DOCUMENT RESUME

ED 246 763 HE 017 486

AUTHOR Stewart, Ian; Johnson, F. Craig
TITLE Applications of Nonlinear Models. AIR 1984 Annual Forum Paper.
PUB DATE May 84
NOTE 31p.; Paper presented at the Annual Forum of the Association for Institutional Research (24th, Fort Worth, TX, May 6-9, 1984).
PUB TYPE Viewpoints (120) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS College Administration; Equations (Mathematics); *Estimation (Mathematics); Higher Education; *Institutional Research; *Mathematical Models; Maximum Likelihood Statistics; *Prediction; *Statistical Studies
IDENTIFIERS *AIR Forum; Catastrophe Theory; *Nonlinear Models

ABSTRACT

Some of the conceptual qualitative ideas needed to test nonlinear models empirically and to modify them are described. Relationships among these ideas and computer applications are also examined to elucidate the general process of nonlinear modeling. Two examples are presented along with a discussion of bifurcation, catastrophe, and maximum likelihood estimate methods. The first example concerns administrators' responses to innovation and uses a verbal description of events. The model is developed based on variables such as the amount of voluntary effort committed to the innovative project and the level of funding agreed to by the institution. An equation consistent with the hypotheses is presented. The second example starts with a mathematical model of promotions within an organization and shows how to go beyond the verbal statements. It is concluded that many observed phenomena in institutions are suggestive of nonlinear dynamics models. A number of standard types of dynamic behavior are well understood mathematics (catastrophe, periodicity, stochastic effects) and may be used to construct plausible models. (Author/SW)

***********************************************************************
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
***********************************************************************

ED246763

# APPLICATIONS OF NONLINEAR MODELS

Ian Stewart
Professor of Mathematics
Mathematics Institute
University of Warwick
Coventry, England CU4 7AL
Ph. (0203) 24-011

F. Craig Johnson
Professor of Education
314 Stone Building
Florida State University
Tallahassee, Florida 32306
Ph. (904) 644-4583

Corrected Copy

U.S. DEPARTMENT OF EDUCATION
NATIONAL INSTITUTE OF EDUCATION
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

This document has been reproduced as received from the person or organization originating it.

Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official NIE position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

AIR

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

This paper was presented at the Twenty-Fourth Annual Forum of the Associa .on for Institutional Research held at the Hyatt-Regency in Fort Worth, Texas, May 6-9, 1984. This paper was reviewed by the AIR Forum Publications Committee and was judged to be of high quality and of interest to others concerned with the research of higher education. It has therefore been selected to be included in the ERIC Collection of Forum papers.

Daniel R. Coleman, Chairman
Forum Publication
Advisory Committee

ABSTRACT

The purpose of this paper is to describe some of the conceptual qualitative ideas needed to test nonlinear models empirically and to modify them. Some relationships among these ideas and some computer applications of them are also examined to elucidate the general process of nonlinear modeling.

Two examples are presented along with a discussion of bifurcation, catastrophe and maximum likelihood estimate methods. It is concluded that many observed phenomena in institutions are suggestive of nonlinear dynamics models. A number of standard types of dynamic behavior are well understood mathematics (catastrophe, periodicity, stochastic effects) and may be used to construct plausible models.



ERIC

# APPLICATION OF NONLINEAR MODELS

## Introduction

Because applications of nonlinear models to the institutional setting are relatively new, the terms "nonlinear" and "model" need to be contrasted with the more familiar linear models which have a lengthy history and a well developed literature. From the mathematical point of view. linear models are defined by equations in which no quantities other than numerical ones occur to a higher power than the first (and there are no product terms). Examples of linear models are:

$$\frac{dy(t)}{dt} = a + by(t) + \left[s \frac{dW}{dt}\right]$$

$$y(t+1) = a + by(t) + \left[s\varepsilon(t)\right]$$

The <u>terms in the square brackets</u> represent stochastic noise and may be omitted fro the model altogether. Solutions are well understood and can often be calculated explicitly. There is a "superposition principle" whereby linear combinations of solutions again form solutions. Equilibrium states are <u>unique</u>. While these features are mathematically desirable and practicable, there is one difficulty. Linearity is extremely special; most equations are nonlinear. Many natural phenomena display features that suggest nonlinearity, for example, multiple equilibria or the absence of any plausible superposition principle. For some purposes, linear models may be perfectly adequate, but it is important not to force apparently nonlinear phenomena into a linear framework.



ERIC

When contemplating nonlinear models it is easy to be bewildered by their apparent variety. The possible forms are limited only by the imgaination of the modeller. However, we should resist the temptation to place too much emphasis on the form in which the equations are written: It is after all the solutions that count, not the symbols used in the equation. Much of the recent progress in understanding nonlinear systems arises by ignoring the form of the equations and investigating the qualitative behavior of solutions. Do these tend to a steady state? Do they oscillate periodically? Do they change quickly or slowly? Are they following a clear pattern, or do they seem random?

One disadvantage of nonlinear equations is that they can seldom be solved explicitly by a formula. They can sometimes be replaced by appropriate linear models, via approximation (ignoring higher order terms) or change of variables. These pseudo-linear equations, however, remain a special, atypical class. Real progress can best be made when the analytical approach is augmented by conceptual ideas such as the qualitative theory of topological dynamics and by numerical simulation. For an introduction to the qualitative theory see Arrowsmith and Place (1982); for a survey see Guckenheimer and Holmes (1983).

The second interest here is in "models" and how to apply them. The traditional models in the physical sciences are mainly differential equations (for continuous data) and difference equations (for discrete data), describing how a system changes its state. This has not been true for social sciences where data are usually collected in order to build a verbal picture of the phenomenon. This verbal picture is also used to state predictions. The advantage of the mathematical model arises when we wish to test the model empirically or when we wish to modify it to take account of additional effects.

Institutional researchers who wish to make real progress dealing with nonlinear phenomena will need to bring to their efforts some qualitative theory of topological dynamics, singularity (catastrophe) theory, time-series analysis, stochastic differential equations and numerical simulations. The verbal models will need to be expressed in a mathematical form for empirical testing and modification. In most cases this will mean using a mathematician as consultant or as a team member on a specific research project.

