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**ABSTRACT**

This issue of "Investigations in Mathematics Education" contains: (1) 12 abstracts of research studies in mathematics education; (2) a list (by EJ number) of mathematics education research studies reported in the July-to-December 1984 issues of "Current Index to Journals in Education" (CIJE); and (3) a list (by ED number) of mathematics education research studies reported in the July-to-December 1984 issues of "Resources in Education" (RIE). The studies abstracted focus on: order and equivalence of rational numbers; relation between cognitions and performance of mathematics anxious students; effects of an instructional systems approach on the concept attainment of sixth-grade Anglo and Hispanic students; use of manipulatives and games in elementary school classrooms; mathematical attitudinal data on eighth-grade Japanese students measured by a semantic differential; performance using drawn, verbal, and telegraphic story problem formats; sex differences in quantitative scholastic aptitude test (SAT) performance; variations in state SAT performance; effectiveness of a cross-age tutoring program in mathematics for elementary school children; solving textbook word problems; the effects of combining cooperative learning and individualized instruction on student mathematics achievement, attitudes, and behaviors; and inducing cognitive growth in concrete operational college students. (JN)

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# MATHEMATICS EDUCATION INFORMATION REPORT

INVESTIGATIONS IN MATHEMATICS EDUCATION  
Volume 18, Number 1 - Winter 1985

THE ERIC SCIENCE, MATHEMATICS AND  
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INVESTIGATIONS IN MATHEMATICS EDUCATION

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Behr, Merlyn J.; Wachsmuth, Ipke; Post, Thomas R.; and Lesh, Richard. ORDER AND EQUIVALENCE OF RATIONAL NUMBERS: A CLINICAL TEACHING EXPERIMENT. Journal for Research in Mathematics Education 15: 323-341; November 1984.

Abstract and comments prepared for I.M.E. by DONALD J. DESSART, The University of Tennessee at Knoxville.

### 1. Purpose

This study investigated some of the understandings of the order and equivalence of rational numbers by 12 fourth-grade students who were each interviewed 11 times during an 18-week teaching experiment. In particular, the students' strategies for comparing three distinct types of fraction pairs were identified.

### 2. Rationale

Recent national assessments have demonstrated that many children have great difficulty learning and applying concepts of rational numbers. Children, for example, add the numerators and denominators of two fractions when finding the sum of these fractions, e.g.,  $\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$ . Many children do not consider the numerator and denominator of a fraction in relation to one another but rather think of them as separate objects, thus giving 19 or 21 as an estimate for  $\frac{12}{13} + \frac{7}{8}$ .

Previous research has concentrated upon children's thinking about rational numbers in ordinary school situations, has investigated students' learning with experimental materials, or has concentrated upon children's handling of the equivalence of fractions in relation to proportional reasoning. These studies have usually used clinical-type research procedures.



This study is a report from the Rational Number Project which was supported by the National Science Foundation at several universities during 1979 through 1983. A major focus of this Project was the role of physical models in learning concepts about rational numbers.

### 3. Research Design and Procedures

An 18-week teaching experiment was conducted in schools of St. Paul, Minnesota and DeKalb, Illinois between October 1980 and March 1981. Twelve fourth-grade students were subjects of the study, with six at each site. The program of studies consisted of 13 lessons, identical at each location. The children worked individually or in groups but did not receive any other formal instruction on rational numbers.

Part-whole interpretations of rational numbers were introduced by circular and rectangular pieces of laminated construction paper. Each unit fraction,  $\frac{1}{n}$ , for  $n$  from 1 to 10, 12, and 15 was presented by different colored paper. Later instruction utilized Cuisenaire rods, paperfolding, and poker chips. The complete instructional phase included five topics: naming fractions, equivalent fractions, comparing fractions, adding fractions of the same denominator, and multiplying fractions.

Each of the 12 children was interviewed 11 times or about every 8 days during the 18-week period. The interviews were audio- or video-taped for later transcription and analysis. The interviews ranged across the major strands of the Project. However, this paper reported only those observations dealing with the order and equivalence of rational numbers, which came up during most of the interviews.



#### 4. Findings

The major concern of the interviewers was to provide an explication of the thought processes of the children from which strategies for determining order and equivalence could be inferred. The strategies were reported according to three classes of fractions: (a) fractions with the same numerators, (b) fractions with the same denominators, and (c) fractions with different numerators and denominators.

In analyzing the interviews covering fractions with the same numerators, five distinct strategies were encountered: (a) Numerator and Denominator: a strategy in which the child decided that if the same numerator was present in both fractions, the smaller fraction had the larger denominator; (b) Denominator Only: a strategy in which the explanation offered by the child only referred to the denominator of the fraction; (c) Reference Point: a strategy in which the child referred both fractions to a third fraction; (d) Manipulative: the child explained his or her decision by using pictures or manipulatives; and (e) Whole Number Dominance: this invalid strategy centered entirely on the size of the denominator--the larger fraction possessed the larger denominator. At both Minneapolis and DeKalb the most prevalent strategy late during instruction was the Denominator Only Strategy; whereas, early during instruction the Whole Number Dominance Strategy was prevalent.

In dealing with the second class of fractions (same denominators), the children used five strategies: Numerator and Denominator, Reference Point, Manipulative, Whole Number Consistent, and Incorrect Numerator and Denominator. The most prevalent strategy used at both DeKalb and Minneapolis was the Numerator and Denominator Strategy.

Six strategies were suggested by the children in dealing with fractions having different numerators and denominators. These were: Application of Ratios, Reference Point, Manipulative, Addition, Incomplete Proportion, and Whole Number Dominance. The most prevalent strategy at DeKalb was Application of Ratios and at Minneapolis was Manipulative.

## 5. Interpretations

The authors provide the following implications of their work:

a. Most children after having been provided adequate instruction deal with questions of order and equivalence of fractions by the end of the fourth grade.

b. Understanding order and equivalence depends upon an understanding of a compensatory relation among the size and the number of equal parts in a partitioned unit.

c. More time should be devoted to instruction in this compensatory relation than is provided in most modern curricula.

d. Children who are insecure in their understandings of rational number concepts often display an interference from their previous understandings of whole number concepts.

e. Some children invented the Reference Point Strategy which may be an outgrowth of or related to estimation skills.

f. Some children remain dependent upon manipulatives to the point of being inefficient.

### Abstractor's Comments

Investigations of this kind often raise numerous questions. For example:

1. The number of children selected at each site was very small (6). Were they randomly selected? Were some criteria used in their selection? Were the children bright, average, slow? These items certainly would affect the observed strategies.

2. What was the role of the classroom teachers? Were they specially trained for this experiment? Were they rated as good, average, or poor teachers?

3. The prevalence of certain strategies at both sites, e.g., "Denominator Only" with Unit Fractions, might have reflected teachers' attempts to instill (drill) some "rule behavior" in the children. Was this done?

4. It appears that the distinction between the "Numerator and Denominator" and the "Denominator Only" strategies with unit fractions is very slight. It seems the second strategy can only instill "bad habits" in children!

Overall, the investigators are to be congratulated on an excellent piece of work that should add significantly to the research literature on childrens' learning of rational number concepts. It is a pity that relatively few elementary school teachers will read the report in JRMF, or this review! The problem of making available "ideas" from research and not necessarily stable, definitive results (Do they ever exist?) to school practitioners is a serious problem which mathematics educators should soon face if the research enterprise is to ever significantly aid in the improvement of instruction in the schools.

Fulkerson, Katherine F.; Galassi, John P.; and Galassi, Merna Dee.  
 RELATION BETWEEN COGNITIONS AND PERFORMANCE IN MATH ANXIOUS STUDENTS:  
 A FAILURE OF COGNITIVE THEORY? Journal of Counseling Psychology  
 31: 376-382; July 1984.

Abstract and comments prepared for I.M.E. by GRACE M. BURTON,  
 University of North Carolina, Wilmington, North Carolina.

### 1. Purpose

This investigation was designed to test three specific questions:

- (a) How do cognitions during mathematical problem solving vary as a function of mathematics anxiety level?
- (b) How do cognitions of men and women differ when they are solving mathematical problems?
- (c) To what extent is variation in mathematics performance accounted for by variation in cognitions?

### 2. Rationale

Mathematics anxiety, sex differences in mathematical achievement and attitude, and the role of "self-talk" in mathematics achievement have been topics of much concern in recent years. Although a person's self-critical and self-congratulatory tendencies have been considered important mediators of behavior and behavior change, empirical evidence for this claim is scanty. In addition, it is not known if differences between "self-talk" patterns of men and women or between math-anxious and non-math-anxious students exist. Delineation of these differences is important if counseling math anxious students is to be maximally effective.

