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ABSTRACT

Described in the context of a computer simulation are lights of a program of research focusing on the storage of mathematics problem solving information in young children's memory and the development of such knowledge structures in older children. Specifically discussed are the problem size effect, the network nature of the memory representation, the development of addition knowledge, and predictions of automatic and conscious retrieval. These topics are broached in relation to a four-stage process model incorporating encoding, search/compute, decision, and response stage processes and two general classes of information stored in memory: a network of stored facts and knowledge about arithmetic. Discussion centers primarily on the network of stored facts, or declarative knowledge component of the model. It was found that the model successfully simulates the pattern of results found in developmental studies of simple arithmetic. The model itself derives from reasonable assumptions about the nature of human memory organization and processes. An especially noteworth feature of the model is that the same fundamental mechanism, learn g, accounts for all four aspects of the model. With learning comes first a representation of arithmetic knowledge in a network-node structure, and then a strengthening of those nodes across development. Retrieval from the network depends directly on node strength, and becomes more automatic as strength reaches asymptotic values. (RH)

Children's Mental Arithmetic: Toward a Model of Retrieval and Problem Solving

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in

The Acquisition of Mathematical Skill: A Look at Addition

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Children's Mental Arithmetic:

Toward a Model of Retrieval and Problem Solving

Figure 1 shows the basic empirical effect in mental arithmetic research which all of us on this symposium are addressing. When children across the developmental span are tested on their knowledge of simple addition problems, we find that their performance slows down considerably as the problem they are working on gets larger. The term for this empirical result is the "problem size effect" -- as the problem gets larger, reaction time (RT) increases. The history of this area of research is largely one of interpretations and reinterpretations of the problem size effect; for example, does the increase in response latency indicate a problem solving strategy based on counting or other rudimentary algorithms? Does the increase reflect a memory retrieval process? Does the obvious age-related change in the slope of this effect indicate a speed-up of a single predominating mental process or strategy, or does this slope change reflect a change in the nature of mental processes across development?

After several years of investigating these sorts of questions, using both addition and multiplication tasks, testing both adults and children at various stages in their schooling, I proposed a model to integrate the various research findings in the literature. Figure 2 illustrates the overall architecture of this model. The figure is a composite of two goals. First, a typical four-stage process model is shown, with the intention of accounting for RT effects in terms of encoding, search/compute, decision, and response stage processes (see Sternberg, 1969). The second goal is addressed by the three-dimensional components diagrammed above the process model. These components are labeled "Network of Stored Facts -- Declarative" and

"Knowledge About Arithmetic — Procedural". The goal here was to specify two general classes of information, both stored in memory, which participate in our performance to arithmetic tasks. At a general level, this model was intended as a framework within which significant questions in the area of mathematical cognition could be asked. The most basic of these questions can be posed in a variety of ways; for instance, how is the subject's inclusive knowledge about arithmetic represented in the human memory system, and how is that knowledge used or applied in various tasks? When we test children on simple addition problems, and find results like those in the first figure, how does the knowledge in these two components account for this pattern of results, or at least help us understand those results at a deeper level? In short, what is it that children learn when they learn arithmetic, and how is that learning stored in memory for later use?

My remarks today will focus heavily on the component labeled "Network of Stored Facts", the declarative knowledge component in the model. Let me spend a few moments justifying this focus. When I began this program of research, the empirical literature in the area struck me as decidedly unbalanced. That is, there were a fair number of studies of very young children's arithmetic performance, and of course extensive reports on other number processes such as counting. The latest theoretical word at that time (Groen & Parkman, 1972) suggested that, by and large, first graders approach simple addition problems by means of a counting process. The imbalance in the field, in my opinion, was that little serious consideration had been given to the mental processes of addition beyond the early elementary school years. In terms of the model in front of you, most of the serious research indicated an involvement of procedural knowledge for first graders' addition performance. To put it bluntly, however, if children follow the advice of their teachers, then they actually "learn their number facts". This memorized information must be stored somewhere in memory, in some component like the declarative store in the illustrated model. Yet hardly any research had concerned itself with the nature of such storage, with the question of "how is this acquired information stored in memory?". Equally neglected in the literature was the development of these knowledge structures in older children. My research over the past seven years has investigated exactly these questions.

I will describe the highlights of this program of research in the context of a computer simulation which I have developed. This simulation effort has forced me to be quite specific about a variety of mechanisms and processes, specifically the notions of network storage, accessibility of information, and automatic processes; such specificity is one of the widely acknowledged virtues of modeling psychological process by computer simulation, of course. Further, this excercise has clearly revealed pockets of ignorance or inattention in the literature -- for instance, we need extensive research on children's acquisition of multiplication, and on the "metacognitive" strategies they devise in the course of arithmetic instruction. Finally, and most importantly, the simulation has generated empirical predictions for a new phase of research, involving the notion of automatic versus conscious memory retrieval across development. We are now beginning this new phase of research, checking on the predictive accuracy of the simulation model.

For today's presentation, I will limit myself to four interrelated aspects of the model. In order, I will discuss 1. the problem size effect, 2. the network nature of the memory representation, 3. the development of addition knowledge, and 4. predictions of automatic and conscious retrieval. An overriding characteristic of the model is its assumption that declarative knowledge about arithmetic is stored in much the same way as any long term memory knowledge, as an interrelated network of facts and relationships. The basic mental process by which these facts are accessed or retrieved is assumed to be one of spreading activation (see Anderson, 1982; Collins & Loftus, 1975).

1. The Problem Size Effect. As I indicated at the outset, the most fundamental empirical result to be explained is the problem size effect, the fact that larger problems are more time consuming in a RT task (see Ashcrat, 1982, for a review), are more prone to error (Siegler & Shrager, 1984), and are judged as more difficult under a variety of rating situations (Ashcraft, Fierman, & Bartolotta, 1984; Stazyk, Ashcraft, & Hamann, 1982). This effect was traditionally indexed by variables like the smaller of two numbers being added, the sum of the two numbers, and so forth. Resnick and Ford's (1981) careful scholarship pointed out that problem difficulty was an issue addressed even as early as the mid 1920's (e.g., Clapp, 1924). For now, assume that the basic addition problems 0 + 0 up through 9 + 9 are inherently different in their level of difficulty, that the larger ones are more difficult than the smaller ones, and that tie problems like 2 + 2 or 9 + 9 are, for some mysterious reason, exceptions to this general rule; this "inherent and mysterious assumption" actually derives from developmental processes to be described below. In other words, it is not an assumption built into the model arbitrarily, but rather is a consequence of acquisition and development processes.