## Purpose

The purpose of this paper is describe some of the conceptual qualitative ideas needed to test nonlinear models empirically and to modify them. Some relationships among these ideas and some computer applications of them are also examined to elucidate the general process of nonlinear modeling.

## Literature Review

A review of the literature yields several examples of nonlinear model applications to institutional problems. Zeeman (1980) provided the broad conceptual framework when he stated the purpose of one nonlinear model (catastrophe) was to give global insight, to reduce arbitrariness of description, to synthesize seemingly unconnected observations, to explain seemingly inexplicable features, and to suggest unsuspected possibilities.

This nonlinear model has been applied for these purposes to problems of financial attrition (Cossu, 1980); faculty vitality, teacher expectations, and student attrition (Staman, 1982); collective bargaining (Johnson, 1980);

and admission policies (Johnson & Lacher, 1982) within the field of institutional research. Parts of the catastrophe model have been used for other purposes. The hysteresis effect has been used to describe a model for faculty rewards (Shapiro, 1978), a difference equation was used in a model describing dynamic budget equilibrium (Hopkins & Massy, 1977), and a discipline-related productivity study used a three-dimensional model to show the weights of graduate and upper divisions in relation to an objective function (Bloom, 1983). While many related applications have been made of catastrophe models, as yet, there exists no case where a model of institutional behavior has been set up and tested against real data; but there appear to be no essential obstacles to this. Experienced institutional researchers now have several case studies to draw on for methodology. These include demographic modeling (Cobb, 1978; Cobb et al., 1983), biological membrane dynamics (Cobb & Zachs, 1983b), effects of alcohol on driving performance (Zeeman, 1977; Cobb & Zachs, 1983b), thyroid dysfunction (Seif, 1979), heterozygotic advantage (Cobb & Zachs, 1983b), multistable perception (Stewart & Peregoy, 1983; Peregoy & Zeeman, 1983), political preferences (Peregoy, 1974), and economic stagflation (Fischer, 1983).

The main problems encountered are (a) adequate definition of variables; (b) selection of suitable class of models; (c) adequate measurement of variables; and (d) design of experiment where possible, or selection of suitable retrospective data where not.

Two examples of how to use nonlinear models are presented here to illustrate some of the potentials and problems.

Example 1: Administrators' Responses to Innovation

The first example takes a verbal description of periodic phenomena found in university administrators' responses to innovative ideas (Walker, 1983).

> Usually, innovation actually occurs in universities because some one person, or occasionally two or three people, catch on fire about an idea and simply will not put it down. They push and test the existing structure to its limits, often arranging end runs that are annoying to those above them in the hierarchy. Basically, they subsidize the new scheme out of their own perspiration and overtime. Having succeeded in attracting acceptance from "the establishment" within the organization, but still sailing under the colors of "innovative verve," they then apply successfully for subsidization. Once they are included in the formal budget structure of the organization, the innovations of the program begin to cost more and more or less and less. Again, the normal tendency in organizations is to spend new monies to make existing programs more comfortable rather than to expand and to continue to innovate. . . .
>
> (An example of a folk festival is discussed)
>
> . . . This scenario is, it seems to me, characteristic of the life cycle of such programs. To complete the description of the scenario at the university with which I am familiar, the demand for expansion and full subsidization occurred in a bad budget year. The director was told that regretfully the university could not expand its commitment and indeed, might have to cut back some.



ERIC

The director with equal regret indicated that the event should be dropped.

Nothing happened for two or three months. Then another individual came to the administration, indicating that she thought the program should and could be rescued by contracting commitments and events, by a return to the principle of calling on volunteers, by limiting the celebration to a weekend in the fall as in the beginning and by relying on faculty families to supply housing to visiting musical groups. Modest postage and telephone subsidization and a small financial stipend for the new organizer were all that was required. The cycle began again. (p. 53)

In the spirit of nonlinear dynamic modeling, we seek to construct a mathematical scenario consistent with these phenomena. It is for explanatory and illustrative purposes only: A serious study might well require modifications. The ingredients of the model are the following two variables:

$E$ = the amount of voluntary effort committed to the project.

$F$ = the level of funding agreed to by the institution.

Both are assumed positive.

Let us assume that there is a critical level of effort $E_{crit}$. Below this level, the administration will offer only minimal support. Above it, however, it will offer funding roughly proportional to the effort sustained.

The effort itself is subject to another "threshold" effect. When official funding rises above a level $F_{crit}$, the voluntary effort becomes hard to sustain (due to increased bureaucracy, administrative chores, a feeling that the event is now 'old hat,' and so on). On the other hand, with official funding below $F_{crit}$, the voluntary effort is stimulated (by the challenge of raising funds).

Finally, we assume that the administration adjusts its funding policy relatively efficiently and quickly, whereas the response of the voluntary effort is less organized and on the whole slower.

The simplest differential equation model consistent with these hypotheses has a phase portrait shown schematically in Figure 1.

Figure 1. A model of oscillatory behavior in innovative ideas. E = level of voluntary effort, F = level of official funding. F moves rapidly in a vertical direction towards the equilibrium curve (solid line). Then there is a slower motion on the curve. Jump behavior is forced at each of the two folds.

The solid line represents the funding level perceived as being appropriate by the administration. There is thus a <u>fast flow</u> in which F adjusts (vertically in the diagram) to the level S. There is also a <u>slow flow</u> on S in which E (and F) adjust relative to $F_{crit}$.

Assume E starts at a low value, with F also low, then $F < F_{crit}$, so E increases. As E passes $E_{crit}$, the administration "recognizes" the new event and supplies funding at a much higher level, so F rises rapidly. However, F is now greater than $F_{crit}$ so the effort E starts to decrease. Owing to the zig-zag shape of S, however, the level of F remains high--though now decreasing--until E drops to a level $E_o$. At this point, the administration withdraws its support and F drops rapidly. Since now $F < F_{crit}$, starts to increase and the cycle starts anew.

A less schematic picture of the flow shows that a limit cycle is responsible for the oscillations, as in Figure 2.

