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AUTHOR Cebulski, Larry A.
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ABSTRACT

Three studies were conducted in order to determine the source and frequency of children's difficulties in subtraction and to examine different approaches to remediation. In the first study, third-grade children were asked to solve subtraction problems and were observed and questioned about their solution processes. Children who had difficulty either attempted to borrow incorrectly or made inversion errors. The second study examined the efficacy of two minimally intrusive methods of remediation. Third-grade children were given either instructions to borrow, promised rewards for accurate performance or no intervention and were asked to solve a series of subtraction problems requiring borrowing. Neither experimental condition resulted in a significant increase in the number of problems solved correctly, and error patterns again indicated that children had difficulty with the borrowing process. In the third study, third- and fourth-grade children were assigned to one of three conditions: training in the component skills required for borrowing; feedback in the form of correctly worked solutions; or a regular classroom control condition. Both treatment conditions resulted in a significant increase in the number of problems solved correctly, and these increases were accompanied by a reduction of errors in the borrowing process. (Author)

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Children's Errors in Subtraction: An Investigation Into Causes and Remediation[1]

Larry A. Cebulski, Ph D.
Department of Psychology
Royal Ottawa Hospital

SE044415

Abstract

Three studies were conducted in order to determine the source and frequency of children's difficulties in subtraction and to examine different approaches to remediation. In the first study, third-grade children were asked to solve subtraction problems and were observed and questioned about their solution processes. Children who had difficulty either attempted to borrow incorrectly or made inversion errors. The second study examined the efficacy of two minimally intrusive methods of remediation. Third-grade children were given either instructions to borrow, promised rewards for accurate performance or no intervention and were asked to solve a series of subtraction problems requiring borrowing. Neither experimental condition resulted in a significant increase in the number of problems solved correctly, and error patterns again indicated that children had difficulty with the borrowing process. In the third study, third- and fourth-grade children were assigned to one of three conditions: training in the component skills required for borrowing, feedback in the form of correctly worked solutions, or a regular classroom control condition. Both treatment conditions resulted in a significant increase in the number of problems solved correctly, and these increases were accompanied by a reduction of errors in the borrowing process.

Children's Errors in Subtraction: An Investigation into Causes and Remediation

Studies examining children's arithmetic performance have shown that subtraction presents more difficulty for elementary school children than does addition, at least in the primary grades (Cox, 1974; Ginsburg, 1977; Reiss, 1943). It also appears that this problem becomes more pronounced when borrowing is required (Cox, 1974; Ellis, 1972; Graeber and Wallace, 1977).

Frequency and Types of Errors

Investigations into the nature of subtraction difficulties usually involve presenting children with different types of subtraction problems and analyzing the written solutions in order to determine the types of errors committed. Cox (1974), Smith (1968) and Ellis (1972) all reported inversion errors to be the most frequent type of error for primary grade children. This error occurs when the child subtracts the smaller number from the larger when the smaller number is located in the minuend of the problem, as in the example:

$$\begin{array}{r} 257 \\ -198 \\ \hline 141 \end{array}$$

Other investigators have implicated failure to recall the correct number fact in subtraction errors (Engelhardt, 1977; Morton, 1925; Williams and Whitaker, 1937). Cox (1974) also presents evidence showing that some children in her sample did not benefit from classroom instruction over the course of a year. She found that these children repeated the same error when retested after a one year interval.

As a preliminary step in an investigation into children's errors in subtraction, Cebulski (1982)[2] conducted a study to determine which type of subtraction error occurs most frequently on problems that require borrowing and to determine whether or not these errors persist over time. This study differed from most earlier ones in that children's verbal descriptions of the procedures they employed as well as their written solutions were used to categorize errors into types. Fifty-six third-grade children from normal classrooms were presented with 20 subtraction problems of the type,

$$\begin{array}{r} 547 \\ -259 \\ \hline \end{array}.$$

These problems were presented one at a time on separate slips of white paper, and children were required to solve the problems while their written solutions were observed. They were then asked how they obtained each digit in the answer. Children were tested individually, and were retested one month later in the above manner. Results showed that, for those eleven children who made errors on every problem, inversion errors occurred more frequently than all other types of errors combined, $t(10)=16.05$, $p<.001$. After a one month interval, all eleven of these subjects again erred on every problem, with inversion again the main source of errors. Inversion errors occur when the smaller number is subtracted from the larger when the smaller number is located in the subtrahend, as in the example,

$$\begin{array}{r} 324 \\ -146 \\ \hline 222 \end{array}.$$

For those children who made errors but solved at least one problem correctly, computational errors caused the most difficulty. That is, these children either counted incorrectly or failed to recall the correct number fact. Of the 27 subjects in this group, 16 again made errors but solved at least one problem correctly after one month. Five others improved their performance to 19/20 or 20/20, and six children solved no problems correctly. Inversion was the main source of errors for these latter six children.

In summary, children who solved no problems correctly tended to make inversion errors on every problem and to repeat this pattern after one month. Computing the correct answer—number fact and counting errors—gave those children who solved at least one problem correctly more trouble than any of the other skills. These children were more likely than the former group to change the number of correct solutions after one month.

