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ABSTRACT

Papers presented for a conference on school mathematics are compiled in this document. The purpose of the conference was: (1) to identify new goals and needed change for school mathematics; and (2) to recommend strategies or describe scenarios whereby these goals and changes can be realized. The document summarizes the deliberations in 20 papers plus discussion and summary reports. Following a background paper by Romberg, the status of mathematics instruction was discussed by Senese, Willoughby, Pollak, and Romberg. The mathematics curriculum was then explored, with the focus on new goals by Pollak, computers by Braun, statistics by Hunter, and needed changes by Usiskin. In the third section, focusing on learning and teaching, research on learning was described by Siegler and on teaching by Peterson. In the same section, Case discussed implications of cognitive science, Carpenter explored learning as a critical variable in curriculum reform, and Lappan discussed implications of research to mathematics teachers. Work group reports were then summarized, followed by papers on policy implications and impediments. Williams discussed the preparation of teachers, and then various perspectives were presented: Makhmaltchi, on publishers; Jones, on test developers; Barclay, on materials and producers; Hala, on mathematics supervisors; and Gawronski, on school administrators. Several annotated references are included. (MNS)

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School Mathematics: Options for the 1990s

Proceedings of the Conference

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SCHOOL MATHEMATICS:

OPTIONS FOR THE 1990s

Proceedings of the Conference

Madison, Wisconsin

December 5-8, 1983

Edited by
Thomas A. Romberg and Deborah M. Stewart

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BACKGROUND DISCUSSION PAPER FOR THE CONFERENCE

Thomas A. Romberg
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Introduction

The Department of Education, the National Council of Teachers of Mathematics, and the Wisconsin Center for Education Research are sponsoring this conference on School Mathematics. The impetus for the conference stems from three sources: (1) an accumulated body of evidence about enrollments, performance, and so on that indicates that serious problems exist in school mathematics, (2) the several recent reports critical of the quality of mathematics instruction in contemporary American schools, and (3) the impact of the current technological revolution on the content of the school mathematics curriculum. The intent of this conference is to provide an invited group of scholars an opportunity to consider the teaching of mathematics in American schools. The purpose of the conference is twofold:

(1) To identify new goals and needed change for school mathematics.

(2) To recommend strategies or describe scenarios whereby these goals and changes can be realized.

The product of the conference will be a report summarizing the deliberations and will include a considered analysis of current problems and trends and outline a set of actions which could be taken by federal, state, and local governments; professional associations such as the National Council of Teachers of Mathematics and Mathematical Association of America; publishers; and foundations. A very tentative outline of the report is the following:

1. The present condition of school mathematics
    a. historical comparison using available data
    b. international comparison using available data
    c. recommendations from recent reports
    d. inferences from data and from experienced observation on what needs to be changed; on causes for unsatisfactory student performance
    e. . . . .

2. Needs and opportunities for changing school mathematics
    a. new demands on education made by business and industry
    b. new technology, which creates new demands and represents a new opportunity
    c. needed changes in what is considered basic mathematics based on new demands and new technology
    d. new knowledge from research on teaching and learning
    e. . . . .

3. What changes are needed
    a. elementary; middle; secondary

b. marginal improvement; radical change
c. . . . .

4. Impediments to change that contribute to stability around present unsatisfactory condition
   a. public understanding?
   b. education policy, planning and development of curriculum?
   c. quality of teaching?
   d. quality of teaching materials?
   e. school culture?
   f. . . . .

5. Strategies for improvement that overcome impediments to needed change
   a. short-term; long-term
   b. new information needed for planning
   c. new research needed for implementation
   d. other activities, including development, training and dissemination (print, media, software, . . .)
   e. "who" needs to do what, in what order
   f. . . . .

From this outline it should be evident that we are not interested in another academic paper, although the analysis and recommendations should have a scholarly basis. What we will provide is a document that gives direction to educators at all levels of how to respond to the pressure for change both by alerting them to needed changes and problems and by providing strategies, a sequence of steps, or senarios to follow.

The Present Condition of School Mathematics

In the last few years, there has been a growing awareness that the current teaching of mathematics in American schools is not good. For example, Ailes and Rushing (1981), Hurd (1982), and Wirszup (1981) made a strong case that the United States is falling seriously behind the Soviet Union, Japan, and much of Europe in training its citizenry in mathematics and science for the technological world of tomorrow.

The scope of the problem has been clearly documented in several position papers prepared to stimulate legislation and federal action (Conference Board of the Mathematical Sciences, 1982; Hurd, 1982; National Science Board Commission on Precollege Education in Mathematics, Science, and Technology, 1982; National Science Foundation and Department of Education, 1980). The evidence takes a number of forms: performance on national tests, participation in mathematics courses, the underrepresentation of women and minorities in careers involving science and technology, the preparation of teachers, and a growing concern that the mathematical content in current school mathematics programs may not provide students with an adequate preparation for the scientific world of the twenty-first century.

In the period from 1961 to 1980, there was a steady decline in scores on the SAT verbal and mathematics tests (Advisory Panel on the Scholastic Aptitude Test Score Decline, 1977). This decline was reflected in performance on other nationally normed tests and in the declines reported for the National Assessment of Educational Progress (NAEP) mathematics assessment (Carpenter, Corbitt, Kepner, Lindquist, and Reyes, 1981). The results from the second NAEP mathematics assessment clearly document that, although most students are reasonably proficient with computational skills, the majority of them do not understand many basic mathematical concepts and are unable to apply the skills they have learned in even simple problem-solving situations (Carpenter et al., 1981).

The data on participation in mathematics courses are equally distressing. Only one-third of the nation's high schools require more than a year of mathematics for graduation and only two-thirds of the nation's students take two or more years of high school mathematics. As a consequence of the low levels of participation in high school mathematics courses, there has been a dramatic increase in the number of remedial mathematics courses at colleges and universities. Between 1975 and 1980, there was a 72 percent increase in enrollment in remedial mathematics courses, and currently 25 percent of the mathematics courses at public four-year colleges are remedial (National Center for Education Statistics, 1982).

The picture is even more bleak for women and minorities. On the average, black students complete approximately one year less high school mathematics than their white counterparts (Anick, Carpenter, & Smith, 1981). Both women and minorities are seriously underrepresented in careers involving science and technology. For example, only 13 percent of the nation's scientists and engineers are women and only 2 percent are black (National Science Foundation, 1982).

If significant changes are going to be made in mathematics instruction, there is a clear need for qualified mathematics teachers. However, during the 1970s, there was a 77 percent decline in the number of high school mathematics teachers being trained. In 1981, 43 states reported a shortage of mathematics teachers, and by some estimates 50 percent of beginning mathematics teachers are not qualified to teach mathematics (Hurd, 1982).

In summary, the evidence that there is a need to change school mathematics is overwhelming. Furthermore, in response to this situation several public and private organizations have recently prepared and issued major reports critical of current educational practices and made significant recommendations for reforming American schools. A summary of major reports related to mathematics and their recommendations, prepared by Marjorie Schneck (1983) for CBMS, is attached.

The recommendations on school mathematics vary from report to report. Nevertheless, school boards, administrators, school staffs, and other educators are or will be attempting to respond to these pressures for change. Although there is general consensus regarding the problems facing mathematics education, and there is some agreement about the nature of the solution, there is no agreement on how to proceed. What is now needed is a set of recommendations of how to proceed. In this conference informed testimony and professional judgment about needed changes will be gathered; then all participants will be expected to suggest options and to consider constraints related to mathematics in American schools. While we will be concerned with the conceptual issues associated with what constitutes mathematics, the technical aspects of learning and teaching, and the socio-political demands of a changing society, we must make recommendations about changes in school mathematics in the coming decade that are realistic and provide practitioners with information about how to proceed.

Changing School Mathematics

The need and opportunities for changing school mathematics are based on changing notions about what mathematics is of fundamental worth for students and on knowledge about the process of schooling.

Some mathematics should be common for all students. Not long ago only arithmetic skills with large and complicated numbers constituted the common curriculum of school mathematics. Today mechanical calculators have replaced human clerks doing calculations in every setting except classrooms. At the same time, the importance of mathematical knowledge is increasing steadily. Its applications are extending into more and more new areas of knowledge and practice. We recognize that some students will be producers of mathematical and scientific knowledge, many students will use mathematics in their adult occupations, and many will (or should be) literate consumers of mathematical knowledge and its applications. The common mathematical content needed by students is neither a list of basic skills such as those advocated in the 1950s, nor the "modern math strands" of the 1960s. Although the recommendations of the above reports do not reflect a consensus about what is fundamental for all students, it is clear that business and industry are expecting tomorrow's employees to have different mathematical knowledge and skills. For example, discrete mathematics, statistics and probability, and computer science should now be regarded as "fundamental," and appropriate topics and techniques from these subjects should be introduced into the curriculum for all students (CBMS, 1982). In particular, the technological revolution brought about by the chip is forcing industry, government, the military, and now schools to reconsider their goals, to change organizational traditions, and to retool. The aspects of mathematics which are now considered important for all students to learn because of the computer are those of the past quarter century. Current school mathematics programs simply fail to reflect this revolution.

Knowledge About Schooling

During the past twenty years our knowledge about the education process has expanded exponentially. Any proposed changes must be cognizant of the research and the implications to school mathematics in at least four areas--learning teaching, curriculum engineering, and school change. For example, there has been a major shift in the direction of research on students' learning and thinking. A new cognitive science is emerging that is beginning to provide real insight into how students learn mathematical concepts and skills. Similarly, traditional research paradigms for the study of teaching are being challenged, and promising new directions of research on instruction are being developed. Thus, both research on students' learning and research on teaching are beginning to provide the kinds of knowledge that may significantly shape the design of instruction in mathematics.

Impediments to Change

In the 1950s, the mathematics education community faced a similar crisis. At that time, the response was to produce mathematics texts that reflected current thinking about mathematics as a discipline. The lessons learned from that experience clearly indicate that, while again we need to rethink the content of the school mathematics program, we need to do more.

One lesson learned during the past two decades is that change is difficult. Schools are stable social institutions. For example, the NACOME report stated that few of the suggested reforms in mathematics teaching of the 1955-1975 era have been extensively implemented in classes (Conference Board of the Mathematical Sciences, 1975, p. 78). They also noted that

> the overwhelming feature of the educational system is its conservatism, inertia, and the imperviousness to sweeping, profound change. It accepts, accommodates, and swallows up all sorts of curricular fashions and practices. (pp. 67-68)

The basic problem facing any reform is the challenge of the traditional characteristics of schooling. One lesson that was learned is that attempts to change schools that only praise the new without challenging the old and the traditions upon which the old rested are doomed to failure.

Only by challenging the current traditions about schooling can significant changes be brought about. That many reform movements have failed is not surprising. The traditional practices of teaching give the participants a sense of order which is essential. Developers or innovators have often failed to recognize the traditions they are challenging.

One way of characterizing proposed changes is to focus on the degree of restructuring they involve. Our interest is in changes which are likely to have a major impact on the amount and nature of mathematics in American schools. Typically innovations are designed (or perceived as designed) to make some ongoing schooling practices better or more efficient (an improvement, not a change), but they do not challenge the traditions associated with the school. For example, the nonprogrammable calculator as a replacement for the slide rule in engineering classes did not challenge how knowledge of engineering classes is defined in that culture, or how teachers are to work.

At the other extreme, some innovations were designed and perceived as challenging in the cultural traditions of schools. For instance, "modern science" texts asked teachers to conceive of knowledge differently; and "team teaching" asked schools to develop new staff relationships. The changes identified and discussed at this conference should have such major impact in mind.

The targets for such innovation should include the following:

Current mathematics curricula--How is mathematics segmented and sequenced? Who is allowed to take courses? What is the text publisher's responsibility?

Mathematics teachers--What are teachers' responsibilities? What background is necessary?

Schools--How much time is allocated to mathematics instruction? What criteria are used to judge students performance? What expectations are held for students?

General public--What should parents expect? What should business and industry expect?

Only from such an examination can realistic but significant changes be proposed.

Recommended Changes and Strategies

Obviously, it is premature to list any set of specific recommendations or strategies at this time. Although recommendations about appropriate mathematical content must precede other considerations, we are aware that significant content changes will not occur as intended without a coordinated effort with respect to other recommendations. Furthermore, we are particularly interested in suggestions about strategies which educators can follow to overcome the impediments so that real change will be possible.

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OPENING SESSION

IMPROVING MATHEMATICS INSTRUCTION:
A KEY TO EXCELLENCE IN EDUCATION

Donald J. Senese
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U.S. Education Department

It is a great pleasure to be here tonight on the University of Wisconsin campus at Madison. As a native of Chicago, I am well aware of the outstanding reputation of the University of Wisconsin in education, and as an educator I am impressed with the quality of scholars at this conference. I am hopeful that in the next few days all of you will have a very productive discussion focusing on improved mathematics instruction.

I'd like to divide my brief remarks tonight into two parts. In the first I will give a short description of the kinds of activities we in the Office of Educational Research and Improvement have supported in the past and continue to undertake now. In the second part I'll discuss what considerations led us to support this meeting and what we hope to accomplish here.

The Office of Educational Research and Improvement, or OERI, is one of the principal offices of the U.S. Education Department. OERI is divided into three operating components: (1) the National Institute of Education, or NIE, which is the main Federal agency for the conduct of education research, (2) the National Center for Education Statistics, which collects statistics on the U.S. educational system, and (3) the Center for Libraries and Education Improvement, which includes Federal support for certain library programs, the National Diffusion Network, and a variety of technology programs designed to assist educators at the state and local levels.

I would like to describe just a few of the projects that have been supported so that you will have an idea of the range of our activities.

Even before the creation of the National Institute of Education or the Cabinet-level Education Department itself, the Office of Education was supporting mathematics education projects at several of the institutions called Labs and Centers. This activity continues presently. In fact, the Center here at the University of Wisconsin is engaged in research on young children's learning of mathematics, as it has been for well over a decade. Indeed, this long history of work in mathematics education, under the leadership of Tom Romberg, Tom Carpenter, and others, is one reason that we are here today. Other significant project activities are being conducted at the Learning Research and Development Center at the University of Pittsburgh; and the Mid-Continental Regional Lab is finishing up work on the Comprehensive School Mathematics Program that was started years ago at another laboratory.

When Mike Smith and others at NIE decided in 1974 that mathematics could join reading as a so-called "essential skill," the Institute sponsored a major national conference called the Conference on Basic Mathematical Skills and Learning, more popularly known as the Euclid Conference. That conference was eight years ago, at a time when many mathematics educators were viewing with alarm the apparent headlong rush to embrace computational skills as the only goal of mathematical education.

The Euclid Conference did not endorse that narrow view of the goals of mathematical education. In fact it led to a follow-up meeting conducted by the National Council for Supervisors of Mathematics, which in turn led to the publication of a now-famous list of ten basic mathematical skills. This list, which included computation as just one of ten basic skills, was very widely disseminated by the supervisors' group and also through the publications of the National Council of Teachers of Mathematics. It has been adopted in many ways by state and local education agencies, and some of the textbook publishers have incorporated the ideas into their texts. I believe it is an excellent example of how the Federal government can act as a catalyst and leader without in any way dictating the content of the curriculum at the local level. I am happy to find that some of the people who were instrumental in that effort are here tonight--in particular Ross Taylor, who is the chief mathematics supervisor in the Minneapolis public schools, and Dorothy Strong of the Chicago schools.

A third conference I should mention is one we jointly supported with the National Science Foundation on hand-held calculators in school mathematics. Recommendations from that conference led to the creation of the Calculator Information Center, which for several years provided advice and assistance to people who wanted to know how to integrate calculators into mathematics instruction. Here again there is continuity between that conference and this one since both Henry Pollak and Fred Weaver are here tonight.

Most recently OERI supported a conference in Pittsburgh, at which several of you were present, entitled "Research on Computers in Education: Realizing the Potential." That conference explored opportunities for research in cognition and computer science aimed at developing substantially more advanced applications of computers than can now be found for improving teaching and learning in reading, writing, mathematics, and science. I believe the publication of the Chairmen's report and papers from that conference will have a great influence on research issues in the field of technology in education. The National Institute of Education recently announced an award to Harvard University for establishing a School T hnology Center, a direct outgrowth of the Pittsburgh conference.

However, we are not content just to conduct conferences. Allow me to describe just a few more of the projects and programs we understake.

Several years ago we initiated support of the Second International Mathematics Study, first through money for the international planning group and later for the U.S. national data gathering (with additional support from the National Science Foundation). Now further analyses of the vast amount of data obtained are being funded through our National Center for Education Statistics. We are pleased that Joe Crosswhite, who is the main author of the U.S. national report, is a member of the Steering Committee of this conference.

We have conducted several grants competitions over the past six or seven years. NIE's Teaching and Learning program supports basic and applied research in mathematics education; many of you have contributed to those either as grantees or as reviewers. We had a special competition, to which Elizabeth Fennema contributed in the planning stages, focusing on women and mathematics. The results of that program are soon to be published in a book from Erlbaum Associates. As another example, the National Science Foundation joined with us for two years in a special competition which resulted in the creation of a number of prototypes of how microcomputers can be used effectively in mathematics instruction.

We also support certain related activities that are not exclusively devoted to mathematics but nonetheless have a mathematical component.

The National Assessment of Educational Progress fits in this category, as does all the basic data-gathering work conducted by our National Center for Education Statistics.

Still another example is a series of contracts devoted to the state of educational software, one of which was directed toward mathematics and science. Tim Barclay is here from that group.

Finally I should mention the National Diffusion Network, which works hard to get exemplary programs, in mathematics and other fields, into the schools. Right now the NDN is supporting ten exemplary projects in mathematics at all grade levels. During the past year these projects have been adopted in almost one thousand schools, and more than one hundred thousand children have benefited from them.

Let me now turn to the second part of my remarks, focusing on the goals of this meeting. It seems to me that now is an especially appropriate time to be having a meeting of this sort, for at least three reasons.

The first is that it's a natural step to be taking after the publication of all of the recent reports on the state of education in this country. Specifically, we are responding in a way recommended by the National Commission on Excellence in Education--that is, providing leadership in getting together all of the various groups concerned with mathematics education.

The second reason is that, even without all the recent criticisms of education, mathematical education in particular is entering a crucial period engendered by incredible technological advances. For example, I am told that there are now computer programs, just beginning to be available on microcomputers, that can do all of the symbolic manipulation usually required in algebra, trigonometry, and calculus, just as the hand-held calculator can do the numerical manipulation associated with arithmetic. Surely that must have implications for what we should or could teach in the secondary schools! I'm confident that that kind of technology will be an important topic of discussion here.

But turning in the other direction, it is also clear that the computer can help with mathematics instruction in ways that no other tool has been able to before. A very simple and short program on a microcomputer can simulate, in just a few minutes, thousands and thousands of tosses of a "fair" coin, for instance. Clearly this must be a powerful tool for strengthening intuitive understanding of probability. The much more complicated simulations that Lynn Steen, for one, has developed even suggest that totally different kinds of mathematics might find a place in our schools.

The third reason is simply that our economy is changing. There is a clear line of debate between those who are the "high-tech" advocates, claiming that soon you will have to have an advanced scientific degree to operate your intelligent refrigerator, and those who may be called the "non-tech" people, claiming that after all most of the new jobs in the next decade will be for janitors, hospital workers, and so on--occupations that currently, at least, don't require much knowledge of mathematics. As this debate goes forward, I believe we have a responsibility to be seriously discussing how mathematics education might respond best to our changing economy. We must maintain a sense of balance, however, realizing that reasons for studying mathematics have always gone beyond its usefulness in high-paying jobs and even in international economic competition.

Certainly we cannot expect to solve all of the difficult problems associated with substantially improved mathematics instruction in a conference as short as this one. I am confident, however, that you will be able to delineate clearly what the issues are and to spell out in some detail what next steps might be taken by all sectors concerned with mathematical education in this country. This particular evening session is the only one open to the public at large, but the conference will result in a full report which we will distribute to everyone who is interested in the discussions that will take place here in the next couple of days. I certainly look forward to reading that report, and I wish you every success in your deliberations. Given the nature and tenor of debate on education, all of you may be participating in a historic event showing new directions for excellence in mathematics instruction.

THE STATUS OF MATHEMATICS TEACHING IN AMERICAN SCHOOLS

Stephen Willoughby
New York University
President, National Council of Teachers of Mathematics

Last January, in testimony for the Committee on Education and Labor of the United States House of Representatives, I proposed the following four-point plan for addressing the long-term problems facing the nation in education. These are unrealistic points. I'm sure they are because I've been told by many of my friends in the government and I received large numbers of letters from NCTM members who explained to me that they were unrealistic, unreasonable, and all sorts of other things. But I'm going to repeat them here today just the same, since I noticed that some variation of them subsequently appeared in the Commission reports that came out after that.

Number One among my four points was to improve conditions within the schools. And this to me is the most important thing that we can do to attract more teachers and get a better education within the schools. The man testifying just before me, the president of the Engineering Society, said that he thought teachers should get more respect and the way to do this was to put a telephone on each teacher's desk. The way he knew that that would work was that coaches got great respect and they all had telephones on their desks. When it came my turn, I said that I personally could do just as well without a telephone on my desk, but if they wanted to do some good they could tear that loud speaker off the wall so the principal could not come on in the middle of my best lesson and announce that somebody's car was misplaced or something of that sort. My feeling is that the conditions within the school, the way in which teachers are treated, the way in which education is treated, make it unattractive for teachers to teach there, and very difficult for them to do the excellent job that many of them try to do in spite of those conditions. In some schools, in New York City where I work, teachers actually go into the schools with some fear that they may not come out alive and in good health. And my feeling is that this is not the way to attract teachers. And it is not the way to help them do a good job once they get there.

Point Number Two that I made was to increase the time children spend on learning. That was widely interpreted to mean that I thought we ought to increase the number of days in the school year. I would have no major objection I suppose to increasing the days in the school year. But to me the first issue is to spend the time that they're already there learning rather than going to pep rallies, going on field trips, collecting milk money, and doing all of the other things that interfere with the education that is supposed to be going on. Testing is one of the principal things that interferes with education, and states all over the country are adding more and more tests that the schools must give so that sooner or later we'll get to the point where we have one day of learning every year and the other 179 testing to see what they learn. If in fact the school year and the school-day are increased we could then put in the pep rallies and the other things the children do. During those extra days and those extra hours we could also have time for children to spend after school studying so that those children who do not have a home environment that makes it easy for them to do homework at home would have an environment within the school to do this. It is not necessary that professional teachers supervise all these activities. It would be much more appropriate for them to spend more time planning lessons, staying up to date in their content areas, and doing the various other things that a professional teacher does. A recent book about the Japanese school systems suggests that, while the children spend a great deal more time in school learning, the teachers spend a good deal less time actually teaching, approximately only one-third of the time that they are in the schools.

Number Three of my points was to improve the standards for becoming and remaining a teacher. By that I don't mean additional degrees. I mean look carefully at what people are studying to become a teacher and make it compare favorably with the guidelines set forth by the National Council of Teachers of Mathematics, by the Mathematical Association of America, and by other professional organizations in other areas. In New York state, for example, I know of one person who failed the Regents' exams in geometry on five separate occasions, never passed a tenth-grade geometry course in his life, never passed a course which could be considered more advanced than tenth-grade geometry, and still managed to get certified to teach high school mathematics so that he could theoretically teach geometry, algebra and trigonometry, calculus, or anything else that might be taught in the high schools. How did he do that? The requirement at that time, 1969-1979 in New York state, was that you must have 18 credits of college mathematics to be certified. Now that sounds almost reasonable, unless you think about some of the courses that are taught at the college level which go under the name of mathematics. He gave me his list. One was called business arithmetic. That was a two semester course--six credits. Another was a mathematics course for the liberal arts major. "Mathematics for the Mathematical Moron" I think it was called. Another was a statistics course which sounds almost reasonable except the particular statistics course did not require ninth-grade algebra in order to take it. And he had one other course which I've forgotten the description of. This is more common around the country than you might suppose. When New York State required a full year of calculus at the end of this eighteen credits of mathematics that would be taken to be a certified mathematics teacher, many of the colleges created special calculus courses which could be taken by prospective teachers so they would not be burdened with the necessity of actually learning any real mathematics.

The Fourth Point that I made was to double the salary of every teacher in the country. That, of course, is the one that everybody considers to be unrealistic. Would I double the salary of the poor teachers as well as the good teachers. Well sure. The reason I would do that very simply is, if there is anybody out there teaching now who is overpaid, that person shouldn't be teaching at all, and if there is anybody who would be overpaid with double the salary, that person is already overpaid. There are indeed many fine teachers who are worth everything that they would be paid under those circumstances, and we would immediately attract large numbers of candidates for teaching jobs. Keep in mind that you can't do that one without raising the standards. And it is crucial that when we raise the standards we think seriously about what we mean when we say raise the standards. It does not mean more degrees. It does not even necessarily mean more course work. It's a question of what course work is involved.

Are all of these things that should be done at the local level or do I think they should be done at the federal level? I believe that 2, 3, and 4 are principally a federal responsibility. From a practical point of view, there is simply no serious hope that local property owners and state tax payers are going to vote the necessary funds to match the major national commitment that has been made by virtually every other civilized country in the world, notably, of course, the Soviet Union, West Germany, and Japan. If the federal government can provide matching funds for highways, surely it can do so for education. The funds for highways come from taxes on users. The individuals and businesses that pay taxes to the federal government are the main users of the products of our educational system. Individual and corporate income taxes are a most reasonable source of funds for education. I must add that since 1950 corporate taxes have dropped from 28.3 percent to 8.1 percent of all federal taxes collected. Corporations pay in the United States 16.1 percent of their domestic earnings, but those same corporations pay 55 percent of their earnings to foreign countries when they are functioning in those foreign countries. So what we have is a situation in which our businesses are out busily supporting the education systems in other countries through very heavy taxes that they are paying to them while paying very low taxes in the United States.

There are two common philosophical arguments against this kind of major federal commitment to education. First "The Constitution reserves control of education to the states." That statement is simply not true. There is not a word in the Constitution that reserves education to the states. The closest that anything comes to that is Section 8 of Article 1 of the Consitution which gives Congress the power to collect taxes to "provide for the common defense and general welfare of the United States." And there is nothing that is more important to the long term common defense and general welfare of the United States today than the education of our children. The need for federal involvement in education is recognized in the Northwest Ordinance of 1787 and has continued to be a tradition ever since. There is simply no truth to the contention that either the Constitution or tradition forbids federal involvement in education. The more serious objection to substantial federal involvement in education is the argument that evil or misguided federal officials could control education for the entire nation (I understand the President is even reviewing textbooks these days) and thus control the hearts and minds of our youth. The events in Germany in the 1930s make this a particularly frightening prospect. Although this danger appears to be a very real one, it is possible to protect against it by limiting the federal government's roles to setting general standards rather than allowing direct influence on the day to day curriculum. Of course such limitations could be abrogated at a later time, but that would be more difficult to do in the future, if such regulations were written now by people who understand and wish to avoid the dangers. If the federal government, in concert with the state and local governments, were to take substantial actions on points 2, 3, and 4, by arranging matching funds on the condition that the time spent on education and the standards for teachers and the salaries for teachers were substantially raised, then it would be easy and natural for local authorities to take substantial steps to improve conditions within the schools. For example, if teachers were paid a reasonable wage it would be as ludicrous that teachers patrol the parking lots and halls of schools as it now would be to suggest that physicians and attorneys patrol the parking lots and the halls of hospitals and court houses.

I'd like to suggest a few specific proposals that this conference might wish to consider to improve mathematics and science education, in particular, and I would suggest the following five points that should be considered. I suppose with the metric system coming in I should have proposed 10, but this is half the metric system anyway--that's about how far we seem to have gotten.

1. It is essential that teachers continue their education after becoming teachers. There are lots of ways of continuing your education. You do not have to go back to a university--not even the University of Wisconsin, though I've no doubt the courses here are excellent--there are other ways as well to continue your education. Go, for example, to professional meetings; be a member of professional organizations and read their journals; discuss those journals and the articles in them with your colleagues. Right now most school systems actively discourage their teachers from going to professional conventions. It's too expensive to hire a substitute. Certainly they wouldn't go so far as to actually pay your way to this convention, but they object to even hiring a substitute, and those teachers who have taken sick pay to go to conventions are, in fact, given a very rough time when it is discovered that they weren't really sick but were instead improving their education.

I suggest that teachers ought to have more free time to visit their colleagues. One of the best ways to get an education on how to teach is to tell a colleague "I'd like to visit you; pick a date." First of all, the teacher who is visited will have a much better lesson that day than almost any day of the year. Secondly, the person who comes and visits will get an idea from this presumably best of the other teacher's lessons and will be able to go back to his or her classroom refreshed and with other ideas of how to do things. I also suggest that we ought to have further institutes like the NSF institute and other such institutes back in the 1960s, and that we ought to encourage continued education simply by taking courses that already exist in the colleges.

2. We must encourage teachers to teach general principles. We cannot continue to put computers in the classroom believing that the latest technology is somehow going to solve the problem of mathematics education. The latest technology will be outdated by the time these children get out into industry and wish to use it. No matter what computer you have, no matter how recent it is, it will not be the one that will be likely to be used by these students when they get out of the schools. I do not think having computers in every school is nearly as important as teaching children to think, to continue to learn, and to be able to deal with new situations and new conditions. I am amazed, for example, at the number of school boards that have passed rules that you cannot have a calculator in an elementary school classroom but you must have computers in those same classrooms. I contend that they are missing the basic point. It is far less important what computer, if any, is in the hands of the students, than what ideas are in the minds of the students.

3. Encourage respect for teachers and education, through your activities, your words, and through the way in which you treat the teachers and children you are trying to educate and who are trying to get an education. When I was asked to propose names for people to serve on the selection committee for the presidential awards in mathematics and science, I proposed the names of five school teachers to select school teachers who were outstanding. I got word back immediately that they wanted important people on the committee. I said, "I thought the purpose of this was to show that teachers are indeed important and I consider those five people five of the most important people in the country."

I think it is unfortunate that people respond almost automatically with the assumption that somebody who is a schoolteacher is not important. As far as I'm concerned, school-teaching is <u>the</u> most important single thing that anybody can do—and it is the most difficult. It is far more difficult than being a physician, for example. Physicians only have to work with the human body. It is the human mind that is the remarkable thing. Put the human body against the body of almost any other animal around and we come out second best. Put the human mind up against any other minds around and we come out first. And it is that mind that the teachers are asked to deal with—in bunches of 30 for 50-minute periods a day; and in that time and in those conditions to do a great job of teaching. My feeling is that teachers deserve all the respect and all the other benefits that we can possibly give them.

4. We must take much more seriously the selection of textbooks. I noticed in some of the introductory material that we got for this conference, as is very common, textbook publishers are raked over the coals for not having done the right thing. It is simply not true that textbook publishers are nasty people out to try to destroy the education of our children. Textbook publishers are in business; they are going to publish what sells. And the way we can improve the quality of textbooks is to see to it that the textbooks that are selected in our schools are the best textbooks available and that they

are used, and changed to better textbooks when better textbooks become available.

One school system in the midwest, some time ago, sent a very nice letter to all the textbook publishers condemning them all for not having done five or six things that are in the Agenda for Action report (NCTM, 1980). It turned out that there are, in fact, four textbook series available that do, in fact, all of the things suggested in that list. I asked the person who wrote the letter, "Why on earth didn't you buy one of the textbooks that does the things you said, rather than critize the publishers for not having produced them?" And the response was, "Oh, our teachers didn't like the format of those books." The format required teachers actually to teach something rather than to do the same thing that they had always done in the past. NCTM Instructional Issues Committee is working on a new set of guidelines to help people in selection of textbooks. There is already a document out by NCTM that does that sort of thing, and it seems to me we can take seriously the guidelines that are proposed in the Conference Board of Mathematical Sciences (1982) report last year, and various other reports, when we go about selecting textbooks.

5. The fifth point that I think might come out of this conference is that people interested in mathematics education should take a serious interest in the action that occurs in Washington, state legislatures, and local school districts. There are, for example, now bills pending before Congress. HR 1310 was passed by the House of Representatives; S 1285 was considered by the appropriate Senate committee. Each of these provides more than 400 million dollars for mathematics and science education and technology education. S 1285 has been hanging there since May--it was not considered important enough to bring up before the last vacation for the Senate; but it is still there and it can be acted upon. I would encourage you as individuals to write to your Senators and encourage them to bring that bill to the floor of the Senate where it will probably pass, preferably without nongermain ammendments which are almost certain to get attached to it.[1] I would also recommend that you try to keep track of what is going on in the world of legislators--federal, state, and local legislators--and influence them.

In summary, in many respects the condition of mathematics education in the United States is probably as good as it has ever been in any country. We still have large numbers of dedicated and competent teachers doing an excellent job of teaching bright, motivated students. Prospects for the future, however, look dim.

[1]S 1285 passed the Senate on June 27, 1984, with a number of amendments that were not attached to HR 1310. The House passed the Senate version of the bill, and it was sent to the President for his signature.

Conditions in the schools are bad and getting worse. Salaries and prestige for teachers are low. Young people who could be excellent teachers are choosing not to enter the profession, and many of our best teachers are choosing to leave. You've all heard, I'm sure, the statement that anybody who is competent to teach mathematics and chooses to do so is truly commited or truly ought to be.

In order to have a chance to live up to their potential, children of this nation must have a better education than they are likely to get under present conditions. A major national committtment is needed to improve the quality of education. At the local level as well, there are many things we can and must do to improve education.

An optimist is one who believes that this is the best of all possible worlds; a pessimist is one who agrees with him. Educators cannot afford to be either optimists or pessimists. We must realize that this is not the best of all possible worlds, but that it can be improved. The education that we are able to give in mathematics is one of the main ways to improve the future of the world. I hope that at this conference we will be able to do something to improve it.

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MATHEMATICS IN AMERICAN SCHOOLS

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Twenty-five years ago, in the summer of 1958, a conference was held at Yale University run by a young associate professor of mathematics by the name of Edward G. Begle. It was the beginning of the SMSG[1] effort. I was very lucky to be there for that conference, and so I am going to start out by reminiscing about this for a little while.

The group at that time consisted of an equal mix of mathematicians and teachers of mathematics working together. It was a tremendously exciting experience, especially the part of it that I was involved in. I have always felt that it was essential to the success of the process that the teachers had an absolute veto power over anything that went in. That is, the last word as to whether something did or did not go into the experimental text was held by the teachers, and we all knew that this was correct.

I learned a lot at that first SMSG writing session. One of the first people that I met was Martha Hildebrandt, whom many of you have, I hope, known. And the very first thing she said to me was, "Now remember, Pollak, you can't teach anything after April." That was my introduction to the reality of teaching, some of the problems that Steve Willoughby was talking about. She would also do the following with us: Someone would get an idea on something we wanted to put in, and we'd go to Martha and explain to her what it was we thought we wanted to do. And then we learned to shut up. Martha would sit there with her eyes closed, silently moving her lips. What she was doing was imagining the class in front of her, and she was teaching this. A few minutes would go by--in which you would try to hold your breath--and she would say, "Pollak, it won't work!" Then she'd tell you exactly where you would run into trouble; and she was right, absolutely.

There were some things I'd get right the first time. On the other hand, I wrote the beginning of one chapter eight times before Martha said, "All right." This was a great experience. People have sometimes talked about mathematicians having foisted off all kinds of high-brow things on education in that kind of meeting and in that period. In response, I would say that we had some exceptionally perceptive teachers who participated in the SMSG writing sessions and other activities; I assure you that they believed in what we were doing just as we did. The excitement was tremendous: Rethinking what was in fact going on in

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1 School Mathematics Study Group. This is one of the curriculum products discussed by Robert Heath in New Curricula (New York: Harper & Row, 1964).

school mathematics teaching. We kept asking, What does it all mean? What is it all about? For example, what are you doing when you solve an equation? When they say, "Simplify," what do they want you to do? How do you know when it's finished? It's a very interesting question. When you get $\sqrt{6} - \sqrt{2}$ you're done, but when you get $\sqrt{8} - \sqrt{2}$, you're not. How do you know? One more example: What really are variables? We found that there was a structure in all of this that made it sensible, coherent, and interesting.

There were some things that we didn't worry about at the initial session, as I remember it, and it's constructive to contemplate those. I don't recall any discussion of "Why does society give you all this time to teach mathematics? Why are mathematics and English the only subjects that many students take every year? I apologize to the history and social studies people, but I think that's true. Later on people started worrying about this, but not at the beginning. Next, nobody was there from any discipline other than mathematics and mathematics education. There were no scientists and no social scientists in attendance--as near as I can recall. There were also projects in the other sciences; one of the interesting features is that, at that time, the last technology disappeared out of the curriculum. When I went to high school we still had the superheterodyne receiver and the refrigerator in the physics course, and we still had the ammonia cycle in the chemistry course. By the time my students took Chem Study and PSSC[2], those were all gone. Right now, we're talking a great deal, wondering how we're going to get some technology back into the curriculum!

Let us return to the first of these issues: Why are we given all this time? Well, there are a lot of reasons given for that. Certainly the beauty of the subject and the structure of it and the exercise of your brains that comes with mathematics are very important. But I suspect that most of society would prefer to believe that they give you this time because of the usefulness of the subject. We didn't take that as seriously then as we do now. Steve Willoughby mentioned the Conference Board report, and I will be talking a fair amount about that tommorrow. "The widespread availability of computers and calculators," it says in that report, "and the increasing reliance of our economy on information processing and transfer are significantly changing the ways in which mathematics is used in our society." And if the ways in which mathematics is used in our society are going to change, and if usefulness of mathematics is a primary reason why we are given so much time, then we'd better think about changing what we teach. And we'd better think about emphasizing this usefulness in connection with our mathematics. Of course, what's useful keeps changing. It's different at different times; it's different in different places. One of the crazy things that happened during the 1960s was the attempt to export

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[2] Chem Study and PSSC, Physical Sciences Study Committee, were also discussed by Heath. See note 1.

curricula lock, stock, and barrel from one country into another. As if, never mind anything else, the usefulness of mathematics was the same at the same time in all different countries!