Figure 2. A dynamical systems model approximating Figure 1 leads to a limit cycle oscillation.

It is now possible to write down a reasonable system of equations for E and F to describe the motion. For example, the following would work:

$$\varepsilon \frac{dF(b)}{dt} = \Phi(E,F)$$

$$\frac{dF(t)}{dt} = \alpha(F_{crit} - F)$$

where $\Phi$ (E,F) is zero on the curve S. For instance, we might take

$$\Phi(E,F) = \beta F + \frac{\ell}{1+(F-\delta)^2} + \sigma - E$$

where $\varepsilon, \alpha, \beta, \ell, \delta, \sigma$ are adjustable parameters and $\varepsilon$ is small. This type of equation is known as a relaxation oscillator and is well known in engineering and in chemistry.

One of the predictions of this model is the occurrence of a second threshold value $E_o$: This arises when we require S to be a continuous curve.

Other predictions may be generated by varying the parameters slightly. For example, consider the effect of $E_{crit}$. When it is much higher (or much lower) the oscillations do not occur.

In other words, for large $F_{crit}$ the growth is as follows: First a slow increase in E, then a sudden increase in F, then a steady increase (decrease is also possible) to a steady state. The innovation becomes a standard fixture. If $F_{crit}$ is low enough, there is only a slow increase of E to a steady state with low F: The innovation "never gets off the ground."

These predictions appear to be in reasonable qualitative agreement with observations. Unlike a purely verbal model, they are at least in principle amenable to quantitative study. We look at general mathematical and practical difficulties of doing this below.



ERIC

Example 2: Promotion within an Organization

The second example starts with a mathematical model of promotions within an organization and shows how to go beyond the verbal statements.

To fix our ideas and introduce a problem of relevance to institutions, we consider some work of Sorensen (1974, 1984) on the promotion structure within an organization. Stripped to its mathematical essentials, this may be stated as follows. We introduce the following variables:

$y(t)$ = an individuals' level of attainment at time $t$

$a$ = his or her level of resources

$b$ = constraints on promotion due to the pyramidal structure of the organization

Here $a$ and $b$ are assumed constant. (Our $a$ here is Sorensen's $z$. To make later analysis more natural we assume $b$ is negative, so a constraint of -10 is more restrictive than -5.) Then Sorensen argues for a model of the time-variation of $y$ taking the form

$$\frac{dy(t)}{dt} = a + by(t) \tag{1}$$

This is an inhomogeneous linear constant-coefficient ordinary differential equation in a single variable $y$. It has the general solution

$$y(g) = (y_o + \frac{a}{b})\, e^{bt} - \frac{a}{b} \tag{2}$$

where $y_o$ is an arbitrary constant representing the initial condition

$$y(O) = y_o \tag{3}$$

Notice that the model predicts the existence of a maximum level of attainment

$$y_m = \lim_{\tau \to \infty} y(t) = \frac{-a}{b} \tag{4}$$

(which we here assume to be greater than $y_o$).

An individual's career is represented as a steady increase towards the maximum $y_m$, at a rate which gets slower as time increases (due to a lack of "room at the top"). Of course, in practice the motion occurs in discrete jumps so the model should be considered as a "smoothed" version of reality. The basic onclusions of Sorensen's model could be summarized verbally as follows: (a) Individuals achieve promotion up to a 'ceiling' level $y_m$; (b) the closer they are to their ceiling, the slower is any further promotion; (c) an individual with higher resources has a higher ceiling; and (d) an individual subject to greater constraints has a lower ceiling.

An immediate question is: to what extent does the mathematical model (1) tell us anything beyond the plausible but somewhat trite verbal picture? It would be easy for a mathematician to prefer (1) just because it is mathematical, whether it really adds anything, and for the non-mathematician to prefer a verbal statement, for the converse reason. The worth of a model, however, should be judged according to more than mere taste.

This issue is confused, rather than clarified, by the extreme simplicity of the mathematical model (1). If something much more complicated were similarly reduced to the verbal level, the imprecision and incompleteness of the verbal description would be more manifest. But that is not the crucial point. As stated earlier, the advantages of a mathematical model over a verbal one, in this kind of exploratory modeling, arise not when we state its predictions (which can always be done verbally) but when we wish to test it empirically, or wish to modify it to take account of additional effects. For example, consider Sorensen's model. By treating the resource variable a or a linear combination of various measureable resource variables (education, background, etc.)

$$a = a_0 + a_1 x_1 + \ \ldots \ + a_k x_k$$

it becomes possible to use empirical data to estimate parameters and test the model (Sorensen, 1979, 1983).

Bifurcation and Catastrophe

Many models contain adjustable parameters, some of which may represent external variables that can influence behavior. If a linear system has such <u>control parameters</u>, its unique steady state will, in general, vary <u>continuously</u> as the parameters change. For example in Sorensen's model, the value $y_m$ - a/b varic   ntinuously with <u>a</u> and <u>b</u>.

For nonlinear systems this is no longer time. Varying parameters can cause the steady state to break up into several states (<u>bifurcation</u>) or change discontinuously (<u>catastrophe</u>). Consider for example, our limit cycle model of innovation, but suppose for the moment that E is arbitrary held fixed. Then there are either 1 or 3 steady states for F, depending on whether $E < E_0$, $E > E_{crit}$ or $E_0 < E < E_{crit}$. (A multiplicity of states is typical of non-linear systems.)

Now consider adjusting E slowly, allowing the system to settle to equilibrium after each adjustment. This is called <u>quasi-static</u> variation. If E starts small and increases there will be a sudden jump in the steady state as E passes $E_{crit}$. This is known as a <u>catastrophe jump</u> or <u>limit-point</u> bifurcation. Note that on reversing the motion of E, the quasistatic hypothesis leads to a jump back at $E_0$, not at $E_{crit}$.