### 3. Research Design and Procedures

During the summer of 1981, five hundred eighty-two students from introductory courses at a large state university were given the

Mathematics Anxiety Rating Scale (MARS), a five-point Likert-type scale of 98 items. From those scoring in the bottom one-third and from those scoring in the top one-third of the MARS distribution, a random selection, stratified by sex, was chosen. The sample consisted of a high-anxious group of 16 men and 18 women and a low-anxious group of 19 men and 18 women. The mean on the MARS for the high-anxious men was 229.38; that of the high-anxious women 254.28. The mean on the MARS for the low-anxious men was 126.63; that of the low-anxious women was 133.78. SAT-Q scores were obtained for each subgroup. Those for the high- and low-anxious men were 494.38 and 591.58, respectively; those for the high- and low-anxious women were 480.56 and 526.11, respectively.

Following a practice session, the students were asked to say aloud whatever thoughts they had as they completed six mathematics problems drawn from the SAT. They then were asked to complete silently a set of 12 problems and the Test Anxiety Inventory. The student verbalizations were independently coded by two trained researchers blind to the hypothesis of the study. The coding categories used were: review of information, strategic plans, conclusions and solutions, attention control, self-facilitation, irrelevancies, self-inhibition, neutral statements, silence, and unclassifiable statements. A percentage of agreement close to 95% was obtained.

Both multivariate analyses and univariate analyses were completed. In the regression using cognition data, sex was entered first, followed by math anxiety, then a sex x anxiety interaction. In that of performance data, the cognition categories of attention control, self-facilitation, irrelevancies, and self-inhibition were entered together first, and then the other seven categories were entered next. Finally, a multiple regression analysis was conducted using all 11 categories as predictor variables.

#### 4. Findings

In the MANOVA, there were no significant differences ( $p > .05$ ) as a function of sex, mathematics anxiety or sex by anxiety interaction on the 11 cognitive variables. On the combined problem set and on the problem sets separately, while neither sex nor anxiety-sex interaction were significant, anxiety approached significance ( $.05 < p < .10$ ).

When the cognitive variables were subjected to univariate analysis, there were only three significant effects disclosed: men reported more irrelevancies and more neutral statements than did women, and women used more strategic calculations than did men. No significant differences were found in performance due to sex, anxiety or their interaction, although the latter approached significance.

In the multivariate regression analysis using performance on the combined problems, the second set of cognitive variables accounted for a significant ( $p < .05$ ) amount of variance (20.2%) beyond that of the first set of cognitive variables. On the think-aloud problems, neither set was significant. On the silent problems, however, both sets accounted for a significant ( $p < .05$ ) part of the variance, with the first set accounting for 14.7% and the second an additional 23.0%.

#### 5. Interpretations

That cognitions as students solve mathematics problems do not vary as a function of math anxiety, sex, or their interaction may point to a general set towards mathematics rather than a particular set towards the specific mathematics task of problem solving. It might also be that the saliency of the cognitions, not merely their number, is important.

Because cognitions were more important in the silent situation, it may be that thinking aloud does impair performance, contrary to the

current literature on this topic. Because of the lack of a strong data base on the basic assumption underlying cognitive theory, much research is needed before effective counseling strategies can be devised.

#### Abstractor's Comments

The area investigated in this study is an important one as data on the self-talk of students as they engage in mathematical tasks is scarce. Such data might help counselors and teachers of mathematics provide effective and efficient assistance to students experiencing a less-than-maximal level of mathematical functioning, whether they are math-anxious or not. Although I acknowledge the importance of the area, I have reservations as to the value of this study on three counts--the adequacy of the bibliography, the nature of the data, and the significance of the findings.

The fact that this article is essentially a report of a doctoral dissertation completed before October 1983 may account for the age of many of the bibliographic entries. It does not account for the paucity of references to related areas such as mathematics education. Mathematics anxiety and sex differences have been the focus of much attention in the last few years; authors of a paper which purports to deal with these issues should, it seems to me, assure their readers of their familiarity with recent studies and summaries of research relating to the topics under discussion.

With respect to the study itself, it appears one must trust, as the authors evidently did, that what one says one is thinking is indeed what is in one's mind. I am not convinced this is the case. Additionally, the chosen categories do not appear to be distinct. Although the raters apparently experienced little difficulty on this score, there are many statements I would have difficulty assigning to one and only one category. I would not know, for example, whether to



code "I've got it!" as inferring a solution (conclusion and solutions) or as positively evaluating knowledge (self-facilitation). "I've seen one like this before" could, it seems to me, be a neutral, an unclassifiable, or even, again, a self-facilitative statement.

Finally, the gathered data appears to have been subjected to extremely detailed treatment, given the size of the sample ( $n = 71$ ). Despite this, few significant differences were found due to sex, anxiety or the interaction of sex with anxiety and those that were found may be less than useful. It is hard to know what to do with the finding, for example, that men used more irrelevancies and more neutral statements than women did.

I can whole-heartedly concur with the authors that "cognitive and cognitive-behavioral approaches to counseling are in the early stages of development" (p. 381). I believe it is an important area to research and I hope that further investigation will draw on the literature of the many fields that impact on this interdisciplinary area and that they will uncover information both statistically and clinically significant.

Hannafin, Michael J. FRUITS AND FALLACIES ON INSTRUCTIONAL SYSTEMS: EFFECTS OF AN INSTRUCTIONAL SYSTEMS APPROACH ON THE CONCEPT ATTAINMENT OF ANGLO AND HISPANIC STUDENTS. American Educational Research Journal 20: 237-249; Summer 1983.

Abstract and comments prepared for I.M.E. by JON M. ENGELHARDT, Arizona State University.

### 1. Purpose

The study purported to compare traditional instruction and an "instructional system" approach to mathematics concept attainment across factors of gender and ethnicity. Concept attainment was defined as relative mastery on three types of mathematics skills (those previously taught, those not previously taught but for which prerequisites have been taught, and those not previously taught and for which previously taught material was not prerequisite).

### 2. Research Design and Procedures

For the sixth-grade sample in this study (four classrooms, suburban middle-class school district, 49 Anglos and 36 Hispanic students, English primary language), these three types of skills (respectively) were whole number computations, adding/subtracting decimals, and adding/subtracting fractions.

Traditional instruction was defined in terms of the materials used -- sixth-grade district-adopted mathematics text, a scope-and-sequence chart of objectives, text-related commercial practice and drill materials, and district-designed supplementary exercises. The instructional system was defined to include the above materials plus individual and class profile sheets, a pretest, and criterion-referenced skill quizzes; it also included a hierarchically sequenced set of computational skills ordinarily taught from grade 2 through 8 and relatively standardized procedures of test-teach (based on test)-test.

There was no indication of the nature of the instruction in either the traditional or instructional system approach. The experiment extended over an eight-month period.

### 3. Findings

It was the thesis of the study that systematic instruction would nullify the supposed influence of gender and ethnicity. The results did, in fact, offer some support for this. Where previous instruction was related to current instruction, differences across ethnic groups (although not type of instruction) were apparent, as expected; but where previous instruction was unrelated to current instruction, effects of ethnicity were greatly reduced and the instructional system approach was generally superior. No gender differences were found.

### 4. Interpretations

The researcher concluded that an instructional systems approach may "help to offset performance gaps often attributed to ethnic group" that presumably are facilitated (or at least not mitigated) by traditional instruction.

### Abstractor's Comments

This investigation represents an important step into examining ethnicity (and gender) in relation to instructional variables, rather than simply looking at achievement. Furthermore, it attempted to look at instruction longitudinally (8 months) and more holistically (traditional versus systems approach).

Although mathematics is used as a content area, the thrust of this study, however, is more general than specific to mathematics education. Unfortunately, despite its positive directions, mathematics educators will find the study problematic from a couple of different angles.

First, and a flaw common to many treatment comparison studies, is the lack of specificity in describing the instructional treatments. Not only is the information so sketchy as to defy understanding of the general nature of the treatments -- traditional versus systems -- but the nature of instruction on whole numbers, decimals, and fractions across the two treatments is indeterminable. Mathematics educators examining this study will be greatly concerned by the lack of specificity about instructional treatments, both generally and specifically. There may have been systematic differences in instruction between the two treatments other than those proposed that offer alternate explanations for observed differences in effect.

A second area of concern for many mathematics educators will be the operational definition of concept attainment, i.e., mastery of various computational skills. Without going headlong into a discussion of concept learning, mathematics educators would generally see the learning tasks of this study as procedure learning. Granted there may well be concept learning involved in learning these procedures, but from the absence of information on instructional treatments, either or both types of instruction may have stressed the mechanical steps to computation without reference to the mathematical structure or basic mathematical concepts out of which they arise. Furthermore, it is likely that a number of relevant mathematical concepts were attained by students regardless of their performance on a measure of computational skill.

This investigation seems to raise more questions for the mathematics educator than it answers. Its results are encouraging in some measure, but a great deal more information is necessary before it will influence future research or begin to have impact on educational practice.

Larson, Carol Novillis and Slaughter, Helen. THE USE OF MANIPULATIVES AND GAMES IN SELECTED ELEMENTARY SCHOOL CLASSROOMS, FROM AN ETHNOGRAPHIC STUDY. Focus on Learning Problems in Mathematics 6: 31-49; Winter and Spring 1984.