Let us examine how the changing technology affects the teaching of mathematics. The first and most obvious effect is the use of the technology in teaching the traditional mathematics, frequently only for drill and practice. More imaginative uses, for example of graphics and of symbol manipulations, are certainly possible. But there are many other things. There are topics that have become more important because of the change in society, because of the technology as we have it. For example, I remind you of estimation. Estimation was always important, but it is even more important now, when you do a computation on the calculator, to have an idea how big the answer should be before you go to it. Another subject whose importance has greatly increased is algorithms. Then there are topics, and this is also important to consider, which are made possible because of technology. These are subjects that we have always wished we could do but which were simply out of bounds because we couldn't handle the mechanics. My favorite subject of this kind is data analysis. This is the point of view on statistics that keeps looking at the numbers themselves rather than just mouthing some formulas that nobody understands. For example, you don't just take a sum of squares, divide by either n or n-1 depending on which side of bed you got out of in the morning, and then take the square root. What do I mean? First, you have to figure out how to take the data. Now that you have the numbers, what are they trying to tell you? How do you get an idea of what is going on? Should you transform them to get a better picture? How can you compare two sets of data and decide whether they are connected with the same phenomenon? How do you draw conclusions from these data? I visualize all of this not with any formulas but as an exercise of arithmetic and of thinking about what's going on, the use of pictures, and transforming of pictures. Many of us have talked for years about wanting to do this kind of exploratory analysis of data. It's terribly useful, but the trouble is that it was simply pedagogically impossible. We just couldn't handle real data. Even our college textbooks in statistics were usually full of fake data--you produce the numbers in the book that the students are going to work with because the answers have got to come out easy. Even a physics course as good as PSSC had many of the inclined planes at 53°26'. Why? That's what you get in a 3,4,5 triangle! You don't let the students take their own data because the arithmetic has got to come out easy! If you try it, you end up with all kinds of messy numbers, it will take an hour for the class to agree on just the average, or anything else about these numbers, and by that time you've forgotten what the question was. And so you simply couldn't do it. But now, with micros in the schools, you can indeed do data analysis, and you have an opportunity for something we've always wanted to do and which is more important than a lot of other material now in the curriculum.

There are also topics which are made less important because of the technology. I was talking to a lawyer in the airplane, coming here, and, as he put it, two-thirds of all elementary school mathematics is taught in order to make calculators and microprocessors obsolete. That was a very good way of putting it. We teach arithmetic in order to replace calculators and micros with paper and pencil! Then, of course, there are topics that are going to be needed later on because of other changes, because of the technology itself that they will be learning. Certainly the subjects of discrete mathematics, of recursion, of algorithms again come up.

There will be a lot of argument about something that I don't know if you'll be able to escape, or want to escape, and that is the teaching of technology itself. Call it computer appreciation courses, if you like; I mean basic technological literacy, which a lot of schools are thinking about requiring for graduation. Who's going to teach that? The answer isn't at all clear to me. At the moment, in most schools, the mathematics teacher is likely to be blessed—or cursed—with having to do it because that person is the closest one. There isn't anybody else as close, even though a lot of technology isn't very mathematical. In the long run, is this sensible? Should there, instead, be teachers specifically trained to be teachers of computers in high school, and perhaps of technology more broadly? People are interested in technology along with science in the schools. Who's going to do it? We have a difficult problem here, because the schools that train a very large proportion of teachers in this country and the schools that have good technology available are two sets whose intersection is almost empty. The great bulk of teachers are trained in liberal arts colleges that have very little engineering and very little computer science on their campuses. The universities and other institutions that have a lot of technology have, for the most part, little teacher training left. So, I don't know how we're going to be able to provide in a sensible way the combination of the two sides of education you want to have in the same institution.

There is also a real change in the incoming students, and I think we might as well face up to that. The problem of access is one that a lot of commissions have talked about. Will what is available in the home in the way of technology make the inequality of access even worse? What happens with poor districts versus well-off districts? You are also going to have little and not-so-little changes in the elementary curriculum because of technology. My favorite one to think about is "What are you going to do about clock arithmetic when clocks are all digital?" The students won't have seen any hands going around on the face, they won't know anything about it!

Another interesting effect, a pedagogic one, is that for the first time that I can think of we are interested in teaching a subject in which a fair fraction of the students in the class is extremely likely to know more than a fair fraction of the teachers. The students coming from a situation where there is technology around the home will simply know more about computers than most elementary teachers. This means that the usual authoritarian method of staying one day ahead of the students is not going to work. They're already six months to a year ahead of you. What are you going to do? You're going to have to learn to teach in a more open-ended way than some teachers instinctively do. You're going to be in the position of running the class as a moderator, or interlocutor, and learning from the students rather than trying to be authoritarian about it. Some years ago, a social sciences researcher at the University of Colorado developed a scale for authoritarianism. It was a scale of four values and, as I remember, they found that 50% of the population as a whole was at the most authoritarian level. Among teachers 80% were in the most authoritarian group. How will such teachers handle computers?

I'd like to point out one other aspect of technology in the schools, one that struck me particularly in chatting with Hugh Burkhardt and Rosemary Fraser from England who visited here last spring, and that is, how people think about using computers and technology in schools. In many cases, in the United States, the idea behind the computer in the classroom is that, if necessary, it can replace a poorly trained or incompetent teacher. If the teacher doesn't know a particular topic, don't worry, the computers will take care of it. In England, the emphasis has been very much on using computers to help turn a mediocre teacher into an excellent one. Use the technology to help get rid of some of that drudgery, to help the teacher do a much more thoughtful an deeper job of teaching. The above generalization is too sweeping, but these tendencies are real. We often think, "If the teacher doesn't kno it, maybe the technology will take care of it," while in England they think very much in terms of using the technology to help someone to become a better teacher.

Let me finish by mentioning some of the changes that I expect will happen in the elementary school in particular. I mentioned data analysis before, which I think will be terribly valuable. It's connected with a much bigger subject, which is the relationship between mathematics and the other disciplines in the elementary school. For example, what is the relationship between elementary mathematics and elementary science, between mathematics and social studies, and so forth in elementary schools? These are interesting to consider. In some sense mathematics has been too strictly and narrowly defined in elementary school. But elementary science has not, in fact, been defined at all: In most states there are no particular requirements about what you do in elementary science. This has been very handy. You can get some nice new materials tried out by calling them science. But it has never been quantitative, because the students, as we said before, couldn't handle the data. Now we're in a position where indeed the point of view in elementary science and elementary social science can be changed; the relationship between them and mathematics can be totally new. There can be a degree of cooperation, of thinking together and working together that we just have never had. This very badly needs work, and I'm very much looking forward to this development. Similarly, we can now teach planning in a quantitative way. Both elementary science and elementary social science can change in a significant way.

CHANGE AND OPTIONS IN SCHOOL MATHEMATICS

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The U.S. Department of Education, the National Council of Teachers of Mathematics, and the Wisconsin Center for Education Research are sponsoring this conference on School Mathematics. The impetus for the conference stems from three sources: (1) an accumulated body of evidence about enrollments, performance, and so on that indicates that serious problems exist in school mathematics; (2) the several recent reports which criticize the quality of mathematics instruction in contemporary American schools and make recommendations for change; and (3) the potential impact of the current technological revolution on the content of the school mathematics curriculum. The intent of this conference is to provide an invited group of scholars an opportunity to consider the problems of teaching mathematics in American schools and to recommend strategies for achieving suggested changes in mathematics instruction.

The product of the conference is to be a report summarizing the deliberations and will include a considered analysis of current problems and trends and outline a set of actions which could be taken. The primary audience of this report is the staffs of local school districts, although of necessity other audiences will be addressed. We realize that school staffs are or will be attempting to respond to the pressures for change. We also realize that actual changes will only occur at the school level, no matter what the wishes of national commissions or politicians at either the national or state levels. Changes will not occur simply because of the recommendations of conference participants. Furthermore, we understand that change will occur in schools only over time. What I hope we produce from this conference is a document that gives direction to educators at all levels of how to respond to the current pressure for change. We hope to do this by alerting them to needed changes and problems and by suggesting strategies, sequences of steps, or senarios to follow.

A Perspective

The title of my talk "Change and Options in School Mathematics" reflects the theme of the conference. First, the object under scrutiny at this conference is "school mathematics" which here will be characterized in terms of:

-- the mathematical content of the curriculum (scope, segmentation, sequencing ...) for all students in grades K-14

-- the work of students in classrooms related to the curriculum

-- the work of teachers in classrooms related to the curriculum

Second, to change something implies that one thing is substituted for another and that there are fundamental differences between the original thing and its substitute. Note, I am emphasizing both substitution and fundamental differences. Providing teachers with a software program or two for every chapter of the current text is only addition not substitution. This point was made clear to me when Madam G. G. Maslova (1979) described one of the principles the Soviets were attempting to follow in developing the new mathematics curriculum they are now implementing. That principle was that "to include new concepts and methods requires a sharp reduction in outdated material while preserving the basic nucleus of the curriculum" (p. 77). We must "replace" not just "add on."

The notion of fundamental differences is also important. One thing we learned from past efforts to change schooling practices is that folkways or traditions of schooling must be directly challenged for fundamental change to occur. The paper that Gary Price and I wrote that was sent to you addresses this issue (Romberg & Price, 1981).

Finally, the word <u>options</u> implies that there are several possible substitutions for current practices which are reasonable. This implies a sequence of decisions (or steps) in changing from current practice to some ideal state. Hence, the listing of possible strategies must be considered.

<u>Current Practice and Change</u>

On pages 3-6 of our chapter for the <u>Third Handbook of Research on Teaching</u>, Tom Carpenter and I described a stereotype of mathematics instruction and its limitations (Romberg & Carpenter, in press).

The instructional stereotype is "extensive teacher-directed explanation and questioning followed by student seatwork on paper-and-pencil assignments" (Fey, 1979, p. 494).

The following remarks by Wayne Welch (1978) are typical:

> In all mathematics classes I visited, the sequence of activities was the same. First, answers were given for the previous day's assignment. The more difficult problems were worked by the teacher or a student at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and problems were assigned for the next day. The remainder of the class was devoted to working on the homework while the teacher moved about the room answering questions. The most noticeable thing about mathematics classes was the repetition of this routine. (p. 6)

Three serious limitations to stereotypical mathematics instruction are described below. First, mathematics is assumed to be a static discipline. The emphasis is on teaching a fixed set of concepts and skills. This traditional view of mathematics sees there is a lot to teach. For schools, the consequences are that mathematics is divorced from science and other disciplines and then separated into topics such as arithmetic, algebra, geometry, trigonometry, and so on, with each treated independently. Within each topic, ideas are selected, separated, and reformulated into a rational order. This fragmentation of mathematics has divorced the subject from reality and from inquiry. Such essential characteristics of mathematics as abstracting, inventing, proving, and applying are often lost.

Second, when one fragments mathematics in this way, the acquisition of the pieces becomes an end in itself. Students spend their time absorbing what other people have done, rather than having experiences of their own. They are treated as pieces of "registering apparatus," which store up information isolated from action and purpose.

Third, the role of teachers in the traditional classroom is managerial or procedural in that "their job is to assign lessons to their classes of students, start and stop the lessons according to some schedule, explain the rules and procedures of each lesson, judge the actions of the students during the lesson, and maintain order and control throughout" (Romberg, in press, p. 4). Furthermore, the individual lessons are provided for teachers via a curriculum guide, a syllabus, or most often a textbook.

This stereotype--where mathematics is seen as a static discipline, learning is viewed as absorbtion, and teaching is considered managing--must be changed if the students who are currently being taught mathematics in school are to have an adequate preparation for the scientific world of the 21st century.

The challenge for participants of this conference is to suggest how changes in school mathematics programs can be accomplished.

We will start by considering changes in the mathematical content of school programs. We have asked four participants (Henry Pollak, Lud Braun, Bill Hunte , and Zalman Usiskin) to focus our thinking on this topic. As Jim Wilson recently said: "Today our elementary programs are designed to train students to compete with a $4.95 calculator; our college-bound program gets students ready for calculus at a time when calculus's pre-eminance is being challenged; and all other high school students are dumped out of mathematics after ninth grade with few marketable mathematical skills" (1983). Such general recommendations as those included in NCTM's Agenda for Action (1980) or CBMS's What is Still Fundamental and What is Not? (1982) now must be taken seriously.

Since we also need to challenge the "absorbtion-management" perspective of learning and teaching, tomorrow afternoon we have asked five others (Robert Siegler, Robbie Case, Tom Carpenter, Penny Peterson, and Glenda Lappan) to summarize recent information about learning and teaching.

Following those presentations we will form four working groups:

-- mathematics for the elementary-junior high schools

-- mathematics for the senior-high school

-- learning and teaching

-- computers and technology

Your task in these groups is twofold: first, to propose recommendations and, second, to decide what implementation would require.

First, as you develop a set of recommendations, remember that the target population is school staffs. Consider hypothetical situations. Suppose a state superintendent asked you to outline the content, scope, and sequence of a "new basics" program for the elementary school. How would you proceed? Or suppose a publisher asked how you would organize materials differently given the new knowledge on learning. What would you suggest? The challenge is to be imaginative, creative, radical and not, at this time, to feel fettered by current constraints (staff, budget, and so on).

Second, after coming to consensus about goals and changes, begin to consider what resources it would take to implement each recommendation. For example, one group may argue that a study group needs to be formed (like CEEB in the 1950s) to suggest a variety of detailed programs for high school students. Another group may suggest that teachers must be really treated as professionals. What is involved in making that possible? Hiring a noninstructional staff responsible for management? Be creative, radical, do not be fettered by constraints.

After we have had a chance to propose solutions, we want to come back to reality and hear testimony from six experts (Robert Williams, Vivian Makhmaltchi, Chancey Jones, William Barclay, Marilyn Hala, and Jane Gawronski) about the problems of change as seen by persons responsible for making change happen. Finally, we will form new groups, examine the feasibility of the previous recommendations, and flesh out the strategies for change. The task is to outline what it would take to make proposals operational.

Following this effort, with the help of the Steering Committee, I will attempt to write a paper summarizing our deliberations. This is to be available in April. I want it clearly understood that your help and input is critical. I can think of no one better suited to do the task than the group assembled here.

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TESTIMONY ON MATHEMATICS

IN THE SCHOOL CURRICULUM

Edward T. Esty, Chair

NEW GOALS FOR MATHEMATICAL SCIENCES EDUCATION

H. O. Pollak
Bell Communications Research, Inc.

Dr. Pollak reported on the recent conference sponsored by the Conference Board of the Mathematical Sciences. The meeting, funded by the National Science Foundation, was held at Airlie House in Warrenton, Virginia, November 13-15, 1983. His actual report is not presented here since the report of that conference has now been published.

In addition to presenting the main recommendations from the CBMS Conference, Dr. Pollak stressed the importance of collaboration and coordination between that conference and this one.

The following summary of topical recommendations is taken from pages 5-7 of the conference report.[1]

I. Recommendations Concerning Curriculum

The fundamentals of mathematics desirable for students at elementary, secondary, and college levels have, in the view of many mathematics educators, changed radically, yet the changes are not reflected in core curricula.

The Conference recommends the establishment on a continuing basis of a Task Force broadly representing appropriate segments of the mathematical sciences community to deal with curricula.

The initial efforts of the Task Force should be:

- To gather information on current practices and alternatives both here and abroad regarding the scope and sequence of mathematical topics in the curriculum.

- To gather recommendations from scholarly groups, industry, and other interested parties on mathematical expectations for all (or some) students K-14.

- To formulate alternative high school programs for students not preparing to continue their mathematical studies at the college level, or intending to pursue college programs not requiring the traditional calculus sequence.

In the long term, the Task Force should provide a number of curricular components which may be assembled into viable curricula.

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[1] Conference Board of the Mathematical Sciences. (1984). New goals for mathematical sciences education. Washington, DC: Author.

II. Recommendations Concerning Teacher Support Networks

With few exceptions there is little contact locally between elementary and secondary teachers in the schools and professional mathematical scientists on the faculties of colleges and universities and in industry.

The Conference recommends the establishment of a nationwide collection of local teacher support networks to link teachers with their colleagues at every level, and to provide ready access to information about all aspects of school mathematics.

III. Recommendations Concerning Communication of Standards and Expectations

There are two distinct issues here. The first concerns the need of secondary students, teachers, counselors, and parents to be kept informed of the standards and expectations of colleges and universities relating to mathematics achievement and the progress of individual students towards meeting these standards and expectations.

The second concerns the need of school systems, schools, and teachers for assistance in setting standards, now and in the future, that will enable their students to meet the expectations of the colleges and universities and of future employers.

The Conference recommends, in response to the first issue, the use of "prognostic" tests designed to measure the progress of students toward fulfilling mathematical prerequisites for college programs, or for employment with or without postsecondary schooling, sufficiently early to allow for remedial and/or additional course work while still in secondary school.

In response to the second issue, the Conference recommends that a Writing Workshop be held to prepare a series of assistance pamphlets and course guides, based on current thinking and curricula, that would have the endorsement of the mathematical sciences community and provide timely assistance to school districts in their efforts to improve the quality of mathematics education.

IV. Recommendations Concerning Mathematical Competence and Achievement

Mathematical skills have become essential in many fields of business, industry and government, not only in technical, but also in non-technical positions, blue collar as well as white collar jobs. As a consequence a much larger fraction of the population must learn more mathematics than ever before in order for society to function and for individuals to function in society.

The Conference recommends that strong efforts be made to increase public awareness of the importance of mathematics, and that more effort go into the identification and encouragement of the mathematically able and gifted, especially among women and m·ᵗ- .ity groups. Care must be taken to ensure that all students, K-14, have equal and adequate access to techno'ogy.

V. Recommendations Concerning Remediation

Although there are reasons to hope that remediation may be a less serious educational problem in the future, it is currently a very serious problem facing secondary and post-secondary institutions in the United States. Current efforts and approaches are inadequate to solve the remediation problem.

The Conference recommends that (1) funding agencies support projects to improve current efforts in remedial education, and (2) a series of regional conferences be called to address the problems and needs of remedial education.

VI. Recommendations Concerning Faculty Renewal

The renewal of mathematics teachers' content knowledge, teaching skills, and enthusiasm for their work is clearly needed at all levels of education.

The Conference recommends new initiatives that address the special situation at each level: Elementary (K-4), Middle School (5-8), High School (9-12), Collegiate, and Two-Year Colleges and Technical Schools. It is recommended that the professional societies in the mathematical sciences, especially NCTM, MAA, and AMATYC seek support as soon as possible for projects to demonstrate effective models of the various faculty renewal activities recommended.

A Recommendation to CBMS

During the closing session of the Conference, the concern of the participants shifted from specific proposals to the question of how to insure appropriate followthrough from the mathematical sciences community. It was agreed that on-going oversight in some form by representatives of the mathematical sciences community will be essential (1) for the recommendations of this Conference to be further developed and implemented, (2) for the establishment of close, mutually rewarding and continuing ties between the research and educational communities as envisioned by the Conference, (3) for a continuing effort to develop a comprehensive view of the needs of mathematical sciences education.

The Conference strongly and unanimously recommends the establishment of a National Mathematical Sciences Education Board, or its equivalent, broadly representative of the mathematical sciences community, and that substantial funding should be sought for this Board to enable it to carry on all activities deemed appropriate.

THE IMPACT OF COMPUTERS ON MATHEMATICS[1]

Lud Braun
New York Institute of Technology

There are three very serious problems in this country in relationship to the use of computers in schools: (1) lack of access to computers, (2) lack of good courseware to use with those computers, and (3) lack of teachers who are trained to use either computers or good courseware. I think the federal government has a very serious role in responding to all three of these problems. Without the federal government playing a significant role in solving these problems, not much will happen that is good.

First, let me underscore the equity issue with computers. There are children in this country who are being denied access to computers because they are female, or live in poor communities, or are black or Hispanic, and so on. Some are simply economically excluded from access to computers. Also, some are excluded by subtle psychological pressures. For example, there are girls in the moderately well-to-do community in which I live who are excluded from access to the computers in the schools with subtle pressures that I do not understand. That problem must be resolved.

Second, much of the courseware that is being developed by commercial organizations is not very good quality. Even though I feel very strongly about computers, I think that computers are only part of the solution to the problems in mathematics education. They are not a panacea. Some people think that computers are going to solve all of our problems; I do not. They can only help us solve some of our problems.

Bill Huggins was the chairman of electrical engineering at Johns Hopkins and probably the most intelligent person I have ever known about how to use computers to create learning environments for kids. He described the computer as a lump of clay which the teachers could mold into any format that they wished. One of the really good examples of molding computers was done by Frank Syndon at Bell Laboratories. He generated a series of films in the 1960s using a computer to generate mathematical ideas visually. An excellent one was on the simulation of planetary systems with force laws that were other than the inverse square law. It is a beautiful example because it illustrates the kind of thing that can be done with a computer. I can create a world in

[1]This paper was edited from a transcript of the proceedings and has not been revised by the author.

which the force between two objects varies as the cube of the distance between them, or as the square root of the distance between them. Here everything behaves according to the inverse square law; there is not anything I can do about it. Within the computer I can play God. I can say, "Thou shalt obey the whatever law." That kind of power is available to you as math educators.

Third, a lot of people think of the computer as a quantitative device; I think of the computer as a qualitative device. It will help us to change the way students understand things. I think that's a lot of what's wrong with mathematics education at the moment. You'll pardon me for saying so, but I think that mathematics education in this country approaches children in very much the wrong way. Most math teachers teach mathematics as if every student in the room were going to become a mathematician. I think that's wrong. I'm not a mathematician, I never was and I never will be. I have learned a lot of mathematics. I have had to to do the things that I have done. There are lots of people like me who need mathematics as a tool. There are also lots of people who are chased away from mathematics by poor teaching and thus cut off from many disciplines. Poor teaching starts at the elementary level. We need to do much more to train elementary school teachers to teach mathematics by getting kids excited. By the time they get into junior high or high school it's too late for many of them--they've already been turned off by mathematics. Teaching kids how to divide one four-digit number by another four-digit number doesn't engender much excitement about mathematics. And it is not mathematics.

Most math teachers do not make mathematics concrete for kids. The way to make it concrete for kids is to build a mathematics laboratory. Within the computer I can build mathematics laboratories for kids of any age and in any subject that you can name. I can get kids to discover what a function is. When I was in high school a teacher once told me, "This is an equation. There is something over here and something over here and there is an equal sign between them." I know what an equation is, but I didn't learn it from that math teacher; I discovered it for myself over a period of years. I learned how to find roots with the best of them in high school, but I thought that only first order and second order polynomial equations had roots--until I was a graduate student. And even then I did not know what a root was, and I didn't know what anybody did with roots, and I didn't know why anybody wanted to know what the roots of an equation were except to get an A on an exam. I can get kids to understand probabalistic ideas in an experimental way within a computer. I can build all kinds of discovery learning experiences in a computer. We have laboratories for kids in physics. No one would dream of offering a physics course or a biology course without having a laboratory experience as part of it.

If we're serious about getting all kids to learn mathematics, we must give them the opportunity to develop intuition about mathematics. That is what's missing in most mathematics education. Lots of kids go through math education and learn how to take exams. I aced almost every math exam I ever took, and I never learned anything about mathematics until I was a doctoral student and was forced by the thesis I picked to learn about lots of the things I had done very well on examinations but never really understood. That, I guess, is why I became an engineer. Engineering involves the application of intuition, bringing science and mathematics together through intuition to solve problems. There are several examples of software which build this relationship: SEMCALC (Sunburst Communications), developed by Juda Schwartz at Harvard; TKSolver (Software Arts), an equation solver; and Rocky's Boots (The Learning Company). I also wanted to mention that there is something new being developed that I find very exciting. TERC is developing a gizmo that you can plug into the back of many microcomputers. Then you can connect the computer to the real world and get data poured into the computer from any kind of a transducer (measuring pressure, temperature). The computer stores all the data and then you can do various kinds of manipulations--you can plot graphs, you can see trends.

With the computer we can focus on higher level thinking skills, on, for example, developing real problem-solving skills, not the kind of problem solving that I learned while I was in high school. Word problems had three variables, one of which was on the left-hand side of the equal sign and the other two on the right-hand side. The only problem was to figure out which of those slots you put the numbers in to solve the problem. That's not serious problem solving. Two things that build real problem-solving skills are already available to us. One of them is teaching kids programming. Another kind of thing that generates problem-solving skills is adventure gaming. Adventure games are fascinating things. Students are excited about them. Students are presented with very rich environments. They are required to read and comprehend what they read. The environments contain lots of information--they need to sort through all that information and pick out those pieces of information that are useful to them. The manual doesn't say, "The problem is to find ten pieces of gold and run out of the building." You have to go through it and figure out what the problem is. That's what real-world problem solving involves.

We need to look at testing very seriously. Testing organizations, like ETS and the Regents in New York State, have a detrimental effect on education. They force teachers to teach certain things in certain orders. They give teachers excuses or reasons for not teaching other things. I do not know how many hundreds of times I have been told in the state of New York, "I can't teach that, because my students don't need to know that to pass this, that, or the other exam." Exams are important. We need to know what our students are learning, how well they are learning it, and what they are not learning. But, in New York State, the teaching stops sometime in the early spring. Teachers have to get the kids ready to pass the Regents' exam. That is important because the teacher is evaluated on the basis of what fraction of the students in the class do well on the exam. And the principal and the superintendent are evaluated on the same basis. I do not care how well kids do on Regents. What I want to know is what they learn. They do not learn all that much. Students learn in spite of rather than because of the schools.

Training of teachers is a very serious problem, especially in the area of computers. We have about five years to get training going for most teachers in this country. Everything that I read suggests that we are at the beginning of an explosion of computers in schools. There are going to be computers essentially in every classroom five years from now. How many teachers are going to be ready to use them in an intelligent way? That is a federal problem, not a state problem, because every state that trains teachers is going to have those teachers stolen by states that aren't spending money training teachers.

We need to develop ways of retaining kids in mathematics. I do not want to see kids drop out of mathematics in ninth grade. I want to see kids excited about mathematics, learning mathematics. We need to make mathematics more concrete, more understandable, and more interesting. If we use computers intelligently we can do that.

In closing, I want to say one other thing. We need to break down the departmental barriers in the high schools. Mathematics is not a discipline unto itself for most people. Mathematics is a tool to learn physics and chemistry and everything else that kids have to learn. There is no reason why mathematics is taught by mathematicians about mathematics and separate from the other things. Why not use genetic examples when you are talking about probability? Why not use physics examples when you are talking about solving equations? Why do these sterile examples seem to fill math textbooks? "How is Farmer McGee going to maximize the number of square feet that his chicken run is going to have, when he only has 150 yards of chicken wire?" Who cares, except Farmer McGee. We need problems that are interesting to kids.

THE IMPORTANCE OF STATISTICS IN SCHOOL MATHEMATICS

William G. Hunter
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I'd like to begin by talking about a Harvard Business School study of air conditioners in the United States and in Japan. They counted the number of assembly line defects per 100 units. In the U.S. they got 63.5 defects as the median number from these different companies. Japan came out better. They had fewer defects. In fact, less than one per 100 units. That's from the New York Times, August 25, 1983. That's a big difference, and the air conditioner manufacturers over here cannot be very happy hearing about these results.

IBM, it was reported in the Toronto Sun, April 25, 1983, as an experiment, ordered some parts from Japan. In the specifications for the parts, IBM said three defective parts per 10,000 would be acceptable, nothing higher than that. When they got their shipment from Japan, there was a letter which said the Japanese had a hard time understanding North American business practices, but the three defective parts per 10,000 had been included; they were wrapped separately.

Many industries in the U.S. are in trouble. Look at ship building, cameras, automobiles, motorcycles, steel, and consumer electronics. The Japanese Sputnik is real. Just talk to people in industry.

Last summer I was at Brigham Young University giving a talk on this sort of thing. A Japanese man came up to me afterward. I had been saying that statistics really plays a part in all of this. He said to me, "The point is more important than you seem to realize. You know the biggest difference between the U.S. and Japan when it comes to statistics? In America the only things I hear are jokes about liars, damned liars, and statisticians. People here don't understand statistics. In Japan, we know what statistics is, we study it in high school, and we study it in grade school. We have a National Statistics Day. Everybody knows about statistics. We know what it can do for us and we use it. And everybody uses it." The more I thought about the point he made, the more profound I thought it was.

This past summer, when I was riding in an elevator at an American Statistical Association Meeting, a man said to me, "Is there a convention in town?" I said, "Yes, it's the American Statistical Association." He had a typical disappointed reaction to this news. But his child, who was about eight years old, very brightly asked me, "Oh, do you do baseball games?" Statisticians do much more important things than figu.. out batting averages.

First of all, what is statistics? It is the study of data: the efficient collection of data and the effective analysis of data. It's really the science of science, the way I look at it, or more broadly, because I'm not just talking about scientists, it's how you learn from data. Statistics therefore has a role in science and government. Government must deal with many numbers--for example, population figures and unemployment figures. The Environmental Protection Agency and other agencies are concerned with problems such as What are we going to do with formaldehyde, pesticides, and chemicals in the environment? What is safe? What should we regulate? What shouldn't we regulate? Here are some numbers that are uncertain; what do we do with them? Such problems point to the role of educated citizens in society. I think the purpose of education is to produce informed and productive citizens who enter the "real world" as critical thinkers. For instance, they should be able to look at TV ads and say, "That's rubbish," or "That's good," or whatever.

Statistics has an especially important role to play in industry. I think in the United States what we really have to have is a transformation of industry, about what people think of when they think of their jobs. If I go out to an industry and say, "What's your job?" somebody will say that it is to produce this product or to deliver this service. Well, that not's the way they should be thinking about their job. It's part of their job, but their job should be something more. (The remarks that follow also apply, perhaps in somewhat modified form, to work in government, education, and elsewhere.) Whenever there's a product being made or a service being delivered, there is also potential information that surrounds this flow. You can either waste it or you can use it. I think it should be used. I think it should be tapped, it should be exploited. Why? To find out how you can do the job better. That's what's going on in Japan in many industries. That's why the number of defects in Japanese cars over the years just keeps steadily going down. And that's why Ford and General Motors, for example, are out telling their suppliers, "Look, you people have to learn statistical methods and use them to get rid of the defects. You have to send us good stuff so we can make good cars." The point is to tap this potential information. I would like to see the day arrive that, when we go out to industry and ask people what their jobs are, they would say, "Part of our job is using our hands. We use appropriate tools that the company has provided to us to make this product or to deliver these services as well as we can. We're also provided with tools to let us tap the potential information that surrounds this process, to use it to try to make things work better around here."

What tools? What tools are you going to use to tap this information? The answer is statistics. That's what it's all about. It's about generating information and analyzing information. In summary, as far as quality and productivity improvement are concerned in industry, government, education, and elsewhere, potential information surrounds all processes. Statistical methods allow everyone (and I mean everyone) to tap and exploit it so that productivity and quality can be improved. What you want to do is engage everybody's brain. When someone carries around a pair of hands, he or she also carries around a brain. Why should it be idle? It shouldn't! The challenge should be to constantly improve all processes.

What are the two big areas of statistics? There is efficient generation of data in the first place and effective analysis in the second place. Those are the two parts. More specifically, in the collection of data, there are things like designing experiments that scientists do, there are observational studies such as sample surveys, and there are censuses to find out about populations. Analysis of data has to do with extracting all the useful information from a set of data.

One thing I like to tell people in industry is the following. Here's a nice simple four-step argument. Suppose you want to improve productivity in quality of products in your company and you want all employees to help. One, you have to make changes in the way you do things. Two, the only rational basis on which to make changes is good data, not opinions, not hunches. Three, what data are you going to collect and how are you going to analyze them? You have to answer this question. Four, that's statistics. The conclusion is that everyone in the organization has to learn statistics. That's what's going on in Japan in many organizations. Everybody is learning statistics. They have these tools to tap the information to improve things. The main message here is that potential information surrounds all industrial processes, all governmental processes, and all educational processes. Statistical methods allow you to tap and exploit it so that productivity and quality can be improved. Engaging the brain as well as the hands of all employees improves participation and profits in industry and service in government. I really do believe we need a new way of thinking about jobs. We have to replace complacency on the job with participation. That means active participation.

People in industry occasionally get uneasy if I don't talk about flipping coins. You, too, may expect me to discuss probability. So I'll tell you the story I sometimes tell when I teach in industry. One day I heard a news broadcast concerning a teacher who gave a true and false test. All the students were working on the test, but one boy was flipping a coin. The teacher asked why he was flipping the coin. He said, "If I flip a coin and it comes out tails, I write down true, and if it comes out heads, I write down false." When she was collecting the exams, the boy in the back was still flipping his coin. She said, "You're supposed to be finished now." He said, "I know, but I'm checking my answers."

Here are some simple statistical ideas: Pareto diagrams, cause-and-effect diagrams, flow diagrams, histograms, stem-and-leaf diagrams, run charts, quality control charts, and scatter plots. They have been used with great success in industry, especially in Japan. They may be appropriate to teach in high school. All of these are listening tools. There are two ways to learn: you can sit and listen, or you can talk with somebody else. Conversational tools include designed experiments and evolutionary operation. Evolutionary operation is a statistical technique that's been used in industry. It's simple. High school students could certainly understand it. It works. It costs very little. You don't need any specialists or special equipment.

Consider productivity in the U.S. and other countries. Is agriculture in good shape in the U.S.? The answer is yes. Do they use statistical experimental design in developing crops (new methods of harvesting, planting, and growing) and raising livestock? The answer is yes. But now look at industry. There are many sectors in which we are not in good shape and we don't use experimental design.

Suppose you have a system with many variables, such as a printed circuit board. Variables include solder type, flux type, and many more. How are you going to figure out which of these variables are critical, and what's the right flux type to use? People in industry face this type of problem all the time and the usual kind of answer is to vary one variable at a time. Hold all of the variables constant, vary one at a time. That's wrong. That's not the right way to do it. But that's the way we teach all of our students to do it. There are more efficient ways to go about problem solving when you have a system with many variables, and this is part of what statistics is about.

Recently, in an undergraduate course the students could run some experiments. One student told me he wanted to run an experiment on a plane. He'd asked people what to do when the engine stalls on takeoff. He got all kinds of conflicting suggestions about what to do. He didn't know who was right, so he said he wanted to run some experiments to find out. I was nervous about this whole thing. But he said he wouldn't do it at ground level, he'd be at 1,000 feet. He planned to simulate a stall on take-off, and then he'd go through this maneuver and try to get back on the runway. There were many questions to answer: What air speed should he maintain? How should he have the flaps? What's the right bank angle? He performed a factorial design and all these and other questions were answered. It is an efficient way to find things out. Let me mention another student. He works as a chef. He had problems with popovers. On Thursday nights they serve them with roast beef. He did a factorial experiment and found out that only one of four variables he studied was important. It wasn't the one he thought it was going to be.

Factorial designs should be used more. In the social sciences, however, there are problems. In many situations you can't do experiments. One social science friend of mine said a definition of social science is hacking one's way through an open door. One big difference between social sciences on the one hand and physical and biological sciences on the other hand is that in the social sciences you often can't experiment, but in physical and biological sciences you really can. I had one student who did an experiment on developing a new piece of equipment using a factorial design. She kept working on the project and I asked her a year or so later, after she was out of the course, how that ever worked out. "Well, actually, it was very successful," she said. She sold her basic idea and with the proceeds bought a house.

You may be thinking that factorial designs are too difficult for high school students. Let me tell you about another application of one of these 2^3 factorial designs on changing variables and making a cake. There are three variables: the amount of baking soda, the temperature in the oven, and the time in the oven. The student who did this experiment measured things like the height of the cakes, the color of the cakes, and consistency and taste. Another thing you should know about this student, Dalia Sredni, is that she was a seventh grader. Not only high school students can do this, seventh graders can do it. Students have done experiments on how to hitch-hike better. They had a crutch and not a crutch, and various other things. A crutch made a difference. Students have gone fishing; they've made popcorn; they've tried various variations of recipes. Basically, the only prerequisite is a curious mind. The key thing is to let the students learn for themselves--by doing experiments on things they care about.

These designs offer a good way to find out things about the real world we're in. People have said our students should learn how to figure things out. That's what I think statistics is about. It seems to me particularly appropriate as we enter 1984 that we should teach students something about statistics and dealing with numbers so we can have more critical thinking by citizens in our society.

NEEDED CHANGES IN MATHEMATICS CURRICULA

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I was asked to speak about needed changes in the mathematics curriculum. The word change implies that we are proceeding from an existing framework and suggests that we are not thinking of starting from scratch. It includes the notion that we have schools, that we have students in schools, that we have a curriculum or curricula in place, and so on. I too am working from that assumption, but one of the options we could entertain at this conference is not to begin with the current curriculum, but to imagine that we are starting from scratch. Then we should begin from day 1 in first grade or perhaps even preschool and ask what we should, could, or would do with children. By not picking this option, I am setting constraints that result in my suggestions being more conservative than they might otherwise be. I say this to appease those who think my suggestions are not bold enough. Tell me to start from scratch and I will be glad to oblige.

We have just heard a wonderfully entertaining and very informative talk by Bill Hunter on statistics. Lud Braun has given us cogent remarks with regard to computers. I disagree with a few points made by each speaker, but I agree with the intent to increase the importance that these topics have in the curriculum. But this is not the first time we have heard such pleas. The NACOME report of 1975 recommends increased attention to this content. So do NCTM's Agenda for Action (1980) and the College Board's Academic Preparation for College (1983). If all we do here is recommend moving in these directions, we will have done nothing new. And if we only recommend moving in these directions, we will have done nothing new. And if we only recommend these directions as options, we will be moving backward.

The other key word in the title of this paper is curricula. This may not be as easy a term as you think. People who study curriculum identify various levels at which the curriculum operates. I understand that the Second International Study of Mathematics Achievement distinguishes three levels: (1) the intended curriculum, consisting of goals, syllabi, and hopes, often seen in reports such as this conference might produce; (2) the implemented curriculum, what is actually taught (e.g., see Stake, Easley, et al. (1978), and (3) the achieved curriculum, which is the focus of discussion when we speak in terms of national assessment or SAT scores.[1]

[1] John Goodlad and his students (Goodlad, 1979) distinguish five levels of curricula, but we can be content with three.

In a background paper for this conference, Tom Romberg stated that the recommendations on school mathematics vary from report to report. I disagree. Not only do the agreements far outweigh the differences, but when the reports differ, it usually is because they didn't consider an issue, not because the opinions are so different. Tom stated that there is no agreement on how to proceed. This is because proceeding means going to the level of the implemented curriculum, and generally reports have ignored not only the problems of implementation, but the different nature that recommendations must take if they are to be relevant to an implemented curriculum.

