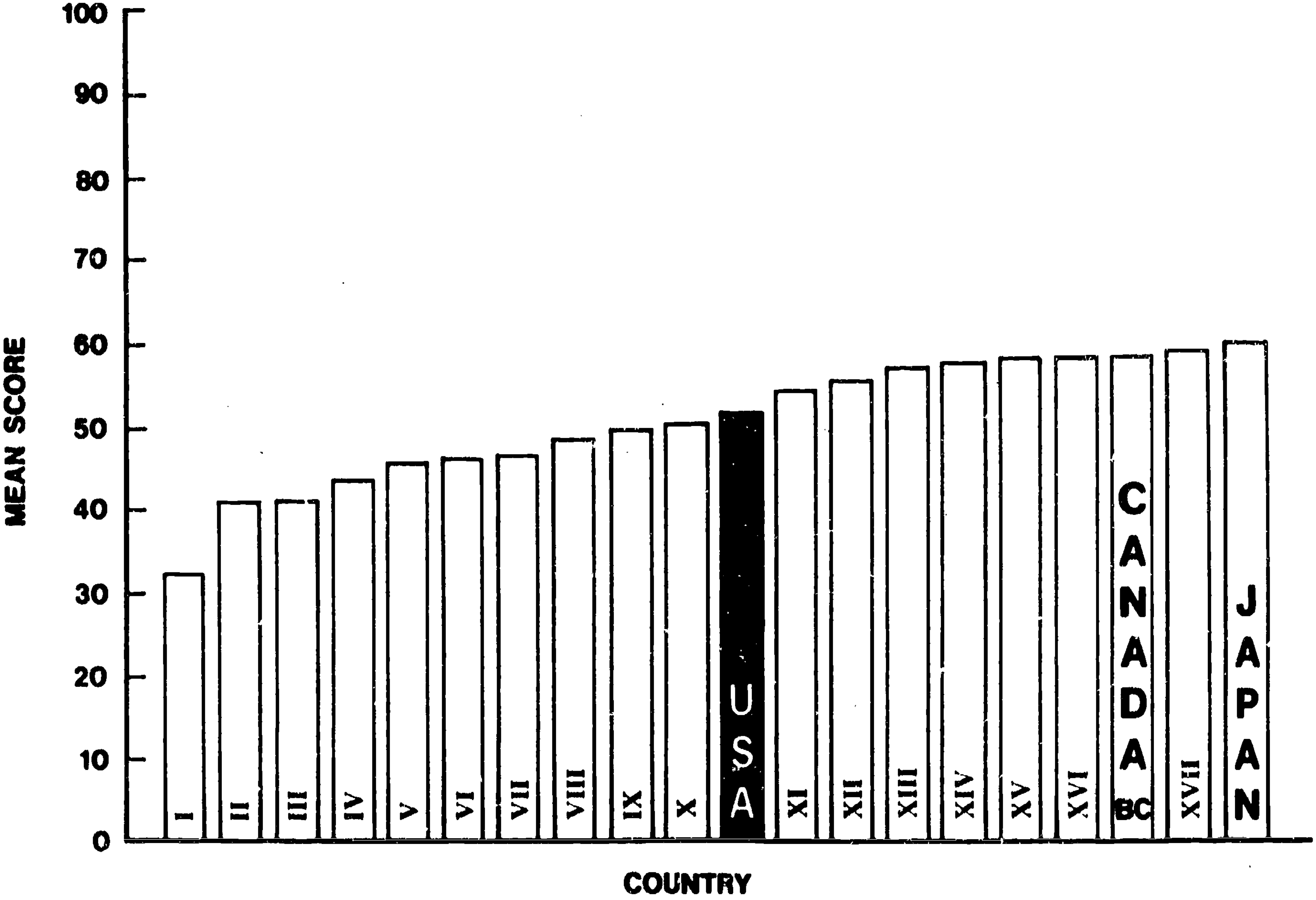


FIGURE 23 MEAN ACHIEVEMENT IN ARITHMETIC FOR POPULATION A (EIGHTH GRADE IN U.S.) FOR 20 COUNTRIES

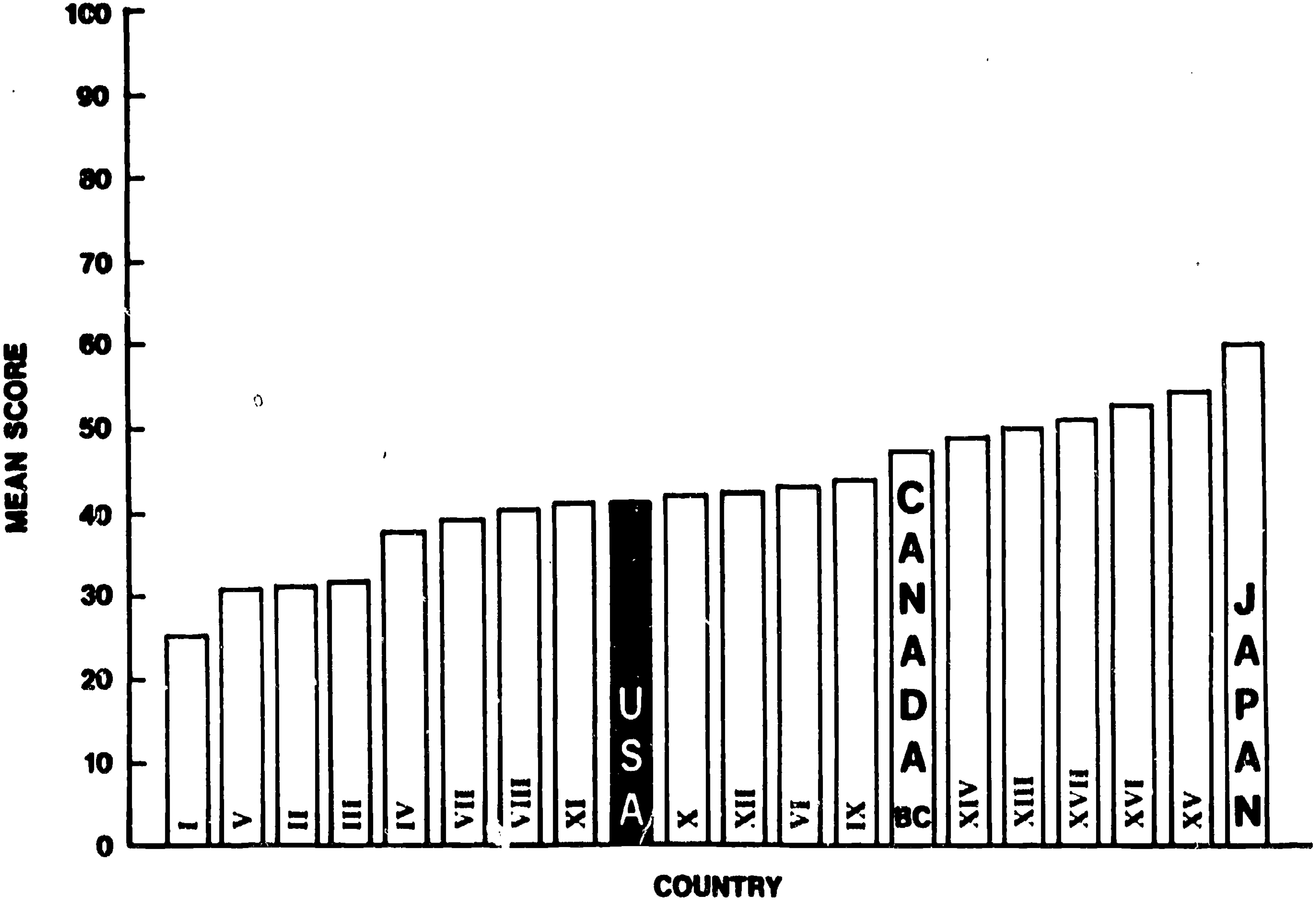


FIGURE 24 MEAN ACHIEVEMENT IN ALGEBRA FOR POPULATION A (EIGHTH GRADE IN U.S.) FOR 20 COUNTRIES

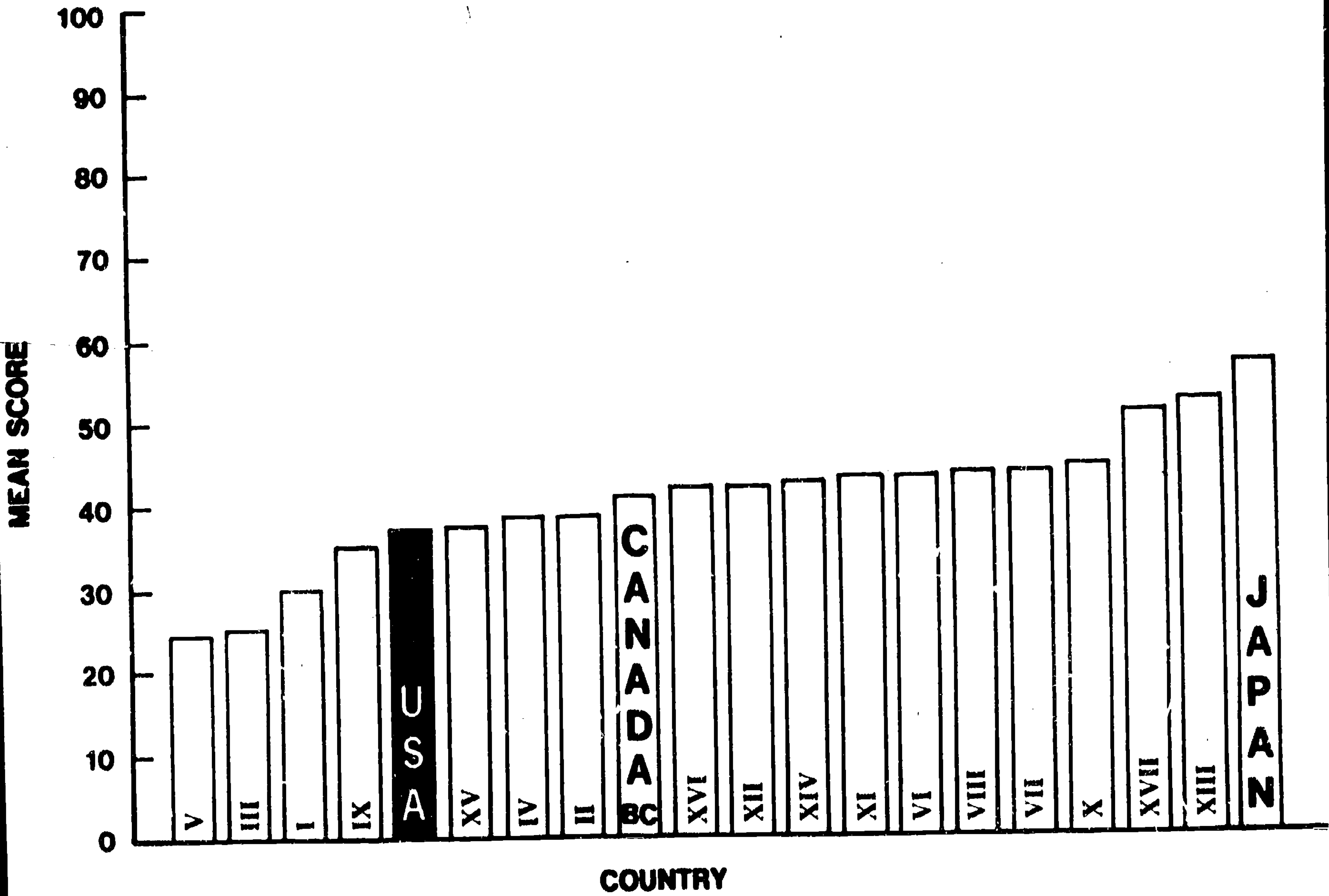


