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ABSTRACT

This issue of "Investigations in Mathematics Education" contains: (1) a review of E. Fischbein's book "The Intuitive Sources of Probabilistic Thinking in Children;" (2) nine abstracts of research studies in mathematics education; (3) a list (by EJ number) of mathematics education research studies reported in the January to March 1984 issues of "Current Index to Journals in Education" (CIJE); and (4) a list (by ED number) of mathematics education research studies reported in the January to March 1984 issues of "Resources in Education" (RIE). The studies abstracted focus on the influence of different styles of textbook use on instructional validity of standardized tests; directional effect in transformational tasks; a comparison of scaling and correlational analysis of perceptions of mathematics objectives; attitudinal differences between students in general mathematics and algebra classes; homework assignments, mathematical ability, and achievement in calculus; engaged student behavior within classroom activities during mathematics classes; the acquisition of addition and subtraction concepts in grades 1 through 3; an evaluation of a process-oriented instructional program in mathematical problem-solving in grades 5 and 7; and mathematics anxiety, instructional method, and achievement in a survey course in college mathematics. (JN)

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Volume 17, Number 3 - Summer 1984

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Summer 1984

Fischbein, E. THE INTUITIVE SOURCES OF PROBABILISTIC THINKING IN CHILDREN. Boston: D. Reidel Publishing Co., 1975.

Review prepared for I.M.E. by THOMAS A. ROMBERG, University of Wisconsin-Madison, USA and DAVID PIMM, The Open University, England.

Introduction

This is a thought-provoking book by an eminent psychologist involved in the study of human thinking. What Professor Fischbein has to say should be of interest to researchers and graduate students in mathematics education for several reasons. First, it is about mathematical intuition, a topic of considerable interest which has not received enough scholarly attention. Second, the ideas grow out of and are an elaboration of aspects of developmental psychology. Furthermore, Fischbein's analyses and findings should be of interest to cognitive scientists. Third, probability is a topic gaining in importance in mathematics. Mathematics educators should be interested in research findings about intuitions related to the learning and teaching of probability. Fourth, Fischbein, in contrast to many psychologists, does not view mathematics simply as a collection of concepts to be mastered. He sees mathematics as a language and set of ideas one uses to reason about problems. Thus, mathematicians will find this a sympathetic book to their ideas. And finally, for these reviewers, this book is a rare document in that it portrays the features of a real research program. For novices, too often research constructs and methods are portrayed in terms of single studies, significant findings, etc. Concentrating on results from single studies distorts the process of inquiry. Real research involves models of phenomenon and a series of studies designed to clarify or answer questions about the phenomena. This book, although not written to portray the features of a research program, does it by example. For all of these reasons, we believe this volume should be in all mathematics education libraries.

Organization and Purpose of the Book

Following a brief laudatory preface by Hans Freudenthal, the book contains eight chapters and four appendices. The latter are not really appendices but are reports of four studies which support the thesis of the acquisition of intuitions and illustrate the process of inquiry.

Preface. Professor Freudenthal contrasts Fischbein's notions about the acquisition of intuitions (learned rather than inbuilt) with the usual concept attainment point of view. He suggests a shifting of stress from concept acquisition to reasoning. He argues that this is desirable within the mathematics education community. Furthermore, he believes Fischbein's views are an important contribution in this change in emphasis.

Chapter I. Introduction. This brief chapter (four pages) presents a number of varied reasons for studying probability. Fischbein discusses the general scientific usage of probability. He argues that behavioral phenomena are stochastic in nature, he points to the increase in the teaching of probabilistic notions at all levels, and he claims that probability is a good way to teach independence and creativity in mathematics. However, his primary interest is with intuitions. As a psychologist he defines intuition as a stabilized action programme which is a hierarchical process in the organism which can control the order in which a sequence of operations is performed (p.20). In addition, "intuition presupposes a set of distinguishing features which confer specificity on it" (p.31). It is these features of instruction that Fischbein is attempting to describe.

Chapter II. Intuition and Intelligence. This is the most important and most interesting chapter of the book, for here the central theoretical constructs are presented. Its title is a misnomer in that its sole concern is with intuitions, although later it is argued that "intuitions cannot be accounted for outside of the mechanisms and tendencies of intelligence as a whole" (p. 65). Fischbein assumes that intuitions are generated and developed rationally. Intuitions are not a priori, but are autonomous cognitive processes. He then discusses various divisions of intuitions into species: a trichotomy into pre-operational, operational, and post-operational, and then two (not necessarily exclusive) dichotomies of primary or secondary intuitions and affirmatory and anticipatory ones. Apart from a couple of examples to illustrate these splits, most of these splits are not used in the rest of the work.

He points out well that intuitions need not be correct, in spite of the fact that they represent stored and therefore verified experience. He claims we have only a meagre intuitive substrate with respect to probability. His prime interest is now made clear and that is the study of intuitions and their relation to action. In fact, he argues that what is critical is "the relationship between intuition and action in that human behavior is itself probabilistic" (p. 17). He goes on to claim:

> The responses of an individual cannot be reduced to either built-in stereotypes, such as instincts, or acquired stereotypes, such as classical conditioned reflexes. The complexity of circumstances frequently compels the individual to respond on the basis of a global intuitive estimate of odds. Such statistical intuitions are an intrinsic feature of behaviour. (p. 17)

Furthermore, he claims that the curriculum of probability learning must take into account the intuitive substrate.

> We therefore believe that the introduction of new curricula in schools should be preceded by research into the primary intuitive substrate of the relevant subject. The primary intuitions may facilitate the assimilation of new knowledge if they correspond to scientific truth, on the other hand, if they do not correspond to scientific truth, they may impede the assimilation of new knowledge. (p. 18)

From this introductory argument the two somewhat disconnected topics are present which comprise the book: probabilistic models of learning, and the organization of conceptual schemas in the domain of probability. Unfortunately for the reader, the language of probability enters the discussion in three ways: first, as the stimulus set of concepts to be learned; second, as a description of intuitive thinking (i.e., intuition is basically probabilistic in its development); and finally, as a mathematical model of the thinking process (i.e., a probability model for response generation). Note that in the last two cases other content could be used to examine questions in those areas. This trichotomy of use of probabilistic terminology, although undoubtedly clear to the author, can be confusing to the reader.

He believes learning about probability concepts is a good place for study of interactions. He then raises the question mentioned by Freudenthal in the preface, namely could conceptual understanding of probability benefit from practical training? He ends with five thought-provoking hypotheses on the ontogenesis of probabilistic behavior:

1. We can hypothesise the existence of a natural intuitive substrate for the notions of chance and probability, because the day-to-day experience of the child comprises stochastic processes. If intuitions provide the mechanism whereby intelligence can rapidly insert itself into the flux of practical or mental action, then we can assume that day-to-day experience would create this adaptive tool in the pre-operational child.

2. If intuitions are syntheses of individual experience, probabilistic behavior should develop in step with general intellectual development.

3. The formation of a natural intuitive substrate must be distinguished from the development of secondary intuitions which are the result of systematic instruction. Since the intuitive substrate of probabilistic thinking is relatively poor (and, as we shall see, contradictory) the problem of the formation of secondary probabilistic intuitions is particularly important from the point of view of mathematics curricula.

4. If the theory of probability is supported by a specific intuitive substrate, and if this substrate is largely to

be acquired through the process of education, then the teaching of probability theory should start at the concrete operational level, or at the latest during the period of organisation of formal operations (12-14 years).

5. It is clear, however, that before the novel intuitions and conceptual system of any branch of science can be imparted through educational procedures, it is necessary to know the primary intuitive substrate underlying the science. (pp. 17-18)

Chapters III, IV, and V constitute a distinct part of the book. Here Fischbein presents the conceptual basis of probability learning theory and examines in detail the spontaneous responses of children to stochastic sequences of stimuli with fixed frequencies.

Chapter III. Probability Learning. In contrast to the last chapter which we found challenging, this one was disappointing. Fischbein found it important to consider a mathematical model of learning (specifically W. K. Estes' probabilistic theory of learning based on stimulus sampling theory). Although Fischbein's review of that theory is adequate, its connection to instruction is not well argued.

Probability learning is a variant on simple conditioning with an intermittent schedule of reinforcement. The intuition of relative frequency is discussed and classified as primary, anticipatory, and not affirmatory and pre-operational. His aim is to use the data on probability learning to formulate and illustrate hypotheses about the nature and development of intuitions as a whole. The only connection to instruction is in terms of probability matching as an expression of a particular intuition, relative frequency.

Chapter IV. Probability Learning in Children. This chapter is a detailed review of literature, organized around four aspects of probability being;

(1) asymptotic and maximising behaviour as a function of age;
(2) the role of reward and punishment;

(3) the role of instructions;

(4) recency effects and sequential analysis.

Overall, this is a well-documented, carefully argued review of the psychological literature on these aspects. Unfortunately, since one concern was curricular, he did not review the teaching or curriculum work on this topic. Nevertheless, in summarizing the literature he posed the following seven findings from the review:

> (1) A tendency to match input probabilities in probability learning tasks is manifest at all age levels studied, down to three years of age.
>
> (2) The rate at which the proportion of choices reaches the input probability level across trials increases with age. The input level is reached, at the latest, by 5-6 years.
>
> (3) Reward induces a maximisation tendency which becomes stronger with age.
>
> (4) Between the ages of 7 and 9 there is a tendency toward stereotyped responses, particularly alternating responses. After the age of 11, however, predictions are determined more by patterns extrapolated from antecedent sequences of events.
>
> (5) Older children increasingly seek more sophisticated strategies, based on the conviction that there are rules determining random sequences.
>
> (6) Probability matching behaviour in children is subject to generalisation in the same way as classical conditioning.
>
> (7) Prior instruction in the concept of chance and probability--as well as in some simple procedures of probability computation--improve probability matching performance in probability learning tasks. This finding supports the hypothesis that there is a rudimentary conceptual organisation underlying probability behaviour and spontaneous probabilistic behaviour in general. (pp. 56-57)

<u>Chapter V. The Intuition of Relative Frequency</u>. Here Fischbein claims that probability matching is the expression of a particular intuition, namely that of relative frequency. Negative recency effect, why it is important, and its use as a strongly-held but erroneous intuition is discussed, overall. This chapter contains a more general

discussion of research and mentions only a few research studies. It concludes with an interesting discussion of the heuristics of availability and representativeness. Here Fischbein argues that

> the intuition of relative frequency, though correct in many situations, is in fact influenced and biased by a variety of conditions. He then suggests that two categories of such disturbing factors should be distinguished. First, there are factors which are extrinsic to the psychological mechanisms of statistical evaluations. Availability is an example. The errors in this case are not due to an incorrect probability judgement as such, but to the initial information on which the judgement is based.
>
> The second category contains errors which are due to the mechanisms of evaluation per se. The heuristic of representativeness belongs to this category. The errors in this category are due primarily to insufficient knowledge of the theory of probability. What is significant is the fact that the errors are not blind errors. They are generally determined by the subject's tendency to interpret randomness as though it were rationally governed. Representativeness, the search for clear interpretable patterns (e.g. the intuition of relative frequency) may be explained as being caused by the effort of human intelligence to make the random more reasonable, in the absence of sufficient mathematical knowledge. (p. 64)

Chapters VI and VII form a second, and for us a more interesting, distinct part of this book. These are two long chapters which describe a series of experiments concerning aspects of the conceptual organization of probability from a neo-Piagetian developmental perspective.