For example, over the past 25 years, there have been many recommendations for and no recommendation against the teaching of geometry at the elementary school level. But we are all aware that geometry is ignored by many teachers in favor of paper and pencil arithmetic. My experiences with the "slow" ninth graders I'm teaching this year have led me to view this problem in an even more dramatic fashion, for it seems that the differences in knowledge between these students and their better-prepared classmates are even more drastic in geometry than on the algebra and arithmetic ideas used to place them in this class. Even in those districts where geometry is supposed to be taught, districts like the suburban one in which I am working this year, geometry seems to be taught only to those elementary school students who already know arithmetic. Because my students never knew the arithmetic well enough, they were taught no geometry.

As a second and perhaps more striking example, I know of no high school that has changed a single day's content because its feeder elementary and junior high schools decided to follow NCTM's Agenda for Action and structure its curriculum around problem-solving.

These examples illustrate that even a topic in everyone's intended curriculum will not reach the implemented curriculum until there is a specific grade level at which people expect it to be taught. A student's teacher one year has to know that the student's teacher next year will be irked if the topic is not taught, and will take advantage of the teaching if she decides to teach it. Consequently I am more than a little frustrated with reports that simply say we should be doing some statistics at every grade level. I agree but I want to know what statistics should be taught, when it should be taught, and how it might be taught.

So I am arguing that our report should differ from others. It should speak to the issues from a standpoint that is meaningful to someone who wishes to implement the curriculum. That requires that four notions, often ignored in other reports, be considered here with respect to anything we recommend:

(1) The population. If we speak of school policy, do we mean all schools? If we recommend something for students, do we mean all students? When we say that there is a problem, for whom is it a problem? When we speak of cities, do we mean all cities? When we speak of college-bound, do we mean all college bound? And so on. Let's try to speak in terms of percentages if we can. Let's not recommend that

the entire nation be above the national median, which seems to be the goal of some reports.

For example, we know that there is a shortage of qualified mathematics teachers, and we often hear it said that this is due to low teacher salaries. In the district in which I live, and in many suburban school districts around Chicago, there are teachers who are making over $40,000 a year. This district has had openings in recent years for mathematics teachers, but I do not know if it has more applicants because its salaries are higher. We say that higher salaries will ultimately affect student performance. However, after socio-economics are accounted for, do students in this district perform better?

If one makes recommendations as if the United States has a homogeneous population of schools, students, and teachers, one is ignoring the extraordinary variability among and even within school districts.

(2) The grade level. When do we do things? When we recommend do such and such with calculators or computers, at what grade level? What statistics? Saying that there should be some at every grade level says nothing at all.

(3) Time and sequencing. Unless we lengthen the school day or school year, or get increased time devoted to mathematics, everything that we recommend putting in requires that we recommend taking something out.[2] If we recommend x be taught at a given level, does that have implications for the teaching of y either before, after, or simultaneously? For example, if we recommend using a calculator for 3-digit long division, surely there will be students wanting to use it for 2-digit long division. And there are students who will want to use calculators for single-digit long division. And there are students who, having a calculator in their hands, will want to use it for addition. What does that do to time available?

(4) The current situation. Is anyone doing what we recommend? Is what we recommend based on research evidence, or is it just based on intuition? For instance, I believe NCTM's Agenda recommendation to base the curriculum on problem solving is based on no research at all, since to my knowledge such a curriculum has never been devised. (This is a classic case of a recommendation made without identification of grade level, time, or population.) Where does one turn to for materials?[3]

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[2] See Usiskin (1980) for an elaboration of this point.

[3] At this point, one participant suggested that there were such curricula and that DMP was one example (Romberg et al., 1974, 75, 76). Another mentioned the Real Math series (Willoughby et al., 1981).

The point is that we should be certain to identify those practices that are, or have been, in consonance with our recommendations.

With that preface, let me turn to what I consider to be needed changes. Obviously, only a broad outline can be given in the time available. I distinguish between the elementary and secondary school levels for reasons that will be evident.

Elementary School (K-6) Level

1. The Calculator Presence. The tragedy of the elementary school mathematics curriculum today is that, thirteen years after the first hand-held calculators appeared, there still does not exist even a prototype elementary school curriculum that assumes their existence and adjusts accordingly.

Regarding the effect of the calculator on paper-and-pencil skills, we would probably agree on the following generalities but we might not agree on the specifics. I'll give one specific for each. (a) Some skills, such as long division, are obsolete. If we recommend only that long division not be taught, we will have freed from one-third of a year for the fastest students to probably a year for the slowest students to learn other things. (b) Other skills, such as partial product multiplication, are not as important as they once were and may be obsolete. (c) Some skills, such as multiplication by integer powers of 10, are more important than they ever were. (d) Some new skills appear, such as the ability to represent expressions with fractions on a single line. This itself requires more facility with parentheses than is now taught.

More than the scope of the curriculum is changed by having calculators (see Usiskin, 1983b). The sequence and timing of virtually all topics have been determined by and organized around paper-and-pencil skill requirements. For example, we carefully sequence addition and subtraction by the number of digits in the terms or by the number of renamings required. With a calculator, there is little reason to delay work with larger numbers.

It seems that if one truly integrated the calculator into today's curriculum even without deleting anything, decimals would have to be discussed in second grade along with money, and a four-function calculator would suffice only for grades 1-4. Then there are too many opportunities for very big and very small numbers to justify getting error messages again and again. Scientific calculators would require an earlier introduction of exponents and rules for multiplying by powers of 10.

The pedagogies and the ways in which we introduce topics have also been determined by paper-and-pencil requirements. For instance, we introduce fractions as ordered pairs or ratios, focusing on numerator and denominator but not on the division represented by the fraction bar, because the paper-and-pencil algorithms for equivalent fractions and for the fundamental operations with fractions get us to look at the parts of the fraction. With calculators, the division idea of fraction is necessary from the start, and it would seem that students might encounter 8/3 before they encounter 2/3. This would reverse the usual order of things. And, given the very poor understanding that today's students have of fractions, this reversal could undo an unwise historical ordering.

Even our research has been affected. We have studies of error analysis in paper-and-pencil subtraction which will be as useful in the future as studies of the best landing support for a dirigible.[4]

If we did no more than get the calculator into the implemented curriculum, we would have done enough at this conference. But there are other areas that deserve our attention.

2. Geometry. We need an elementary school curriculum in geometry. That is, we need to identify grade levels at which certain skills are expected to be mastered and certain concepts are expected to be discussed. Until we have such a curriculum, we can assume that geometry will continue to appear in school books and continue to be taught primarily as a topic for good students to cover while others are remediating arithmetic.

3. Applications. What was said about geometry applies to applications as well. There is no curriculum for applications. Lists of objectives will often be so specific with regard to skills--e.g., multiply a one-digit whole number by a two-digit whole number--and then, in the last line, say "Be able to solve word problems using multiplication".

But the situation is worse for applications than for geometry, because whereas we could consider Euclidean geometry to be the realm from which we choose almost all elementary school geometry content, there is no such commonly known or considered realm for applications. Max Bell and I have tried to remedy this in a 500-page manuscript on applying arithmetic (Usiskin & Bell, 1983). We have tried to elucidate all major uses of the operations, symbols, and maneuvers of arithmetic (such as estimating, rewriting, etc.).

There is no way I can describe this work to you. I have brought only a single copy. However, I can whet your appetite. The students who have gone through it in a course, ranging from math-anxious future elementary school teachers to people very capable in mathematics, uniformly report that it has changed their view of arithmetic. They now have a context in which to place applications somewhat similar to the

[4]In fairness, it should be noted that not long after the conference there appeared an item in newspapers regarding the possible resurrection of the use of dirigibles because they might be the best vehicles for sightseeing.

real number field context we have for the mathematical properties of the numbers of arithmetic.

4. Grouping. A quote from a similarly-titled paper of a generation ago seems just as applicable today. "In attempting to sugar-coat the course in mathematics for the slow-learning students, we are, at the same time, lowering the standards of accomplishment for the more gifted and, as a result, the bright student becomes the most retarded of all" (Reeve, 1955). At the elementary school level, it is not just the gifted who become retarded but even the average students who have learned what they were taught the preceding year.

Consider the following. (1) There has been almost no change in the arithmetic curriculum of the primary schools despite students coming in with far more knowledge than their counterparts of a generation ago. As a result, comparative test scores show students at early grades ahead of those counterparts, but students at later grades not ahead. (2) Classes are rarely grouped in grades 1-5, and when they are, it is by reading rather than mathematics performance. (3) From one-third to one-half of every year of elementary school mathematics is review of previous years.

It does not take much to see that profound changes might occur in the amount of elementary school mathematics that an average or above average students might learn were such students grouped by mathematics performance. We might even be surprised at what would happen with slower students where teachers did not simultaneously have to work with gifted students.

5. Deletions. We should make a concerted effort to change the view of the elementary school arithmetic curriculum as sacrosanct. Long division (i.e., paper-and-pencil division with a divisor of more than one significant digit) is the first topic mentioned as a potential topic for deletion from the elementary school curriculum, but all complicated arithmetic falls in the same category. Because the Cockcroft report in Great Britain (1982) recommends it go completely, I've looked into this point in some detail. There seem to be only two places in which long division seems to have use beyond getting answers to division problems. First, we use it to explain why every rational number has a repeating decimal. Second, we generalize it to the division of polynomials. Now the question is: Are these things important enough to keep long division in the curriculum?

In general, we must look at the implications of deleting what we suggest for deletion. Often, examination of the implications shows that the idea had little value in the first place and provides even stronger arguments for the deletion than one might have before such examination.

Secondary School (7-12) Level

I have remarked here more about the elementary curriculum because earlier this year I wrote a paper detailing a proposal for reforming the secondary school curriculum (Usiskin, 1983a). I have brought a half dozen copies of that paper and will refer to a brief summary that has been duplicated in enough copies for all. However, let me again begin with some general remarks.

A fundamental problem with the secondary level (and to a lesser extent at the elementary level as well) is a widely-held belief that mathematics is not good unless it is hard. Yet the history of mathematics is, in part, a history of finding easier methods for doing problems. And so today we have a fundamental conflict between the belief that mathematics is hard and the existence of easy, if not automatic, ways of doing problems on the calculator or computer.[5]

For many reasons, the secondary level is different from the elementary level. The obvious differences are in training of teachers, the subject-matter compartmentalization, the size of schools, the elective nature of some of the courses, and the dual-track (college-bound and non-college-bound) curriculum.

Two other differences are seldom cited. (1) Whereas we know how many students take each grade of elementary school, and so know to some extent what students are taught, we do not have reliable data concerning what courses students take at secondary school grade levels. For instance, we do not know what percentage of eighth-grade students take algebra. In fact, we do not know the percentage of ninth-grade students who take first-year algebra. While the NSF studies (Weiss, 1977) gave an indication of the most widely used books at the elementary school level, they did little for high school. Thus we are sitting here potentially making recommendations about changing mathematics education at the secondary level, but we have not much data regarding the present curriculum from which to work.

(2) The mathematics taught at the secondary level is not tested as it is at the elementary school level. The most commonly cited barometer of performance, the SAT-Ms admittedly do not require higher level mathematics, and even if they did, they do not constitute a random sample of students taking various courses.[6]

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[5]See Usiskin (1984) for a longer essay on this point.

[6]The deceptiveness of our scanty knowledge can be seen by just a small additional examination of SAT-M scores. The population of students taking the SATs is rather interesting and more complex than is reported in the media. The media reports the mean SAT score only: for 1982-83 that mean was 467 on the SAT-M. However, the mean score for the sub-population of seniors was 455. That's quite a difference. Also, the mean score of 1965-66 juniors on the SAT-M was 505, and more juniors are taking the test now than then. Have the scores of juniors remained relatively constant while others have declined? If so, why? Furthermore, some 142,000 students took the SAT in 1982-83 who were neither juniors or seniors and their mean was a very low 430. There were only 78,000 such students in 1965-66 and their mean was 488. What is this group whose scores have declined so much and what has been their effect on the overall mean through the years?

Nor does a random sample take the ACTs, tests supposedly based more on the actual content of high school courses. Those states with required mathematics competency exams for graduation from grade 12 do not provide helpful data with regard to secondary school mathematics; they cannot test an idea that first is taught in grade 9 or above, because nowhere does everyone take algebra. Since the number of high school graduates in recent years is only 75% of the age cohort (see Table 1), even National Assessment data from 17-year-olds do not give us a reliable picture.

So our evaluations of what is going on in high schools are based upon much shoddier evidence than our evaluations of elementary school mathematics. We do not really know what is taught and we do not know what is learned of what is taught. Oh, I have directed studies in algebra and geometry and have some data that I could share with you from fairly large samples, so I may have more educated guesses than some of you. However it is sad but true that many people tend to believe studies of performance only when they are from national carefully stratified or random samples, unless they are done on your campus, in which case studies are trusted and cited.

When I made up this proposal, I tried to get some hard data on enrollments. Some of what I found is in Table 1. The most significant points: (1) Geometry study is about equal with planning to go to college. (2) The number of mathematics majors is only about one-seventh what it was in 1966, but the slack has more than been taken up by computer science. Thus there has been a great change in the kinds of mathematics students will study and need at the college level but there has not been a corresponding change in the curriculum. (3) Only about 15% of the age cohort will major in those areas (natural sciences or engineering) in which calculus plays a more important mathematical role than other subjects such as linear algebra or statistics. In this percentage are included the computer science majors, but I have been told by some that they should not be so placed.

The proposal for re-forming the secondary school curriculum (see Figure 1) is based on several assumptions. One is to be line with recommendations of national reports over the past ten years (e.g., the NACOME report (1975), NCTM's <u>Agenda for Action</u> (1980), the College Board's <u>Academic Preparation for College</u> (1983), and the recent CBMS recommendations (1983)). A second is to keep those things that people think are going well, so as to minimize implementation barriers. (It's hard enough to get rid of things that are going poorly.) Specifically, that means keeping the courses for our best students about as is.

The proposal itself can be summarized by seven recommendations. (1) The curriculum is differentiated for three populations of students: Population I, the college-bound planning to major in engineering or natural sciences; Population II, the college-bound or college-hopeful planning to major in all other areas; and Population III, the non-college-bound. (2) All students take a semester of computer mathematics. (3) For all students this is followed by a semester of statistics.

With these recommendations, for populations I and II the high school mathematics curriculum has become five years long. Since one cannot expect to put five years into four, this requires either going to summer school, finishing the curriculum in college, or starting early. For various reasons, it seems wisest to (4) put algebra in eighth grade. This recommendation is helped by following the advice of the various committees to (5) give greater attention to applications. The algebra and geometry courses so envisioned are entitled World Algebra and World Geometry (to indicate a broader conception than just the mathematician's algebra or Euclid's geometry). (6) In these courses and in World Algebra II, formal manipulation and formal proof are given reduced roles. Computers are given a stronger role, particularly in World Algebra II. Table 2 summarizes differences between these and existing courses.

(7) Even for the non-college bound, there is a semester of algebra and a semester of geometry. This, coupled with the semester of computers and semester of statistics, is necessary in my opinion if we wish to educate all students for the 21st century. Consumer mathematics is kept for these students not only because it is important that they learn applications of this type, but also because that course seems to be the only widely implemented course that has been successful with these students. The courses in grades 9-11 for these students might be done in an order different from the one given here.

For many reasons, a unified integrated curriculum at the secondary level is not recommended here for any students. One reason predominates; we have had a unified curriculum at the elementary school level for at least a generation and we haven't been able to get anything universally taught except arithmetic. The fact is that unified curricula work only in those places (e.g., New York State with its Regent's exams or foreign countries with their uniform college entrance exams) where there is a test at the end that serves to pressure teachers to teach everything. A second important reason is that, while mathematics is in theory a unified subject in which all branches can be deduced from a common origin, in practice and in technique the ideas learned in one branch often do not apply to another branch, and the problems that motivate a particular branch are almost always unique to that branch. Again we have here the difference between intended and implemented curricula; what seems to be a good idea at the intended level is a poor idea to implement. If it were so good, then why don't college mathematics departments teach courses to their mathematics majors that unify the various branches? Instead of a unified integrated curriculum, the recommendation is that what is learned in one course be used in the next. That is, in contrast to today's practice of ignoring geometry in second-year algebra (or vice-versa, when the algebra is taught first), all courses should make use of ideas and techniques from all previous courses.

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Table 1
SOME RELEVANT DATA
(All U.S. nationwide)

1. BC & NCHS: There were 4,268,000 babies born in 1961; about 2.3% of these died before age 19. There were 3,134,000 h.s. graduates in 1978-79. [Thus about 75% of an age cohort graduates h.s.]

2. BC: In October 1981, about 16% of 18- and 19-year-olds had not graduated high school and were not enrolled in school. [This datum, together with the above, suggests that 9% of the age cohort gets GED's.]

3. NAEP: 17-year-olds in school in spring, 1978, reportedly taking at least 1/2 year of each course:

Algebra 1	72%
Geometry	51%

4. NCES: seniors in May 1980 reportedly taking each course:

Algebra I	79%
Geometry	56%

[These data are not inconsistent. CDASSG: About 5% of geometry students are seniors. Some students who do not take Algebra 1 may drop out after their junior year. Combining with the above, about 2/3 of an age cohort presently takes Algebra 1, and about 1/2 takes Geometry.]

5. NCES: Of May 1980 h.s. seniors, 46% plan to get a college degree; 15% plan to go to college but stop short of a degree; 19% plan to go to a vocational, trade, or business school; 19% plan no postsecondary ed. [Combined with above data, 52% of the age cohort plans on some college education; 68% plans some postsecondary education or training.]

6. Adapted from NCES: The planned majors of May 1980 h.s. seniors planning on college:

Engineering	10%
Natl. Sciences	16%
Business	22%
Soc. Sciences	5%
Health Services	8%
Hum or Arts	16%
Education	6%
Other	17%

7. Adapted from Astin et al.: The probable majors of September 1980 college freshmen:

Engineering	12%
Natl. Sciences	17%
Business	24%
Soc. Sciences	7%
Health Services	8% (est.)
Hum or Arts	9%
Education	8%
Other, Undecided	15%

[Except for humanities or arts, these data are consistent. Mathematics, statistics, and computers are under natural sciences.]

8. Astin et al.: 4.9% of entering 1980 freshmen indicated a probable major in computer science, data processing, or computer programming, compared to 0.6% indicating a major in mathematics or statistics, down from 4.5% in 1966.

BC = Bureau of the Census, as reported in Information Please Almanac 1983.
CDASSG = Cognitive Development and Achievement in Secondary School Geometry Project, University of Chicago, 1982.
NCES = National Center for Education Statistics, High School and Beyond, 1981.
NCHS = National Center for Health Statistics, as reported in Information Please Almanac 1983.
NAEP = National Assessment of Educational Progress, Results from the Second Mathematics Assessment, NCTM, 1981.
Astin et al., The American Freshman: National Norms for Fall 1980, as reported in the Conference Board of the Mathematical Sciences Report of the Survey Committee, Volume VI, 1981.

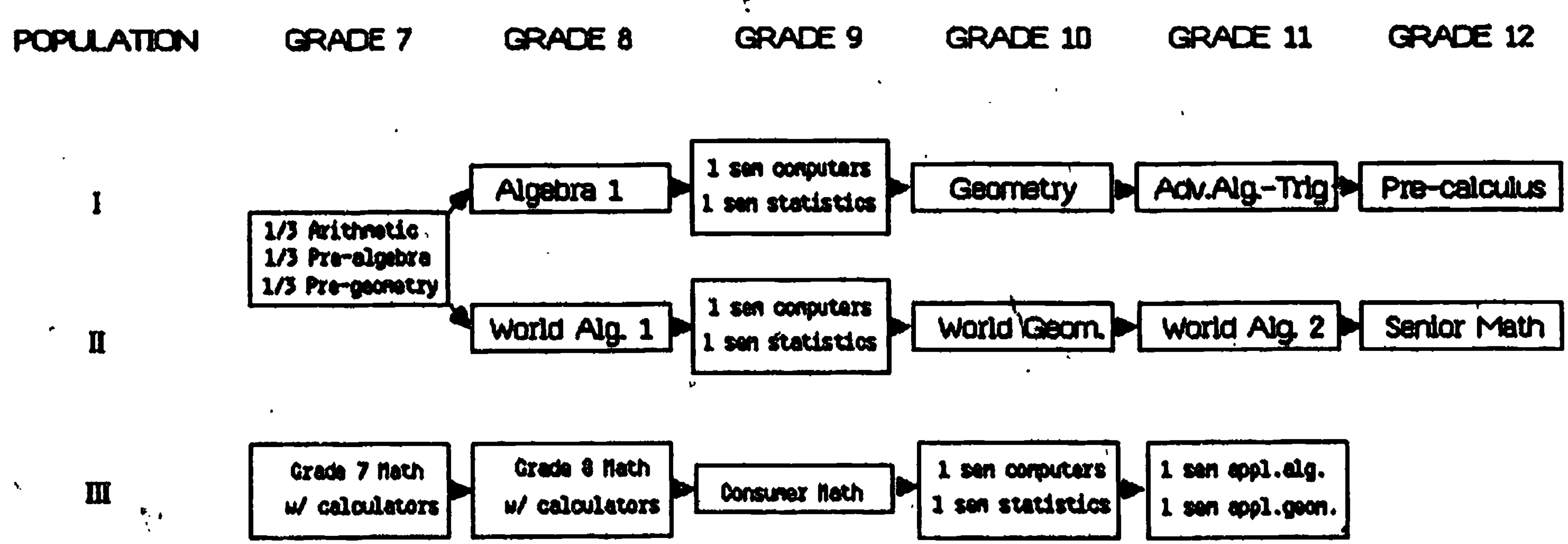
Table 2

DIFFERENCES BETWEEN COURSES for POPULATIONS I AND II

Change	Out	Into both courses
Algebra I to World Algebra I	Contrived word problems Trinomial factoring Complicated polynomial and fractional expression manipulation	Applications Calculators
Geometry to World Geometry	Rigorous approach from first postulates Anything > 6 weeks solely devoted to proof Advanced theorems on circles and triangles	Transformations for congruence & similarity Symmetry More applications More coordinates More 3-dimensional geometry
Algebra II to World Algebra II	Complicated manipulations done by hand Log tables Intricate graphing Permutations and combinations (to stat.)	Complicated manipulations done by computer Graphing via computer General graphing theorems Matrices More applications

Figure 1

A SECONDARY SCHOOL MATHEMATICS CURRICULUM TO ACCOMMODATE RECENT PROPOSALS



DISCUSSION

The comments by the conference participants following these presentations were on a variety of topics. While many issues were raised, the participants inevitably returned to five major themes related to mathematics in the school curriculum:

1) the intended mathematics curriculum,

2) computer literacy,

3) the role of professional organizations,

4) testing and the curriculum, and

5) equity.

The Intended Mathematics Curriculum

The discussion about the content of the intended curriculum, focused on the needed direction of change.

All the recommendations of recent reports can be split into two types. The types that say increase requirements, increase the length of the school day, do more. And these have as a fundamental assumption that what is currently being taught is fine. Students just are not taking enough mathematics, or the schools are not designed to do it. We have to make the schools better. The second type of recommendation in the reports implies that what we are doing now is not right. Times have changed, and even if the students learned everything that we now are teaching them, it would not be right. These recommendations are quite different. The first type of recommendation is easy to change. If you want to increase requirements, you legislate. That is what we see happening all over the country. Implementing such recommendations may get in the way of implementing the other. Where is that computer course going to come in? Where is statistics? We must look back at the assumptions that underlie the conclusions that we make and really look back at them carefully.

There was almost immediate consensus that "what we are now doing is not right." In fact, the recommendations of CBMS (1982) in What is Still Fundamental and What is Not and of NCTM (1980) in its Agenda for Action are excellent. The problem now is to develop curricula based on those recommendations and have them implemented.

There was a strong feeling that we should not follow the "new math" approach to curriculum development.

The new math did a serious disservice in that it gave us a curriculum predominant model. It eclipsed some of the second generation new math efforts looking at learning and teaching.

The new math was not just new topics but a new way of thinking. As long as we have a curriculum that is topic controlled, it seems to me we are going to continue to misdevelop in the same way. For example, if manipulative materials are used well in the early grades, they have incredibly strong effects. But teachers are using them less and less. This trend needs to be reversed. Incidentally, the microcomputer is a marvelous manipulative device.

The question perhaps which is the most important is What is success?

We want students to be able to understand what they are doing so they can apply it elsewhere. We know what skill is, we don't know what understanding is. And I believe that it might be helpful to think of four domains. The first is algorithms. Algorithms are not just skills. They are understandings associated with skilled performance. We tend to think of following algorithms as a low level activity, when in fact very high level processes are often used on algorithms. A second domain involves understanding. For many teachers, students understand something if they can do the algorithms. That is incorrect. Mathematicians, when they get together, say somebody understands if they know the mathematical underpinnings (e.g., numeration, the whole numbers). But the goal for teaching is not mathematical underpinnings but utility. A person only understands something if they can apply it to the real world. This is the third domain of understanding. Until about 1957 we only taught the first of these domains. Then we changed to make mathematical underpinnings a primary goal. Then around 1967 when we found that the second did not help the first as much as we thought, we came to the third. But there is a fourth domain, representations or metaphors. The use of concrete materials, or of the computer to simulate, gives still a different kind of understanding. It is not a real world, it is not of mathematical underpinnings. We say somebody understands only if they can give us a picture, if they can draw a graph on the blackboard or the chalkboard. To me this is a fourth kind of understanding.

These points are similar to those raised by the National Science Board's commission last year. They said that "basics" mean four different things. The ability to do the thing with paper and pencil is one. Understanding why it works is the second. Knowing when to use which operation is the third. And the fourth is knowing how to do it on a calculator and in other ways.

This issue is really the process vs. product question. It is very easy to identify content. It is not so easy to identify process. It is easier to see if content is in materials than process. It is easier to test content than it is to test process. Yet, the ultimate goal is usually that you do want process, the learning to learn.

There was a great deal of discussion about the intended curriculum, even though in reality the intended curriculum is not what is being taught.

Teachers modify or change the intended curriculum not for different levels of understanding but because of perceived needs of different kids. When we talk about an intended curriculum we must indicate for which kids, which teachers, and which schools. There is some evidence that situation decisions are much more different than reports would have us believe. When someone recommends doing something different in tenth-grade geometry, that one dictum does not apply to all of the geometry classes in the country. What is appropriate for one place may not be appropriate for another.

Although participants agreed that they want to produce critical thinkers, there is a part of our society that does not want schools to produce critical thinkers.

We have realize that we have a very complex society in the United States and that there are many people in the society who feel that schools are for perpetuating a certain doctrine, whether that doctrine be freedom or whatever. We argue for critical thinking. We are already making an assumption about the nature of schools and the reasons for schooling that is not accepted by everybody. There are people who don't want processes taught because they do not want children to think creatively and originally.

Computer Literacy

There is no question that technology (calculators, computers, etc.) will effect school mathematics, but there was no consensus on details of that effect.

Everybody is saying that we should teach computers and computer literacy. How do we know that that's so? Do people have feelings about what should be taught or what shouldn't be taught and when? More specificity is needed about when, what grade level, to what kids, in what fashion.

Some kids find the computer a tool and most kids ignore it. For those who find it a tool, it is just fantastic. They use it repeatedly to get roots of polynomials; they use it to make graphs in three dimensions and look at the graph from different positions so they can see what surfaces look like. Some of them have made games that are really quite respectable video games, as good as the typical ones. What's bothering me is that it seems to me that we would make a mistake if we tried to make everybody do this. I do not understand why we would want to do that. It's very important that somebody play the cello well, but do we want to make everybody play the cello? I think we are making a mistake by saying How can you teach computer science so that everybody is going to get to be good at it?

It bothers me that we are talking about injecting a child into an environment in which mathematics is used and that environment is in fact the computer. Most of the kids are going to ultimately have to use mathematics, not just in connection with computers but in connection with the real world. And it seems to me one of the difficulties that we have had with mathematics teaching in the past is that children have seen mathematics as something that is done in schools; and then there's something else that you do outside the school which is to solve your real every day problems. There's a story that Max Beaberman used to tell about his second-grade daughter. She was getting the wrong answer that many children get in the second grade because she had been taught to add numbers without carrying and then she got to ones with carrying and she added an extra digit. He convinced her with popsicle sticks what the correct answer would be and he then asked her. "Isn't that right?" And she said, "Yes, with popsicle sticks the answer is 62 and in school the answer is 512." I fear that if we substitute the computer environment for the school environment we will get the same effect. I am delighted to use the computer as a part of the environment, but I do not want children growing up believing that a computer has all the answers anymore than I want them growing up believing that the teacher has all the answers.

There is one important aspect of technology in connection with mathematical content. We need to take a look at the traditional content and ask ourselves Why is it there? What was it optimizing? We will find a very large amount of material which is there in order to prepare for hand computation. That is the only reason it is there. We can now ask ourselves Is that a sensible optimization? The answer is apparent.

There are really two separate issues here. One of them is teaching about computers, teaching what people call computer science, and the other is to use the computer as an environment within which kids can learn. We need to keep those two very carefully separated.

I would like first to separate out what people call computer literacy from the rest of the conversation. Computer literacy is a very large bag of worms which is separate from mathematics However, I would argue that there is beginning to be some evidence that teaching programming to people increases their problem-solving skills significantly. Although these people were college students, the examples they've looked at have been traditional kinds of word problems. When you take a problem and try to use a computer to solve it, what you have to do is identify the problem, express it in a particular way, develop an algorithm for solving it, and then convert that algorithm into a language which the computer you're going to use will understand, and then you have to do another very interesting intellectual thing, you have to debug it not only syntactically but also logically. All of those things are higher level skills which are part of problem solving, real problem solving. Also there are lots of ideas in mathematics that children are given in inadequate ways now that can be enhanced dramatically by using computers. Computers are wonderful graph devices. They're wonderfully interactive devices. The combination of interactivity and graphics plus the computational things that they can do make them really good tools for immersing kids in learning environments.

I also had a comment about problem-solving skills in programming. I think the reason that students who take programming do better in problem solving is that that's one of the few courses where they're taught problem skills. It's not because of the programming itself. It's because they're taught problem solving-skills in the course.

Unfortunately what is most frequently taught is the least intellectually challenging aspect of programming. The essential ingredient is development of an algorithm, in my opinion. What is frequently taught in computer science courses is how to you convert a given algorithm into a language that the computer understands. That's what is often called programming. It involves understanding the syntax of a language, and it is the most trivial part of getting a computer to solve a problem. We need to address that issue when we look at curriculum. How do we get teachers to teach algorithmical problem solving, formulation of problems, understanding of what the problem is, and debugging, which are much more intellectually difficult than the expression of the algorithm in Basic or Pascal.

The computer is only part of the solution, it's not a panacea. I don't want to see kids not have popsicle sticks, but I do want kids to be able, for example, to run the lemonade stand program and learn about economics and numbers. There are times when one child should be at a computer alone to learn some things, but there are times when the class ought to be together working with the computer to learn some things. There will be times when the whole class together has a thousand popsicle sticks in the room, and there ought to be times when one kid is in a corner with popsicle sticks. Computers are different from popsicle sticks, they are interactive.

The Role of Professional Organizations

Of particular concern was how curricular change could occur. How effective have the professional mathematics organizations been? Have NCTM and CBMS, for example, improved mathematics education in the schools over the past twenty or thirty years?

The recommendations on the preparation of teachers of mathematics (both elementary and secondary) that were prepared by NCTM and MAA (CUPM) were effective. There were conferences held in every single state on teacher training. In many states the requirements were changed as a result of those conferences.

On the other hand, NCATE, which certifies educational programs, ignored what was said by NCTM and by other professional organizations.

If there is to be change and improvement in mathematics education, it can be done with the help of professional organizations. Such organizations include the mathematics organizations, publishers, and teacher preparation institutions.

Change comes from without and within. The real question is to what extent do these changes come from within, that is within the professional organizations and within the professional networks. If we go back to 1923, 60 years ago, there was a report of a Mathematical Association of America out of which NCTM was formed. For the next dozen years, every textbook would state that it followed the recommendations of that report. Now, that's pretty weighty, since our evidence is that teachers follow textbooks. In the 1960s, people would say they followed the recommendations of either SMSG or UICSM or CEEB, but they needed some nationally recognized symbol to substantiate the changes they were making because the changes were coming from within.

On the other hand, with computers, most of the changes are coming from without. I feel that they are not coming from organizations. Organizations are reacting to the marketplace, to parental demands, to parental pressure. Change goes both ways, and it's hard to separate these things. Although it starts without, it becomes within. Many people are here because they represent various professional organizations or various groups or they found out about each other through professional organizations or groups. The networks that were created over the last generation have really helped mathematics education in this country without question. They make it a lot easier to do things.

I draw an analogy with engineering. In engineering, if I am a president of a college and I want to offer an engineering curriculum and I get my state to approve that engineering curriculum, I still must go to an organization which accredits colleges in engineering. There is a tremendous pressure on engineering colleges to get that accreditation. There is a visitation which is made periodically to every engineering school which looks at the facilities that are available, the curriculum structure, the preparation of the faculty, and the commitment of the institution and its resources to the engineering programs. If all of those things are not in place, the institution is put on notice. Maybe NCTM or CBMS could form an accrediting agency for mathematics curricula and publish those accreditations so that the colleges know which schools are offering good mathematics programs. The mathematics professional organizations which have a vested interest in the quality of mathematics instruction should have the leverage to either accredit or not accredit mathematics programs on an individual basis.

The problem with accreditation is convincing a school district we are correct. When we teach experimental design to our students, we typically introduce it to them by saying the focus of this is to inform or convince reasonable critics that ideas they didn't believe before are in fact correct. There is a flavor of preaching to the initiated in our comments. A lot of statements have been made that problem-solving skills are important, we ought to have faculty renewal, it is not important that kids learn long division. How do we know any of these statements? Is there evidence behind these statements? What evidence would convince people that didn't already agree with us that what we say is right.

Testing and the Curriculum

One way to change what is being taught is to change what is being tested.

There is some consensus that the national tests have a powerful influence in shaping curriculum. It may be that those tests are not good. The question then is whether the national tests should be changed as a way of changing mathematics education or tentatively whether there should be some increased focus on testing to improve teaching and learning in the classroom.

I think it's important that this group consider taking a different position than what had been called for by other national groups, namely that there be a national enterprise in testing.

Testing is a two-edged sword. We don't get geometry taught in the elementary school because the teachers don't think it's on the test. And so the tests do have some purpose. If we told the teachers from now on there is going to be such and such on those tests, we could get some changes. Some states have gone to competence testing which includes a skill part and a problem-solving part. They are now getting some problem solving taught. I do not know what they call the problem solving, but I suppose they are word problems. They're getting it taught because they're testing it.

Testing helps create the learning environment. I do not know where you begin to talk to the people who create tests about ethics because they just say, "We're just testing what is. We're not creating it." But in a sense they also have created it.

It seems to me that the way we do our testing in this country is absurd. We decide that we have large numbers of students whose tests need to be evaluated and that can only be done by machine. Because that can only be done by machine that requires a certain kind of question to be asked. That is exactly the wrong way to design examinations. I always tell my students that my exams have two purposes. One of them is to let me know what I'm doing well and what I'm doing poorly. The other is to let the students know what they're doing well and what they're doing poorly. Most of the examinations that we give to our students in the United States do neither. They are examinations which are easy to administer and easy to grade. And that's the principal basis on which they're designed.

I want to remind ourselves of something that we're all familiar with but I need to remind myself of it periodically. There's a natural tendency to identify testing with the most routine and manipulative kinds of skills. But in fact it is possible to test (even make multiple-choice tests) for all levels of mathematical thinking and performance.

One of the things that surprises me in discussions about testing is the realization that the new technology has also reached testing. There is a lot of experimental work going on on ways in which reasonable data can be gathered in more effective techniques than the paper-and-pencil standard tests. If we're going to talk about a reasonable testing program, we ought to then start looking at the new technology.

We need to reorient ourselves more to the idea that the teacher is in fact a good judge of what mathematics children can learn and that we need to assist that teacher in making that judgement. The easiest thing in the world to do is to mandate at the state level that we have another achievement test for some other purpose. I do not think that's going to address the problem nearly as effectively as helping teachers become better assessors themselves of mathematics learning.

Equity

Throughout the meeting there was concern about differential access, treatment, and curricula for different students.

There is some evidence that in well-to-do suburban schools, kids who are using computers are using computers in ways which they are controlling the computer. They're using simulations and problem solving packages. And in poorer communities where kids have access to computers, they are being given the computer in an environment in which the computer controls the kids. They are being given drill and practice in which the kids learn the computers are smarter than they are. That's perhaps a more serious element of the equity issue than simple access to equipment.

I want to go back to the issue of selective curriculum and tie that to the equity issue. Before we can have selective curriculum for different segments of the population, we must first what they would select if we, in fact, appropriately educated.

I am very much afraid of our establishing a curriculum and saying this is for one group and this is one for another group. In particular, should there be a curriculum for those who are on their way to college and those who are not?

I want to make a brief comment about remediation and how that ties into equity, and to try to help the teacher and the student address the issue of the things that I always have problems trying to understand. What do people mean when they say <u>equity</u>? It's used in so many different ways. During the '50s and '60s and '70s when we talked about equity, we always talked about opportunity. Now we're beginning also to understand that access has something to do with equity. I do not see how we can disregard the question of remediation. I think we've probably come up with about the same number of definitions for remediation. The reports have talked about the percent of remediation that the colleges have absorbed over the past 10 to 15 years and relating that to the kind of people going into high school who are also in the process of remediation. The national reports are pushing for change in graduation requirements. It seems as though there's a race among the states to see what they can do about change in the state requirements for graduation of students relative to mathematics and science. What they are not doing is looking at the question of teacher preparation and the quality of those who are teaching as they change these recommendations.

On remediation, we will have an increased number of dropouts in high school in the next 3-5 years before we can even talk about an increase in graduation.

Raise the requirements and we'll increase the number of dropouts.

TESTIMONY ON LEARNING AND TEACHING

F. Joe Crosswhite, Chair

RESEARCH ON LEARNING[1]

Robert Siegler
Carnegie-Mellon University

The basic premise of my talk is that to teach effectively you need to understand the knowledge that people already have and how to build upon that. The talk is going to be divided into five very brief sections. First, I'm going to discuss how a good part of people's knowledge is rule governed. Second, I'm going to talk about how knowing the rules that learners are currently using can help us know what to teach. The third point that I'm going to make, one that I suspect might be a bit unpopular, is that associative knowledge is also important, that we ignore it at our peril. The fourth idea, which I suspect people will cheer whether I have any evidence for it or not, is that it's a bad idea for teachers to discourage children from using backup strategies, for example, telling children not to use their fingers. And finally I'm going to discuss a fifth, the role of computers in aiding instruction based on the use of rule-governed and associative knowledge.