FIGURE 25 MEAN ACHIEVEMENT IN GEOMETRY FOR POPULATION A (EIGHTH GRADE IN U.S) FOR 20 COUNTRIES

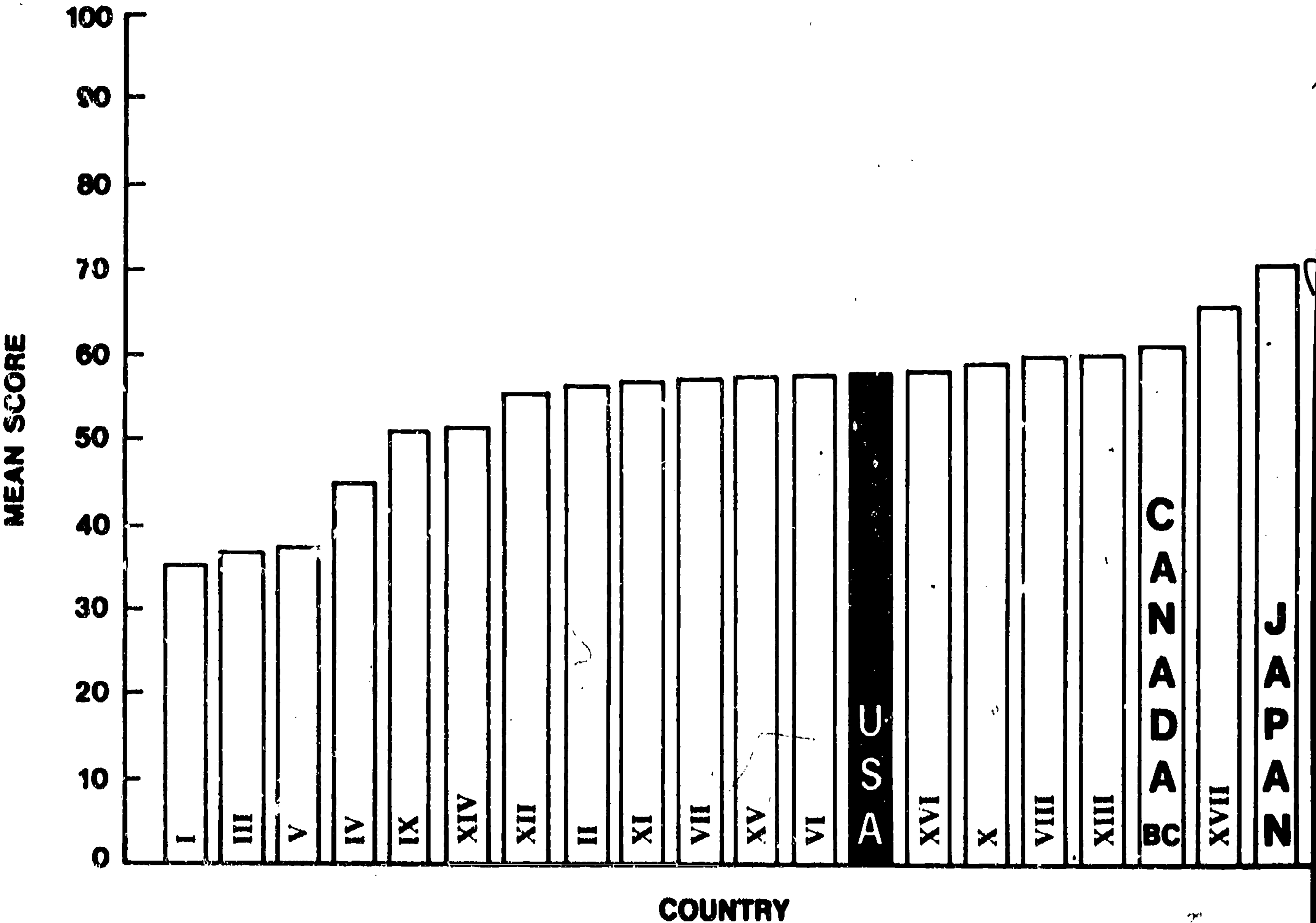


FIGURE 26 MEAN ACHIEVEMENT IN STATISTICS FOR POPULATION A (EIGHTH GRADE IN U.S.) FOR 20 COUNTRIES

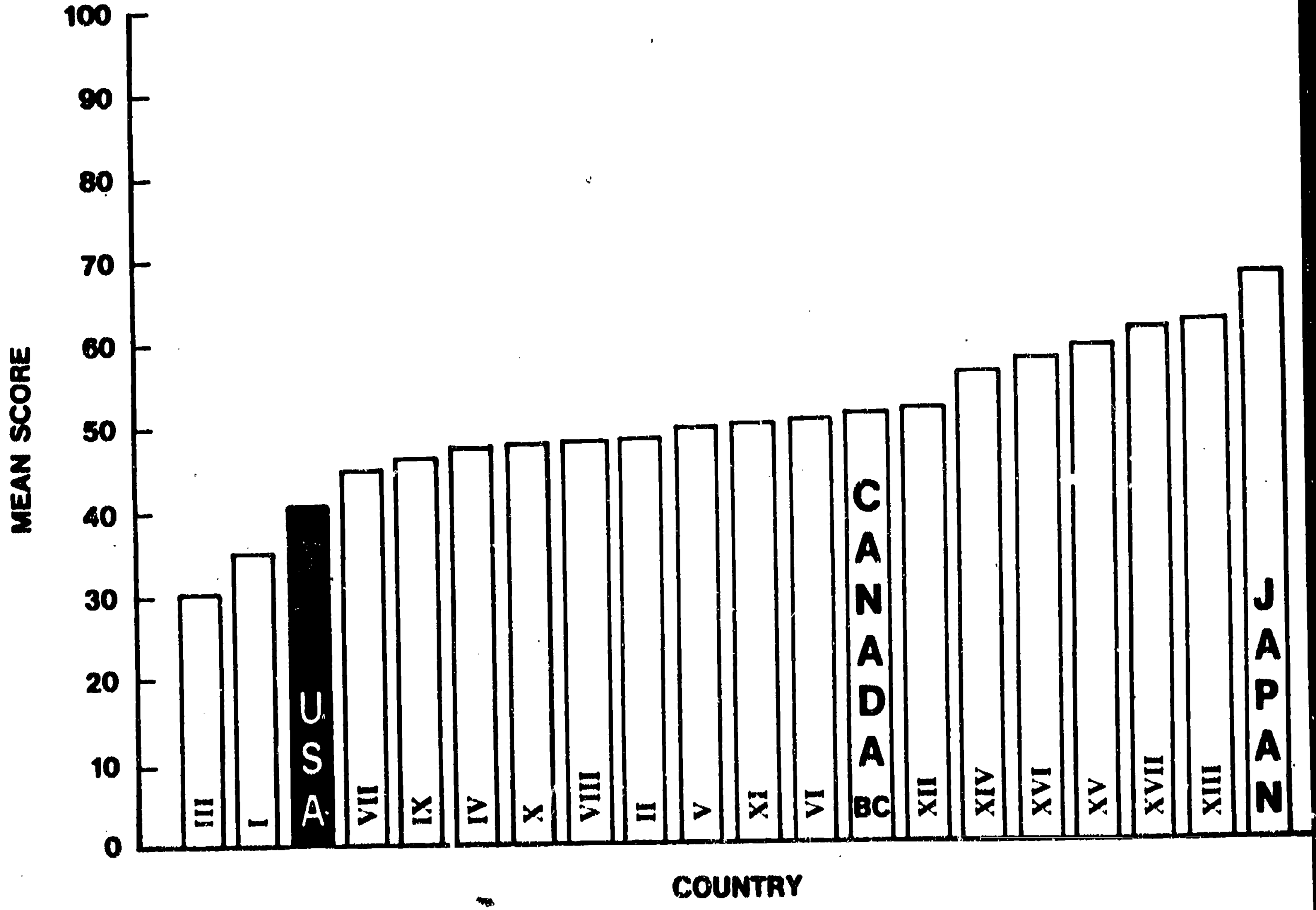


FIGURE 27 MEAN ACHIEVEMENT IN MEASUREMENT FOR POPULATION A (EIGHTH GRADE IN U. S.) FOR 20 COUNTRIES