Chapter VI. Estimating Odds and the Concept of Probability. In this chapter, several experiments are discussed. First, chance and necessity are seen as a pair of polar intuitions. Here Fischbein reviews Piaget's classic work on the concepts change and necessity and then extends that research (Piaget and Inhelder, 1951). Fischbein and his associates report two results from a well-done study reported in Appendix 1 that

> Well before the operational stage, the child possesses an intuition of chance, and carries out intuitive estimation of odds, although

the absence of operationally structured thought preclude the conceptual structuring of this intuition, which is complementary to the intuition of necessity.

At the level of formal operations, according to Piaget and Inhelder, there will be an improvement in the estimation of probabilities. In fact, however, as our experiments have shown, with increasing age the estimations become poorer: pre-school children give the highest percentage of correct responses, when compared with 12-13 year-olds, in situations with equiprobable outcomes. With increasing age, the responses became more erratic, more hesitant, and more frequently incorrect. (p. 22)

Furthermore, they explain these findings in terms of instruction by arguing that schools inculcate the notion of univocal determinism. At the operational level, the child looks for causal relations which will permit univocal predictions, even when the objective situation provides no evidence of such relations. Evidently, chance implies to older children nothing but ambiguity and uncertainty, and thus denotes the failure of cognitive efforts. The pre-school child is less disturbed by ambiguity. The child approaching adolescence is in the habit (inculcated by instruction in physics, chemistry, mathematics, and even history and geography) of seeking causal relations which can justify univocal explanations (pp. 72-73).

Second, on estimation of odds, Fischbein makes the distinction between making predictions while "knowing the structure of the conditions" and predictions based on estimation. He then presents an excellent review of a series of studies on the topic of systematic instruction. His summary of the studies on estimation of odds shows the clear relationship between instruction and level of reasoning.

Pre-school children possess a natural intuition of chance and the quantification of chance, but, at this age, only estimations based on binary comparisons are possible. Instruction does not bring about any significant improvement in this respect.

If appropriate instruction is given at the level of concrete operations, children can learn to compare odds by means of a quantitative comparison of ratios.

At the level of formal operations, these estimations are carried out directly. The difficulties encountered by the

> intelligence in acquiring and using probabilistic concepts are explained in part by certain fundamental lacunae within the set of intuitions relevant to probability, and in part by an increasing tendency of maturing intelligence to seek univocal causal explanations. (p. 98)

In addition, one serendipitous discovery was that teachers wrongly advised students. In fact, they found it was more difficult to make teachers understand the concepts of probability than to make their pupils understand them. The teachers wrongly corrected tests on several occasions; the children had given the correct responses, but the teachers had interpreted the questions wrongly. This is an important finding, since it demonstrates the loss with age of certain intuitive faculties. An adolescent has better chances of rebuilding an intuitive structure than an adult (p. 92).

Chapter VII. Combinatorial Analysis. In this chapter, Fischbein relates combinatorial ability to logical thought. He begins by challenging the conclusion of Piaget and Inhelder that combinations are not available until the level of formal thought. From his detailed analysis of the results obtained by Piaget and Inhelder he made the following conclusions.

First, not all subjects at the level of formal operations were able to discover the method of constructing combinations. Subjects were not able to deal satisfactorily with arrangements until the age of 13, and they did not find a method for dealing with permutations until the age of 14-15 years. Fischbein concluded that

> This indicates that during the stage of formal operations (12-15 years) the intellectual capacities required for combinatorial operations are continuing gradually to develop, and this development is not, in fact, completed during this stage. (p. 105)

Second, he claims

> the experimental design used by Piaget and Inhelder incorporated a learning factor, since the gradual increase in the set size of elements suggested a particular method to the subjects. It is

> therefore quite natural to wonder what would happen if one intended in the developmental process (which, in its natural form, seems to be quite slow and laborious) by offering the adolescent a systematic combination technique. (p. 105)

From this analysis he poses the following problem: is it possible that systematic instruction could accelerate the acquisition of the set of operational schemas needed? An important aspect of this process would be that it would require the acquisition of structures, and not of specific information or particular procedures.

The teaching strategy Fischbein chose to follow to answer this question he called the "prefiguration of structures strategy." This strategy expresses the necessity (not merely the possibility) of preparing for the assimilation of abstract structures by prefiguring these structures in the previous stage of intellectual development to that in which they are normally assimilated, but which uses the methods appropriate to this prior stage (p. 109). He then argues that by using adequate methods of prefiguration, it is possible not only to prepare for the next stage of development, but to accelerate development toward the new stage.

Such prefiguration can be accomplished by creating generative models which have the following properties:

> (1) If with a limited number of elements and rules for their combination, it can correctly represent an unlimited number of different situations.
>
> (2) It must be heuristic. It must lead to solutions which must be valid for the original as a result of the genuine isomorphism between the two realities involved (i.e., the model and the original).
>
> (3) It must be capable of self-reproduction, in that its image-concept coding is sufficiently general for it to be able to suggest new models (p. 110).

A good example of such a model is the tree diagram used in combinatorial analysis and probability theory. Fischbein then refers to a study (reported in Appendix IV) on the extent to which children's assimilation of tree diagrams could accelerate their acquisition of combinatorial operations. At all ages they got spectacular results. In fact, they found that even at the level of formal operations, combinatorial techniques were not spontaneously acquired. Instruction was necessary (p. 115).

<u>Chapter VIII. Summary and Conclusions</u>. This, too, is an excellent chapter. The summary starts by examining the notions of chance, relative frequency, estimation of odds, effect of instruction, and combinatorial operations for three developmental levels of reasoning: pre-operational, concrete operations, and formal operations. Fischbein then concludes that:

> (1) Intuitions are cognitive components of intelligent behaviour which are adapted, in their function and properties, to ensure the efficiency of behaviour. They are stable, structural schemas which select, assimilate and store everything in the experience of the individual which has been found to enhance rapidity, adaptability, and efficiency of action. (p. 125)

> (2) In the contemporary world, scientific education cannot be profitably reduced to a univocal, deterministic interpretation of events. An efficient scientific culture calls for education in statistical and probabilistic thinking. Probabilistic intuitions do not develop spontaneously, except within very narrow limits. The understanding, interpretation, evaluation, and prediction of probabilistic phenomena cannot be entrusted to primary intuitions which have been neglected, forgotten, and abandoned in a rudimentary state of development under the pressure of operational schemas which cannot articulate with them.

> But in order for this requirement of an efficient scientific culture to be met, it is necessary to train, from early childhood, the complex intuitive base relevant to probabilistic thinking; in this way a genuine and constructive balance between the possible and the determined can be achieved. (p. 131)

> (3) In order to be effective, the teaching of a subject should be preceded by a survey of the intuitive ground, just as the construction of a building is preceded by a survey of the nature and potential resistance of the ground on which it is proposed to build it. (p. 139)

Chapter VIII is then followed by the complete reprints of the reports of four related and well-done studies which support the overall thesis of the book. These studies are:

Appendix I — Fischbein, E., Barbat, I., & Minzat, I. (1971). Primary and secondary intuitions in the introduction of probability. Educational Studies in Mathematics, 4, 264-280.

Appendix II — Fischbein, E., Pampu, I., & Minzat, I. (1967). The child's intuition of probability. Enfance, 2, 193-206.

Appendix III — Fischbein, E., Pampu, I., & Minzat, I. (1970). Comparison of rations and the chance concept in children. Child Development, 41, 377-389.

Appendix IV — Fischbein, E., Pampu, I., & Minzat, I. (1970). Effects of age and instruction on combinatory ability in children. The British Journal of Educational Psychology, 40, Part 3.

Final General Comments

This is an excellent but not totally coherent book. It is a collection of chapters with some, but not enough, continuity between the different parts. A large proportion of the work is an extensive review of several literatures. The important core of the book is Chapter II.

The audience for the book is developmental psychologists (not mathematicians or mathematics educators). For example, Fischbein provides lengthy discussions of the probabilistic settings which could have been omitted for mathematicians. He expects the reader to be thoroughly versed with both the conceptual frameworks and methodology of European developmental research and assumes the readers will be familiar with that tradition. Unfortunately, for these reasons American readers are likely to find it a difficult book to read.

Nevertheless, Fischbein has a lot to say, particularly to today's researchers interested in the relationship between cognitive processing and instruction.

Reference

Piaget, J. and Inhelder, B. (1951). La genese de l'idee de hasard chez l'enfant. Paris: PUF.

Freeman, Donald J.; Belli, Gabriella M.; Porter, Andrew C.; Floden, Robert E.; Schmidt, William H.; and Schwille, John R. THE INFLUENCE OF DIFFERENT STYLES OF TEXTBOOK USE ON INSTRUCTIONAL VALIDITY OF STANDARDIZED TESTS. Journal of Educational Measurement 20: 259-270; Fall 1983.

Abstract and comments prepared for I.M.E. by RANDALL I. CHARLES, Illinois State University.

1. Purpose

The purpose of this study was to examine the degree to which the match in textbook-test content varies as a function of how a teacher uses the book.

2. Rationale

In a content analysis of textbooks and tests of fourth-grade mathematics (Freeman, Kuhs, Porter, Floden, Schmidt, & Schwille, 1983), the authors found that the match between the content covered by texts and the content on standardized tests was better for some textbook-test pairs than for others. In other words, a student's opportunity to learn tested content varied according to the text used. Subsequent case studies by the authors found five different styles of textbook use. Since teachers use texts in different ways, the authors conjectured that a student's opportunity to learn tested material might also vary according to the teacher's style of using the text.

3. Research Design and Procedures

The authors identified five styles of text usage from year-long case studies of seven elementary school teachers (grade levels are not reported.)

I. Textbook bound. Here the teacher would start the school year on page one and progress page-by-page through the book over the course of the year.

2. Selective omission. The teacher progresses lesson-by-lesson with this approach but completely omits some chapters, most typically geometry, advanced work with fractions, and topics they believe will be emphasized in later grades (e.g., decimals).

3. & 4. The basics with and without measurement. Here the teacher introduces students only to the content he or she believes to be the "basics." The "basics" for this study included a review of addition and subtraction, introduction or refinement of skills for multiplication and division, and introductory work with fractions. Some teachers included measurement among the basics.

5. Management by objectives (MBO). The content delivered with this approach is determined by a list that correlates specific textbook exercises with specific instructional objectives. Twenty-three objectives for the fourth-grade mathematics program were identified for this study by examining a particular school district's objectives. These objectives reflected minimum competencies in mathematics.

One fourth-grade text was used in this study [Holt School Mathematics (Nichols, Anderson, Dwight, Flournoy, Kalin, Schluep, & Simon, 1978)], and five standardized tests of fourth-grade mathematics were selected for analysis: (a) Comprehensive Test of Basic Skills (CTBS-I & CTBS-II), Level I/Grades 2.5-4.9 and Level II/Grades 4.5-6.9, McGraw-Hill, 1976; (b) Iowa Test of Basic Skills, Level 10/Grade 4, Houghton Mifflin, 1978; Metropolitan Achievement Tests, Elementary Level/Grades 3.5-4.9, Harcourt Brace Jovanovich, 1978; and the Stanford Achievement Test, Intermediate Level/Grades 4.5-5.6, Harcourt Brace Jovanovich, 1973.

