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ABSTRACT

This paper reviews some of the basic issues and concerns which are reflected in the literature and which either directly or indirectly influenced the development of a syllabus for a mathematics methods course for preservice elementary teachers. These issues include: (1) What should be the content in mathematics for elementary school children? (2) What should be the content in mathematics for elementary school teachers? (3) What should be the goals of the mathematics methods course? (4) What objectives should be included in the mathematics methods course? (5) What is the role of individualized instruction in the mathematics methods course? (6) What is the competency-based mathematics methods course? (7) What is the content-methods approach to the mathematics preparation of preservice elementary teachers? (8) What is the laboratory approach to teaching a methods course? (9) What is the role of professional laboratory experiences in a methods course? and (10) Who should teach the methods course? The syllabus, prefaced by a set of explanatory statements, is included in an appendix. (JN)

THE DEVELOPMENT OF A MATHEMATICS METHODS COURSE FOR PRE-SERVICE ELEMENTARY TEACHERS: SOME CONSIDERATIONS

by

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THE DEVELOPMENT OF A MATHEMATICS METHODS COURSE FOR PRE-SERVICE ELEMENTARY TEACHERS: SOME CONSIDERATIONS

Introduction

This paper reports a review of some basic issues and concerns which are reflected in the literature and which influenced, either directly or indirectly, the development of a syllabus for a mathematics methods course for pre-service elementary teachers.

The syllabus, which is included in the appendix, is admittedly a preliminary and rudimentary draft and is prefaced by a set of explanatory statements for the reader.

Historical Background

The methods component of mathematics education in the United States has seldom been static. Burns (1970) traced the development of elementary school mathematics teaching in this country from its early origins to modern times.

> After gaining a place in the curriculum, arithmetic at times represented one-half of the school time (1850), about one-quarter (1890), and about one-eighth (1963). Influences responsible for reduction included studies of time allotments and achievement, together with the demands of other curricular areas.
>
> Beginning with major emphasis upon social utility aspects of the subject, due to needs of the time, shifts are noted (1850-1900) toward considering it valuable training in mental discipline, as suggested by "faculty

psychology"; back again to social uses (1900-35) with the advent of new theories of learning; and then to a movement toward structure of the subject (logical organization and meaning theory) with greatest impetus by the post-Sputnik experimental programs.

From rules-and-examples, the inductive approach received brief attention (1921-50); returned to rules-and-examples, supported by the connectionist theory of learning, and then moved toward meaning and understanding, as influenced by scientific studies indicating comparable achievement under such programs.

Instructional materials evolved from the "master's book" (1788), to topically organized texts for each pupil (1821), to a spiral text for every two or three grades (1890), to one book for each of the grades (1927). Since 1950, the use of media and programs of specialized materials has become an increasing part of teacher education programs.

A professional text first appeared in 1880. Issues raised by research on elementary school mathematics provided a basis for discussion and recommendations on methods of teaching in professional texts used by teacher education programs. Elementary school teacher preparation began slowly (1839) and moved toward its professionalization in the normal schools in 1915--progressing from a rather formalized approach to attention to more professional problems through demonstrations, field work, projects, readings, laboratory work, and participation in elementary school mathematics classes. Strong indication as to which type of course is best is still lacking: separate method and content courses, combined content-method course, CAI course, remedial course, course with or without discussions [underlining mine]. The four years of college requirement for certification has been reached by nearly all states at present, and a fifth year is required in some states for permanent certification.

Finally, teacher education for elementary school mathematics during the period of its professionalization of the subject matter (1915 to present), which corresponds closely with the life span of NCTM, has been affected by many factors: mathematics methods textbooks; children's mathematics textbooks; materials and media; research in mathematics; mathematics guides by states, cities, and counties; yearbooks of the National Society for the Study of Education and the

> National Council of Teachers of Mathematics; the official journals of the NCTM; and pronouncements from commissions and committees of the NCTM. (p. 434)

Clearly, the mathematics which is relevant today is different from the mathematics which was relevant in the early history of our country (Dienes, 1970). The challenge for teacher training institutions is evident. We must prepare teachers, both in content and in methodology, for entering the elementary classroom in which learning how to learn is a meaningful goal for each student so that s/he can keep pace with our rapidly advancing technological society (Elliott, 1976).

Selected Issues and Concerns in the Development of a Syllabus for a Mathematics Methods Course for Pre-Service Elementary Teachers

What Should be the Content in Mathematics for Elementary School Children?

The goals of mathematics for elementary school students in the United States have been influenced by the psychological and cultural developments of each period (Glennon, 1965), and have evolved from divergent, even polar, concerns. Hershkowitz, Shami, and Rowan (1975) stated:

> Mathematicians would select their objectives based upon the content and structure of mathematics . . . . Educators would place considerably more emphasis on learning theory--the way children learn. Politicians would accept either view as long as success could be "accounted" for and a dollar figure could be attached. (p. 723)

These authors suggested:

> One way to establishing a framwork within which objectives could be designed is by

> determining what is important to most people in the general public. This should be a central concern of the schools since the schools exist because and for that group. (p. 723)

This approach to the determination of goals for mathematics education has not been widely utilized; the literature reveals goals developed by individuals and groups at different periods to meet the perceived needs of the times.

Banks (1959) included in his professional text for arithmetic content and methods courses a check list of twenty-nine items; this listing was compiled by the Commission on the Post-War Plans of the National Council of Teachers of Mathematics and was first published in 1947. This was considered by Banks to be an ambitious check list for the first eight grades, yet an ideal to be sought in the elementary school program.

1. Computation. Can you add, subtract, multiply, and divide effectively with whole numbers, common fractions, and decimals?
2. Per cents. Can you use per cents understandingly and accurately?
3. Ratio. Do you have a clear understanding of ratio?
4. Estimating. Before you perform a computation, do you estimate the result for the purpose of checking your answer?
5. Rounding numbers. Do you know the meaning of significant figures? Can you round numbers properly?
6. Tables. Can you find correct values in tables; e.g., interest and income tax?
7. Graphs. Can you read ordinary graphs: bar, line, and circle graphs? the graph of a formula?
8. Statistics. Do you know the main guides that one should follow in collecting and interpreting data; can you use averages (mean, median, mode): can you draw and interpret a graph?

9. The nature of a measurement. Do you know the meaning of a measurement, of a standard unit, of the largest permissible error, of tolerance, and of the statement that "a measurement is an approximation"?
10. Use of measuring devices. Can you use certain measuring devices, such as the ordinary ruler, other rulers (graduated to thirty-seconds, to tenths of an inch, and to millimeters), protractors, graph paper, tape, caliper micrometer, and thermometer?
11. Square root. Can you find the square root of a number by table, or by division?
12. Angles. Can you estimate, read, and construct an angle?
13. Geometric concepts. Do you have an understanding of point, line, angle, parallel lines, perpendicular lines, triangle (right, scalene, isosceles, and equilateral), parallelogram (including square and rectangle), trapezoid, circle, regular polygon, prism, cylinder, cone, and sphere?
14. The 3-4-5 relation. Can you use the Pythagorean relationship in a right triangle?
15. Constructions. Can you with ruler and compass construct a circle, a square, and a rectangle, transfer a line segment and an angle, bisect a line segment and an angle, copy a triangle, divide a line segment into more than two equal parts, draw a tangent to a circle, and draw a geometric figure to scale?
16. Drawings. Can you read and interpret reasonably well, maps, floor plans, mechanical drawings, and blueprints? Can you find the distance between two points on a map?
17. Vectors. Do you understand the meaning of vector, and can you find the resultant of two forces?
18. Metric system. Do you know how to use the most important metric units (metric, centimeter, millimeter, kilometer, gram, kilogram).
19. Conversion. In measuring length, area, volume, weight, time, temperature, angle, and speed, can you shift from one commonly used standard unit to another widely used standard unit; e.g., do you know the relation between yard and foot, inch and centimeter, etc.?
20. Algebraic symbolism. Can you use letters to represent numbers; i.e., do you understand the symbolism of algebra--do you know the meaning of exponent and coefficient?

21. Formulas. Do you know the meaning of a formula--can you, for example, write an arithmetic rule as a formula, and can you substitute given values in order to find the value for a required unknown?
22. Signed numbers. Do you understand signed numbers and can you use them?
23. Using the axioms. Do you understand what you are doing when you use the axioms to change the form of a formula or when you find the value of an unknown in a simple equation?
24. Practical formulas. Do you know from memory certain widely used formulas relating to areas, volumes, and interest, and to discount, rate, and time?
25. Similar triangles and proportion. Do you understand the meaning of similar triangles, and do you know how to use the fact that in similar triangles the ratios of corresponding sides are equal? Can you manage a proportion?
26. Trigonometry. Do you know the meaning of tangent, sine, cosine? Can you develop their meanings by means of scale drawings?
27. First steps in business arithmetic. Are you mathematically conditioned for satisfactory adjustment to a first job in business; e.g., have you a start in understanding the keeping of a simple account, making change, and the arithmetic that illustrates the most common problems of communications and everyday affairs?
28. Stretching the dollar. Do you have a basis for dealing intelligently with the main problems of the consumer; e.g., the cost of borrowing money, insurance to secure adequate protection against the numerous hazards of life, the wise management of money, and buying with a given income so as to get good values as regards both quantity and quality?
29. Proceeding from hypothesis to conclusion. Can you analyze a statement in a newspaper and determine what is assumed, and whether the suggested conclusions really follow from the given facts or assumptions? (pp. 13-15)

Rauff (1979) proposed that the questions related to mathematical competencies must be answered within the context of general educational goals.

> In this regard, I view American education as a social-cultural-political process of rearing children to desired intellectual and moral levels so that they can become confident and competent adults. (p. 50)

Rauff listed four categories of competencies:

1. Academic mathematics (real arithmetic with particular emphasis on the order of operations; basic algebra; use of formulas; formulation of algebraic sentences to solve applications; mensuration)

2. Consumer mathematics (in the marketplace; interest rates; tax schedules; ratio and proportion; percentages; following intelligently a mathematical explanation; computing the dimensions of required materials)

3. Machine mathematics (uses of the digital computer and handheld calculator; constructing and implementing algorithms; interpreting binary arithmetic; approximating answers)

4. Political mathematics (critical thinking; statistical topics)

Close scrutiny of the goals of mathematics education as expressed by the NCTM in 1947 and those proposed in 1979 by Rauff to reflect his view of mathematics as an integral part of a broad foundation in modern culture is invited. It is the observation of this writer that, with the exception of Rauff's inclusion of machine mathematics (reflective of recent technological advances), few critical differences betweeen the two lists are evident.

Such congruence is not necessarily evidenced in other sets of goals for mathematics education. Ragan and Shepherd (1977) reflected upon the goals of the "new math" of the 1960s and compared them with the goals of programs of the 1970s.

> The goal of programs in the 1960s was for the pupil to experience and think about mathematics in ways familiar to the mathematician. The goal of programs in the 1970s is for the pupil to experience and think about mathematics in ways which the average citizen does when producing, adapting, and functioning. (p. 355)

Both sets of programs stress the needs for logical reasoning and for refined performance standards.

Piaget (1975) was critical of traditional mathematical practices, and also warned against the "formalization" of the modern mathematics curricula for children before they are developmentally ready.

> In traditional mathematics it was often necessary for children to solve quantities of problems, some of them quite absurd, and this would mean a huge number of numerical or metrical calculations. In this case, the only way to succeed with children who were not particularly talented in mathematics was to proceed in two stages (but this was often forgotten): the first stage was purely qualitative and dealt with the logical structure of the problem and only afterwards in a second step were numerical or metrical facts introduced with the additional difficulties this type of calculation would create. With modern mathematics programmes the problem is less acute as they are basically qualitative. However, in this case, the problem can be found at another level--the teacher is often tempted to present far too early notions and operations in a framework that is already very formal. In this case, the procedure that would seem indispensable would be to take as the starting point the qualitative concrete levels: in other words, the representations of models used should correspond

> to the natural logic of the levels of the pupils in question, and formalisation should be kept for a later moment as a type of systematisation of the notions already acquired. (p. 9)

Whatever the goals, one of the most outstanding things Piaget teaches us is that mathematics teaching must have infinite variety, for each individual child needs to learn in his own way and at his own pace for maximum value from the learning (Sime, 1977).

