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ABSTRACT

These instructional materials were produced as part of the project, Developing Computational Estimation Materials for the Middle Grades. The introduction to these seventh grade materials covers the following: why teach estimation; how the materials were developed; and how the lessons are organized. The 15 lessons that follow are designed to teach such estimation strategies as front-end estimation, compatible numbers, clustering, and rounding in lessons with whole numbers, fractions, mixed numbers, decimals, and percents. Each lesson plan includes objectives, teacher background, suggestions for teaching the lesson, acceptable answers for exercises, and six worksheets for student use. (MNS)

Why Teach Computational Estimation?

Estimation has long been recognized as a valuable, useful skill in many vocations and in daily life. With the growing use of calculators and computers it is vital that people be able to judge the reasonableness of an answer. Also there are many instances where an estimate is all that is required to make an important decision. Despite the importance of estimation it has seldom received serious attention in curriculum materials and teachers have had few resources available for supplementing their own ideas. Evidence of students' performance on estimation indicates that most students do not have high proficiency with it, nor do even good estimators have a strong level of confidence in their ability to estimate.

How Were These Materials Developed?

In recent years there has been a renewed interest in this topic, including an increase in research on students' thinking on estimation tasks and on learning specific estimation strategies. This curriculum development project has been built upon this growing body of research. These materials were developed as part of a National Science Foundation project to provide a teaching resource for middle grades and junior high school. This particular set of lessons is designed to provide systematic instruction of effective estimation strategies in Grade 7. Other sets are available for Grades 6 and 8.

These lessons have been successfully used in schools. (A report documenting their effectiveness is available from any of the authors.) This package of lessons has been field tested and reflects helpful suggestions that have been provided by many teachers and students.

The extensive field testing of these materials revealed that there are many ingredients necessary to helping students become proficient estimators. One of these ingredients which you will want to keep in mind as you use these materials is the development of a proper mental set for estimation. This includes:

1. Recognition that estimation is [illegible]tant and useful.
2. Awareness that many situations require only an estimate.
3. Recognition that there are many ways to obtain reasonable estimates.

How Are the Materials Organized?

Fifteen lessons have been written for each grade. Each lesson has the following components:

1. Objectives The objective(s) for each lesson is stated at the top of the first page of teacher notes.

2. Teacher Background This section discusses the strategies taught in the lesson in detail. Sometimes it also provides some additional background notes to help teachers better understand the approaches used.

3. Teaching the Lesson Brief suggestions for teaching the lesson are provided. The major portion of each lesson is developed through overhead transparencies. You will need to make the transparencies from the paper copies provided in these materials. The transparencies often provide real-world settings requiring estimation. They also present key steps highlighting each strategy along with examples for students to try under your direction. We think you will find the transparencies very useful in your teaching. They highlight the main ideas and focus students' attention on the key steps.

4. Using the Exercises Brief comments and suggestions for using the assignment sheets are given.

5. Student Assignment Sheets A two-page worksheet is provided for each lesson. Each worksheet also provides some real-world applications of estimation. These should be started in class and completed as homework. As time permits, discussion of selected exercises the following day will promote estimation thinking and awareness of the many ways of obtaining a reasonable estimate.

What Are The Lessons?

The lesson titles for the seventh grade materials are given here. In most cases the titles are descriptive, however please refer to the specific lessons for a more comprehensive explanation of the topic.

GRADE 7

TABLE OF CONTENTS

USING THE MATERIALS

The process of developing students' estimation competency is a long one. As they have repeated contacts with estimating and as they develop competence with specific techniques for obtaining an estimate, students will gain skill and confidence. Although your students may not reach a high level of competency in one year, progress will be made through systematic instruction.

You have an important role to play in developing students' ability to estimate. Initially many students may show resistence toward estimating. Other students will welcome the opportunity to share self-developed estimation strategies. Through discussion of thinking strategies with students and the encouragement of students' sharing their own thinking for a problem, you can help them gain new appreciation for the estimation process.

We think these lessons emphasize the important components of estimation skill and will be most interested in learning about your experience in using them. Good luck to you!

OBJECTIVES: To create an awareness and appreciation for the usefulness of estimation.

To introduce the front-end strategy for addition and subtraction with informal adjusting of the initial estimate.

TEACHER BACKGROUND:

The lesson introduces the nature and usefulness of estimation and presents the front-end strategy as one effective way to estimate. In the lesson and throughout the program, students should recognize and accept that:

- estimation is important;
- only an estimate is often required;
- there are many ways of obtaining an appropriate estimate;
- any estimate within a reasonable range is acceptable;
- estimation should be done quickly and mentally, and produce reasonable answers.

Students need to gain confidence in their ability to estimate and become flexible in their estimation thinking. To attain these goals, it is important to engage them in discussion and listen to how they think.

This is the first of two lessons on front-end estimation in addition and subtraction. In this lesson the emphasis is on getting an initial or ballpark estimate by finding the sum or difference of the front (or lead) digits and then informally adjusting the estimate. In the next lesson students are shown more precise ways of adjusting estimates. This two-stage development should help students feel comfortable with estimating this way and develop confidence in their ability to estimate.

Front-End Addition Estimation

FRONT-END estimation begins by finding the sum of the front (or lead) digits. At the right this sum is $9, which is an initial estimate.

$2.73
4.21
+3.47

FRONT-END SUM: $2 + $4 + $3 = $9

INITIAL ESTIMATE: $9 or $9+

The total of cents is more than $1.00, so the initial estimate is ADJUSTED upward. It is possible to be more precise at this point, which is the emphasis of Lesson 2.

$2.73
4.21
+3.47

ADJUSTED ESTIMATE: $10, $10+ , over $10

Front-End Subtraction Estimation

Again, one begins with the front digits. The difference of the thousands is 4, so the initial estimate is 4000.

6382
- 2546

FRONT-END DIFFERENCE: 6000 - 2000 = 4000

Now the rest of the number is examined to adjust the estimate. In Lesson 1 students write 4000- to adjust.

~~6~~382
- ~~2~~546

382 is less than 546, so the answer is less than 4000.
ESTIMATE: 4000-

Note: 4000 is an overestimate since the answer to 6382-2546 is less than the estimate. To show that the answer is less than the estimate a - sign is written after the estimate.

In the addition example (page 1), $10 is an underestimate since the answer is greater than the estimate. This is shown by the + sign written after the estimate.

Horizontal Form

Students often make mistakes when examples are written horizontally by adding the lead digit of each number, regardless of its place value. When exercises are written horizontally students must make sure that the lead digits they use have the same place value.

57 + 246 + 832

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

Each lesson begins with a transparency that is designed to help students think about the nature and process of estimation. TR #1 provides an introduction to the program.

Before showing the transparency, ask students to tell what they think estimation is and to give words that are used when estimating. Some words are:

about	almost	about
around	close to	a little more (less) than
approximately	over	under.

Use the top half of TR #1 to present and discuss some characteristics of estimation. Then have students discuss the situations at the bottom of the transparency. Have students think of other situations where they use estimation.

ANSWERS:

1. The calculator answer is not sensible. 97 x 1 equals 97, so the product is a little over 97.

2. You can not buy 3 rolls of film. 3 x $2 equals $6 and the cents are over $.50. The 3 rolls will cost more than $7.

3. The car averages less than 30 miles per gallon. If the car got 30 miles per gallon, on 11 gallons it would go 330 miles. Also, when 300 is divided by 11 the quotient is in the 20s.

FRONT-END ESTIMATION

TR #2: Tell students that they are going to learn a way to estimate that may be new for them. Present the top half of the transparency and discuss the FRONT-END strategy. Emphasize that estimation begins by using the lead digits. Here the "getting closer" process is handled informally. It is acceptable if students adjust more precisely, but it is not necessary.

Discuss the TRY THESE exercises with students.

ANSWERS:

1. $5; $6.00^{+}
2. $13; $14.00^{+}
3. $16; $17.00^{-}

TR #3: Discuss the problem situation at the top and the FRONT-END strategy for subtraction. Point out that the "-" sign after $4000 means that the answer is <u>under</u> $4000.

Discuss the first row of TRY THESE exercises with students. Then have them try the second row independently before discussing them.

ANSWERS:
1. under 5000 (5000^-)
2. under 400 (400^-)
3. over 8000 (8000^+)
4. under 300 (300^-)
5. under 4000 (4000^-)
6. over 600 (600^+)

TR #4: Carefully discuss the two examples at the top of the transparency. Since students often make errors when examples are written horizontally, stress that the front-digits used must have the same place value.

ANSWERS:
1. 10,000; $10{,}000^+$ or $11{,}000^-$
2. 600; 700^+ or 800^+
3. 300; 400^+ or 500^+
4. 800; 800^+ or 900^-

USING THE EXERCISES

With the lesson development as a background, students should be ready for independent work. You might do one example of each type with them so they will know what is expected. Emphasize that they do not need to find exact answers and encourage them to estimate.

Suggested answers are given below. Be flexible in evaluating students' work and accept alternate answers if they seem plausible.

ANSWERS:

1. 900; 900^+ or 1000
2. \$15.00; $\$15.00^+$
3. 17,000; $18{,}000^+$ or $19{,}000^-$
4. \$170.00; $\$180.00^+$
5. 1500^+
6. $24{,}000^+$
7. 1700^+
8. 1900^+
9. 4000^+
10. 300^-
11. 5000^+
12. 500^-
13. 2000^-
14. 3000^+
15. $13{,}000^+$
16. 2000^-
17. 4000^+
18. $50{,}000^+$ or $57{,}000^+$
19. 3652 & 1,257
20. \$0.19
21. 107 & 959
22. 1,877
23. $40{,}000^+$ to $50{,}000^-$
24. 800^+ to 900^-
25. $\$70^+$ to $\$80^-$
26. $\$8.00^+$ to $\$9.00^-$
27. $20{,}000^+$ to $23{,}000^-$
28. 100^-
29. 2000^+
30. $50{,}000^-$

GET YOUR MIND IN GEAR

Estimation is:

Quick: FASTER THAN PAPER AND PENCIL.

FASTER THAN PUNCHING CALCULATOR NUMBERS.

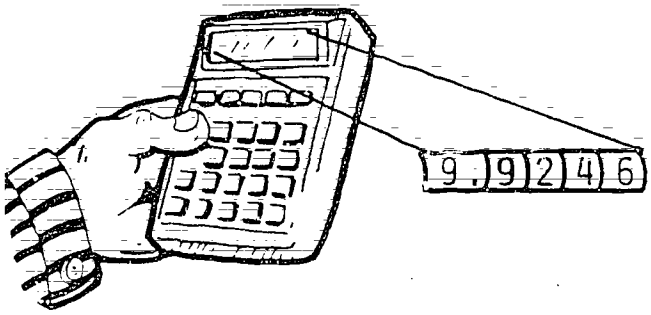
Reasonable: THE ESTIMATE IS "CLOSE ENOUGH" FOR WHAT YOU NEED TO DO.

Done in your head: YOU WORK WITH NUMBERS YOU CAN MANAGE.

Useful:

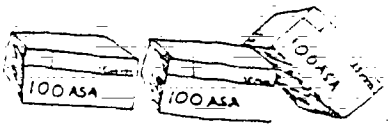
IS THE RESULT SENSIBLE?

1.02 X 97.3

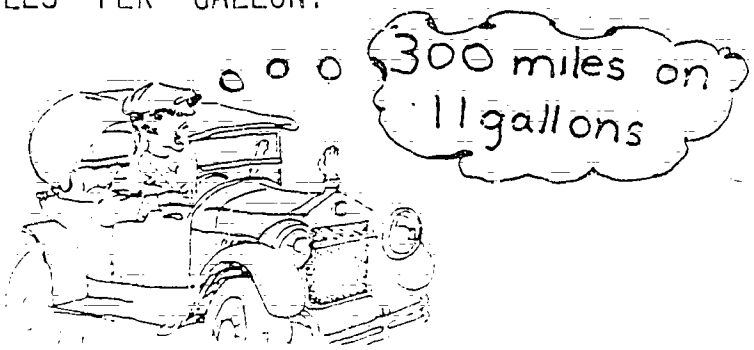


YOU HAVE $6.50.

CAN YOU BUY THESE?

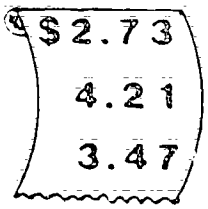


DID THE CAR AVERAGE MORE OR LESS THAN 30 MILES PER GALLON?



7-1-TR1

FRONT-END ESTIMATION



Front-End Sum

1. ADD THE FRONT-END

$2.73
4.21
+ 3.47
_____

Getting Closer

2. LOOK AT THE REST

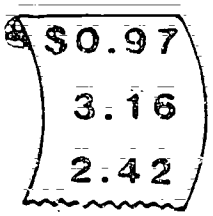
~~$2.~~ 73
~~4.~~ 21
~~3.~~ 47

the cents make it go over $10.

Estimate:

over $10.00
OR $10.00+

Try these!

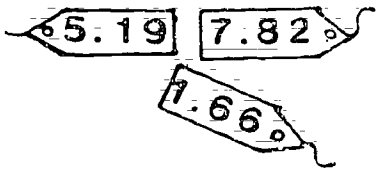


Front-End Sum

$ ___.00

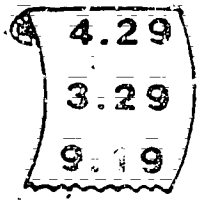
Getting Closer
IS IT OVER ANOTHER DOLLAR?

$6.00+ OR $6.00-?



$ ___.00

$14.00+ OR $14.00-?



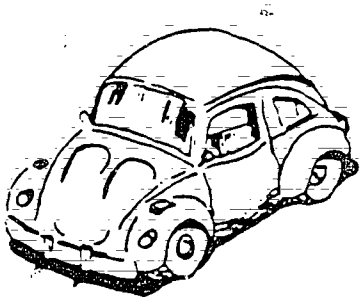
$ ___.00

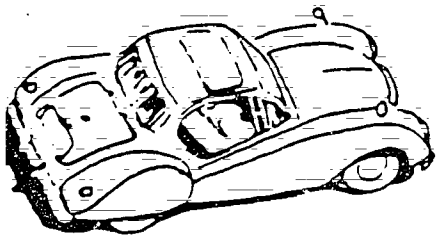
$17.00+ OR $17.00-?

7-1-TR2

FRONT-END SUBTRACTION

ESTIMATE THE DIFFERENCE IN COST FROM 1960 TO 1980.





1960
TOTAL $25 46

1980
TOTAL $6382

FRONT-END DIFFERENCE

$6382
−2546
$4000

COMPARE THE REST

6382
−2546

382 is Less than 546

FINAL ESTIMATE

UNDER $4000
OR 4000

TRY THESE

6283
− 1526
//////
under 5000
OR 5000

837
− 466
/////
____ 400
OR 400

9286
− 1162
//////
____ 8000
OR 8000

458
− 167
/////
____ 300
OR 300

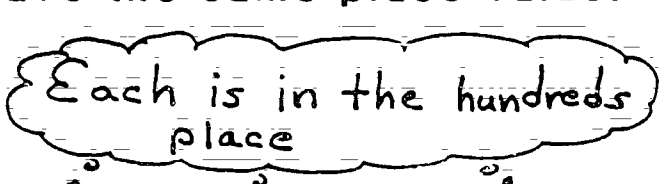
9276
− 5672
//////
____ 4000
OR 4000

873
− 238
/////
____ 600
OR 600

7-1-TR3

HORIZONTAL FORM

Be sure the "front-ends" you use have the same place value.

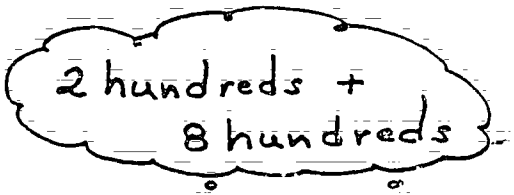


$\underline{4}27 + \underline{3}64 + \underline{1}92$

FRONT-END SUM: _____

GETTING CLOSER: 900^{+}?

OR 900^{-}?



$57 + \underline{2}46 + \underline{8}32$

FRONT-END SUM: _____

GETTING CLOSER: 1100^{+}?

OR 1100^{-}?

TRY THESE:

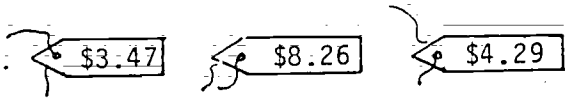
	Front-end Estimate	Getting Closer
8642 + 2347	_____	_____
82 + 426 + 296	_____	_____
398 + 75 + 86	_____	_____
428 + 109 + 317	_____	_____

7-1-TR4

Name _____

Estimate using the front-end strategy.

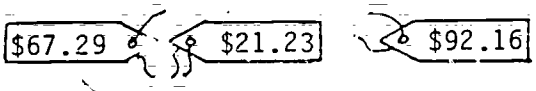
1. 427 Front-end sum: _____
129
431
+15 Adjusted Estimate: _____

2. 

Front-end Sum: _____

Adjusted Estimate: _____

3. 3256 Front-end Sum: _____
4126
9982
+ 1232 Adjusted Estimate: _____

4. 

Front-end Sum: _____

Adjusted Estimate: _____

Circle the closest estimate:

5. 364 1400^{+}
829 1500^{-}
+345 1500^{+}

6. 9063 $24,000^{-}$
8112 $24,000^{+}$
1223 $25,000^{+}$
+6062

7. 264 1600^{+}
264
374 1700^{-}
+906 1700^{+}

8. 961 1800^{+}
844 1900^{+}
+112 2000^{+}

Estimate using the front-end strategy. Use + or - to adjust.

9. 8621
-4139
?
Estimate: _____

10. 627
-389
?
Estimate: _____

11. 7391
-2016
?
Estimate: _____

12. 621-156
Estimate: _____

13. 3,196 - 1,984
Estimate: _____

7-1-p.1

Name _____

Estimate the difference in size for each pair.

Planet	Diameter
Earth	7,927 mi
Mars	4,219 mi
Jupiter	88,700 mi
Saturn	75,100 mi
Uranus	29,300 mi
Neptune	31,200 mi
Pluto	3,700 mi

14. Earth and Mars _____

15. Jupiter and Saturn _____

16. Uranus and Neptune _____

17. Pluto and Earth _____

18. Jupiter and Neptune _____

If you were to estimate the following totals, which numbers could you ignore and still make a good estimate? Make a check by those numbers.

19.
3,652
44,867
108,396
+ 1,257
///////

20.
$4.67
3.82
0.19
+ 1.45
////

21.
3,208
107
959
+ 6,708
/////

22.
24,062
1,877
+ 9,002
/////

Write an estimate for each.

23. 14,682 + 2,146 + 28,677

Estimate: _____

24. 397 + 426 + 28

Estimate: _____

25. $14.56 + $63.79 + $1.25

Estimate: _____

26. $.89 + $6.79 + $1.23

Estimate: _____

27. 44,127 - 21,362

Estimate: _____

28. 387 - 293

Estimate: _____

29. 4,698 - 2,677

Estimate: _____

30. 81,928 - 35,679

Estimate: _____

NSF ESTIMATION
GRADE 7 - LESSON 2

OBJECTIVES: To estimate by grouping numbers whose sum is close to $1.00.

To adjust front-end estimates for addition and subtraction.

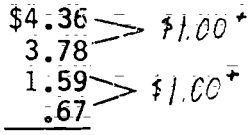
TEACHER BACKGROUND:

Estimation is a flexible process in which the approach used often depends on the numbers and situation. It cannot be reduced to a single step-by-step process. You can expect that many students will initially feel uncomfortable with estimation. However, as they gain more experience with various approaches and have the opportunity to discuss the work, they will become more comfortable.

The major focus of the lesson is adjusting the front-end sum in order to get a more precise estimate. Three approaches for adjusting the front-end sum in addition are presented.

1. Group to make nice numbers.

   In the example at the right. The cents are grouped to make dollars. This amount is added to the front-end sum to get the estimate.



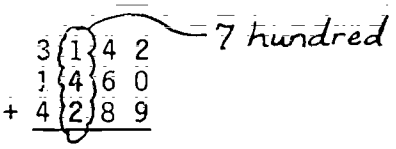
FRONT-END SUM: $8.00

ADJUST UP: $2.00+

ESTIMATE: $10.00+

2. Add the digits in the second column.

   In the example, the sum of the hundreds digits is found. This is a very useful approach that can always be used. But other ways of adjusting should also be explored.



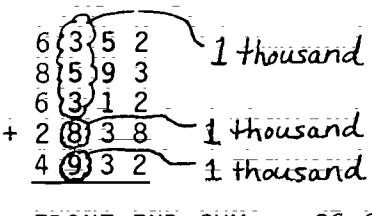
FRONT-END SUM: 8000

ADJUST UP: 700

ESTIMATE: 8700

3. Group the digits in the second column to make nice numbers with which to work.

   This approach, illustrated at the right, is useful when there are several addends.



FRONT-END SUM: 26,000

ADJUST UP: 3,000

ESTIMATE: 29,000

Additional Notes:

1. The emphasis of the lesson is getting closer through adjusting the front-end sum. However, avoid pressing for estimates that are too precise and recognize that there will be a wide range of individual differences in ability to refine initial estimates. Also, the use of "+" or "-" to indicate an underestimate or overestimate is not essential in this lesson.

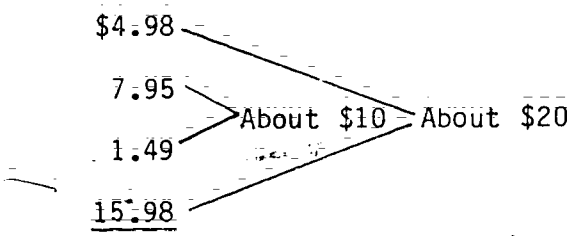
2. Work with grouping strategies and the use of nice numbers can be extended. The two examples below show ways of going further with these ideas.

   a. In the example shown, two amounts whose sum is close to $1.00 are found. Then this is adjusted upward with the third addend. Any estimate between $1.20 and $1.30 is good.

a. $.38 and $.64 are about $1.00

b. $.23 more makes it about $1.20 to $1.25

   b. Grouping numbers to "nice" dollar amounts is an extension of grouping cents to dollars. One looks at the numbers and groups them to whole dollar amounts that are easy to work with.



ESTIMATE: $30.00

Students are also shown how to get closer in subtraction in the lesson. For many students this will be a natural extension of the initial work in subtraction estimation. However, do not require that all students do it in the future work.

In the example at the right, the initial estimate is adjusted by looking at the hundreds digits. To adjust, the student thinks: 13 hundred - 5 hundred = 800. So the estimate is 5800.

```
  ↓
  7 3 8 2
- 1 5 7 9
```

INITIAL ESTIMATE: 6000-

ADJUSTED ESTIMATE: 5800

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 addresses the theme of a rough initial estimate (close enough) versus the need to refine the initial estimate (getting closer). Students need to recognize when "eyeballing" is enough and when they need to get closer.

Have students begin each time by finding the sum of the dollars. Encourage them to discuss their thinking.

ANSWERS: Top Left: Not enough - tell at a glance (eyeball)

Top Right: Enough - tell at a glance (eyeball)

Bottom Left: Not enough - need to get closer (refine)

Bottom Right: Enough - tell at a glance (eyeball)

ADJUSTING IN ADDITION ESTIMATION

TR #2: Present the situation at the top of TR #2. Then present and dis-
TR #3: cuss the three ways of adjusting on TR #2 and TR #3. Discuss each of the TRY THESE exercises with students.

ANSWERS: $2.00 $2.80 - $3.00 $2.00+ - $2.50

8700 - 9000 105,000 - 106,000 11,700 - 12,000+

ADJUSTING IN SUBTRACTION ESTIMATION

TR #4: Present the problem and discuss the adjusting step. Since renaming is required the difference is less than 1000. 12 hundred - 4 hundred = 800, so a good estimate is 800.

ANSWERS: 5800 430 2200 160

USING THE EXERCISES:

Read each set of directions with the class. You may wish to do the first one in each set orally with students.

ANSWERS:

1. 34 and 65
2. 28 and 73
3. 78 and 23
4. 57 and 44
5. 13 and 85
6. 59 and 39
7. $1.60 - $2.00
8. $2.00+ - $2.20
9. $1.00+ - $1.20
10. $25.40 - $26.00
11. $39.00 - $41.00
12. $18.00 - $18.50
13. $7.00 - $7.30
14. $6.80 - $7.00
15. $4.80 - $5.00
16. 23,000 - 24,000
17. $220.00 - $230.00
18. 2,400,000 - 2,500,000
19. 81,000 - 84,000
20. 10,000 - 10,500
21. No, because the front-end sum is $20.00
22. Yes
23. Yes, the total is probably a little less than $5.00
24. Yes

GET YOUR MIND IN GEAR

Sometimes you only need to "eyeball" an estimate ... sometimes you need a more refined estimate.

DECIDE:

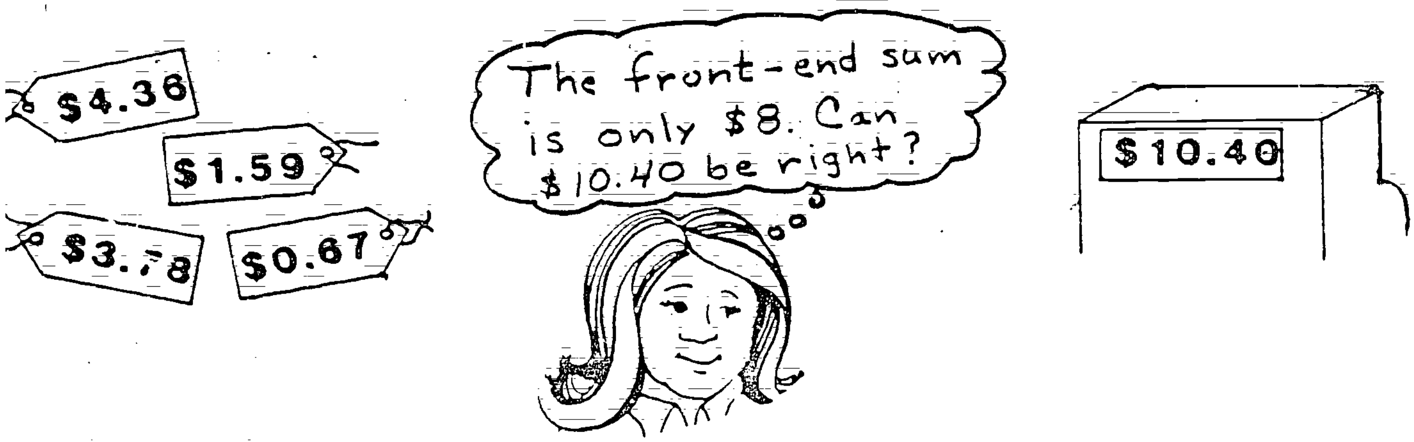
EYEBALL: Can you tell if you have enough in a glance?

OR

REFINE: Do you need to get a little bit closer?

7-2-TR1

GETTING CLOSER IN ADDITION



Often you need to get closer when estimating.

You need to adjust the initial estimate.

1. Group to make nice numbers.

```
$ 4.36 \
  3.78 /  $1.00+
  1.59 \
+  .67 /  $1.00+
```

Front-End Sum: $8.00
Adjust Up: $2.00+
Estimate: ______

```
  4 367 \
  8 923 /  1000+
  1 383 \
  1 225  > 1000+
+ 7 637 /
```

Front-End Sum: 21,000
Adjust Up: ______
Estimate: ______

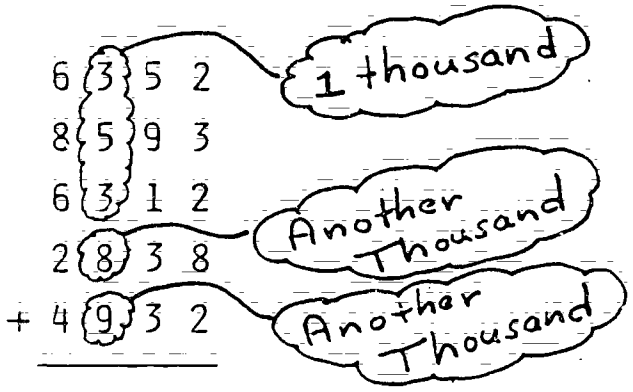
2. Add the digits in the second column.

```
3,142   7 hundred
1,460
4,289
```

Front-End Sum: 8,000
Adjust Up: 700
Estimate: ______

7-2-TR2

3. Group the digits in the second column to make NICE NUMBERS.



Front-End Sum: 26,000
Adjust Up: 3,000
Estimate: _____

TRY THESE: Group cents to make dollars.