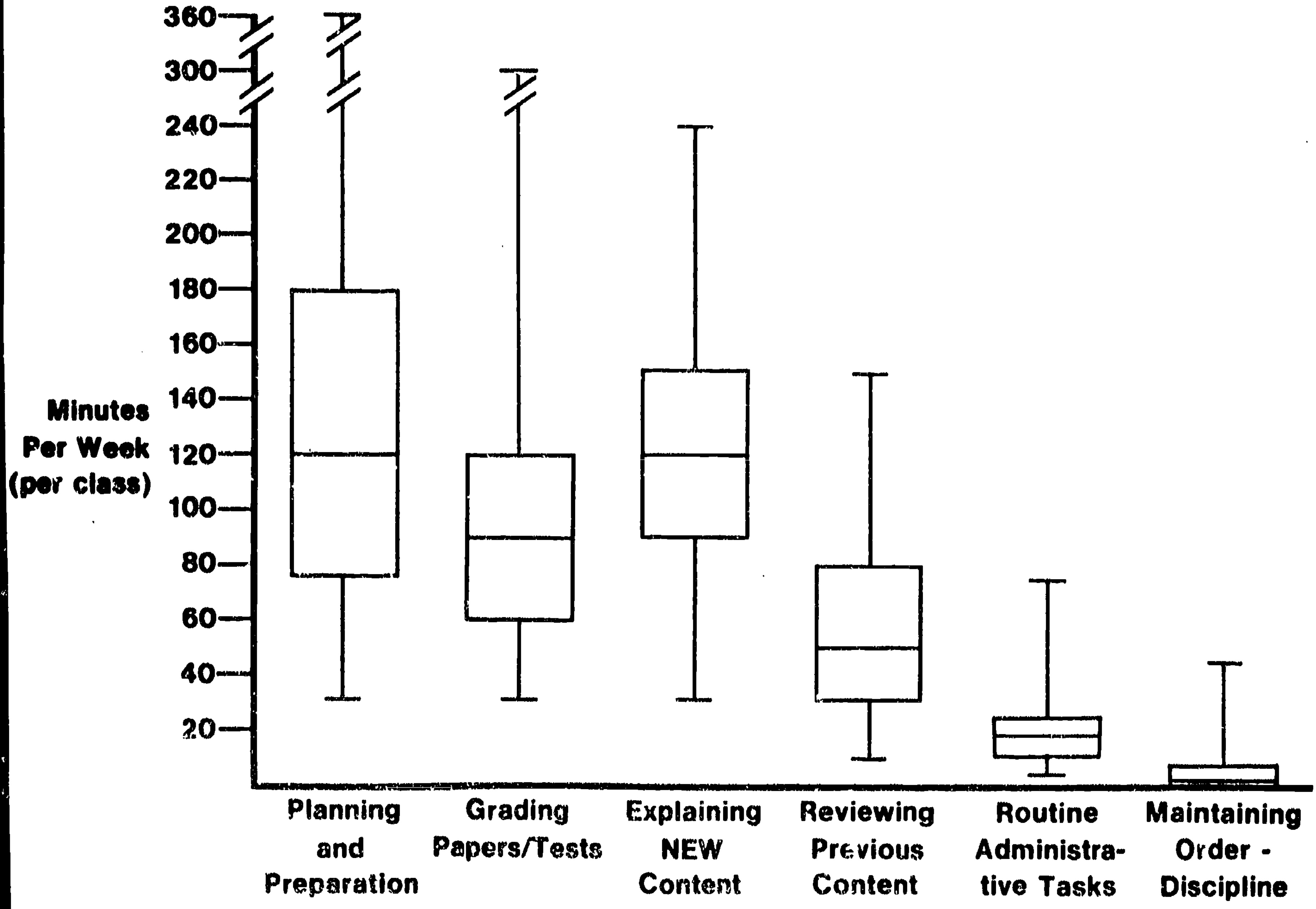


Figure 28 How Teachers' Time was Spent on Selected Teaching Activities in Twelfth Grade Mathematics

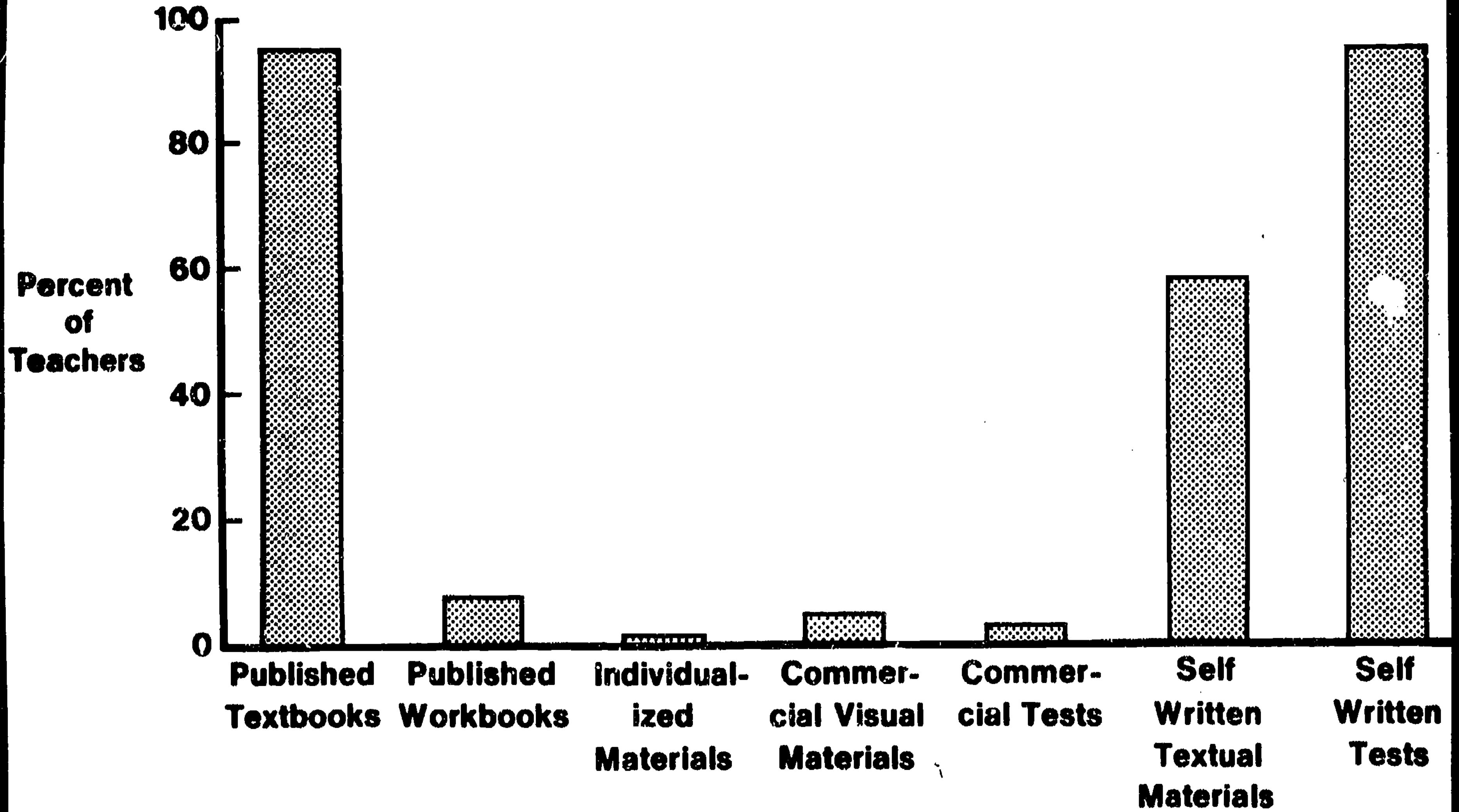


Figure 29 Percent of Teachers Using Various Instructional Resources in Twelfth Grade Mathematics Classes