First, let's talk about a lot of knowledge being rule governed. There's quite a bit of evidence on this that many of you are probably familiar with, and a fair amount of it comes from people in this room. For example, Robbie Case has done some very nice studies in the area of missing-addend problems as well as a whole host of Piagetian tasks that I will talk about later on, indicating that these domains are rule-governed. Tom Carpenter along with some of his colleagues has also done work to show very similar points. John Seely Brown and Kurt Van Lehn have also done a good deal of work on subtraction. I have done some work myself on tasks like balance scales and time, speed, and distance. Knowledge in all these areas is rule-governed.

Now, to give you a feel for what rule-governed means, I'm going to show you a simple subtraction problem of the type kids often encounter in third grade and a pattern of answers that often emerges. In fact I saw my son produce these very patterns of answers just this past week. In addition to demonstrating what rule-governed means, these particular illustrations are interesting because they point out how an analysis of errors can help. What rule was used to answer these problems?

$$\begin{array}{r} 804 \\ -\underline{579} \\ 135 \end{array} \qquad \begin{array}{r} 402 \\ -\underline{125} \\ 187 \end{array}$$

[1]This paper was edited from a transcript of the proceedings and has not been revised by the author.

Yes, the child is borrowing incorrectly across the zero in the ten's place. This is a very common kind of error. What struck me is that, when my son was doing this kind of problem for three or four days, the teacher did not identify this kind of error. It is a rule-governed error. In fact it was easy to explain to him what rule he was using and why it was incorrect. The point of this illustration is to argue that it's a good idea to know what kinds of rules people are using.

There is a lot of experimental evidence--moving on to the second point--that rules tell teachers what to teach. They are found in a variety of tasks, among them balance scales and concept of time tasks. If you can isolate the particular rule that kids are using to answer incorrectly, you can find out what kinds of problems can help them and what kinds of problems will have no effect. And also some kind of problems will have a long-term effect but will not have a short-term effect. For example, in teaching kids about the balance scale, we first isolated a group of children (third graders and kindergartners) who were using a very common rule, building on weight to make one side of the balance go down. We presented children with one of three kinds of problems, those that their existing rule would answer correctly all the time, those that would be answered not by their existing rule but by the rule children ordinarily adopt next, and finally those that could be answered correctly with either rule, their existing rule or the rule that they would ordinarily adopt next. When the children were presented with problems that their rules already answered, they did very well but they did not learn anything. Given a posttest they used exactly the same rule they started out with. When given problems that were one rule more advanced from where they started, kindergartners particularly learned something, and they often moved to the rule one more advanced.

When kindergartners were given problems that were two rules more advanced, they did not learn anything. They abandoned their initial rule. When they tried the rule they intuited was the other reasonable one, it didn't work. So they sort of figuratively shrugged and gave up. On the other hand we found that for some of these children the experience of encountering very hard problems started to change the way they encoded some of the problems. They started to look more broadly at what the problem was. Some of the older kids, the third graders, did master the rule that was two rules more advanced. We have to evaluate carefully the knowledge of the person relative to the kinds of problems that they are getting. We also have to look at the kinds of encoding of the problem that people do in order to figure out what kind of effect instruction is likely to have.

Moving on to the third point, we believe that associative knowledge is considerably more important than people frequently acknowledge. It's kind of fashionable, certainly if you are in Cambridge, Massachusetts, to mock associative knowledge by saying that it is boring stuff or that a chimpanzee can be trained to do better stuff. However, one of the basic lessons to come out of artificial intelligence is that in the absence of a great deal of specific knowledge about the world, machines are almost helpless. In order to get machines to do sensible things, and in order to get people to do sensible things, knowledge about particular domains is essential. All of the expert systems--whether medical diagnosis, prospecting for oil, whatever--demand a great deal of very specific knowledge. I do not think that any of us operate well in domains in which we have no knowledge whatsoever, no matter how intelligent we may think ourselves. We have developed a model that's basically associative, although it has rules operating on the associations in the domain of addition and subtraction. What got us interested in this domain is a question that I think will be a very popular one in the coming years in psychology at least. That is, how do people decide what strategies to use?

It is the case that people can use a wide variety of different ways to solve problems. Usually we formulate models that tell us that this is the way that people do this task. But, in fact almost any task worth doing we can do in multiple ways. When we have a problem spelling the word <u>accommodate</u>, for example, sometimes we retrieve the answer, sometimes we write out alternatives and try to recognize which is the right one, sometimes we try to form mental images, and sometimes we look it up in the dictionary. And all of these are strategies that people can and do use. And the basic questions that we're interested in are How do people decide to use them? How do they attack the problem? In the domain of addition, we believe it works like the following:

Problem	Processes Retrieve Represent . . .	Answer

The middle is the guts of the system. Over on the left are inputs that build up the system, that help it develop. On the right hand side is the output side, the behavior you get out of the system, but the basic processes that one has are in the middle.

The processes that people use in a wide variety of situations, among them addition and subtraction, are as follows. First, one tries to retrieve the rules. When you run into a problem like 9 - 4, the first thing you do is think, "Do I know the answer to this problem?" If you don't know any answer with sufficient confidence to state it, then you create a more elaborate representation. In this particular domain, what that means is either putting up your fingers or forming a mental image of the objects of the problem. Through a kinesthetic or visual process you put out the 9 and the 4 and say, "Aha, that's 5." But this might not work, and there are a variety of backup strategies, such as counting in the domain of addition, to give you the answer you still are unable to get. This process operates on the distribution of associations. Each problem involves associations to that problem and various possible answers. So, for 9 - 4, the answer could be 3, it could be 4, it could be 5, it could be 6, it could be 7. You have varying associations and varying stresses such as time to choose the answer to the problem. The distribution of the associations turns out to be crucial. If you have a very peaked distribution of associations where almost all of the associative strength is in one answer, you're very likely to retrieve that particular answer. If you have a very flat distribution, if you have some associative strength for several different answers, you're unlikely to retrieve any given answer, or whatever answer you do retrieve is unlikely to have sufficient strength for you to state it.

In building up a distribution of associations the input seems to be important, in particular frequency of exposure to the problem. We brought parents into our lab to teach their four- and five-year-olds about addition. We gave them some problems and watched how they taught. We looked at which problems the parents presented, and it turned out that the problems that were easy were more frequently presented. Knowledge from other domains also turned out to be important. The domain that turned out to be very influential with these four- and five-year-olds was knowledge of counting; it is the one other numeric operation that they knew very well. On all ascending series problems, like 3 + 4 and 4 + 5, and tie problems, 2 + 2 and 4 + 4, the most frequent incorrect answer, sometimes the most frequent answer, was the answer one greater than the second number. Children said 3 + 4 = 5, 2 + 3 = 4, 4 + 5 = 6. The most straightforward explanation is that they have a next-connection between 5 and 6, 4 and 5. They hear the ascending series all the time, and they just jump into that numeric operation rather than into another one.

The third important factor is the nature of the backup strategies. In multiplication, adults make errors and have a hard time given a true or false problem like 7 x 8 = 48. The reason they do is that they have a backup strategy of repeated addition. It's very easy to have one eight too few or one eight too many. The nature of the backup strategy determines the kind of errors you're going to see. In addition and subtraction you actually get a lot of errors, not as many as the counting errors, being close to the right answers. What happens is that you learn the answers by mistake. Either you put up the wrong number of fingers or you count them wrong. You're likely to either count one too many or one too few. You skip an object, skip a number, or count twice on a number--it turns out that if you put these three types of input into a regression equation you can account for 85 percent of the variance in the number of errors that children make on plus-25 addition problems.

Finally, what kind of outputs do you get from this study? First of all you get the four strategies that we observed in addition and subtraction: putting up fingers and counting them, putting up fingers and not counting them, counting aloud without any obvious external reference, and simply retrieving the answer. Second of all you get the kind of error patterns that we observed. Certain problems, such as 3 + 4, are very hard problems. Other problems, such as 4 + 1, turn out to be very easy. It all depends on the factors that we talked about before: frequency of exposure, associations from other domains, and the nature of the backup strategies. And you can account for the solution times, relations among strategies, and errors.

Perhaps the most interesting instructional implication to come out of this model is that we may be making a serious mistake if we discourage children from using backup strategies, such as counting their fingers. Some of you may be familiar with the history of research in reading. It was discovered around the turn of the century that good readers had different patterns of eye movements than bad readers. One of the first impulses of educators, and this was carried out on a wide scale, was to try to train eye movements. It sounds laughable, but it was tried quite seriously with good intentions, and it was completely ineffective. They tried very hard to get the pupils to do it and when they did get them to do it, they read terribly.

Interviewing six teachers in a school district known for excellent education, we found that at least three of the six openly said that they discouraged pupils in second and third grade from putting up their fingers to add. Children only use the finger strategies when they are unable to retrieve an answer of sufficient associative strength to state it. On the easy problems like 1 + 1, 2 + 1, and 2 + 2, they do not use their fingers anyway. They only use them when they come to the hard problems like 8 + 9 and 7 + 5. If you discourage pupils from using the backup techniques on problems where they really do need them, the odds are that they're going to state wrong answers because they have developed a flat distribution of associations. If they state wrong answers, our research indicates they will learn those wrong answers. Think of a spelling example—for me <u>accommodate</u>. I misspelled <u>accommodate</u> early on a couple of times; forever after I've had difficulty with <u>accommodate</u>. I know the possibilities, I spell it this way or that way, and one of them is dead wrong. Remembering which one is dead wrong has been the bane of my existence. Especially after John Flavell told me that you could tell people who didn't understand Piaget by their misspellings of <u>accommodate</u>.

Computers can help us in this kind of domain in three different ways. First, they can help us in assessing the rules that childr n use. In any of the 15 or so tasks that we have studied, children use limited number of rules. They use two or three or four different rules, but they don't use 20 different rules. It's not a difficult task for a computer program to indicate what rules children are using and present problems that deal with discriminative patterns. You can find out exactly what the source of the child's mistake is, and then you can go about explaining to them the source of their error. As I indicated in the example with my son earlier, you can save an incredible amount of time, and teaching is much more satisfying if you're teaching to a specific source of error. Second of all, computers can individualize problems to meet the demands of particular rules. Very often you know exactly which one. The computer can generate just the problem to discriminate between two rules, something that would be a quite arduous task for teachers, ordinarily.

Finally, in keeping with my belief that associative knowledge is important, we can build up associations through practice. This morning I heard a number of people define the use of computers to just give pupils drill and practice. I do not think that this is the only thing that computers should be used for, but I certainly think it's one good thing. It's tedious, laborious, and unpleasant for teachers to do many of these tasks, and they just don't get done as much as they'd like a lot of the time. If our model is even in the right ballpark, and if other people's models are even in the right ballpark, the kind of associative knowledge that you acquire by going over problems many times is indispensible for solving more sophisticated problems as well as for performing effortlessly and quickly on these kinds of simple problems.

IMPLICATIONS OF COGNITIVE SCIENCE TO INSTRUCTION[1]

Robbie Case
Ontario Institute for the Study of Education

The first point I want to make is that Piaget was right--sort of. We've seen in developmental psychology, both in examinations of mathematics and in examinations of insights in science and other areas, that there's a regular change in the strategies that children employ with age. Second, recent research is tending to show that the different strategies children use stem from different forms of representation for the particular problem they're facing. Third point, a very Piagetian point, higher order strategies and representations emerge out of lower ones. Finally, the rate of change is often surprisingly, though not necessarily, slow. Let me illustrate some different strategies and, one presumes, the different ways of representing the problem that children use at different ages with a math problem.

The problem is a simple sort of word problem that children are taught how to do in grade 6: For $5 you get 6 pieces, for $13 how many do you get? They're presented with these kinds of tasks in the guise of chemistry problems, physics problems, or arithmetic word problems. Although they're taught at grade 6, you still see, for many years afterwards, a change in the way they represent the problems and in the strategies they use. At the first level, children recognize you're getting a little bit more the first time and the way they uncover the problem is in terms of addition and subtraction. Their strategy is to equate the differences. So, if you got one extra the first time, you'll get one extra the second. So the answer would be that for $13 you get 14 pieces.

Up at the next level, children can handle multiplication and use unit ratios if they're given, as in this problem: For $1 you get 5 pieces. How many for $4? They will multiply and give you five times that many as the answer. But if they have to do anything more complicated, like first of all determining that unit ratio, they will fall back on a different strategy. Now, you get a whole lot of other strategies from children. You get bizarre combinations of subtraction and division which indicate they don't know what they're doing at all. But these are the sort of responses you get when you ask them to reason too carefully and you probe them; you get the kind of answer that they're most confident with. At a next level they'll be able to handle

[1]This paper was edited from a transcript of the proceedings and has not been revised by the author.

problems like this but they'll first reduce it to a unit ratio and then, having found how much one cost, multiply it up to find out how much the whole bunch cost. And then at final levels you get formations of equations, cross-multiplication, and so on. It's really very often not until grade 12 that children are using spontaneously the strategies we're teaching them in grade 6.

Now, the second point that I wanted to make has to do with the possibility of teaching children to use the expert strategy. We know what experts do: they solve those problems by setting up a pair of equations and cross-multiplying. We've designed programs that very carefully look at the cognitive components and insights that the experts use in solving ratio problems. These are the standard arithmetic problems that we were using; this kind of thing comes into a number of areas of physics and chemistry and is drawn on by people in other areas. We said, "What's involved in extracting two rates from the verbal description?" and we broke that down into parts. This is a standard kind of learning hierarchy approach where we knew the children could do the lowest level skills. We got an overall kind of cognitive map of the domain, and then we brought the children from where they were right up through the hierarchy.

We found that, in fact, the better children learned very well from this, they scored very well on the posttest. We normally give our posttests in a sort of developmental tradition about a month after we've taught the children; we figure that's most important. When we analyzed the actual strategy they were using on the posttest, we discovered that none of them was setting up the equations and cross-multiplying. They were figuring out how much one cost; once they had that they were multiplying. So the bright children had found a way, you might say, in spite of our instructions, to make it meaningful to themselves and to do it in a sensible way which felt natural to them.

Now the third point is that developmentally based instruction works. Siegler was giving you examples of instruction I would call developmentally based in that he was choosing the kind of problem he gave to the child according to the type of strategy or rule the child was using to begin with. There are a variety of developmental approaches like this; really there is a family of them but they are all very similar. We start with a problem that is meaningful to the students, has some relevance, and where they have some basis for telling whether they are right or not. Then we like to help them explore the limits of their approach and gradually, bit-by-bit, elaborate that approach so that it will end up turning into a fancier approach which will work for a broader range of problems. One can do it simply by giving them problems that are one level above, as Siegler was suggesting. Our procedure is a little bit more elaborated but has a similar rationale.

For ratio problems, since the lowest level strategy is thinking in terms of addition and subtraction and since we were going to be dealing with children at grades 5 and 6, we started off with some problems like this: We have 2 pieces in this box. How many pieces do you think we have over there in those two boxes? None of the children has any trouble understanding that problem. They solve it by repeated addition and give the right answer. We're doing this with manipulatives so that they can also check. After they guess four, we ask them to check the pieces and see whether four pieces for the two boxes in fact is right. Having started with this sort of problem, which the children get right, we then start doing what Siegler was mentioning, give them a problem they'll be apt to get wrong. One way to give them a problem they would be apt to get wrong is simply to lay out a number of boxes and a number of pieces on one side and say, "We want to have the same number of pieces for each box over there." Now the children do not find it all nicely tidied up per box to begin with. They start to use an incorrect strategy. They start to say, "You have two more pieces than boxes here, so you'll need two more pieces than the boxes you have over there." Because they understand the situation, you can say, "Well, put them in and see if that works." Very quickly they see the mistake they are making, and they realize they have to figure out how many per box first. We give students a situation which they really understand so they can see what's right, and then we gradually work them up with slightly tougher problems, which forces them to modify the strategy they were originally using, forces them to modify the way they represent the problems. Ultimately, the 15- to 18-year-olds can solve the most complicated problems.

We have two groups of children, normal children and children who were already having a good deal of trouble in math. Our posttest, given from four to eight weeks afterwards, included a variety of transfer problems they had never seen before. The children who had been brought up to this very meaningful approach were doing quite well. The ones with a hierarchy that teaches all the skills did rather well, particularly the normal children. But I would stress that these normal children were not doing what they were taught; they solved problems by figuring how many for one and then multiplying it up. So, that the program succeeds is to its credit, but why one has to fight the children's representations isn't clear. Also the children who end up being the remedial children aren't able to do that. If you teach them an expert strategy, without the intervention of situations they clearly understand and can represent at their own level, they're in trouble. They can't make the leap and invent one for themselves that makes sense--in the absence of a form of representation that you're giving them.

Manipulatives per se aren't the key. The key thing, as I see it, is that the child is given a situation that he can already represent at his own level of understanding so that he is able to shuttle back and forth between that representation and a symbolic representation which you're hoping that he'll cope with. Manipulatives could be extremely important but also be irrelevant. We have some nice demonstrations with adults. We have problems that are too difficult for adults to solve. We give them the appropriate manipulatives and say, "Work it out." They get no better--in fact they get quite irritated with us. It depends on how the manipulatives are used and whether or not that representation is one students can easily work with. In one experiment we talked about <u>x</u>s and <u>y</u>s instead of using gum and gumboxes, saying "For every one <u>x</u> you always get two <u>y</u>s. We found no difference in the success rate. In another experiment, we kept the gumboxes but we did not let adults physically check it out, we just asked them to do it visually. Again we found no difference. So the key thing wasn't whether they could physically manipulate the objects, rather it was that the way of representing a problem was one which is intuitively very easily understandable. As I say, I'm a great fan of manipulatives and things like Cuisinaire rods; trays of popsicle sticks can do wonders for giving children who don't have it a sense of place value and so on. But the reason they work is because they're something the children can already understand. Furthermore they will only work if you make their relevance to the symbolic representation apparent to the children.

Now, back to Siegler's unpopular point. Although this is the general procedure we use, the need for practice and drill is not obviated by it. In one study we got adults to do some problems in an artificial universe. We created a universe in which physical things were happening according to laws that we decided. The person's task was to induce what physical law was operating. The only way they could induce this law was to have encountered a foreign language in which the counting principles were somewhat different. They were forced to be like children, in that they were doing something that wasn't highly automatic for them but was a vital step in order to acquire the higher order insight. What we discovered was that, when adults were forced to count as slow as six-year-olds spontaneously count, the level of rule they used, that they induced, was a six-year-old rule. When we prepracticed them for three weeks, every single day, counting in this foreign language until they could count as fast as ten-year-olds, a level of rule they were able to induce was a ten-year-old rule.

We should not be over-dosing children with drill and practice, but some minimum amount of practice on the things you already know is going to be absolutely vital if you are going to free up the attention or memory space to handle higher order insights. I think when phrased that way there wouldn't be too much disagreement. One could also point out that the practice doesn't have to come by straight drill, it can come in the context of problems that require trying out a mass of calculations to get to something that interests you. You get the children very involved with that kind of problem-solving activity and, incidently, they will be getting scores and scores of trials on the lower level operations if you want them to.

Now, one last point which may, in fact, be even less popular than that one is some potential caution with regard to the excitement with computers. We've read about the problems with bringing real math into the schools. There are many social forces which go against change in any system. We must confront those folkways in the schools or we'll never get any change. I sure wouldn't want to be the one doing that. When you bump your head against a strong tradition or something people have a vested interest in, you're in a difficult situation.

Another problem with the new math may have been that teachers will do what they can do--what works with the children. If they're given things that are exciting to mathematicians but don't work with the children, then that's going to be an additional factor that's going to militate against change. Although certain teachers did put their stamp of approval on new math, many children, maybe even a majority, found it very difficult to relate to the set theories they were being asked to learn maybe 10 years before children do that spontaneously. We may encounter that sort of problem with computers. Kurland has worked with bright children taught by people out at MIT. His studies of children' understanding in LOGO of recursion, which is supposed to be one of the deep principles they come to understand, suggest that those children understand nothing about recursion. What they're doing is totally different from what the designers intended. We're in the process of doing a few clinical studies on LOGO with some children in Toronto from one of our top schools. They're selected by intelligence tests in addition to money in order to get in. They've all mastered some LOGO. What we found is that, when we gave them some fun little design problems and looked at the structure of what they're doing at any given age, the children are using rules identical to the sort of rules that Siegler mentioned on a balance beam. That is to say, six-year-olds are capable of solving problems that are very much like six-year-olds solve in other domains. They may in fact be acquiring all sorts of new knowledge, but in addition any learning they are doing is being, if you will, filtered through the six-year-old mind. When we start teaching statistics, when we start teaching whatever we're going to be teaching geometrically on the computer, the caution would be that we're going to have to make sure that the instruction works for the children at an age where we're giving it to them and that we're setting realistic expectations. We must try to understand how children represent that sort of situation. We must pitch our instruction in the new domains to the natural ways that children interpret things when they are novices in that particular domain.

There are different levels of rules or strategies that have more and more powerful representations that children use at different ages. You can go in teaching the exciting expert stuff that is new, and some of the children will in fact profit from it. Even then they're apt to assimilate it to their own way of functioning. An improvement is to pre-diagnose, whether you're using Siegler's suggestion of computers or some other way, where they are and come in with something that they'll understand easily to begin with and move up from there. Manipulatives may be a great aid in that; the reason they're working is probably because of their match to the children's representational system and the ability they give them to move from that to more symbolic ways of representing things. Need for practice and drill will not be obviated by this sort of thing, although it can be done in a fashion that won't bore the pants off the children and dull them to the higher sorts of things that they're learning. And finally, the computer caution. As we move into these new areas we must not expect everything to be radically different for the children. The children will be the same children, they'll do the same stuff to this new material that they do to the other. Although the end product may, in fact, be quite different, we've got to meet them where they are and move them up from there.

CHILDREN'S LEARNING: A CRITICAL VARIABLE IN CURRICULUM REFORM IN MATHEMATICS

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In the early part of the century, the psychology of learning had a significant influence on the mathematics curriculum. In the 1920s, Thorndike's theories of learning were directly translated into practice in arithmetic instruction (Cronbach & Suppes, 1969). Since that time, the impact of psychological theories on the mathematics curriculum has declined.

By the time of the curriculum reforms of the 1950s and 1960s, psychologists were conspicuous by their absence. In the major curriculum recommendations of the time, some lip service was paid to how children learn mathematics, and several psychologists, notably Jerome Bruner, participated in the debate. But the discussions of children's learning generally were based on philosophical consideration, not carefully researched theories of learning. In both the proposed recommendations and the curriculum that was implemented, content and sequencing decisions were based on mathematical structure not on a careful analysis of how students learn mathematics. It clearly would be overstating the case to say that the new mathematics failed because it was not based on learning theory. But it is also now clear that many of the changes implemented during the period were not consistent with how children actually learn and think about mathematics.

A number of reasons could be cited for the minimal influenoe of learning theory on the curricular reforms of 1950s and 1960s. The prominent learning research of the day focused on performance on laboratory tasks that could be carefully controlled but were not clearly related to the types of tasks involved in the learning of mathematics. And educators have become disillusioned with the mechanirtic nature of learning suggested by behavioral theories.

In the last ten to twenty years, research on learning has focused to an increasing degree on how children and adults acquire complex concepts and skills like those in the school curriculum. A great deal of the research has focused on the explicit analysis of how specific mathematical concepts and skills are acquired. Thus, there is emerging a viable body of research on children's learning of mathematics that has clear implications for the mathematics curriculum and can help us avoid some of the mistakes of the past.

The research that currently appears to have the clearest implications for the mathematics curriculum has focused on the explicit analysis of the learning of specific mathematical concepts rather than the development of broad general principles of learning. But some generalizations regarding the nature of the potential contribution of this research can be identified.

Learning as a Constructive Activity

One of the most fundamental contributions of current research is the conception of the learner that it provides. Most mathematics instruction has tacitly assumed that students learn what they are taught, or at least some subset of what they are taught. But current research indicates that students actively construct knowledge for themselves. Although instruction clearly affects what students learn, it does not determine it. Students are not passive recipients of knowledge; they interpret it, put structure on it, and assimilate it in light of their own mental framework. There is a growing body of research that suggests that children actually invent a great deal of their own mathematics (Resnick, 1976).

For example, children enter school with highly developed informal systems of arithmetic (Fuson & Hall, 1983; Gelman & Gallistel, 1978; Ginsburg, 1977). Before they receive any formal instruction in arithmetic, they can solve simple addition and subtraction word problems by modeling the problem with physical objects or using a variety of counting strategies (Carpenter & Moser, 1983). These solutions suggest quite well developed conceptions of addition and subtraction even though the children have not learned the formal terminology associated with the operations. Children also invent a variety of strategies for adding and subtracting. For example, although counting-on[1] is not explicitly taught, most children go through a stage in which they count-on to solve simple addition problems (Carpenter & Moser, 1984; Groen & Resnick, 1977).

The strategies that children invent to solve addition and subtraction problems are more efficient and require a deeper understanding of the operations than the procedures that generally are taught. Similarly, the problem-solving analysis that children naturally apply to simple word problems provides a much better model of problem-solving behavior than many of the superficial tricks for solving word problems that are often taught. Thus, children's informal knowledge of arithmetic provides a substantial basis for developing number concepts and problem-solving skills. Currently the curriculum fails to capitalize on this knowledge.

The perspective that children actively construct knowledge also provides fresh insights into children's errors and misconceptions. John Seeley Brown and his associates (Brown & Burton, 1978; Brown & VanLehn, 1982) argue that many errors result not from failing to learn a

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[1]To count-on, children start counting with one of the addends in the problem rather than starting at one. For example, to solve 8 + 5, a child would count 8 - 9, 10, 11, 12, 13.

particular algorithm but from learning the wrong algorithm, which they call a "buggy algorithm." They hypthosize that buggy algorithms are constructed by students when they are confronted with problems for which the algorithms they have learned are inadequate. To resolve this impasse, they modify their existing algorithm to fit the new problem situation. Their modifications often result in a buggy algorithm. Brown and his colleagues start by decomposing a skill into its primitive elements or rules. By deleting one of the rules, they model the situation of a student who has forgotten or failed to learn a specific procedure.

Brown and his associates have applied the theory to the analysis of errors in subtraction of whole numbers (Brown & Burton, 1978; Brown & VanLehn, 1982), and Matz (1980) has demonstrated how it could be applied to errors in algebra. The theory provides a much clearer specification of buggy algorithms and the underlying source of errors than is provided by other analyses of students' errors. By analyzing the buggy algorithm, it is possible to identify the specific procedural rules that were not available and resulted in the bug. Successful remediation could focus on the acquisition of those rules. Furthermore, the theory provides a language for analyzing errors and discussing them with the children making them. Without such a precise formulation it is often difficult to communicate to children the reasons for their errors, even after a systematic error has been identified.

The Acquisition of Concepts

The curriculum programs developed in the 1950s and 1960s generally were based on the assumption that the instructional sequence of a topic should follow the logical-mathematical development of the content. Current research indicates that children do not necessarily acquire concepts by building up from the logical foundations of the concepts. Research is beginning to provide a picture of how concepts and skills actually develop in children.

For example, many primary mathematics programs written in the 1960s introduced basic number concepts through activities that involved constructing one-to-one correspondences between sets. This is consistent with one way in which number could be derived logically, but it is not consistent with how young children acquire number concepts. Counting is a more basic operation to children than one-to-one correspondence (Gelman & Gallistel, 1978), and children enter school with reasonably advanced counting schemes (Ginsburg, 1977). There are other critical differences between the logical development of number concepts and the way children learn them. Counting strategies like counting-on are not derived from a formal logical analysis of mathematical operations, but they play a prominent role in children's learning to add and subtract (Carpenter & Moser, 1983, 1984).

Studies in other domains in mathematics also indicate that the development of concepts in children is not always consistent with a logical analysis of the subject (c.f. Case & Bereiter, in press). Case and Bereiter argue that instruction should be based on the developmental sequences observed in children. They propose that instruction is most effective if it reflects the stages that children pass through in acquiring a concept or skill.

Understanding Understanding

Prevailing theories of instruction in mathematics have often been based on assumptions about whether it is more important to develop understanding or teach skills. Current research is beginning to provide some perspective on the intricate relationship between understanding and skill development (Resnick & Ford, 1981). It is also beginning to sort out exactly what constitutes understanding. In the curriculum of the 1960s, the understanding issue was addressed by the use of precise language, the specification of basic principles like commutativity and associativity, and the reliance on formal mathematical justification or proof. The research on children's learning of mathematics indicates that to develop understanding one needs to consider how the learner thinks about a problem or concept. Understanding involves fitting information to the learner's existing cognitive framework. This means taking into account the knowledge of the mathematics under consideration that the learner brings to the situation, connecting semantic knowledge and procedural skills, and encouraging integration of related concepts.

Thinking About Thinking

A major reason that current research on children's learning of mathematics has made significant advances is that it has gone beyond simply looking at scores on tests or whether a problem was correct or incorrect. It has focused on the processes that children use to solve problems and has attempted to unveil the nature of children's mathematical concepts. However, a growing body of research suggests that students' mathematics learning cannot be understood strictly in terms of the processes they use to solve problems or the concepts they have formed. This research suggests that it is also important to take into account how decisions are made. The solution of any problem involves a number of executive decisions about what to attend to in the problem, how to decide between competing approaches, how to monitor the solution process, how to allocate time, and so on. These decisions generally are not made explicit in instruction, but they play a critical part in the solution. There is evidence that instruction that encourages students to monitor their own thinking and decision processes is effective in improving problem solving (Schoenfeld, 1983; Silver, Branca, & Adams, 1980).

Evaluation

Research on children's learning of mathematics suggests that different children have very different conceptions of mathematics. To optimize instruction, we need to assess what knowledge students have in order to build upon it when it is sound and address the misconceptions when they exist. One of the major contributions of research on learning is the powerful tool it provides for evaluating students' knowledge and the effects of instruction.

From a broad perspective, the research clearly documents that it is critical to look beyond right and wrong answers, to consider the processes that students use to solve problems, and to analyze the errors they make. The research goes one step further, however, in that it provides explicit frameworks for analyzing processes and errors within specific domains. John Seeley Brown's work provides the most obvious example of how research can be applied to evaluation, but most research on children's cognitive processes provides a basis for assessing their knowledge and misconceptions.

Conclusion

Most research on children's learning does not provide radical new conceptions of how children acquire mathematical concepts and skills. Many of the findings are consistent with the intuition and observations of experienced teachers and curriculum writers. But current research provides a level of precision and rigor that offers some hope of really moving things forward in the development of the mathematics curriculum rather than riding another swing of the pendulum. It is not a new idea that structure and understanding are important. But understanding generally has been a very amorphous concept. Current research offers some hope of developing an operational definition of understanding within different domains, and the research on expert knowledge in a variety of areas is beginning to explicate the nature and importance of structure. It is not a new idea that many errors are caused by the systematic application of an invalid algorithm. But the work of John Seeley Brown and his associates identifies the causes of the error in terms of explicit procedures. This provides a tool for developing diagnostic tests that can discriminate between errors and a framework and language for remediating them.

The kind of precision offered by cognitive science is critical in applying technology to instruction. Technology offers fantastic power for instruction, but most of the available courseware is based on behavioristic assumptions about learning that are inconsistent with the perspective portrayed in this paper. Many applications of technology require a very precise formulation of how machine and learner will interact. Courseware developers often have fallen back on simplistic models of learning based on behavioristic principles. Cognitive science is beginning to provide explicit models of competence, principles to infer students' knowledge and misconceptions from responses, and principles for interacting with students that make it possible to develop reasonably sophisticated tutor-critic programs that can do more than reinforce correct responses and identify incorrect ones. Burton and Brown's (1982) tutor-critic program for "How the West Was Won" illustrates the power of integrating cognitive science and technology.

There are no simple formulas for applying learning theory to instruction. Prescriptions for instruction do not follow immediately from research on learning and cognition, and additional research is needed to determine the most effective way to make the connection. There is no single ideal program of instruction that will come out of this effort. However, instruction needs to be consistent with what we know about how children learn and think. If we are going to make real progress in curriculum reform, we need to build upon this knowledge.

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RESEARCH ON TEACHING

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In reading the recent reports on education, one is led to believe that determining what constitutes excellence in education and then using these findings to improve education is a new idea. In fact, for more than 30 years, researchers on teaching have grappled with the problem of what constitutes effective teaching and effective classroom processes (see, for example, Gage, 1963; Rosenshine, 1979). In the past decade, research on teaching has produced several findings that have implications for the improvement of educational practice. In particular, the following four areas of research need to be considered as educators plan for mathematics teaching in the 1990s: (1) time as a variable in mathematics learning and teaching; (2) the student as an active information processor; (3) small-group learning as an alternative to whole-class mathematics instruction; (4) the teacher as a thoughtful professional.

Time as a Variable

Findings from the Beginning Teacher Evaluation Study showed that the amount of time that elementary teachers allocated to mathematics and to a particular topic in mathematics varied considerably from school to school and from classroom to classroom (Berliner, 1979). Moreover, subsequent analyses showed that students had higher mathematics achievement in classes in which more time was allocated to mathematics (Borg, 1980).

Although time allocated to mathematics was shown to be related to students' mathematics achievement, an even stronger relationship was found between student engagement in mathematics and mathematics achievement (Borg, 1980). Student engagement has been defined as the amount of time or the percentage of time that a student appears to be attending to, thinking about, or actively working on mathematics tasks, as judged by classroom observers. In the Beginning Teacher Evaluation Study, Fisher et al. (1978) reported a mean engagement rate of 73% for fifth-grade mathematics classes. Similarly, in a recent study of 36 fourth-grade mathematics classes, Peterson and Fennema (in press) found that students were engaged in mathematics about 76% of the time. Findings from the Beginning Teacher Evaluation Study showed a significant positive relationship between student engagement in mathematics and mathematics achievement (Borg, 1980). While most researchers on teaching agree that the relationship exists between student engagement in mathematics and mathematics achievement, some researchers have asserted that the relationship is "weak" (Karweit, 1983), while others have indicated that the research shows "low to moderate correlations between attention and learning when ability is statistically controlled" (Good, 1983).

The above findings suggest the importance of student engaged time as a variable in mathematics lear ing, but they do not necessarily suggest that increasing the time spent on mathematics will lead to higher mathematics achievement without giving consideration to the quality of time. That is, one must consider not only the quantity of time that students spend on mathematics but also the quality of time. For example, Peterson and Fennema (in press) found that the global variable of student engagement/non-engagement in mathematics did not adequately explain sex-related differences in mathematics achievement. Rather, the relationship between engagement and mathematics achievement depended on the specific type of activity in which girls and boys were engaged in during mathematics class and whether mathematics achievement was defined as achievement of computational skills or higher-level problem solving abilities. Thus, a knee-jerk reaction of increasing the time spent on mathematics would be inappropriate without giving consideration to the quality of time. Examination of the quality of time spent in mathematics would involve attention to such variables as the type and kind of mathematics task in which the student is engaged and the difficulty of the mathematics task, as well a^ attention to individual differences in students.

A further limitation of the research on student engagement is that quantitative measures of engagement have been based on observers' judgments of apparent student attention. However, research has shown that students as young as second grade are able to "fake attention" (Brophy & Evertson, 1976). Moreover, some research has shown that students' reports of their cognitive processes during instruction--the kinds of things that students report thinking about and the kinds of information they are processing--are actually better predictors of student achievement than are observers' judgments of students' apparent attention (Peterson, Swing, Braverman, & Buss, 1982; Peterson & Swing, 1982; Peterson, Swing, Stark, & Waas, 1984). We turn now to the second area of research which has focused on the student as an active processor of information.

The Student as an Active Processor of Information

As Carpenter (1984) points out, current research in cognitive learning has indicated that students actively construct knowledge and should be considered active "processors" of information. Carpenter focuses on how children actively construct knowledge about mathematics concepts and skills. Researchers on teaching have added the perspective of how students actively construe the teaching and learning situation in the classroom. For example, research has focused on what students perceive to be the purpose of a classroom task (Anderson, 1981); what students' perceive to be the teacher's intent (Winne & Marx, 1982); students' perceptions of teacher behavior (Weinstein, Marshall, Brattesani, & Middlestadt, 1982); and students' reports of their understanding of the mathematics content as well as the kinds of cognitive processes and strategies that they report engaging during mathematics class (Peterson et al., 1982; Peterson & Swing, 1982; Peterson, Swing, Stark, & Waas, 1984). (See Weinstein, 1983, and Wittrock, in press, for complete reviews of this research).

For example, in two studies of fifth-grade students' mathematics learning, Peterson et al. (1982) and Peterson, Swing, Stark, and Waas (1984) found that student ability and student mathematics achievement were significantly related to students' reports of their thoughts during classroom instruction in mathematics, including students' reports of attending to the lesson, understanding the mathematics lesson, and either engaging in a variety of specific cognitive processes or engaging in them more frequently. In the second study, student engagement in mathematics as assessed by classroom observer was found to be unrelated to student achievement. Thus, students' reports of their understanding of the mathematics lesson and their cognitive processes during mathematics instruction may be more reliable and more valid indicators of students' classroom learning in mathematics than observers' judgments of students' attention. In other words, this research suggests again that the quality of time that students spend attending to the mathematics task--the actual cognitive processes involved in processing the mathematics information presented during classroom instruction--may be equally important or possibly even more important than the quantity of time that students spend engaged in the mathematics task.

In addition, research on teaching has pointed out the importance of focusing not only on the cognitive aspects of how students construe the classroom task but also on personal and social aspects. For example, Doyle (1979) suggested that for many students classroom tasks may be construed as an "exchange of performance for grades." Doyle has argued that "a student's perception of a classroom task structure will determine how the information is processed and that the information-processing strategies selected will in turn, determine what the student is capable of doing on the teacher's test. Comprehension may in fact be detrimental in a performance-grade exchange that requires exact reproduction of previously encountered answers" (p. 200).

In sum, then, a cognitive view of the learner goes beyond focusing merely on student behavior as an index of apparent student engagement and attempts to determine the kinds of processes that are "going on inside the students' head." Similarly, a cognitive view of the teacher would go beyond merely examining teachers' behavior, such as giving praise or asking a "higher-order question," and would attempt to determine the kinds of thought processes and decisions going on "inside a teacher's head." In the last decade, some researchers on teaching have turned toward examining the act of teaching from this new cognitive perspective.