A three-dimensional taxonomy of elementary school mathematics was used to analyze the content of the text and the five tests (Kuhs, Schmidt, Porter, Floden, Freeman, & Schwille, 1979). All interrater correlation coefficients were greater than .94.

Two measures of "instructional validity" and one measure of "instructional focus" were defined and calculated for each pairing of textbook usage style and standardized test. Instructional validity is a measure of the "opportunity to learn" the content of the tests. The measure is the percent of items on a test that would be covered with a particular style of text use. Instructional validity was calculated at two levels, content covered (at least 3 items in the text) and content emphasized (at least 20 items in the text). Instructional focus reflects the relative emphasis that topics included on a test receive in instruction. The measure is the percent of textbook problems covered by the particular method that are represented by the items on the test.

4. Findings

(a) Both measures of instructional validity were far lower for the MBO approach than any of the other styles of text use.

(b) The selective omission and basics with measurement approaches had almost the same and the highest instructional validity for content coverage and the basics with measurement approach had the highest validity for content emphasized.

(c) For three of the tests (CTBS-I, Iowa, Stanford), instructional validity was not generally affected by the other four styles of text use (excluding MBO). For two of the tests, instructional validity was affected by the style of text use.

(d) The MBO approach ~~devotes~~ the highest proportion of instructional time to tested content across all five tests (i.e., instructional focus). Approximately one to two full lessons of additional practice was provided on each topic with the MBO approach.

(e) The other four styles of text use did not differ significantly in the level of instructional focus.

5. Interpretations

(a) "Although the MBO system provides greater depth of coverage of the test topics it considers, it is clearly inferior to the other four styles of textbook use in the match it provides in content taught and content tested" (p. 268). "To the extent that the district is concerned about performance on standardized tests of achievement, steps must be taken to ensure that all students receive instruction in mathematics beyond the curriculum defined by minumum competency objectives" (p. 269).

(b) "For some, but not all, standardized tests of achievement, the match in content taught and content tested will vary across the other four styles of textbook use considered in this investigation" (p. 268).

Abstractor's Comments

The authors of this study should be commended for addressing an interesting and important research question. Teachers should know that the way in which they use their mathematics text may affect a student's opportunity to learn the variety of content included in school mathematics programs and may affect a student's performance on a standardized test of achievement. However, beyond this statement, it is difficult to draw any conclusions from this research report.

Because of the limitations of this research, the authors even note that the data summarized in this report should be viewed as "illustrative, rather than definitive evidence, of variation in the level of instructional validity of tests that may result from differences in how a textbook is used" (p. 268). Three of the important limitations of this study are that only one text was used, one grade level was examined, and "definitions of instructional validity were based on arbitrary standards for describing the content of instruction" (p. 268).

In addition to these limitations, five specific styles of textbook use were used to select content domains and these types may not generalize to all teachers. For example, many teachers I work with use a "selective omission" approach where specific lessons are omitted, not always or only entire chapters as in this study. Many textbooks contain optional lessons and even optional chapters to assist teachers in making decisions about which content may be omitted. Also, the authors' definition of an MBO system must be considered when interpreting what appears to be fairly definitive conclusions about the failings of minimum competency programs.

The authors' conclusions seem rather strong considering the design of this study. The fact that the five instructional approaches used in this study came from case studies of only seven teachers should be considered when assessing the generalizability of these approaches. A final concern I have about the design of this study is the content analysis taxonomy used for the texts and tests. There are some cells in the taxonomy that I cannot interpret. For example, what kind of problem would be in the place value-geometry cell? A different scheme for categorizing the content of texts and tests may produce quite different findings from those reported in this research.

In spite of my concerns about this particular study, this investigation can serve as a starting point for more definitive research concerning the types of content decisions teachers make relative to their textbook, how and why they make these decisions, and what the effects of these decisions are on students' understanding and mastery of mathematics.

References

Freeman, D. J., Kuhs, T. M., Porter, A. C., Floden, R. E., Schmidt, W. H., & Schwille, J. R. (1983). Do textbooks and tests define a national curriculum in elementary school mathematics. Elementary School Journal, 83: 501-513.

Kuhs, T. M., Schmidt, W. H., Porter, A. C., Floden, R. E., Freeman, D. J., & Schwille, J. R. (1979). A taxonomy for classifying elementary school mathematics content (Research Series No. 4). East Lansing, MI: Institute for Research on Teaching, Michigan State University.

Nichols, E. D., Anderson, P. A., Dwight, L. A., Flournoy, F., Kalin, R., Schluep, J., & Simon, L. (1978). Holt School Mathematics. New York: Holt, Rinehart & Winston.

Schultz, Karen A. and Austin, Joe Dan. DIRECTIONAL EFFECTS IN TRANSFORMATION TASKS. Journal for Research in Mathematics Education 14: 95-101; March 1983.

Abstract and comments prepared for I.M.E. by JOHN G. HARVEY, University of Wisconsin-Madison.

1. Purpose

This study determined the effect of direction of movement on the difficulty of slide, flip, and turn transformation tasks for first-, third-, and fifth-grade students. Because the tasks were spatial visualization tasks, sex-related performance differences were investigated.

2. Rationale

Content from transformation geometry is included in some mathematics textbooks, tests, and research tasks. The direction in which the transformed object is moved may affect childrens' understandings (Schultz, 1978). This finding provided the impetus for the study of the effect of the direction of movement on performance.

It is unclear whether there are sex-related differences in performance on spatial visualization tasks. The available, conflicting evidence motivated the search for sex-related differences.

3. Research Design and Procedures

The subjects were all but 26 of the 131 first-, third-, and fifth-grade students enrolled in a metropolitan Atlanta (Georgia) school. Thirty subjects (13 female, 17 male) were in first grade; 35 subjects (18 female, 17 male), in third grade; and 40 subjects (23 female, 17 male), in fifth grade. The teachers in the school indicated that these students had received no formal instruction in transformation geometry.

Twenty-six students (18 in first grade; 7 in third grade; 1 in fifth grade) were excluded because they were unable to complete successfully an initial task: copying a fixed object in several different orientations.

The materials used were two 8-cm square sheets of Plexiglas and two sets of sailboat pieces (i.e., hull and sail). One set of pieces was glued onto one of the Plexiglas sheets to form the shape of a sailboat.

Each task consisted of the interviewer (a) placing the sailboat-attached Plexiglas sheet with the other (clear) Plexiglas sheet atop it in front of the subject, (b) as the subject watched, transforming the clear sheet using a slide, flip, or turn and a movement; (c) asking the subject to place the unattached sailboat pieces on the clear sheet to show the result, on the sailboat, of the transformation; and (d) recording the subject's placement of the pieces. Subjects were not told whether their placements of the sailboat pieces were correct

Fifteen tasks (3 transformations x 5 directions) were presented to each subject during the 20- to 30-minute individual interview session. The five directions were horizontal-right (HR), horizontal-left (HL), vertical-up (VU), diagonal-up-right (DUR), and diagonal-up-left (DUL). The response to each task was scored on a scale from 0 to 4. A score of 0, 1, or 2 was assigned to the subject's placement of each sailboat piece; these two scores were summed to obtain the score for that task. The score given to the placement of a piece was as follows: (a) a score of 2, if the piece was both correctly located and correctly oriented; (b) a score of 1, if the piece was correctly located or correctly oriented; and (c) a score of 0, if the piece was neither correctly located nor correctly oriented.

All of the responses of 12 randomly selected subjects were scored both by one of the investigators and a university faculty member not otherwise involved in the study. The correleation coefficient of these independently determined scores was 1.00.

The task response scores were summed across type of transformation and across all of the tasks to produce three subscores and a total score. Coefficient alpha internal consistency reliability estimates were computed, by grade, for each subscore and the total score. With one exception, the reliabilities were greater than 0.60; the first-grade flip subscore reliability was 0.11.

A repeated measures ANOVA was used to analyze the data. Each response of each subject was used as a data point. The 15 measures were classified using two factors: type of transformation and direction of movement. Grade and sex were grouping variables. When the ANOVA indicated there were significant interactions, the Bonferroni t-test was used to examine further the interactions between the variables.

4. Findings

There were significant differences in performance between grade-levels ($F = 10.55$, $p < .001$, $df = 2$), types of transformations ($F = 139.15$, $p < .001$, $df = 2$), and direction of movement ($F = 48.08$, $p < .001$, $df = 4$). There were no statistically significant sex-related effects. In general, mean performance improved as the grade level increased, was highest on slide tasks, and was lowest on turn tasks. There was no clearcut trend in the direction of movement data across grades and transformation tasks, but in general, mean performance was highest on either the horizontal or vertical movement tasks and lowest on the diagonal movement tasks.

Of greater interest were the three significant interactions between the variables. One was the interaction between the type of transformation and grade level (T x G) ($F = 3.00$, $p < .05$, $df = 4$); a second, the interaction between type of transformation and direction of movement (T x D) ($F = 46.83$, $p < .001$, $df = 8$); and the third, the interaction between direction of movement, grade level, and sex (D x G x S) ($F = 3.20$, $p < .01$, $df = 8$). The result of the Bonferroni comparisons of the T x G and T x D interactions are reported in Tables 1 and 2.

Table 1
Bonferroni Comparisons for
Transformation-Type-by-Grade Interaction

Grade 1:	Slide	Flip	Turn
Grade 3:	Slide	Flip	Turn
Grade 5:	Slide	Flip	Turn
Slide:	Grade 5	Grade 3	Grade 1
Flip:	Grade 5	Grade 3	Grade 1
Turn:	Grade 5	Grade 3	Grade 1

Note. In each row, entries are in declining order of unweighted means. Underlining joins entries whose unweighted means did not differ significantly.

(Schultz & Austin, 1983, p. 100)

Table 2
Bonferroni Comparisons for
Transformation-Type-by-Direction Interaction

Slide:	H-L	D-U-L	V-U	D-U-R	H-R
Flip:	H-L	V-U	H-R	D-U-L	D-U-R
Turn:	D-U-L	H-L	V-U	H-R	D-U-R
H-L:	Slide	Flip	Turn		
D-U-L:	Slide	Turn	Flip		
V-U:	Slide	Flip	Turn		
D-U-R:	Slide	Turn	Flip		
H-R:	Slide	Flip	Turn		

Note. In each row, entries are in declining order of unweighted means. Underlining joins entries whose unweighted means did not differ significantly.

(Schultz & Austin, 1983, p. 100)

Analysis of variance of the D x G x S interaction by grade showed that the direction of movement by sex interaction was significant ($F = 4.43$; $p < .01$; $df = 4$) at Grade 1; this significant interaction appeared to be due to higher performance by boys on VU tasks and by girls on HR and HL tasks. Bonferroni comparisons indicated that these sex-related differences on VU, HR, and HL were not statistically significant.