The simulation model represents the basic addition problems as nodes in a memory network, each node having its own particular level of difficulty or accessibility. The basic phenomenon of memory retrieval is a time consuming mental process of spreading activation, a process which accesses the network of memory nodes. Time for access is a function of the node's strength or accessibility, where these values are merely the empirical ratings of difficulty collected from a sample of college adults. The success of this scheme, representing the addition facts as nodes of varying strengths, is revealed in the next figure (3); on the left you see adults' RT data, plotted across the sum of the problem. The exponential increase in RT across sum is represented here by the predictor variable sum^2, and the curve in the left panel is the best fitting regression line to the data. On the right you see the performance of the simulation model, based on the accessibility values I described, with the same empirical regression line drawn through the predicted points. The fit of the simulated data to the empirical regression curve is quite good; the systematic exceptions to this fit are the circled points, tie problems, and points flanked with dashes, problems containing an addend of zero (e.g., 8+0). So, the simulation successfully predicts the familiar problem size effect, with the prediction based on node accessibility or strength.

2. Network effects. In the simulation, each of the digits 0 through 9 is a memory node, representing as it were the column and row headings of a printed addition table. These entry nodes are termed "parent nodes", since they are the initially activated memory locations during search. When two addends are presented, say 4 + 3, the simulation retrieves the answer to this problem by means of a spreading activation search. That is, each of the two

parent nodes begins passing activation to its "family members", nodes connected to the parent nodes by the network pathways. The amount of activation accruing at the family members is a function of two factors, the node's accessibility and its distance from the parent nodes which activate it. (If the stimulus problem states an answer, e.g., 4 + 3 = 7, then each such answer node also receives activation; this feature of the model is largely irrelevant to the present explanation.) Each of the now activated nodes in the network (family members as well as answer nodes; i.e., nodes which coincidentally represent the same answer, but for a different problem; e.g., 2 + 5 = 7), continues to spread activation through the network, in what might be called the "second phase" of the search process (this label is actually quite arbitrary, in that the entire activation process is a seamless procedure which makes no distinction between "first" and "second" generation search; the label is a convenient device for explaining the search stage activity, however). At the end of this search, the simulation selects that node with the highest level of activation to be the answer to the stimulus problem.

While several important retrieval effects are simulated by this search procedure, one in particular constitutes strong evidence for the assumption of a network structure. In our adult data, subjects given a multiplication problem like 7 X 5 = 28 were particularly slow in their judgments, presumably since 28 is a multiple of 7 (figure 4). Multiples seem to be very confusable with the correct answers. The spreading activation process just described generates exactly this confusion effect in the simulation (figure 5, see Ashcraft, 1983; Stazyk et al., 1982); the so-called "confusion answers" in our empirical report, the incorrect 28 for instance, turn out to be the family members which receive the greatest amount of activation during search in the simulation model. To continue with the 7 X 5 = 28 example, the spread of activation from 7 and 5 primes the adjacent 28 node to a great degree, and as such this node competes aggressively with the correct answer when a decision is finally made about the stimulus problem. In short, network "relatedness" effects, quite analogous to those found in the semantic memory literature, are both characteristic of the empirical results and characteristic of the simulation model's performance.

3. The Development of Addition Knowledge. In the simulation, I assume that each addition problem has some level of strength associated with it, where the strength indicates the accessibility of that node of information. We must address the issue of how these strengths accumulate, first because this is the developmental heart of the matter, and second because this is where much of the controversy in this area of research resides; parenthetically, it is also important to discuss this since it is the "inherent and mysterious" assumption that I promised to justify. Let me give what will sound like a distressingly oversimplified answer to the question, then explain how that answer meshes with the empirical literature and with the simulation. In answer to the question "how do the different strengths accumulate?", I respond "practice". By the answer "practice", I include any encounter with the arithmetic problem, whether informal or formal. This answer specifically includes the "invented addition" knowledge so compellingly described by Siegler and Shrager (1984), where incorrect solutions may even temporarily elevate a wrong answer to the status of most likely to be retrieved. "Practice" also specifically includes any and all classroom exposure to the problems, whether in the form of drill,

instruction on the structure or other meaningful aspects of arithmetic, or individual problem solution.

A particularly important source of node strength, notice, is the child's own reconstructed answer; in other words, feedback from performance strengthens the memory nodes. Thus, in the simulation, if a problem is of such low accessibility that it cannot be retrieved, then processing shifts to the procedural component illustrated earlier. In this component, the child's use of a counting strategy is simulated, with the result that the problem is solved correctly but with considerably longer RT (Siegler & Shrager, 1984). I assume that this correct solution adds an increment of strength to the memory representation for that problem. Across time, then, successful reconstructions of addition problems should "feed" the declarative network, much the way that successful mnemonic devices strengthen a piece of information to the point that it is retrievable without assistance from the mnemonic. While I believe this to be an important aspect of the model, I make no claim that the idea is novel in any way -- it is, after all, merely a mechanism by which we learn from our experiences.

There is other evidence related to this "practice" answer which suggests how the particular node strengths come about. Siegler and Shrager (1984) have documented the differential strengths of simple addition facts for a group of kindergartners, from performance on simple addition tasks, and from estimated frequencies of presentation in parent-child interactions. There was also an important project by Hamann (1983), in which elementary grade arithmetic books were examined. Based on Hamann's results (figure 6), it seems clear that the smaller addition problems enjoy higher frequencies of presentation in classroom materials. Equally interesting is the fact that the larger problems seem never to catch up to the smaller problems, even across several subsequent years of schooling. Smaller problems, thus, are more likely to be invented by the pre-schooler, are more likely to be presented by the parents, and then benefit from more frequent presentation in the classroom. Beyond a rather global improvement for all problems later in schooling, no special developmental assumptions need be made to forsee the typical adult's pattern of node accessibilities — problems which were initially stronger remain so, and those initially weaker, remain weak. (see also Lachman, Shaffer, & Hennrikus, 1974; Whaley, 1978) Figure 7 illustrates the developmental problem size effects which are generated by the simulation. Clearly, there is a very decent fit between these simulated data on the right and the empirical results on the left.