Abstract and comments prepared for I.M.E. by LOYE Y. "MICKEY" HOLLIS, University of Houston-University Park.

### 1. Purpose

The study describes how manipulatives and games were used in nine elementary school classrooms over an extended period of time.

### 2. Rationale

Detailed description of the classroom setting has been sorely lacking. Some of the areas related to the teaching of mathematics in which not enough is known are: (1) the amount of time the teachers allocate to mathematics instruction and the manner in which this time is used; (2) the extent to which teachers differentiate instruction; (3) the extent and nature of teachers' use of manipulative materials; and (4) the identification of the factors that determine teachers' use of non-text learning materials.

### 3. Research Design and Procedures

This ethnographic study used a sample of nine third- and fifth-grade teachers in eight sites. They were chosen from 27 Title I teachers who volunteered to receive classroom services from the mathematics project specialist. Target students who were low achievers in mathematics were identified in each classroom.

Ten to 15 ethnographic observations were conducted in each classroom over a period of approximately five weeks by four trained assistants. Observations were scheduled before, during, and following

the delivery of classroom services by a mathematics project specialist. After each observation period of approximately one hour, the ethnographic assistants wrote narrative accounts (protocols) of the classroom events from their field notes. All data were reviewed by teacher collaborators during a two-day research colloquium at the end of the school year.

#### 4. Findings

The reported findings are in three categories: description of classrooms, manipulatives and games.

##### Classrooms

Three different organizational patterns (with some variation) were observed in the first week's observation prior to the time when the mathematics project specialist began. The organizational patterns were: (1) whole-group instruction with "extra help" for children having problems; (2) some type of small-group instruction; and (3) individual contracts.

The organizational pattern changed in three of the classrooms when the mathematics program specialist was teaching the classroom. This change was to small-group instruction.

##### Manipulatives

The study provided a number of insights into the contribution of manipulatives to the mathematics learning of low-achieving students. Often low achievers were more successful doing manipulative activities than other mathematics activities.

There was a lack of discussion between teachers and students relating the mathematical symbols and equations to the manipulatives. In some cases after teachers used manipulatives for two or more weeks in a unit, they were surprised and disappointed that students were not successful on traditional abstract tasks.

### Games

Children were observed playing mathematical games as a regular part of the instructional program in six of the research classrooms. Two types of games were described in the protocols: games that provided practice of previous instruction and games that were used as vehicles for instruction to help children develop new mathematical concepts.

Management of games appeared to be an important factor in the successful use of games in the classroom. Game groups were more easily managed when more than one adult was working in the classroom and an adult led the game groups. Some of the problems that occurred in single teacher rooms were: (1) some games did not hold students' interest for the period of time required for the teacher's instructional group and the independent group to finish their activities; and (2) the social demands of the game activity, such as conflict resolution, abiding by rules, fair treatment of others, and negotiating first turn, dominated the groups' behavior.

### 5. Interpretations

Two main problems were evident: the difficulty low-achieving children had with transfer from the concrete to the abstract level and the management of peer game groups. Both of these problems seem to be related to a lack of individualized instruction.



The small-group organizational patterns were chosen by the teachers not for the purpose of varying instruction, but so that they could more easily use the manipulatives and games. Despite this, instruction in small groups especially benefited the lower achievers, who participated more actively in the smaller groups than in the whole-group instruction.

The mere use of manipulative materials in the classroom does not guarantee that mathematical learning is taking place. It would seem that in order for manipulatives to be effective teaching tools, the teacher must have (1) a good understanding of the relationship of models to mathematics, and (2) a good understanding of how to promote students' thinking about what they are doing with the models and how the models relate to mathematics.

Some suggestions for using games are:

1. Games should be chosen that reinforce present instruction.
2. Consideration must be given to the complexity of the game.
3. When selecting games, teachers should try to foresee problems such as children's disruptive behavior.
4. An answer key of some type is required if children are to play games without an adult present in the group.
5. An adult should play at least one round of a new game with the children.
6. The game group is a good spot for a teacher aide or a parent helper.

#### Abstractor's Comments

This report should be of interest to mathematics educators, mathematics supervisors, and teachers of mathematics. The report is informative and identifies some critical aspects of using manipulatives and games. They are similar or identical to some of my own observations. When a study supports your own beliefs, you do have a tendency to view it positively. Such is the case with this report.

The report describes enough of the ethnographic procedures to give the reader some insights into the procedure. More studies of this type are needed. I would hope this one would encourage others.

Minato, Saburoh. SOME MATHEMATICAL ATTITUDINAL DATA ON EIGHTH GRADE STUDENTS IN JAPAN MEASURED BY A SEMANTIC DIFFERENTIAL. Educational Studies in Mathematics 14: 19-38; February 1983.

Abstract and comments prepared for I.M.E. by LEWIS R. AIKEN, Pepperdine University, Malibu.

1. Purpose

The purpose of this study was to construct and validate a semantic differential scale of attitudes toward mathematics for Japanese school children, and to provide additional statistical data on sex-related differences in attitudes toward mathematics.

2. Rationale

Likert-type, Thurstone-type, and semantic differential scales of attitude toward mathematics developed in English are briefly described. The author maintains, however, that existing semantic differential scales of attitude toward mathematics are not culture-free, thus justifying the construction and validation of an instrument of this kind in Japan.

3. Research Design and Procedures

The author's working definition of attitude is: "Attitude is a learned implicit process which is potentially bipolar, varies in its intensity, and is part of the internal mediational activity that operates between a stimulus and the individual's more overt evaluative response pattern." Using this definition as a starting point, the author proceeded to develop a semantic differential scale of attitude toward mathematics. The instrument, referred to as MSD, requires the respondent to rate the concept SCHOOL MATHEMATICS on 17 seven-point bipolar adjectival scales. Scores on each scale range from -3 (extremely unfavorable) to +3 (extremely favorable). Total score on the MSD is the sum of scores on the 17 scales.

Participants in the study were 175 eighth-grade students (87 boys and 88 girls) in a public lower secondary school in a small Japanese city, Akita Prefecture. In addition to the MSD, two other mathematics attitude instruments, Japanese translations of the Dutton Attitude Scale and another semantic differential scale devised by Anttonen, were administered. Scores on a mathematics achievement test measuring computation with the four fundamental operations were also analyzed. The attitude instruments were administered at the beginning of the school year (April) to students in five classes containing approximately equal numbers of boys and girls. Each class was taught by one of two experienced women teachers licensed to teach mathematics in Japan.

#### 4. Findings

An analysis of the ability of the 17 MSD items to discriminate between high and low scorers on the MSD instrument as a whole showed that every item possessed significant discrimination ability. An alternate item analysis procedure devised by Brinton confirmed the findings of the first item discrimination analysis.

With respect to the validity of the MSD, total scores on the instrument correlated .67 with the Dutton Attitude scale and .47 with the Anttonen semantic differential scale; both correlations are statistically significant. Correlations of MSD scores with teacher's estimates of students' attitudes were .51 for boys, .55 for girls, and .52 for both groups. Test-retest reliability coefficients of the MSD over an interval of ten months were .67 for girls, .61 for boys, and .64 for combined groups.

A centroid factor analysis was performed on the MSD scores of a group of elementary school teachers in a previous study. Four factors, labeled Evaluation, Potency, Beauty, and Clarity, were obtained. The correlations between MSD scores and scores on the mathematics

achievement test were .50 for boys, .53 for girls, and .51 for both groups combined. The mean MSD score was significantly higher for girls than for boys, but the mean achievement test scores of the two groups were not significantly different. Comparisons of boys' with girls' scores on the MSD items revealed several item means in favor of girls, especially items having high loadings on the Evaluation factor.

##### 5. Interpretation

The author of this study concluded from the findings that: (1) overall, the MSD is superior to translated American attitude scales as a measure of attitude toward mathematics in Japan; (2) the reliability of the MSD is lower than might be expected; (3) the validity of the MSD is higher than that of comparable American instruments; (4) girls reveal more positive attitudes than boys on the MSD.

##### Abstractor's Comments

The article is fairly well written, although a bit long for an instrument-development study. The author also has a tendency to "jump around" from the current study of eighth-grade students to his previous studies of elementary teachers.

The rather modest test-retest reliability of the MSD may be attributable to the age of the respondents, the relatively long (10 months) test-retest period, and the brevity of the instrument. Internal consistency coefficients (Kuder-Richardson, Cronbach alpha) should also have been computed.

The higher-than-typical (at least in the U.S.A.) correlations of the MSD with achievement tests scores may be a reflection of the fact that the achievement test was on arithmetic fundamentals and was administered before rather than after the MSD. Also, the reader is not

told how much feedback the students received about their mathematics achievement scores or what the precise time intervals were. Finally, the fact that girls had more positive mathematics attitudes than boys is not particularly surprising at the eighth-grade level. It may be that changing social conditions in Japan, and perhaps the U.S. as well, are causing girls to develop more positive attitudes toward mathematics. Be that as it may, girls' attitudes toward mathematics are usually less positive than those of boys in high school and college than in the earlier grades, at least in the United States. Finally, the results of this study reinforce the writer's belief that one must be cautious in making cross-cultural generalizations when dealing with educational and psychological matters.