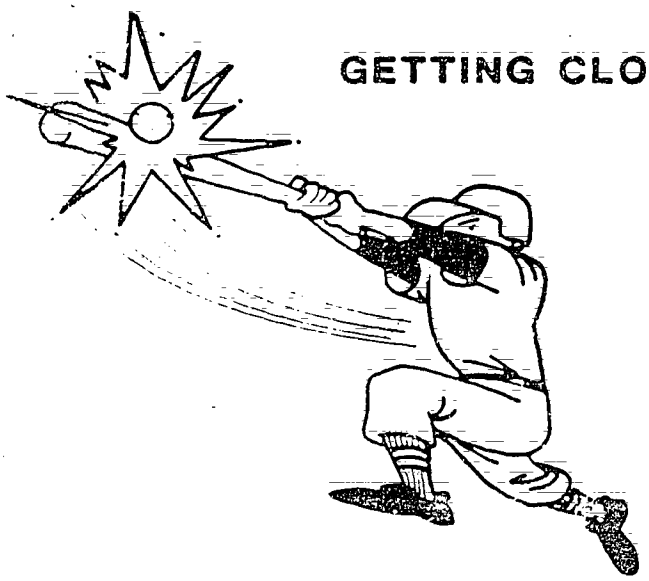
$ .12	$ .39	$ .84
.63	.79	.57
.85	.62	.42
+ .39	.12	.18
//////////	+ .89	+ .31
	//////////	//////////
ESTIMATE: ( )	( )	( )

Think how you can adjust up.

4263	24,367	3467
3418	+ 81,408	2982
+ 1192	//////////////	1556
//////////		2268
		+ 1792
		//////////
ESTIMATE: ( )	( )	( )

7-2-TR3

GETTING CLOSER IN SUBTRACTION



MICKEY MANTLE AND HANK AARON WERE FAMOUS BASEBALL PLAYERS. ABOUT HOW MANY MORE GAMES DID HANK AARON PLAY?

PLAYER	GAMES PLAYED
AARON	3298
MANTLE	2401

SOMETIMES IT'S GOOD TO GET CLOSER.

$$\begin{array}{r} 3\,2\,9\,8 \\ -\,2\,4\,0\,1 \\ \hline /\!/\!/\!/\!/\!/\!/\!/ \end{array}$$

298 IS LESS THAN 400, SO YOU MUST RENAME TO SUBTRACT HUNDREDS.

Initial Estimate: 1000⁻

Getting Closer: 800

TRY THESE:

$$\begin{array}{r} 7\,\overset{\downarrow}{3}\,8\,2 \\ -\,1\,5\,7\,9 \\ \hline /\!/\!/\!/\!/\!/\!/\!/ \end{array}$$

Subtract Hundreds
(13-5) hundreds

CLOSE ENOUGH: 6000⁻

GETTING CLOSER: _____

	$\begin{array}{r} 671 \\ -\,249 \\ \hline /\!/\!/\!/\!/ \end{array}$	$\begin{array}{r} 4127 \\ -\,1963 \\ \hline /\!/\!/\!/\!/\!/\!/\!/ \end{array}$	$\begin{array}{r} 526 \\ -\,368 \\ \hline /\!/\!/\!/\!/ \end{array}$
CLOSE ENOUGH:	400⁺	3000⁻	200⁻
GETTING CLOSER:	_____	_____	_____

7-2-TR4

Name_____

Write the two numbers whose sum is closest to 100.

1. 34, 18, 65 _____

2. 49, 16, 28, 73 _____

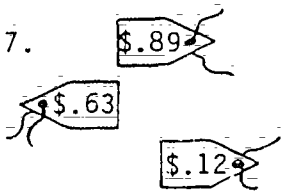
3. 78, 23, 46 _____

4. 12, 57, 44, 32 _____

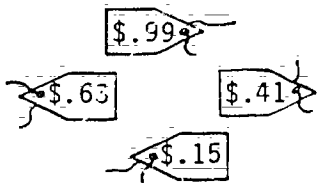
5. 52, 13, 85 _____

6. 11, 39, 59, 81 _____

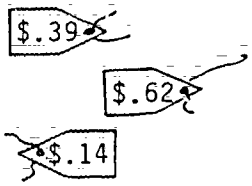
Group the cents to make dollars. Write your estimate.

7. 

Estimate:_____

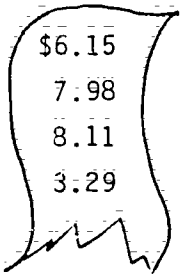
8. 

Estimate:_____

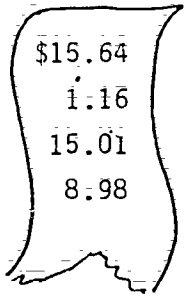
9. 

Estimate:_____

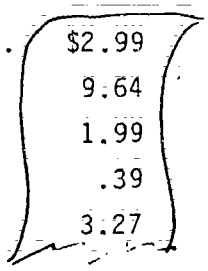
Estimate. Find the front-end sum, and then adjust by grouping the rest to dollars.

10. 

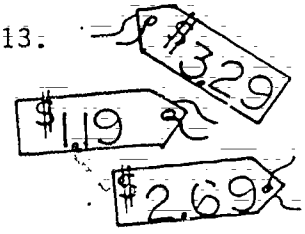
Estimate:_____

11. 

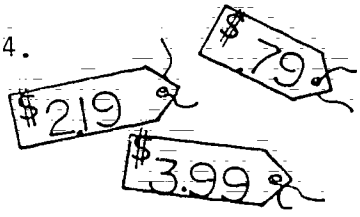
Estimate:_____

12. 

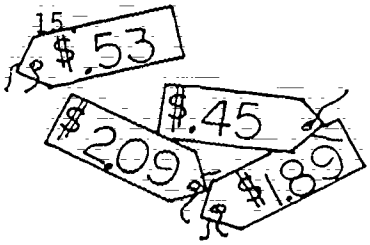
Estimate:_____

13. 

Estimate:_____

14. 

Estimate:_____

15. 

Estimate:_____

Name _____

Estimate. Find the front-end sum and then adjust by using any method you want.

16.
3,674
4,217
8,926
4,528
+ 1,808
?

Front-end Sum: _____

Estimate: _____

17.
$62.18
79.14
1.95
+ 32.79
?

Front-end Sum: _____

Estimate: _____

18.
346,492
827,006
400,608
912,153
+ 11,863
?

Front-end Sum: _____

Estimate: _____

19. 31,675 + 21,647 + 29,876

Front-end Sum: _____

Estimate: _____

20. 3,684 + 6,275 + 438

Front-end Sum: _____

Estimate: _____

21. Robin has $20. Can she buy a bird and a bird cage?

_____

22. Arlyn has $5. Can she buy 2 goldfish and some fish food?

_____

23. Art has $5. Can he buy a hampster, a goldfish, and goldfish food?

_____

24. Ed has $25. Can he buy a bird, a bird cage, a fish, and some fish food?

_____

7-2-p.2

NSF ESTIMATION
GRADE 7 - LESSON 3

OBJECTIVES: To estimate sums using the clustering strategy.

To estimate using large numbers.

TEACHER BACKGROUND:

The clustering strategy can be used when a group of numbers centers (clusters) about a number (average). In the example, the numbers cluster around 70,000, which can be considered to be a rough average for these numbers.

World's Fair Attendance	
Mon.	72,250
Tues.	63,819
Wed.	67,490
Thurs.	73,180
Fri.	74,918
Sat.	68,490
ESTIMATE:	420,000

To estimate:

a. Select a central number that is easy to work with:

70,000

b. Multiply the number by the total number of values:

6 x 70,000 = 420,000

When the clustering strategy is used it is usually not possible to adjust the estimate.

Estimation with larger numbers is often viewed by students as more difficult. However, the same strategies can be used as before. Students need to be flexible and realize that being off by a few thousand is not very much in relation to the size of the numbers involved. Two strategies are particularly effective with larger numbers.

1. Grouping to make nice numbers with which to work.

   As shown in the example, numbers are grouped to make partial sums which are easy to compute mentally.

32,432 > 50,000+
20,672 > 50,000+
21,362 > 50,000+
+ 29,432 > 50,000+

ESTIMATE: 100,000+

7-3-1

2. Using compatible numbers in subtraction.

Since 23 thousand + 23 thousand equals 46 thousand, 23 thousand is a good estimate for the example. In this example, students need to think of finding a missing addend.

47,125
- 23,685

ESTIMATE: 23,000

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 addresses the theme of recognizing sensible answers. The ability to tell whether an answer is sensible or not is an important outcome for the study of estimation. Display each problem and have students select the sensible amount of change. Have them tell why they selected the answer they did.

ANSWERS: 1. $35.98 2. $.79 3. $6.08 4. $4.41

THE CLUSTERING STRATEGY

Display the problem at the top of TR #2 and discuss the steps involved in using this strategy. Help students see:

a. Why 80 was selected. (The numbers cluster around 80 and 80 is a nice number with which to work.)

b. Why they multiply by 7. (There are 7 numbers and 80 is being used in place of each one.)

Discuss the first two TRY THESE exercises with students. Then have them try the next three independently before discussing them.

ANSWERS: 5 x 400 = 2000 4 x 70,000 = 280,000

6 x 40 = 240 4 x 50,000 = 200,000 3 x 6000 = 18,000

ESTIMATING WITH LARGER NUMBERS

TR #3: Present the problem at the top and have students suggest estimates. Then show the two approaches in the middle of TR #3. Give particular attention to the use of grouping. Then discuss the TRY THESE exercises with students.

ANSWERS: 90,000+ 140,000 - 155,000

90,000 - 20,000

TR #4: Present the problem at the top of TR #4 and have students suggest estimates for it. Then present the four approaches on TR #4, giving particular attention to the use of nice numbers and addition. Let students use other methods or refinements. Do the TRY THESE exercises with students.

ANSWERS: 20,000 - 27,000 18,000 - 20,000

15,000 - 20,000

USING THE EXERCISES:

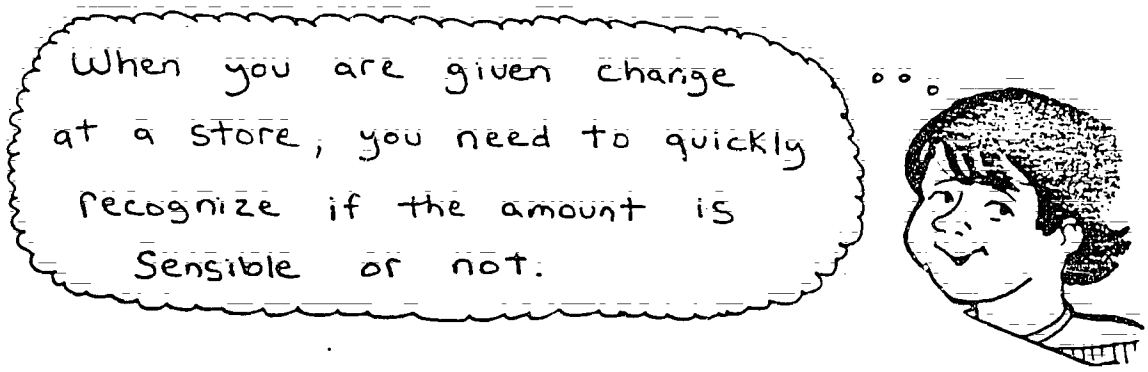
Page 1 centers on the use of clustering. On exercises 7 - 10 students just need to indicate whether the clustering strategy would be useful. Page 2 focuses on estimating with larger numbers.

ANSWERS:

1. 2000
2. 60,000
3. 100,000
4. 1500
5. 8000
6. 1,200,000 (-)
7. yes
8. no
9. no
10. yes
11. 160
12. $25.00
13. $320,000 - $350,000
14. $300,000 - $350,000
15. $340,000 - $353,000
16. $260,000 - $270,000
17. $40,000,000 - $43,000,000
18. $40,000,000 - $42,000,000
19. $10,000,000
20. 9,000,000 - 10,000,000
21. 11,000,000 - 12,000,000
22. 40,000,000 - 45,000,000
23. 130,000,000 - 135,000,000

GET YOUR MIND IN GEAR

RECOGNIZE SENSIBLE AMOUNTS



Circle the sensible amount of change.

THE BILL IS $14.02. GIVE THE CLERK $50.

$35.98 $36.98 $46.98

THE BILL IS $4.21. GIVE THE CLERK $5.

$.79 $.89 $1.79

THE BILL IS $23.92. GIVE THE CLERK $30.

$17.08 $7.08 $6.08

THE BILL IS $.59. GIVE THE CLERK $5.

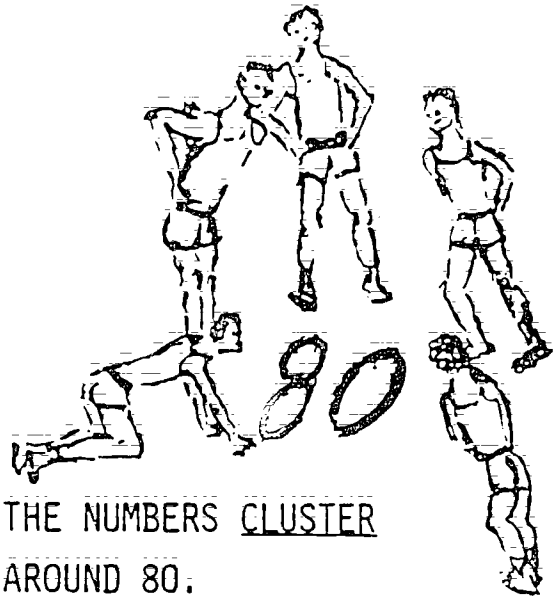
$3.51 $3.41 $4.41

7-3-TR1

THE CLUSTERING STRATEGY

THE NUMBER OF GAMES PLAYED BY SEVEN PROFESSIONAL BASKETBALL PLAYERS IS SHOWN. ESTIMATE THE TOTAL NUMBER OF GAMES THEY PLAYED.

PLAYER	GAMES
L. Bird	82
H. Catchings	77
A. Gilmore	82
M. Johnson	76
N. Nixon	79
D. Thompson	78
J. Wilkes	81



THE NUMBERS CLUSTER AROUND 80.

TO ESTIMATE:

1. Find the "cluster" number: 80
2. Multiply by 7: 7 x 80 560

TRY THESE:

```
  396
  391
  411
  407
+ 389
-------
///////
```

5 x ____

```
  68,673
  71,526
  70,201
+ 69,291
----------
//////////
```

4 x ____

36 + 42 + 39 + 43 + 41 + 40

51,637 + 47,832 + 49,356 + 52,827

6637 + 5952 + 5848

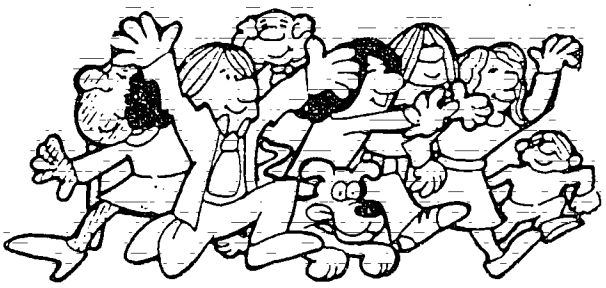
____ x ____

____ x ____

____ x ____

7-3-TR2

ADDITION ESTIMATION WITH LARGER NUMBERS



Summerfest Attendance

Wednesday	32,432
Thursday	20,672
Friday	21,362
Saturday	18,106

1. Use Front-End

3 2
2 0
2 1
1 8

Front-End Sum: 80,000
Adjust Up: 11,000
Estimate:

2. Use Grouping

3 2
2 0
2 1
1 8

Estimate:

TRY THESE: Use Both Front-End And Grouping.

21,367	13,932
82,467	15,086
11,928	32,157
38,167	31,292
//////////	//////////

ESTIMATE: _____ ESTIMATE: _____

7-3-TR3

SUBTRACTION ESTIMATION WITH LARGER NUMBERS

CITY	POPULATION
Gatorsville	37,187
Hodgeton	19,261

THERE ARE MANY WAYS TO ESTIMATE WITH LARGE NUMBERS. KEEP LOOKING FOR NICE NUMBERS TO WORK WITH.

About how much larger is Gatorsville?

1. Front-End

37,187
- 19,261

30 thousand - 10 thousand = 20 thousand
7187 is less than 9261
So, 20,000 - ... or about 18,000

2. Rounding

37,187
- 19,261

40,000
-20,000

3. Use Nice Numbers

19,261 → 20,000
37,000 - 20,000 = 17,000

4. Use Addition

19,000 + ? = 37,000
19 + 20 = 39
I'll say 20,000 ... that's close enough

TRY THESE:

52,365
- 26,126

31,000
- 12,637

66,324 - 49,138

7-3-TR4

Name_____

Estimate by using the clustering strategy.

1. 463 + 501 + 538 + 512

"Cluster" Number:_____

Multiply:_____x_____

Estimate:_____

2. 21,392 + 18,678 + 19,997

"Cluster" Number:_____

Multiply:_____x_____

Estimate:_____

3.
23,677
24,392
27,866
+ 26,001

____x_____

Estimate:_____

4.
269
319
341
298
+ 288

___x_____

Estimate:_____

5.
2,119
2,008
1,977
+ 2,010

____x_____

Estimate:_____

6.
316,927
290,502
276,116
+ 299,308

____x_____

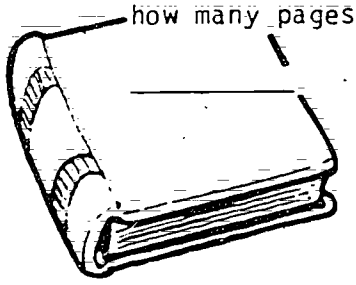
Estimate:_____

Would the clustering strategy be a good strategy to use here?

7. 1,256 + 986 + 1,096 + 1,119	yes	no
8. 132 + 12,467 + 3,821 + 24,368	yes	no
9. 89 + 867 + 376 + 4,343	yes	no
10. 5,398 + 4,988 + 4,672 + 4,756	yes	no

11. Jack read a book for English class in 4 days. He read 42 pages on Monday, 38 pages on Tuesday, 45 pages on Wednesday, and 36 pages on Friday. About how many pages are in the book?

_____



12. Carlysle has a job after school. He made $5.29 on Monday, $4.75 on Tuesday, $4.95 on Wednesday, $5.20 on Thursday, and $4.85 on Friday. About how much did he make that week?

_____

7-3-p.1

Name _____

Here are the prices of homes that a few realtors sold during July. Estimate the total sales for the month for each realtor.

13. Elaine Rodgers

$159,630
$ 78,450
$108,700

Estimate: _____

14. Wanda Vogel

$ 67,650
$ 89,500
$115,082
$ 72,632

Estimate: _____

15. Jerry Munoz

$215,500
$135,650

Estimate: _____

16. Allen Vargas

$92,600
$84,250
$89,627

Estimate: _____

The amount of money made on some movies is shown. Find the difference between the given pairs of movies.

Jaws	$118,727,000
The Sting	$ 72,160,000
The Godfather	$ 82,015,000
Gone With the Wind	$ 76,700,000
M*A*S*H	$ 40,850,000

17. Jaws and Gone With the Wind

_____

18. M*A*S*H and The Godfather

_____

19. The Sting and The Godfather

_____

20. The Mexico City area has a population of 31,616,000, while the New York area has a population of 22,212,000. About how many more people live in the Mexico City area than the New York area?

_____

21. The United Nations projects that in the year 2000, there will be 19,663,000 people living in Calcutta. There were 8,077,000 in 1975. By about how much is the population expected to increase over those 25 years?

_____

22. In 1974, The United States had 14,123,000 tourists. In 1975, there were 13,986,000 and in 1976, there were 14,426,000. About how many tourists visited the U.S. in those 3 years?

_____

23. In Australia there are 145,304,000 sheep and 13,339,000 people. About how many more sheep are there than people?

_____

NSF ESTIMATION
GRADE 7 - LESSON 4

OBJECTIVES: To estimate products using the rounding strategy.

To estimate products by rounding one factor to 10, 100 or 1000.

To indicate whether an estimate is an overestimate or underestimate.

TEACHER BACKGROUND:

When one or both factors have at least two digits, it is quick and efficient to use rounding to estimate.

4 x 2876

ESTIMATE: 4 x 3000 = 12,000

Some students will profit from a brief review of rounding numbers to the lead digit before the lesson.

71 x 238

ESTIMATE: 70 x 200 = 14,000

A variation occurs when one factor is close to 10, 100, or 1000. Here just one factor is rounded and the other factor is multiplied by 10, 100 or 1000. This strategy produces a closer estimate than rounding. both factors.

645 x 94

ESTIMATE: 645 x 100 = 64,500

As shown in the above examples, students need to be competent with some basic mental computation skills (6 x 4000, 70 x 200, 645 x 100). Many students have not had much experience with them and the skills may need additional attention. Thus, you may need to spend additional time on these prerequisites. The approach shown below is useful when multiplying by multiples of ten, one hundred and one thousand.

40 x 600

1. Multiply the lead digits — 4 0 x 6 0 0 = 2 4

2. Write as many zeros as there are in both factors. — 4 0 x 6 0 0 = 2 4,0 0 0

The lesson encourages students to compare estimates to actual answers by indicating whether the estimate is an overestimate or underestimate. The emphasis on adjusting estimates needs to be an ongoing part of the program. Examples of adjusting are shown below:

a. 4 x 2876 ESTIMATE: 12,000- Since 2876 is rounded up, 12,000 is an overestimate and 4 x 2876 is less than 12,000. To show this, a "-" sign is written after the estimate.

b. 28 x 56 ESTIMATE: 1800- Since both factors are rounded up, 1800 is an overestimate and 28 x 56 is less than the estimate.

c. 62 x 23 ESTIMATE: 1200+ Since both factors are rounded down, 1200 is an underestimate and 62 x 23 is greater than the estimate.

d. 62 x 78 ESTIMATE: 4800 One factor is rounded up and one factor is rounded down. When this occurs, there is no general rule that can be used to adjust the estimate.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 stresses decision-making and flexibility in estimation. Students need to examine an example before they start estimating to find the approach that is quick and appropriate for that situation. Present the four strategies shown on TR #1 and ask students to look at each example and first decide what strategy works best. Discuss the various suggestions and estimates that they give.

ANSWERS: It is often difficult to state which approach is "best" for an example. Probable choices for the exercises are given.

Rounding Front-End	Front-End	Grouping
Clustering	Front-End Rounding	Front-End Rounding

MULTIPLICATION ESTIMATION AND ADJUSTING

TR #2: Begin by reviewing the mental computation skills discussed in the previous section. Have students quickly give answers for the following exercises:

5 x 70	6 x 800	7 x 8000	6 x 500
30 x 80	700 x 20	900 x 300	50 x 400
47 x 100	83 x 1000	10 x 645	286 x 100

Present the two examples on TR #2 and discuss the procedure for estimating using rounding. Do the TRY THESE exercises with students. Write the rounded numbers in the bubbles so that students can see the numbers they are multiplying.

ANSWERS:	5600	24,000	1900
	18,000	24,000	80,000

TR #3: Present and discuss the four examples of adjusting estimates. Point out that it is useful to know whether the answer is more or less than the estimate. Do the TRY THESE exercises with students.

ANSWERS: 1800^{+} 15,000 $12{,}000^{-}$ 1600^{-} 4200^{-}

TR #4: Develop the use of using 10, 100 or 1000 in place of a factor that is close to one of those numbers. Have students tell why the estimate is written as 3400^{-}. (100 x 34 is greater than 92 x 34. So, 3400 is an overestimate and the answer to 34 x 92 is less than 3400.)

Then compare rounding both factors to rounding one factor to show how the latter often produces a closer estimate.

92 x 34 = 3128 ESTIMATE 1: 90 x 30 = 2700

ESTIMATE 2: 100 x 34 = 3400

You might also point out the greater ease of using 10, 100 or 1000.

ANSWERS: 6700^{-} 4700^{+} $46{,}000^{-}$

2460^{+} $38{,}100^{-}$ $641{,}000^{+}$

$\$360^{+}$ $\$1400^{-}$ $\$120^{-}$

USING THE EXERCISES:

The exercises, which begin with a review of the mental computation skills, are a straightforward application of the strategies taught in the lesson. You might look at the directions for #23 - 26 and do the first one orally with students.

ANSWERS:

1. 32,000	2. 3600	3. 1800
4. 3600	5. 4000	6. 140,000
7. 2100	8. 35,000	9. 540,000
10. 6800	11. 3470	12. 64,000
13. 6 x 400	14. 50 x 30	15. 50 x 80
16. 80 x 300	17. 3500	18. 1400
19. 3500	20. 4000	21. 24,000
22. 540,000	23. 1500^{-}	24. 2100^{+}
25. 3500 (?)	26. $12{,}000^{+}$	27. 460^{-}
28. $39{,}400^{-}$	29. 420^{+}	30. 6800^{+}
31. $68{,}000^{-}$	32. $291{,}000^{+}$	33. $\$20.00^{-}$
34. $20,000	35. $\$8100^{+}$	36. $200
37. $590 - $620	38. $\$2000^{-}$ or $\$1600$	

GET YOUR MIND IN GEAR

LOOK BEFORE YOU LEAP

There are lots of ways to estimate, and good estimators stop and think before they start. Tell <u>which</u> <u>strategy</u> you would use and <u>why</u>.

Front-End ?

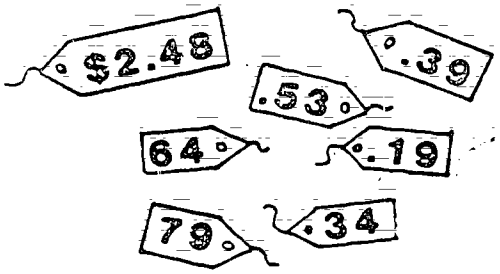
Rounding ?

Grouping ?

Clustering ?

$$\begin{array}{r} \$98.95 \\ 49.50 \\ 29.75 \\ \hline ? \end{array}$$

$$\begin{array}{r} 392 \\ 1417 \\ 84 \\ +\ 3198 \\ \hline ? \end{array}$$



TOTAL: ?

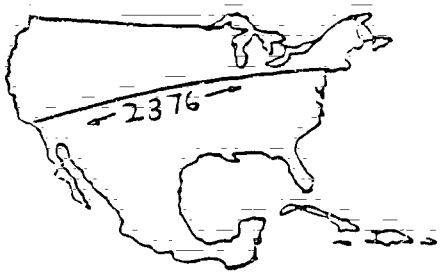
$$\begin{array}{r} \$3.55 \\ \$2.90 \\ \$3.08 \\ \$3.19 \\ \$2.40 \\ \$2.99 \\ \hline ? \end{array}$$

$$\begin{array}{r} 697 \\ 299 \\ 392 \\ +\ 189 \\ \hline ? \end{array}$$

$$\begin{array}{r} 4467 \\ -\ 2698 \\ \hline ? \end{array}$$

7-4-TR1

USING ROUNDING IN MULTIPLICATION ESTIMATION



MS. LEONARD MADE 2 ROUND TRIPS. ABOUT HOW MANY MILES DID SHE FLY?

4 x 2876 = ?

1. Round One Factor.

3000

4 x 2876

2. Multiply

4 x 3000 = 12,000

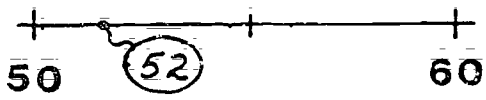
About 12,000 miles.

BILL EARNS $275 EACH WEEK. ABOUT HOW MUCH IS THAT A YEAR?

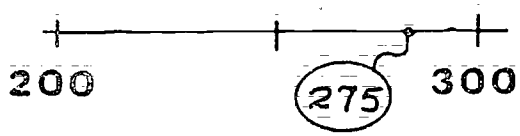
52 x $275 = ?

1. Round Both Factors.

52 is closer to 50



275 is closer to 300



2. Multiply.

50 x 300 = 15,000

52 x $275

About $15,000 per year.

TRY THESE:

8 X 729

34 X 827

88 X 23

326 X 58

4287 X 6

423 X 227

7-4-TR2

ADJUSTING ESTIMATES

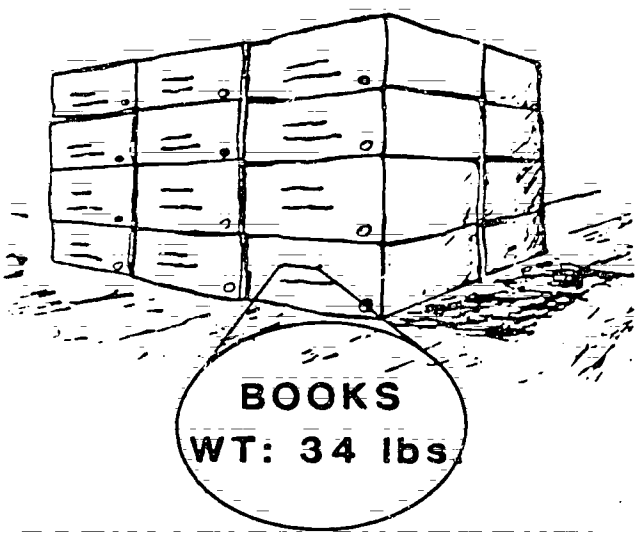
		Adjusted Estimate
6×427 → (400)	427 is rounded down. So 2400 is an underestimate. I'll adjust up!	2400^{+}
(30) (60) ↑ ↑ 28×56	Both factors round up. So 1800 is an overestimate. I'll adjust down.	1800^{-}
62×23 → (60) (20)	Both factors round down. So 1200 is an underestimate. I'll adjust up	1200^{+}
(800) ↑ 42×790 → (40)	Here you really can't tell. I'll just say... 32,000	32,000

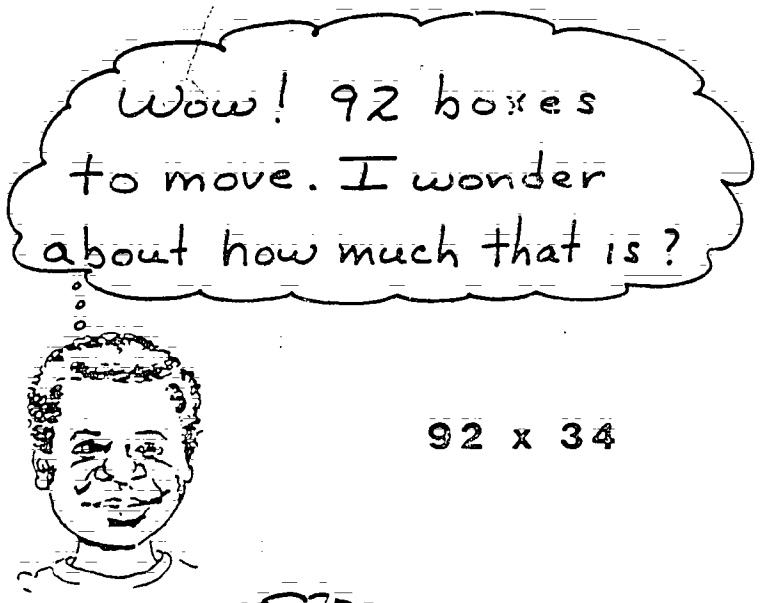
TRY THESE: Choose the best estimate.