5. Interpretations

The investigators concluded that their results support the following conclusions:

1. The scorer reliability and internal consistency estimates for the three subscales and the test were high enough "to warrant further use and study of this evaluation procedure" (p. 98).

2. The type of transformation by direction of movement interaction suggested that the direction of movement affected student performance. This effect was particularly evident for diagonal transformations.

3. The easiest transformations for students to visualize seemed to be slides. Direction of movement influences the difficulty of flip and turn transformations.

4. There were no significant sex-related differences. The D x G x S interaction seemed due to differences in first-grade performance. Since the first-grade flip score data had a low reliability (i.e., 0.11), this three-way interaction would need replication to make possible a clearer understanding of this result.

Abstractor's Comments

This is a well-planned, well-executed study of elementary school students' performance on transformation geometry tasks. It clearly shows, for the students examined, that the direction in which an object is moved during a transformation must be considered as well as whether the task is a slide, flip, or turn. Like most studies, the conclusion could be doubted if one engaged in "what if". For example, what if the object used had not been a sailboat or what if the student had been permitted to manipulate one or both sheets of Plexiglas instead placing the sailboat pieces on the transformed, clear Plexiglas sheet? In the first instance, it can be argued that a sailboat of the kind used is a common object in the child's environment and that the two pieces are very dissimilar from each other and so were probably not confused with each other by the subjects. In the second instance, it is possible that performance would have been higher had children been permitted to move the clear Plexiglas sheet (e.g., to return it to its original position atop the other sheet and then, to repeat the transformation made by the interviewer). However, it would have been difficult to define clearly to subjects what they were and were not permitted to do and so to obtain valid data had they been permitted to manipulate one or both of the Plexiglas sheets. In addition, it is difficult to believe that, even if performance did improve, the important outcomes (cf., "Findings") would have been altered.

Therefore, there is little to criticize about this study. The following questions should be raised in connection with it.

1. This is a status study of students' performance. As the authors point out, an important next step would be to investigate the effect of instruction on that performance.

2. If students are permitted to manipulate one or both of the Plexiglas sheets, is performance affected? In particular, do the significant differences between grade-level, type of transformation, and direction of movement performance and the significant type of task by direction of movement interaction still occur?

3. Are the subjects in this study typical of elementary school students elsewhere? The authors did not characterize them well enough to tell.

4. Were there status variables which should have been measured and used in the analyses of the data? In particular, are there differences in performance between students of differing mathematical achievement?

5. Why are there such noticeable differences between the vertical/horizontal movement tasks and the diagonal movement ones?

6. What algorithms, if any, are students using to determine the effect of the transformation on the object?

Reference

Schultz, K. A. (1978). Variables influencing the difficulty of rigid transformations during the transition between the concrete and formal operational stages of cognitive development. In R. A. Lesh & D. B. Mierkiewicz (Eds.), Recent research concerning the development of spatial and geometric concepts. Columbus, OH: ERIC/SMEAC.

Nagy, Philip and Drost, Dale. A COMPARISON OF SCALING AND CORRELATIONAL ANALYSIS OF PERCEPTIONS OF MATHEMATICS OBJECTIVES. Alberta Journal of Educational Research 29: 180-195; September 1983.

Abstract and comments prepared for I.M.E. by PHILLIP HUGHES, The University of Tasmania, Hobart, Tasmania, Australia.

1. Purpose

The study has both methodological and substantive purposes. In methodology, the purpose is to investigate the feasibility of multi-dimensional scaling to analyse perceptions of the priorities among curricular objectives. The second purpose is to apply that approach to a particular curriculum area, high school mathematics.

2. Rationale

The study arises from the familiar problem for those involved in curriculum reform: obtaining what they see as appropriate changes in teaching in schools. A possible reason for this problem was seen as differences in perceptions on curriculum priorities between teachers and curriculum designers.

3. Research Design and Procedures

Data were collected by a preference questionnaire, and perceptions of priorities were analysed in two distinct ways: by correlational and by multi-dimensional scaling techniques. The instrument was derived from nine major content areas in mathematics, with each area contributing two objectives, one at a lower level of cognitive skill, the other at a higher level. These were used to develop a questionnaire, composed of all possible pairs of objectives, placed in random order, with a random half of the pairs reversed. Subjects were asked to indicate which number of each pair they considered more important. Three groups of teachers were asked to respond, high school teachers, vocational teachers, and university teachers. Unfortunately, response rates were not high.

4. Findings

In the analysis, results of the within and between group differences based on a correlational analysis were compared with the results of a multi-dimensional scaling analysis, using the SINDSCAL algorithm. The correlational analysis exhibited both commonalities and differences in perception but without a clear pattern emerging. There was, both in this and in the scaling analysis, evidence of preference for higher cognitive levels. The two approaches showed considerable agreement.

Abstractor's Comments

The substantive results of the study can only be described as disappointing. They would not, on their own, justify the complexity of the methods used. The approaches, however, have sufficient intrinsic interest to justify the paper. It is to be hoped that these approaches might be further developed, as the objective of the study is worthwhile even though the specific results are inconclusive.

Prawat, Richard S.; Lanier, Perry E.; Byers, Joe L.; and Anderson, Ariel L. H. ATTITUDINAL DIFFERENCES BETWEEN STUDENTS IN GENERAL MATHEMATICS AND ALGEBRA CLASSES. Journal of Educational Research 76: 215-220; March/April 1983.

Abstract and comments prepared for I.M.E. by JERRY P. BECKER, Southern Illinois University at Carbondale.

1. Purpose

The purpose of the study was to explore the differences in attitude that characterize students in general mathematics classes and those in algebra classes.

2. Rationale

The researchers explore attitudinal differences using two kinds of measures: one assesses some status variables (e.g., self-esteem) and the other assesses how individuals view the group. The former variables are associated with mathematics tracking in previous research. The second kind of measure has been used in research showing that measures of students' attitudes towards classroom environment are related to achievement. Further, the researchers assert that classroom "climate" may exert a strong influence on individual attitudes. In particular, it was thought by the researchers that algebra students would view the learning climate in much more positive terms than general math students.

3. Research Design and Procedures

Subjects were 209 ninth-grade students in 13 different general math and algebra sections (7 in a middle-upper class high school and 6 in a lower to middle class junior high school). The two samples were representative of their community populations. Subjects were pretested in the fall of the school year and posttested in the spring using a battery of individually linked attitudinal measures. Testing took place over a two-day period in intact classrooms. The following instruments were used: Mathematics Attitudes, Conceptions of Mathematics, Self-esteem, Locus of Control, and Achievement Motivation.

A Learning Environment Inventory, measuring social psychological climate, was also administered at posttesting time, but not at pretesting because students had insufficient time to form opinions about their classes.

4. Findings

A series of analyses of variance was done on the status variables, with time of measurement, school, sex, and curriculum as factors. The only significant main effect emerged on the measure of self-esteem with, not surprisingly, algebra students responding more positively than those in general mathematics. There was no change in the group differences from pretest to posttest. Time of testing effects were hugely positive for mathematics attitudes and conceptions of mathematics from pretest to posttest.

A multivariate analysis of variance was performed on the Learning Environment Inventory data, with 15 climate dimensions as dependent variables. School, sex, and curriculum were factors. There was a significant main effect on the curriculum factor (general math and algebra) and there was a significant multivariate curriculum x school interaction. Cohesiveness, Diversity, and Cliqueness were prime contributors in the interaction. More cohesiveness was perceived in algebra classes. Regarding Diversity and Cliqueness, differences between algebra and general math students were relatively greater in the higher social economic school compared to the lower, and algebra students perceived their classes to be more diverse and cliquish than general math students. Regarding the main effect for curriculum, univariate significance was reached for Apathy, Friction, Difficulty, Speed, Goal Direction, Cohesiveness, and Diversity. This main effect is discussed in detail by the researchers, but will not be further reported here.

5. Interpretations

In short, and after looking at all the data, the researchers cautiously report that attitudes relating to the classroom learning environment are more affected by curricular manipulation than are attitudes relating to self. Concerning the overall pattern of results, the researchers wonder why lower-track students appear so similar to higher-track students in their school attitudes and in their attitudes towards mathematics. The similarity in individually linked attitudes between both groups is difficult to reconcile, say the researchers, with the divergent views of the two groups regarding classroom learning environment.

Data in the study are absent of evidence bearing on the connection between lower-track students' negative appraisal of the classroom atmosphere and their own achievement motivation. Finally, the researchers comment that the best explanation of the overall pattern of results for this study is that lower-track students seem to unquestionably accept their fate, which is not a very healthy response.

Abstractor's Comments

This investigation was well-planned and certainly reported very well. For example, the results are discussed at length and in detail and are tied in with findings in studies done by other researchers. It seems as though the researchers have a pretty good grasp of related research. Data were examined from virtually all directions and all the possible interpretations of the results of analyses were fleshed out, reported, and discussed. And what does it all boil down to? As the researchers comment, lower-track students just seem to accept their fate. And that isn't good.

Hirsch, Christian R.; Kapoor, S. F.; and Laing, Robert A. HOMEWORK ASSIGNMENTS, MATHEMATICAL ABILITY, AND ACHIEVEMENT IN CALCULUS. Mathematics and Computer Education 17: 51-57; Winter 1983.

Abstract and comments prepared for I.M.E. by GERALD D. BRAZIER, Pan American University, Edinburg, Texas.

1. Purpose

This paper investigates the effect that the structure of homework assignments has on achievement in a college calculus course. In particular, the "distributive pattern" of making assignments is compared to the more standard pattern.

2. Rationale

In the distributive pattern, daily review of past topics is incorporated into the homework assignment schedule. As the authors point out, the effectiveness of distributive assignments has been well established (even by the authors!) in many settings, but no published research exists based on a college calculus setting.

3. Research Design and Procedures

The students in two intact sections of first-semester calculus were the participants in the study. Items taken from the Cooperative Mathematics Tests (ETS) on Algebra II, Algebra III, and Analytic Geometry were used as a measure of pre-calculus competence. Each of the sections was taught by the same instructor with the same content and using the same lecture-discussion method. Assignments were made daily, consisting of 8-10 exercises with about half of them having answers available to the students. These assignments were collected about twice a week and selected exercises were graded. What differed between the sections were the actual exercises assigned. For the control group, the assigned exercises were related only to the topic covered that day. For the experimental group, half of the exercises for a unit (except for the first and last days) were directly related to that day's topic and the rest were related to previously taught topics of that unit.

An investigator-constructed test was given at the end of each of the four units. During the last week an unannounced comprehensive test consisting of selected items from Cooperative Mathematics Tests (ETS) on Calculus, Parts I and II, and Vervoort's Calculus Test was administered.

4. Findings

Achievement data were analyzed using a linear regression model with analysis of covariance. The pre-calculus achievement test was the covariate for each of the five analyses (four unit tests and the comprehensive test). For Test 3 and the comprehensive test, homogeneity of regression was satisfied, but the analysis of covariance did not show a significant difference between the control and experimental groups on the adjusted means. For Tests 1, 2, and 4, the slopes of the regression lines were significantly different, indicating a significant ($p < .03$) interaction between pre-calculus achievement and the treatment. In each of the three cases, the regression line for the control group rose more sharply than for the experimental group.