<u>Why Children Commit Errors</u>

When children make errors on every problem they attempt, these errors tend to be procedural in nature. Specifically, the children do not attempt to implement borrowing procedures. Instead, they subtract the smaller number from the larger when the larger number is located in the subtrahend, as in this example:

$$\begin{array}{r} 324 \\ -145 \\ \hline 221 \end{array} \quad \text{versus} \quad \begin{array}{r} 2\,11\;\; \\ \not{3}\,\not{2}14 \\ -1\;\,4\;\,5 \\ \hline 1\;\,7\;\,9 \end{array} .$$

The consistent pattern of inversion errors observed in children who fail to solve problems correctly suggests three possible causes. The children may have gaps in their knowledge about borrowing procedures and, therefore, use the only available alternative when confronted with problems which require borrowing. Alternatively, these children may not have recognized the conditions where borrowing was appropriate. They may be able to borrow correctly when explicitly told to do so, but invert when not provided with specific instructions or contextual cues.

A third possible explanation for inversion errors suggests that these children are not motivated to apply their borrowing skills. This situation could occur when the contingency structure existing in the classroom reinforces problem completion rather than accuracy. Children may apply an easier (inversion) strategy because it helps them finish faster and therefore facilitates escape from an unpleasant or boring task. Low motivation may also explain the erratic performance of children whose errors are primarily computational in nature. Some evidence is available suggesting that performance may be enhanced when reinforcement is made contingent upon accuracy (Marholin and Steinman, 1977; Ferritor et al., 1972; Copeland et al., 1974; Harris and Sherman, 1973; Lovitt and Esveldt, 1970; Chadwick and Day, 1971). In addition, visual displays available in some of these reports indicate that performance often improved prior to the first receipt of reinforcement, suggesting that an increase in motivation was responsible for some of the reported improvements.

Whether errors in subtraction were the result of failure to recognize when to borrow, or the result of low levels of motivation was examined in a study of third graders (Cebulski, 1983)[3]. In this study, children were randomly assigned to one of two treatment conditions or to a no-treatment control condition. In the Instructions condition, children were directed to apply borrowing procedures to every problem, but were not provided with information concerning how to apply these procedures. In the Motivation condition, children were promised the opportunity to select a tangible reward of their choice from an array of small toys in exchange for a specified number of correct solutions.

Results showed that neither instructions to borrow on every problem nor the promise of a prize for correct solutions resulted in an increase in the number of subtraction problems solved correctly relative to no-treatment controls. However, for children who originally made procedural errors on every problem attempted (inversion errors or errors in the application of borrowing procedures), instructions to borrow resulted in a significant decrease in the number of inversion errors committed relative to controls, Bonferroni $t(27)=2.5$, $p<.025$, one tailed. Those in the Instructions condition also committed fewer inversion errors than those in the Motivation condition, Bonferroni $t(27)=2.45$, $p<.025$, one tailed. Accompanying a reduction in the number of inversion errors committed was a significant increase in the number of borrowing errors committed by children in the Instructions condition relative to controls, Bonferroni $t(27)=2.5$, $p<.025$, one tailed. No significant difference in the number of borrowing errors was found between the Instructions and Motivation conditions.

Taken together, these findings suggest that children who have difficulty with subtraction requiring borrowing tend to commit inversion errors at every opportunity. Why they do this is unclear, but it does appear that these children do not have the skills required for borrowing. These results also suggest that, in order to be effective, remediation may require an instructional component.

Improving Subtraction Performance

Reports of remedial approaches to subtraction are difficult to evaluate because they often fail to provide specific details of the procedures advocated or tested. Some studies, however, have shown that various types of remedial procedures involving instruction can be effective in improving performance (Harris and Sherman, 1973; Smith, Lovitt and Kidder, 1973). Performance feedback has also been shown to be effective (Conlon, Hall and Hanley, 1973; Sagotsky, Patterson and Lepper, 1978; Kirby and Shields, 1972; Fink and Carnine, 1975; Baxter, 1973; Blankenship, 1976), although some studies combined feedback with additional reinforcement, demonstrations, etc. None of the above studies compared instruction with feedback. In addition, many of the above studies used small samples of children. This means that the question of whether or not the techniques used were effective for all children having difficulty with subtraction could not be addressed.

Cebulski and Bucher (1983)[4] compared the effectiveness of two types of remedial programs for subtraction. Subjects were grade three and four children selected from nine normal classrooms.

All of the children in these classes were given a pretest by their classroom teachers. This pretest consisted of 24 subtraction problems which the children were required to solve. Those children who solved fewer than 60 percent of the problems correctly were categorized as Unsatisfactory, while those who solved between 60 percent and 80 percent of the problems correctly were classified as Satisfactory. The 60 percent criterion was selected as it is a commonly used standard of acceptable academic performance. On the basis of these criteria, 67 subjects were selected for further study.

Children from Unsatisfactory and Satisfactory groups were randomly assigned to either the Component Skills Training, the Criterion Training, or the Control conditions. In Component Skills Training, children were given a series of 5 training booklets which presented the skills required to solve subtraction problems requiring borrowing. These skills were introduced in a step by step fashion. In the program, a sentence containing some information about subtraction was presented on each page, followed by a question based on that sentence. A space was provided for the child to enter his/her response. The child was then instructed to correct his/her answer by referring to the right answer provided on the next page, and to record whether or not the question was answered correctly. If the child's response was correct, he/she placed a coloured star beside the answer. The individual components included in this training package were selected on the basis of a task analysis of the solution of a subtraction problem requiring borrowing.

