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ABSTRACT

This monograph presents 13 papers dealing with various aspects of elementary problem solving. They are: (1) "Training for Effective Problem Solving" (Gary A. Davis); (2) "Patterns of Problem Solving--A Campus-Wide Course at UCLA" (Moshe F. Rubinstein, L. Robin Keller, Edward A. Kazmarek); (3) "A Taxonomy of Problem-Solving Activities and Its Implications for Teaching" (H. L. Plants, R. K. Dean, J. T. Sears, W. S. Venable); (4) "What Is the Problem in Teaching Problem Solving" (Donald R. Woods, Cameron M. Crowe, Terrence W. Hoffman, Joseph D. Wright); (5) "Structure and Process in Problem Solving" (Maynard Fuller, Geza Kardos); (6) "Solving Physics Problems" (J. L. Aubel); (7) "Engineering Student Problem Solving" (Lois B. Greenfield); (8) "A Backward-Reasoning Model of Problem Solving" (D. L. Marples); (9) "Learning Skills as an Overplay in Elementary Calculus" (Jeffrey M. Brown); (10) "Learning Skills" (P. J. Black, Joan Bliss, Barbara Hodgson, Jon Ogborn, P. J. Unsworth); (11) "Learner and Instructional Variables Affecting Problem Solving" (Jerry K. Stonewater); (12) "The Representation and Solutions of Problems in Applied Mathematics: An Artificial Approach" (George F. Luger, Alan Bundy, Martha Palmer); and (13) "Problem Solving in Physics or Engineering: Human Information Processing and Some Teaching Suggestions" (F. Reif). (JN)

THE TEACHING OF ELEMENTARY PROBLEM SOLVING IN ENGINEERING AND RELATED FIELDS

Edited by

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THE TEACHING OF ELEMENTARY PROBLEM SOLVING IN ENGINEERING AND RELATED FIELDS

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EDITOR'S INTRODUCTION

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EDITOR'S INTRODUCTION

For five years I have been interested in educational innovation and the improvement of college-level engineering and science teaching. I have read about and played with "PSI", self-paced instruction, the Keller plan, individualized instruction, mixtures of traditional and innovative methods, and so forth. I have even contributed to what I consider to be one of the "wavelets of the future": computer-assisted homework and test construction, a finely-adjustable method of individualization which blends well with both traditional and innovative methods.

All of these are exciting, and properly managed, all can help. But even when I have created as rational an environment for learning as I could, it was still clear that the teaching of problem solving was not as simple as I thought. Most students still could not teach themselves what had to be learned.

This monograph grew out of the difficulties which remained. I had several purposes in mind when I asked the present authors to make their contributions:

1. I wished to assemble in one place a number of current papers dealing with problem solving. Through these papers and their references, I wished to point to the growing literature of this important field. I especially sought authors from a variety of disciplines, with a variety of approaches.
2. Much attention is now being focused on open-ended or creative problem solving ("design"). Excellent papers and books address this subject. Unfortunately, many students cannot correctly solve the elementary subproblems which are inherent in the design process. The teaching of elementary problem solving *has* to be the place to start, so I asked the authors to concentrate on this less fashionable but fundamentally important area.
3. I also assembled these papers in order to be educated on how to teach elementary problem solving. Lest anyone think that your editor is an expert in this field, let me hasten to disabuse you. I simply researched out some of the real experts and asked them to write articles for me, the archetype of the "willing-but-igno-

rant" engineering educator, as a representative of a large audience ripe for guidance. I am pleased to report that the authors have generally avoided the trade jargon which might put their ideas beyond the reach of the typical educator in science and engineering.

Here are my first conclusions from the educational process which I sought in these papers:

* They have clarified the many meanings of the word "problem" and the disarmingly simple expression "problem solving". We must define our terms very carefully before we begin to talk about remedies and methodology.
* They have made quite clear what is missing in much of our teaching, i.e., what we are doing wrong, and the gap between what we teachers think we are doing and what we actually are doing. Put more bluntly, few educators in engineering and science have been taught how to teach, and few students reach them knowing how to learn. In particular, few teachers realize that it is part of their job to teach students how to learn.
* The authors have made it clear that time spent in teaching students how to learn is not necessarily time lost from the course. (Those who prefer not to change their ways always have facile excuses, including "time lost" and "they were supposed to learn that in high school.") Teaching students how to learn makes them more independent, and able to learn more quickly and with greater confidence thereafter. Time put into the early courses of a curriculum to teach students how to learn will probably permit more effective learning in the later courses of the same curriculum.
* Inevitably, some of the authors' recipes and suggestions leave the reader with a vague feeling of dissatisfaction. The papers make the difficulties clear, but the proposed cures may seem blurred and imprecise. It is then that you realize that teaching and learning are human activities which are not easily codified. Part of the imprecision arises because the subject is still in its infancy. The teaching of abstract reasoning and logical thinking is a difficult exercise in human psychology. An enormous variety of approaches are possible and relevant. Accordingly, some of the suggestions will strike a responsive chord for one teacher and repel another. Some of them will work for one teacher personality and not for another. But collectively, all of the papers do offer enough patterns, procedures, templates and ideas to help most teachers start exploring and experimenting with their own courses. I will be quite satisfied if this monograph makes the reader hungry to get started.

The primary topic of this monograph is elementary problem solving. Some of the papers, particularly the first, do not adhere strictly to this topic. However, this serves the very valuable purpose of establishing the position of elementary problem solving in relation to problem solving in general. Beyond this, please do not look for any organizational pattern in the grouping of the papers. There is none.

James L. Lubkin

January, 1978

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TRAINING FOR EFFECTIVE PROBLEM SOLVING

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ABSTRACT

Emphasizing the complexity of human problem solving, this article begins with a brief review of several taxonomies of problem solving, eac of which identifies different types of problems and different thinking processes. The report then summarized a five-step analysis of the problem-solving process: (a) fact finding, (b) problem finding, (c) idea finding, (d) solution finding, and (e) acceptance finding. The analysis helps clarify human problem solving and also serves as an heuristic to guide individual or group problem solving. The idea-generation stage includes a brief summary of some creative thinking techniques.

One difficulty in training for problem solving is that there are many different kinds of "problems." One taxonomy of problems (Samson, 1970) identifies, first, situations requiring a <u>discovery</u> process, such as detective mysteries, matchstick or number sequence puzzles, and many mathematical problems. A second type of problem involves <u>planning</u>, for example, transporting 300 children to the King Tut exhibit, with minimal casualties, or organizing a new college course. Samson's third category was <u>creative</u> problems, which requires a free-wheeling gush of imaginative possibilities. Some examples would be listing ideas for a 60-second TV commercial, brainstorming ideas for a traffic safety problem, or just listing unusual uses for a brick.

With another taxonomy Lewis (1977) identified four types of problems which differ according to whether the <u>means</u> to the solution is clear or ill-defined, and whether the <u>end</u> itself is clear or ill-defined. For example, an arithmetic problem (What is the square of 5,678?) has both a clear means and a clear end. These are called <u>procedural problems</u>. Choosing a vacation spot, a movie, or selecting wallpaper represent problems with a clear means, but an unclear end (<u>decision problems</u>). Common puzzles (crosswords, riddles, etc.) and such problems as designing a better mouse trap would be problems with a clear end, but an unclear means (<u>solutional problems</u>). Finally, economic, political, and even some personal problems represent situations with both an unclear goal and ill-defined means (e.g., how do we halt inflation, settle the middle-east situation, increase personal self-confidence), which Lewis calls <u>generational problems</u>. Across all four combinations, problem solving is defined as the "...process of clarifying both means and ends, that is, creating <u>procedural problems</u> from the other three types."

Looking only at laboratory tasks of experimental psychology, Davis' (1966) two-part taxonomy divided tasks into, first, those requiring observable trial-and-error problem solving. For example, trial-and-error is used in figuring out which combination of switches will produce a desired pattern of lights, in learning to correctly classify arbitrarily-related stimulus objects, in learning a finger-maze blindfolded, or in the real world to find a lost checkbook. The second category of problems includes those solvable by implicit mental processes, which may involved some "mental" trial and error. In this category are chess problems, anagrams (scrambled letters), missionary-cannibal problems (ferrying three missionaries and three cannibals across a river in a two-person boat without ever allowing missionaries to be outnumbered), number-series problems, and others. Davis notes that an "implicit" type problem will become a trial-and-error to solve matchstick problems, and a person solving anagram problems will use trial-and-error if he is given letter blocks.

In still one more taxonomy, Fuller (1973) notes that "problems" can vary in (a) having one or a multiplicity of possible solutions, (b) the degree of uncertainty in either the problem data or the problem outcomes, (c) the quantitative/mathematical nature of the problem, (d) the degree of the social, people-related character, (e) the abstract (symbolic vs. concrete) character of the needed solution, (f) the complexity of the solution, or (g) the immediacy of the solution evaluation.

The upshot of this introduction is simply that "problem solving" is a difficult, catch-all concept which includes countless types of situations and, of present importance, innumerable sorts of mental activities and processes. At the very least we find such processes as trial-and-error searching, logical deduction, diagnosis, extrapolation, classification, metaphorical thinking, step-by-step planning, idea retrieval, idea synthesis, discovery, means-end analysis, abstracting commonalities, transferring old solutions to new problems, evaluation, and many more. Small wonder that some sceptics have questioned whether problem solving skills, whatever they may be, can be taught at all.

Existing strategies for training problem solving show tremendous variation in the scope of problems for which the training is appropriate. Perhaps the most general approach is the Upton and Samson (1963) Creative Analysis workbook, which promises to strengthen thinking, creating, problem solving, and even intelligence itself. The program provides broad-based exercises in structure analysis, qualitative analysis, classification, abstracting similarities, discovery, using symbols and tree diagrams, metaphorical thinking, analysis of operations, and much more. Other efforts at teaching problem solving focus on a smaller number of helpful, verbalizable principles. For example, Simon and Reed (1976) emphasized the importance of means-end analysis, which gives direction to a problem-solving search. Fuller (1974) recommended his "special vocabulary" for problem solving, which calls attention to the "principal parts": the data set, algoriths (rules), unknowns, and constraints. Wickelgren's (1974) strategy for teaching mathematical problem solving, which optimistically "...guarantees that you will never again have a blank mind in such (problem solving) circumstances," includes specifying givens, classifying action sequences, evaluating states, defining subgoals, searching for contradictions, working backward from the goal, looking for relationships (similarities) between problems, and others.

The remainder of the present essay will focus specifically on solving "creative problems," those situations requiring new ideas for designing a better bumper, keeping costs down and efficiency up, marketing Hula Hoops, and infinitely more.

Creative Problem Solving

Skills, strategies and attitudes for creative problem solving are taught daily in universities and large corporations through America and the world. The effectiveness of such training naturally varies with the course and with the individual participant. The most common and predictable outcome is a solid change in "creativity consciousness." Participants come to understand creative thinking and problem solving better, they become more confident in their own creative ability, they become more likely to use a creative approach in solving professional problems, and they are ready to take a more creative approach to life in general. Hard evidence of program effectiveness, which is not often available, has taken the form of higher scores on divergent thinking tests (such as listing unusual uses for a brick) and higher scores on personality tests measuring self-confidence, initiative, and leadership potential (Parnes, 1962).

A recurrent strategy for teaching creative problem solving is the stage approach, which forms the core of the *Creative Actionbook*, prepared by Creative Education Foundation leaders Noller, Parnes and Biondi (1977). The reader may be acquainted with Wallas' (1926) stages of preparation, incubation, illumination and verification, which very generally summarize the sequence of events in many problem solving episodes. The Noller et al steps of (a) fact finding, (b) problem finding, (c) idea finding, (d) solution finding and (e) acceptance finding represent an up-dating of the 1926 steps (see Figure 1). In the 1977 model each of the five stages involves both a divergent, idea-

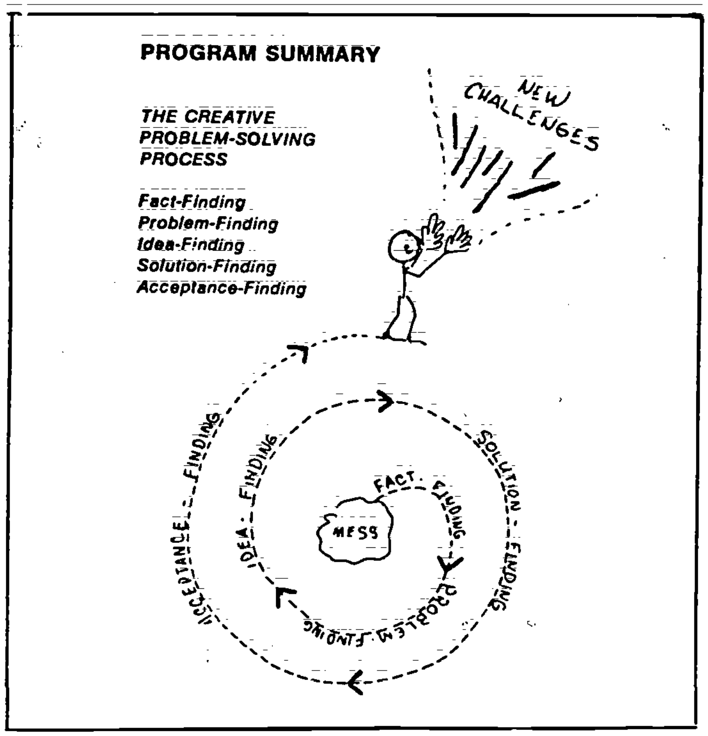

Figure 1. Five stages in creative problem solving. From Creative Actionbook, by Ruth B. Noller, Sidney J. Parnes, and Angelo M. Biondi, Charles Scribner's Sons, 1977. Reprinted by permission.

generation phase followed by a convergent, evaluative phase. An assumption of the model seems to be that an understanding of the dynamics of problem solving, as represented by the five steps, will lead to more systematic, effective problem solving. In the following summary, most space will be given to step (c), idea finding, since much has been written about idea-generation techniques.

FACT FINDING

The first step i gathering pertinent information. Activities in this stage will include listing relevant facts and raising questions, some of which may need to be researched. For example, let's say the problem at hand is designing packaging for a new line of Mrs. Plum's Pickled Kumquats. A group perhaps composed of food processing engineers, executives, and layout artists would need to gather information, for example, on relevant biochemical reactions, deterioration rates, contamination dangers, taste factors, and costs, availability and market research data on various packaging alternatives. After the free-flowing idea-production period the most significant facts would be sifted out and perhaps clarified and elaborated. Quite often, ideas listed in this stage will relate directly to ideas produced in later stages.

PROBLEM FINDING

Problem finding amounts to identifying and defining the problem(s) to be attacked. One does this by (a) listing many possible problem statements, (b) by repeatedly rephrasing a particular statement, and/or (c) by defining the problem in a broader, more general fashion, which has the effect of opening new avenues of thought. Problem statements often begin with "How might we..." or "In what ways might we..." For example, "How might we present attractive 'units' of pickled kumquats?" "How might we keep costs down?" "In what ways might we prevent loss of flavor over time?" And so on.

One would consider variations of a single problem statement. "How might we present attractive units?" can be rephrased as "How can we make people like our package?" "How can we make the package say 'Try me! I'm good!'?" or "How can we make people put our package in their shopping carts?"

Some more general problem statements, which usually provide new problem viewpoints, could be "In what creative ways can our product attract attention at the grocery store?" or "How can a biological product be made to last forever?"

Problem listing is followed by the evaluative process of selecting the most productive problem statement. Of course, different problem definitions will result in different lists of solutions. Frequently, many important subproblems will be identified, each of which will require separate attention.

IDEA FINDING

After a problem is selected, the individual or group should generate a list of solution possibilities. The list will be longer and more imaginative if the thinkers observe the deferred judgment principle. That is, even "wild" ideas should be freely suggested and recorded with no immediate evaluation or criticism. Far-fetched ideas may suggest realistic, creative problem solutions.

In addition to using ones intuition there are a number of deliberate, supplementary strategies for producing new idea combinations. The following techniques are taught in virtually every professional creative thinking course.

Attribute listing. The attribute-listing technique (Crawford, 1954, 1971) involves either (a) listing important attributes (characteristics, dimensions) of the problem and then listing ideas for improving each of those attributes, or (b) transferring an attribute (or problem solution) from one situation to a new problem context. "Packages" of pickled kumquats, for example, have attributes of material, size, shape, color or color patterns, pickling medium, vitamin additives, appeals to different groups (e.g., sugarless kumquats; dried kumquats for backpackers; Kiddie Kwats; kumquats with prune juice; etc.), product names, cartoon characters (as on cereal boxes), and more. Each dimension will spur ideas--lots of them--related to that attribute.

As for transferring attributes, our thinkers would seek inspiration from other attractive and successful forms of packaging/promotion. For example, the catchy 7 Up and Levi's TV commercials or the unique L'Eggs pantyhose displays might suggest packaging ideas for Mrs. Plum's kumquats. The reader might recognize the process of metaphorical thinking as central to this attribute transfer strategy.

Matrix methods. As extension of attribute listing is the matrix approach, sometimes known as morphological synthesis or morphological analysis (Allen, 1962, 1966). Ideas for one problem dimension (or attribute) are listed along the horizontal axis of a matrix, ideas for a second dimension are listed along the vertical axis. This system forces the problem solver to examine the very large number of solution combinations found in the cells of the matrix, some of which are likely to be creative, practical, or with a little luck, both.

Idea checklists. Idea checklists also may be used to prod the imagination. The late Alex Osborn, inventor of brainstorming, co-founder of the successful advertising agency, Batten, Barten, Dursten and Osborn, and founder of the Creative Education Foundation, devised a set of "73 Idea-Spurring Questions" which may be applied to virtually any problem-solving task for which creative solutions are sought. A condensed form of the list (from Davis, 1974) includes:

Put to other uses? New ways to use as is? If modified?
Adapt? What else is like this? What other person, place or thing does this suggest?
Modify? Change meaning, color, motion, sound, odor, form, shape?

Magnify? More time? Greater frequency? Stronger? Higher? Longer? Thicker? Plus ingredient? Multiply? Exaggerate?

Minify? Smaller? Lower? Shorter? Lighter? Split up? Understate?

Substitute? Who or what else instead?

Rearrange? Interchange components? Other layout? Other sequence? Transpose cause and effect?

Reverse? Transpose opposites? Turn it backward? Upside down? Inside out? Turn tables? Turn other cheek?

Combine? How about a blend? An assortment? Combine units? Purposes? Appeals?

The reader might wish to think of the kumquat-packaging problem (or something more relevant) while considering each item on the list. While such a list should be used only to supplement ones intuitive idea supply, the list almost guarantees the production of some ideas which otherwise would not have occured to the thinker.

Synectics methods. Space will not permit a fair review of the very amazing and amusing synectics strategies which are used both in professional creative problem solving (Gordon, 1961; Prince, 1968, 1971) and for strengthening imaginations in the schools (Gordon, 1968, 1971). In brief, the techniques teach systematic, metaphorical thinking. The *Direct Analogy* method asks the thinker to look to nature for metaphorically-related problem ideas. What, for example, is "packaged" by seagulls, spiders, bees, salmon, oak trees, rose bushes, and so on.

The *Personal Analogy* method asks the thinker to become part of the problem objects. The reader might think about what it's like being a flavorful, well-preserved and attractively-packaged bunch of kumquats. How do you feel? What makes you happy? How could you be improved?

With *Fantasy Analogy* the problem solver searches for fanci[illegible], perhaps ideal solutions, for example, by asking how the problem might solve itself. How might pickled kumquats preserve themselves? Maintain their own flavor? Become more and more attractive to shoppers? In the world of consumer products refrigerators defrost themselves, tires patch their own leaks, ovens clean themselves, cameras adjust their own shutter speeds, turkeys baste themselves, a new chain saw sharpens itself, and automobile engines diagnose their own problems.

Whether the thinking strategy is intuitive or "forced," the long list of creative, sometimes preposterous ideas must be reduced to the potentially most fruitful ones for further development. Additional information on creative thinking techniques may be found in Davis (1973), Davis and Scott (1971), Biondi (1974), Gordon (1961), and Stein (1976).

SOLUTION FINDING

Quite often a good solution will need no formal evaluation (solution-finding) stage--When it's right, you know! Most of the time, however, it's worthwhile to systematically list criteria for evaluating the goodness of each of the solution possibilities. The reader might imagine a matrix with specific solution ideas listed vertically along the left side (vertical axis) and evaluation criteria listed across the top (horizontal axis). Working from left to right, each cell would contain a rating (poor, fair, good, great) for each idea for each criterion. Some useful criteria are: Will it work? Will the public accept it? Does it require too much time? Too much money? Are materials available? Reliability? Durability? Safety? Would my mother like it?

As with all other stages, a longer list of criteria can be shortened to those which seem most promising.

ACCEPTANCE FINDING

"Acceptance finding" is a slight misnomer, since this final stage includes both gaining acceptance (selling the idea) and devising ways to implement and assure effectiveness of the problem solution. Noller et al. (1977) recommend a self-test, asking such questions as: (a) What might I do to gain enthusiasm for the idea? How? When? Where? Why? (b) What might I do to insure effectiveness? How? When? Where? Why? And others.

Conclusions

Most problem solvers do not consciously follow separate stages in problem solving. There also is no iron-clad assurance that training with this stage approach will guarantee more successful thinking. However, the five-step model does draw attention to necessary components and subskills in problem solving and suggests clear, conscious means for coping with these components. Also, the model implicitly fosters the development of appropriate attitudes by encouraging flexibility, originality, and openmindedness, and by generally reinforcing a more creative approach to effective thinking and problem solving.

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PATTERNS OF PROBLEM SOLVING - A CAMPUS-WIDE COURSE AT UCLA

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ABSTRACT

Patterns of Problem Solving is a four-unit course offered in twelve sections each quarter of the academic year. The course discusses tools and concepts useful in problem solving with a balance sought between modeling techniques and attributes of human problem solvers. Problem solving is presented as a dynamic open-ended process encompassing diverse academic disciplines.

The course is sponsored by the School of Engineering and Applied Science at UCLA, but it has an appeal to students campuswide. The more than 4000 students who have taken the course represent more than thirty major fields of study and all levels at the University from freshmen to graduate students. The students have repeatedly rated the course as an outstanding educational experience that helped them consolidate past experiences and set the stage for easier assimilation of subsequent learning.

The diversity of both student backgrounds and the course subject matter led to the development of a unique peer teaching program. The peer teachers are a link between the instructor and the students, providing assistance to students and feedback to the instructor. This makes it possible to maintain continuity in classes with students of diverse backgrounds and interests. Peers and instructors are available daily in the course learning laboratory for consultation with students.

BRIEF HISTORY

Patterns of Problem Solving was developed at UCLA and offered for the first time in the fall of 1969 to a class of thirty-two students. It was announced as an elective campuswide interdisciplinary course. By 1973 the course grew to three sections per quarter with an enrollment of 250 students for the year. The steady state enrollment in 1976-77 reached 1600 students, with twelve sections of the course offered each quarter and two sections in the summer session.

As enrollments kept increasing, a special peer program was introduced in 1974. Peer teachers are outstanding students who have completed the course and undergo an extensive summer training program to prepare them for their role. For the past three years the peer program has stabilized in the number of peers and their duties. Generally, there are two peers per course section. The peer teachers are available in the course learning laboratory to assist students and provide the instructors with important feedback. Peer teachers are paid for eight hours per week although they often put in much more time on their own. The involvement of the peer teachers in the learning process makes it possible to conduct classes for students with diverse backgrounds and interests and maintain a continuity that is otherwise difficult to achieve. The peer program makes assistance available to the students at all times with no need for appointments, and reduces the demands on the time of the instructors.

Sections of the course have been taught by faculty members from different disciplines, such as Engineering, Psychology, Law, Business, Philosophy, Architecture, Mathematics and Computer Science. These faculty members are outstanding teachers and have been noted for their diverse interests. Some instructors are recruited, others offer to teach the course as an extra load and consider the undertaking a valuable educational experience. Since it is not possible to cover the thirty-eight sections of the course offered each year with professors only, they are joined by outstanding teaching assistants who are trained to teach their own sections of the course. The teaching assistants are selected on the basis of talent, personality, and interest in students.

Funding for the course comes from various sources -- the School of Engineering, the Office of Undergraduate Affairs, and the Chancellor's Office. The idea that led to the creation of this course was sparked by Dr. Chauncey Starr, former Dean of the School of Engineering and Applied Science at UCLA. Dr. Starr and Professor O'Neill, the present Dean, have supported the course in both deed and spirit from its inception.

Modified versions of this course have been offered since 1973 in the National Science Foundation's Chautauqua Type Short Courses conducted by the American Association for Advancement of Science for college teachers across the country. A public lecture series was offered under the auspices of UCLA Extension to 300 participants from the community which included many professionals from medicine, law, business, education, and industry. Aspects of the course have been presented to various professional societies, executives, and at the annual University of the Young Presidents Organization. A lecture series was also given at the University of Tel Aviv and at the University of Belgrade under the auspices of a Fulbright-Hayes lectureship.

COURSE OBJECTIVES

The primary objectives of the course are:

- To develop a general foundation of problem solving approaches, including some specific techniques.
- To emphasize the thinking processes at all stages of the problem solving activity.
- To expose students to both objective and subjective aspects of problem solving.
- To provide a framework for a better appreciation of the role of tools and concepts that the students may have acquired or will acquire.
- To bring together students from diverse backgrounds so they can observe different attitudes and problem solving styles, and learn from each other.

COURSE CONTENT

Patterns of Problem Solving was designed to provide the foundation for attitudes and skills productive in dealing with complex problems in the context of human values. The most significant feature of the course is its interdisciplinary approach. This is manifested in the diverse background of students in the course and the broad range of subjects covered. The ten chapters of the text Patterns of Problem Solving, Prentice-Hall 1975, which was developed by Professor Rubinstein specifically for the course, reflect this approach:

Chapter 1:	Problem Solving: culture, values and models of problem solving; difficulties, guides and attitudes.
Chapter 2:	Language and Communication: from evolution of writing to computer language, symbolic representation
Chapter 3:	Computers — Fundamental Concepts: computers, their structure, their use in problem solving, how they work.
Chapter 4:	Probability and the Will to Doubt: information, its relevance, credibility and measurement, entropy.
Chapter 5:	Models and Modeling: purpose and nature of models, models in history, behavioral science, and engineering.
Chapter 6:	Probabilistic Models: samples, distributions, errors of omission and commission, simulation.
Chapter 7.	Decision-making Models: decision criteria, utility theory, game theory, group decisions.

Chapter 8: Optimization Models — Selecting the Best Possible: linear and nonlinear programming, dynamic programming.

Chapter 9: Dynamic Systems Models: cybernetics, its ubiquity, history, and applications; simulation of dynamic systems.

Chapter 10: Values and Models of Behavior: value judgement, social preferences, consensus, assessment of values.

The entire subject matter of the text was not intended to be taught in a ten-week quarter. The subjects taught, and the extent to which they are covered depend on the instructor's prerogative. However, there is a core of subject matter upon which every instructor focuses. This core comprises the major part of the text and course and includes Chapters 1, 2, 4, 5, 7, and parts of 10. The material of Chapters 3, 6, 8, and 9 is dealt with only to the extent that each instructor desires. The text is not the only source of subject material. Some topics covered during the quarter are those which an instructor introduces as a result of individual background.

The emphasis on practical application reflects both classroom and nonclassroom experiences. Instructors often use real world examples in their lectures and assignments, and most instructors require the students to apply the tools they have learned to a personal problem. This becomes the class project. Projects in the past have covered such diverse subjects as selecting a career, buying a car, and finding a place to live. These projects are submitted in written form, and in addition, some are presented orally, shown on film, or illustrated by use of slides. Most of the students enjoy working on their projects and consider their efforts valuable.

STAFF TRAINING

Each June teaching assistants and peer teachers undergo intensive training in preparation for their teaching roles the ensuing Fall. The training program consists of a series of meetings in which basic course content is reviewed, supplementary material is presented, teaching skills are developed, and course policies are discussed. Teaching assistants are videotaped while conducting simulated class lectures. They plan a sample course syllabus and share ideas on course content, lecture styles, and grading systems. Peer teachers review course material by preparing notebooks of lecture outlines, answers to textbook problems, and additional reference material. Peers also practice explaining course concepts in role-playing sessions in the learning laboratory. Perhaps most important, the staff learns to work together and develops a sense of community which continues to grow throughout the academic year. The training program has been extremely successful. Participants in the program feel that it is very worthwhile and that they have acquired knowledge and confidence in preparation for the new academic year.

In addition to the summer training program, the course staff of about thirty people, gets together each quarter at the Rubinstein home to share experiences and suggest innovations for the program. These get-togethers serve as forums for promoting the growth of the sense of community among the staff and students. For example, instructors and peers have planned end-of-quarter class parties, organized Mastermind tournaments, and formed an intramural coed football team. A recent addition is a Patterns of Problem Solving Tee shirt displaying the cover of the textbook. Also, one peer teacher presented the program with a log book for the lab room in which students, peers, instructors, and visitors are encouraged to enter comments, puzzles, and suggestions. This sense of community is displayed by the many times that students and staff plan sessions with food, so they can "eat together while learning together."

THE FIRST DAY OF THE COURSE -- ONE EXAMPLE

> "Welcome to Patterns of Problem Solving. During this introductory course we will explore many problem solving styles and techniques. But first, here is a problem for you....
>
> "You are lost on the moon, your ship has just crash landed and you and your crewmates have been able to save the following items:
>
> > a box of matches, food concentrate, nylon rope, parachute silk, solar powered portable heating unit, two .35 caliber pistols, one case of dehydrated milk, two 100 pound tanks of oxygen, a stellar map, a self-inflating life raft, a magnetic compass, five gallons of water, signal flares, a first aid kit containing injection needles, solar powered FM receiver-transmitter.
>
> "The mother ship is 200 miles away on the lighted surface of the moon. Rank the items according to their potential for helping you survive."[1]

Immediately the students, who earlier were divided into small groups, become immersed in a problem. How should we decide? How could we rank the items? How can a group reach consensus? How can we try to overcome the uncertainties in the situation? Will a flare work in the moon's atmosphere? What is more important to us: food or shelter? What is more important: food or water?.....

This "lost on the moon" exercise is then used to illustrate many of the topics which will be covered during the Patterns of

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[1]This "Lost on the Moon" exercise is from Psychology Today, Nov. 1971.

Problem Solving course. Students realize that they have a problem since there is a difference between where they are (initial state) and the mother ship 200 miles away (the goal state). They are trying to decide how to solve their problem. Thus, they are attempting to determine a specific procedure for reaching their goal state from the initial state. Each group must analyze the alternatives available to it and make some sort of group decision. The students also will want to develop a means for coping with the uncertainties inherent in the problem. They may not know, for example, how long it would take them to walk to the mother ship, but they might be able to estimate probabilities. As they attempt to rank the fifteen salvaged items they would profit from the course discussions on utility theory. Throughout their work on this problem, students will realize that values play an important part in the way we view problems. In such a life-and-death situation, the supreme value of survival will probably overshadow the values of comfort or beauty. Finally, this simulated problem solving situation is just a model of a real situation. Its purpose is to approximate a real problem so students can practice their skills and analyze their own problem solving styles.

THE CONDUCT OF THE COURSE AND THE LEARNING LABORATORY

Patterns of Problem Solving is unique in that along with its diverse academic content, many alternate learning modes are provided. Each section of the course has around forty students, so there is ample opportunity for close interactions among the students, instructor, and peer teachers. The format of class meetings varies, including lectures, group exercises, movies, and problem solving sessions. Assignments may include homework problems, journals, individual problem solving projects, take-home quizzes, and in-class exams.

Outside of class, students are encouraged to go to the course learning laboratory when they need to talk about their homework. The lab room is staffed five hours daily by peer teachers and instructors and provides a meeting ground for students to study and receive help or feedback. Often students come to the lab room during office hours of the peer teacher who is assigned to their class section, but they may ask questions of any staff member. Students can leave messages for peer teachers or instructors in their mailboxes in the lab. Also, answers to homework assignments and additional information are posted on the bulletin board in the lab room. Many students are amazed that the staff is so available and so helpful. Students especially appreciate the fact that undergraduate peer teachers are available to help others master the course material, and some students express interest in becoming peers themselves.

The undergraduate students who become peer teachers have been outstanding students in the course and are invited by their instructors to join the peer teaching program. For these students, being a peer teacher is more than just a job. From a peer teacher's viewpoint, perhaps the most rewarding aspect of peer teaching is the opportunity to actively contribute to the educational system. Often university students become passive receptacles of facts; peer teaching combats this passivity. Peers focus attention on the educational process and think about alternate methods to teach certain concepts. They become more aware of what occurs in the classroom, and provide needed feedback to the course instructor.

Peer teachers assist in a specific course section in a number of ways. They attend class lectures; read homework, exams, and projects; and assist in running the course. As previously mentioned, peers conduct office hours in the course lab room. In the lab room, students and peers interact on a one-to-one basis. Students realize that the peer cares about them and wants them to understand the concepts. Peer teachers gain a solid understanding of the fundamental academic aspects of the course and also become proficient at explaining these concepts. Peers also gain confidence in dealing with people. Often a peer finds that a student really wants friendly, helpful reassurance as well as clarification of a point from the class lecture. Some peers become very interested in teaching and adopt that as one of their lifetime goals, but all peers have found the problem solving training very beneficial as they embarked on a career or continued their studies.

REWARDING EXPERIENCE

Patterns of Problem Solving has been a rewarding experience for individuals involved in all aspects of the program. Students feel the course is very enligntening. They appreciate the practical course content and the personal attention. Outstanding students often become very interested in the material and continue to take related courses. Twice a year the follow-up seminar course, Applied Patterns of Problem Solving, is offered to a select group of excellent Patterns of Problem Solving students. These students are specially invited by their instructors to take the seminar. Students consider it an honor to be invited to participate in the seminar. In this course, more advanced topics are studied and students conduct in-depth group projects. One group designed a "UCLA GAME" which is used at UCLA's incoming student orientation to introduce the campus and its regulations to newcomers. Another group analyzed the Planning, Programming, and Budgeting System (P.P.B.S.) at a Southern California high school. Other topics have included improving the UCLA fraternity and sorority system and forming a small business. Students sometimes continue these projects by taking independent study courses.

Not only outstanding students find the course appealing. Many students who have not been exposed to problem solving concepts before find them fascinating. One student, for example, worked very hard throughout the quarter attempting to understand concepts that were alien to him. He was very proud to earn a grade of "C" at the end of the class and thanked the teacher for the only course he'd enjoyed at the university! This student was especially aided by the daily availability of help in the course learning laboratory. Patterns of Problem Solving may be unique in that both instructors and students consider the course to be a valuable learning experience. This fact is at the very heart of the reasons for the phenomenal success of the course.

There are many benefits for course instructors. Instructors often share the same offices. They maintain common office hours in the learning laboratory and meet to discuss their experiences in social get-togethers several times a year. The effects of this constant fraternization are several. The instructors develop close personal friendships. (Two of the instructors even got married!) They exchange information on the course, based on their own area of expertise; share amusing (or frustrating) experiences; develop new problems, examination questions, and class examples; and help each other in some of the more difficult aspects of the course content. They also discuss some of the more philosophical aspects of teaching: the role of the peer teacher; the best ways to make use of student skills; the proper amount of discussion versus lecture; and so on.

An informal tradition exists that some aspects of the course content, emphasis, examples, and so on, are always changing. Part of this results from the interchange of ideas among the instructors; as new ideas are developed and are successful, they are communicated to the other instructors and adopted. But more than this, the instructors fundamentally believe in one of the bases of the course content: one must maintain an open mind, a will to doubt, and flexibility. They believe that, in a dynamic world, it makes little sense to set one's plans in concrete, especially in a course that purports to teach the skills necessary to solve real-world problems. Furthermore, the instructors are taught, and believe, that all classes are different; courses have personalities just as people do and one does not treat all classes alike any more than he would treat all people alike. Oftentimes an instructor will modify his teaching technique or the course content to accommodate a particular class of students.

Instructors maintain an honest respect for their students. In a sense, this is a necessity, because in a course that covers subjects from psychology to linguistics to physics to engineering, there may be students who know more about some given subject than the instructor.

Finally, and perhaps most important, if there exists a philosophy common to the instructors it is that the course, and teaching it, ought to be fun. While instructors are not hired because of their senses of humor, it never seems to fail that the type of people who enjoy the course enough to pursue it further and to eventually end up teaching it are the kind of people who enjoy life and are happy. They neither take themselves, nor the world, too seriously and they know that students do not learn much in a course which they dread attending.

CONCLUSION

Patterns of Problem Solving has had a record of success virtually unheard of in academia. The reasons for this success are the course content itself, which is flexible and designed to instill in the student the skills and attitudes productive for dealing with complex problems; the preparation and training of the staff involved in teaching the course; the varied format of the actual teaching; the availability and helpfulness of the staff, including the existence of a "learning laboratory," where students can receive individual attention directed towards their particular needs; and the enthusiasm, and concomitant effectiveness, of the instructors.

We feel that as the teaching of problem solving becomes more widespread, it is fundamental that the organizers of such courses examine not only the content of the course but the atmosphere in which it is conducted.

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Moshe Rubinstein created the campus-wide course "Patterns of Problem Solving." He is author of four textbooks and more than sixty publications in the areas of problem solving, decision theory, dynamic response, and structural systems. He has received many awards for his teaching excellence. He has lectured widely throughout the country and abroad. Moshe Rubinstein is listed in Who's Who in America and is a graduate of UCLA, having received his B.S. in 1954, M.S. in 1957, and Ph.D. in 1961.

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A TAXONOMY OF PROBLEM-SOLVING ACTIVITIES AND ITS IMPLICATIONS FOR TEACHING

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A Rationale for a Taxonomy

"A Taxonomy of Problem-Solving Activities"--what is it and what use is it? The dictionary defines taxonomy as classification. In the world of biology and elsewhere, it has come to mean a classification for the purpose of study. We create a taxonomy to enable us to study the parts of a subject which is too large or too diffuse to be studied as a whole.

Problem-solving is just such a topic. Solving a problem is an activity which can consume days, months, or years, or can take place in a matter of seconds. It can subsume many behaviors or very few. It can be extremely complex or very simple. Consequently, it is almost impossible to talk or even to think about it as a whole. Discussion of problem-solving tends to degenerate to a discussion of one phase of problem-solving or even of solving a particular class of problems. Thus, general statements about problem-solving are often made which would be better addressed to a particular part of problem-solving.

The development and use of a taxonomy of problem-solving activities can help with these problems. By breaking problem-solving into its component activities it makes it possible to consider each activity separately without the mental haze which results from trying to think about too many related activities at one time. It enables the thinker to examine a problem-solving system for the presence or absence of appropriate activities and to take corrective measures. Last, it allows the thinker to describe more accurately the problem-solving process and thus communicate it to another.

Since a taxonomy is an aid to description and communication, it is of more use to the person who wishes to think about and talk about the problem-solving process than it is to the person who wishes to do problem-solving. Thus it becomes a tool of utmost utility to the teacher who struggles to transmit the problem-solving process to the student rather than teaching the solution of specific problems.

A Problem-Solving Taxonomy (PST)

Most, if not all, problem-solving activities can be divided into five classifications: routines, diagnosis, strategy, interpretation[1], and generation.

There is no particular sequence for these classes of activity and in solving an actual problem the student will move back and forth among them according to the dictates of the particular problem. The following working definitions have been evolved for these activities:

Routines are those operations which, once begun, afford no opportunity for decision but proceed by simple or complex mathematical steps to a unique solution. Long division is a routine. The evaluation of a complex integral is a routine. The solution of a quadratic equation is a routine. The determination of the moment of inertia of a composite area about a centroidal axis is a routine. All of these depend only on the correct execution of a number of steps. The student may find it necessary to recall mathematical or physical facts in order to perform a routine but no decisions are necessary.

Diagnosis is the selection of the correct routine or routines for the solution of a particular problem. Diagnosis is sorting out correct routines from incorrect routines. Deciding on the flexure formula to find the stresses at a given point in a beam is diagnosis. Deciding on integration by parts for a given integration problem is diagnosis. In both cases there is only one way to go, but the student must find it. He must examine the problem until he finds a correct routine.

Strategy is the choice of a particular routine for the solution of a problem which may be solved by several routines or variations of routines, all of which are known to the student. Strategy is choosing among correct routines. The selection of a point about which to take moments is a strategy decision. The decision to use polar rather than cartesian coordinates is strategy. The use of the method of sections or of the method of joints in analyzing a truss is a matter of strategy.

Interpretation is the reduction of a real-world situation to data which can be used in a routine, and the expansion of a problem solution to determine its implications in the real world. It includes the making of appropriate assumptions and the interpretation of results.

Generation is the development of routines which are new to the problem-solver. It may simply be laying out a number of routines to put them together in new ways, in which case it is probably a matter of pure recall. It may be the bringing together of previously unrelated ideas to spark a new attack, in which case it is highly creative. It may be somewhere between these two extremes. It must result in an activity which is completely new to the problem-solver and which he has never been taught.

These are the five dimensions of problem-solving as the taxonomy defines them. A student enters with certain skill levels in each and exits with a different set of skill levels. The difference in these skill levels is a measure of what has been learned about solving problems. To teach problem-solving the teacher must address each of the five dimensions.