5. Interpretations

The authors point out that the significant interaction between pretest and treatment fits the classic ATI pattern in which a certain kind of instruction "levels out" the effect of an aptitude or background on achievement. In such a situation, the weaker student is helped by the special instruction while the stronger student gains more benefit from the standard approach. The results of the study point clearly to the potential benefit of a distributive pattern of homework assignments for the average or below-average student.

Abstractor's Comments

The study is technically quite good. The problem is well-defined and related to previous research. The design and experimental procedures are acceptable and the data analysis follows the recommended pattern for ATI studies.

There are no theoretical issues raised by the study and there is no reason to believe a priori that the effectiveness of distributive pattern homework assignments would be different for a calculus setting. This is not a criticism of the study, because extension of results to different settings is an important contribution to mathematics education. What is true, however, is that the study breaks no new ground and does not provoke interesting questions, at least for me.

The only difficulty I found in reading the study was a lack of detail on the implementation of a distributive assignment pattern--how to spread out the exercises, etc. There is ample literature referred to by the authors that supplies that information for anyone interested in replication or simply employing the technique in their own teaching.

In summary, the study, though small in scope and significance, is very well done and contributes to our knowledge--if only more published research could say the same.

McIntyre, D. John; Copenhaver, Ron W.; Byrd, David M.; and Norris, William R. A STUDY OF ENGAGED STUDENT BEHAVIOR WITHIN CLASSROOM ACTIVITIES DURING MATHEMATICS CLASS. Journal of Educational Research 77: 55-59; September/October 1983.

Abstract and comments prepared for I.M.E. by MARILYN N. SUYDAM and doctoral students, The Ohio State University.

1. Purpose

The purpose was to examine student engaged and non-engaged behaviors in mathematics classes within various instructional activities in order to ascertain the activities in which students exhibit the most engaged behaviors. Four research questions were formulated:

a. How do student engagement rates differ across grade levels?
b. How do student engagement rates vary for selected activities?
c. How do student engagement rates for on-task behaviors vary for selected activities across grade levels?
d. Do student engagement rates fluctuate through the week?

2. Rationale

Studies have demonstrated that students who remain on-task during instruction have higher achievement than students who are off-task. Moreover, high achievers appear to be actively involved for more time than low achievers. Researchers have been urged to examine other relationships, such as the distribution of time across activities. The teacher could manipulate those activities and behaviors which result in more student engaged time, thus possibly enhancing achievement.

3. Research Design and Procedures

The sample included all mathematics classes in grades 3, 5, and 7 in a small midwestern school district: 10 classes in grade 3, 12 classes in grade 5, and 7 classes in grade 7. Six students from each class were randomly selected; one seventh grader was omitted because of insufficient observation data, so the study involved 173 students, none of whom was mainstreamed.

The study was conducted during a six-week period from February to April. Each class was observed five times, once each day of the school week. Teachers were not aware of when observations would occur, nor of the identity of observed students. A student was observed for a 10-second interval, during which the behavior and the classroom activity was recorded by a trained observer. Thus, each student was observed once each minute throughout the mathematics class.

An observation form, based on earlier work by others, was developed and utilized to record student behaviors. Engaged behaviors included:

a. attending
b. writing
c. reading
d. raising hand
e. answering questions
f. asking questions
g. talking to peer (regarding subject matter)

Non-engaged behaviors included:

h. walking
i. playing
j. talking to teacher (not regarding subject matter)
k. talking to peer (not regarding subject matter)
l. waiting stalled
m. non-cooperative
n. not attending (not paying attention or listening to instruction)
o. outside distraction (announcement over intercom, student called out of class, etc.)

Observers also recorded the classroom activity occurring simultaneously with the observed behavior:

a. teacher-led
b. seat work
c. small group
d. organizational activities (taking roll, announcements, etc.)
e. activities other than mathematics that occurred during the scheduled class time (reading a library book, studying another subject, etc.)

The basic unit of analysis, student engagement rate, was defined as the percentage resulting from dividing student on-task time by time allocated for mathematics.

4. Findings

There was a slight decline in engagement rate from grades 3 to 5 to 7. Allocated time (47, 43, and 42 minutes, respectively) dropped by five minutes or 11% between grades 3 and 7, while student on-task behaviors dropped by almost 6 minutes (36.19, 32.25, 30.66 minutes) or 15%. With less allocated time and students spending less time on task, lower rates of engagement across grade levels were found (77%, 75%, 73%).

Direct teacher instruction and seat work accounted for over 90% of the allocated time. An additional 7% of the time was devoted to organizational activities.

Engagement rates for teacher-led activities ranged across grade levels from 79% in grade 3 to 77% in grade 5 to 76% in grade 7, and 80%, 77%, and 76% for seat work. The amount of time committed to teacher-led activities (15.98, 15.05, 23.10 minutes) and seat work (26.32, 26.23, 15.54 minutes) in grades 3 and 5 was approximately the reverse of that for grade 7; however, this shift did not significantly alter the engagement rates because on-task behavior time shifted (teacher-led activities - 12.69, 11.61, 17.64 minutes; seat work - 21.15, 20.21, 11.76 minutes).

The pattern of student engagement rates associated with on-task behavior appears to be quite similar for teacher-led and for seat work activities across grades 3, 5, and 7. During teacher-led activities, students had substantial engagement rates (47%, 49%, 44%) for attending to the teacher. Engagement rates for writing (8%, 11%, 11%) and reading (12%, 8%, 14%) were also similar for each grade level. Engagement rates for asking questions and talking to peers were almost non-existent during teacher-led activities (0% to 3%) and low during seat work (2% to 3%).

The pattern of engagement rates during seat work shifted to emphasis on writing behavior (48%, 44%, 43%), while attending rates dropped (13%, 10%, 11%) and reading rose slightly (16%, 15%, 16%). (Raising hand accounted for 5% to 6% of the behavior during teacher-led activities (0% to 3%) and low during seat work (2% to 3%). for 2% to 3% during teacher-led activities and 0% during seat work.)

As to off-task behaviors, the general category of not attending had the highest engagement rates in either teacher-led activities (12%, 11%, 13%) or seat work (11%, 10%, 8%). Talking to a peer and walking around the room seem to increase during seat work, also. (Talking to a peer during teacher-led activities was done by 3%, 3%, and 2%, and during seat work by 3%, 3%, and 5%; for walking around the room, the data were 0%, 3%, and 2% for teacher-led activities and 3% at each level for seat work.) Students were waiting or stalled "a considerable amount" of time (for teacher-led activities, 6%, 6%, 5%; for seat work, 2%, 5%, 8%).

No consistent patterns in engagement rates were found across the days of the week. The rates tend to be higher Monday through Thursday in grades 3 and 5, declining on Fridays. Engagement rates in grade 7 are in marked contrast to the other two grades for Thursday and Friday. While teacher-led activities and seat work had over 75% engagement rates in grades 3 and 5, they fell to below 60% for grade 7 on Thursday. On Fridays, however, the engagement rates for grades 3 and 5 fell to below 75%, while the grade 7 rates rose to 95%.

Five general findings emerged:

1. The dominant classroom activity during mathematics is seat work in grades 3 and 5, while teacher-led activities dominate in grade 7.
2. Attending, writing, and reading are the predominant engaged student behaviors in mathematics classes.
3. Percentage of waiting/stalled behavior appears to increase in each grade during seat work.
4. No consistent pattern or fluctuation was found across the days of the week.
5. As one progresses from grades 3 through 7, an increase in teacher-led activities and a decrease in seat work is paralleled by a decrease in overall engagement rate in mathematics classrooms.

5. Interpretations

(1) That the dominant classroom activity during mathematics is seat work corroborates findings in the Beginning Teacher Evaluation Study (BTES). Grades 3 and 5 are responsible for seat work being the dominant activity; in grade 7, teacher-led activities dominate. Engagement rates for these two activities are consistent across grade levels; despite differences in activity time and on-task behavior time for each activity, engagement rates were the same (77%). Engagement rates decreased slightly as grade level increased, but the pattern is consistent despite the shift in times devoted to the two types of activities in grade 7.

These findings are supported by previous research which indicated that teacher-led activities seem to dominate in the secondary grades. The BTES only examined grades 2 and 5 and therefore did not detect changes between elementary and secondary school. Future studies should examine this factor more closely, determining the activities that generate the most engaged time for elementary and secondary students in mathematics classes.

(2) Data from BTES for grade 2 and from this study for grade 3 differ markedly on the minutes allocated to mathematics, engagement rate, and engaged minutes, as well as the time spent in teacher-led activites and seat work. Further study can determine if actual differences exist. There is remarkable stability in the fifth grade across both studies.

(3) Further research is needed on the relationship between teacher effectiveness characteristics and activities used in the classroom.

(4) Attending appears to be the dominant behavior during teacher-led activities, while writing is dominant during seat work. The amount of time devoted to reading remains fairly constant no matter what the classroom activity. Future studies should investigate the possibility that a relationship between reading, writing, and gain scores in mathematics may exist, as they do in reading studies.

(5) The percentage for waiting/stalled behavior seems to increase in each grade level during seat work. It may be that the problems assigned during seat work become progressively more difficult as grade level increases, resulting in more waiting for teacher assistance.

(6) Although the authors had conjectured that the engagement rate would peak on Wednesday and then decline through Friday, this did not occur.

(7) Does a relationship actually exist between engagement rate and type of activity? Previous research indicates that direct instruction is positively related to achievement, while this study implies that as teacher-led activities increase, engaged student time decreases. Is the decline in engagement rate a function of the grade level? That is, does the approach to instruction at elementary and secondary levels differ enough to affect the engagement rate?

(8) The results of this study contribute to knowledge of what is occurring in classrooms, and should prove useful in teacher training at both pre- and in-service levels.

Abstractor's Comments

[When an IME reviewer declined to review this study, with stated reasons, I (having only skimmed the study at that point) was puzzled about his comments. So I had a seminar group of doctoral students analyze it. The following is a compilation of the comments of Claire Crook, Alfinio Flores, Carol Fry, Patrick Kent, Peter Larsen, Jeanette Palmiter, Endang Russeffendi, Dennis Shaw, and Margaret Sooy.]

When a study purports to examine student behaviors during mathematics instruction, the attention of mathematics educators is drawn. That attention is hardly warranted by this study. Failure to designate the mathematics content that was being taught is a serious omission. The authors are not mathematics educators; they were clearly not sensitive to the differing demands on both teachers and students effected by varying content. This flaw alone makes the study of little value to teachers in general. [As the reviewer who declined to review the study stated, "I would hypothesize that the various student engagement rates described in the report would be vastly different for teacher-led seat work in a class period that had been devoted to the guided discovery of an enrichment topic as opposed to a lesson dealing with maintenance activities in various kinds of computation. The fact that this study did not control for the variable of content causes me grave concern about the meaning of any of the results from the study."]