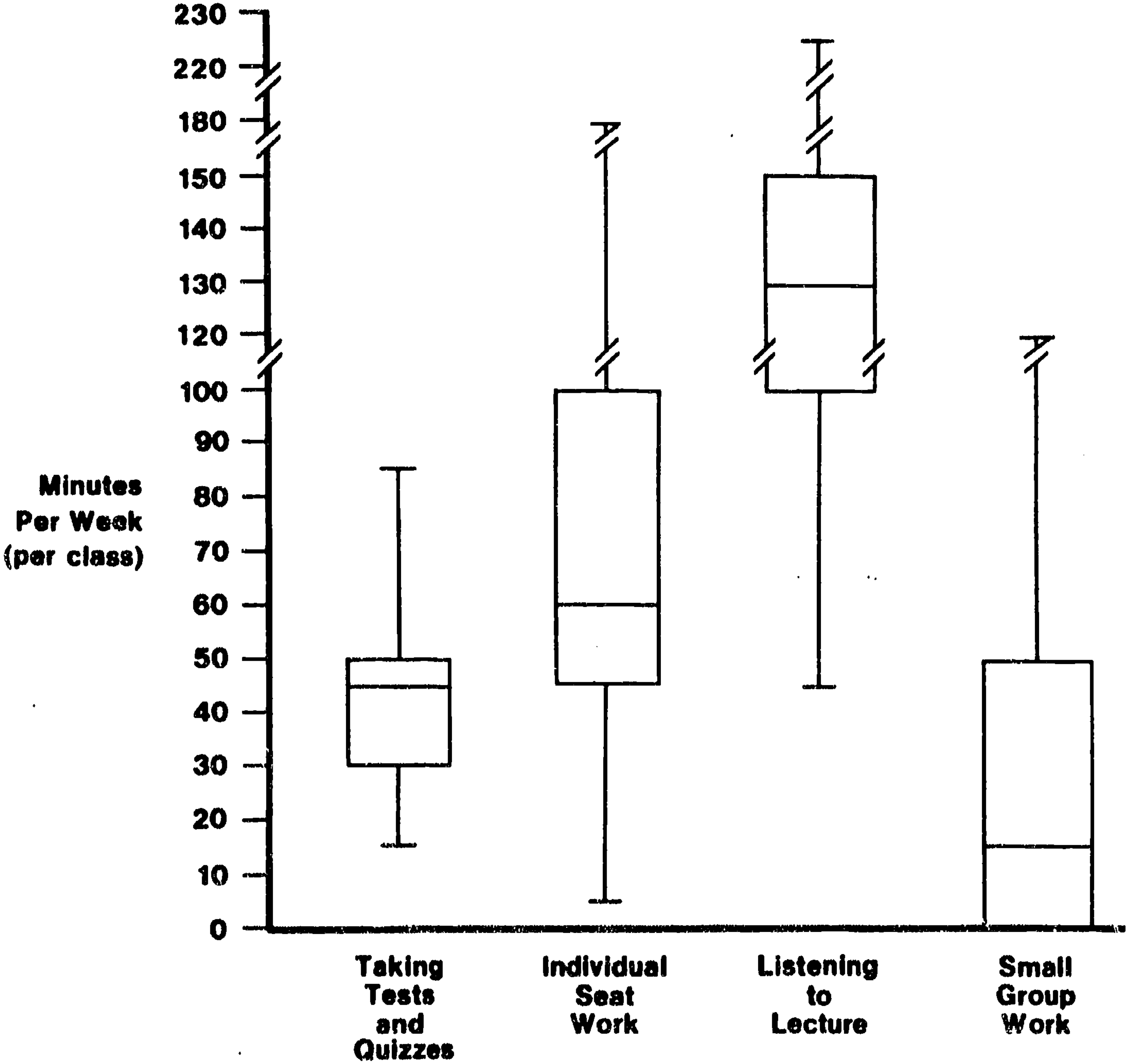


Figure 30 Amount of Student Time Spent in Selected Activities in Twelfth Grade Mathematics Classes.

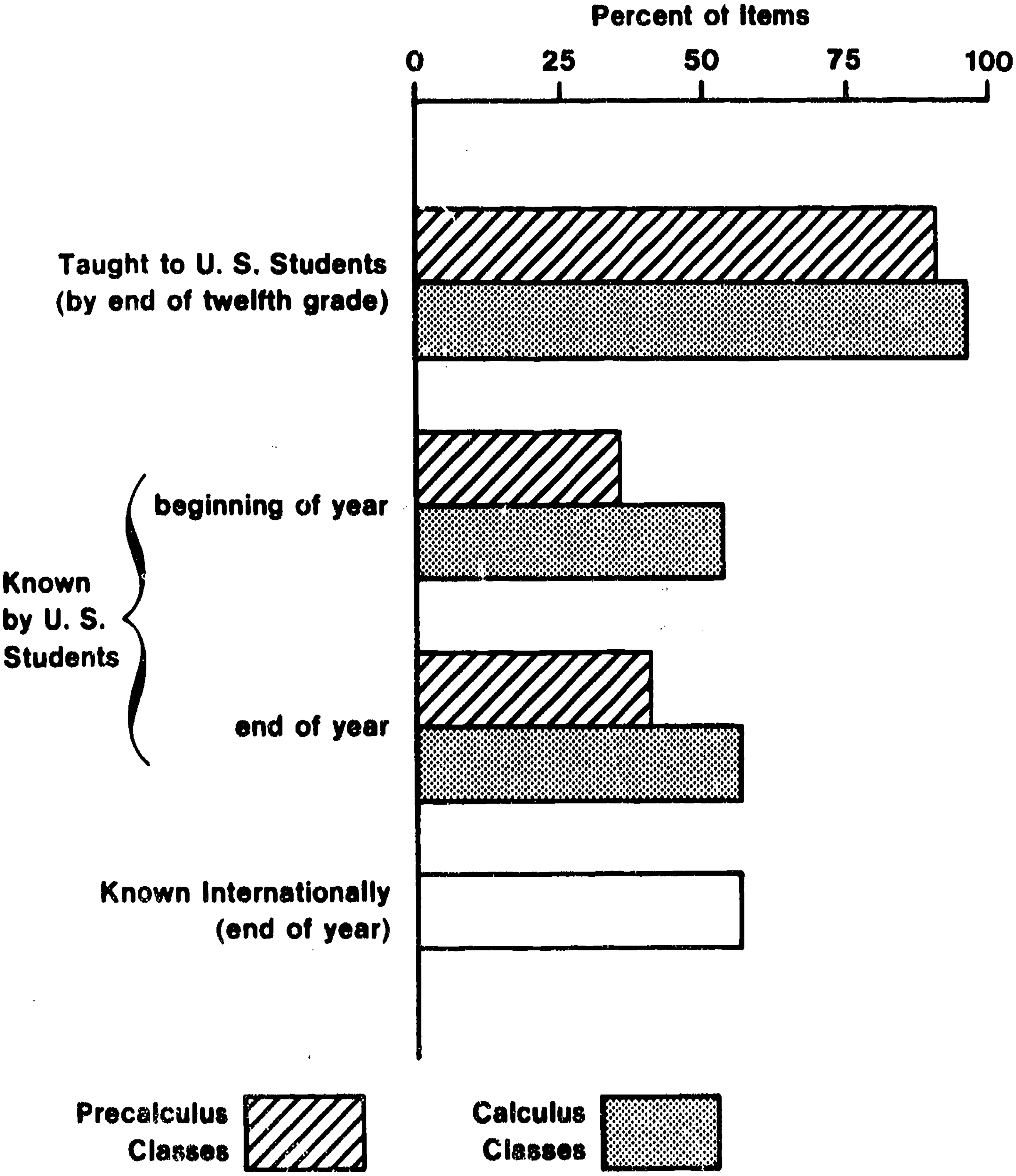


Figure 31 Average Percent of ALGEBRA Items on International Test Taught and Learned at End of Secondary School