In addition, some components were obtained from those included in computer programs designed to simulate children's solutions to subtraction problems (Young and O'Shea, 1981; Brown and Burton, 1978). A listing of the program components included in the Component Skills Training program is provided in Figure 1.

Insert Figure 1 about here

The format for Criterion Training was similar to that described above. Subjects were presented with 4 training booklets with 1 to 10 problems on each page. The child was instructed to solve the problem(s) on each page and then to correct his/her solution(s). Feedback consisted of the correctly worked problem, as in this example:

$$\begin{array}{r} 4 \\ 3\,\not{5}17 \\ -1\;2\;9 \\ \hline 2\;2\;8 \end{array}$$

At the end of both training programs, 10 subtraction problems were presented. These were used as a criterion index of how well the child had acquired the target skill. Subjects in the Control condition received regular classroom instruction.

Training was provided in three sessions which occurred once a day for three consecutive days. Children worked on the training booklets in groups of 4 to 6, and each training session lasted about 45 minutes. Two to three days following training, the classroom teachers administered a posttest in the classroom. The posttest again consisted of 24 subtraction problems. Posttest #2 was administered two to three weeks later.

For Unsatisfactory subjects at posttest #1, a main effect for treatment condition was obtained, Kruskall Wallace $H(2)= 6.55$, $p<.05$. Between cell comparisons using the protected

Figure 1: Program Components for Component Skills Training

Booklet #1

1. Judgements of relative magnititude of numerals.
2. Sign recognition (+,-)
3. Column identification (ones', tens', hundreds').
4. Identification of the number of ones, tens and hundreds in each column.
5. Review.

Booklet #2

1. Order of operations.
2. Location of largest and smallest number within a column.
3. Subtract number in subtrahend from number in minuend.
4. Identify when largest number is on the bottom (subtrahend) in ones' column.
5. Review.

Booklet #3

1. Identify when the largest number is on the bottom in ones' column.
2. Same as above for tens' column.
3. Identify when a problem requires borrowing (presence of 1 and/or 2).
4. Review.

Booklet #4

1. Review identification of borrowing problems.
2. Reduce numeral in minuend of tens' column.
3. Add ten to numeral in minuend of ones' column.
4. Identify problems requiring borrowing from tens' column and apply 2 and 3.

Figure 1 (continued)

5. Review.
6. Heuristics for identification of borrowing problems and execution of borrowing procedures in tens' column.
7. A mnemonic aid for above heuristics.
8. Review.

Booklet #5

1. Review above heuristics.
2. Identification of problems requiring borrowing from hundreds' column.
3. Application of heuristics to hundreds' column.
4. Review.
5. Identification of problems requiring borrowing from tens' and hundreds' columns.
6. Application of heuristics to 5.
7. Review and practice.
8. Criterion.

rank sum test indicated that subjects in both Component Skills Training and Criterion Training conditions solved more borrowing problems correctly than controls, $z=2.32$, $p<.02$ and $z=1.97$, $p<.05$ respectively. Treatment means are presented in Table 1.

Insert Table 1 about here

Examination of the means at posttest #2 indicates that maintenance of treatment gains was obtained over the two week interval. Failure to obtain a significant treatment effect was the result of an increase in the number of problems solved correctly by three control subjects. These three children had been selected from the same classroom and apparently had received additional instruction from the classroom teacher in the procedures required for borrowing during the 2 week interval between posttests #1 and #2. None of the other subjects in the control group demonstrated an improvement in performance during this interval.

No significant changes in the number of inversion errors, computational errors or errors in the application of borrowing procedures were obtained as a result of either training condition. Trends in the expected direction were observed, however, for both inversion and borrowing errors in both experimental conditions. Also, no effects of training on the number of problems solved correctly or on the number of errors committed were obtained for Satisfactory subjects.

Remedial Program Performance

Although it was found that the remedial programs resulted in improvements in the performance of Unsatisfactory subjects as a group, some children did not do very well during training.

Table 1: Mean Number of Borrowing Problems Solved Correctly

		pretest	posttest #1	posttest #2
Unsatisfactory				
CST	mean	1.27	8.27	11.27
	sd	2.97	5.92	7.20
	n	11		
CRT	mean	.91	7.64	6.64
	sd	2.07	6.53	7.42
	n	11		
Control	mean	.80	2.20	7.10
	sd	2.53	4.54	7.72
	n	10		
Satisfactory				
CST	mean	11.80	14.80	15.80
	sd	1.14	1.75	2.35
	n	10		
CRT	mean	10.55	11.81	13.18
	sd	3.59	6.40	5.10
	n	11		
Control	mean	9.55	10.55	11.55
	sd	3.91	4.52	5.97
	n	11		

It was decided, therefore, to examine the performance of these children in some detail. In order to identify those who had difficulty with remedial training, a median split was done on the basis of the number of criterion problems solved correctly. Recall that the criterion measure consisted of the last 10 problems within each training program.