The present taxonomy does not deal with problem definition, because in most cases the engineer or the engineering student is set to solve a particular problem, large or small, rather than to discover the problem to be solved.

Other Learning Taxonomies

The reader of educational papers may wonder whether there is a need for a taxonomy of problem-solving. After all, excellent taxonomies of learning already exist. That is quite true. However, a closer reading will show that most of the existing taxonomies of learning behavior end where a taxonomy of problem solving begins.

The best known of all educational taxonomies is undoubtedly that of Bloom.

His Taxonomy of Educational Objectives categorizes all cognitive behaviors as knowledge, comprehension, application, analysis, synthesis and evaluation. It is an extremely powerful tool, but its very rigor makes it difficult to use in teaching problem-solving. It is quite possible for two teachers to argue for hours over whether a given objective is actually comprehension or application, and to end up agreeing that it is really a bit of both. By focusing on groups of behaviors leading to a particular outcome, rather than on individual behaviors, the Problem-Solving Taxonomy cuts across Bloom's Taxonomy and groups behaviors as they occur in the solution of problems. For instance, diagnosis, an activity in the Problem-Solving Taxonomy, may combine knowledge, comprehension, and application as identified by Bloom.

In his eight types of learning, Gagné lays out a hierarchy which culminates in problem-solving. Problem-Solving as Gagné sees it is a, far narrower activity than that envisioned by the engineer. The Gagné problem-solving is quite analagous to the activity designated as "Routine" in the Problem-Solving Taxonomy. The Gagné hierarchy does not deal with the more complex activities involved in problem-solving.

In a later work, Gagné has delved somewhat deeper into problem-solving and has somewhat extended his range. An activity which he calls "rule learning" corresponds well with our "routines", and he has divided his problem-solving into four main areas: presentation of the problem, definition of the problem, formulation of hypotheses (both correct and incorrect), and verification. This approach looks at generation and analysis but ignores the areas of diagnosis and strategy.

In a very recent attempt to assemble and integrate various taxonomies, Holland and his co-workers have evolved a taxonomy with three main divisions: psychomotor learning, memory learning and complex cognitive learning. A subdivision of memory learning, algorithms, bears a considerable resemblance to routines. The remainder of the activities catalogued in PST are treated under "complex cognitive". There, under the heading of "principles", they consider an activity much like diagnosis, and their "strategies" grouping includes a mixture of strategy, generation, and application as identified in PST.

The existing learning taxonomies are thus seen to be much more general and diffuse than PST, and require the teacher to utilize many different levels and even different taxonomies, in order to completely describe and analyze problem-solving activities as they are seen in the practice of engineering. It may be argued that the present Problem-Solving Taxonomy can be used to complement the more general learning taxonomies already in use, and can provide a useful specialized tool for the teacher whose primary concern is the teaching of problem solution. Other taxonomies are perhaps more useful in teaching the solution of one particular problem or class of problems. PST is most useful in teaching an approach to problem-solving in general.

Principal Approaches to Problem-Solving

It may be of some interest at this time to examine the approaches of several current schools of thought on problem-solving, and to describe them by means of the Problem-Solving Taxonomy (PST). For instance, brainstorming and synectics are aimed almost entirely at generation. Both are designed to facilitate the development of many alternative ideas for problem solution. The working out and evaluation of the ideas has no place in either system, but are saved for a later day.

On the other hand, Polya maps presume the generation to have already taken place and concentrate on the logical development of strategy based upon analysis. Process synthesis and computer simulations of human thought also emphasize strategy but base it upon some generation activity. The role of strategy in both approaches considerably outweighs the other activities.

Inquiry learning of all sorts is based upon meticulous questioning and thus can be characterized in PST-terms as primarily concerned with interpretation.

The cognitive and gestalt theories explain human behavior in terms of conscious, strategic purpose. "The organism perceives, thinks about, and analyzes its environment." In these theories, problem-solving is primarily seen as those activities which PST describes as diagnosis and strategy.

Behavioral psychology sees problem-solving, like all learning, as resulting from the reinforcement of correct solutions. It is not concerned with the mental mechanism by which problems are solved but with increasing the frequency with which problems are solved. In practice it becomes a powerful method for teaching routines and may produce some proficiency in diagnosis. It does not address generalized problem-solving skills.

There are many variations on these various schools of thought about learning in general and problem-solving in particular. These give varying degrees of emphasis to the activities described by the Problem-

Solving Taxonomy. However, the PST appears to be equal to the task of describing any of them and may indeed provide a useful tool for comparing and contrasting the various approaches.

Types of Problems

The foregoing discussions should serve to help place the Problem-Solving Taxonomy in perspective with current theories about problem-solving. The remainder of this paper will be devoted to the use of PST in understanding and enhancing the development of the problem-solving activities of students. Before undertaking to use PST to classify the problem-solving activities of students, it is advisable to examine the sorts of problems they are expected to solve.

Problems can be classified as simple close-ended, complex close-ended, or open-ended. In all cases the problem solver combines ideas to produce an answer to a previously unanswered question. Often the combination of ideas is a new one for the individual problem solver, but this is not always so.

A simple closed-ended problem is one which has one right answer and one set method by which that answer may be obtained. Taking the derivative of an algebraic expression is such a problem. In terms of the PST, simple closed-ended problems are solved primarily by diagnosis and routine.

A complex closed-ended problem is one which has one right answer but several methods by which the answer may be obtained. For example, many problems in dynamics may be solved by the use of Newton's laws, by energy methods, or by applying the principles of impulse and momentum, but the final answer will be the same no matter which method is chosen. The taxonomy would describe the solution of such problems as consisting of routines, diagnosis, and the use of considerable strategy.

Open-ended problems are those for which more than one correct solution can be found. However, an open-ended problem can be broken down into a cluster of close-ended problems. The correct solution is inherent once a method of attack is determined and appropriate assumptions are made. Different solutions are obtained by changing either the attack or the assumptions. Developing the attack is described in PST as generation, and choosing assumptions is interpretation. Thus, the open-ended problem emphasizes generation and interpretation at the same time that it requires all the routines, diagnosis and strategy used in close-ended problems.

The engineering curriculum attempts to develop in the engineering student the ability to solve all three types of problems. It meets with variable success. Often its successes and its failures seem to be more a matter of luck than good management. Nevertheless, engineering education does succeed. Engineering students do become problem solvers. The next section of the paper will be devoted to looking closely at how this occurs, describing the students progress by means of PST.

Problem-Solving and the Beginning Student

Generalization about students' skills in any area is a dangerous occupation. Nowhere is this more true than in the assessment of freshmen. The effects of their varying backgrounds are still very strong. Nevertheless, most teachers will agree that a freshman is more like other freshmen than he is like a senior. It is this broad common pattern we shall examine.

What is the entrance profile of the freshman engineering student? There is tremendous variation in individuals and in institutions. There is quite probably a sex-related difference, although our observations of females have been too few to include. Nevertheless, let us examine the fictitious average entering male. What are his problem-solving skills?

He is essentially a specialist in routines. Most of his previous educational experience has been directed to teaching him more and more complicated routines. However, his most sophisticated experience with routines has been with multi-step single-path operations such as long division. He is not only good at routines; he is good at learning routines. He tries to reduce all of problem solving to the application of routines.

His skill in diagnosis is limited. He can select a formula such as the ideal gas law in order to initiate a routine, but his repertoire of such formulas is very small in any given area so that selection is relatively easy. He has had the most opportunity to develop diagnostic skills in mathematics, where he has had considerable practice in matching the method to the problem.

His skill in strategy is rudimentary. It is limited to choosing between orders of operation in a single routine. In other words, he can decide whether to take one arithmetic or algebraic step before another. The capability of his calculator has frequently taught him to make some strategy decisions in order to use it efficiently.

His skill in interpretation is almost non-existent. It consists almost entirely of the identification of knowns and unknowns in a problem statement so that he can use them in the routine he has selected. He is really at the stage of recognizing that a quantity given in units of psi is pressure and goes into the gas law as P, while a quantity given in cc is volume and goes in as V. He probably also knows that something will have to be done about the units. He has had no experience in making initial assumptions or in evaluating results.

His skill in generation is yet unborn. He will brand as unfair any problem which is dissimilar to those he has been taught to do.

Problem-Solving at the Midpoint

At the end of the sophomore year the student is halfway to his bachelor's degree insofar as course work is concerned. He is ready to leave the generalized instruction of the underclassman and enter upper-class specialization. What are his problem-solving skills at this point?

To describe the students' problem-solving skills at the end of two years of instruction we must once more generalize. Obviously some students will have made far greater strides than others. Sex-based differences will probably have diminished. However, the average student will have made some progress in all areas although he has not advanced equally in all.

He has added a great many routines to his repertoire and has learned to handle more complex kinds of routines. He is able to handle chaining routines where he must complete one routine to get to the beginning of a second, and must complete the second to begin the third, and so on until he reaches the final answer. He has also learned to work with interlocking routines where one routine must be completed and the result stored while second and third routines are completed and stored in their turn, until the results of all can be used together in a final routine. He has, in fact, advanced to the final stage of proficiency in using routines. Although he will probably learn additional routines throughout his professional life, he is unlikely to encounter any new patterns for routine calculation.

In the area of diagnosis the student has made comparable progress. He can now select a set of routines and order them so that the solution of one provides the starting point for the next or, in the case of an interlocking routine, break it down into the necessary subroutines. He has learned to incorporate feedback into the diagnosis. That is, at the end of one routine he can use its results to choose the next appropriate routine. He can also carry out parallel routines and, as a final step, compare their results and select the correct answer. In the area of diagnosis, as in routine, he has gone about as far as he can go. He will continue to practice his diagnostic skills and will become more proficient, but he has acquired the complete groundwork.

In the areas of strategy the battle has just begun. Coming in with essentially no skill in problem-solving strategy, he has learned a little but he still has a long way to go. He has learned to accept the existence of more than one acceptable approach to a problem. He can select an approach from several possibilities and is beginning to develop a rational basis for some selections. He can select a starting point for his work and he can evaluate the efficiency of alternative orders of operation in complex routines. None of these skills is really well developed, but he can handle strategies for ordering work within a routine better than he can handle strategies for selecting routines.

Students enter the sophomore year with very little skill in interpretation. They leave it with little more. They are able to translate more complex problem statements and drawings into usable data. They have been exposed to some information on the applicability of the material they are learning, but they have not yet practiced interpretation. That is probably as it should be, since interpretation must deal primarily with open-ended problems while the sophomore problem is almost entirely closed-ended.

In the area of generation a start has been made. The student has become accustomed to the idea of working "new" problems, using routines in situations where he has not been specifically taught to use them, or putting routines together in a way which he has never seen before. Mechanics courses generally have provided such practice and have forced the student to a realization that he will be repeatedly forced to solve such problems, unfair as he may view them.

Problem-Solving and the Upper Classman

During the first two years the student has become expert in routines and diagnosis and has taken the first steps in strategy, interpretation, and generation. During his final two years he will develop his abilities in the last three areas. The precise emphasis shifts from curriculum to curriculum, but all curricula develop these skills.

The junior year focuses primarily on the development of strategy. There is an emphasis on seeking the best way to solve a given problem. Routines and diagnosis are still taught but only in the sense of increasing the students' repertoire. Interpretation begins to be of considerable importance, as the students' attention is focused more and more on the real-world implications of his work. The ability to generate solutions continues to develop, as again and again the student is forced to face unfamiliar problems.

During the senior year all the processes already in motion continue. Routines, diagnosis, and strategy continue to be practiced with new material and new situations. It is in the design courses that application and generation become the primary focus of the teaching effort as the teacher tries to show the student how to bring all his previous work to bear on truly open-ended problems.

Implications for Teaching

This is the developmental pattern for problem-solving skills in engineering students as it can be observed in most engineering schools. Is it inevitable? Can it be changed by changing teaching techniques? Can the more complex skills be introduced earlier?

It would appear that this can indeed be accomplished by a teacher who becomes aware of what he is teaching in terms of generalized skills rather than of particular subject matter.

For example, most teachers are quite competent at providing practice by means of assigned homework problems. Homework problems focus primarily on learning and using routines and this may be the reason that students seem to be so much more proficient in this area of problem-solving than in any other. The typical homework problem requires a very simple interpretation step as the student reads the problem, a simple diagnosis that leads to the selection of a routine, and two pages of routine calculation. Thus students become far more expert in routine calculations than in interpretation or diagnosis. Obviously they learn best what they practice most.

How can the teacher increase the students' practice in the other areas of problem-solving? By devising activities, possibly homework, where the focus is on the non-routine areas. For instance, rather than asking that a problem be solved for an answer, the same problem could be posed and the student asked to:

(1) tell how he would solve it,
(2) why he chose that method, and
(3) the order in which he would perform the routines in the solution.

Fluency in strategy might be increased by posing a problem and asking the student to describe several possible plans of attack with the advantages and disadvantages of each, and to decide which he would choose and why. Attention should be paid to making the student conscious of the decisions he makes and the reasons for them.

If the emphasis of the lesson is on these questions rather than on working out the details, more problems can be posed and examined in a given period of time and the students' attention is directed to the importance of this part of problem-solving.

Thus, it would appear that by carefully examining the particular problem-solving activities involved in an instructional episode, instruction can be fine-tuned to develop a particular problem-solving skill. The last sections of this chapter will be devoted to suggesting some ways of developing each of the skills in the taxonomy.

Teaching Routines

There are obviously many ways to teach routines since such a large proportion of teaching effort is devoted to teaching routines. Some activities and media which seem particularly appropriate are listed in Table I. The list is by no means exhaustive, but includes those items the authors have found to be effective.

The student activities and media columns are probably self explanatory, but some amplification may be in order for the items listed as teaching techniques.

The identification of routines is an important first step. The teacher should make sure in his own mind that the item to be taught is a routine and then teach it as such. He should not glorify the use of a simple equation into some higher-sounding teaching objective. Instead, he should show the students the proper use of the routine as a tool and tell them that he expects them to learn to use it accurately and quickly rather than worrying about the more intellectual issues he might raise.

Teaching routines in a formalized fashion is a direct outgrowth of the first technique. The teacher should help the student develop rules and formalized methods wherever possible. A good example of such formalization is the development of a tabular solution for finding moments of inertia of a composite body. If the table is properly laid out the solution becomes extremely easy.

By always devoting a portion of every test to routine problems, the teacher impresses the student with the value and necessity of routines and rewards the student for learning them. "Mastery" techniques are particularly useful here since it is easy to grade a routine on a pass or restudy basis, and thus to insist that important routines be performed at a very high level of accuracy.

Teaching Diagnosis

Table II shows a number of suggestions for enhancing the teaching of diagnosis. Most of the suggestions may be summarized as making sure that the teacher teaches diagnosis rather than merely expecting the student to learn it. This seems to consist of calling the students' attention to the diagnostic process and making sure that the student has an adequate opportunity to practice it under some supervision.

The authors have found it useful to make use of rather heavy prompting when the student first begins to learn the diagnosis process. This has the effect of making fairly sure that his initial diagnoses are correct, so that the student develops confidence in his own diagnostic ability and is not afraid of the process.

A student's repertoire of routines is rather small in the beginning stages. It is usually pretty well restricted to what he has learned in the particular course. The insertion of frequent unmarked review problems forces the students to sort repeatedly through his bag of tricks to find the one applicable to the problem in hand. Inclusion of several topics on each exam forces the same sort of sorting and rewards success in it. As the students' repertoire fills with material from other courses, it becomes less necessary to consciously provide opportunities for sorting. They become inherent in the problems posed.

Teaching Strategy

Table III shows a number of means of enhancing skills in strategy. Again the emphasis is on conscious instruction by the teacher on ways to select strategies, and adequate practice by the student in making strategy decisions. It is important that the teacher realize he is teaching strategy and that the student realize he is learning it.

The teacher who teaches strategy must make sure that the student has valid alternatives among which to choose. This means teaching several routines to achieve the same result, as well as teaching the student to follow parallel routines to different results among which results the student must finally choose. A classic example of the latter is the friction problem which determines whether a given object will tip or slip under loading.

Probably the best way to teach strategy decisions is for the teacher to explain the mental steps that lead to the choice of a given strategy. The teacher should act out or model the thought process that leads to a decision, or actually think aloud before the class as he or she solves a new problem.

Similarly, the student activities which seem to be most useful are those which make the student lay out his thinking in some form or another. The student seems to learn best if he is required to do his thinking aloud or on paper, since this forces his attention to logical development rather than intuitive leaps, and makes him conscious of the thought process as well as the end result.

Teaching Interpretation

Table IV gives suggestions for strengthening interpretive skills. They are aimed primarily at giving the student a wealth of data to interpret and at presenting the data in as many forms as possible. It is somewhat easier to provide occasions for the interpretation of data leading to the beginning of a routine than to provide occasions for interpreting the outcome of the routine in real-world terms, but attention should be devoted to both aspects.

Once again the teacher has an important role as he models the interpretation of data. He is particulary helpful in the beginning as he shows students how to convert observations into the basic data for a routine.

Teaching Generation

Table V suggests a very few ideas for teaching generation. Generation is particularly difficult to teach because very few people understand the means which they use to generate new ideas. Apparently the best plan is to provide opportunities for the students to attempt generation together with encouragement. About the only elementary activity in generation which the authors have been able to devise is the sort which asks a student to derive in polar coordinate an expression he knows in cartesian coordinates.

Until the teacher learns to produce new ideas on demand, he is in a poor situation to teach others to do so. However, modeling his own difficulties and their solution may be of some benefit to his students.

Summary

This paper has presented a simple and potentially-useful taxonomy of problem-solving activities. By its use it is possible to break problem-solving into its separate hierarchical but non-sequential activities, so that the attention can be focused on a specific skill. The paper has discussed the place of the taxonomy among similar taxonomies, and has used it to look at several schools of thought on problem-solving.

Table I

Suggestions for Teaching Routines

Teaching Techniques	Student Activities	Media
Identify routines as such	Homework	Texts
Teach formalized routines	Practice problems	Programmed instruction
Put routine problems on tests	Chalkboard work	Audiovisuals
Use "mastery" approach		

Table II

Suggestions for Teaching Diagnosis

Teaching Techniques	Student Activities	Media
Teach criteria for diagnosis	Practice problems which emphasize choosing the correct method	Texts
Prompt student toward correct choice in early diagnosis problems		Programmed instruction
Include review problems throughout course without identification as review	Practice problems which emphasize recognizing diagnostic criteria	Audiovisuals
Cover several topics on each hour examination		

Table III

Suggestions for Teaching Strategy

Teaching Techniques	Student Activities	Media
Teach multiple routines for same result Teach parallel routines to alternative solutions Develop standards of comparison Describe relative merits of routines Explain why teacher chooses a particular routine	Practice problems involving strategy decisions Lay out steps in a solution Verbalize reasons for choice Polya maps	Texts Programmed instruction Case Studies Design problems

Table IV

Suggestions for Teaching Interpretation

Teaching Techniques	Student Activities	Media
Provide problems with excess information Give data in many forms, (verbal, drawings, etc.) Give some data in "real" form (complete tables, graph, etc.) Work with actual objects Model teacher's own interpretive process	Building models Building prototypes Collection of field data Collection of library data Laboratories Projects Design problems	Audio-tutorials Special notes Lab manuals Handbooks Case studies

Table V

Suggestions for Teaching Generation

Teaching Techniques	Student Activities	Media
Give practice problems requiring novel use of a familiar routine Give practice problems involving new combinations of routines Model teacher's own way of attacking new problem	Devising a new path to a known result Brainstorming Design problems	Current literature Special notes Case studies

The taxonomy has also been used to describe a student's progress as he learns problem-solving skills, and has suggested methods for expediting that progress.

The taxonomy results in a simple and pragmatic approach to teaching problem-solving and for that reason is is believed that it may prove useful to others. It is presented not as a solution, but as a starting point upon which others may successfully elaborate.

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[1]This was called "application" in an earlier writing on the Taxonomy.

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4

WHAT IS THE PROBLEM IN TEACHING PROBLEM SOLVING

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This paper does not describe a course in problem solving, although for many years we have offered courses that we thought effectively taught problem solving.[1,2]. Instead we summarize the results of a five-year project to define the problem of how to teach problem solving.

WHAT IS PROBLEM SOLVING?

Defining what we mean by solving problems is not easy. A problem could be defined as a stimulus situation for which an organism does not have a ready response(3) or more formally as "a specific situation or set of related situations to which a person must respond in order to function effectively in his environment. The problem arises when the individual cannot immediately and effectively respond to the situation".[4] We could formally define problem solving as a "behavioral process, whether overt or cognitive in nature, which

- makes available a potentially effective variety of responses for dealing with the problematic situations, and
- increases the probability of selecting the most effective response from among these various alternatives".[4]

For the purposes of this paper we define problem solving simply as the activity whereby a best value is determined for an unknown, subject to a specific set of conditions. Within this activity we can identify (1) a strategy, procedure or set of steps by which we perform the activity, and (2) elements or skills that are necessary to carry out the strategy. Some of these elements we will call prerequisite skills and others we will consider to be integral parts of what we call problem solving.

Now that we have attempted to define problem solving, let us use what we know about it to try to define the problem of how to teach it.

The first step, according to most strategies[5], is to motivate the problem solver to accept the need to solve the problem. Most of us accept that our students should be adept at solving problems. We realize that we should have training in solving problems in our educational programs. We believe that most teachers are motivated to try to improve their students' ability to solve problems.

The second step is to define the problem. This means that we need to identify[6,7] the unknown, the knowns, the system, the diagram, the criteria, and the constraints. The stated problem to be solved is "how can we teach problem-solving skills to our students?" In Table 1 we have identified the subpoints of this problem.

In thinking about this stated problem we could ask:
- What prerequisite skills are needed by the students?
- Is learning "problem solving" the <u>real</u> problem or is it that the students just do not have the basic knowledge needed to solve problems?
- What is the expected hierarchy of skills included in the term problem solving?
- Do we really know the meanings of all the words used in Table 1?

COLLECTING DATA.

Our approach was to collect data to try to answer some of the questions and to discover where a select volunteer group of students fits into the above problem definition. From this we then hoped to propose alternatives for solving the problem; to evaluate these alternatives; and to select and implement the best ones.

We collected four types of data.

First, we observed expert problem solvers solving a variety of problems. Second, we surveyed how others taught problem solving[5,8]. Third, we attended different courses on problem solving. Fourth, we sent one professor back to school to be a student and attend all the required classes together with the regular students. This professor was:

1. to observe (as a student) the problem-solving training to which our students were exposed.

2. to identify the major difficulties our students were having in acquiring skill in solving problems.

3. to identify the necessary problem-solving skills needed by professional engineers; and to develop the teaching techniques and integrate the training in these techniques into our four-year program.

4. to improve the skills of those students who were helping us gain the information and experience outlined in objectives 1, 2 and 3.

One of us started as a "freshman" in 1974 and is currently in his "senior" year, following the same group of students through their program. We use a voluntary, non-credit, two-hour-per-week tutorial to meet these four objectives. Student volunteers meet once a week in a special room for problem solving skills to discuss, work out and discover how they solve the ordinary homework problems assigned in the credit courses. To a very large extent, the students themselves decide what occurs during the tutorial; they select the problems they want to work on. The professor's role is supportive, as a specialist in how to solve problems, but not in the subject material of the courses being taken. His purpose is to assist the students to discover their difficulties and how to overcome them. To achieve this, the walls of the problem solving skills room are projection screens, the tables are arranged in an open U formation and each student has an overhead projector with which to project his or her ideas about a homework problem onto the screens. Thus all can see each other's ideas. All work at the same stage of solving the same problem. We call this an "Everybody-Share" tutorial approach. For example, all might be "drawing a diagram of the system" for problem 22.1 in the freshman chemistry course.

In collecting this information through the problem-solving skills session we are walking a tightrope. On the one hand we want to observe students solving the assigned problems in their required courses. We are not offering a well-thought-out, well-planned course on problem solving. On the other hand, if we do not provide help to the students on how to solve problems they would not come to the voluntary tutorial so that we could not test possible ideas on how to teach problem solving.

For the freshman year this duality presented no challenge because, by means of the everybody share[6] arrangement and because we were focussing on how to define the problem, the students could learn from each other and we could observe the differences, the difficulties and the improvement.

For the sophomore and junior years when the students had progressed to more complex behavior in solving problems, the students lacked the basic knowledge of terms, procedures and processes pertinent to problem solving. Now the students expected that some training be provided directly by the instructor. With the help of Dr. Alan Blizzard of the McMaster University Instructional Development Center we were able to introduce the hint sheet[9,10] to briefly introduce and illustrate new ideas on how to solve problems. By this means we were able to satisfy both the needs of attracting a group of volunteers to the skills sessions and of being able to observe students solve problems.

WHAT HAVE WE DISCOVERED SO FAR ABOUT PROBLEM SOLVING?

First, many we talk to suggest that the real reason students cannot solve problems is because the students just do not have the knowledge needed to solve the problem. For example, students cannot "design a heat exchanger for a given duty" because the students just do not know enough about heat transfer. This suggestion is partly true. On the one hand there are many prerequisite skills needed to be able to solve problems but on the other hand there is a set of knowledge called problem solving that can be learned. The prerequisites that we would identify include:

1. basic knowledge the students must know ("the students must know enough about heat transfer");

2. memorized experience factors (they need to know reasonable values for velocities of fluids in the heat exchanger, the usual sizes of commercially-available units so that they can judge when they have a reasonable answer;

3. communication skill (they are required to communicate the answers to the receiver);

4. learning skills (students have to be able to obtain any missing information and overcome any weakness in basic knowledge they might have);

5. group skills (the students must be able to work effectively in groups as well as individually);

6. motivation (the student is willing to try to design the heat exchanger).

Concerning problem solving, we classify it into two activities: the strategy (or sequence of steps that one follows to travel from the unknown to the best answer for the unknown), and the elements or skills (other than the prerequisite skills) that are needed to effectively apply the strategy. Included in the elements are an ability to

1. analyze

2. synthesize

3. make decisions

4. generalize

Since synthesis is a combination of creativity, analysis and decision-making, we modify the list to: (1) analyze, (2) create, (3) make decisions and generalize. Details of this are available elsewhere[11].

We can now focus on each of these activities and identify levels of development. Such can be done following either Bloom's taxonomy[12] or Sister Austin Doherty's excellent identification of the developmental levels.[13] For problem-solving strategies, for example, these levels could be those listed in Table 2. For the problem-solving element creativity, these levels could be as given in Table 3.

Concerning the steps in a strategy, we asked expert problem solvers to apply Polya's four[14] step strategy to solve engineering problems. The experts used another step -- something in between defining the problem (Polya's step 1) and planning (Polya's step 2). In this additional step the experts tried to put the whole problem into perspective; sometimes they would make many simplifying assumptions and solve a simplified version of the problem just "to see what the problem was all about". Fuller[15] describes this activity as "poking the problem to see how it responds"; Crowe[16] talks about rumination of the problem while Rief[17] talks of solving the problem by successive refinements[18]. This prompted us to break Polya's first step, define, into two steps: define the problem (as it is given to us) and think about it (mull it over, identify the real problem, identify the sensitive parts of the problem, kick it around and see what happens).[6]

In summary, we perceive problem solving:

1. as having six prerequisite skills
2. as consisting of a strategy plus four elements: creativity, analysis, decision-making and generalization
3. as having at least five levels of developmental skill in applying the strategy and each of the elements
4. as including a five-step strategy.

WHAT HAVE WE DISCOVERED SO FAR ABOUT STUDENTS?

Our data about the students came from a variety of sources. The prime source was the dozen volunteers who came to the problem solving skills sessions each week for the past three years. They were about half the students registered in the course and represented a spectrum of abilities as measured by the grades they received in their courses. A second source of information came from the non-volunteer group of students. Some of the tutorials in the required courses were run jointly by the instructor in charge of the course and by the problem-solving skills tutor. In this way we could compare in-class activities in solving problems between those who attended the skills sessions and those who did not. A third source of information came from observation sessions run for educators, graduate students, professional engineers, and student groups in which no one had participated in any problem solving skills prior to the observation session. A fourth source of information came from the responses to our questionnaire and from our reading of the literature.

Here are some of our observations.

1. Our senior students and freshman and the graduate students and sophomore students, educators, and graduate engineers from other institutions have common weaknesses in solving problems.

2. Students do not have an organized method of defining or thinking about problem solving. They just "do their thing". To many this meant starting to solve a problem with whatever came into their head first. To others this meant that they tried to locate a worked example problem. Still others found they just did not have confidence to do anything; getting started on solving any problem was just a traumatic experience. When they got "stuck" on solving a problem, few could describe where they were, what was causing the difficulty and what obstacle they were trying to overcome. In trying to develop plans for solving problems, too many students try to solve problems by "playing around with the given symbols and data" until they find an equation that "uses up all the given information". Such a dependence means that the students cannot function when they are given problems where too much or insufficient data are given.

   In summary "doing their own thing" did not provide satisfaction to most of the problem solvers and much of what they were doing was reinforcing what we would identify as bad habits. Our observations were confirmed by the correspondence of Doyle(19), Fuller(15) and Van Wie(20). Counterbalancing this, when we gave students specific assistance in improving their problem solving skills these are their comments: "this experience opened my mind", "now I have a new awareness of how to look at a problem" and "the best thing I gained from this experience was confidence; I no longer panic; when I see a new problem I just patiently start to apply the problem solving strategy". These illustrate the deficiencies as perceived by the students.

3. Students cannot describe what they do when they solve problems. If students are asked "What are you doing?" they usually can say little more than "solving this problem". Few have been exposed to any formal strategy for solving problems, hints on how to improve problem-solving skill or learning skills. We assume that the students know these skills and few do. Yet this is not a difficult task for educators. After about 10 hours of problem solving skills sessions, the volunteer group identified as the most significant thing that had happened was their ability to describe what they were doing, to talk

to each other about how they were solving problems, to talk the same language, and to be able to identify where they were stuck in the problem solving process.

4. Students were weak in many of the prerequisite skills. Learning skills - such as how to budget time, how to separate important information from unimportant information, how to take lecture notes, how to see the "structure" in a subject - were weak among freshman students. Bad habits acquired here persisted in our senior students. Good lecture note-taking is a skill that few students in science and engineering seem to have. How students learn new knowledge could be a key to improving problem solving. Our work with the students to identify a structure within the subjects being learned, the difficulties our students had in identifying pertinent fundamentals needed to solve a problem; the role of symbols in problem solving and the discoveries of Rief and Larkin(21,22) - these all confirm the importance of how new knowledge is acquired. Our observations were confirmed in part by the responses of Snyder(23), Walters(24), Aubel(25), and Fuller(15).

5. Students were weak in at least two of the four elements: creativity and analysis (we have insufficient evidence on decision-making and generalization). Students did not know many techniques for improving their creativity. Often students confused creativity with critical thinking. By doing so, they hastily rejected a novel and imaginative idea without giving that idea a chance to be explored.

   Most suffered from mental rigidity when they get stuck on a problem. They could not approach the problem from a different direction very easily.

   Examples may illustrate some of what we observed.

Case 1. In the skills session the students were stuck on a "simple" integration problem. One who was able to see a method was not able to describe how he had thought of the method, while all the rest could not think of any method at all. We had introduced brainstorming earlier. When we tried a brainstorming session together with a follow-up analysis session, the students discovered three methods of performing the integration. Thus, when the students tried a new technique they discovered ways of solving the problem.

Case 2. In the skills session the students were stuck on a problem in electricity and magnetism. The hint sheet for that day had described de Bono's juxtaposition technique(26) in which three random words are placed into the context of the problem and some relationship sought between those words and the problem. The students were dubious. Yet when we then tried the method using the randomly chosen words "monster, airplane and problem" we were able to identify several possible methods of solving the problem. Here again the students were stuck and could not see where to go. They lacked confidence in applying a new technique.

Case 3. In a required sophomore class tutorial which included both students who had attended the skills sessions and students who had not, one who had not attended the sessions was stuck. He could not see what the problem was all about. We suggested that he try "personal imagination" in which he imagined that he was part of the problem and described how he felt. He could neither do this nor gain assistance from it. One might argue that perhaps this "personal imagination" technique is not important to this student. However, "personal imagination" was later used by two professors in two different courses in the junior year, in order to explain complex phenomena. Students in these classes were asked to "visualize that you are riding along on a streamline", or "visualize that you are in the bubble in this fluidized bed". Thus, a technique important for solving problems was also used by instructors as a learning device. Hence the student who was unfamiliar with this or unable to apply it was at a disadvantage.

Case 4. In the skills sessions the students could solve simple analogy problems[27] very well but when they were given analogy problems written in technical terms, they had difficulty. These observations are confirmed by those of Wallach.[28]

In summary, the students did not know how to create new ways of viewing situations nor how to analyze situations well. To overcome this requires the conscious introduction of a variety of procedures and practice in applying each.

6. The students as a group have a variety of needs that change markedly throughout their college careers.[9] This we detected because of the students' responses to the question that opened each problem solving skills session, "What do you want to work on today?" The responses ranged from "What is expected in our up-coming laboratory report?" to "how can I go about getting summer employment?". This has important implications. We cannot just insert a course on problem solving skills at any place in the curriculum. Whatever experience is introduced should be done when the students need it. This spectrum of needs we have tried to identify[9,10].

7. The students as individuals have a variety of thinking preferences. Some prefer to visualize concepts, ideas and problems. They see the ideal gas law as gases in cylinders and visualize how the volume and pressure in that cylinder change with changing conditions. When faced with a problem they start drawing diagrams and imagining experiments that they have seen or done.

Some prefer to "mathematize" concepts, ideas and problems. They see the ideal gas law as an equation $pV=nRT$. When faced with a problem they start by converting everything into symbols and setting up equations.

Some prefer to verbalize concepts, ideas and problems. They see the ideal gas law as a definition in words. When faced with a problem they try to relate word definition.

Thus students are visualizers, mathematizers or verbalizers.(4,8) In another sense the students have different preferences - they need to have an opportunity to develop their own style - to discover what problem solving strategies and hints work best for them. For example, in developing a plan some students prefer to explore working backwards tactics first while others prefer to work forward via the sub problem approach(29). For some juxtaposition(26) works; for others, it does not.

The approach we took in the problem solving skills session was to introduce the students to a wide variety of suggestions and hints and to encourage the students to develop their own style(9,30).

8. The students would freely discuss their difficulties and the struggles with problem solving in a non-credit setting such as we used in the skills sessions. Yet they felt that they could not do so if the evaluator in the course was running the session.

9. Only problems assigned in the required courses would be solved in the skills session - that was the original intent. But some important types of problems were not assigned - some to test certain creative skills, some to test analytical skills, some team project problems and some lengthy projects. When such "extra" problems were introduced in the skills sessions to provide the variety of skills we wished to observe, some students were reticent, some stopped coming, and eventually these types of problems were either abandoned or modified. Thus, the students wanted to voluntarily work on problems that were directly for course credit even though they accepted and recognized the eventual benefit to be gained from working on the other type of extra problems. This reaction is very understandable because of the weekly pressures from assignments and the relatively heavy time demands that we place on our students.

10. Students have difficulty with apparently simple tasks that most instructors take for granted that the students can do well. These include drawing good diagrams, locating a

coordinate axis system, identifying the system and choosing symbols to represent concepts, variables or unknowns.

In summary, the students

1. have common weaknesses that can persist well into their professional career.

2. do not have an organized method of thinking about problem solving.

3. cannot describe what they do when they solve problems.

4. are weak in prerequisite skills - especially learning skills.

5. are not strong in creativity or analysis.

6. have changing needs as they progress through a college program.

7. have a variety of thinking and problem solving preferences.

8. prefer a non-evaluative atmosphere to discuss difficulties with solving problems.

9. prefer, in a voluntary activity, to work on required problems for which they will receive credit.

10. had difficulty drawing diagrams, locating coordinate axis systems, identifying the system and choosing symbols.

WHAT HAVE WE DISCOVERED SO FAR ABOUT TEACHING PROBLEM SOLVING TO STUDENTS?

So far we have described the data collected to help us define the meanings of the terms "problem solving" and "students". What have we learned about the word "teach?"

Since September 1974, one of us has sat in over 2000 hours of lectures that constitute all the required courses in the first three years of our undergraduate Chemical Engineering program.

What type of specific assistance is given the students to improve their problem solving skills?

1. Only one professor consistently and clearly identified the strategy steps he used to solve problems. While others used strategies, they did not say such highlight words as "let's <u>define</u> the problem first", "now let's develop a <u>plan of attack</u>" etc.

2. Many do an outstanding job of verbally thinking out loud while they are working problems at the board, although they do not highlight what strategy step they are working on. However, these details are not recorded on the board. Because the students in science and engineering usually copy down only what is written on the board, these ideas are missed.

3. A few instructors ask the students to offer suggestions on how to go about solving problems. Concerning this approach,
(a) if the students keep quiet for 30 seconds they know that the instructor will answer the posed question himself or else he will rephrase the question into a simpler, leading question.
(b) if a suggestion is given then rarely are all the alternative suggestions sought and then evaluated. Usually the instructor criticizes each suggestion as it is posed and stops when he gets the suggestion he is looking for.

4. Each instructor worked problems with which he was familiar. Hence, the instructor never did nor had to demonstate what he did when he got stuck on a problem.

5. Few instructors explain the very, very simple steps such as how to draw a diagram, how to locate a coordinate axis system and how to simplify a problem.

6. The students are not exposed to problems that have different degrees of difficulty. They experience, in our judgment, too many of the very elementary type, "show-me-that-you-know-a-concept." Yet we expect them to be proficient at solving the most complex synthesis problems with very little practice at solving problems with intermediate levels of difficulty. To illustrate this, we have coded all the homework problems our students were asked to do in all courses over the three years. Bloom's taxonomy[12] was used to code the problems. In three years the students were asked to solve about

   2200 problems of Bloom's levels 1 and 2 (Cognition and Comprehension: given a familiar situation, the student will recall information and use that to solve a recognizable problem).

   30 problems of Bloom's level 3 (Application: given an unfamiliar yet well-specified situation, the student will identify the pertinent knowledge and then exhibit comprehension).

   510 problems of Bloom's level 4 (Analysis: given an unfamiliar and ill-specified situation that contains

missing data, unstated assumptions or inconsistent data, the student will identify the omissions and inconsistencies, will recognize the relevant particulars, will collect the required data and make the necessary assumptions, and finally will exhibit application).

10 problems of Bloom's level 5 (Synthesis: given an unfamiliar and ill-specified situation, the student will create alternative solutions that pertain to a selected criterion and then exhibit analysis).

This concern for a structured building-up of experience has been described by Plants and Dean. (30A)

7. Across the whole spectrum of teaching-learning interaction the student does not get the impression from instructors:

   (a) that problem solving skill *per se* is a skill to be learned. We hear instructors exhort "you must be able to communicate", "you must know these fundamentals", but we do not hear "you must polish up your skill in solving problems". Indeed, the failure by professors and students alike to recognize that problem solving in itself is a legitimate educational goal has been emphasized elsewhere by Neufeld(31).

   (b) that all instructors (when they work examples) are demonstrating a common skill.

8. Our colleagues could not believe or appreciate the degree or type of difficulty the students were encountering until they tried the "Everybody Share" tutorial format.

9. We can identify sufficient processes and terminology that the student needs to learn about problem solving that a separate course on problem solving might seem appropriate. Yet the weakness in having a separate course is that the students seem to have difficulty translating the skills learned in such a course to solve problems outside the course(32). Although we have not tried such an approach directly based on our observations, we would concur that a major challenge is to get the students to be able to transfer skills learned in a separate course to problems outside the course.

As a sidenote, we have naturally observed how well various teaching techniques have worked in general. Most of these reinforce ideas already presented in such books as McKeachie's "Teaching Tips"(33) but we felt that it was useful to reinforce some of the more dominant factors as we have experienced them.

These include

(i) Clearly-stated course objectives are important. However, these objectives have to be reinforced throughout the program to gain the optimum effectiveness. It is not sufficient to hand out a list of course objectives at the beginning of the term.

(ii) An overview is necessary. Course outlines are helpful, but the questions students have are -- why is this important? where does this fit into my program? what is the practical significance? what will be the major themes in the next set of lectures? where are we going?

We should assign the reading for the next lecture so that the students have the opportunity to come to lecture prepared.

(iii) For classes of 150 or more - if one uses an overhead projector then two projectors should be used so that there is continuity about the presentation. If one uses the blackboard, TV chalk should be used to provide a thicker mark. A good chest-microphone amplification system may be necessary. For all questions asked, the question should be repeated so that everyone in the auditorium can hear. For the large classes we should anticipate that some time, someone is going to throw paper airplanes; one needs to decide ahead of time how to handle this.

(iv) Some lecturers help the students recognize what is important by emphasis during lecture, or by marking a "review sheet" that the students prepare and bring to tests.

(v) What happened to the demonstrations and in-class experiments?

(vi) For note-taking in the sciences and engineering, the students quickly develop the habit of writing down only what is recorded on the board or transparency.

(vii) With rare exceptions restlessness starts about 20 minutes after a lecture begins.

SOME INTERIM IDEAS.