Moyer, John C.; Sowder, Larry; Threadgill-Sowder, Judith; and Moyer, Margaret B. STORY PROBLEM FORMATS: DRAWN VERSUS VERBAL VERSUS TELEGRAPHIC. Journal for Research in Mathematics Education 15: 342-351; November 1984.

Abstract and comments prepared for I.M.E. by GLEN BLUME, University of Iowa.

1. Purpose

The purpose of this study was to compare children's performance on story problems in three contexts: drawn (picture labeled with brief phrases), verbal (full sentences), and telegraphic (reduced verbiage). The study also examined the relative difficulty of the three formats for children with high or low reading ability.

2. Rationale

The authors note that a reduction of working-memory overload can be hypothesized as support for the use of the telegraphic rather than the verbal format. However, previous research by the authors found that the telegraphic format was not superior to the verbal one and that high-ability students did somewhat better on problems in the verbal format, perhaps due to the richer contextual cues provided by complete sentences.

The authors' previous research also found performance on the drawn format to be superior to the verbal format at Grade 5. Four arguments are given suggesting superiority of the drawn format: the drawing can reduce reading-related memory overload, the drawn format may help with recall of similar situations and may encourage the construction of appropriate images, it may induce students to link the meanings of the telegraphic phrases with the drawing and thereby aid understanding of the problem, and it may help organize information in the problem.



### 3. Research Design and Procedures

The sample consisted of 854 students from three cities. Eight classes chosen by administrators were used in each of Grades 3 through 7.

Three parallel forms of a 24-item test were developed for each grade and randomly assigned to subjects. Each form contained eight drawn, eight telegraphic and eight verbal items that were balanced for number size, operations required, and the like. Problems were one-step, multi-step, and extraneous data story problems requiring operations on whole numbers, fractions, and decimals, as appropriate for each grade. The eight items in each format required 12 operations; one point was scored for each correct operation-operand pair disregarding correctness of computation. Reliability (Cronbach's alpha) ranged from .79 to .88 for the three format subtests at the five grades.

The Syntactic Similarities subtest of the Test of Reading Comprehension was administered to measure reading ability. Subjects were classified as "high" or "low" based on whether they scored above or below their grade-level mean.

The 24-item story problem test was administered in two 25-minute sessions on two days, with the reading test given just prior to the second session.

### 4. Findings

For each reading level within each grade the mean for the eight drawn problems was higher than that for either of the other two formats. Across the grades, means in the three formats for the low readers were approximately four to six out of a possible score of 12. For high readers the means were approximately seven to ten.

ANOVA with format scores as repeated measures indicated significant format, grade, and reading level effects and a significant format-by-reading-level interaction. Simple format effects were significant for both reading levels, with means for the drawn format significantly higher than means for the other two formats, which did not differ significantly from each other. The grade effect seemed to be due to easier tests at Grades 4 and 5, and the reading level effect reflected better performance by the high readers.

##### 5. Interpretations

This study replicated the authors' previous studies in that the drawn format was superior and no significant difference was found between the telegraphic and verbal formats. However, unlike the previous study, the verbal format was not easier than the telegraphic for the high-ability students (high readers). The authors contend that the superiority of the high readers over the low readers for all grades and formats suggests that "some difficulty in story problem solving may be due to reading-related memory overload" (p. 349).

Low readers appear to profit more than high readers from the drawn format. The authors suggest that, although the drawn format helps both good and poor readers by making semantic processing easier, it helps poor readers even further by relieving memory overload.

The authors question that use of the telegraphic format and suggest use of the drawn format as a pedagogical aid. They argue that other formats such as oral presentation or motion portrayed by photography or computer graphics also might reduce memory overload and assist with representing and organizing problem data.

Abstractor's Comments

The study was a "clean" study in that it used an adequate sample, dealt with clearly-defined constructs such as problem formats, and extended the results of other studies in this chain of inquiry. The article was clearly written, making it easy for the reader to grasp exactly what was done in the study.

I raise two questions concerning procedures of the study. One concerns the use of only two categories, either "high" or "low," for reading ability. Would the results be the same if the upper and lower quartiles were used to define high and low reading ability?

A second question concerns the lack of information about subjects' prior experience with the three problem formats. To what extent did instruction focus on one or more of the three formats? Presumably this would influence performance on problems in those formats. Although I suspect that typical textbooks were used and the drawn format was encountered least frequently, a description of relative emphases on the three formats would further clarify the findings.

Only one suggestion for further research is offered in the article, that being inferred from the authors' discussion that other formats may offer advantages similar to those of the drawn format. I think another extension of this research would be a study designed to gather information on why the drawn format was superior. Carefully designed clinical interviews might be used to determine specific aspects of the drawn format that contribute to improved performance. This would help to clarify the extent to which working memory overload is related to problem difficulty in the various formats.

Another issue which this study raises is the appropriateness of these formats for instruction at various stages in students' work with story problems. Is the drawn format superior to the others when initially presenting new and difficult story problems? After students have had substantial experience with story problems of a certain type? The evidence provided by the simpler tests at Grades 4 and 5 seems to suggest that the format effect may be similar for problems at different levels of difficulty.

This study is a likely one to cite in answer to critics who contend that mathematics education researchers have nothing to offer to practitioners. The findings of this study shed light on an issue directly related to pedagogy and the design of instructional materials. Despite its frequent use in textbooks, the telegraphic format appears not to enhance student performance, even for students classified as below average in reading ability. Furthermore, this study suggests to teachers and materials developers that the drawn format can enhance student performance, making a difficult topic somewhat easier for students.

Pallas, Aaron M. and Alexander, Karl L. SEX DIFFERENCES IN QUANTITATIVE SAT PERFORMANCE: NEW EVIDENCE ON THE DIFFERENTIAL COURSE HYPOTHESIS. American Educational Research Journal 20: 165-182; Summer 1983.

Abstract and comments prepared for I.M.E. by C. JAMES LOVETT, Brooklyn College of the City University of New York.

### 1. Purpose

This study was motivated for the most part by the Benbow and Stanley report in Science (210: 1262-1264; December 1980). The purpose was to "provide a direct test of the hypothesis that the sex difference in quantitative SAT performance may be due to differences in the pattern of quantitative coursework taken by males and females in high school" (p. 165).

### 2. Rationale

The report notes the sex differences on tests of mathematical aptitude and achievement documented by various studies, and the attention given to social-psychological factors to account for these differences. Primary emphasis is placed on the Benbow and Stanley report which challenged the differential coursework hypothesis as an explanation for sex differences in mathematical ability. The report is criticized mainly because (1) it was not a direct test of the hypothesis in that it considered students prior to the opportunity for differential coursework and (2) because of the dubious generalizability of findings based on "a self-selected group of extremely able, highly motivated students" (p. 169). The need for the study is based on these weaknesses of the Benbow and Stanley study and the further observation that "no one has come forth with convincing evidence supporting the differential coursework hypothesis" (p. 167).

### 3. Research Design and Procedures

The data were derived from an earlier study (conducted by the Educational Testing Service), and consisted of: student sex, parent's occupations, race, 9th grade SCAT-Q score (obtained in 1965), enrollment in high school mathematics courses (13 categories, obtained from academic transcripts), enrollment in three non-mathematics quantitative courses (physics, business/commercial, industrial arts), grade point average for the above courses, and 12th grade SAT-M score (obtained in 1968). The sample consisted of 1842 females and 1770 males. The statistical method was a multiple regression analysis of the SAT-M scores on the background and quantitative coursework variables.

Citing a personal communication, the authors note a disagreement between themselves and Benbow and Stanley regarding the appropriateness of the SCAT-Q as a "control for pre-high school differences in standardized test performance when SAT performance is used as the criterion" (p. 172). Benbow and Stanley apparently take the position that the characteristic measured by the SAT-M ("mathematical reasoning ability") is not measured adequately by the SCAT-Q. Citing CEEB and ETS studies, the authors provide a detailed argument to support their position that the SCAT-Q is indeed a suitable control variable.

### 4. Findings

#### A. Findings from descriptive statistics

1. With respect to parent's education, race, and ninth-grade SCAT-Q scores, no significant sex differences were found. It is noted in particular that pre-high school quantitative performance (measured by the SCAT-Q) of males and females is about the same (males 290.1; females 289.7) in contrast to the Benbow and Stanley result (using the SAT-M).

2. "... Females are much less likely than males to take certain higher level mathematics and quantitative science courses." (p. 175)

3. The grade point average for the courses they did take were "somewhat" higher for females than for males.

4. There was a significant sex difference for the twelfth-grade SAT-M (males 425.2; females 388.5).

#### B. Findings from the regression analyses.

Three equations were considered: (1) a base-line equation with mother's and father's education, race, sex, and SCAT-Q as predictors; (2) an equation with the five previous predictors together with the 16 coursework variables; (3) the previous 21 variables together with grade point average.