This median split yielded two subsamples of children within each of the Satisfactory and Unsatisfactory conditions. Those above the median, considered to have successfully completed the programs, were labelled Tutorial-High, while those below the median, considered to have been unsuccessful, were labelled Tutorial-Low.

Criterion Training

In an attempt to understand why some children did not complete the tutorial programs successfully, performance throughout the programs was examined. Problems in the Criterion Training program were divided into 9 blocks of 10 problems each, with 30 problems (3 blocks) presented each day.

Figure 2 shows the performance of Unsatisfactory subjects in the Criterion Training condition. The number of problems solved correctly by Tutorial-High subjects jumped at day 2 of training and was maintained on day 3, as described by a significant linear, $F(1, 4)=10.56$, $p=.0314$ and quadratic, $F(1, 4)=12.39$, $p=.0224$ trend. The number of problems solved correctly for Tutorial-Low subjects tended to decrease during each day, showing some recovery between days. These data are described by a significant cubic trend, $F(1, 5)=13.87$, $p=.0136$. As shown in Figure 3, Satisfactory subjects in the Tutorial-

High condition maintained a high level of accuracy throughout training. Those in the Tutorial-Low condition demonstrated a pattern of performance similar to that of Unsatisfactory, Tutorial-Low subjects and was described by significant quadratic, $F(1, 3)=19.12$, $p=.0221$ and cubic, $F(1, 3)=37.87$, $p=.0086$ trends.

Insert Figs. 2 and 3 about here

While Unsatisfactory, Tutorial-High subjects appeared to "catch on" to what was required by about the middle of day 2, and while Satisfactory Tutorial-High subjects did well from the start, the performance of Tutorial-Low subjects in both Unsatisfactory and Satisfactory groups deteriorated each day.

Despite this deterioration in training performance, pretest and posttest scores for these children did not differ significantly. In the Criterion Training condition, examination of the errors of Tutorial-Low children suggest that: 1) for some children, contingencies were not sufficient to maintain initially moderate to high levels of accuracy or to maintain improvements in performance, and 2) some subjects did not appear to be utilizing feedback.

Component Skills Training

In order to evaluate performance during the Component Skills Training program, each booklet was divided into 2 blocks, with the first block containing the first half of the questions and with the second block containing the second half of the questions. Each block, therefore, was composed of a unique set of items.

Figures 4 and 5 show performance trends for the Component Skills Training program. Both Satisfactory and Unsatisfactory

Figure 2: Mean Number of Problems Solved Correctly Across Training Blocks in the Criterion Training Condition, Unsatisfactory Group.

