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ABSTRACT

A new procedure is proposed for developing, evaluating, and implementing routing procedures for use with individualized instruction programs. An item response theory model for describing the relationship between performance on sequentially arranged units of instruction, and procedures for using the model to evaluate sequential relationships and for making routing decisions are discussed. A procedure for estimating the parameters of the model is described. Empirical data are presented to support the validity of the model. The model and procedures described appear to be useful and to merit continued research efforts directed toward their development. (DWH)

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A Latent Trait Model for Use with Sequentially Arranged Units of Instruction

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One of the fastest growing areas in the field of education today is the area of individualized instruction - instruction in which content, organization, or pacing is modified for each individual. Although individualized instruction comes in many forms (e.g., personalized systems of instruction, computer assisted instruction, individually prescribed instruction, and programmed instruction), all of these forms share the same basic design. All are basically sequences of instructional units through which subjects are routed by means of a series of tests.

The way in which the units of instruction are sequenced and the routing decisions made are two of the more crucical components of any individualized instruction program. While they have been the topic of considerable research, as yet no generally accepted procedures have been developed for these components. The purpose of this paper is to propose a new procedure for developing, evaluating, and implementing routing procedures for use with individualized instruction programs. Specifically, a model will be proposed for describing the relationship between performance on sequentially arranged units of instruction, and procedures for using the model to evaluate sequential relationships and for making routing decisions will be discussed. Then a procedure for estimating the parameters of the model will be discussed. Finally, empirical data will be presented to support the validity of the model. Before beginning the discussion of this procedure, however, some theory about the nature of sequential units of instruction will be presented as a basis for the procedure.

Sequential Units of Instruction

Underlying Theory

The basic assumption underlying the sequential arrangement of units of instruction is that performance on module 2 requires the prior knowledge of the material contained in module 1. It might be true that all material in

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module 1 must be mastered before any (or at least any appreciable amount) of the material in module 2 can be mastered, or it may be the case that certain sections of module 1 are prerequisite for certain sections of module 2. For this paper, the former will be assumed to be the case. It will also be assumed that the tests that measure the skills taught in the modules are unidimensional.

To say that a sequential relationship exists means that a certain level of performance is required on module 1 (m_1) before learning on module 2 (m_2) can begin. Once that level (c_1) is achieved, learning on m_2 can begin. Improvement above level c_1 on m_1 facilitates improvement on m_2. Once the mastery level on m_1 (c_2) is achieved, additional learning on m_1 does not facilitate learning on m_2. This relationship is illustrated in Figure 1. This type of figure is called a module characteristic curve, or MCC.

For the relationship shown in Figure 1, the vertical axis is the proportion of examinees passing m_2. The horizontal axis is the examinee's status (level of achievement) on m_1. As can be seen, the relation is horizontal until the level of achievement on m_1 designated by c_1 is reached. At that point a linear relationship between status on m_1 and performance on m_2 is depicted. When the m_1 mastery level, c_2, is reached, a horizontal relation is again present, indicating that further improvement on m_1 does not aid performance on m_2. Of course, the relationship in the range from c_1 to c_2 need not be a linear one, and in reality examinees would be expected to fall in a scatter around the curve shown in Figure 1.

The low end of the curve shown in Figure 1 is not at zero, nor is the top at one. It would be expected that some small portion of examinees might pass m_2 even with very little learning on m_1. This would be due to chance or other factors, and would generally be a small proportion of the total number of examinees. It would also be expected that some portion of examinees who had mastered m_1 would fail m_2, simply because of failure to master the m_2 material not included in m_1.

An Illustration

In order to illustrate the processes described above, simulation data were generated according to the following process. Item parameters for the three-parameter logistic (3PL) model (Birnbaum, 1968) were selected for two thirty item modules. Examinee m_1 achievement levels were randomly selected from a normal distribution with a mean of zero and a standard deviation of 1.5. The c_1 value was set equal to θ (achievement level) = -1.0, and c_2 was set equal to $\theta = 0.5$. Mastery of m_2 was arbitrarily defined as seventeen correct out of thirty items.

For each examinee, an m_2 achievement level was selected as follows. If the examinee's m_1 achievement level (θ_1) was less than c_1, the examinee's m_2 achievement level (θ_2) was randomly selected from a normal distribution with a mean of -1.0, and a standard deviation of 0.5. If $\theta_1 > c_1$, but $\theta_1 < c_2$, θ_2 was randomly selected from a normal distribution with a mean of θ_1 and a standard deviation of 0.5. If $\theta_1 > c_2$, θ_2 was randomly selected from a normal distribution with a mean of 0.5 and a standard deviation of 0.5. Table 1 presents a summary of the relationship between θ_1 and θ_2. Using θ_2 and the module 2 item parameters, response data were generated for module 2 according to the 3PL model for 1000 examinees.

Figure 2 shows a plot of θ_1 by θ_2 for the 1000 simulated examinees. As can be seen, below -1.0 on the θ_1 scale there is a correlation of about zero between θ_1 and θ_2. Between $\theta_1 = -1.0$ and $\theta_1 = 0.5$ there is a positive correlation between θ_1 and θ_2. Above $\theta_1 = 0.5$, there is again no correlation between θ_1 and θ_2.

Figure 3 shows an empirical MCC for the generated data. The empirical MCC was computed by grouping examinees into intervals of the ability scale on the basis of θ_1. For each interval, the proportion of examinees in that interval who passed m_2 was computed and plotted against the interval midpoint. As can be seen, the plotted values form a rough approximation to the curve shown in Figure 1.

The Procedure

The Model

The procedure proposed for use with sequential units of instruction is based on the notion of the MCC. An MCC describes the probability of passing a unit of instruction (module) conditional on latent ability (achievement level)

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on the prerequisite module. The form of the MCC proposed in this paper is the four-parameter logistic (4PL) model, which is given by

$$P_j(\theta_{ik}) = c_j + (1 - c_j - e_j)\,[1 + \mathrm{EXP}\,(-Da_j\,(\theta_{ik} - b_j))]^{-1}\;, \qquad (1)$$

where θ_{ik} is the latent ability of examinee k on module i (the prerequisite module), $P_j(\theta_{ik})$ is the probability of passing module j given ability θ_{ik}, a_j is a discrimination parameter associated with module j, b_j is a difficulty parameter associated with module j, c_j is a lower asymptote parameter for module j, e_j is an upper asymptote parameter for module j, D = 1.7, and EXP(x) = e^x. The c_j term is used to account for the nonzero probability of passing module j for examinees with very low ability on module i, and the e_j term accounts for the nonunity probability of passing module j for examinees of very high ability on module i.

Figure 4 shows a 4PL MCC. The a-parameter is related to the slope of the MCC at the point of inflection, while the b-parameter serves to locate the point of inflection on the ability scale.

Using the Model

Interpreting the Parameters. Using the 4PL model in conjunction with sequential units of instruction involves estimating and interpreting the parameters of the model. The slope of the MCC, as indicated by the a-parameter, represents the strength of the sequential relationship. A steep slope indicates that small increases in achievement on the prerequisite module yield large increases in performance on the subsequent module. This would be indicative of a strong sequential relationship. A relatively flat MCC indicates that even large increases in achievement on the first module do not yield substantial improvement in performance on the second module. This would be indicative of a weak sequential relationship. Thus, the a-parameter serves as an indicant of the strength of the sequential relationship.

