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ABSTRACT

An extensive body of research indicates that men on the average achieve higher scores in mathematics than women. Despite this extensive research, conclusions about sex differences in mathematics achievement have suffered from various inadequacies in the use of measures of association among coursework experiences, sex, and other correlates of mathematics achievement. This paper addresses these issues by estimating a latent-construct causal model of mathematics achievement with a mixed matrix of tetrachoric, polyserial, and product-moment correlations. This model of mathematics achievement is a block-recursive model. Mathematics achievement of high school seniors is considered to be a function of sophomore mathematics and verbal abilities, attitudes toward mathematics, and exposure to mathematics. The model was tested using data from the first follow-up of the nationwide, longitudinal study "High School and Beyond." Based on a covariance-structures model of the process of mathematics achievement and appropriate measures of the associations among the variables in the model, it was found that this process differs for men and women. (Author/BW)

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A STRUCTURAL MODEL OF SEX DIFFERENCES IN MATHEMATICS ACHIEVEMENT USING TETRACHORIC AND POLYSERIAL MEASURES OF ASSOCIATION

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An extensive body of research indicates that men on the average achieve higher scores in mathematics than women. Despite this extensive research, conclusions about sex differences in mathematics achievement have suffered from various inadequacies in the use of measures of association among coursework experiences, sex, and other correlates of mathematics achievement. This paper addresses these issues by estimating a latent-construct causal model of mathematics achievement with a mixed matrix of tetrachoric, polyserial, and product-moment correlations.

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On tests of mathematics achievement, men, on the average, achieve higher scores than women, but men and women typically start their high school mathematics programs with equal mathematical abilities (Armstrong, 1981; Pallas and Alexander, 1983). Efforts to explain why this difference develops have resulted in an extensive body of research with no consistent conclusions. Maccoby (1966) and Fennema and Sherman (1977) propose that this difference is due to differential socialization processes. This argument is supported by Sherman (1980), who shows the importance of sex-role factors, as indexed by attitudinal variables, in the development of sex-related differences in mathematics achievement. Benbow and Stanley (1982), however, find few sex differences in attitudes and little relationship between attitudes and achievement. Others have concluded the difference is due to the pattern of quantitative coursework taken by men and women (Pallas and Alexander, 1983; Wise, Steel, and MacDonald, 1979), yet Armstrong (1981) and Benbow and Stanley (1980, 1983) reject this hypothesis. Benbow and Stanley (1980) favor the hypothesis that sex differences in achievement result from superior male mathematical ability and are apparently supported in their conclusions by the work of Geschwind (1982).

An analytic approach often taken in this realm of research has been to examine differences between men and women in the extent to which explanatory variables are correlated with mathematics achievement

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(e.g., Fennema and Sherman, 1977; Pallas and Alexander, 1983). These correlational and regression analyses have been conducted using Pearson product-moment correlations and yet, at times, have included dichotomous or polychotomous variables. Thus, conclusions resulting from these studies have suffered from various inadequacies in the use of zero-order measures of association. One such inadequacy involves the measure of association between sex and variables such as those indicating enrollment in mathematics courses. These latter variables are included in studies as measures of the extent of exposure to mathematical concepts, and as such, are dichotomous variables with assumed underlying continuities. Sex, however, has no underlying continuous distribution; as a result, there is no ideal measure of the association between these variables and sex.

A second inadequacy involves measures of association among dichotomous and ordered polychotomous variables even with assumed underlying continuities. Carroll (1961) and Muthen (1983), among others, have shown that Pearson product-moment correlations underestimate the relationships among such variables, particularly when they are highly skewed, and suggest tetrachoric and polychoric correlations as more appropriate measures. Olsson, Drasgow, and Dorans (1982) support the use of tetrachoric and polychoric correlations in such situations, and go on to discuss a third area in which inadequate measures of association have been used. Their studies indicate that Pearson correlations also underestimate the true relationship between categorical variables with underlying continuities and continuous variables measured on interval scales. Again, the degree of underestimation is greatest for associations involving highly skewed variables. Olsson, et al. (1982) propose the use of polyserial correlations as more appropriate measures of association in these instances.