The first equation indicated an adjusted average difference on the SAT-M scores of 35 points in favor of the males. Given the similarities noted between males and females on the five predictor variables, the authors expressed "...considerable confidence that the differences... in the 12th grade SAT-M performance did in fact emerge during high school" (p. 177). The resulting  $R^2$  was .569. The second equation reduced the gap in SAT-M performance to 14 points, and raised  $R^2$  to .692. "This indicates that most of the initial 35-point differential may be due to sex-linked differences in student's high school programs" (p. 178). The result indicated the plausibility of effect due to the pattern of coursework with larger benefits noted for enrollment in more advanced courses. The third equation indicated a substantial effect for grades suggesting that "...level of mastery of the material encountered in quantitative coursework has an important influence on SAT-M performance over and above that simply coming from having taken such courses" (p. 179). This equation indicated an adjusted average difference in SAT-M scores of 20.5 points and  $R^2$  of .707.



## 5. Interpretations

Considerable support is given for the hypothesis that sex differences in high school programs of study are "highly relevant" to the SAT-M differences which emerge by the end of high school. While differential coursework and grade performance do not fully explain the gap in SAT-M scores, it is premature to conclude that the residual difference is not due to socialization processes.

### Abstractor's Comments

This study is reasonably well designed, conducted, and reported. I am bothered by the use of data collected 15 to 20 years ago, but my concern is somewhat balanced by the apparent soundness of the data and the fact that large-sample, longitudinal data is difficult and expensive to obtain. Based on the methodology and findings presented, I find myself in general agreement with the authors' conclusions.

The report is a worthwhile contribution to the research literature on gender/mathematics-learning issues. I think, however, that for the overall improvement of mathematics teaching as well as for better understanding of the underlying learning processes, further research along this particular line (testing of the differential coursework hypothesis) is not necessary. The real rebuttal to Benbow and Stanley (1980) lies not in further research, but in pointing out the inadequacies of their methodology and, much more important, the interpretations they give to their findings. This, I think, has been accomplished (See Letters, Science, 212: 114-121, April 1981); now let us turn to more critical matters.

Powell, B. and Steelman, L. C. VARIATIONS IN STATE SAT PERFORMANCE: MEANINGFUL OR MISLEADING? Harvard Educational Review 54: 389-412; November 1984.

Abstract and comments prepared for I.M.E. by ALAN OSBORNE and MARGARET SOOY, Ohio State University.

### 1. Purpose

The primary objective of this research is the examination of state and regional variations in SAT performance. Factors such as percent of seniors taking the test; demographic/compositional factors such as sex, racial composition, and economic status; and school structural characteristics such as number of years of coursework in social sciences, natural sciences, and humanities, public expenditures per student, and public vs. private school are examined for their contribution to variation in SAT performance. Will taking them into account alter state or regional comparisons of performance?

### 2. Rationale

Not since the launching of Sputnik in 1957 has so much attention been focused on the American educational system. Particular attention has been paid to the decrease in scores on the Scholastic Aptitude Test (SAT) that measure preparedness for college and are used as admission criteria by some colleges. In the past twenty years the average total SAT scores have decreased from 973 to 893 out of a possible 1,600 points.

Standardized test scores are used as "objective" indicators of academic growth and potential, to evaluate differences among individuals, and as an aggregate measure of the national academic health. State and federal officials compare state scores and speculate about causes and correlates.

Since educational institutions are governed chiefly at state and local levels, an analysis of test performance at the state and regional levels is needed. With the publication of state "report cards" in educational performance by the media and state and federal officials, implications of state variation in test performance extend beyond the realm of academic assessment and into the economic sphere.

The SAT was analyzed because (a) it is taken by about two-thirds of the entering students, and (b) it is the test most frequently used by scholars, politicians, and the lay public to assess school quality.

### 3. Research Design and Procedures

Statistical reports, including information on the average SAT scores, verbal and mathematical sections of the SAT, and statistical descriptions of the test-takers for each of the fifty states, were requested and received from the Educational Testing Service. These reports included student self-reports on academic and financial background.

The state was the unit of analysis. There were fifty equally weighted observations in this study. The dependent variable was the total SAT average, but mathematical and verbal scores were evaluated separately. The operational definitions of the independent variables were:

percentage - percentage of eligible high school seniors in a state taking the SAT.

sex composition - percentage of test-takers in a state who were female.

racial composition - percentage of test-takers in a state who were black.

median income - the median family income of the test-takers in a state.

percent public - percentage of students who reported attendance in public as opposed to "other than public" schools at the time of the examination in a state.

academic years - the mean number of years spent studying social sciences, natural sciences, and humanities.

expenditures - public expenditures per student in a state.

The data were analyzed in sequential stages:

Stage 1. The impact of the percentage of test-takers on SAT scores.

Stage 2. The impact of features of the test-taking population such as sex composition, racial composition, and median income.

Stage 3. The impact of private versus public education, time spent in basic subjects, and public expenditures.

The data were reanalyzed to examine verbal and mathematical scores separately. The effects of these variables were estimated using ordinary least-squares regression equations.

#### 4. Findings

Nearly three-fourths of the state variation in SAT scores can be attributed to the percentage of students in each state who take the test. Examination of a scattergram that relates the percent of graduating seniors in each state taking the SAT and the average total SAT scores of each state showed a non-linear relation. Adding the square root of the percent to the model increased the explained variation to 81 percent.

Unadjusted SAT averages underestimated the South/non-South variation. The disparity between the averages of southern and non-southern states increased from thirty-four points to forty-six points with the corrections.

In the second stage of analysis, adding the variables sex composition, racial composition, and family income bolstered the explained variance to .922. The effect of racial composition was the most pronounced.

The relative position of southern states improved if these factors were controlled, but the regional discrepancy remained significant beyond the .01 level.

Using school/structural variables, the explained variance increased to .944. All demographic variables remained significant with the exception of income. Of the structural characteristics, only expenditures attained significance. Results did not indicate that there is a beneficial impact of private schooling on test performance. The regional disparity was reduced to a nonsignificant level.

Analyses of the verbal and mathematical scores demonstrated results similar to the total SAT analyses. The only structural variable with significant effect is expenditures.

For mathematics scores, the South/non-South difference increased with percentage controlled. Sex was a significant predictor if all demographic variables were included. Regional difference remained significant. The only school/structural characteristic that remained significant was expenditures. The South/non-South difference persisted if all of the variables were controlled.

##### 5. Interpretations

The larger part of the variance in state differences in SAT scores is attributable to the percentage as well as the compositional features of test-takers and does not reflect differences in the organization of state school systems. Comparisons of states not accounting for these factors may be seriously misleading.

Most state variation is a function of one factor, the percentage of eligible students taking the exam. The strength of this factor may be a consequence of a combination of selection biases related to percentage including ability, socioeconomic background, and educational aspirations. Sex composition affects a state's level of SAT performance only through its influence on mathematical proficiency, while racial composition affects both verbal and quantitative performance. States that encourage women and minorities to attend college may unwittingly dilute their uncorrected scores.

The number of years spent in academic coursework is not a significant predictor of state differences. These findings do not support the National Commission on Excellence's (1983) recommendation. There may be several reasons for this. First, students devoting more time to basic coursework may be those with motivation and ability who take the exam. Second, the average number of years in basic subjects may not vary enough to result in state differences. Finally, since the SAT evaluates competency on skills that are usually taught in early high school years, the addition of academic years beyond this point may be inconsequential in terms of state SAT differences.

The only school/structural characteristic tested in this model that seems to influence state/regional disparities in SAT performance is public expenditures. The greater the per student funding, the higher the state's level of SAT performance. Both verbal and mathematical performance appears to be associated positively with this factor. This contradicts recent claims that expenditures have no effect and attests to the need for using corrected SAT scores. As long as states vary in their capacity or willingness to allocate funds to education the disparity in state SAT scores may continue or be made worse.

The regional gap on total SAT performance lessens if demographic and structural characteristics are taken into account. These findings imply that if the credentials or skills required to enter occupations continue to shift to a more technical base, then students educated in the South may compare poorly to others in the labor force. If the population of the United States continues to drift southward, a further decline in SAT performance may be expected.

Educational performance is a multidimensional concept. Overreliance on any one indicator, such as the SAT test, to draw sweeping conclusions is unwise. Future researchers should broaden this research with the inclusion of other variables and with the assessment of other measures of educational effectiveness.

#### Abstractor's Comments

Evaluation is ultimately a political act. Each year the popular press considers the results of the SATs, making comparisons and comments about the nature of schools and schooling. Moreover, reactions to the SAT data are not limited to lay people and the popular press; frequently our colleagues in education are puzzled about the meaning of SAT scores, the appropriate interpretations to place on them, and what the implications are for schools and schooling. Mathematics educators are frequently called on to comment on the nature of testing results.