4. Predictions of Automatic and Conscious Retreival. A serial and exhaustive scan of the literature on automatic and conscious mental processes reveals the following generalization: the circumstance under which conscious processes become automatic is practice, massive, repetitive, consistent practice (e.g., Posner & Snyder, 1975; Shiffrin & Schneider, 1977). You will no doubt notice that the "psychoactive" ingredient for automaticity is precisely the same ingredient assumed by the simulation to account for item accessibility and developmental effects. Indeed, the simulation makes a strong prediction concerning automatic retrieval of addition facts — once a fact achieves some criterion level of strength, such that it can be retrieved consistently without recourse to counting procedures, then further strengthening of the node renders the search and retrieval process more automatic. A remarkably straightforward prediction from the simulation, then, is that the smaller addition

problems, those of higher accessibility, will show the effects of automatic retrieval earlier in development. No radical shift from conscious to automatic is predicted, notice; the progression is predicted to be a smooth and continuous one. Automaticity, in the simulation, is merely a by-product of the gradual strengthening of memory nodes across development. Figure 8 shows the developmental progression of automatic retrieval, as predicted by the simulation.

In conclusion, the model presented here successfully simulates the pattern of results found in developmental studies of simple arithmetic. The model itself derives from reasonable assumptions about the nature of human memory organization and processes. An especially noteworthy feature of the model is that the same fundamental mechanism, learning, accounts for all four aspects of the model I've discussed. With learning comes first a representation of arithmetic knowledge in a network-node structure, and then a strengthening of those nodes across development. Retrieval from the network depends directly on node strength, and become more automatic as strength reaches asymptotic values. We are currently testing some of the predictions made available through this simulation effort.

References

Anderson, J.R. (1982) Acquisition of cognitive skill. Psychological Review, 89, 369-406.

Ashcraft, M.H. (1982) The development of mental arithmetic: A chronometric approach. Developmental Review, 2, 213-236.

Ashcraft, M.H. (1983) Simulating network retrieval of arithmetic facts. Learning, Research, and Development Center, University of Pittsburgh, 1983/10.

Ashcraft, M.H., Fierman, B.A., & Bartolotta, R. (1984) The production and verification tasks in mental addition: An empirical comparison. Developmental Review, 4, 157-170.

Clapp, F.L. (1924) The number combinations: Their relative difficulty and frequency of their occurrence in textbooks. Bureau of Educational Research Bulletin No.1, Madison, Wi.

Collins, A.M. & Loftus, E.F. (1975) A spreading-activation theory of semantic processing. Psychological Review, 82, 407-428.

Groen, G.J. & Parkman, J.M. (1972) A chronometric analysis of simple addition. Psychological Review, 79, 329-343.

Hamann, M.S. (1983) Simple addition problems in elementary texts: A frequency study. Unpublished manuscript, cited in Siegler & Shrager, 1984.

Lachman, R., Shaffer, J.P., & Hennrikus, D. (1974) Langauge and cognition: Effects of stimulus codability, name-word frequency, and age of acquisition on lexical reaction time. Journal of Verbal Learning and Verbal Behavior, 13, 613-625.

Posner, M. I & Snyder, C.R.R. (1975) Facilitation and inhibition in the processing of signals. In P.M.A. Rabbitt & S. Dornic (Eds.) Attention and performance V. New York: Academic Press.

Resnick, L.b. & Ford, W.W. (1981) The psychology of mathematics for instruction. Hillsdale, N.J.: Erlbaum.

Shiffrin, R.M. & Schneider, W. (1977) Controlled and automatic human information processing: II. Perceptual learning, automatic attending, and a general theory. Psychological Review, 84, 127-190.

Siegler, R. & Shrager, J. (1984) Strategy choices in addition: How do children know what to do? In C. Sophian (Ed.) Origins of cognitive skills. Hillsdale, N.J.: Erlbaum.

Stazyk, E.H., Ashcraft, M.H., & Hamann, M.S. (1982) A network approach to mental multiplication. Journal of Experimental Psychology: Learning, memory, and Cognition, 8, 320-335.

Sternberg, S. (1969) The discovery of processing stages: Extensions of Donders' method. In W.G. Koster (Ed.) Attention and performance II. Acta Psychologica, 30, 276-315.

Whaley, C.P. (1978) Word-nonword classification time. Journal of Verbal Learning and Verbal Behavior, 17, 143-154.