The b-parameter helps to indicate what level of performance is required on the first module to attain a given level of performance on the second module. If the b-parameter for the MCC shown in Figure 4 were increased, the curve would be shifted to the right. If this were the case, a greater level of ability would be required on module 1 to attain the same level of performance on module 2 as was the case before the curve was shifted. Thus, the b-parameter locates the module on the achievement scale.

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The c-parameter is a 'pseudo-guessing' parameter. It represents the probability of passing module 2 even when little or none of the material of module 1 has been mastered. A large value for c indicates that much of the material of module 2 can be learned without knowledge of the material in module 1. Thus, the c-parameter is an indicant of the degree to which all of module 2 actually requires knowledge of module 1 material.

The e-parameter is a reflection of the fact that module 2 contains instruction and material beyond those in module 1. Perfect mastery of module 1 does not guarantee mastery of module 2. That is, module 1 is necessary but not sufficient for module 2. The greater the value of e, the more that module 2 requires knowledge beyond what is required for mastery of module 1.

<u>Setting a Pass/Fail Score.</u> The goal of setting a pass/fail cut score for module 1 is to minimize the number of examinees failing module 2 and to minimize the number of examinees who could have passed module 2 but are held back. If these two types of errors are considered equally serious, then the most obvious procedure for setting a cut score for module 1 is to determine the level of ability on module 1 for which the predicted probability of success on module 2 is 0.5. Setting equation 1 equal to 0.5 and solving for θ yields

$$\theta_c = \left(\ln \left(\frac{0.5 - c}{0.5 - e} \right) / \, Da \right) + b \; , \qquad (2)$$

where θ_c is the pass/fail cut score for module 1, $\ln(x)$ is the log to the base e of x, and the other terms are as previously defined.

Once estimates of the MCC parameters and examinee ability parameters have been obtained, θ_c is calculated from (2). Examinees for whom $\hat{\theta}$ (estimated achievement) > θ_c considered masters of module 1 and are routed to module 2. Examinees with $\hat{\theta} < \theta_c$ are considered nonmasters and are not allowed to proceed to module 2.

Parameter Estimation

The procedure for estimating the item parameters of the 4PL model selected for this research is based on a maximum likelihood estimation technique. An iterative procedure based on the Newton-Raphson approach to solving simultaneous nonlinear equations is employed.

Criterion Function

The estimation procedure is designed to maximize the criterion function given by

$$L = \prod_{j=1}^{N} P_j^{u_j} Q_j^{1-u_j} , \tag{3}$$

where L is the likelihood of the string of observed outcomes (passes and failures) for a module, N is the number of examinees, u_j is the module outcome (zero for fail, one for pass) for examinee j, and Q_j is $1-P_j$. P_j is given by (1). In practice, (3) is maximized by minimizing the negative of the logarithm to the base e (natural logarithm) of L. That is, L^* is minimized where

$$L^* = - \log_e(L) . \tag{4}$$

Estimation Procedure

The Newton-Raphson procedure employed requires the first and second partial derivatives of (4), taken with respect to the item parameters. If $\underline{f}'$ is a column vector of first derivatives, and $\underline{f}''$ is the matrix of second derivatives, then for any set of provisional item parameter estimates, updated estimates are obtained using the following formula:

$$\underline{f}^{i+1} = \underline{f}^{i} + [(\underline{f}'')^{-1}\underline{f}']|_{\underline{f}^{i}} , \tag{5}$$

where $\underline{f}^i$ is the vector of item parameter estimates after iteration i, and $\underline{f}^{i+1}$ is the vector of item parameter estimates after iteration i + 1. The first and second derivatives of (4) are given in the Appendix. In a given iteration, these derivatives are evaluated using the estimates from the previous iteration.

One problem which is encountered in a procedure like this occurs when the matrix of second derivatives, given by $\underline{f}''$, is not positive definite. The

Newton-Raphson procedure guarantees convergence only when $\underline{f}''$ is always positive definite. When a model such as the 4PL model is used, the matrix of second derivatives, evaluated at the provisional item parameter estimates, very often is not positive definite. Therefore, it is necessary to check $\underline{f}''$ for positive definiteness. If it is not positive definite, it should be forced to be positive definite. A number of procedures for doing this have been proposed.

Work is currently underway on a program implementing the above estimation procedure. At this point research is underway to determine the optimal procedure for forcing the matrix of second derivatives to be positive definite. It is hoped that a working version of the program will be available shortly.

Example

In order to illustrate the operation of the estimation procedure just described, a preliminary version of the 4PL estimation program was applied to the simulation data generated in the previous section of this paper and for which the empirical MCC is shown in Figure 3. The true m_l achievement levels were used as input to the estimation program.

Table 2 shows the item parameter estimates which resulted from the application of the 4PL estimation program to the simulation data. Figure 5 shows the empirical MCC shown in Figure 3, with an overlay of the theoretical MCC computed using the item parameter estimates shown in Table 2. As can be seen, the theoretical curve shown in Figure 5 provides a reasonable description of the observed data.

Evidence for the Validity of the Model

Method

For the purposes of acquiring evidence to either support or discredit the 4PL model and the MCC concept, real response data were collected for a two-part arithmetic test. It was hypothesized that the two parts of the test were such that the skills required for performance on the first part would be prerequisite to performance on the second part. Using these two parts as modules, empirical MCCs were plotted for various pass/fail cutoffs on the second module. These plots were then examined as evidence of the usefulness of the 4PL model for use with these data. Details of the process follow.

Data. The test used for these analyses was the Numerical Skills subtest of the Career Placement Program (CPP) test (The American College Testing Program, 1983). The first part of the test, module 1, is comprised of nineteen four-choice multiple-choice arithmetic computation problems, while the second part, module 2, is comprised of thirteen four-choice multiple-choice word problems that require arithmetic computation skills and problem-solving skills. Response data for these items were collected for 3768 cases from the 1983 norming administrations of the test.

Since there is no already determined pass/fail cutoffs for the CPP subtests, the analyses performed in this stage of the research were repeated for a number of different cutoffs for module 2, so as to avoid any capitalization on chance from the cutoff selection. Using a given pass/fail cutoff for module 2, each examinee was assigned a score of 0 (fail) or 1 (pass) depending on whether the examinee's raw score on module 2 exceeded the cutoff for module 2. These 0, 1 data, along with examinees' achievement level estimates from module 1, were the input for these analyses.

Ability Estimation. The achievement level estimates on module 1 for the examinees were obtained through the application of the 3PL model to the examinees' response data for module 1. The LOGIST estimation program (Wingersky, Barton, and Lord, 1982) was used to estimate the parameters of the 3PL model.

Plotting MCCs. The initial step in these analyses was the division of the achievement scale into a number of narrow intervals (0.1 width). Examinees were then sorted into these intervals on the basis of their module 1 achievement level estimates. For a given module 2 pass/fail cutoff, the proportion of examinees within each interval passing module 2 was computed. For each module 2 cutoff, the proportions passing module 2 were plotted against the interval midpoints, thus forming an empirically derived MCC. Adjacent intervals were collapsed to assure an interval sample size of at least ten. These MCCs were examined to assess the reasonableness of the 4PL model for describing the form of the resulting curve.

Results

Figures 6 through 12 show the empirical MCCs obtained for the CPP data for pass/fail cutoffs on module 2 of three through nine correct out of the thirteen items, respectively. Table 3 shows the obtained proportions passing plotted in Figures 6 through 12. Table 3 also shows the numbers of examinees in the different intervals.