Powell and Steelman have provided a major service in making explicit some of the statistical artifacts inherent in SAT testing and comparisons. Their careful analysis and commentary should make mathematics educators more sensitive to sampling problems in research or evaluation projects. Some of the variables we might like to ignore do make a difference when we attempt to draw conclusions.



The fact that public expenditures was the only school/structural characteristic that seemed to influence disparities in SAT performance is an important difference. Examination of unadjusted scores does not reveal an effect of such expenditures. The consequences of erroneous conclusions that can result from consideration of unadjusted scores are evident.

This was a well-done analysis of existing data. Such secondary analysis is useful to mathematics educators in dealing with the nature of evaluation in its political aspects. We think that the "score card of the states" that is used to assess state-by-state performance across the country by the Department of Education (see the February, 1985 issue of Phi Delta Kappan) indicates clearly the political stake for mathematics educators.

#### Reference

Lewis, A. C. (1985). Washington Report. Phi Delta Kappan. 66: 387-388.

Sharpley, Anna M.; Irvine, James W.; and Sharpley, Christopher F.  
AN EXAMINATION OF THE EFFECTIVENESS OF A CROSS-AGE TUTORING PROGRAM IN  
MATHEMATICS FOR ELEMENTARY SCHOOL CHILDREN. American Educational  
Research Journal 20: 103-111; Spring 1983.

Abstract and comments prepared for I.M.E. by BARBARA SIGNER, University  
of Massachusetts-Boston.

### 1. Purpose

The purpose of this study was to conduct empirical research on the cognitive and affective effects of cross-age tutoring in mathematics with elementary school children. Both the tutors and tutees were studied for possible consequences as a result of such a program. While other mathematics learning was examined, the study focused on effects on learning "operations" of mathematics. In addition, the study sought to investigate whether there are significant differences among the tutees based on the predetermined mathematics achievement level (high or low) of the tutors.

### 2. Rationale

The rationale for this study is that most of the peer-teaching evaluation studies reported in the literature are based on anecdotal reports rather than the analysis of objective data. With the positive claims made in these previous studies, the authors sought to conduct a rigorous experimental study to investigate these findings.

In regard to the few experimental studies conducted on the efficacy of academic and social benefits of cross-age tutoring, the authors' review of the literature found many committed violations of empirical research design. Therefore, the authors were doubtful of the authenticity of the positive claims reported and designed a rigorous experimental study to substantiate claims or report the absence of experimental support.

### 3. Research Design and Procedures

Four experimental groups were formed for the study. They were based on the premeasured mathematics achievement levels of both the tutors (fifth and sixth graders) and tutees (second and third graders):

- (1) high achieving tutor/high achieving tutee
- (2) high achieving tutor/low achieving tutee
- (3) low achieving tutor/high achieving tutee
- (4) low achieving tutor/low achieving tutee.

Subjects that participated in the experimental groups came from two different schools. Four classes from one school (experimental school) and two classes from a second school (experimental/control) provided both male and female tutor/tutees.

Subjects for a control group also were chosen from two separate schools. Two classes in the experimental/control school also provided a nontutoring class of sixth graders and a nontutored class of third graders to control for within-school effects. A third independent school (noncontact control school) provided four nontutoring classes. They consisted of classes in the following grades: sixth and fifth, third and second.

### 4. Findings

Results of the study were the following:

- (1) Mathematics achievement of the tutors increased significantly more than that of nontutors.
- (2) Mathematics achievement of the tutees increased significantly more than that of the nontutees.
- (3) The predetermined mathematics achievement level of the tutors did not affect the mathematics achievement gains of the tutees.

- (4) There were no significant increases in self-esteem scores for tutors over nontutors.
- (5) Tutors increased the "operations" and "other" (estimation, geometry, fractions and time) scores significantly more than nontutors.
- (6) Tutees showed significant gains in "operations" but not in "other" scores over nontutees.

### 5. Interpretations

The primary interpretation of the results of this study is that empirical research does support the previous anecdotal claims of the mathematics achievement benefits of cross-age tutoring for tutors and tutees in the tutored areas. In addition, gains in achievement for tutors in the nontutored areas can be documented. The researchers interpret this to be the result of an experience that increased motivation to study mathematics.

### Abstractor's Comments

The strength of this study lies in its rigorous planning and execution. The research design carefully eliminates intervening variables and conservatively documents all claims. The benefits of cross-age tutoring to both tutors and tutees as shown in this study are important for mathematics educators to be aware of and to research further.

While both males and females participated in the study, as both tutors and tutees, there is no indication that pairings by sex was investigated. It would be of interest to know the following: the number of pairings of tutor and tutees by sex (same sex, male tutor - female tutee, and female tutor - male tutee). In addition to the numerical breakdown of the pairings by sex, the gains in mathematics achievement according to the specified pairings would be important to know. This could be a possible area to investigate in the future.

Sherrill, James M. SOLVING TEXTBOOK WORD PROBLEMS. Alberta Journal of Educational Research 29: 140-152; June 1983.

Abstract and comments prepared for I.M.E. by JOHN C. MOYER, Marquette University, Milwaukee.

### 1. Purpose

The purpose of this research is stated only in a very general way: "...to look at problem solving as it relates to certain types of word problems found in elementary school mathematics." The design of the study, however, belies a more specific purpose, viz to investigate causes for the performance of those students who are very good at solving one-step word problems but are very poor at solving similar multi-step problems.

### 2. Rationale

The impetus for this study seems to have been provided by the 1977 and 1981 British Columbia Mathematics assessments. In particular, the author deduced from the results of the B.C. assessments that there is a large group of students who are able to solve one-step, textbook word problems, but are unable to solve related multi-step problems. He then designed this experiment in which he identifies such students and compares their problem-solving performances to those of students who are able to solve multi-step problems.

### 3. Research Design and Procedures

The study was conducted in two parts. The first part was an identification phase in which 18 elementary school students were selected for individual interviews. The second part was an interview phase.

For the first phase all grade seven students enrolled in seven different British Columbia elementary schools ( $N = 283$ ) were given a 42-item computation test and a 12-item word problem test. For the word problem test, 600 seventh-grade items were collected by the author. From these items, 120 one-step and related multi-step problems appropriate to grade seven were derived and field tested. Three forms of the test were created. Each form was composed of four sets of three problems. Each set consisted of: (1) a one-step problem, (2) a related multi-step problem whose statement explicitly contained all the data needed for the computation, and (3) a related multi-step problem in which one of the numbers needed in the computation had to be generated from the given numbers. The computation test was designed to assess the ability of the students to perform the computation needed to solve the word problems.

The following criteria were used to select students for the interview phase of the study: each student selected was required to correctly complete all four one-step problems, to have a score of 75% or better on the computation test, and to have a reading level of at least 7.3. These criteria eliminated all but 54 of the 283 students. The further criterion that the students score either extremely poorly (0-3) or extremely well (7-8) on the multi-step problems eliminated another 25 students. From the remaining 29 students, those with the 9 lowest scores on the multi-step problems were placed in group A and those with the 9 highest scores were placed in group B. Both groups averaged 83% on the computation test, had a reading level of 8.7, and scored a perfect 4 on the one-step problems. Group A, however, averaged 3.0 on the multi-step problems, while group B averaged 7.1.

During the interview phase each of the 18 chosen students was asked to think aloud as he or she worked the problems. Three sets of three related problems were used. However, each student was given only two of the three sets to solve. The presentation of problem sets was systematically varied so that each problem set was given to six

students in group A and six students in group B. The students received a one-step problem followed by two of the first type of multi-step problems, then the other one-step problem followed by the other two multi-step problems. Data collected for each problem can be deduced from the results section which follows.

#### 4. Results

- (a) Group B outperformed group A on both parts of the test (100% vs. 88.9% on the one-step problems and 86.1% vs. 58.3% on the multi-step problems).
- (b) Average time spent (ignoring outliers) on all but two of the problems was greater for group A than for group B students.
- (c) Group A students re-read problems more frequently than group B students (Group A: 47 rereads; Group B: 20 rereads).
- (d) Group A students re-attempted 7 of their 17 missed problems.
- (e) Only three missed problems involved computation errors.
- (f) Group A students checked only two of their answers and group B students checked only nine of their answers; and 11 checks resulted in correct solutions in every case.
- (g) Of eight commonly observed heuristics (diagrams, trial and error, related facts, related method, mnemonics and equations, algorithms, special cases, and successive approximations), only "uses algorithms" was observed to any great extent, but its use was "unanimous."

#### 5. Interpretations

In the discussion of these results the author comes to four main conclusions:

- (a) The all-pervasive strategy of searching for key words, selecting an algorithm (or sequence thereof) and applying the algorithm(s) was full of pitfalls: the incorrect algorithm(s) were chosen (7 instances), incorrect inputs were used (9



instances), and only part of a multi-part sequence of algorithms was identified and/or completed (8 instances). It is to this last pitfall that the author attributes the observed discrepancy between performance on one-step and multi-step problems. He asserts that 40% of the incorrect answers to the multi-step problems came about because the students treated the problems as one-step problems. He feels that such errors could be avoided by encouraging students to re-read the entire problem once key words are found.