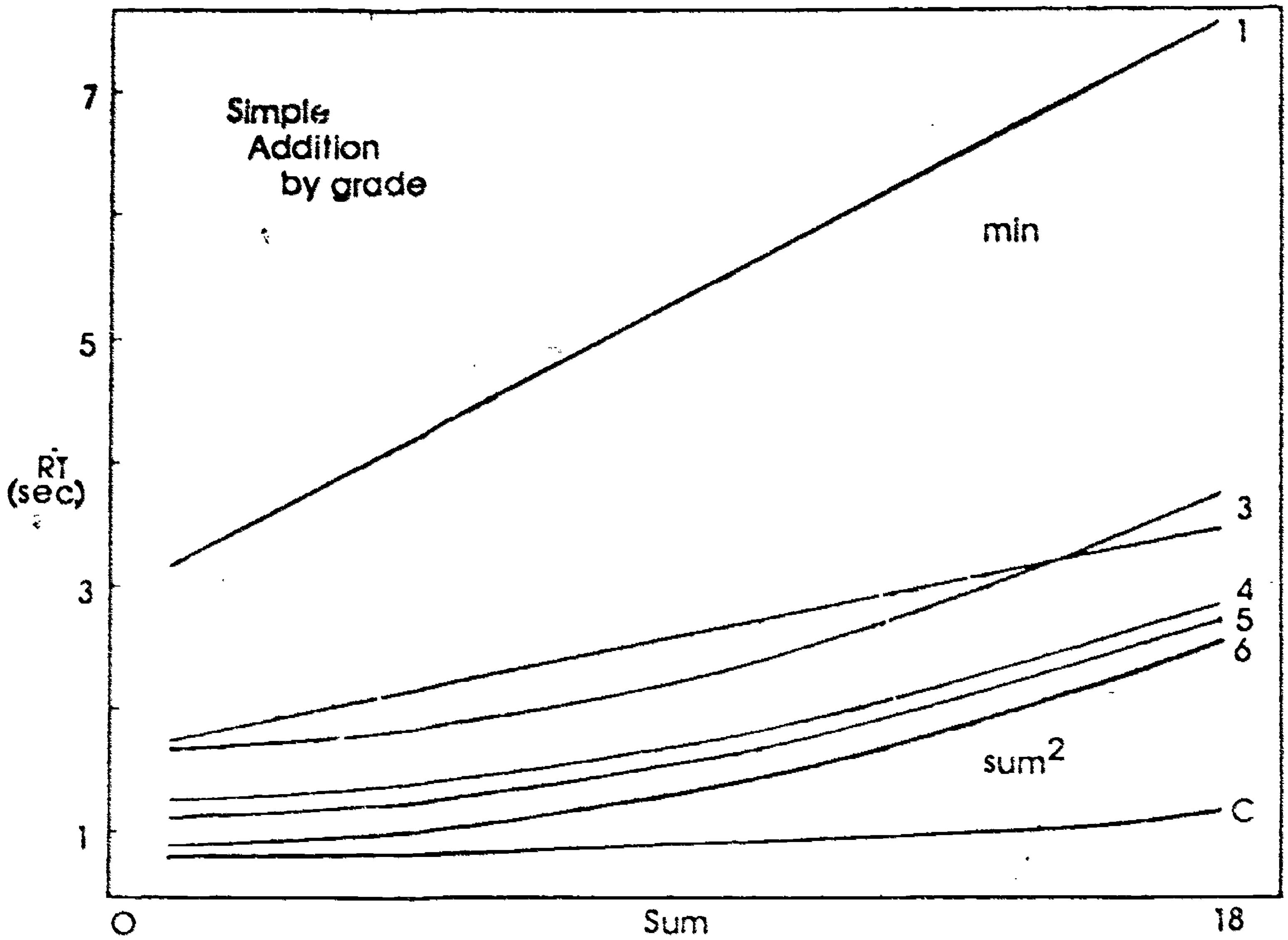
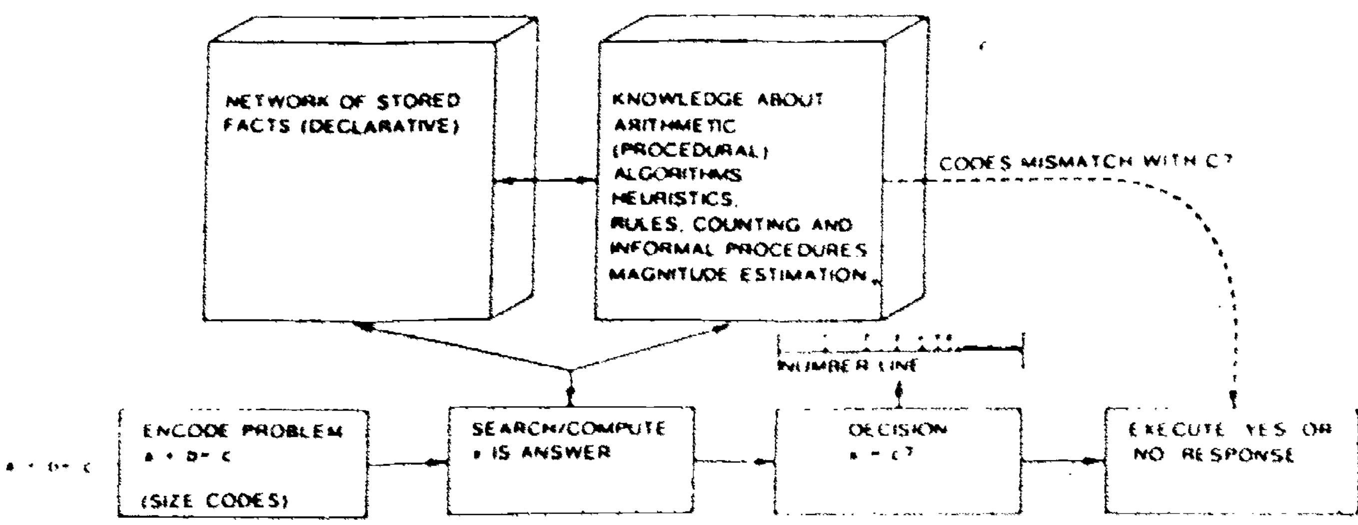


Fig 1



$RT = t_{ENCODE} + t_{SEARCH/COMPUTE} + t_{DECISION} + t_{RESPONSE}$

WHERE $t_{S/C} = f$ (NETWORK DISTANCE AND PROCEDURAL INVOLVEMENT)

AND t_D = NEGATIVE f(SPLIT)

AND $t_E + t_R = K$(y - INTERCEPT)