The Teacher as a Thoughtful Professional

The rationale for this cognitive perspective of the teacher was presented most clearly in a report produced by Panel 6 as part of the National Conference on Studies in Teaching that was convened by the

National Institute of Education in June 1974. The panelists argued that:

> It is obvious that what teachers do is directed in no small measure by what they think. Moreover, it will be necessary for any innovation in the context, practices, and technology of teaching to be mediated through the minds and motives of teachers. To the extent that observed or intended teacher behavior is "thoughtless," it makes no use of the human teacher's most unique attributes. In so doing, it becomes mechanical and might well be done by a machine. If, however, teaching is done and, in all likelihood, will continue to be done by human teachers, the question of the relationships between thought and action becomes crucial. (National Institute of Education, 1975, p. 1)

Research on teachers' thought processes and decisions has burgeoned in the last decade since the publication of the Panel 6 Report, and comprehensive reviews of this research have been done by Shavelson and Stern (1981) and more recently by Clark and Peterson (in press). Rather than attempt to provide an exhaustive review of this research here, we will briefly summarize Clark and Peterson's conclusions.

Clark and Peterson concluded first, that the research shows that thinking plays an important part in teaching and that the image of a teacher as a reflective professional, which was proposed originally by Panel 6 (National Institute of Education, 1975), is not farfetched. As thoughtful professionals, thus, teachers have more in common with physicians and lawyers than they have in common with technicians. Secondly, the research shows that teachers plan for instruction in a rich variety of ways, and these plans have real consequences in the classroom. Third, during interactive teaching, teachers are continually thinking, and the research shows that teachers report making decisions quite frequently--one every two minutes. Fourth, teachers have theories and belief systems that influence their perceptions, plans, and actions in the classroom.

In sum, the research on teachers' thought processes to date substantiates a professional view of the teacher as a reflective, thoughtful individual. Moreover, the research documents that teaching is a complex and cognitively demanding human process. Furthermore, one might infer from the research that any reform in mathematics teaching and education that is to take place in the 1990s needs to take the perspective that teachers are active, thoughtful, individuals who must be actively involved in the process of reform for it to be effective. Such a perspective also suggest that reform movements, such as "teacher-proof" curricula, that view the teacher as a passive recipient are likely to fail. In contrast, reform efforts that take into account teachers' beliefs and perspectives and actively involve teachers in planning and decision making, while treating the teacher as a reflective professional, are more likely to succeed.

Small-Group Learning as an Alternative to Whole-Class Instruction

In addition to providing us with a new perspective on the rich mental life of the teacher and student during classroom teaching and learning, researchers on teaching have also identified effective classroom processes that might be introduced into mathematics classrooms. Romberg and Carpenter (in press) presented data from several studies that indicated that the mathematics classroom today is very much like the mathematics classroom that we remember being in when we were in school. The picture is one of extensive teacher-directed explaining and questioning in the context of whole-group instruction followed by students working on paper-and-pencil assignments at their seats. Similarly, in a recent study of 36 fourth-grade mathematics classrooms in Wisconsin, Peterson and Fennema (in press) found that 43% of the class time in mathematics was spent in whole-group instruction, and 47% of the time was spent with students doing seatwork. Thus, the picture of mathematics instruction that is presented is one of whole-group instruction followed by individual seatwork by students.

Recent research by Webb (1982) and Peterson (Peterson & Janicki, 1979; Peterson, Janicki & Swing, 1981; and Peterson, Wilkinson, Swing, & Spinelli, 1984) has suggested an effective alternative or adjunct to whole-class instruction--that is, having students work together in small cooperative groups on seatwork problems. For example, Peterson has adapted small-group cooperative learning techniques to the format of mathematics instruction typically used in elementary classrooms. In this approach, the classroom teacher teaches the day's mathematics lesson for approximately 20 minutes, and then students work together on their mathematics seatwork assignments in small mixed-ability groups of four students. Peterson and Webb have found that the positive effects on mathematics achievement seem to depend on the task-related interaction that occurs in the small-group. What happens is that students learn by explaining an answer or explaining why an answer is incorrect to another student or by helping another student with their work. Each student works on his/her own mathematics seatwork, but when a student has a problem another student helps. Research indicates that the students learn by explaining why the answer is incorrect and by helping the student come to see the correct answer. In addition, the receiver of the explanation may benefit from receiving an explanation that describes the kinds of strategies and processes that a student should use to solve the problem. (See for example, Webb, 1983, and Peterson, Wilkinson, Swing, & Spinelli, 1984).

If one takes the perspective of the student as an active information processor, one might argue that students learn effectively in small cooperative groups because they become active information processors rather than passive recipients of information being presented to them by the teacher. The following example of second- and third-grade students working together in a cooperative math group on their seatwork presents such a picture of active information processing. In this example, the small-group members were told to check their answers with one another after doing 10 or 12 problems. Johnny, the high-ability student in the group, learns during the course of answer checking that his answer is incorrect:

Katie: (reading the answer from her paper) Dollar sign zero point forty-four.

Johnny: What? Whaddya mean "zero point forty-four"?

Katie: (pointing on Johnny's paper) Zero point forty-four.

Johnny: What? Eight nickels and four pennies equals thirty-six.

Katie: Eight nickels.

Johnny: Eight nickels. Eight times four equals thirty-two. Thirty-two plus four equals thirty-six.

Anne: No, it's forty-four, Johnny.

Katie: Let's go on with it.

Johnny: Which one are we on?

Anne: We're on five.

Katie: Five.

Johnny: (to Anne) Whaddya mean forty-four?

Anne: It's the eight nickels--forty-four.

Johnny: Ah, yeah. Wait a minute. Wait a minute.

Anne: It's forty-five.

Johnny: No, wait, it's not even thirty or forty-four. Naw, God, it's forty-nine.

Katie: Yeah.

Johnny: Forty-nine. No, wait a minute, it's forty-eight?

Anne: It's forty-four.

Johnny: It's forty-eight. Eight times . . .

Katie: Okay. (counting on fingers) 5, 10, 15, 20, 25, 30, 35, 40, 41, 42, 43, 44.

Johnny: No, wait, wait a minute. Okay, Okay, eight . . .

Anne: (counts on fingers to show Johnny) 5, 10, 15, 20, 25, 30, 35, 40, 1, 2, 3, 4

Johnny: 5, 10, 15, 20, 25, 30, 35, 40. Okay, 40 + 4 = 44.

In the above example Johnny is convinced, or has to be convinced, that his answer is incorrect. The process that the other students in the group go through, basically through the steps of working the problem, makes their thought processes explicit to Johnny to convince him that his answer is wrong. Also, Johnny himself must think aloud and go through the problem solving steps of his own thought processes before he is convinced that the answer 0.44 is indeed the correct answer. One might also hypothesize that not only are students learning the correct mathematics answer from such small-group interaction, but they are also more likely to learn the different strategies for arriving at answers to mathematics problems as well as possible skills and strategies for diagnosing and monitoring their own mathematics learning.

Unfortunately, small-group learning is used much less frequently than whole class instruction in mathematics classrooms. For example, Stodolosky (1984) reported that, in her research on fourth-grade and fifth-grade mathematics classrooms, small peer work groups were used only a out 4% of the time. Similarly, Peterson and Fennema (in press) found that students were working together in small-groups only about 4% of the time in fourth-grade mathematics classes. Lockheed and Harris (1984) reported a slightly higher figure of 10% in their study of fourth-grade and fifth-grade mathematics classrooms. Thus, although research suggests that small-group cooperative learning techniques can be effective in increasing students' mathematics learning, particularly when effective task-related interaction in mathematics occurs in a small group, surveys of current classroom practices suggest that such small-group techniques are not being used currently in elementary mathematics classrooms. Strategies would need to be developed for encouraging the increased use of small-group cooperative learning techniques in mathematics classrooms or for incorporating small-group cooperative learning techniques into the predominant traditional mode of whole-class instruction.

Conclusion

Findings from recent research on teaching cannot and probably should not provide a detailed recipe for how mathematics teaching should proceed in the 1990s. Indeed, provision of a "recipe" for mathematics teaching in the 1990s would be in direct contradiction to the perspective of the teacher as an active, thoughtful professional that has been put forth in the last decade by researchers on teaching. However, research on the quality of instructional time, the student and the teacher as active information processors during classroom instruction, and research on small-group learning can provide useful concepts and findings that should be considered as we plan for mathematics instruction for the 1990s. These several areas of research offer hope not only for improvement of students' mathematics learning in the next decade but also for greater understanding of the processes of teaching and learning that are occurring in our mathematics classrooms.

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IMPLICATIONS OF RESEARCH TO MATHEMATICS TEACHERS

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As with all technologically advanced nations, we must give continued attention to the mathematical competence of young people if we hope to maintain a strong and vital future. The United States continues to increase the percentage of its population that completes high school. However, large numbers of high school graduates have inadequate mathematical preparation to allow full participation in our modern society. Many students are denied access to a wide range of career options because of their lack of mathematics. The National Assessment of Educational Progress (NAEP) from 1973 to 1978 found a slight decrease in the average performance of 9-year-olds, a slightly larger decrease for 13-year-olds, and an "appreciable" decline for 17-year-olds over the five year interval (Carpenter, 1980). The Scholastic Aptitude Test scores dropped from a mean of 502 in 1963 to 466 in 1980. Paul DeHart Hurd, in a paper given at the National Convocation on PreCollege Education in Mathematics and Science in 1982, observed that "the fastest growing minority group in the United States is the scientifically and technologically illiterate" (Hurd, 1982). The American Association for the Advancement of Science (1982) stated in Education in the Sciences: A Developing Crisis that the "American educational enterprise is not preparing our young people to live in a society increasingly defined, supported, enriched--and sometimes endangered--by science and technology."

Articulate statements of the crisis in school mathematics have proliferated in the wave of reports and studies of American schools in 1983. Suddenly education is a "hot" political topic. Politicians, legislators, corporate officials, university presidents, among others, are crusading to improve schools. For those of us who have spent our professional lives trying to find ways to improve schools, and in particular to improve mathematics teaching and learning, the times are welcome. But with these times and this broad expression of concern from many corners of our society comes a particular responsibility for professional educators. We must examine where we are in developing our knowledge base, what the most salient outstanding problems are, and how we can redirect our talents and energies to make "real" gains during this time of national interest. This conference is a most timely response to this need.

I have been asked to present evidence to the conference participants on "The Implications of Research to Mathematics Teachers". I have chosen to focus my remarks on the following questions:

Why has research had relatively little effect on schooling?
How can we make research relevant to mathematics teachers?

Why Has Research Had Relatively Little Effect on Schooling

> Many suppliers and users of social research are dissatisfied, the former because they are not listened to, and the latter because they do not hear much they want to listen to. (Lindblom and Cohen, 1979)

This quote from Lindblom and Cohen, (1979) in their book Usable Knowledge, states the case succinctly. We are precisely in a situation where our knowledge base about how children learn mathematics and about the complexities of schools and teaching is growing. Hard earned research results have some real promise for improving schooling. At the same time our credibility with schools and teachers in general is not growing. There is a suspicion on the part of teachers that what mathematics education research is all about is irrelevant to the classroom and to themselves in particular.

One possible reason for this mistrust of teachers for researchers is the lack of a personal researcher-client relationship. For the most part teachers (or schools) do not go to a researcher and ask for help on a particular problem. With research on mathematics teaching and learning we seem to expect teachers to simply "draw on published work of faceless social scientists" (Lindblom & Cohen, 1979). We must find more personal ways of informing teachers of research findings and working with them to use the results to improve mathematical experiences of children.

Another important contributor to the mistrust of teachers for research is the fundamental conflict between the professional researchers' view of inquiry and the teachers' on-the-firing-line forms of problem solving analysis. Teachers (and many university faculty from scientific disciplines) often dismiss educational research with a wave of the hand, a sneer, and the comment, "They have wasted all this time proving what common sense would have told them before they started!" Educational researchers must realize that teachers attack problems, as they arise in the classroom, with ordinary knowledge, common sense, casual empiricism, thoughtful speculation and analysis and by so doing keep schools going. "Despite the professional development of specialized investigative techniques, especially quantitative, most practitioners of professional social inquiry, . . ., inevitably rely heavily on the same ordinary techniques of speculation, definition, conceptualization, hypothesis formulation, and verification as are practiced by persons who are not social scientists. . . (Lindblom and Cohen, 1979). Mathematics education researchers use these same techniques. It is the deliberateness and control of the process of observation that distinguishes scientific research.

"Research" in the mind of society is generally associated with solutions to problems. The research that lead to the Sabin oral vaccine for poliomelitis is a case in point. Dramatic breakthroughs or final solutions are rare indeed in mathematics education research. I believe that it is more honest and ultimately more important to view mathematics education research as a process of refining ordinary knowledge in moving toward a solution to a problem. There may not exist ideal or final solutions, but there are improvements on the existing situation.

Another parameter of the implication of research to the teacher is the question of whether or not, even if a hypothesis is scientifically verified, other demands of schools will allow anyone to act on the assumption that the research is true. For example, even though we "know" that very large schools are not educationally better for our children, we still close school buildings and consolidate into larger and larger high schools and middle schools because the economics of the situation become the overriding concern. I am suggesting that the mathematics education research community must carefully direct at least part of its efforts into studying ways to implement innovations rather than continue to put effort into studies that advocate principles and changes that the existing social structure of schools either reject as contrary to ordinary knowledge or reject in favor of other competing "facts."

How Can We Make Research Relevant to Mathematics Teachers?

One answer seems obvious. Involve teachers as full partners in applied research efforts. To do so will affect both the selection of problems to be researched and the collected ordinary knowledge to be used in attacking the problems. Teachers know schools and classrooms in ways that outside researchers do not. Through these kinds of partnerships teachers can come to understand the nature and promise of research on learning and teaching mathematics. Mathematical researchers engaged in such collaborations are unlikely to suggest impractical, radical innovations, likely to be rejected, but rather to suggest changes that are useful refinements of ordinary knowledge. We must accept ordinary knowledge and work on instances where research can validate, change, or explain general knowledge.

The Institute for Research on Teaching at Michigan State University, which has been funded by the National Institute of Education since 1976, has demonstrated the benefits to research, to teachers, and to the dissemination of research results of having Teacher Collaborators involved in the research process. In an interview for the Communication Quarterly of the IRT, Teacher Collaborator Barbara Diamond says that involvement in research has stimulated her to think about why she does what she does in her classroom. She now thinks about what her decisions about teaching are based on. "It was great to have a chance to step back from the classroom and reflect on teaching, both my own and that of others, and its effects on students and student learning" (IRT, 1982).

The IRT's Teacher Collaborators are taken seriously by researchers. They present papers at national conferences, give inservice workshops, and write articles. They gain confidence in their own teaching, in their decision-making, and in their ability to "research" problems in their own classrooms. As these teachers come to understand more about how research can be applied to the classroom, as they learn to look to research as a source of information, they become very valuable disseminators of these ideas in their own schools. Jean Medick is an excellent example. She was one of the first IRT Teacher Collaborators. Even though she has been back as a full-time teacher for several years, she still helps to improve her school through research. The teachers in her building used new strategies for beginning the school year as a result of Medick's distributing a summary of relevant research to her colleagues. This personal contact between school and research through a teacher collaborator has provided a vehicle for published research results to impact the school.

This notion of a close professional partnership between researchers and teachers has pervaded the work of mathematics and science educators at Michigan State University. As additional evidence I will give an update on four projects at MSU which exemplify close public school-university ties. The projects are, the Middle Grades Mathematics Project, the General Mathematics Project, the Science Teaching Project, and the MSU Task Force for the Improvement of Mathematics and Science education, K-12.

Figure 1 provides a sort of advanced organizer for these projects. It is meant to convey the notion that research can try to influence schools through curriculum (broadly defined to include program materials and instructional strategies) or through more general less content-specific concerns. The ellipse represents the latter emphasis, the inner path the former. I have placed the four projects to show their major emphasis.

In developing the mathematical content of the MGMP Units we started from the premise that the material presented must be a mathematically sound, important collection of related concepts, principles, and skills. To help children process the information learned we presented problems in an organizing story context, whenever possible. We have also made an effort to build in opportunities for children to see problems with very different surface characteristics that in fact have the same mathematical structure.

While this collection of activities for the students is extremely important, the heart of an MGMP unit is the detailed teacher guide. The instructional model imbedded in the teacher guide presents each activity in three phases, launching, exploring and summarizing. The teacher and student roles as an activity proceeds through its three phases are shown in the schematic diagram (Figure 2).

Many of the activities in MGMP units are built around a specific mathematical challenge. During the first phase the teacher launches the challenge. The launching consists of introducing new concepts, clarifying definitions, reviewing old concepts, working through a minichallenge, and finally issuing the challenge.

The second phase of instruction is the class exploration. During the exploration the students work individually or in small groups. The students may be gathering data, sharing ideas, looking for patterns, making conjectures, or developing other types of problem solving strategies. The teacher's role during exploration is to encourage the students to persevere in seeking a solution to the challenge. The

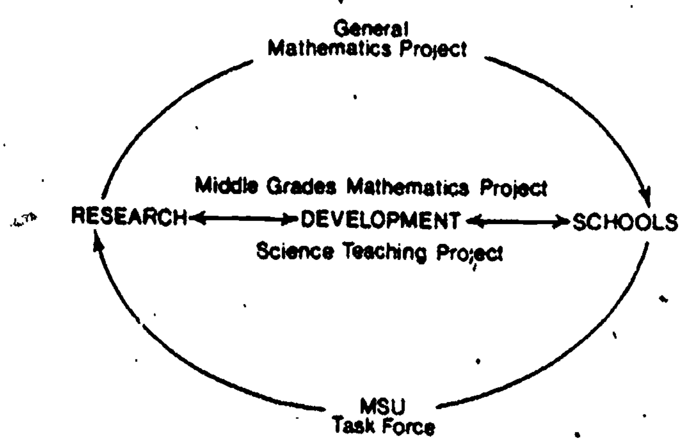

Figure 1. University-schools partnership

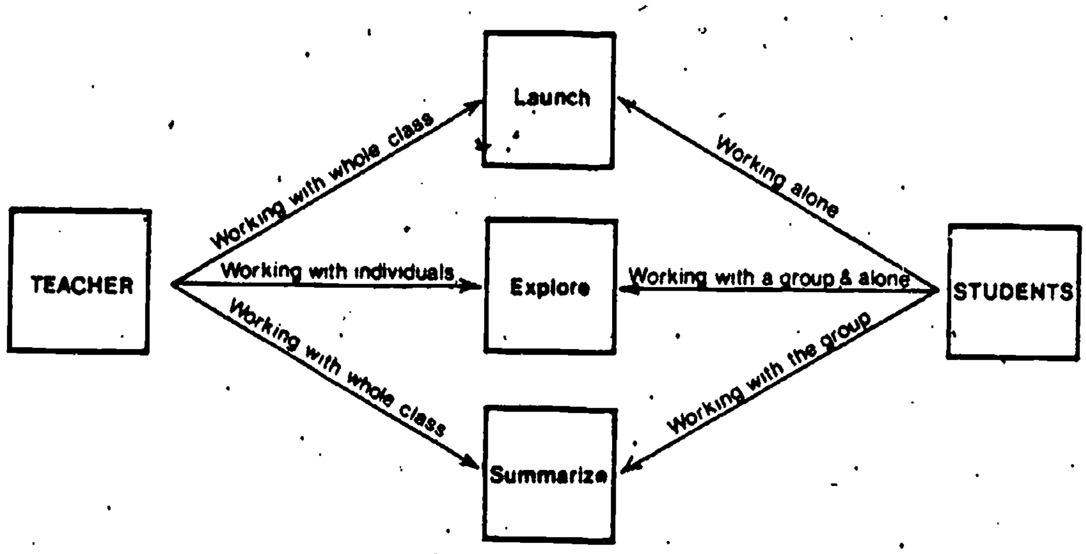

Figure 2. MGMP instructional model.

teacher does this by asking appropriate questions, encouraging and redirecting where needed. For the more able students, the teacher provides extra challenges related to the ideas being studied. In this way individual needs are responded to within a whole class activity.

When most of the children have gathered sufficient data, the class returns to a whole class mode (often beginning the next day) for the final phase of instruction--summarizing. Here the teacher has an opportunity to demonstrate ways to organize data so that patterns and related rules become more obvious. Discussing the strategies used by the children helps the teacher to guide the students in refining these strategies into efficient, effective problem solving techniques.

The teacher plays a central role in this instructional model. First the teacher provides and motivates the challenge and then joins the students in exploring the problem. The teacher asks appropriate questions, encouraging and redirecting where needed. Finally, through the summary, the teacher helps the students to deepen their understanding of both the mathematical ideas involved in the challenge and the strategies used to solve it.

Each unit was developed and evaluated with the help of eight affiliated teachers. These teachers were involved individually with the development and trials of the first version of each unit. As a group they were involved in a summer institute to produce a more polished version of the unit for field testing.

Each summer institute involved the entire project staff, eight affiliated teachers, and 40 children from grades 5-8 who met for two weeks at a nearby middle school. Two units were evaluated each summer. The children were divided into two groups: grades 5-6, and grades 7-8. Each group was taught each of the two units by one of the staff. In each classroom, another staff member observed and videotaped the session. Four experienced teachers also observed and participated in the exploration phase of each activity. Each day the teachers and staff met for two hours after the children had left, and discussed the activity observed in each unit that day.

The summer institutes were extremely important to the development of each unit. The teachers came from different types of schools--city, suburban, and rural. Each teacher had recruited 5 students from his/her school for the program; these children had very different backgrounds and abilities. The teachers were actively interested in the learning of each child. The teachers made many comments and observations which helped the staff to find errors or omissions in the sequences of activities and in particular in the unit guide. Test items were also generated and evaluated by the staff and teachers during these institutes. These teachers came to own the unit and the process of curriculum development in a way that made them excellent resource personnel for their buildings.

The Michigan State University Task Force for the Improvement of Mathematics and Science Education, K-12

Co-chaired by Glenda Lappan and Glenn D. Berkheimer

On June 8, 1983, Provost Clarence Winder appointed a 25-member Task Force to organize MSU's response to the present mathematics-science crisis. Two aspects of the work of the Task Force are relevant to this conference. First the Task Force takes the view that university-school partnerships hold the best promise for building working models to improve aspects of mathematics and science schooling. We have begun the process of exploring with local school districts what such university-school partnerships might look like. Certainly one area of concern in such a partnership would be research.

We have identified three areas in which outstanding progress has been made during the last decade. Two of those areas involve research and development which has led to better understanding of teaching and learning in science and mathematics. The third involves advances in technology which have led to new instructional possibilities inside and outside of classrooms.

1. Understanding of Classroom Teaching

During the past decade the field of classroom research has expanded enormously, and the results of that research have become increasingly powerful and useful. Today we have a useful understanding of how teachers think and behave in classrooms, how they use prepared curriculum materials, and what knowledge forms the basis for their performance.

2. Advances in Cognitive Psychology

Cognitive psychology has undergone a revolution in the last twenty years; one of the effects of this revolution has been a vastly improved understanding of the difficulties that students have with scientific and mathematical reasoning.

3. Improvements in Instructional Technology

Improvements in instructional technology have opened up possibilities that did not exist previously. Calculators and microcomputers are now available at prices that schools can afford. These devices are not only transforming society in ways that bring parts of the old curriculum into question, they are also making it possible to pursue objectives or to use methods of instruction that previously were impossible. Videocassette or videodisc technology may ultimately have important effects on the curriculum; we must understand those effects in classroom contexts and attempt to assure that they are beneficial.

The General Mathematics Project

Director Perry Lanier

The Project research questions are:

1. What do teachers see as the central problems in teaching general mathematics? What approaches have they used in dealing with the problems? What effect do they perceive they have had?

2. How do teachers alter their views about general mathematics teaching and general mathematics students as a result of (a) exposure to literature and (b) systematic trial of new approaches to teaching?

3. What concepts, strategies, and research results from the literature are seen by teachers as applicable to the task of improving their general mathematics classes? Through what process do teachers make use of new insights and skills?

4. What happens in classrooms when teachers systematically alter their approaches to general mathematics? What evidence of student improvement can be found?

The Project has identified the following deterrents to success--in student terms--

a) a history of poor mathematical achievement/attitude,

b) a repertoire of fragmented mathematical concepts, algorithmic skills, and problem solving strategies;

c) student interaction problems;

d) perception of mathematics as irrelevant to the present or future;

e) school habits--attendance, study, etc.;

f) resistance to instruction--particularly that which was somewhat familiar; and

g) the clamor for seatwork--mundane assignments.

It has also posited these portents of success:

1. using social organization to facilitate instruction,

2. improving content communication--its quantity and quality, and

3. modifying mathematics content/tasks.

The Project is currently working with four collaborating teachers in designing and implementing intervention activities. Lewin's (1947) change model--unfreezing, changing, refreezing--is the model selected to guide the intervention.

Lanier serves as a consultant to the teachers as part of the unfreezing-changing-refreezing model. He summarized his views on the role of consultation as he uses it in his project.

> Consultation is an interchange between colleagues who share their respective expertise in a collaborative effort to improve the teaching and learning of general mathematics. One implication of such a view of consultation is that the consultant avoids telling the teacher what to do.
>
> . . . the teacher must have the freedom to accept or reject the consultant's recommendation.

Lanier's work exemplifies a very personal research-client relationship.

Science Teaching Project

Charles Anderson, Kathleen Roth, Edward Smith.

The Science Teaching Project focuses on the three-way interaction among teachers, students, and science program materials. The project looks at the point of view of the materials, the point of view of the teacher, and the point of view of the children as they come to a particular lesson. The teacher decision making, what's going on in the mind of the teacher; the misconceptions and preconceptions that that child brings to the lesson; the point of view of the material itself--three distinct points of view and the interaction between those three is where the ballgame is at. This project has identified four kinds of teachers--the activity-driven teacher that does the activity for the activity's sake and never really thinks about long-term planning or where this fits in the scheme of things; the didactic teacher who thinks very carefully about the material and the logic of presenting it but does not attend to the students, where they are, what they know, what kinds of things they are misconstruing about what is being taught; the discovery teacher (and I must confess, in my experience, I have not run into any mathematics teacher that I would put into this category) who feels that the important thing is what the children discover for themselves--the teacher who is carrying this to the extreme never redirects, never encourages the children to refine their strategies to come to better understanding; and, the kind of teacher that I think most of us would like teachers to become, one teaching for conceptual change. The point of this project, and the point of our work with MGMP, is that if we are asking that teachers become conceptual change teachers then we must give them some help.

The Science Teaching Project is an excellent example of practical applied research that deals with the complex interactions between the points of view of the students, the teacher, and the curriculum materials. Research projects of this sort have the potential for great payoff in units written both to help students give up their naive misconceptions in favor of more scientific principles and to help teachers become more sensitive to what their students actually believe is true.

While the teachers in this project have not been teacher collaborators in the IRT sense, they have had significant opportunity to affect the directions of the research. They are not just objects of research but are partners involved in trying to find ways to improve the effectiveness of instruction.

> The very act of excluding teachers from the research and development process may explain why research findings heretofore have been difficult to implant in classroom instruction (Tikunoff & Mergendoller, 1983).

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DISCUSSION

The discussion which followed the testimony on learning and teaching focused on five issues:

1) the strategies children use to solve problems,

2) instruction on strategies,

3) metacognitive (affective) aspects of learning,

4) teachers' knowledge of learning processes, and

5) the interpretation of research.

Strategies Children Use

There was consensus that we need to understand the strategies children use to solve problems (including strategies that lead to errors), and knowledge about strategies should eventually help teachers. But there was concern over what it is we know about strategies.

In the analysis of errors in subtraction, for example 804 - 579, the error was borrowing from the hundreds place rather than the tens place since there was a zero there. The answer, which was 135, shows that the child's real problem has nothing to do with borrowing from the hundreds vs. borrowing from the tens. The child's real problem is simply that he or she has no understanding of what is a reasonable answer. I mean, 135 is very far off from what makes any sense at all in this problem; your answer must be bigger than 200. We are telling teachers that they ought to be trying to figure out how the child found that answer. Was it from borrowing from the hundreds? Instead the teacher ought to be saying Does this answer make sense? The teachers ought to be encouraging the child to say, after getting an answer of 135, Is this answer anywhere in the ball park? Does it make sense? And if the kid looks at it and thinks this is about 800 minus 600 and the answer should be about 200, more than 200, then the child should be saying what I did is wrong.

I have to disagree with that, although I feel very strongly we need to develop estimation skills with children. The child's procedure is wrong. The child does not understand the procedure and that needs to be discovered.

I am not saying that that procedure should not be discovered. All I am saying is that, if you have a procedure that gives outrageously wrong answers, it would seem to me that that fact alone should stop the child. The child should be encouraged to stop and try to figure out what's going on rather than simply carrying on.

When the child is doing an exercise like that, he does not stop and look at it. The child does not write the answer down, he writes down the digits of the answer in reverse order. It is like the programmer of the computer. If the computer program runs to completion, you are satisfied, you go on to the next exercise.

Another thing one might note is that a lot of errors fall under general categories which math teachers know are general categories to be taken care of. One of them of course is borrowing and the other is the zero. Children treat zero as nothing and therefore, it's an eminently reasonable and rational thing to do because zero is nothing, it's not there. A lot of errors are explicable that way. If we group these types of errors into major categories, then it gives us a sense of where we want to aim our next major thrust. Procedures should follow from genuine insights. So we better work on zero and an understanding that they can get so that when they even scan their procedure, they know that that part of it is nonsensical because zero no longer just means nothing.

Teachers often tell the child, "Look. This answer's ridiculous. How can you say this answer is 135. It doesn't make sense." And the child will say, "You're right." And the teacher says, "Did you believe that answer when you wrote it?" And the child will say, "No," and the teacher will say, "Well, why did you write it?" The child will shrug and say, "Well, it was the best I could do." Knowing that your procedure is yielding an answer that you do not have a lot of confidence in is insufficient. You also need the ability to recognize an unreasonable answer and discriminate it from a reasonable one. You also need an understanding of what the procedure is based on so that you can take that procedure and say all right, what did I do wrong in terms of the sense of this procedure. There's an understanding not only that the answer is unreasonable, but also of what the procedure is about, what borrowing and subtraction really mean.

On verbal problems, the way children addressed their problems was the safe way. You're sure to come up with the correct answer if you can count correctly. I wonder whether teachers in the classroom are just getting kids to make as sure as they can that they give us the right answer. The good way to look at a problem is to look at alternative solution strategies and pick the one that's the best one.

Children always like to get the right answer. They always like to look good for the teacher. Maybe what we do in getting them always to give us the right answer is discourage them from thinking about how to solve the problem. Do we encourage kids in the classroom to look at solution strategies and alternative solution strategies and pick one that's a good one in whatever sense we mean good?

There are two parts to that question. First, it was interesting to me that their safe answer was counting. Considering the fact that the initial intent of the instruction was on one-to-one correspondence, the children were beyond that. Second, safe responses are normal. It takes a while to get the children to trust you, and to get them to talk about the strategies that they use. In fact, we do develop socially acceptable kinds of responses. For example, counting on fingers is a very normal thing that students do. But there is a lot of social stigma against doing that. Children are very reluctant to admit that they count on their fingers.

Instruction on Strategies

The connection between knowledge of strategies and instruction has already been alluded to, but explicit comments were as follows.

Shouldn't we, as a group, as part of our report encourage teachers to actually develop the skill of looking at alternative solution strategies and chosing one that in some sense is best. Isn't that really part of problem solving? Should not that be addressed in whatever curriculum recommendations we make?

Thinking about your own thinking is important, as is recognizing what the additional available strategies are. Frequently, although children make choices between strategies, their choices are not always particularly conscious. The evidence is that if you can get kids to think about strategies, that in fact is productive.

I was interested in Case's ratio example from the perspective of mathematics. This topic has for a number of years been poorly taught. It was intriguing that your good problem solvers seem to simply do it the way it should have been taught from the beginning. That is to connect it with division. One implication is that we have to wait until the students reach a particular developmental level. The other implication is that performance was not due to developmental level, but in fact only good instruction. How do you put those implications together?

That's a standard conflict; it's one of the best ways to pull apart developmentalists. We got all students to 100% performance, but I do not feel that what we did was get them operating at the same abstract level that you or I would use. They were not setting up a set of equal ratios and cross multiplying. What we do is give children a way of understanding a task at their level so that they can get answers, check themselves, and give you reasonable justification for that answer without operating at a high level of abstraction.

When comparing sixth graders with eighteen-year-olds, there's a lot of instruction that occurs apart from what you were doing in your studies. Some of that other instruction may contaminate those students and make it harder for them to learn the concept. I think this has happened with ratio. You focused on a particular strategy which relates to what they were instructed, perhaps outside of your experimental setting. You have to consider that they've actually had more work with these ratios outside your experimental setting than inside, even if that was not that productive.

There are short term changes and there are long term changes. The sort of changes we were looking for--and we got--were very short term. We measured a month later, and we were instructing for a maximum of forty minutes. When you get that rapid kind of effect that lasts, my sense is that you've hooked into a way they can think of it easily at their own level and they don't get interference. There are other sorts of changes that take a long time. As yet, we don't fully understand the relationship between them. A nice feature I see of teaching this way is that you don't have to solve that problem theoretically, whether you have to wait and so on. If you have a sense of how children at different age levels are functioning spontaneously, then you can give them lots of rich, creative and noncreative problems at that level. Then if you know what the next level is from time to time you can offer them that.

Metacognitive (Affective) Aspects of Learning

We've been talking about behavior as if it was totally under the control of a rationality that wanted us to behave in a cognitively organized way, and that such behaviors depend upon development. What about internal motivation? Why did the child want to get the answer?

What other influences were working on the child that had nothing to do with his rationality? How do you take into account affective variables which effect whether the child even wants to get the answer, or if the child is an independent thinker and wants to think through the answer or if he wants the teacher to give him the answer. So what are we going to do about that whole other body of behavior out there that's not rational?

If you don't study affect, that doesn't mean that you don't think that it's important. However, with the young children a vast proportion of mistakes are rational, if you take their point of view. They are thinking about math. They are thinking very sensibly. And they are making very sensible mistakes. Now if you imagine them being submitted to a program for a number of years which doesn't take account of that and doesn't enable them to draw on their intuitions, to do numerical computations and make that numerical world a separate world, one would expect a certain set of affective changes to take place.

Affect variables are important. In a study we tried to code children's thoughts. In the course of the interview, we found that kids would report thinking things that really had an affective loading to them. We called one category negative evaluative self-thoughts. They would spontaneously say things like, "I can't do this," "I'm dumb." We also had a category called positive self-thoughts. In addition, a lot of kids just said, "I was just trying to get done." These categories of responses we found were extremely important because they were highly related to kids' reports of their understanding. And the most significant of these three categories was negative evaluative self-thoughts. Positive evaluative self-thoughts did not seem to be related to achievement. However, the more negative self-thoughts they made the less they reported understanding the material, the less they were able to say what it was or why they did not understand. I can't really say that it's the negative self-thoughts that cause the lack of understanding. Maybe the lack of understanding causes the negative self-thoughts. But we have to recognize that the affective things that we think about or say to ourselves are clearly important.

Teacher's Knowledge of Learning Process

I can see why coding of strategies is important in an interview situation. But, I'm interested in trying to transfer this to classrooms. If a teacher has to figure out where the child is in order to help the child learn, what can we say. I am going to go back to subtraction and the error patterns. If teachers do find the error pattern (and we tell the teacher about the error patterns), what are we going to do given classrooms with teachers who have minimal mathematics backgrounds. How are we going to make this transferable into the situation where it is going to have an effect on the classroom? Until this research is transferred into teacher training nothing will happen at grade 12. And I think if you're going to have an effect, that's the other part of the whole thing, we've got to go beyond grade 12. Error patterns are just another bit of information to frustrate that teacher who doesn't know what to do about it.

It seems to me that one of the things you can do about teachers who don't have the tools to explain well to the children what to do about their mistakes is to develop simple cassettes for microcomputers that do the explaining for the teacher and that diagnose the errors. You can get the error patterns being diagnosed off of problems that are presented on the computer. Even if teachers are attuned to finding the errors, it's time consuming and difficult to do in the classroom. The microcomputer offers us a wonderful tool here for overcoming those difficulties.

I am asking for reality. To tell me to use the microcomputer is to tell me to go and get a cup of coffee. I do not have microcomputers in all of my schools in an adequate number for them to be available for diagnosing error patterns.

Microcomputers right now are not all that expensive. Especially when compared to the amount of money that a school district spends per pupil on other things. It may well be worth a teacher's while to accept one extra student in each classroom in return for having a microcomputer in that classroom. They make the teacher's job easier even with that one extra student.

We can always raise the objection that because of reality constraints we can not implement something. What is offered by the analysis of errors is in fact something the teachers can understand and can implement. Teachers at the elementary level can learn those things. Clear analysis provides a precise language and a precise framework. That's a big step. It provides a framework for remediating errors. It identifies the kind of problems one can give students and the kind of rules they are missing.

I'm not disagreeing, but we have to design the mechanism for making it work. We have to tailor the programs to meet the needs of urban children.

I would operate from the perspective that we've got to start by understanding how one student would solves a problem, under ideal situations diagnose what the student was doing. We can develop a fabulous delivery system for training teachers and if we don't have anything to train them about, then it isn't going to do any good. What the cognitive research is providing is the first step. It is a step in the right direction that will respond to what teachers need.

The greatest payoff for improved achievement is improving teacher motivation. The problems are political. If we paid teachers in part according to how well their students achieved that would do wonders for students' achievement. The needs for teacher support networks and retraining of teachers would solve themselves if there was something in it for the teacher. If their children learned more, then teachers could benefit in a material way. It's a part of the whole educational puzzle. We're not going to influence teachers' unions, we're not going to influence boards of education, we're not going to influence state legislatures with cognitive research.

I was struck by the phenomenal number of decisions that a teacher has to make. There are apparently no studies which try to get teachers to improve or to change their decisions. What is needed is a practitioner's theory so that teachers are equipped to make the kind of decisions that they need to make on the spot. A lot of the research we do, valuable though it is, is more like the linguistic research: after the fact you can make a nice analysis if you can replay the video tape a couple of times and look at it carefully. The practitioner doesn't have that luxury. The teacher needs a theory.