92 X 21	1800^{+}	1800^{-}	1800
48 X 327	$15,000^{+}$	$15,000^{-}$	15,000
371 X 28	$12,000^{+}$	$12,000^{-}$	12,000
78 X 19	1600^{+}	1600^{-}	1600
589 X 7	4200^{+}	4200^{-}	4200

7-4-TR3

USE 10, 100, & 1000





92 x 34

100

92 x 34

ESTIMATION SHORTCUT!

WHEN ONE FACTOR IS CLOSE TO 10, 100, OR 1000, JUST CHANGE THAT FACTOR AND MULTIPLY.

Estimate: 3400 lbs.

92 was rounded up.

So 3400 is an overestimate.

3400 lbs.

IT'S ALSO EASY TO SHOW IF THE ESTIMATE IS AN OVERESTIMATE OR AN UNDERESTIMATE.

TRY THESE: Estimate and adjust.

67 X 98	47 X 103	988 X 46
11 X 246	381 X 96	1037 X 641
$10.29 X 36	14 X $94.72	12 X $9.62

7-4-TR4

Name_____

Use mental computation to find the products.

1. 8 x 4000	2. 60 x 60	3. 300 x 6
4. 40 x 90	5. 80 x 50	6. 7 x 20,000
7. 30 x 70	8. 50 x 700	9. 600 x 900
10. 68 x 100	11. 347 x 10	12. 1000 x 64

Circle the one you would use to estimate by rounding.

13. 6 x 395	6 x 300	6 x 400	6 x 500	6 x 600
14. 49 x 32	40 x 30	40 x 40	50 x 30	50 x 40
15. 53 x 81	50 x 80	50 x 90	60 x 80	60 x 90
16. 77 x 286	70 x 200	70 x 300	80 x 200	80 x 300

Round one factor or both factors. Write the rounded numbers in the bubbles. Then multiply to get the estimate.

17. 48 x 72 Est._____

18. 7 x 242 Est._____

19. 684 x 5 Est._____

20. 49 x 83 Est._____

21. 34 x 792 Est._____

22. 646 x 893 Est._____

Name_____

Estimate. Then decide how your estimate should be adjusted.

23. 49 x 28 _____ + ? -

24. 328 x 7 _____ + ? -

25. 73 x 47 _____ + ? -

26. 209 x 62 _____ + ? -

Use 10, 100 or 1000 to help you estimate quickly. Write + or - to adjust your estimate.

27. 9 x 46 _____

28. 394 x 92 _____

29. 42 x 11 _____

30. 68 x 104 _____

31. 68 x 981 _____

32. 1068 x 291 _____

Estimate.

33. Juan earns $3.75 per hour. He worked 5 hours on Saturday. About how much did he earn?

Estimate:_____

34. Mr. Hurd earns $382 per week. About how much does he earn in a 52 week year?

Estimate:_____

35. Caroline earns $927 per month. She works 9 months per year. How much does she make per year?

Estimate:_____

36. La Verne earns $4.95 per hour. She worked 42 hours last week. About how much did she make?

Estimate:_____

37. Neri makes $269 per week on one job and $338 per week on another job. About how much does he make per week?

Estimate:_____

38. Fred was offered two jobs. One paid $27,350 per year and the other paid $25,765. What was the difference in salary for the two jobs?

Estimate:_____

NSF ESTIMATION
GRADE 7 - LESSON 5

OBJECTIVES: To make closer estimates with two-digit factors.

To use front-end estimation to estimate products.

TEACHER BACKGROUND:

In this lesson students learn a variety of strategies for getting closer estimates when both factors have two or more digits. These strategies are a supplement to the rounding strategy presented in Lesson 4 and are useful for many students in some situations. Students need to be aware of the many options available when estimating and be able to use them when they are needed. Three strategies are presented:

1. Round one factor.

   In the example 31 is rounded to 30 and the product (30 x 82) is computed in horizontal form. 3 x 82 is easy to compute and 30 x 82 is ten times as great. This approach is most useful when the product is easy to compute mentally.

   (30)
   31 x 82

   3 x 82 = 246

   so 30 x 82 = 2460

   ESTIMATE: 2460

2. Multiply by 25 and 50 mentally.

   When one factor is close to 25 or 50, it is rounded to 25 or 50 and the product is computed using a mental shortcut. While not all students will make this part of their repertoire of strategies. Some will find it interesting and useful.

   (25)
   26 x 88

   To find 25 x 88 think:

   a. 100 x 88 = 8800

   b. 25 is 1-fourth of 100.

   c. $1/4$ of 8800 = 2200

   d. So, 25 x 88 = 2200

   ESTIMATE: 2200

   (50)
   49 x 88

   To find 50 x 88, think:

   a. 100 x 88 = 8800

   b. 50 is 1-half of 100.

   c. $1/2$ of 8800 = 4400.

   d. So, 50 x 88 = 4400

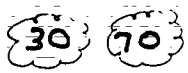
   ESTIMATE: 4400

3. Round Up and Down

When both factors are close to the middle of the range, a more precise estimate is obtained by rounding one factor up and the other factor down.

Both estimates are closer estimates than occurs when both factors are rounded to the nearest ten (40 x 70 = 2800)

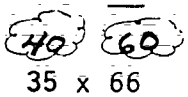
Students should feel comfortable with this approach and should be encouraged to see if it can be applied in exercises.



35 x 66

ESTIMATE: 2100

OR



35 x 66

ESTIMATE: 2400

The lesson also presents a FRONT-END approach to multiplication estimation when one factor is less than 10. The conventional algorithm starts at the "back-end" of the problem and works to the front, as shown below.

```
  3          1 3
6 4 9        6 4 9
  x 4          x 4
-----        -----
    6        2 5 9 6
```

4 x 649

a. 4 x 600 = 2400

4 x 649

b. 4 x 50 = 200

ESTIMATE: 2400 + 200 = 2600

In FRONT-END estimation, multiplying starts at the front end as shown in the example. 2400 is an estimate for 4 x 649. It is a lower bound. That is, the product is at least 2400.

In the second-step the intital estimate is adjusted up to get a closer estimate. Since 49 is close to 50 and 4 x 50 is easy to compute, 49 is rounded to 50. The two parts are then combined to get the final estimate. This is a useful approach for many examples, particularly ones involving money.

In presenting the lesson, remind students that they have learned acceptable ways to estimate products. The purpose of this lesson is to examine ways of getting closer when it seems appropriate and easy to do so. In future work, students should be allowed to use the basic rounding strategy.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

The focus of TR #1 is sensible answers. One long term value of work on escimation should be greater sensitivity to answers that are not reasonable or sensible. Discuss each exercise with students. In the exercises at the bottom, you may want to have students look up information to determine reasonable answers for some of the problems.

ANSWERS: TOP: $6500 70,000 425 lbs

BOTTOM: Answers will vary.

GETTING CLOSER IN MULTIPLICATION

TR #2: Present the top half of TR #2 and have students estimate by
TR #3: rounding both factors to the nearest ten. Then present the three strategies along with the TRY THESE exercises. Compare the estimates obtained using the new methods with the original ones. You may wish to provide additional practice in multiplying by 25 and 50. If so, select numbers that are easy for students to work with.

ANSWERS:				
	1.	1260	2480	1680
		1920	1350	1520
	2.	800	32,000	9000
	3.	2800	3000	4200 or 4000
		10,000	18,000	3600 or 4000

FRONT-END MULITIPLICATION

TR #4: Present and discuss the front-end approach at the top of the transparency. Show the two ways of doing the second step. Stress that students should choose the way that seems easiest for them.

ANSWERS:	3720	1560 (16[illegible])	9600	
	$24.90	($25.20)	16,800 (17,000)	$168 ($175)

USING THE EXERCISES:

Guidance is provided for students in the exercises to make the work easier for them. You may wish to do page 1 with some classes. On page 2, students should be permitted to use any valid method to estimate. The THINK IT THROUGH exercises provide a mental computation skill for capable students. It can be assigned to these students to be done independently or discussed with the class.

ANSWERS:

1. 840	2. 1890	3. 960	4. 740
5. 2100	6. 1800 (2400)	7. 16,000	8. 3200 (3500)
9. 400	10. 1200	11. 3100	12. 1600

13. $18.60 - $19.20

14. 2880 - 2920

15. 4800 - 5200

16. 7200 - 7800

17. 3600 - 3900

18. 9600 - 10,400

19. Yes

20. $3.50 - $4.00

21. $48.00 - $51.00

22. Yes

23. 10

24. Less than $5.00

25. $8.00

THINK IT THROUGH

1. $3.92 2. $2.07 3. $7.96 4. $9.75 5. $5.96

GET YOUR MIND IN GEAR

WHAT'S SENSIBLE?

Choose the reasonable number:

A NEW SUBCOMPACT CAR COSTS ABOUT:

$650 $6500 $65,000

THE NUMBER OF PEOPLE ATTENDING A SUPER BOWL FOOTBALL GAME IS:

700 7000 70,000

THE TOTAL WEIGHT OF 5 SEVENTH GRADE BOYS IS:

425 LBS. 850 LBS. 1000 LBS.

Fill in a reasonable amount:

THERE ARE ABOUT _____ STUDENTS IN OUR SCHOOL.

IN ONE MINUTE YOU CAN DO ABOUT _____ MULTIPLICATION FACTS.

YOU ARE IN SCHOOL ABOUT _____ HOURS A WEEK.

THE HEIGHT OF THE CLASSROOM IS ABOUT _____ FEET.

ABOUT _____ PEOPLE LIVE IN OUR CITY.

GETTING CLOSER IN MULTIPLICATION

82 per day
31 days

88 per day
26 days

66 per day
35 days

How many newspapers did each one deliver?

Use rounding to get a BALLPARK ESTIMATE.

BEN _____ ANGIE _____ DOT _____

Sometimes you need to GET CLOSER.
Here are some ways to do it.

1. Round one factor.

(30)
31 x 82

30 x 82 is easy to do in my head.
3 x 82 = 246
So 30 x 82 = 2460

TRY THESE:

ROUND ONE FACTOR, THEN MULTIPLY IN YOUR HEAD.

21 X 63

38 X 62

84 X 18

79 X 24

32 X 45

23 X 76

7-5-TR2

2. Multiply by 25 and 50.

25

26 x 88

100 x 88 = 8800
25 is 1-fourth of 100
$\frac{1}{4}$ of 8800 = 2200
25 x 88 = 2200

50

49 x 88

50 is half of 100
$\frac{1}{2}$ of 8800 = 4400
50 x 88 = 4400

TRY THESE:

25

24 X 32

50

52 X 640

25

27 X 360

3. Round Up - Round Down.

30 70

35 x 66

WHEN BOTH NUMBERS ARE CLOSE TO THE MIDDLE, ROUND ONE UP AND ONE DOWN TO GET A CLOSER ESTIMATE.

TRY THESE:

40 70

45 X 65

50 60

45 X 65

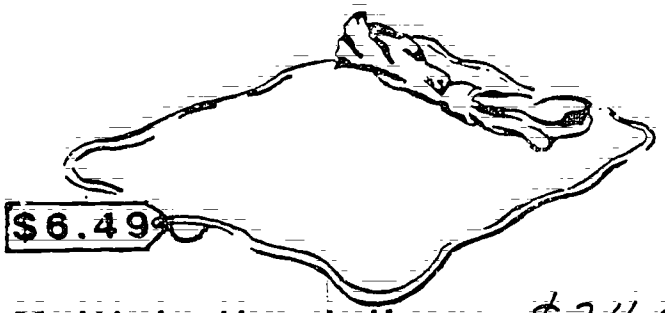
56 X 75

257 X 35

67 X 281

86 X 46

GETTING CLOSER WITH 1-DIGIT FACTORS



About how much will 4 table settings cost?

You can GET CLOSER by using Front-End Estimation.

4 × $6.49

4 × $6.49

Multiply the dollars: $24.00

Multiply the dimes: $ 1.60

Add: $25.60 Est.

IN STEP 2, YOU COULD ROUND TO DIMES FIRST:

$6.50

4 X $6 = $24

4 X $.50 = $ 2

$26

TRY USING FRONT-END ESTIMATING TO GET CLOSER:

4 X 928 (1. 2.)

(1) 3600

(2) 120

ESTIMATE

6 X 269 (1. 2.)

ESTIMATE

8 X 1239

ESTIMATE

3 X $8.39

ESTIMATE

2 X 8468

ESTIMATE

7 X $24.75

ESTIMATE

7-5-TR4

Name _____

Practice ways of getting closer when you estimate.

Round 1 Factor: Ex. 19 x 64 (20) — 2 x 64 = 128, so 20 x 64 = 1280

1. 42 x 21 (20) Est: _____

2. 63 x 31 Est: _____

3. 37 x 24 Est: _____

4. 37 x 19 Est: _____

Round Up and Round Down: Ex. 35 x 75 Est: 2800

5. 67 x 35 (70) (30) Est: _____

6. 86 x 26 Est: _____

7. 374 x 45 Est: _____

8. 44 x 74 Est: _____

Multiply by 25 and 50: Ex. 32 x 24 (25) Multiply by 100: 3200 Divide by 4: 800

9. 24 x 16 (25) Est: _____

10. 26 x 48 (25) Est: _____

11. 49 x 62 (50) Est: _____

12. 51 x 32 (50) Est: _____

Use front-end Multiplication: Ex. 3 x $2.49 1) 3 x $2.00 = $6.00 2) 3 x $0.50 = $1.50 3) Estimate: $7.50

13. 6 x $3.19

1) _____

2) _____

3) _____

14. 4 x 726

1) _____

2) _____

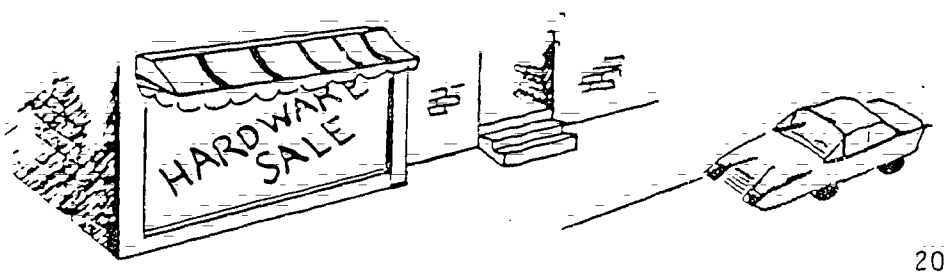
3) _____

1275 nails in each box

	Estimate
15. Buy 4 boxes	_____
16. Buy 6 boxes	_____
17. Buy 3 boxes	_____
18. Buy 8 boxes	_____

7-5-p.1

Name_____

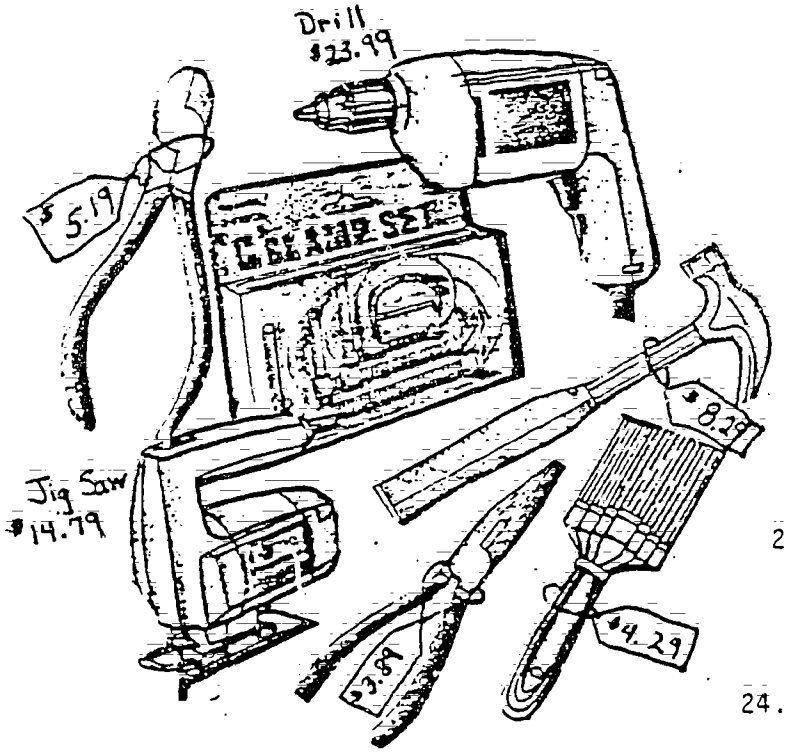


Practice your estimation skills.

19. Mr. Teague buys two brushes to paint his house. Is $10.00 enough? _____

20. Mrs. Johnson has seen the same hammer for $10.19. About how much would she save if she bought two hammers at this sale price?

_____



21. Carpenter Ken buys a hammer for each of his six sons. About how much will he spend?

_____

22. If the clamp set costs $3.99, will Jim be able to buy it and the big pliers with a 10-dollar bill? _____

23. Mr. Jackson has $159.00 to spend on jig saws for his school's shop course. About how many can he buy?

_____

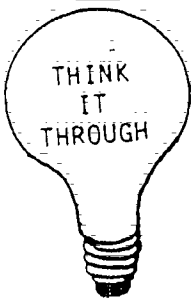
24. Aaron borrowed the money to buy a drill. He will pay back the money over a 5-month period. Will he pay more or less than $5.00 a month?

_____

25. Is the cost of two small pliers closer to $7 or $8?

_____

Work out the exact answer in your head.



Ex. 3 x $.49

3 x $.50 = $1.50
so 3 x $.49 is 3¢ less.
3 x $.49 = $1.47

1. 4 x $.98 = _____
2. 3 x $.69 = _____
3. 2 x $3.98 = _____
4. 5 x $1.95 = _____
5. 4 x $1.49 = _____

NSF ESTIMATION
GRADE 7 - LESSON 6

OBJECTIVES: To find the size of the quotient.

To estimate quotients using compatible numbers.

TEACHING THE LESSON:

The lesson focuses on division estimation with 1-digit divisors. Two aspects of division estimation are presented.

1) Find the size of the quotient. Students find the location of the first quotient digit, which tells how many digits there are in the quotient. This indicates whether the quotient is in the ones, tens, hundreds, etc. In the example at the right there are 4 digits in the quotient, which means the quotient must be in the thousands. Thus, the quotient must be 2000, rather than 200 or 20,000.

7 / 16,281

7 / 16,281

7 / 16,281

200 (2000) 20,000

Finding the size of the quotient is a form of estimation and helps students recognize whether a quotient is sensible in terms of its place value. This approach is useful throughout work with long division, particularly when there are 0's in the quotient.

2) Use compatible numbers to estimate. For 6/352, we know there are two digits in the quotient - the quotient is in the tens. One way of obtaining an estimate is to simply find the first digit of the quotient. As shown at the right, the first digit is 5, so an estimate is 50. While this approach does not always give the closest estimate, it is an acceptable way to estimate.

Find the first quotient digit.

6 / 352 (first quotient digit: 5)

ESTIMATE: 50

At this level students are taught to find the closer number of tens, hundreds, thousands, etc. in the quotient using compatible numbers. 6 and 36 are compatible numbers since 36 is a multiple of 6.

6 / 352

6 / 36 (quotient: 6)

In the example, students need to recognize which multiple of 6 (30 or 36) 35 is closer to. 35 is closer to 36, thus, the closer estimate is 60. Some students may recognize that the quotient is a little less than 60. (60^-).

ESTIMATE: 60

In the first example at the top of page 1, the use of compatible numbers gives the same estimate as finding the first quotient digits, although the thinking may differ slightly.

Some students may initially have difficulty with finding compatible numbers. Do not expect all students to master the work at this time. In the lesson, exercises have been selected in which the closer multiple is more easily seen (that is, the use of 7 / 268 rather than 7 / 248.)

In this lesson, students should find the closer estimate. In other situations accept both the lower and upper estimates (for 6 / 2852 , 400 and 500 are both acceptable).

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 provides practice with multiplication estimation. Let students discuss different ways in which the estimating could be done and encourage them to adjust initial estimates by changing them to ones that are closer to the actual product.

ANSWERS:

$65 - $72	$55 - $70	$60 - $70
$70 - $75	$60 - $70	$12 - $15
$24 - $30		

DIVISION ESTIMATION: RECOGNIZE SENSIBLE ANSWERS

TR #2: Present the problem at the top and the steps for finding the number of digits in the quotient. Then have students select the correct estimate. 2000 is correct since it has 4-digits (it is in the thousands). Do the TRY THESE exercises with students, first having them indicate the number of digits in the quotient and then having them select the correct estimate.

ANSWERS: 30 $200 600

ESTIMATE THE QUOTIENT:

TR #3: Put a table like the one at the right on the board and have students identify multiples of 6. Circle the multiples of 6.

Develop the two approaches for estimating the quotient for the problem at the top of TR #3. Emphasize the language and concept of compatible numbers.

Do the TRY THESE exercises with students.

MULTIPLES OF 6					
43			48		
	24	63			
13			18		
	32			30	
54		42			
	49		36		
25		20			

ANSWERS:

7 / 49	8 / 64	4 / 24	
5 / 40	9 / 36	3 / 27	7-6-2

MORE ABOUT COMPATIBLES

TR #4: The top provides one more example of using compatible numbers to get the closer estimate. Emphasize the two steps shown. Then have students do the two steps for the TRY THESE exercises.

ANSWERS:

1.	Way Off:	$.40	2.	Way Off:	$100
	Closer:	$.30		Closer:	$ 90
3.	Way Off:	30			
	Closer:	40			

USING THE EXERCISES:

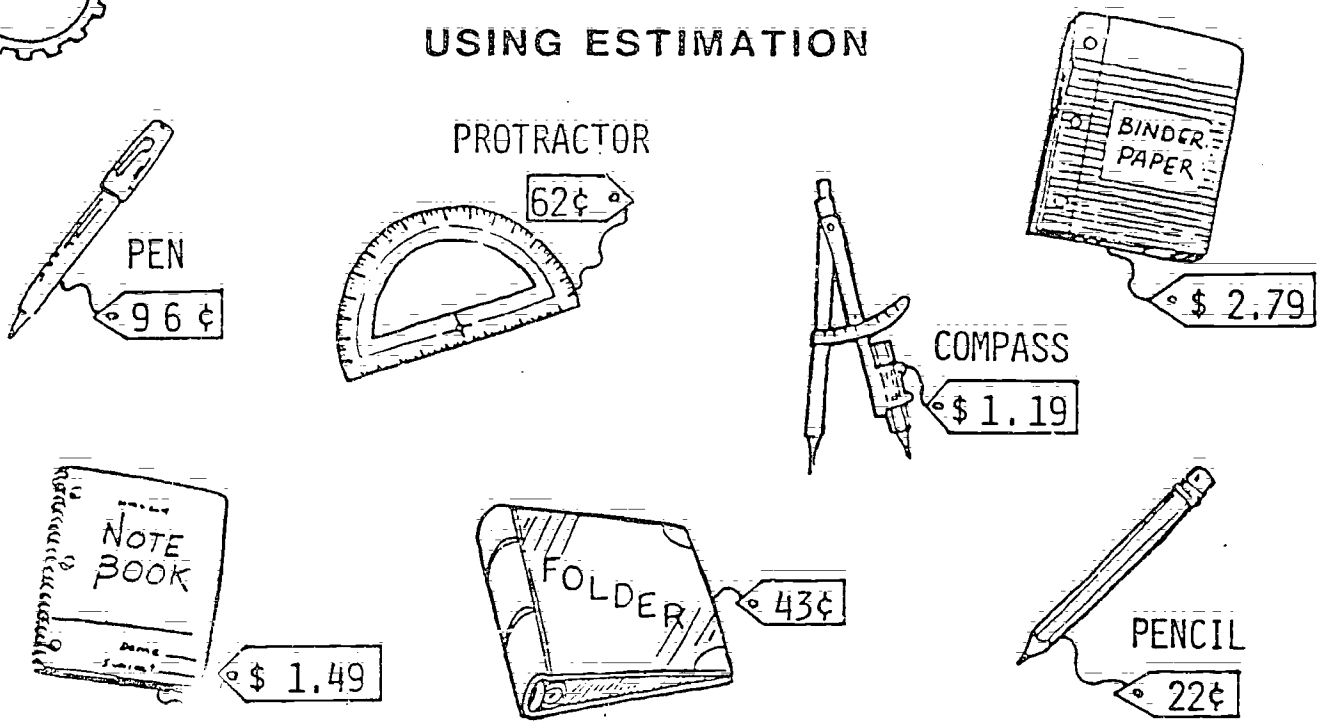
Read the directions for each set of exercises with the students and discuss the sample exercises with them or do the first exercise of each type orally. Exercises 31-36 are more challenging and require more sensitive estimating and judgment. You may wish to do these as a group.

ANSWERS:

1. 3 digits (hundreds)	2. 4 digits (thousands)	3. 2 digits (tens)
4. 3 digits (hundreds)	5. 2 digits (tens)	6. 4 digits (thousands)
7. 40	8. 300	9. 600
10. 7000	11. 7000	12. 500
13. 5 / $\overline{400}$	14. 7 / $\overline{490}$	15. 8 / $\overline{320}$
16. 6 / $\overline{4200}$	17. 4 / $\overline{3200}$	18. 9 / $\overline{7200}$
19. 200	20. 300	21. 4000
22. 40	23. 700	24. 2000
25. 500	26. 2000	27. 600
28. $9.00	29. $.30	30. $4.00
31. $.30 (+) $.30 (-)	32. $.03 (+) $.02 (+)	33. $.30 (-) $.30 (+)
34. $.60 (-) $.60 (+)	35. $.20 (-) $.20 (+)	36. $.10 (+) $.09 (-)

GET YOUR MIND IN GEAR

USING ESTIMATION



Estimate the cost for the School Bookstore to buy:

A BOX OF 72 PENS: _____

A CASE OF 144 FOLDERS: _____

A BOX OF 288 PENCILS: _____

A CASE OF 48 NOTEBOOKS: _____

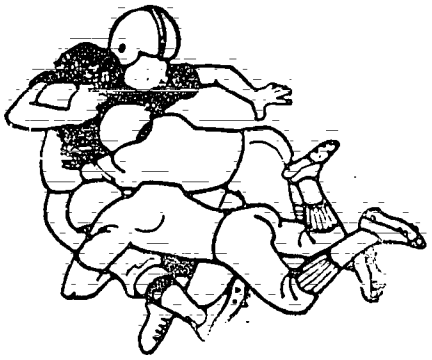
A CASE OF 24 PACKAGES OF PAPER: _____

A BOX OF 24 PROTRACTORS: _____

A BOX OF 24 COMPASSES: _____

7-6-TR1

DIVISION ESTIMATION: RECOGNIZE SENSIBLE ANSWERS



THE TOTAL ATTENDANCE FOR 7 GAMES WAS 16,281. THE AVERAGE ATTENDANCE PER GAME WAS ABOUT ___?___

(200, 2000, 20,000)

$7\overline{)16{,}281}$

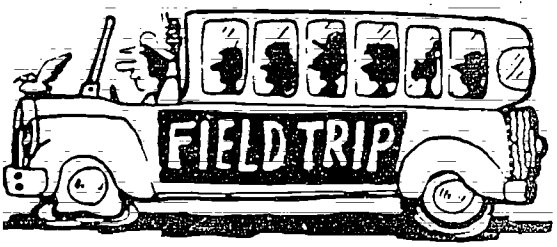
Are there enough ten-thousands to divide? $7\overline{)1}$ ////

Are there enough thousands to divide? $7\overline{)16,}$ ///

The quotient is in the thousands. It has 4 digits. $7\overline{)16,}$ ////

2000 IS A SENSIBLE ESTIMATE.

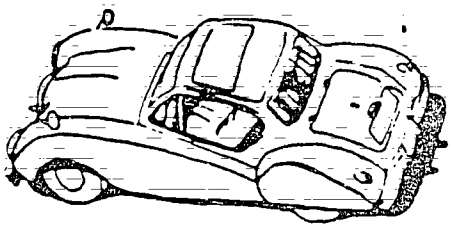
TRY THESE: MAKE MARKS TO SHOW HOW MANY DIGITS IN THE QUOTIENT. CHOOSE THE SENSIBLE ESTIMATE.