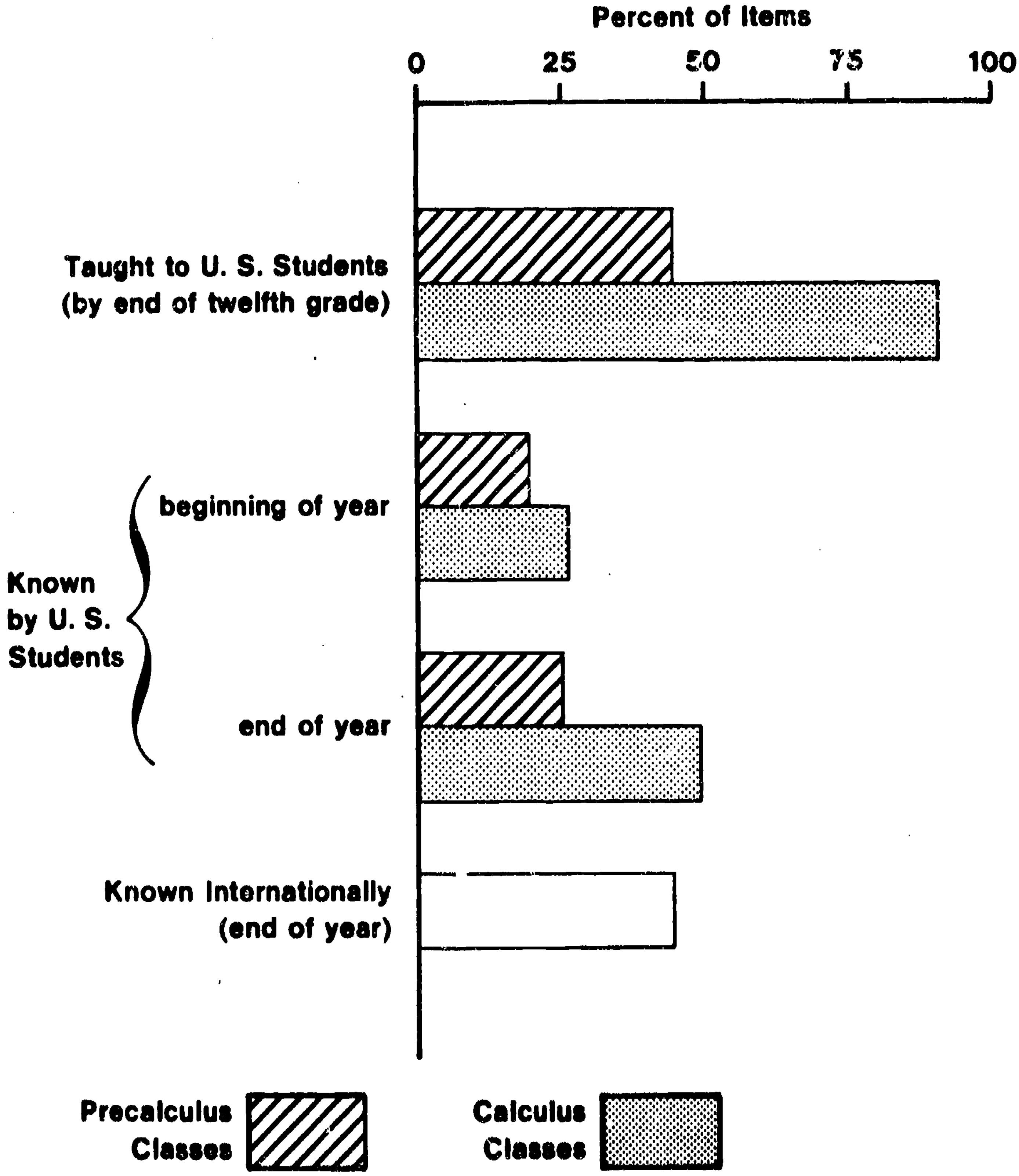


Figure 32 Average Percent of ELEMENTARY FUNCTIONS/CALCULUS Items on International Test Taught and Learned at End of Secondary School

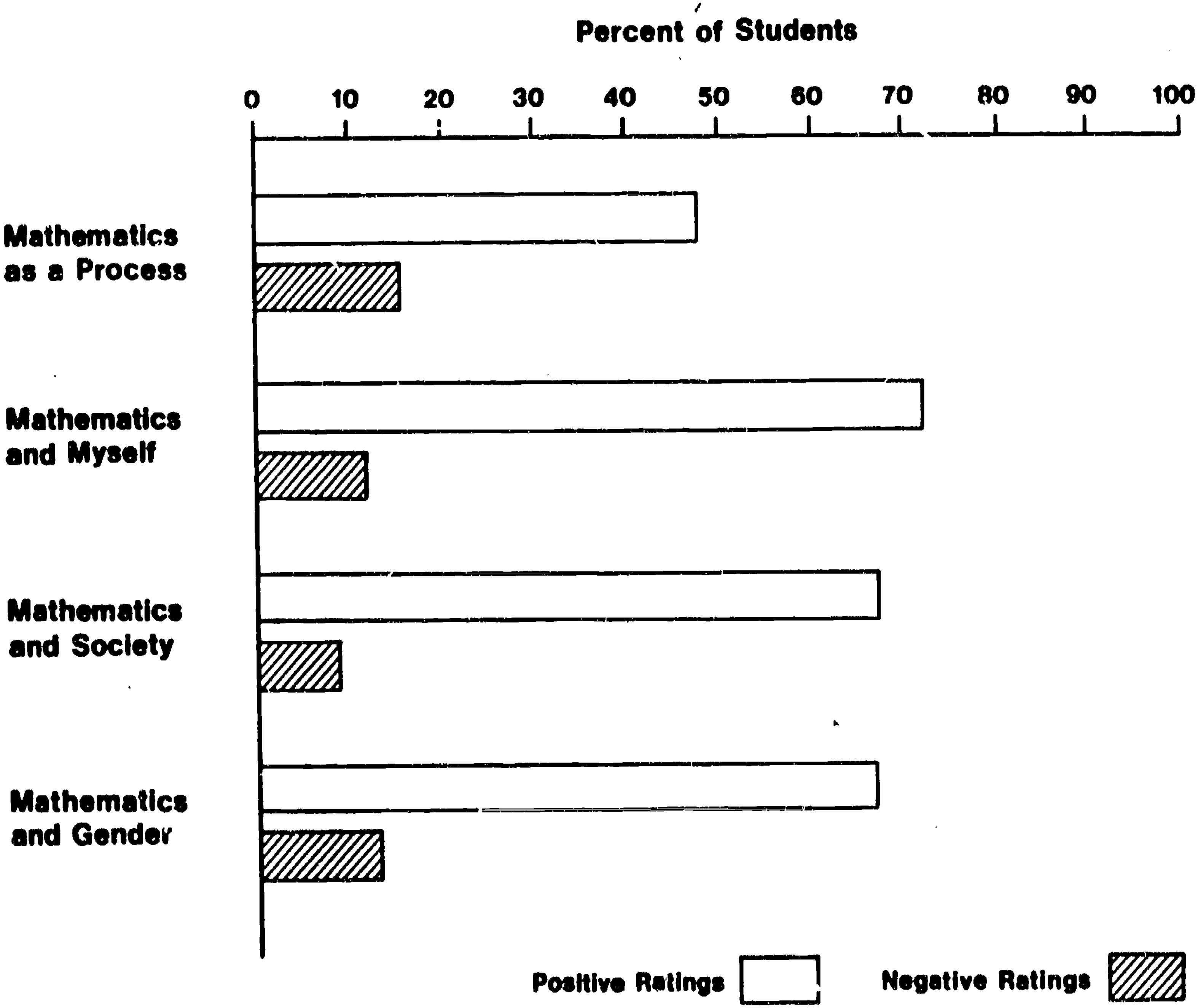
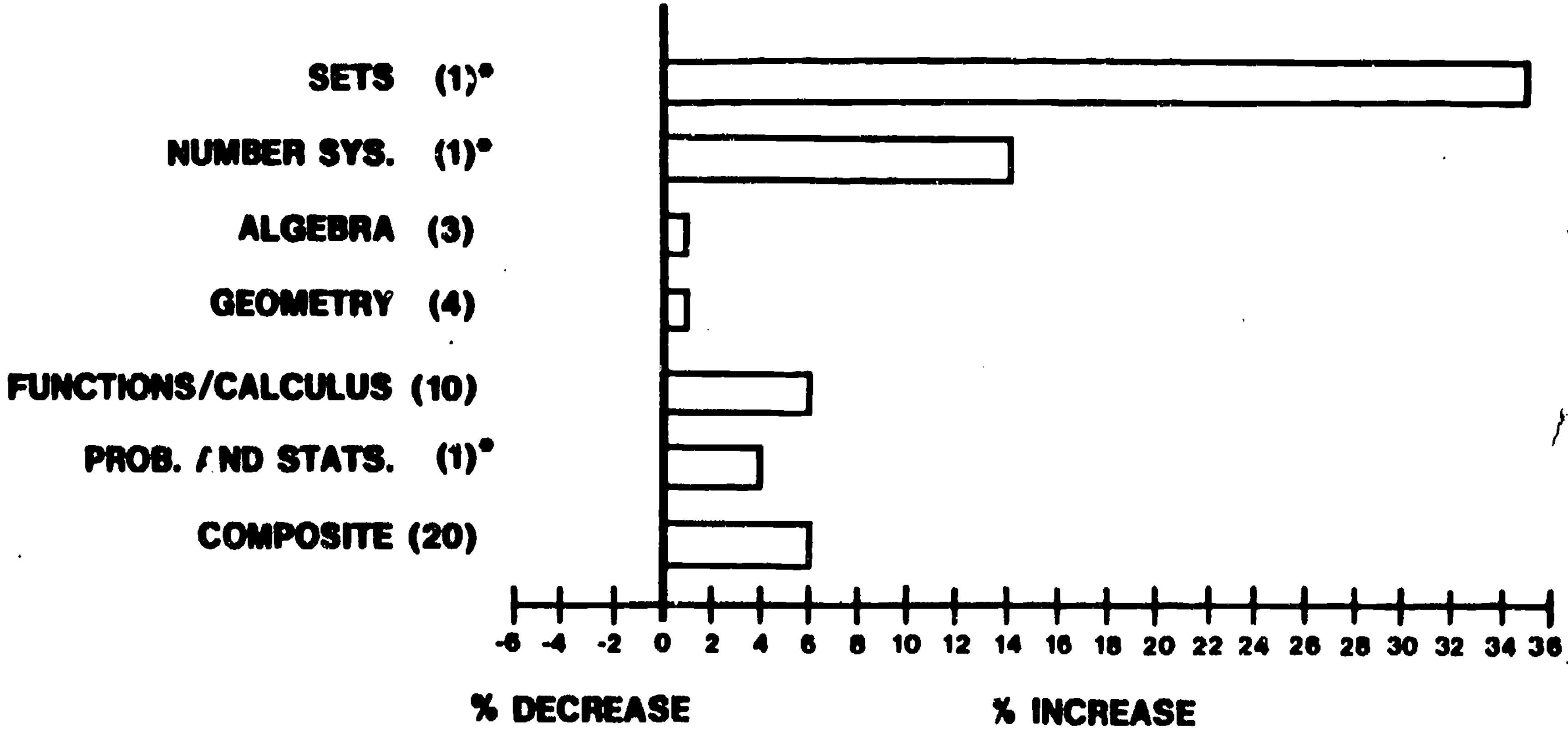