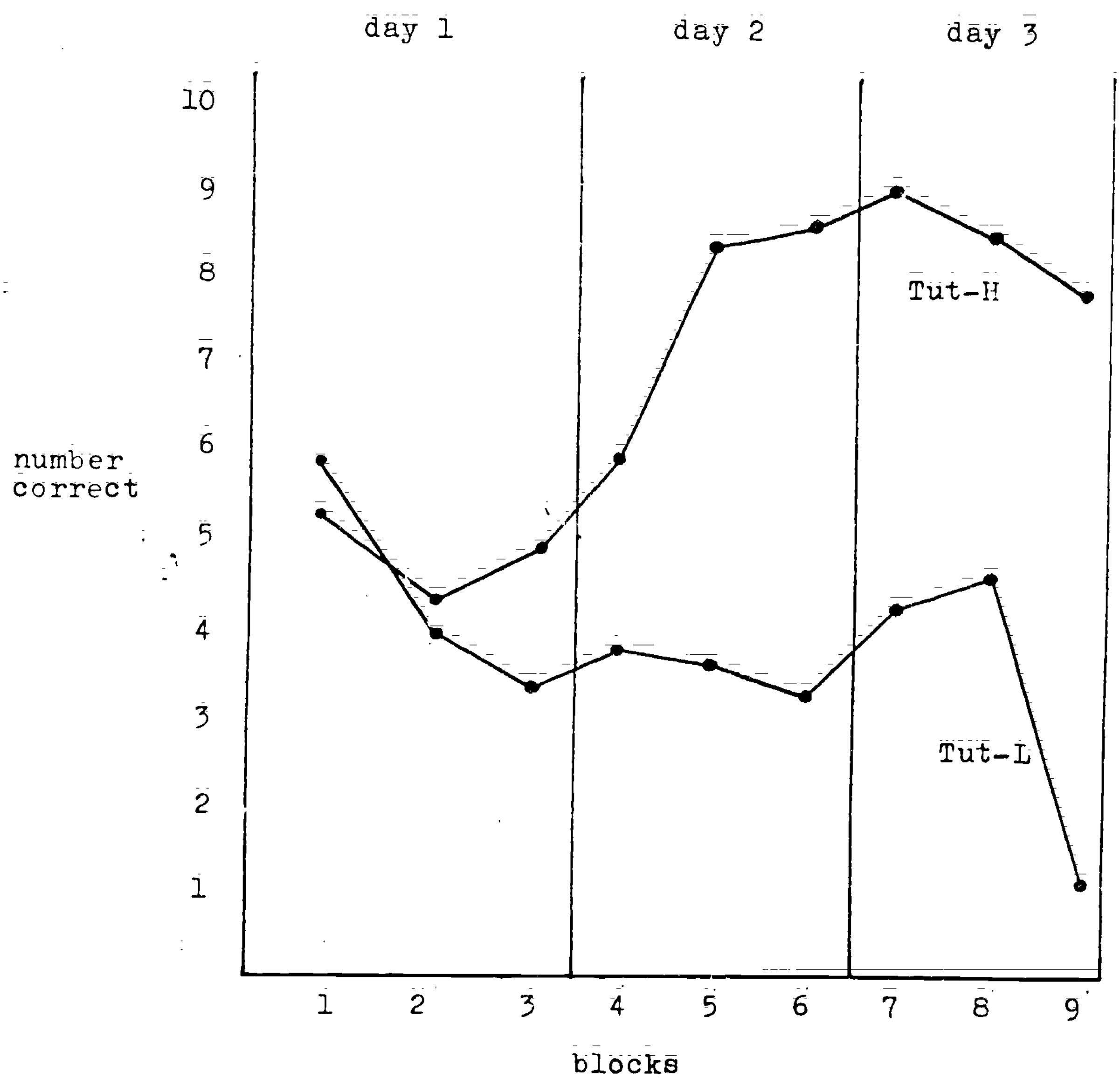
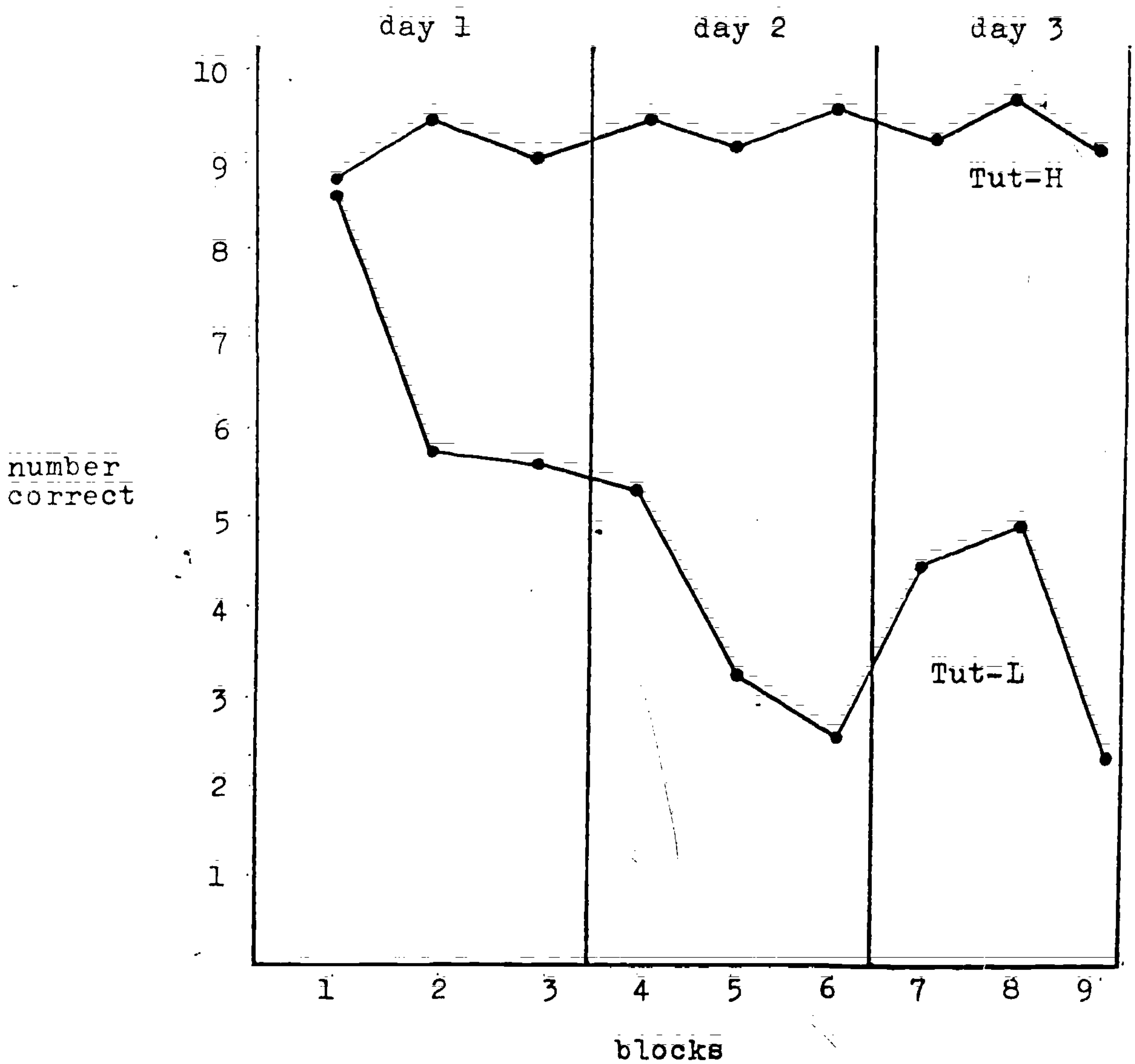