215 STUDENTS
6 BUSES
ABOUT ___?___ STUDENTS PER BUS.

$6\overline{)215}$

3 30 300



$8500
48 MONTHS TO PAY
ABOUT ___?___ PER MONTH.

$48\overline{)\$8500}$

$20 $200 $2000

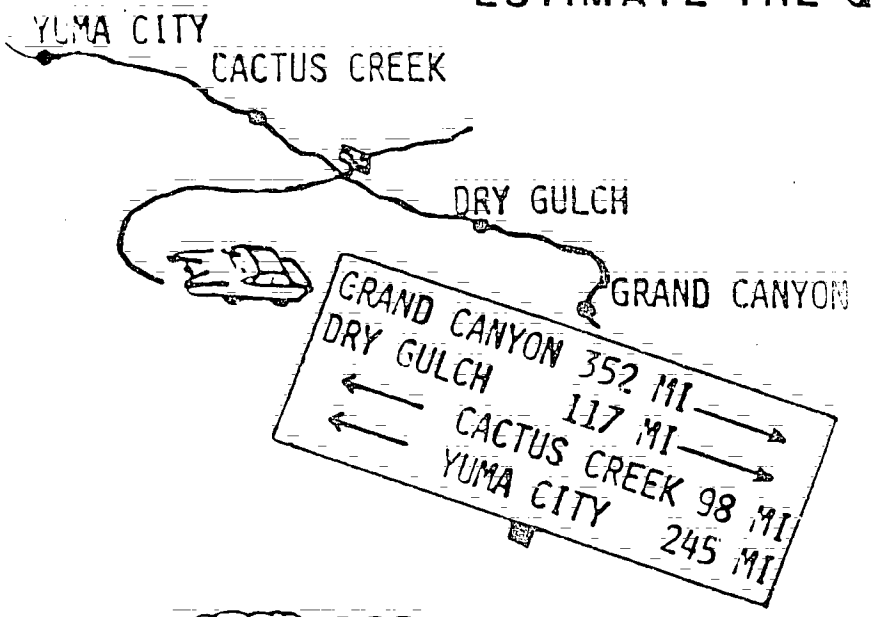
4215 STUDENTS
7 SCHOOLS
ABOUT ___?___ STUDENTS PER SCHOOL.

$7\overline{)4215}$

6 60 600

7-6-TR2

ESTIMATE THE QUOTIENT



IT TOOK THE YEE FAMILY 6 HOURS TO GET TO THE GRAND CANYON. ABOUT HOW MANY MILES AN HOUR DID THEY AVERAGE?

$6 \overline{)352}$ → $6 \overline{)360}$ (6 → 60)

LEE USED COMPATIBLE NUMBERS.
35 IS CLOSE TO 36, AND
36 IS A MULTIPLE OF 6.
LEE HAS THE CLOSER ESTIMATE.

$6 \overline{)352}$ (5 → 50)

DAVID FOUND THE FIRST DIGIT IN THE QUOTIENT.

6 and 36 are Compatible Numbers because 6 divides 36 evenly.

TRY THESE:

Use compatible numbers to find the closer estimate.

$7 \overline{)48}$ — $7 \overline{)42}$, $7 \overline{)49}$

$8 \overline{)62}$ — $8 \overline{)56}$, $8 \overline{)64}$

$4 \overline{)25}$ — $4 \overline{)24}$, $4 \overline{)28}$

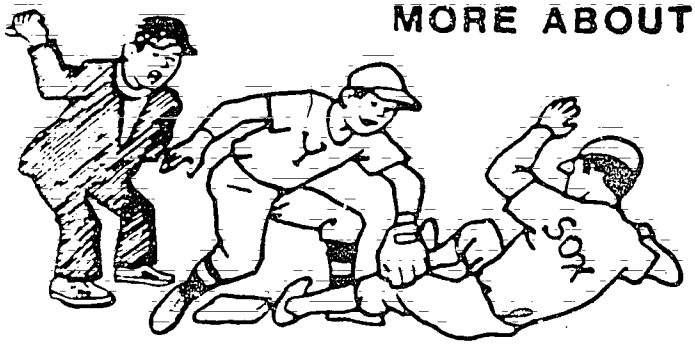
$5 \overline{)38}$ — $5 \overline{)35}$, $5 \overline{)40}$

$9 \overline{)38}$ — $9 \overline{)36}$, $9 \overline{)45}$

$3 \overline{)26}$ — $3 \overline{)24}$, $3 \overline{)27}$

7-6-TR3

MORE ABOUT COMPATIBLES



152,652 ATTENDED A 4-GAME BASEBALL SERIES. ESTIMATE THE AVERAGE ATTENDANCE.

1. Find the size of the quotient. $4\overline{)152{,}652}$

In the ten-thousands

2. Use Compatible Numbers to estimate.

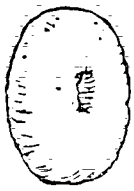
$4\overline{)15{,}000} \longrightarrow 4\overline{)16{,}000}$

15 is close to 16, and 4 and 16 are compatible.

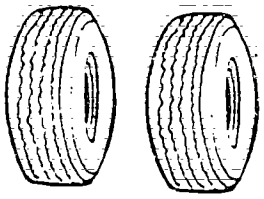
ESTIMATE: _____

TRY THESE:

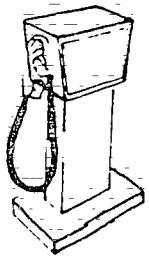
1. TELL WHICH ESTIMATE IS "WAY OFF."
2. TELL WHICH OF THE OTHER TWO IS CLOSER.



6 FOR $1.69
ONE DOUGHNUT COSTS:
(20¢, 30¢, 40¢)



4 TIRES: $352.99
ONE TIRE COSTS:
($80, $90, $100)



332 MI
8 GALS
THE NUMBER OF MILES ON ONE GALLON IS:
(30, 40, 50)

7-6-TR4

Name_____

Make marks to show the number of digits in each quotient. Then tell if the quotient is in the tens, hundreds, or thousands.

Ex: 9 / 2 3 8

in the tens

1. 6 / 1 4 6 2

in the_____

2. 4 / 6 2 9 0

in the_____

3. 8 / 4 3 2

in the_____

4. 7 / 4 9 1 4

in the_____

5. 6 / 5 9 2

in the_____

6. 3 / 1 3, 4 6 2

in the_____

Circle the better estimate for each example.

7. 7 / 2 8 4

a. 40

b. 400

8. 4 / 1 2 2 4

a. 30

b. 300

9. 8 / 4 8 0 6

a. 600

b. 6000

10. 9 / 6 3, 1 0 4

a. 7000

b. 70,000

11. 4 / 2 9, 8 4 2

a. 700

b. 7000

12. 5 / 2 5 1 5

a. 50

b. 500

Circle the example that gives the closer estimate.

13. 5 / 3 9 2

a. 5 / 3 5 0

b. 5 / 4 0 0

14. 7 / 5 0 1

a. 7 / 4 9 0

b. 7 / 5 6 0

15. 8 / 3 0 3

a. 8 / 2 4 0

b. 8 / 3 2 0

16. 6 / 4 0 8 7

a. 6 / 3 6 0 0

b. 6 / 4 2 0 0

17. 4 / 3 3 0 4

a. 4 / 3 2 0 0

b. 4 / 3 6 0 0

18. 9 / 7 0 3 8

a. 9 / 6 3 0 0

b. 9 / 7 2 0 0

Name _____

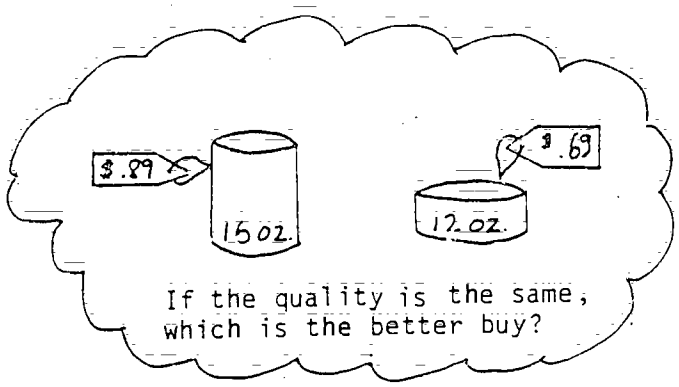
Estimate. Use compatible numbers if you can.

	Estimate		Estimate		Estimate
19. $7 \overline{)1532}$	_____	20. $3 \overline{)899}$	_____	21. $4 \overline{)17,640}$	_____
22. $8 \overline{)312}$	_____	23. $5 \overline{)3625}$	_____	24. $2 \overline{)3897}$	_____
25. $4721 \div 9$	_____	26. $18,425 \div 9$	_____	27. $3487 \div 6$	_____
28. $\$43.95 \div 5$	_____	29. $\$2.73 \div 9$	_____	30. $\$26.49 \div 7$	_____

BE A SMART SHOPPER!

Estimate the cost of one.
Put a check next to the best buy.

Ex. BATTERIES	Approx. cost per item
2 for $1.44	$.70 or $.72
5 for $3.49	$.70 ✓



31. LIGHTBULBS	Approx. cost per item
4 for $1.27	_____
2 for $ .59	_____

32. PLASTIC SILVERWARE	Approx. cost per item
24 pieces for $ .75	_____
50 pieces for $1.09	_____

33. GLUE	Approx. cost per item
4 oz. for $1.19	_____
6 oz. for $1.99	_____

34. DRILLS	Approx. cost per item
7 for $3.99	_____
9 for $5.49	_____

35. TOOTHPASTE	Approx. cost per item
6 oz. for $1.19	_____
9 oz. for $1.81	_____

36. CLEANSER	Approx. cost per item
5 oz. for 52¢	_____
8 oz. for 70¢	_____

7-6-p.2

NSF ESTIMATION
GRADE 7 - LESSON 7

OBJECTIVES: To estimate quotients using compatible numbers when dividing by a 2-digit number.

To estimate quotients by first rounding the divisor when dividing by a 2-digit number.

TEACHER BACKGROUND:

Two approaches to estimating quotients when dividing by 2-digit numbers are presented.

1. Use Compatible Numbers. Here the whole divisor is used as shown at the right. One thinks of multiples of the divisor and adjusts the dividend. 12 / 37 is changed to 12 / 36 since 36 is a multiple of 12.

12 / 3748
↓
12 x 3 = 36
↓
12 / 36

ESTIMATE: 300

Both the dividend and the divisor can be changed as shown in the samples below.

2 3 / 4 7 6

23 / 46 24 / 48 25 / 50

Compatible numbers is a valuable estimation tool for many division situations, particularly real life settings where the numbers are not set up in the usual / form. It requires the ability to recognize pairs of numbers that are multiples and works best when the quotient is 2, 3, 4 or 5. You can expect a variety of ways of finding compatible numbers and this is reflected in the exercises.

2. Round the Divisor. Compatible numbers (using the whole divisor) are not always appropriate. In these situations it is best to begin by rounding the divisor and then proceeding in one of two ways:

6 3 / 4 3 7 5

a. 6 0 / 4 3 7 5

round to tens — truncate

a. Find the first quotient digit. This parallels the work of Lesson 6. While this may not give the closer estimate, it it produces a reasonable estimate in an efficient way.

7
6 0 / 4 3 7 5

ESTIMATE: 70

7-7-1

b. Use compatible numbers. After rounding the divisor and truncating the dividend, look for compatible numbers to get the closer estimate.

b. 30 / 2016

7
30 / 2016 (3 / 21)

ESTIMATE: 70

Encourage students to take a few moments to examine an example to determine the best way to proceed. Also, emphasize the importance of first finding the number of digits in the quotient, so their estimates will have the correct place value,

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 emphasizes recognizing sensible answers. Have students scan the examples to find the ones that do not have sensible answers. Discourage them from trying to work the problems. Have students tell why the ones they identify don't have sensible answers.

ANSWERS: The answers to #3, #6 and #7 are not sensible.
In #3, 100 x 24 = 2400 and the product of 2 two digit numbers can only have 3 or 4 digits.

In #6, the quotient has 3 digits and is in the hundreds.

In #7, 6000 + 3000 = 9000, so the sum must be around 10,000. Susie probably added all the lead digits without looking at their place values.

COMPATIBLE NUMBER ESTIMATION (USE THE WHOLE DIVISOR)

TR #2: Present the problem and development of the use of compatible
TR #3: numbers using the whole divisor. Then use the work at the bottom to to provide practice in finding numbers compatible to the number given.

The top of TR #3 shows ways of changing the divisor and dividend to make compatible numbers. Have students do the exercises at the top. There are a variety of ways of changing the ones where the numbers are not compatible.

Do the TRY THESE exercises with students. Real world situations have been used to encourage the use of the whole divisor.

ANSWERS: 200 miles per day 400 miles per hour

30 miles per gallon 200 boxes per girl

40 seats per row

ROUND THE DIVISOR

TR #4: Show just the situation at the top. Emphasize that another way to estimate is to first round the divisor. Then develop the two approaches for completing the work. Use the work in the middle to reemphasize the importance of first finding the number of digits in the quotient.

Do the TRY THESE exercises with students. You might have the estimates done both ways.

ANSWERS:	about 200	about 40 (30)	about 700
	about 300 (200)	about 90 (80)	about 600

USING THE EXERCISES:

Page 1 has been constructed to reinforce the development of the lesson with emphasis on recognizing the size of the quotient and the use of compatible numbers with the whole divisor. On page 2, students should recognize that they may use any of the approaches to the exercises. On page 1, do the sample example before #9 - 20 with the students.

ANSWERS:

1. 60	2. 30	3. 3	4. 200
5. 700	6. 5	7. 100	8. 2000
9. 200	10. 30	11. 3000	
12. 300	13. 300	14. 2000	
15. 70	16. 700	17. 40	
18. 50	19. 800	20. 50	
21. 2000	22. 3 - 4	23. 60	
24. 300	25. 20	26. 60 - 70	
27. 20	28. 400 - 500	29. 900 - 1000	
30. $.10 - $.12	31. $.30 - $.35	32. $.09 - $.10	
33. $.20 - $.22	34. $.04 - $.06	35. $.02 - $.03	
36. 20 - 25	37. 7000 - 8000		
38. $400 - $450	39. $200 - $220		
40. 24,000	41. 150 - 160		

GET YOUR MIND IN GEAR

SENSIBLE ANSWERS

GOOD ESTIMATORS can scan their test sheets to pick out unreasonable answers.

MATH CHECK-UP QUIZ Name Susie

1. $\begin{array}{r} 249 \\ 27 \\ +416 \\ \hline 692 \end{array}$

2. $\begin{array}{r} 4663 \\ -\ 167 \\ \hline 4,496 \end{array}$

3. $\begin{array}{r} 98 \\ \times 24 \\ \hline 23,502 \end{array}$

4. $5\overline{)475}$ = 95

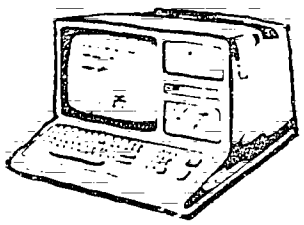
5. $\begin{array}{r} 247 \\ \times\ 6 \\ \hline 1,482 \end{array}$

6. $6\overline{)1320}$ = 22

7. 6157 + 700 + 3478 = 16,635

7-7-TR 1

TWO-DIGIT DIVISORS: USE THE WHOLE DIVISOR



$3748
12 MO. TO PAY

ABOUT HOW MUCH IS EACH PAYMENT?

1. Find the size of the Quotient.

$12\overline{)3748}$

THE QUOTIENT HAS 3 DIGITS. IT'S IN THE HUNDREDS.

2. Use Compatible Numbers.

$12\overline{)3648}$

12 AND 36 ARE COMPATIBLE.

3. Put it together.

$$12\overline{)3748}^{\$3\_\_}$$

ABOUT $300 PER PAYMENT.

MAKE COMPATIBLE NUMBERS.

	DOUBLE	TRIPLE	QUADRUPLE
15	30		
21			
42			
24			
16			

7-7-TR2

Compatible numbers divide evenly!

$13\overline{)22}$ $13\overline{)26}$ $11\overline{)22}$ $12\overline{)24}$

NOT COMPATIBLE COMPATIBLE

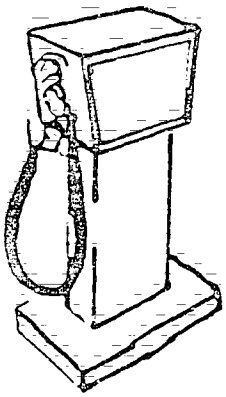
FIND COMPATIBLE NUMBERS. Change the others to Compatible Numbers.

$33\overline{)64}$ $12\overline{)48}$ $14\overline{)42}$ $26\overline{)51}$

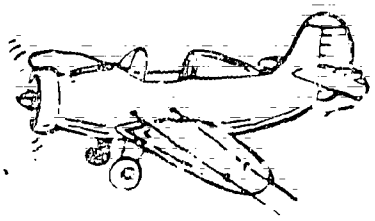
$52\overline{)158}$ $23\overline{)67}$ $41\overline{)85}$ $43\overline{)129}$

TRY THESE: Estimate. Use the whole divisor.

3325 MILES
16 DAYS
? MILES PER DAY.



432 MILES
14 GALLONS
? MILES PER GALLON.



9582 MILES
24 HOURS
? MILES PER HOUR.

7452 BOXES
36 GIRLS
? BOXES PER GIRL.

THEATER
1150 SEATS
31 ROWS
? SEATS PER ROW.

7-7-THE

TWO-DIGIT DIVISORS:ROUND THE DIVISOR

Sometimes using the whole divisor doesn't seem workable.

$67\overline{)4097}$

SO... ROUND THE DIVISOR

$70\overline{)4097}$

and

1. Find the first digit of the quotient.

$$70\overline{)4097}^{\,5\_}$$

About 50

OR

2. Look for Compatible Numbers.

$$7\overline{)42}^{\,6\_}$$

About 60

BE CAREFUL ABOUT PLACE VALUE.

$34\overline{)2369}$

About 70

TRY THESE: Round the divisor to estimate.

$28\overline{)7316}$ ABOUT _____

$72\overline{)2753}$ ABOUT _____

$59\overline{)43,287}$ ABOUT _____

$47\overline{)13,671}$ ABOUT _____

$34\overline{)2687}$ ABOUT _____

$78\overline{)46,300}$ ABOUT _____

7-7-TR4

Name_____

Circle the best estimate. Think about the size of the quotient.

1.	62 / 3872	6	60	600	2.	47 / 1470	3	30	300
3.	78 / 262	3	30	300	4.	19 / 3942	2	20	200
5.	36 / 25874	7	70	700	6.	406 ÷ 84	5	50	500
7.	1986 ÷ 20	1	10	100	8.	74,831 ÷ 36	20	200	2000

Rewrite using compatible numbers. Then estimate.

Example: 26 / 5125 → 25 / 50 Estimate: 200

9. 13 / 2563 Estimate:_____

10. 49 / 1624 Estimate:_____

11. 32 / 9517 Estimate:_____

12. 24 / 7332 Estimate:_____

13. 12 / 3263 Estimate:_____

14. 44 / 89712 Estimate:_____

15. 30 / 2041 Estimate:_____

16. 60 / 43241 Estimate:_____

17. 80 / 2997 Estimate:_____

18. 28 / 1497 Estimate:_____

19. 50 / 43872 Estimate:_____

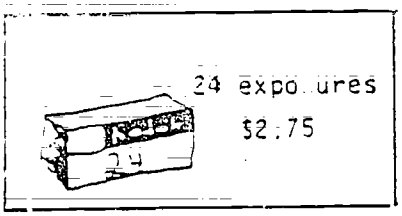
20. 67 / 3789 Estimate:_____

Name _____

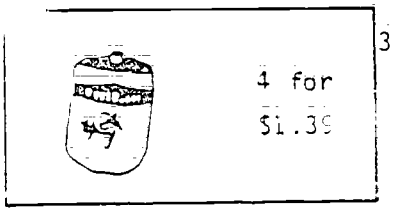
Estimate. You can use any method.

		Estimate			Estimate			Estimate
21.	33 / 67,145	_____	22.	83 / 298	_____	23.	46 / 3075	_____
24.	53 / 15,724	_____	25.	62 / 1259	_____	26.	28 / 2074	_____
27.	1637 ÷ 84	_____	28.	8376 ÷ 16	_____	29.	43,720 ÷ 44	_____

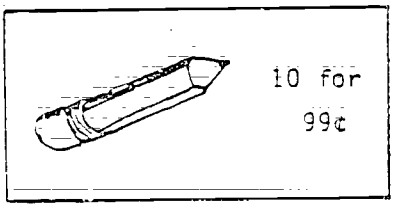
Estimate.

30. 

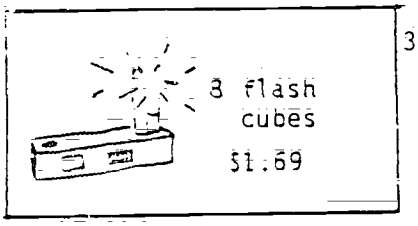
cost per exposure: _____

31. 

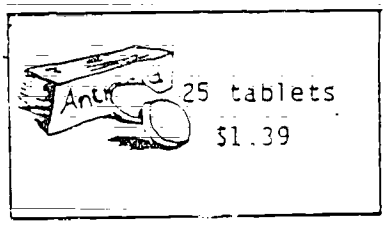
cost per battery: _____

32. 

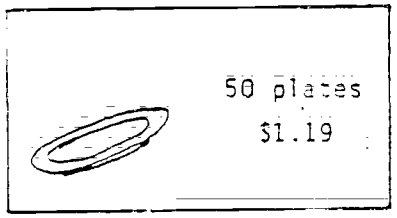
cost per pencil: _____

33. 

cost per flash cube: _____

34. 

cost per tablet: _____

35. 

cost per plate: _____

36. There are 152 seventh graders at Woods Middle School. There are 7 homerooms. About how many students are in each homeroom?

_____ students

37. In May 235,352 imported cars were sold in the United States. There are 31 days in May. About how many imported cars were sold each day in May?

_____ cars

38. To pay off a loan of $5000 in 12 months, each payment would be a little over $_____.

39. To pay off the same loan in 24 months (see ex. 38) each payment would be about $_____.

40. A fireman said every house needs 2 smoke detectors. If a town has about 42 houses on a street and 319 streets, about how many smoke detectors would the town need?

_____ smoke detectors

41. There are 206 bones in the human body. If there are 29 in the head and 24 in the wrist and hand, about how many bones are in the rest of the body?

_____ bones

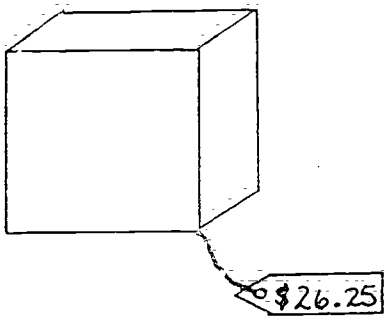
7-7-p.2

NSF ESTIMATION
GRADE 7 - LESSON 8

OBJECTIVES: To estimate a fractional part of a number.

To adjust estimates.

TEACHER BACKGROUND:



Finding fractional parts has wide social applications and is an important use of fractions. The lesson is placed at this point in the program because of the mathematical and pedagogical value of relating the work to division. There are four parts to the lesson.

1. First students review finding a fractional part of the whole. Finding 1/3 of 27 is related to splitting 27 into 3 equal parts, or dividing 27 by 3.

2. To estimate a fractional part, the compatible number strategy is used. Since 26 is close to 27, and 27 and 3 are compatible numbers, the estimate is found by dividing 27 by 3.

3. It is natural to adjust the estimate. Since $26.25 is rounded up, the answer to 1/3 of $26.25 is less than $9.00. $9.00 is too much.

4. $9.00 is an estimate of the part saved. To estimate the part owed, two approaches can be used:

1/3 of $26.25

ESTIMATE: $9.00

ADJUSTED ESTIMATE: $9.00

a. Use Nice Numbers to Subtract.

Total		Part Saved		Part Owed
$27	-	$9	=	$18

b. Use Multiplication.

You save 1/3, so you pay 2/3.

1/3 of $26.25 is about $9.00.

2/3 of $26.25 is twice as much.

You pay 2 x $9 or about $18.00.

It is important to develop this last point, but recognize that not all students will be able to become proficient. This work receives additional attention in Grade 8 material.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 emphasizes that an estimate is often sufficient. Also, estimation is useful as a check on computation when an exact answer is required. Discuss these ideas as you use the transparency.

ANSWERS: 1. No; No; Yes; Probably Yes

2. Yes; No

3. No; Yes

ESTIMATE A PART

TR #2 The meaning of finding a fractional part is reviewed. Present the diagram and example at the top and link finding 1/3 of 27 to 27 ÷ 3. Then present the work on estimating a fractional part of a number at the bottom of TR #2. Discuss the reasoning behind adjusting the estimate. (Since $117 is rounded up to $120, the estimate is too high. You would save a little less than $40 or $40⁻.)

TR #3: TR #3 presents practice in estimating the part saved. Have students estimate and adjust.

ANSWERS: $15 - $20 $9.00 - $10.00 $30 - $33

$11.00 - $12.00

$ .00⁻ $50.00⁺ - $60.00⁻ $22⁺ - $25⁻

ESTIMATE THE AMOUNT PAID

TR #4: Develop the two approaches to estimating the amount paid or owed. Then do the TRY THESE exercises with students.

ANSWERS: $40 - $43; $80 - $86

$38 - $40; $150 - $170

$18 - $20; $54 - $60

$110 - $120; $220 - $240

USING THE EXERCISES:

There is a careful progression of exercises on page 1 to reinforce the estimation process. While adjusting estimates is not required, students should feel free to do so . Discuss the work at the top of page 2 and consider doing #28 with students orally.

ANSWERS:

1. 22 2. 20 3. 15 4. 8 5. 70 6. [illegible]

7. 28 or 32 8. 36 9. 400 or 410 10. 140

11. 150 12. 320 13. 540 14. 240 or 246

15. $2[illegible] 16. $3- 17. $30-

18. $10 - $11 19. $20 - $21

20. 50 - 52 21. 14 - 15

22. 60 - 62 23. 30 -31

24. 350 - 400 25. 45 - 50

26. 45 - 50 27. 40 -42

28. under,over 29. over,under

30. over,under 31. under,over

32. under,under

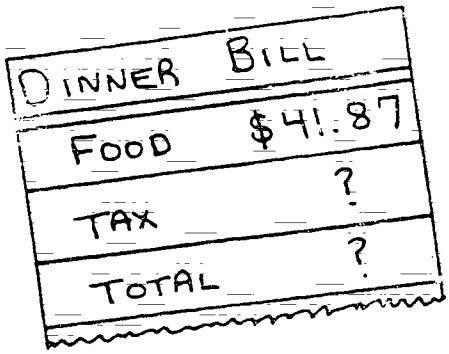
33. $44 - $45; [illegible]4 - $45 34. $45 - $50; $180 - $205

35. $14 - $15; $28 - $30 36. $145 - $15[illegible]; $420 - $450

37. $6.75 - $7; $60 - $63

YOUR MIND IN GEAR

WHEN IS AN ESTIMATE ENOUGH?



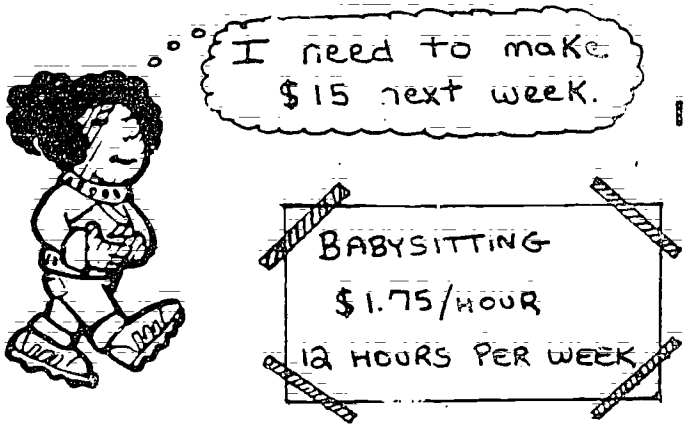
Is an estimate enough when:

THE WAITRESS FIGURES 5% TAX?

THE WAITRESS FINDS THE TOTAL?

THE CUSTOMER FIGURES A 15% TIP?

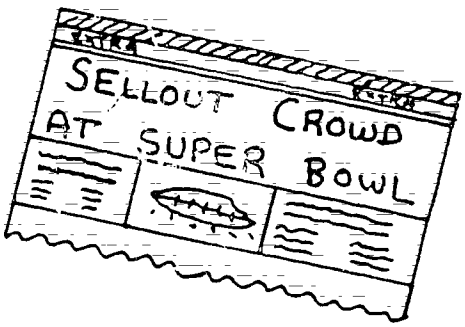
THE CUSTOMER CHECKS THE BILL?



Is an estimate enough when:

CAROL DECIDES IF SHE'LL MAKE ENOUGH MONEY IN ONE WEEK?

THE PERSON WHO HIRES CAROL FIGURES OUT HOW MUCH TO PAY HER?