Fig 2

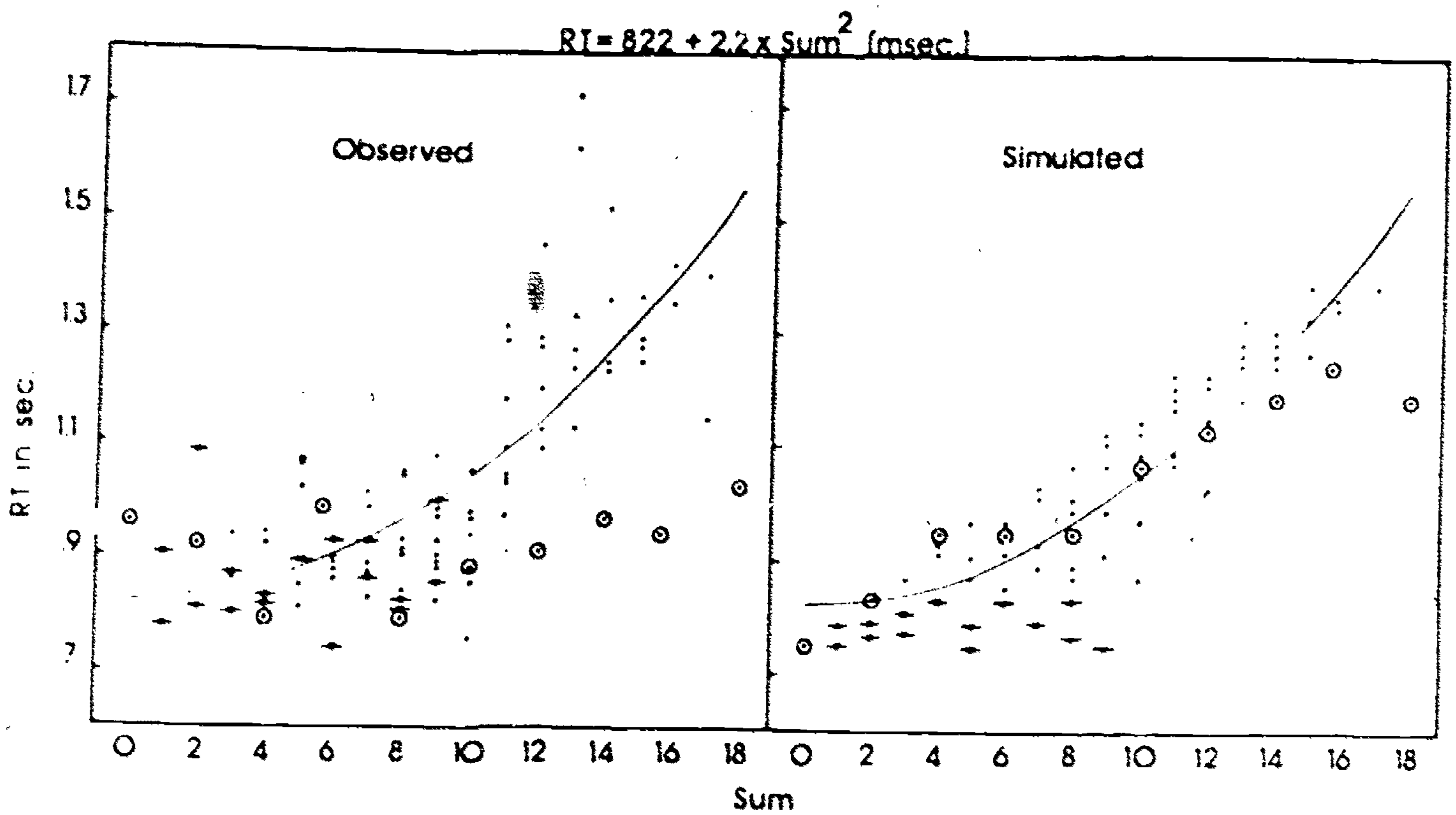


Fig 3

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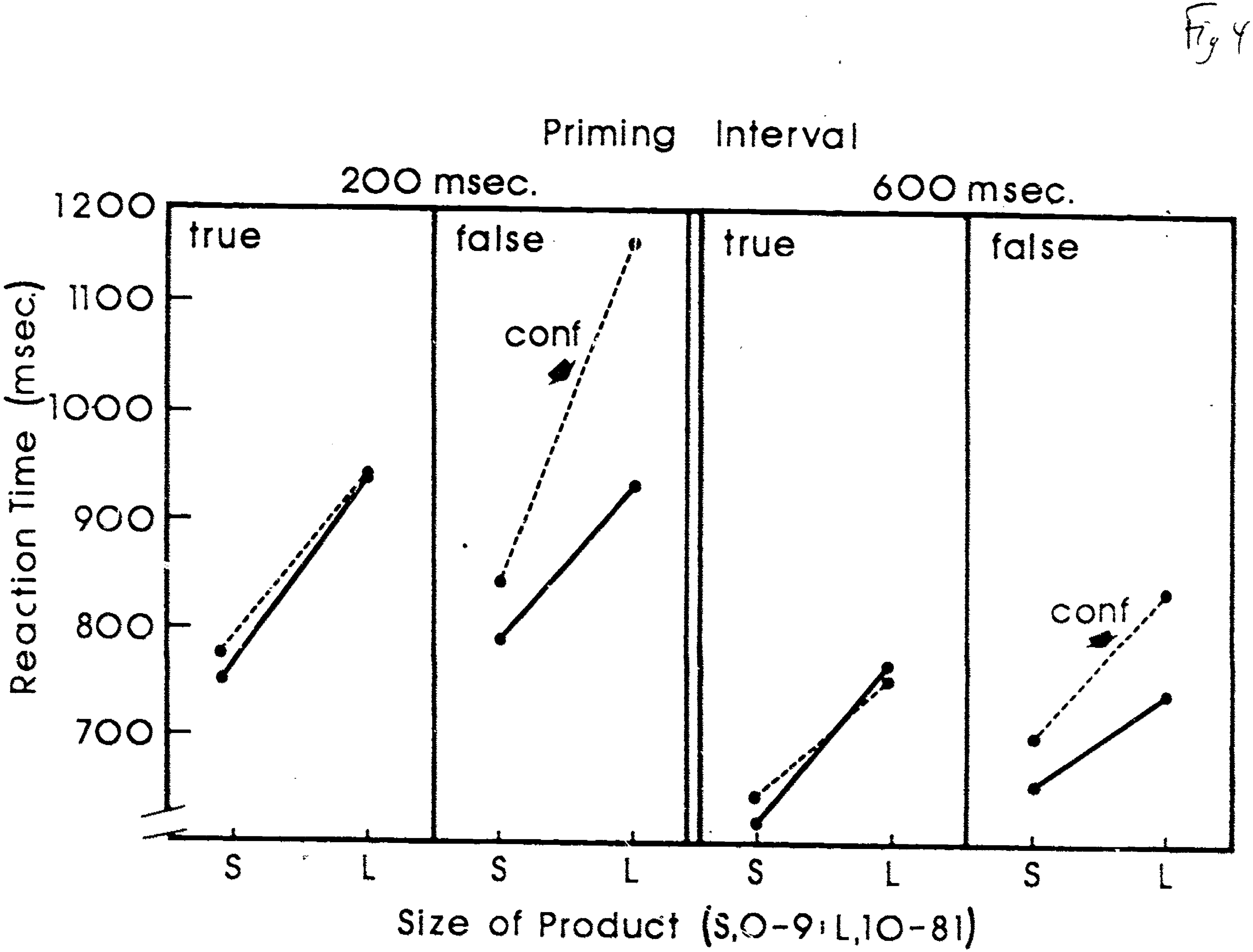

