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ABSTRACT

Arguing that a major re-thinking of the mathematics curriculum is needed, this paper urges two-year colleges to take the lead in curriculum revision. Section I suggests that the pre-calculus orientation of high school mathematics may be inappropriate, viewing mathematics related to computers and dependent on computers for computation as more widely applicable. In section II, two-year college instructors are urged to take the initiative in developing new curriculum and teaching modes, taking advantage of their training, the classroom environment, and the absence of standardized tests. Section III offers principles for the development of a new curriculum, arguing that it should emphasize problem solving, meaningful mathematical topics, practical applications, and proper motivation of mathematics. Two ways of naturally motivating algebra are highlighted: through computer programming and by trying to do problems without using algebra. Section IV focuses on the content of the new curriculum, indicating that it should cover algebra taught in conjunction with computer programming, as well as probability/statistics, logic, and matrices and linear models. Section V provides examples of ways of introducing these new topics. In section VI, concluding remarks urge the Sloan Foundation and National Science Foundation to support two-year college efforts to develop new mathematics curricula and recommend that two-year colleges obtain the backing of the principal transfer institutions of their students. (LAL)

A NEW START FOR MATHEMATICS CURRICULUM

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ABSTRACT: This writer advances the thesis that a major re-thinking of the mathematics curriculum is needed at secondary school and upper-division college levels. Two-year colleges suffer the most from the problems in the current curriculum and may be the best place to initiate change.

I. IT IS TIME FOR SOME CHANGES

The two dominant issues in mathematics education today appear to be, first, curriculum and student performance in secondary school (7-12) mathematics, and second, the role of computers in mathematics education. This writer believes that the curriculum can be improved by the use of computers and mathematics related to computers.

The academic community and employers are not complaining about collegiate mathematics curriculum or the mathematical skills of college graduates. Rather most problems facing colleges, two-year and four-year, seem to be as the result of problems in the secondary schools. The lack of basic skills in high-school graduates especially affects two-year colleges, where over 40% of mathematics enrollments are at the level of intermediate algebra or below. These remedial courses are often a trying experience for both students and faculty because the pace is too fast (remedial courses tend to have a lot of ground to cover, but if students did not learn the material when taught at a normal pace, teaching it faster is not going to help) and the students are unmotivated (the courses are a general studies requirement forced on the students).

A basic conclusion of the recent CBMS Conference Report, New Goals for Mathematical Sciences Education, was "the current organization of topics, courses, and course content needs to be reformulated for all students [for pre-college mathematics]". In particular, this writer suggests that the pre-calculus orientation of the high school mathematics, and hence of remedial collegiate mathematics, may now be all wrong. The concepts of calculus that are needed by the average student who does take calculus, say, for an economics major, are minimal. On the other hand, the mathematics related to computers and mathematics dependent for computation on computers (such as statistics and linear models) provide topics for study that are intellectually rigorous and much more widely applicable.

Calculus is something engineers and physical scientists use extensively, but which now has as its primary purpose for other students, being a benchmark course for measuring academic rigor in mathematics. It is required of all pre-meds but never used in medical school. "Calculus develops mathematical maturity" is the phrase heard so often. People are impressed when a college student comes home and tells his friends and parents that he/she is taking calculus. But the calculus course of today is much more superficial than the calculus course of thirty years ago, the course that gave calculus its good name.

Ralston and others have argued persuasively that calculus is not the only way to develop mathematical maturity. The discrete mathematics of computer science and other new applied mathematics has equally attractive maturity-building content. The case for changing from a calculus focus seems strongest for those who will never even take calculus, but are forced to take preparatory courses leading up to calculus.

Other college departments often encourage mathematics departments to keep calculus in its focal position, but these same departments are nibbling away important new areas of the mathematical sciences by teaching their own courses in statistics, computing, or mathematical modeling. Many mathematicians find this vote of support for calculus very assuring. This writer does not view efforts to lock lower-division mathematics into pre-1900 topics very reassuring (while others teach the mathematical sciences of the 20th century).