We have emphasized throughout this paper that we are trying to define the problem of how to teach problem solving; we are not indicating what method should be used to teach it. Yet some of the results so far have been so surprising and disturbing to us that we have already made a number of changes in how we teach our usual courses. Here is how the data summarized in this paper have influenced us already and some ideas about how to try to overcome some of the challenges to teaching problem solving.

Our first revelation was how much we have learned about problem solving. Each of us when we started this project though knew quite a lot about it. Yet in hindsight we knew very little compared with what we know now. This has been a very humbling and rewarding experience. Of great benefit to us has been our continued analysis of what problem solving really is. A second idea concerns the strategy of solving problems. We acknowledge that most of the focus on teaching problem solving will be on the strategy. Nevertheless, although there are apparent differences among the various strategies proposed, we think that it is vital for an instructor to identify one strategy and highlight it as being the backbone behind all examples he works. We have done this now in two of our courses. We emphasize the application of a consistent problem solving strategy of: define, think about it, plan, do it and look back. We try to identify and label the steps in each example problem worked. We use consistent terminology. Sometimes it helps to have worksheet transparencies made up ahead of time that are then completed in class for every problem.

A third result is that we have added a junior level course in "Process Model Formulation and Solution" that blends problem solving more explicitly with numerical methods.

Concerning the prerequisites, we now ask our students to collect and complete experience factor charts.

We are extending the sections on creativity and analysis that are given in a senior level required course and have changed many of the problem assignments so that they test different elementary skills: some assignments require that the students list the steps to be taken to solve a problem (they do not detail the steps nor actually calculate an answer); some assignments test analogies; others test analysis in general.

Another activity that is not easy to apply concerns the "visualizer" versus "mathematizer" in each of us. One step that we have taken is to try first to identify our own preference.

This can usually be discovered easily by writing down the definitions or ideas behind some basic laws in our discipline. If all our definitions are equations; then we look for "mathematizer" tendencies in our teaching. If all our definitions are diagrams and graphs, then we look for "visualizer" tendencies. Once we have identified our own tendencies the next step is to identify those few fundamental concepts that are cornerstones for our course. For these cornerstone ideas, we are trying to present these from all three viewpoints. For example, one would describe the ideal gas law by all three definitions:

- graphs and demonstrations as to what happens
- an equation $pV = nRT$, and
- word definitions and relationship to Boyle's and Charles' law.

SUMMARY

To define "how do we teach problem solving skills to our students?" we collected data from questionnaires, from the literature, from expert problem solvers and primarily from a group of student volunteers who we are observing as they progress through our four-year undergraduate program. To observe the students one of us has become a fellow student, has attended all the required classes along with the students and has offered a separate problem solving skills session which, on the one hand, allows us to try to improve the students' skills and, on the other hand, allows us to observe the students difficulties.

We perceive problem solving as having six prerequisite skills: basic knowledge, memorized experience factors, communication skills, learning skills, group skills and motivation. Problem solving consists of a strategy and the four elements: creativity, analysis decision-making and generalization. For each of these later activities, we can identify at least five levels of development.

The students have weaknesses that are common to all levels of undergraduate students, to graduate students and to practicing engineers. Most students do not have an organized method of thinking about problem solving nor can they describe what they do when they are solving problems. Prerequisite skills, creativity and analysis -- these are all areas where the students showed weaknesses. The students had needs that changed throughout the program. Each student had his own preferred way of solving problems. Students had difficulty with such tasks as drawing diagrams, locating the coordinate axis, identifying the system and choosing symbols. The non-evaluative atmosphere worked well in that the students were quite willing to talk about their difficulties in solving problems. However, for any voluntary activity they strongly preferred not to work on problems unless they were problems for credit within the credit course structure.

We summarized a number of observations about teaching problem solving and listed some interim ideas about what might be done to improve the teaching of problem solving.

Table 1: An Attempt to Define the Problem.

Unknown: "how do we teach problem solving skills to our students?"

or

"how can we provide an environment so that our students learn the skills necessary to solve problems?"

Known: - age of students.

- numerical grades and formal titles of background experiences

Verbal examples:

Instructors solve problems.

Written examples:

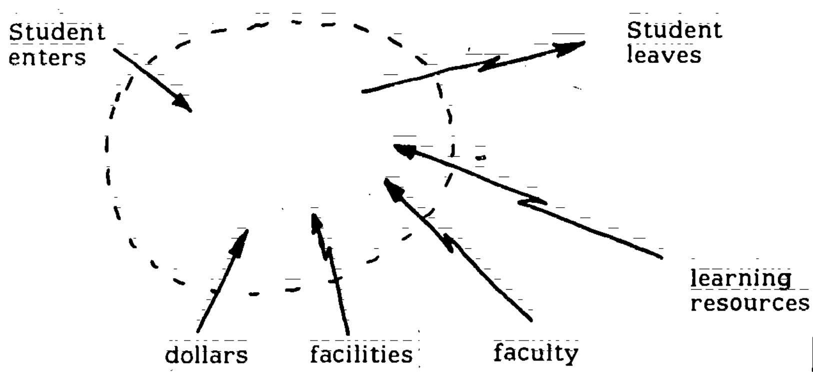
Textbooks have sample solutions.

- resources available.

- time available.

System: Four year program consisting of 104 weeks. The faculty available can be identified. The University resources available can be identified.

Diagram:



Criteria: Competence tests at various levels.

Constraints: dollars, facilities, time, faculty, learning resources available.

Table 2: Some Suggested Competence Levels for Problem Solving

Level 1
- state the steps and substeps in a strategy to solve problems.
- state the limitations to a serialistic application of such a strategy.
- state the relationship between analysis, creativity, decision-making and generalization and the steps and substeps.
- state the characteristics by which we classify the different types of problems.
- state the relationship between the type of problem to be solved and the steps in the strategy.
- state the prerequisites.

Level 2
- Given a problem to be solved, recall the stated problem solving strategy and elements and apply these to solve the problem. All the data necessary to solve the problem should be given in the problem statement.

Level 3
- Given a situation where it is not evident that a problem solving strategy is required, identify when the strategy and elements should be applied, then show comprehension as in Level 2. Examples include:
  - detection problems,
  - personal problems,
  - community problems,
  - trouble-shooting problems (where the strategy needs to be applied several times),
  - plant improvement situations,
  - writing a report,
  - group problem solving.

Level 4
- Analyze what you do when you apply the given strategy and identify personal preferences about steps and elements.

Level 5
- Develop your own strategy for solving problems.

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Table 3. Some Suggested Competence Levels for Creativity.

Level 1.	State the definition of - the brainstorming atmosphere, - state the definition of a trigger, - list at least ten different triggers, their definitions and their attributes.
Level 2.	As a group, use the knowledge of Level 1 to generate at least fifty attributes or uses of an object.
Level 3.	As a group, use the knowledge of Level 1 and the experience of Level 2 to generate at least fifty ideas for a given situation.
Level 4.	As an individual, generate at least fifty attributes or uses of an object.
Level 5.	As an individual, generate at least fifty ideas for a given situation.
Level 6.	Analyze what you do when you brainstorm and identify personal preferences.
Level 7.	Develop your own method of thinking up ideas.

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5

STRUCTURE AND PROCESS IN PROBLEM SOLVING*

Maynard Fuller
McGill University

Geza Kardos
Carleton University

Introduction

Why should engineering students learn problem solving? The question is like asking, "Why should birds learn to fly?" One of the dominant patterns of engineering thinking involves composing a statement of purpose in the form of a problem, modifying the problem to give new problems that are solvable by available means, solving these, and then selecting one or some combination of these as a solution to the original problem. A vast amount of experience shows that this pattern of thinking is a good way of achieving goals. This is why engineering students should learn problem solving.

How can the student learn this thinking pattern? Well, we all know from experience that simply giving a student a stack of problems and telling him to solve them works, but is less than universally effective. Here we propose a more effective method for teaching the process of problem solving. The method applies to the common single-answer problems that are usually given as practice of the concepts, laws and rules that students study in their courses. These are "Determination problems." We are not advocating that this class of problems should be more prominent than it already is. Indeed, if most students were more proficient in solving determination problems, there would be more room for other classes of problems.

The method presented here is based upon one of two kinds of problem-structure mapping suggested by George Polya (1), hence the name "Polya Map." The use of the Polya Map was developed at McGill in 1972-73 (4) and has since been used in courses in creative problem solving and machine design at Carleton, and in an elementary mechanics course at Marianopolis College ("Math in Motion," Dr. J. M. Brown, Marianopolis College, Montreal). The method is thus relatively new, and has been used for eight classes of students, in four different courses, by three teachers. It works!

*Adapted from a paper presented to the 85th Annual Conference of ASEE, 1977.

What Is A Polya Map?

In his classic, "how to Solve It," Polya (2) points out that the solution process should have four stages:

1. Understanding the problem.
2. Devising a plan.
3. Carrying out the plan.
4. Looking back.

The Polya map is an aid for stage two. It is a graphical way of presenting the problem structure and it becomes a plan for the solution of the problem.

The Preliminaries

Before a plan for the solution can be formulated the problem must be dissected and understood.

Most problem statements do not explicitly give nor identify the principal parts of the problem. They are usually mixed with various kinds of background information and extraneous data. The first requirement, therefore, is to separate and identify the principal parts of the problem. This may seem trivial, but it is vital.

Understanding the problem is essentially the same for any problem-solving method. After carefully reading over the problem statement it is dissected by asking these questions: -- What are the unknowns? Identify and write them down. -- What data are needed? Make a list of all data likely to be involved. -- What data are given? -- What relations (algorithms) are available that relate the data to the unknowns? --What are the constraints? -- Can we draw a diagram with the data and the unknowns identified? -- Can we now express the problem more simply? -- Are there any concepts that are new to us in the problem? Look them up to understand them and integrate them into the problem. -- Are there any relations missing? If so, where can they be obtained? -- Are the constraints contradictory or redundant?

After this it is worth reviewing what we have. We may want to modify our information. We should be certain we understand the problem sufficiently to know generally what we want to achieve and how we can get there. Complete, detailed understanding and identification of every bit of data and every relation is not essential; one function of the Polya map is that it allows us to proceed and identify the missing data and relations as we go along. If the problem is understood, the most efficient thing to do is to first search our memory to see if we have solved this kind of problem before. If not, then we can proceed with the plan for solving the problem using a Polya map.

Components of a Polya Map

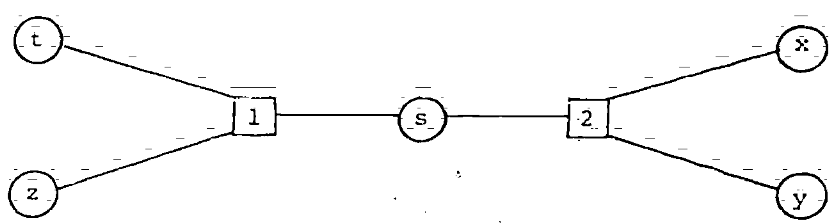
To generate a problem-solving plan we need a way of concisely expressing the structure of our problem. The plan must display the interrelationship of the various principal parts of the problem without being cluttered with the details of the necessary manipulations. Once we are assured that the structure is complete and will give us a solution, we can then introduce the details. With a properly-constructed plan, the puzzling aspects of the problem disappear. All that remains is to carry out the orderly routine calculations or transformations to obtain the desired solutions. The Polya map is the device we use to display this problem structure.

In the Polya map a variable or data is represented by a letter symbol in a small circle, relation between variables is represented by an identifying number in a small square. To show which variables are involved in which relations, lines are drawn joining the relevant circles and squares. Thus our map consists of circles and squares joined by lines in a pattern that shows the interrelations within the problem. Note that a circle cannot be joined to a circle and a square cannot be joined to a square.

If we have the variables x, y, z, s, t and they are related by the following functions,

1. $z = f(s,t)$
2. $y = g(x,s)$

then the structure can be represented by the following Polya map:



There is no implication that the functions 1 and 2 are explicit. The only requirement is that, in principle, if we knew the values of all but one of the variables involved we could determine the unknown one, by some operation. The structure thus tells us that if we know x, y and t we can determine z.

Here we see one of the advantages of the Polya map. We do not have to establish the exact forms of the algorithms relating the variables to be used in the solution at the outset. It is sufficient to know that such relations exist; this can often be done by simple physical reasoning. Thus the problem structure can be formulated without establishing all of the detailed operations.

Similarly, there is no requirement that a variable be a single-valued function of the others. If it is not, an additional step will be needed to distinguish between the values. The recognition of this adds to the understanding of the problem.

Constructing the Polya map focuses our attention on the relations between variables. In putting the map together we discover whether intermediate variables are necessary and where additional algorithms may be needed. We also discover what information is redundant.
At first glance a finished Polya map looks a bit like an information flow diagram for a computer program, but the resemblance is only superficial. Most Polya map links are reversible and there is no inherent sequence in Polya map operations. Rarely are branching and looping operations used as in computer programs. Polya Maps are for people, not machines.

Constructing a Polya Map

Although each Polya map is unique, their construction can be generalized. The particular characteristics of a specific problem will lead only to differences in construction details. Since it is not necessary to start with complete information, the construction can be expected to be iterative. It may often be necessary to redraw the map several times, rearranging the spatial relationships in order to see the problem more clearly.

We start by identifying and listing all the variables and data that appear to be relevant to the problem. For simplicity they are usually identified by the same symbol used in the mathematical model. Next we list and number all relations and algorithms which we think will be needed.

Assembling the Polya map then becomes much like solving a jig-saw puzzle. We fit the elements together to give a consistent whole, with each variable and each relation appearing only once. A good start is to first map each relation separately. Then they can be linked through their common variables.

The construction of the Polya map can produce additional insights. It helps identify the intermediate variables and reveals if additional relations or data are needed. By examining the structure we see if the solution will be a specific value of the unknown or a relation between the unknown and a variable. These insights make it possible to modify the map to give a more desirable form of solution, or to make the solution process more efficient.

Meta-Operations

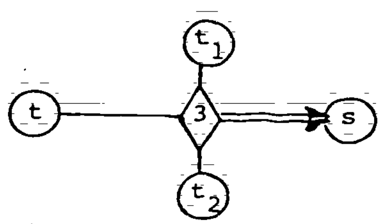
The map elements to this point consist of either variables or relations, and the operations are assumed to be completely reversible. The relations are procedures for obtaining the value of one variable from the other known variables, e.g., solving an equation, reading values from a graph, looking up values in a set of tables, etc. However, experience has shown that some problems require operations which are not reversible. There exist well-defined operational procedures where an unknown can be determined uniquely from known variables, but the operations are not reversible. For the Polya Map these special procedures are identified as "meta-operations." Meta-operations are represented by diamond-shaped boxes joined to the diagram by a double-lined arrow. The arrow indicates the direction in which the meta-operation must take place to give the specific result required.

s = variable

t = variable

t_1, t_2 = specific values of the variable

3 $\quad s = \int_{t_1}^{t_2} t^3 dt$



Polya Map

In general, meta-operations are used to indicate operations that are essentially non-arithmetic. Some arithmetic may be necessary to carry out a meta-operation, but arithmetic alone is not always sufficient. Some examples of meta-operations are: integrate between limits; combine two algorithms; fit a polynomial to a set of data by least squares; trial-and-error computations; iterate; optimize; choose maximum values; derive an algorithm for a set of assumptions; test an algorithm for a set of assumptions; test an algorithm against a set of data; graph relations between two variables and find a maximum. This is not an exhaustive list, but it gives an idea of the other kinds of problem-solving operations that can be introduced into the Polya Map.

The meta-operations extend the usefulness of the Polya map beyond simple mathematical problems. Unlike our other operations, one meta-operation may be connected to another meta-operation, although the end of a chain of meta-operations will be a variable.

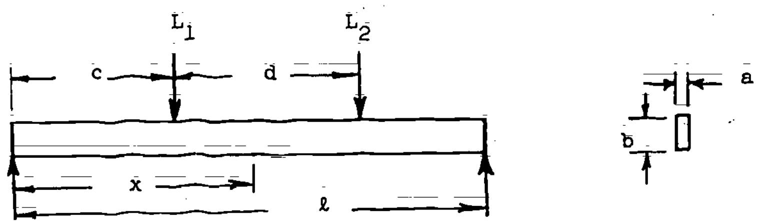
Sample Problem

To demonstrate how a Polya map is produced we will construct a map for the following problem:

Problem Statement: Determine the maximum bending stress in a simply-supported beam of rectangular cross-section, "a" by "b." There are two concentrated loads, L_1 located "c" from the left end, and L_2 located "d" to the right of L_1. The distance between supports is ℓ.

The first three steps in preparing a Polya map are carried out concurrently: "identify and list unknowns, identify and list data, and draw a diagram."

Diagram of Problem



Data Set (given)

a = width of beam

b = depth of beam
L_1 = load at c
L_2 = load at c + d
c = location of L_1 from left end
d = distance between L_1 and L_2
ℓ = distance between supports

Additional Data Set (intermediate)

This data set will be added to as problem develops.

x = location from left-hand end of beam
σ = outer fibre stress due to bending at x
x_m = location of maximum bending stress
R_1 = reaction at x = 0
R_2 = reaction at x = ℓ

Required Unknown (to be found)

σ_m = maximum bending stress

We now identify the relations that may be useful. Our problem is one in which we wish to find bending stresses in a beam. Therefore we can look for relations that give stresses due to bending and relations that give bending moments due to loads and geometry.

Data Set (additions)

M = Bending moment at x

I = Moment of inertia of section

Z = Section modulus

1. $\alpha = Mb/2I = M/Z$

2. $I = ab^3/12$

3. $Z = b/2I = ab^2/6$

4. $M = -R_1 x + \Sigma Lx$

4a. $M = -R_1 x$: $o<x<c$

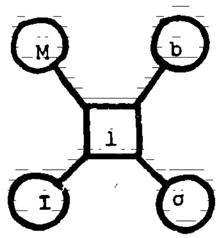
4b. $M = -R_1 x + L_1(x-c)$: $c<x<d$

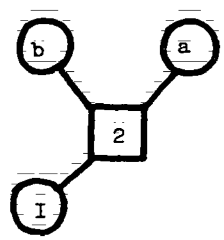
4c. $M = -R_1 x + L_1(x-c) + L_2(x-c-d)$: $d<x<\ell$

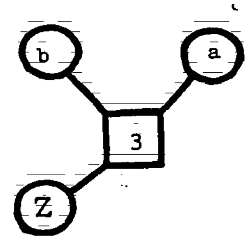
5. $R_2 = [L_1 c + L_2(c+d)]/\ell$

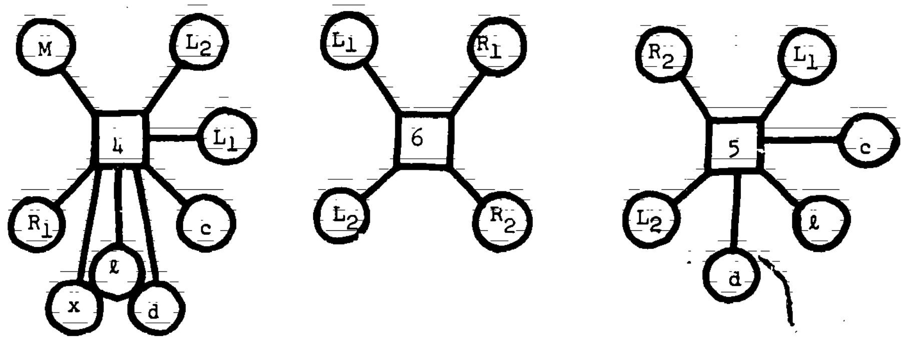
6. $R_1 = L_1 + L_2 - R_2$

We now have 5 independent relations and 6 unknowns. This will not give a single answer, but will give us a relation between two unknowns. Let us start to draw a Polya map, first mapping each relation independently.



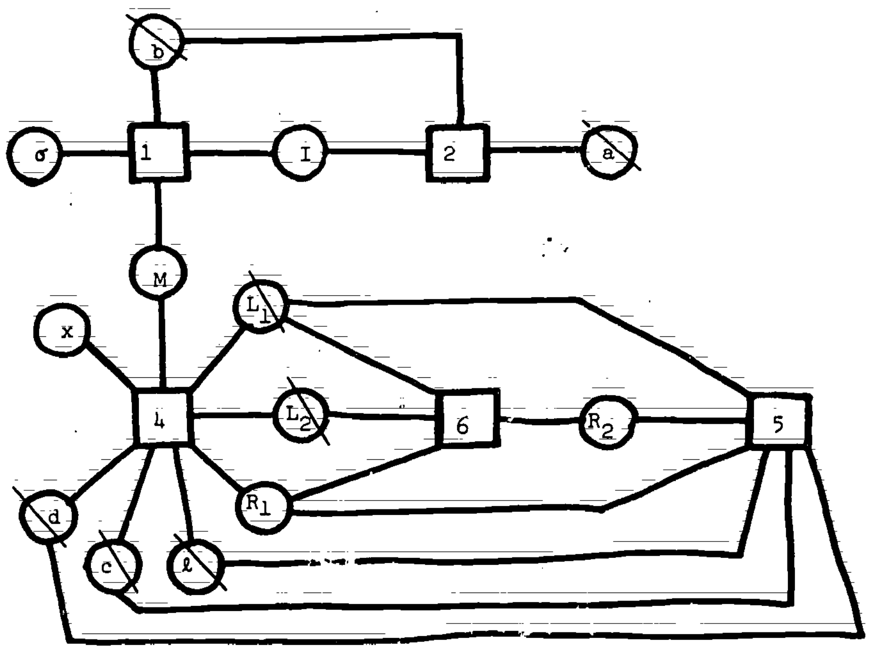






We note that some of the relations, e.g. 2 and 3, are alternate ways of arriving at the same result.

We can now combine these individual Polya maps to the map for the whole problem.



By drawing strokes through the given data in our problems we can see how the variables are related. The only variables which are not given or are not intermediate variables are x and σ. Therefore this Polya map gives us the value of σ with respect to x.

Examination of the Polya map shows that it has two parts linked together by variable M. The bottom half gives the moment distribution M with respect to x. The top half gives the stress σ with respect to the moment M. For any value of x we can determine M and thereby σ.

We note that R_1, R_2, M and I are intermediate variables and need not be determined specifically unless it is convenient for computation purposes.

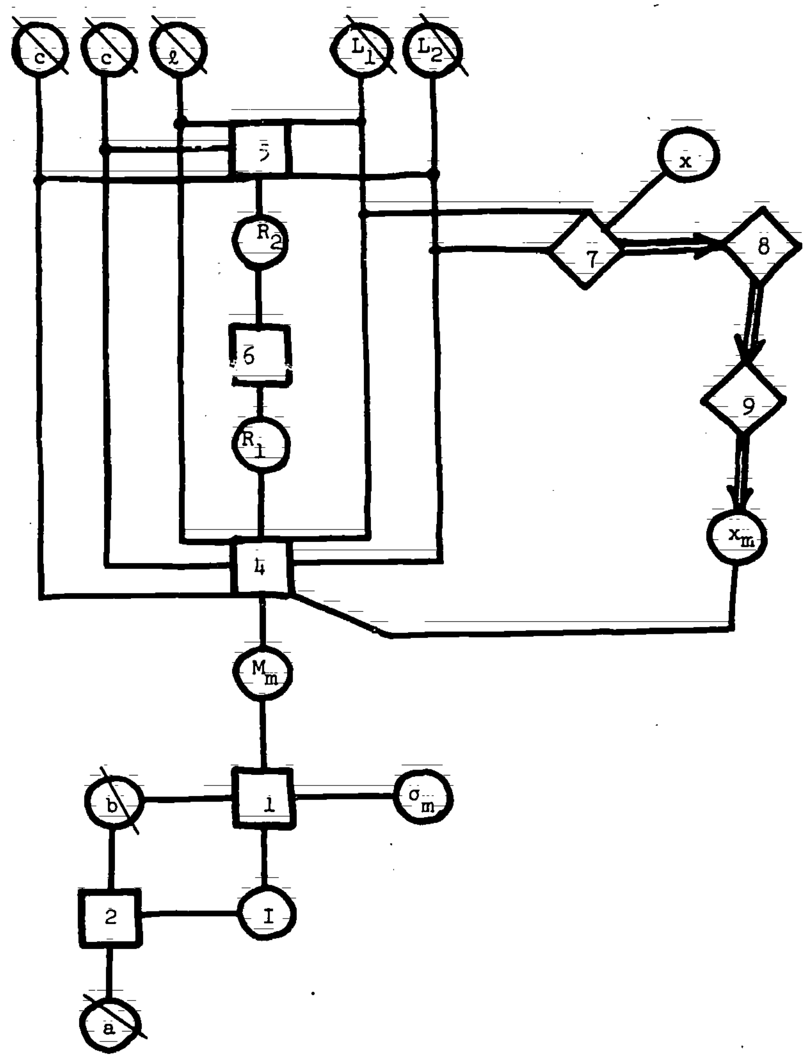
But our objective is not simply to find σ with respect to x, but also to determine the maximum stress $\sigma_{\bar{m}}$. We can define $x_{\bar{m}}$ as the point along the beam at which $\sigma_{\bar{m}}$ occurs. Thus we must introduce some way of determining where $x_{\bar{m}}$ and $\sigma_{\bar{m}}$ occur. Mathematical means are available, but they are cumbersome. A simple semigraphical method will be used.

Our diagram has shown that σ is dependent only on M. Therefore, if we can determine the location of the maximum value of M, we can move directly to $\sigma_{\bar{m}}$. Thus $M_{\bar{m}}$, the maximum bending moment, represents intermediate data that we have to introduce.

From experience we know that we can sketch a moment diagram and by inspection determine the location of the maximum bending moment. For this we introduce the following meta-operations

7	draw shear force diagram
8	draw bending moment diagram
9	by inspection of bending moment diagram determine x_m

Now we redraw the Polya map, introducing the meta-operations.



We have, thus, diagrammed our problem to give us σ_m, the maximum stress with the data set available.

A second example that is taken from a real design problem is given as Appendix B.

Problem Composition

We teachers usually have little difficulty in solving the determination problems that we set for our students. Why is that? In addition to our intelligence and good looks, maybe the fact that we compose problems is a contributing factor. This line of thinking leads to the idea that problem composing would be a good way to teach students about solving determination problems.

When faced with a problem the beginner often assumes that there are a very large number of possible approaches to solving it; whether he finds a good approach in his search is often a matter of luck. On the other hand, anyone who has composed problems knows that there are relatively few structures that make any sense; he discards most of the superficially-possible approaches on the basis of structural clues.

The standard format of the Polya map and its principal parts make possible the assignment of problem composition to students, and provides a medium of communication. The student-composed problems can be evaluated to see that all of the variables, algorithms, and meta-operations are being used in appropriate ways, that nothing has been omitted, and that the problem is solvable in principle. It would be virtually impossible for the teacher to evaluate all of the student problems without the Polya map as a common form of expression.

The Polya map and principal parts facilitate exercises in problem composing for both student and teacher.

Here are some examples of problem composition exercises that have been used.

1. For the problem given below,

    (a) identify and list the components of the principal parts of the problem
    (b) construct the Polya map of the problem
    (c) compose a variation in the given problem that has the same Polya map as the given problem (describe this variation in terms of its principal parts), and
    (d) write a stepwise solution procedure for your composition.

(These instructions are followed by a typical determination problem.)

2. (Exam) Nitric oxide is being reduced by carbon monoxide in a porous catalyst:

$$2NO + 2CO \rightarrow N_2 + 2CO_2$$

The reaction occurs at 600° K and 1 atmosphere. The density of the solid in the catalyst is 2.5 gm/cm^3. The specific volume in macropores is 0.04 cm^3/gm and the typical pore radius is 400 A. The specific volume in macropores is 0.11 cm3/gm and the typical pore radius is 35 Å. The specific surface area is 65 m^2/gm. The molecular collision diameter of NO is about 3.5 Å.

Compose and solve (numerical answer required) a problem involving the diffusion of NO and the pore structure of the catalyst. You may, of course, select additional equations and assume additional data so long as the set of data and equations is consistent and does not violate laws of physics or chemistry.

In Appendix A, there is an example of a completed problem-composition exercise that has been used.

Problem composition exercises may require the inclusion of specified elements in the composed problem. In this way, the teacher controls the content of the exercise in terms of concepts, algorithms and techniques. By specifying few or many elements the teacher can vary the exercise from the single-answer type to one that is open-ended.

Suggestions for Practice

The use of Polya Maps is new and we feel we have only touched the surface of the possible classroom use of Polya maps. The following suggests some uses for Polya maps; we are certain other uses can be found. We suggest that you try.

1. Problem-solving exercises for students that require the use of a Polya map.

2. Problem composition by the teacher where the objective is to avoid under- and over-specification of data.

3. Problem-composition exercises by the student.

4. Lectures in which the teacher is solving a sample problem. A Polya map may help to focus attention on problem solving and may help the student see the relation of one part of a problem to the next, or of one lecture to the next.

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Appendix A

A Problem Composition Exercise from Chemical Reaction Engineering

Compose a problem involving:

a) A reversible, first-order reaction
b) A fixed-bed reactor
c) Effectiveness factor
d. Production rate

Solution

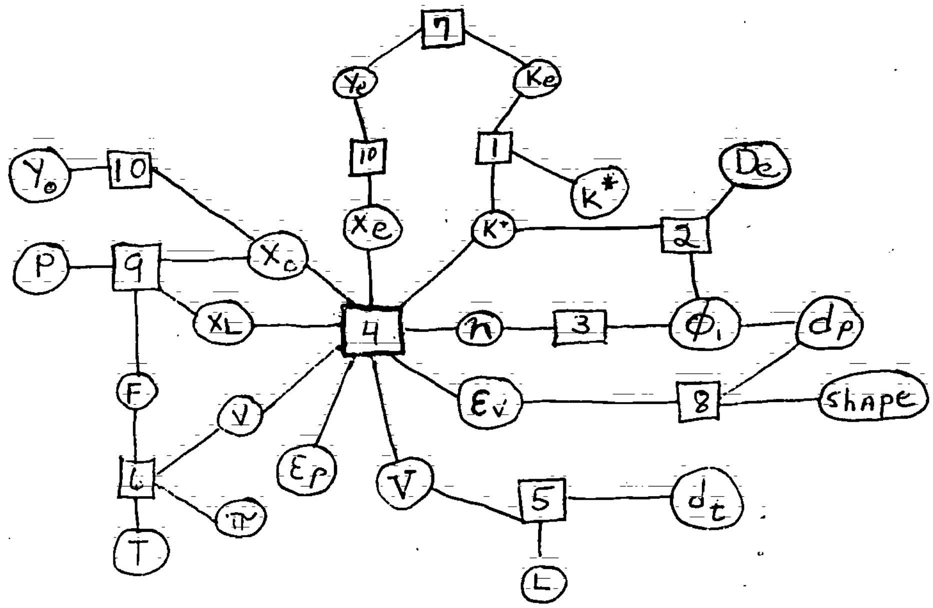
A. Principal parts:

Data: d_t, L, d_p, F, y_o, K_e, k^*, T, π, D_e, ρ_p, ϵ_p

Unknowns: x_L, p

Algorithm set:

1) $k^+ = k^* (K_e+1)/K_e$
2) $\phi = (d_p/2)(k^+/D_e)^{1/2}$
3) $\Omega = \eta = 3(\phi_1 \coth \phi_1 - 1)/\phi_1^2$
4) $\ln |(x_e-x_L)/(x_e-x_o)| = -\eta\epsilon_p(1-\epsilon_v)k^+ V/v$
5) $V = \pi d_t^2 L/4$
6) $v = FR_g T/\pi$
7) $y_e = 1/(K_e+1)$
8) $\epsilon_v = \mathrm{fcn}(d_p/d_t, \mathrm{shape})$ (Table look-up)
9) $P = F(x_L-x_o)$
10) $x = 1-y$



C. Outline of solution:

1. Find k^+ from k^* and K_e by 1
2. Find ϕ from d_p, k^+, D_e by 2
3. Find n from ϕ by 3
4. Find y_e from K_e by 7
5. Find ε_V from d_p, d_t, shape by 8
6. Find V from L, d_t by 5
7. Find v from F, π, T by 6
8. Find x_o from y_o by 10
9. Find x_e from y_e by 10
10. Find x_L from x_o, x_e, k^+, n, ε_V, ε_p, V, v by 4
11. Find P from F, s_L, x_o by 9

Appendix B

Problem Statement: Determine the time required for an induction motor driving a fan to come up to speed from rest, given the moment of inertia of the motor, the fan, the rated horsepower and speed of the motor, the rated horsepower and speed of the fan. The torque-speed characteristics of the motor running free are defined by the starting torque, the maximum torque and the synchronous speed. The power drawn by the fan in moving air is a power-law function of the speed.

(Note: This problem statement is drawn from the natural context described in ECL 171 "Time-To-Start," ASEE Engineering Case Library)

What is required is the time for the motor-fan combination to come up to speed. The motor torque-speed curve can be approximated from motor ratings. The power drawn by the fan will be some function of the speed. These two need not be explicitly expressed mathematically, but they can be given graphically or as functionals.

Data

H_m = motor rate H.P.	a = system angular acceleration
S_m = motor rated speed	T_m = motor driving torque
S_s = motor synchronous speed	T_f = torque drawn by fan
T_s = motor starting torque %	I_m = moment of inertia of motor
T_b = motor breakdown torque%	I_f = moment of inertia of fan
H_f = fan rated H.P.	T = accelerating torque
S_f = fan rated speed	t = elapsed time
w = system angular velocity	t_s = time to start

Relations

1. $T_m = f(H_m, S_m, S_s, T_s, T_b, w)$
2. $T_f = f(w, H_f, S_f)$
3. $T_t = (I_m + I_t)a$
4. $dw = adt$
5. $T_t = T_m - T_f$

Relations

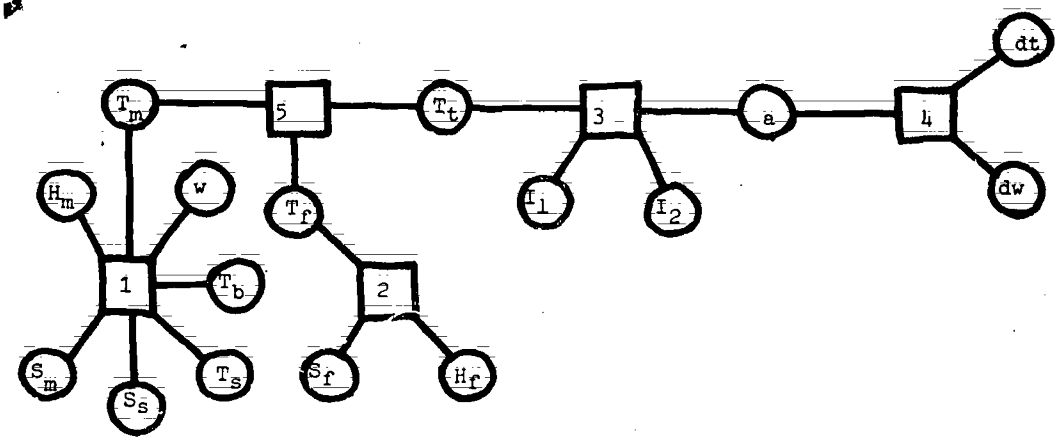
1. $T_m = f(H_m, S_m, S_s, T_s, T_b, w)$

2. $T_f = f(w, H_f, S_f)$

3. $T_t = (I_m + I_t)a$

4. $dw = adt$

5. $T_t = T_m - T_f$



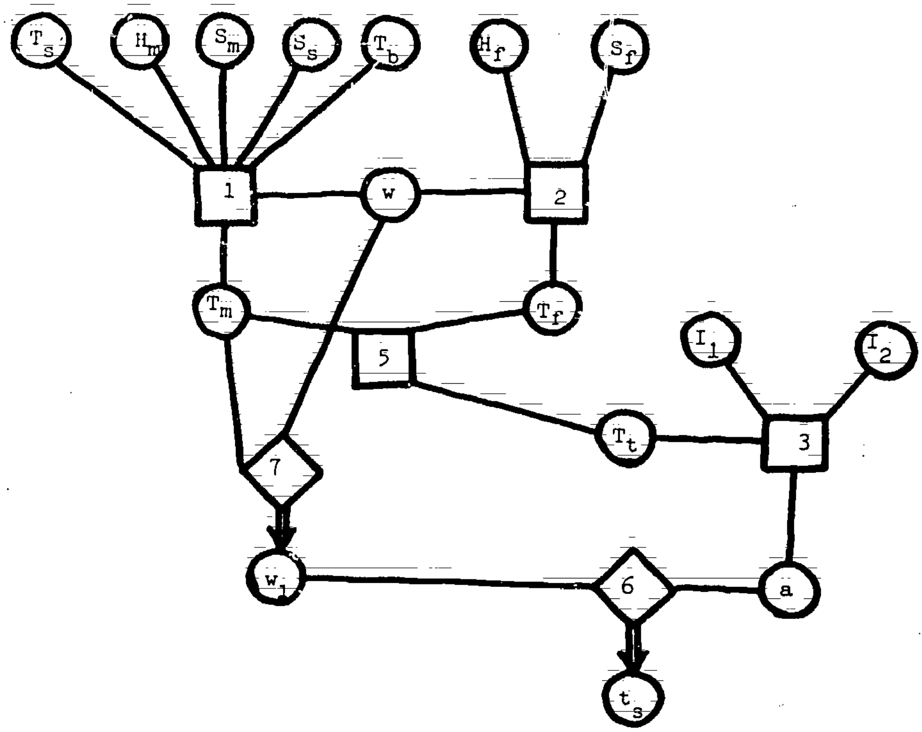
By inspection of 4, we see that we can express angular acceleration "a" as a function of w.

To find the total time to start we must integrate 4 [illegible] suitable limits, and must introduce the meta-operation:

⟨6⟩ $t_s = \int_0^{w_1} \frac{1}{a}\, dw$

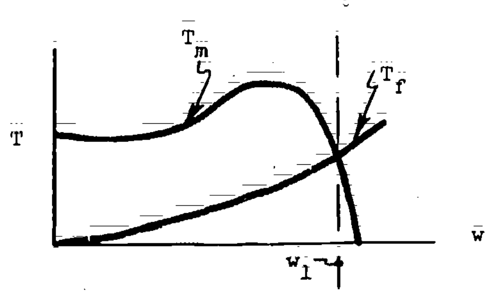
where w_1 is new data which is the final angular velocity of the system. This final velocity will be reached when $T_t = 0$. If the rated horse-power of the motor and fan are the same, then $w_1 = S_m = S_f$. But in the more general case where $S_m \neq S_f$, then $T_t = 0$ when $T_m - T_f = 0$. To determine w_1 we must introduce another meta-operation.

⟨7⟩ Plot T_m and T_f against w and find w_1 at the point where $T_m = T_f$.

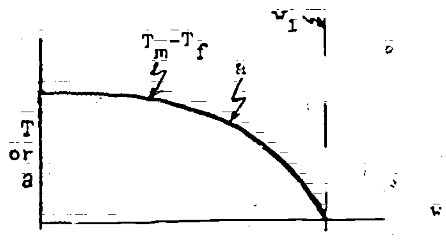


Operation 4 is not needed and can be dropped from our Polya map.

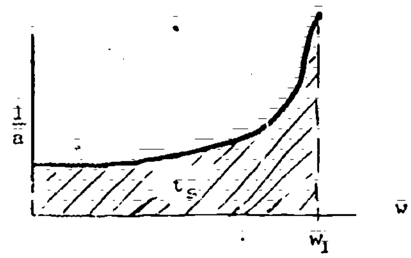
Inspection shows that the angular velocity w is the key to the problem solution. If we plot all our characteristics against w, we should be able to solve the problem.



Plot motor torque and fan torque against w.



Plot $T_m - T_f$ against w.
Change scale to give a.



Plot 1/a against w and
measure area under the curve
up to $w = w_1$ to find t_s.

Solution can be found semi-graphically if motor torque and fan torque characteristics are specified.

6

SOLVING PHYSICS PROBLEMS

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ABSTRACT

A detailed strategy for solving general physics problems is discussed. Starting from general problem-solving techniques, a rationale is developed that enables students who have mastered simple algebra to improve greatly their problem-solving abilities.

INTRODUCTION

Most teachers of general physics will agree with the objective of teaching their students to think more analytically. But there is widespread uncertainty and disagreement as to how this can be done, or even if it can be done. As a consequence, courses have been developed which describe physics with little or no mathematics. Such courses can greatly expand the student's awareness of various physical relationships, but only through experimentation or memorization -- not deduction. It is argued that the students for whom such courses are developed are either unable or unwilling to develop analytical problem-solving skills.

For the purpose of this paper, let us define analytical problem-solving skills as the skills needed to permit deductive solutions of problems which are intrinsically new to the student. Far fewer physics teachers accept development of this ability as an objective of their course, usually because their past efforts to teach this ability have been so markedly unsuccessful. Indeed my own teaching experience demonstrated that result. For seven years, I struggled with various modifications of my lectures, the text, the homework, and the tests, to try to teach students better analytical problem-solvng skills. The results were never very encouraging. There was always a small percentage of "A" students who "had it" when they entered my course, and I could see their proficiency grow. But most students remained outside of this group. One obvious solution is to limit the objective, either by going the non-mathematical route or by restricting tests to questions similar to those which the students have been shown how to do. The latter solution is the more popular one for students of science and engineering who will be expected to "use" the physics they are learning. The non-mathematical approach is more common for the remainder of the student population.