Figure 3: Mean Number of Problems Solved Correctly Across Training Blocks in the Criterion Training Condition, Satisfactory Group.



children in the Tutorial-High group maintained a fairly high level of accuracy throughout all 5 training booklets. Bonferroni

Insert Figures 4 and 5 about here

t comparisons indicated that, for Tutorial-Low children in both Satisfactory and Unsatisfactory groups, the percentage of items answered correctly tended to drop significantly from booklets 3 to 5, $t(6)=3.92$, $p<.05$ for Unsatisfactory and $t(4)=4.67$, $p<.05$ for Satisfactory subjects. It appeared that, for Tutorial-Low children, booklets 4 and 5, which presented the actual steps required for borrowing, were more difficult than the earlier booklets which instructed children in pre-borrowing skills.

Summary and Conclusions

A consistent feature of children who have difficulty with subtraction was difficulty with borrowing procedures. It appeared that these children experienced difficulty because they did not have the skills necessary to solve borrowing problems correctly. For this reason, remedial techniques which did not incorporate instruction failed. Neither manipulating stimulus conditions by instructing children to borrow, nor offering prizes for correct solutions resulted in performance gains. When an instructional component was introduced, however, children who had difficulty with subtraction improved significantly. Both feedback consisting of correctly worked solutions and systematic instruction in component skills were effective remedial strategies.

However, the failure of some subjects to complete the training programs successfully highlights the fact that remediation is not always effective for all children. Why

Figure 4: Mean Percentage of Items Answered Correctly Across Training Blocks in the Component Skills Training Condition, Unsatisfactory Group.