ERIC

Figure 3. Hysteresis in the model of innovative ideas when E is allowed to vary quasistatically. The funding level F jumps in different places according to the direction of motion of E

This phenomenon, known as hysterisis, cannot occur in linear models. It implies a "history-dependent" behavior of some theoretical interest, since numerous social phenomena exhibit a similar affect. One of the best known classes of static bifurcations is the elementary catastrophes of Thom (1975) which classify the multi-parameter bifurcations of steady states of gradient differential equations. The best known of all is the cusp catastrophe

$$x^3 + \alpha x + \beta = 0 \tag{5}$$

where both $\alpha$ and $\beta$ are "control parameters." This represents a surface which is either 1- or 3-sheeted over the $(\alpha,\beta)$ control space. The cusp differential equation is

$$\begin{aligned} \frac{dx}{dt} &= -(x^3 + \alpha x + \beta) \\ &= - \text{grad} \quad V(x) \end{aligned}$$

where the potential V is given by (6)

$$V(x) = \frac{x^4}{4} + \frac{\alpha x^2}{2} + \beta x.$$

We assume the reader has some familiarity with the basic ideas of this theory. If not see Zeeman (1977), Poston and Stewart (1978), Saunders (1980), Gilmore (1981), Thompson (1982). For an introduction aimed towards institutional modeling, see Johnson and Lacher (1983) and Zeeman (1980).

An alternative source of models, with many mathematical similarities, is the theory of time series discrete dynamic models, either deterministic or stochastic. We lack space here to discuss these ideas, which is not to deny their interest or importance. (See Gregson, 1983.)

## Maximum Likelihood Estimation

An objection sometimes raised to the use of multi-state models in data-filting may be phrased thus: Allowing more than one predicted state automatically enlarges the chances of a fit, but in a trivial way. Two guesses are better than one!

Figure 4. How multivalued curves apparently lead to trivially 'better' fits to data. (a) poor fit, (b) improved fit, (c) close fit. Consideration of probability densities resolves this difficulty.

The objection is best dealt with by re-interpreting the data-fitting exercise as one involving not <u>curves</u>, but probabilities. The issues are most clearly seen if we consider the prediction of a simple observation. The classic probabilist's example is the toss of a fair coin. There are two outcomes, <u>H</u> and <u>T</u>. A single-state prediction (say <u>H</u>) will be correct about half the time. A two-state prediction (<u>H or T</u>) is of course always right! So it is automatically "better" to make two predictions rather than one. Howeve , consider the corresponding probability densities applied to the result of a series of measurements.

Figure 5. Probability histograms for (a) a biased coin, (b) a fair coin. With a fixed total probability of 1 to be assigned, there is no automatic advantage in having two states (H,T) rather than one (H). Which fits best depends on the coin.

First, imagine a fair coin. Clearly (b) will provide a better fit to the empirical frequency histogram than (a) will. However, now imagine a biased coin, which always shows heads. Now (a) gives the better fit. It is <u>not</u> true that adding an additional state leads to an automatic improvement.

On the contrary, it can worsen the fit. The reason is that in fitting a probability density, the object is to distribute a total probability of 1 over the possible states. If a new state is added, it "steals" probability from the old ones. In other words, on the level of probability densities, there is no advantage in assuming multiple states (unless they are actually occurring).

The same reasoning holds for catastrophe-theoretic models (or others predicting multiple states) where the prediction depends on control parameters. For each control value there is a probability density $P_a(x)$ for the observation x. Thus, we are fitting a family of densities, not a curve or surface.

In cusp-type models, the relevant densities can be found by replacing the cusp equation (6) by a related stochastic differential equation and seeking the "stationary probability density" of the result. (See Cobb, 1978; Cobb & Zachs, 1983.) If the noise term is a Wiener Process ("White noise") then one is led to the exponential family.

Figure 6. Family of stationary probability densities for the cusp-type stochasti differential equation. Here $\alpha$ is fixed and the curves show the transition to bimodality as $\beta$ varies. (Courtest of Cobb.)

$$p_{ab}(x) = K_{a,b,} \exp\left[\left(\frac{x^4}{4} + \frac{ax^2}{2} + bx\right)\Big/ c\right]$$

where $K_{abc}$ is a normalizing constant. Typically, a and b are (approximately) linear functions of exogenous variables

$x_1 \ldots x_k$

$a = a_0 + a_1x_1 + . + a_kx_k$

$b = b_0 + b_1x_1 + . + b_kx_k$

and x represents the deviation of a measured state y from some reference state $\lambda$:

$x = y - \lambda.$

The problem is to estimate $\lambda$, $a_i$, $b_i$. It is solved by computing a maximum likelihood estimate. This produces the parameter values which render it most likely that the observations come from a population with the corresponding family of densities. A computer program to perform this estimate (by an itetrative method) has been developed by Cobb (in a more general form).

Figure 7. A catastrophe jump in a noisy system: "stochastic tunnelling can lead to a jump earlier than the fold point. (Courtesy of Cobb.)

Figure 8. An example of cusp catastrophe data-fitting. The data from Napanstek et al (1974), demonstrated phase transitions in biological membranes. See Cobb and Zachs (1983) for details.

It provides a print-out of the best-fitting parameters, along with other information on the goodness of fit.

## Conclusions

Many observed phenomena in institutions are suggestive of nonlinear dynamic models. It is especially important <u>not</u> to select data or experimental methods that "design away" multimodel behavior, hysteresis, etc. Many standard methods do this, sometimes in "hidden" ways (e.g., averaging, smoothing linear regression, analysis of variance). A number of standard types of dynamic behavior are well understood mathematically (catastrophe, periodicity, stochastic effects) and may be used to construct plausible models. In suitable cases there exist methods to fit these to data and hence, to make useful predictions. This exercise has been carried out in full, with reasonable success, in a number of areas in the social and biological sciences. In the physical sciences, with an appropriately more mathematical methodology, similar models have proved highly successful. While no extensive work along these lines yet exists in institutional research, the prospects are good and the necessary techniques exist. They require cautious use, and some expert advice is worth seeking to ensure a reliable approach, but they tackle an important and novel area: the search for genuinely nonlinear affects and models.

References

Arrowsmith, D. K., & Place, C. M. (1982). Ordinary differential equations. London: Chapman & Hall.

Bloom, A. M. (1983). Differential productivity indices. Research in Higher Education, 18(2), 179-193.