Figure 33 Percent Positive and Negative Ratings of Students for Attitude Scales in Twelfth Grade College-Preparatory Mathematics: U.S., 1981-82



*ONLY ONE ITEM IN THIS TOPIC AREA

FIGURE 34 POPULATION B - (TWELFTH GRADE IN USA)
CHANGES IN MEAN SCORES FROM 1964 TO 1982

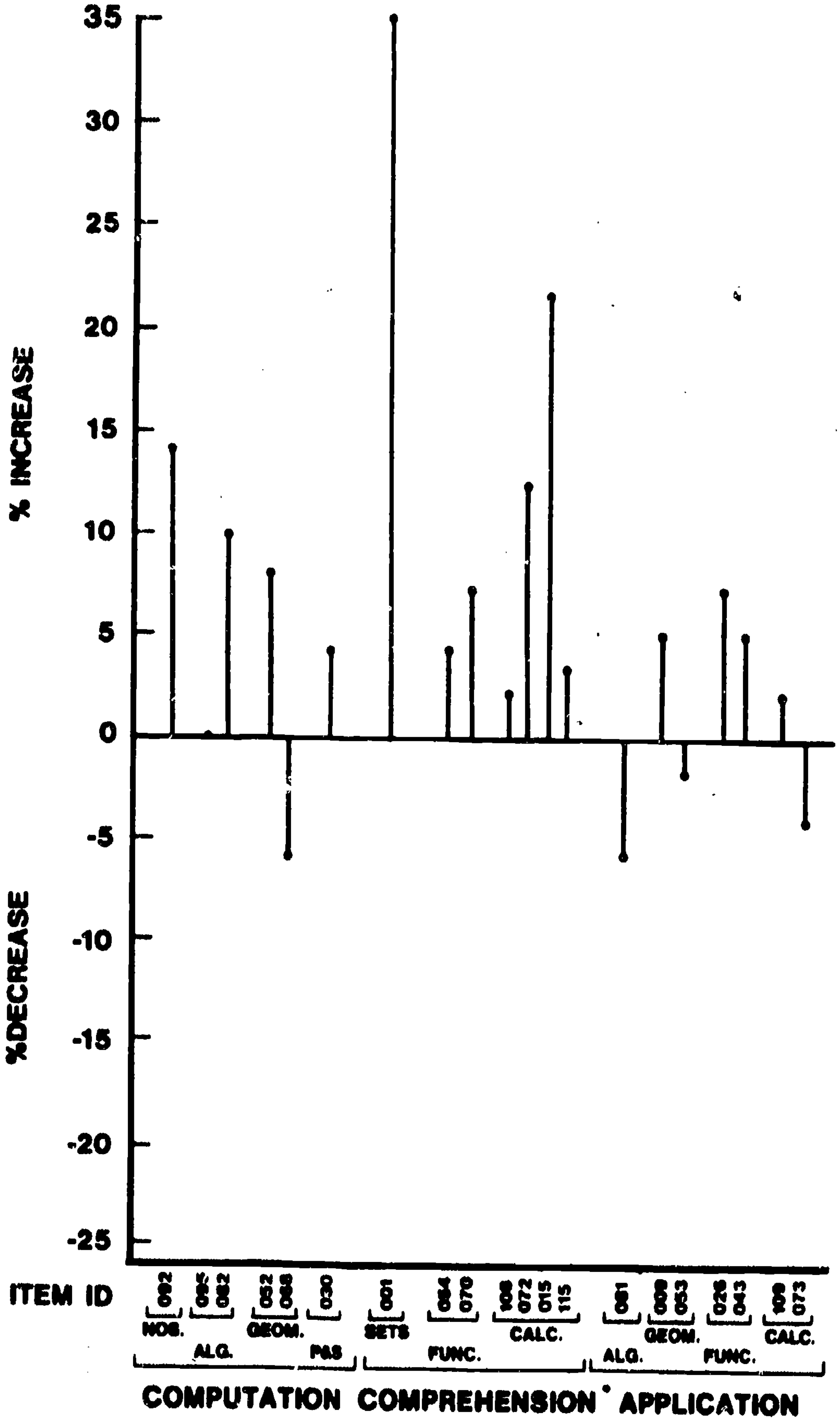


FIGURE 35 TWELFTH GRADE (USA)
STUDENT ACHIEVEMENT BY COGNITIVE LEVEL
CHANGE FROM 1964 TO 1982

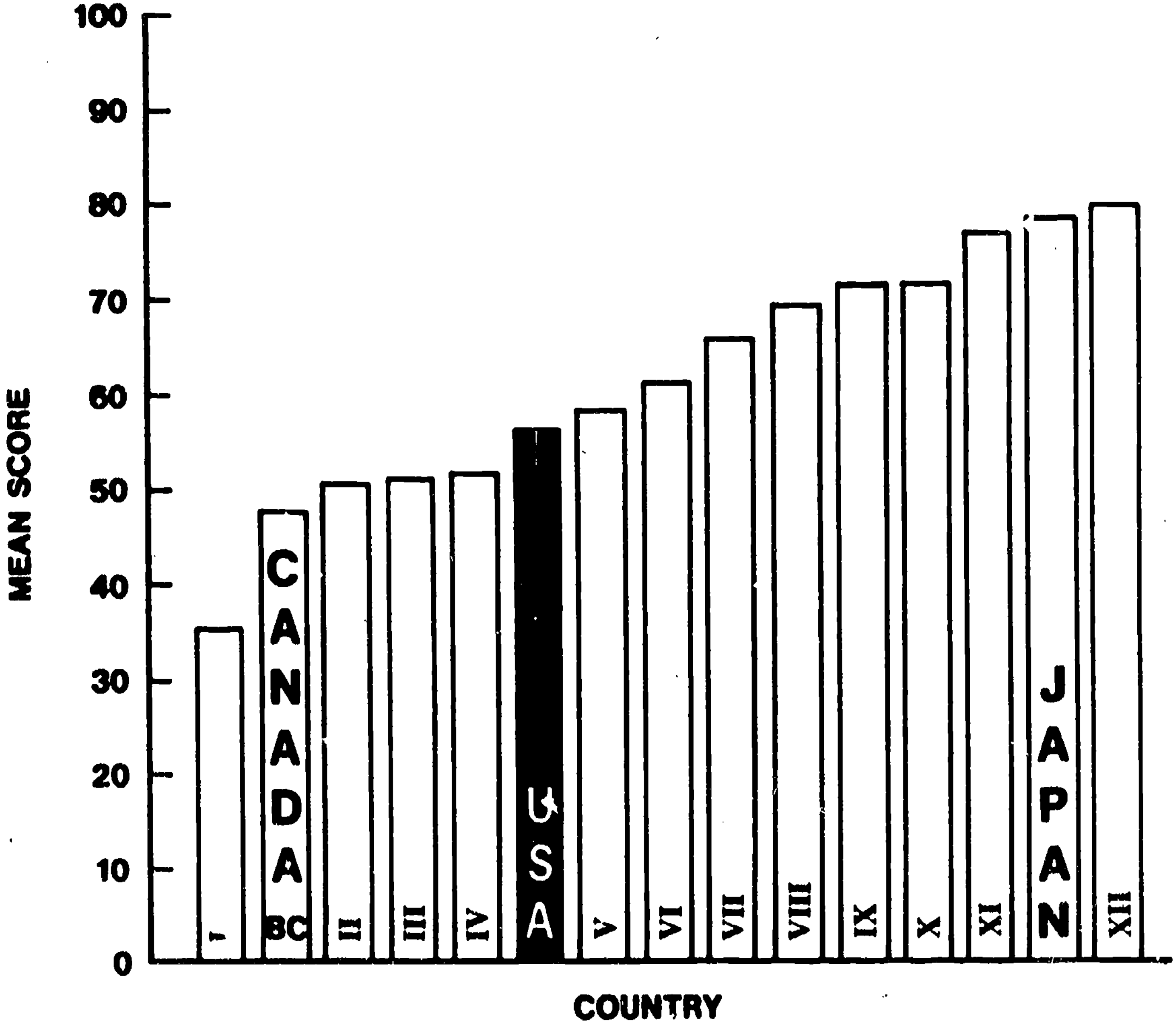