In recent years, there has been growing evidence that it is not necessary to give up the goal; analytical problem-solving skills can be taught. Indeed a large part of our difficulty in teaching them in the past has been our tacit assumption that they do not need to be taught; i.e., that any student who has the ability to learn analytical problem-solving skills will pick them up from seeing problems worked in the text and by the teacher. That assumption is clearly incorrect. Although there is that small group of "A" students who "get it" no matter what we do, most students must be taught explicitly.

How then does one go about teaching analytical problem-solving skills? The answer is disarmingly simple: break up this complex set of skills into pieces, i.e. concepts small enough for the student to handle. Then give the student the encouragement, the time and the exercise which is needed to master them. This is not unlike our usual approach to teaching any complex skill! It has not often been used to teach analytical problem-solving skills but only because of our historical inability to see them as the complex set of skills which they are -- either because they have become subconscious for us, or because we avoid them.

The remainder of this paper will be devoted to a discussion of one problem-solving strategy for general physics problems. It assumes that the student understands fundamental algebra; e.g. that he sees $x = a\,y$ and $y = x/a$ as intrinsically the same relationship. Work by Bauman[1] and others[2,3] has shown that this is not a fundamental limitation; these concepts too can be taught, but I have not yet had the time and other resources necessary to do so

General theories of problem-solving have been discussed by Polya[4] and others[5]. Although there are many elements in common among these discussions, they are usually different in detail and in the names associated with the various facets of the problem-solving task. The strategy reported here, although less general than those cited, has proven useful in my teaching in the sense that a significantly larger percentage of my students have demonstrated development of analytical problem-solving skills. An additional pedagogical benefit is its separation of the math from the physics. As divided, each of its three major sections is of roughly equal difficulty when averaged over the student population.

A PROBLEM-SOLVING STRATEGY

The strategy has three major divisions. The overall objective of part I, SETUP, is to convey the nature of the problem, including all given information and desired quantities, with a minimum of English, using meaningful symbols and pictures that can be used in part II. The objective in part II, ANALYSIS -- PHYSICS, is to find sufficient independent equations so that they may be solved for the desired unknowns. In part III, ANALYSIS -- MATH, mathematical answers are obtained for the desired unknowns, and checked for reasonableness.

Part of the reason for the diversity of labels given to the subdivisions of any problem-solving strategy is the fact that no set of subdivisions is universal. There are situations where one finds in part III a need to return to part II or even part I. So one should view the whole process as cyclical, continuing until satisfactory answers are obtained. Furthermore, there is nothing sacred about the order of events in the strategy. If one can see the answer without explicit consideration of parts I or II, fine. If part II seems easier than drawing the picture for a particular problem, then part II should be done first! Indeed the strategy is not to be viewed as a straightjacket approach to problem-solving, but rather as a guide to what to think about next when one is stumped a particular problem. On the other hand, beginning students should be encouraged to practice the strategy on problems which seem easy to them so that they will become familiar enough with its rationale to use it on more difficult problems. What follows is essentially addressed to the student, interspersed with discussion for the benefit of the reader.

I SETUP

The first and most obvious task in problem-solving is to decide what it is you are trying to find and what information you have to work with. It is awfully hard to get right answers if you are doing the wrong problem!

If a problem has been properly set up it will contain a minimum of English -- often none at all, but at most a few words. But you, or others with similar training, will be able to return to it months later and figure out what the problem was all about without referring to the original English statement of the problem. Part I can be thought of as a translation task from English to pictures and symbols.

A. LIST GIVEN QUANTITIES with appropriate, meaningful symbols.

It is useful to be able to reject irrelevant information by simply not listing it, but if you are not sure, list it.

Why assign a symbol to something which is already known as a number? There are several reasons.

1. It is usually easier, e.g., it is easier to write m than 4 kg.

2. It is often more meaningful. If the symbol KE is recognized as kinetic energy, that is much more helpful than 20 J, which could be any energy.

3. It helps avoid the trauma of not knowing a number to insert for an unknown quantity which may cancel out later in the problem.

When more than one quantity of the same type is involved, such as two or more moving masses, or velocity at different times, it is often useful to assign an appropriate symbol to one of them and list the others as multiples of the quantity given a symbol. For example, in a problem involving masses of 2 kg, 6kg and 8kg, label them (in a sketch) as M, 3M and 4M respectively, and list M = 2 kg.

In making up an appropriate symbol for certain unknowns it may be appropriate to combine several more common symbols. Thus in a problem which talks about 4 bullets per second leaving a gun, that information could be listed as N/t = 4/s.

B. ASSIGN reasonable SYMBOLS to the requested UNKNOWNS.

Again, it may take more than one common symbol to make up a symbol for a desired quantity.

At this point it is wise to go back over your lists to make sure that the dimensions which you would associate with each symbol are consistent with the units which were given or desired. Thus if you were asked to find the rate at which liquid leaves a system in gallons per minute, an appropriate symbol would [illegible] but certainly not V alone.

C. SKETCH A PICTURE for each unique time implied in the problem, and label with the symbols from parts A and B.

This is one of the most difficult yet crucial skills in the entire problem-solving strategy. The trick is to draw a sketch complete enough to show the important ideas yet simple enough to aid rather than confuse the analysis. Thus one must learn that (if rotation or rotational equilibrium is not implied in the problem) it is more appropriate to draw a point or a small box rather than the man or car or whatever is named in the problem.

The primary goals of the sketch(es) are to show all important spatial relationships and help define all symbols listed in parts A and B. Thus if two or more times are implied in the problem, a sketch for each time must be drawn in such a way that distances moved between sequential times can be clearly indicated. It is often convenient to show a "final time" sketch superimposed upon an "initial time" sketch, using dashed lines for one or the other.

Before leaving part I, you should reread the problem statement to make sure that every relevant phrase is indicated in your setup. Think in terms of the overall goal of this section: could someone else with your training, who had not seen the English version of the problem, decipher it from your setup?

II ANALYSIS -- PHYSICS

The objective here is to find sufficient independent equations to solve for the desired unknowns. Suppose there are N unknowns which the problem asks you to find. You may need to introduce an additional number U of unknowns while writing physical [illegible]ons. The mathematicians tell us that you will need a[illegible] and at most N + U independent equations in order to [illegible] for the desired unknowns. To put it another way, if there are K unknowns which CAN be uniquely determined from the given information, it will take K independent equations to do so. There is admittedly some uncertainty involved in the preceding statements; one cannot usually predict at the outset how many equations will be necessary. Thus an alternative approach is to find as many independent relations between the various symbols as you can! On the other hand, it is useful to know the minimum and maximum number of equations needed.

In general, when writing equations from part II, they must be expressed in terms of the symbols listed in part I. The beginning student may find it useful to first write equations in the form in which he remembers them. This should be done in the margin or in a special box, since those equations do not count toward the ones needed for part II. Only equations which have been translated into the specific symbolism of part I count in part II. This will often require adding one or more symbols to the sketches in part I. These symbols, for unknowns which were not required in the problem, should be listed near (but distinct from) the list from part I B.

A. Consider CONSTRAINTS.

These are spe[illegible]ific conditions set forth in the statement of the problem [illegible] deductible from the sketch(es), which sometimes can be w[illegible]tten as equation[illegible] involving some of the listed symb[illegible]. [illegible]onstraints are often considered automatically in part I. For [illegible] object on an incline, motion will be parallel to the [illegible]line. Similarly, two objects tied together by an ideal st[illegible] over a pulley will have an acceleration of the same magn[illegible] But some problems have more subtle constraints. Inc[illegible]tting down the proper constraint relation CAN be the most difficult part of the problem. It is mentioned here at the beginning of Part II because if a simple constraints is present and not recognized at this point in the problem, it can seriously hinder Part II.

In general physics problems, constraints are usually geometric in nature. They also frequently involve approximations, such as:

-- strings, ropes, etc. are assumed not to stretch.

-- the arc subtended by a small angle is essentially a straight line.

-- solid objects are assumed not to deform under the action of forces.

Figure 1 gives examples of constraint equations.

B. Consider DEFINITIONS of knowns and unknowns.

If a definition of any one of the symbols in your lists from part I relates two or more of the quantities in those lists, it will probably be useful -- write it down. Remember to add any new unknowns introduced by these definitions to your sketch and second list in part I B. Anytime a new variable is introduced, you should consider ITS definition.

C. Consider CONSERVATION LAWS

Since conservation laws imply the existence of a time interval during which the distribution of some conserved quantity has changed, it follows that you must have at least two sketches (possibly superimposed) if conservation laws are to be applicable. If so:

1. CHOOSE a clearly defined SYSTEM; often the whole system implied in the problem.

2. CHOOSE an appropriate TIME INTERVAL over which the conservation law is valid. You should have sketches for both ends of that time interval.

3. CHOOSE a convenient COORDINATE SYSTEM for each moving object.

4. WRITE the conservation LAW:

The total in the system at the beginning of the time interval + any entering the system during the time interval

= the total in the system at the end of the time interval + any leaving the system during the time interval.

D. Consider KINEMATICS equations.

For people with a good calculus background, these are nothing more than definitions. But for most students they need to be considered separately as known relations between displacement, velocity, acceleration and time.

E. Consider NEWTON'S SECOND LAW.

1. Isolate an object so that its mass and its vector acceleration are clearly definable.

2. Draw a FREE-BODY DIAGRAM for that object:

a). Count the number (i) of objects which exert forces directly ON the chosen object. Do NOT count forces internal to your object or which it exerts on other objects. Do not count indirect forces. Normally i will equal the number of things which actually touch the chosen object, plus one for the gravitational force (if any), plus one for the electromagnetic force, if present.

b). Draw the i forces on a free-body diagram. Do not draw components of forces, except in the case of contact forces between surfaces, where it is usually best to draw the component normal to the surface, N, and the component parallel to the surface, f.

3. CHOOSE A COORDINATE SYSTEM with the positive x axis along the direction of the acceleration of the object, if known.

4. WRITE NEWTON'S SECOND LAW in scalar form for each significant axis.

5. If there are other moving masses in the problem which affect the answers, you will need to repeat E for each of those masses.

F. Consider OTHER physics relations.

There is little to be said here without going through an entire physics course. The student who has been studying properly usually has little or no difficulty in realizing which specialized relations apply in a given problem.

III ANALYSIS -- MATH

At this point we should have a set of independent equations involving only symbols listed in part I. Obviously the symbol for every requested unknown must be included at least once; if not, go back to part II.

A. Obtain an ALGEBRAIC SOLUTION.

Solve algebraically for each desired quantity in terms of the symbols for the given quantities (or previously "found" unknowns). Use numbers only in those relatively rare situations where it is definitely easier to deal with the numbers than the symbols.

B. TEST the algebraic solution for REASONABLENESS:

1. Check for correct physical dimensions. Dimensions must obey the ordinary rules of algebra. For example, terms which add or subtract or equate must have identical dimensions; if they do not, the equation cannot be correct. Thus an equation which asks you to add a mass and a length is obviously in error. In that case, check dimensions in each equation from part II and then redo part III.

2. Check for the expected behavior in intuitively-obvious situations. Do the relations seem plausible? For example, in the result $x = a\ b/\cos\theta$:

-- is is reasonable that "x" is proportional to "a" to "b"?

-- should $x = a\ b$ when $\theta = 0$?

-- should x approach infinity as θ approaches 90°?

C. PLUG IN NUMBERS in a one-to-one relation with your algebraic solution. If some of the given information is not in a consistent set of units, check to see whether those quantities occur in two or more positions such that the conversion factors would cancel out. If not, you must INSERT CONVERSION FACTORS until the units left are consistent. For example, in the equation $\theta = s/r$, which is dimensionally correct, you cannot use "s" in inches and "r" in feet unless you supply the appropriate conversion factor. Thus, if s = 3" and r = 2', θ = (3"/2') (1'/12") = 1/8 (radian).

Do the arithmetic to one significant figure in your head or on scratch paper. Then do it more carefully, cancelling as much as possible and using other math shortcuts such as calculators. If there is not reasonable agreement between your two answers, check them both again.

D. CHECK your answer for physical reasonableness and appropriate units.

Having read this much about one problem-solving strategy, it should no longer be any surprise to find that most students do not pick up a good problem-solving strategy by simply seeing a few problems done by the text and/or teacher, especially when these are not fitted into an overall plan.

In summary, we have discussed one useful problem-solving strategy having the following overall structure:

I. Set up a concise statement of the problem in terms of labeled sketches and algebraic symbols.

II. Do the physics; i.e., write relations until there are sufficient independent equations (in terms of the symbols from part I) to solve for the desired unknowns.

III. Do the math--first the algebra, then the conversion factors and arithmetic, and finally check for reasonableness.

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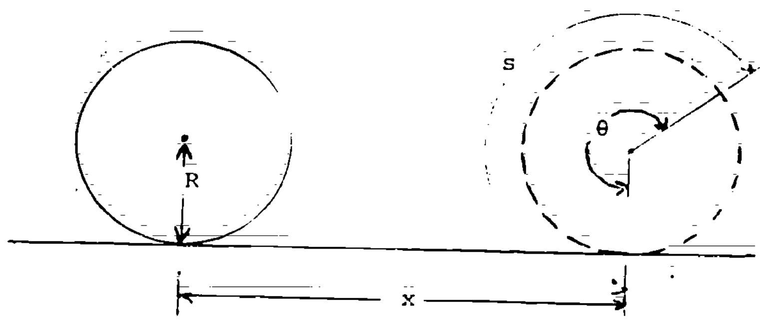
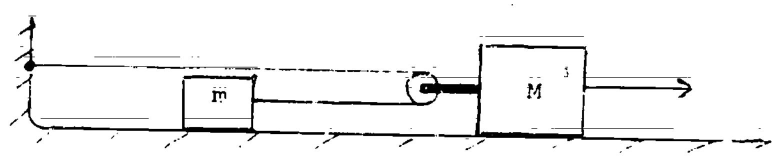


If the wheel rolls without slipping, then s = x.



If M moves a distance X while m moves a distance x, then x = 2 X

FIGURE 1

Examples of Constraint Equations

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ENGINEERING STUDENT PROBLEM SOLVING

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The engineer is a problem solver.

Certainly, if you were to look in on the majority of engineering classrooms in the country, you would find both students and teachers concerned with problem solutions, which are the products of problem solving. How then do engineering educators teach problem solving skills to their students? Do they, in fact, make a special effort to teach such skills?

It is at this point that I would like to emphasize the difference between the product of problem solving, i.e. the answer or solution, and the process, or method of attack. The answer or solution to the problem is readily observed and quantified and is easy to deal with. The engineering student's homework solutions can be graded, and marked right or wrong if the answer alone is considered. Emphasis is placed on accuracy, where method may really be equally important. In the "real world", a variety of answers may satisfy the problem conditions--indeed, two engineers may look at a problem, and develop two completely different solutions to what they have seen as two completely different problems.

Although it is possible to infer the process or method of attack used in solving a problem from the product obtained, this may be misleading. For example, if a student gets the wrong answer to a problem, can we ascertain the reason? Do we know whether the student has used the wrong formula, made an error in arithmetic, omitted an important variable, or completely misinterpreted the nature of the problem he was asked to solve?

One way to ascertain the process of problem solving is to ask the problem solver to think aloud as he solves the problem. This solution, recorded as it is developed, may not be totally complete, but it is more likely to reflect the actual process used than if a later, retrospective account is obtained, which is likely to be cleaned up, edited, and made more coherent. The protocols obtained by having students think aloud as they solve problems offer a close approximation to the problem solver's method of attack on problems. (1)

In the current literature, there have been reports of problem solving procedures, as well as descriptions or remedial efforts made to teach students more effective methods of solving problems These paths and precepts are developed logically, and have been shown to be helpful, but analysis of the protocols of students or of experts thinking aloud do not conform to these step-by-step rational, logical patterns.

For example, Polya's How to Solve It (7) outlines a procedure for solving problems.

> "First you have to understand the problem... Second, find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of solution... Third, carry out your plan... Fourth, examine the solution obtained."

In this analysis, much depends on luck and good guessing.

Rubenstein (9) described general precepts of problem solving as:

- Get the total picture
- Withhold your judgment
- Use models
- Change representation
- Ask the right questions
- Have a will to doubt

He further suggests paths to generating a solution:

- Work backwards
- Generalize or specialize
- Explore directions when they appear plausible
- Use stable, substructures in the solution process (modules)
- Use analogies and metaphors
- Be guided by emotional signs of success

Leibold, et al (5) describe an adaptation of Polya's approach used to teach problem solving to freshman engineering students in which they divided the definition of the problem steps into "Define" and "Think About It". Their breakdown of the "Define" step includes:

- Define the unknown
- Define the system
- List knowns, concepts, and choose symbols

- Define the constraints
- Define the criteria

However, the student group reports difficulty in using these skills learned in the special class in solving their regular homework assignments.

As part of the same program, Woods et al (12) have attempted to observe the problem solving training to which engineering students were exposed, to identify major difficulties the students were having in solving problems, to identify necessary problem solving skills and to teach these to the students. They identified a set of steps combining creative and analytical thinking, and used these as a basis for developing and teaching a strategy of problem solving to the students in a tutorial program. The strategy is outlined below. (12)

DEFINE	Identify the actual problem
THINK ABOUT IT	What are the attributes?
	Identify area of knowledge
	Collect information
	Flowchart solution
PLAN	Think up alternative plans
	Translate
CARRY OUT PLAN	Solve
LOOK BACK	Check reasonableness & math
	Check criteria & constraints
	Study related problems
	Identify applications in engineering, everyday behavior & deserted island
	Identify & memorize order-of-magnitude numbers
	Develop successive approximation strategies
	Study problem-solving skills learned
	Communicate results

The strategy is devised rationally. The procedure for improving student skills requires the students to focus on a particular step, simultaneously, and then to discuss each other's ideas. The problem definition step has had greatest emphasis thus far. This method, although developed on the basis of a rational analysis of problem solving, does place emphasis on the actual process. An interesting observation made by the professors assigned to sit in on all the required courses with the students was the discovery that the lecturers actually presented a large number of hints for solving problems as well as numerous example solutions, yet the students did not capitalize on these hints or examples. They failed to note them in class (presumably because they were verbal and not written on the board).

Stonewater (10) points out that, although, engineering instructors are well able to specify the problem solving processes they use, they may have difficulty specifying the processes a student should use, since through experience and practice the instructors have internalized so much of what they do. He points out that step-by-step procedures instructors present to students as the blackboard problem solutions may not be ordered in the way that the problem was solved by the instructor initially, but rather may be edited to be more elegant. Again, the strategy used for problem solution may not be identified.

Stonewater developed a course, "Introduction to Reasoning and Problem Solving", using a task analysis procedure to develop strategies for problem solving. In the eight module course, three modules were devoted to the preparation phase:

1. Preparing for Problem Solving - discriminating between relevant and irrelevant information, specifying solution derived, visualizing problem

2. Drawing Diagrams

3. Organizing Data Tables

and five to strategies for solving problems

1. Sub problem strategy - identify unknowns and sequence the order in which they must be solved

2. Sub problem strategy - develop an organized method to solve the problem

3. Contradiction strategy - state an assumption which is the logical negation of what is to be proved and use this to contradict given information

4. Inference - infer additional information from what is given

5. Working backwards - start at solution, rather than with givens.

Stonewater reports three areas of concern in teaching problem solving. These are finding techniques to help students improve their organizational ability; improve their abstract reasoning skills; and improve transfer of learning to other courses.

Stonewater's instructional applications include use of a diagnostic test to determine which students have mastered a particular strategy; design of self-paced materials to implement learning and pairing students to study particular materials.

The latter method of pairing students to teach problem solving skills has been described and used successfully by Whimbey (11) as adapted from a method described by Bloom and Broder (1). Whimbey developed a program where students, working on a one-to-one basis with other students think out loud as they solve problems to try to determine what is hindering their success as problem solvers. The students work a series of exercises designed to increase their ability to read and understand technical and scientific writing as well as to solve mathematically based reading problems. Initially, students contrast their methods of solving problems with the methods used on the same problems by good problem solvers. Characteristics in which good problem solvers differ from poor problem solvers included:

- Motivation and attitude toward problem solving
- Concern for accuracy
- Breaking problems into parts
- Amount of guessing
- Activeness in problem solution

Researchers at the University of Massachusetts Heuristics Laboratory and Department of Physics and Astronomy and School of Engineering have been cooperating in investigating a variety of instructional techniques for developing students' cognitive skills, and for teaching analytical reasoning (2,3)(6). Asking students to think aloud, Lochhead and Clement study individual student's cognitive processes to determine what learning strategies they employ, the basic concepts from which they operate, and the techniques they use in problem solving. They use this information to make the students better problem solvers. Lochhead (6) reports:

> "We find that what students usually learn from a physics course is not at all what we believed. For example, a few months ago I gave a student a ride into the University. He asked me what I did and I told him I taught problem solving to introductory physics students. He replied that he had taken a physics course the previous year (Physics for biologists): it had been OK but of no lasting value. He had tried to understand the material but that took too much time and wasn't any use as far as the grade was concerned. So after a couple of weeks he settled in on memorizing formulas

and found that the homework and exam problems could always be solved by plugging the given variables into whatever equation happened to involve those variables. He got some practice in algebra and also in trigonometry but the physics he learned was just rote formulas which less than a year later he had completely forgotten. This approach to learning and problem solving is an example of a syndrome we call 'formula-fixation'."

"The student is neither allegorical nor unusual. He is perhaps more perceptive than many of his classmates, but by no means unique. Last winter Robert Gray, who is directing the experimental physics course, was visited by an angry student who had obtained a B+ in introductory physics. The problem was that she had understood none of it. The purpose of her visit was to set up an independent study course for the January term in which she could try to understand the material she had mastered..."

"There is a popular myth that students cannot understand physics because they are weak in mathematics. The above examples show that the inverse is often the case, namely; an ability to do mathematics makes understanding the physics unnecessary. But students are not the only people skilled at the use of algebra to avoid thinking. We all do it most of the time; and we regularly continue the practice when we teach. With rare exception textbooks and teachers emphasize the mathematical manipulations and spend little effort on explaining the physical concepts or on explaining why the mathematics is an appropriate representation of those concepts."

Clement (3) comments:

"It is easy for a student to memorize a law and to recite it faithfully when given the appropriate prompt. With a little practice the student may also be able to apply the law in a limited class of problems such as those typically found on tests. However, these abilities in no way imply the un-learning of contradictory intuitions. In fact, a careful investigation of how students solve problems shows that in most cases they are operating with an inconsistent system--a collage of newly learned principles and old intuitive concepts. The old intuitive concepts are remarkably resistant to change and this presents a difficult challenge for teachers."

"One area of physics where these intuitions are particularly strong is Newton's first law. The first law states that a body in motion will remain in motion unless acted on by a force. It is a strange law because it directly contradicts our own perception. Our everyday experience shows that bodies in motion come to rest without the application of a visable force. Furthermore to keep a body in motion requires the application of a force. Thus learning Newton's first law implies the unlearning of certain intuitive concepts."

These points are illustrated with student protocols, generated as the students think aloud. Research and experiences with teaching analytical reasoning in programs fashioned after those of Arthur Whimbey (11) focus on five aspects of analytical reasoning,

- critical thinking applied to understanding complex instructions
- analyzing of errors in reasoning
- solving word problems
- analyzing trends and patterns
- using analogies in formal contexts

Based on this, faculty members at the University of Massachusetts have proposed the development of an Analytical Skills Center, headquartered in the Department of Rhetoric, which will attempt to diagnose the causes of students' weaknesses in analytical reasoning ability, and provide instructional programs that teach these skills. The Project will be a joint effort of people from the departments in engineering, physics, mathematics and rhetoric.

Larkin and Reif (4) and others at the University of California Department of Physics and Group in Science and Mathematics Education have contrasted the method of solution of experts (professor of physics) as they solve physics problems thinking aloud, with the method of solution of a novice (a student who had completed one quarter of physics) as he solved problems the same way. The record of the novice shows a direct approach, that of simply applying various physical principles to the problem in order to produce equations. The equations are then combined to produce the desired quantitative solution.

In contrast, the experts do not jump directly into a quantitative solution but rather seem first to redescribe the problem in qualitative terms. The qualitative description is close to the quantitative equations which ultimately complete the solution. The qualitative analysis seemingly reduces the chances for error since it is easily checked against the original problem, and it outlines an easily remembered description of the global features of the original problem.

In addition to the interpolation of the qualitative description, novice and expert seem to differ in the way they store physical principles in their memory. The novice seems to store such principles individually while the expert groups principles which are connected, and stores them as "chunks". Thus, seemingly, when the expert accesses one principle from memory, the other associated principles become available.

Larkin (4) applied this research to her teaching of a calculus based physics course. Ten students were trained to apply individually seven physical principles needed to solve a DC circuit problem. After this, the students worked three problems, which could be solved by systematically applying the learned principles. Then five of the students were given additional training (one hour) in qualitative analysis and chunking. Then all students were given three additional problems, working individually, with the instructor, and thinking aloud as they solved the problems. In the experimental group three students solved all three problems, and two solved two. In the other group four of the five students solved at most one problem. Larkin stresses that "if one is serious about trying to enable students to solve problems in physics more effectively, the following procedures seem promising. (1) Observe in detail what experts do in solving problems. (2) Abstract from these observations the processes which seem most helpful. (3) Teach these processes explicitly to students."

Now, based on this description of work going on in the field, what is it that you as an engineering educator can do to enhance the problem solving capabilities of your engineering students?

Well, you can continue to do as you have done in the past, assigning problems, and either correcting them or providing the correct solution, assuming that, by this method, your students are learning to be problem solvers. And indeed, many of them do learn engineering problem solving skills in this way.

Or, you can present and discuss a logical strategy for problem solving, similar to that described earlier being carried on at McMaster University (5,12) with special guidance on problem solving aspects of homework assignments carried on independently of the classroom presentation. In the classroom, you can present hints for solving problems, and offer solutions to example problems, remembering to call attention to these hints or examples, or write them on the board, so that the students will attend to them.

The problem solving course developed by Stonewater (10), at Michigan State, offers a different technique, a separate mastery learning self-paced model course which teaches students such things as how to prepare for problem solving, to draw diagrams, develop data tables, and use sub-problem strategies. The special course developed by Woods, et al (12) at McMaster also uses a rational approach to the teaching of problem solving skills.

In my opinion, one of the least expensive and least disruptive things you can do is to reduce emphasis on the products of problem solving, on always getting the "right answer", and stress the process of problem solving as you present material in the classroom. Let the students watch and listen as you, the expert, tackle a problem. Let students in on the way you discriminate between the relevant and irrevelant information in your problem solving process; the way in which you translate the given problem description into a more workable form, how you redefine the problem into terms such as force, moment, etc., for which you can develop equations, or how you draw diagrams or data tables or graphs to help in this process. Call the student's attention to the process by which you break a problem into more manageable sub parts. Permit the students to listen in on your process of developing a plan, even where it leads to false starts, and to learn how it was you recognized that you had chosen the wrong path, and how you check for consistency. In other words, let the students see the scratch paper you have discarded, rather than just your elegant final solution! In elementary courses, try, as Lochhead and Clement have done, to make sure students understand the relationships in equations and are not merely plugging in numbers.

According to Reif (8) and Larkin (4), it is helpful to students if they can organize their knowledge base qualitatively, using verbal descriptions to group principles, to "chunk" relationships. Point out that students should examine problems for such relationships rather than immediately spewing forth equations (which may or may not be relevant).

Reif (8) points out that some common teaching practices used in engineering courses, such as emphasis on mathematical formations, may be deletrious to students' skill at problem solving, while shunning verbal or pictorial descriptions may also hinder the students. Students should probably be encouraged to use verbal descriptions and arguments. Stressing linear procedures as in combining equations should be down played, while hierarchial relations should be stressed.

In summary, pay more attention to the processes of problem solving your students use rather than the products, and you will probably do a better job of teaching engineering problem solving skills.

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A BACKWARD-REASONING MODEL OF PROBLEM SOLVING

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Introduction

This paper presents a description of a typical problem used in engineering education in universities and of the kind of argument and technique which the solution of such problems require. While it has drawn upon an examination of real problems, real cribs and the protocols of real problem solvers, it is not an analysis of actual problem-solving behaviour. An idealised solver is assumed whose knowledge of engineering theory is supposed to be complete. There is no discussion of the difficulties that may beset a problem solver who does not know the theory or the technique required for solving a given problem. Similarly, real problems, as set, may have omissions, ambiguities or obscure phraseology. Many of them are curiously truncated to fulfill the purposes of education and examinations. Furthermore, not every problem will conform to the classic type discussed here. Real solutions too, reveal all kinds of behaviour on which no comment is made. Cribs are assumed to reveal solutions in which the swiftest logical progression from problem to answer is represented. But they, too, may have imperfections of argument and technique and the statements in them are selected from all those which have been made by the crib writer in the course of his wrestling with the problem.

The paper provides a relatively simple[2] model of problem solving with an emphasis on focusing and backward reasoning, and concludes with a brief discussion of its possible value to teachers and students.

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[2] See the last page of this paper.

The Analytic Engineering Problem

The origin of the educational problem

In industrial practice, the prototype analytic engineering problem is provided by an artefact* about whose nature enough is known to be able to answer questions about its behaviour when it is subjected to certain conditions. A bridge is subject to dead and wind loads; what are the stresses in the members? A cooling system has been designed for a nuclear reactor; what will be the pressure drop at various rates of flow? An electronic circuit has been put together to achieve certain objectives; how well is it likely to succeed? In each case the design has been conceived and given sufficient form for the question to be asked and answered**. The conditions under which it is expected to perform are specified by edict and by nature and the engineer is now required to comment on its behaviour. Calculations of this kind are required in every branch of industry to "test" designs. All designs are put forward in the expectation that they will be subjected to such calculations.

The questions are not always directed at discovering the behaviour of the artefact. Sometimes the behaviour is specified and an attribute of the artefact must be discovered which will produce that behaviour. For example, a railway vehicle is subjected to certain loads; with how much camber must the frame be built so that it will be straight and level when loaded? Sometimes the behaviour is specified and the conditions under which this behaviour will occur are required. A large range of vibration problems come under this heading.

Speaking broadly, the problems used to train engineers in universities have the same characteristics except that the description of what is happening to the artefact is pruned to the point at which only those aspects having reference to the answers are specified. In the classic form, an artefact is described first, then what is happening to it and then the question is put which requires the student to calculate, with the aid of engineering theory, the value of a specified attribute of either the artefact or the happening. In fact, the world is divided into two parts, the artefact and the rest of the world. The happening specifies what the rest of the world is doing to the artefact but only just as far as is necessary to enable the student to answer the question asked. Certain attributes of the

*We use the term "artefact" rather than "system" because "system" has much wider connotations and also because it has special meanings in Thermodynamics and Control Engineering.

*Engineering science is concerned with knowing which questions need to be asked and how they may be answered. Engineering design is concerned with imagining artefacts that will perform the taks for which they were conceived and will also stand up to the subsequent catechism of engineering science.

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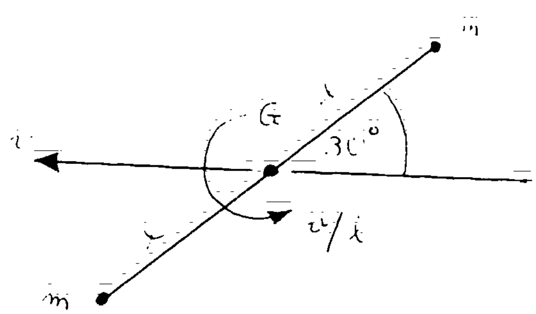
world, such as gravitational pull, can usually be taken for granted in any problem in which they are relevant, and usually are not specified as part of the conditions of the happening. The unasked question is how the world manages to do it:- this is irrelevant; a mighty but unseen hand is always at the disposal of the problem setter.

Usually, the question asked refers to an attribute of just a part of the artefact, and the solver is obliged to focus his attention on this part and to consider its behaviour when it is subjected to the actions both of the rest of the world and the rest of the artefact. The ability to replace the rest of the artefact with a set of interactions couched in engineering terms relevant to the question asked is one of the important skills of the analytic engineer.

This broad analysis may now be illustrated by reference to the problem.

Fig. 1: The problem

A light rod, of length 2ℓ, has small masses attached at each end which slide on a rough horizontal plane of friction coefficient "[illegible]". At a particular time, the centre of the rod G is moving with velocity v and the rod is orientated as shown in the figure, at 30° to the direction of motion of G. If the angular velocity of the rod is then v/ℓ, find the acceleration of G.



Artefact	A light rod connecting two small masses on a rough horizontal plane.
The happening	The masses slide on the plane with initial translational and rotational velocities in a given orientation.
Question	Find the acceleration of the centre of the rod.

The artefact

All artefacts have more than one component. Each component is described by a noun, but may be qualified by adjectives or adjectival clauses which define its attributes either qualitatively or quantitatively. Such attributes have great significance for the problem solver. Given two masses, a rod and a plane, we could not be sure whether we are concerned with statics or dynamics, or even a problem in projective geometry. However, the masses are "small" so that the moment of inertia of each mass about an axis through it can be ignored in comparison with the moment of inertia of the mass about any other axis. The rod is "light" and "of length 2ℓ" so that the mass of the rod can be neglected compared with those of the masses, and the distance between the masses enables the solver to calculate moments of inertia (and so forth). The plane is "rough", "horizontal" and has a "friction coefficient μ" so that "sliding" will produce friction forces equal to μ times the reaction between the surfaces, and the reaction will be mg.

These qualifications can be classified under two other headings; those that go to the root of the problem and specify the nature of the artefact which causes the behaviour, e.g. the plane is rough, horizontal and has a coefficient of friction μ, and those that, under the "house rules" of the teaching situation, signal appropriate behaviour on the part of the problem solver, e.g. "the rod is light" equals "ignore its mass" and the "mass is small" equals "treat as being concentrated at a point".

This discussion demonstrates that the description of the artefact, and in particular, the manner in which the description of the components are qualified, may lead to important inferences before the question asked is stated. As we shall see, the question asked tells us which of these inferences must be brought into play in order to answer it. Furthermore, the nature of the components and of their attributes, and the relationships specified between them, give rise to expectations about the branch of engineering which is relevant to the analysis of the artefact's behaviour.

The happening

The happening* is usually specified by a verb suitably qualified by adverbs or adverbial caluses which define the conditions of the happening either qualitatively or quantitatively. In this instance the rod and masses slide on the plane and the sliding is qualified by the conditions that, at a particular time, the rod and masses have particular translational and rotational velocities.

If we add the description of the happening to that of the artefact we find that our expectations about the branch of

*We chose the word "happening" rather than "event", "circumstance", "situation" or "action" because problem solvers so frequently ask themselves, "What is actually happening?".

engineering theory involved will be confirmed and, possibly narrowed. We shall also find that more attributes of the artefact or of the components can be deduced which will only be true for the given happening. So, given the initial velocities, we can calculate the initial kinetic energies of translation and rotation and the angular momentum of the rod and masses.

Usually enough attributes of the artefact and happening are specified in symbolic or numerical terms to enable the engineering reasoning which the question requires to be expressed in mathematical form and a symbolic or numerical answer obtained. It will be convenient to refer to the sum total of the numerical or symbolic quantities provided in the description of the artefact and of the happening as the "givens". In this instance the givens are 2ℓ, m, μ, v, $30°$ and, assumed, g.

The question and the unknowns

Given the description of the artefact and the happening, a number of different questions are possible. The broadest possible question is of the kind: "Discuss the ensuing motion". If this question were asked, we might expect the solver to find the initial linear and angular accelerations, the time and distances to rest and the path of G. He might even go on to find the force in the rod.

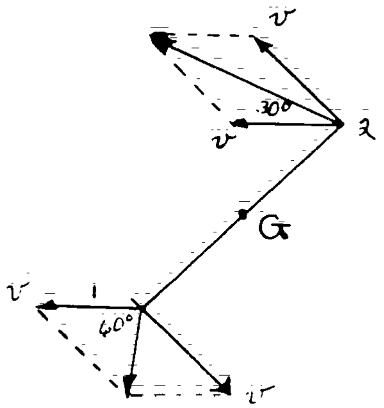
In fact, he is asked for the acceleration of G so that the question is a means of limiting the calculations required of the solver once the artefact and the happening are given. If this is the case, a solver will work more efficiently by asking what calculations are necessary to provide the answer required than by asking what calculations are possible.

The question does not specify explicitly the component of the artefact whose attribute is to be valued. The component can be identified by recognising that G is not only the centre of the rod but also the centre of gravity of the masses. Consequently, the solver must focus on the rod and masses and replace the plane by the vertical and horizontal forces due to gravity and friction.

Again the question asked enables the solver to test whether his preliminary hypothesis about the relevance of a particular branch of engineering theory to the analysis of this happening is reasonable. If, to the masses and forces defined by the description of the artefact, we add the sliding and velocities defined by the happening we are not altogether surprised to be asked for the acceleration of the masses. All this data supports the view that the propositions of particle dynamics will be required for the solution.

The Solution

Fig. 2. The crib solution



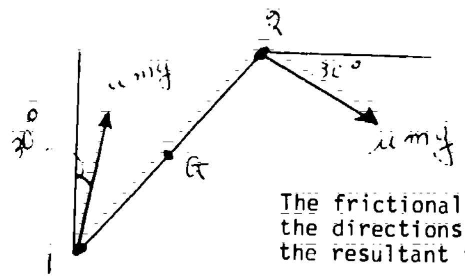
velocities

The velocity of each mass is the velocity of G plus the velocity of the mass relative to G.

For the rod and masses together

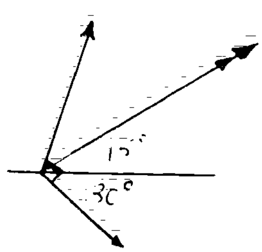
The frictional forces are mutually $\perp$. Hence the total accel. of G, the centre of mass

$$= \frac{\mu m g \sqrt{2}}{2m} = \mu g / \sqrt{2}$$



The frictional forces act in the directions opposite to the resultant velocities

forces



(Acceleration)
Resultant

If we look at the crib we observe that it begins by drawing vector diagrams for the velocities of each of the masses. The question asks for the acceleration of G. Clearly, an argument is missing from the crib which enables the solver to know that by startin where the crib does, he is making the best first move towards obtainin the value of the unknown*. We put forward the following explanation for this behaviour.

Intermediate unknowns

Our discussion of the artefact and the happening shown that a number of their attributes can be calculated from the givens using engineering theory. Each such attribute is an unknown prior to its calculation, but is may not be the unknown asked for by the question. When it has been calculated, its value can be added to the list of givens to determine more unknowns which will be attributes either of the artefact or of the happening. Such a process cannot be continued indefinitely however, and, sooner or later the value of the unknown required by the question will be obtained. Unless the required unknown can be found by the first application of the engineering theory to the givens, the necessary calculations will provide values for a number of unknowns which are not asked for. These unwanted values we call intermediate unknowns.

For the given problem, it is not possible in a single step to apply a process to the givens and thereby determine the acceleration of G. We know therefore that there are a number of processes applied successively to a series of givens to derive the value of the unknown from the initial givens. Each process determines some unknown which is rejected as not being the unknown required but which at least reduces by one the number of unknowns which must be found before finding the unknown required. At worst, it neither moves nearer to the unknown required nor helps us in our understanding of the problem. If, therefore, we can start with the unknown required and specify what unknowns must be found in order that it can be calculated from these unknowns, we will have reduced by one the number of steps, unknowns and processes required to move from the initial givens to the required answer. Clearly this backward step may have to be repeated, the new unknown now identified leading to the identification of still further unknowns, each associated with successively-earlier sets of givens, until we arrive at unknowns whose values can be determined in terms of the initial givens. Reasoning which asks and finds an answer to the question "What unknowns must I find to calculate the unknown I want?" will be called "backward reasoning". The process of calculating an unknown from givens will be called "forward processing".