The purpose of this paper is to estimate a latent-variable structural equation model of the process o mathematics achievement, including variables identified in earlier studies as being significantly related to mathematics achievement. To test for sex differences in the process of mathematics achievement, the model is estimated separately for men and women, and comparisons made between corresponding parameter estimates. The model used here does not approach the complexity of those of previous studies, but addresses the question of sex differences in mathematics achievement in a new light, using an analysis that takes into account the correlational problem and the interpretive value of a structural equation analysis. The model is estimated using the unweighted least squares (ULS) method of LISREL (Joreskog and Sorbom, 1983) to analyze a mixed matrix of tetrachoric, polyserial, and Pearson correlation coefficients.

THE STRUCTURAL MODEL

The model of the process of mathematics achievement proposed in this paper is a block-recursive model, which is diagrammed in Figure 1. Mathematics achievement of high school seniors is considered to be a function of mathematics and verbal abilities measured when the respondents were sophomores, attitudes toward mathematics, and exposure to mathematics. The mathematics and verbal ability variables are considered exogenous variables, which are correlated for reasons unanalyzed in this model. Attitudes toward mathematics and exposure to mathematics form a block of endogenous variables, each considered dependent on the exogenous mathematics and verbal ability variables and a residual disturbance term uncorrelated with the independent variables. No causal nexus is specified between mathematics attitudes and exposure. While one might argue that enrollment in math courses is likely to affect attitudes toward mathematics, an equally plausible argument may be made that these attitudes affect decisions to enroll in math courses. Thus, specifying any unidirectional causal relationship between these factors would be inappropriate. Their disturbance terms, however, are allowed to covary, reflecting the extent to which the association between them is not explained by their mutual causes explicit in the model. In any event, the effects of mathematics and verbal abilities on both of these variables are expected to be positive.

Insert Figure 1 About Here

Mathematics achievement is seen to be dependent on the two exogenous variables, the preceding endogenous variables, and a disturbance term that is specified to be uncorrelated with the independent variables. Positive effects on achievement are expected from all predetermined variables. Mathematics exposure and mathematics ability are expected to have the strongest influence on mathematics achievement.

THE DATA

Data for this study were drawn from the first follow-up of "High School and Beyond" (HSB), a nationwide, longitudinal study of high school sophomores and seniors in 1980 sponsored by the National Center for Education Statistics. For this investigation, only the sophomore cohort has been used. These data are particularly appropriate for this study in that they provide measures of verbal and mathematics ability when the respondents were sophomores, prior to enrollment in higher level mathematics courses, and also provide a measure of mathematics achievement taken when the same respondents were seniors, after completing most of their high school mathematics program. A description of these data can be found in the user's guide prepared by the National Opinion Research Center (1983). The test battery for HSB was designed in part to allow for the measurement of school effects and cognitive change in a longitudinal framework (Heyns and Hilton, 1982). All test scores used in these analyses represent formula scores that have been corrected for guessing.

In the causal model analyzed here, the first exogenous variable, mathematics ability, had a single manifest indicator, SOPHMATH, and was specified to be measured with perfect reliability. SOPHMATH is the formula score of a 28-item Mathematics, Part 1, test taken in 1980 by high school sophomores. It is a measure of mathematics ability for sophomore respondents prior to the typical completion of not more than one year of algebra. Formula scores for this test ranged from -9.34

to 28. The second exogenous variable, verbal ability, was indexed by two manifest variables, SOREAD and SOVOCAB, which are formula scores of tests measuring reading and verbal abilities of the 1980 sophomore respondents. SOREAD is a 20-item test with scores ranging from -4.75 to 19, and SOVOCAB is a 21-item test with scores ranging from -5.25 to 21.

The factor representing exposure to mathematics was measured by four dichotomous variables indicating enrollment in high school courses in algebra II, geometry, trigonometry, and calculus. These variables were recoded such that a value of 1 indicated that they had enrolled in the course, and 0 indicated that they had not. Thus, higher scores on the latent factor would indicate greater exposure to mathematical concepts, while lower scores would indicate less exposure.