- (b) Students need to become convinced of the importance of checking their solutions. Of the 22 problems worked incorrectly, none was checked.
- (c) Students do not make much use of heuristics other than "uses algorithm." However, for these type problems it is possible that nothing else is necessary.
- (d) The students were willing to work hard and long on the problems, perhaps because the problems were well within their ability level, the arithmetic was easy, and/or the students liked the interviewer.

#### Abstractor's Comments

Much talk recently has centered around the need to spend less time in the elementary school on computation and more time on problem solving (as well as other topics). Unfortunately, problem solving is not currently held in unanimously high regard by teachers or students. Part of the reason lies in the lack of success that is evidenced when problem solving is taught. So, it seems reasonable to conjecture that studies that help unlock the mysteries surrounding word problems will go a long way toward getting problem solving into the classroom.

The fact that multi-step problems are more difficult for elementary school students to solve than similar one-step problems is a well-accepted fact. However, as Sherrill rightly points out, it is quite disconcerting to discover that students who are very good at solving one-step problems can be so very bad at solving their multi-step counterparts. The existence of such students points to some type of discontinuity in the way these problems are being processed. The whole is more difficult than the sum of the individual parts.

Sherrill has designed a very nice study to investigate this phenomenon. With the exception of a somewhat sketchy description of the interviews (did he have hints, ask questions, etc?), he has clearly and succinctly reported the method he used. He even included a listing of the problems he used during the interview phase of the study.

In carrying out the study, he has admirably achieved his goal of selecting two groups of students who are matched almost perfectly on the important traits of reading ability, computation ability, and ability to solve one-step problems, yet are at opposite ends of the spectrum on ability to solve multi-step problems.

He has reported appropriately the data regarding student success (right/wrong), time to completion, number of re-reads, and number of re-attempts. The raw data are presented in tabular format broken down by problem and group. For student success, he further breaks down the groups, tabulating right or wrong for each individual on each problem.

The data on the arithmetic errors, frequency of answer checking, and use of heuristics is less complete. He no longer presents the data in tabular format. Further, he does not break down the data by problem. Further still, except in the case of the arithmetic errors, he does not give separate results for Groups A and B. The reader

begins to wonder why the author has chosen to abandon the major strength of the design--the ability to compare two matched groups of students on the important variable where they differ--ability to solve multi-step problems.

In the discussion section, the reader wonders even more about the reason for abandoning the two-group comparison. At no point in the discussion or in the subsequent conclusions does the author even mention the existence of the two groups, let alone come to any conclusions regarding reasons for the difference between their abilities to solve multi-step problems. Furthermore, the discussion centers around that data which were less completely presented.

For example, Sherrill's main conclusion is that multi-step problems are missed because they are treated as one-step problems. The evidence he uses to support this conclusion is that 40% (8 out of 20) of the incorrect solutions to the multi-step problems were incorrect because the students treated the problems as one-step problems. Now we know from the data that Group A students missed 15 multi-step problems and Group B students missed 5. We are not given a similar breakdown by group with respect to the 8 problems that support the main conclusion. Even if one were to agree that 40% is a large enough percentage to support Sherrill's conclusion, one is still left wondering how much of the 40% was attributable to group A students. Perhaps even more puzzling is the reason for Sherrill's decision to omit the information.

Sherrill suggests that if students were encouraged to re-read problems, then they would notice that the problems were multi-step and treat them as such. He does not cite anything from the study to support this suggestion. Rather, he seems to be drawing on conventional wisdom. Unfortunately, the data from the study do not support this suggestion, since group A students re-read every multi-step problem more often than Group B students, yet Group A was outperformed by Group B.

Sherrill also suggests that students would correctly solve multi-step problems if they checked their work. He states:

Of the 108 problems used in the interviews, 11 were checked. Of the 11 problems checked, all were solved; so, of course, of the 22 problems worked incorrectly, none were checked. The data suggest strongly that students must be convinced of the importance of checking their work and the final solution.

Are we to infer that Sherrill is basing this recommendation on the fact that better students check their answers more often than poorer students? If so, why doesn't he mention this and remind us of the data to support the recommendation? In this instance Sherrill has previously given us data which tell us that Group A students checked 2 problems and Group B students checked 9 problems. Unfortunately we are not told how many of the problems checked were multi-step problems. It would also be helpful to know how many times the checking revealed errors that were subsequently corrected.

These are very curious omissions, for without such information it is difficult to judge the force of Sherrill's recommendation. For example, we might ask how Sherrill explains Group B's ability to solve multi-step problems without checking. This question is critical only if Group B was, in fact, able to solve lots of multi-step problems without checking. But we do not know whether Group B students checked any of the multi-step problems at all. If they did not, then they correctly worked 31 problems without checking. If we assume that all the checked problems were multi-step, then they only worked 22 problems without checking. Even if we assume that all the checked problems were multi-step, we don't know whether the checking really had any effect on the eventual outcome. So the reader is left wondering whether the Group B students were able to solve 22/36, 31/36, or ?/36 multi-step problems without checking them.

In the beginning of the article, Sherrill makes a distinction between the two different types of multi-step problems that are used in the study: the first type had all the data needed for the

computation explicitly given; the second type required the solver to generate some data before it could be used in the computation of the final answer. The distinction seemed well worth considering. The distinction, however, seems all the more worthwhile in the light of Sherrill's major conclusion that the main difficulty with multi-step problems is that they are treated as if they are one-step. If such is indeed the case, one might expect that there would be differential performance on the two types of multi-step problems, with the second type being more likely to be treated as if they were one-step. Unfortunately, Sherrill not only fails to discuss the implications of the differential performance; he does not even acknowledge the existence of two types of problems once he begins the Results section.

It is very unfortunate that Sherrill does not report more of the qualitative differences between Group A and Group B students, rather than these somewhat incomplete quantitative differences. In studies of this type, with such a small number of students, much more can be gained by reporting on the qualitative ways that good students approach problems and compare them to the qualitative approach used by the poorer students. This study is too well conceived, designed, and carried out to allow that information to go unreported. Even if the result was that there is no identifiable difference between the groups, we would know that we should look elsewhere. Perhaps improved performance on multi-step problems is simply a matter of time spent and experience gained.

Slavin; Robert E.; Levey, Marshall B.; and Madden, Nancy A.  
**COMBINING COOPERATIVE LEARNING AND INDIVIDUALIZED INSTRUCTION:  
 EFFECTS ON STUDENT MATHEMATICS ACHIEVEMENT, ATTITUDES, AND BEHAVIORS.**  
Elementary School Journal 84: 409-421; March 1984.

Abstract and comments prepared for I.M.E. by ROBERTA L. DEES, Purdue University Calumet, Hammond, Indiana.

### 1. Purpose

The authors ask, "How can it be that individualized, programmed instruction does not increase student achievement?" (p. 410) and "Can the problems of programmed instruction be solved in a way that would make this strategy effective?" (p. 411).

The two studies described compare a program in which student-managed individualized instruction is provided, with "traditional methods" for teaching mathematics.

### 2. Rationale

The problems that individualization, or programmed instruction, attempted to address are still with us and are getting worse as students are being mainstreamed and/or desegregated. Programmed instruction, while logically compelling, has never been shown to be effective, apparently because of difficulties in management and motivation. Since cooperative learning methods have been found to be effective, a method called Team-Assisted Individualization (TAI) was designed. This program is managed by the students themselves, and motivation is fostered by team recognition based on team members' progress through the program.

The curriculum materials themselves were constructed to be checked by teammates with answer keys provided; skills were subdivided so that a check was required every four problems. Students presented themselves for a test on a skill only if they passed the "checkout" for that skill.



### 3. Research Design and Procedures

This article reports two studies in elementary schools. In Study 1, the TAI system is compared with a second treatment and with a control. The second treatment consisted of Individualized Instruction (II), exactly the same curriculum materials and management program, but without the teams. That is, the students also received answer keys with their units but did their own checking individually. Tests were still given by rotating student monitors.

In Study 2, the TAI system is compared with a control. In both studies, the control group "used traditional methods for teaching mathematics, which consisted in every case of traditional texts, whole-class lectures, and group-paced instruction, supplemented by small, homogenous, teacher-directed math groups." (p. 413)

Study 1: The three treatments were implemented for eight weeks in the spring of 1981 in a third-, fourth-, and fifth-grade class in each of six schools in a middle-class suburban Maryland school district. Of the 504 students, 23% were receiving special education, reading instruction or other special educational services.

Study 2: Subjects were 375 students in grades 4, 5, and 6 in another suburban Maryland school district; 27% were receiving special educational services. Two TAI schools with a total of 10 classes were matched with six control classes in two schools. In this study, TAI was implemented for 10 weeks in Spring 1981, and compared to the control.