The impact of Piagetian developmental psychology on the education of children has been much more strongly felt in the British schools than in the United States (NCTM, 1971). The Nuffield Mathematics Project (Mathematics--the first three years, 1976) and other exemplary programs incorporate working applications of an understanding of the developmental stages as observed by Piaget and lean heavily on environmental and concrete experiences in the mathematics education of the young child. These programs are activity-based with emphasis upon the manipulation of objects and the use of language to foster and enhance the mental activity which leads to the development of cognitive structures by the child (Picard, 1969). Of interest, then, is a listing of skills by British mathematics educators Gardner, Glenn, and Renton (1973) entitled "Mathematics for life: an essential minimum."

1. Addition, subtraction, and one digit multiplication for numbers up to two digits.
2. The use of money in daily life--in effect number work to two decimal places, but handled mentally as pounds and pence.

3. All common aspects of time and date, including timetables and the twenty-four-hour clock.
4. Familiarity with the use of (not computation with) the recommended metric units and such imperial measures as continue to be met.
5. Meaning (not computation) of percentages and averages.
6. Understanding very simple statistical graphs as used, for example, by newspapers.
7. Rough estimates of sizes, distances, and costs.
8. Rounding-off measurements.
9. Reading graduated scales. (p. 30)

It is emphasized that the above is a minimum and indeed a very bare minimum of essential topics without which no child should leave school--at the very least, every attempt should be made for each child to cover these basic skills. In no way are the needs of an educated person to be confused with this listing. The reader may observe that an implementation of a mathematics program based on theoretical developments advanced by Piaget does not preclude some basic expectations of the school curriculum.

The goals of current mathematics education programs as viewed by Nerbovig and Klausmeier (1974) reflect the influence of modern mathematics and Piaget's developmental theory: (a) discovering meaning through involvement; (b) understanding the relatedness of mathematical concepts; (c) thinking creatively about mathematics; and (e) developing favorable attitudes toward mathematics.

Wolfe (1976) expressed the concern that, while new math programs emphasized mathematical concepts, many well-intentioned teachers have taken this to mean that they should not emphasize computational skills. He stated the need "to achieve a more desirable balance between the _understanding_ of mathematics and the _doing_ of mathematics" (p. 91) and called for a renewed emphasis on fundamental skills "_within_ the new math programs rather than _in place of_ them" (p. 96).

In a position statement on basic skills, the National Council of Teachers of Mathematics (1977) voiced strong support for school programs which promote computational competence within good mathematics programs, but at the same time indicated concern that the "back to basics" movement might eliminate teaching for mathematical understanding. The NCTM encouraged the stressing of basics in the context of total mathematics instruction and identifed ten basic skills areas:

1. Problem solving--the process of applying previously acquired knowledge to new and unfamiliar situations--as the principal reason for studying mathematics

2. Applying mathematics to everyday situations

3. Alertness to the reasonableness of results--with the increase of the use of calculators, this skill is essential.

4. Estimation and approximation

5. Appropriate computational skills--facility with addition, subtraction, multiplication, and division of whole numbers and decimals; knowledge of single-digit number facts; mental arithmetic

6. Geometry

7. Measurement

8. Reading, interpreting, and constructing tables, charts, and graphs

9. Using mathematics to predict--elementary notions of probability

10. Computer literacy--what computers can and cannot do.

A desirable level of computational skill was described by Hamrick and McKillop (1978) as: immediate recall of the 390 basic facts; performance of addition, subtraction, multiplication, and division with understanding and at a moderate rate of speed; and skills in estimating, rounding, mental computation, and judging the reasonableness of an answer. The authors listed four reasons for advocating the attainment of this level of skill: (a) it facilitates the learning of subsequent related topics; (b) computational skill helps pupils to understand the meaning and significance of arithmetic operations and to apply these operations appropriately; (c) it facilitates exploration of various topics; and (d) some aspects of computational skill continue to have social utility.

From the previous skills listings, it is obvious that writers in the field of mathematics education for elementary school children are not generally proposing mathematics for the sake of mathematics; neither is there a widely supported movement which limits the subject to purely "immediately applicable to everyday life" concerns. Perhaps there appears to be a considerable case for the proposal for an integrated new mathematics to replace new mathematics as it has been generally interpreted in the elementary school classroom (Kapur, 1977), incorporating those elements of traditional mathematics and new mathematics which will enhance the integration of mathematics education within the totality of all education. Whatever mathematics programs are created, introduced, and supported in the years to come, "neither teachers, educational administrators, parents, nor the general public should allow themselves to be manipulated into false choices between

> The old and the new in mathematics
> Skills and concepts
> The concrete and the abstract
> Intuition and formalism
> Structure and problem solving
> Induction and deduction" (Hill, 1975, p. 136).

What Should be the Content in Mathematics for Elementary School Teachers?

Elementary school teachers have the very important responsibility of helping children to develop an understanding of and an interest in mathematics. The common sense thesis proposed by Rappaport in 1958--that in order to adequately fulfill their responsibility, teachers must themselves understand the basic concepts of mathematics--is just as applicable to teaching today.

The mathematics education of elementary teachers is not a new problem; neither is it a problem of limited or superficial concern (Weaver, 1965). As early as 1938, recommendations were being made for substantial undergraduate preparation in mathematics as a prerequisite for elementary school teaching (Taylor, 1938). However, numerous studies have found elementary school teachers to be woefully lacking in mathematics skills. Newsom (1951) found that many teachers were only one step ahead of their good students. Morton (1953) disclosed that 13.6% of the students in his education classes were below the eighth grade level in arithmetic skills, and some were even below the sixth grade level. Glennon (1949) and Weaver (1956) conducted research in this area and concluded that there was overwhelming evidence that teachers did not have needed arithmetic competencies.

To provide the needed background in arithmetic, Newsom (1951) recommended the following program for teacher trainees:

1. Evolution of arithmetical concepts and notions
2. Number--one-to-one correspondence
3. Positional notation
4. Properties of integers
5. Four basic arithmetical operations
6. The fractions
   a. Terminology
   b. Rational numbers
   c. Common fractions
   d. Decimal fractions
7. The arithmetic of measurement
   a. The process of measurement
   b. Systems of measurement
   c. Computation with approximate numbers
8. Applications
   a. Evaluation of formulas
   b. Ratio and proportion
   c. Business arithmetic
   d. Statistical concepts
   e. Probability (p. 249)

Schaaf (1953) recommended a slightly different program for the preservice mathematical content for elementary teachers.

I. Number Concepts and Numeration

   A. Historical Development
   B. Theory of Numeration

II. Nature of Number

   A. Psychological Considerations
   B. Number Systems of Algebra
   C. Logical Foundations of Arithmetic

III. Computation

   A. Historical Development
   B. Analysis of Theory of Computation

IV. Measurement

   A. Direct Measurement
   B. Indirect Measurement
   C. Elements of Statistics

V. Socio-economic Applications

   A. Arithmetic in the Home
   B. Arithmetic in the Market Place
   C. Arithmetic and Finance

Rappaport (1958) concluded that there was general agreement among the writers on teacher training--although some variations among specific programs--that the background course (note singular form) should deal with the concept and nature of number, the fundamental operations, fractions, decimals, and measurement. However, prior to 1961 college mathematics courses were not generally required; a course in methods of teaching arithmetic made up the average program. Such a course usually had a small amount of mathematical content and concentrated on the "mechanics of teaching" (Dubisch, 1970, p. 287).

In 1960, during the early stages of mathematics curriculum reform, the Committee on the Undergraduate Program in Mathematics of the Mathematical Association of America (through the CUPM Panel on Teacher Training) developed recommendations for five levels of teachers of mathematics. These levels were:

> I. Teachers of elementary school mathematics--grades K through 6
> II. Teachers of the elements of algebra and geometry
> III. Teachers of high school mathematics
> IV. Teachers of the elements of calculus, linear algebra, probability, etc.
> V. Teachers of college mathematics. (CUPM, 1971, p. 1)

The recommendations suggested the type and amount of mathematical training which should be required of teachers of mathematics at each of the five levels. As a prerequisite for the college training of elementary teachers, CUPM recommended at least two years of mathematics at the high school level, consisting of one year of algebra and one year of geometry, or the same material in integrated courses. Then, for their college training, CUPM recommended the following courses or their equivalents: (a) a two-semester sequence devoted to the structure of the number system and its subsystems; (b) a semester course devoted to the basic concepts of algebra; and (c) a semester course in informal geometry.

> During the years 1961-62 CUPM published "Course Guides for the Training of Teachers of Elementary School Mathematics" . . . .
> When it was proposed, the Level I

curriculum received widespread attention and approval. It was approved formally by the Mathematical Association of America, and it was endorsed by three conferences held by the National Association of State Directors of Teacher Education and Certification (NASDTEC) and the American Association for the Advancement of Science (AAAS). It formed a part of the "Guidelines for Science and Mathematics in the Preparation Program of Elementary School Teachers," published by NASDTEC-AAAS in 1963.

In the years 1962-1966 CUPM made an intensive effort to explain its proposed Level I program to that part of the educational community especially concerned with the mathematics preparation of elementary teachers. Forty-one conferences were held for this purpose, covering all fifty states. Participants in these conferences represented college mathematics departments and departments of education, state departments of education, and the school systems. At these conferences the details of CUPM proposals were discussed and an effort was made to identify the realistic problems of implementation of the recommendations . . . .

As a result of these conferences and of other forces for change, there has been a marked increase in the level of mathematics training required for the elementary teacher. In 1966 CUPM repeated a study it had made in 1962 of the graduation requirements in the various colleges having programs for training elementary teachers. A summary of this study is given . . . , but two of its important results are revealed in the following table:

	1962	1966
Per cent of colleges requiring no mathematics of prospective elementary school teachers	22.7	8.1
Per cent of colleges requiring five or more semester hours of mathematics of these students	31.8	51.1

(CUPM, 1971, pp. 1-2)

Throughout the decade of the 1960s CUPM continued to expend considerable effort on problems associated with the preparation of teachers, and in 1966 made minor revisions in the original 1961 recommendations. Continued study by the CUPM led to new recommendations for the minimal preparation of teachers of mathematics. The recommendation for an integrated course sequence for prospective Level I (K-6) teachers was as follows:

> We propose that the traditional subdivision of courses for prospective elementary school teachers into arithmetic, algebra, and geometry be replaced by an integrated sequence of courses in which the essential interrelations of mathematics, as well as its interactions with other fields, are emphasized. We recommend for all such students a twelve semester-hour sequence that includes development of the following: number systems, algebra, geometry, probability, statistics, functions, mathematical systems, and the role of deductive and inductive reasoning. The recommended sequence is based on at least two years of high school mathematics that includes elementary algebra and geometry. (CUPM, 1971, p. 10)

Other groups of mathematicians and/or mathematics educators proposed content requirements for elementary teachers. Among these was the Cambridge Conference on Teacher Training (1967), which produced its Goals for the Mathematics Education of Elementary School Teachers, a widely discussed report which required a substantial mathematics background for all generalists. Several years later, the NCTM Commission on Preservice Education of Teachers of Mathematics developed its Guidelines for the Preparation of Teachers of Mathematics (NCTM, 1973). Emphasized in these guidelines were: (a) the academic and professional knowledge a prospective teacher should possess; (b) the professional competencies and attitudes a prospective teacher should exhibit; and (c) the responsibilities of the institution providing the teacher education programs. The guidelines were designed to provide latitude for testing and experimentation in the establishing of new approaches to teacher education.

