DOCUMENT RESUME

ED 241 353 SE 044 290

AUTHOR Tobin, Catherine D.
TITLE hm Math Study Skills Program. Teacher's Guide.
INSTITUTION National Association of Secondary School Principals, Reston, Va.; National Science Teachers Association, Washington, D.C.
REPORT NO ISBN-0-88210-124-2
PUB DATE 80
NOTE 66p.; Developed by the Study Skills Group.
AVAILABLE FROM National Association of Secondary School Principals, 1904 Association Drive, Reston, VA 22091 (Item #3308103, $3.50).
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052)

EDRS PRICE MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS Estimation (Mathematics); Junior High Schools; Listening Skills; Mathematics Education; *Mathematics Instruction; *Mathematics Skills; Problem Solving; *Secondary School Mathematics; *Skill Development; *Study Skills; Teaching Methods; *Test Wiseness; Units of Study

ABSTRACT

This program, which provides an introduction to study skills in mathematics for pre-algebra students through a series of 10 activity-oriented units, is structured on the assumption that activity-oriented lessons are the most effective way to teach study skills. By completing activities in the units, students learn about study skills needed for mathematics and practice those skills in a mathematics environment. The program addresses a wide range of student needs, providing an introduction to specific skills for students who have little sense of a particular study skill, providing a learning experience of initial mastery for students who are ready to acquire a skill, and offering review and reinforcement for students who have mastered a given skill. Topic areas of the units include: listening as a mathematical skill, problem solving, understanding the language of mathematics, learning from homework, using formulas, estimation, preparing for a test, and taking a mathematics test. This guide contains strategies for teaching these units, suggestions for further instruction in each study skill area, and an overview of study skills. It is strongly recommended that the units be taught within the context of an ongoing mathematics course rather than in separate settings. (JN)

math study skills program

TEACHER'S GUIDE

nassp

THIS BOOK IS THE PROPERTY OF:

STATE _____
COUNTY _____
PARISH _____
SCHOOL DISTRICT _____
OTHER _____

Book No. _____
Enter information in spaces to the left as instructed

ISSUED TO	Year Used	CONDITION	
		ISSUED	RETURNED

PUPILS to whom this textbook is issued must not write on any page or mark any part of it in any way, consumable textbooks excepted.

1 Teachers should see that the pupil's name is clearly written in ink in the spaces above in every book issued

2 The following terms should be used in recording the condition of the book: New, Good, Fair, Poor, Bad

MATH STUDY SKILLS PROGRAM

Developed by The Study Skills Group

Author: Catherine D. Tobin

Senior Editor: David Marshak
Bureau of Study Counsel
Harvard University

Editors: Kiyo Morimoto, *Director*
Bureau of Study Counsel
Harvard University

Jerome Pieh, *Headmaster*
Milton Academy

James J. McGuinn

National Council of Teachers of Mathematics
Reston, Virginia 22091

The National Association of Secondary School Principals
Reston, Virginia 22091

ISBN 0-88210-124-2

TABLE OF CONTENTS

Introduction

Study skills in mathematics are essential for effective learning. They include a wide array of procedures or methods which are necessary for good academic performance. Study skills help the student make good use of classroom and study time.

Mathematics has its own vocabulary, notation, and procedures. Just as working with integers is a learned skill, so are the abilities to listen, problem solve, locate information, and take a test. We need to take the responsibility for teaching math study skills because they are critical to our students' success in learning.

Many students who are doing well in mathematics can develop more efficient study skills. Others who are doing poorly often think they do not have the necessary "math ability". In these cases, the problem is often the combination of a lack of skills with which to begin or analyze a problem and a minimum of self confidence. When students learn effective study skills, they become both more competent and more confident as problem solvers and learners.

Study Skills in Mathematics: What Are They?

Study skills are procedures for acquiring knowledge and competence. In mathematics they involve such skills as: identifying missing information; organizing materials and time; estimating; and understanding vocabulary.

In a larger sense study skills are strategies and methods for solving problems of any sort. The student who gains mastery of mathematics study skills is really discovering more about how to learn effectively in any situation.

Study Skills and Independent Learning

When students develop a repertoire of study skills, they become more independent as learners. For example, many students who can follow a set of directions when the teacher is present experience great difficulty when they must work on their own, either in school or at home. Students who know how to follow directions, use resources, develop a study plan, and analyze problems will be more successful as independent learners.

Some students have begun to develop study skills. Yet they have not learned to organize these skills into an efficient system. Many students lack any proficiency in study skills and need to learn both the skills and ways of using them systematically.

Study Skills And Basic Skills

Study skills are basic skills. The acquisition of a repertoire of effective study skills is a necessity for a good basic skills education.