There appear to be some points in favor of the study: it looks at actual classrooms, observes student behaviors, and provides some descriptive information. However, it does so within decided limits, and fails to address adequately the research questions posed. The set of student behaviors and especially the set of classroom activities are so generalized as to be meaningless. What teacher-led activities or seat work entail was never defined. There are many kinds of teacher-led activities, some of which might be more effective in engaging the students' attention than others. Seat work also comes in many varieties --drill and practice, problem solving, and so on--and these, too, might have different engaged rates. Knowing the percentages of time spent on teacher-led activities or seat work will do little to help teachers plan more effectively so that students ~~will~~ exhibit valued engaged behaviors.

Moreover, there are a number of other flaws in the design, procedures, and interpretations of the results.

* Additional information about the sample would have been helpful. Was the district urban, suburban, or rural? Were there district characteristics that might have affected the results? How were the students assigned to classes? What was the overall ability and achievement of the students? (Were they a representative sample of an identifiable population?) What were the teachers like--their experience, teaching approach, and capability?

* The observation process was not clearly described. What was the reliability of the instrument and of the observers? How well an observer can evaluate what is seen is vital. A student seemingly attentive may in fact be day-dreaming; whether a student is "waiting/stalled" appears

clearly judgmental; and talking to peers, unless the conversation can be heard, could be either on- or off-task.

* Furthermore, what is the difference between "not attending" and "waiting/stalled" behavior? Does "not attending" result in "waiting/stalled"? Does "waiting/stalled" become "not attending"? Do the two behaviors merge so that it is impossible to tell the difference? How does a student get out of the waiting/stalled mode during teacher-led activities, since the data indicate that students do not ask questions during that time? Are they instructed to wait until the seat work time? Again, more information is needed about the nature of the classroom activities in order to see any significance in the data.

* Ten seconds per observation has been used by a variety of observation schedules--but perhaps whether this is a viable length of time should be questioned. For some of the factors under study, the number of observations was also far too small; for example, to determine whether a pattern of behavior exists for different days of the week, one observation each day is not sufficient. One also wonders if the observation form was actually designed to answer the questions posed by the researchers.

* No indication, beyond the use of words such as "significant," was given of whether or not differences were actually significant, either statistically or educationally. Consistency in describing data fluctuations is similarly lacking. Thus, a 3% difference is termed "very similar," while a 2% difference is elsewhere termed "significant."

* Calling attention to the higher rates of question-asking and talking to peers during seat work seems implausible when the highest percentage was 3%. Other data are similarly exaggerated or misinterpreted.

* Changing percentages back to times sometimes gives a different perspective on the data. The statement "talking to one's peer and walking around the room seem to increase during seat work activities" is softened by the knowledge that the increases range from less than half a minute to just over a minute. Similarly, that "students were waiting/stalled a considerable amount of time" is hard to justify when the percent is changed to actual time; that is, to about 1 to 3 minutes.

* The percentage reported for time spent on organizational activities was 7%, but no data are given for the individual grade levels. Presumably, small-group instruction and non-mathematical activities accounted for the remaining 3% of the total time, but these are not mentioned aside from the initial listing.

* It is curious to observe that the difference between allocated time and on-task behavior time is consistently about 11 minutes for each grade level.

* There is an error in the statement of the results of the BTES study: if the fifth graders were allocated 44 minutes for mathematics and had a 74% engagement rate, then they were engaged for 32.6 minutes not 35.

* Minutes are expressed to the hundredths' place, conferring an unrealistic level of precision on the data.

* Why was "waiting/stalled" included in the table on off-task behaviors, with a footnote to indicate that it is an academic behavior? Which is it? Interpretation of some data might change based on the placement of this category, yet no rationale is given for its placement.

* The conclusion that the percentage of waiting/stalled behavior appears to increase in each grade during seat work is not supported by the data: only in grade 7 is it higher.

* Alternative conjectures arise at several points in the discussion. For instance, the statement is made that seat work becomes more difficult at higher grade levels. It could just as easily be concluded that students have learned that it is easier to wait for the teacher to show them how to work the problem than it is to attempt it themselves.

* There is little conjecturing about the variance in engagement rates across the days of the week. In particular, the engagement rates of grade 7 on Thursday (59% and 58%) and Friday (94% and 95%) ~~are~~ suspicious. What mathematics content was being taught in these lessons, and how was it being taught? Were different materials being used? Was a test being given? Were extracurricular activities, an assembly, or other "outside" factors affecting in-class behaviors?

* Was the time of day each class was observed controlled?

* The researchers appeared to be trying to convince the reader of the significance of the research through the use of the words "interesting" and "interestingly" when discussing the results of the study. They obviously did not appear to be used appropriately in many places.

And there is a final comment from the reviewer who declined to review the study; it is included as a general caution to writers: "The authors of this study telegraph what to me is the meager worth of this investigation when they state at least five times in the article that further study or research is needed to determine the actual effect of various variables. They seem to be saying that their work asks more questions than it answers. If this is the case, and I think it is, I fail to see the relevance of this study."

Carpenter, Thomas P. and Moser, James M. THE ACQUISITION OF ADDITION AND SUBTRACTION CONCEPTS IN GRADES ONE THROUGH THREE. Journal for Research in Mathematics Education 15: 179-202; May 1984.

Abstract and comments prepared for I.M.E. by CHARLES E. LAMB, The University of Texas at Austin.

1. Purpose

A primary goal of this research is to describe the major stages in the development of addition and subtraction concepts and skills. A three-year longitudinal study of children's solutions on simple addition and subtraction word problems provides a test of the assumptions underlying recently proposed models of the knowledge and procedures underlying children's solutions to simple word problems.

2. Rationale

The study of children's work on addition and subtraction has been popular since the turn of the century. From these studies, a reasonably well-defined set of children's strategies has emerged. In general, children tend to operate in a manner that models the actions or relationships described in a problem. Basic addition strategies (use of fingers and objects, counting sequences, and memorization of basic facts) and subtraction strategies (separation, missing addend, and comparison) are discussed in detail. Details of proposed models for skill development are also described. Although the study was started prior to the generation of these models, the data do provide an empirical test of the assumptions underlying the models and the models provide a good conceptual framework for analyzing the data collected.

3. Research Design and Procedures

"A three-year longitudinal study was designed to study the processes that young children use to solve simple addition and subtraction word problems and how these processes evolve over time." Clinical interviews were used to collect data. Children were interviewed three times each year in first and second grades (beginning, middle, and end) and twice in the third grade (beginning and middle). The study followed pupils from a point prior to addition and subtraction instruction to a point following algorithmic instruction.

Six basic types of problems were chosen for the study. The problems were administered under six different conditions over the course of the study. Conditions varied due to number size and the availability of manipulative materials. Number triples were assigned to problem types using a 6 x 6 Latin-square design. This gave six sets of problems for each problem condition. The problems were randomly assigned to students.

The subjects were 144 first-grade children in Madison, Wisconsin. All schools used a modified version of Developing Mathematical Processes for their curriculum. Eighty-eight children were in the final sample and all data are presented from this sample. Classroom activities for the period of the study are briefly outlined.

4. Findings

The data indicate that children are not entirely consistent in choice of solution strategies. However, certain patterns did tend to emerge across subjects. The effect of magnitude of numbers did not appear to be significant unless it caused a change in strategy choice for solution of the problem.

In terms of addition performance, the results are clearly suggestive of the fact that children initially solve problems with a count-all strategy, then move to counting-on, and finally employ the use of number facts. There is somewhat less compelling evidence for a separation of stages for counting on from first number and counting on from larger number.

With regard to subtraction, children employ additive actions to look at join-missing addend problems. Analogous results were obtained for separate problems. Combine and compare problems produced ambiguous results.

Number facts (memorized) and derived number facts played an important part in problem solution. Most children were using number facts in some way by the end of the study.

Five levels of development were identified:

(1) Level 0 - unable to solve any problems
(2) Level 1 - direct modeling
(3) Level 2 - transitional period
(4) Level 3 - counting strategies
(5) Level 4 - number facts

5. <u>Interpretations</u>

"The characterization of children's performance that is proposed in this paper is not as precise as the models developed by Briars and Larkin (in press) and by Riley et al. (1983). However, the data presented call into question whether such specific models can capture the variability in children's performance." Some alterations in the model are suggested by these results.

The study indicates that it is not necessary to save word problems until computational skills are mastered. Instructional activities should capitalize on children's natural problem-solving capabilities rather than setting up artificial situations. Instruction could, in the future, more closely follow the progression of skills identified here.

Abstactor's Comments

(1) The empirical verification of hypothesized models in mathematics education is an important research activity.
(2) The study looks at basic skill development and makes comments with instructional implications.
(3) The report is extremely thorough and detailed. So much so, in fact, that it is a little hard to follow at times.
(4) The background section refers to other studies for a deeper review of the literature. More elaboration on studies could have been done, such as, for example, work done by Le Blanc, Steffe, Ginsburg, and others.
(5) The presentation of data is well-done (both tables and figures).
(6) In general, the study is a fine one. More in-depth, longitudinal, clinical work of this type is badly needed in mathematics education.

Charles, Randall I. and Lester, Frank K., Jr. AN EVALUATION OF A PROCESS-ORIENTED INSTRUCTIONAL PROGRAM IN MATHEMATICAL PROBLEM SOLVING IN GRADES 5 AND 7. Journal for Research in Mathematics Education 15: 15-34; January 1984.

Abstract and comments prepared for I.M.E. by WALTER SZETELA, The University of British Columbia.

1. Purpose

The effectiveness of a Mathematical Problem Solving (MPS) program was evaluated at grade levels 5 and 7 on the following basis:

a) A comparison of the problem-solving performance of students participating in the program with students whose problem-solving instruction was limited to regular textbook material.
b) The nature of changes in students' problem-solving performance over three periods of about 8 weeks each.
c) The attitudes of teachers toward problem-solving and the MPS program.

2. Rationale

Other instructional programs on problem-solving have promoted the learning of problem-solving strategies, emphasized solving problems and encouraged an active role for the teacher. In addition to these characteristics of problem-solving programs, the MPS program focused upon each phase of Polya's four-stage model of problem-solving, emphasized extensive experience with process problems, sought to develop students' abilities to select and use a variety of strategies, and incorporated a specific teaching strategy for problem-solving.

3. Research Design and Procedures

The MPS program was a curriculum research and development project sponsored by the West Virginia Department of Education. The program consisted of (a) instructional materials for problem-solving;

(b) guidelines suggesting ways to create a classroom atmosphere to enhance problem-solving success; and (c) a teaching strategy for problem-solving. From 36 schools in four counties in West Virginia, 23 seventh-grade teachers were asked to participate in a problem-solving project. Teachers' classes were assigned to treatment or control groups to maintain "roughly equal mean achievement on the Comprehensive Test of Basic Skills (CTBS, 1973)". Complete data were available for 10 grade 5 treatment classes and 11 control classes and in grade 7 for 10 treatment and 13 control classes.

Teachers in treatment classes received 3 hours of training on use of the MPS program prior to the pretest. During the 23 weeks of the study, each treatment class was observed at least three times "to assure proper implementation of the program." Treatment classes had the regular mathematics program and the MPS program in the same period. By the end of the study, treatment and control classes had covered the same number of pages in the textbook.

Three mathematics educators wrote problems for two item pools, one for fifth grade and one for seventh grade. Problems were to be at three levels of difficulty and such that "a student of average ability should be able to solve after participating in a good problem-solving program for one year." At each of grades 5 and 7, four forms of a test were developed. Each form contained two complex translation problems and two process problems.