The attitude factor was indexed by four variables that are responses to statements concerning feelings toward mathematics classes and assignments. The respondents were asked if they felt at ease in mathematics classes, if doing math assignments makes them feel tense, if math classes don't scare them, and if they dread mathematics classes. These dichotomous variables, ATEASE, TENSE, NOSCARE, and DREAD, respectively, were coded or recoded such that 1 represented a positive attitude, and 0 represented a negative one.

Finally, mathematics achievement was measured by a single manifest indicator, SRMATH, which is the formula score on the 10-item Mathematics, Part 2, test from the sophomore cohort test battery. The test was taken during the 1982 follow-up survey, when the respondents were high school seniors. Designed to measure achievement, this test had scores ranging from -3.33 to 10.

The analyses reported here were based on 16,555 respondents who did not have learning disabilities and who had complete reports for all of the variables used in the analysis. There were 7643 men in the sample and 8912 women.

METHODOLOGY

The covariance-structures causal model of the process of mathematics achievement analyzed here is a general LISREL model defined by three sets of equations:

Structural Equation Model: $\eta = B\eta + \Gamma\xi + \zeta$

Measurement Model for y: $y = \Lambda_y\eta + \varepsilon$

Measurement Model for x: $x = \Lambda_x\xi + \delta$

In order to determine if the process of mathematics achievement was the same for men and women, the structural portion of the model was compared across groups. The usual approach used to compare such models is first to estimate the model separately for the groups, applying maximum-likelihood procedures to analyze covariance matrices and obtaining the structural parameter estimates therefrom.[1] A comparison of the corresponding parameter estimates can then be made by progressively applying equality constraints, and examining the likelihood-ratio chi-square statistic for evidence of deterioration in the fit of the model. The use of maximum-likelihood procedures, however, requires the assumption that the observed variables have a multivariate normal distribution. For the model proposed here, several manifest variables

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are highly skewed, dichotomous variables, which necessarily make the usual approach inappropriate.

Thus, to determine if the process of mathematics achievement was the same for men and women, we had to employ an alternative method of comparing coefficients across groups. We first generated matrices consisting of tetrachoric, polyserial, and Pearson product-moment correlations, obtaining more accurate measures of the associations among the observed variables than those provided by Pearson correlations alone (Carroll, 1961; Muthen, 1983; Olsson, Drasgow, and Dorans, 1982). Using these matrices, the model was then estimated separately for men and women, utilizing the unweighted least squares (ULS) method in LISREL. ULS estimates are obtained by an iterative procedure that minimizes a definite fitting function, which, unlike the fitting function for maximum likelihood, is justified without distributional assumptions for the observed variables. The only assumption necessary is that the observed variables are measures of underlying latent normally distributed variables. An examination of the goodness-of-fit index, root mean square residual, and residual matrix for each group gave an indication of the apparent fit of the model for each group. To compare parameter estimates across groups, the usual procedure would call for successively applying equality constraints to corresponding elements of the structural parameter matrices for men and women, and examining the likelihood-ratio chi-square statistic for indications of equality of slopes. Using ULS estimates, however, precludes this approach since methods are not available for testing the significance of changes in ULS measures of goodness-of-fit. Thus, at this point we found it necessary to apply an ad hoc procedure for comparing the two groups.

Using the ULS estimates of the variances and covariances of the latent factors, the correlations among the latent variables were calculated for both men and women. The model was then re-estimated for both groups using these correlations and ignoring the measurement portion of the model. Assuming a multivariate normal distribution among the latent factors, maximum-likelihood estimates of the parameters and the associated likelihood-ratio chi-square statistic were obtained. Equality constraints were then applied to corresponding elements of the structural parameter matrices. Points at which significant changes appeared in the chi-square statistic indicated that parameters constrained at those points were different for men and women. It must be emphasized that this was an ad hoc procedure, testing <u>not</u> for equality between parameter estimates for the full model, but only for those estimates obtained by analyzing the matrices containing the correlations among the latent factors.