Students in both studies were measured in pre- and posttests on mathematics achievement, attitudes, and behavior ratings. For mathematics achievement, the computation subscale of the Comprehensive Test of Basic Skills (CTBS), level 2 (1975) was used. For attitudes, two eight-item scales were given, one on "liking of math class" and



the other on "self-concept in math." Teachers rated subsamples of their students on behavior, using these subscales: classroom behavior, confidence, friendships, and negative peer behavior.

## 6. Findings

Outcomes on seven criteria were reported: mathematics achievement as measured by gain on the CTBS, gains in self-scores on "liking of math class" and in "self-concept in math," and decreases in the number of problems reported in the four teacher-rated behavior subscales.

Students in the individualized instruction programs significantly outperformed the control groups on the CTBS and on the self-confidence and friendship subscales of the behavioral rating. Other results were mixed.

In Study 1, a marginally significant overall treatment effect was found on the CTBS ( $p < .07$ ). The TAI group gained significantly more than the control group ( $p < .03$ ), while the II group gained marginally more ( $p < .09$ ). When compared pairwise, there were no significant differences between the TAI and II groups. In Study 2, the TAI group performed significantly better than the control group on the CTBS ( $p < .03$ ).

In the six affective measures, students in the treatment groups in Study 1 were significantly higher than the control groups. Between the two treatments (TAI or II alone), there were no significant differences except in one scale, the self-confidence subscale of the behavior rating ( $p < .03$ ). In Study 2, a similar analysis showed significant differences between the treatment group and control group on only two of the affective scales, both subscales of the teacher-rated behavioral scale: self-confidence ( $p < .02$ ) and friendships ( $p < .01$ ).

## 5. Interpretations

The authors feel that the evidence supporting the positive effects of the TAI program on mathematics achievement is "relatively unambiguous" (p.418). Nevertheless they cannot conclude that the use of teams adds to the achievement effects of the II program alone.

The improvement in students' attitudes toward mathematics in the first study was not replicated in the second study.

The authors suggest caution in interpreting the results of the behavioral ratings of the teachers, who may have had trouble maintaining objectivity during the experiment.

They note that the curriculum materials were finely subdivided (20 items for each step) and provisions were made for feedback early and often (students worked on a block of four items and proceeded to the next block only if those four items were correct.)

Perhaps the most important finding of this study was that an individualized mathematics program was developed that could be managed by a single teacher without an aide, relying on the students themselves to perform the routine checking and management procedures of the program. (p.421)

They further conclude that students themselves may be enlisted to help solve the management and motivational problems inherent in an individualized method.

### Abstractor's Comments

The authors have suggested the limitations of the study (restricted generalizability, due to using the individual as the unit of analysis; possible Hawthorne effect; and the short duration of the experiment). I make three substantive comments.

1. The title is misleading, because the authors use "cooperative learning" when they mean "cooperative management." It is not a typographical error, but a misunderstanding of the term. In describing their method in some detail, they explain that though the students are working in pairs, "students read their instruction sheets individually, asking teammates or the teacher for help if necessary." (p. 412). Thus the purpose of working in pairs is to exchange answer sheets and check each other's work. In fact, the students are generally not working together nor even necessarily working on the same thing. This is not considered to be "using cooperation as a learning method." In such a method, students would be working together on the same thing, and would produce a common end product (for example, they would agree on the answers before checking with the answer key) (see Johnson and Johnson, 1975).

The authors' misunderstanding of the method is further exemplified in this statement: ". . . cooperative learning methods are group paced; the entire class studies the same material at the same rate." (p. 411) While that could happen, it is by no means a necessary characteristic of a cooperative learning method; an advantage of the method is that different small groups can be working on different things.

Had the authors used a true cooperative learning method, they might have seen even more phenomenal gains, as I have when I have insisted that students work together and discuss mathematics. Talking about mathematics can enable students to clarify concepts and increase their problem-solving ability (as opposed to computational skills) (Dees, 1983).

2. It is a shame that these researchers feel the need to reward students for doing what they should be doing and should want to be doing, namely learning. The chance to work in a social atmosphere with peers was probably motivating enough. And research by

psychologists has shown that intrinsic motivation, when it exists, may actually be lessened or destroyed if external rewards are added. Therefore I question the need for and the use of the certificates for teams with the highest scores.

3. I feel that programmed instruction was abandoned too soon. I was involved in a successful individualized program for several years. Students often gained two grade levels in one year. But, as the authors observe, students find its continual use tedious, boring, and lacking in human contact. Students sometimes manufactured questions to ask, just to get a chance to talk to another human being. The authors are right when they try to get student interaction back into the learning process in mathematics.

Our problem is looking for the one method that works best. We need to use a combination of methods; one method works better for one goal and for some students, another for others. And no student wants to do the same thing day in and day out for the entire school year. The authors have demonstrated that it is possible to design an individualized instruction program in which students can help to manage their own learning. Such a program should be included in teachers' lists of effective teaching strategies.

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Thomas, William E. and Grouws, Douglas A. INDUCING COGNITIVE GROWTH  
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Mathematics 84: 223-243; March 1984.

Abstract and comments prepared for I.M.E. by ROBERT C. CLARK, Florida  
State University, Tallahassee.

### 1. Purpose

The purpose of this study was to investigate a method for stimulating "the intellectual development of college students which takes only a short time to administer and which is largely content-free" (p. 234).

### 2. Rationale

Typically, fewer than forty percent of college freshmen demonstrate the thinking strategies normally associated with formal operational reasoning. The demonstration of these strategies is thought to be directly related to success and inversely related to anxiety in courses which require formal reasoning. Previous studies have shown that inducing cognitive growth in college students is possible, but have required the students to enroll in a course to receive the necessary experiences. This approach takes an extended period of time and the content of the course (e.g., science) may cause such anxiety that the student will not enroll.

The authors felt that what was needed was a method for inducing the necessary cognitive growth through meaningful experiences which were: "largely content-free, challenging to the student without being overwhelming, and brief" (p. 234).

The method selected was to have the students play the game of Master Mind with an observer. A previous study had demonstrated that Master Mind was an effective tool "for teaching deductive reasoning" (p. 235).

### 3. Research Design and Procedures

A pretest of cognitive development (covering combinatorial analysis, proportional reasoning, and separation and control of variables) selected from materials developed by Renner, Prickett, and Renner was used as a pretest with a total of 185 college students. Test-retest reliability of the instrument was tested with seventy students not participating in the experiment and was found to be .88.

On the pretest, seventy-six students were found to be concrete operational and thirty-nine of these students agreed to participate in the study. The students were randomly assigned to one of three groups:

- a. a structured interaction (SI) treatment group,
- b. a neutral interaction (NI) treatment group, or
- c. a Hawthorne control group.

Each student met with an observer one hour per week for four weeks. Students in the SI group were asked questions which would cause them to reflect upon their problem-solving strategies while playing Master Mind with the observer. Students in the NI group played Master Mind with the observer, but without help or questioning. Students in the Hawthorne group played checkers with the observer. The same test of cognitive development was administered to each student during the fifth week of the experiment.

### 4. Findings

A single factor analysis of covariance was used to compare results of the pretest and posttest for the three groups. A statistically significant value of  $F$  ( $\alpha = .05$ ) resulted in the use of the Newman-Keuls test to make pairwise comparisons of adjusted group means. The only differences found to be significant were those between the SI group and each of the other groups. Seventy-eight



percent of the subjects in the SI group changed from concrete operational reasoning to either transitional (57%) or formal operational (21%) reasoning.

##### 5. Interpretations

The authors conclude: "although playing Master Mind can have a positive effect on the cognitive development of concrete-operational college students, problem-related interaction while playing the game is essential" (p. 239). The authors provide an analogy between science and Master Mind as evidence of the importance of the method. They also indicate a need to examine further how Master Mind may be played to maximize cognitive growth and to examine the long-term effects of the treatment.

##### Abstractor's Comments

The most striking facet of this study is that the authors actually did an adequate power analysis prior to beginning the experiment. The authors chose to set limits on type I ( $\alpha$ ) errors, type II ( $\beta$ ) errors, and effect size (ES). A power analysis based upon these limits indicates that the required sample size is 11. Differences which are statistically significant may be of no practical significance. By selecting an effect size, the authors indicated how big a difference would be "big enough" to be considered practical. This study indicates that it may be possible to raise a student's level of cognitive development in a short period of time. Since many teachers find formal operational thinking to be a necessary prerequisite for success in their courses, the method explored by the authors may be a practical way to provide success for more students in these courses without resorting to time-consuming and expensive remedial courses.



Several questions remain concerning the use of Master Mind to raise a student's level of cognitive development. The authors identify the student's role in playing the game (code maker or code breaker) and the long-term effects as factors requiring further study. Other factors for study include:

1. What skills are required of the "observer" to provide the most effective interaction?
2. What types of questions are the most effective in causing students to reflect upon their own problem-solving strategies?
3. What effect will such training have on student performance in college classes?

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