As can be seen from these figures, the relationship between module 1 ability and module 2 performance does appear to be at least a monotonically increasing one. Also, for several of the plots, there appears to be a non-unity upper asymptote. It is, however, difficult to discern a lower asymptote in these plots. Of course, a lower asymptote of zero is a special case of the 4PL model. It may eventually be fruitful to drop the lower asymptote, but as yet there is little evidence to support such a step.

There are a couple of interesting trends evident in Figures 6 through 12. As the pass/fail cutoff score on module 2 increases, of course, fewer examinees of low achievement level on module 1 pass module 2. If the material in module 2 requires the knowledge of module 1 material, clearly requiring more module 2 material for passing will require more module 1 material.

As the module 2 pass/fail cutoff increases, the upper asymptote of the MCC decreases (the e term increases in value). This is an indication that complete knowledge of module 1 is not sufficient for guaranteed success on module 2. Another way of saying this is that word problems require more knowledge than simply mastering arithmetic operations.

The patterns evident in these figures suggest that, if module 2 were still easier to pass than was the case with the pass/fail score of 3, there

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would be a nonzero lower asymptote to the MCC. Unfortunately, for this particular test lower cutoffs yielded an almost flat MCC near unity. Almost all examinees got at least two items correct on module 2, regardless of their module 1 ability.

Summary and Conclusions

While this research project is still incomplete, it has yielded encouraging results. A theory relating performance on sequentially arranged units of instruction was derived, and a model for describing that relationship was formulated. Procedures for using the model to evaluate sequential relationships and for making routing decisions were described. A procedure for estimating the parameters of the model was outlined, and data supporting the validity of the model were presented. All things considered, the model and procedures described appear to be useful ones, and they appear to merit continued research efforts directed toward their development.

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Appendix

Derivatives of the Negative of the Natural Logarithm of the Criterion Function

The negative of the natural logarithm of the criterion function, denoted by L^*, was given by (4). The vector of first derivatives with respect to the item parameters, denoted by $\underline{f}'$, is given by

$$\underline{f}' = \begin{bmatrix} \frac{\partial}{\partial a} L^* \\ \frac{\partial}{\partial b} L^* \\ \frac{\partial}{\partial c} L^* \\ \frac{\partial}{\partial e} L^* \end{bmatrix}$$

$$= \begin{bmatrix} D \sum_{j=1}^{N} (\theta_j - b) \frac{(P_j - c)(u_j - P_j)}{P_j\, Q_j \left[1 + \mathrm{EXP}\,(X_j)\right]} \\ -Da \sum_{j=1}^{N} \frac{(P_j - c)(u_j - P_j)}{P_j\, Q_j \left[1 + \mathrm{EXP}\,(X_j)\right]} \\ \sum_{j=1}^{N} \frac{u_j - P_j}{P_j\, Q_j \left[1 + \mathrm{EXP}\,(X_j)\right]} \\ - \sum_{j=1}^{N} \frac{(u_j - P_j)\,\mathrm{EXP}\,(X_j)}{P_j\, Q_j \left[1 + \mathrm{EXP}\,(X_j)\right]} \end{bmatrix},$$

where $P_j = P_j(\theta_{ij})$; $Q_j = 1 - P_j$; $x_j = Da(\theta_j - b)$, and the remaining terms are as defined for (1). The matrix of second derivatives, denoted by $\underline{f}''$, is given by

$$\underline{f}'' = \begin{bmatrix} \frac{\partial^2}{\partial a^2} L^* & \frac{\partial^2}{\partial a \partial b} L^* & \frac{\partial^2}{\partial a \partial c} L^* & \frac{\partial^2}{\partial a \partial e} L^* \\ & \frac{\partial^2}{\partial b^2} L^* & \frac{\partial^2}{\partial b \partial c} L^* & \frac{\partial^2}{\partial b \partial e} L^* \\ & & \frac{\partial^2}{\partial c^2} L^* & \frac{\partial^2}{\partial c \partial e} L^* \\ & & & \frac{\partial^2}{\partial e^2} L^* \end{bmatrix} .$$

The matrix is symmetric. The individual terms in the matrix are given by:

$$\frac{\partial^2}{\partial a^2} L^* = -D^2 \sum_{j=1}^{N} (\theta_j - b)^2 (P_j - c) \frac{Q u_j c - P_j^2 Q_j - P_j e \, EXP(x_j)(u_j - P_j)}{P_j^2 \, Q_j^2 \, [1 + EXP\,(x_j)]^2} ;$$

$$\frac{\partial^2}{\partial b^2} L^* = D^2 a^2 \sum_{j=1}^{N} (P_j - c) \frac{P_j^2 Q_j - Q_j c u_j + P_j e \, EXP(x_j)(u_j - P_j)}{P_j^2 \, Q_j^2 \, [1 + EXP\,(x_j)]^2} ;$$

$$\frac{\partial^2}{\partial c^2} L^* = \sum_{j=1}^{N} \frac{u_j - 2P_j u_j + P_j^2}{P_j^2 \, Q_j^2 [1 + EXP\,(x_j)]^2} ;$$

$$\frac{\partial^2}{\partial e^2} L^* = \sum_{j=1}^{N} EXP(2x_j) \frac{P_j^2 - 2P_j u_j + u_j}{P_j^2 \, Q_j^2 [1 + EXP(x_j)]^2} ;$$

$$\frac{\partial^2}{\partial a \partial b} L^* = D \sum_{j=1}^{N} (P_j - c) \left\{ \frac{x_j \left[Q_j c u_j - P_j^{2} Q_j - P_j e \ \mathrm{EXP}(x_j)(u_j - P_j)\right]}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} + \frac{u_j - P_j}{P_j \ Q_j \left[1 + \mathrm{EXP}\ (x_j)\right]} \right\} ;$$

$$\frac{\partial^2}{\partial a \partial c} L^* = D \sum_{j=1}^{N} (\theta_j - b) \frac{P_j Q_j u_j - Q_j c u_j + P_j e \ \mathrm{EXP}\ (x_j)\ (u_j - P_j)}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} ;$$

$$\frac{\partial^2}{\partial a \partial e} L^* = -D \sum_{j=1}^{N} (\theta_j - b) \ \mathrm{EXP}(x_j) \frac{P_j^{2} - P_j^{2} c - P_j^{2} u_j - Q_j c u_j + P_j c u_j}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} ;$$

$$\frac{\partial^2}{\partial b \partial c} L^* = -Da \sum_{j=1}^{N} \frac{P_j \ Q_j u_j - Q_j c u_j + P_j e \ \mathrm{EXP}\ (x_j)(u_j - P_j)}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} ;$$

$$\frac{\partial^2}{\partial b \partial e} L^* = Da \sum_{j=1}^{N} \mathrm{EXP}\ (x_j) \frac{P_j^{2} - P_j^{2} u_j - P_j^{2} c + P_j c u_j - Q_j c u_j}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} ; \text{ and}$$

$$\frac{\partial^2}{\partial b \partial e} L^* = - \sum_{j=1}^{N} \mathrm{EXP}\ (x_j) \frac{P_j^{2} + P_j u_j + Q_j u_j}{P_j^{2} \ Q_j^{2} \left[1 + \mathrm{EXP}\ (x_j)\right]^2} .$$