FIGURE 36 MEAN ACHIEVEMENT IN SETS AND RELATIONS FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 15 COUNTRIES

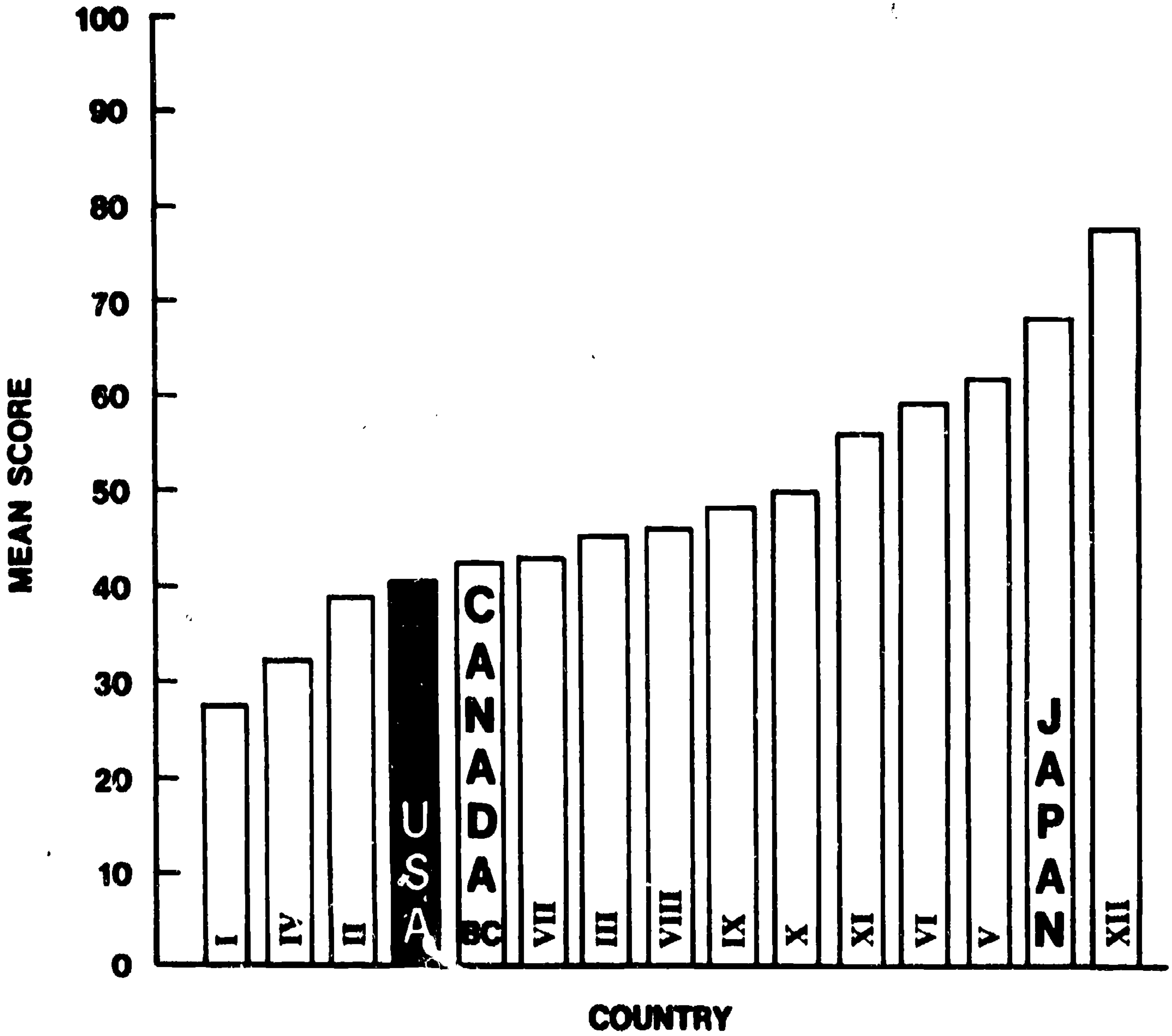


FIGURE 37 MEAN ACHIEVEMENT IN NUMBER SYSTEMS FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 15 COUNTRIES

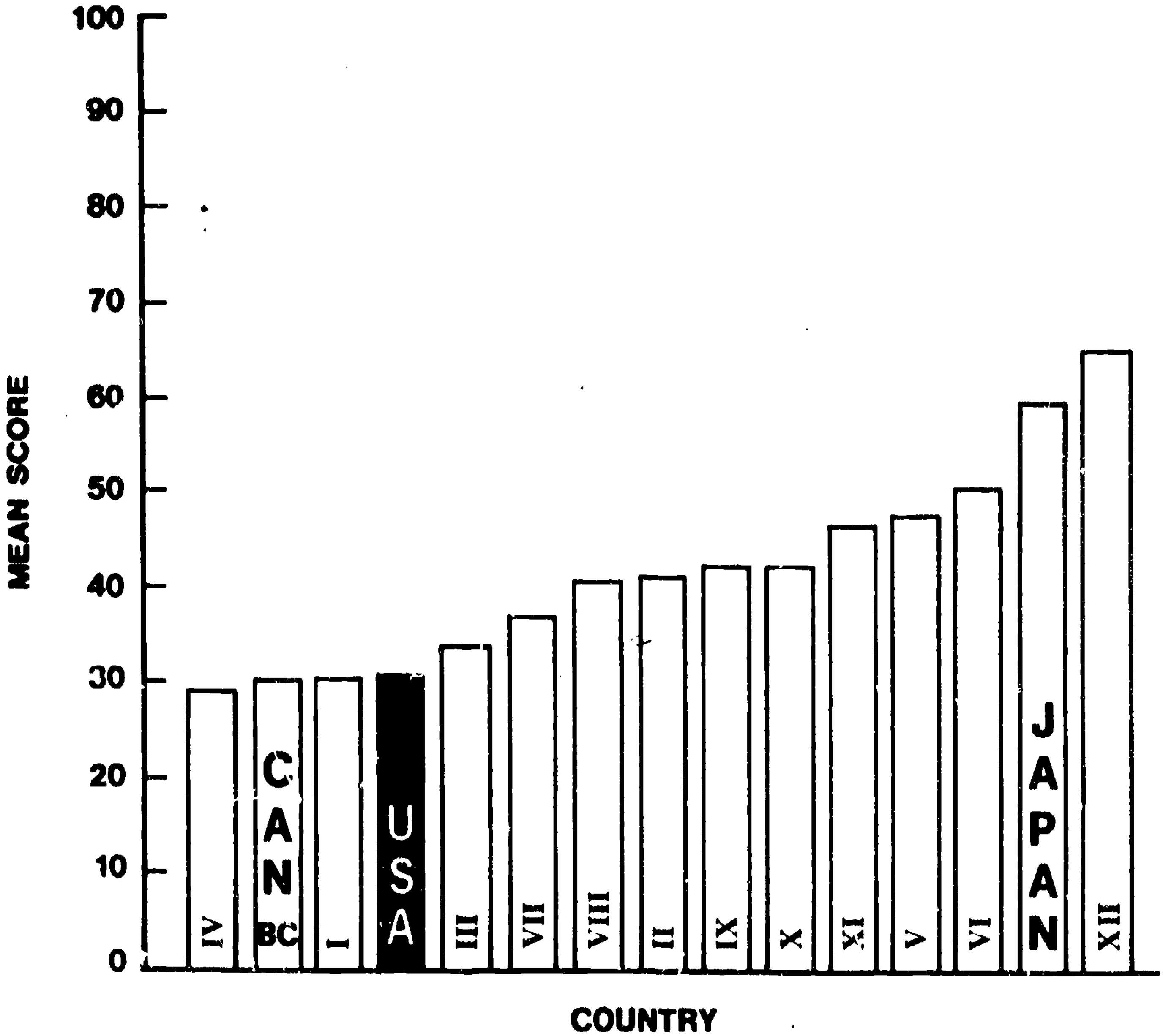


FIGURE 38 MEAN ACHIEVEMENT IN GEOMETRY FOR POPULATION B (TWELTH GRADE IN U.S.) FOR 15 COUNTRIES

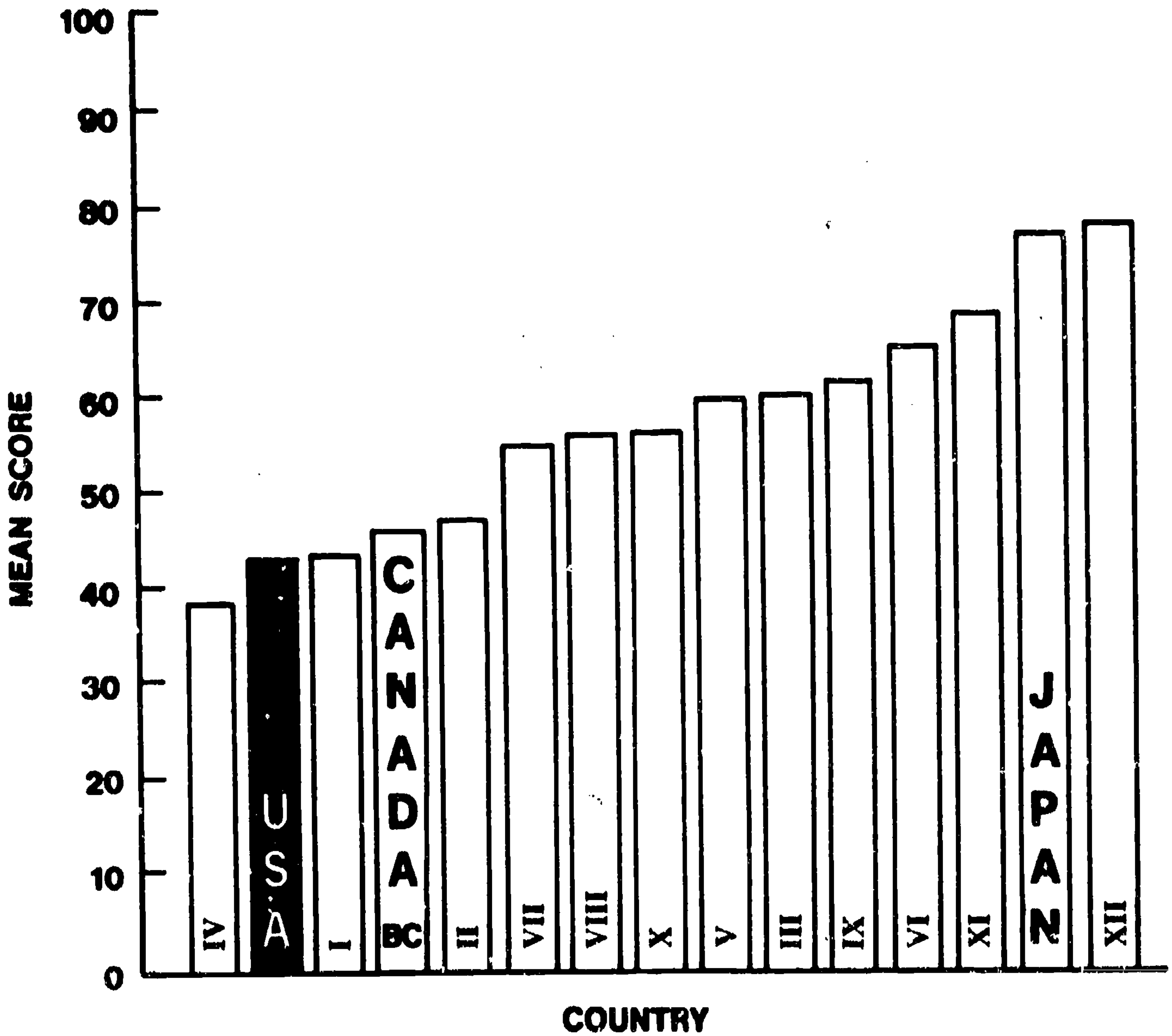