Is an estimate enough when:

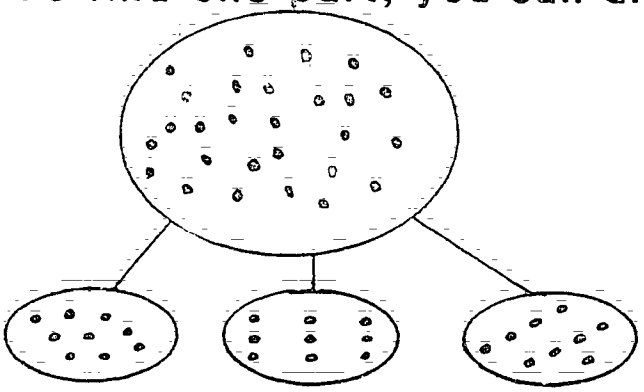
THE ACCOUNTANT FIGURES OUT HOW MUCH MONEY WAS MADE ON TICKET SALES?

THE NEWSPAPER REPORTS HOW MANY PEOPLE ATTENDED THE GAME?

7-8-TR1

ESTIMATE A PART

To find one part, you can divide!

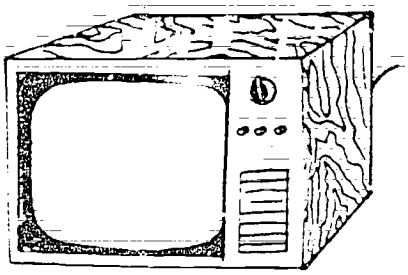


$\frac{1}{3}$ OF 27

To find one of 3 equal parts ... divide.

$\frac{1}{3}$ OF 27 = 9 ... 27 ÷ 3

To estimate one part, use Compatible Numbers!



$117

$\frac{1}{3}$ OFF

How much is saved?

$\frac{1}{3}$ OF 117

Think: 117 is close to 120

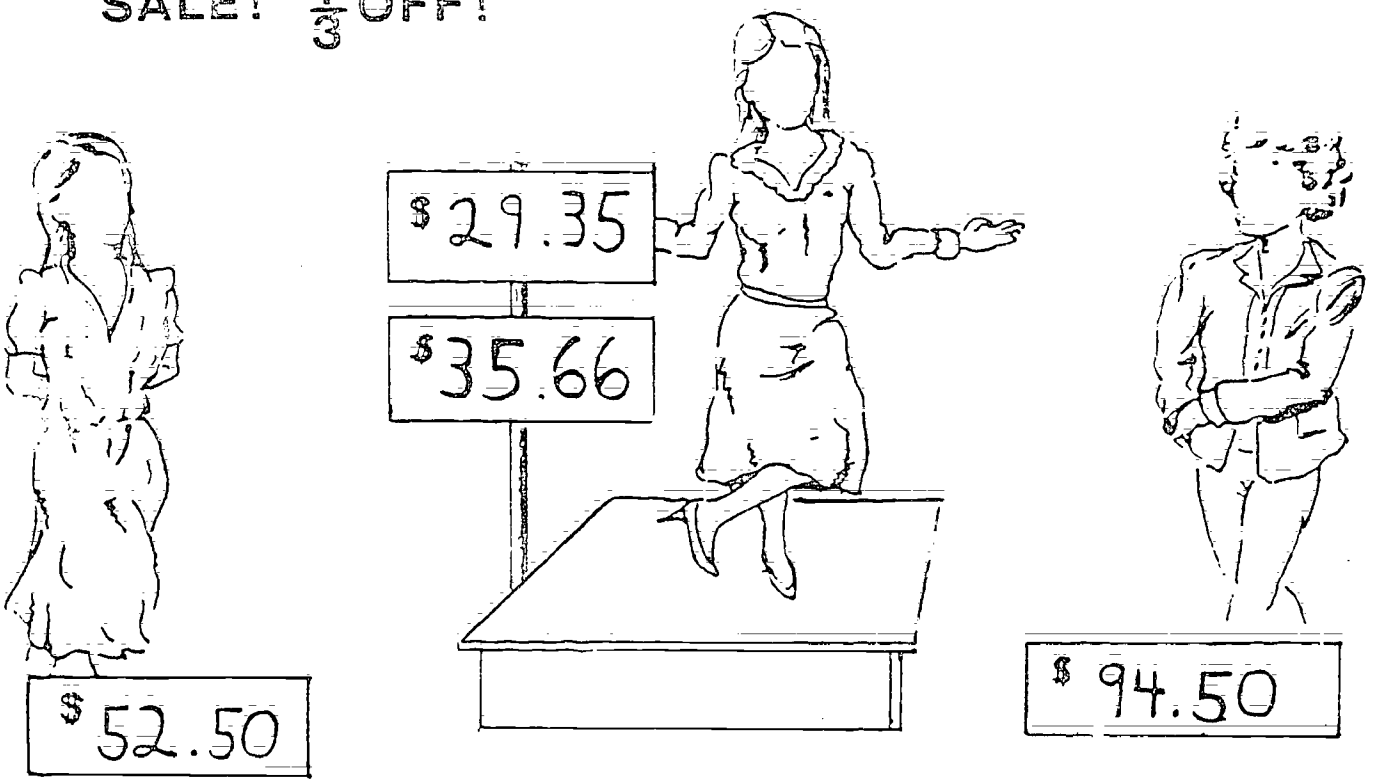
$\frac{1}{3}$ of ~~117~~ 120 is 40

So, about $40 .... or a little less.

$40⁻

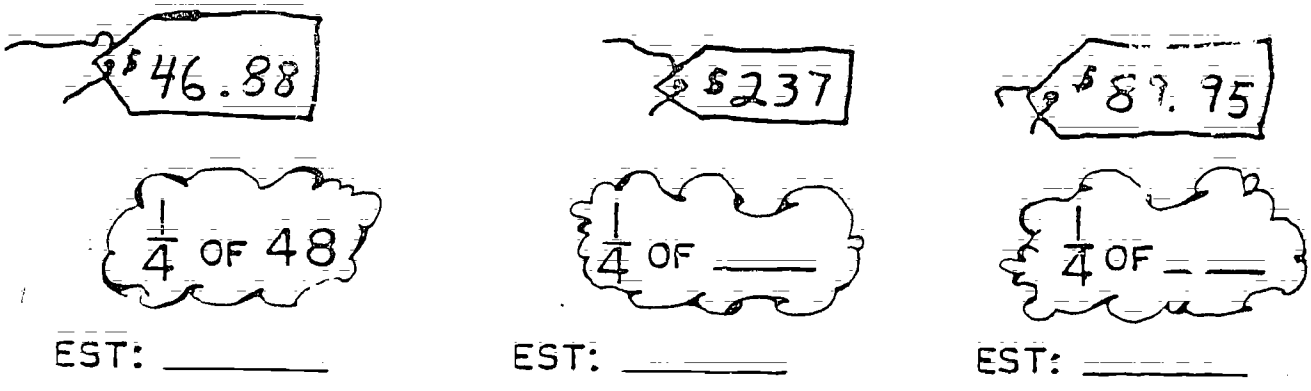
7-8-TR2

SALE! $\frac{1}{3}$ OFF!



Try these!

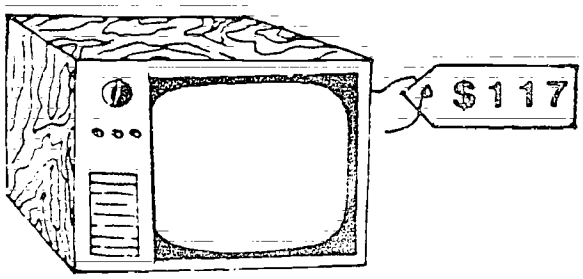
SALE! $\frac{1}{4}$ OFF!



EST: _____ EST: _____ EST: _____

7-8-TR3

ESTIMATE THE AMOUNT PAID



Amount Saved: $30

Amount Paid: ?

Use Nice Numbers to subtract:

TOTAL	−	PART SAVED	=	PART OWED
$120	−	$30	=	$90

Use multiplication:

You save $\frac{1}{4}$, so you pay $\frac{3}{4}$.

$\frac{1}{4}$ IS $30

$\frac{3}{4}$ IS 3 TIMES AS MUCH.

SO, 3 X 30 = $90

TRY THESE:

			SAVE	PAY
$127.49	$\frac{1}{3}$	OFF		
$195.99	$\frac{1}{5}$	OFF		
$78.95	$\frac{1}{4}$	OFF		
$354	$\frac{1}{3}$	OFF		

7-8-TR4

Name _____

Write the answer.

1. $\frac{1}{4}$ of 88 = _____ 2. $\frac{1}{6}$ of 120 = _____ 3. $\frac{1}{5}$ of 75 = _____

4. $\frac{1}{8}$ of 64 = _____ 5. $\frac{1}{4}$ of 280 = _____ 6. $\frac{1}{3}$ of 330 = _____

Rewrite using a compatible number.

7. $\frac{1}{4}$ of 31 — $\frac{1}{4}$ of _____

8. $\frac{1}{3}$ of 37 — $\frac{1}{3}$ of _____

9. $\frac{1}{5}$ of 409 — $\frac{1}{5}$ of _____

10. $\frac{1}{7}$ of 139 — $\frac{1}{7}$ of _____

11. $\frac{1}{3}$ of 148 — $\frac{1}{3}$ of _____

12. $\frac{1}{8}$ of 331 — $\frac{1}{8}$ of _____

13. $\frac{1}{6}$ of 531 — $\frac{1}{6}$ of _____

14. $\frac{1}{3}$ of 245 — $\frac{1}{3}$ of _____

Rewrite using a compatible number. Then estimate.

15. $\frac{1}{4}$ of \$89.95 — $\frac{1}{4}$ of _____ Estimate: _____

16. $\frac{1}{9}$ of \$25.89 — $\frac{1}{9}$ of _____ Estimate: _____

17. $\frac{1}{5}$ of \$146.15 — $\frac{1}{5}$ of _____ Estimate: _____

18. $\frac{1}{3}$ of \$31.19 — $\frac{1}{3}$ of _____ Estimate: _____

19. $\frac{1}{8}$ of \$163.45 — $\frac{1}{8}$ of _____ Estimate: _____

The school board did a survey to find out how students come to school.

	# students
Lincoln School	207
Cany School	147
Ress School	186
West Jr. High	723
Muir School	121

Estimate the number of students who walk.

20. $\frac{1}{4}$ of Lincoln: _____ 21. $\frac{1}{10}$ of Cany: _____

22. $\frac{1}{3}$ of Ress: _____ 23. $\frac{1}{4}$ of Muir: _____

Estimate the number of students who ride in car pools.

24. $\frac{1}{2}$ of West Jr. High: _____ 25. $\frac{1}{3}$ of Cany: _____

26. $\frac{1}{4}$ of Ress: _____ 27. $\frac{1}{5}$ of Lincoln: _____

Name _____

Two good estimates are shown for each problem. If the estimate is more than the exact answer, circle 'over'. If it is less than the exact answer, circle 'under'.

Example:

$\frac{1}{2}$ of \$12.85 — $\frac{1}{2}$ of \$12.00 is \$6.00; Estimate: \$6.00 — This estimate is less than the exact answer so circle 'under'. — over (under)

28.	$\frac{1}{4}$ of \$89.95	Tom's estimate:	\$22.00	over	under
		Al's estimate:	\$25.00	over	under
29.	$\frac{1}{5}$ of \$24.97	Sue's estimate:	\$ 5.00	over	under
		Kim's estimate:	\$ 4.00	over	under
30.	$\frac{1}{3}$ of \$35.98	Beth's estimate:	\$12.00	over	under
		Jack's estimate:	\$11.00	over	under
31.	$\frac{1}{5}$ of \$96.49	Sam's estimate:	\$19.00	over	under
		Ken's estimate:		over	under
32.	$\frac{1}{4}$ of \$49.95	's estimate:	\$11.00	over	under
		Jill's estimate:	\$12.00	over	under

Estimate the amount saved and the amount paid.

			Amount Saved	Amount Paid
33.	\$ 89.98	$\frac{1}{2}$ off	_____	_____
34.	\$247.78	$\frac{1}{5}$ off	_____	_____
35.	\$ 44.49	$\frac{1}{3}$ off	_____	_____
36.	\$599.00	$\frac{1}{4}$ off	_____	_____
37.	\$ 68.95	$\frac{1}{10}$ off	_____	_____

OBJECTIVES: To identify fractions that are close to 1,0 and 1/2.

To estimate a fraction.

To estimate the sum of fractions.

TEACHER BACKGROUND:

Recognizing fractions that are close to 1, 1/2, or 0 is important in estimating the sum of fractions and mixed numbers. It also encourages students to think about the amounts represented by fractions. The idea is easy to learn and contributes to their understanding of fractions.

Fractions Close to 1	Fractions Close to $\frac{1}{2}$	Fractions Close to 0
$\frac{9}{10}$ $\frac{13}{12}$ $\frac{17}{20}$ $\frac{4}{5}$ $\frac{6}{7}$	$\frac{7}{15}$ $\frac{5}{8}$ $\frac{5}{12}$ $\frac{4}{}$ $\frac{13}{24}$	$\frac{4}{30}$ $\frac{3}{19}$ $\frac{1}{6}$ $\frac{3}{17}$ $\frac{1}{8}$
A fraction is close to 1 when the numerator and denominator are about the same size.	A fraction is close to $\frac{1}{2}$ when the denominator is about twice as large as the numerator.	A fraction is close to 0 when the numerator is very small in comparison to the denominator.

Students then use the above ideas to estimate the sum of fractions as shown at the right. The strategy is an easy and enjoyable one for students and they should pick up quickly on it.

$\frac{12}{13} + \frac{4}{9}$ (1, $\frac{1}{2}$)

ESTIMATE: $1\frac{1}{2}$

Many times it is possible to adjust estimates. In the above example, the sum is a little less than $1\frac{1}{2}$ ($1\frac{1}{2}$-) since 12/13 is less than 1 and 4/9 is less than 1/2. In the example at the right, the sum is a little less than 2.

$\frac{7}{8} + \frac{5}{6}$ (1, 1)

ESTIMATE: 2 or 2-

The lesson also includes estimating the fraction for real world situations. In the situation shown at the right, Terry had base hits about one-third of the time, since the denominator is a little more than 3 times as large as the numerator. Also, 11/33 equals 1/3.

TERRY

11 base hits
36 times at bat

$\frac{1}{4}$?

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

The game (TR #1) encourages estimation thinking and encourages thinking about adjusting estimates in division. In each problem, students are to make a guess about the missing divisor. Then the estimate is checked with a calculator. Finally, the estimate is adjusted to get closer to the target number for the quotient.

Example:

$2357 \div \underline{?} \approx 61$

OR

$\underline{?} \overline{)2357}$ — $\overline{61}$

A good first choice would be 40, since 40 x 60 equals 2400 or $2400 \div 61$ is about 40. When 40 is the divisor the quotient is 59.

To get a larger quotient, a good choice would be to decrease the divisor.

Try 39: Quotient: 60^+ or $60\ 1/2$

Try 38: Quotient: 62^+

Thus the closest one can get using whole numbers is 38.

Discuss the first problem with students to help them understand the game. Then do the other 3 problems in the same way.

ANSWERS: 1. 38 2. 105 (104) 3. 34 4. 31 (30)

ESTIMATE THE FRACTION

TR #2: Develop each of the 3 situations. Bring out through discussion:

a. that a fraction is close to 1 when the numerator and denominator are about the same size.

b. that a fraction is close to 0 when the numerator is very small in comparison to the denominator.

c. that a fraction is close to $1/2$ when the denominator is about twice as large as the numerator (or when the numerator is about half as large as the denominator).

Then have students select the fractions at the bottom in the TRY THESE exercises.

ANSWERS:

Close to 1:	$3/4$	$4/5$	$7/9$
Close to 0:	$3/17$	$1/8$	$2/13$
Close to $1/2$:	$7/13$	$4/9$	$6/11$

TR #3: Present the problem at the top and discuss why 8 of 25 is about $\frac{1}{4}$. Then discuss the TRY THESE exercises with the students, having them give their reasoning for each one.

ANSWERS: Jeff: $\frac{1}{2}$ Charlene: $\frac{2}{3}$

Terry: $\frac{1}{3}$ Alisa: $\frac{1}{4}$

Kelly: $\frac{1}{2}$ Chris: $\frac{3}{4}$

ESTIMATING SUMS

TR #4: Present the problem at the top of the page and discuss the approach to estimating the sum. Then develop the two problems in the middle of the transparency. Emphasize that students should first determine if the fraction is close to 0, $\frac{1}{2}$ or 1 and then estimate the sum.

Discuss the first two of the TRY THESE exercises with students. Then have students try the next row independently before discussing them.

ANSWERS: Middle: 2 1 $\frac{1}{2}$

Bottom: 1 1 -

1 $\frac{1}{2}$ 1 $\frac{1}{2}$ 2 $\frac{1}{2}$

USING THE EXERCISES:

Look over each set of exercises with students. Give attention to #13 - 17. Point out that the diagrams will help them find whether the sum is greater or less than 1.

ANSWERS:

1. $\frac{3}{7}$, $\frac{8}{17}$, $\frac{18}{34}$ 2. $\frac{13}{16}$; $\frac{7}{6}$; $\frac{17}{19}$

3. $\frac{1}{3}$ 4. -/3 5. $\frac{2}{3}$ 6. $\frac{1}{2}$

7. $\frac{1}{5}$ 8. $\frac{1}{6}$

9. Lions 10. Tigers Muskrats 11. Cougars Wildcats 12. Bears

13. less 14. greater 15. greater 16. less

17. less

18. about 2 19. about $\frac{1}{2}$ 20. about 1 21. about 2

22. 1 23. $\frac{1}{2}$ 24. 2 25. 1

26. 2 27. 2 $\frac{1}{2}$ 28. 1 29. 1 $\frac{1}{2}$

GET YOUR MIND IN GEAR

ESTIMATION BEAN BAG

1. Estimate to find a divisor that gives a quotient close to the target number.
2. Check with a calculater.
3. Adjust the divisor to get closer.

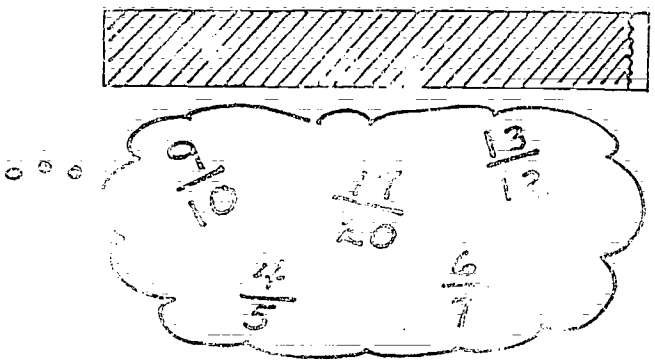
	QUOTIENT	ESTIMATE	QUOTIENT
2357 ÷ ?	61	1. 40	59 (2357 ÷ 40)
		2.	
		3.	
4925 ÷ ?	47	1.	
		2.	
		3.	
6730 ÷ ?	200	1.	
		2.	
		3.	
769 ÷ ?	25	1.	
		2.	
		3.	

7-9-TR1

FRACTIONS CLOSE TO 1, 0, AND $\frac{1}{2}$

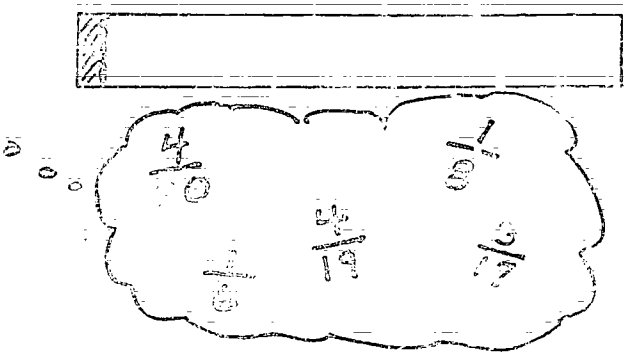
Close to 1

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO 1?



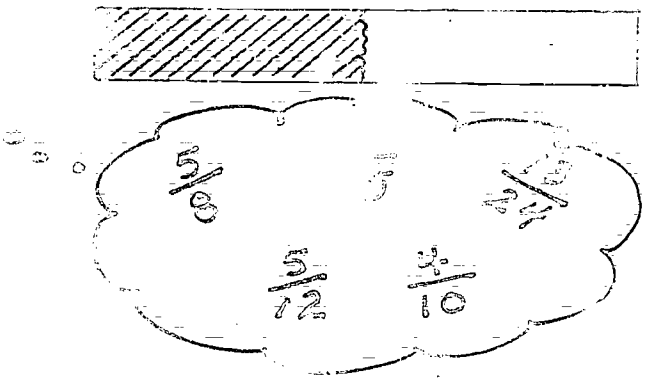
Close to 0

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO 0?



Close to $\frac{1}{2}$

HOW CAN YOU TELL WHEN A FRACTION IS CLOSE TO $\frac{1}{2}$?

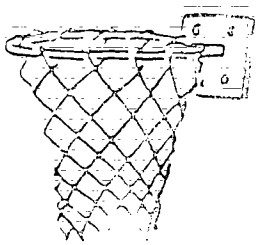


TRY THESE

CLOSE TO 1 ?	$\frac{3}{8}$	$\frac{3}{4}$	$\frac{7}{12}$	$\frac{4}{5}$	$\frac{7}{9}$
CLOSE TO 0 ?	$\frac{3}{5}$	$\frac{3}{17}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{2}{13}$
CLOSE TO $\frac{1}{2}$?	$\frac{7}{13}$	$\frac{2}{7}$	$\frac{4}{9}$	$\frac{6}{11}$	$\frac{8}{9}$

7-9-TR2

ESTIMATE THE FRACTIONS



JOHN MADE 6 OF 25 FREE THROWS. HE MADE ABOUT ? OF HIS SHOTS.

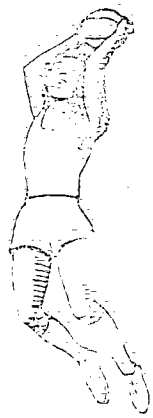
$\frac{6}{25}$ is close to $\frac{6}{24}$

and

$\frac{6}{24}$ equals $\frac{1}{4}$

He made about $\frac{1}{4}$ of his shots.

TRY THESE: Estimate the fractions.



JEFF

8 BASKETS
17 SHOTS

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

CHARLENE

19 BASKETS
31 SHOTS

$\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$

TERRY

11 BASE HITS
36 TIMES AT BAT

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

ALISA

12 BASE HITS
50 TIMES AT BAT

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

KELLY

16 COMPLETED PASSES
33 ATTEMPTS

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$

CHRIS

13 COMPLETED PASSES
16 ATTEMPTS

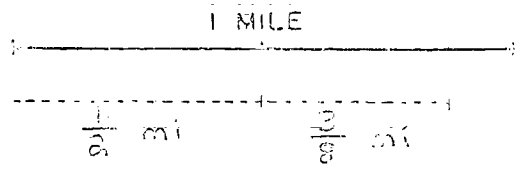
$\frac{1}{2}$ $\frac{4}{5}$ $\frac{3}{4}$

7-9-TRS

ESTIMATING SUMS OF FRACTIONS

CLAUDIA JOGGED $\frac{1}{2}$ MILE IN THE MORNING AND $\frac{3}{8}$ MILE IN THE AFTERNOON.

DID SHE JOG AT LEAST 1 MILE?



$\frac{3}{8}$ is close to $\frac{1}{2}$.
So, Claudia jogged
almost 1 mile.

Estimate Sums

$\frac{7}{8} + \frac{6}{10} + \frac{1}{2}$

$\frac{1}{4} + \frac{1}{9} + \frac{3}{19} + \frac{16}{33}$

ABOUT _____

ABOUT _____

TRY THESE

$\frac{14}{15} + \frac{1}{2}$

$\frac{1}{19} + \frac{6}{13}$

$\frac{19}{41} + \frac{7}{8} + \frac{12}{21}$

$\frac{11}{12} + \frac{}{15}$

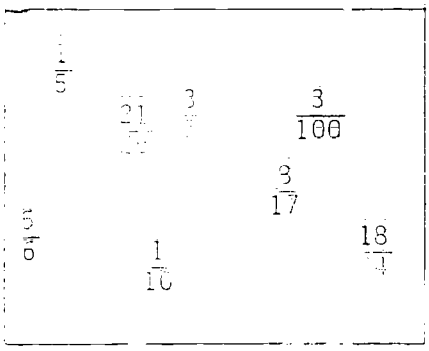
$\frac{}{} + \frac{10}{15}$

$\frac{21}{20} + \frac{}{15} + \frac{3}{7}$

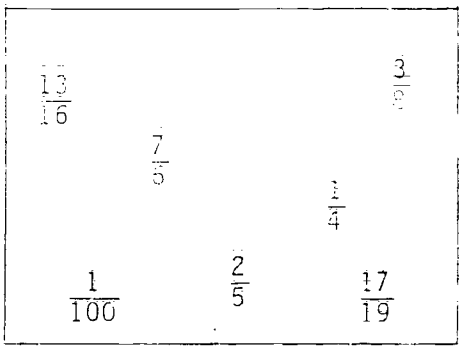
7-9-TR4

Name _____

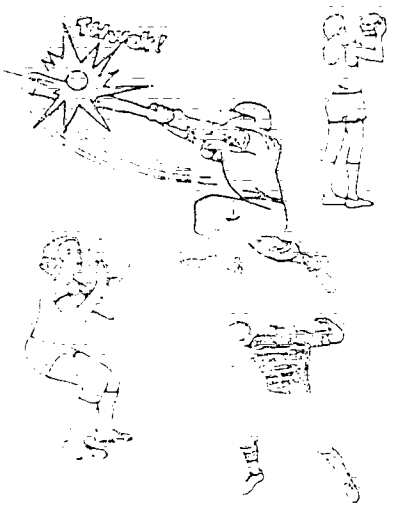
1. Circle the fractions that are close to $\frac{1}{2}$.



2. Circle the fractions that are close to 1.



Estimate the fraction.



3. Travis ___ baskets / 25 shots

4. Mary Lou 24 base hits / 74 times at bat

5. Laura 11 free throws / 18 attempts

6. Jason 21 completions / 43 passes

7. Alan 15 base hits / 74 times at ___

8. ___my 6 goals / 37 shots

Team	Games Played	Games Won
Tigers	21	10
Lions	18	11
Muskrats	27	13
Cougars	19	6
Bears	21	5
Wildcats	29	9

9. Which team has won more than half of its games? _____

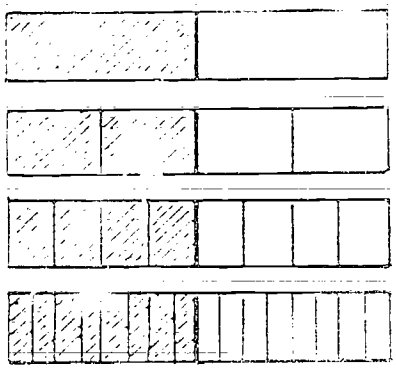
10. Which teams have won just under half of their games? _____

11. Which teams have won about one-third of their games? _____

12. Which team has only won about one-fourth of their games? _____

Name _____

Circle the best answer. Think whether each fraction is greater than or less than 1.



13. $\frac{1}{2} + \frac{1}{4}$ greater than 1 less than 1

14. $\frac{1}{2} + \frac{3}{4}$ greater than 1 less than 1

15. $\frac{1}{2} + \frac{5}{8}$ greater than 1 less than 1

16. $\frac{9}{10} + \frac{1}{8}$ greater than 1 less than 1

17. $\frac{3}{8} + \frac{1}{16}$ greater than 1 less than 1

Estimate the sum. Circle the best choice.

18. $\frac{3}{4} + \frac{9}{10}$ about $\frac{1}{2}$ about 1 about 2

19. $\frac{1}{4} + \frac{3}{15}$ about $\frac{1}{2}$ about 1 about 2

20. $\frac{3}{16} + \frac{1}{8}$ about $\frac{1}{2}$ about 1 about 2

21. $\frac{2}{8} + \frac{5}{6}$ about $\frac{1}{2}$ about 1 about 2

Estimate:

22. $\frac{3}{5} + \frac{4}{9}$ _____

23. $\frac{5}{12} + \frac{1}{15}$ _____

24. $\frac{9}{10} + \frac{14}{15}$ _____

25. $\frac{1}{16} + \frac{9}{10}$ _____

26. $\frac{2}{7} + \frac{1}{15} + \frac{8}{9}$ _____

27. $\frac{12}{13} + \frac{4}{7} + \frac{19}{20}$ _____

28. $\frac{1}{16} + \frac{3}{11} + \frac{3}{4}$ _____

29. $\frac{7}{15} + \frac{5}{12} + \frac{6}{11}$ _____

NSF ESTIMATION
GRADE 7 - LESSON 10

OBJECTIVES: To estimate the sum of mixed numbers using the FRONT-END strategy.

To estimate the sum and difference of mixed numbers by rounding one factor.

To estimate the sum and difference of mixed numbers using nice numbers.

TEACHER BACKGROUND:

Three approaches for addition and subtraction estimation with mixed numbers are presented. Each is an extension of strategies students have used before.