Even though there has been wide variation and lack of unanimity as to the specific mathematics background needed by elementary teachers (Cambridge Conference . . . , 1967; Combs, 1963; CUPM, 1960; Dubisch, 1970; Glennon, 1949; Grossnickle, 1951; Hicks & Perrodin, 1967; Layton, 1951; NCTM, 1973; Rosenberg, 1959; Taylor, 1938; and others), few would deny that "a common theme of all programs, past and present, is that elementary school teachers should possess the ability to compute" (Eisenberg, 1974). Evidence indicates that many programs are falling far short of the goals recommended by the CUPM and other recommending bodies, in number and rigor of courses offered and in computational efficiency and understanding of mathematical concepts by the students involved. In the mathematics methods courses taught by Rising (1967) and Catanzano (1977), students who have completed their mathematical content classes were assessed to determine if they possessed the mathematics competencies required of top sixth graders; if not, then that level of competency was aimed for by the end of the methods course.

Englehardt (1974) reflected the concern for the improvement of the mathematics preparation of elementary teachers:

> Although significant improvements have been realized in mathematics instruction at the elementary and secondary levels, disappointingly little progress seems to have been made toward upgrading the mathematics preparation of prospective elementary teachers. In recent years the number of required semester hours of mathematics in college programs for elementary teachers has increased . . . ; yet it is still being reported that teachers lack the necessary comprehension of mathematics . . . . It thus appears that increased exposure to mathematics content is not sufficient for improving prospective elementary teachers' competence in mathematics. (p. 10)

He challenged the content-methods dichotomy and suggested the integration of content and methods instruction as a vehicle for providing more effective and meaningful mathematics education for prospective elementary teachers. (The content-methods course is discussed in some detail later in this paper.)

Catanzano (1977) questioned whether increasing the number of hours or topics required during teacher preparation would improve the teaching of mathematics to elementary school students: "What prospective elementary teachers need is mathematics that can be used by them at the level at which they will be teaching; that is, useful mathematics" (p. 6).

One may detect from the preceding discussion that there is less than solid agreement as to what or how much mathematics the teacher at the elementary school level should know. There is evidence, however, that the mathematics training of the elementary teacher is generally less than adequate by whatever standards are used. Most mathematics educators of today would concur with Weaver (1956) in his statement of the crucial problem relating to the inadequacy of the preparation of elementary teachers in the area of mathematics: "The main requirement . . . is . . . that you understand . . . mathematics . You cannot teach what you do not know" (p. 255).

What Should be the Goals of the Mathematics Methods Course?

In a journal article by Crittenden (1974), a "prepared" teacher was defined as one who had:

> *achieved a prescribed level of mastery of mathematics content;
> *accumulated a theoretical and empirical repertoire of teaching strategies, techniques, aids, and activities; and
> *exhibited a positive attitude toward mathematics as a field of study. (p. 428)

It is obvious to the reader that the time allotment for the mathematics methods component of the elementary teacher education program--usually one three-semester hour course--limits the achievement of the goals implied by the above definition, if all are expected to be accomplished in the one course. Fortunately, certification requirements have been strengthened, and many teacher preparation institutions have increased the requirements for mathematics content in the elementary education program over the last several years, although few have fully implemented the CUPM standards (CUPM, 1971).

Rising (1967) emphasized that the basic goal of the methods course should be "teaching . . . how to teach"

(p. 413). He warned that the course should not be subverted to peripheral goals. He further stated:

> Certainly one of the basic requirements of good teaching is subject mastery; but it is, in mathematical terms, a necessary but not sufficient condition for good teaching. Content goals must play a subservient role in the methods course . . . . (p. 413)

There is a strong case for the combined content-methods course, as discussed later in this paper. According to Morley (1969), to completely dissociate discussion of aspects of teaching mathematics (and vitally, work with children) from teaching the mathematical content is to invite trouble because it cuts off the college students from their main source of motivation. Also, there is the movement in some sectors as discussed by Smith (1973) to replace courses in teaching in special academic areas such as mathematics with courses in general methods. This practice is frowned upon by the CUPM (1960), Smith (1973), and others in the field of mathematics education. For the discussion of the purposes and objectives of the methods course for the teacher of elementary school mathematics, however, attention is directed to the usual teaching arrangement of the methods course in mathematics as a separate course taught by teacher educators.

Brown (1954) found the following to be characteristic of the aims of a functional methods course in teaching elementary school arithmetic: (a) basic mathematical understandings; (b) fundamental principles of learning as related to arithmetic teaching; (c) recognized techniques of successful instruction; and (d) pertinent desirable attitudes.

Ray (1967) stated as the primary purpose of the methods course: to acquaint students with modern methods of teaching with emphasis on purposes, content, activities, vocabulary, and evaluation.

According to Dienes (1970), there should be no goals related to the learning of principles of methodology; any principles arrived at should be learned by the trainees themselves as the result of their experiences with materials, with children, and among themselves.

Inskeep (1972) related the goals of the mathematics methods course to the goals of mathematics education in the classroom:

> Mathematics education in the classroom must include methods (the means to teach), consideration of appropriate content (both scope and sequence), and the effect of the interaction of content and method in the learner (specifically considering both the characteristics of children and the psychology of learning and knowing mathematics). (p. 255)

Murtha (1977) further related the goals of the mathematics methods class to the goals of teaching:

> Given the difference in environment, responsibilities and expectations there seem to be some basic principles . . . . To be effective, any teacher must, among other things, have the ability to (1) plan and organize material, (2) convey a sense of enthusiasm toward the subject, (3) evaluate the learners' progress, and (4) cope with a range of personalities. It seems . . . that . . . experience in these fundamentals is not only valuable but transferable from one level of teaching to another. (p. 476)

Whatever the proclaimed goals of mathematics methods courses, this writer agrees with the following statement:

> The training of good teachers is far more important than the curriculum. Such teachers can do wonders with any curriculum. Witness the number of good mathematicians we have trained under the traditional curriculum, which is decidedly unsatisfactory. A poor teacher and a good curriculum will teach poorly whereas a good teacher will overcome the deficiencies of any curriculum. (Kline, 1973, p. 170)

Further, this writer accepts the following statement of four basics in mathematics teaching (Trivett, 1977) as guidelines in the formulation of goals for mathematics methods classes:

1. Mathematics lessons are human activities--mathematics is the study of relationships, their dynamics, and their crucial role in our understanding of ourselves.

2. Teachers must know well the mathematics they are teaching.

3. Teachers must know that their pupils are capable of learning and enjoying what is offered.

4. Teachers must know how such mathematics is communicated between human beings in classroom situations.

What Objectives Should be Included in the Mathematics Methods Course?

Perhaps the most apparent finding resulting from an examination of the objectives to be included in a mathematics methods course for elementary pre-service teachers is the lack of consensus as to what those objectives should be.

--In an analysis of six textbooks commonly employed for methods courses, it was impossible to determine any systematic direction of modern mathematics education; much variation was found in topics covered. Specific teaching methodology appeared to be the predominant topic. "There is little consensus in the essential thought set forth in the books" (Cruikshank, 1969, p. 48).

--Henson (1971) observed that there was lack of uniformity in the methods course from section to section within an institution; how much more diversity might there be among institutions?

--In a discussion of the work of the International Congress for Mathematics Educators, Egsgard (1978) stated that the group was unable to find or develop an effective example of teacher education which could be used universally.

Various groups and individuals have made recommendations for the objectives and content of the mathematics methods course.

The Committee on the Undergraduate Program in Mathematics indicated that effective mathematics teachers must be familiar with such items as:

A. The objectives and content of the many proposals for change in our curriculum and texts.
B. The techniques, relative merits, and roles of such teaching procedures as the inductive and deductive approaches to new ideas.

C. The literature of mathematics and its teaching.
D. The underlying ideas of elementary mathematics and the manner in which they may provide a rational basis for teaching . . . .
E. The chief applications which have given rise to various mathematical subjects. These applications will depend upon the level of mathematics to be taught, and are an essential part of the equipment for all mathematics teachers. (NCTM, 1970, p. 340)

LeBlanc (1970) stated that the mathematics methods courses for the general elementary school teacher should include careful work on:

Identifying performance objectives--scope and sequence--of a typical modern program.
Identifying "nice-to-know" concepts as differentiated from "need-to-know" concepts.
Becoming familiar, through use, with aids and materials appropriate for mathematics centers or for teaching. (p. 609)

Houston (1971) asked college professors (mathematics educators) and elementary school teachers to rate thirty-seven objectives in terms of their importance for the prospective elementary teacher. The ten most highly rated objectives (not necessarily in order of importance) related to the following:

1. Introduction of a lesson to elicit active pupil participation

2. Provision for discovery

3. Use of inductive teaching techniques

4. Understanding of a variety of teaching strategies

5. Use of manipulative devices

6. Interpretation of test data

7. Use of the diagnostic interview
8. Construction of tests
9. Discussion of the function of a testing program
10. Use of accurate and appropriate mathematical language.

In view of the current effort to convert to the use of the metric system in everyday life, Trent (1975) stated the need for increased emphasis on methodology for the teaching of the metric system to be incorporated into the elementary mathematics methods course.

The need for a strong diagnostic component was related by Rexroat (1972), Hollis and Houston (1973), and Catanzano (1977). This emphasis is compatible with the current trend to diagnostic and prescriptive teaching.

Various reports of the needs of practicing elementary teachers (Collea & Pagni, 1973; Fowler, 1973; Muzzey, 1974; Reys, 1967; Weiss, 1978) indicate that there are sizable numbers of teachers who are less than satisfied with the preparation provided by their undergraduate mathematics methods courses. In a study by Fowler, over 70% of the teachers surveyed indicated that more specific details on presenting lessons, more training in innovative techniques, more observations, and more opportunities to try out methods with children would have made methods courses more effective. Teachers surveyed by Muzzey stated that the effectiveness of the mathematics methods course would be enhanced by limiting discussion in the area of mastery of basic skills and by increasing the discussion devoted to diagnostic, remedial, and evaluation procedures. Weiss found that a sizable number of teachers would like additional assistance in obtaining information about instructional materials, learning new teaching methods, implementing the discovery/inquiry approach, and using manipulative materials.

In light of the abundance and diversity of objectives for the methods component of the education of the teacher of elementary school mathematics, it is essential that the methods teacher recognizes the limitations inherent in a three semester-hour course: "A methods course cannot do everything. It is best to concentrate on a few objectives, objectives which instructors and students select" (Leake, 1976, p. 193).

In an article by Hansen (1978) entitled "Returning to the Basics--Or Should We Have Ever Left Them?" basic and foundational teaching skills were identified and clustered around general instructional components for teaching success:

> Planning and Preparation. The ability to construct adequately, organize, and implement the goals and objectives of instruciton.
>
> Classroom Management and Organization. The ability to effectively and efficiently institute and maintain the operation of classroom practices.
>
> Communication Skills and Strategies. The ability to utilize various and appropriate communication techniques and procedures for optimal teaching-learning experiences.

Interpersonal Regard Skills. The ability to demonstrate concern, feelings, emotion, and understanding for others and provide a classroom climate where these transactions may occur.

Assessment and Evaluation Skills. The ability to determine the needs, means, and processes to determine the level and extent of instruction and the success of that activity.

Teaching Strategies and Techniques. The ability to move logically and psychologically in a classroom activity so that the process results in a successful learning experience.

Teaching Mode and Style. The ability to appraise and to utilize the preferences, personality, and behaviors of an individual, respecting his integrity, and allowing him to function and influence the classroom.

Skills in the Effective Use of Materials and Resources. The ability to identify and to utilize appropriate resources to demonstrate, illustrate, enhance, and amplify the learning objectives.

Skills in Group Behavior. The ability to understand and to utilize the processes and dynamics of transactions between individuals and groups to promote a worthwhile classroom experience.

Information Processing Skills. The ability to present the content and subject area, acquire new and useful information, and construct activities and opportunities for successful student participation with the content.