Cobb, L. (1978). Stochastic catastrophe models and multimodel dis ibutions. Behavioral Science, 23, 360-374.

Cobb, L. P., Koppstein, P., & Chen, N. H. (1983). Estimation and moment envision relations for multimodal distributions of the exponential family. Journal of American Statistics Association, 78, 124-130.

Cobb, L., & Zacks, S. (1983). Applications of statistical catastrophe theory in the biosciences. Preprint, Biometry Department, Medical University of South Carolina, Charleston, S.C.

Cossu, C. (1980, November). Financial attrition, fixity of variable costs . . . and catastrophe theory. Paper presented at the meeting of the European Association for Institutional Research, London, England.

Fischer, E. O. (1983). Empirical investigation of a catastrophe theory extension of the Phillips-curve. Preprint, Department of Business Studies, University of Graz, Austria.

Hopkins, D.S.P., & Massy, W. F. (1977). Long range budgeting in private colleges and universities. In D.S.P. Hop.ins and R. G. Schroeder (Eds.), New Direction for Institutional Research: No. 13. Applying alytical Methods to Planning and Management. San Francisco: Jossey-Bass, 43-66.

Gilmore, R. (1981). Catastrophe theory for scients and engineers. New York: Wiley.

Gregson, R.A.M. (1983). Tone series in psychology. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Guckenheimer, J., & Holmes, P. (1983). Nonlinear oscillations, dynamical systems, and bifurcations of sector fields. Springer, New York.

Johnson, F. C., & Lacher, R. C. (1982). Contributions of mathematics to qualitative research methods. In S. E. Kuhns & S. V. Martorana (Eds.), New Directions for Institutional Research: No. 34. Qualitative Methods for Institutional Research. San Francisco: Jossey-Bass, 27-41.

Johnson, F. C., & Lacher, R. C. (1983). Catastrophe theory: State of the art and potential applications. Arlington, VA: Office of Naval Research.

Naparstek, A., Romette, J. L., Kerehez, J. P., & Thomas, D. (1974). Memory in enzyme membranes. Nature, 249, 490-491.

Peregoy, P. (1974). In preparation.

Peregoy, P., & Zeeman, E. C. (1983). The etiology of sudden perceptual change. Preprint, Psychology Department, University of Pittsburgh at Johnston.

Poston, T., & Stewart, I. N. (1978). Catastrophe theory and its applications. London: Putnam.

Saunders, P. T. (1980). An introduction to catastrophe theory. Massachusetts: Cambridge University Press.

Seif, F. J. (1979). Cusp bifurcation in pituitary thyrotropin secretin. In W. Guttinger, & H. Eikemeier (Eds.), Structural Stability in Physics. New York: Sponger.

Shapiro, E. (1979). Effects on teaching of changes in relative rewards for research and teaching. Research in Higher Education, 9(1), 43-67.

Sorensen, A. B. (1979). A model and a metric for the analysis of the intragenerational status attainment process. American Journal of Sociology, 95, 361-384.

Sorensen, A. B. (1983). Interpreting time dependency in career processes. IRP Discussion Paper. University of Wisconsin-Madison.

Sorensen, A. B. (1984). Inequality of Opportunity in Internal and Competitive Labor Markets. Preprint, University of Wisconsin-Madison.

Stamen, E. M. (1982). Catastrophe theory in higher education research. Research in Higher Education, 16(1), 41-53.

Stewart, I. N., & Peregorey, P. (1983). Theory modeling in psychology. Psychology Bulletin, 94, 336-362.

Thom, R. (1975). Structural stability and morphogenesis. New York: Benjamin-Addison Wesley.

Thompson, J.M.T. (1982). Instabilities and catastrophies in science and engineering. New York: Wiley.

Walker, D. E. (1983). Theorems about administration and/or catastrophies. In Johnson & Lacher, Catastrophe theory: State of the art and potential applications. Arlington, VA: Office of Naval Research, 47-53.

Zeeman, E. C. (1977). Catastrophe theory: Selected papers, 1972-77. Reading, MA: Addison Wesley.

Zeeman, E. C. (1980, April). Catastrophe models in administration. Paper presented at the Forum of the Association for Institutional Research, Atlanta, GA.