II. TWO-YEAR COLLEGES SHOULD TAKE THE LEAD IN CHANGING MATHEMATICS CURRICULUM

It seems to this writer that the current interaction between colleges and secondary schools in trying to change mathematics curriculum is a classic example of negative feedback, with "negative" used here in the sense of encouraging each other to do something the wrong way. Secondary schools look to colleges for guidance in their curriculum, since they are supposed to prepare students for college courses. But the colleges now are looking to the secondary schools for their curriculum: remedial curriculum is based on high school courses. There are many reasons to believe that secondary school mathematics and the remedial college curriculum are stuck in a rut and each is digging the hole a little deeper for the other.

The two-year colleges are even worse off, since they look both to four-year colleges and high schools for guidance. They are not expected to set standards or take any initiative about the training of students they admit from high schools or students they pass on to four-year colleges.

This writer thinks that two-year colleges should turn the tables around and take the initiative in developing new curriculum and teaching modes. Two-year college faculty have a much greater interest than the college faculty (whose training was research-based) in the transition high school-college mathematics curriculum and its teaching. Most universities handle lower-division classes in huge lectures and/or TA-run sections, where innovative teaching is impossible. TYC faculty also have many advantages over high school teachers, in their training, the classroom environment, and the absence of standardized tests such as College Board exams (which stifle curriculum innovation). The problem of poor motivation found in two-year college students is even worse in the high schools.

Two-year college students are typically older and more mature than the four-year students. Most are in two-year college for a purpose, not forced to be there by law or parents, although they may be forced to take in a particular math course as a requirement of their major or as a general studies requirement. The mathematics courses for these students should respect their maturity and motivation. They should have clearly-explained, meaningful goals. There are so many jobs in society today that depend on quantitative reasoning that it should be easy to design mathematics courses that develop useful (marketable) skills and/or modes of reasoning and challenge students' maturity.

Thus, two-year colleges should take the initiative in mathematics curriculum changes that in turn will change the high school curriculum. High schools certainly cannot be expected to initiate change. They worry about training students for the College Board exams that are written by college people. Some four-year colleges are starting to make changes, but the colleges involved are the most selective colleges and their students are so different from the typical high-school graduate that their curriculum ideas may take years to reach two-year colleges and high schools.

The question is what mathematics should the typical two-year college student know? Closely related is the question, what mathematics should the typical high-school graduate know? Instead of a watered down version of what a pre-engineering student needs, our answers should put the typical TYC student in center stage. We may give the pre-engineering student a separate curriculum or maybe an advanced version of what the typical student gets.

III. PRINCIPLES FOR A NEW CURRICULUM

The first step in revising the mathematics curriculum is deciding what the fundamental objectives of the curriculum are. This decision brings us immediately to the inevitable trade-off between reasoning ability and problem-solving versus mathematical facts and technical skills. The NCTM Agenda for the 1980's gave problem-solving its top priority. This writer agrees with that priority. But problem-solving takes time. Whatever material is taught, it will have to be taught more slowly to make time for problem-solving. Implicit in a problem-solving emphasis is that the problem-solving should involve relevant applied mathematics: statistics, systems analysis annd other modeling, and most of all computing. One further tenet of the writer is that the mathematical concepts and techniques should be well motivated and meaningful to the student. This also takes additional time.

A curriculum of meaningful mathematics will include new topics, but it will in large measure be current topics taught with a different point of view. There are many underlying mathematical skills needed in applied problem-solving, in particular the symbolic reasoning and manipulation of algebra. But these skills should not be taught in isolation of their uses. Instead, some applications should come first. The usefulness of symbolic manipulation must be made clear. If a course just teaches students to genuinely appreciate the power of symbolic reasoning and manipulation, then that alone would be a major achievement.

There are two ways in which algebra is naturally motivated. One way in through computer programming. Programming the solution to a set of numerical problem is mostly algebra. The other way is by trying to do without algebra. Let students work problems that can be done "the long way" by trial-and-error or tedious calculations, but which are quickly solved with algebra. Students should see that algebra makes problems easier, that moving away from the specific numbers of a real-world application into a more abstract setting can be the best (fastest, most insightful) way to solve that practical problem. Of course, formulating one general solution method for a class of problems is also what computer programming is all about.