Fig 4
Priming Interval
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true
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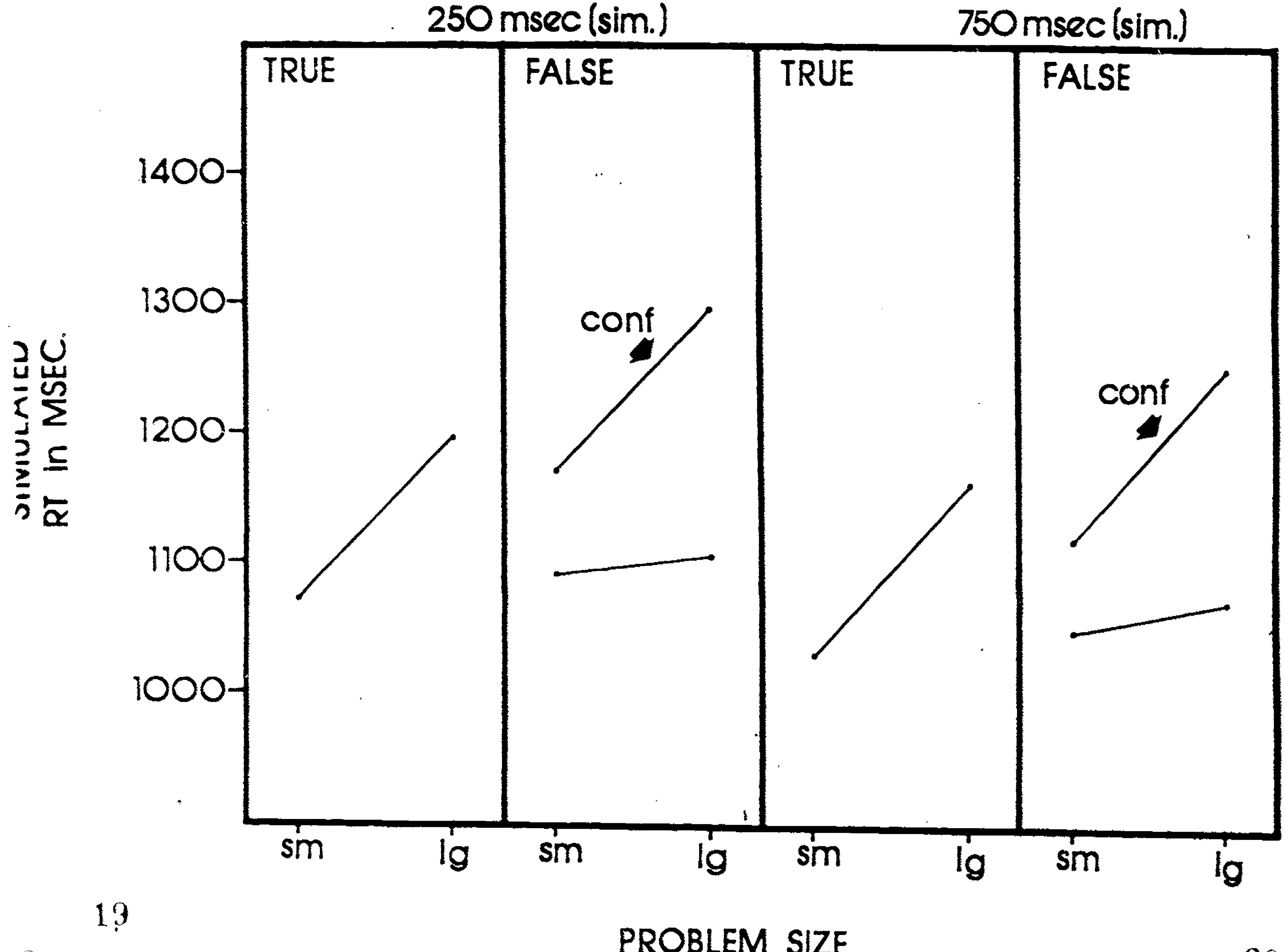