1. Front-End Addition. This is the key approach and involves rounding fractions to 0, $\frac{1}{2}$ or 1.

$2\frac{3}{8} + 4\frac{5}{9}$

a. Add the whole numbers: $2 + 4 = 6$

b. Estimate the rest: $\frac{3}{8} + \frac{5}{9}$ is about 1, since each fraction is close to $\frac{1}{2}$

c. ESTIMATE: 7

There are a variety of ways that estimates can be expressed. For example, at the right the estimate is given three ways. Encourage students to use the + or - sign to adjust when it is possible. It can be used when both fractions have been rounded up or rounded down.

$6\frac{2}{3} + 5\frac{3}{4}$

a. 13 or about 13

b. 13- (a little less than 13)

c. $12\frac{1}{2}$ or about $12\frac{1}{2}$

2. Round One Number. The two examples at the right illustrate this approach. In subtraction, it is important that the second number be rounded. If the first number is rounded, students are likely to make the following error:

$6 - 2\frac{3}{8} = 4\frac{3}{8}$

Students may need to practice adding and subtracting when one number is a mixed number.

$4\frac{3}{4} + 7\frac{2}{3}$

ESTIMATE: $4\frac{3}{4} + 8 = 12\frac{3}{4}$

$14\frac{1}{3} - 1\frac{9}{10}$

ESTIMATE: $14\frac{1}{3} - 2 = 12\frac{1}{3}$

It is also possible to estimate by rounding each mixed number to a whole number. While this is not included in the lesson, you might discuss it with students.

3. Use Nice Numbers. When numbers are relatively large (addends greater than 10), the fractions are relatively insignificant. Here students are encouraged to look for whole numbers that are easy to work with, as shown at the right. When nice numbers are used it is usually difficult to adjust the estimate.

$$18\ 7/8 + 26\ 1/4 + 21\ 3/4$$

ESTIMATE: $20 + 25 + 20 = 65$

Students should be encouraged to be flexible in how they estimate with mixed numbers. This lesson is a good place to stress this idea. Any estimate within a reasonable range is acceptable.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 emphasizes examining an answer to see if it is reasonable. For example, in 3013 - 2995, both numbers are about 3000, so the number shown on the calculator, 1018, does not make sense. Have students look at the numbers as a whole rather than attempting to compute the exact answer.

ANSWERS: The answers to 3013 - 2995, 2087 + 3652 + 197, and 22 x 48 are not reasonable.

ESTIMATE WITH MIXED NUMBERS:

TR #2: Develop the front end approach with students. Give attention to the adjusting step (since $7/8$ and $3/4$ are each less than 1; the exact sum is less than 11). Do the first row of TRY THESE exercises with students. Then have them try the second row independently before discussing the exercises. Encourage students to share their thinking and ways of adjusting.

ANSWERS: 9^- ($8\ 1/2 = 9$) 8^+ ($8 - 8\ 1/2$) 7

18^- (18^-) 14 7 (7^-)

TR #3: Discuss the use of rounding one number at the top. Include a subtraction example such as $6\ 2/3 - 3\ 1/3$. Point out that in subtraction it is important to round the second fraction. It is possible to round the first fraction, but there is the danger of making a common error. For example, the answer to $6 - 2\ 1/3$ is $3\ 2/3$, not $4\ 1/3$.

The exercises in the box provide practice in adding a whole number to a mixed number or subtracting a whole number from a mixed number. It may help some students to use pairs of examples as shown below. You also can illustrate the work using a number line.

$5 + 4 = 9$ $6 - 2 = 4$

so $5\ 2/3 + 4 = 9\ 2/3$ so $6\ 1/4 - 2 = 4\ 1/4$

If any students raise the possibility of rounding both numbers discuss this as another way to estimate and permit students to use this method as well.

Do the first row of the TRY THESE with students. Then have them try the remaining ones independently before discussing them.

ANSWERS: $12\frac{1}{7}$ (12^+) $\frac{2}{3}$ (17^-) $29\frac{3}{4}$ (30^-)

$12\frac{1}{3}$ (12) $\frac{7}{8}$ (6^+) $8\frac{11}{12}$ (9)

$4\frac{7}{9}$ (5) $4\frac{1}{12}$ (4^+) $4\frac{6}{7}$ (5^-)

TR #4: Present the problem at the top and look at the example. Point out that the fractional part of each mixed number does not affect the size of the number very much. Also point out that with these numbers the best way to estimate is to choose nice numbers to work with. Discuss the solution and ask students if there are other ways to estimate. Do the TRY THESE exercises with students.

ANSWERS: $50 = 51$ $145 = 150$

$44 = 46$ $35 = 36$ $45 = 50$

USING THE EXERCISES:

Discuss the sample example at the top of page 1, pointing out that answers may be expressed in a variety of ways. Also point out the directions at the top of page 2, where students are asked to estimate by rounding one number. The THINK IT THROUGH exercises at the bottom of page 2 present a mental computation strategy for obtaining exact answers and should be used for additional challenge for capable students.

ANSWERS:

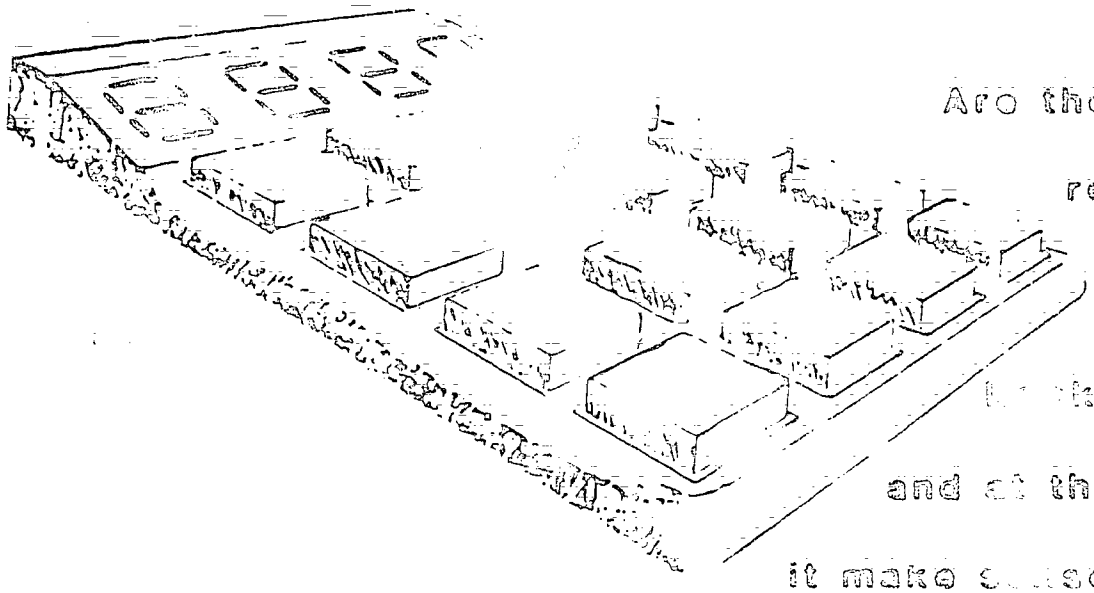
1. $16\frac{1}{2}$ - 17
2. 12 - 12^+
3. 23 - 24^-
4. $15\frac{1}{2}$ - 16
5. 24 - $24\frac{1}{4}$
6. 11 - $11\frac{1}{2}$
7. $13\frac{1}{2}$ - 14
8. 14 - $14\frac{1}{2}$
9. 10 - 11
10. Yes
11. No
12. over 6
13. under 12
14. under 8
15. over 4
16. $12\frac{3}{7}$ or $12\frac{1}{5}$
17. $16\frac{2}{3}$ or $16\frac{9}{10}$
18. $29\frac{3}{7}$ or $29\frac{1}{6}$
19. $12\frac{1}{3}$
20. $\frac{7}{8}$
21. $8\frac{5}{8}$
22. 49 - 50
23. 69 - 70^+
24. 19 - 22
25. 55 - 60
26. 120 - 125
27. 175 - 180
28. 29 - 32
29. 55 - 57
30. 65 - 70

THINK IT THROUGH

1. $4\frac{2}{3}$
2. $7\frac{1}{2}$
3. $5\frac{3}{4}$
4. $6\frac{3}{4}$
5. $2\frac{2}{3}$
6. $3\frac{7}{8}$

GET YOUR MIND IN GEAR

SENSIBLE ANSWERS



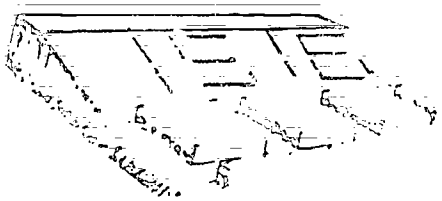
Are those answers reasonable? Use estimation. Look at the problem and at the answer. Does it make sense?

1[illegible] ÷ 15 = 32

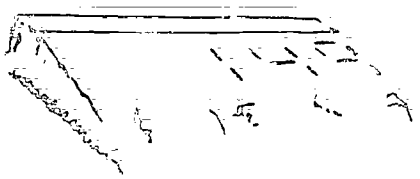
3013 - 2995

2087 + 1552 + 197

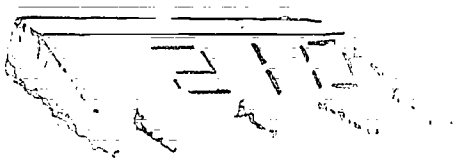
22 x 43



346 - 191

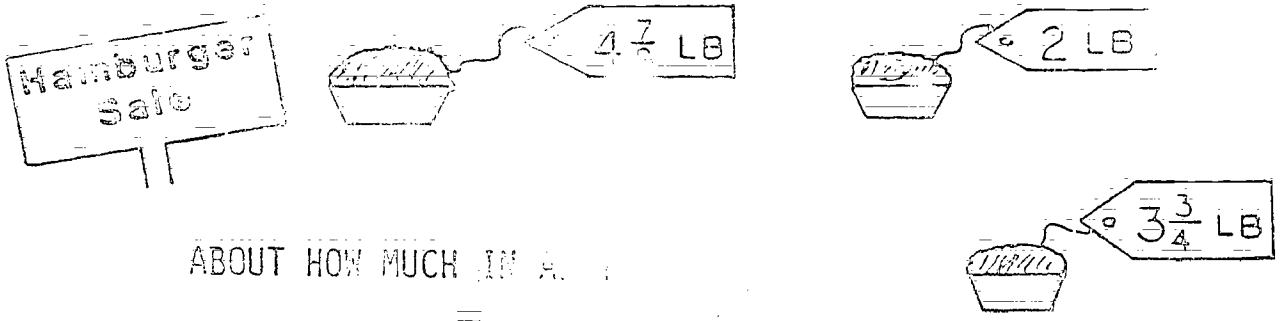


6720 ÷ 32



7-10-TR1

ESTIMATE THE SUM OF MIXED NUMBERS



ABOUT HOW MUCH IN ALL?

$$4\frac{7}{8} + 2 + 3\frac{3}{4}$$

Add the wholes: $4 + 2 + 3 =$ ____

Estimate the rest: $\frac{7}{8} + \frac{3}{4}$ (1) (1)

almost 2 more

Put it together: 9 wholes . . . 10, 11

Estimate: 11 pounds

TRY THESE: ADJUST IF YOU CAN.

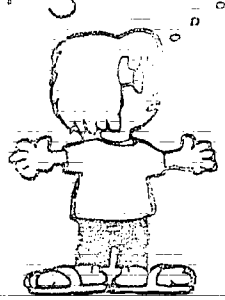
$4\frac{2}{3} + 3\frac{7}{8}$ $\quad 6\frac{1}{8} + 2\frac{2}{11}$ $\quad 2\frac{1}{9} + 1\frac{2}{15} + 3\frac{3}{4}$

$8\frac{2}{3} + 9\frac{1}{6}$ $\quad 6\frac{1}{12} + 7\frac{9}{10}$ $\quad \frac{11}{12} + 3\frac{3}{4} + 1\frac{1}{8}$

7-10-TR2

ROUND ONE NUMBER

$4\frac{3}{4} + 7\frac{2}{3}$



Another way to estimate sums and differences is to just ROUND ONE, and then add:

$4\frac{3}{4} + \boxed{8}$ about 12

PRACTICE ADDING AND SUBTRACTING IN YOUR HEAD.

$2\frac{3}{4} + 4$	$3\frac{1}{3} - 5$	$5\frac{2}{3} + 4$	$5\frac{2}{3} + 6$
$6\frac{3}{5} - 3$	$12\frac{1}{9} - 8$	$7\frac{7}{8} - 6$	$11\frac{1}{3} - 5$

TRY THESE:

$4\frac{1}{7} + 8\frac{1}{5}$	$9\frac{2}{3} + 6\frac{9}{10}$	$21\frac{3}{4} + 8\frac{1}{6}$
$14\frac{1}{3} - 1\frac{9}{10}$	$8\frac{7}{8} - 2\frac{1}{6}$	$13\frac{11}{12} - 4\frac{7}{8}$
$6\frac{7}{9} - 2\frac{1}{3}$	$9\frac{1}{12} - 4\frac{2}{3}$	$13\frac{6}{7} - 8\frac{4}{5}$

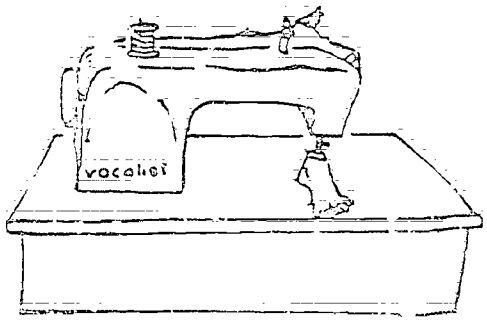
7-10-TR3

Pep Club Project:

SCHOOL BANNER

Entrance	$18\frac{7}{8}$	yd
Gymnasium	$26\frac{1}{4}$	yd
Auditorium	$21\frac{3}{4}$	yd

I'll change these to NICE NUMBERS that are easy to compute in my head.



$18\frac{7}{8} + 26\frac{1}{4} + 21\frac{3}{4}$

20 + 25 + 20

ABOUT 65 YARDS

CAN YOU THINK OF ANOTHER WAY?

Try these!

$34\frac{9}{10} + 16\frac{1}{4}$

$21\frac{3}{5} + 82\frac{1}{3} + 45\frac{5}{7}$

$49\frac{5}{9} - 5\frac{1}{6}$

$45\frac{9}{16} - 9\frac{8}{9}$

$50\frac{1}{3} - 12\frac{1}{2}$

T-10-TR4

Name _____

Estimate. Use the Front-End method. Use + or - to adjust your estimate when you can.

Example: $8\frac{1}{9} + 4\frac{5}{6}$

Sue: 13 Jaun: $12\frac{1}{2}$

Helene: Between 12 and 13

1. $7\frac{9}{10} + 8\frac{11}{13}$ Estimate: _____

2. $9\frac{9}{16} + 2\frac{13}{24}$ Estimate: _____

3. $12\frac{2}{3} + 10\frac{1}{2}$ Estimate: _____

4. $6\frac{5}{8} + 9\frac{1}{7}$ Estimate: _____

5. $12\frac{1}{10} + 12\frac{1}{16}$ Estimate: _____

6. $7\frac{3}{5} + 3\frac{5}{9}$ Estimate: _____

7. $8\frac{11}{12} + 3\frac{10}{13} + \frac{15}{16}$ Estimate: _____

8. $6\frac{3}{8} + 5\frac{1}{4} + 2\frac{8}{15}$ Estimate: _____

9. $3\frac{1}{8} + 4\frac{1}{10} + 3\frac{3}{10}$ Estimate: _____

10. Bob needs $10\frac{1}{2}$ pounds of lunch meat for a picnic. He bought:

$3\frac{1}{4}$ pounds of ham

$3\frac{7}{8}$ pounds of salami

$2\frac{15}{16}$ pounds of bologna

Does Bob have enough meat?

_____ (YES or NO)

11. Tammy said it would take less than 5 hours to drive from Chicago to St. Louis. She drove:

$2\frac{5}{12}$ hours before lunch, and

$3\frac{1}{4}$ hours after lunch.

Did she make the trip in less than 5 hours?

_____ (YES or NO)

Circle the better estimate.

12. $3\frac{6}{7} + 2\frac{3}{8}$ over 6 under 6

13. $7\frac{1}{10} + 4\frac{1}{2}$ over 12 under 12

14. $6\frac{3}{8} + 1\frac{1}{4}$ over 8 under 8

15. $3\frac{5}{8} + \frac{5}{8}$ over 4 under 4

7-10-p.1

Name_____

Estimate by rounding one number to a whole number.

Example: $14\frac{2}{5} - 1\frac{9}{10}$

Round $1\frac{9}{10}$ to 2

$14\frac{2}{5} - 2 = 12\frac{2}{5}$

16. $4\frac{3}{7} + 8\frac{1}{5}$ Estimate:_____

17. $9\frac{2}{3} + 6\frac{9}{10}$ Estimate:_____

18. $21\frac{3}{7} + 8\frac{1}{6}$ Estimate:_____

19. $14\frac{1}{3} + 1\frac{9}{10}$ Estimate:_____

20. $8\frac{7}{8} - 2\frac{2}{7}$ Estimate:_____

21. $13\frac{5}{8} - 4\frac{7}{8}$ Estimate:_____

Estimate. Use numbers that are easy to work with.

22. $16\frac{3}{8} + 28\frac{1}{3}$ Estimate:_____

23. $18\frac{3}{4} + 51\frac{7}{8}$ Estimate:_____

24. $\frac{1}{8} + 4\frac{5}{6} + 15\frac{1}{8}$ Estimate:_____

25. $38\frac{2}{3} + \frac{2}{5} + 19\frac{1}{4}$ Estimate:_____

26. $82\frac{3}{8} + 39\frac{1}{5} + 1\frac{2}{3}$ Estimate:_____

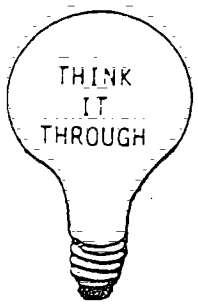
27. $148\frac{3}{4} + 28\frac{1}{5}$ Estimate:_____

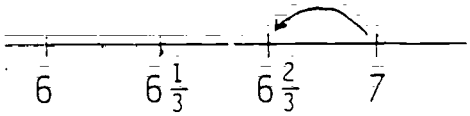
28. $49\frac{7}{8} - 18\frac{3}{4}$ Estimate:_____

29. $66\frac{3}{7} - 9\frac{1}{8}$ Estimate:_____

30. $128\frac{2}{3} - 58\frac{7}{9}$ Estimate:_____

SUBTRACT IN YOUR HEAD





Think: $\frac{1}{3}$ less than 7

$7 - \frac{1}{3} = 6\frac{2}{3}$

1. $5 - \frac{1}{3}$

2. $8 - \frac{1}{2}$

3. $6 - \frac{1}{4}$

4. $7 - \frac{1}{4}$

5. $3 - \frac{1}{3}$

6. $4 - \frac{1}{8}$

7-10-p.2

NSF ESTIMATION
GRADE 7 - LESSON 11

OBJECTIVES: To estimate products and quotients for mixed-number problems.

TEACHER BACKGROUND:

For multiplication and division estimation involving mixed numbers, the primary strategy is to round both numbers as shown below:

MULTIPLICATION

$11\ 3/4 \times 8\ 1/3$

$\downarrow \qquad \downarrow$

12×8

ESTIMATE: 96

DIVISION

$6\ 2/3 \div 4\ 7/9$

$\downarrow \qquad \downarrow$

$7 \div 5$

ESTIMATE:

A. $5 \div 5 = 1$, so $7 \div 5$ is a little more than 1 (1^+).

$1\ 1/2$ is also a good choice.

B. $7 \div 5 = 7/5$ and $7/5 = 1\ 2/5$

The estimate is $1\ 2/5$.

Rounding is acceptable for mixed number work with multiplication and division. However, it is also important to make students aware of the variety of techniques that are sometimes possible. This contributes to developing mental flexibility, important in estimation, problem solving and all mathematical thinking. While all students should be aware of these techniques, only some students may feel comfortable with the alternative approaches. The other approaches are:

1. Round One Factor (Multiplication)

In the example at the right, $11\ 3/4$ is rounded to 12 since it is easy to compute $12 \times 8\ 1/3$ mentally.

The shortcut for multiplying a mixed number by a whole number is then used. The shortcut, illustrated here, is based on the distributive property of multiplication over addition. The estimate is a little too high since $11\ 3/4$ was rounded up. The estimate can be written: 100-.

$11\ 3/4 \times 8\ 1/3$

$\swarrow \qquad \swarrow$

$12 \times 8\ 1/3$

$12 \times 8 = 96$

$1/3$ of $12 = 4$

ESTIMATE: 100

2. Use Compatible Numbers (Division)

Since 30 is a multiple of 6, 30 and 6 can be used to get an estimate of 5.

For $46\ 3/4 \div 7\ 1/5$, the student could think: $48 \div 8$.

$29\ 1/8 \div 6\ 2/5$

$30 \div 5$

ESTIMATE: 6

3. Compare the Quotient to 1

In the example, the divisor ($3\ 1/4$) is greater than the dividend ($2\ 3/8$) so the quotient is less than 1.

While this approach does not produce a numerical estimate, it does help a student know if an answer is reasonable.

$2\ 3/8 \div 3\ 1/4$

ESTIMATE: Less than 1

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 stresses flexibility in estimation. There are several ways to approach the two grocery lists shown on the transparency.

$2.78
1.19
.07
2.48
.42
3.95

For the example at the left, one could begin by using front-end estimation and then adjust up. The adjusting could be done by adding the dimes or grouping cents to dollars.

Another approach is to group the first two amounts to $4.00, the next three amounts to $3.00 and round $3.95 to $4.00

Let students share their approaches to the exercises.

ANSWERS: $10.00 - $11.00 $5.00 - $6.00

MULTIPLICATION ESTIMATION

TR #2: Develop the rounding strategy. Discuss why the estimate is less than 48 miles (48^-). Since both factors are rounded up, 48 is too high (an overestimate). Do the TRY THESE exercises with students, encouraging them to adjust their estimates when it is possible.

ANSWERS:			
	12	84	45^-
	42^+	90^+	6^-
	$49^+ - 56^-$	$20^+ - 22$	$24 - 25$

TR #3: Introduce the rounding of one factor. Point out that they need to multiply twice (12×8 and $1/3$ of 12) and then add. Do additional examples using the mental computation procedure ($6 \times 4\ 1/2$, $9 \times 5\ 1/3$, $11\ 1/2 \times 7$). Point out that this is a good strategy when the computation can easily be done mentally.

As you use the TRY THESE exercises, it would be good to let students estimate using both methods (rounding both factors and rounding one factor) and comparing the two estimates.

ANSWERS: 42 29 95

DIVISION ESTIMATION

TR #4: Develop the two strategies at the top of the page and discuss when they might be useful. Then present and develop the rounding strategy. Point out that students can choose how to estimate depending on the example. Do the TRY THESE exercises orally, having students try different strategies for each exercise.

ANSWERS: $5 - 6^-$ $3^+ - 4^-$ 4

2^+ $2/3$ $6 - 7$

USING THE EXERCISES:

The exercises are a straightforward application of the strategies presented in the lesson. Read each set of directions with students before they start to work.

ANSWERS:

1.	35^-	2.	32	3.	6^+	4.	63				
5.	48	6.	45^-	7.	28	8.	18				
9.	26	10.	22	11.	56	12.	$12\ 1/2$	13.	26		
14.	84 (88^-)	15.	66 (66^+)	16.	96 - 100	17.	38 (38^-)				
18.	greater than 1			19.	less than 1						
20.	less than 1			21.	greater than 1						
22.	6 - 7			23.	1 - 2			24.	3 - 4		
25.	2 - 3			26.	5 - 6			27.	1 - 2		
28.	81^- sq. ft.			29.	90 - 99^- sq. ft.			30.	75 - 81 sq. ft.		
31.	30 - 33 sq. ft.			32.	110 - 120 sq. ft.			33.	57 - 60 sq. ft.		

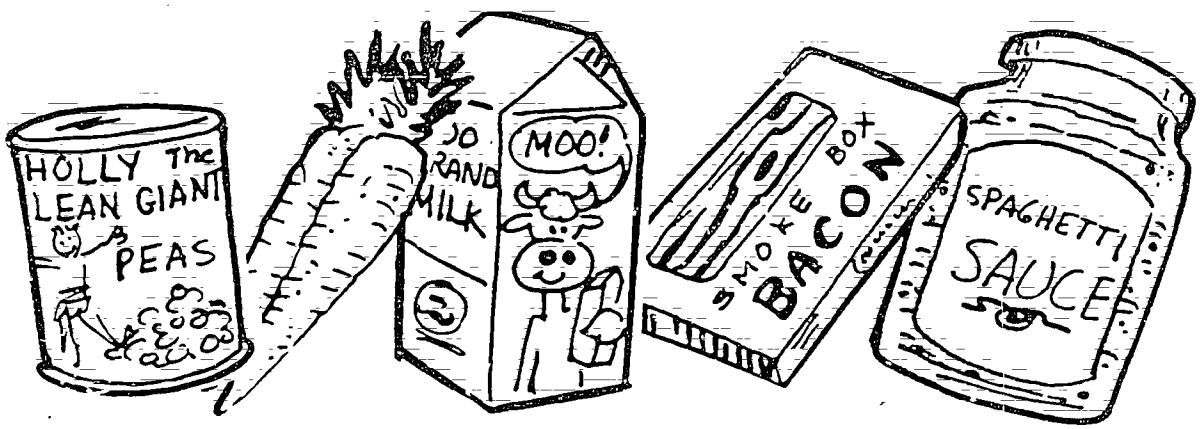
GET YOUR MIND IN GEAR

ESTIMATE. TELL HOW YOU THINK.

BE FLEXIBLE. USE A WAY THAT IS QUICK AND EASY TO DO IN YOUR HEAD. MAKE SURE YOUR ESTIMATE IS "IN THE BALLPARK."

$2.78
1.19
.07
2.48
.42
3.95

$ .49
.10
.53
1.55
.57
2.30

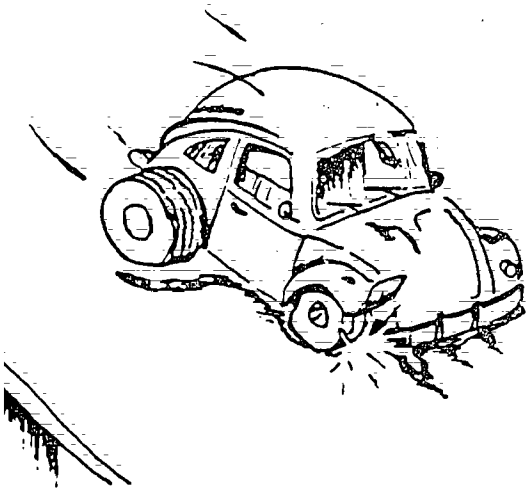


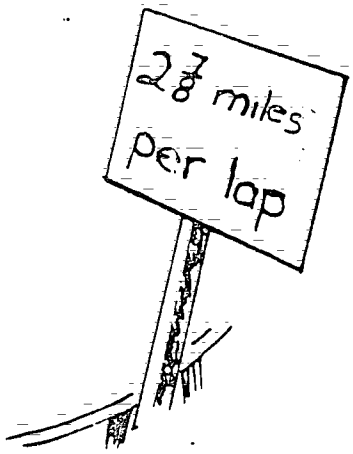
7-11-TR1

ESTIMATING PRODUCTS

DIRK HAD A FLAT TIRE AFTER MAKING $15\frac{3}{4}$ LAPS.

HOW MANY MILES HAD HE RACED?





$2\frac{7}{8} \times 15\frac{3}{4}$

Round both factors: 3 × 16

Multiply: 3 × 16 = 48

LESS THAN 48 MILES.

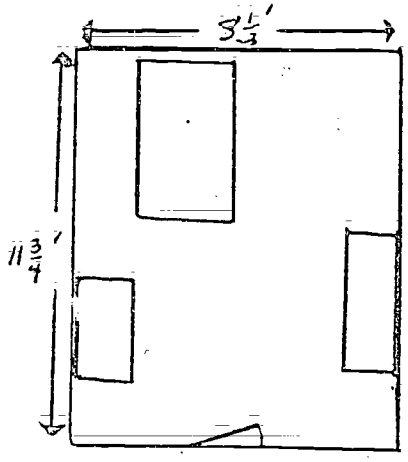
Try these!

$4\frac{1}{3} \times 2\frac{7}{8}$	$6\frac{9}{10} \times 12\frac{1}{8}$	$4\frac{9}{10} \times 8\frac{2}{3}$
$6\frac{1}{9} \times 7\frac{2}{15}$	$10\frac{1}{6} \times 9\frac{1}{5}$	$2\frac{3}{4} \times 1\frac{7}{8}$
$7 \times 7\frac{4}{7}$	$10 \times 2\frac{1}{5}$	$4 \times 6\frac{1}{4}$

ROUND ONE FACTOR

TO ESTIMATE THE AREA OF HIS BEDROOM, MIKE ROUNDED ONE FACTOR AND THEN MULTIPLIED.