Implementation of Learning Theory. The ability to demonstrate and to initiate principles of learning and motivation into a practical classroom experience.

Skills in School, Staff, and Community Relationships. The ability to identify one's presence and role in terms of others within the system, organization, and community.

Skills in Dealing with Pupil Behavior. The ability to understand and to relate the levels of development, individual needs, and behavioral patterns. (pp. 90-91)

It is the view of this writer that, through careful selection and implementation of objectives, the mathematics methods course has potential for contributing significantly to the development of these "'old hat' concepts of instructional practice" (p. 91) which appear to incorporate the essence of good teaching for now and for the future.

What is the Role of Individualized Instruction in the Mathematics Methods Course?

The literature appears to support at least two strong contentions for providing for individualized instruction in the methods course:

1. The methods course should respond to the individual needs of the pre-service teacher, since each learner is different. "Professional educators offering courses in the teaching of arithmetic must . . . provide instruction for prospective teachers which is based upon individualized . . . goals" (Dutton and Cheney, 1964, p. 198).

2. If the methods student is to learn to individualize instruction in the elementary classroom, s/he needs to be involved in a model of that approach in the metnods class.

> An effective methods course needs to be taught with a variety of methods so students can see in action what they are supposed to be learning. A university professor espousing individualized instruction in a methods course conducted with a traditional lecture-textbook approach is less than reassuring. (Leake, 1976, p. 193)

In providing for individualized learning in a mathematics methods course at California State University, Sacramento, Arnsdorf (1977) organized the course around six objectives in which there were provisions for options:

1. Reading in a methods textbook

2. Peer microteaching ten concepts found in elementary school mathematics

3. Investigating mathematics laboratory materials

4. Completing a written examination to demonstrate competency in working with mathematical content (items on the assessment were taken directly from the sixth grade textbook under statewide adoption in California; the methods student is expected to perform on this test at a minimum of 85% competency)

5. Reading from journals pertaining to elementary mathematics education

6. Creating an aid, device, or game to be used in teaching elementary mathematics.

Not all students completed all objectives; these objectives were satisfactorily completed by students earning a grade of "A" or "B".

Houston and Hollis (1972) advocated a personalized mathematics methods course:

> Personalized instruction extends beyond individualized instruction. Individualized instruction implies differentiated pacing, differentiated content and varied modes of instruction. However, the attention is upon instruction. The teacher maintains control of the parameters of instruction; the student maneuvers within these already prescribed restrictions. His choices are

> limited by tradition, by discipline structure and by teacher precept. The personalized program recognizes the student as a participant in decisions which affect him, not just as a recipient. Personalization assumes that the student has the following opportunities: to negotiate that which is studied in his program; to assess independence and responsibility; and to understand himself as he relates to his environment. Thus, personalization of instruction requires its individualization; humanization of instruction requires its personalization. (p. 48)

Through a personalized, criterion-referenced, modular instructional program, prospective teachers were exposed to experiences involving directed discovery, programmed instruction, multimedia presentations, group discussions, simulated teaching sequences, actual experiences with children, and continual feedback.

Computer Assisted Instruction (CAI) was utilized by Heimer (1973) to provide individualized instruction on the theoretical concerns of the methods course. Advantages of the use of CAI reported by Heimer were: (a) effective individualization of instruction relative to theoretical aspects of the course; (b) the ability inherent in the computer-based program to update and refine as necessary; and (c) the utilization of the CAI model as a basis for research.

Regardless of the apparent dearth of models presented in the literature, it is the position of this writer that a basic truth concerning individualization is evident: learning <u>will be individual</u> whether or not the methods teacher actively plans for individualized instruction.

Because each learner is different and because each person must learn for himself/herself, it seems that students need to at least partially determine what they are to learn and how and when they are to learn it.

<u>What is the Competency-based Mathematics Methods Course?</u>

Closely akin to an individualized approach to the teaching of the mathematics methods course is the competency-based course. McKillop (1975) communicated the focus of competency-based teacher education in elementary mathematics methodology:

> Methods courses in CBTE programs may be expected to contain a balance of theoretical and applied information. Theory is presented because it supports and explains the performances expected. It supplies the information needed for decisions as to when, how, and with whom to exercise the observable performance. Inferences from the theory should be made clear: Do this, this, and this to obtain that result. Methods in CBTE would be eclectic, using psychological theory, the nature of the knowledge being taught, research findings, conclusions based on observations of teachers, and whatever other source of information produces descriptions of valid competencies.
>
> Methods courses in CBTE are not necessarily coordinated with internship or student teaching experiences. The attainment of some competencies can be demonstrated without actually teaching children. Simulated teaching, presenting lessons to other college students who play the part of students, can to some extent substitute for an internship experience. It is my experience, however, that these courses are most effective when the students are concurrently working with a class (or better yet a small group) of children in a normal public school setting. The main advantage is improved learning of the teaching

> techniques. Interns start out with a real group, diagnose their needs, use a specific teaching technique, and observe that the students do in fact learn what they intended. This not only demonstrates convincingly that the intern has acquired the competency but the experience of successfully teaching reinforces and validates the technique for the intern. Without the final step of using it and seeing it work, the competency remains at the level of "theory," easily overlooked in the chaotic experience of beginning teaching. (pp. 10-11)

Many competency-based mathematics methods programs were reported in the literature (Brent, 1973; Justice, 1975; McGregor, 1976; Rexroat, 1972; Sowell, 1973; Woodworth, undated). A program developed by Brown and others (1974) incorporated a set of 15 modules, each on either a content or a methodological problem.

1. Problem-solving in elementary mathematics
2. Using drill activities
3. Using the text
4. Teaching mathematical ideas
5. Teaching by discovery
6. Teaching reading in mathematics
7. Teaching concepts of fractional numbers
8. Teaching addition and subtraction of fractional numbers
9. Teaching multiplication and division of fractional numbers
10. Geometry: content for grades 1 through 6
11. Geometry: activities for grades 1 through 6
12. Teaching measurement in the primary grades

13. Whole number concepts: learning stages

14. Whole number concepts: teaching procedures

15. Teaching numeration in the primary grades.

The authors reminded the reader that the set of modules was subject to further development.

Each module was designed to have certain features in common:

> 1. Each module is based on a specific set of minimum competencies that the inservice or preservice teacher must attain in order to complete the module. These are the competencies deemed to be necessary (but certainly not sufficient) for acceptable teaching of elementary school mathematics. Where possible, these competencies are expressed in behavioral items.
>
> Example: "Given a problem in multiplication of fractions, the teacher can draw a diagram or picture to illustrate the problem and its solution."
>
> 2. Each module has a pretest that covers the objectives of the module. Thus a teacher who has already attained a particular competency is not required to study the related portion of the module.
>
> 3. The modules make use of existing instructional material--elementary school mathematics textbooks, textbooks on methods and content designed for teachers, films, and so forth. Where possible, alternate routes to the attainment of objectives are provided.
>
> 4. The modules make regular use of either real or simulated teaching performance. Paper-and-pencil test performance is necessary but not sufficient for completion of these modules.
>
> 5. The modules are so designed that local educational personnel can use them with minimal training. They have some built-in self-study avenues although the presence of a knowledgeable instructor is an advantage.

> 6. Each module has a posttest, often a parallel form of the pretest. Because the objectives define what have been judged to be minimum competencies, a teacher is not considered to have completed a module until she has attained all of the listed competencies. (Brown, et al, p. 221)

Mueller (1977) developed a four semester-hour combined science and mathematics program; the following content was reported for the mathematics methods component:

> 1. The Methodological Core--a psychological and methodological basis for mathematics teaching
> 2. Understanding Numbers and Numeration
> 3. Teaching Whole Number Algorithms and Rational Number Concepts and Algorithms
> 4. Introducing Measurement and the Metric System
> 5. Geometry for Elementary School Children. (p. 185)

Each topic was introduced by establishing a rationale, a list of objectives, a statement of assignment steps to follow, and a statement illustrating mastery.

> Topic one directed the student to a variety of readings, tapes, and slides. Topics two through five were organized into modules or "unipaks." These were semi-programmed auto-tutorial assignments which required the student to "think as an elementary teacher must think." Manipulation of physical materials, i.e., blocks, popsicle sticks, Cuisenaire, rods, geoboards, attribute pieces, tanagrams, mirror cards, plus rocks, bottle caps, lengths of string, paper, and wood, were required to complete the assigned activities.
>
> Student competence was determined by successful completion of the assignments and 80% mastery as demonstrated on a criterion-reference measurement for each topic. (p. 185)

The current emphasis on humanistic objectives in teacher education has led to a decline in the competency-based movement as reported in the literature. The competency-based thrust has left its impact, however, on the methodology employed by the teacher educator, particularly in relation to the specification of student outcomes. This writer has utilized elements of the philosophy underlying competency-based education in the development of a syllabus for a mathematics methods course; the syllabus is included in the appendix of this paper.

What is the Content-Methods Approach to the Mathematics Preparation of Pre-service Elementary Teachers?

One possible approach to the mathematics preparation of teachers is the integration of content and methods. Although this approach has not been implemented on a widespread basis for various reasons, there is a strong case evident in the literature for such a plan for teacher preparation. Reys (1968) questioned the pedagogical soundness of offering separate courses in mathematics content and methodology for elementary school teachers, and Van Engen (1972) emphasized that in the mathematics courses for elementary teachers, there must be some relationship--both mathematical and pedagogical--between what preservice teachers learn and what they are going to teach. The combined content-methods courses discussed in the following paragraphs varied in the number of credit hours assigned; in some instances the amount of credit was not specified.

Phillips (1960) observed that in teaching separate courses, the elementary teachers usually did not gain a deep understanding of the fundamentals of mathematics and the principles of learning. He gave a very convincing rationale for the preparation of elementary school teachers through the combined content-methods approach. The following reasons or goals for this approach were stated:

> 1. Efficiency in learning.
> The combined content-methods approach brings about a greater amount, depth, and integration of knowledge in the amount of time we can allocate in teacher training of elementary mathematics.
> 2. Retention of learning.
> The greater depth and integration of knowledge achieved by the combined content-methods course results in better retention. This is important since the prospective teacher may teach next year or three years later.
> 3. Application (or transfer) of learning.
> The combined content-methods course with its emphasis on the integration of the three categories will result in better teaching in the actual classroom. This outcome is due to the fact that in the actual classroom there is a fusion of the three aspects (mathematical, social, and psychological.)
> 4. Attitude of the learner.
> A combined content-methods course will bring about a better attitude toward the teaching of elementary mathematics. One of the reasons that elementary teachers are afraid to teach elementary mathematics is that they don't understand it themselves. The combined course with its emphasis on presenting a vertical development from the concrete to the abstract (which at times includes algebraic representations, generalizations, principles, and relationships) will result in understanding elementary mathematics. Understanding aids in confidence, interest, and attitude. Confidence, interest and attitude of the elementary teacher is contagious. We want the children to like mathematics and to have an interest in continuing in mathematics as they go to high school and college. (p. 158)

Phillips (1968) reported a successful program at the

University of Illinois which combined the mathematics and methods courses into one; this course which was equivalent in hours of credit to both separate courses, was characterized by the simultaneous development of both content and teaching and was taught in either the mathematics or the education department by a staff member qualified in both fields.

The term conceptual mathematical methodology was introduced by Brousseau (1971) to represent the integration of mathematics concepts with the methods employed to teach mathematics in the elementary schools. Brousseau emphasized the need to rely heavily on combining into integrated courses the methods of teaching elementary school mathematics and the math concepts one would expect elementary teachers to master.

The Indiana University Mathematics-Methods Program (1972) completely integrated mathematics content and methods courses for undergraduate elementary education majors. Each instructional unit in the program consisted of activity-lessons in which the mathematical content, the related elementary school learnings, and appropriate pedagogical techniques were developed using a laboratory strategy. In conjunction with the two semester, twelve credit hour programs, the students visited an elementary school each week for the purpose of learning how children think and reason about mathematical ideas.