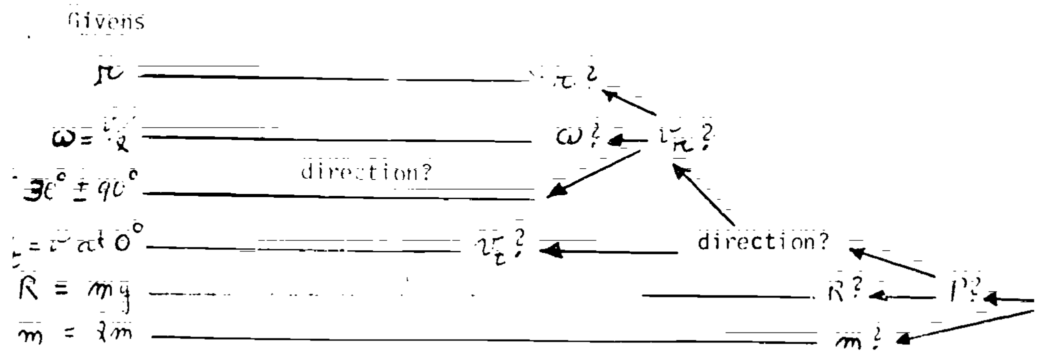
Consider the problem again. The question is: find the acceleration of G. To which a possible response is: "What do we

*An inspection of the cribs of a large number of problems shows that a high proportion begin with the calculation of unknowns which have no immediate connection with the unknown required.

need to know in order to calculate the acceleration of G?" Let us suppose we see that Newton's Law is applicable here in the form $a = P/m$. Then we need to know P, the applied force and m, the mass. We see that m is known in terms of the givens (= 2m). The variable P, however, is still not given and is the new unknown. If we see that P is the sum of the horizontal forces on the artefact and is therefore the sum of the friction forces, we know that P = μR for each mass, where R is the vertical reaction between a single mass and the plane, and that it acts in a direction opposite to the direction of motion of each mass. The new intermediate unknowns are therefore R and the directions of motion of the masses. Since the plane is horizontal, R = mg. For the directions of motion we need to see that these are the resultants of the velocity of G and of the velocities due to rotation about G. The velocity of G is one of the givens. The new unknowns are the magnitude and direction of the velocity due to rotation about G. We see that the magnitude can be determined by multiplying the given angular velocity by the given half-length of the rod and that its direction is at right angles to the given orientation of the rod, and these are given. Since we have shown how all the unknowns whose values need to be calculated in order to evaluate the acceleration of G can be determined in terms of the givens, we are now in a position to perform the "forward processing" which will carry out these calculations. "Backward reasoning" appears to carry out the necessary calculations in reverse only where the steps are so simple that their formal setting out is unnecessary. So, for example, seeing that "a" can be obtained from $a = P/m$ (if we know P and m) is a backward reasoning step whose forward processing is the precise reverse, viz. given P and m, $a = P/m$. But the appreciation that the velocities of translation and rotation can be added vectorially or that the two forces can be added vectorially is quite different from their actual addition by forward processing. Each operation of this kind requires the knowledge of a specific technique -- in this case vector addition -- which has been devised, taught and learnt. Thus a capacity for planning in some such fashion provides a possible explanation for the time interval which frequently occurs between a solver reading the question and beginning to write. Furthermore, it explains how and why he begins his solution by drawing velocity diagrams, and then goes on to draw force diagrams for the addition of the friction forces. These are the two steps which can be planned backwards but only carried out forwards.

However, from the discussion on the description of the artefact we discovered that the existence of the horizontal forces $P = \mu mg$ for each mass can be inferred without reference to the question asked and this part of the argument could have been determined by the solver and be held at the "ready", as it were, before the backward reasoning begins. But he would have no reason for doing this as opposed to calculating the values of the kinetic energy, etc., unless he had already decided that it was a problem based upon Newton's Law. The latter implies that forces are required and results in the planning set out above, unless the solver used a house rule of this kind; "The value of μ would not be given unless the value of

Fig. 3. Backward-Reasoning Diagram



the friction force is required. Therefore note that the friction force is μmg".

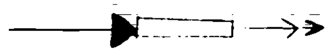
If we assume that he reads the whole question before making any calculations we may suppose that his backward reasoning is restrained by his knowledge of the givens, and the forward processing he may be tempted to apply to the givens will be restrained by the knowledge he has of the unknown required. In difficult cases he might have to work from both ends and then find that some of the properties specified in the artefact enabled him to calculate a middle step in the argument as well. What seems certain is that, one way or another, the chain of argument must be completed. All the operations and arguments set out must be accomplished even if some are so trivial as to seem scarcely worth the mention.

Particular-general analysis

In the discussion of backward reasoning it was necessary to assume the capacity on the part of the problem solver to see that Newton's Law, the Friction Laws, the Laws of Vector Addition and so forth were the relevant bits of engineering theory required for the solution of the problem, and at what point each bit should be brought in. The recognition processes whereby each relevant bit of theory is invoked at the appropriate point are subtle and complex, and the description put forward here is no more than a first step towards their analysis.

If we consider the crib, we see that the solution consists of a set of particular statements of general propositions. The velocity diagrams are particular examples of the law of vector addition and of the general proposition (written alongside them) concerning the relationship of the velocity of any point on a body to the velocity of the centre of gravity of the body. The force diagram is a particular example of the laws of vector addition and of the general proposition (written alongside it) that frictional forces oppose the direction of motion. The final equation

"Acceleration of G = $\mu mg\sqrt{2}/2m$" is a particular statement of Newton's Law whose general statement specifies that a body free to move under the action of a system of externally applied forces experiences an acceleration equal to the resultant force divided by the total mass. More briefly we say and remember "Force equals mass times acceleration". Even more briefly we remember "$P = ma$". Whatever the form in which it is remembered, it is not associated with a particular force, a particular mass or a particular acceleration. If anything is envisaged it is a rectangular block with two arrows, one representing force and the other -- possibly double-headed -- to represent acceleration.



The mass can be anything: an aeroplane, a car, a quantity of fluid, what you will, and the origins of the forces and the magnitude of the acceleration change accordingly. In this sense $P = ma$ applies to all artefacts which are free to move and is, therefore, of great generality.

The law $F = \mu R$ describes the relationship between the tangential and normal forces on two bodies in sliding contact and applies to all such circumstances. When we remember it in this form we have neither the particular surfaces in mind nor the circumstance, which press them together and make them slide upon one another.

It would appear that we store our working version of engineering theory in a series of shorthand statements, many of them of the kind $P = ma$ and $F = \mu R$, and that it is in these forms that the theory is brought into play to solve problems. Behind each shorthand lies a more comprehensive statement of the proposition which recognises its limitations and defines its terms with more precision. Behind these statements lie physical descriptions which enable the user to represent the circumstances associated with each proposition pictorially or diagramatically. The engineering model of the real circumstances has a pictorial representation, a symbolic or mathematical representation and a representation in precise English. The use of the symbolic representation implies the use of the pictorial representation even if this is not set down explicitly. And this must be so since the various general propositions used in the course of solving a problem are true only with respect to particular parts of the artefact. In fact, the focusing on which we have already commented in connection with the form in which the question is asked occurs each time the solver's efforts are directed towards determining another unknown. Focusing is an inevitable concomitant of each general proposition invoked and each intermediate unknown pursued.

Consider the problem again. The question as put directs attention to the rod and masses in association with $P = ma$. The quest for a is replaced by the quest for P, and $F = \mu R$ is thereby invoked. But $F = \mu R$ is associated with a focus on the point of contact of either of the masses with the table. The two velocity diagrams are associated with a focus on each mass in turn. The determination of the linear velocity of each mass due to the angular velocity associates the simple formula $v = \omega r$ with a focus on the length of the rod.

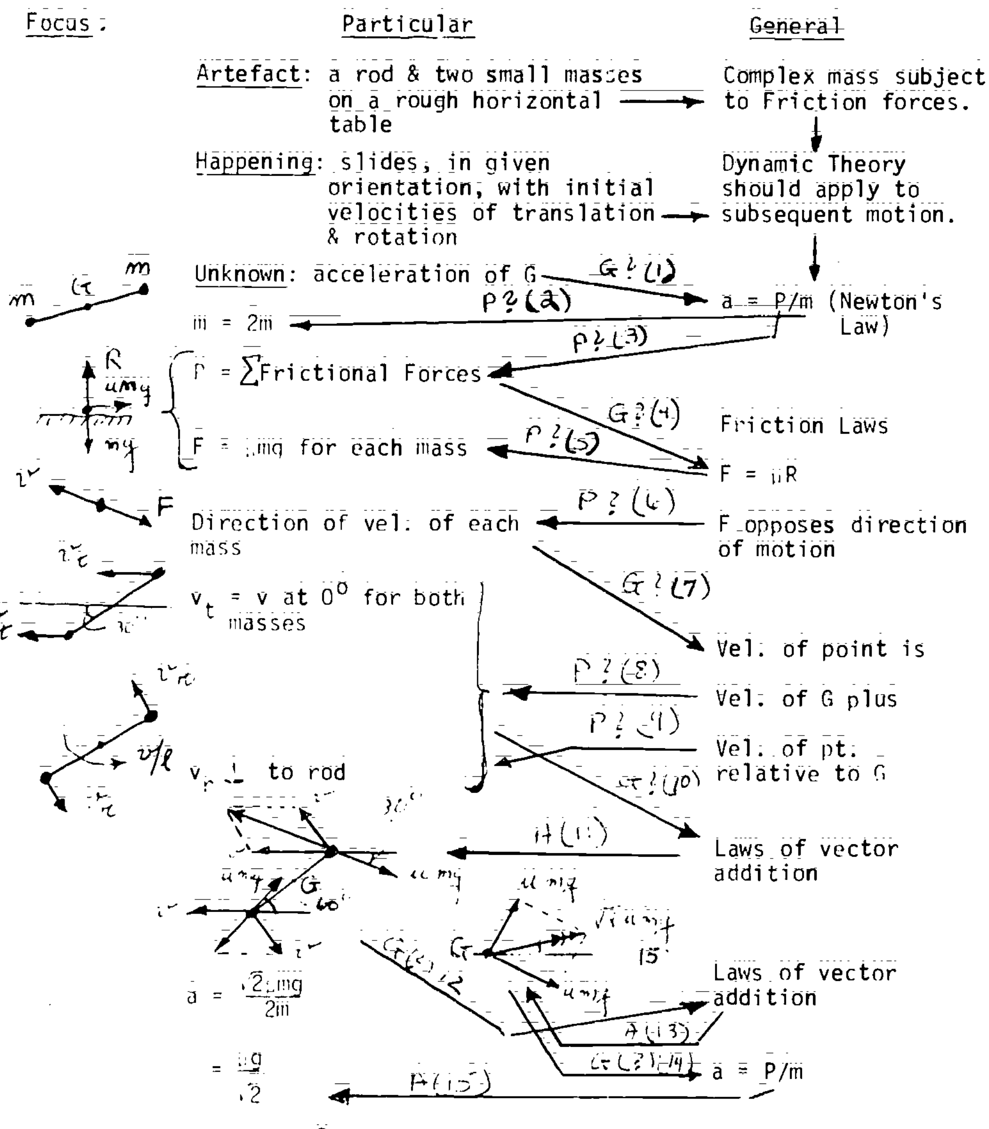
In most cases, general engineering propositions are true only of components of an artefact. It seems apparent, therefore, that the process of solving this problem involves the recognition of a series of subsystems, each appropriate to the proposition required for the determination of an intermediate unknown. The joint recognition of the subsystems on which the solver should focus and of the propositions that go with it is an important part of the total process. The previous discussions on making inferences from the description of the artefact, and on the way in which backward reasoning directs the search for relevant subsystems and their properties, explain in some degree how these acts of recognition are made.

So, for example, if the applicability of Newton's Law to the rod and masses leads us to attempt the evaluation of P, we need then to recognise that in this particular case P is the resultant of the frictional forces. We then generalise the friction situation to recognise that the laws of sliding friction apply. This leads us to recognize the need to find the direction of motion of each mass, which in turn leads us to invoke the general rule for the direction of motion of any point on a body and then to determine this motion in the particular instance, and so on.

In fact the generalization process begins with the artefact, the happening, and the question. The rod and masses must be perceived as a complex mass, the friction as a source of applied force, and the initial velocities and the request for acceleration as an invitation to apply the laws of dynamics to the ensuing motion. The impact of each interpretation is cumulative. The interpretation must be hypothetical in the first instance, but as each additional component of the problem description confirms the original hypothesis, the solver becomes more certain that his first surmise is correct. If such confirmation is not forthcoming, the solver raises further hypotheses and tests these against each bit of the problem description to ensure that no ambiguities or inconsistencies exist. The process is like any other recognition process, and since the information is very dense, there are a large number of bits of information in a single problem description, all of which must be consistent with the chosen interpretation.

Figure 4 shows the relationship of the particular to the general at all stages of the solution, and the way in which the focus changes as each particular question draws attention to a new intermediate unknown. Generalizations from the artefact, the happening and the unknown build up to the recognition that Newton's Law is the generalization to invoke. Arrows labeled G? with their heads in the general column imply the question "What generalization must we invoke in order to evaluate the particular unknown?" Arrows labeled P? and having their heads in the particular column imply the question "What, in this particular case, is the value of the

Fig. 4. Particular-General-Focus Chart



symbol identified in the general proposition?" If we number off each of these questions as in the figure, we see that numbers 1-10 inclusive reproduce most of the particular questions whose sequence is set out in Fig. 3. In addition, these also specify the general questions which identify the propositions that enable the solver to replace his quest for one unknown by a quest for another nearer to the givens.

As soon as the solver finds himself in a position to draw the velocity diagrams, he can begin forward processing by using his general law of vector addition and going forward along his planned sequence. The translation from general to particular is no longer in the form of a question, but an application of the general law to the particular instance. In fact, the move labeled A(II) (A for application) processes all the planning moves from G?(4) to G?(10). Its result is the values of the friction forces at right angles to each other. Their summation P?(3) must be recognized to require the vector addition of forces G?(12), and A(13) actually does this in the particular case. Finally, the solver invokes G?(1) again at G?(14), and applies this and P?(2) in A(15) to get the answer.

It seems clear that this collection of acts of recognition, of focusing, of reasoning and of processing must be undertaken by any solver who succeeds. It is also clear that they do not have to be undertaken in the precise order given by Fig. 4.

On the other hand, the order given seems to be the most efficient and logical one possible. Any order which involves working forwards from the givens without recognizing that a first objective must be to find the friction forces because they are needed for substitution in Newton's Equation, may well result in calculating other unknowns which turn out not to be intermediate between the givens and the answer.

Summary to this Point

This description of a particular problem and of the processes required for its solution emphasises three main aspects of problem solving. First it provides an analysis of the problem statement which shows how useful inferences and moves towards the solution can be made even before the question is asked, and which establishes a basis for the recognition of all the possible foci to which a solver can direct his attention.

Second, it shows how a focus, a generalization and intermediate unknowns are always associated and that a number of instances of such association are required to produce a solution. Each such occasion, therefore, provides the possibility of three forms of attack. The solver can ask "Given an intermediate unknown, what focus and generalization is true of it and what intermediate unknown is then raised?" or "If this generalization is true of the happening, what focus is required to express it and what intermediate unknowns will then be raised?" or "Taking this focus, what generalization is true of it and what intermediate unknowns will then be raised?".

Third, it demonstrates how, because the solution involves a forward processing technique, a qualitative plan using backward reasoning is a necessary part of the process. The backward reasoning diagram, of course, is not drawn but its equivalent occurs in the form of some such argument as "The friction forces are the cause of the deceleration; friction acts as a direction opposing motion so I shall have to find the direction of the velocities of the masses". In most problems such plans are not essential but a backward reasoning approach provides one means of entering upon a solution when a complete plan for it is still hidden from the solver. He asks first, "Given the unknown required, what focus and what generalization will produce an equation containing it and what intermediate unknowns will this equation reveal?". He then repeats the question with one of the intermediate unknowns as the unknown required until no more intermediate unknowns turn up in his equations. The elimination of the intermediate unknowns from these equations will then provide the answer.

The implications for the teacher

For teachers the important question is not how well the model describes actual problem solving behaviour (although we have some corroborative data from the analysis of the errors in 122 examination scripts and protocols of the solutions from six staff members), but how useful such a model might be for teaching students to solve problems and for the students themselves. Usefulness depends upon the width of the range of problems to which it is applicable, how significant an increase in the knowledge required for problem solving it represents and whether it can be used as a pedagogic aid. The next three sections discuss each of these questions briefly.

The range of problems

It is clear that the model is concerned with the way principles and procedures are put together to provide solutions and is in no sense a replacement for them. Since procedures are always forward-processing sequences, solutions which depend heavily upon them will only involve backward reasoning to the extent of recognizing that to obtain the unknown required a particular procedure must be used. Procedures however, include not only the generalizations to be applied to each focus, but also rules for selecting foci and the order in which they are taken. This characteristic of procedures is most easily seen in all those problems which can be solved by the use of vector diagrams. The so-called "method of sections" of statics is a good example of a change of focus using the same generalization when the rules for the selection of foci fail to provide values of the intermediate unknowns required next. It might be better recognized as such.

The model applies equally to all subjects, which differ not only in the generalizations involved but also in the character of the foci. Some of them show interesting complementary foci. For example, joints and members in structures and mechanics, states and processes in thermodynamics, of course, considerable emphasis is laid upon the necessity of defining the "system" or the "control volume" as a preliminary to the calculations. In mechanics a similar emphasis is often laid upon the drawing of "free-body diagrams".

It seems that all problem statements provide scope for solvers to invoke a wider range of focus, generalization and intermediate unknown than is needed to answer the questions asked. Difficulty is experienced by the solver if the cues for the right selection are not clear. Backward reasoning is one of the aids to making such selections. If it is not clear what principle is applicable to the happening, he can ask what equation will contain the unknown required. If the generalization is known, he can ask what focus will introduce the unknown required. When the particular version of the generalization is written the relevant intermediate unknowns are disclosed and the same questions can be asked about them.

The model as an addition to problem-solving knowledge

It has to be recognized that the process knowledge embodied in the model is already possessed by most engineering students when they arrive at the University, even if informally and implicitly. Somehow, they manage to carry out the operations suggested by the model.

When teaching a specific element of engineering theory, the procedures necessary for its use tend to be made explicit and to be taught as part of the subject. The use of the "Method of Sections" in Structures and the emphasis on defining the system in Thermodynamics are cases in point. Furthermore, any procedure that is invented to handle a specific class of problem specifies the processes to be used as well as the theory to be applied. The model, therefore, is likely to be most useful in handling problems which are off the "beaten track".

It is also well-known that students have difficulty in assimilating into their handling of a variety of subjects a technique which is common to all of them and taught separately. The use of differential equations is a case in point. Similar difficulties would have to be expected if the use of the model were included in the curriculum.

The model as an aid to pedagogy

The student solver turns to the teacher when he is "stuck". The teacher can then adopt one of two strategies. He can either demonstrate the solution by talking it through in front of the student; or he can attempt a Socratic dialogue to discover just where the student's difficulty lies. A teacher who adopts the second strategy frequently finds that if his questions are relatively "open", the student cannot see what he is driving at while, if he "closes" them to the point at which the student understands him, he has, in effect, done the student's thinking for him.

If the teacher knows the model and the student doesn't, he will have a further set of open questions at his disposal which the student will fail to understand. If both know the model, the teacher is provided with a new range of open questions which can be understood by the student and which will direct the student's attention to particular aspects of the solution and discover what part of it is not available to him.

Typical questions could be: "What is really happening here?", "In what terms are you going to describe the happening?", "What principle is applicable to this happening?"; "Of which component of the artefact is the unknown required an attribute?", "What generalization applies to this component which will include the unknown required?", "What are the possible foci in the problem?", "Have you considered focus X?", "What inference can you make from a consideration of focus X?", "Which foci will introduce the unknown required?".

Our expectations are, therefore, that the model would be relevant to examples of the classic analytic engineering problem in all subjects; that its formal exposition is not likely to improve students' problem-solving abilities noticeably, but that it provides an additional set of terms in which teachers can discuss the processes of problem solving, provided both teacher and student are familiar with them.

Acknowledgments

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Notes

1. Backward reasoning is discussed in:

Polya, G.	"How to solve it". Princeton. Princeton University Press, 1945.
Minsky, M.	"Steps towards artificial intelligence".
Nevins, A.	"Plane geometry theorem-proving using forward chaining". M.I.T. Artificial Intelligence Laboratory. Memo 303.

Bhasker, R. & Simon, H.A. — "Problem solving in semantically-rich domains: an example from engineering thermodynamics". C.I.P. Working Paper 314. Department of Psychology C-MU, 1976.

Luger, G.F. & Bundy, A. — "Concepts, inferences and search in mechanics problems". Department of Artificial Intelligence, University of Edinburgh, 1977.

2. A more comprehensive account of the complexity of the knowledge and processes involved in problem solving is given in:

Simpson, P.J. — "Teaching for expertise". Surrey University. (A paper given at the Third International Conference on Improving University Education, June, 1977).

9

LEARNING SKILLS AS AN OVERLAY IN ELEMENTARY CALCULUS

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INTRODUCTION

When one considers the magnitude of the task involved in pursuing an education in engineering or in the sciences, it is obvious that gross inefficiencies in the learning process cannot be tolerated. Yet in a very real sense "on-the-job training" for university students has been nonexistent. A student entering a university is presented with the job of learning, and is provided with all the necessary materials and tools to do the job. The product called knowledge, which must then begin to be produced, is inspected by a quality-control department which detects only the flaws in the product and not the difficulties experienced or the inefficiencies present in the student's execution of the learning process. In effect, there are no shop foremen in the brain factory.

In a recent program intended to provide engineering students with instruction in problem-solving skills, Professor D. R. Woods (Chemical Engineering Department, McMaster's University) found that little if any progress could be made in this area without first providing instruction in learning skills (3). This is not surprising because the results of the learning process form much of the input to the problem-solving process. The need for improved powers of perception and analysis, for skills in scheduling and planning, and the requirement that knowledge be carefully structured, make a program in learning skills a prerequisite to instruction in problem solving.

For some time the author has incorporated a program of learning skills into courses in Elementary Calculus in an overlay format, an approach suggested by Professor O. M. Fuller (Chemical Engineering Department, McGill University). This approach has several advantages over a separate program. Because it is incorporated into an existing course, it does not make an additional demand on a student's time. Since the students in the program are studying calculus in the same class, the two programs can be combined to develop both learning skills and mathematical skills simultaneously. In particular, exercises in learning skills can be based upon existing needs of the students rather than on certain additional material introduced in the interest of uniformity. Finally, as the term progresses, the content and principles of the learning-skills program can be reinforced and repeatedly applied within the context of calculus.

There are, of course, several disadvantages to the overlay approach. Because time is taken from instruction in calculus, the content of the learning skills program must be limited to the most essential topics. This format also places restrictions on the amount of time available for open discussion of the content. Finally, it suffers from the fact that the student is left to interpret and apply the principles of the program to other disciplines.

In spite of these weaknesses, on the basis of subsequent student evaluations, the content of the program was considered helpful by a substantial majority of students and they have urged that the program be continued.

The learning-skills program deals with five specific topics:

1. Planning -- suggestions for managing time.

2. Memory -- the structure of the human memory and the implications of this structure for the learning process.

3. Objectives -- a discussion of the meaning of comprehension and expertise.

4. Content -- an analysis of the material presented in a typical course in engineering or science.

5. Methods -- a presentation of a structured program of study activities.

The program of instruction in learning skills is accomplished by devoting the first five hours of the calculus course to a detailed discussion of these topics. This discussion provides a foundation upon which the calculus program can be constructed. The list of study objectives and the analysis of course content furnishes a frame of reference in which each new segment of the calculus program can be located, while simulation of the suggested study methods in the lectures provides not only a model for the students, but a means for communicating many of the fundamental concepts of the calculus program. In addition, the introduction of the learning-skills program has had a noticeably beneficial effect on the author's teaching methods.

PLANNING

The first topic for discussion is the scheduling of time. Students must be convinced that the most important factor in scholastic success is the time spent in study. Though one student may be more successful than another for a given time input, an individual's degree of success will vary almost directly with the time spent in study. It is therefore essential that control be exercised over time, the student's most important resource.

In the book, How to Study in College, Pauk (1) describes several approaches to time management. A number of these techniques are discussed with the class, of which the most significant is the need for study objectives. It is stressed that a student have a well-defined immediate objective when studying, and a method for evaluating progress toward this objective. The student who sets immediate study objectives will find it easier to begin to study and much of the time wasted in aimless wandering through notes and textbooks (in the absence of specific assignments) will be saved. On the other hand, setting such objectives is a difficult task for many students. One aim of this program was to provide the students with a carefully structured collection of learning activities from which to choose.

MEMORY

One criterion for a learning activity is that it be consistent with current theories of forgetting and learning. As in the case of planning, much of the class discussion on memory is based on the presentation in Pauk (1). In discussing the structure of the memory and the present theories on forgetting, the following points are emphasized:

1. Tree Structure -- The student should strive to structure new information in the form of trees having at most six branches at any node. This type of structure appears to be most compatible with the structure of the memory as revealed through psychological studies. It facilitates the recall of specific information by increasing the number of paths of approach, and enhances the transfer of information from the long-term memory (used for storage) to the active memory (used for thought), by promoting the transfer of blocks of related facts rather than the transfer of one fact at a time.

2. Staggered Recall -- In order to learn and retain information one must force the periodic recall of the information at intervals of several days. The work of K. Gordon as described in Pauk (1) substantiates the wisdon of this approach.

3. Recitation -- Oral recitation of information to be learned results in a significant improvement in learning efficiency. It might be speculated that the sounds perceived by the ear produce a separate trace of the information at a different location in memory.

4. Written Notes -- Written notations produce yet another trace of the information in memory. Moreover, in order to stagger recall over longer periods of time written notations become an important source of cues for future recall.

5. Creativity -- A student must strive to contribute original content to the learning process because information formulated by the learner makes a stronger trace in the memory than does information passed on by others.

It is stressed that constant awareness and proper implementation of these five major principles are essential to an efficient learning process. Students are guided into a program of study activities which focuses on these principles.

OBJECTIVES

In addition to considering the current theories on learning, a student must also choose short-term study objectives and learning activities which are consistent with the broader objectives of learning. As the discussion turns to the objectives of learning, students generally agree that the goal of their education is to develop an ability to perceive, formulate and solve problems. To place learning objectives in proper context the discussion therefore begins with a brief introduction to the six phases of the problem-solving process. These are:

1. Analysis -- to divide the problem into its component subproblems.

2. Interpretation -- to transform or restate the subproblems in terms of familiar concepts, ideas and facts.

3. Recognition -- to locate each subproblem within the appropriate discipline.

4. Procedures -- to apply or modify established procedures for application to the subproblems.

5. Synthesis -- to combine the solutions of the various subproblems into a total solution.

6. Evaluation -- to evaluate this solution in the light of reality.

Several examples of practical problems are discussed in order to illustrate these phases. It is pointed out during the discussion that interpretation, recognition and procedures are areas in which individual courses make contributions to problem-solving skills. Analysis, synthesis and evaluation are skills which develop as a result of one's total intellectual experience.

After having acquainted the students with the role to be played by individual courses in the development of problem-solving skills, the program continues with a consideration of objectives for learning in a course in engineering or science. To begin the discussion, students are asked to contribute possible objectives for learning in such a course. The most common responses are "to understand" and "to be able to apply", which are termed Comprehension and Expertise respectively. Because the students seem to have difficulty being more explicit, it is necessary to discuss both objectives in greater detail. To accomplish this the first objective Comprehension, is interpreted to mean the capacity to participate in meaningful dialogue on the subject. It is emphasized that dialogue implies the involvement of two parties, both conversant with the subject. Five specific requirements for dialogue are set forth:

1. Overview -- to have a broad mental view of the content and the scope of the subject so that all aspects of the dialogue can be kept in proper perspective.

2. Facts -- to be able to recall and state significant facts accurately.

3. Interpretation -- to be able to rephrase a statement for the purposes of clarification or simplification.

4. Justification -- to be able to support statements with examples, or with arguments framed by linking facts and examples together in a logical fashion.

5. Evaluation -- to be able to appraise the accuracy and relevance of what is heard through the recall of facts and the spontaneous formulation of interpretations, examples, and counterexamples.

This development of comprehension supersedes the students' abstract idea of understanding and replaces it with a collection of tangible skills by means of which a measure of the level of comprehension is made possible, a requisite if this objective is to be pursued through specific study activities.

Prior to describing the requirements of Expertise, it becomes necessary to distinguish between a problem and an exercise. "Problem" is used when referring to a question having no known definitive solution, while "exercise" is used in referring to a question for which the answer is known to be obtainable through the application of established procedures.

In order to clarify Expertise it is defined as the ability to perform the exercises typically with a study of the subject. In a sense Expertise is defined as a limited form of problem-solving skill, restricted in scope to a single discipline, and requiring little skill in analysis or synthesis. Furthermore, Expertise does not imply the ability to modify established procedures for application in new situations. Hence Expertise as defined here demands far less Comprehension than does skill in problem-solving. Five requirements are enunciated:

1. Catalogue -- to have in mind a complete list of the types of exercises which are typically included in a study of the subject.

2. Interpretation -- to be able to rephrase the statement of an unfamiliar exercise in a variety of ways.

3. Recognition -- to be able to match one of the interpretations of the unfamiliar exercise with one of the listings in the catalogue.

4. Procedures -- to be able to apply accurately the appropriate established procedure.

5. Evaluation -- to be able to appraise the validity of the proposed solution in terms of expectations and past experience.

This discussion of Comprehension and Expertise results in a noticeable improvement in student-teacher communication, especially when dealing with a student's weaknesses or when discussing the objectives of homework assignments or examination questions. Furthermore, this discussion pinpoints two specific skills which normally would not have been addressed by the student. The importance of skill in interpretation and skill in evaluation is punctuated by the fact that they alone are requirements of both Comprehension and Expertise, not to mention problem-solving itself. Finally this development of learning objectives provides the students with a frame of reference in which to locate the objectives of chosen study activities.

CONTENT

When choosing study activities the student must be concerned with the general classification of course content and with the role to be played by each of the various types of information in a manifestation of Comprehension or of Expertise. In addition to assisting students in choosing study activities, an awarenes of the nature of course content has a positive effect on perception. During the previously mentioned program, Professor Woods became acutely aware of the difficulties which students have in perceiving the content of a lecture. Perception, more than any other skill necessary for learning, is dependent upon the student's total experience and expectations of the future. For information to register on an individual's mind it must align with some bit of information already stored in the mind. For it to lock in it must fit like a key. The difficulty associated with perception cannot be eliminated by reshaping the information. The problem is not the shape of the key, it is the absence of the keyhole. This problem can be lessened appreciably through a detailed study of the structure and purpose of course content in general.

To begin the discussion of content, the students are asked to contribute some general headings. The responses include discussion, theory, examples and demonstrations. These are then grouped into two general categories, Concepts (discussion and theory) and Experience (examples and demonstrations). To carry this study further, the following classification of Content is presented:

I. Concepts

A. Theory

1. Definitions -- the established terminology of the subject.
2. Facts -- theorems, laws, principles, important formulas, etc.
3. Sequentials -- proofs, derivations, arguments, physical or biological processes, etc.

B. Discussion

1. Interpretations -- descriptions and simplifications of definitions, facts and sequentials.
2. Insights -- examples, counterexamples and antiexamples.

II. Experience

A. Examples

1. Classification -- a generalization of the statement of an example exercise.
2. General procedure -- a general statement of the procedure employed in the solution.
3. Likely errors -- a description of the context of any type of exercise or step in a procedure which frequently results in an error, and a description of the methods necessary to avoid the mistake.
4. Special cases -- a description of situations which require special procedures, and a description of these procedures.

B. Perspectives -- information to be used in forming expectations about the solution of exercises, or to be used in the evaluation of proposed solutions.

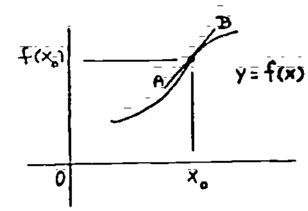
The three types of information under Theory are self-explanatory. In contrast both interpretations and insights are discussed in some detail. In order to acquire further understanding of interpretations, the students are introduced to the six media used for interpretation. As an example, six interpretations of the derivative are presented here for the benefit of the reader.

1. Verbal -- using words.

The derivative of $f(x)$ at a point $x = x_0$ is the instantaneous rate of change of $f(x)$ at $x = x_0$.

2. Visual -- using pictures of graphs.

$f'(x_0)$ is the slope of the line AB tangent to the graph of $y = f(x)$ at the point $(x_0, f(x_0))$.



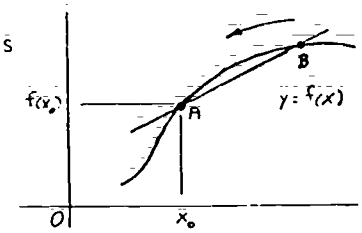
3. Symbolic -- using established symbolism.

$$f'(x_0) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

(In mathematics, the symbolic interpretation usually _is_ the definition).

4. Dynamic -- in terms of moving objects

$f'(x_0)$ is the limit of the slope of the line AB as the point B slides along the curve $y = f(x)$ and merges with point A.



5. Analogical -- through an analogy with familiar concepts.

$f'(x_0)$ is to $f(x_0)$ as the steepnes of a hill at a certain point is to the elevation at that point.

6. Combinational -- as a combination of more familiar concepts.

$f'(x_0)$ is a combination of

1. The slope of the line joining two points
2. The theory of limits.

Of course, these examples cannot be used in the initial discussion of interpretations because the students are not yet familiar with the notion of the derivative. The author interprets the various trigonometric functions instead. The importance of having facility in interpretation cannot be overemphasized in any program on learning skills. Only through interpretations are the concepts of one discipline applicable to another, especially in quantitative subjects where there is an endless dialogue between mathematics and the real world. Interpretations also contribute to recall because they provide a number of distinct paths of approach to a definition or fact which is only partially remembered. Through interpretations and reasoning the forgotten details frequently can be reconstructed. Since interpretations are simplified versions of the original definition or fact, interpretations often can be recalled over a much longer period of time than the sophisticated original information.

The discussion of concepts concludes with several remarks concerning the value of insights. In order to evaluate the various contributions made to a dialogue, a participant must have access to a wealth of examples, counterexamples and antiexamples. As in the case of interpretations, the ability to recall and formulate insights is a fundamental skill seldom addressed by students who have had no formal exposure to learning skills. The importance of this skill is illustrated repeatedly during the calculus course, as the usual multitude of questions from students are resolved using simple insights. For example, $x = 0$ is a critical point of $y = x^3$, but neither a maximum nor a minimum. It is also stressed that insights are a valuable aid to recall, because information partially recalled often can be reconstructed through knowledge of a simple example or counterexample. Furthermore, it is shown that important information which is recalled may be evaluated for accuracy through the use of simple insights. Thus the location of the sign in the quotient rule for differentiation can be checked by computing the derivative of $y = x^{-1}$ two ways, even if a textbook is not on hand.

It is also pointed out that one reason for the students' lack of knowledge in the field of insights is because such insights usually are given in response to a student's question; often neither the question nor the insight is presented on the board. Certainly some study activities should [illegible]s their objective the development of knowledge of this type.

As the discussion moves on into Experience, the author emphasizes that much of the valuable content of a course which relates to Experience is not explicit, as is the case in Concepts, and has to be obtained through analysis and generalization. It is stressed that the statement and solution of a specific example worked in class or in the text is of little value until the student has abstracted the content of the statement and solution, and has analyzed the procedure employed in the solution. In addition, it is emphasized that the processes of generalization and analysis must become part of the study activities of every student. One study habit referred to as "the two-handed bandit" is specifically criticized in this regard. The two-handed bandit is the title given to the study practice which sees the student placing one hand on a given example while the other hand transcribes it, making appropriate changes in symbolism. It is repeatedly stressed that this method, which for many students is standard practice, runs precisely counter to the perception of experience because it circumvents every aspect of analysis and generalization.

Such information as the graphs of the powers of x; the trigonometric functions; of $|x|$; of the families $y = a^x$ and $y = \log_a x$; and the way that these graphs change as a result of algebraic operations -- these constitute typical perspectives in Elementary Calculus. In Mechanics, typical perspectives would include a spectrum of values for the various physical quantities, and approximate conversion factors between the Metric and English systems. In Biology, taxonomy would fall under perspectives, while species of specific interest would be classified as insights.

Though at times it seems that students have no appreciation for the distinction between "wrong" and "ridiculous", the actual difficulty is that few students anticipate solutions or evaluate solutions in terms of these expectations. The cause of this is a significant lack of knowledge of perspectives. As a store of this type of information builds, anticipation and evaluation become spontaneous. Because of the overlay format of our program of learning skills, we often get a chance to illustrate the importance of perspectives, especially during the study of curve-sketching and the applications of differentiation.

METHODS

Having completed the study of planning, memory, objectives and content, the program turns to a discussion of study methods. In this section the objective is to propose a unified program of study activities which would reflect the foregoing development. Three areas are considered: homework, unassigned study, and self-evaluation.

In discussing homework, several points are stressed in addition to the obvious requirement that homework must be done. In the following listing of these points the justification for the adoption of the technique is indicated in parenthesis.

1. Time allocation (efficient utilization of time) -- determine before beginning an assignment the maximum amount of time which can be allowed for completion, on the basis of established priorities. Avoid spending hours attempting to solve what should be a relatively easy exercise. Usually a fellow student or teacher can resolve the difficulty in a very short time.

2. Stop thief (analysis and generalization) -- avoid the "two-handed bandit" by first taking notes on the procedures employed in solving the given examples. Then put the examples aside and attempt to solve the exercises using only the notes that have been taken. If necessary, return to the worked examples and make further notations, but never resort to the "bandit".

3. Variety (classification, recognition and staggered recall) -- when an assignment covers several distinct types of exercises, skip from one type to another without doing two exercises of the same type in succession.

4. Spacing (staggered recall) -- when an assignment is given for submission after several days, complete part of the work each day.

5. Programmed review (classification, recognition, staggered recall) -- set aside in a special file several exercises

of each type and turn to this file for the purpose of review. If exercises are chosen from the text when reviewing, the chapter and the other exercises give away the classification and thus provide little practice in recognition.

Though homework is fundamental to study, it is stressed that homework is for the most part directed at the development of Expertise. Thus, the student who does homework but little other study will probably develop very little comprehension of the subject.

In addition to homework, a student must become involved in a substantial amount of unassigned study. For many students, unassigned study reduces to staring at the printed page. In this program, unassigned study centers on the development of two notebooks, the Concepts Book and the Experience Book. These books represent at least a partial realization of the "Memory Board" and "Experience Board" introduced by D. R. Woods. The Concepts Book and Experience Book are constructed from information recorded in class notes, from texts, assignments, and original contributions from the student.

The <u>Concepts Book</u> consists of a <u>Table of Contents</u> and two major sections, the first entitled <u>Theory</u> and the second <u>Discussion</u>. Clearly this reflects the analysis of concepts presented in the discussion of Content.

The section entitled Theory contains a presentation of the definitions, facts, and sequentials in the order in which they were presented in the lectures. It is emphasized that definitions, facts and sequentials usually bear a relationship to one another which is broken in the course of a lecture in order to introduce interpretations, insights and other content aimed at Expertise. It is suggested that the theory be stripped of these unnecessary encumbrances prior to in-depth study, so that the inherent interrelationships can be grasped more readily.

Within the theory section, sequentials require additional attention. By its nature a sequential consists of a sequence of statements in a specific order. Since it is desirable to structure information as trees rather than sequences, it is necessary to identify structure within the sequentials. Students are informed that the process of identifying structure within an object is analysis, and that this is the same process as that referred to in the study of problem solving.

It is advantageous to make a brief diversion at this point. According to Upton and Samson (2), there are three types of analysis:

1. Classification -- dealing with "what sort of?" (the analysis of course content is a classification analysis).
2. Structure Analysis -- dealing with "what part of?" (the discussion of the structure of the Concepts Book is a structure analysis).

3. Operation Analysis -- dealing with "what stage of?" (the discussion of the problem-solving process is an operation analysis).

The process of performing an analysis is not only informative, but also creative. Because of the time, effort, and conceptual input which the analyst must contribute, the act of analysis often is more valuable than the resulting analysis. While the act of analyzing an object can be extremely stimulating to the analyst, a second-hand analysis (i.e. a completed analysis handed to another interested party) seldom produces much of an impact.

Although it was not possible to include an in-depth discussion of analysis in the learning-skills program, several of the suggested study activities are analytical in nature. Certainly, any activity which facilitates the acquisition of Comprehension or Expertise and provides practice in the fundamental skill of analysis must receive a high priority.

Returning to the presentation of sequentials in the Theory section of the Concepts book, students in the program are told that every sequential should be accompanied by an analysis. Several examples of analyses are presented, including the following analysis of the derivation of the Quadratic Formula:

$ax^2 + bx + c = 0$	subtract c	COMPLETE THE SQUARE
$ax^2 + bx = -c$	divide by a	
$x^2 + \frac{bx}{a} = \frac{-c}{a}$	add $\frac{b^2}{4a^2}$	
$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$	factor	SOLVE FOR X
$(x + \frac{b}{2a})^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$	take $\sqrt{\ }$ *	
$(x + \frac{b}{2a} = \pm\sqrt{\frac{-c}{a} + \frac{b^2}{4a^2}}$	solve for x	
$x = \frac{-b}{2a} \pm \sqrt{\frac{-c}{a} + \frac{b^2}{4a^2}}$	L.C.D. in $\sqrt{\ }$	SIMPLIFY
$x = \frac{-b}{2a} \pm \sqrt{\frac{-4ac + b^2}{4a^2}}$	L.C.D. on left	
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		

*Don't forget the $\pm$

When using this format, the number of entries encompassed by any bracket should be limited to six whenever possible. This process of analysis seems to be quite foreign to most students, but by requiring that the students analyse every proof presented in the calculus program this situation is soon improved.

The second major section of the Concepts Book, entitled Discussion, consists of a collection of work sheets, one for each important fact or definition contained in the theory section. On such a sheet there should appear the statement of the definition or fact under consideration. Beneath this should be one of each of the six types of interpretations. This implies that the student will have to formulate a number of these interpretations, because few instructors present this many in a lecture. The interpretations should be followed by a collection of insights. These should include an example of the concept under consideration, an antiexample of the concept, and a counterexample to any hypothesis concerning the concept which the student feels might be reasonable but which in fact is false. For example, consider the fact:

Theorem — If $f'(a)$ exists then $f(x)$ is continuous at $x = a$.