RESULTS

Raw data for the 7643 men and 8912 women in the HSB survey who had complete reports for the variables were used in LISREL to generate a matrix of tetrachoric, polyserial, and Pearson product-moment correlations for each group. These matrices, shown in Table 1, were subsequently used as input into LISREL, and ULS estimates of the covariance-structures model of the process of mathematics achievement were obtained for both men and women. Goodness-of-fit indices of .993

and .990 together with root mean square residuals of .045 and .050, for men and women, respectively, indicated a fairly good fit of the model for both men and women. The correlations among the latent variables were then computed for both men and women. These correlations, shown in Table 2, were used to re-estimate the models, ignoring the measurement portion. Subsequent results reported here are in relation to the maximum-likelihood estimates among the latent variables.

Insert Tables 1 and 2 About Here

The structural parameter estimates, shown in Table 3, indicate as expected that mathematics ability and exposure to mathematics were the most influential predictors of mathematics achievement. With regard to the predictors of mathematics exposure and attitudes, it had been hypothesized that the effects from both mathematics and verbal ability would be positive. In the event, it was found for both men and women that verbal ability had a negative effect on attitudes toward mathematics. Apparently higher verbal abilities lead to less favorable attitudes toward mathematics net of the influence of mathematics ability.

Insert Table 3 About Here

While all effects from predetermined variables on mathematics achievement were significant for both men and women, the parameter estimates were different for the two groups. At issue, however, is whether these differences were due to sampling error or to real differences between men and women in the process of mathematics achievement. To answer this question, each of the eight structural coefficients was successively constrained to be equal between the groups. Following each constraint, the difference in likelihood-ratio chi-square statistics from the more constrained to the less constrained model was computed, and when a significant difference occurred, the parameters constrained at that point were considered to be different. Those variables would therefore be considered to interact with sex in the explanation of mathematics achievement.

The above procedure is admittedly ad hoc. In order to substantiate our conclusions about interactions in the process of mathematics achievement, we went back to the original measurement and structural equation model, which had been estimated by the method of unweighted least squares. Equality constraints for men's and women's structural parameter estimates were then successively applied to the full model. Since the chi-square test is inappropriate for ULS estimates, changes in the root mean square residuals were examined. In all cases where previously we had found significant chi-square changes in the model of latent factors, the root mean square residuals for the full model were found to increase for both men and women. Both methods, therefore, led to the same conclusions about differences in structural parameters between men and women.

As a result of applying equality constraints across groups, four structural coefficients were found to be different for men and women.

These were the effects of verbal ability on exposure to mathematics, of verbal ability on attitudes toward mathematics, of attitudes toward mathematics on mathematics achievement, and of mathematics ability on mathematics achievement. The influences of verbal ability in this model are particularly interesting, for there is an equally strong positive effect of verbal ability on mathematics achievement for both sexes, but higher verbal abilities lead to more positive attitudes toward mathematics for men than for women. In turn, attitudes also have a stronger influence on achievement for men than women. Thus, taken in total, higher verbal abilities lead to greater increases in mathematical ability for men than for women.

CONCLUSIONS

Based on a covariance-structures model of the process of mathematics achievement and appropriate measures of the associations among the variables in the model, we have found that this process differs for men and women. While the variables included in this model do not exhaust the possible experiential differences between men and women, the interactions found between sex and the variables included indicate that the process for men and women is not simply additive, and may be more complicated than previous researchers have assumed.

The role of attitudes in the process of mathematics achievement is an example of the complexities involved. We found attitudes toward mathematics to be less negatively influenced by verbal abilities for men than for women, and, unlike Benbow and Stanley (1982), found attitudes to have a significant influence on achievement that is stronger for men than for women. We also found that men appear to take advantage of prior mathematics ability to a greater extent than do women. These results imply that questions about male-female differences in mathematics achievement may have no meaning unless one asks the question in relation to specific values of prior ability and educational experiential variables.