Shakrani (1973) described an experimental program in which the mathematics component was composed of two courses which integrated the study of mathematics with the methodology of teaching mathematics in a laboratory setting. The courses were developed by a team of mathematics educators and elementary school teachers. Each topic was covered in one week (eight class hours) and was implemented in an elementary school during the succeeding week. This activity-oriented, integrated content-methods approach with concurrent clinical experiences resulted in significant positive effects on the achievement and attitudes of the experimental group of college students.

Englehardt (1974) strongly supported the integration of content and methods in the preparation of elementary school teachers, and identified the following advantages:

> One advantage to this scheme is that the instructor would have the opportunity to present subject matter using instructional methods and procedures consonant with those recommended for children. Of course, this is not to imply that the content would be presented at the same level of abstraction as that in the elementary mathematics curriculum; much can be said of teachers' need for a broader understanding of the mathematical structure underlying the elementary curriculum. A second advantage to this scheme is the continuity of content and methods instruction. By having such a prolonged exposure to a given mathematical topic, a more thorough mastery of the content may be encouraged and less loss of conceptual understanding due to the interference of intervening time or concepts might be experienced. A third advantage is that content and methods instruction may be mutually motivating. For each topic, those students "turned on" by their exposure to the content may be motivated for the methods instruction and those students positively anticipating the methods instruction may be motivated sufficiently to attempt

> mastery of the content. In general, this scheme may provide continuity and consistency in the preparation of elementary teachers of mathematics. (p. 497)

If the dichotomy between mathematics content and methods courses were left open to question, it is the contention of Englehardt that one could view mathematics content-methods instruction as a nine or twelve hour block and devise new and creative ways of approaching the preparation of elementary teachers of mathematics.

The case for the content-methods course has been investigated through dissertation study. Young (1969) studied the effectiveness of three approaches to the teaching of the mathematics methods course: one approach emphasized the mathematical content of the elementary school curriculum; the second was a combined content-methods approach; and the third emphasized the psychology and methods of teaching elementary school mathematics including classroom visits and working with elementary school children but excluding the study of separate topics of content. The tests of significance were not conclusive in determining which approach was best for all types of students. Students with different backgrounds were found to have varying degrees of success with different approaches.

Knodel's comparative study of two approaches to teaching mathematics methods to prospective elementary teachers--an integrated content-methods course and a sequence with separate content and methods courses--indicated that the course in which methods and content were integrated was more effective than separate courses (1971).

Fithian (1972) studied the relative effect of coordinated sequence of mathematics content and methods on the attitudes toward mathematics, achievement in mathematics, and achievement in the methods of teaching mathematics of prospective elementary teachers. It was concluded that the coordinated sequence of courses in mathematics content and methods was successful in improving attitudes and achievement.

This writer agrees with Catanzano's statement (1977) that teachers of prospective teachers should integrate as much as possible method with content into the course sequences. The feasibility of implementing a combined content-methods course in her current setting, however, is another issue. Because of administrative difficulties (the methods course is taught in the College of Education and the content courses are taught in the College of Science and Technology), the decision was made to design a syllabus for a methods course within the guidelines of current administrative arrangements.

What is the Laboratory Approach to Teaching a Methods Course?

The laboratory method is defined as activity by students primarily with materials other than chalkboard, paper and pencil, textbooks, or library reference materials. It involves the use of materials, models, instruments, equipment, and various manipulative aids "with the aim of deducing and abstracting therefrom certain mathematical concepts and understandings" (NCTM, 1954, p. 214). In teacher education, a methods course utilizing laboratory procedures provides actual experiences with a wide variety of such laboratory materials which are of types appropriate to future teaching needs. Because the variety of such materials is so broad, there is considerable lack of uniformity among such courses from one teacher preparation institution to another.

Influenced to a great extent by Piagetian developmental psychology, the theme of British preparation of elementary school teachers in mathematics education is that "if teachers are to be convinced that children can learn mathematics through their own activity and discovery, they must first experience discovery of mathematical concepts for themselves" (McGlone, 1972, p. 5). Therefore British pre-service and in-service education strongly emphasizes independent exploration and discovery. There is limited use of laboratory situations in the training of elementary school teachers in the United States.

The value of mathematics laboratories for prospective elementary school teachers has been emphasized by many individuals. The first strong influences in the United States date back to John Perry in 1901, and to E. H. Moore in 1902, who advocated "a shift from the purely abstract teaching of mathematics to the graphic approach and the use of models and equipment to discover the principles as well as portray the applications of the subject" (NCTM, 1954, p. 212).

More currently, the writings of several individuals have been of some influence.

The impact of the British movement in mathematics education has been felt in a large part through the work of Edith Biggs (1968), who emphasized these three aims in mathematics teaching at any level: (a) let people think for themselves; (b) let them discover the mathematical patterns which are to be found everywhere in the man-made and natural environments; and (c) give people the skills they need.

Arthur Morley wrote in 1969 that "laboratory type courses are much more successful for many prospective elementary school teachers. An important aim of this work is to give the student experience in using materials to set up problem situations" (p. 59).

John LeBlanc (1970) noted the need for the elementary school teacher to know what materials are available and how to use them in mathematics laboratories: "It is just as appropriate to have a math lab center for preparing teachers as it is to have a lab for science methods" (p. 607).

R. E. Reys suggested the following statements as the basic foundation underlying the rationale for the use of the laboratory approach in learning mathematics:

1. Concept formation is the essence of learning mathematics.
2. Learning is based on experience.
3. Sensory learning is the foundation of all experience and thus the heart of learning.
4. Learning is a growth process and is developmental in nature.
5. Learning is characterized by distinct, developmental stages.
6. Learning is enhanced by motivation.
7. Learning proceeds from the concrete to the abstract.
8. Learning requires active participation by the learner.
9. Formulation of a mathematical abstraction is a long process. (Callahan and Glennon, 1975, p. 118)

In a study of the use of manipulative materials in a combined mathematics content-methods class, Fuson (1975) found "that teachers can learn mathematics in a way that is relevant to, and very useful in, teaching mathematics to children" (p. 61).

In a study of the use of the mathematics laboratory approach with pre-service teachers, Flexor (1978) found the following:

1. Manipulating physical objects in an inquiry-discovery mode is a nonintimidating context in which prospective teachers can discover, conceptualize, and verify important mathematical concepts for themselves.

2. Confidence gained from understanding mathematical concepts is reflected in improved attitudes toward mathematics.

3. Students who have themselves learned some mathematics in the inquiry-discovery mode are more likely to use this method when they become teachers.

This writer agrees with Eidth Biggs' (1968) statement that "there is need for us to shift the emphasis from teaching to learning, from our world to the children's world" (p. 105). There are various manipulative materials housed in an alcove of the Educational Resources Center of her university; methods students will be encouraged to explore and utilize these materials in planning and implementing practicum activities.

What is the Role of Professional Laboratory Experiences in the Methods Course?

Professional laboratory experiences have been defined to include all those contacts with children which make a direct contribution to an understanding of individuals and how to guide them in the teaching-learning process (NCTM, 1954). A methods course in the teaching of mathematics can be made meaningful through the utilization of professional laboratory experiences. The experiences provide an opportunity for college students to relate the theory of the college classroom to practice by giving them a responsible role in an actual teaching-learning situation and by guiding them in the defining and study of problems involved in teaching the content of mathematics.

Suggested aims for professional laboratory experiences were included in the National Council of Teachers of Mathematics Yearbook in 1954, and have remained current to date. They are:

1. To define and study problems arising in a teaching-learning situation

2. To study boys and girls as groups and as individuals
3. To relate concepts developed in the professional program to those which exist in practice
4. To become effective in relationships with the boys and girls, the teacher, and the school staff
5. To become familiar with planning for teaching-learning experiences
6. To stimulate the boys and girls to think critically
7. To become effective in securing and using to advantage material for teaching-learning situations. (p. 190)

The commitment to professional laboratory experiences was reiterated by the NCTM in its Guidelines for the Preparation of Teachers of Mathematics (1973).

> The prospective teacher of mathematics should study the theories of teaching and learning concurrently with laboratory and clinical experiences, direct and simulated, so as to be able to relate theory and practice. This combined study and experience should begin as early as practicable . . . in the preparation of the teacher . . . . This study and activity should integrate what the prospective teacher has learned about the mathematical, humanistic, and behavioral sciences. (p. 15)

Recognition of the need for the utilization of professional laboratory experiences in the mathematical methods course is clearly indicated in the literature.

Rising (1969) urged the methods teacher to get his/her students into the classroom setting. "The methods class is not a substitute for the one more valuable teacher training experience: internship. You will, however, be teaching in limbo if you do not get your students thinking about young people" (p. 416).

Brousseau (1971) urged the methods teacher to provide each elementary major with the opportunity to actually teach the material to the level of his/her own choosing, K-6.

Unkel (1971) found that tutoring elementary students over a period of a quarter (50 to 120 minutes biweekly) resulted in a statistically significant increase in knowledge of basic mathematical concepts.

The Indiana University Mathematics-Methods Program (1972) involved the methods students in a visit to an elementary school each week.

Merwin and Templeton (1973) emphasized the need to place some experiences in methods courses in the field: "One can hypothesize that if teacher education programs would provide earlier and greater involvement in the public school that at least the teacher trainees would perceive the course offerings to be of greater value" (p. 159).

Shakrani (1973) reported that in a content-methods course which was activity-oriented and provided clinical experiences with groups of elementary school children, there was a significant positive effect on achievement and attitudes of the college students.

Hope and Aikenhead (1974) utilized the miniature teaching episode in an elementary science and mathematics methods class--the students taught a small group of children for approximately one hour.

Green (1976) recommended that laboratory experiences--experiences with children--should be required simultaneously with elementary methods courses so that day-by-day concerns and problems may be solved immediately.

Redwine and Wojtowicz (1976) stated the need to structure field experiences in relation to methods classes in such a way that the program leads to a synergistic system involving pre-service teachers, teacher education, and co-operating teachers.

Thornton (1977) observed the following:

> There is an indication that regular, planned school experiences in conjunction with the mathematics preparation of preservice elementary teachers may have an impact on teacher competency to produce mathematical learning in children. (p. 24)

This writer is of the conviction that properly planned for, implemented, and evaluated professional laboratory experiences can be of great value in aiding the pre-service elementary teacher in developing competencies in content and in methodology and in integrating theory and practice. Because of her commitment to the involvement of the pre-service teacher in the elementary classroom, she has planned a mathematics methods course in which 15 of the 40 class sessions are practicum experiences (see course syllabus).

Who Should Teach the Methods Course?

The Committee on the Undergrate Program in Mathematics made recommendations for the qualifications of the teacher of the mathematics methods course:

> We would like to stress that adequate teaching of such courses can be done only by persons who are well informed both as to the basic mathematical concepts and as to the nature of American public schools--and as to the concepts, problems, and literature of mathematics education. In particular, we do not feel that this can be done effectively at either the elementary level in the context of "general" methods courses, or by persons who have had at least the training of level IV [equivalent to the M.A. in mathematics]. (NCTM, 1970, p. 340)

Rising (1967) stressed the implications of the difficulty of teaching yet the potential value of classroom teachers of the mathematics course.

> Many can do, but cannot teach;
> Fewer still can teach teachers. (p. 412)

Servais and Varga (1971) emphasized the need for teacher trainees to be instructed by highly competent teachers who display in their own teaching the characteristics and qualities required of the future teacher.

In practice, the mathematics methods course is taught most commonly as a component of the teacher education department of colleges and universities, although in some instances it is a function of the mathematics department. It is taught by instructors who hold varying degrees of preparation for and commitment to their instructional assignment--in many institutions methods courses are low in prestige and priority.

Throughout the literature the mathematics methods teacher is urged to recognize the importance of the methods course in shaping the attitudes and future performance of the pre-service elementary teacher; further the methods teacher is encouraged to "exemplify what s/he explicates" in planning, organizing, and managing the learning environment for the methods student.