Reasonable Hypothesis — If $f(x)$ is continuous at $x = a$ then $f'(a)$ exists.

Counterexample — $|x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

Again, the student is required to contribute many of these insights because lectures usually do not present all of this information. Finally, the work sheet should provide room for further comments on the subject, including references to the types of exercises which require the concept for their solution.

The Table of Contents should closely resemble an outline of the Theory section of the book. Phrasing in this outline should be concise and suggestive of the content of the theory section, but should not be an excerpt from the content. A section of the Table of Contents of a Concepts Book in Elementary Calculus should resemble the following:

DERIVATIVES

Def. -- The definition of $f'(x)$
Theorem -- the theorem relating differentiability and continuity.
Proof --
Def. -- The definitions of $f'_-(x)$ and $f'_+(x)$.

Theorem -- on one-sided derivatives and the derivative.
Proof --
Theorem -- the rules for differentiation.

Such an outline serves as a listing of cues for the recall of the theory while assisting the student in developing an overview of the theory, the first requirement of Comprehension.

The Experience Book consists of a Table of Contents and two main sections entitled Examples and Perspectives. The first section, dealing with Examples, should consist of a collection of exercises with their solutions. These examples should be chosen from the totality of examples worked in class, in the text, in assignments, or independently by the student. There should be a representative of each type of problem presented in the course, chosen on the basis of completeness of content. These examples should illustrate any Likely Error or Special Case.

In order to best utilize the examples, the statement should be accompanied by an abstraction and the solution by an analysis. The format here is similar to that used in analyzing a Sequential. The statement and solution are presented on the left while the abstraction and analysis appear on the right. The analysis represents a General Procedure for the class of problems defined by the abstract statement. The analysis should be footnoted regarding Likely Errors and Special Cases.

The Table of Contents of the Experience Book consists of a listing of the types of problems contained in the section entitled Examples. The statements in this listing should be suggestive of the abstracted problem statements but should not repeat them. This listing constitutes the Catalogue required by Expertise. A portion of this listing should resemble the following:

1. $f'(x)$ using the definition

   (a) poly's (b) $(\text{poly's})^{-1}$ (c) $\sqrt{\text{poly's}}$ (d) $(\text{poly's})^{-\frac{1}{2}}$

2. $f'(x)$ for a P.W.D.F. at a joint.

3. $f'(x)$ using rules.

4. Implicit differentiation.

The second main section of the Experience Book should be a recording of perspectives, a topic already discussed at some length in the section on Content.

In addition to homework and unassigned study, the student must engage in some form of self-evaluation. In order to evaluate progress toward Comprehension, the student must become involved in dialogue on the subject. On the basis of the discussion of Comprehension, this is the ultimate measure of a student's understanding. On the other hand, to evaluate the level of Expertise the student should set an appropriate examination covering the requisite topics. In order to compose the exercises for such an examination it is necessary to have a commanding knowledge of the catalogue of exercises, and of the subtleties inherent in each exercise type. Two students might exchange such examinations in order to maximize the amount of feedback in this activity.

The benefits of such a program of study activities are apparent in terms of the foregoing discussion. First, this program provides students with a large number of study activities from which to choose when setting short-term objectives. From the point of view of the section on memory, the proposed activities clearly address or facilitate each of the established points: Tree Structures, Staggered Recall, Recitation, Written Notes and Creativity. There are activities which are intended to develop each of the requirements of Comprehension and Expertise, and activities which address each type of content.

CLOSING REMARKS

The learning skills program begins with a presentation of the material described in this paper. This requires that about five class hours be taken from the seventy-five hours provided for instruction in Calculus during the first term. Five more class hours may well be dispersed throughout the Calculus program but need not all be counted as time taken from Calculus because both topics are profiting during these activities. The loss of class time is subsequently compensated for by the higher levels of perception and the more efficient study habits of the students. In fact, the content of the Calculus program has expanded to include a detailed study of several proofs previously omitted; a number of exercises usually avoided; and a more complete development of interpretations, insights and perspectives. In this sense the Calculus program has assumed a better balance of Concepts and Experience than was previously possible. Though it would be premature to comment on any long-term effect of the program, the author can state emphatically that a number of students who have previously experienced difficulty in science courses and who have attempted to follow the methods described here have shown marked improvement.

It should be noted that learning skills may be introduced as part of the content of any introductory course in engineering or science. No matter how limited in scope or content, any effort made by instructors to assist the students in improving their learning techniques will be appreciated by the students. It is important to note that students are offended initially by the idea of being told how to study. It must be stressed in any program that learning skills are not a part of natural talents and that poor learning skills mask native ability. Finally, it must be emphasized that students are reluctant to change their study habits. For this reason, it is essential that any instructor embarking on the presentation of learning methods in an overlay format be prepared to require the adoption of the recommended study activities, at least for short periods of time, if any lasting effect is to result.

The author does not anticipate that a student from this program must continue to employ the formal techniques introduced. Rather, it is believed that by having to approach at least one course in this way, a fundamental and irreversible improvement will occur in the student's approach to learning.

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10

LEARNING SKILLS*

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1. INTRODUCTION: EXAMPLE OF A SKILL SESSION

A skill session is a kind of group teaching which differs significantly from a conventional tutorial, both in method and content. The idea is first introduced by an example; then section 2 describes the method and section 3 the selection of content. The outcome of the use of skill sessions in several institutions is considered in section 4, and possible developments in 5. The remainder of the chapter, section 6, is a portfolio of outlines of skill sessions which have been tried.

If one could eavesdrop on a skill session, one might see something as follows:

> Sixteen students arrive, and the tutor tells them that they are going to do some order-of-magnitude estimates, giving as an example working out whether the solar energy falling on the roof of a house could provide all, or a half, or just a tiny part of its energy requirements. He says how useful a physicist finds it to be able to do this kind of thing, and mentions an example from his recent research.
>
> The tutor then writes three problems on the blackboard:
>
> > 'What is the rate of growth, in metres per second, of your hair?'
> > 'How many words did this morning's lecturer say? How many words are there in your notes?'
> > 'How much fare money does a 'bus in this town collect in a day?'
>
> He divides the sixteen students into four groups, and makes each group of four sit round its own table, telling them to agree on an answer to the first problem, and to go on to the others only if they finish. One student in each group

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is told that in half an hour's time he will be asked to tell the rest what his group agreed. All this takes about five minutes.

For the next half hour the groups work alone in a buzz of conversation. The tutor stands apart, listening generally but leaving the groups mostly alone. One group falls silent, and he goes over and suggests an idea to them. He hears another group arguing about the exact value of a quantity, and reminds them that a rough value will do.

When the half hour is up, he gets the students to come and sit in front of the blackboard, even though one group is deep in the middle of an argument. One by one, the spokesmen are called up to report (having been reminded about this a little before the groups were stopped).

Two of the answers to the hair-growth problem are abo 10^{-8}m s^{-1}; one is much larger, and one much smaller. On going through the calculations, one contains a slip in working out a power of ten, and another an absurd estimate which is first defended, then abandoned by the group that made it. The tutor has some trouble getting them to agree that answers differing by a factor of two are really the same. One group has tried all three, two have done one and most of the second, and one has barely finished the first problem.

The hour is up before the third problem is discussed, and the tutor sums things up, saying that next time they will try some similar problems with a bit more physics in them. As a parting shot, he suggests that it would be interesting to work out how many atoms are added each second to a growing hair.

The essential features of the method are:

- a set of prepared problems.
- a group large enough to be divided for part of the time into about four sub-groups of about four.
- sub-groups working more or less alone for about half the total time.
- spokesmen to report the work of each sub-group.
- discussion with the whole class based on the reports.

The content can, but need not, be something like making order-of-magnitude estimates, sketching graphs from an equation, or planning the main outline of an experiment, which can be seen as skills needed by a working scientist.

The problems set do have to be ones that groups of students can get somewhere with in half an hour on their own, and which throw up useful material for discussion.

2. A METHOD OF WORKING

As a method of working, skill sessions imply a belief in the value of students discussing problems amongst themselves. To achieve that, attention has to be paid to the structure of the session: to having suitable material; to having a workable organisation of groups; and to using time to the best advantage. These points are expanded on below.

It can be seen as a virtue of the method that it leads to students talking with one another about physics, if such talk develops their ideas and their ability to express them. That the talk is with peers can have several advantages: people often then talk more freely than they do with superiors, and are more receptive and self-critical; in addition the pace and level is kept within students' abilities.

For the discussion to have these virtues, it is essential both that the groups work alone, and that the work set is simple enough to allow participation without seeming trivial or boring. Experience has shown that this can only be done effectively within a carefully organised framework, so that it is important to pay close attention to the sort of detail given here.

The introductory phase needs to be kept short, to allow time for the rest, and so that the tutor does not seem to dominate. But it does need to convey explicitly what the session is about, and why it is important. It may help to do one example quickly. An instance of the importance of the topic from the tutor's own experience is usually telling.

It is important to appoint spokesmen for each group at the start, and to explain their job. They should be asked to keep notes, and to report on progress, difficulties, agreements and disagreements, as well as on results. It can be better to have some system for appointing spokesmen in rotation, though some leave the choice to groups.

The same problem can be given to all groups, different problems to each group, or groups can be asked to choose one from a set. The choice depends on the topic: the first plan is best when useful differences may arise between groups which can be exploited in the final discussion; while the other two are better if the value of the topic lies in a variety of examples. A design problem might be of the first kind, and the interpretation of graphs one of the second.

Students and furniture will need to be moved around during the session. Sub-groups work best round a small table, or round the end of a large one. It is usually necessary firmly to insist on moving chairs and tables; otherwise students stay where they are and try to work while sitting in a row, for example. For the full group discussion at the end, it is again necessary to insist on moving into a circle or arc, around a blackboard or projector screen. The temptation to avoid this seeming fuss, or to mention it mildly without seeing that it is done, is strong and needs to be overcome.

So far as is reasonable, leave the sub-groups alone. If the tutor attempts to help, neither he nor they may discover the real capabilities or problems of a group. At this stage, the tutor's job is to ensure that groups know what they are supposed to be doing, and to give information or advice when asked. It is, however, useful to listen in from a distance, so as to plan the timing, and pick up issues which ought to be brought out in the full group discussion.

The final discussion is the part most tutors find hardest, but is the part where much of the value of the session emerges. The value comes when students have to abandon previously agreed positions, modify agreed views, accept other equally good alternative solutions, and see others getting into the same trouble as they did.

The temptation is always to cut short this part, by letting subgroups go on with a lively argument, or by letting every group finish all that it is doing. A firm calling of a halt is needed, not less than twenty minutes before the end.

Ideally, in the final discussion, students will challenge one another, ask questions, and get nearer resolving difficulties amongst themselves. In reality, the discussion presents the tutor with many dilemmas.

One dilemma is that students make mistakes. Some need correcting at once, but if the tutor corrects or improves on points frequently and immediately, students will soon learn to keep quiet. Often, it is best to make a note of such things and bring them up later. Quite often, the mistake does get spotted in due course.

Another dilemma is that other groups do not react to the various reports, seeming content to report and leave it at that, despite glaring differences. It may be important to go round insisting on one question from each of the other groups about every report. Done at the first few meetings, this can gather its own momentum in time.

The tutor has two roles in the final discussion: chairman and expert commentator. It is useful to bear them both clearly in mind, saying when one proposes to make a comment before joining in with it.

The tutor as expert has much to offer in the final discussion. Students can see him reacting on his feet to suggestions, and from that learn standards of skillful performance. The dilemma for the tutor is of striking a balance between displaying his skill and encouraging students to contribute. Perhaps the best advice is to have patience, saving up one's own points for a time, and then to give them as a clarification of what has been said.

It is important to sum up at the end, saying what has been achieved and commenting on unresolved problems. Students easily lose sight of the aim of the exercise, and can be confused and disheartened if it tails off in a tangle of unclear arguments as time runs out.

Despite the above insistence on ground rules, the format can be used flexibly. The cycle of: introduction-work in subgroups-discussion of reports, can be gone round twice in an hour by using shorter tasks, for example. Numbers are less flexible. A subgroup of less than four can easily run out of ideas, or not offer enough criticism, thus reaching premature and invalid conclusions. A subgroup larger than four can easily have a silent passenger. If there are more than four subgroups it takes too long to hear and discuss all the reports. Thus twelve to sixteen seems to be the optimum number.

Staff consistently overestimate what can be achieved by a subgroup in the time available, so problems must be simple and few in number. If they are not, they take time from the final discussion and the tutor is forced to take it over and rush through it, giving all the comments himself. Indeed, experience suggests that one hour is shorter than the ideal time, so that timetabling which allows sessions to run on can be useful.

Finally, do not expect too much or try to inject too many ideas. The expert has acquired his skills gradually over many years, to the point where they may seem trivial to him. One hour spent on (say) order of magnitude estimates will not work magic. It can be a signal that such a skill is valued, and can perhaps help confidence to grow a little. Indeed the tutor may find himself surprised by the frailty of students' skill, and learn useful lessons about what it is reasonable to expect. A flood of expertise may wash away the seedling, where a drop of water might encourage it to grow.

3. WHY SKILLS?

The content of sessions described, as distinct from their format, might be called skills: that is, things a working scientist can do which are not directly to do with subject matter. Teaching courses tend to be organised around subject areas, assuming that skills will be developed by the way, in doing problems. The original impetus for skill sessions was a hope that there would be value in something more direct (Black, Griffith, and Powell 1974).

One may or may not think that such skill can be analysed into components. In any case, if such a session is to have a clear focus, some particular skills have to be identified and suitable work on them devised. Section 6 gives a set of such sessions which have been tried. It is not complete or final: new ideas continue to arise, some from thinking about problems of learning in general and others from experience of difficulties revealed in students' work.

It has proved easy for staff to think of ideas for such sessions, but hard for them to produce clear and suitable work for them. Having such work, usually in the form of specific problems, is crucial, especially if colleagues will have to put the sessions into effect. It is no use just having a general idea for a session.

A range of graded problems is needed, to allow choice according to preference, and to allow adaptation when a chosen problem proves too hard. A good working rule is to include some which seem trivial; often they prove not to be, and if they are a group disposes of them quickly and little harm is done.

Sessions can with value be arranged in some developing sequence. One might be a series on problem solving, including: selecting main principles, formulating a model and an equation, and thinking of alternatives. Another might be related to project work, including orders of magnitude, selecting instruments, and deciding what to measure.

It cannot, of course, be automatically assumed that skill is readily built up in this way from parts, even though there is evidence enough that students do lack the skills looked at here as components. While the sessions reveal a good deal about students' needs, they do not solve the problem of doing more than a little about them. Experiments with sequences of skills are worth trying, perhaps culminating in sessions intended to synthesise them, but their success cannot be guaranteed. For this sort of reason, the sessions that exist are based on the instincts and intuitions of practioners.

4. WHAT HAPPENS WHEN YOU TRY SKILL SESSIONS?

An innovation can be judged in part by its hardiness. Skill sessions began and survived in the physics department at the University of Birmingham, and have since been transplanted without withering to other departments and subjects. That is, staff have been satisfied with them and have not abandoned them.

The results of questionnaires in more than one department show a clear majority of students finding them interesting and valuable, and finding the level of difficulty of sessions in section 6 about right. They agreed that the sessions 'help one to think like a physicist', and an unanticipated spin-off that they helped them to get to know other students early in the course. They did not accept that tutors gave insufficient help, but did feel that final discussions were too often left 'in the air'. They were not always convinced of their relevance to the rest of the course, or of their value for examinations. One department found that the last opinion shifted when a sequence of sessions was explained, and reflected in specific examination questions. Some students say they gain in self-confidence and ability to solve problems, more from skill sessions than from problem classes or tutorials. Finally, the sessions are clearly enjoyed.

Given adequate outlines, staff have proved willing to run the sessions. Later, many think up their own material. The way they run them varies considerably: some doing in two or three sessions what others do in one; and varying in emphasis, time, and conduct of aspects of the sessions. Personal style is important; and success has been achieved with very diverse styles. In general, tutors comment favourably on the sessions, notably on the active participation of students.

Despite such favourable opinion, there is not-nor could there easily be-any firm evidence that students actually acquire the skills in question. Indeed, realism suggests that in an hour one could hardly hope to do more than focus attention on a skill and the need for it.

5. REFLECTIONS AND DEVELOPMENTS

It is a matter for conjecture whether the format of skill sessions can be used for other purposes such as problem classes, and whether their content could as well or better be taught in another fashion, perhaps using written material. The format is certainly usable, and indeed has been used for many purposes before being applied to skill sessions. Ideas from chapter 6 suggest that it strikes a good balance between the various problems that constrain effective small group work.

It can be argued that the format is a good one for the content. Learning new skills may involve a change of attitude and a degree of self-criticism, for which working with one's peers is likely to be helpful.

Perhaps a good part of the interest of skill sessions is in the way they focus on an important problem of undergraduate courses. They may be seen as a small contribution to fresh thinking about it, and as a way of exposing, if not of disposing of, difficulties of students in this area.

Those who wish to try the sessions will need to consider how to introduce them. In a small department, a few can easily be tried on the whole class, even as an optional extra. In a large department, agreement of colleagues and a timetable slot is necessary. Some have made a trial replacement of some tutorials or problem classes; others have taken time from laboratory work and concentrated on laboratory skills. The method has been used at levels from the sixth form to the final year at university, and in engineering, zoology, genetics and other subjects.

A trial can be short and need not interfare with other courses. This obvious advantage is offset by the consequence that students may see the work as unrelated to other things. However, it would seem to be wise to try it first and worry about this later, thinking then about the part the sessions should play in the whole teaching programme. Indeed, the debate will acquire a needed vigour when skill sessions compete with other things.

6. SKILL SESSION OUTLINES

This section consists of outlines for skill sessions:

1. ESTIMATING ORDERS OF MAGNITUDE
2. SCALING
3. TRANSLATION: WORDS AND GRAPHS
4. TRANSLATION: SYMBOLS AND GRAPHS
5. USING ALGEBRA IN ARGUMENT
6. THE ART OF NEGLIGENCE
7. FIRST STEPS IN PLANNING AN INVESTIGATION
8. DESIGNING AN EXPERIMENT
9. SELECTION OF INSTRUMENTS
10. THINKING OF ALTERNATIVES
11. THINK ABOUT IT FIRST
12. WHAT IS THE PRINCIPLE?
13. WHEN IS IT TRUE?
14. WHAT ARE THE RELEVANT VARIABLES?
15. SPOTTING THE FALLACY

The list of titles is in no way exhaustive, complete or balanced. The outlines are intended to serve two purposes:

to provide a starting point; something to try at first, or something to have on hand when time presses.

to suggest other possibilities by example.

The outlines offered here have, however, been tried in a number of departments by a variety of tutors and have been modified in the light of that experience. In particular, questions which at first sight look trivial are in general found not to be. Indeed, the outlines probably still contain too many problems which are too difficult.

Most of the outlines are written with first year students in mind, because most trial work was done with first year students. Some can easily be adapted to other years by changing the problems, the session on order of magnitude estimates being an obvious example.

While wanting to encourage tutors to produce their own material, it is important to say that producing successful material is not easy. Most ideas have to be modified after trial.

Each of the following outlines gives a brief introduction to the idea of the session, and some advice about running it. Then a number of possible problems are suggested, from which the tutor can select a few, or which may suggest to him some similar problems. In general, the first one or two are easy.

6.1 ESTIMATING ORDERS OF MAGNITUDE

Problems 1 to 5 are simple non-physical ones to introduce the idea. For a single session, one can be chosen, followed by another from problems 6 to 12. There is work enough for two sessions, so if time permits more of the first few questions can be asked in the first session, followed by others in the second. It is useful to start groups on different questions, but to tell them to go on to questions set to other groups.

PROBLEMS

1. What is the rate of growth of your hair, in metres per second? How many atomic layers per second? How many atoms per second per hair?
2. How many bricks are there in Birmingham?
3. How much fare-money does a city bus collect in a day?
4. What thickness of your sleeve is worn off by one rub across the table? (Could radioactive tracers detect the wear?)
5. How many words were uttered by this morning's lecturer? How many words are written in your notes?
6. If I give a man 10 joules:
   to heat his coffee, will it scald him?
   in a punch on the nose, will it hurt him?
   How much energy in joules is dissipated:
   by burning a match?
   by stopping a car at the traffic lights?
7. How does the cost of a joule of electrical energy from a dry cell compare with the cost of a joule from the mains?
8. Roughly what error is made by using the volume of a block of brass at room temperature, in working out the density of liquid nitrogen from a measurement of the weight of the block immersed in the nitrogen?
9. An anarchist says he has just dissolved one curie of iodine-131 in the city reservoir. Do you laugh it off or raise the alarm? (The half-life of iodine-131 is 8 days, and the maximum dose in the thyroid gland is

0.7 microcurie.)

10. What is the chance that you are now breathing in one of the molecules that Caesar exhaled with his dying breath?
11. What is the recoil velocity of the Earth when a child bounces a ball on the ground?
12. What is the daily consumption of electrical energy in the United Kingdom?

6.2 SCALING

Why are things the size they are? Some are determined by intrinsic factors, others by the size of man. Questions of size relate to questions of design-how big to make the apparatus-and to the use of scale models. The session includes some practical dimensional analysis.

Questions 1 and 2 are intended to show that there may be problems connected with scale. One might be done as an introductory example. Problem 5 is probably best given to all groups at the same time, since different assumptions lead to different approaches.

PROBLEMS

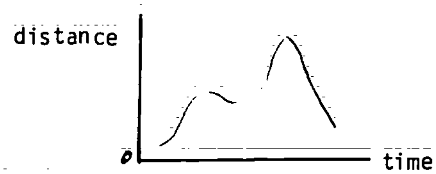
1. How tall is a flea, and how high can a flea jump? How tall is a horse? So-how high can a horse jump?
2. Which of two teapots keeps the tea warm longer, if one is twice as large in all linear dimensions as the other?
3. What effects would scaling up all the linear dimensions by a factor of two have on the performance of : an electric heater; a bicycle generator; a telescope; a nuclear reactor; and an apparatus to measure specific heat capacity electrically?
4. Why are these items the size they are: cameras; mercury-in-glass thermometers; and chemical balances?
5. A man has driven in a small nail with a small hammer. The next nail has twice the linear dimensions of the first, so he picks a hammer of twice the size too. Can he now drive in the nail more, less, or equally quickly?
6. Of two cars with all linear dimensions including the engine in the ratio two to one, which has the better power to weight ratio? (Or, could Lilliputians have cars?)

6.3 TRANSLATION: WORDS AND GRAPHS

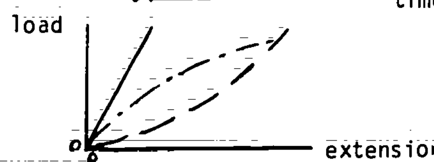
Problems 1 to 4 asks for graphs to be put into words. Problems 5 to 8 ask for graphs to be sketched from a description. The problems are relatively easy, especially 1 to 4, and several can be given. They can be extended by asking for further plots, for example of power against time for problems 3 or 5, and by asking for information contained in slopes, intercepts and areas.

PROBLEMS

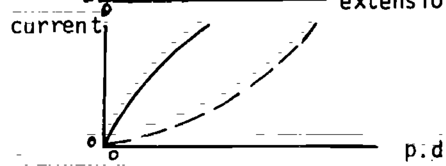
1. Describe this journey:



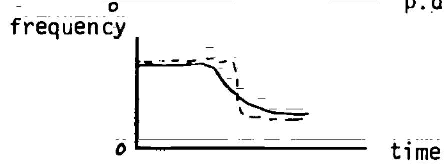
2. Say what it would feel like to pull the wires whose load-extension graphs are as shown:



3. How does the resistance change with p.d. for these curves? What sort of thing might each be?



4. If two people hear the frequency of the whistle of a train going past on a straight track change as shown, what can be said about how far they are from the track?



5-8 Sketch graphs of:

5. Current against time, after the normal working p.d. has been suddenly applied across a flash-light bulb.
6. The temperature of a room against time from the time the heating is switched on, if the heating is controlled by a thermostat.
7. The net gravitational force against distance, for a moon probe travelling directly from the earth to the Moon.
8. Acceleration against time for a train going from one station to the next.

6.4 TRANSLATION: SYMBOLS AND GRAPHS

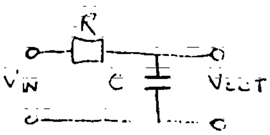
This session is more difficult than the last. Equations are given from which graphs are to be sketched. It is advisable to select expressions from recent lectures; even so, prepare to be disappointed! The first question is a simple introductory example.

PROBLEMS

1. Draw the graph of E against ν if $E=h\nu-W$, where h and W are positive constants. How does the graph change if

h and W are made larger?

2. Sketch graphs of amplitude against time for:
   a travelling wave $A(t)= A_o\cos(kx \pm \omega t)$
   a standing wave $A(t)= A_o\cos kx \cos pt$
   a modulated sinusiodal oscillation $A(t)= A_o(1+b \cos\omega_1 t)\cos\omega_2 t$
   a damped harmonic oscillation $A(t)= A_o\exp(-\gamma t)\sin\omega t$
3. Sketch the graph of the Maxwell-Boltzmann distribution $N(u)=u^2\exp(-\lambda u^2)$ if λ is small and positive. Where is the maximum? Which term dominates at large and at small values of u?
4. The equation $|v|_{out}/|v|_{in}=1/\sqrt{1+\omega^2 C^2 R^2}$ gives the ratio of the amplitudes of output and input voltages, when an alternating voltage, angular frequency ω, is applied to the RC filter shown. Sketch the variation of the ratio with ω. How does the curve cut the axis at $\omega=o$?



5. Sketch curves of amplitude against angular frequency ω, at various values of ω_o, k, and F, for forced oscillations given by $A(\omega)=F/\sqrt{(\omega^2-\omega_o^2)^2 + 4k^2\omega^2}$
6. Sketch the variation of intensity I_t with phase difference δ for $o<\delta<\pi$, for r=0.5 and r=0.9, in a Fabry-Perot interferometer, for which:

$$I_T = I_o/(1 + \frac{4r^2}{(1-r^2)^2} \sin^2 \frac{\delta}{2})$$

6.5 USING ALGEBRA IN ARGUMENT

It is not trivial to produce a quantitative model of a situation, deciding on assumptions, deciding on variables, and getting an equation between the variables. The problems here use only a few simple ideas, so as to focus on the process of setting up a model. It is possible to put numbers in at the end, but this is not the main aim. However, students in difficulties can be advised to try the problem with numbers to start with, to find first how things fit together. Because answers depend on assumptions, it is probably best if all groups have the same task.

PROBLEMS

1. What is the carrying capacity of a motorway in passengers per unit time, in terms of the speed of vehicles, passengers per vehicle, and number of vehicles per unit distance?
2. Suppose that a city centre is a circle of radius r, with n people per unit area working in it. They all commute in and out in rush hours lasting for time t, with p people per unit time able to cross per metre of the perimeter. How are these variables related? Why is there an upper limit to r ?
3. As skyscrapers get taller, more of the ground space is used up with elevators and stairs, and with thicker supporting columns. Make some assumptions and:

express free ground floor space as a function of height.
express free floor space on any floor as a function of its height above the ground.
express total free floor space in a skyscraper as a function of its height.

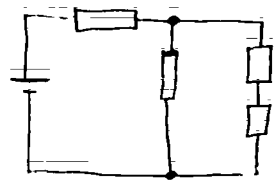
4. The sky is dark at night. If there were a uniform distribution of stars in space extending to infinity, show that this would not be so.
5. What can be deduced about nuclear binding energies from simple facts about nuclei (sizes, numbers of protons and neutrons) and knowledge of electrical forces and of simple wave mechanics?
6. Stellar bodies might be formed by accretion. Suppose matter adheres to a rotating sphere. If the angular velocity stayed constant, why would bits fly off again when the sphere reached a certain size? Now argue that the angular velocity will not stay constant.
7. If neutron irradiation produces nucleons of a short-lived isotope, at a rate proportional to the neutron flux and to the number of nucleons being irradiated, how much of the isotope will there be after a long irradiation?

6.6 THE ART OF NEGLIGENCE

Scientists neglect all sorts of aspects of problems they tackle. They approximate, oversimplify, and idealise situations. The difficulty is to do it enough to make a problem tractable without making it trivial. The problems suggested ask for lists of approximations or simplifications, and for some statements of their consequences.

PROBLEMS

1. List as many approximations as possible that are made in using $s=ut + \frac{1}{2}gt^2$ to find the time of fall of an object under gravity. Explain why they make the problem easier to solve. Suggest at least one case where the answer would be badly wrong.
2. List approximations made in calculating the amplitude of the alternating current I drawn from a signal generator, voltage V, angular frequency ω, by a parallel-plate capacitor, using $V/I=1/\omega c$ and $C= \varepsilon_0 A/d$. What effect do they have on the answer?
3. List approximations and simplifications made in calculating the temperature of the Earth from the temperature of the Sun, using the Stefan radiation law and the inverse square law.
4. List as many ways as possible in which one is idealising in making ordinary calculations of the currents and p.d.'s in a network such as:



6.7 FIRST STEPS IN PLANNING AN INVESTIGATION

The idea is to decide what one would do first, in starting an investigation of some phenomenon: noting the relevant theory; deciding on relevant factors; and suggesting where to make a start experimentally. For problems like those suggested, students can be asked to imagine that they will start on it next week, so that they now need to select some first ideas to test, and specify what sort of apparatus they need for that.

It may be desirable to specify topics actually used in teaching laboratories.

PROBLEMS

1. The variation of pressure in a rubber balloon as it is blown up.
2. The size of craters made by dropping hard spheres on dry sand.
3. Ionisation in flames.
4. Condensation of liquid from a vapour on cold walls of a chamber.
5. Variation of conductivity of a semiconductor with temperature.
6. The strength of glued joints.

6.8 DESIGNING AN EXPERIMENT

The problems below involve choosing one of several approaches and also making the design quantitative. Students find the latter difficult, and may not attempt it without prompting. Indeed, a full discussion of one of the problems needs more than a hour, so that it may be best to limit them further.

PROBLEMS

1. Design an experiment for a teaching laboratory, to illustrate Fraunhofer diffraction. It should allow patterns from one slit to be seen, and changes as further slits are opened, so that a slit system large enough to have a moving shutter is wanted. Give a quantitative specification for the components and their spacing and dimensions.
2. Design an electric field deflection system for an oscilloscope tube. Assume that the accelerating

voltage is about 2 kV. A 20 mm deflection on the screen is to correspond to a 20 V signal across the deflection plates. Details are wanted of the length, width and spacing of the plates, and their position in the tube.

3. Propose a method to detect the deflection of an electron beam under gravity, and work out whether the deflection will be detectable or not.
4. Design an experiment to measure the slowing down of a rifle bullet by air resistance, considering what precision might be needed to detect any change in speed.

6.9 SELECTION OF INSTRUMENTS

There are reasons, sometimes conflicting, for preferring one instrument to another. Two aspects matter: the physics behind the choice and the balance of arguments about the choice. The whole class can discuss an initial example, for instance listing features of a good ammeter such as range, sensitivity, resolution, accuracy, low resistance, speed of response, cost, size, robustness, and so on. Then sub-groups can make similar lists for other instruments, being asked to identify all important characteristics, not to design or choose the instrument. The groups are likely to go through a good number of instruments, so it is important to leave plenty of time for reporting back.

PROBLEMS

1. A balance including a set of appropriate weights for:
   weighing coins in a bank.
   dispensing dangerous drugs.
   finding the density of oil used in the Millikan experiment.
   determining the density of air.
   weighing rocks on the Moon, without bringing them back to Earth.
   determinging the mass (about 1g) of balls used in a precision measurement of the acceleration of free fall by timing the fall.
   studying corrosion of metal plates left exposed to a corroding environment for several years.
2. A thermometer for:
   obtaining meteorological records in the U.K.
   identifying organic chemicals by measuring their melting points.
   determining molecular weights by measuring the change in freezing point when a substance is added to solvent.
   measuring the temperature drop when a compressed gas is released into a larger volume.
   measuring the specific heat capacity of diamond.
   measuring the thermal conductivity of silver wire.
   measuring the temperature of the melt in a blast

furnace so as to be able to control the temperature automatically.

6.10 THINKING OF ALTERNATIVES

This session is about divergent thinking; about having many and varied ideas, not about analysing each. Such an exercise can seem frivolous if pursued for too long, and it may be worth switching to a closer comparison of some of the alternatives after a time.

PROBLEMS

How many different ways can you think of to:

1. measure the thickness of tissue paper?
2. measure the height of a tower?
3. estimate, or measure, the weight of a bus?
4. measure the duration of the light from a flash bulb?
5. estimate the energy arriving at the Earth from the Sun?
6. estimate the power of an average car engine?
7. measure the acceleration of free fall?
8. measure the speed of rotation of dentist's drill?
9. estimate, or measure, the rainfall in a week?
10. measure the frequency of a.c. power lines?

6.11 THINK ABOUT IT FIRST

This session is about looking at a problem as a whole, before starting detailed work on particular aspects of it, so as not to waste time working on false assumptions. It can start with a question from an examples sheet, and proceed in two stages. First, a number of particular queries are raised and noted down. For example, if a question involves an electron beam, what is the beam current or the number of electrons per second?

Second, sub-groups are given these queries to answer, while the final discussion is used to check the answers and see how they affect the whole problem.

The point is that in doing many problems, it is necessary to check first on relevant conditions: such as whether flow can be assumed to be streamline, whether an effect is in a linear region, whether an effect is negligible, and so on.

6.12 WHAT IS THE PRINCIPLE?

It is often hard for students to say what basic principles are involved in something, so that they may give every possible detail or none at all. The phrase itself seems clear but very possibly is not. So the task in this session is to analyse phenomena not for the details, but for the basic principles which govern them. Much of the final discussion may need to distinguish principles which are more or are less basic.

PROBLEMS

What physical principles are involved in:

1. the action of a diffraction grating?
2. the Geiger-Marsden experiment, and Rutherford's inferences from it?
3. measuring the Planck constant using the photo-electric electric effect; using electron diffraction?
4. explaining the binding energy of a nucleus?
5. the working of a mass spectrometer?
6. driving cars fast but safely round corners?
7. going to the Moon?
8. the efficiency of a nuclear power station?
9. the working of a powerful electromagnet?
10. a very high vacuum system for an accelerator?

6.3 WHEN IS IT TRUE*

Each group is given a law or laws to discuss, and is asked to produce three statements for each:

what does the law say is generally true?
what circumstances can alter, and the law still be true?
what circumstances invalidate the law, or make it true only with qualifications?

It is useful to start with an example discussed with the whole class.

PROBLEMS

1. Ohm's law.
2. Hooke's law.
3. The laws of electromagnetic induction.
4. The Lorentz transformations.
5. The mass-energy relation.
6. Newton's laws of motion.
7. The uncertainty principle.
8. The inverse square law for gravitation.
9. The inverse square law for electric charge.
10. The inverse square law for a source of light.
11. The gas laws.
12. Conservation of energy.
13. Conservation of momentum.
14. The Boltzmann Factor.

*See Shockley W. (1966) 'Articulated science teaching and balanced emphasis', in New Trends in Physics Teaching, Vol II, UNESCO 1972.

6.14 WHAT ARE THE RELEVANT VARIABLES?

The task here is to take a phenomenon and decide, using common sense and simple physical or dimensional arguments, what physical quantities determine the effect, not forgetting those which might do but in fact do not. The aim is to practice simple verbal physical reasoning.

PROBLEMS

1. The speed of big waves out at sea.
2. The frequency of oscillation of water slopping (in the tilting mode) in a shallow tray.
3. The speed at which a rotating flywheel is torn apart.
4. The time of contact of a golf ball with a golf club.
5. The power that can be obtained from a dynamo.
6. The frequency of oscillation of a steel strip clamped at one end.
7. The radius at which a glass plate bent in a curve snaps.
8. The brightness of a television screen.

6.15 SPOTTING THE FALLACY

Arguments (not least in examination answers) go wrong for several reasons, including:

defining a symbol one way and using it differently.
treating a variable as constant.
treating a vector as a scalar.
applying a principle when it does not apply.
making an assumption and then violating it.
treating coupled quantities as independent.
ignoring a relevant effect.
having a physical quantity equal to unity, and losing it.

Some of these can be introduced, perhaps with examples from recent work. Sub-groups can then be given written arguments containg fallacies, and be asked to identify them. It is best that all groups have the same material and look at all the examples, even if they cannot do some.

PROBLEMS

1. If the charge CV on the dome of an electrostatic generator is connected to ground through a resistance R, the current is V/R and so the charge will be gone in a time $t=\frac{CV}{V/R}=RC$.
2. The force on a charged particle moving at right angles to a magnetic field is $F=Bqv$ because:

   the force on a current element, $F=BI\delta\ell$

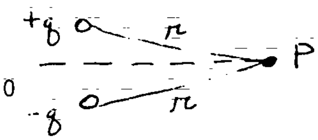
   current, $I=\delta q/\delta t$

   force, $F=B(\delta q/\delta t)\delta\ell$

   $F=B\delta q(\delta\ell/\delta t)$

   $F=Bqv$

3. The electric field at point P, due to equal and opposite charges $+q$ and $-q$ is $+q/4\pi\varepsilon_0 r^2 - q/4\pi\varepsilon_0 r^2 = 0$ Similarly, the potential is zero, so that $E=-\text{grad}\ V=0$.



4. Resistance to the motion of a satellite in a circular orbit reduces the kinetic energy $T=\frac{1}{2}mv^2$, and so the speed v. The angular momentum $J=mvr$ is conserved, so as v decreases the radius gradually increases.
5. Since the resolving power of a grating is equal to Nn, the resolution of a spectroscope can be made as large as required by using a grating with the lines ruled more closely.
6. The Emperor of China refused to be measured, so 10^8 Chinese were asked to guess his height. Their estimates varied from 1.5 to 1.9m, the mean being 1.71322 with a standard deviation of about 0.1 m. The standard error of the mean is smaller by the square root of the number of observations, so this method gives the height of the emperor to a precision of 10 microns. (What happens if he has his hair cut?)

REFERENCES

The first reference below describes skill sessions. The others contain questions which may be useful in developing one's own material for sessions.

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11

LEARNER AND INSTRUCTIONAL VARIABLES AFFECTING PROBLEM SOLVING

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INTRODUCTION

Anyone involved in the teaching of the mathematical sciences teaches problem solving. By its very nature engineering is a discipline of problem solving and problem solvers. Professors teach it, practitioners do it, and students learn it. Yet whether we are professor, practitioner, or student, we struggle with the "how to's" of the problem solving: how to teach it, how to do it, and how to learn it.

Our difficulties with the teaching and learning of problem solving can be diminished by careful attention to principles of teaching and learning drawn from the psychology of learning and the theory of instructional design. In this article, I will discuss four problems in the teaching and learning of problem solving, describe the elements from learning psychology and instructional design that would help in solving these problems, and provide examples of "how to's" from my own course, Introduction to Reasoning and Problem Solving, which was my attempt to address these difficulties in problem-solving instruction.

MAJOR DIFFICULTIES IN PROBLEM-SOLVING INSTRUCTION

There are two sources of difficulty in problem-solving instruction: instructional variables, which affect how we teach, and learner variables, which affect how the student learns. For example, an instructional variable may be the method of delivery: lecture, individualized learning, or discussion. Learner variables may include attitude, ability, or perhaps general motivation.

In problem-solving instruction, there are learner and instructional variables which limit our effectiveness in teaching problem solving. However, these variables can be controlled in order to improve teaching effectiveness. First, the variables and how they are limiting effectiveness.

MAJOR LEARNER AND INSTRUCTIONAL VARIABLES

There is one learner variable and three instructional variables to consider:

1. A large percentage of college students do not have the cognitive ability to learn how to solve logical, abstract problems. (Learner Variable)
2. We tend to teach problem solving in a way that describes how we solve the problem, rather than how the students should solve the problem. (Instructional Variable)
3. We tend to teach solutions to problems by showing the steps in an elegant order rather than in the order used when the problem was originally solved. (Instructional Variable)
4. We tend to teach the content of solutions to problems rather than the underlying strategy. (Instructional Variable)

The fact that a large percentage of students cannot learn to solve abstract problems is based upon current research findings exploring Piaget's theory of cognitive development. In his theory, Piaget specifies that in order to be an abstract thinker, one must be able to draw

> ... conclusions, not from a fact given in immediate observation, nor from a judgment one holds to be true without any qualifications, but in a judgment which one simply assumes, i.e., which one admits without believing it, just to see what it will lead to. (Piaget, 1952, p. 69)

Essentially, Piaget was describing the kind of problem-solving ability an engineering student must possess in order to reason from "A" to "B", even though "A" may be entirely hypothetical (Stonewater, 1977, p. 3). The current research, however, points out that anywhere from 42% to 78% of the college students studied cannot reason at this level (Elkind, 1962; Karplus & Karplus, 1970; Lawson & Renner, 1974; McKinnon, 1970, 1971). Based upon my own research, as many as 85% of minority engineering freshmen could not reason at high enough levels to solve engineering-type problems (Stonewater, 1977). Thus, without the ability to reason abstractly, students cannot solve the kinds of problems we expect them to. Hence, we need to develop instructional procedures that facilitate the development of abstract reasoning.