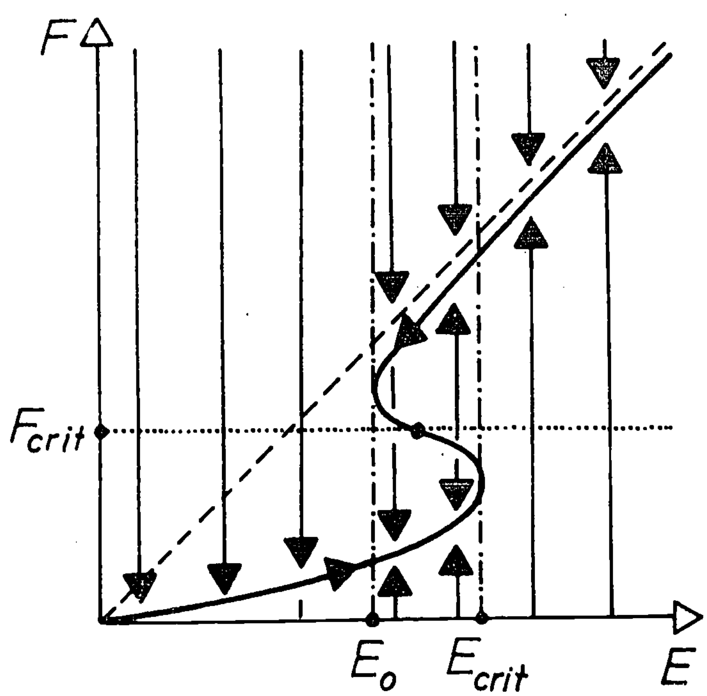



Figure 1

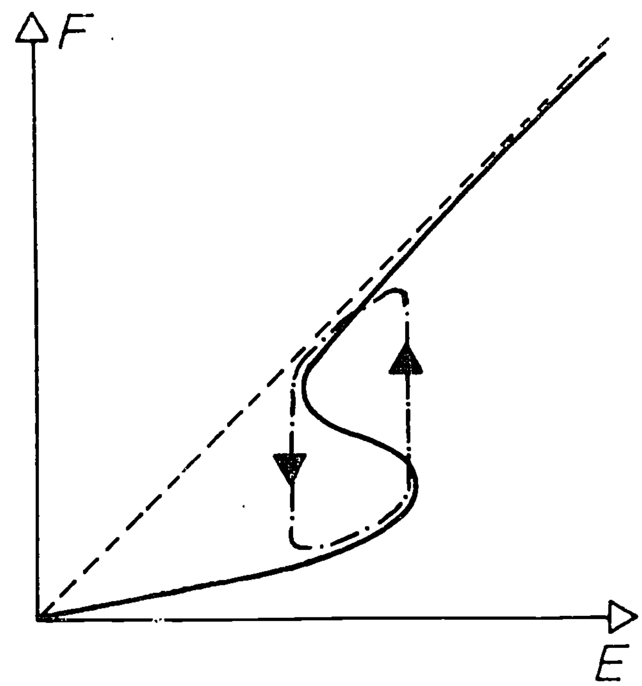



Figure 2

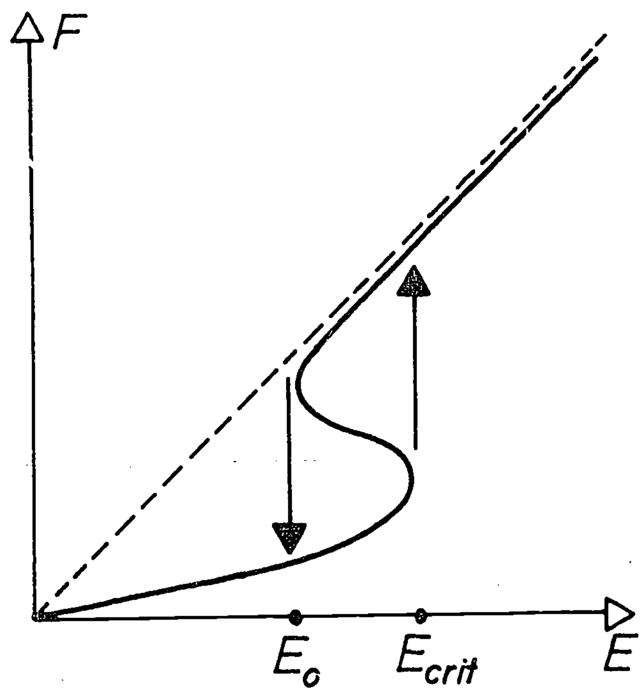



Figure 3

ERIC