$11\frac{3}{4} \times 8\frac{1}{3}$



Round one factor: $12 \times 8\frac{1}{3}$

Multiply: $12 \times 8 \longrightarrow 96$

$\frac{1}{3}$ of 12 ... $12 \times \frac{1}{3} \longrightarrow 4$

Add: ABOUT 100 SQUARE FEET

Work these out:

$8 \times 5\frac{1}{4} =$ _____

8 × 5 = ___
$\frac{1}{4}$ of 8 = ___

$4\frac{1}{2} \times 6 =$ _____

4 × 6 = ___
$\frac{1}{2}$ of 6 = ___

$2\frac{1}{3} \times 15 =$ _____ $6\frac{1}{2} \times 10 =$ _____ $3 \times 6\frac{1}{3} =$ _____

Try these!

$4\frac{1}{5} \times 9\frac{7}{8}$ $6\frac{2}{3} \times 4\frac{1}{7}$ $14\frac{2}{3} \times 6\frac{1}{3}$

7-11-TR3

ESTIMATING QUOTIENTS

To estimate in division, you can

Use compatible numbers:

$$29\frac{1}{8} \div 6\frac{2}{5}$$

30 and 6 are compatible.

$30 \div 6 = 5$

Compare the quotient to one:

$$2\frac{3}{8} \div 3\frac{1}{4}$$

Because $2\frac{3}{8}$ is less than $3\frac{1}{4}$, the quotient must be less than one!

Round to wholes:

$$6\frac{2}{3} \div 4\frac{7}{9}$$

$$7 \quad 5$$

5 divided by 5 is 1. So, $7 \div 5$ is a little more than one.

1^+

7 divided by 5 means $\frac{7}{5}$... and $\frac{7}{5}$ equals $1\frac{2}{5}$

Try these!

$18\frac{1}{4} \div 3\frac{4}{5}$

$9\frac{5}{6} \div 3\frac{1}{10}$

$14\frac{7}{8} \div 3\frac{9}{10}$

$11\frac{1}{8} \div 5\frac{1}{6}$

$3\frac{7}{8} \div 5\frac{15}{16}$

$20\frac{1}{7} \div 3\frac{2}{10}$

7-11-TR4

Name _____

Estimate by rounding both factors. Write + or - to adjust the estimate when you can.

1. $4\frac{2}{3} \times 6\frac{5}{7}$ _____

2. $8\frac{3}{8} \times 3\frac{3}{4}$ _____

3. $3\frac{1}{8} \times 2\frac{1}{3}$ _____

4. $9\frac{1}{10} \times 6\frac{2}{3}$ _____

5. $6\frac{3}{8} \times 7\frac{9}{10}$ _____

6. $4\frac{5}{7} \times 8\frac{9}{10}$ _____

7. $3\frac{4}{5} \times 7\frac{1}{9}$ _____

8. $2\frac{9}{10} \times 6\frac{1}{3}$ _____

Multiply in your head. Find the exact answer.

EX. $5\frac{1}{4} \times 8 \rightarrow$ (5×8) 40 + 2 ($\frac{1}{4}$×8) → _____

9. $6\frac{1}{2} \times 4 \rightarrow$ _____ + _____ → _____

10. $10 \times 2\frac{1}{5} \rightarrow$ _____ + _____ → _____

11. $9\frac{1}{3} \times 6 \rightarrow$ _____ + _____ → _____

12. $5 \times 2\frac{1}{2} \rightarrow$ _____ + _____ → _____

13. $8 \times 3\frac{1}{4} \rightarrow$ _____ + _____ → _____

Estimate by rounding one factor.

14. $7\frac{1}{3} \times 11\frac{2}{3}$ (12) _____ Estimate

15. $21\frac{1}{10} \times 3\frac{1}{7}$ (21) _____ Estimate

16. $4\frac{1}{6} \times 23\frac{2}{3}$ (24) _____ Estimate

17. $5\frac{8}{9} \times 6\frac{1}{3}$ (6) _____ Estimate

Name_____

Circle your answer. Ex. $3\frac{1}{8} \div 4\frac{1}{8}$ $4\frac{1}{8}$ is greater than $3\frac{1}{2}$ so the quotient is less than 1.

18. $2\frac{1}{2} \div 1\frac{5}{8}$ Greater than 1 Less than

19. $2\frac{2}{3} \div 3\frac{1}{5}$ Greater than 1 Less than 1

20. $7\frac{5}{8} \div 3\frac{1}{4}$ Greater than 1 Less than 1

21. $1 \div \frac{3}{4}$ Greater than 1 Less than 1

Estimate the quotient.

22. $42\frac{1}{6} \div 6\frac{2}{3}$ Estimate:_____

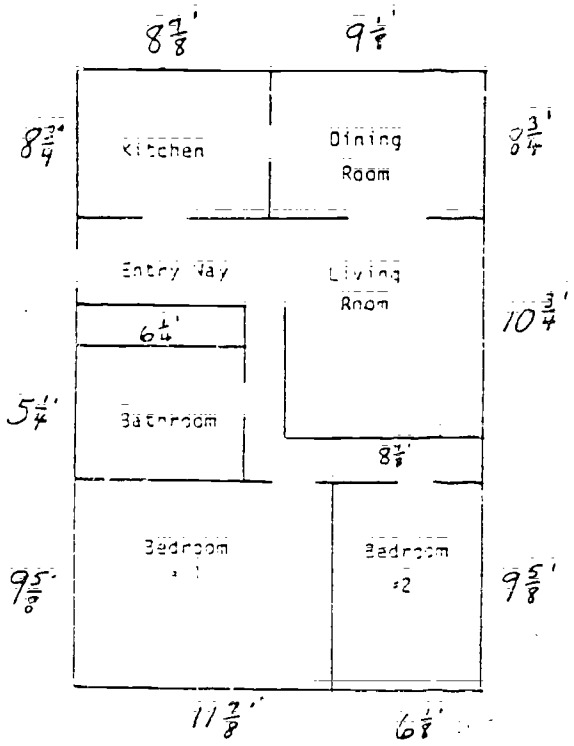
23. $7\frac{2}{3} \div 5\frac{3}{4}$ Estimate:_____

24. $18\frac{1}{3} \div 5\frac{3}{10}$ Estimate:_____

25. $24\frac{2}{7} \div 9\frac{3}{5}$ Estimate:_____

26. $33\frac{1}{4} \div 6\frac{1}{3}$ Estimate:_____

27. $9\frac{1}{4} \div 6\frac{2}{3}$ Estimate:_____



Estimate the area of each room.

28. Kitchen: 9ft. x 9ft. 81 sq. ft.

29. Living Room:_____

30. Dining Room:_____

31. Bathroom:_____

32. Bedroom #1:_____

33. Bedroom #2:_____

NSF ESTIMATION
GRADE 7 - LESSON 12

OBJECTIVE: To estimate sums and differences with decimals.

TEACHER BACKGROUND:

Estimation with decimals is developed for two cases: 1) estimation with "smaller" numbers (numbers between 0 and 10 or in the low tens) and 2) estimation with "larger" numbers (numbers with 2 or more digits). With smaller numbers, the decimal portion is important because of its effect on the estimate. With larger numbers, the decimal portion contributes little and can be ignored.

The strategies employed were taught with whole numbers and are extended to decimals. Students are encouraged to select a strategy that works well for them.

Decimal estimation is useful in reducing errors when working with different numbers of decimal places. Students often line up the right-hand digits, ignoring the place value of the digits.

$$\begin{array}{r} 3.8 \\ .27 \\ +\ 1.5 \end{array}$$

"Smaller Number" Strategies

A. FRONT-END

$0.2 + 2.8 + 9.7 + 7$	$15.3 - 7.8$
1) $0 + 2 + 9 + 7 = 18$	1) $15 - 7 = 8$
2) $.2 + .8 + .7$ is between 1 and 2	2) $.3$ is less than $.8$
3) ESTIMATE: 19^+, 19.5, etc.	3) ESTIMATE: 8^-, 7.5, etc.

B. ROUNDING

$0.2 + 2.8 + 9.7 + 7$	$15.3 - 7.8$
1) $0 + 3 + 10 + 7 = 20$	1) $15 - 8 = 7$
2) ESTIMATE: 20	2) ESTIMATE: 7

"Larger Number" Strategies

Students are encouraged to choose numbers that are easy to use, including compatible numbers and grouping of addends to nice numbers.

$92.7 + 48.5 + 63.72$	$14.27 + 6.83 + 34.6 + 7.2$
1) $100 + 50 + 60$	1) $20 + 40$
2) ESTIMATE: 210	2) ESTIMATE: 60

Encourage students to be flexible in the approach they choose.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

In many instances it is important to adjust the estimate by changing the estimate to get a closer estimate. This is the focus of TR #1. In each example a "ballpark" estimate is given. Students are to give an adjusted estimate that is closer. Encourage a variety of ways of thinking by having several students give adjusted estimates.

ANSWERS: 6.00^+ ($6.60) 7.00^+ ($8.00)

8.00^+ ($9.00) 4.50^+ ($5.00)

DECIMAL ESTIMATION

TR #2: Develop the ROUNDING AND FRONT-END approaches. Then have students do the TRY THESE exercises using both approaches.

ANSWERS: 9.4 - 10 19 - 20

2 (2^-) 18 - 18.5

TR #3: Present the two strategies for larger numbers. Ask students if there are other ways to estimate.

ADJUSTING ESTIMATES:

TR #4: Discuss the 3 examples of adjusting. Remind students that while it isn't always necessary to adjust an estimate, it is often useful to do so and they should have a sense of the relationship between the estimate and the exact answer when possible.

Have students do the TRY THESE exercises independently. Then discuss their estimates. You will likely get a range of estimates for each problem.

ANSWERS: 650 - 660 160 -170 180 - 200

760 - 800 48 - 50 100 - 110

230 - 250 290 - 310

USING THE EXERCISES:

Go over each set of directions with students before they start to work. The THINK IT THROUGH exercises (bottom of page 2) involve mixed operations and are intended to challenge more capable students.

ANSWERS:

1. 10^+ - 11 2. 8 - 9 3. 20 - 21.5

4. 2.6 - 3^- 5. 3 - 4^- 6. 13 - 14

7. 11.0 = 11.5	8. 19.0 = 19.3	9. 31 = 32
10. 21 = 22	11. 2.4 = 3	12. 3.5 = 4
13. 5 = 6	14. 1.5 = 2	15. 6 = 6.2
16. 10 = 11	17. 2	18. 7.5 = 9.0
19. 5.5 = 7	20. 9.8 = 11	21. 54 = 60
22. 5 = 5		
23. 120 - 124	24. 450 - 500	25. 264 - 280
26. 20 - 30	27. 40	28. 490 - 500
29. 285 - 290	30. 45 - 50	31. 700 - 740
32. 380 - 450	33. 800 - 850	34. 680 - 750
35. 100 - 120	36. 200 - 250	37. 180 - 200
38. 350 - 360		

THINK IT THROUGH

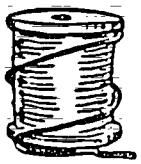
1. 20 - 23	2. 150 - 200
3. 450 - 460	4. 30 - 31

GET YOUR MIND IN GEAR

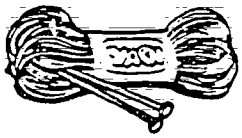
ADJUSTING

Good estimators can usually tell how their estimate compares to the actual amount. You can adjust by writing a + or −.

THREAD
$ 1.09 EACH



REMNANTS
$ 1.29 PER YARD



ARE YOU A GOOD ADJUSTER? SEE IF YOU CAN GET CLOSER WITH THESE.

GETTING CLOSER

BUY $2\frac{3}{4}$ YARDS OF REMNANTS AND 3 SPOOLS OF THREAD.

ESTIMATE: $6.00 _____

BUY $5\frac{7}{8}$ YARDS OF REMNANTS AND 1 SPOOL OF THREAD.

ESTIMATE: $7.00 _____

BUY $4\frac{1}{8}$ YARDS OF REMNANTS AND 4 SPOOLS OF THREAD.

ESTIMATE: $8.00 _____

BUY $1\frac{1}{2}$ YARDS OF REMNANTS AND 3 SPOOLS OF THREAD.

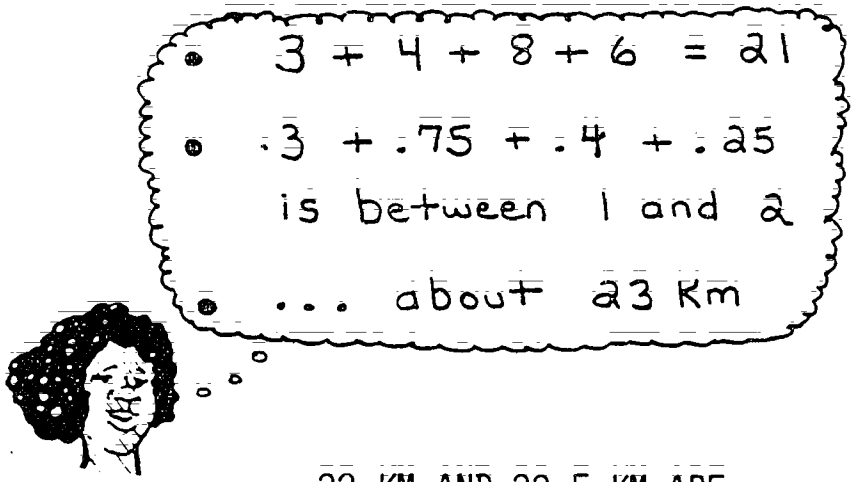
ESTIMATE: $4.50 _____

7-12-TR1

ESTIMATE WITH DECIMALS

There are many ways to estimate when you add or subtract decimals.

1. Use Front-End.



JOGGING CHART	
DAY	DISTANCE
MONDAY	3.3 KM
TUESDAY	4.75 KM
WEDNESDAY	8.4 KM
THURSDAY	6.25 KM

22 KM AND 22.5 KM ARE ALSO GOOD ESTIMATES.

TOTAL: ? KM

2. Use Rounding.

3 5 8 6

3.3 + 4.75 + 8.4 + 6.25

ABOUT: 22 KM

TRY THESE:

16.8 - 7.43

4.62 + 0.6 + 12.1 + 1.85

7 - 5.17

9.9 + 3 + 2.1 + 3.45

7-12-TR2

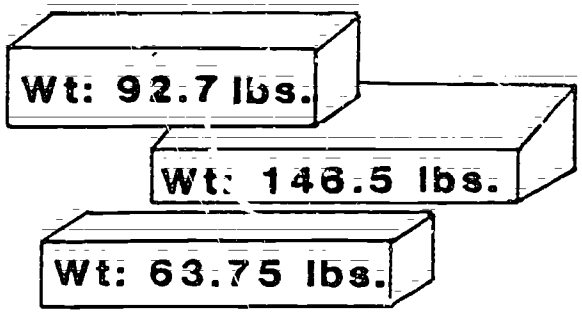
ESTIMATING WITH LARGER NUMBERS

3. Use numbers that are easy to add or subtract in your head.

- 92.7 + 146.5 + 63.75

100 + 150 + 60

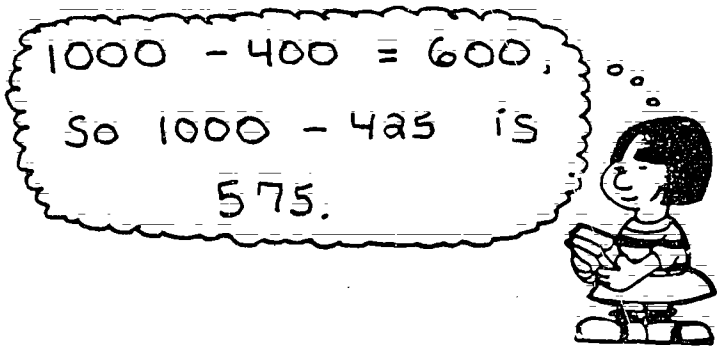
ESTIMATE: 310 LBS.



- 985.62 - 427.3

1000 - 425

ESTIMATE: 575



4. Look for compatible numbers.

- 14.27 + 6.83 + 34.6 + 7.2

About 20 About 40

ABOUT 60 IN ALL.

- 6.29 + 34.7 + 4.55 + 15.02

About 10 About 50

ABOUT 60 IN ALL.

7-12-TR3

ADJUSTING ESTIMATES

5. Adjust estimates when you can.

0.89 + 0.92

ESTIMATE: 2^-

19.3 + 4.932

ESTIMATE: 24^+

47.37 − 21.9

ESTIMATE: 27^-

Each addend is less than 1, so the sum is less than 2.

19 + 4 = 23
.3 + .932 > 1
so 24^+

47 − 20 = 27, so
47 − 21 < 27

TRY THESE: ESTIMATE. ADJUST WHEN YOU CAN.

	ESTIMATE
639.7 + 9.78 + 4.2	
89.93 + 62.3 + 14.28	
376.52 − 181.7	
863.8 − 97.34	
1.27 + 8.33 + 14.9 + 25.1	
24.7 + 1.7 + 6.8 + 72.5	
(289.1 + 43.8) − 94.5	
149.321 + 61.3 + 39.483 + 0.027 + 48.37	

7-12-TR4

NAME _____

ESTIMATE WITH SMALLER NUMBERS

Use the Front-End method for 1-6. Write + or - to adjust your estimate when you can.

1. 2 + 6.8 + 1.7

Front-End Sum: _____

Final Estimate: _____

2. 0.02 + 1.1 + 7.39

Front-End Sum: _____

Final Estimate: _____

3. 16.265 + 1.9 + 2.82

Front-End Sum: _____

Final Estimate: _____

4. 4.2 - 1.67

Front-End Difference: _____

Final Estimate: _____

5. 1 - 10.47

Front-End Difference: _____

Final Estimate: _____

6. 19.65 - 6.1

Front-End Difference: _____

Final Estimate: _____

Estimate. Use Front-End or Rounding. Write + or - to adjust your estimate when you can.

7. 3.408 + 7.82 _____

8. 6.08 + 4 + 9.2 _____

9. 9.74 + 6 + 16.009 _____

10. 0.9 + 6 + 4.73 + 10.074 _____

11. 4.6 - 2.13 _____

12. 13 - 9.263 _____

13. 8.1 - 2.69 _____

14. 4.684 - 3.04 _____

15. 13.2 - 7.004 _____

16. 20.63 - 9.904 _____

Growth in Height in 1 Year

Barbara	2.6 cm
Bob	0.71 cm
Paul	1.26 cm
Judi	3.8 cm
Lynn	1.68 cm

17. Estimate the total growth of the boys. _____ cm

18. Estimate the total growth of the girls _____ cm

19. Estimate the difference in the total growth of boys and girls. _____ cm

20. Estimate the total growth of all five students. _____ cm

April Hiking Record

April 2	8.85 km
April 9	12.25 km
April 16	10.9 km
April 23	9.65 km
April 30	14.4 km

Jessica belonged to a hiking club. She kept a record of how far she hiked each Saturday in April.

21. About how many kilometers did she hike in April? _____ km

22. About how much farther did she hike on April 30th than April 2nd? _____ km

7-12-p.1

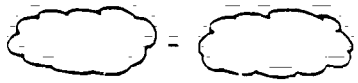
ESTIMATE WITH LARGER NUMBERS

Change each number to a number that is easy to work with.
Then estimate the sum or difference.

23. 62.35 + 51.6 + 9.28

Estimate: _____

24. 936.32 - 445.6

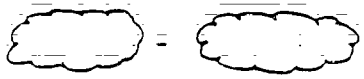


Estimate: _____

25. 243.8 + 22 + 3.695

Estimate: _____

26. 44.65 - 16.527



Estimate: _____

Pairs of compatible numbers have been marked for you.
Estimate each partial sum and then estimate the total.

27. 6.7 + 3.96 + 21.4 + 8.5

About _____ About _____

Estimate: _____

28. 342.1 + 22.7 + 30.08 + 96.7

About _____

About _____

Estimate: _____

Estimate.

29. 269.4 + 0.0047 + 18.99 _____

30. 24.2 + 16.2 + 5.7 _____

31. 497.2 + 1.6 + 237.41 _____

32. 264.3 + 14.86 + 35.2 + 85.31 _____

33. 486.09 + 149.3 + 204 _____

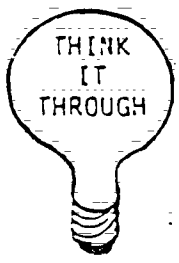
34. 8.97 + 214.6 + 41.34 + 462.7 _____

35. 239.02 - 127.9 _____

36. 437.06 - 198.35 _____

37. 647.4 - 452.36 _____

38. 371.84 - 12.93 _____



Estimate

1. (14.7 + 22.6) - 15.3 _____
2. (392.1 + 5.003) - (129.3 + 122.8) _____
3. (423.51 - 2.006) + 37.78 _____
4. (84.1 + 0.0083 + 37.56) ÷ 4 _____

NSF ESTIMATION
GRADE 7 - LESSON 13

OBJECTIVE: To estimate products and quotients with decimals.

TEACHER BACKGROUND:

To estimate for multiplication and divison with decimals, students apply strategies learned for whole numbers and fractions. The strategies that are emphasized are:

1. Multiply or Divide by 1, 10, 100 and 1000.

   395 x 0.97

   ESTIMATE: 395 x 1 = 395

   To use this strategy, students need to be able to recognize decimals close to 1. (0.97 is close to 1, while 0.097 is not close to 1).

   They also need to know how to multiply and divide a number by 10, 100, and 1000.

   Both of these prerequisites are reviewed in the lesson.

2. Front-End

   3.7 x 1.49 ESTIMATE: 6 or 6^-

   Change to 4 x 1.5.
   4 x 1 = 4 and
   4 x 0.5 = 2.

3. Rounding

   61.48 x 7.3 ESTIMATE: 60 x 7 = 420

   (Since both numbers were rounded down, 420 is too low. The estimate can be written as 420^+.)

   0.297 x 0.41 ESTIMATE: 0.3 x 0.4 = 0.12

4. Compatible Numbers

   127.5 ÷ 39.96 ESTIMATE: 120 ÷ 40 = 3

5. Compare the Quotient to 1.

   3.5 ÷ 5.21 ESTIMATE: Less Than 1

   3.5 is less than 5.21.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 encourages students to be flexible when estimating. The multiple step problems and real world setting require students to select numbers that can be handled easily and still produce a realistic estimate. Estimates will vary from "in the ballpark" to quite close.

ANSWERS: 1. $2.50 - $3.20 2. $1.00 - $1.60

3. $44.00 - $52.00 4. $0.40 - $0.45

5. Answers will vary.

PREREQUISITE SKILLS

TR #2: Two prerequisite skills are reviewed. Develop each one with students and do the TRY THESE exercises orally. For multiplying and dividing by powers of 10, give attention to examples where additional zeros must be written.

Examples: $61.3 \times 1000 = 61{,}300.$

$4.5 \div 100 = 0.045$

ANSWERS:				
	Set 1	1.062	0.897	0.9021
	Set 2	1360	24.696	3700

MULTIPLICATION AND DIVISION ESTIMATION

TR #3: Present and discuss the various strategies. Remind students
TR #4: that they have used the strategies with whole numbers or fractions. Point out that they need to be flexible when estimating and take a few moments to decide what is a good way to estimate for each problem.

TR #3

ANSWERS:			
	2.4^{+}	47.2^{-}	35.1^{-}
	395	90.84^{+}	4.762^{-}

TR #4

ANSWERS:				
	Set 1	24 - 28	0.27 - 0.32	1600
		9 - 10	4 - 5	20
	Set 2	Greater	Less	Greater
		Greater	Less	Greater

USING THE EXERCISES:

Go over the set of directions with the students. Some sets of exercises require the use of specific strategies.

ANSWERS:

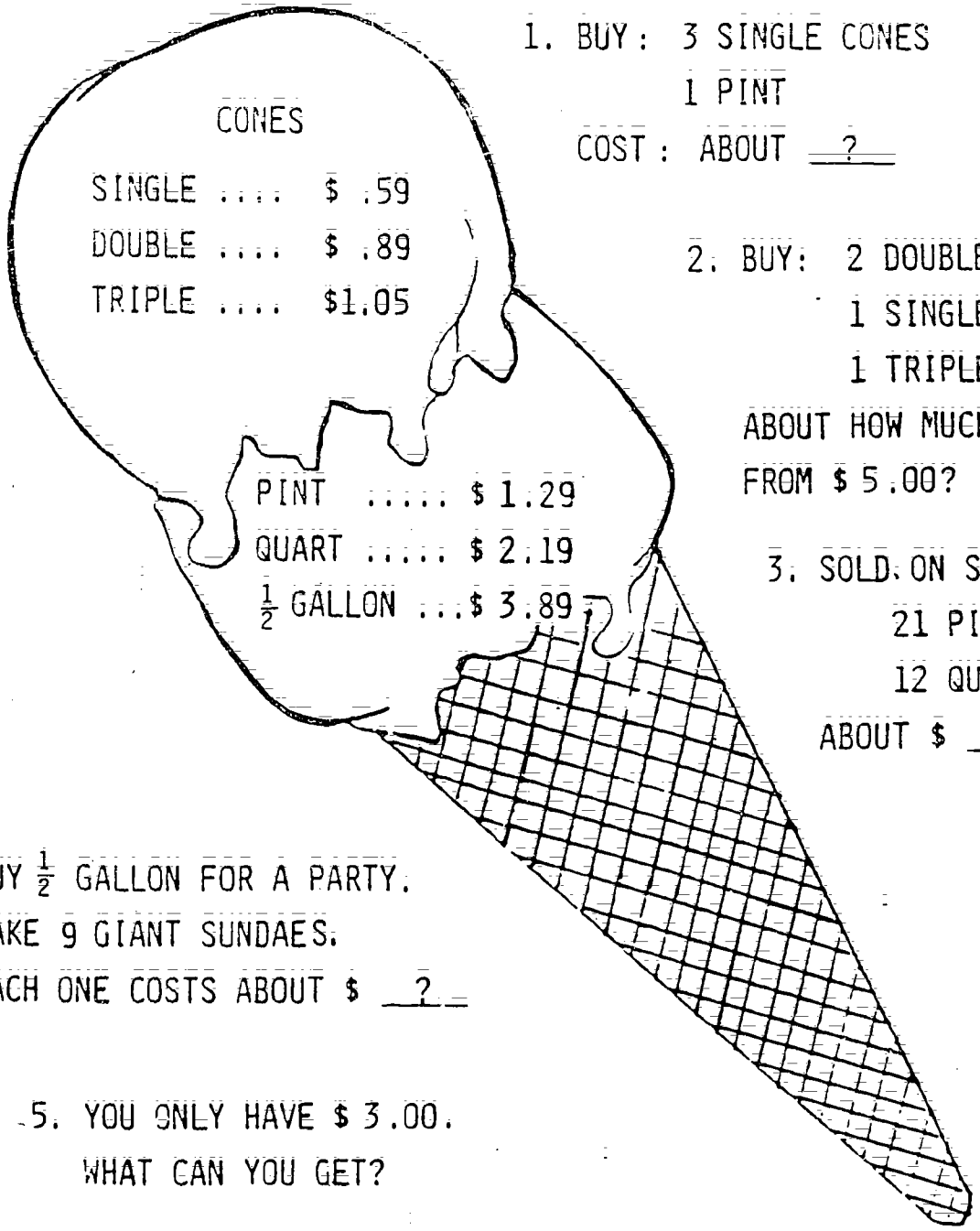
Top: 0.92 1.038 0.894 0.9002

1.	400 - 480	2.	800	3.	$6 = 8^-$
4.	0.2 - 0.24	5.	0.2 - 0.5	6.	800 - 1000
7.	9 - 10	8.	80 - 90	9.	4 - 5
10.	5 - 6	11.	7 - 8	12.	7 - 9
13.	4.67	14.	216	15.	41.628 (41.6)
16.	3.429^+ (3.4)	17.	$32{,}600^-$	18.	4.6374^+ (4.6)
19.	Less than 1	20.	Greater than 1	21.	Greater than 1
22.	Less than 1	23.	Less than 1	24.	Greater than 1
25.	54.37	26.	0.30	27.	6.49
28.	0.82	29.	0.88		
30.	can't tell (or, lower)	31.	lower	32.	higher
33.	lower				
34.	254^+	35.	342^-	36.	3600 - 3900 (3120^+)

GET YOUR MIND IN GEAR

MORE THAN ONE STEP

Ice Cream Shoppe



1. BUY: 3 SINGLE CONES
   1 PINT
   COST: ABOUT __?__

2. BUY: 2 DOUBLE CONES
   1 SINGLE CONE
   1 TRIPLE CONE
   ABOUT HOW MUCH CHANGE
   FROM $ 5.00?

3. SOLD ON SATURDAY:
   21 PINTS
   12 QUARTS
   ABOUT $ __?__ IN ALL?

4. BUY ½ GALLON FOR A PARTY.
   MAKE 9 GIANT SUNDAES.
   EACH ONE COSTS ABOUT $ __?__

5. YOU ONLY HAVE $ 3.00.
   WHAT CAN YOU GET?

7-13-TR1

SOME IMPORTANT DECIMAL SKILLS

- **Recognize decimals close to 1.**

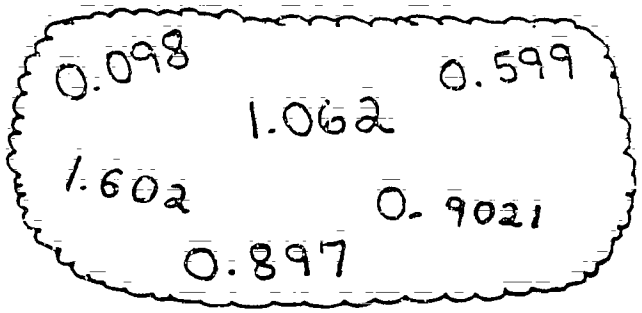
LOOK AT THE NUMBER OF TENTHS.