The second problem - that we tend to teach in a way we, rather than the student, solve a particular problem - is difficult. It is not easy for the sophisticated instructor to describe a thinking process that the student must use to attack a particular class of problems. This is not to say that instructors cannot specify the processes they use; we all do this every day in our lectures and conversations with students. It is, however, an entirely different matter to specify a process the students should use. The point is that instructors with a vast amount of experience to draw upon and with years of practice and hindsight to guide them have internalized a process. The process may be internalized to such an extent that we may not even be aware that when we go from step one to step two in solving a problem, the student may go through many more steps, decisions, and false starts to get from step one to two. What to us may be an easy and obvious procedure may be a complex sequence of events for the student. Thus, as teachers of problem solving, we need to explain problem solving from the viewpoint of the novice rather than the expert.

The third issue instructors face occurs when we show students how to solve a particular problem. The procedures we write on the blackboard in step-by-step fashion may not all be presented in the order we actually used when solving the problem the first time. Such reorganization of a solution to put it into an elegant form is instinctual for the seasoned problem solver. Once the problem is solved, one invariably reorganizes the solution into a more elegant form. This is good problem solving but not necessarily good instruction, for the student sees an elegant finished product rather than a thought process. Thus we need to focus our instruction on the process one actually goes through when solving the problem, not the elegant solution that looks nice.

The last instructional issue is that we often stop short of pointing out elements of the underlying strategy used in solving a particular class of problems. Insuring that students are trained in applying such strategies may not enable them to solve all problems, but it does provide them with a set of skills that they can call upon in attempting to solve various classes of problems. Our efforts should be to instruct students in the underlying strategies involved in problem solving.

Hence there are four major issues involved in the instruction of problem solving which we must face if we are to improve our efforts: Many students cannot reason at the level of abstraction required of them for successful mastery of engineering subjects, we tend to teach problem solving as we do it, rather than how students should learn it, we tend to teach elegant solutions rather than thought processes, and we tend to teach content rather than process.

LEARNING PSYCHOLOGY AND INSTRUCTIONAL DESIGN

My attempt to control the four learner and instructional variables to facilitate improved problem-solving ability was to utilize learning psychology and the principles of instructional design.

First, I drew upon Piaget's theory of learning psychology to determine how problem-solving instruction should be designed to improve students' abstract reasoning ability so that they could develop abstract reasoning skills. Instruction was designed around the following "rules" in order to affect this learner variable:

1. Design instruction so that students have the opportunity to organize and adapt to their perceived environment. To accomplish this, I had to find a way of presenting the material so students would organize their approach to solving a problem. Two methods were used to accomplish this. First, each problem-solving strategy to be learned was presented as an algorithm which outlined a step-by-step procedure to follow when attempting a solution to a problem. In short, the organization was provided for them. The second method used was to include material that instructed students in how to set up a problem before attempting a solution. This will be discussed later when the "preparation" phase of the course is described.

2. Design instruction so that increased social interaction results. Piaget maintains that increased interaction with others over a topic that is to be learned will facilitate the development of abstract reasoning. To accomplish this, a self-paced system of instruction was used with a student-tutor ratio of approximately 10 to 1.

3. Design instruction so that students have the opportunity to act rather than be passive receivers of information. Once again, this "rule" lead to the use of a self-paced system of instruction. In order for students to act, they must solve the problem, rather than the instructor. It was feared that in a lecture situation the instructor rather than the student was most likely to be solving the problems.

Thus, learning psychology - especially Piaget's theory - lead me to a self-paced, algorithmic approach to attacking the learner variable of lack of abstract reasoning skill.

To address the three instructional variables, the instructional design procedure of a task analysis was utilized. Simply, a task analysis is a careful analysis and sequencing of the steps, procedures, thoughts and decisions one must move through in using a particular strategy. Once an original analysis is conducted, one must iterate through it again and again to insure it is complete. A complete task analysis is very similar to a computer program, in that every step used must be clearly and specifically delineated.

Using a task analysis to develop a strategy for solving a particular class of problems controls the instructional variables previously mentioned. First, the difficulty of teaching problem solving as we do it rather than as how the student should do it is eliminated because a complete task analysis specifies all of the steps involved in solving the problem, including the ones we have internalized and fail to point out to students. The problem of the elegant solution is also eliminated because by definition the algorithm that results from the task analysis is the procedure used in solving the problem, rather than the reworked "nice" solution.

Lastly, forgetting to point out the elements or underlying strategy involved in the solution process is also avoided when a complete and careful task analysis is conducted prior to instruction. Once again the task analysis, by definition, is a strategy to use when solving the problem it was derived from.

Thus, utilizing task analysis procedures and deriving means to improve abstract reasoning skills should control the four learner and instructional variables. I will now describe my attempt to accomplish this.

"HOW TO'S" - A COURSE IN PROBLEM SOLVING

"Problem Solving" was designed to be a part of a curriculum development project in the College of Engineering at Michigan State University. Funded by the Alfred P. Sloan Foundation, the intent of the grant was to provide instructional and support programs for minority students so that an increased number would complete an engineering degree. The problem-solving course was designed as a separate course so that students could concentrate on building skills that should help them in other engineering, math, and science work. Copies of the text, which includes all of the modules described later, are available at cost by contacting the author.

A word of caution at this point. Keep in mind that the comments presented here are based upon a minority-student population which is primarily enrolled in remedial mathematics, which scores in the lowest third on ACT/SAT tests (MSU norms), and which at best has a very weak background in high school math and science.

As was pointed out, the instructional format used in the course was the self-paced mastery model. The course was divided into eight modules each including objectives, instructional readings, intermittent practice frames with feedback, and homework problems to be handed in. After successful completion of the homework, students took module exams which required 80% mastery for passing. Retesting without penalty was allowed until the mastery level was attained. Including a midterm and final exam, all tests were computer-generated. This permitted individualization of the testing process and collection of data for item-analysis purposes, to improve test content and instruction.

The content of the course falls into two areas: (1) the "preparation phase" of problem solving (modules 1-3), and (2) four strategies for solving problems (modules 4-8). The preparation phase taught students how to set up problems before attempting a solution. As was discussed, the strategies in the remaining modules were each taught as an algorithmic process. Each of the modules are described below:

<u>Module 1 - Preparing for Problem Solving</u> Students learned to discriminate between relevant and irrelevant information in a problem statement, to specify the given information and the solution desired, and to visualize the problem. Although these skills seem very elementary and basic, it was surprising how many students were not proficient with them. On the pretest given at the beginning of the course, one-half of the students could not list all of the given information in a problem and only two out of 18 could specify the solution desired.

In addition to the fact that students needed instruction in preparing for problem solving, properly-written given information and solution statements eliminate irrelevant stimuli which often lead problem solvers astray. Specifically listing given information and solution statements helps the student organize the problem into a manageable form, focuses attention, and provides a goal toward which to direct the solution. Failure to write solution statements often results in the wrong problem being solved.

<u>Module 2 - Drawing Diagrams</u> The criteria for drawing and labeling diagrams to assist students in visualizing a problem were outlined in this module. Further emphasis was placed upon specifying the given information and solution statement and including only this information in the diagram. Hopefully, the process of working from the given and solution statement to the creation of a diagram insures that irrelevant information is omitted from the diagram. Thus, when the diagram is finished, students have a picture clearly depicting the problem situation and the solution needed.

Module 3 - Data Tables. The purpose of this module was to teach students to organize data into an efficient and easily-read table from which the desired calculations could be computed. In more complex problems requiring multiple solutions, it was necessary for students to think about what computations were necessary for all the solutions. This encouraged students to analyze the problem before trying to solve it and to translate this analysis into a workable data table.

Module 4 - Subproblem Strategy. The subproblem strategy utilized an algorithm that helped students identify parts of a given problem which must be solved before the major variable could be found. The following is a simple example of this type of problem:

> What is the total income for the day if 3 hats were sold in the morning, grossing $18, and twice as many hats were sold in the afternoon for $1 less than the morning price?

In order to solve this problem, the student must add morning income to afternoon income to obtain the answer. However, the value for "afternoon income" is not directly available from the problem statement. One must first figure out the cost per hat in the morning (X) and from this determine the cost per hat in the afternoon (Y). X and Y are each examples of subproblems. The major aim of this strategy was to provide students with a procedure for analyzing problems in order to identify unknowns and sequence the order in which they must be solved. The procedure results in an organized plan for solving the problem.

Module 5 - Subproblem Strategy, ~~Part~~ II. The subproblem strategy was used in this module to solve problems like "How many rectangles are there in a 5 x 7 rectangle?" (i.e., count all i x j rectangles for all combinations of i and j, i = 1 to 5, j = 1 to 7). Although such problems are meaningless in themselves, they were used because they require the problem solver to develop an organized method to solve the problem. Which rectangles have been counted and how many of them there were must be kept track of. Thus, students who traditionally approach problems in a disorganized manner must develop a system to organize the solution before they can solve this type of problem.

In order to help students identify which rectangles should be counted in a given n x m rectangle, the double-summation formula summing over n and m was taught. Not only did this help them identify what the rectangles were, but it also provided the experience of utilizing a fairly sophisticated mathematical formula in solving a problem.

After students were able to solve the simpler problems, more complex ones which introduced a condition on the type of rectangle to count were used. For example, problems such as the following were included: "In a 3 x 4 rectangle, how many rectangles are there such that the number of rows is three or more greater than the number of columns?" Here, the student must first be able to translate the English expression "number of rows is three or more greater than the number of columns" into the mathematical relationship $i \geq j + 3$. Unfortunately, a fair number of students had difficulty translating such expressions into mathematical relationships and the modules did not specifically address this issue. Subsequent revisions of the course will include a new unit on translating English into mathematics and will most likely be included in Module 1.

Module 6 - Contradiction Strategy This strategy was conceptually the most difficult for students to understand. The algorithm told students to state an assumption that was the logical negation of what was to be proved and then use this assumption in their solution in attempting to contradict some given information. In order to insure that students could write correct assumptions that were negations of what was to be proved, part of the module discussed the rules of logic for negating "and" and "or" statements.

Module 7 - Inference Strategy In module 7, students learned how to infer additional information from the givens. Problems were like the following:

> A business office has a manager, assistant, cashier, etc. Their names are given and facts about each are specified. (e.g.: The assistant manager is the manager's grandson.) The problem is to match positions with people.

Inferences need to be made such as "the assistant manager is male," "the manager is married and old enough to have a grandson," etc. Many students recognized on their own that once the inferences were drawn, the problem often required the subproblem or contradiction strategy to solve it.

Module 8 - Working Backwards The working backwards strategy taught students how to solve problems starting at the solution and working backwards to the givens as opposed to the traditional approach of starting with the givens. For example, the following problem is much easier to solve working backwards:

> Describe a sequence of emptyings and fillings of a 3-qt. jar and a 7-qt. jar to obtain 5 qt. of water.

As opposed to starting with three and seven quarts of water and describing how five quarts are obtained, this algorithm directed students to begin with five quarts and work backwards until three and seven quarts are left.

RESULTS

To what extent did the course control the four instructional and learner variables?

Concerning the effects of the course on increasing abstract reasoning ability, the results are mixed. Students taking the course did increase their level of abstract reasoning as measured by the Equilibrium in the Balance test, but not by the Pendulum Problem.[1] Before taking the course, only from 6-19% (depending upon the test used) of the students were abstract thinkers; after the course 31% were classified as abstract. The change in level of reasoning ability is significant for $p \leq .05$, indicating that for the group taking the course, there was a statistically significant increase in abstract reasoning ability.

For the control group - students not enrolled in the problem-solving course - about the same percentage of students as in the experimental group were abstract thinkers at the beginning of the term (9-18%). However, by the end of the term, there was no increase in the percentage range of abstract thinkers. The figures remained at 9-18%. Obviously, there was no statistically significant change in level of abstract reasoning for the control group.

Even though the experimental group did evidence a significant increase in level of abstract reasoning with respect to the Equilibrium in the Balance test, this increase was not significantly different from that of the control group. Put in simpler terms, the experimental group did improve with respect to itself, but not with respect to the change observed in the control group.

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[1] These two tests are outlined by Inhelder and Piaget (1958, pp. 67-69; 164-181) and summarized by Stonewater (1977, p. 60). Essentially, they are both clinical experiments which the student performs. Inferences are drawn from the student's behavior that classify the student in Piaget's hierarchy of abstract reasoning.

These results indicate that the course did have some effect on level of abstract reasoning, but that the effect was not sufficiently greater than the effect of no treatment at all.

Turning to the effects of the algorithmic approach in addressing the three instructional variables, it appears that the approach did facilitate learning. As measured by grades, students did learn the material. The average grade earned each term was over 3.0. Most students finished all eight modules with at least 80% mastery. Students also did reasonably well on the homework and evidenced an understanding of the application of the various strategies. Additionally, although no specific follow-up was conducted on the students who took the course, a number of them reported that the course did help in subsequent math and science courses. These students found that setting up the given information and solution statement prior to solving a problem was most beneficial. It seems that these skills were helpful in discriminating relevant and irrelevant problem information.

DIFFICULTIES WITH THE COURSE

In assessing the effectiveness of the problem-solving course, three difficulties should be mentioned. First, even though the course had some effect on facilitating abstract thinking, additional instructional procedures need to be developed. Second, even though the course emphasized organizational skills, students still had difficulty in this area. Lastly, more emphasis should be placed on discrimination learning as a means of clarifying concepts.

Concerning the development of abstract reasoning, McKinnon (1970) pointed out that "inquiry/discovery" sessions facilitated the development of formal reasoning. He defined this approach to include practice at "questioning, hypothesizing, verifying, restructuring, interpreting, synthesizing, and predicting..." (1970, p. 37). Students met in small groups to discuss and research a particular topic and were asked to examine a particular aspect of a problem, to find out what was known, and to suggest ways to interpret data to either arrive at a solution or better understand the problem. Thus, the inquiry emphasis of McKinnon's course was not oriented to facts about science, but rather oriented to the process of problem solving in science.

Even in a self-paced course, one class session per week could be used as an instructor-led inquiry session. A problem could be presented for the class and the instructor could utilize inquiry/discovery techniques to assist students in discovering solutions and processes of solving different classes of problems. An additional factor that may help some students is that with this approach, not only would they be discovering solutions but they would also be able to see how others attempt to solve problems. In the event of a correct solution, students would have a model to follow; for incorrect procedures students could see why they do not work and possibly receive feedback on their own methods of attack.

A second change to increase abstract reasoning would be to introduce laboratory-type "hands-on" experiences. Many of the problems in the course are the kind that can actually be worked as an experiment, providing the student with the opportunity to handle and manipulate concrete rather than hypothetical or verbal problems.

A third change would be to include more intermittent practice frames with immediate feedback into the content of each module. This would increase the amount of time students spend practicing the skills that are descriptive of the formal stage (e.g. hypothesis building and testing, "If..., then..., therefore" reasoning, etc.). As was pointed out earlier, this would provide greater opportunity for the student to be active rather than passive, an important factor in developing abstract reasoning skills.

The second area of the course which needs improvement is that additional techniques need to be found to assist students in developing better organizational skills. A large number of students had difficulty writing down solutions to problems that were organized, that included enough information to enable me to interpret their thinking processes, and that included the reasons for the statements made. This is not to say that they necessarily could not solve problems, they simply could not write down solutions. Often students would verbally "walk me through" a solution, but had no idea how to write down a solution. Thus when facing a problem they could not solve, their attempts at a solution were not written down; hence neither I nor the student could analyze what they had done in order to find an alternative approach or an error. Instruction in problem solving must somehow teach students that an organized attempt at a solution can often give information that will help in finding the correct solution.

Tied in with the problem of organizing a problem solution was a general tendency to avoid writing down the givens and solution statement, and to avoid following the particular algorithm for a strategy. Even though I tried to stress the importance of these steps in setting up a structure within which they could solve the problem, students were reluctant to do so. One can speculate on the reasons for this reluctance: The algorithmic, structured approach did not match their cognitive style, the algorithms did not really describe the way the problems are solved, or students did not understand the algorithm. Although any of these could be reasons, it is my hunch that students avoided the structured approach because it took too much time. It is much easier not to write the solution in a detailed fashion. Whatever the reason, however, techniques need to be found to overcome students' reluctance to follow a strategy.

I also noticed that some topics in the course were learned "better" than other topics. Although there are usually a multitude of reasons for this, the instructional approach of discrimination learning seeme' to increase the quality and efficiency of learning. Disc imination learning is the process of being able to select the appropriate example of a particular concept from a list of positive and negative examples of the concept, i.e., the student must discriminate correct from incorrect examples of the concept. This process implies that the instructor must first develop precise definitions of the concept under study and then devise positive and negative examples of the concept for the students to discriminate. I found this approach very useful in teaching some of the basic concepts in this course, such as given information, solution statements, tables, and diagrams. Students seemed to benefit from this instructional technique. Although not tried, it could also be used in helping students identify appropriate and inappropriate examples of a problem solved by a particular strategy.

INSTRUCTIONAL APPLICATIONS

This leads me to the last area I wish to discuss: ways others can utilize modules from the problem-solving text. First, I think the best way to teach problem solving is to address it within a particular course, not teach it as a separate course. Thus, my suggestion below will be related to ways you can utilize the problem-solving modules within courses you currently teach. It goes without saying that you would develop examples of the application of a particular strategy from your course and write such applications into the existing modules.

Before instruction, the class should be diagnosed to determine which students require instruction in a particular strategy. Although I have a diagnostic test for my course, it is based upon non-mathematical problems. For courses requiring more mathematical sophistication as a prerequisite, one or two problems could be developed which are based upon the level of mathematics required in your course.

Once you have diagnosed students' abilities to apply the strategy, you know who can apply which strategies. If a large segment of your class cannot utilize the necessary strategies it is advisable that instruction on the strategy be included in the course. If only a few students cannot apply the strategy, it is most likely that they need to spend out-of-class time learning it, rather than waste the time of the entire class.

If in-class instruction is necessary, two instructional alternatives are available: the mini-self-paced frame or the learning-cell approach.

With the mini-self-paced approach, since the modules are designed to be used as independent, self-paced materials, it would be feasible to set aside a number of days for the students to study the materials on their own. No lectures would be given during this period of time. Students could either study the materials outside of class or a modified self-paced class could be run using the students who passed the diagnostic as tutors. For example, suppose your class meets three days a week for one hour. One week could be set aside to teach two of the strategies. Students would come to class and study the modules, work problems, and consult with the tutors or the instructor when they needed help. (Since individualized, computer-generated exams were available for my particular course, students could test at any time during the week when they were ready.) Exams and tutoring can take place only during the hour the course is scheduled to meet, or can be expanded to additional hours depending upon the availability of time, space and personnel. Of course, an alternative to the individualized-exam system is that an in-class exam covering the material can be given to all students at the same time at the end of the week.

If the mini-self-paced session sounds like too much confusion and effort on your part, variations of the traditional lecture could be considered. One such variation called the learning cell has been reported tobe effective by Alexander (1973). Essentially, the learning cell is a method to use within a lecture. Students are paired so that they can study a particular instructional package with each other. For example, if the contradiction strategy is to be taught within a lecture, the module can be given to the students with directions to select one partner to study and work problems with. If you would prefer to lecture instead of asking the students to spend class time reading the module, the materials can be handed out ahead of time so that students can prepare. Once in class, the instructor could present a lecture over the strategy and then break up the class into learning cells for practice on applying the strategy to actual problem solutions.

Alexander reports that when using a learning-cell approach to instruction, students learn more than when studying individually. They learn to analyze, synthesize and apply their knowledge, and their feelings of isolation/alienation and test anxiety are decreased. He attributes these positive outcomes of the learning cell to the active practice involved, to the immediate feedback between peers in the cell, and to the responsibility participants develop not only for their own learning, but also for the learning of a valued partner. Thus the learning cell could be used within a traditional lecture structure.

The learning cell and the mini-self-paced course within a traditional lecture are two ways to utilize the problem-solving modules within traditional instruction.

SUMMARY

The four learner and instructional difficulties discussed here are major sources of concern in problem-solving instruction. Ways to improve abstract reasoning ability and an algorithmic approach to the design of problem-solving instruction were found helpful, but additional approaches need to be developed to further reduce the effects of the learner and instructional variables.

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THE REPRESENTATION AND SOLUTIONS OF PROBLEMS IN APPLIED MATHEMATICS: AN ARTIFICIAL APPROACH*

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A. Introduction

During the past ten years several groups of researchers in the field of Artificial Intelligence have addressed the issues arising in solving applied mathematics problems by computer. In Section B of this paper the work of four of the major projects in this area will be outlined. In Section C, the authors' work on the MECHO* project will be considered. Finally, some of the ... ;ults of these projects and possible implications for educating engineers will be discussed.

B. Four Projects

B.1. Computer-Aided Circuit Analysis

Stallman and Sussman at MIT (1976) have designed and implemented a system for computer-aided circuit analysis. The system consists of a set of rules for electronic circuit analysis. This set of rules encodes physical laws such as Kirchoff's Law and Ohm's Law, as well as models of complex devices such as transistors. Facts, which may be given to or deduced by the system, represent information such as circuit topology, device parameters, voltages and currents.

The system works by forward reasoning. That is, the facts of the problem situation, combined with the rules encoding the physical laws that apply to this situation, drive the reasoning system. New deduced facts are tagged with justifications for deducing them, which include the problem facts and the inference rules used in their deduction. The justifications may then be examined by the user of the

*A Science Research Council Project, funded through the Department of Artificial Intelligence of the University of Edinburgh--Dr. A. Bundy, grant holder, Dr. G. Luger and Mrs. M. Palmer assisting.

system to gain insight into the operation of the rule system as it applies to the problem. This is helpful for correction (debugging) of the rule system when it arrives at erroneous conclusions.

Furthermore, the justifications for new deductions are employed by the system in the analysis of fruitless search or blind alleys. This allows the system to avoid these situations in future reasoning.

The application of each rule in the system implements a one-step deduction. Four examples of these deductions, resulting from application of rules in the domain of resistive network analysis, are:

1. If the voltage on one terminal of a voltage source is given, then one can assign the voltage on the other terminal.

2. If the voltage on both terminals of a resistor is given and the resistance is known, then the current through the resistor can be assigned.

3. If the current through a resistor, the voltage on one of its terminals, and the resistance of the resistor are given, then the voltage on the other terminal can be assigned.

4. If all but one of the currents into a node are given, then the remaining current can be assigned.

Thus circuit-specific knowledge is represented by assertions in the data base and general knowledge about circuits is represented by laws or rules. Some laws represent knowledge as equalities, such as the laws for resistors stating that the current going into one terminal of the resistor must come out the other, or the laws for nodes stating that the currents must sum to zero. Other laws handle knowledge in the form of inequalities, such as the law that a diode can have a forward current if and only if it is ON, and can never have a backward current.

When a circuit-specific assertion (e.g., the voltage on a collector has values 3.4 volts) is added to the data base, several rules representing general circuit knowledge may match it and thus be activated (in the example, all the other elements terminals connected to the collector will be known to have 3.4 volts). The names of the activated rules will be put on a queue, together with information such as the place in the circuit that the rule is applied. Eventually this information will be taken from the queue and processed, perhaps making new deductions and starting the cycle over again.

When each general rule is processed it can do two useful things: make new assertions, or detect a contradiction. The new assertion, together with its antecedents, is entered into the data base. These antecedents, the asserting rule together with all the other rules asserted or used by the asserting rule, become useful when a contradiction is to be handled. This contradiction can arise when some previously-made arbitrary choice (for example, assuming some linear operating region for some non-linear component) was incorrect. The system then scans backward along the chains of deductions from the scene of the contradiction to find those choices that contributed to it. These choices are labelled NOGOOD and recorded in the system so that the same combination is not tried again. An example of a NOGOOD deduction could be one that says it cannot be simultaneously true that a transistor is cut off and a diode is conducting if the two are connected in series.

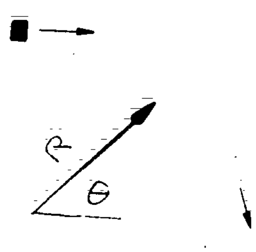
The forward reasoning together with the intelligent reduction of the possible search space effected by the NOGOOD assertions gives the system a flavor suggestive of the behaviour of the circuit expert. The justifications for deduced facts allow the user to examine the bases for their deduction. This is useful both for understanding the operation of the circuit, as well as for overcoming any problems arising within the set of general rules. For example, a device parameter not mentioned in the derivation of the value for a voltage has no part in determining that value. If some part of the circuit specification is changed (a device parameter or an imposed voltage or current) only those facts depending on the changed fact need be removed and rededuced, so small changes in the circuit may require only a small amount of new analysis.

For more details of the work see Sussman et al., 1975, and Stallman et al., 1976.

B.2. Quantitative and Qualitative Reasoning

Also at MIT, de Kleer has written a computer program to solve problems involving the motion of a particle under gravity on a variety of paths (de Kleer, 1975). He calls these "roller coaster" problems. E.g.,:

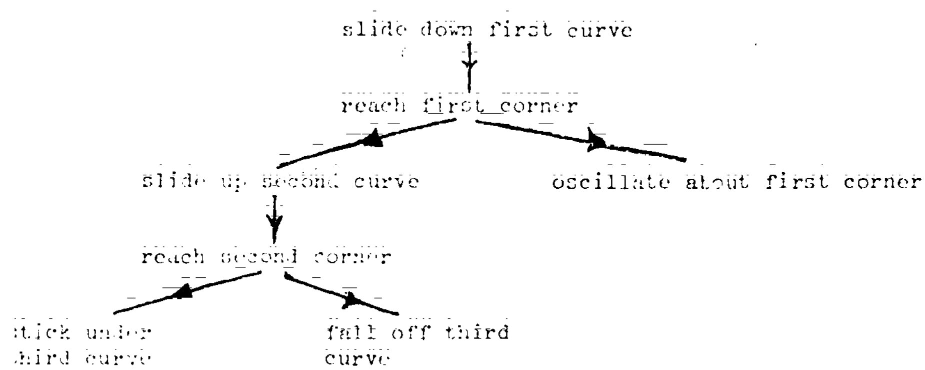
What is the minimum height h for which the particle will still loop the loop?



At what angle θ will the particle leave the circle?

These problems call for a mixture of qualitative and quantitative reasoning. The qualitative reasoning is responsible for deciding what kind of motion can take place, for instance in the loop-the-loop problem the particle might: oscillate about point a; fall off at some point b; or loop the loop. The quantitative reasoning is responsible for deciding precisely under what conditions each of these possibilities will occur. In de Kleer's program these two kinds of reasoning are clearly separated, with the qualitative reasoner proposing possibilities which are later checked out by the quantitative reasoner. This rigid separation eventually proves a liability since it hampers the flexible interaction of the two components.

The contribution of de Kleer's work lies in the design of the qualitative reasoner, which works by a process he calls "envisionment". For each shape of curve the program has a list of possible behaviours, e.g. a particle travelling uphill can reach the top and pass to the next curve, or it can slide back down again. Each of these possible behaviours puts it in a new situation from which further possibilities arise. Thus the program builds up a tree of possible behaviours, for instance in the loop-the-loop example:



etc.

This tree is then passed to the quantitative reasoner which calculates what conditions have to hold for the particle to take the branches which lead to the desired state of looping the loop.

B.3. Reasoning in Semantically-Rich Domains

Two groups of researchers at Carnegie-Mellon University are studying reasoning patterns in areas of applied mathematics. Hinsley, Hayes, and Simon are studying the reasoning and solutions to algebra word problems, and Bhaskar and Simon the solutions to problems in chemical engineering thermodynamics.

These researchers describe problem-solving domains such as the above as semantically-rich. This seems a good characterization in that large yet fairly well-defined amounts of prior semantic knowledge and task-related information are necessary for solving such problems. For example, it takes much more than an intelligent person and a "textbook" of relevant information to solve problems in thermodynamics. It is not information as available to the problem solver that is important, it is rather how the information is organized and stored, that is, information as useful.

As an example of a system without complete semantic information available, consider Bobrow's STUDENT. This system, designed to solve algebra word problems, attempts to solve these problems by a "direct translation" process which attempts to translate sentences of the problem directly into equations, and then to solve these equations. "The distance between Boston and New York is 250 miles" becomes "(the-distance-between-Boston-and-New-York) = 250 x miles". STUDENT also recognizes key words such as "Distance" and can respond by adding "Distance = Rate x Time" to the equation list. This direct translation process and recognition of key words offers a good first approximation to human problem solving in these domains, but it is unable to deal effectively with the semantic information which is necessary to expose as nonsensical "The value of N nickels and D dimes is 93 cents".

The study of the semantics of a problem domain is very important for designing a computer program to solve problems, as well as for the human engineer solving problems. Several studies have shown (Marples, 1976; Marples and Simpson, 1975; and the authors' own work with problem-solving subjects, Luger, 1977) that it isn't what information is available to the problem-solving subject, but rather how this information is used, that brings success in problem solving. For example, knowing that a resolution-of-forces equation is relevant in determining accelerations of weights hanging over pulleys is only a small part of solving the problem. Much more important is the knowledge of how friction in a pulley may affect the tension in the string over the pulley, and how fixed contacts between the weights and string and the extensibility of the string may affect the acceleration of the particles and strings. This is the semantic content of the problem domain: it must be carefully specified for any computer program that would be of any interest, and it certainly marks the expertise of the successful problem solver.

Hinsley, Hayes, and Simon discuss the notion of problem schemata. These are sets of facts, relations and heuristics present in the problem-understanding process that allow the semantics of the problem domain to be properly processed. In the money example cited earlier, these facts and heuristics would determine that nickels and dimes were nondivisible units of money worth five and ten cents repsectively, and that no sum of them could equal 93 cents.

The Hinsley, Hayes, and Simon study ran five experiments to determine when and how human subjects employed problem-type schemata in problem solving, that is, how the humans organized and structured semantic information in the process of understanding and solving algebra word problems. In particular, they demonstrated (1) how subjects recognize problem categories; (2) that this categorization often occurs very early in reading the problem; (3) that subjects possess a body of information about each problem type which is potentially useful for formulating problems of that type for solution; and (4) how this category information is actually used to formulate problems in the process of their solution.

The Bhaskar and Simon and Hinsley, Hayes and Simon research has not, as yet, led to their successful creation of a computer system to solve problems in different areas of applied mathematics. It is best to understand this work as a "prolegomena" to future problem-solving systems. This, indeed, is the main reason for including their work in this survey - not because it itself provides a useful model for machine or human problem solving, but because it provides a framework for future work in mechanical problem solving as well as an important key to the expertise and failings of human problem solvers.

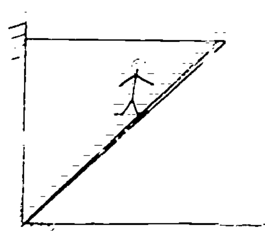
The next important step in designing a mechanical problem-solving system is to specify the contents of the problem-type schema. That is, to select a problem domain and to attempt to fully specify the semantic information necessary to solve an interesting class of problems within this domain. The ISSAC system has done this for equilibrium problems (B.4) and the MECHO project has done it in the domain of pulley problems (C).

B.4. Solving Equilibrium Problems

Novak at the University of Texas, Austin, has developed a program called ISSAC for solving physics problems. ISSAC takes several simple statics problems stated in English, translates the English into several internal representations and solves the problem. Novak claims that is is necessary to use commonsense knowledge and 'hidden' laws of physics to infer the relationships needed for solving the problem.

To investigate the Novak system, it is best to examine a problem in detail: "The foot of a ladder rests against a vertical wall and on a horizontal floor. The top of the ladder is supported from the wall by a horizontal rope 30 ft long. The ladder is 50 ft long and weighs 100 lb, with its centre of gravity 20 ft. from the foot, and a 150-lb man is 10 ft from the top. Determine the tension in the rope."

ISSAC uses syntactic and semantic information to parse the English sentences into a representation more amenable to automatic problem solving. Novak defined several categories in com[illegible] every possible type of object that could be mentioned. A ladder is a PHYSICAL ENTITY; the top and the foot of a ladder are LOCATION PARTS (meaning that the use of the word 'top' allows one to designate a particular area of the ladder), the weight and length of a ladder are ATTRIBUTES, a rung of a ladder is a PART, and 'by the wall' indicates a LOCATION for a physical entity. In the program the general categories are defined as SFRAMES. Each SFRAME contains specific instructions about satisfactorily completing itself. For instance 'top' will trigger a LOCATION PART SFRAME, which will know that 'top' must be connected to a PHYSICAL ENTITY such as a 'ladder'. This information is very necessary to correctly associate all the 'tops' and 'feet' of ladders mentioned in the above problem. It may seem painfully obvious that the 'top' in the second sentence and the 'top' in the third sentence refer to the same place and that the man is therefore 10 ft. from the point at which the rope is connected, but this is the type of inference that a program could easily fail to make, resulting in a misunderstood problem situation. When the parsing has been completed, all of these 'simple' inferences have been made, thanks to the SFRAMES, and the program translates its abstract model of the ladder into more concise geometric form and presents it on a graphics screen. The picture produced is similar to the one below:



Producing a picture tests reference ambiguities such as the one mentioned above about the 'top'. Obviously, if the spatial relations cannot be worked out sensibly, something must have gone wrong in the parsing.

The program is still not ready to generate equations. At this point the problem has only been understood in commonsense terms of physical entities and their locations with respect to each other. Now the effects these physical entities have on each other need to be accounted for in terms of forces. This requires more specific physical information, such as the knowledge that because of gravity any object with a mass exerts a force downwards, and that any force exerted on an object causes a reaction of equal and opposite force to be exerted, assuming that the objects remain stationary. Using this information, ISSAC assumes forces exist everywhere two objects are in contact with each other. Again, these 'laws' may seem painfully obvious but realizing how carefully they need to be spelled out for the computer can give insights into possible problems students could have. ISSAC is also given another type of problem-solving information, this time relating to idealizations of real-world objects as they are commonly used in statics problems. ISSAC must recognize that the ladder can be idealized as a LEVER while the wall and floor are frictionless plane SURFACES and the man is a WEIGHT. ISSAC has been told that ladders are idealized as LEVERS.

Because of the limited domain, there is no reason for a ladder to be anything else, although it could easily be a WEIGHT in another type of problem. The man presents more of a problem, because even in this domain, men can be given more than one idealization, i.e. WEIGHT or PIVOT. To resolve this ambiguity, ISSAC makes use of the commonsense knowledge that a WEIGHT is usually supported and a PIVOT usually supports something. In this problem the ladder supports the man, so the man must be a WEIGHT. It is clear that choosing an appropriate idealization is not always trivial, and that in complicated problem-solving areas it could present a serious deductive problem.

Since all of the problems ISSAC deals with are simple lever problems, once the forces have been identified generating equations is trivial. The sum of moments about a point must simply be set to zero. In mechanically writing equations for all moments, ISSAC generates several equations that a human problem solver would leave out. For instance, in the ladder problem ISSAC creates variables to represent certain horizontal forces exerted by the ladder, only to set those variables equal to zero in the next step. A competent problem solver should not need to go through such a step explicitly. However, Novak suggests that this is exactly the kind of unconscious leap that might confuse a poor student. In more complicated problem-solving situations, especially where motion is involved, taking note of all existing forces is only the first step. It is at this point that serious problem solving begins. Novak recommends that ISSAC, or a program with a similar approach, be extended to deal with dynamics problems. The authors have done this and discuss it in section C.

In summary, ISSAC solves twenty equilibrium problems competently. The program illustrates a sufficient semantic understanding of the problem situation to resolve referential ambiguities as in the 'top' example, and to interpret all objects and their relationships to each other correctly. In achieving such a level of understanding, certain necessary inferences are brought to light that can easily be overlooked in a classroom, and that might fill the gaps in a student's understanding.

C. The MECHO System

The MECHO project consists of writing a computer program to solve problems in applied mathematics. The scope of the project is broad: to take the English statement of a mechanics problem, give it to a computer, and receive in return answers to the questions asked in the problem. Three problem domains within the general area of mechanics have so far been considered: (1) acceleration, velocity, distance problems such as might arise with trains traveling between two stations (Bundy, Luger, Stone, and Welham, 1976); (2) the motion of particles over complex paths, such as the "roller coaster" problems tackled by de Kleer (Bundy, 1977); and (3) the domain of pulley systems (Luger, 1977). A simple problem in the third domain, in fact one of the first problems considered by the MECHO group, is:

> A man of 12 stone and a weight of 10 stone are connected by a light rope passing over a pulley. Find the acceleration of the man.

The thrust of the MECHO project research is pragmatic in that its primary goal is to design a computer program that can solve a wide class of problems. A further, but very important, goal of the project is the study of the running computer program as a model of human problem-solving activity. The trace of the program can be compared with the data of human protocols. The MECHO group has found this comparison fruitful, both as a source of new ideas which may be incorporated into the computer program itself, as well as to clarify important differences between the human and the mechanical problem-solving systems.

One of the important insights gained from studying human problem-solving protocols (Marples, 1975; Luger, 1977) has been to design the MECHO system as a forward or problem-driven and a backward or goal-driven problem-solving system. The remainder of this section will be spent clarifying this approach to problem solving and describing its implementation in the MECHO system.

Like the computer-aided circuit analysis described in section B1, the MECHO system employs problem-driven forward reasoning. This is accomplished by the creation and assertion of problem-type schemata. The word schema is used, following Bartlett and Piaget, to refer to a structuring of information loose confederation of relations which represent the capacity to form some task or function. In the terminology of Hinsley, Hayes, and Simon the problem-type schema contains the semantic information present in a problem situation, together with the ability to use this information for solving a particular problem.

The problem type schema itself is composed of three parts: the declaration of entities, the set of facts and inference rules describing the problem situation, and a set of default facts and explicitly stated in the protocols we took of expert problem solvers: "We'll treat the man and weight both as particles, point masses .... I'll put these two dots on the paper and join them by this rope looped over a pulley". The entities declared in this situation are two particles, a pulley, and a rope over the pulley joining the two particles.

The set of facts and inferences relating these entities are similar to the following:

(a) an angle is assigned to the string between the pulley point and the left end

(b) an angle is assigned to the string between the pulley point and the right end

(c) fixed contact of particle 1 to the left end of the string

(d) fixed contact of particle 2 to the right end of the string

(e) the tension in the left section of the string is the same as the tension in the right end if the pulley is smooth (frictionless)

(f) the acceleration of the system is constant if the particles are in fixed contact with the string.

(a), (b), (c), and (d) above are examples of facts, and (e) and (f) are inferences that represent part of the semantic content of the pulley-system domain.

Finally, the pulley-system schema contains a set of default values. These values are facts and inferences such as:

(a) if the pulley is underspecified assume it to be smooth

(b) a rope is assumed to have constant length unless specified as elastic

(c) a rope has fixed contact with objects at its end points unless the problem states otherwise

(d) the pulley itself is fixed unless specified as movable.

The MECHO system's data base is ordered so that a problem-type schema, when it is invoked, is able to create new entities and assert new facts and inferences at the "top" of the data base. It can also assert the default values at the "bottom". Thus when a call is made to the data base the facts and inferences about entities are checked first. Finally, after every other check is made, the default values are assumed. In the pulley problem above, when a resolution-of-forces formula is attempting to assign a tension to the string, it will need to know whether the pulley is smooth. (If it is, a uniform tension will be assigned to the entire string.) When no information about the friction of the pulley can be found, as in this problem, the default value of a pulley without friction will be asserted. Similarly, when the angle of the string is sought, the default value of the string with the weight hanging vertically downwards will be asserted.

So it is that the problem-type schema, representing the semantic information of the problem situation, is asserted. This represents the forward or problem-driven aspect of the MECHO problem solver. The goal-driven aspect is represented by the "Marples" algorithm for equation extraction.

The Marples algorithm was suggested by D. Marples from his work with engineering students at Cambridge (Marples, 1976). It is a procedure which starts from the desired unknown of the problem and works "backward", attempting to instantiate equations until a set of simultaneous equations sufficient to solve the problem is determined. The MECHO system has a focusing technique that "forces" the Marples algorithm to consider equations appropriate to the problem type, rather than to thrash about through lists of all possible equations. The focusing technique in the pulley system domain forces the Marples algorithm to consider first the general resolution-of-forces equations at the contact points of the string and weights.

Furthermore, the Marples algorithm is able to create "intermediate unknowns" in the process of solving the desired unknowns of the problems. In the pulley problem, for example, the desired unknown is the acceleration of the man. But it is impossible to determine the man's acceleration (using the resolution of forces) from the givens of the problem. Thus the Marples algorithm creates an intermediate unknown, the tension in the string at each end point of the string. When the inferences in the problem-type schema, (e) above, assign the same tension to each end of the string, the Marples algorithm produces the following equations:

$$T - 10g = 10a \qquad \& \qquad -(T - 12g) = 12a$$

These simultaneous equations are sufficient for solving the pulley problem.