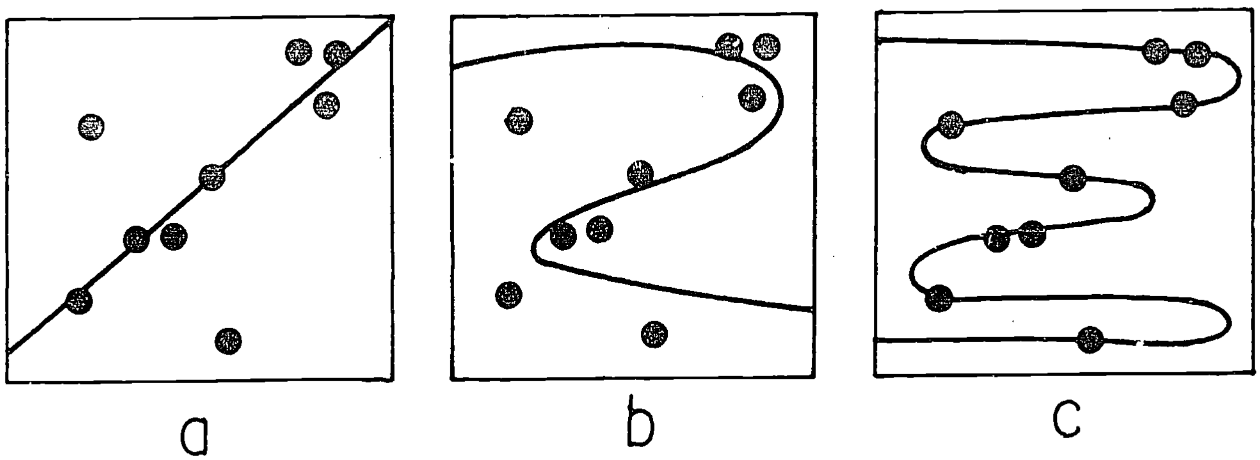



Figure 4

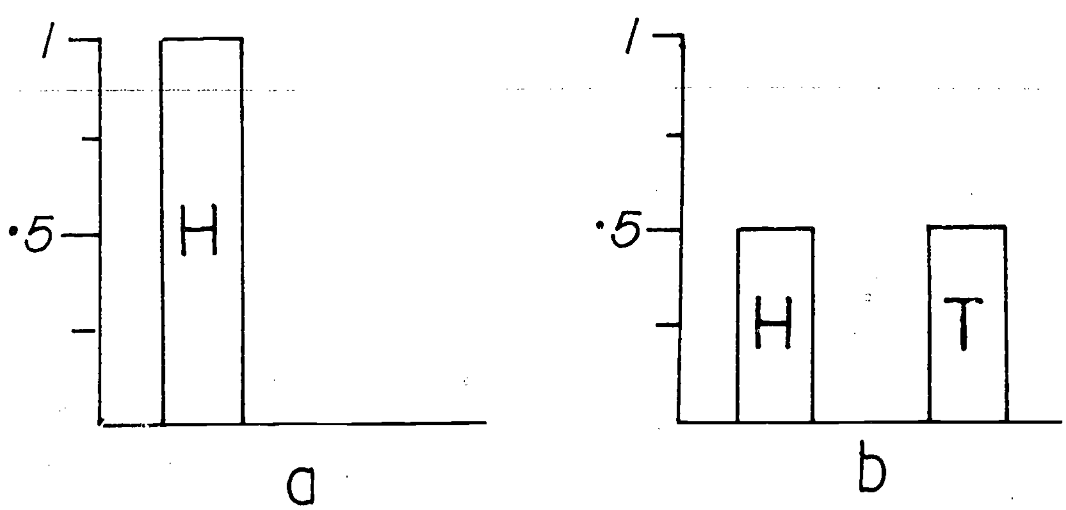



Figure 5

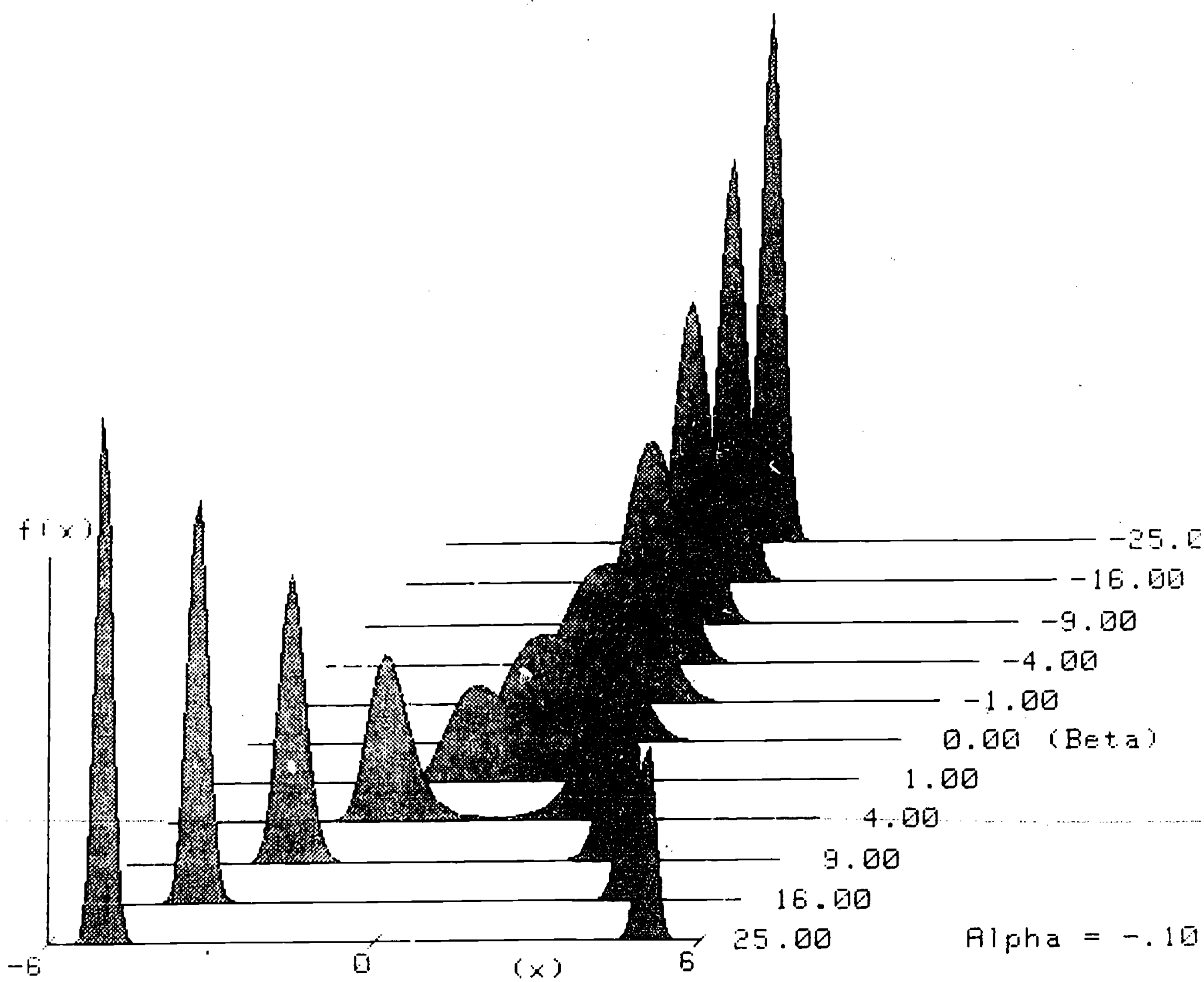



Figure 6

ERIC