Unfortunately, we do not have much research on the relationship between the kinds of interactive decisions the teachers make and their effects on student achievement. One of the things we ought to be looking at is alternative instructional organizations in classrooms. To get children actively involved in cognitive processing may require grouping children differently. What happens in group behavior may be different and effective. Then what is the effect of group behavior upon the development of the cognitive processing?

The Interpretation of Research

One of the problems faced by teachers involves how to read and interpret research. Funding research which we feel is important should make an impact. However, results must be usable by the practitioner. What can we suggest so that researchers can address the critical issues, have a bigger impact, and give teachers more help.

The problem is that interpreting research so that it is useful is hard work. For example, in a science teaching project at Michigan State a particular idea in science photosynthesis was studied. They spent two years trying to help teachers unravel kids' misconceptions. They finally developed two sets of materials: one that addressed the misconceptions that kids have and the other on teachers' misconceptions of where the children are. Trying to make teachers sensitive to the things that cognitive scientists are saying to us about children is extremely time consuming. But it must be done.

SUMMARY OF WORK GROUP REPORTS

Elementary and Junior High Mathematics
Jane Gawronski, Chair

We first looked at the general areas in which there may be problems or in which we want to develop recommendations. Initially we thought about the curriculum, instruction, and textbooks. One of our group members last night telephoned mathematics supervisors in three cities of 500,000 or more and got a preliminary reading on what the receptivity would be for some of our ideas. That information helped us to structure our comments. We identified three problem areas and developed two proposals for their solution.

First, we saw that there was a real mismatch between the arithmetic children learn in the schools and the mathematics used by adults. The content that is being recommended by other groups, for example by NCTM in the Agenda for Action, is not being implemented. There is consensus among us that these are valued recommendations, but we do not see much evidence of the implementation.

Second, performance on applying arithmetic to problems is not what it should be. Performance on computation is much better.

Our primary suggestion for solving those problems by the 1990s would be to have mathematics specialists in elementary schools. A mathematics specialist would function as a leader in one of two ways. That person could function as the mathematics specialist in a unit, a group of teachers, or the mathematics specialist could be separate from the general classroom teacher and just teach mathematics.

Third, there is the sense of failure on the part of many students because of their lack of achievement in computation, and the classroom response is actually detrimental. Many students are typically kept doing the same computation before they are exposed to different content in mathematics. Teaching long division to mastery with two or three digits takes time away from the mathematical content that we want children to achieve. We are not saying that mastery of some concepts and some skills is unimportant. What we are saying is that there is an inordinate amount of time being spent trying to get mastery on some skills, when it is not clear that mastery of those skills is really that important.

Our response to these problems is to vastly limit or curtail the paper-and-pencil arithmetic in the schools. Further, we suggest that some content be resequenced: decimals earlier, operations with rational numbers later. Less time should be spent on multiplication and division algorithms with whole numbers. More time should be spent on mental arithmetic, on estimation, and on problem solving.

Senior High Mathematics
Edward T. Esty, Chair

First, we have consensus about the importance of probability, statistics, applications, and problem solving. Second, there has to be more unification, more intermingling of topics throughout the curriculum.

Our long range view is that someday all children will have access to videodisc players hooked up to powerful microcomputers. The new math, symbolic algebra, will be just as easy as numerical calculation is now with hand held calculators. Our short range view is somewhere between now and then. We see the population of secondary students in four groups: a top group, middle group, lower group, and a lowest group. The same body of mathematics should be taught to all with differences in approach, depth, breadth, pace, and context of applications. We need more intermingling of math topics, perhaps by quarters or nine-week modules. This is assuming that students can get a module in the form of a videodisc. We do not foresee that kids would be interacting only with computers. There must be teachers and there must be group work. But the groups do not have to be people within the same school. One can be interacting in a group where one person is in this school and another person is in a school 50 miles away and another person is in some other place.

We recommend that a "core" group, somewhat like the Cambridge conference, be formed to look at the K-14 curriculum. We cannot treat secondary school mathematics in isolation from previous instruction. This core group would have mathematical scientists, mathematics education researchers, psychologists who were doing work in mathematical learning, teachers, supervisors, and appliers of mathematics and science. The group would be augmented from time to time for a sequence of conferences including vocational education people, psychologists, inservice educators, publishers, test makers, special education people, post-secondary education people. Conferences in the sequence should be very closely interrelated. A planned core group might keep people on reasonably consistent target.

We envisioned that group as also having the flexibility to respond more immediately in certain target areas. The core group would have a five-year lifetime at least, funded with at least $200,000 annually. It should not be a federally funded operation. It would be good to have some sort of consortium of industries or foundations fund a group.

Learning and Teaching
Thomas P. Carpenter, Chair

First, we need to recognize and make very clear that there exists a body of research on the learning of mathematics that makes explicit, and perhaps more useful, some of the things that many teachers have already known. Second, there exists a promising line of research on teaching. There is a need to create ongoing mechanisms to transfer this knowledge into teacher education and teacher reward programs. The knowledge exists but there has not been a great deal of implementation of that knowledge.

Recognizing that any kinds of decisions that we make are not going to be final decisions, we came up with a goal which on the surface seems fairly obvious--the goals of mathematics are to get children to learn mathematics, to be in a position to use mathematics, and to continue to learn mathematics. That may seem trivial, but a part of our discussion was on incentives. The system now is not designed to reinforce goals. Teachers are not rewarded for meeting those goals. School administrators and students also are not rewarded. We thought that it was important to recognize that these are critical problems that we have been talking about. We are not going to resolve them in three days. A substantial amount of time is needed to discuss those issues.

We need a continuous dialogue involving a broad spectrum of people who are involved in the teaching of mathematics. We came up with a slightly different recommendation about how to implement that. We believe a series of substantial conferences should be held to address particular topics. A conference would last at least one and perhaps two months and would be preceded by approximately a year of lead-time for setting up the conference and clearly establishing goals of the conference.

Computers and Technology
Arthur Melmed, Chair

Our discussions followed the flight of the bumble [illegible] but I will try to capture as much of it as possible. We started [illegible] four categories. One was improving mathematics education by improving the content of mathematics education. The second was improving mathematics education by the use of certain tools which I'll describe. The third was improving mathematics education by the use of another set of tools which I'll try to distinguish from the first set. And the last was improving mathematics education by the increased use of diagnosis, testing, and instructional management.

On improving mathematics education by improving the content of mathematics education, we discussed the particular intersection between mathematics education and computer science education or computer literacy as it might be done in the secondary school. We did not go into that very deeply. We hoped that the Senior High Mathematics group would deal with that.

There is a set of tools which one might use in the classroom to learn about other things, to learn about other mathematical objects. We identified programming, which is used by students both to improve their math knowledge and also to instantiate solutions to mathematical problems and thereby not only improve their knowledge of mathematical facts but also improve their general problem solving skills. Another use of tools by the student is the use of a computer lab to learn from data, that is, experimental mathematics. There are also tools for the teacher to use. The tools for the teacher are clearly for individual use by the teacher. The tools for use by the student might be used by individual students, or groups of students, or groups of students with the teacher. The teacher would use tools to demonstrate these things dynamically in a way that is now very difficult for the teacher to do.

As opposed to the use of tools by the student or teacher to learn about other mathematical objects and facts, the second class of tools is instructional software for student learning. And that software is distinguished from the first in that perhaps there are higher costs for getting it in the first place and distinguished also possibly by the fact that the student has to work within the constraints of that software and doesn't learn about mathematical objects or facts that are outside the constraints of that software. We defined single-concept software, adventure games, and microworlds as being species of that class.

Finally, there are diagnosis, testing, and instructional management. We do not know as much about these as we should. We distinguished between nationally normed tests and tests in the classroom to assist the teacher in improving student achievement. For classroom testing, we distinguished between the teacher knowing in a timely way what the student had learned of what had been taught, or even what had not been taught, as opposed to what the student had not learned of what had been taught. And then of course there was sort of the derivative of that which is, if the student didn't know something, in what way did he not know it, which gets to the rule-governed procedural knowledge. Unfortunately, we do not know how to elicit with a computer program the skill that a good interviewer has or a good teacher has in understanding what has been learned.

After discussing the four categories, we considered what's more important, what's less important, what do we do first, what do we do second, what's more costly. We broke this into training, hardware, and software. The point was strongly emphasized that inservice training was training in the short term for people already in the system. Inservice training for teachers and administrators was very important. Do we know how to do it in sufficient amounts so that we could train a large fraction of the elementary and secondary school teachers? Hardware simply seemed to be a matter of money. Nobody quarreled very much about the availability of hardware, just that it was a matter of having enough money around to buy the hardware. We did not agree how much hardware was absolutely necessary. I also have the sense that the number of hardware units in the classrooms will grow. Obviously, the operative question is how fast they should grow relative to other things, relative to training and relative to the availability of software.

Software seemed to be more problemmatic. We cannot rely on the existing school publishers for that. We discussed briefly the possibility that schools might be able to develop software. We should not rely very much upon the schools to develop their own software; however, networks might be developed involving institutions of higher learning, school people, or perhaps the new developers or even existing school publishers. How to devise an incentive structure that would make that likely to happen is not clear. One option is that the federal government might provide the money for that. Centers could be set up to provide a nucleus for the establishment of these networks. The money would act to increase the probability that those networks would be established.

Finally, there are two considerations that ought to be kept in mind as proposals are prepared. Equity is the first consideration that we need to keep in mind. Training for change is the second. Somehow colleges of education have to change what it is they teach teachers so they can take into account needed changes, as well as how teachers are to teach based on new knowledge, research, and cognitive science. Thus, technology provides the possibility for some neat new things coming down the line, like intelligent videodiscs. Computers might acquire some new capacities in the future, such as speech recognition and the capacity of presenting good quality pictures in a way that is not presently the case.

TESTIMONY ON POLICY IMPLICATIONS

AND IMPEDIMENTS

Arthur Melmed, Chair

THE PROBLEMS OF CHANGE IN RELATIONSHIP TO THE PREPARATION OF MATHEMATICS TEACHERS

Robert T. Williams
North Carolina State University

I am amazed that many of the things I planned to say today have already been touched on in the past several days. It's fascinating to see the ideas come out. I have attempted to overcome the time constraint by preparing a five-page appendix of my thinking. The working conference format will provide some opportunities for these ideas to be expanded, improved upon, picked thin, or discarded.

Now, just a few comments about my background, so you'll know my context. I took mathematics methods under Veryl Schult and did my student teaching under her in Washington, D.C. I was a mathematics teacher for 6½ years, at the high school and technical institute levels. For the past 9 years, I have been Associate Dean at the School of Education at North Carolina State University.

For the past 3½ years, my research and publications have been focused on the mathematics and physical science teacher shortages in North Carolina, the related issue of out-of-field teaching, and on strategies for solving these problems.

At NCSU, we have a Department of Mathematics and Science Education, with three science educators and four mathematics educators. Our B.S. production in mathematics education has fallen from an annual average of 45 ten years ago to an annual average of 16 for the past three years. We have leveled off and may increase slightly because we have a larger than usual sophomore and freshman enrollment in mathematics education. We have a secondary track for persons preparing for grades 9-12 and a middle grades track for persons preparing for grades 6-9. We do not have an elementary education program.

I am not a member of NCTM and therefore am not up-to-date on its emphases and activities. So if my appendix or remarks offer some suggestions that have already been implemented, just ignore them. My appendix lays out four questions that tie together the status of mathematics teacher preparation, the changes that need to be made, some problems that seem to prevent us from making these changes, and some suggestions for overcoming these problems.

I have addressed only four aspects of the preparation of mathematics teachers in the appendix. There are others that need addressing, and I have listed some of them at the end of the appendix. Still others may surface during our discussions. Rather than come up with an extensive laundry list, I thought that a better contribution would be to develop four of them in a way that would enable us to discuss some possible solutions.

The four aspects I have developed are these:

1. Many elementary education majors are deficient in the quantity and quality of college mathematics courses they take and in their knowledge of mathematics methods.

   I have examined the catalogs and colleges in North Carolina to see how much mathematics is required of majors in primary (K-3) education. Not much. A couple of the institutions require none. A few require as many as three, but all of these would not be what we would call college-level mathematics. I' you were to see comparable data for your state, what would the picture look like? How can you teach these people mathematics methods when their content background is so weak?

   Then I examined the catalogs to compare the mathematics/science requirements against the humanities/social science requirements for primary teachers. It comes out to about 6-8 courses versus 18-24 courses. Is it any wonder that mathematics and science get shortchanged both quantitatively and qualitatively in elementary curriculum? If you had this kind of college preparation, what would be your implemented curriculum? Are these teachers going to be receptive to geometry in the elementary grades? Statistics in the elementary grades? Probability in the elementary grades?

2. States and colleges need to have a different curriculum for preparing middle grades/junior high teachers than they do for preparing high school mathematics teachers. We may be contributing to the teacher shortage if we unnecessarily "scare off" some potential middle grade mathematics teachers with unrealistic curriculum requirements. At the same time, if our mathematics major is too strong, we are enhancing the likelihood that some of our secondary-level graduates will be recruited into nonteaching jobs because we have overtrained them.

3. From my limited experience, it seems that state and local NCTM affiliates are seldom involved in decision-making processes affecting teacher education, including mathematics education. If mathematics educators and people representing NCTM affiliates are not involved, this allows higher education and state agency people not familiar with or sensitive to the real world of teachers to set standards and policies that affect the training, assignment, and working conditions of mathematics teachers.

4. Until the B.S. production in mathematics education increases greatly, a substantial number of mathematics teachers are going to receive their content knowledge and/or their knowledge of mathematics methods <u>after</u> they become teachers of mathematics, rather than before--or they won't receive this knowledge at all! I'm referring to those hundreds of teachers from social

studies, physical education, and other fields who are teaching mathematics. This has tremendous implications for mathematics educators and for local school districts.

I prefer to think about these issues, and the other problems that we are addressing at this conference, not as national problems, but as state and local problems that are rather consistent throughout the country. If I were to think of these issues as being national problems, then I tend to expect a national solution. But that's not the way most things get done. Look at where we stand with respect to S 1285. Even when we get federal initiatives such as in school desegregation, NDEA, and the several vocational education acts, we get impact but not solutions.

Because we accept education to be primarily a state function, you will see in my appendix a heavy reliance on state and local action. It just seems more fruitful for 50 states and 17,000 colleges and local school districts to be taking initiatives simultaneously rather than to be waiting for a federal initiative that may never come and is likely to be insufficient if it does come. There is a federal role, but not as the initiator.

Many of you are familiar with the television show Different Strokes. In North Carolina, we have a phrase that goes "different strokes for different folks." In a nation as large and as diverse as the United States, state and local initiatives can better provide for differences between states, within states, and among the several teacher education programs in a state. For the most part, our governments at all levels operate by disjointed incrementalism, rather than by grand-plan methods or national decision-making. Problems are attacked piecemeal. Progress is characterized by hit-and-miss efforts. Let's take advantage of the way things really work, and move when and where the opportunity presents itself.

In North Carolina, the initiative to strengthen mathematics has come from the governor's office. In Houston and Philadelphia, it has come from large city school districts. In Massachusetts, it is coming from two institutions of higher education. In 1980, it came from NCTM in the form of An Agenda for Action.

I see NCTM as the unifying thread that has national, state, and local contacts, as well as contacts in teacher education. It is the only such organization I know of that has the improvement of elementary and secondary mathematics as a primary mission. In my appendix, you will see that I am calling for NCTM to formally involve its state and local affiliates in the decision-making processes that affect the preparation of mathematics teachers in K-12. If you're not sitting at the table, you can't expect to get a piece of the pie.

APPENDIX

The Problems of Change in Relationship to the Preparation of Mathematics Teachers

A. What is the status of mathematics teacher preparation?	B. What changes need to be made?	C. What problems are preventing these changes?	D. What can be done to overcome these problems?
1a. Elementary education majors take an insufficient number of college mathematics courses.	a. A predetermined achievement level should replace a predetermined number of credit hours, in order to achieve competence in mathematics.	a. The absence of an agreed-upon standard of mathematics achievement for elementary education majors.	a. A commitment by NCTM and other appropriate national advocates to address this issue.
b. Many of the college mathematics courses taken by elementary education majors are high-school level courses.	b. Elementary education students should take a mathematics methods course (not a shared course with other subjects).	b. The absence of documentation that shows the competency level of elementary education graduates.	b. The development and dissemination of standards of competence in mathematics for elementary teachers.
c. It is believed that many elementary education majors are not ready for college level mathematics courses.	c. Elementary education students should not take the mathematics methods course before attaining the desired level of mathematics proficiency.	c. An imbalance of influence over elementary education curriculum change, with language arts-social studies types far outweighing math-science types.	c. The development of a program of action by these groups, to be carried out at the state, local, and IHE levels.
	d. Institutions that do not have the will, resources or faculty to carry out these recommendations should lose the authorization to offer elementary education.	d. The unwillingness of mathematics and mathematics education faculty to involve themselves in elementary education issues.	d. Support of the plan of action by the national advocates, through publications, the dissemination of models for documentation of the problem, and suggestions for modes of affecting the decision-making process.
		e. The uninvolvement of state NCTM units and other appropriate advocates in the decision-making process to address this issue.	

A. What is the status of mathematics teacher preparation?

2a. States and colleges need to have a different curriculum for middle school/junior high mathematics education majors than for high school mathematics education majors.

b. At both levels, the weight on content-focus is often too heavy, versus the weight on student-focus.

c. At both levels, vocational applications need to be included.

B. What changes need to be made?

a. Courses in the major for middle school/junior high teachers of mathematics should replace some of the post-calculus "depth" with breadth.

b. After (or while) competence in the major is acquired, research findings from ed., psyc., learning theory, etc., must be discussed and applied in the methods course, and while student teaching.

c. Because rookie teachers are often assigned to the "least desirable" courses and students, they need all the help they can get to relate to these students, whereas more than likely, we have prepared them for high school college-prep classes.

Two examples:
1. "A can do a piece of work in 12 days ..." problems should be replaced by Ohm's Laws for DC parallel circuits;
2. The volume of a cylinder can be discussed in the context of an automobile engine's compression and displacement.

C. What problems are preventing these changes?

a. A belief that this could dilute the quality of the program.

b. Lack of recognition that grades 6-9 have taken the brunt of the mathematics teacher shortage.

c. It's easier for Mohammed to go to the mountain than for the mountain to go to Mohammed.

d. Inadequate number of mathematics educators who could provide the leadership for curriculum modification.

e. Inadequate number of mathematics educators who are knowledgeable about relevant research in education, psychology, and mathematics education.

f. College mathematics and mathematics education faculty are not sensitive to the assignments their new graduates are likely to get, are not knowledgable of vocational applications, or simply choose to avoid these issues.

D. What can be done to overcome these problems?

a. A commitment by NCTM and other appropriate national advocates to address these issues.

b. The development and dissemination of standards and models for middle school/junior high mathematics education curricula and vocational applications.

c. The development of a program of action by NCTM and other appropriate national advocates, to be carried out at the state, local, and IHE levels.

d. Support of the plan of action.

This page is written from a North Carolina perspective. It is presumed that it reflects at least some other states, and would not apply to those states where NCTM affiliates are politically active.

A. What is the status of mathematics teacher preparation?	B. What changes need to be made?	C. What problems are preventing these changes?	D. What can be done to overcome these problems?
3a. State and local NCTM affiliates seldom are involved in the decision-making process affecting teacher education (as opposed to individual members of NCTM and its state-affiliates who may be involved, but don't speak for the organization).	a. State and local NCTM affiliates need to become visible and vocal regarding issues in mathematics education.	a. Affiliates do not have staff members to devote time to this.	a. A commitment by NCTM to develop political action capabilities in its affiliates.
b. The absence of an NCTM voice at the state and local levels allows non-members to use divide-and-conquer and absence-of-information strategies, to the detriment of mathematics education.		b. This effort would require an adjustment to the traditional programs and activities of affiliates.	b. Initial activities can be data collection, analysis, and dissemination.
c. The absence of an NCTM voice at the state and local levels contributes to a feeling of helplessness, and then resentment, on the part of members who are not able to participate in decision-making on professional issues.		c. Mathematics teachers might have to rethink their purposes for affiliating with NEA/AFT units and their expectations for the NCTM affiliates and the NEA/AFT units.	c. Initial participation in the decision-making process can be restricted to position statements, without specifying recommendations.
d. NCTM state and local affiliates are not perceived by decision-makers as organizations that should be contacted for input.		d. Mathematics teachers may find themselves "in competition with" teachers in other fields.	d. Involvement with recommendations can be reactive.
		e. Mathematics teachers may find their ranks split on some issues, between those with appropriate certification and those without appropriate certification.	e. Initiation of recommendations can start with those judged to be least controversial.
			f. Affiliates can work behind the scenes to support leaders and groups which will speak the affiliates' interests and also take the flak.

A. What is the status of mathematics teacher preparation?	B. What changes need to be made?	C. What problems are preventing these changes?	D. What can be done to overcome these problems?
4a. Until B. S. production increases greatly, a substantial number of mathematics teachers are going to receive either their content knowledge or their math methods knowledge <u>after</u> they become teachers of mathematics, rather than before -- or they won't receive this knowledge at all.	a. Teacher education institutions will have to readjust the use of their present mathematics education resources in order to service the market of out-of-field teachers.	a. Decision-makers in teacher education have not seen the need for this extensive involvement.	a. Commitments must be made to bring out-of-field teachers of mathematics up to acceptable levels of mathematics and mathematics education knowledge.
b. This will apply to elementary grades teachers, middle/school junior high teachers, teachers of remedial mathematics, and to a lesser extent, teachers of high school non-college prep mathematics courses.	b. Teacher education institutions which can continue to effectively use their resources in pre-service education may have to get additional resources if they are going to also service the large numbers of out-of-field teachers.	b. Decision-makers in teacher education have not accepted the responsibility for addressing this problem.	b. A documentation of the extent and degree of the problem needs to be done.
	c. School districts and states will have to devote more resources to get out-of-field teachers appropriately certified than they have traditionally devoted to in-service education.	c. Decision-makers in teacher education have not been assertive in seeking and obtaining resources to address this problem.	c. Collaborative decisions need to be made as to which agencies will take responsibility for the training.
	d. School districts and states may lay sanctions and penalties on out-of-field teachers who don't obtain appropriate certification.	d. School districts and states have been reluctant to admit that they have this problem.	d. Dollars need to be made available to conduct the training.
		e. Dollars for expanded in-service programs are reportedly hard to come by (or are simply a low priority) in school district and state budgets.	
		f. Many inappropriately certified teachers of mathematics are not ready for college-level mathematics courses.	

Other Change Issues Relating to the Preparation of Mathematics Teachers

5. The need for mathematics teachers to be able to participate in instructional computing.

6. Decisions regarding trade-offs between the quantity of new mathematics teachers and the quality of new mathematics teachers.

7. The dichotomy between research in mathematics education and other research findings in education and psychology that can be applied to mathematics education.

8. The absence of mathematics educators from the preparation programs of some mathematics teachers, both elementary and secondary.

9. How much (and which) mathematics and mathematics methods is necessary for elementary grade teachers?

10. How much (and which) mathematics and mathematics methods is necessary for teachers of mathematics in middle grades/junior high.

11.

12.

THE PROBLEMS OF CHANGE FROM THE PUBLISHER'S PERSPECTIVE

Vivian Makhmaltchi
Macmillan Publishing Company

I am Executive Editor of Mathematics, Elementary Science, and Music at Macmillan. Macmillan publishes elementary and secondary texts in language arts, mathematics, social studies, and music. Our major series are in reading, English, and mathematics. In mathematics our senior authors are Tina Thoburn and Jack Forbes, who I'm sure have been members of panels similar to this one. We do not publish in the high school mathematics area at this point, but naturally we have some plans.

First of all, since it does seem that generally both publishers and test producing people are in somewhat of an adversary relationship with some of the rest of the folks in this group, I'd like to point out some of our similarities. Almost everybody--at least everybody that I know in school publishing, whether or not they're editors like I am--has been teaching predominantly in the subject area in which they are now working. In my case, this is mathematics. Most of our sales force were once teachers. Certainly our marketing people were teachers or sales people or editors previously. We do have some advantages. We do have frequent professional contact at NCTM meetings. We certainly have all publications available to us. You know we have access to conferences. A large part of our job is to be aware of conferences like this, to be aware of the literature out there, and to be aware of the competition. And we have our own mechanisms within publishing companies for dealing with trends from the grass roots. We have, and I'm sure almost every publishing company has something very similar, what's called an ad hoc committee, made up of three or four sales people and consultants from each region of the country. These people are given specific tasks to do. They go out into the schools. They talk to teachers, supervisors, administrators. And they report back. They are given real homework assignments. We rely very heavily on their information. They are very important to the way we do business.

Most of our authors are people involved in education like yourselves, who have been teachers and who now spend time writing. They have ready access to the classroom, both with our sales people and through their own contacts. So we do have lots of similarities in terms of the people we are dealing with. We also have similar problems. I remember working under lots of the same mandates. In order to sell books, we have to abide by the requirements of states, cities, districts, whatever. We read the proclamations, the curriculum guides, the minimal competency lists, lists of objectives. I sometimes think we spend more time, paper, and so forth on curriculum correlation than on manuscripts. Anyway, we also have a lot of the social problems that all the teachers face. I was thinking the other day, when Steve Willoughby was pointing out the role of teachers as policemen, janitors, and parking lot attendants, that we've got a lot of that same kind of thing.

But ours is a little bit different. We have to deal constantly with all the guidelines for sex, race, and ethnic groups, and with various readabilities such as Dale-Chall and Fry. And whenever we've done one of them, somebody wants another. We have the handicapped, the aged, the single parent to deal with. There was a time when you wouldn't dare show a mother in the kitchen. It had to be a father. We have debates over whether, in fact, we have too many hearing aids. People say, "Well, they don't show any more. And students wear contact lenses." But this is constant with respect to artwork, with respect to story problems. These are the kinds of things by which every word that goes on that page is judged.

There are lots of other things besides pure mathematics. Our lunch today would not have been accepted in one of our textbooks. It contains sugar. I saw those lovely little petits fours. You can't count cookies in a mathematics book any more. California won't let you. We debate whether or not pizza is junk food. Soda, cupcakes, and cookies are good old manipulatives. I sometimes have the feeling when I look at our textbooks and other people's textbooks that the only ones who would enjoy the illustrations would be rabbits. We have gorgeous carrots and broccoli.

I've attended some of the discussions here about the new technology. Of course, we are all very aware of the software. And we've all seen some of the very exciting things that people do with the disks like the adventure games. Well, they're very nice, but we can't have exploding rockets, dungeons, and dragons. We've been told little pictures on the page are too scary. It's a real problem and certainly, by the time you get to junior high school, where you've got pretty sophisticated students, it gets a little rough to deal with the bunnies and ducks. I think we share a lot of the same mathematics concerns. But I wanted to point out some of the other kinds of things that publishers are constantly dealing with.

I guess probably our biggest problems with respect to all of this are judgement, editorial time, authorship time, field testing--and money. All publishers that I know of have some sort of a forum for convincing management that yes, in fact, we should spend somewhere in excess of $10 million to do a mathematics series. And every time you start one, obviously due to inflation and everything else, it ends up being more expensive. There are more parts. Our Series M in mathematics, for example, had at last count 142 different things you can order--and we don't have the most. That's not including possible Spanish translations. It's different teacher resource books, work books, challenges, computer management systems; all that sort of thing. So you have to make a good case to the management of any private company to convince them that you're going to make a profit with the money that you're going to spend to develop all these things. And we don't really have very high profit margins. The school publishing business as a whole doesn't. So you're dealing with a very small piece of profit and you're cutting it closer and closer. That is a very real concern to us. Part of our documents to convince management that we should go ahead and do this marvelous project is something called estimated sales figures. And we have to have some pretty good reasons to believe, by gosh, that when we're done some people are going to buy our books. We at Macmillan, in particular, believe that the teacher in the classroom is our end user. Well, they and the students are our end users. But we do believe that teachers are the people we really have to convince.

We have had some talks today, right before me, about the training that has gone on for, in particular, the primary and the intermediate teachers. And we're very hesitant to use sales figures that indicate that these folks are going to be able to accept a lot of change very quickly. We're basically asking for help from you. We certainly, as mathematics educators ourselves, agree. I have no problem agreeing that a lot of time in elementary school is wasted on paper-and-pencil calculation. But I'm also very nervous about convincing some teachers, those who are already fairly unhappy about mathematics and finally have a few little areas in which they've found some success, to give up these sort of safe and comfortable pencil-and-paper calculations and move into other areas. I'm not saying that it can't be done. But I don't think it can be done overnight. I tend to want to stick some kind of quantitative--10% or 20% in a given year or in a given time period. Just actually go right through the curriculum and assign the kinds of things you can start doing in small steps. I think that is the kind of thing that we would be able to convince our management of. We'd make an appeal to teachers and eventually to supervisors and administrators.

Let's see what else we have to do. One of the other problem areas with respect to publishing in mathematics is that right now everyone is very concerned about software. And software is an expensive proposition. Most major publishers have not made money in software. All of us constantly get proposals and suggestions. We get letters asking, "What are you going to do?" We get the most interest in management systems, for our elementary products basically. We get some for tutorial diagnostic kinds of things and some for computer-assisted instruction of various types. But all of this costs money. We have to decide where the money is going to go.

I think that basically there is a different problem in the high school area. You can effect change a little faster there. The subjects are somewhat more discrete. The production of a high school mathematics series is not as costly. Maybe that's where we ought to start. I don't know. It's hard, there are various opinions. Some people think you should start in high school; some people think you should build up. But basically, if you're going to start in elementary school, we just need some warning. Most of the major publishers with elementary mathematics series out there have large investments in what they already have. We can't just turn it all over at once. But we can work at it. We can take small steps, and we're asking for help along those lines.

I guess I want to quote Steve Willoughby again. I was very impressed with his remark on the beginning night about particular guidelines issued, I believe on problem solving, by some city in the midwest. Steve pointed out that in his opinion three or four texts did meet these guidelines. But when it came down to what the teachers and the school districts would, I guess, adopt, the teachers were much more comfortable with a traditional teacher's edition that was then in place.

You had your objectives, your lesson plan, your manipulative activities, your answers, your extensions, your reinforcements, your diagnosis. And you know, from my very own experience with the letters we get, most letters say, "On page 57, exercise 37 has the wrong answer." I've never yet gotten a letter on the philosophy of a mathematics program from the teachers. The worst mistake we ever made was that our sixth-grade book had three pages printed in the teacher's edition without the red answers. Boy, did we get letters. That's the kind of concern that is evidenced by the teachers, to us at least. And if it should be something else, we need teacher help in getting some proof that it can be done.

I guess this is a plea for help. If the texts are going to change, we need some time. We need time because we have to plan them. First of all you folks have to tell us what we should be planning. We've got to plan them; we've got to test them. We've got to write, rewrite, edit, and specify all the art and copy that will meet all those other guidelines I mentioned. Then we've got to check them. And even when that's all done, it still takes six months just to physically produce the materials. So we'll need some firm guidelines that show some promise of being implemented. I do think that both this meeting, and I really am very interested in it, and the meeting that was held at the NCTM in Detroit--and there may have been some others where you have asked representatives of the publishing community to participate in some of your deliberations--are really helpful. At least let us get a head start. Most of us, as I said before are educators. Most of us are also parents. I have two students in the New York City public school system, and that's an education in itself.

We want to improve our books, and we want to improve education. We've still got to sell books. That's the bottom line for us. So I hope we can get on with it and all work together.

THE PROBLEMS OF CHANGE FROM THE TEST DEVELOPER'S PERSPECTIVE

Chancey O. Jones
Educational Testing Service

I am pleased to have been invited to participate in this conference. It seems eminently appropriate for someone who is involved in testing to be a part of a conference on curriculum since education and testing go hand in hand. One of the catalysts, in fact, for the recent national concern about the plight of secondary school education was the Report of the Advisory Panel on the Scholastic Aptitude Test Score Decline entitled <u>On Further Examination</u>. This 1977 report was the culmination of an intensive two-year investigation into a decline of S... scores for more than a decade. The panel found the decline to be a complex phenomenon, yielding neither simple explanations nor easy solutions. Since this report, at least a dozen projects or studies concerned with secondary education and funded at a level of one million or more dollars have been undertaken. The findings and recommendations of several of these projects are the stimulus for this conference.

Test scores have helped to identify the nature and extent of the current problems in secondary education. It is likely that such data will be used in the future to judge the extent to which actions taken and programs implemented in response to these national studies and projects have been successful in overcoming and solving the current problems.

Thus it is essential that the tests and questions used to gauge student performance adequately reflect the outcomes as well as the content of the secondary school mathematics curriculum. This will occur, however, only if test specifications are based on a curriculum that adequately mirrors the aspirations of the nation for a program of excellence in precollege education in mathematics that is accessible to all schools that desire to implement such a program.

The setting of test specifications--content, ability and, to a lesser extent, statistical--is therefore the most significant task faced by the test developer. It is vital that the decision-making process by which test specifications are developed rest primarily with those who are best qualified to make such important decisions: professionals who are trained and experienced in mathematics and mathematics education. This is not to say that others should not provide input, but it is reasonable to expect that the primary responsibility for establishing content specifications should be entrusted to college and secondary school mathematics teachers and to persons who apply their mathematical training such as engineers, computer scientists, physicists, and applied mathematicians.

One challenge that must be met by test developers is the selection of professionals to be involved with them in the test development process. It is important that individuals selected possess the training, experience, and ability to interact with others that will allow them to contribute to the process. Perhaps as important is the need to balance the membership of a six- or seven-person national committee to represent the following: colleges and secondary schools, courses or grade levels taught, public and private sector, gender, regional and ethnic groups, and appropriate professional organizations. Such committees have tremendous responsibilities: they establish or update specifications, write and review questions, and review and approve final tests. It is a demanding challenge to attempt to assemble a committee that will have the necessary balance and the ability and insight to ascertain with accuracy which mathematics programs are being taught and the extent to which any given program has been adopted.

A key problem for the test developer and test committees is to determine when a new curriculum or program has been implemented to such an extent that test specifications should be changed. It is one thing for professional organizations or curriculum reform groups to recommend curriculum changes; it is another matter to convince school boards and educators to adopt and implement the changes. Therefore one of the greatest challenges that face the test developer is to determine when the time is right for a change in content specifications. Test makers do not wish to impose a particular curriculum on the secondary schools, nor do they wish to impede change in curriculum. There is always the question of whether to let the tests evolve gradually from current specifications as topics are added to or deleted from the curriculum, or whether it is necessary to make a radical change from the established specifications.

An example of a significant change in mathematics tests and test specifications took place in the late 1960s when the College Board replaced the old Intermediate and Advanced Mathematics tests with the current Level I and Level II exams. This change resulted from recommendations made by the Commission on Mathematics and the implementation of "modern" mathematics materials written in the early 1960s by various curriculum reform groups.

When such a major change is made in the specifications, it is usually necessary to take the time to conduct studies that will help to ensure that scores on the new tests will be comparable to scores on the previous tests. This is essential if the scores of students who have taken the different tests are likely to be compared. The test makers do not want to have a student either advantaged or disadvantaged by having taken one test or another.

On the other hand, test specifications may evolve over time without such concerns, provided only minor modifications are made in the specifications at any one time. Such was the case in 1978 when the specifications for the Mathematics Level I examination were revised to decrease the amount and coverage of trigonometry. However, even in this case, all existing forms of the Level I test that were to be used in the future were revised to reflect that change. Also there would have been more concern if the Committee had wished to add topics to the content coverage.

Another problem facing the test developer is the one- to two-year lead time necessary in the test development process. It is therefore vital that committees keep abreast of changes and accurately anticipate future directions in the curriculum so that they can recommend changes to the test makers early enough to allow for the test development and production time and to communicate expected changes to the educational community. A recent change that illustrates this principle is the decision to allow the use of hand-held calculators on the Advanced Placement Calculus examinations. Over a period of several years students and teachers were surveyed regarding the use of calculators in taking tests. Once the decision was made, attempts were made to communicate to schools and teachers that calculators would be allowed, but not required, beginning in a particular year. Now the committee has become increasingly concerned that, because of the rapid change in technology, students who cannot afford sophisticated calculators may be considerably disadvantaged in a few years. Consequently the committee is recommending that beginning in 1985 the use of calculators no longer be allowed. Because the tests for 1984 have already been constructed and the descriptive materials indicate that calculators can be used on those tests, the policy could not be changed for the 1984 examinations.

For the seventeen years that I have worked with College Board committees concerned with the development of the total array of mathematics test offerings, it has always been the case that each committee has first wished to determine what content is being taught and, secondly, from time to time to introduce topics into the tests that they have thought should be taught. Such topics have been introduced only with the approval of the particular Development Committee and, since 1974, with the approval of the Mathematical Sciences Advisory Committee as well.

Currently there are more than 50 persons either serving on the various Mathematics Committees of the Board or interacting with Educational Testing Service Test Development staff about Board tests. The dedication and professional commitment with which these persons have served over the years, as well as their willingness to take on difficult and challenging projects, leads me to believe that, whatever the outcomes of the current national interest and concern about secondary education and its impact on mathematics preparation, the Board's Committees will advise it well about the future configuration of its mathematics test offerings.

Since 1980 the Mathematical Sciences Advisory Committee has been deeply involved with the Board's Educational EQuality Project, a 10-year effort of the Board to encourage the strengthening of the academic quality of secondary education and to foster the equality of access to post-secondary education for all students. The Advisory Committee first refined the mathematics competencies and then prepared the outcomes for basic academic preparation in mathematics. These statements are contained in Academic Preparation for College: What Students Need To Know and Be Able To Do. (Jeremy Kilpatrick was one of the major authors of the Mathematics Outcomes, and Dorothy Strong is on the Council of Academic Affairs which has overall responsibility for the EQuality Project.) Jeremy and I will be leaving here to go to a meeting of the Advisory Committee to discuss strategies and means by which the recommendations contained in this report can be implemented. Certainly the deliberations of this Conference as they relate to the Advisory Committee's agenda, as well as the forthcoming Conference Report, will be conveyed to that Committee.

The 1955 Commission on Mathematics and the 1980 Education EQuality Project are two examples of the College Board's attempts to help strengthen curriculum and maintain high academic standards. I am confident that the overlap of concerns and interests that exist between those involved in mathematics education and the Board and ETS will continue to provide grounds for productive dialogues and interactions that will help to maintain strong mathematical curriculums and high academic standards. Such collaboration is likely to lead to the progressive updating of test specifications in order to keep national tests appropriate to the desired content and outcomes established by mathematicians and mathematics educators.