Fig 5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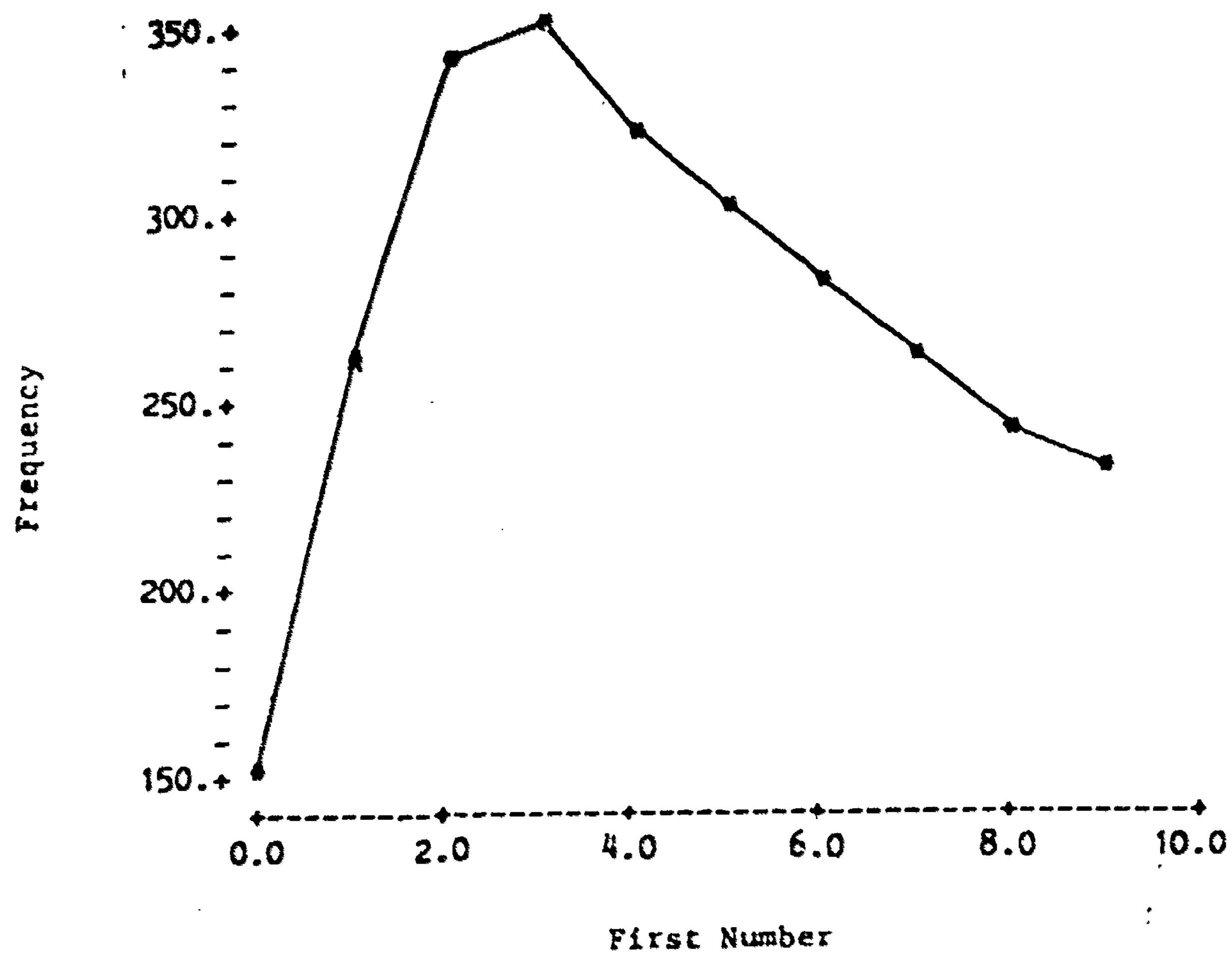
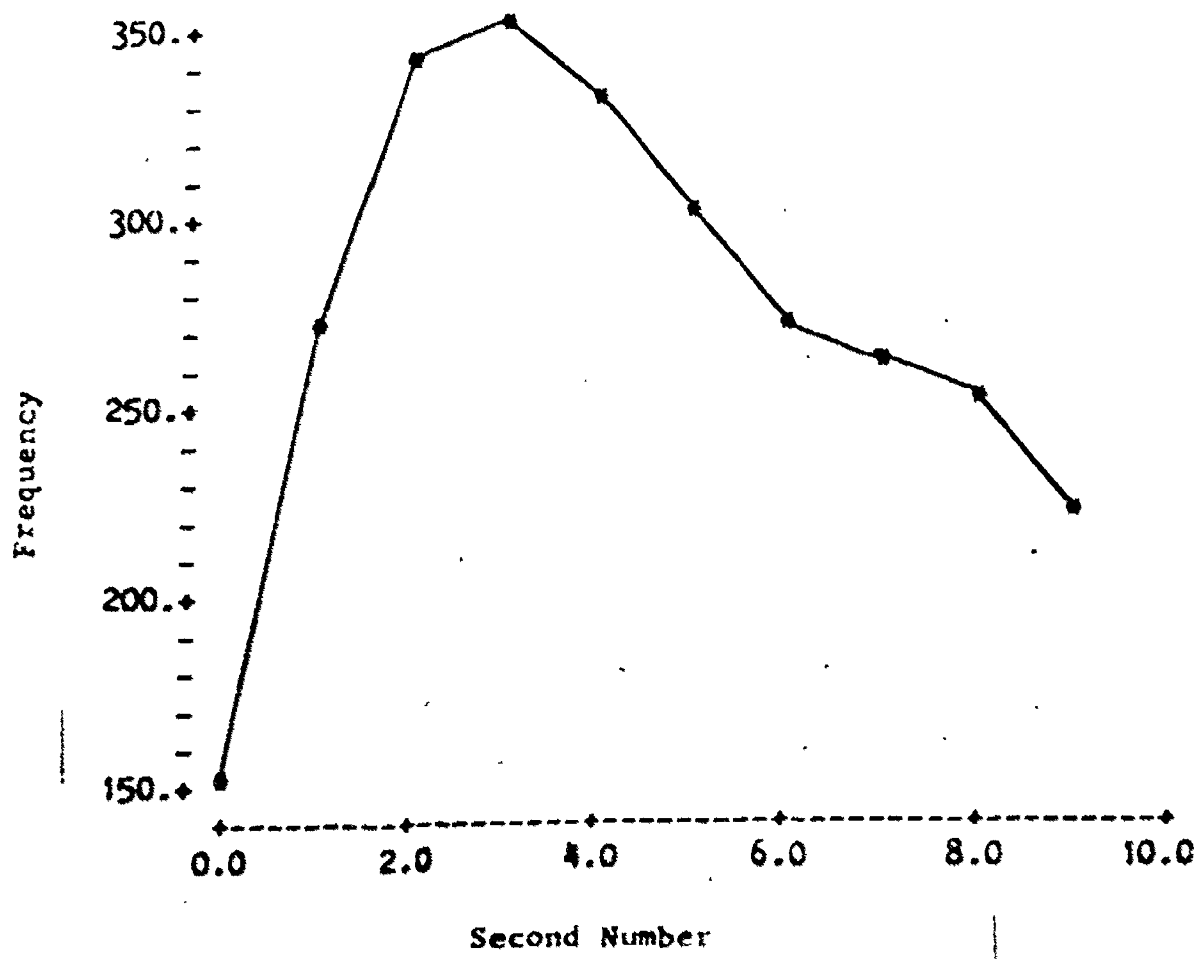
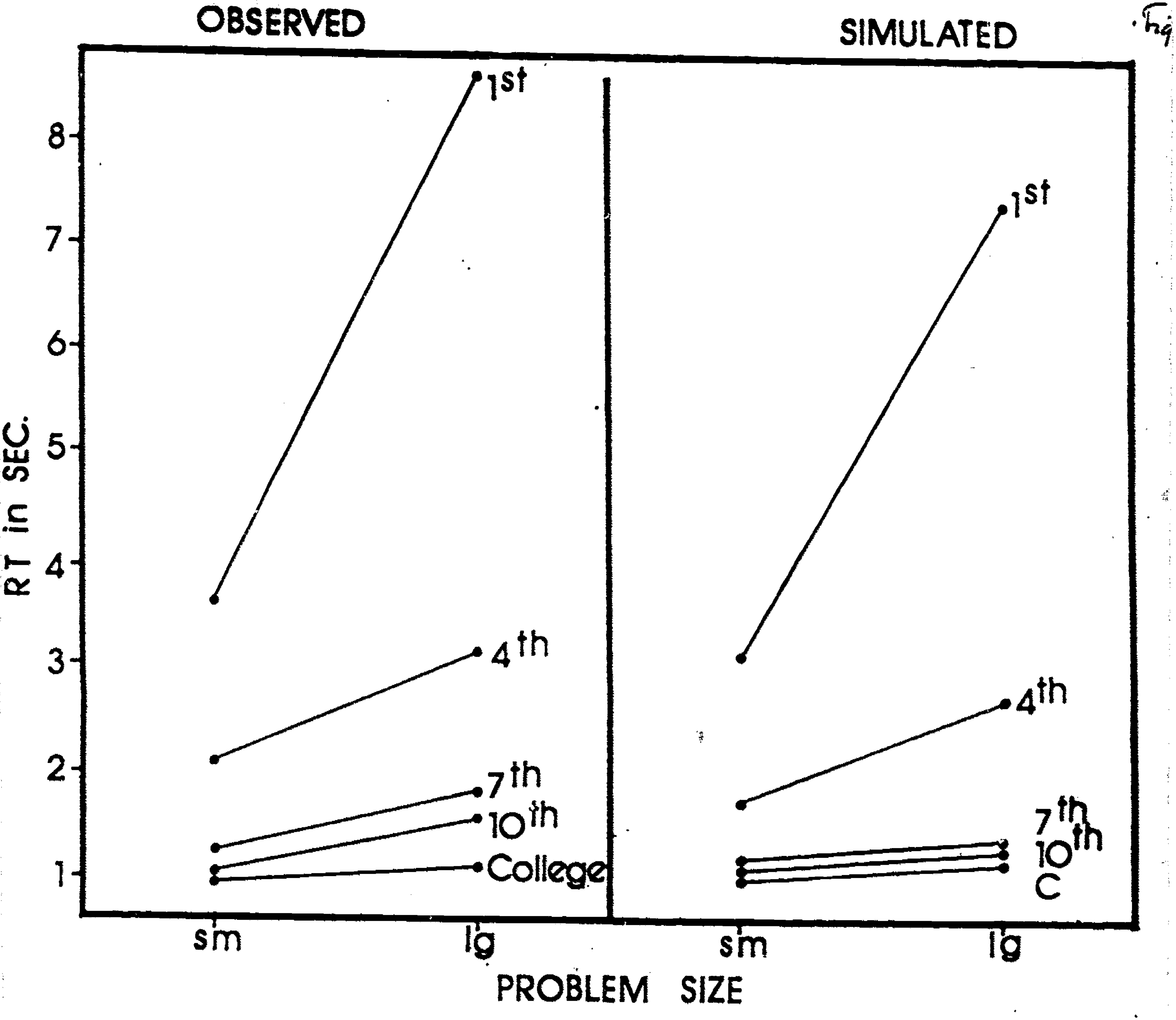
PROBLEM SIZE