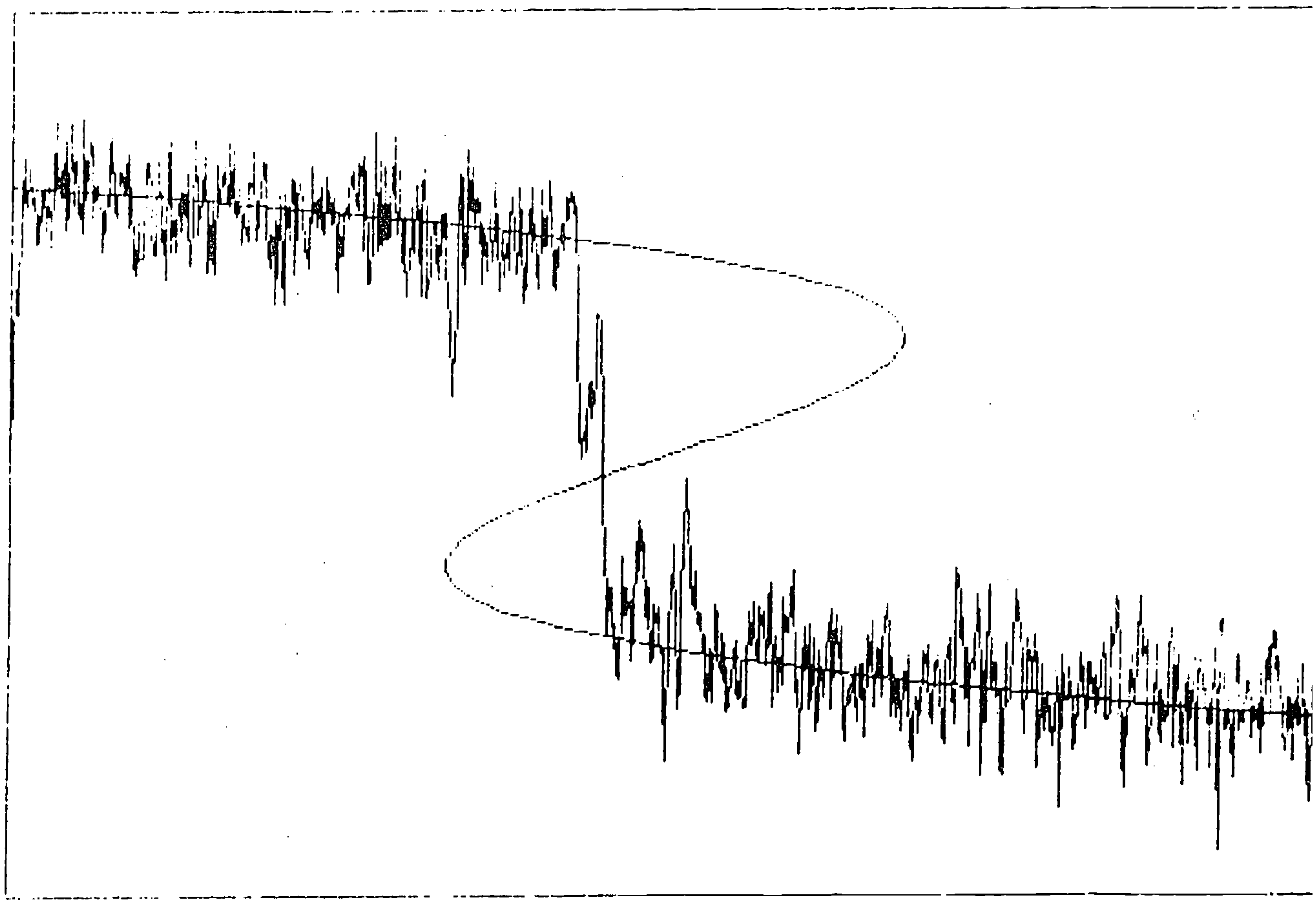
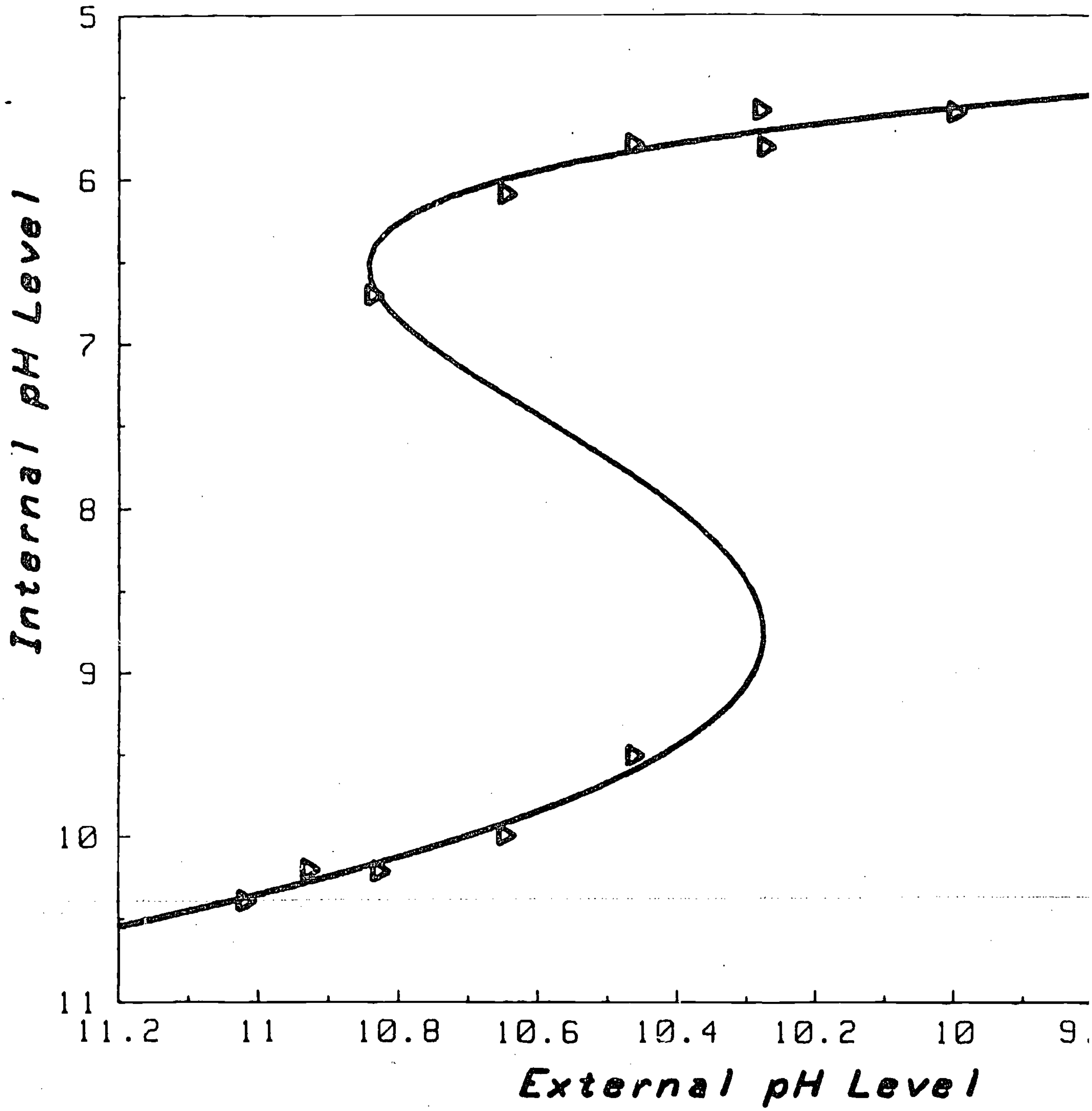


Figure 7



5
6
7
8
9
10
11
Internal pH Level
11.2
11
10.8
10.6
10.4
10.2
10
External pH Level


Figure 8