In summary, there are four major concerns for the test developer when significant curriculum changes are implemented.

(1) Surveys and studies may need to be conducted to support the need for changes in test specifications or to ensure comparability of scores on different tests.

(2) Committees of professionals will need to be appointed to provide advice.

(3) New test specifications will need to be established.

(4) The educational community will need to be informed in advance about any significant changes that are to be made.

In closing I would like to reiterate that the need for concern for educational quality must also be expressed in a commitment to quality for all students. In this vein I am hopeful that this Conference will give some attention to the need for providing a means by which students who enter secondary school with deficiencies in their mathematics preparation can be afforded the opportunity in secondary school to make up the deficiencies and to complete the training necessary for college entrance, without having to do remedial work at the college level.

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THE PROBLEMS OF CHANGE FROM THE MATERIALS PRODUCER'S PERSPECTIVE

William L. Barclay, III
Technical Education Research Centers

TERC is a non-profit research and development organization focusing on using microcomputers in education. Our activities have fallen into three general categories: workshops for teachers, software development, and work supported by federal grants. One of those grants was from the U.S. Department of Education to do a survey of mathematics and science software at the pre-college level. This involved finding what software is available and also interviewing teachers and developers personally and by questionnaire to ask what software they were using, what kinds of uses of microcomputers they were emphasizing, and what sorts of software and use did they wish for the future. Some of the findings from this survey are of interest here. We located 1,873 different software packages, more in mathematics than in science. We estimated that this was probably better than 80 percent of the commercial software in existence at that time--spring of 1983.

The distribution by age level was interesting: 78 percent of the mathematics software is for grades K-6, while in science there is very little at the elementary level. A breakdown of the elementary mathematics software showed that slightly more than 50 percent of it deals with adding, subtraction, multiplying, and dividing--drill and practice on the four arithmetic operations. Another 20 percent is drill and practice disguised in a game format, and another 19 percent is tutorial (but often what is advertised as tutorial software is really mostly drill and practice also). That means 92 percent of all the elementary level mathematics software falls into a category which we are calling explicit software. By explicit we mean that all the decisions about what is going to happen are made in advance by the developer: what is to be learned, what are the right answers, what kind of help will be given, and what feedback there will be. This is in contrast to implicit software where the user learns within the context of the activity, by seeing the consequences of his or her actions, rather than having to be told by the program. All drill and practice and tutorial programs are examples of explicit software; Darts (now available as Fractions or as Decimals from Control Data) and Green Globs (on Graphing Equations from CONDUIT) are good examples of implicit software.

We were not able to sample all of the software, but we did read reviews where they were available, and we have a library of software of our own, so we were already familiar with some of it. But a problem is that the descriptions of software, and sometimes the reviews, are often much more positive than our own view. It was not our task within the survey to rate the software, but let me give you a list of my own current favorites.

- SemCALC by Judah Schwartz, available from Sunburst.

- Darts, an old Plato program now available from Control Data.

- Graphing Equations by Sharon Dugdale, available from CONDUIT.

- Various graphing utilities such as CactusPlot from Cactus Software.

Beyond these, there are a number of developmental projects which are working on good software.

An important aspect of this list of favorites is that they all have come from projects which had extensive outside support, either from foundations or the federal government. Software that includes implicit learning, is most interactive, and has the most flexibility also tends to be the most difficult to produce. LOGO (Terrapin, Inc.) is another good example of exciting software that was dependent upon federal support for its development. Rocky's Boots (The Learning Company) is another example. By and large, however, developing software is a cottage industry. We identified 160 different developers and vendors, and we found that there are about 100 new titles coming on to the market every month, twice the rate of two years ago.

This cottage industry aspect of software development is both a strength and a weakness. It's strength lies in the diversity of input that comes without the domination of large companies, as is more the case in the textbook field. It's weakness is seen in the predominance of explicit software. The reason must be because such software is much easier to write--the danger is that we will fail to use the micro in the classroom for more than this workbook-type task.

It is interesting to look at teachers' wish lists we got in our survey. There was a strong desire for individualized instructional material--which usually meant drill and practice and tutorial type programs. There was also a desire for better motivational material, although it was not clear what was often meant by this--math in an arcade game format? Need for new curriculum ideas was also stated, especially in the areas of problem solving and applied mathematics. Finally, about a quarter of the respondees mentioned the need for more support: funding from their schools and other agencies for equipment, space, software, and training.

There are a lot of problems which any developer faces these days, however. One is the issue of which machine. At the moment the Apple is the dominant educational microcomputer, but there are real questions about the impact of some of the new entries into the field, such as the Commodore-64 and IBM, and of other micros which are in the wings. Bigger software developers are tending to make versions for several machines; smaller, one-person shops often write for only one machine.

Another problem is marketing. In the commercial market you need to make a product that will sell. Many of the software ideas which I would give the highest ratings to are unfamiliar to teachers. Without exposure to such products, many teachers do not have a sense of the exciting potential that the microcomputer can offer. This forms a depressingly negative loop--teachers often think of the micro as a remedial, drill and practice aid, so they buy software of this type, which is a reason for developers to make more of the same.

Finally, there is the question of the role which the textbook publishers are going to play in the software marketplace. The publishers are all starting to get into software, and although in many cases they are still feeling their way about what exactly they should do, they all seem to feel that it is important to do something. An example of the pressure which is pushing them to act is the Texas Approved Textbook List. It is now a requirement that any mathematics text series have supporting software. As a result, publishers are developing software packages which are keyed to their texts and typically include extensive classroom management systems. This does not seem to me to be a direction which will exploit some of the exciting and imaginative ways in which microcomputers can be used to enhance learning. But the publishing houses, with their extensive marketing capabilities, are in a position to overwhalm the market and make it increasingly difficult for innovative software to find a place in the classroom. On the positive side, we have been giving workshops for publishers, and there is a real sense of openness to exploring just what is possible and what directions they should be pursuing.

Perhaps the most encouraging factor in this whole area of using microcomputers in the mathematics class is the students. We are fast approaching the day when teachers will be working with classes of computer literate children. What are the implications for the mathematics teacher at the junior high level when all the students will have already had two, or three, or even more, years of Logo? This is an important force that we need to recognize and capitalize on. Then there are teachers who are excited, and they are creating a ferment for innovative and creative uses of the microcomputer for learning and teaching. We have seen reflections of that excitement here in this conference, and surely our mandate is to map ways which capitalize upon and increase just such a feeling in all our mathematics classrooms.

THE PROBLEMS OF CHANGE FROM THE MATHEMATICS SUPERVISOR'S PERSPECTIVE

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Mathematics supervisors in state education agencies are becoming extinct, with fewer than 35 states claiming mathematics specialists per se. Many who formerly held these positions have been reassigned to a Basic Skills, Chapter I, or testing program within state offices. Others, although they carry the title of mathematics specialist, have add-on responsibilities in such areas as science or microcomputer education.

By looking at the way state education departments are funded, we begin to see why the shifts have been made. About 80 percent of the state mathematics supervisors positions are federally funded. When federal money is not available or is reallocated within the state, the state level position is dropped or the person is reassigned so that their responsibilities include those that can be federally funded in another program and mathematics. This is, budget rather than need seems to determine positions. Unfortunately, the public is told that a number of state workers have been cut and are given the mistaken impression that state tax dollars are being saved.

One of the groups at this meeting has recommended a mathematics curriculum specialist in each school. I am the only mathematics supervisor at the state level and, furthermore, the only full-time mathematics curriculum director in South Dakota. Not one of the 196 school districts in the state has a full-time mathematics curriculum specialist. In fact, fewer than 10 schools have full-time general curriculum directors in the K-12 systems. So, if schools are to have mathematics curriculum specialists, we have at least 196 openings in South Dakota.

Some details are available about mathematics teachers in South Dakota. Of those teaching, 45 percent have majored in mathematics; 52 percent are teaching outside their major and have eighteen hours of academic preparation in mathematics; and 1 percent have provisional certification. A provisional certificate in South Dakota means the teacher doesn't have eighteen hours or is not recertifiable in the state. In our state, like many of our neighboring states, about half of the mathematics teachers do not have a major in mathematics.

How do these people get certified? What are the stages that they go through? In our state, and many of the states in the nation, the teacher training institutions work with the state teacher certification offices to get their programs approved. Once the program is approved by the state, students who complete this program are automatically certifiable. The program is approved through a visitation team that reviews and verifies a self-study done by the teacher training institution to see how the standards identified by the state certification office are being met. The state certification office identifies standards from those listed by the Association of State Teacher Education Certification Offices.

In a college we reviewed recently, mathematics majors were required to complete a one-credit methods course, a five-credit education block including student teaching, and one week of sophomore experience. They technically met the standards, but they were very weak. For elementary teachers it's quite typical to have a three-credit content course in mathematics and a three-credit mathematics and science methods course combined. As we were talking to the people who were teaching these courses, we began to ask an idle question, "How many of your staff happen to have had experiences in local school education?" We found about half in the major field and the education department combined; the education department alone was much, much higher. I don't have national statistics, but I think it's something we must look at as we consider recommendations from this meeting.

In most states teachers are certified to teach all subjects and all grades K-8. People who are subject area specialists are being taken from the junior high school to teach in grades 9-12. Then that junior high position is being filled by someone certified to teach K-8. Recall that these are the people required to take <u>one</u> content course and a methods course shared with science. With our former college entrance requirement of one year of mathematics, this same person may have had only one high school course.

Now, when it comes to courses with titles like consumer mathematics, certification goes up for grabs. I asked colleagues in the certification offices who would be certified to teach business mathematics or consumer mathematics for homemakers. And I thought that I would really make it far out and asked, "What about mathematics in music? Who would be certified to teach this course?" The answer I received was, "Either person!" That is, the mathematics in music could be taught by either the mathematics department or the music department. The business mathematics by either a business major or a mathematics major. Consumer mathematics, the same.

Where states really vary is in requirements for recertification (continuing certification). The time before a teaching certificate expires varies from state to state. One neighboring state said three hours of coursework are required after three years of teaching. There are some states that require none. Typically what is required is about 10 semester hours of coursework within 10 years of teaching. When the question is raised about acceptable coursework for recertification, there is no uniform response. Usually as long as the course is broadly related to education or to areas being taught, the credits are accepted for recertification. Another way of keeping up-to-date is through inservice work. We are fortunate that in our state inservice days are required to be a part of the school calendar. What we have done in South Dakota is to divide the state into 10 regions and one school within each region sponsors an inservice day. Typically, the day is planned by a committee of teachers and administrators from within the region working with a member of the state education agency. This regional inservice day has become a powerful informational tool in terms of sharing changes in content and in methods for instruction. Another benefit comes from the dialogue established between school district teachers and college and university level instructors.

This same dialogue is an important feature of another program that we call Summer-in-Depth Grants. Some of the federal monies are set aside, and schools write grants for summer work they'd like to do in curriculum development. This will range from one to two weeks. Personnel from the state education agency and/or from colleges and universities go to work with the staff from the local district. What happened in one of the projects, for example, was a review of the total mathematics curriculum in grades two, three, and four. They have redefined their entire content in terms of strands very much like the NCSM 10 basic skill areas. Now this school is looking at a unified second, third, and fourth grade, and teachers have to learn how to teach in a setting other than a self-contained classroom. The teachers requested help from one of our state universities and have set up coursework that would address their particular need of restructuring a classroom and setting up a new management system. And the plan goes on. This school has already started a plan to work next summer on materials to enrich the curriculum. The point that I want to make here is that it is possible for higher education to work with a single school district in restructuring the current program in mathematics.

There's something I have not seen research on but would like to find out more about. I am thoroughly convinced that teachers go through developmental stages as they learn how to teach. I am certain that we start off at a very concrete level as teachers and develop to a formalized or an abstract level. When we examine our inservice program or the work we do with teachers, we try to talk in a formal style to someone who is operating at the concrete level.

Most of our neighboring states espouse local autonomy in the area of curriculum. The states have guidelines developed in state education agencies, but <u>the</u> curriculum is developed at the local level. Most states have a cycle of reviewing their guidelines; our particular cycle happens to be every five years. We have some subject area developing guidelines each year so that not all guides come out at once.

Now, in terms of high school graduation requirements, we have just gone from a one-year to a two-year mathematics requirement. Our joint boards have decided that they would like to have more information. Instead of a time requirement, what should be included? This question has resulted in a set of accreditation hearings. I must admit that it is no coincidence that the mathematics requirements parallel those recommended by NCSM.

To change what is required in South Dakota, there are several boards to go to. To change college requirements for students, one goes through the Board of Regents. To change high school graduation requirements, one goes through the State Board of Education. To change requirements for entrance to technical schools, there is a Vocational Education Board. A single board doesn't determine what students need to enter the next level of education.

One other thing I would like to mention briefly occurred last July when we reviewed the papers submitted by the nominees for the Presidential Award for Excellence in Science and Mathematics Teaching. It was interesting to read the responses to what the critical issues and problems are in the teaching of mathematics. Some named a need for curriculum K-12. Others mentioned they needed microcomputers. Some of the others said they needed to learn more about mathematics in general. One of the problems that came up very often was student absenteeism. In fact, one person mentioned that they had a student who was absent 22 times in a nine-week period for one school activity or another. The student was in school every day so their report card would not show that they were gone from algebra class 22 days. That school system is now reporting the number of days absent from school and the absences from each class.

An interesting idea about how change occurs was written up in the November issue of Educational Leadership. The article suggests that change doesn't have to come, as other research is saying, from getting the users involved in developing the curriculum or making materials. Change does occur by getting administrators involved and letting teachers see that a program will work. Teachers will buy into a credible program even though they have not developed it. In fact, this kind of program has more of an inclination to be institutionalized than some other programs developed locally.

One thing I see as the most important part of my office is to encourage teacher leadership development. Most teachers need to hear from somebody who is perceived to be in a higher position than they are that they are doing a good job and that their ideas are worthwhile. That is why I like to get teachers on committees for conference planning or involved in something, be it no more than driving a car from one place to another. I hope that I'm saying to that person, "You are important enough to help. I want you as part of the team."

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THE PROBLEMS OF CHANGE FROM THE SCHOOL ADMINISTRATOR'S PERSPECTIVE

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I've taught mathematics in both public and private schools and at the elementary, the middle school, and the high school level in three different states--New Hampshire, Massachusetts, and Minnesota. I have also been a curriculum consultant in mathematics and computer applications with the San Diego County Department of Education, and later was the Director of Planning, Research, and Evaluation there. Currently, I'm the Assistant Superintendent for Educational Services and Programs.

I will not talk about problems that face mathematics education today, but I will talk about challenges. As school administrators attempt to influence school mathematics K-12, we have some very real challenges facing us. But there are also wonderful opportunities available to us. Those of you who know me know I tend to look at the world in a positive way. I think each and every one of us can make a difference as long as we believe that we can make a difference. Sometimes you make a difference by providing and allowing the opportunity for others to do that. That's the way that I interpret my role as Assistant Superintendent in the school district. There's a need in my role to be the person who climbs the flagpole and looks down at things to get the big picture, to get a better perspective on what's happening. As a result of that, some materials that are not directly related to mathematics education are materials that affect what I do.

There were three major works this last year that have influenced activities in school districts in the country. One was A Nation At Risk (National Commission on Excellence in Education, 1982) with its description of the status of schooling in this country. The retoric that they used, "a rising tide of mediocrity," has certainly been quoted widely and had an impact. One other was Megatrends, written by John Naisbitt (1982). He described the United States as having reached the point in schooling that youngsters who graduate now are less educated than their parents. That had an impact on us. He identified California as one of the bellweather states. You look to see what is happening in a bellweather state because what happens there has a potential of happening in the rest of the country. For example, California was the first state to pass tax reform legislation--Proposition 13--and what happened after that? Tax reform went all the way to Massachusetts where they passed Proposition 2-1/2. We heard additional evidence of California's being a bellweather state from Vivian Makhmaltchi this morning when she said, "We cannot put certain pictures in our textbooks because we aren't allowed, if we do, to sell them in California." She's right about that. Our curriculum commission will not approve textbooks that have pictures of junk food.

The third report or book that influenced schools was In Search of Excellence by Thomas Peters and Robert Waterman (1983). In that book, they examined the major characteristics of successful companies in the private sector. In the private sector the bottom line is profit. In the schools, we don't have a bottom line of profit, but we certainly do have a product and we do have a clientele that we need to serve. In their review of successful organizations, they looked at the chief executive officer and the influence that that one individual had on the company. The individual at the top sets the culture and determines the shared value system of the organization. This observation has tremendous impact for the perception of the role of the principal in a school or the superintendent in a district. Another characteristic they found was that successful companies paid attention to their client group. And in the school business, who is our client group? It's the taxpayers, the community. The fast response you might want to give is "the students" because we provide services to the students. But they're not our client. Our client group is the taxpayer, the community, the parents of those children. The product we are preparing for them are students.

What happens as a result of reports like In Search of Excellence, Megatrends, and A Nation At Risk is that states and school districts begin to respond with reform legislation. In California a few years ago on the finance front the reform legislation was Proposition 13. On the education front this year, it's Senate Bill 813. For example, Senate Bill 813, for the first time since the late 1960s, has set state graduation requirements. The graduating class of 1987 in all districts will be required to have two years of mathematics and two years of science. In addition to the graduation requirements, the State Board of Education developed a model curriculum and identified course outlines. The model curriculum of the State Department of Education, which is not mandatory, requires three years of mathematics in an algebra-geometry oriented sequence. The reform legislation also relates teacher evaluation to grade level achievement. In addition, clinical supervision is identified as the method for teacher evaluation, and school boards have to certify that their administrators are all competent to utilize the clinical supervision model in evaluation of teachers. The reform legislation also provides, although it was not funded, for those school districts that raise test scores in the California Assessment Program to be rewarded financially. The California Assessment Program is based on the objectives of the California state curriculum frameworks, and test items are based on the objectives in the frameworks. It tends to be a criterion referenced test but does not give any individual student data. It is a matrix sampling of test items at grades 3, 6, and 12.

Other current reforms include a mechanism to raise beginning teachers' salaries to $18,000. A five-period day is also required at the high school level. One of the cutbacks as a result of Proposition 13 was to a four-period day at some high schools, especially for seniors. Another reform measure is the mentor teacher program that provides an additional $4,000 stipend for up to 5 percent of the certified teaching staff in the district. They have to be involved 60 percent of the time in direct instruction with students; the other 40 percent of the time can be used for curriculum development work, for peer observation, for improving the mathematics program, or any other area. I share all of this information because, if Proposition 13 experience is any indication, some of the other states might begin to move in this same direction. California may also be a bellweather state for educational reform measures.

Change in school programs is possible, and I want to illustrate by telling you how we implemented computers into the program at Walnut Valley. Walnut Valley Unified School District is a K-12 school district in the greater Los Angeles area. We have 8,500 students, two comprehensive high schools with 1500-1600 students in each, two middle schools grades 6-8 with about 800-900 each, and seven elementary schools. Computers for instructional purposes first started to be used by the mathematics department, which is not unusual. The equipment that we have at one comprehensive high school includes 30 Commodores. These are owned by the Regional Occupational Program (ROP). We use them during the day, and ROP uses them in the later afternoon and on Saturdays. It is mutually beneficial. We were fortunate enough--ETS had the good judgment to select us--to be one of the sites awarded 15 IBM PCs with peripheral equipment on a competitive basis. At the other high school, we have a classroom set of Apples that we bought with district money and an additional laboratory of Apples that were purchased with a computer-based business education grant that the state awarded on a competitive basis. These computers are used daily for instruction. The Commodores, for example, are scheduled every period of the day. The IBMs are not. So, someone teaching a calculus course who wants to use those IBM PCs third period for two weeks can schedule them.

In the Walnut Valley Unified School District our Superintendent, Dr. David Brown, has a computer on his desk that he uses daily. He models the behavior. We figured if we really want to move the elementary schools to the uses of computers, the elementary teachers had to see the administrators using and modeling the use of computers. Last year every elementary principal was given a computer for administrative purposes. We provided workshops to train them in word processing and in the use of Visicalc as an electronic spread sheet. We did have some complaints about spending money on computers from the elementary principals because they didn't see that they'd ever have a use for them. Those same people this year are "born again" converts to the uses of computers. We are getting requests for that second computer. What's happening at the elementary level now is greater interest from the elementary teaching staff.

We involved teachers last year in a curriculum development effort K-12 to develop our scope and sequence. For the K-6 staff, we identified the K-6 objectives and instructional activities matched with those objectives. We also cross referenced those onto the existing curriculum. At the elementary level, we cannot present computer science or computer applications as an "add on," so we cross referenced to reading, language arts, mathematics, the arts, and so on. We identified what we thought were cognitive prerequisites to learning with computers, such as learning to follow directions. Some of the activities in the kindergarten and first grade classrooms very naturally are matched to that notion of teaching children to follow directions. In the game

Simon Says, what are the children learning? They're learning how to follow very exact directions--a prerequisite skill for computer programming. We also matched activities of that type that we considered prerequisites to working with computers. Because of the administrative involvement, we're getting much more commitment and, we think, at a much faster rate than we would have had if we had used some other kind of strategy. Our superintendent set the tone for the use of computers with his own use. The "culture" became computer oriented.

California's curriculum framework in mathematics was developed by the leaders in mathematics education in the state. We're really very fortunate. Do you remember the old Madison Project? Bob Davis certainly remembers the Madison Project. It had tremendous influence and still does in California. The Madison Project was funded through legislation that provided Miller/Unruh mathematics teachers who were specialist teachers able to provide inservice, demonstrate lessons, and function as lead teachers in mathematics. Where are they today? They are officers of the California Mathematics Council; they are in mathematics curriculum positions in some of the largest districts in California and in mathematics supervisor positions in county offices. They've infiltrated the entire power structure in terms of influencing mathematics education in California. We have very fine talented trained people in mathematics because of that earlier legislation.

Problem solving is the basis for the curriculum framework in mathematics. As I said, we are used to having extremely good people and insightful people working on the framework. Who do you think we would use as a consultant to help us plan for problem solving? We had George Polya come to our meetings about four years ago when we were putting together the materials. He was almost blind and had a very frail body. We were nervous about his being there. But when he sat down and starting talking, it was pure magic. It was an unbelievable, magnificent experience. The man was very bright, very articulate. He was able to explain and illustrate his problem-solving strategies.

In addition to the framework and the testing system that's matched back to the framework, there is a handbook for planning and for implementing a quality mathematics program. In looking at change in a mathematics program, we look at three areas: the content, the methods, and the support for implementation of a quality program.

The content for a mathematics program must include the language of mathematics--how children talk about mathematics, how they use the terminology appropriately. It also addresses the comprehensive mathematics curriculum. Does it have gaps? Does it have variety? Does it include computing as well as the problem-solving skills. From my vantage point, the major challenges are in the textbooks and in the curriculum materials. I really applaud the effort of meetings like this to attempt to bring together representatives of the publishing industry, the testing industry, the curriculum developers, and the supervisors. Because if we are really going to make a change in the content as it's represented in the curriculum materials and textbooks and the standardized tests, a mutually collaborative effort among all of the parties affected is required. I know we shouldn't be spending all that time on the long division algorithm, but that content still appears in textbooks and on tests. I probably get asked about our test scores more by real estate sales persons than by anybody else. When people are moving into district, they want to know. We need a better match between culturally relevant content and curriculum materials and tests.

Attention to methods must include providing for learning styles and attitudes as well as calculators and computers. What we need there, of course, is the equipment, and the constraints are largely money. However, we still need research. I think we aren't really clear on the cognitive consequences of learning with computers. We need a much better developed research base in that area to support what we're doing with computers. Our challenges here also include a need to be working on the attitude and community expectations with the use of the calculator. Calculators are more prevalent in calculus and chemistry classes than they are in a consumer mathematics classroom. There's still a real hesitancy to use calculators for general mathematics or the seventh- and eighth-grade mathematics curriculum. It doesn't make sense. That attitude has to be changed. The PRISM (NCTM, 1981) data told us there was a much higher acceptance of computers than of calculators. We still have that same attitude in many communities about calculator use in general.

In developing support for a quality mathematics program, the areas in the handbook include school climate and staff development. There it's a problem of the teacher's role, the school day, and the school year, particularly at the elementary level. Reading is sacred in the elementary curriculum. There is very little time left after that. We must consider both the quality of time and also the amount of time that we have in the school day and the school year and work toward increasing both.

Changing--improving--school mathematics programs is possible with cooperation among the practitioners, the curriculum developers, test constructors, and teacher educators. It will require revising of the content of school mathematics, improving methodologies used, and generating support for the quality and amount of time spent in learning mathematics. It's an exciting challenge in which each one of us has a critical role.

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DISCUSSION

The discussion that followed the six presentations on impediments to change was extremely varied and was less clearly focused than the previous discussions. Nevertheless, four themes emerged in the discussion:

1) inertia and professional risk taking,

2) community/parent response,

3) testing, and

4) quality software.

Inertia and Professional Risk Taking

Bringing about change is a serious problem. Each one waits for others. The test developers say, "We don't want to get too far out ahead because it's not fair to test students on something they haven't been taught." Textbook publishers say, "We don't want to write great books that nobody's going to buy because we'd go out of business and we wouldn't be able to make incremental improvements." And the teachers say, "We can't teach what isn't in the textbook." And therefore, change does not occur. It requires somebody with the guts to get out and lead.

I have been to more than one meeting in the last year where a lot of people within the mathematics and the mathematics education community have said pretty much what you've said, but they've not been willing to take the risk. We've been timid souls.

If we start blaming people, let me tell you who really should get the blame. Much inertia is caused by mathematics educators. If mathematics educators did not agree to put their names on textbooks, maybe they would change. The mathematics education community has been afraid for the last decade to take any risk. There was a time as mathematics educators when we looked at a situation and said, "If it's good for students, we're going to do it." Now we want to protect ourselves the same way textbook publishers want to protect their money. We have to take the risk.

I see four inert forces that are going to make it very very difficult to implement anything. The first is test publishers. The second is textbook publishers. The third one is text review and selection committees in states. And the fourth, and probably the most critical one, is the teachers themselves. All of these forces tend to keep things going in the directions they are currently going and to prevent change.

Sometimes we make platitudinous observations. We can get depressed if we look at the whole U.S. system as a totality and say nothing ever really changes. In fact if you view it as 50 separate state systems and 16,000 local systems, change then is possible. Change will only occur within local systems.

Reliance on individuals is too simplistic. Some individuals can make things happen within their districts. Although new curriculum is needed, in the long run we need committees and administrators committed to change.

I'm an administrator in one of these school districts. I want to report from a group of professional people who know the field very well, spent years of research involved in it, and have some ideas about the direction that this nation ought to go in the area of mathematics education. Don't worry about what the parents are going to say; might be good to stir them up a little bit. Do not be concerned with what other supervisors are going to say; it might stir them up. I need a crutch. If I believe in the report then I can argue that this is what we believe and this is where we think the nation ought to be going in terms of mathematics education. The time is right. I am much more influential in my district if I can say, "The National Council of Teachers of Mathematics, or The Madison Group, or this body of experts has recommended. . . . It's not just something that I invented because I think it's a good thing for the students. It's something that came from a group of learned people together, collectively." It's a very helpful crutch.

I don't think of these as inert forces, I think of these as driving forces or restraining forces. At the moment, they tend to be restraining forces. But, they produce change. For example, you do not see any junk food in the textbooks today because California said no. If you tell publishers, "Hey, this stuff is sexist. We're not going to buy it," they suddenly become believers. And teachers have a big effect in terms of selection of texts, and so on. I think if we can train our teachers to demand better things, we'll get it.

Community/Parent Response

I think we're still dealing with a bandaid. I think that one of the things we've got to do is to get the people's attention. Then we have to have the courage to hold it when we get it. I suspect that it is going to be very critical to move the nonmovers. Somebody has to move them. Right now it's too expensive to write a good textbook. We can create the atmosphere in which it will be too expensive not to write good textbooks. That's the kind of influence we need. We've got to get the support from a community. Once the parents say, "Hey, it's our fault," we have won. We need to say to society that the reason we are not educating is that you don't want us to educate.

A lot of times we forget that about 60% of the public now does not have children in the schools. Often criticism comes from those who do not have children in the school system and do not know what is going on.

They often are operating under false information as they make their criticisms. How do we deal with a public and keep them informed when they do not have children in the schools?

Part of the general public we want to include happens to be state legislators. And I don't know what's happening in your state, but I can assure you that if we don't do something in mathematics education, the state legislatures are going to do it for us. And the kind of legislation I see being written in many states now scares me. They are not well thought out bills. For the most part, they seem designed to get legislators some attention rather than to solve our problems. That makes them detrimental to society.

Certainly, there are parents who say, "Do not use handheld calculators." Parents often are restraining forces. Part of the problem is competition. What competition does is produce an effect like primetime TV. It is very hard to get a good show on primetime TV, not only because it's hard to do a good show, but because there's so much competition from stuff that isn't good. This is true with textbooks, tests, and software. The good is competing with a lot of trash.

Testing

I think that testing is the toughest nut to crack. The cycle for revision for norm-referenced tests is seven to ten years. One possibility is to get a major testing company to come out with an alternative. From the school district perspective there is pressure to do well on the standardized tests. I know that long division does not make any sense. But as long as our students are tested on it and those results are in the paper, we're going to be teaching it.

From a local district point of view, it's going to take more reliance on locally developed tests and test norms rather than on standardized test norms. We can affect curriculum change faster by relying on locally developed tests than on standardized national norm tests. I see the growth of a new bureaucracy that has to do with creation of tests at the state level for licensing teachers, for minimum competency, and so on. That is potentially a bad situation. Unlike the achievement test publishers who have competition, there's no quality control related to the construction of these instruments. I'm concerned about the inhibiting force of these testing agencies.

Quality Software

Numbers of good software packages in proportion to the total software packages are estimated to range from 3% up to 10%. For example, the state of Virginia looked at 4,000 courseware packages and found that 3% of them were of worth. Also, there are about a hundred new programs being introduced each month. There are between 10 and 20 reviews of courseware published every month, about half of which are educational courseware. So what's happening is that in the review materials we are falling behind at a rate which is unbelievable.

STRATEGY GROUP REPORTS

On the last day of the conference, participants were organized into seven strategy groups. Members of these groups discussed and developed ideas presented earlier on the following topics: a National Steering Committee, the elementary school curriculum, the secondary school curriculum, courseware, elementary mathematics specialists, practitioner training, and linking research and practice. Strategy group reports are summarized below.

A National Steering Committee
Edward T. Esty and Robert B. Davis, Chairs

The establishment of a National Steering Committee for Mathematics Education is seen as a necessary step in the improvement of school mathematics. The Committee should include mathematicians, mathematics educators, psychologists working with the learning and teaching of mathematics, teachers, supervisors, and appliers of mathematics to science. A respected mathematics educator should serve as the group's leader. Membership on the Committee should be determined by nominations from the professional organizations.

The National Steering Committee should have long term base funding from private sources such as the Ford Foundation or the Carnegie Foundation. These base funds would be for establishing an institutional home; supporting the Director, an administrative assistant, and any other needed staff; and holding quarterly meetings of the Committee. In addition, it is assumed that one of the activities of the Committee will be to solicit funds from the Department of Education and from the National Science Foundation to carry out several of the other recommended functions. Such functions include monitoring national progress toward meeting standards, communicating with groups that have a role in or a concern for meeting the standards, and establishing task forces on curriculum and on training. These task forces should not be limited in the choice of personnel. Persons with a wide variety of backgrounds and interests should be included such as teachers, administrators, and persons from industry, in addition to mathematicians, educators, and psychologists.

Monitoring. The National Steering Committee's responsibility for monitoring the progress toward meeting established standards is critical. Standards are measures of how one meets goals. They are desirable because they operationalize the procedures for determining whether a goal is being met.

Initial information should be derived by operating in conjunction with existing data sources (ERIC, NAEP, NCES, etc.). Syntheses of existing information should be solicited (e.g., research on cognitive learning). From this effort we should be able to identify gaps in both our knowledge about aspects of school mathematics and our ability to set reasonable standards. Also, we should be able to identify needed information for future decisions.

A continuing concern of the Committee should be the inertia in schools and the constraints which inhibit change.

Communication. One major task of the National Steering Committee will be to communicate (explain and enter into a dialogue) our concerns and standards to a variety of interest groups. For example, a three-day meeting might be scheduled with publishers. The intent of the meeting would be for the National Steering Committee to explain our standards for school mathematics, to clarify emerging topics, to encourage publishers to express their concerns, and to outline next steps to coordinate activities toward meeting those standards. Similar meetings should be held with state school officers (so that the many state efforts are not at cross purposes), test publishers, computer software developers, and other groups.

The National Steering Committee should inform parents of the amount of mathematics that their children need to be prepared upon graduation to undertake college level studies, to perform meaningful work in the market place, or both, and to take their places in our free society as informed and participating citizens.

Curriculum task forces. These groups should meet for at least three weeks (and possibly for as long as eight weeks) for two consecutive years. The interim period would allow for an initial dissemination of the standards and provide for feedback from a variety of groups. The second meeting would then be to revise and polish each prospectus for final dissemination.

Prior to the first meeting of each task force the Steering Committee should gather information about both current practices and alternatives in this and other countries with respect to the scope and sequence of mathematical topics in the curricula. They should also gather the recommendations from all scholarly groups, industry, and interested parties on their mathematical expectations for students K-14.

The expectation is that the various task forces will not begin from scratch, but from a considerable data base derived from the deliberations and recommendations of others. We see the task not in terms of stating new goals for school mathematics but of establishing standards for those goals and means for reaching those goals.

Training task force. This group should meet for two to three weeks for two consecutive years. The product of the first year would be a handbook for preservice and inservice training based on a model (or models). The second year should involve revisions of the handbook based on feedback and on evolving curriculum standards.

Specialists[1]
Glenda Lappan and James M. Moser, Chairs

Improved curriculum and instruction in mathematics at the elementary school level require staff who are trained in mathematics as well as the teaching and learning of mathematics. We presently have a real mismatch between our expectations for the teaching of mathematics at the elementary school level and the preparation provided for elementary teachers. The designation of special mathematics teachers at the elementary school level is a realistic approach to solving this problem.

Existing teachers who have background and interest in the teaching and learning of mathematics will have a new opportunity for responsibility in an area of expertise and interest. This position will also motivate new or prospective teachers to select specialized training in an area of interest and skill.

The present demands on a typical self-contained classroom teacher at the fifth-grade level, for example, are too diverse and intensive to also allow time for providing adequate instruction in mathematics. The level of sophistication of the content and subject matter taught requires individuals who have the capabilities of a special mathematics teacher.

It is an often found research result that children learn what you teach them. Staff who know the content and structure of mathematics, value it for its relevance and beauty, and teach it with care and expertise are needed. With such special mathematics teachers, children will in fact have the opportunity to learn mathematics.

Administrative support. For the full benefit of such a program for special mathematics teachers to be realized, there must be commitment on the part of the school administration to the teachers involved. The administration must be willing to support the special mathematics teachers by providing time and resources for these teachers to develop, coordinate, and monitor the mathematics programs in their buildings.

While each special mathematics teacher is expected to provide direct instruction for some students, each must also have sufficient time allocated for the other aspects of the job. Since these teachers must be accountable for the overall mathematics program in their schools, it is not unreasonable to expect that 30 to 40 percent of their time would be spent on staff development, demonstration teaching, diagnosis or remediation, curriculum planning, personal professional growth, and monitoring the overall mathematics program.

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[1] In the steering committee meeting which followed the conference, it was decided that the argument for special mathematics teachers at the elementary school level was so strong and important that an extrapolation to the secondary school was warranted. In particular, with efforts to minimally train teachers certified in other areas to teach mathematics, a parallel differentiation of staff responsibilities between "master teachers" of mathematics and "regular teachers" should be considered.

The administration must provide the opportunity and financial support for special mathematics teachers to belong to and participate in professional organizations such as NCTM. This involvement is essential to the continued professional growth and development of the special mathematics teachers themselves. The promise this holds is that the mathematics curriculum and instruction will, through these special teachers, become a system that is more responsive to new ideas and developments.

Sources and preparation. Two sources of special mathematics teachers for the elementary school are inservice teachers and people who are preparing to become teachers. If such positions were commonly available, people who would not otherwise choose to teach elementary school would probably be attracted.

Inservice teachers who have both the preparation and interest should be reassigned to be special mathematics teachers with relatively little additional cost. Most prospective special mathematics teachers should receive special preparation and support, at least for the first several years, to assure maximum benefits. Such preparation and support should include (but not be limited to) special course work, cooperation from the administrator of the school building, and support for the necessary activities and for continued professional growth.

One possible model for identifying and preparing special mathematics teachers would involve NSF-type institutes.[2] To assure local support and continuity, teacher participants would be identified early in a school year and would be required to have the cooperation and support of their local administrators and school districts. This support would include a commitment on the part of the principal to attend certain sessions and support the special mathematics teacher's activities in the school. The district would be required to commit an appropriate amount of the teacher's time to developing resource activities for other teachers.

The institute might start with visits from the institute faculty to the participants' schools for discussions with the participants and principals and observations of the participants' classes. Then institute classes might meet every second week from February through June. Principals would be required to attend certain designated sessions.

In the summer, a six- to eight-week inservice institute considering methods and content of elementary school mathematics would take place. Again, principals would be required to attend certain designated sessions.

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[2]Although they focused on the NSF summer institutes, Hillier Krieghbaum and Hugh Rawson (An investment in knowledge. New York: New York University Press, 1969) also discussed the NSF inservice institutes.

The following academic year, the participants would act as special mathematics teachers with the full support of the principal and school system. They would also continue taking institute classes every second week with principals attending some designated sessions. During the year, the institute faculty would again visit participants' schools to document change and to propose possible further modifications.

At the preservice level, teacher education institutions should be encouraged to provide concentrations in either language arts or mathematics for prospective elementary school teachers, with appropriate courses in other areas still required, of course. A mathematics concentration ought to include at least 12 semester hours of mathematics (along the line of CUPM and NCTM proposals[3]) and at least 6 hours of methods courses, including techniques for identifying and remediating children's difficulties, procedures for choosing curricular materials, procedures for helping other teachers, and the other duties identified in the job description section.

State departments of education ~~should be~~ encouraged to provide special certification appropriate to a special mathematics teacher.