Limitations and Concluding Remarks Regarding the Syllabus

The reader is asked to consider the syllabus (see Appendix) as a preliminary and rudimentary draft and to keep in mind the following:

1. It is not reasonable, in the writer's opinion, to ask college students to purchase three textbooks for a three-hour course; therefore, textbook choice will require further consideration. The textbook by Weiss is currently used by most teachers of Elementary Education 305 at Western Kentucky University.
2. The fifth and sixth year class was chosen for practicum involvement because one group of students (multiaged) at this level is assigned to the writer.
3. Probably more activities have been written into the course than can reasonably be expected to be completed in the time available. These activities need to be prioritized further and perhaps some need to be eliminated.
4. At this point there is little planned strategy for student involvement in the course design. This is a shortcoming which can be remedied when the writer has had enough experience to establish a baseline of reasonable expectancies for student participation.

5. The writer will need to locate, develop, or adapt an instrument which can be used for comparing and contrasting elementary school mathematics texts.

6. The demonstration lessons will be planned to exemplify specific teaching strategies and may be on videotape or live. Details for these are yet to be completed; hopefully lessons in primary mathematics will be included.

7. The reference list is only a starting place for reading. It may or may not include sources which will meet the individual needs of the college student. An excellently equipped Educational Resource Center in the College of Education houses mathematics manipulatives and audio-visual aids as well as books and journals.

8. Two assessments of basic mathematics skills are included. They were adapted by this writer from sixth grade materials currently utilized for instruction. One assessment is to be used as a pretest and is to be administered early in the methods course so that college students can identify their weaknesses and seek remediation through individual pursuits. The second assessment is to be used as a posttest near the end of the course.

9. Accompanying the syllabus are some additional materials developed or adapted by this writer to be used as needed in implementing elements of the methods course.

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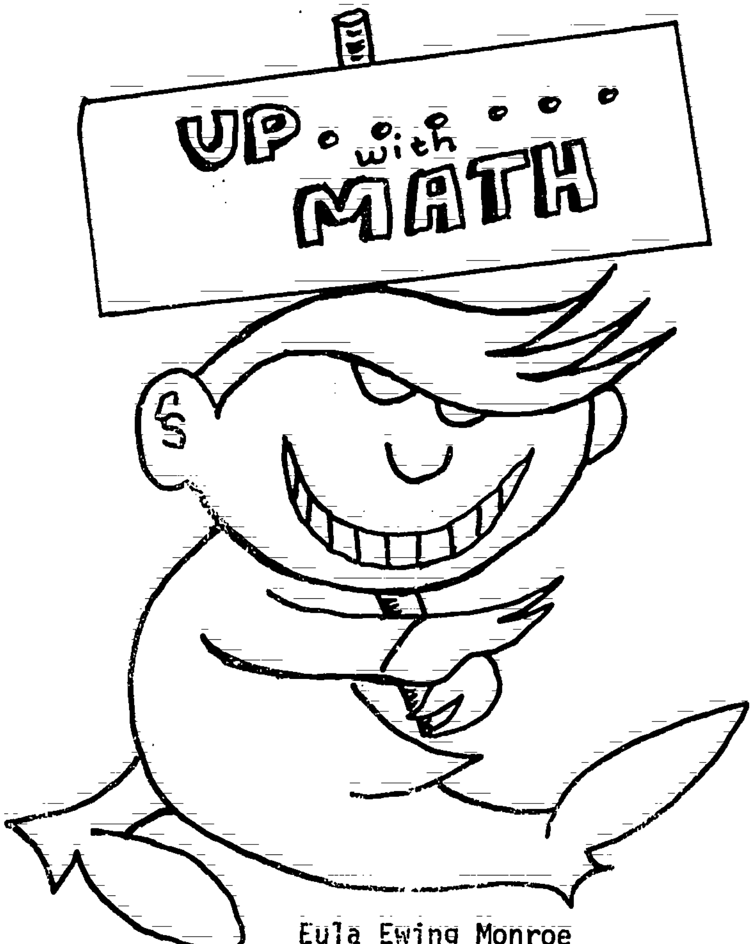
APPENDIX

TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL

SYLLABUS

Elementary Education 305
TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL
(3 hours)



Eula Ewing Monroe
Jones-Jaggers Laboratory School
Western Kentucky University
Bowling Green, Kentucky 42101
502/745-4844

PREREQUISITES

Mathematics 101 and 102

DESCRIPTION

Materials and methods of instruction in elementary school mathematics with emphasis upon creative utilization of available materials and techniques

TEXTBOOKS

Weiss, Elementary mathematics: Teaching suggestions and strategies, 1978

AND/or

Copeland, How children learn Mathematics, 1979

and/or

Hollis and Houston, Acquiring competencies to teach mathematics in elementary schools, 1973

OBJECTIVES

Upon completion of the course Elementary Education 305 the participant should have:

1. assessed own skills in basic mathematics and remediated as necessary.
2. examined in detail elementary school mathematics practices.
3. employed realistic procedures for assessing children's mathematics achievement, needs, and interests as bases for classroom organization and instruction.
4. analyzed and implemented various approaches to teaching mathematics.
5. examined mathematics skills needed in other subject areas.
6. utilized appropriate materials, equipment, and media for the teaching of mathematics at the elementary school level.

PRACTICUM

General Information

In intermediate classroom at Jones-Jaggers Laboratory School
During regularly scheduled college class period
For approximately fifteen experiences
Primarily individual or small group instruction of fifth and/or sixth year students
Includes diagnostic and prescriptive activities
Lesson plan completed specified time ahead of each teaching experience
Very difficult to reschedule teaching experience if college student is absent

Anticipated Performance of College Student

1. Identifies learner's needs and interests
2. Identifies and/or specifies instructional objectives based on learner's needs and interests
3. Designs instruction appropriate to objectives and to the children being taught
4. Implements instruction consistent with preplanning activities
5. Designs and implements evaluation procedures of lessons taught which focus on (a) learner achievement of specified objective(s) and (b) instructional effectiveness
6. Demonstrates a repertoire of mathematics skills and teaching skills appropriate to specified objectives and to particular learners
7. Uses instructional materials appropriate to objectives
8. Promotes effective patterns of communication
9. Modifies instruction on the basis of learner's written work and verbal and nonverbal feedback during instruction
10. Uses organizational and management skills to establish an effective learning environment
11. Identifies and reacts with sensitivity to the needs and feelings of self and others
12. Exhibits openness and flexibility
13. Works effectively as a member of a professional team

*EVALUATION

(Activities herein specified are to be designed for utilization with students at the intermediate level.)

I. Students may earn a "D" grade by effectively completing II, Parts A, B, C, E, F, H, and J.

II. Students may obtain a "C" grade by:
   A. Attending and participating regularly in class
   B. Scoring 80% or higher on post-assessment of basic mathematics skills
   C. Presenting written work promptly and in clear, proofread, and well-organized format
   D. Observing demonstration lessons
   E. Comparing and contrasting two basal series at specified grade levels
   F. Constructing, administering, and analyzing the results of a survey test (Informal Inventory) in mathematics
   G. Completing additional diagnosis as necessary
   H. Participating regularly, effectively, and in a <u>planned</u> way in practicum experiences (lesson plans prepared and available to instructor a specified time before teaching)
   I. Effectively utilizing course textbook(s) and additional sources for reference
   J. Recording diagnostic, prescriptive, and instructional activities

III. Students may obtain a "B" grade by:
   A. Meeting the requirements for a "C" grade
   B. Observing and critiquing in writing observations of demonstration lessons
   C. Developing, administering, and analyzing the results of a diagnostic test in mathematics
   D. Selecting manipulative materials for a specific grade level within a hypothetical budget

IV. Students may obtain an "A" grade by:
   A. Meeting the requirements for a "B" grade
   B. Designing, constructing, and utilizing an effective teaching game in a specific mathematics skills area
   C. Developing and effectively implementing a learning center (this can be a small group project)
   D. Scoring 85% or higher on final test over selected topics from textbook(s) and other readings

*Substitutions may be made through negotiation with the instructor.

EVALUATION (continued)

The student selects a grade goal in this course; however, it must be kept in mind that the instructor decides whether the project or activity is of high enough quality to meet the level selected.

The student may in some instances have the opportunity to rework requirements until they meet the approval of the instructor.

Syllabus guidelines are to followed at all times unless there has been a negotiation with instructor which changes that guideline.

Students not meeting established deadlines for assignments risk having their assignments rejected.

For the student's own use and security, s/he should retain a copy of his/her written work.

ELEMENTARY EDUCATION 305

TEACHING MATHEMATICS IN THE ELEMENTARY SCHOOL

ACCORDING TO MY RECORDS YOU HAVE COMPLETED THE FOLLOWING:

I. Items II A, B, C, E, F, H, and J ____________

II. "C" level
- A. Attended and participated regularly ____________
- B. Scored 80% or higher--Basic Mathematics Skills Assessment ____________
- C. Presented all written work according to guidelines ____________
- D. Observed demonstration lessons ____________
- E. Compared and contrasted two basal series ____________
- F. Completed diagnostic work (F and G on EVALUATION) ____________
- G. Participated fully in practicum ____________
- H. Utilized appropriate references ____________
- I. Recorded activities with child(ren) ____________

III. "B" level
- A. See above ____________
- B. Observed and critiqued demonstration lessons ____________
- C. Developed, administered, analyzed diagnostic test ____________
- D. Selected manipulative materials ____________

IV. "A" level
- A. See above ____________
- B. Designed, constructed, utilized teaching game ____________
- C. Developed and implemented learning center ____________
- D. Scored 85% or higher--test over textbook topics ____________

IF YOUR RECORDS AGREE WITH MINE, PLEASE SIGN YOUR NAME AND DATE AND RETURN THIS SHEET TO ME. IF YOUR RECORDS DO NOT AGREE WITH MINE, PLEASE SEE ME AS SOON AS POSSIBLE.

NAME____ DATE____________________

SOME SUGGESTED REFERENCES

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Journals

Arithmetic Teacher
Elementary School Journal
Learning
Mathematics Teacher
School Science and Mathematics Teacher

OBSERVATION FORM

NAME OF OBSERVER____________________

DATE____________ TIME____________ GRADE LEVEL____________

TEACHER____________ FIRST NAME OF CHILD____________

****

1. Did your student appear actively involved in what was going on in the classroom?

2. In what ways did s/he show interest or disinterest?

3. Did you notive any unusual responses?

4. Did s/he interact with other students?

5. Describe the general classroom atmosphere.

6. Describe any positive/negative reinforcement of this particular child.

7. Was there anything done or said during the period which left a question in your mind, caused concern or impressed you? If there is, please write about it on the back of this sheet.

(Adapted from form devised by C. Simmons)

OBSERVATION

Name____________________ Date____________ School____________________ Level______

Skill	Procedures	Material	Comments

ELEMENTARY EDUCATION 305

Lesson Plan

Date____________________

Student's name____________________________________ Student(s)_______________

Objective	Detailed Procedure	Materials	Evaluation

Games:

1. Promote student-centered learning
2. Are highly motivating
3. Provide immediate feedback
4. Reduce the risk of failure
5. By their very nature promote cooperation and social learning

HOW TO DESIGN A GAME

The design process can be reduced to six essential steps:

(1) DEFINE THE OBJECTIVES

A good classroom game, like a good lecture or classroom discussion, should teach something worthwhile. If a game teaches only facts or provides something amusing to do on Monday, it is not a very good game.

(2) LIMIT THE SCOPE OF THE GAME

Decide exactly what the game is to teach. Determine if the game is to be played in just one class period, or several. Also, determine if teams will be needed and, if so, how many.

(3) OUTLINE THE SEQUENCE OF EVENTS

Decide exactly what steps are involved in playing the game and in what order.