FIGURE 39 MEAN ACHIEVEMENT IN ALGEBRA FOR POPULATION B (TWELFTH GRADE IN U.S.) FOR 15 COUNTRIES

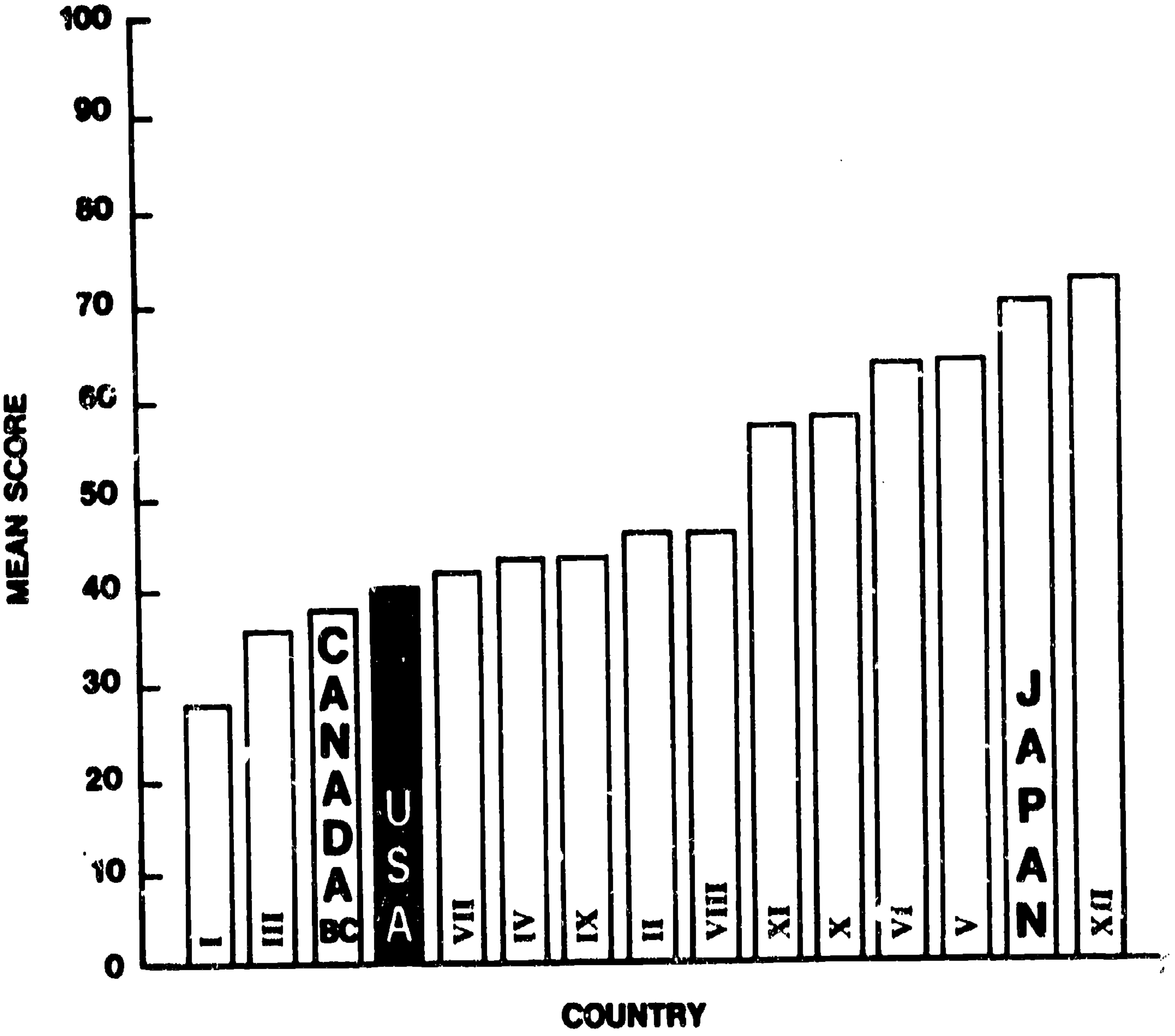


FIGURE 40 MEAN ACHIEVEMENT IN PROBABILITY AND STATISTICS FOR POPULATION B (TWELFTH GRADE IN U. S.) FOR 15 COUNTRIES

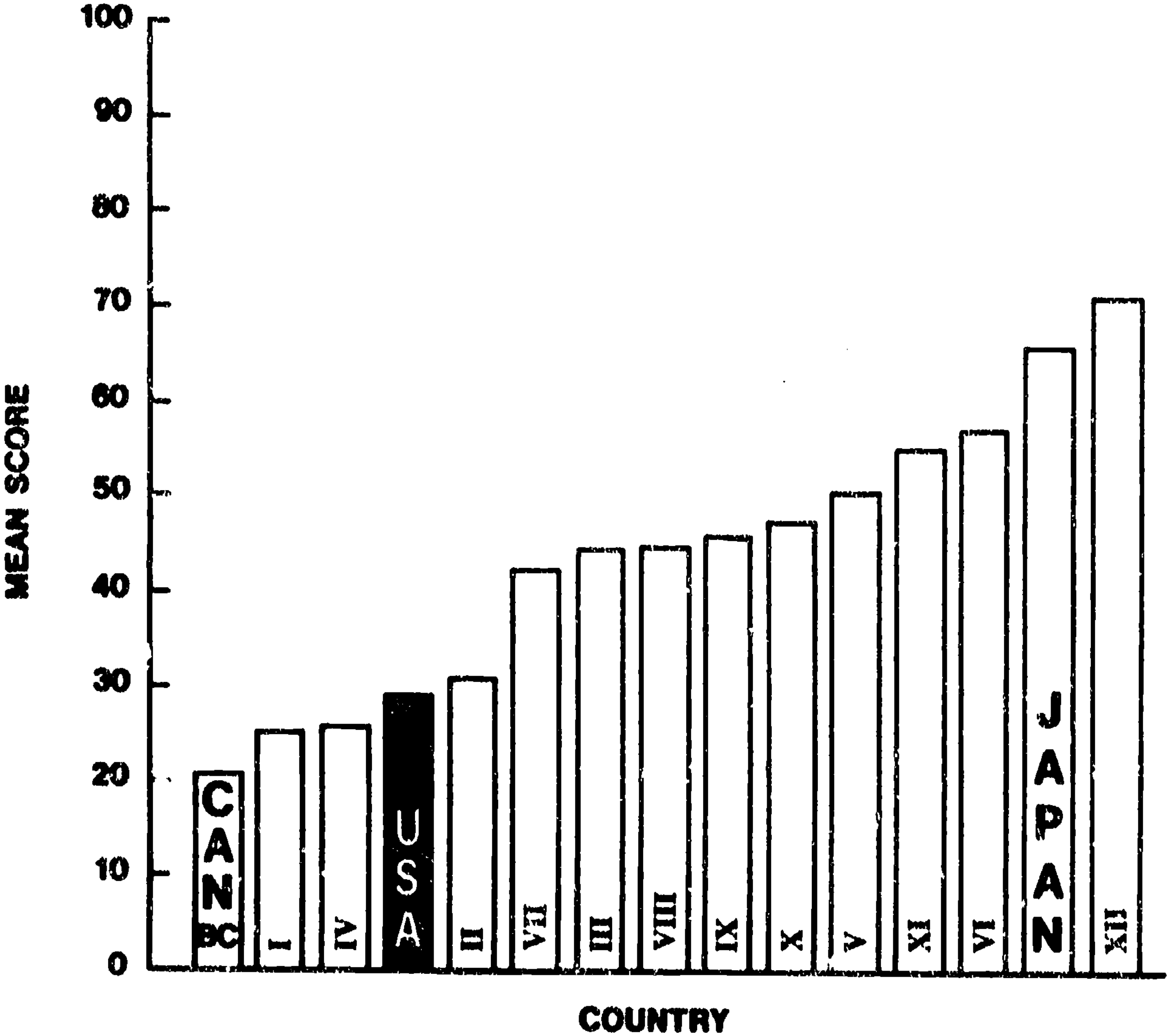


FIGURE 41 MEAN ACHIEVEMENT IN ELEMENTARY FUNCTIONS/CALCULUS FOR POPULATION B (TWELFTH GRADE IN U.S) FOR 15 COUNTRIES

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