An analogous situation is the way grade schoolers accept the need to learn multiplication tables. Initially they compute simple products like 3×4 by repeated addition. But long multiplication problems like 334×52 win students over to the advantages of memorizing products. In a similar way, students should be encouraged to experiment with "the long way" solutions, even using computers. The abstract formulations should come when students see that they need them to solve problems. The obvious dividend of this approach is that students would leave these mathematics courses with a positive attitude towards mathematics and, more generally, abstract reasoning. They would be more accepting of mathematical approaches in other subjects and would be more willing to take more mathematics courses later in their careers.

Summarizing, the two-year curriculum should emphasize:

* Problem Solving
* Meaningful Mathematical Topics
* Practical Applications
* Proper Motivation of the Mathematics

IV CONTENT OF NEW CURRICULUM

As noted in the previous section, algebra must continue to play the central role in the high school curriculum. Algebra should be taught in conjunction with computer programming and it should be used to organize and simplify the solution of applied problems.

Probability/statistics and logic are two other basic topics, which also should be taught with a heavily applied emphasis. For example, propositional logic can be embedded in many programming questions: what combinations of circumstances will cause the program to come to a certain statement? Probability and statistics have limitless applicability, and an empirical approach based on applications should be used.

Finally, the curriculum should include matrices and linear models. The writer's interest in this topic is based on a very deep philosophical theme. In the 19th century, calculus was the model of what a scientific theory should be. All other scientific theories were expected to have the simple, elegant structure and predictive power of calculus. Today, we know that this is a very naive view. Now we expect that most interesting phenomena are inherently complex and have many input variables and many output variables. Think of the molecular biology of a cell or of just a DNA molecule, think of the design of the spacecraft, think of an airline's computerized reservation system.

Matrices are the simplest mathematical model for such systems of organized complexity. A linear model that hypothesizes linear relationships between a set of inputs and a set of outputs (in the form of a matrix of input/output coefficients) is easy to build for almost any complex system, and its behavior is easy to simulate on a computer. A matrix of input/output coefficients is the first-order approximation of any activity that can be measured numerically. This linear-systems way of thinking about activities around us is the basic quantitative philosophy of the modern world, and as such, should have an important role in the curriculum.

These topics will develop students' reasoning abilities of students in a way that students can respect, and that will lead to a general college math requirement that can be considered just as essential as a general college English (writing) requirement.

V EXAMPLES OF NEW WAYS OF TEACHING MATHEMATICS

* As noted above, algebra would be introduced in connection with computer programs to do numerical computation, and logic could be taught in connection with logical possibilities for program flow; logic and Boolean algebra are also used heavily in finite probability and combinatorial analysis.

* The current canonical algebra example of two equations in two unknowns involves the speed of a canoe going upstream and downstream. The new canonical example could involve two companies A and B with current sales of s_A and s_B with $s_A > s_B$ and current growth rates (for sales) of g_A and g_B, respectively--specific values would be given by s_A, s_B, g_A, g_B. We want to know when (if ever) B's sales will surpass A's sales.

* Geometry should arise in the context of computer graphics problems to draw shapes and move them around a screen (this would be almost wholy analytic geometry; Euclidean geometry would have a minor role, in secondary schools, in this curriculum).

* Logarithms and the exponential function would be taught in connection with exponential growth models and binary-tree data structures (for dictionary searches).

* Probability and statistics should have an exploratory spirit, using statistical computer packages early, to look for trends in data.

* The role of trigonometric functions needs to be re-thought. Trig is only used by students in engineering or physical sciences, and there in connection with Fourier series and complex-valued solutions to differential equations. Perhaps Fourier series should be introduced early in calculus, and symbolic integration programs used to compute Fourier coefficients. Then one could plot (on a computer) approximations to a function f(x) using the first k terms of the Fourier series for f(x). Elementary time series analysis might be attempted.

VI Concluding Remarks

The suggestions here for a new curriculum are fairly radical. No texts now exist. As was done with four-year colleges, the Sloan Foundation (and National Science Fondation) should support the efforts of a few two-year colleges to develop new mathematics curricula. The two-year colleges should obtain the backing and professional cooperation of the principal four-year institutions to which their graduates transfer for continued college study.

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