The causal model used in this study is by no means the definitive model of the process of mathematics achievement. Other, equally plausible, models have been and will be used to explain why men and women differ in mathematics achievement. Future researchers, however, should take particular note of the possibility that the process of mathematics achievement is not an additive one, and accordingly need to take into account possible interactions between sex and other explanatory variables in the process of mathematics achievement.

There is also the question of choice of measures of association. Since models of the process of mathematics achievement often include dichotomous or ordered polychotomous variables, the use of Pearson product-moment correlation coefficients may underestimate the relationships among the variables, and tetrachoric, polychoric, and polyserial correlation coefficients are the more appropriate choices for measures of association. The use of these correlations with LISREL at present, however, requires the application of ad hoc procedures in order to compare structural coefficients across groups. Work in progress, however, should alleviate this deficiency in the near future.

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FOOTNOTES

1. Although the desired procedure involves the comparison of metric slopes across groups, we have here based our analysis on standardized regression coefficients calculated from a mixed matrix of product-moment, tetrachoric, and polyserial correlations. The reason should be obvious, since there is no direct relationship between tetrachoric correlations and their corresponding covariances. One may wonder whether analyzing standardized slopes has affected the results reported here. We do not think so. The standard deviations of the variables included in the model do not differ very much between men and women; consequently, the relative magnitudes of the standardized and metric coefficients across groups would be almost exactly the same.

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Table 1. Product-Moment, Tetrachoric, and Polyserial Correlations for Variables in Model of Mathematics Achievement; 1980 High School Sophomores

	SOPHMATH	SOREAD	SOVOCAB	ALG2	GEO	TRIG	CALC	SRMATH	ATEASE	TENSE	NOSCARE	DREAD
SOPHMATH	---	.678	.643	.607	.668	.673	.660	.712	.281	.221	.113	.270
SOREAD	.666	---	.721	.484	.544	.551	.526	.562	.195	.149	.037	.180
SOVOCAB	.643	.713	---	.472	.542	.544	.529	.532	.142	.104	-.005	.126
ALG2	.571	.458	.447	---	.809	.842	.691	.574	.275	.226	.101	.280
GEO	.638	.534	.537	.780	---	.879	.749	.627	.189	.178	.020	.229
TRIG	.617	.494	.485	.823	.855	---	.868	.657	.335	.284	.164	.360
CALC	.616	.490	.483	.659	.734	.869	---	.630	.347	.300	.198	.354
SRMATH	.658	.534	.513	.535	.578	.599	.581	---	.275	.218	.117	.266
ATEASE	.189	.086	.057	.193	.101	.258	.335	.201	---	.657	.676	.644
TENSE	.151	.065	.024	.120	.065	.196	.281	.143	.695	---	.564	.633
NOSCARE	.077	-.019	-.031	.076	-.009	.125	.205	.082	.722	.654	---	.473
DREAD	.183	.091	.048	.219	.154	.310	.329	.199	.670	.634	.534	---

Note: Correlations below the main diagonal are for women (N = 8912); correlations above the main diagonal are for men (N = 7643).

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Table 2. Correlations Among Latent Factors in Model of Mathematics Achievement

	SOPHMATH	SOPHVERB	MATHATT	MATHEXP	MATHACH
SOPHMATH	---	.778	.295	.695	.712
SOPHVERB	.775	---	.195	.663	.645
MATHATT	.190	.067	---	.368	.324
MATHEXP	.646	.610	.271	---	.673
MATHACH	.658	.620	.197	.621	---

Note: Correlations below the main diagonal are for women; correlations above the main diagonal are for men.

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Table 3. Structural Coefficients for Model of Mathematics Achievement for Men and Women

Predetermined Variables	Dependent Variables					
	Men			Women		
	MATHATT	MATHEXP	MATHACH	MATHATT	MATHEXP	MATHACH
SOPHMATH	.363	.453	.374	.344	.434	.304
SOPHVERB	-.088	.311	.149	-.199	.274	.207
MATHATT			.080			.049
MATHEXP			.285			.285
Coefficient of Determination	.090	.520	.581	.052	.448	.515

All coefficients are at least twice their standard errors.