Fig 6





Hamann (1983)


OBSERVED
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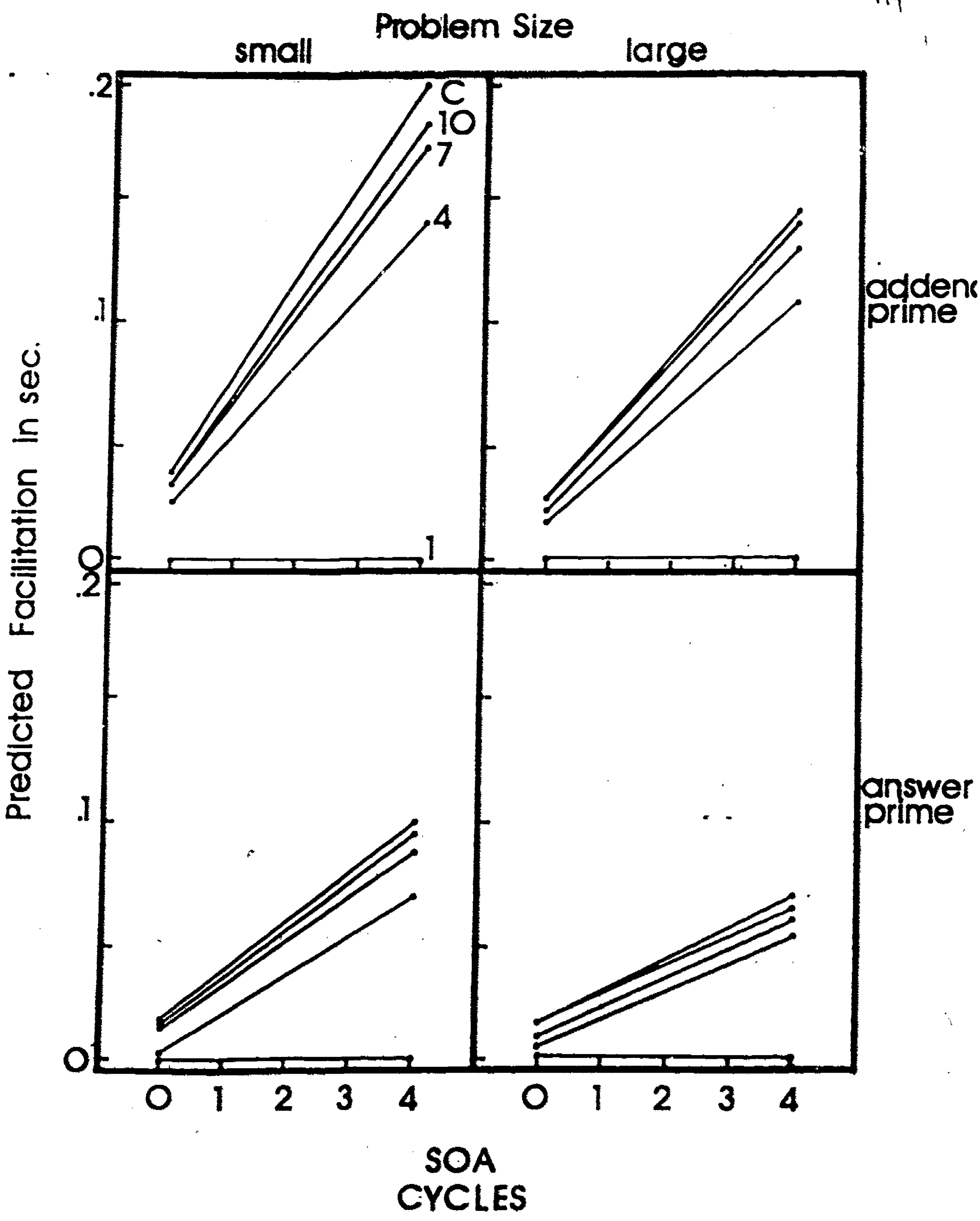

Problem Size
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Predicted Facilitation in sec.
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SOA CYCLES