Job description. The special mathematics teacher should have overall responsibility and be accountable for the program of mathematics instruction in an elementary school. In schools and/or districts of sufficient size to warrant them, this responsibility and accountability will be shared with other special mathematics teachers in the same school building or designated mathematics coordinators of the school district. In particular, we envision the following five components of the job as worthy of mention.

1. Classroom teaching. The special mathematics teacher will carry out direct and daily mathematics instruction for all children beginning no later than fourth grade. This is not to suggest that this direct instruction cannot begin in earlier grades. In fact, in many situations we could imagine that such instruction could start as early as second grade. By direct and daily instruction, we mean the type that is presently carried out in most schools by a teacher in a self-contained classroom. This includes planning and execution of daily lessons, diagnosis and remediation of learning difficulties, pupil assessment, and reporting to and consulting parents or parent surrogates.

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[3]Committee on the Undergraduate Program in Mathematics Panel on Teacher Training. (1983). Recommendations on the mathematical preparation of teachers. Washington, DC: Mathematical Association of America.

National Council of Teachers of Mathematics. (1981). Guidelines for the preparation of teachers of mathematics. Reston, VA: Author.

As a part of staff development, about which more will be said later, the special mathematics teacher will also perform demonstration teaching lessons in the lower grade-level classes where ordinary instruction is not carried out by a special mathematics teacher.

While direct classroom instruction is seen as the primary responsibility of the special mathematics teacher, we recommend that this aspect of the special mathematics teacher's job not demand a disproportionate amount of time, so that ample opportunity is available for performance of other duties. Thus, we suggest that a maximum of four lessons per day be taught. If a special mathematics teacher is teaching at the primary level where the duration of an individual lesson is appreciably shorter, then perhaps five lessons a day is not an unreasonable expectation. In either event, we would hope that at least one-third of the working day be devoted to nonteaching responsibilities.

2. Curriculum development. Under this rubric we intend to include all activities related to programs and materials of instruction. The special mathematics teachers should become familiar with the various textbooks on the market, know their strengths and weaknesses, and be able to offer expert advice at the time of text adoptions. Furthermore, there are currently many nontextbook materials, both of a soft- and hardware nature, that are available to supplement and enrich a standard textbook approach to teaching and learning. We take the position that such materials are essential to a program of instruction in order to meet the particular needs and interests of an individual child or of a neighborhood or locale that is served by the school. The special mathematics teachers should be aware of the availability and practicality of such materials. But more than simply being knowledgeable about the strengths and weaknesses of text and nontext materials, the special mathematics teachers must play an active role in the school in suggesting ways that these materials can be integrated into a curricular program. This role should not be simply advisory in nature. In addition to the "whats" of curricular materials, the special mathematics teachers should be ready to demonstrate the "hows" of them through demonstration lessons and inservice sessions.

Before turning to another aspect of the special mathematics teacher's job description, special mention should be made here of technological devices such as the calculator and computer. In other sections greater detail is given to the reaffirmation of recommendations made by other groups and conferences that computers and calculators can and should play an increased and important role in the improvement of mathematical teaching and learning. We feel these recommendations have direct implications for a special mathematics teacher. We therefore suggest that the special mathematics teachers become sufficiently computer literate to serve as a resource person in a school for computer-related activities in mathematics. We do not intend to imply that mathematics is the only subject area in which the computer can be used to good advantage. Other subject areas are equally fertile grounds for utilization of the capabilities of the computer, particularly language arts. We envision the special mathematics teachers as facilitators along with other school staff members in the effective utilization of these technological devices.

After the time allocated for direct instruction, we see the special mathematics teacher's responsibility as a curriculum specialist as requiring the second largest amount of time. However, we definitely are of the opinion that the insistence that a special mathematics teacher be a curriculum innovator is no less important than that she or he be responsible for some direct instruction. In fact, in making the recommendation that there be at least one special mathematics teacher in every elementary school, we are not implying that large proportions of current teachers are doing a poor job of teaching their mathematics programs. Rather, we see the key advantage will come from the role as mathematics curriculum specialist wherein more quality programs in mathematics--not just arithmetic computation, but complete programs including problem solving, measurement, geometry, probability and statistics--will be developed and implemented.

Finally, the curricular role of the special mathematics teachers should not be conceived as a solitary one. We expect the special mathematics teachers to act in conjunction with other special mathematics teachers in the same building or other buildings within the school district, with building principals, and with school district subject matter specialists.

3. Professional development. We propose that the professional development aspect of the special mathematics teachers be twofold: first, for the special mathematics teachers and second for fellow staff members. Working on the assumption that, in the first 5 to 10 years of implementation of this recommendation, the majority of the special mathematics teachers will come from the ranks of already certified teachers, we propose that the professional development of the special mathematics teachers be accomplished through an inservice program that includes both mathematical content and pedagogy. We are not necessarily suggesting that this be accomplished solely through advanced degree programs or graduate courses from colleges and universities. We recognize that many school districts and other educational agencies offer excellent courses, programs, institutes, and workshops that could contribute to the professional improvement of a special mathematics teacher. We simply state that once a person is identified as a special mathematics teacher, part of the job responsibility is to embark on a planned program of improvement.

Furthermore, it is necessary that the special mathematics teachers share knowledge and expertise with other staff members with the aim of improving classroom practice. There are many ways in which this can be accomplished--demonstration teaching, inservice workshops or staff meetings, distribution of printed information on new or available materials, or organization of presentations by other mathematics resource persons (college professors, state mathematics consultants, textbook authors, etc.).

4. Individual pupil diagnosis. Recent research has clearly demonstrated the validity and practicality of individual pupil interviews as a method of identifying pupil thought processes and problem-solving procedures. The information gleaned from such interviews could be invaluable to a classroom teacher providing

instruction or attempting to remediate certain learning difficulties a child might have. Unfortunately, a busy elementary teacher with full-time responsibility in a self-contained classroom rarely has time to conduct systematic interviewing with all students. Thus, we see this as an area in which a special mathematics teacher can provide a valuable service. Certainly we would expect the special mathematics teacher to do such interviewing with the students for which she/he has responsibility for teaching mathematics directly. But it should also be possible to carry out diagnostic interviewing of pupils of other teachers who may be teaching mathematics.

The skill of good diagnosis through individual interviews does not come automatically simply because one has been designated as a special mathematics teacher. However, this skill could be acquired as part of the professional development of the special mathematics teacher.

5. Monitoring progress. The special mathematics teacher must also be responsible for monitoring and assessing the progress or improvement of the school's mathematics program. If additional resources are to be provided to a school's mathematics program in the form of a special mathematics teacher, it is not unreasonable to hold the school staff accountable for continued high levels of student achievement in mathematics. The special mathematics teachers can be of direct assistance in program improvement by providing and analyzing evaluative data for program improvement, identifying program strengths and weaknesses, and providing information for decisions related to the mathematics program. This systematic feedback concerning program effectiveness is needed to assure continued growth and improvement. It provides a strategy for schools and school districts to insure they are teaching appropriate mathematical skills in effective and affective ways so children can learn in today's schools the skills that will be useful in tomorrow's society.

Elementary School Curriculum

Claude Mayberry and Zalman Usiskin, Chairs

Previous groups and commissions have identified a number of major problems confronting elementary school mathematics. Among them are the following:

1. mismatch between the way arithmetic is done in school (always with paper and pencil) and the way arithmetic is done by adults (mentally or by calculator more often than with paper and pencil);

2. a content that is too restricted to low level arithmetic skills at the expense of applications, geometry, problem solving, and dealings with numerical information;

3. a dismaying performance on the ability to apply arithmetic despite quite good performance on whole number arithmetic;

4. a lack of time to teach, in the current schedule for most schools, all of the mathematics that children should know before entering senior high school; and

5. a sense of failure among many students who do not master paper-and-pencil arithmetic.

To correct these problems, these groups and commissions have recommended that calculators be used at all levels; that problem solving, applications, geometry, estimation, and working with data be given high priority; and that time be increased for mathematics study.

These recommendations are not being implemented in most schools in our nation. Some states implement competency tests to increase performance among slower students. However, these tests tend to be highly restrictive in content and only exacerbate some. There is one underlying cause to the above problems not given enough attention in prior reports. That cause is the dependence of the elementary school mathematics curriculum on paper-and-pencil arithmetic, much of which is today obsolete. This dependence is also a major barrier to the implementation of other recommendations that have been made for school mathematics. That is, a decreased dependence upon paper-and-pencil arithmetic would seem to be a prerequisite for updating and improving the elementary school mathematics curriculum in ways suggested by other committees and groups. We can only conclude that paper-and-pencil arithmetic in the elementary school must be vastly curtailed.

We recognize the importance that doing arithmetic with paper and pencil has had in schools. It has been claimed that paper-and-pencil arithmetic is essential for the brain, orderly thinking, the ability to apply mathematics, and the ability to understand mathematics. These purported values are either false or exaggerated.

Adults not familiar with the teaching of arithmetic in school tend to underestimate the amount of time that is spent on certain paper-and-pencil skills. Replacing these skills with the corresponding calculator facility, we can free considerable time in the elementary school mathematics curriculum for teaching applications, problem solving, estimation, mental arithmetic, geometry, and work with numerical information that so many have desired.

The dominance of the curriculum by paper-and-pencil skills applies more than just to what is taught. Topics are sequenced to be available for paper-and-pencil computation.[4] Topics are often approached pedagogically in ways affected by computation. Thus freeing up the curriculum from paper-and-pencil dominance could have profound implications for the timing and approaches given to topics that remain.

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[4]See Usiskin, Z. (1983). One point of view: Arithmetic in a calculator age. Arithmetic Teacher, 30(9), 2.

It goes without saying that we want children to be able to get answers to arithmetic problems. We could not make many of the above statements were it not for the existence of the calculator. We expect calculator facility to replace paper-and-pencil facility for all complicated arithmetic in school. The replacement has already taken place in business and in the marketplace. However, we expect increased attention to mental arithmetic and to ways of checking answers by estimation and other techniques.

An important task for the elementary curriculum task force will be to outline what arithmetic is (algorithmic routines, basic facts, uses, mathematical properties, numeration, objects, operations, etc.) and what we mean by students' understanding arithmetic (able to represent, knowing when to use, being able to use, knowing why it works, etc.).

Content specifications should include what should be eliminated from current curricula (long division, square roots, partial product multiplication, fraction calculations with large numbers, decimal calculations). What should be added is not totally clear, but the recommendations of other reports should be the starting place. Rather than being driven by algorithms, the sequence should be determined by the readiness for applications. The placement of topics will need to be reconsidered.

Techniques of evaluating student progress need to be incorporated at every level. Tests should reveal both how and whether students achieve.

Secondary School Curriculum

David R. Johnson and Marilyn Hala, Chairs

The school mathematics program for grades 7-14 should have the following characteristics:

1. At least three alternate programs should be developed with enough flexibility and common topics so that students can reasonably move from one to another.

2. At least six years of mathematics should be outlined for all students.

3. All students should be able to enroll in mathematics during grades 12-14.

4. Although alternate programs are important, no program should lead to a dead end.

5. No program should accelerate students out of mathematics.

6. The topics should be unified and integrated. Some should be common across programs.

7. It is assumed that calculators and computers will be available for all students and that they will be used when appropriate.

8. Concept of proof should be integrated throughout all phases of programs, not just in geometry.

All students should study the following topics:

1. Algebra, at least 1 year, a Usiskin-type course with application and statistics.[5]

2. Statistics, at least 1/2 year, again integrated with other areas such as algebra.

3. Geometry, at least 1/2 year, including right triangle trigonometry using calculators, analytic geometry, and measurement.

4. Computer package, including discrete mathematics, algorithm development, programming. Discrete mathematics includes logic (truth tables, switching networks), combinatorials, discrete probability, and graph theory.

Ratio/proportion should be included in discrete mathematics, algebra, and geometry.

Numeration skills (rational numbers, including percent) should also be included in the mathematics courses for all students.

Six additional topics should be provided for top students: one-half year of geometry, continued work in discrete mathematics, elementary functions including trigonometry and logarithms, algebra necessary for calculus, introduction to calculus, and exploratory data analysis.

There are also topics which should be deemphasized. First, the extreme writing of proofs as the geometry course for all students should be deleted. The structure of proof should be examined and explained. Second, there should be a deemphasis on the study of logarithms for calculation. Third, much of the traditional trigonometry course (e.g., the "solving of triangles," the use of tables, and interpolation), should also be deemphasized.

The recommendation of increasing the amount of mathematics for all students should not result in keeping all students in the same traditional pre-calculus courses. This intent cannot be met by adding more "general mathematics" courses or shifting lower track students into a two-year beginning algebra course. Alternate programs need to be

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[5]Usiskin, Z. (1979). Algebra through application with probability and statistics (2 vols.). Reston, VA: National Council of Teachers of Mathematics.

defined and materials identified or developed for such alternatives. One suggestion would be to catalog alternate programs now in place (or on the drawing board), to evaluate the appropriateness of elements of these programs, to select appropriate elements, and to identify gaps. From such an analysis, a description of appropriate alternatives should be possible, and guidelines for the development of prototypic materials prepared.

Courseware
Lud Braun and William L. Barclay, III, Chairs

There are four basic problems that the task force on courseware should address:

1. Which mathematics can be well supported with the microcomputer, and how can appropriate software be developed?

2. What types of software are appropriate for different modes of instruction (teacher demonstration, whole class interaction, small groups, individual)?

3. How does one evaluate different possible uses of existing software and relate them to the curriculum? This problem is complex because many programs have been written so that they are usable at many levels (e.g., Rocky's Boots from The Learning Company and Green Globs from Conduit).

4. How can we deal with being knowledgeable about what is possible and available now, informed about what is coming next, and continually open to learning and renewing as future generations of software, language, expert systems, and hardware are developed?

One serious problem which is tangential to the goal of developing standards for courseware is the differential access to computers. All students should have adequate access to computers to enhance their mathematical learning and to do their mathematical work. Having access to a computer is not of itself an education. There must be appropriately trained staff and adequate materials to integrate the technological advances with the school mathematics program.

Practitioner Training
Dorothy Strong and Ross Taylor, Chairs

To improve the quality of mathematics teaching in K-14, the highest priority should be placed on efforts to improve the professional status of teachers and the school environment in which they work. Putting teachers on a 12-month contract; giving them time to study, reflect, and plan; expecting them to collaborate with other teachers and with the mathematics community; and reducing noninstructional responsibilities are all important aspects of such an effort.

The level of competence in the classrooms of this nation has always been a reflection of society at large. The improvement of that competence will result only from an improved perception of teacher roles and teacher expectations by the many segments of that society. Hence, addressing the question of improving the practitioners' art (or science) is dependent upon changing understandings on the part of all of society. Any improvement program must also address administrators, school board members, teacher educators, and parents. Addressing these audiences separately and specifically, in addition to the teacher audience, is an essential aspect of any successful effort to improve instruction.

Since employment opportunities in the 1990s will be tied to the possession of mathematically related skills and knowledge, our schools are today determining the haves and have nots on the basis of mathematics experiences provided. The quantity and quality of mathematics education available to students is directly related to the competence of their teachers. The number of mathematics classes taught by teachers without adequate preparation is a major contributor to the problem. Students receiving instruction from teachers who are inadequately prepared mathematically are at a disadvantage.

Practitioners accept employment with different levels of adequacy of preparation for their positions. For instance, two first-year teachers may be teaching fourth-grade in adjacent classrooms, but possess vastly different competency levels in mathematics and vastly different repertoires of strategies for teaching fourth-grade mathematics. Similarly, two first-year principals in adjacent middle schools may possess vastly different understandings of the mathematics curriculum and of effective strategies for teaching it. Similar discrepancies occur with other positions, e.g., teacher educators, state supervisors of mathematics, and school district supervisors and administrators. As long as practitioners stay in their positions, changing conditions and a changing knowledge base require that they continue to learn on the job.

We believe that the current public interest in improving K-12 mathematics provides the support for addressing personnel shortcomings openly, if we are willing to responsibly remedy these shortcomings. The annual evaluations that principals are required to do of their teachers is one existing mechanism that could be tapped to identify items for a teacher's professional development plan in mathematics. This process should address both content and methods (process). The principal-teacher evaluative interaction could be replicated for all other practitioners whose positions impinge upon K-12 mathematics.

It is assumed that all teachers need to be cognizant of the computer and its capability for instruction. An immediate pressing problem is the retraining of all teachers. This probably can best be done by training a few teachers (specialists and master teachers) who in turn can teach other teachers in the system.

We identified the following priorities:

1. Training trainees--develop training models.
2. Training administrators.
3. Initiating planning at all levels.
4. Training curriculum leaders at district and school levels.
5. Upgrading formality of mathematics.
6. Developing competence in preservice education.

One aspect of the teacher training need is the magnitude. In the past NSF summer institutes have been effective with the training of a few hundred teachers at a time.[6] We now are talking about probably tens of hundreds each year. At the present $5,000/teacher cost of NSF summer institutes, this is a lot of money. Alternative means of teacher training using technology (computers, video, telecommunications), which can be much cheaper, must be considered.

We do not begin at point zero. Much has been accomplished over the past two or three decades and much is currently underway that has important implications for our efforts. There are teachers and other practitioners with mathematics experiences from the post-Sputnik era who are in a unique position at this time to provide real leadership. The findings of recent research are also available to improve instruction. The past decade has witnessed the completion of several studies with direct and useful implications for practice. Certainly, organizations and agencies at the national and state levels provide many useful resources.

Another vehicle for retraining and reform is the development of networks. A mathematical sciences community can be developed by involving mathematics educators at all levels from elementary schools through the university, mathematicians working in industry, representatives of the media and government, and others. Mathematics teachers at all levels should have access to a professional network for information and support. Networks of professionals could readily begin in geographical areas.

Membership in national and state organizations and participation in their activities is a resource that should be encouraged. Finally, on the state level, including colleges and universities, expert resources abound. It is within and through these organizations that improvement efforts need to be funneled.

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[6] For a study of the first twelve years of NSF summer institutes, see Kreighbaum, H., & Rawson, H. (1969). An investment in knowledge. New York: New York University Press.

Outline of Approach

A. The problems must be addressed at each level.
   1. Individual
   2. School
   3. District
   4. State education agency
   5. Teacher education institutions
   6. National

B. Planning should take place at each level and address the following components.
   1. Statement of major goals.
   2. Assessment of needs.
   3. Statement of specific objectives.
   4. Development of plans that address objectives.
   5. Implementation.
   6. Evaluation that focuses on objectives.
   7. Revision based on evaluation results.

C. Desirable characteristics of plan.
   1. Major input for development of the plan should be by the recipients of the training.
   2. There should be an emphasis on the use of peers as trainers.
   3. Build upon what we know from research and experience.
   4. The plans at the various levels should be designed to compliment each other.
   5. Maximum use should be made of existing resources and structures.
   6. There should be a blending of resources.
   7. It should be a cooperative effort.
   8. Successful practices should be identified and replicated.
   9. Where possible, follow-up activities should take place.
   10. The training should be ongoing.

D. Specific importance of computer training.
   1. General awareness of educational uses of computers.
   2. Hands-on experience, including opportunities to take computer home.
   3. Overview of existing software.
   4. Process for reviewing software.
   5. Factors to consider in purchasing hardware (e.g., software, continued support from vendor, flexibility of use, reliability).
   6. Factors to consider in implementation (e.g., location, scheduling, monitoring of use, security, maintenance).
   7. Incorporation of computers into the instructional process.

Linking Research-based Knowledge to Practice
Jeremy Kilpatrick and Thomas P. Carpenter, Chairs

As a part of increased professionalization of mathematics teachers, they should be involved as equal partners in ongoing research related to the teaching and learning of mathematics. The formulation of viable research is a two-way street. Practitioners and researchers share the responsibility of identifying phenomena of interest, of clarifying ideas, and of formulating questions to be investigated. Collaboration is critical. Furthermore, it should yield more relevant outcomes than past research, and it should make the findings easier to disseminate.

The training and retraining of teachers (and particularly specialists and master teachers) should involve training in research. Teachers should be able to be mediators of research knowledge and be able to incorporate research knowledge into practice. Obviously, we need teacher training programs, or guidelines (models), to be developed for this purpose. Also, we need research on such programs to evaluate their effectiveness.

An additional training program in research and research collaboration needs to be developed for specialists and master teachers. Familiarity with research in mathematics education should be seen as an important aspect of this career ladder position.

It is the responsibility of the professional associations associated with mathematics education (e.g., NCTM, MAA) to respect, encourage, and support research on the teaching and learning of mathematics. In addition, the organizations should help in providing a forum for research through their publications and meetings.

The agencies of the federal government responsible for mathematics education (NSF, Department of Education) should provide funds both for basic research on the teaching and learning of mathematics and for research which links research-based knowledge with its implementation in classrooms.

The National Steering Committee on Mathematics Education should establish a mechanism for a continuing dialogue on the critical problems for research.

Several examples of needed research were mentioned during the conference:

1. An in-depth survey of estimates of how mathematics is likely to be used in the 21st century should be carried out.

2. An analysis of the implications of research on teaching and learning for each curriculum prospectus. (Researchers should be members of each curriculum task force.)

ANNOTATED KEY REFERENCES

The following references were considered by the steering committee and participants of the Conference in their deliberations. They are separated into two categories: references provided to all the participants prior to the conference and major reports.

References Provided to Participants

Lesgold, A. M., & Reif, F. (1982, November). Computers in education: Realizing the potential (Chairmen's report of a research conference). Washington, DC: U.S. Government Printing Office.

This report tries to answer some of the questions raised with the rapid adoption of microcomputers in schools. Conference participants identified a number of educational opportunities presented by the computer, including the following:

1. The computer can be an excellent teacher.

2. Computers can be used to provide new "learning environments."

3. Computers can be used to diagnose an individual student's current knowledge, thinking strategies.

4. Through new telecommunications technologies, it is possible to create intellectual communities without regard to participants' physical locations.

5. Computers help with administrative tasks.

6. Computers are powerful intellectual tools.

Striking improvement in the quality and productivity of education through computer-based instructional systems is attainable, but only with a national investment that continued reliably for several years.

It was recommended that at least two research centers be established, dedicated to applying technology and new knowledge of human cognition to improving education. One of these centers should be predominately for research on new applications of the computer in reading and writing. The other, predominantely for comparable research in mathematics and science.

National Council of Teachers of Mathematics. (1980). An agenda for action: Recommendations for school mathematics of the 1980s. Reston, VA: Author.

The National Council of Teachers of Mathematics made the following recommendations for school mathematics of the 1980s:

1. Problem solving be the focus of school mathematics in the 1980s.

2. The definition of basic skills be expanded to include problem solving; applications; alertness to reasonableness of answer; estimation and approximation; computations; geometry; measurement; graphs; prediction; and computer literacy.

3. Full advantage taken of the power of calculators and computers at all grade levels.

4. High standards of efficiency be applied to the teaching of mathematics.

5. Mathematics programs and learning be evaluated through measures other than testing.

6. More mathematics study be required for all students, and a flexible curriculum be designed to provide for different individual needs and interests of students.

7. Mathematics teachers demand of themselves and their colleagues a high level of professionalism.

8. Public support for mathematics instruction be raised to a level which reflects the significance of the subject to individuals and society.

Romberg, T. A. (1983). A common curriculum for mathematics. In G. D. Fenstermacher & J. I. Goodlad (Eds.), Individual differences and the common curriculum: Eighty-second yearbook of the National Society for the Study of Education, Part I (pp. 121-159). Chicago: University of Chicago Press.

This chapter's purpose was to give direction for the development of mathematics curriculum. The chapter addressed the following four points: the nature of mathematical knowledge, the choice of and rationale for mathematics tasks, the principles upon which to base instructional activities, and the consideration of individual differences.

The author distinguished between two types of mathematical knowledge: the record of past knowledge and the active construction of or "doing" of mathematics. He advocated the latter and developed the basic mathematical activities of abstracting, inventing, proving, and applying. Based on this conception of mathematics, Romberg presented the choice of and rationale for the mathematics tasks from the following three perspectives: the discipline itself, psychology, and sociology.

In the third section, he recommended instructional activities which are process-oriented and based on curriculum units, and he described seven major content strands of mathematics. The final part of the chapter related the common mathematics curriculum to the individual differences of students as based on the perspectives of five different interest groups.

Romberg's summary recommendations stated that "there should be a 'core' set of mathematics activities in a curriculum unit based on a strand of mathematics that all students should experience and master." He further defined the core as "the basis for differential and additional study."

Romberg, T. A., & Carpenter, T. P. (in press). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. Wittrock (Ed.), The third handbook of research on teaching. New York: Macmillan.

The paper examines the current state of instruction in mathematics, the directions taken by current research on children's learning, the potential contribution of that research for making decisions about mathematics, and finally the approaches (scientific and field-based) to the studies of mathematics teaching. The general finding was that studies related to the teaching of mathematics have failed to provide teachers with a list of tested behaviors that will make them competent teachers and ensure that their students will learn. Specific findings include the following:

1. Mathematics is taught as a static bounded discipline.

2. Instruction assumes students absorb information rather than construct it.

3. Teachers are more concerned with management (maintaining order and control) than with learning.

4. Instruction that capitalizes on children's natural problem-solving abilities (especially on addition and subtraction strategies) is effective.

5. Classes in which less time is allocated to mathematics instruction or to developmental portions of lessons have relatively poorer achievement in the subject.

6. Teachers spend little time planning. However, the largest proportion of their planning time is on content (subject matter) to be taught and then on instructional processes (strategies and activities).

Plans also focused on large-group routines and not small groups or individuals.

7. The textbook is found to be the sole authority for knowledge.

Among the authors' recommendations are the following:

1. The scope of research on students' learning must be expanded.

2. Teaching research should consider how learning proceeds.

3. Models which bridge the learning-teaching gap need to be constructed.

4. Mathematical content should be seriously included in such models.

5. The role of computers and technology must be considered.

6. New assessment tools must be developed.

Romberg, T. A., & Price, G. G. (1981, February). Assimilation of innovation into the culture of schools: Impediments to radical change. Paper presented at NIE Conference on Issues Related to the Implementation of Computer Technology in Schools, Washington, DC.

Beginning with the premise that attempts to change schooling practices must be viewed as natural phenomena, these authors examined the problems involved in introducing a new program into schools. They began by characterizing the culture of schools in terms of work, knowledge, and professionalism. Next they argued that innovations can be described along a continuum from ameliorative to radical. Where a particular innovation is situated on this continuum depends on the amount of restructuring of work, knowledge, and professionalism it involves.

The authors then examined responses to radical change in schools in terms of nominal change and actual change. Nominal change involves changing labels but not changing routines. Actual change occurs where the school staff understands that a radical innovation is expected and attempts to implement it. Actual change breaks down into three types: mechanical, constructive, and illusory. Mechanical change involves adopting the rituals and routines of a new program without fully grasping the intent of the program. Constructive change, on the other hand, involves an understanding of its underlying values and principles. Illusory change has all the trappings of radical innovation without conviction.

Based on this analysis, the authors concluded with three recommendations for developers of educational innovations:

1. Prepare a dissemination plan and develop materials to support dissemination of the innovation.

2. Identify the cultural traditions that will be challenged by the innovation.

3. Plan and implement a systematic monitoring procedure for any innovation.

Schneck, M. (1983, November). Summary of recent reports and recom. a-dations on the improvement of mathematical sciences education. Report prepared for the Conference Board of the Mathematical Sciences.

As the title of this paper states this is a summary, an organized summary, of 12 recent reports which have examined schooling and mathematics in schools. Comments from the reports are summarized in terms of the curriculum (course content, graduation requirements, time, and materials and technology); college entrance requirements; student performance (grades, standardized tests, and attitudes); and teachers (preparation, certification, career ladders, extended contracts, teacher education, salaries, and performance).

Major Reports

National Commission on Excellence in Education. (1983, April). A nation at risk: The imperative for educational reform. Washington, DC: U.S. Government Printing Office.

On August 26, 1981, T. H. Bell, the Secretary of Education, created the National Commission on Excellence in Education to examine the quality of education in the United States. The charges given the Commission included assessing the quality of teaching and learning across the educational spectrum, comparing the American educational system with those of other advanced nations, and specifying problem areas that must be dealt with in order to successfully pursue a course of excellence in education.

The Commission found that the declines in educational performance today are mainly the result of disturbing inadequacies in the way the educational process is conducted. Secondary school curricula have been diluted and diffused until they no longer have a central purpose. Grades have risen as student achievement and effort have been declining. Compared to students in other nations, American students spend much less time in the classroom and on schoolwork, and the time they do spend is often used ineffectively. The Commission found that the academically able students are, in general, not being attracted to the teaching profession, and that teacher preparation programs need considerable improvement. The members also concluded that certain aspects of the professional working life of teachers are unacceptable, and that a serious shortage of qualified teachers in such areas as mathematics and the sciences exists.

The recommendations put forth by the Commission in their report are as follows: that high school graduation requirements be strengthened to include four years of English, three years of mathematics, three years of science, three years of social studies, and one-half year of computer science; that standards and expectations at all levels of education be raised; that more student time be devoted to educational pursuits; that teaching be made a more rewarding and respected profession; that the preparation of teachers be improved; and that educators and elected officials exhibit the leadership necessary to achieve these reforms.

The National Science Board Commission on Precollege Education in Mathematics, Science and Technology. (1983). Educating Americans for the 21st century: A report to the American people and the National Science Board. Washington, DC: Author.

In this report, the Commission is concerned with the inadequacy of present elementary and secondary education, particularly in mathematics, science, and technology, in preparing young Americans to work in, contribute to, profit from, and enjoy the increasingly technological society. This report is an outline of actions directed at achieving the objective that "by 1995, the Nation must provide, for all its youth, a level of mathematics, science, and technology education that is the finest in the world, without sacrificing personal choice, equity, and opportunity."

The Commission proposed that this goal can be achieved through a strategy of building a strong and lasting commitment to quality mathematics, science, and technology education for all students; providing earlier and increased exposure to these fields; providing a system for measuring student achievement and participation; retraining current teachers, retaining excellent teachers, and attracting new teachers of the highest quality and the strongest commitment; improving the quality and usefulness of courses that are taught; establishing exemplary programs; utilizing all available resources; and establishing a procedure to determine the costs of required improvements and how to pay for them.

Conference Board of the Mathematical Sciences. (1982). The mathematical sciences curriculum K-12: What is still fundamental and what is not. Washington, DC: National Science Foundation.

This report contains the recommendations submitted by the Conference Board of the Mathematical Sciences (CBMS) to the National Science Board (NSB) Commission on Precollege Education in Mathematics, Science, and Technology on needed changes in school mathematics curricula. The conferees' main conclusion was that the ways in which mathematics is used in the American society are significantly changing due to the fantastic technological developments and to the mounting need for information processing and transfer capabilities. Therefore, mathematics education in the schools should be altered to meet these changes. Following are condensed versions of some of the recommendations referred to above.

1. With regard to grades K-8, it was recommended that:

   a. Calculators and computers be utilized in the classrooms for enhancing understanding and problem-solving ability.

b. Mental arithmetic, estimation, and approximation be emphasized, and paper and pencil work deemphasized.

c. Students be familiarized with data collection and analysis.

2. With regard to grades 9-12, it was recommended that:

a. The traditional component of the curriculum be streamlined, and the curricula of the currently taught subjects be reexamined in light of new technology.

b. Appropriate topics from discrete mathematics, statistics and probability, and computer science be considered fundamental.

3. With regard to mathematics teachers, it was recommended that those in the service should be retrained in the new topics, approaches, and technology. And prospective teachers should take in the course of their studies modern topics and approaches as deemed necessary according to CBMS's recommendations.

Task Force on Education for Economic Growth. (1983, June). Action for excellence: A comprehensive plan to improve our nation's schools. Denver, CO: Education Commission of the States.

The Task Force on Education for Economic Growth was formed to link education to the economic well-being of the individual states and the nation as a whole. The Task Force concluded not only that the level of illiteracy in our nation at the present is unacceptable but also that the overall performance in higher-order skills such as inference, analysis, interpretation, and problem-solving has declined, particularly in the most able students. The members found a severe shortage of qualified teachers in such critical subjects as mathematics and science. The Task Force also found a large educational management and leadership void in our public school systems. The members felt that the greatest overall educational deficiency in our school systems is the absence of clear, compelling, and widely agreed-upon goals for improving performance.

The recommendations put forth by the Task Force are as follows: immediately develop state plans for improving K-12 education; create more effective business, labor, and teaching partnerships; make the best possible use of existing educational resources; improve the methods of recruiting, training, paying, and retaining top quality teachers; strengthen the curriculum, upgrade the requirements, and increase effective educational time; provide quality assurance in education through improved certification procedures; develop more effective educational management techniques; and serve better those students who are now unserved or underserved.

Fey, J., Good, R., Heid, K. M., Johnson, J., & Kantowski, M. (1982, October). Computing and mathematics: The impact on secondary school curricula: Report of a conference at College Park, Maryland. Reston, VA: National Council of Teachers of Mathematics.

Ideas in this report represent the imagination, analysis, and experiences of many people involved in recent curricular studies. Study groups focused on the impact of computing on algebra, geometry, calculus, and "new topics." Each group addressed six questions:

1. What is the rationale for curricular change in various facets of secondary school mathematics?

2. What are the main themes or central ideas in the newly conceived version of the subject?

3. How might those themes or central ideas be elaborated in more detail as sample syllabi, courses, or content strands in a total program? What sequence and/or emphasis seem plausable?

4. How do the proposed curricula differ from current predominant patterns and how are the changes justified?

5. For major new directions or content, what would sample instructional materials or approaches look like?

6. What underlying research questions and exploratory curriculum development activities are suggested by the anticipated new curricular direction?

There was conviction among the participants that computing is here to stay and provides a unique opportunity and s 'mulus for real change in mathematics curricula. But careful research, development, demonstration, and teacher education must precede any attempt to integrate computing into the mathematics curriculum. They set an agenda of research and development tasks ahead.

National Assessment of Educational Progress. (1983). The third national mathematics assessment: Results, trends and issues (Report No. 13-MA-01). Denver, CO: Education Commission of the States.

This report, which was prepared by the National Assessment staff and members of the National Council of Teachers of Mathematics (NCTM), documents, analyses, and discusses the major findings of the third national mathematics assessment which was conducted in 1982. It also documents the changes that took place in the mathematical performance of American 9-, 13-, and 17-year-olds between 1978 (when the second national mathematics assessment was conducted) and 1982. Out of the many major findings of the third assessment, one may single out the following three findings for special mention:

1. The 13-year-olds have improved considerably between 1978 and 1982, but much of this improvement was on routine items. Very little gain was made on items assessing deep understanding or application.

2. The mean performance of black and Hispanic students continued to be below the national mean. But the 13-year-old black and Hispanic students attained more positive changes between 1978 and 1982 than their white counterparts did.

3. At ages 9 and 13, very little difference in mathematical performance was observed between males and females.

The report gives explanations for the detected changes and points out their implications. Moreover, it gives specific suggestions aimed at enhancing the mathematics programs addressed to all concerned in designing, preparation, and implementation of these programs.

Conference Board of the Mathematical Sciences. (1984). New goals for mathematical sciences education. Washington, DC: Author.

This is the report of a two day conference of leaders of the various segments of the mathematical sciences community in November 1983. The intent of the meeting was to examine the recent reports critical of mathematical education, to seek consensus on the problems and obstacles to quality mathematics in the schools, to set goals, and to make recommendations about how to improve mathematical sciences education. Participants made the following seven recommendations:

1. A Task Force that broadly represents appropriate segments of the mathematical sciences community should be established on a continuing basis to deal with curricula.

2. A nationwide collection of local teacher support networks should be established to link teachers with their colleagues at every level and to provide ready access to information about all aspects of school mathematics.

3. Prognostic tests should be used to measure the progress of students.

4. A writing workshop should be held to prepare a series of assistance pamphlets and course guides for schools.

5. Strong efforts are needed to increase public awareness of the importance of mathematics.

6. Funding agencies should support projects to improve remedial education and should hold a series of regional conferences to address the problems of remedial education.

7. New initiatives are needed that address the special problems of faculty renewal at each level.

The participants recognized that the above recommendations needed appropriate coordination and follow through; thus they agreed that a National Mathematical Sciences Education Board should be established.

CONFERENCE SCHEDULE

MONDAY DECEMBER 5
8:00 - 10:00 PM

Opening Session
Thomas A. Romberg, Chair

Welcoming Remarks
John R. Palmer
Improving Mathematics Instruction: A Key to Excellence in Education
Donald J. Senese
The Status of Mathematics Teaching in American Schools
Stephen Willoughby
Mathematics in American Schools
Henry Pollak
Change and Options in School Mathematics
Thomas A. Romberg
Discussion

TUESDAY DECEMBER 6
8:30 - 12:00

Testimony on Mathematics in the School Curriculum
Edward Esty, Chair

New Goals for Mathematical Sciences Education
Henry Pollak
The Impact of Computers on Mathematics
Lud Braun
The Importance of Statistics in School Mathematics
Bill Hunter
Needed Changes in Mathematics Curricula
Zalman Usiskin

1:00 - 4:30

Testimony on Learning and Teaching
F. Joe Crosswhite, Chair

Research on Learning
Robert Siegler
Implications of Cognitive Science to Instruction
Robbie Case
Implications of Learning Research to School Mathematics
Thomas Carpenter
Research on Teaching
Penelope Peterson
Implications of Research to Mathematics Teachers
Glenda Lappan

4:30 - 5:30

Formation of Working Groups

Elementary Junior High Mathematics
Jane Gawronski, Chair
Senior High Mathematics
Edward Esty, Chair
Learning and Teaching
Thomas Carpenter, Chair
Computers and Technology
Arthur Melmed, Chair

WEDNESDAY DECEMBER 7
8:30 - 10:45

Working-Group Meetings Continued

10:45 - 11:00

Break

11:00 - 12:00

Report of Working Groups
Thomas A. Romberg, Chair

1:00 - 5:30
Room 259

Testimony on Policy Implications and Impediments
Jim Gates, Chair

The Problems of Change in Relationship to the Preparation of Mathematics Teachers
Robert Williams
The Problems of Change from the Publishers Perspective
Vivian Makhmaltchi
The Problems of Change from the Test Developer's Perspective
Chancey Jones
The Problems of Change from the Materials Producer's Perspective
William Barclay
The Problems of Change from the Perspective of a Mathematics Supervisor
Marilyn Hala
The Problems of Change from the Perspective of a School Administrator
Jane Gawronski

THURSDAY DECEMBER 8
8:30 - 4:00

Strategy Group Meetings