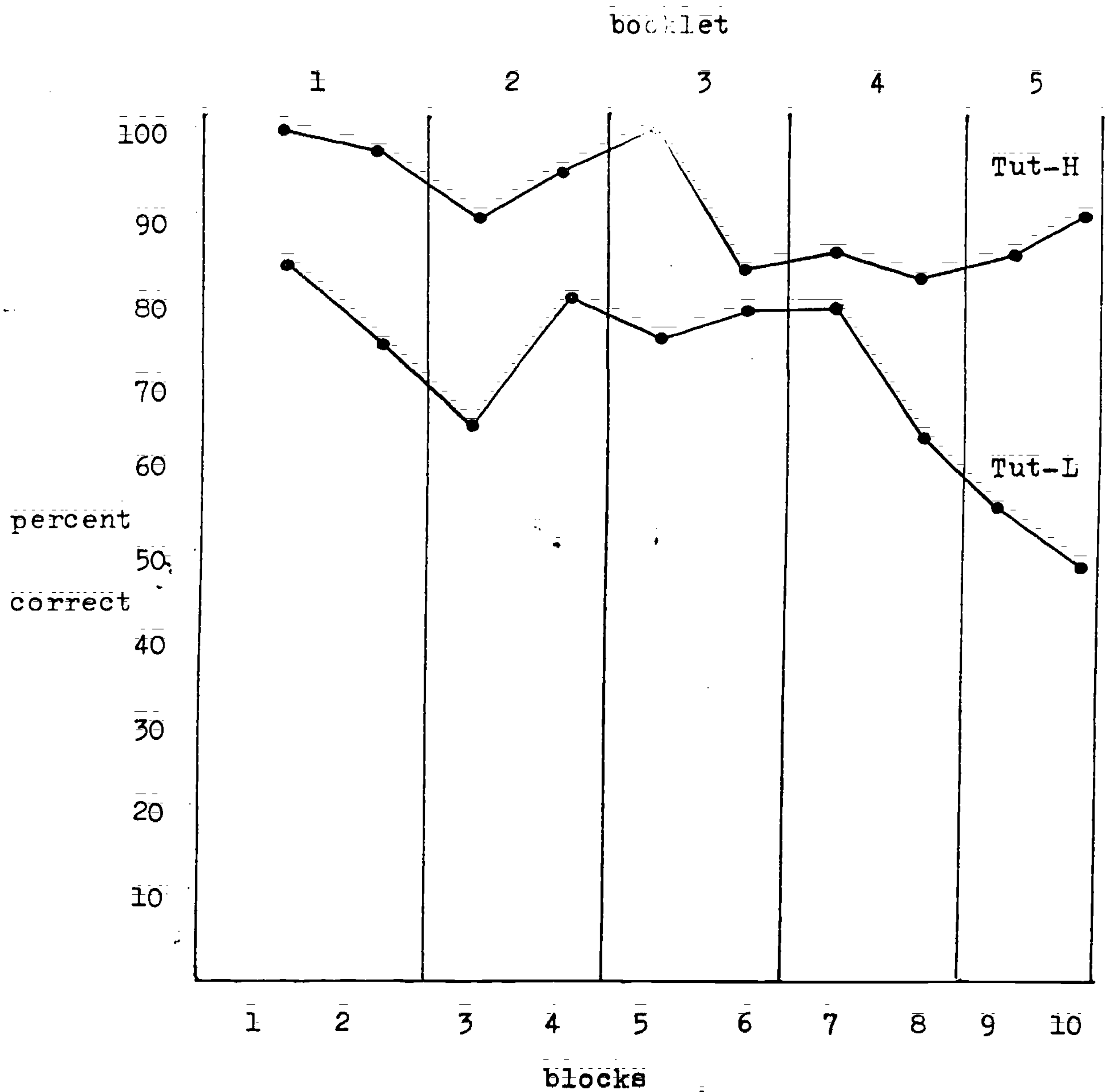
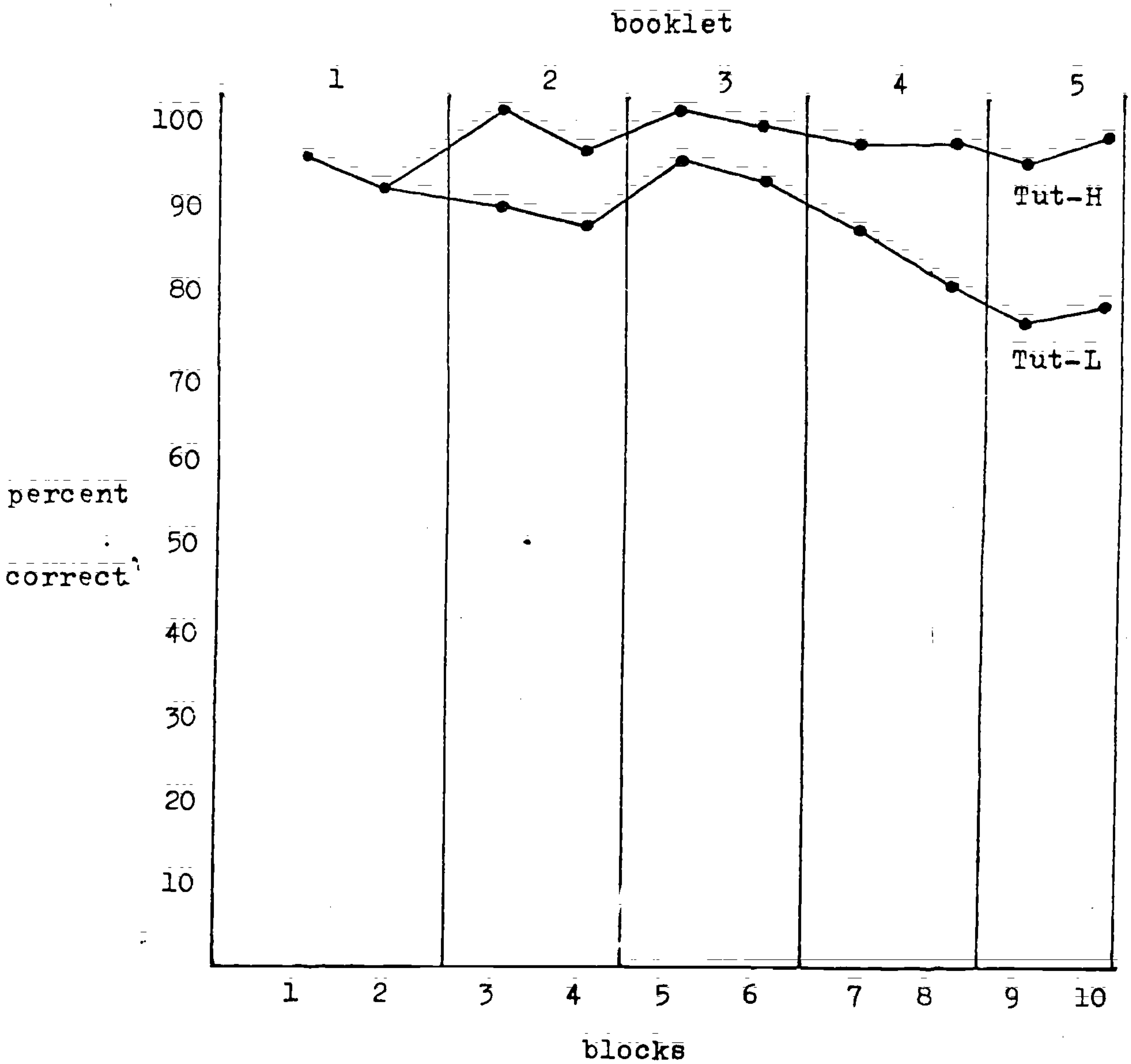


Figure 5: Mean Percentage of Items Answered Correctly Across Training Blocks in the Component Skills Training Condition, Satisfactory Group.



Tutorial-Low children deteriorated in performance during the programs but did not show pretest-posttest changes is unclear. Fatigue or boredom are possibilities. Explaining these observed declines in accuracy during training may be important in the understanding of what makes those who fail to learn different from their more successful counterparts.

The types of remedial programs examined here were also not effective in significantly improving the subtraction performance of children who were already relatively accurate (Satisfactory group). It may be that, for these children, re-instruction and/or reinforcement in the form of feedback are not potent motivators. That is, they may not command the additional attention and effort required to further elevate performance.

The results of these studies suggest two goals for future research. The first is to better identify and understand individual differences among children who appear to have similar difficulties. The second is to discover how to elevate and maintain performance at high levels. This implies a need to develop better programs which will work equally well for different types of children. With the current emphasis on meeting the special needs of children, attaining the first goal is paramount. Pursuit of the second, however, may provide some answers which could make the education of those with learning difficulties a more successful venture.