One form of the test was administered to each student in September as a pretest. After 23 weeks, a different form of the test was given as a posttest. Students in the treatment group took two other forms of the test, one after 8 weeks and one after 16 weeks of instruction.

Problem-solving performance was assessed on three dimensions: understanding the problem, using strategies in planning to solve the problem, and the result of work on the problem. Each dimension was scored 0, 1, or 2 for poor, fair, and good performance, respectively. Scores for the two complex problems were combined, and scores for the two process problems were combined. Thus, on three dimensions evaluated, there were six scores for each test. Intercoder reliabilities for two trained scorers ranged from 0.77 to 0.95.

4. Results

Test data were analyzed by analyses of covariance, and teachers were interviewed for comments about the MPS program.

On all measures except for result for complex translation problems at both grade levels, treatment classes scored significantly higher than control classes.

For classes in the MPS program, on the dimension of understanding, the greatest growth occurred during the first 8-week period, at both grade levels, for both translation and process problems.

On the planning dimension, the investigators discuss the steadiness of growth by grade 5 classes and some variability for grade 7 classes. However, graphs displaying the growth patterns show strikingly similar results for understanding and planning at both grade levels and both types of problems.

Results of interviews with teachers indicated that teachers grew more positive about problem-solving and their ability to teach it, found clearly structured guidelines for implementing the program valuable, and felt low achievers were motivated by the program; most of the teachers also found the MPS program "teachable."

5. Interpretations

The authors concluded that the MPS program did succeed in improving students' success in problem solving on dimensions of understanding and planning, and that the MPS program is more successful with process problems than translation problems. The program did not substantially improve success on obtaining correct results. The MPS program had a positive influence on teachers and has the strength of "being organized in a way that teachers can use it with very little in-service training."

The authors recommend a change of focus from instruction variables to student and process variables through observations of students as they solve problems.

Abstractor's Comments

Charles and Lester have conducted an important study, the type which is critical for any hope of achieving the problem-solving goals stated in the NCTM's Agenda for Action. Without problem-solving programs for teachers which are "teachable," there may be more talk about problem solving but less action in the classroom. The MPS program appears to be a sound and well-developed program which can be integrated with regular mathematics programs in classrooms with relatively little in-service training.

The authors have recognized some limitations of the study such as a lack of observation of students actually working on problems. This practical problem does not diminish positive effects noted from paper-and-pencil tests. The authors also recognize the limitations of using volunteer teachers when making generalizations.

The following are additional comments and limitations:

1. Although the authors do point out the limitations concerning the effects of administering two problem-solving tests at intermediate stages to the treatment groups, their justification on the grounds that the "test problems replaced the instructional problems on the days of the tests" and that the students were unaware that the test problems were being used for test purposes is tenuous. The MPS classes may well have gained advantages from the extra testing. The authors were interested in changes occurring as the MPS program developed. It would have been useful to determine as well what changes were taking place with classes following a regular problem-solving program.

2. The authors mentioned three levels of difficulty in the item pool of problems and also a statement about the desirability of an average student being able to solve the problems. The conditions seem contradictory. Other than referring to problems as complex translation or process problems, the nature of the problems actually used is a mystery for the reader. It would have been very helpful to provide examples of problems at the two grade levels reported.

3. There is a contradiction between a graph that shows the trends for planning in the grade 7 process problems and the statement that "students in grade 7 exhibited ... similar improvement rates during the first and third periods and a much slower rate during the middle period." The graph indicates that the greatest growth occurred in the first period and the slowest growth in the third period. From the graph it appears that growth rates were about 0.6, 0.25, and 0.10 approximately.

4. Suggested daily time allotments for problem-solving were as low as 3 minutes and as much as 25 minutes. It would have been useful to obtain information on the approximate amount of time devoted to problem-solving by both treatment and control group teachers. A questionnaire to obtain such information would have enhanced the information gathered from interviews of teachers in the treatment group.

5. The evaluation scoring scheme gave equal weight to understanding, planning, and result in the problems. The means obtained on dimensions of understanding and planning are so similar that one cannot help but ask the question: Are they really measuring the same thing? For the translation problems the means are almost identical. In 8 comparisons the means for understanding and planning differ at most by 0.04 (unadjusted means). For process problems the greatest difference between means for the same two dimensions is 0.19. This observation illustrates the difficulties of evaluation in problem-solving, s point not unnoticed by the authors, who suggest the need to develop valid and reliable problem-solving instruments.

Despite these observations and limitations, overall, Charles and Lester have demonstrated a much needed model for classroom implementation of problem solving which appears to be a sterling example of practical research within reach of any interested teacher.

Clute, Pamela S. MATHEMATICS ANXIETY, INSTRUCTIONAL METHOD, AND ACHIEVEMENT IN A SURVEY COURSE IN COLLEGE MATHEMATICS. Journal for Research in Mathematics Education 15: 50-58; January 1984.

Abstract and comments prepared for I.M.E. by RUTH ANN MEYER, Western Michigan University.

1. Purpose

The stated purpose of this study was to look at the relationship of anxiety and teaching method, and their interaction to mathematics achievement.

2. Rationale

The author comments that although research has shown that mathematics achievement is related to mathematics anxiety and that research also supports the notion that teaching behaviors and techniques such as direct instruction make a difference in student achievement, no one had previously investigated the relationship of the two factors, mathematics anxiety and instructional method, to mathematics achievement. Consequently, she designed a study for which she hypothesized that "college students with low mathematics anxiety would perform higher on a mathematics achievement test when taught using a discovery approach, whereas students with high anxiety would find an expository approach more conducive to learning."

3. Research Design and Procedures

Subjects: The subjects were 44 college students at the University of California, Riverside, and 37 students at California State College, San Bernardino, who enrolled in a mathematics survey course. The purpose of this survey course was to teach logical, problem-solving, and critical thinking aspects of various mathematics topics.

Procedures: Subjects were separated into high, medium, and low anxiety groups as determined by their scores on the Mathematics Anxiety Rating Scale (MARS). At each college, students were rank-ordered according to their MARS scores and randomly assigned to one of two treatment groups, direct instruction discovery or direct instruction expository. The University of California and California State University Mathematics Test (UC/CSU) was then administered to the sample to assess students' ability to handle high-school-level algebraic computations and to assess the equivalence of the two treatment groups within and across colleges with respect to these skills.

Following the test administration, the author taught the same survey topics to the two treatment groups at each of the colleges. Her role in the discovery method was "to facilitate the lesson and guide students toward a discovery of the daily objectives through questioning strategies." In the expository method, her goal was to present a well-organized lecture that would present the daily objectives in a clear manner. At the end of the quarter, she administered a multiple-choice mathematics achievement test (MAT) that she had developed to measure how well the students had acquired the survey course content.

4. Findings

Since there were no significant college effects for the descriptive data of the MARS and the UC/CSU Mathematics Test for the two treatment groups, data from the two colleges were pooled. An analysis of variance of the pooled data showed a significant anxiety effect ($p < .01$). Students with high mathematics anxiety scored lower on the mathematics achievement test than did those students with low mathematics anxiety. The difference between the two treatment methods was not significant; however, there was a significant interaction between method and anxiety level. Groups with a high level of mathematics anxiety seemed to score higher when taught by the expository method, whereas low and medium anxiety level groups seemed to score higher when taught by the discovery method.

When the items of MAT were classified into low and high cognitive levels, an analysis of variance of the low item scores showed significant effects for anxiety and for the method-by-anxiety interaction, with the differences in the same direction as before. An analysis of variance for the high-level items showed a significant main effect for method (discovery groups performed better), a significant main effect for anxiety (higher performance was associated with lower anxiety), and no significant interaction between method and anxiety.

5. Interpretations

"The results of the study provide new evidence that high-anxiety students may benefit more in terms of achievement when taught using an expository method whereas low-anxiety students may benefit more when taught by a discovery method. If the desired outcome is correct answers to high-level questions, a discovery method may benefit students at all levels of anxiety. It seems highly likely that another variable, confidence, could interact with instructional method and affect achievement." (p. 57)

Abstactor's Comments

The investigator's findings seem to support an hypothesis that I have accepted for some time in college classroom situations. Students with high anxiety do seem to learn better when taught using an expository method, whereas low or moderate anxiety students prefer a guided discovery approach. Since I cannot separate students by anxiety levels, I adjust by varying and combining methods of instruction.

One questions, however, the credence that should be placed in a study such as this. The anxiety-level groups were relatively small; the high anxiety level at San Bernardino had only five students for each instruction method. Moreover, one may question the procedure that was used to form the anxiety groups. The investigator adjusted cut-off scores of MARS for the low, medium, and high groups in order to have approximately equal groups. One questions what high, medium, or low anxiety really mean. How can there ever be cross-study comparisons if investigators continue to change cut-off scores for the anxiety levels?

Two minor errors are:

In Table 1, the first anxiety level for the UC/CSU test should be High.

For Tables 4 and 5, the maximum scores are interchanged.

MATHEMATICS EDUCATION RESEARCH STUDIES REPORTED IN JOURNALS AS INDEXED BY CURRENT INDEX TO JOURNALS IN EDUCATION
January - March 1984

EJ 286 766 Hunting, Robert P. Emerging Methodologies for Understanding Internal Processes Governing Children's Mathematical Behaviour. Australian Journal of Education, v27 n1, 45-61, April 1983.

EJ 287 154 Nagy, Philip; Drost, Dale. A Comparison of Scaling and Correlational Analysis of Perceptions of Mathematics Objectives. Alberta Journal of Educational Research, v29 n3, 180-95, September 1983.

EJ 287 180 Smith, Richard L. Bibliography: Secondary/College Mathematics. Journal of Computers in Mathematics and Science Teaching, v2 n4, 40-41, Summer 1983.

EJ 287 193 Wagner, Sigrid. What Are These Things Called Variables? Mathematics Teacher, v76 n7, 474-79, October 1983.

EJ 287 203 Becker, Henry Jay. How Schools Use Micro-Computers. Classroom Computer Learning, v4 n2, 41-42, 44, September 1983.

EJ 287 236 Collis, Kevin F. Development of a Group Test of Mathematical Understanding Using Superitem/SOLO Technique. Journal of Science and Mathematics Education in Southeast Asia, v6 n1, 5-14, July 1983.

EJ 287 237 Ibe, Milagros D. Setting Specific Criteria for Scoring Word Problems in Mathematics: Effects on Test Validity and Reliability. Journal of Science and Mathematics Education in Southeast Asia, v6 n1, 15-18, July 1983.

EJ 287 271 Thompson, Richard B. Competency Unit Testing with Limited Repetitions. Mathematics in College, 6-15, Winter 1983.

EJ 287 275 Pallmann, Margot. Verbal Language Processes in Support of Learning Mathematics. Mathematics in College, 49-55, Winter 1983.

EJ 287 278 Jacobowitz, Tina. Relationship of Sex, Achievement, and Science Self-Concept to the Science Career Preferences of Black Students. Journal of Research in Science Teaching, v20 n7, 621-28, October 1983.