Table 1

Summary of Relationship Between θ_1 and θ_2

θ_1	θ_2
$\theta_1 > c_1$	$\theta_2 \sim N(-1.0, 0.5)$
$c_1 < \theta_1 < c_2$	$\theta_2 \sim N(\theta_1, 0.5)$
$c_2 < \theta_1$	$\theta_2 \sim N(0.5, 0.5)$

Table 2

Item Parameter Estimates
for Simulated 4PL Data

Parameter	Estimate
a	1.175
b	-0.160
c	0.021
e	0.076

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Table 3

Sample Sizes and Proportions Passing
for Each Achievement Interval on M_1

Interval	Sample Size	Cutoff on M_2: 3	4	5	6	7	8	9
1	11	0.091	0.000	0.000	0.000	0.000	0.000	0.000
2	10	0.100	0.000	0.000	0.000	0.000	0.000	0.000
3	15	0.333	0.133	0.067	0.000	0.000	0.000	0.000
4	16	0.500	0.188	0.062	0.062	0.000	0.000	0.000
5	20	0.250	0.200	0.100	0.000	0.000	0.000	0.000
6	16	0.250	0.125	0.000	0.000	0.000	0.000	0.000
7	13	0.231	0.077	0.000	0.000	0.000	0.000	0.000
8	26	0.423	0.192	0.038	0.000	0.000	0.000	0.000
9	34	0.382	0.265	0.088	0.059	0.029	0.000	0.000
10	29	0.414	0.207	0.103	0.069	0.034	0.000	0.000
11	34	0.294	0.118	0.000	0.000	0.000	0.000	0.000
12	32	0.375	0.250	0.094	0.063	0.031	0.031	0.000
13	43	0.558	0.395	0.186	0.047	0.047	0.023	0.000
14	60	0.400	0.200	0.150	0.067	0.000	0.000	0.000
15	59	0.525	0.305	0.119	0.051	0.017	0.000	0.000
16	65	0.631	0.477	0.323	0.185	0.092	0.046	0.015
17	70	0.471	0.300	0.200	0.157	0.043	0.029	0.029
18	87	0.655	0.414	0.195	0.103	0.069	0.034	0.023
19	89	0.517	0.360	0.169	0.090	0.022	0.011	0.011
20	71	0.606	0.479	0.296	0.155	0.113	0.056	0.000
21	79	0.684	0.519	0.329	0.177	0.114	0.025	0.025
22	101	0.653	0.465	0.317	0.228	0.089	0.050	0.030
23	89	0.629	0.416	0.326	0.213	0.112	0.045	0.030
24	103	0.709	0.583	0.398	0.223	0.097	0.068	0.000
25	114	0.789	0.605	0.456	0.333	0.211	0.149	0.029
26	99	0.808	0.737	0.657	0.444	0.323	0.192	0.061
27	143	0.790	0.657	0.497	0.371	0.259	0.189	0.091
28	112	0.777	0.625	0.545	0.438	0.277	0.161	0.116
29	122	0.820	0.730	0.590	0.467	0.320	0.189	0.074
30	120	0.800	0.717	0.667	0.517	0.350	0.250	0.208
31	155	0.845	0.742	0.671	0.606	0.445	0.368	0.297
32	116	0.914	0.767	0.724	0.612	0.431	0.293	0.181
33	102	0.912	0.833	0.775	0.667	0.598	0.431	0.343
34	112	0.884	0.821	0.741	0.643	0.571	0.455	0.339
35	142	0.923	0.852	0.768	0.599	0.507	0.437	0.268
36	78	0.949	0.859	0.782	0.679	0.538	0.410	0.308
37	49	0.837	0.735	0.592	0.490	0.429	0.388	0.265
38	72	0.889	0.833	0.722	0.667	0.583	0.458	0.375
39	54	0.963	0.889	0.778	0.667	0.630	0.444	0.370
40	129	0.938	0.899	0.845	0.752	0.705	0.636	0.496
41	122	0.918	0.869	0.836	0.779	0.697	0.549	0.402
42	82	0.951	0.878	0.780	0.659	0.524	0.488	0.488
43	20	1.000	0.900	0.850	0.700	0.550	0.500	0.400
44	29	1.000	1.000	1.000	0.966	0.828	0.690	0.552