(4) IDENTIFY KEY PLAYERS AND THEIR OBJECTIVES

Will the game be played individually or in teams? How many players are on a team? Are all players working toward a common goal (as in Monopoly, all players have the same goal: accumulating wealth)? Are there clear cut winners and losers?

(5) DECIDE ON RULES FOR WINNING AND LOSING

(6) DEVELOP THE FINAL FORMAT OF THE GAME

Before a final version is made, be sure to play through the game yourself to be rid of things that don't quite fit. Is the timing off? Has something been left out? Any suggestions for improvement? Once the game has had a trial run, you're ready to build the final format.

Making the final version has three main steps:

(A) An overview of the game--introduce the game and describe its objectives. The test of clarity is: Could a colleague who was not present when the game was designed play it?

(B) Rules for playing

(C) Ma[illegible]me materials

LEARNING CENTERS

Definition of a Learning or Interest Center:

A learning center is an area in the classroom which contains a collection of activities and materials to teach, reinforce, and/or enrich a skill or concept.

Begin with a few centers. As students know how to use them and are comfortable with them you can add additional centers as they are required. Discuss and demonstrate the possiblities and limitations of each center.

Combine centers to save space, eliminating those that are of little interest. Avoid having so many materials in the classroom that they become confusing and cluttered.

For the student, the learning center is used as a self-selected activity for independent study, follow-up for a teacher-taught lesson, an activity in place of a regular assignment, or an individual activity.

A Step-By-Step Approach to Creating a Learning Center

1. Select a subject area. Example--mathematics

2. Determine the skill or concept to be taught, reinforced, or enriched: Example--to teach skills of linear measurement using the metric system

3. Develop the skill into a learning activity: manipulating, experimenting (observing, charting, keeping a log), listening, or viewing Example--students will learn about linear measurement in the metric system by measuring and recording the lengths of various objects and distances using centimeters and meters

4. Incorporate the skill into an extending activity: Example--students will extend their skill of linear measurement in the metric system by constructing a scale drawing of a room in the building, using metric measurement

5. Place all the games, worksheets, charts, etc., together in one area of the room for children to use in a self-selected manner

Teacher Learning Center Checklist:

The teacher should prepare the learning tools, such as worksheets and games, and collect all available resources for the center so that it contains all the necessary equipment for students to discover, learn, and apply the concept or skill for which it was developed.

The teacher should thoroughly introduce the learning center to the students so that they can clearly understand the answers to these questions:

1. What can be done at the center?
2. How is each activity, game, etc., used?
3. Where are the materials necessary for production kept?
4. Where are the finished products to be stored?

The teacher should motivate and encourage students to use the learning center by doing the following:

1. Adding new activities or materials to the center.
2. Letting students create their own activities at the center.
3. Having teacher-directed lessons in small or large groups at the center.
4. Providing opportunities for students to share who have worked at the center.

Learning Centers Evaluation

I. Orientation

A. Train children to use learning center
B. Care of materials, returning materials

II. Objectives

A. Stated simply and clearly
B. Limited
C. Displayed prominently

III. Materials -- varied

A. Written
B. Games
C. Puzzles
D. Audio-visual
E. Manipulative materials
F. Task cards

IV. Appearance

A. Attractive
B. Colorful

V. Evaluation

A. Self-checking
B. Student/teacher evaluation
C. Students tell how to improve center, what they like, what they dislike
D. Record keeping and check list

VI. Tasks

A. Well-organized
B. Independent and group work
C. Legibly printed
D. Easily accessible

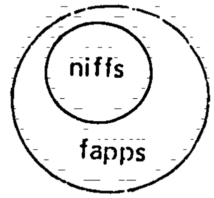
ASSESSMENT OF MATH SKILLS

PART I

I. Sets, bases, factors, and exponents

Study the diagrams. Then fill in the blank with <u>all</u>, <u>some</u>, or <u>none (no)</u>

1. ______________ niffs are fapps.



2. Find the solution set for n if the replacement set is (1, 2, 3, 4, 5, 6, 7, 8)

   $4 \times n < 30$ $\qquad n = \{ \qquad\qquad \}$

Write E if the equations are equivalent and N if they are not equivalent.

3. $7 + n = 15$

   $n = 15 - 7$ ________

4. Rename as a numeral in standard form.

   Six million, four hundred twenty-nine thousand, three hundred sixty-four.

   ____

5. Rename the numeral in the given base.

   $15_{ten} = \underline{\qquad}_{six}$

6. Write a complete (prime) factorization for the whole number 36.

   (The factors may be listed in any order.) ______________________

7. Find the greatest common factor of the pair of whole numbers. 42, 63

   ____

8. Write the product in exponential form. $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 =$

   ____

II. Whole numbers: basic operations

1. $\begin{array}{r} 5,172 \\ 3,657 \\ 4,832 \\ +\ 2,716 \\ \hline \end{array}$

2. $\begin{array}{r} 6,005 \\ -\ 1,486 \\ \hline \end{array}$

3. $\begin{array}{r} 53,862 \\ \times\ \ 341 \\ \hline \end{array}$

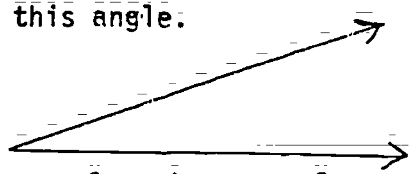
4. $734 \overline{)\,3\,[illegible]\,7,828}$

Write an equation for the word problem; then solve it. Show your work.

5. The boys' baseball league purchased 240 baseballs and 54 bats. There are 6 teams and each team receives the same number of balls and bats. How many baseballs and how many bats will each team receive?

III. Geometry: measurement

1. Estimate the size of this angle.



This angle is closer to 30°, 85°, or 150°? ______

2. If $\triangle DFG \cong \triangle RST$ and $m(\angle D) = 60^\circ$, then $m(\angle R) =$ ______

Use $\pi = 3.14$ to find the circumference.

3. radius = 20 feet

   C = ______

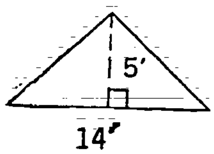
Use $\pi = 3.14$ to find the area.

4. diameter = 10 inches

   A = ______

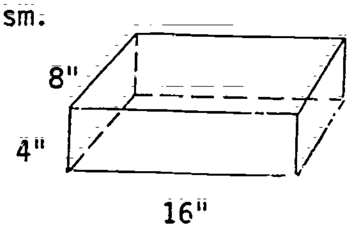
Find the area of the figure.

5.



A = ______

6. Find the surface area of the rectangular prism. Show your work.

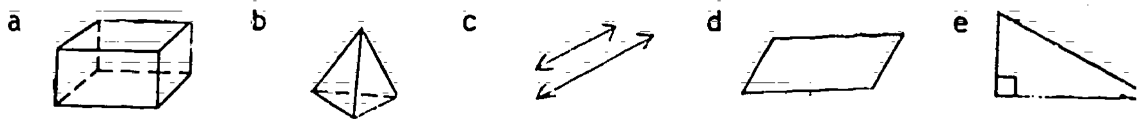


______

7. Find the volume of the rectangular prism. Show your work. ______

IV. Geometric principles

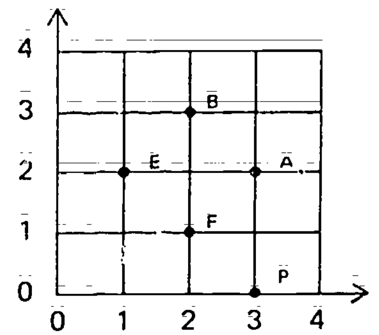
Match the letter of the figure with its name.



1. parallelogram. ____________

Give the coordinates for the point.

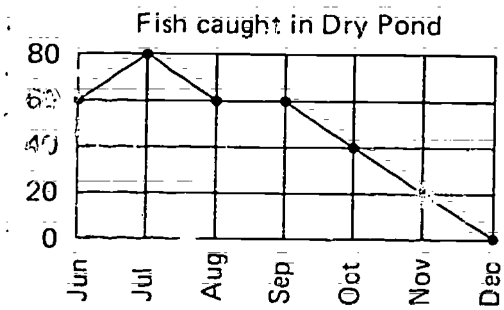
2. E = (________, ________)



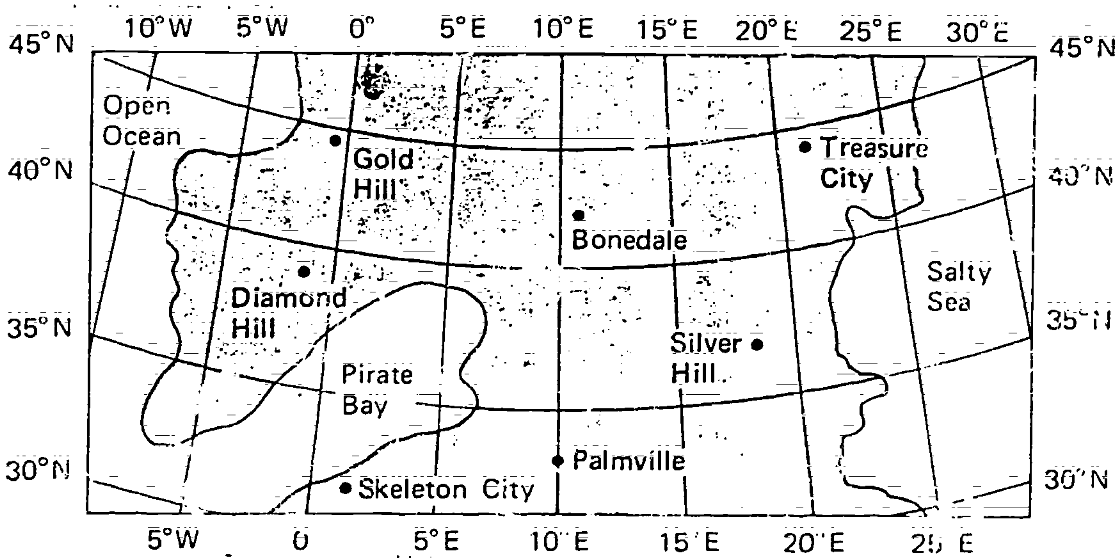
Use the line graph to solve the problem.

3. How many more fish were caught in August than in November?

____



Study this pirate map.



Name the place closest to the given longitude and latitude.

4. Longitude 10 E
Latitude 33 N ____________________

V. Fractions: Addition and subtraction

1. Write as a mixed numeral.

   19/4 =

2. Subtract. Express your answer in simplest form.

$$\begin{array}{r} 17\ 3/4 \\ \underline{-\ 8\ 4/5} \end{array}$$

ASSESSMENT OF MATH SKILLS

PART II

VI. Fractions: Multiplication and division

1. Find the product. Express your answer in simplest form. Circle your answer.

   2 1/17 x 3/4 =

2. Find the quotient. Express your answer in simplest form. Circle your answer.

   2 1/2 ÷ 5 1/3 =

3. Solve the word problem. Show your work.

   Dan and David brought home a piece of copper wire for a science project. The piece of wire was 11 2/3 feet long. They planned to cut it into pieces 1 2/3 feet long. Into how many pieces could they cut the wire?

VII. Decimals: Basic operations

1. Rename the decimal as a fraction. Express your answer in lowest terms.

   3.2 =

2. Find the difference.

   .63 - .088 =

3. Find the product.

$$\begin{array}{r} 28.137 \\ \times\ \ .35 \\ \hline \end{array}$$

4. Solve the word problem. Show your work.

   Charlie is getting more track for his miniature railroad. The new track needs to cover 73.5 inches. Each piece of track measures 10.5 inches. How many pieces of track will he need?

VIII. Decimals and percents

1. Rename the fraction as a percent.

   3/9 = ________

2. Rename the percent as a fraction. Express your answer in lowest terms.

   24% = ________

3. Complete the equation.

   5% of 135 = ________

Solve the problems.