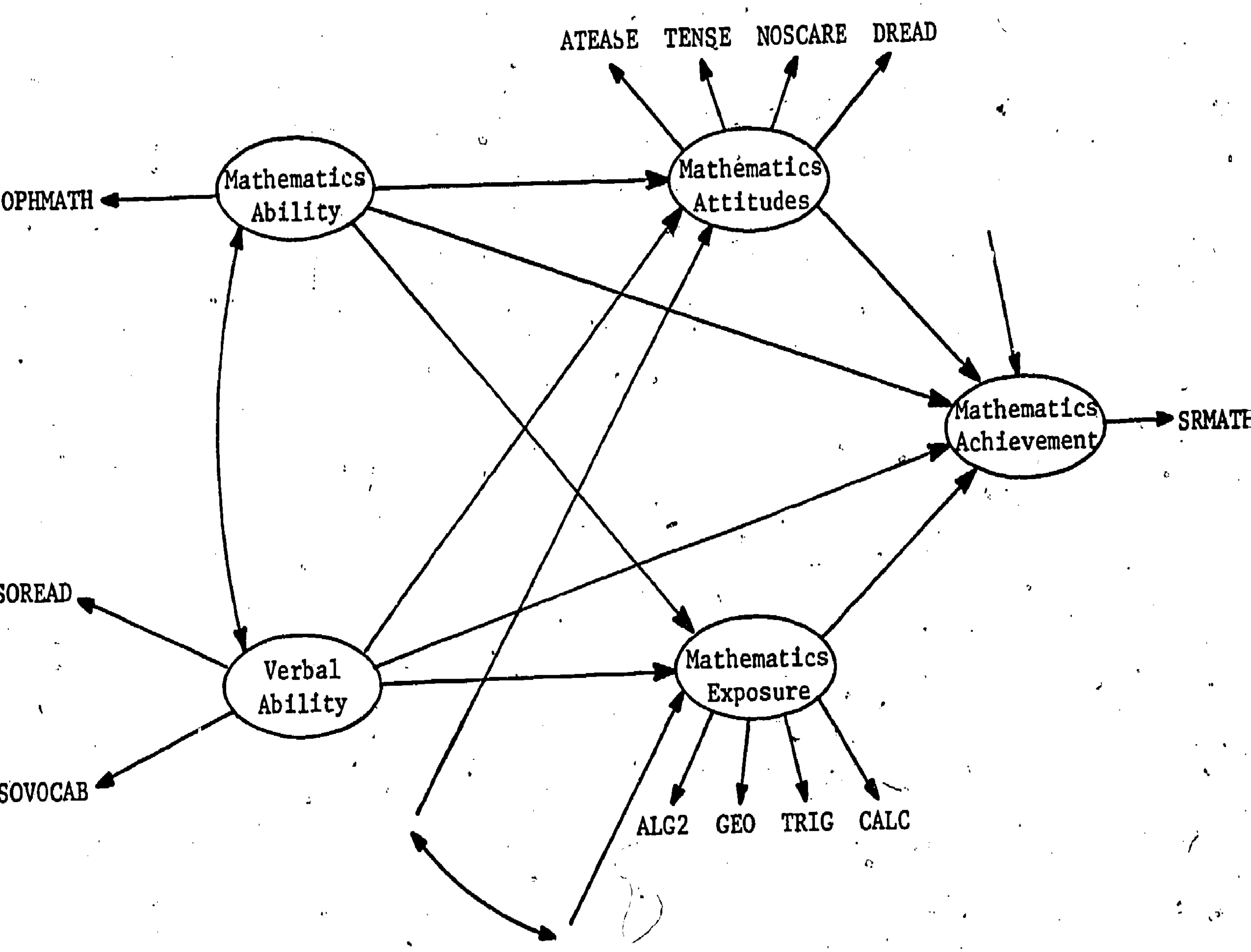

Figure 1. Structural Equation and Measurement Models of Mathematics Achievement