Figure 1

Theoretical Relationship
between Performance on Two Modules

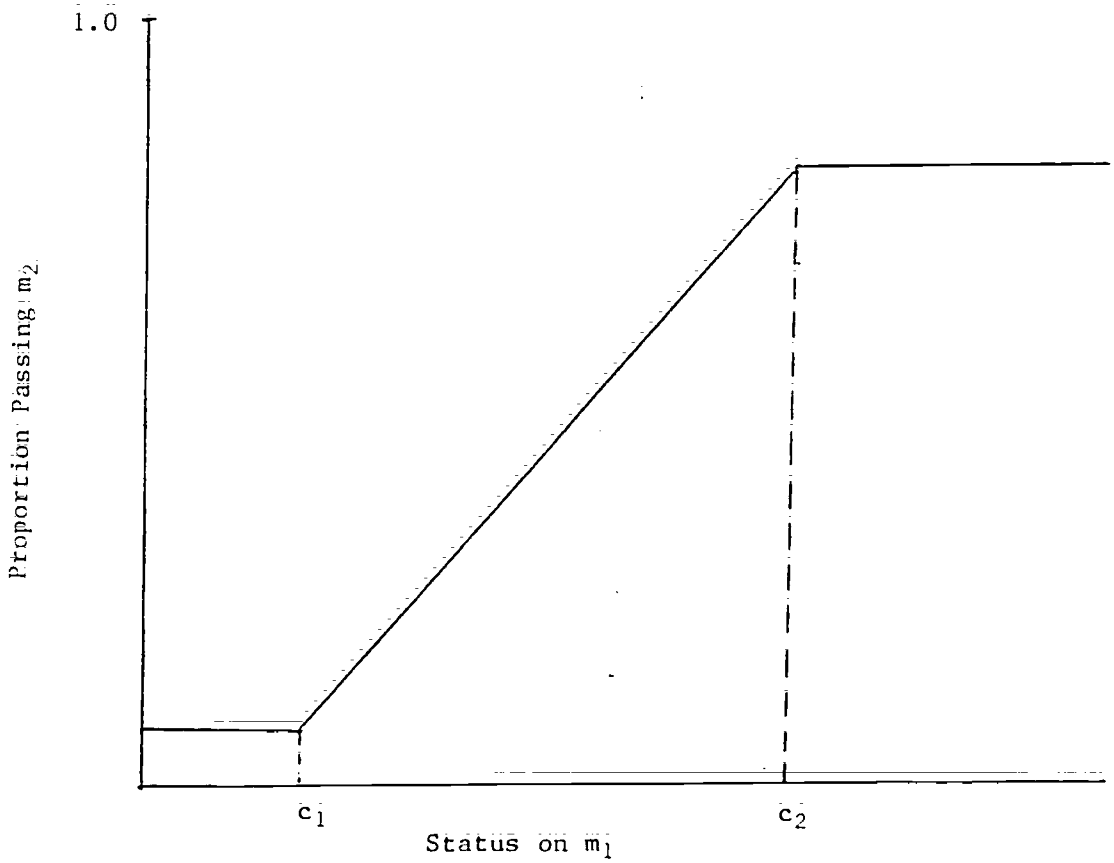

Figure 2

Relationship between Achievement Levels on Modules 1 and 2

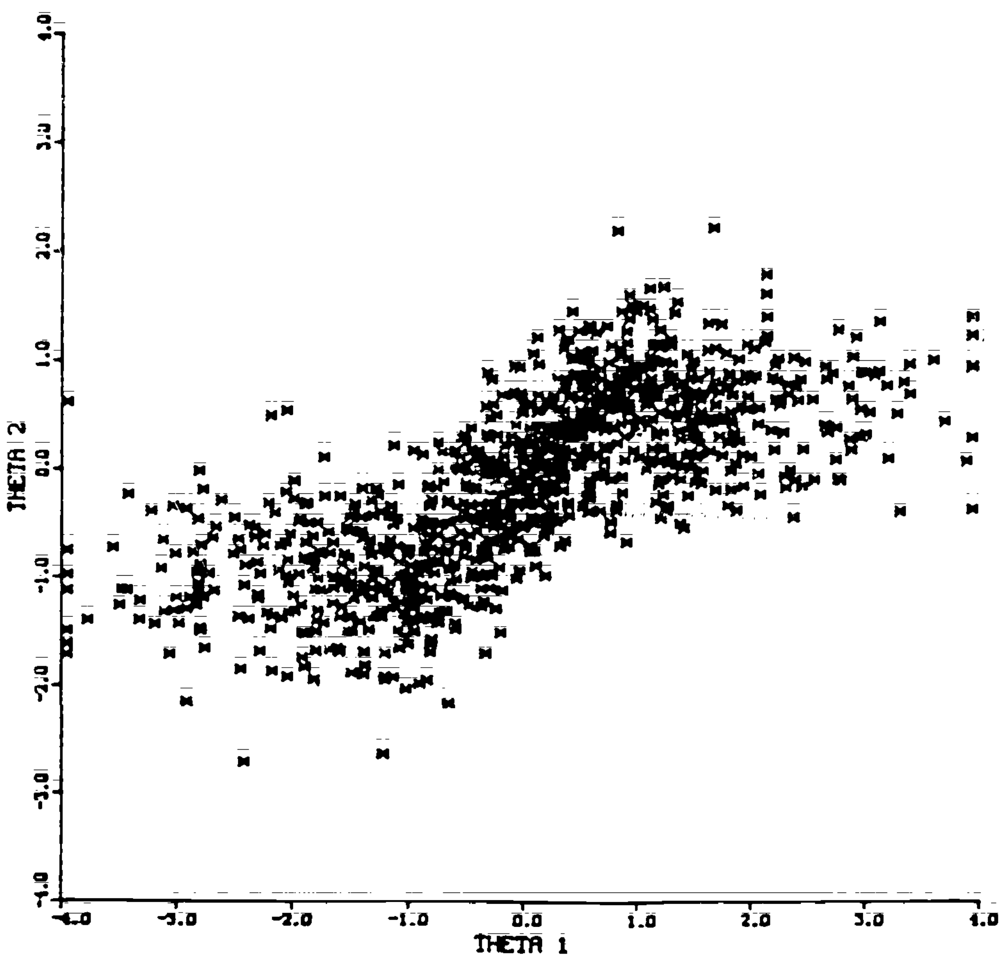

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Figure 3

Empirical MCC for Generated Data

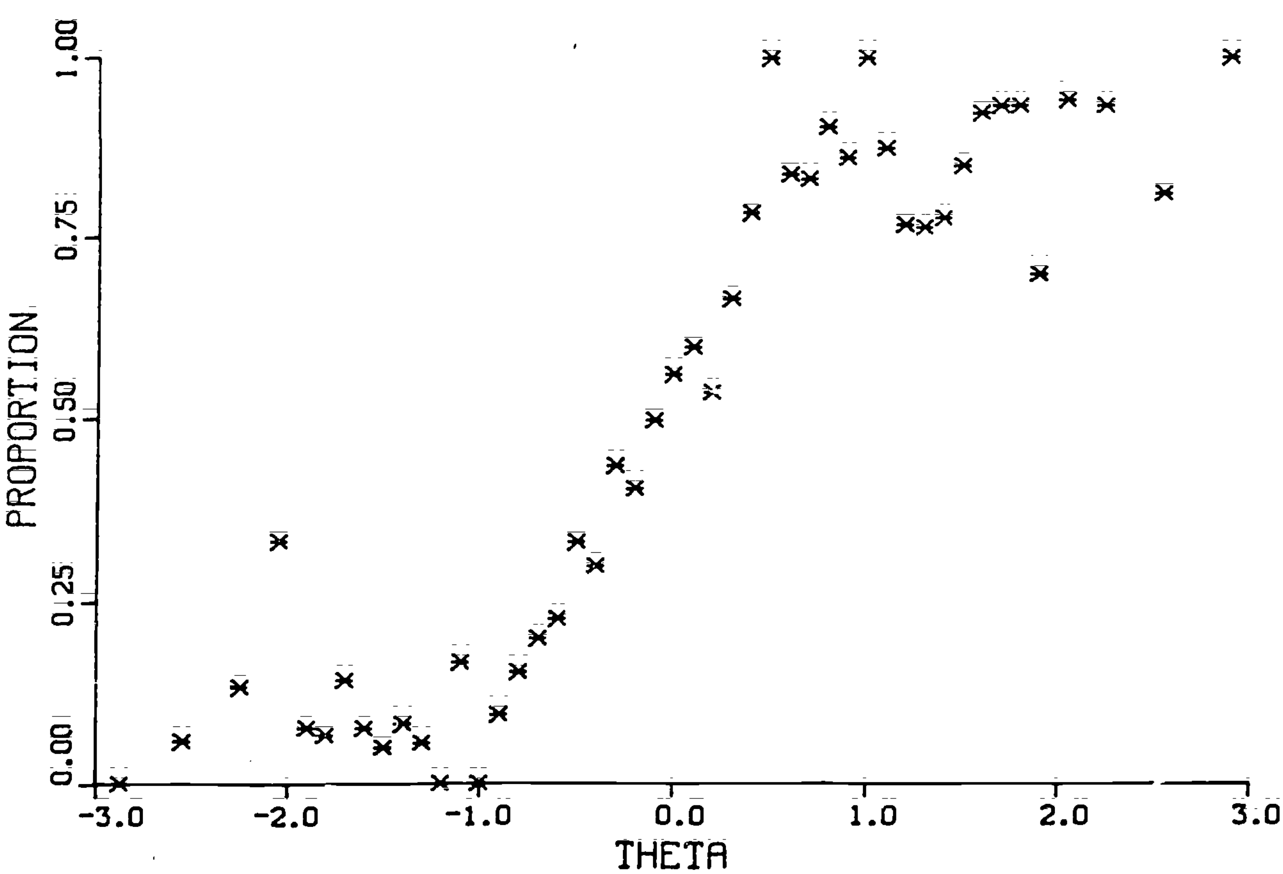


Figure 4

A 4PL MCC

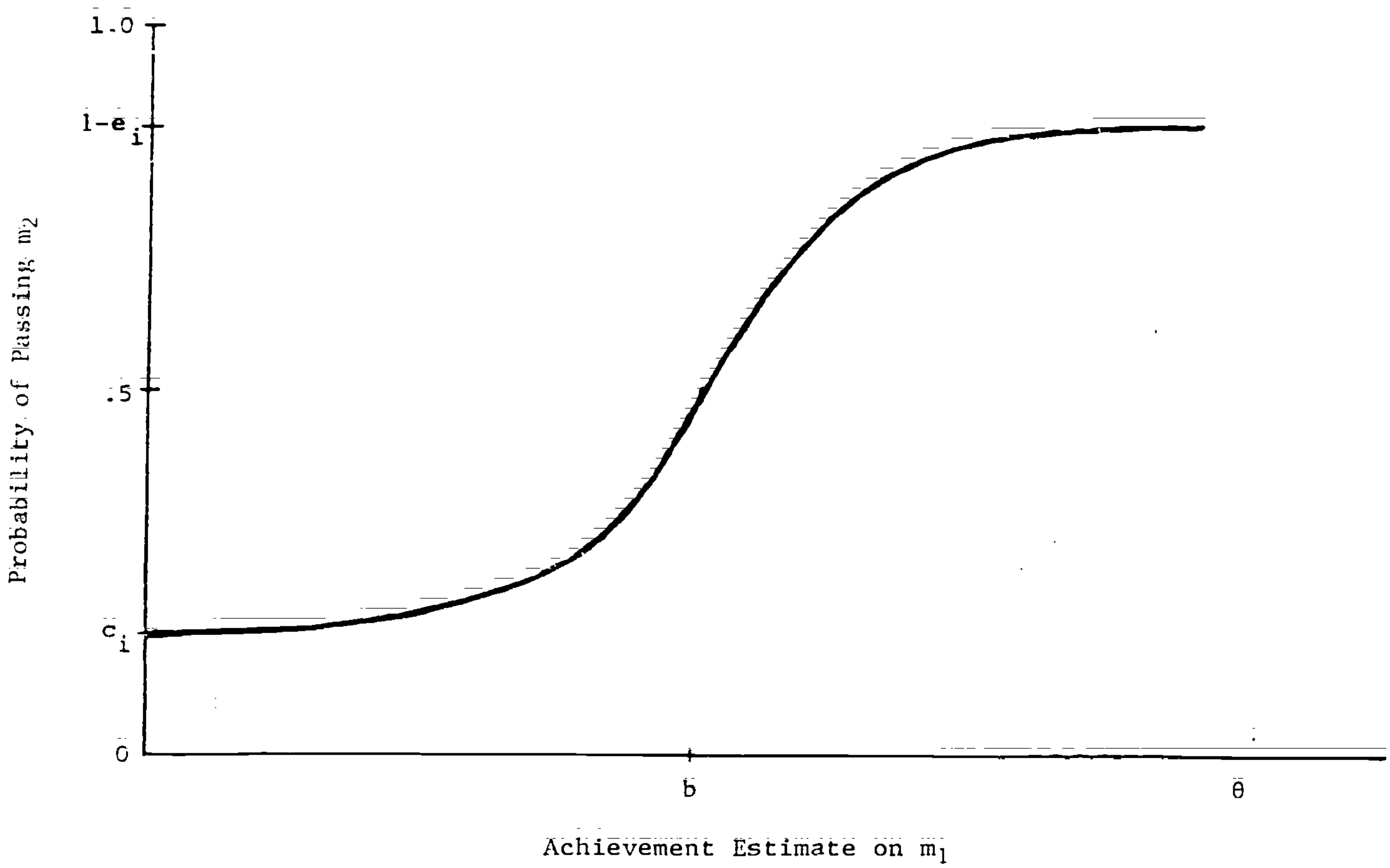


Figure 5

Empirical MCC for Generated Data