0.92 (9 tenths) 0.92 IS CLOSE TO 1.

0.09 (0 tenths) 0.09 IS NOT CLOSE TO 1.

TRY THESE:

TELL WHICH DECIMALS ARE CLOSE TO 1.



- **Multiply and divide by 10, 100, or 1000.**

$4.78 \times 100 = 478$

THE ANSWER WILL BE GREATER THAN 4.78. MOVE THE DECIMAL POINT TWO PLACES TO THE RIGHT.

$376.2 \div 10 = 37.62$

THE ANSWER WILL BE LESS THAN 376.2. MOVE THE DECIMAL POINT ONE PLACE TO THE LEFT.

TRY THESE:

13.6×100	$246.96 \div 10$	3.7×1000
$842 \div 100$	$6.35 \div 1000$	$892 \div 10$

7-13-TR2

MULTIPLICATION AND DIVISION ESTIMATION

1. Use 1, 10, 100 or 1000.

ROUND ONE NUMBER, THEN MULTIPLY OR DIVIDE.

100	10
32.8 X 97.76	89.6 ÷ 0.97
ESTIMATE: 3280	ESTIMATE: 89.6 OR 90

TRY THESE:

1.099 X 2.4 4.72 X 9.37 35.1 X 0.908

395.2 ÷ 0.97 908.4 ÷ 9.871 476.2 ÷ 102.87

2. Use numbers that are easy to compute in your head.

A. 3.7 X 1.49

4 X 1.5
4 x 1 = 4, and a half of 4 is 2. So, 0.5 x 4 = 2
I'll say about 6 in all.

B. 127.5 ÷ 39.36

That's close to 120 ÷ 40.
About 3.

C. 0.297 X 0.41

I'll use 0.3 x 0.4.
The answer is close to 0.12.

7-13-TR3

D. 41.6 ÷ 4.9

40 ÷ 5 is 8. So 8 is a good estimate.

TRY THESE:

3.46 X 7.9 　　0.89 X 0.32 　　39.8 X 42.72

81.9 ÷ 8.6 　　359.4 ÷ 86.7 　　647.2 ÷ 32.8

3. See if the quotient is greater or less than 1.

$4\overline{)3.1}$

3.1 is less than 4. So, the quotient is less than 1.

8 ÷ 7.2

8 is greater than 7.2. So, the quotient is greater than 1.

TRY THESE:

TELL WHETHER THE QUOTIENT IS GREATER OR LESS THAN 1.

$4.3\overline{)6.8}$ 　　$2\overline{)1.7}$ 　　7 ÷ 6

5 ÷ 3.7 　　$\frac{7}{8.9}$ 　　$\frac{63}{1.01}$

7-13-TR4

Name_____

Circle the decimals that are close to 1.

0.399	0.92	0.099
1.038	0.894	0.9002
0.198	.089	0.083

Estimate. Use numbers that are easy to multiply mentally.

1. 4.3 x 119.2 Estimate:_____

2. 38.9 x 21.72 Estimate:_____

3. 3.91 x 1.68 Estimate:_____

4. 0.426 x 0.597 Estimate:_____

5. 2.42 x 0.192 Estimate:_____

6. 4.296 x 243.5 Estimate:_____

Estimate. Use numbers that are easy to divide mentally.

7. 28.9 ÷ 2.8 Estimate:_____

8. 638.7 ÷ 7.92 Estimate:_____

9. 246.41 ÷ 52.7 Estimate:_____

10. 17.934 ÷ 3.29 Estimate:_____

11. $\frac{423.1}{57.5}$ Estimate:_____

12. $\frac{27.5}{3.623}$ Estimate:_____

Estimate. Change one number to 1, 10, 100 or 1000.

13. 4.67 ÷ 0.982 Estimate:_____

14. 9.148 x 21.6 Estimate:_____

15. 416.28 ÷ 11.07 Estimate:_____

16. 342.9 ÷ 97.68 Estimate:_____

17. 32.6 x 968.1 Estimate:_____

18. $\frac{4637.4}{958.92}$ Estimate:_____

Name_____

Circle the examples where the quotient is greater than 1.

19. $3.6 / \overline{2.54}$

20. $4 / \overline{5.13}$

21. $3.9 \div 2.78$

22. $16.13 \div 20.4$

23. $\frac{5.3}{5.892}$

24. $\frac{3.14}{0.8}$

Estimate to find the most sensible answer.

25. 3.17×17.15	5.44	54.37	543.66
26. 0.624×0.482	0.30	3.01	30.08
27. 3.009×2.156	6.49	64.87	648.74
28. $2.6 \div 3.17$	0.82	8.2	82.02
29. $4.68 \div 5.3$	0.88	8.83	88.3

Circle whether the estimate is higher or lower than the exact answer. If you cannot tell, circle "can't tell".

PROBLEM	ESTIMATE	THE ESTIMATE IS ? THAN THE EXACT ANSWER		
30. 3.17×17.5	$3 \times 20 = 60$	higher	lower	can't tell
31. 42.3×413.6	$40 \times 400 = 16{,}000$	higher	lower	can't tell
32. 68.2×1.98	$70 \times 2 = 140$	higher	lower	can't tell
33. 10.096×37.4	$10 \times 37.4 = 374$	higher	lower	can't tell

The following chart gives the density for some elements. Density is the number of grams per cubic centimeter. Estimate the total mass of each element.

ELEMENT	DENSITY (grams per cm^3)	VOLUME	TOTAL MASS (density x volume)
34. Neon	1.21 g/cm^3	254 cm^3	
35. Sodium	0.97 g/cm^3	342 cm^3	
36. Bromine	3.12 g/cm^3	1265 cm^3	

NSF ESTIMATION
GRADE 7 - LESSON 14

OBJECTIVE: To estimate using 1%, 10% and 100% of a number.

TEACHER BACKGROUND:

Three examples of estimating using 1%, 10% and 100% of a number are shown at the right. This is a highly useful estimation tool and strengthens understanding of finding a per cent of a number.

100% of $2518 = $2518

97.8% of $251

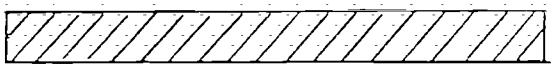
10% of $6.75 is about $0.68

9.2% of $6.75

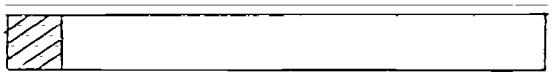
1% of $525 = $5.25

1.2% of $525

To use this strategy students need to understand the meaning of these percents and be able to find 1%, 10% and 100% of a number mentally.

a. 

100% is the Whole

b. 

10% is 1-tenth of the whole

So, to find 10% divide by 10.

c. 

1% is 1-hundredth of the whole

So, to find 1%, divide by 100.

To estimate, students must recognize that a number is close to 1%, 10% and 100% and be able to do the appropriate computation mentally.

Two other ideas are also stressed in the lesson.

1. Adjusting: The adjusting is of two types. First, when finding 10% of $.65, the exact answer is $.065. However, it is often more useful to give the answer as $.06 or $.07. Thus as students estimate, have them give their estimates in round numbers.

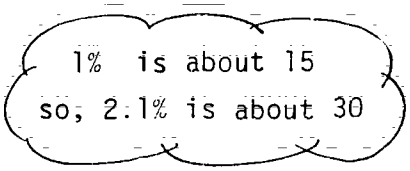
Second, students can adjust estimates by indicating whether they are overestimates or underestimates. Thus, the estimate for 9.5% of $250 can be written as $25-.

2. Estimate 2% and 20% of a number.

Here, the basic thrust of the lesson is extended. The student first estimates 1% and then doubles it to estimate 2%.

Students should be exposed to this extension of the basic lesson, but should not be required to master it at this time.

Estimate: 2.1% of 1450



TEACHING THE LESSON:

GET YOUR MIND IN GEAR

The emphasis on TR #1 is again sensible answers. Point out to students that knowing if the amount charged is reasonable is one important use of estimation skill. Have them select the sensible amount for each problem.

ANSWERS: $5 $2 $12 $12

FIND 1%, 10% AND 100%

TR #2: Develop the meaning of these per cents and the mental shortcuts for computing them. Do the TRY THESE exercises orally.

ANSWERS:			
	450	86.9	3845
	45	8.69	384.5
	4.5	0.869	38.45
	65	2.6	750
	0.28	385	58.9

ESTIMATE AND ADJUST WITH 1%, 10%, 100%

TR #3: Develop the three situations involving 1%, 10%, and 100%. Give attention to the adjusting. Point out that since they are estimating, it is more reasonable to give an estimate as $6.30, rather than $6.29. They also can consider adjusting by increasing or decreasing the estimate.

TR #4: Students are given practice in estimating using real world situations. Discuss the problems with students.

ANSWERS:		
	$1400 - $1560	12,000 - 12,500
	$.85 - $1.00	$850 - $1000

USING THE EXERCISES:

Page 1 contains the exercises that are basic to the lesson. Go over the directions for exercises #9 - 16 with students. Page 2 works with estimating 2% and 20%. These exercises can be done orally if you wish. The THINK IT THROUGH exercises at the bottom of page 2 are intended to provide an introduction to a useful estimation skill for more capable students.

ANSWERS:

1. 700, 70, 7
2. 400, 40, 4
3. 350, 35, 3.5
4. 670, 67, 6.7
5. 285, 28.5, 2.85
6. 162, 16.2, 1.62
7. 59, 5.9, .59
8. 34, 3.4, .34

9. $.55
10. $540
11. $.50
12. $8.50
13. $32.58
14. $.16
15. $6429
16. $.30
17. 61
18. 25
19. 1430
20. 5
21. 0.15
22. 345 = 350
23. 4 - 5
24. 33 -35
25. $.24 - $.26
26. $.38 = $.40
27. $.46 - $.50
28. $.14 - $.15
29. $7.60 - $8.00
30. $5.90 - $6.00
31. $42.00 - $44.00
32. $13.00 - $14.00

THINK IT THROUGH

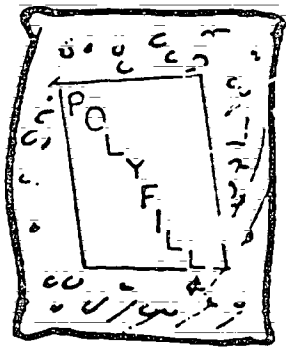
1. $.60 - $.70
2. $1.20 = $1.50
3. $2.00 - $2.50
4. $6.00 = $7.00

GET YOUR MIND IN GEAR

RECOGNIZE SENSIBLE ANSWERS

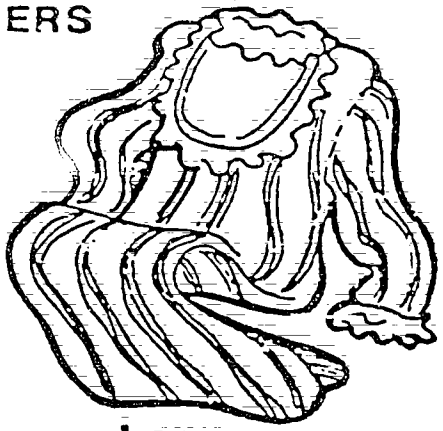
Material

$1.89 PER YARD



Stuffing

49¢ PER OUNCE



Lace

$2.29 PER YARD

Circle the sensible amounts.

BUY $2\frac{5}{8}$ YARDS OF MATERIAL.

$2 $5 $9

BUY $\frac{7}{8}$ YARDS OF LACE.

$2 $9 $14

BUY 24 OUNCES OF STUFFING.

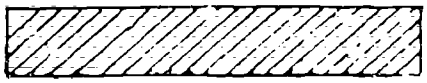
$0.80 $8.00 $12.00

BUY $6\frac{3}{8}$ YARDS OF MATERIAL.

$6 $9 $12

FIND 1%, 10%, 100%

100% is the Whole Thing:



SO, 100% OF 85.2 IS 85.2

10% is 1-tenth of the whole:

So, to find 10% ... divide by 10

10% OF 64.7 = 6.47

1% is 1-hundredth of the whole:

So, to find 1% ... divide by 100

1% OF 64.7 = 0.647

TRY THESE:

100% OF 450	100% OF 86.9	100% OF 3845
10% OF 450	10% OF 86.9	10% OF 3845
1% OF 450	1% OF 86.9	1% OF 3845
10% OF 650	1% OF 260	100% OF 750
1% OF 28	100% OF 385	10% OF 589

7-14-TR2

ESTIMATE AND ADJUST WITH 1%, 10%, 100%

.9% of $629

1% OF $629

ESTIMATE: $6.29 OR $6.30

ADJUST: $6.30⁻ OR $6.00

97.8% of 2518

100% OF 2518

ESTIMATE: 2518 OR 2520

ADJUST: 2500⁻ OR 2450

11.5% of $6.70

10% OF $6.70

ESTIMATE: $.67 OR $.70

ADJUST: $.70⁺ OR $.75

ESTIMATE 2% and 20%

2.1% of 1450

1% OF 1450 IS $14.50
SO 2% IS TWICE AS MUCH.

ESTIMATE: 29 OR 30

19.5% of $440

10% OF $440 IS $44
SO 20% IS TWICE AS MUCH

ESTIMATE: $88 OR $88⁻

7-14-TR3

ESTIMATING REAL WORLD PROBLEMS

in the news

Teachers Get 9.4% Pay Raise

THE AVERAGE PAY FOR THESE TEACHERS IS $15,600. ABOUT HOW MUCH IS THE AVERAGE RAISE?

Poll Says 97.6% Support the Mayor

THERE ARE 12,500 PEOPLE IN THE TOWN. ABOUT HOW MANY SUPPORT THE MAYOR?

Food Prices Down 1.2% in June

AN AVERAGE FAMILY OF 4 SPENDS $85.00 ON FOOD PER WEEK. ABOUT HOW MUCH WILL EACH FAMILY SAVE PER WEEK?

Car Prices Rise 11.3% This Year

THE AVERAGE PRICE OF A CAR LAST YEAR WAS $8,500. ABOUT HOW MUCH IS THE PRICE INCREASING?

7-14-TR4

Name_____

Find 1%, 10% and 100% of each number.

	100%	10%	1%
1. 700			
2. 400			
3. 350			
4. 670			

	100%	10%	1%
5. 285			
6. 162			
7. 59			
8. 34			

Find 1%, 10% or 100% of each number. Give the answer to the next higher cent.

Ex. 10% of $84.95: $8.50

9. 10% of $5.49 _____
10. 100% of $540 _____
11. 1% of $49.98 _____
12. 1% of $849.30 _____
13. 10% of $325.75 _____
14. 1% of $15.50 _____
15. 100% of $6429 _____
16. 10% of $ 2.98 _____

Choose the best estimate.

17. 9% of 670 a. 610 b. 61 c. 6.1
18. 11% of 2500 a. 2500 b. 250 c. 25
19. 98.9% of 1450 a. 1430 b. 143 c. 14.3
20. 11% of 49 a. 50 b. 5 c. 0.5
21. 0.9% of 16.25 a. 15 b. 1.5 c. .15

Estimate.

22. There are 350 students in Hawthorne School One day 99% of the students were present. About how many students were at school? _____

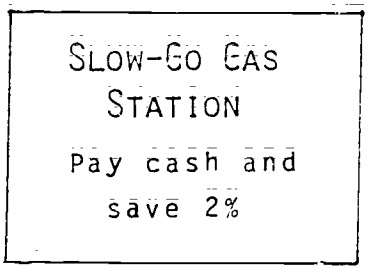
23. 1.2% of the 350 students forgot to bring their lunches. About how many students was this? _____

24. On a snowy day, 9.7% of the 350 students were absent. How many students were absent? _____

Name _____

Use 1% to help you estimate.
Use nice numbers to work with.

Ex. 2% of $15.25
1% is about $.15
so 2% is about $.30



25. $12.75
Estimate: _____

26. $19.89
Estimate: _____

27. $23.42
Estimate: _____

28. $ .73
Estimate: _____

Use 10% to help you estimate.
Use nice numbers.

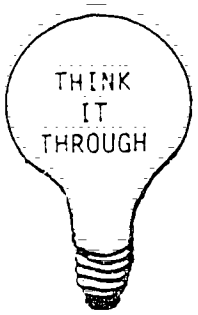
Ex. 20% of $23.75
10% is about $2.40
so 20% is about $4.80

29. Shoes: Were: $ 38.00
Save: _____

30. Sweaters: Were: $ 29.95
Save: _____

31. Suits: Were: $219.20
Save: _____

32. Dresses Were: $ 68.50
Save: _____



Total: $7.84
10% of $7.84 is about $.80
5% of $7.84 is half of that.
so a 15% tip is $.80 + $.40 or $1.20

Estimate a 15% tip.

		Bill	Tip
1.	Breakfast	$ 4.32	_____
2.	Lunch	$ 8.95	_____
3.	Dinner	$14.85	_____
4.	Dinner Party	$43.75	_____

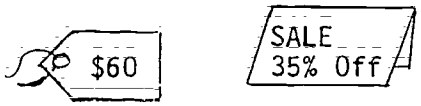
NSF ESTIMATION
GRADE 7 - LESSON 15

OBJECTIVE: To estimate the percent of a number for commonly used percents.

TEACHER BACKGROUND:

This lesson builds on the previous one, extending the work to 25%. 33 $1/3$ % and 50%. It involves changing percents to equivalent or nearly equivalent fractions. Students then use compatible numbers to obtain an estimate.

At the conclusion of the lesson, students are introduced to estimating the per cent saved. For example, \$5 off from an original price of \$21 is about $1/4$ or 25%. This work builds on the fraction work of Lesson 9. Estimating the percent should be viewed as exploratory work only.



How much will you save ?

a. 35% is about $\frac{1}{3}$

b. $\frac{1}{3}$ of \$60 = \$20

ESTIMATE: \$20, or a little more since 35% is more than $1/3$.

TEACHING THE LESSON:

GET YOUR MIND IN GEAR

TR #1 stresses flexibility in estimation thinking by showing a variety of approaches that can be used. For each situation, three thinking strategies are displayed. Have students tell what the person did in each case. At the bottom, students get additional experience in adjusting an estimate.

ESTIMATE THE PERCENT OF A NUMBER

TR #2: Review the basic equivalents at the top of TR #2. Then show how they can be used to find the part. Do the TRY THESE exercises with students.

ANSWERS:		
	20	15
	22	6
	24	40
	100	220
	9	12

TR #3: Here, the emphasis is on estimating. Develop the two examples at the top of TR #3. Point out that first, a rough fractional equivalent is found for the percent. Then, the whole is changed to a compatible number.

Do the TRY THESE exercises with students.

ANSWERS:	$1/5$	$1/3$	$1/4$	$1/10$	$1/2$
	$35 - $45		$12 - $13		
	19 - 22		65 - 70		11 - 13
	57 - 60		25 - 30		13 - 15

TR #4: Here students do exploratory work with estimating the percent saved. Discuss the two examples at the top, then do the TRY THESE exercises together. Students should look for a familiar fraction and then give the percent for that fraction.

ANSWERS:	10%	$33\ 1/3$ %
	20%	$33\ 1/3$ %

USING THE EXERCISES:

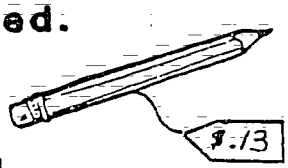
The exercises cover the work of lessons 14 and 15. Do the first exercise in each set with students. The THINK IT THROUGH exercises should be reserved for more capable students.

ANSWERS:

1. 2.346	2. 4.27	3. 84.2
4. 4.67	5. 46.82	6. 9.256
7. 9	8. 15	9. .125
10. 35	11. 200	12. 105
13. $33\ 1/3$ %	14. 10%	15. 50%
16. 1%	17. 25% of 160	18. $33\ 1/3$ % of 120
19. 25% of 48 or 20% of 50	20. 50% of 36 (40)	21. 10%
22. 73 - 78	23. 3.0 - 3.3	24. 22 - 24
25. 10 - 12	26. 10 - 12	27. 2.6 - 3.0
28. 5 - 6	29. 45 - 50	30. 60 - 65
31. 0.6 - 0.7	32. 24 - 25	33. 370 - 400
34. $210 - $220	35. $.28 - $.30	36. $7.90 - $8.00
37. $8 - $9	38. $.35 - $.50	39. $24 - $27
40. $2.40 - $2.50	41. $.55 - $.60	42. $12 - $13
THINK IT THROUGH	$12.00	$195 - $200
	$24.00	$585 - $600

GET YOUR MIND IN GEAR

here are many ways to estimate.
ell how each person estimated.



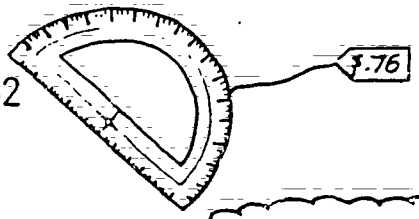
ABOUT HOW MUCH WILL
48 PENCILS COST?

$\frac{1}{8}$ OF 48 = 6

So $6.00

100 x $.13 = $13.00
So, 50 x $.13 = $6.50

$.10 x 50 = $5.00



ABOUT HOW MUCH WILL 42
PROTRACTORS COST?

$\frac{1}{4}$ OF 40 IS 10
So, $\frac{3}{4}$ OF 40 IS 30
$30.00

4 x $.75 = $3.00
So, 40 x $.75 = $30.00

80 x 40 = 3200
So, $32.00

IF 40 PROTRACTORS COST ABOUT $30.00,
ABOUT HOW MUCH WILL 42 COST?

7-15-TR1

USE FRACTIONS TO FIND A PART

$10\% = \frac{1}{10}$ $25\% = \frac{1}{4}$

$33\frac{1}{3}\% = \frac{1}{3}$ $20\% = \frac{1}{5}$ $50\% = \frac{1}{2}$

TO ESTIMATE THE SAVINGS ON THE COAT:

Think: $33\frac{1}{3}\%$ of \$120
$\frac{1}{3}$ of 120
\$40 off

Try these!

$\frac{1}{4}$

25% of 80

$33\frac{1}{3}\%$ of 66

50% of 48

$33\frac{1}{3}\%$ of 300

25% of 36

$\frac{1}{2}$

50% of 30

25% of 24

25% of 160

50% of 440

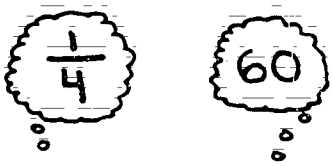
$33\frac{1}{3}\%$ of 36

7-15-TR2

ESTIMATE THE PART

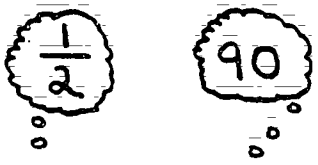
Use a "nice" fraction for the percent, then estimate the part.

TELL HOW THEY ESTIMATED.



24% of 62

ESTIMATE: 15



49% of 89

ESTIMATE: 45

TRY THESE:

TELL THE FRACTION YOU SHOULD USE.

19% 34% 26% 12% 47%

ESTIMATE.

36% OF 129

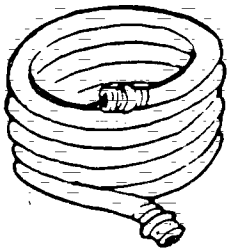
$24\frac{1}{2}$% OF 51

31% OF 65 54% OF 133 27% OF 45

48% OF 119 19% OF $148\frac{1}{3}$ 23.4% OF 61.3

7-15-TR3

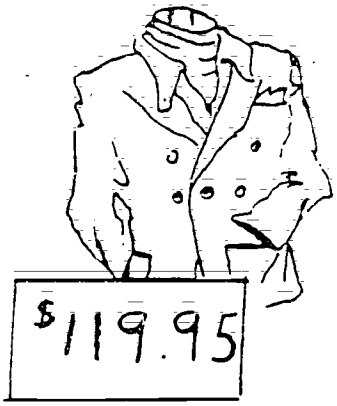
WHAT PERCENT DO YOU SAVE?



WAS: $21
NOW: $5 OFF
% SAVED: ____ %

$\frac{5}{21}$ is about $\frac{1}{4}$
and $\frac{1}{4}$ is 25%

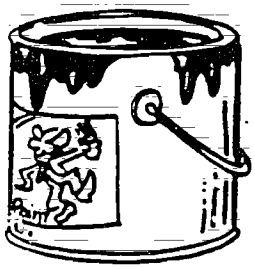
WAS: $119
NOW: $40 OFF
% SAVED: ____ %



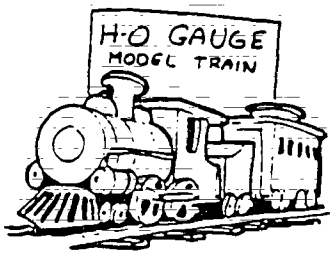
$\frac{40}{119} \approx \frac{40}{120}$
or about $\frac{1}{3}$

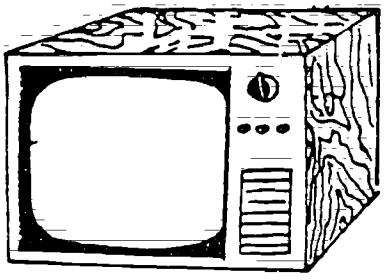
TRY THESE:

WAS: $19
NOW: $2 OFF
% SAVED: ____ %



WAS: $78
NOW: $24 OFF
% SAVED: ____ %





WAS: $498
NOW SAVE $95
% SAVED: ____ %

WAS: $59.95
NOW SAVE $19.95
% SAVED: ____ %

7-15-TR4

NAME _____

Compute the exact answer in your head.

1. 1% of 234.6 = _____
2. 1% of 427 = _____
3. 10% of 342 = _____
4. 10% of 46.7 = _____
5. 1% of 4682 = _____
6. 10% of 92.56 = _____
7. 25% of 36 = _____
8. $33\frac{1}{3}$% of 45 = _____
9. 50% of 250 = _____
10. 50% of 70 = _____
11. $33\frac{1}{3}$% of 600 = _____
12. 25% of 420 = _____

Rewrite each example by changing the percents and/or numbers to ones easy to compute mentally.

13. 34.7% of 60
_____ of 60

14. 9.5% of 120
_____ of 120

15. 48½% of 250
_____ of 250

16. 1½% of 27
_____ of 27

17. 26.7% of 158.92
_____ of _____

18. 31% of 118.6
_____ of _____

19. $22\frac{3}{4}$% of 47.6
_____ of _____

20. 52.5% of 37
_____ of _____

21. 11.5% of 134.62
_____ of 134.62

Change one or both numbers in your head to estimate each problem.

22. $3\frac{1}{3}$% of 147.3
Estimate: _____

23. 0.92% of 326.4
Estimate: _____

24. 35% of 65.9
Estimate: _____

25. 11% of 106.2
Estimate: _____

26. 22.9% of 49½
Estimate: _____

27. 9.86% of 27
Estimate: _____

28. 32½% of 16.8
Estimate: _____

29. 1.1% of 4560
Estimate: _____

30. 27.3% of 237.5
Estimate: _____

31. $\frac{7}{3}$% of 67
Estimate: _____

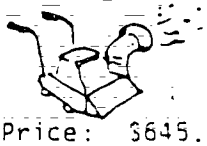
32. 33% of 72.8
Estimate: _____

33. 47.6% of 769
Estimate: _____

COUPON CORNER

Estimate the savings on each item.

34. SNOW BLOWER



Reg. Price: $645.89

35% OFF!

Estimate: _____

35. NOTEBOOK PAPER

Reg. Price: $2.89

10% OFF!

Estimate: _____

36. BASKET BALL

Reg. Price: $15.95

50% OFF!

Estimate: _____

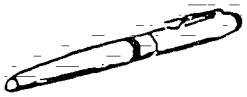
37. 14K GOLD NECKLACE

Reg. Price: $25.89

$33\frac{1}{3}$% OFF!

Estimate: _____

38. FELT TIP PEN



Reg. Price: $1.49

25% OFF!

Estimate: _____

39. ALUMINUM COOKWARE SET

Reg. Price: $79.99

30% OFF!

Estimate: _____

40. FOLDING CHAIR

Reg. Price: $24.86

10% OFF!

Estimate: _____

41. COFFEE MUG

Reg. Price: $2.39

25% OFF!

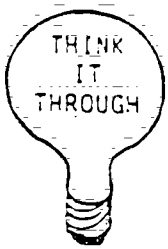
Estimate: _____

42. MEN'S SWEATER

Reg. Price: $37.89

35% OFF!

Estimate: _____



When you save 25% on an item, you pay the remaining 75%.

Here is an example of estimating how much you SAVE and PAY!

Example: COAT $90 25% OFF

Save: $\frac{1}{4}$ of 90 ≈ $22

Pay: $\frac{3}{4}$ of 90 ≈ $66

22 x 3

BLOUSE: $36.95 Save $33\frac{1}{3}$%

Save: $\frac{1}{3}$ of $36.95 ≈ _____

Pay: $\frac{2}{3}$ of $36.95 ≈ _____

x 2

SOFA: $789 Save 25%

Save: $\frac{1}{4}$ of $789 ≈ _____

Pay: $\frac{3}{4}$ of $789 ≈ _____