The equations that may be applied to a problem situation are each encoded in a special format. The MECHO system, in asserting a particular equation, for example the resolution-of-forces equations above, specifies exactly the situations in which the equation is to asserted. This includes the specification of each variable, the possible boundary values, and all semantic information necessary for the equation to be asserted.

The general resolution-of-forces equation, for example, sums all forces acting at a point. This would be trivially satisfied in the problem above but would have to include the friction on a plane, the angle of the plane and other forces that could be acting in different pulley problems -- such as a pulley at the top of an inclined plane. Marples has stated (Marples, 1977) that the lack of this exact specification in using equations and misunderstanding the situation of their application is a major cause of mistakes for engineering students: it is not that the relevant equations are forgotten or ignored in problem solving, it is rather that the conditions of their use are misunderstood. The MECHO system specifies each equation exactly, and the conditions under which it may be used.

Finally, the MECHO system includes automated procedures for solving sets of simultaneous equations. These are described in Bundy, 1975. As noted above, a more complete description of the MECHO system may be found in Bundy, 1977; Luger, 1977; and Bundy, Luger, Stone, and Welham, 1976; Stone, 1976.

D. Summary and Conclusions

This paper has attempted to summarize several Artificial Intelligence research projects in the areas of applied mathematics. These projects were intended to demonstrate the design and use of computer systems both as interactive engineering aids and as models for solving problems in certain well-defined domains of applied mathematics. These systems may serve as models of how engineering problems may be understood and represented in the process of their solution -- both by man and machine.

Many of the same problems that arise for humans solving problems in applied mathematics are exactly those encountered by researchers attempting to design a computer system to solve problems: specifically, to design a system not limited to small sets of problems, but at the same time able to deal with the idiosyncracies of individual problems.

To deal with this conflict, each system creator has had to design an inferencing system. These include, for the computer-aided circuit analysis, the forward reasoning from the problem situation and the tagging of each new fact with the conditions and rules used in its assertion. The de Kleer system attempts to control search by the quantitative vs. qualitative reasoning distinction and the use of envisionment. The Hinsley, Hayes and Simon and Baskar and Simon studies, while not explicitly constructing a problem-solving system, have outlined conditions that could apply to the creation of a successful problem solver. The Novak program solves twenty equilibrium problems from their English-language statement. It has sufficient semantic understanding to resolve all referential and relational ambiguities.

The MECHO system creates a forward or problem-driven and a backward or goal-driven inference system. The problem-driven aspect is represented by full specification of the problem-type shcema. This includes the assertion of facts and inferences, both relating to the entities within the problem domain and sets of default values. The goal-driven aspect is represented by the Marples algorithm, which works backward from the desired unknown to the given facts of the problem. This often necessitates the creation of intermediate unknowns to link the desired unknown with the problem facts. The MECHO system also attempts to represent the semantics of each possible equation that may be asserted. This is intended to guarantee that the equation is only asserted in the manner and at the time appropriate.

The overall aim of this paper has been to provide a summary of some current work in artificial intelligence research in the areas of applied mathem[illegible] This summary is meant not merely to make engineers awa[illegible] some interactive aids available (the computer-aided circuit analysi[illegible] but more importantly to give some idea of the representation of information and control of inferencing necessary for the successful problem solver in these domains. The reader is recommended to consult the references for more complete descriptions of each system surveyed.

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13

PROBLEM SOLVING IN PHYSICS OR ENGINEERING: HUMAN INFORMATION PROCESSING AND SOME TEACHING SUGGESTIONS

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There are both practical and intrinsic intellectual reasons why problem solving is of central interest to engineering and the physical sciences. The practical reasons are fairly obvious: (1) Problem solving is crucially important in most engineering and scientific tasks. (2) Furthermore, many students have substantial difficulties learning how to solve problems. Thus the teaching of effective problem-solving skills is an essential, and often troublesome, part of education in engineering or the sciences.

In addition, there are substantive intellectual reasons why problem solving presents significant challenges worthy of study: (1) Modern engineering has become increasingly concerned with developing and exploiting new techniques of information processing. This trend is illustrated by the rapidly growing engineering fields of computer science and "artificial intelligence" (the utilization of machines to perform tasks that would be deemed "intelligent"). A basic understanding of problem-solving processes is of major importance in these new fields. (2) The modes of analysis used in these fields have stimulated substantial progress in the study of human information processing. Indeed, the recently emerging field of "cognitive science", which spans both computer science and cognitive psychology, emphasizes the similarities of information processing common to machines and persons. Thus human problem solving deserves at least as much careful attention as problem solving by computers. Furthermore, the study of problem solving by men and machines can be mutually stimulating and illuminating. (3) Finally, new theoretical insights about problem solving are likely to lead to practical applications useful for enhancing human problem solving and for improving the teaching of problem-solving skills.

In the following pages I shall discuss work done by me at Berkeley, partly with the collaboration of Dr. Jill Larkin, to study systematically some aspects of human problem solving relevant to physics or engineering. Our main aim has been to obtain a better basic understanding of problem-solving processes, and ultimately to use the resulting insights to design instruction for enhancing students' problem-solving skills. The problems which we have chosen for study have been sufficiently simple so that we could hope to make some progress, yet sufficiently complex to be of practical significance. Accordingly, we have focused our attention on the kinds of problems commonly encountered in basic undergraduate physics or engineering courses.

HOW WE STUDY PROBLEM SOLVING

Since we are interested in understanding how effective problem solving is achieved, we have striven to obtain insights about underlying mechanisms responsible for problem-solving performance. Thus we have tried to formulate some explicit theoretical models which, even if hypothetical or primitive, offer the potential of unifying and predicting various observations. As usual, it is important to choose the level of description of such models judiciously in accordance with one's aims. Thus a study of problem solving need no more concern itself with microscopic neural or psychological processes than computer science need deal with the intricacies of transistors, or than circuit theory need deal with the quantum mechanics of electronic conduction in materials. But useful models of problem solving can appropriately describe observations at the level of the underlying information processing used by persons solving problems. (For example, they may specify how information can be organized usefully and described symbolically to facilitate optimum retrieval in complex contexts; what strategies can be used to effect such retrieval successfully; and similar issues.) The systematic analysis of information processing has, of course, been successfully pursued in artifical intelligence (Winston, 1977) and in studying human problem solving in relatively simple domains (Newell and Simon, 1972). Attempts to extend models of human information processing to more complex problems, at least at some level which is not unduly detailed, seem thus potentially promising (Bhaskar and Simon, 1977).

In trying to observe problem-solving performance, we usually avoid collecting extensive statistical data on many persons, since the information thus obtained is ordinarily too gross to provide specific insights about underlying mechanisms of problem solving. (As Aaron Levenstein once said, with considerable justification, "Statistics are like a bathing suit. What they reveal is suggestive, but what they conceal is vital".) Instead, we strive to make very detailed observations of relatively few persons in controlled experimental settings. In particular, we often observe such persons solving problems after they have been instructed to talk out loud about their thought processes. Considerable detailed information can then be obtained from the resulting "protocol" consisting of a transcript of a person's tape-recorded comments, together with the person's written work during the problem-solving process. The persons thus observed in detail have include both "experts" (persons who are experienced and successful problem solvers) and "novices" (e.g., students with only limited problem-solving experience).

Needless to say, there is an intimate interplay between the formulation of hypothetical models and detailed experimental observations. Thus the validity or utility of any model must be assessed by its ability to relate or predict various observations. Conversely, judicious observations may often suggest useful models.

OVERVIEW OF THE PROBLEM-SOLVING PROCESS

To provide a framework for the following discussion, let me briefly outline a particular point of view (or model) about the problem-solving process:

There exist some problems which are so simple or familiar that they are readily "solvable" (i.e., that they can be solved by available methods whenever desired).

A more complex problem can then be decomposed into one or more "subproblems" (where a "subproblem" is any problem whose solution facilitates the solution of the original problem). Any such problem which is solvable can be solved; any such subproblem which is not readily solvable can, in turn, be decomposed into further subproblems. In this way one can proceed successively until the original problem is completely decomposed into solvable subproblems. The original problem can then clearly be solved.

What are the prerequisites necessary to implement the preceding process of problem solving by successive decomposition into subproblems? One necessary prerequisite is obviously the existence of some repertoire of basic solvable problems whose solutions are widely useful to cope with more complex problems. Accordingly, I shall devote the next section of this article to discuss briefly some work concerned with acquiring such a repertoire of basic solvable problems useful for physics and engineering problems.

But such a basic repertoire of solvable problems is not sufficient to implement the problem-solving process efficiently, if at all. The difficulty is that a moderately complex problem can be decomposed into possible subproblems in many possible ways, but only very few of these decompositions lead to solvable problems and thus to a solution of the original problem. Hence it is imperative to proceed systematically so as to select subproblems judiciously. A practical implementation of the problem-solving process requires thus the following essential ingredients: (1) A strategy for efficiently decomposing a problem into judiciously selected subproblems. (2) A "knowledge base", consisting of basic solvable problems and other information, carefully organized and symbolically described so as to facilitate the implementation of the strategy. Accordingly, I shall devote most of this article to discussing these centrally-important aspects of problem solving.

In the course of the discussion, I shall several times point out some practical implications for teaching students improved problem-solving skills. However, I should warn the reader that some of the work I shall describe, particularly in the latter part of the article, is still in progress. Hence some of my conclusions and implications must be regarded as tentative. (Indeed, at the end of the article I shall spell out explicitly some of the major gaps and limitations of this work.) Nevertheless, I hope that the reader, properly forewarned, may find the discussion of some current ideas more stimulating to his or her own thinking than a report dealing exclusively with work solidly buttressed by experimental evidence.

BASIC INFORMATION UNITS USED FOR PROBLEM SOLVING

In physics or engineering certain key relations (i.e., statements or equations summarizing [illegible] laws or definitions) constitute the basic informational b[illegible] used to construct the solutions of many different probl[illegible] any such relation is equivalent to a class of primitive [illegible] problems. For example, Ohm's law $V = RI$ relating the potential [illegible] V, the current I, and the electric resistance R) is equivalent to the class of basic solvable problems allowing one to find any of the three quantities V, I, and R from the other two. An adequate knowledge of such basic relations is thus a necessary condition for problem solving, although it is definitely not a sufficient condition.

Experts do, indeed, use basic relations effectively as basic building blocks for constructing solutions to more complex problems. In particular, observations suggest that experts have the following abilities: (1) Experts "understand" any relation with which they are familiar, in the sense that they [illegible] use this relation reliably and flexibly to solve corresponding basic subproblems in various contexts. (2) Experts have a general "learning skill" whereby, after being presented with a description of a new relation unfamiliar to them, they subsequently "understand" the relation so that they can use it appropriately.

[illegible], our observations (discussed more fully later) show that [illegible] students have neither of these abilities. Instead, for many [illegible] known relation is merely a memorized formula, which can be quoted [illegible] used.

[illegible] is to formulate an explicit model specifying what [illegible] are required to "understand" any relation well [illegible] class of solvable subproblems that can be used [illegible] in diverse [illegible] contexts. Our analysis identified [illegible] following specific abilities:

[illegible] Properties: Knowing the properties of the quantities in the relation so that these quantities can be properly recognized and identified. For example, knowing whether each quantity is a scalar or vector; knowing its units, its typical magnitude, ...

(2) Discriminations: Making appropriate discriminations between symbols (either quantities or entire relations) and their referents (i.e., the things to which they refer). Thus a particular symbol should be correctly assigned to a particular referent, without being either misassigned to inappropriate referents or confused with other symbols. (See Fig. 1.) There are two types of such discriminations: (a) Discrimination of quantities, so that each quantity in a relation is correctly assigned to its referent (i.e., correctly interpreted) and not confused with other quantities. For example, in the case of Newton's gravitational force law $F = G m_1 m_2/R^2$; the quantity G should be properly interpreted as a universal constant of nature; nor should it be confused with g, the gravitational acceleration at a particular position. (b) Discrimination of relations, so that a particular relation is correctly assigned to its referent (i.e., correctly applied to an appropriate situation) and not confused with other quantities. For example, Newton's gravitational force law should be applied only to particles rather than to large objects of arbitrary shape; nor should it be confused with the relation F = mg for the gravitational force near the earth.

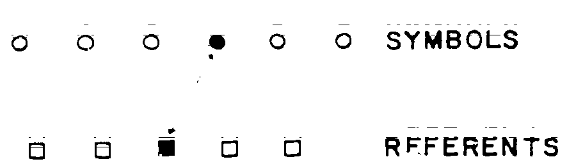

Fig. 1. Diagram indicating schematically the connection between a particular symbol (black circle) and a particular referent (black square).

(3) Alternative symbolic representations: Being able to use interchangeably different symbolic representations of a relation (e.g., expressed in algebraic symbols, in words, in pictures, in graphs, ...), since some of these representations may be more useful than others in various contexts.

(4) Application to solve basic problems: Being able to apply the relation to find any quantity from the others, or to infer functional relationships (such as proportionality or scaling relationships).

Experts seem to "understand" any relation familiar to them, in the sense that they possess for it all the preceding abilities. On the other hand, some of our carefully designed experiments showed that many students do not possess these abilities about relations which they have studied. Furthermore, even after a couple of months spent in a physics course where students learn many relations, most students do not acquire the learning skill which would permit them to gain independently such abilities for any new relation explained to them.

On the other hand, on the basis of our analysis of the abilities requisite for understanding a relation, we have been able to design an instructional program whereby students could be taught how to learn relations more effectively. This instructional program made students explicitly aware of the abilities necessary for understanding a relation; then it provided these students with practice and feedback on a limited set of physics relations. After such instruction, it was found that students had substantially improved their general learning skill. Thus they could now better (1) gain independently an understanding of new physics relations presented to them in a realistic course context; (2) gain an understnading of new relations outside of physics; and (3) gain such an understanding even without elaborate teaching materials.

These investigations, which are described much more fully elsewhere (Reif et al. 1976; Larkin and Reif, 1976), show that an explicit analysis of prerequisite abilities can identify some essential requirements for problem solving and can be used to teach a generally-useful learning skill. Furthermore, these investigations have the following implications for practical instruction: (1) They show that some seemingly simple but important learning skills are often not automatically acquired (at least, not efficiently) as a result of ordinary instruction in science courses. (2) They indicate that some general learning skills can be taught successfully, if they are viewed as explicit instructional goals and taught deliberately. (3) They suggest that such efforts, designed to teach generally-useful learning skills, can have wide utility to students in similar contexts, and even in more remote contexts. (4) Furthermore, such efforts may facilitate future instruction: once students have acquired better independent-learning skills, they can subsequently be taught more economically with less extensive reliance on elaborate teaching materials.

GENERAL FEATURES OF EFFECTIVE PROBLEM SOLVING

The preceding section dealt with some of the basic information units (relations) which must necessarily be available to permit any problem solving. However, as emphasized previously, the mere existence of such information units is not sufficient to insure effective problem solving, even when they provide a useful repertoire of basic solvable problems. Indeed, except in the case of quite simple problems, success in solving problems requires much more than a knowledge of facts and principles. Success depends crucially on the organization of this knowledge and the strategies for using it. The rest of this article will be devoted to discussing some of these central issues.

By observing in detail how novices and experts solve problems, we have come to identify the following salient differences between their problem-solving behavior (even when the novices, as well as the experts, have a very good command of all the basic knowledge relevant to a problem):

A novice, such as a relatively inexperienced physics student, usually strives to retrieve various individual formulas and then tries to combine them one-by-one until he or she attains a solution. If the problem is moderately complex, the novice frequently fails to attain a solution, not so much because he makes mistakes but because he "gets stuck" and does not know what to do next.

By contrast, an expert seems to approach a problem in a surefooted way that almost always leads to a solution. He or she tends initially to engage in seemingly imprecise verbal or pictorial arguments, without getting involved in details [illegible] resorting to any equations. He then proceeds by a process of successive elaborations, wherein equations are used only sparingly in [illegible] stages of the solution process. Furthermore, his arguments seem [illegible] proceed smoothly and continuously with only minor hesitations between [illegible]terrupted chains of reasoning.

To give a trivial example, consider the problem illustrated in Fig. 2 where the information to be found is the electric potential V at the junction of the wires. Many novice students find this seemingly simple problem difficult and fail to solve it. Typically, they attempt to write down various equations involving several unknowns, and then get lost in the morass. On the other hand, an expert typically starts the problem by making verbal statements such as: "The current produces a potential drop in each wire. These potential drops are related to the resistances of these wires, and these resistances are related to the geometry." Having made these statements, the expert then quickly solves the problem with a minimum of equations.

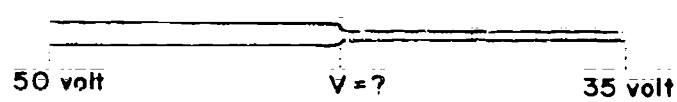

Fig. 2. Current flowing through two joined wires of the same material and length. The diameter of the first wire is twice as large as that of the second.

Let me then make a few general theoretical comments to indicate that the experts' problem-solving behavior makes sense. As pointed out previously, problem solving can be conceived as a process of successive decompositions into simpler subproblems. It is then crucially important to select judiciously, among the many possible subproblems, those relatively few subproblems most likely to be useful. A good strategy for achieving this aim consists of subdividing the problem-solving process into successive stages such that each stage involves only a selection among very few alternatives. A person's "knowledge base" (i.e., the store of knowledge which he or she brings to the problem) should then also be appropriately organized and symbolically described, so that the preceding strategy can be implemented effectively.

How can such a good strategy be realized in practice? One obvious way is to organize one's knowledge base in such a way that individual principles and methods are appropriately grouped together (or "chunked") into a relatively small number of coherent methods which can be used to solve typical major subproblems. The resulting advantage is that the problem solver then merely needs to select among a few major methods, instead of having to choose among many separate principles or methods for solving minor subproblems. For example, in the domain of mechanics, it is useful if the knowledge base contains a coherent method for applying the equation of motion $ma = F$ and not merely separate principles subsumed under this method (i.e., separate principles such as the equation of motion $ma = F$, the superposition principle stating that F is the vector sum of individual forces, force laws specifying the properties of various common forces, ...)

Even more important, the desirable characteristics of a good strategy can be achieved by executing the problem-solving process through a series of successive refinements. The early stages of the process then merely involve choices among alternatives described at a global level, unencumbered by excessive detail; while the later stages involve choices at a more detailed level. The advantage of such a strategy (which is similar to the "top-down" approach useful for complex computer-programming tasks) is that only a few decisions need be made at any one time. Major decisions are made first, each of them implying a selection among a whole class of more detailed alternatives; and more detailed decisions are made later (with full awareness of the major features of the entire problem).

This problem-solving strategy of successive refinements is very powerful and deserves further comment. First, different symbolic descriptions may be most useful at various stages of refinement. Thus verbal and pictorial descriptions are often useful for the early solution stages to achieve global descriptions, imprecise but unobscured by distracting detail. On the other hand, descriptions in terms of mathematical symbols and equations can be most useful at later stages, to achieve more refined descriptions of specific details. Note that these comments agree with our observations of experts' problem-solving behavior.

Second, it is worth pointing out that the implementation of a problem-solving strategy of successive refinements is helped by an appropriately-structured knowledge base. In particular, this knowledge base should be hierarchically organized into multiple levels which describe the same situation with different degrees of detail and with correspondingly different symbolic representations. For only then can the strategy retrieve information which is appropriately described with the degree of refinement suitable for a particular stage of the problem-solving process.

As an elementary example, consider again the simple problem of Fig. 2. As mentioned earlier, when starting work on the problem, an expert may make a statement such as "the resistances of the wires depend on their geometry". The word "geometry" is quite vague and refers loosely to various geometric factors such as length, diameter, shape, ... Early in the solution, such a vague statement about dependence on geometry is much more useful for planning a solution than would be a distractingly precise statement (specifying proportionality to length and inverse proportionality to area). Only later in the solution, when details need to be worked out, will the problem solver want to invoke the specific equation R = ρL/A which relates the resistance R precisely to resistivity ρ, length L, and cross-sectional area A. However, note that the global statement "resistance depends on geometry" can only be made if this statement is part of the person's knowledge base. On the other hand, the vocabulary of many novice students does not even include the word "geometry", used in the experts' vague sense of "miscellaneous geometric factors."

What experimental evidence can we adduce to confirm that the preceding characteristics of a problem-solving strategy are indeed important? One kind of evidence is provided by noting that these characteristics are exhibited in the problem-solving protocols of experts. In addition, some specially-designed experiments carried out by Jill Larkin (Larkin, 1977) provide more specific evidence. For example, in one such experiment Larkin measured the times elapsed between successive statements made by experts or students solving a particular mechanics problem. In the case of an expert, unlike that of a novice, Larkin found that most elapsed times are short and that there are only a few longer times corresponding to hesitations between major trends of thought. These results are quantitatively consistent with a model that novices retrieve many small information units independently, while experts retrieve their information in larger coherent units.

In another of Larkin's experiments, students first received instruction to gain a good understanding of all the principles of DC circuit analysis and to apply these principles individually. When these students were afterwards given some circuit problems to solve, they were largely unsuccessful. Then one group of these students was shown how the principles can be visualized pictorially and how they can be applied jointly in some coherent methods. The second group of students was given equivalent training on how these principles can be combined algebraically. Neither group was given any practice in problem solving. Yet afterwards the first group (but not the second group) was much more successful in solving circuit problems. This experiment shows that mere organization of existing knowledge base, even without any practice, can effect major improvements in problem-solving performance.

The various kinds of evidence lend support to the comments made in this section about some important general characteristics of effective problem solving. In the next section I shall use these general characteristics to propose a specific problem-solving strategy. But the general comments of this section already suffice to suggest some practical implications for teaching students improved problem-solving skills.

Some of the implications are negative, warning us what not to do, and indicating that some teaching practices common in science and engineering courses may be deleterious to students' problem-solving skills. The reason is that many instructors (myself included) often emphasize mathematical formalism unduly and shun seemingly vague verbal or pictorial descriptions. Thus many students, even though they may initially find words more congenial than mathematical symbolism, come to believe that verbal arguments should be disparaged as imprecise and inappropriate for scientific work. As our studies indicate, nothing could be further from the truth. Seemingly vague verbal and pictorial descriptions are invaluable for the crucial early design decisions in a problem-solving process, and are commonly used by expert problem solvers.

Thus students should not be led to suppress their natural verbal inclinations. Instead, they should be taught how to use qualitative verbal or pictorial arguments effectively, and how to translate them into more precise mathematical form when appropriate.

On the more positive side, the general remarks of this section i: ly the following suggestions for teaching improved problem-solving skills: (1) Providing students with hierarchically-organized knowledge described at multiple levels of detail. (2) Teaching students to approach problem solving by a process of successive refinements, from more global to more detailed aspects of a problem (instead of proceeding linearly by combining individual equations). Indeed, students might profit by being taught explicitly how to design problem solutions, without necessarily implementing them in detail.

A DETAILED PROBLEM-SOLVING STRATEGY

The preceding section discussed some general features of effective problem solving. Let me now attempt to translate these general ideas into a more specific strategy useful for solving physics or engineering problems. This strategy reflects my current thinking, not to be blamed on Jill Larkin, and is thus subject to further revisions, elaborations, and experimental testing.

strategy seeks to decompose any problem into a sequence of simpler subproblems described at successively more detailed levels of refinement. In particular, the strategy first seeks to decompose any moderately complex problem into a sequence of five major "standard subproblems" of general utility. The first two major subproblems seek to bring the original problem into a form facilitating the third and usually most difficult subproblem, the actual "design" of the solution. The last two major subproblems then seek to exploit this design to produce a f al solution. Let me now briefly outline these five major subproblems. In the next section I shall then exemplify the strategy by applying it to solve an illustrative problem in physics.

The original form of a problem is usually determined by various extrinsic factors (such as brevity of statement in a textbook), rather than by intrinsic utility to a problem solver. Thus the first helpful task is to start from the original problem statement to generate a problem description most convenient the problem solver. We may call this task "translation". This problem description still incorporates very little of the general or theoretical knowledge which the problem solver can fruitfully bring to bear on the problem. Thus the next helpful task is to start from the previous convenient problem description expressed optimally in terms of theoretical background knowledge. (We may call this task "analysis".) The problem is now in rm facilitating the search for a solution. Accordingly, the next task starts from this useful problem formulation to generate a "schematic solution", i.e., a decomposition of the problem into subproblems which are deemed solvable, although not yet explicitly solved. (We may call this task "design".) Clearly the next task then involves actually solving these various subproblems in order to generate a tentative solution of the problem. (We may call this task "implementation".) Finally, the last task consists of assessing and revising this tentative solution in order to generate a good solution which is both correct and optimal.

The preceding five tasks constitute the five major subproblems of the proposed problem-solving strategy, as illustrated in Fig. 3. Note that each such subproblem is completely specified by its input and desired output (e.g., by "convenient description" and "useful formulation"), and that the name given to a subproblem (e.g., "analysis") is merely a convenient label without any intrinsic significance.

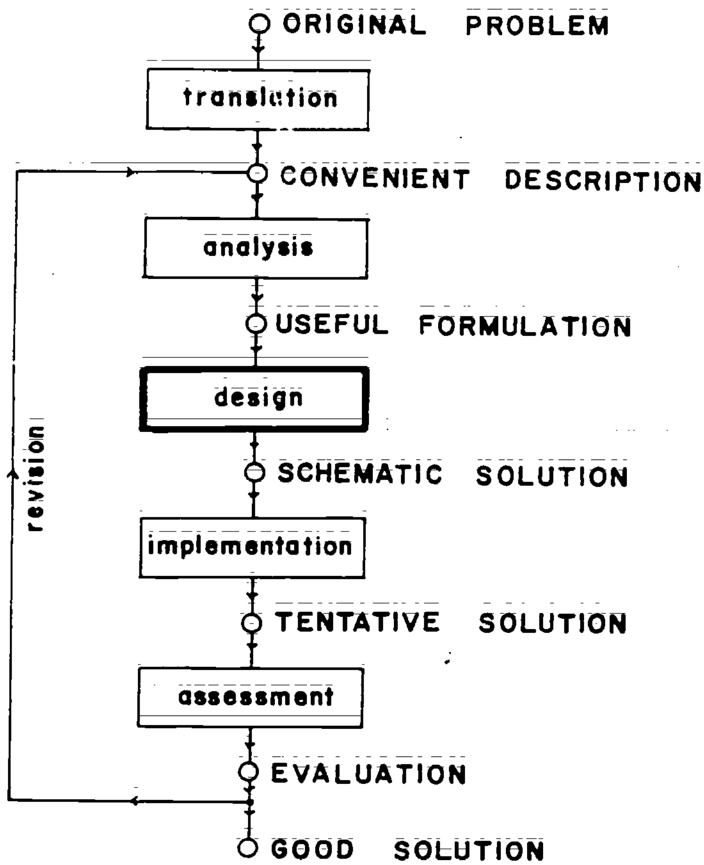

Fig. 3. Major subproblems of the problem-solving strategy.

Let me now indicate briefly how each of these major subproblems is to be solved.

Translation

To find a convenient problem description:

(1) Introduce a convenient symbolic representation (draw a design and choose labeling symbols)

(2) List specified and desired information

Analysis

To find a useful problem formulation:

(1) Retrieve relevant basic knowledge

(2) Generate a theoretical problem description by
   - (a) expressing problem statements in theoretical form
   - (b) considering significant problem features (systems, stages of a process) and applying theoretical knowledge to them

(3) Find global information about problem by
   - (a) identifying general properties (e.g., invariants)
   - (b) identifying possible types of solutions (including limiting and critical cases)

Note that all these things are to be done at a high level of description, unencumbered by distracting details.

Design

To find a schematic problem solution:

(1) Search for solution by using the "generate-and-modify" strategy described below

(2) Do this at successively more detailed levels of refinement, if necessary

The search strategy mentioned (a form of "means-ends analysis") tests the problem at any stage to identify the difficulties involved in finding the desired solution. These difficulties are of two kinds: lacks, i.e., desirable features (such as needed information) which are absent; and flaws, i.e., undesirable features (such as extraneous information) which are present. The general strategy is then to carry out appropriate actions to generate (fairly uncritically) useful information likely to supply the lacks; and then to modify this information judiciously by removing the flaws in it. Typically, useful information can be generated by applying some relevant principle or method to the problem; information containing flaws then can be modified suitably by combining it with other information which relates such flaws to more desirable information elements. This cycle of generation and subsequent modification can be applied repeatedly until one attains the solution (i.e., all the desired information without any remaining flaws).

Implementation

To find a tentative solution, solve in detail the various (presumably solvable) subproblems identified in the schematic solution.

This solution process is not really different from the one used in design, except that the problems to be addressed are now more straightforward and are solved at a more detailed level.

Assessment and revision

To find a good solution, assess the tentative solution by examining the following questions and making appropriate revisions until the answers to these questions are affirmative:

(1) Is the solution unambiguous?
(2) Is it internally and externally consistent (i.e., self-consistent and in agreement with other known information)?
(3) Is it complete?
(4) Is it optimal (e.g., might it be simpler)?

The preceding description of the strategy is general and fairly abstract. To show that the strategy has quite specific implications, it is instructive to apply it in detail to a particular simple example. After that we shall be in a better position to point out the general implications and limitations of such a strategy

ILLUSTRATIVE EXAMPLE

Table 1 states a physics problem typical of the kind encountered in a basic college physics course. To illustrate our proposed problem-solving strategy in greater detail, let us apply it systematically to this problem.

After descending a mountain slope, a sled moves up another hill whose top, which has a radius of curvature R, is at a height h above a valley. Assuming that the sled moves over the snow-covered ground with negligible friction, at what height above the valley must this sled start from rest on the mountain slope so that the sled just leaves the ground at the top of the hill?

Table 1. Original statement of a physics problem.

Translation

According to the preceding section, the strategy leads one first to generate a convenient problem description which consists of the diagram of Fig. 4; of some convenient symbols (such as the initial height y of the sled); and of Table 2 which lists the relevant information about the problem. Note that this information is now available in much more usable form than in the original Table 1. This is beacause it is represented pictorially, corresponding more closely to the observable physical situation, and because it is listed succinctly in the order of the specified motion process.

Specified information

Start of sled motion:

speed $v = 0$

unknown height y

Sled moves without friction (1)

Top of hill:

height h

radius of curvature R

sled just leaves the ground (2)

Desired information

$y = ?$

Table 2. Convenient description of the problem of Table 1. (To be supplemented by Fig. 4).

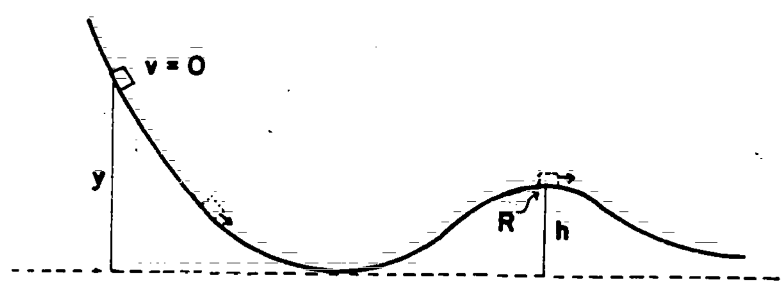

Fig. 4: Diagram representing the information provided in the problem of Table 1.

Analysis

The "analysis" part of the strategy, as outlined in the preceding section, leads to the following information:

(1) The relevant basic knowledge concerns mechanics, i.e., principles involving motion, forces, and energy. A theoretical description of the problem should thus involve these concepts.

(2a) Statement 1 in Table 2 specifies that the sl . moves without friction. Thus the frictional force is zero, i.e., the surface force F_s exerted on the sled by the ground is always perpendicular to the ground surface.

(2a, continued) Statement 2 in Table 2 specifies that the sled just leaves the ground at the top of the hill. The word "just" indicates a critical case between two types of solutions, one type where the sled still touches the ground at the top of the hill (so that the surface force $F_s \neq 0$ at the top of the hill), and the other type where the sled has left the ground before it reaches the top (so that $F_s = 0$). The critical case between these two types of solution is then that where the sled at the top of the hill still barely touches the ground and $F_s = 0$.

(2b) The only relevant system is the sled. The motion process can be subdivided into two stages, the entire initial motion until the sled reaches the top of the hill, and the stage where the sled is at the very top of the hill. Consider first the initial motion. Application of force arguments (and the equation of motion) leads to the conclusion that the sled moves as a result of the downward gravitational force $F_g = mg$ and the surface force F_s on the sled. Application of energy arguments leads to the conclusion that, since there is no frictional force, energy is conserved. Thus the kinetic energy and the speed of the sled vary simply with its corresponding gravitational potential energy (or height).

(2b, continued) At the top of the hill, the surface force $F_s = 0$ so that only the gravitational force acts on the sled.

(3) As already mentioned, the energy of the sled is a simplifying invariant. We have also mentioned types of solutions and shall not do more along these lines (although we could).

The results of this analysis lead to the useful problem formuation partially summarized in Table 3. Note that the previous statements describing the problem in Table 2 have been reformulated into theoretical statements about the surface force. Also the crucial importance of the gravitational force (never even mentioned in the original problem statement) is now fully apparent. The problem formulation of Table 3 is clearly much easier to solve than the earlier problem descriptions of Tables 2 or 1. Indeed, if Table 3 were transcribed into prose form and were given as a textbook problem to students, it would be much simpler for them than the original problem of Table 1.

Design

A schematic problem solution, obtained by the "generate-and-modify" strategy of the last section, is succinctly summarized in Table 4. This table indicates explicitly how the problem is decomposed into successive subproblems and what information results from the solution of each solvable subproblem. Such information is described by global statements which disregard distracting detail in order to specify merely an existing relation between key quantities. (In Table 4, the quantities in these statements

Specified information
Start of sled motion:
speed v = 0
unknown height y
Sled motion
Surface force $\underset{\sim}{F}_s \perp$ surface
Mass of sled m
Downward gravity force $\underset{\sim}{F}_g = m\underset{\sim}{g}$
Energy is constant
Top of hill
height h
radius of curvature R
$\underset{\sim}{F}_s = 0$
Desired information
y = ?

Table 3. Useful formulation of the problem of Table 1. (To be supplemented by Fig. 4.)

are conveniently indicated by single-letter abbreviations arranged in separate columns. However, without much sacrifice in transparency one could also express such statements in words. For example, in designing the solution of the problem of Fig. 2 one might use the statement "resistance, geometry" to indicate that a particular subproblem yields information relating a resistance to geometrical factors.)

To explain the schematic solution of Table 4, let me express it in words I might use as a problem-solver. I start with the well-formulated original problem P and note that it lacks adequate information to attempt a solution. Hence I undertake subproblem P1 to find relevant information relating the sled's motion to its interaction with other objects. To solve this problem, my previous problem analysis suggests that I try to solve subproblem P1.1, applying the conservation of energy to the initial sled motion, and thus obtain a relation between the sled's initial height y and its speed v

Prob#	Problems	Results	#
P	Well-formulated problem		
P1	FIND mechanics information		
.1	APPLY E=const: initial motion	y, v	1
.2	APPLY ma=F: top of hill	a F_g	2
P2	ELIM a		
.1	APPLY centrip accel	v, a	3
.2	ELIM a: 2,3	v F_g	4
P3	ELIM v: 1,4	y, F_g	5

Table 4. Schematic solution of the problem of Table 1. (The circles and lines in the last column indicate pictorially which statements are combined to yield new statements.)

at the top of the hill. The previous problem analysis also suggests an attack on subproblem P1.2, applying the equation of motion m$\underline{a}$ = $\underline{F}$ to the sled at the top of the hill. This yields a relation between the acceleration $\underline{a}$ of the sled and the gravitational force $\underline{F}_g$ acting on it. Hence I have now schematically solved (and may thus check off) the original subproblem P1 of finding relevant mechanics information. Now I note that the acceleration $\underline{a}$ (circled in Table 4) is a flaw in my available information since it is not wanted for the solution of the original problem. Hence I undertake subproblem P2 to eliminate this acceleration. To do this, I first solve subproblem P2.1, applying the relation for centripetal acceleration. Thus I find a relation between the acceleration $\underline{a}$ and speed $\underline{v}$ of the sled at the top of the hill. I then tackle subproblem P2.2 to eliminate the acceleration $\underline{a}$ by combining statements 2 and 3, thereby finding a relation between $\underline{v}$ and $\underline{F}_g$. I have now solved the original subproblem P2 of eliminating the acceleration. Finally I note that the speed $\underline{v}$ (circled in Table 4) is a last remaining flaw. Hence I undertake subproblem P3 of eliminating $\underline{v}$ by combining the previous information contained in statements 1 and 4. In this way I obtain a relation between the initial height $\underline{y}$ and the known gravitational force $\underline{F}_g$. Hence my entire original problem P has now been solved.

Implementation

The task of obtaining the actual solution of the problem is now quite straightforward since it only remains to solve in detail the simple subproblems specified in the schematic solution of Table

4. Thus subproblem P1.1 (applying energy conservation) yields $mgy = mgh + mv^2/2$, or $y = h + v^2/2g$. Subproblem P1.2 (applying the equation of motion) yields simply $ma = F_g = mg$, or $a = g$. Subproblem P2.1 yields $a = v^2/R$ and subproblem P2.2 then produces $v^2 = gR$. From subproblem P3 we get simply $y = h + R/2$, which is the answer to the entire original problem P.

The last step of the strategy, involving assessment and revision, is so straightforward that it needs no illustration in this particular case.

DISCUSSION

The preceding illustrative example has hopefully clarified and made more concrete some of the basic ideas discussed earlier. Hence it is now possible to point out some of the general implications and limitations of this work.

The problem-solving strategy proposed in the last couple of sections can be regarded as a specific model for effectively solving problems in physics. This strategy strives to achieve the following advantages: (1) It systematically decomposes any problem into a sequence of simpler subproblems. (2) It directs one to proceed by a series of successive refinements. Thus only a few decisions need to be made at any one stage. Furthermore, major global decisions are made first, unobscured by distracting detail. These can then guide subsequent decisions at a more specific level. (3) The fact that a problem solution is described at a global level without burdensome detail (e.g., as illustrated by the schematic solution in Table 4) helps to reveal the essential features of a solution; and to modify a solution easily when a problem is changed. (4) Finally, the symbolic representation illustrated in Table 4 can itself be a useful aid in designing a solution, in helping students design solutions, or in summarizing someone else's solution (e.g., in summarizing the solution process inferred from a person's problem-solving protocol).

The problem-solving model described by the strategy is intended to simulate some of the central features of the problem-solving processes of experts. However, it should be noted that the actually observed problem-solving behavior of an expert may conceal much implicit information processing which is not explicitly apparent. The reason is that an expert can often use very few words to invoke fairly complex solvable subproblems which he has accumulated in his repertoire as a result of his extensive past problem-solving experience. The familiarity with such major solvable subproblems makes problem solving easier for an expert than for a novice, who must solve the same problem by piecing together a larger number of more primitive subproblems.

As emphasized earlier, the work I have described is still very much in progress and much more needs to be done along the following lines:

(1) The problem-solving model needs to be refined and made more explicit. In particular, it is important to specify more precisely the hierarchical structure of a knowledge base, described at multiple levels of detail, which permits efficient retrieval of

relevant information by the proposed problem-solving strategy. Models of such a structure could then be tested by examining or constructing knowledge bases for some specific subfields of physics or engineering.

(2) Detailed experiments are necessary to ascertain to what extent someone, proceeding in accordance with the problem-solving strategy, does attain significantly improved problem-solving performance. Some aspects of the problem-solving model might also be checked by computer simulation.

(3) If such experiments substantiate the merits of the problem-solving strategy, one should then be able to use this strategy as the basis of a systematic and practical instructional procedure for teaching students improved problem-solving skills.

(4) The present domain of applicability of the problem-solving model is limited to relatively simple problems of the kind encountered in basic physics or engineering courses. In the case of more complex problems, the design phase of the strategy must be amplified by incorporating in it more elaborate heuristic procedures for redescribing problems or searching for their solution. Examples of such heuristic procedures are discussed in Polya (1957) or Wickelgren (1974). Furthermore, one should explore to what extent the problem-solving model can be extended to areas beyond physics.

CONCLUDING REMARKS

I have tried to show that serious concerns with problem solving can benefit significantly from detailed studies of the information processing underlying successful problem solving. Such studies, pursued in depth and with the formulation of detailed theoretical models, can lead to substantial intellectual challenges and touch on fundamental questions of substantive interest both to modern engineering fields (such as computer science and artificial intelligence) and to cognitive psychology. Furthermore, on a less fundamental and more practical level, the analysis of human information processing can have far-reaching implications for teaching students improved problem-solving skills and for furthering the development of an effective applied science of education.

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CAPTIONS

Fig. 1: Diagram indicating schematically the connection between a particular symbol (black circle) and a particular referent (black square).

Fig. 2: Current flowing through two joined wires of the same material and length. The diameter of the first wire is twice as large as that of the second.

Fig. 3: Major subproblems of the problem-solving strategy.

Fig. 4: Diagram representing the information provided in the problem of Table 1.

Table 1: Original statement of a physics problem.

Table 2: Convenient description of the problem of Table 1. (To be supplemented by Fig. 4.)

Table 3: Useful information of the problem of Table 1. (To be supplemented by Fig. 4.)

Table 4: Schematic solution of the problem of Table 1. (The circles and lines in the last column indicate pictorially which statements are combined to yield new statements.)