with an Overlay of the Theoretical MCC

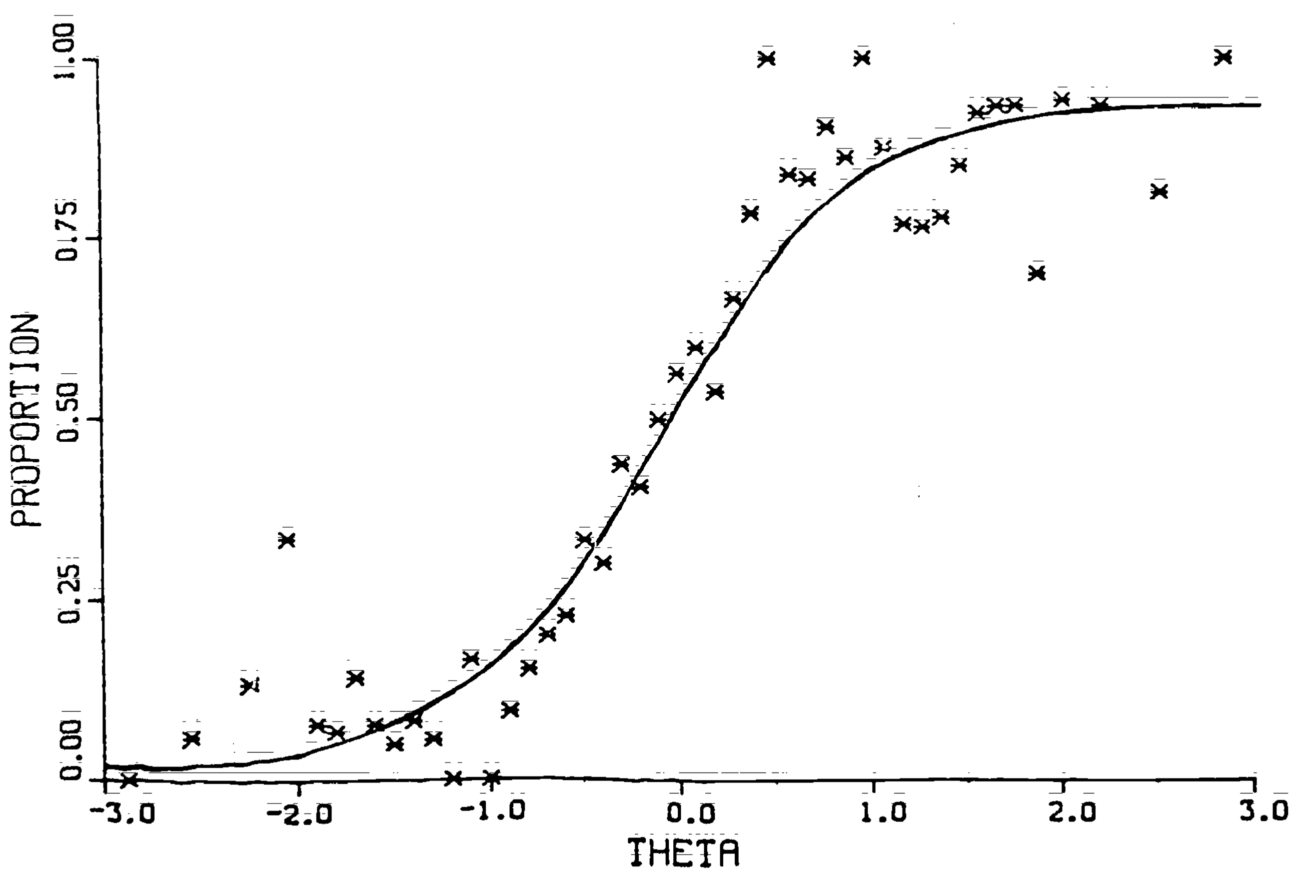


Figure 6

Empirical MCC for CPP Data
Cutoff = 3

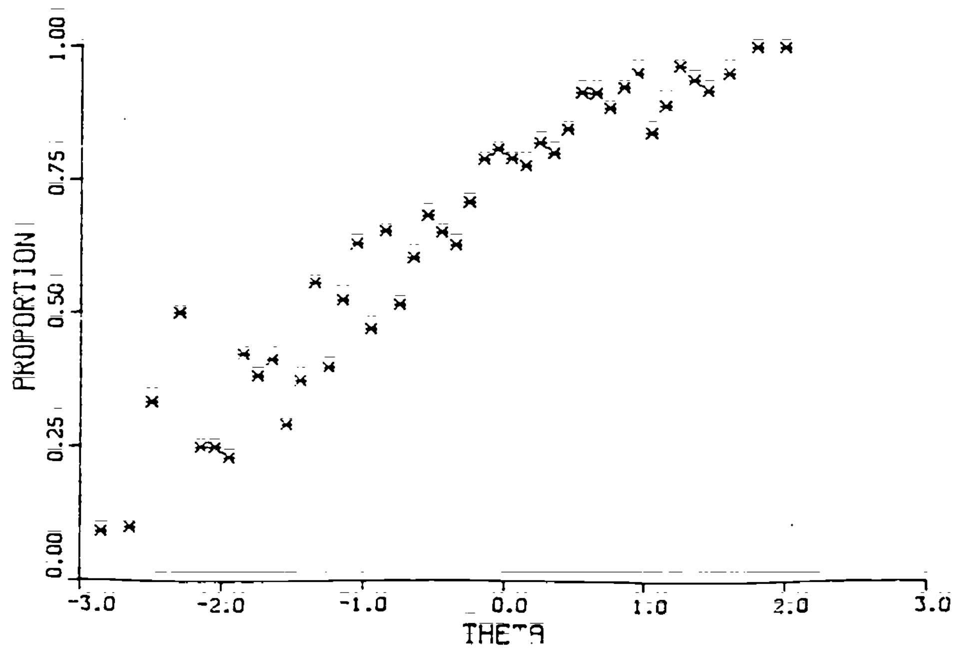


Figure 7

Empirical MCC for CPP Data
Cutoff = 4

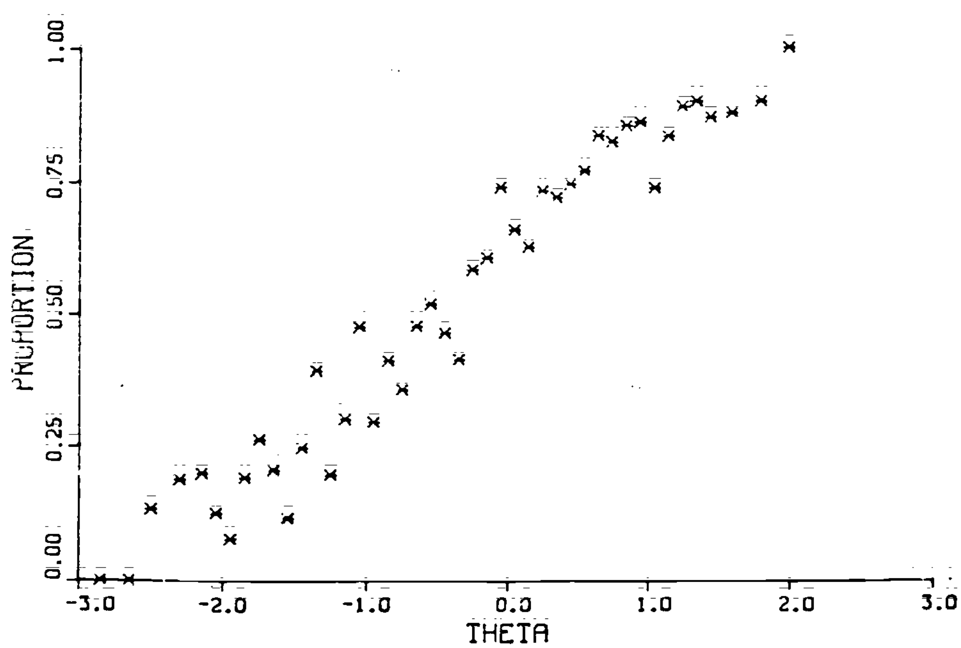


Figure 8

Empirical MCC for CPP Data
Cutoff = 5

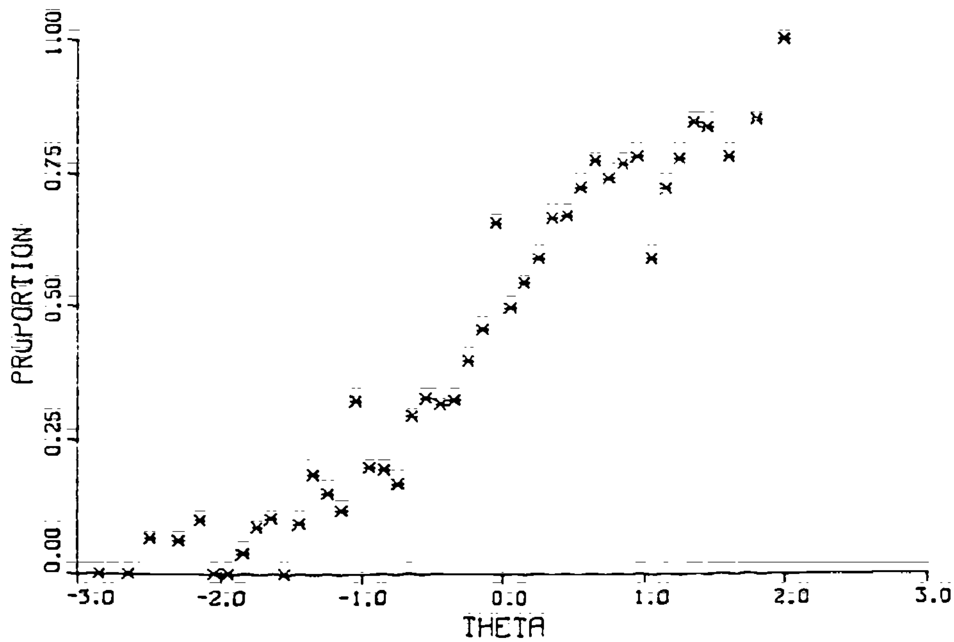


Figure 9

Empirical MCC for CPP Data
Cutoff = 6

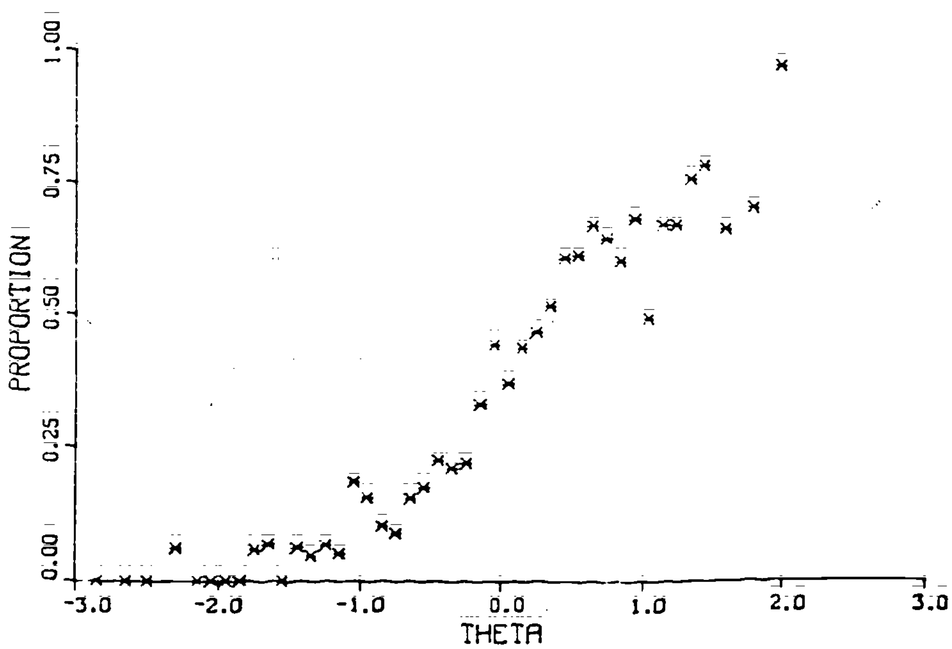


Figure 10

Empirical MCC for CPP Data