EJ 287 310 Kalin, Robert. How Students Do Their Division Facts. Arithmetic Teacher, v31 n3, 16-20, November 1983.

EJ 287 311 Suydam, Marilyn N. Research Report: Achieving with Calculators. Arithmetic Teacher, v31 n3, 20, November 1983.

EJ 287 315 Clements, Douglas H.; Callahan, Leroy G. Number or Pre-number Foundational Experiences for Young Children: Must We Choose? Arithmetic Teacher, v31 n3, 34-37, November 1983.

EJ 287 320 Williams, Paul D. Discovery Learning in Remedial Mathematics: Multiple-Choice versus Written Generalization. Mathematics and Computer Education, v17 n3, 171-77, Fall 1983.

EJ 287 354 Pothier, Yvonne; Sawada, Daiyo. Partitioning; The Emergence of Rational Number Ideas in Young Children. Journal for Research in Mathematics Education, v14 n5, 307-17, November 1983.

EJ 287 355 Llabre, Maria M.; Cuevas, Gilberto. The Effects of Test Language and Mathematical Skills Assessed on the Scores of Bilingual Hispanic Students. Journal for Research in Mathematics Education, v14 n5, 318-24, November 1983.

EJ 287 356 Marshall, Sandra P. Sex Differences in Mathematical Errors: An Analysis of Distracter Choices. Journal for Research in Mathematics Education, v14 n5, 325-36, November 1983.

EJ 287 357 Jackson, Michael B.; Phillips, E. Ray. Vocabulary Instruction in Ratio and Proportion for Seventh Graders. Journal for Research in Mathematics Education, v14 n5, 337-43, November 1983.

EJ 287 358 Roberge, James J.; Flexer, Barbara K. Cognitive Style, Operativity, and Mathematics Achievement. Journal for Research in Mathematics Education, v14 n5, 344-53, November 1983.

EJ 287 359 Turnau, Stefan. Dissertations in Mathematics Education at the Higher School of Education in Cracow, 1968 to 1981: An Annotated Bibliography. Journal for Research in Mathematics Education, v14 n5, 354-60, November 1983.

EJ 287 360 Baroody, Arthur J.; Price, Joyce. The Development of the Number-Word Sequence in the Counting of Three-Year-Olds. Journal for Research in Mathematics Education, v14 n5, 361-68, November 1983.

EJ 287 362 Green, David. From Thumbtacks to Inference. School Science and Mathematics, v83 n7, 541-51, November 1983.

EJ 287 364 Bright, George W. Applying a Sociological Concept about Sex Differences to Science and Mathematics Teaching. School Science and Mathematics, v83 n7, 568-75, November 1983.

EJ 287 367 Clark, H. Clifford; Richmond, Alan. Seven Years Since the Metric Conversion Act: Metric Achievement and Attitudes in Simi Valley, California. School Science and Mathematics, v83 n7, 596-600, November 1983.

EJ 287 660 Gorman, Henry, Jr.; Bourne, Lyle E., Jr. Learning to Think by Learning LOGO: Rule Learning in Third-Grade Computer Programmers. Bulletin of the Psychonomic Society, v21 n3, 165-67, 1983.

EJ 288 243 Beattie, John; Algozzine, Bob. Improving Basic Academic Skills of Educable Mentally Retarded Adolescents. Education and Training of the Mentally Retarded, v17 n3, 255-58, October 1982.

EJ 288 602 Swetz, Frank J.; And Others. Attitudes Toward Mathematics and School Learning in Malaysia and Indonesia: Urban-Rural and Male Female Dichotomies. Comparative Education Review, v27 n3, 394-402, October 1983.

EJ 288 622 Baker, Dale R. Research in College Science Teaching: Can the Difference Between Male and Female Science Majors Account for the Low Number of Women at the Doctoral Level in Science? Journal of College Science Teaching, v13 n2, 102-07, November 1983.

EJ 288 650 Dawe, Lloyd. Bilingualism and Mathematical Reasoning in English as a Second Language. Educational Studies in Mathematics. v14 n4, 325-53, November 1983.

EJ 288 651 Clarkson, Philip. Types of Errors Made by Papua New Guinean Students. Educational Studies in Mathematics, v14 n4, 355-67, November 1983.

EJ 288 652 Af Ekenstam, Adolf; Greger, Karl. Some Aspects of Children's Ability to Solve Mathematical Problems. Educational Studies in Mathematics, v14 n4, 369-84, November 1983.

EJ 288 653 Mevarech, Zemira R. A Deep Structure Model of Students' Statistical Misconceptions. Educational Studies in Mathematics, v14 n4, 415-29, November 1983.

EJ 288 668 Hativa, Nira. What Makes Mathematics Lessons Easy to Follow, Understand, and Remember? Two-Year College Mathematics Journal, v14 n5, 398-406, November 1983.

EJ 288 676 Menis, Joseph. Attitudes towards Chemistry as Compared with Those towards Mathematics, among Tenth Grade Pupils (aged 15) in High Level Secondary Schools in Israel. Research in Science and Technological Education, v1 n2, 185-91, 1983.

EJ 288 729 Barr, George. The Operation of Division and the Embedded Zero. Mathematics in School, v12 n4, 4-5, September 1983.

EJ 288 735 Carpenter, Thomas P.; And Others. Results of the Third NAEP Mathematics Assessment: Secondary School. Mathematics Teacher, v76 n9, 652-59, December 1983.

EJ 288 746 Linquist, Mary Montgomery; And Others. The Third National Mathematics Assessment: Results and Implications for Elementary and Middle Schools. Arithmetic Teacher, v31 n4, 14-19, December 1983.

EJ 288 753 Suydam, Marilyn N. Research Report: Low Achievers. Arithmetic Teacher, v31 n4, 40, December 1983.

EJ 289 064 Pallas, Aaron M.; Alexander, Karl L. Sex Differences in Quantitative SAT Performance: New Evidence on the Differential Coursework Hypothesis. American Educational Research Journal, v20 n2, 165-82, Summer 1983.

EJ 289 068 Hannafin, Michael J. Fruits and Fallacies of Instructional Systems: Effects of an Instructional Systems Approach on the Concept Attainment of Anglo and Hispanic Students. American Educational Research Journal, v20 n2, 237-49, Summer 1983.

EJ 289 077 Willig, Ann C.; And Others. Sociocultural and Educational Correlates of Success-Failure Attributions and Evaluation Anxiety in the School Setting for Black, Hispanic, and Anglo Children. American Educational Research Journal, v20 n3, 385-410, Fall 1983.

EJ 289 078 Webb, Noreen M.; Cullian, Linda K. Group Interaction and Achievement in Small Groups: Stability over Time. American Educational Research Journal, v20 n3, 411-23, Fall 1983.

EJ 289 081 Shumway, Richard J.; And Others. Feature Frequency and Negative Instances in Concept Learning. American Educational Research Journal, v20 n3, 451-59, Fall 1983.

EJ 289 356 Dew, Kathleen Michie Harriss; And Others. Mathematics Anxiety: Some Basic Issues. Journal of Counseling Psychology, v30 n3, 443-46, July 1983.

EJ 289 753 Garnett, Katherine; Fleischner, Jeannette E. Automatization and Basic Fact Performance of Normal and Learning Disabled Children. Learning Disability Quarterly, v6 n2, 223-30, Spring 1983.

EJ 289 903 Roach, Jacquelyn C.; And Others. The Comparative Effects of Peer Tutoring in Math by and for Secondary Special Needs Students. Pointer, v27 n4, 20-24, Summer 1983.

EJ 290 189 Kulik, James A.; Bangert-Drowns, Robert L. Effectiveness of Technology in Precollege Mathematics and Science Teaching. Journal of Educational Technology Systems, v12 n2, 137-58, 1983-84.

EJ 290 190 Ragosta, Marjorie. Computer-Assisted Instruction and Compensatory Education: A Longitudinal Analysis. Machine-Mediated Learning, v1 n1, 97-127, 1983.

EJ 290 272 Baroody, Arthur J.; Ginsburg, Herbert P. The Effects of Instruction on Children's Understanding of the "Equals" Sign. Elementary School Journal, v84 n2, 199-212, November 1983.

EJ 290 300 Beeson, B. F.; Adams, Sam. Using Audiotapes to Increase Student Achievement. Small School Forum, v5 n2, 3-4, Fall 1983.

EJ 290 326 Benbow, Camilla Persson; Stanley, Julian C. Sex Differences in Mathematical Reasoning Ability: More Facts. Science, v222 n4627, 1029-031, Decemb34 2, 1983.

EJ 290 365 Siegler, Robert S. How Knowledge Influences Learning. American Scientist, v71 n6, 631-38, November-December 1983.

EJ 290 408 Bidwell, James K. Calculation Skills vs. Problem Solving: Scotland vs. Michigan. School Science and Mathematics, v83 n8, 682-93, December 1983.

EJ 290 447 The Classroom Environment Study. ACER Newsletter, n49, 1, 6, November 1983.

EJ 290 474 Suydam, Marilyn N. Research Report: Manipulative Materials. Arithmetic Teacher, v31 n5, 27, January 1984.

EJ 290 479 Sadowski, Barbara R.; McIlveen, Delayne Houston. Diagnosis and Remediation of Sentence-solving Error Patterns. Arithmetic Teacher, v31 n5, 42-45, January 1984.

EJ 290 480 Lessen, Elliott I.; Cumblad, Carla L. Alternatives for Teaching Multiplication Facts. Arithmetic Teacher, v31 n5, 46-48, January 1984.

EJ 290 485 A Plan for Assessing the Impact of the NCTM's Agenda for Action. Journal for Research in Mathematics Education, v15 n1, 3-14, January 1984.

EJ 290 486 Charles, Randall I.; Lester, Frank K., Jr. An Evaluation of a Process-Oriented Instructional Program in Mathematical Problem Solving in Grades 5 and 7. Journal for Research in Mathematics Education, v15 n1, 15-34, January 1984.

EJ 290 487 Burton, Leone. Mathematical Thinking: The Struggle for Meaning. Journal for Research in Mathematics Education, v15 n1, 35-49, January 1984.

EJ 290 488 Clute, Pamela S. Mathematics Anxiety Instructional Method, and Achievement in a Survey Course in College Mathematics. Journal for Research in Mathematics Education, v15 n1, 50-58, January 1984.

EJ 290 489 Campbell, Patricia F. Measuring Agreement among Coders Who Use a Process-Sequence System. Journal for Research in Mathematics Education, v15 n1, 59-63, January 1984.

EJ 290 490 Moyer, John C.; And Others. Story Problem Formats: Verbal Versus Telegraphic. Journal for Research in Mathematics Education, v15 n1, 64-68, January 1984.

EJ 290 516 Battista, Michael T.; Krockover, Gerald H. The Effects of Computer Use in Science and Mathematics Education Upon the Computer Literacy of Preservice Elementary Teachers. Journal of Research in Science Teaching, v21 n1, 39-46, January 1984.

EJ 290 522 DeBoer, George E. A Study of Gender Effects in the Science and Mathematics Course-Taking Behavior of a Group of Students Who Graduated from College in the Late 1970s. Journal of Research in Science Teaching, v21 n1, 95-105, January 1984.

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