Reference Notes

1. This paper is based on a doctoral dissertation prepared by the author. Studies conducted by the author and cited in this paper are presently in preparation and will be available as separate articles at a later date. This paper is intended as a brief summary only. Copies of this paper and others cited in this article may be obtained from the author, c/o Royal Ottawa Hospital, 1145 Carling Ave., Ottawa, Ontario, Canada, K1Z 7K4.

2. Cebulski, L. Types and frequency of children's errors in subtraction. Paper in preparation, University of Western Ontario, London, Canada, 1982.

3. Cebulski, L. Prompts and rewards as remedial tactics for children with difficulty in subtraction. Paper in preparation, University of Western Ontario, London, Canada, 1983.

4. Cebulski, L. and Bucher, B. A comparison of two instructional approaches to remedial subtraction. Paper in preparation, University of Western Ontario, London, Canada, 1983.

Baxter, M.M. Prediction of errors and error type in computations of sixth grade mathematics students. Doctoral dissertation, Penn State University, 1973.

Blankenship, C.S. Reduction of systematic inversion errors in the subtraction algorithm. Doctoral dissertation, University of Washington, 1976.

Brown, J.S. and Burton, R.R. Diagnostic models for procedural bugs in basic mathematical skills. Cognitive Science, 1978, 2, 155-192.

Chadwick, B.A. and Day, R.C. Systematic reinforcement: Academic performance of underachieving students. Journal of Applied Behavior Analysis, 1971, 4, 311-319.

Conlon, M.F., Hall, C. and Hanley, E.M. The effects of a peer correction procedure on the arithmetic accuracy of two elementary school children. In G. Semb and H.W. Hopkins (eds) A new direction for education: Behavior analysis. Lawrence, Kansas, 1973.

Copeland, R.E., Brown, R.E. and Hall, R.V. The effects of principal implemented techniques on the behavior of pupils. Journal of Applied Behavior Analysis, 1974, 7, 77-86.

Cox, L.S. Analysis, classification and frequency of systematic error computational patterns in the addition, subtraction, multiplication and division vertical algorithms for grades 2-6 and special education classes. Kansas City Medical Centre, 1974.

Ellis, L.C. A diagnostic study of whole number computations of certain elementary students. Doctoral dissertation, Louisianna State University and Agricultural and Mechanical College, 1972.

Engelhardt, J.M. Analysis of children's computational errors: A qualitative approach. British Journal of Educational Psychology, 1977, 47, 149-154.

Fink, W.T. and Carnine, D.W. Control of arithmetic errors using informational feedback and graphing. Journal of Applied Behavior Analysis, 1975, 8, 461.

Ferritor, D.E., Buckholdt, D., Hamblin, R.L. and Smith, L. The noneffects of contingent reinforcement for attending behavior on work accomplished. Journal of Applied Behavior Analysis, 1972, 5, 7-17.

Ginsburg, H. Children's arithmetic: The learning process. Toronto: Van Nostrand Co., 1977.

Graeber, A.D. and Wallace, L. Identification of systematic errors: Final report. Research for better schools inc., Philadelphia, 1977, National Institute for Education (DHEW), Washington, DC.

Harris, V.W. and Sherman, J.A. Effects of peer tutoring and consequences on the math performance of elementary classroom students. Journal of Applied Behavior Analysis, 1973, 6, 587-597.

Kirby, F.D. and Shields, F. Modification of arithmetic response rate and attending behavior in a seventh grade student. Journal of Applied Behavior Analysis, 1972, 5, 79-84.

Lovitt, T.C. and Esveldt, K.A. The relative effects on math performance of single versus multiple ratio schedules: A case study. Journal of Applied Behavior Analysis, 1970, 3, 261-270.

Marholin, D. and Steinman, W.M. Stimulus control in the classroom as a function of the behavior reinforced. Journal of Applied Behavior Analysis, 1977, 10, 465-478.

Morton, R.L. An analysis of errors in the solution of arithmetic problems. Educational Research Bulletin, 1925, 4, 187-190.

Reiss, A. An analysis of children's number responses. Harvard Educational Review, 1943, 13, 149-162.

Sagotsky, G., Patterson, C.J. and Lepper, M.R. Training children's self control: A field experiment in self monitoring and goal setting in the classroom. Journal of Experimental Child Psychology, 1978, 25, 242-253.

Smith, C.W., Jr. A study of constant errors in subtraction and in the application of selected principles in the decimal numeration system made by third and fourth graders. Doctoral dissertation, Wayne State University, 1968.

Smith, D.D., Lovitt, T.C. and Kidder, J.D. Using reinforcement contingencies and teaching aids to alter subtraction performance of children with learning difficulties. In G. Semb and W.L. Hopkins (eds) A new direction for education: Behavior analysis. Lawrence, Kansas: University of Kansas, 1973.

Williams, C.L. and Whitaker, R.L. Diagnosis of arithmetic difficulties. Elementary School Journal, 1937, 37, 592-600.

Young, R.M. and O'Shea, T. Errors in children's subtraction. Cognitive Science, 1981, 5, 153-177.