Cutoff = 7

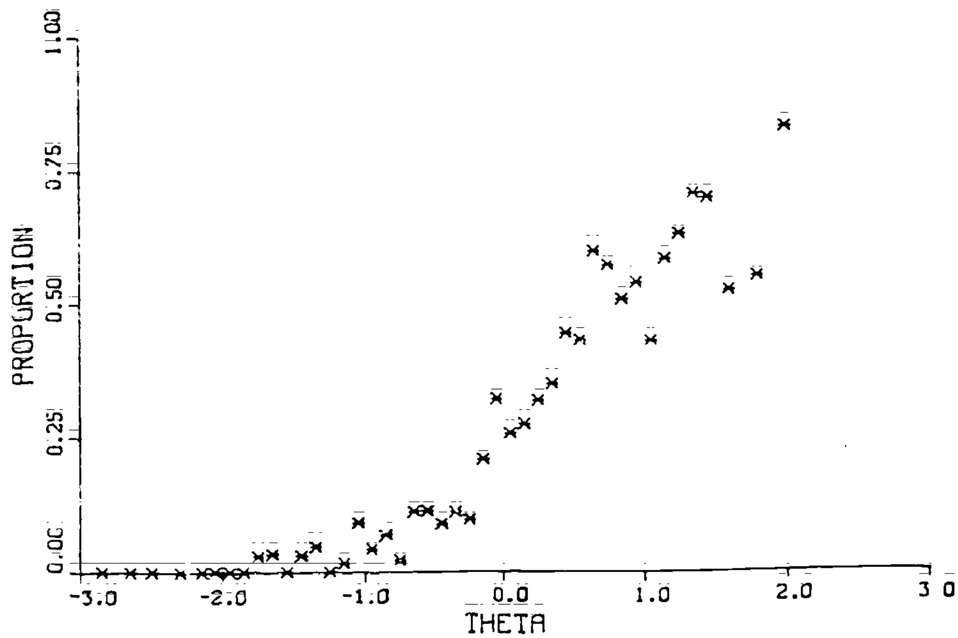


Figure 11

Empirical MCC for CPP Data
Cutoff = 8

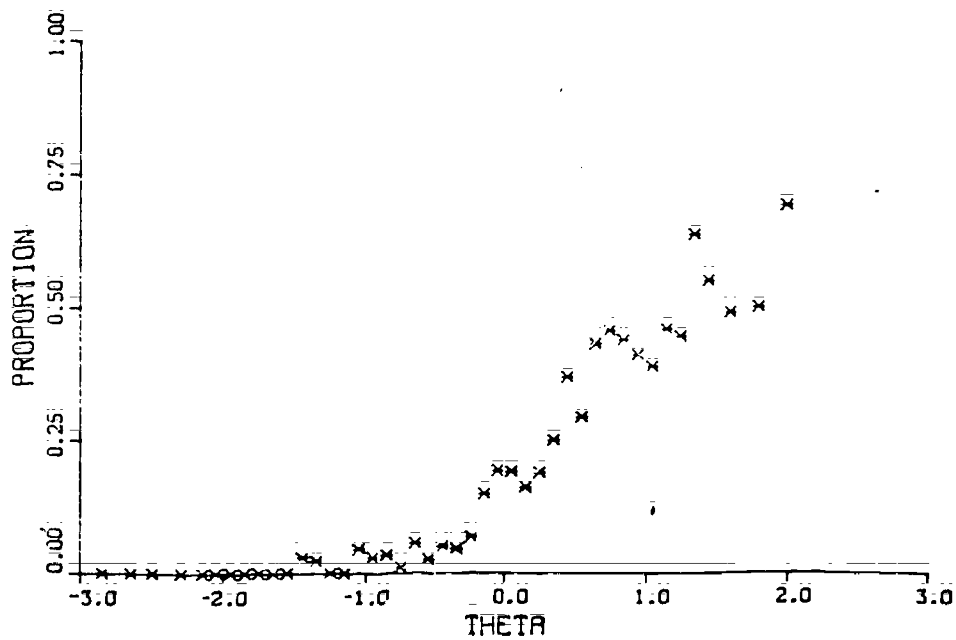
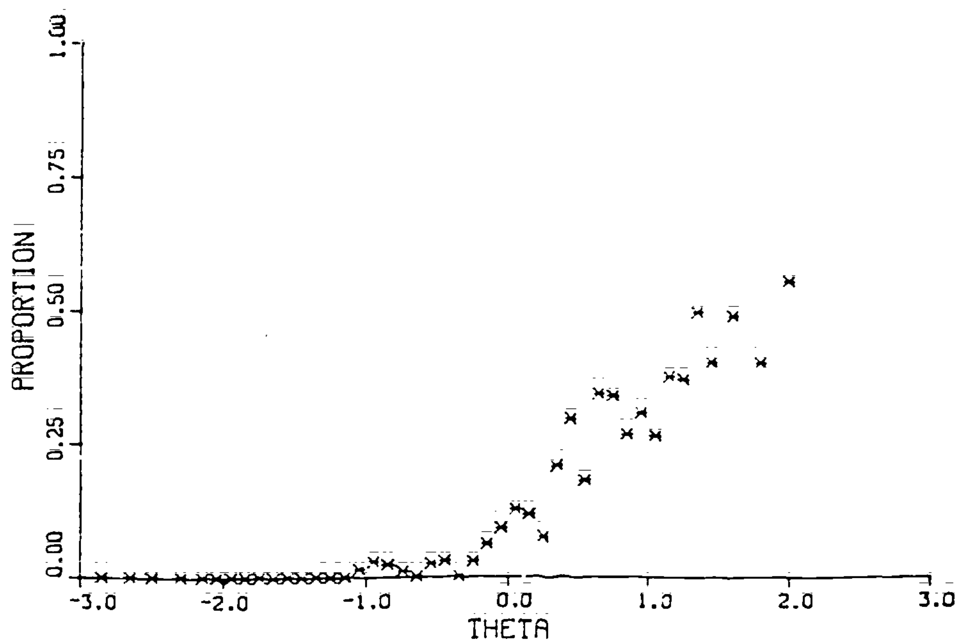


Figure 12

Empirical MCC for CPP Data
Cutoff = 9



A Latent Trait Model for Use with Sequentially Arranged Units of Instruction

Abstract

A theory relating performance on sequentially arranged units of instruction was derived, and an item response theory model for describing that relationship was formulated. Procedures for using the model to evaluate sequential relationships and for making routing decisions were described. A procedure for estimating the parameters of the model was outlined, and data supporting the validity of the model were presented. Overall, the model and procedures appeared to be useful ones, and they appeared to merit continued research efforts directed toward their further development.

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