4. What number is 25% of 160?

5. Mr. Randall has an annual income of $12,000. In 1 year, he paid 15% of his annual income in taxes. How much of his income did he pay in taxes?

IX. Measurement

1. Jean weighs 40 kilograms.
   About how many pounds does she weigh; 85 lb., 120 lb., or 135 lb.?

   ____

Find the difference and quotient. Express your answer in simplest form.

2. 6 lb. 3 oz.
   -2 lb. 8 oz.

3. $5\overline{)18 \text{ yd. } 1 \text{ ft.}}$

4. Circle the measurement having the greatest possible error.

   greatest possible error is 1/14 (7 1/14" 14 3/28" 1 4/7")

X. Integers and rational numbers

1. Complete the equation to make proportions.

   7:8 = 49: ______

2. Complete the equation.

   $^{-}15 \div 3 =$ ______

3. Find the quotient.

   $128 \div {}^{-}4 =$ ______

4. Find the difference by adding the opposites.

   $^{-}16 - 24 =$ ______ + ______ = ______

5. If one number is selected at random from set S, what is the probability of it being a prime number?

   $S = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ ______

6. Find the average of this set. $\{42, 43, 46, 50, 52\}$ ______

7. Find the median of this set. $\{10, 11, 12, 13, 14\}$ ______

8. Find the range of this set. $\{37, 38, 39, 40, 45, 70, 82\}$ ______

9. Find the square root. $\sqrt{10,000} =$ ______

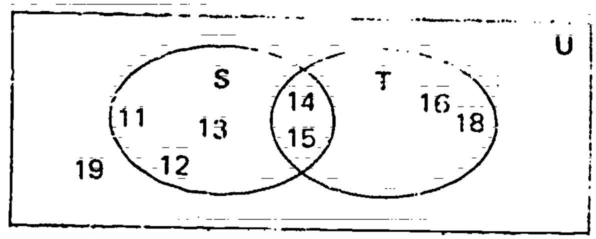
ASSESSMENT OF MATH SKILLS

PART I

I. Sets, bases, factors, and exponents

1. Use the diagram to solve the problem.

   S ∩ T = { }



2. Find the cross product of the sets.

   A = {21, 22, 23, 24}   B = {r, s}   C = {t}

   A ⊗ C =

3. Write E if the equations are equivalent and N if they are not equivalent.

   18 - n = 9

   n = 18 + 9 __________

4. Rename as a numeral in standard form.

   Nine million, six hundred thousand,
   three hundred four ______________

5. Rename the numeral in the given base.

   55_{twelve} = __________$_{ten}$

6. Write a complete (prime) factorization for the whole number 40. (The factors may be listed in any order.)

   ____

7. Find the greatest common factor of the pair of whole numbers 56, 64. ________

8. Write the base ten numeral in exponential form. 10,000 = __________

II. Whole numbers, basic operations

1. Find the sum.

$$\begin{array}{r} 78,362 \\ +\ 89,289 \\ \hline \end{array}$$

2. Find the difference.

$$\begin{array}{r} 37,365 \\ -\ 16,531 \\ \hline \end{array}$$

3. Find the product. 74,401
x 29

4. Find the quotient. 734 ⟌ 397,828

5. Write an equation for the word problem; then solve it. Show your work.

Ann purchased a 10-speed bicycle for $123.00. She earned $200.00 last summer working as a swim teacher. How much of her summer earnings did she have left after purchasing the bike?

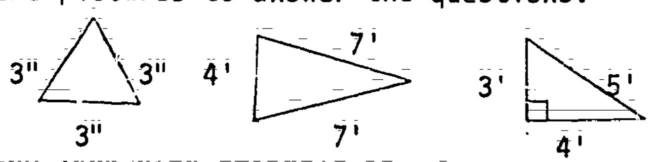
III. Geometric measurement

1. Estimate the size of this angle.

This angle is closer to 60°, 90°, 130°

____

2. Use the pictures to answer the questions.



a. The isosceles triangle is △ ________.

b. The equilateral triangle is △ ________.

3. Use $\pi = 3.14$ to find the circumference.

diameter = 12 inches

C = ____________

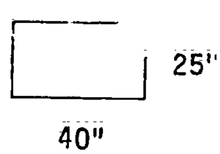
4. Use $\pi = 3.14$ to find the area.

radius = 8 feet

A = __________

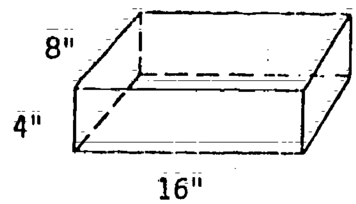
5. Find the area of the figure.

A = ______________



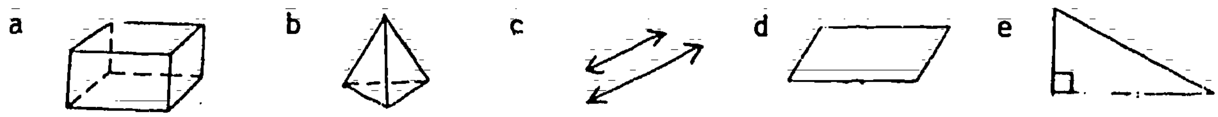
6. Find the volume of the figure.
Show your work.

____



IV. Geometric principles

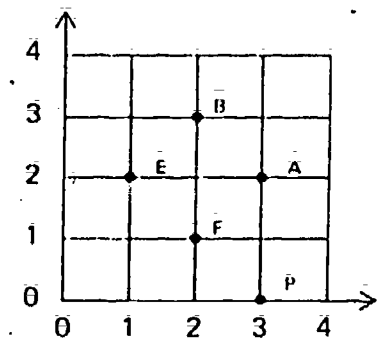
1. Match the letter of the figure with its name.



rectangular prism ________

2. Match the point with its coordinates.

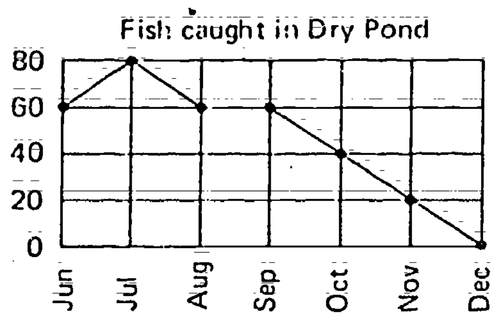
________ = (3, 2)



3. Use the line graph to solve the problem.

How many fish were caught in Dry Pond in July?

____



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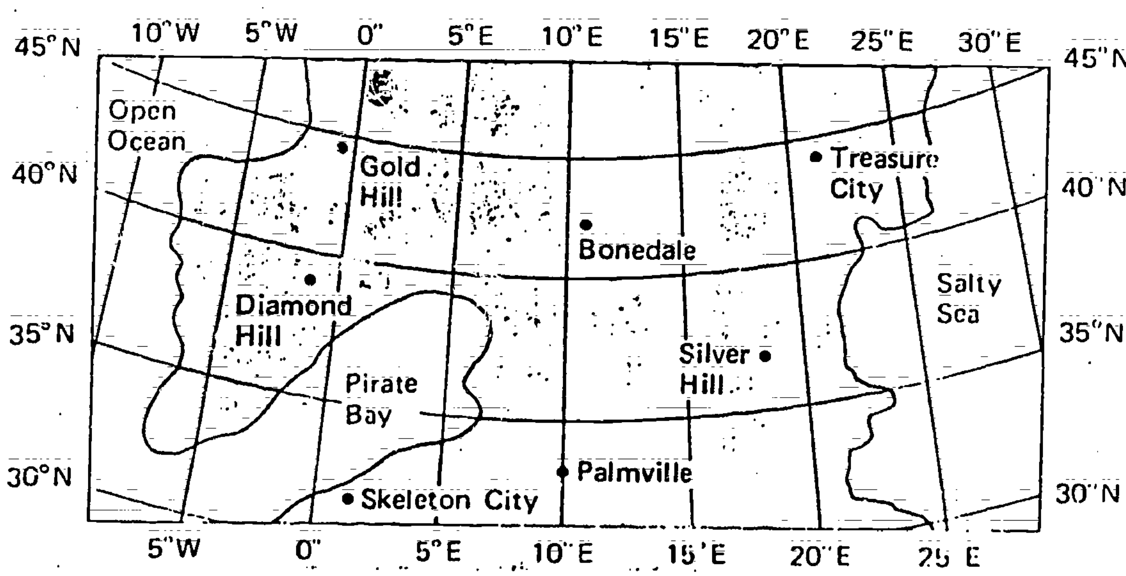

4. Name the place closest to the given longitude and latitude.

   Longitude 1° W
   Latitude 39° N ______________________

V. Fractions: Addition and subtraction

1. Write as an improper fraction.

   $4\ 7/10 = \frac{\_\_}{10}$

2. Subtract. Express your answer in simplest form.

   $\begin{array}{r} 2\ 2/3 \\ -\ 1\ 1/4 \\ \hline \end{array}$

ASSESSMENT OF MATH SKILLS

PART II

VI. Fractions: multiplication and division

1. Find the product. Express your answer in simplest form. Circle your answer.

   5 1/6 x 2 2/5 =

2. Find the quotient. Express your answer in simplest form. Circle your answer.

   6 1/2 - 3 =

3. Solve the word problem. Show your work.

   Professor Scientific stores his new formula in test tubes. If 2½ ounces can be stored in each test tube, how many ounces of formula can he store all together in his 22 new test tubes?

VII. Decimals: basic operations

1. Rename the decimal as a fraction. Express your answer in lowest term.

   .06 =

2. Find the difference.

   32.3 - 21.5 =

3. Find the product.

   $$\begin{array}{r} 28.137 \\ \underline{\times\ \ .35} \end{array}$$

4. Solve the word problem. Show your work.

   The jet airplane had used .8 of its fuel when it landed. If it carried 250 gallons of fuel at takeoff, how much did it have when it landed?

VII. Decimals and percents

1. Rename the fraction as a percent. 3/4 = ________________

2. Rename the percent as a fraction. Express your answer in lowest terms.

   33 1/3% =

3. Complete the equation.

   52% of 52 = ______________

Solve the problems.

4. What percent of 56 is 7?

5. 20 out of every 50 jet airliners arrive later than scheduled. 15 out of every 50 arrive earlier than scheduled. What percent of the aircraft arrive on schedule?

IX. Measurement

Use this table to help solve the problem.

1 meter $\approx$ 39 inches = $3\frac{1}{4}$ feet = $1\frac{1}{12}$ yards
1 liter $\approx$ $1\frac{1}{20}$ quarts
1 kilogram $\approx$ $2\frac{1}{5}$ pounds = $35\frac{1}{5}$ ounces

1. Which is larger, 1 gallon or 4 liters? ______________

2. Find the difference. Express your answer in simplest form.

$$\begin{array}{r} 7 \text{ ft. } 2 \text{ in.} \\ -\ 5 \text{ ft. } 7 \text{ in.} \\ \hline \end{array}$$

3. Find the quotient. Express your answer in simplest form.

$$6\overline{)19 \text{ yd.}}$$

4. Circle the measurement having the greatest possible error.

greatest possible error is 1/14" { 7 1/14" 14 3/28" 1 4/7" }

X. Integers and rational numbers

1. Complete the equation to make a proportion.

6:8 = __________:24

2. Complete the equation. 11 + ______ = 0

3. Find the quotient. -132 ÷ 3 = ________

4. Find the difference by adding the opposites.

-3 - 5 = -3 + -5 = ________

PROFILE

Name ____________________ Date ____________

Unit	Problem Number 1	2	3	4	5	6	7	8	9
I. Sets, bases, factors, and exponents									
II. Whole numbers: basic operations									
III. Geometry: measurement									
IV. Geometric principles									
V. Fractions: addition and subtraction									
VI. Fractions: multiplication and division									
VII. Decimals: basic operations									
VIII. Decimals and percents									
IX. Measurement									
X. Integers and rational numbers									