The development of basic skills in this Program centers around the ten basic skill areas as identified by the National Council of Supervisors of Mathematics:

problem solving
applying mathematics to everyday situations
alertness to the reasonableness of results
estimation and approximation
appropriate computational skills
geometry
measurement
reading, interpreting, and constructing tables, charts, and graphs
using mathematics to predict
computer literacy

Study Skills And Problem Solving

"Priority in classroom time should be devoted to involving students in meaningful problem solving activities."[1] Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. This process provides the foundation for the development of study skills. Students learn these skills best when they have the opportunity to try a variety of study or learning strategies, decide which ones are most effective for them, and gradually refine these study strategies into an efficient study system.

1. *An Agenda for Action: Recommendations for School Mathematics of the 1980s.* National Council of Teachers of Mathematics: Reston, Virginia, 1980.

THE hm MATH STUDY SKILLS PROGRAM

The *hm Math Study Skills Program* is designed to provide *an introduction to study skills in mathematics* for pre-Algebra students through a series of ten activity-oriented units. Some of the units will require one period of class time. Others will need two periods.

The hm Program is structured on the assumption that activity-oriented lessons are the most effective way to teach study skills; more succinctly, that "learning by doing" is the best way to master study skills. In the activities in the hm Program, students will both learn about study skills needed for mathematics and will practice the skills in a mathematics environment.

The Program is deliberately designed to address a wide range of student needs:

1. For the student who has little sense of a particular study skill, it provides an introduction to that skill.
2. For the student who is ready to acquire a skill, it provides a learning experience of initial mastery.
3. For the student who has mastered a skill, participation offers review and reinforcement.

Thus, the Program's exercises allow for the participation of students with a diversity of skills and promote learning on various levels of competence.

The *hm Math Study Skills Program* will provide you with a focus on the nature and value of study skills in mathematics. It will give you ten initial units for teaching study skills and suggestions for further instruction in each study skill area.

The units in the *hm Math Study Skills Program* include:

I. Listening Is A Mathematical Skill
II. Problem Solving (1)
III. Understanding The Language Of Mathematics (1)
IV. Understanding The Language Of Mathematics (2)
V. Problem Solving (2)
VI. Learning From Your Homework
VII. Using Formulas
VIII. Estimation
IX. Preparing For A Test
X. Taking A Math Test

SUGGESTIONS FOR TEACHING THE hm MATH STUDY SKILLS PROGRAM

Suggested Directions

1. The Teacher's Guide offers "Suggested Directions" for teaching each unit in the Program. Our classroom testing has shown these methods to be useful. Of course, we invite you to adapt them in ways which you see fit.
2. We suggest that you examine both the Student Text and the Teacher's Guide carefully prior to your teaching of the various units.
3. Unit IX (Preparing For A Test) can be used most effectively when a test is imminent in your class. This unit as well as Unit X (Taking A Math Test) can follow your classroom sequence rather than the order of units in the Program. We suggest that Unit X *not* be studied in preparation for the same test as Unit IX but in conjunction with a later testing situation.
4. Student use of a calculator is suggested for some of the units, as follows: Unit II: Problem Solving (1); Unit V: Problem Solving (2); Unit VII: Using Formulas; and Unit VIII: Estimation. You may want to have your students use calculators for other units as well.

Suggested Times

1. The units in this Program are structured to accommodate the teacher who meets with his or her class for one period in a given day.
2. Each section of the "Suggested Directions" in the Teacher's Guide includes approximate times for the various parts of the units. We caution you that these times are approximates based on a variety of classroom testing of the material. All of the units range from 45–70 minutes with one exception:

   > Unit VII (Estimation) will take two full class periods. You may wish to teach this unit on successive days or teach the second half after working with Unit IX.

   Our classroom testing experience has shown us that the wide variation in teaching style results in an equally wide range in the pacing of instruction. We strongly suggest that you examine the Program units carefully and gauge your planning of instructional time according to your knowledge of how things actually work in your classroom.

Unit Summaries

1. Each unit includes a summary as its final page. While the use of the summaries has not been integrated formally into the suggested directions for instruction, we recommend that you bring the summaries to the attention of your students.
2. You will also find each summary included with its unit in the Teacher's Guide.

Additional Suggestions

1. Students will need additional activities beyond the scope of this Program to practice and reinforce their acquisition of math study skills. Some ideas for the extension of the various units are included with each section of this Teacher's Guide and are entitled "Additional Suggestions."

2. You will find a selective bibliography about problem solving at the end of Unit II in this Guide.

Grading

1. Given the grade-oriented reality of most classrooms, we suggest that the student's involvement with the hm Program be evaluated in some fair and concrete manner. Your standard of evaluation ought to keep in mind the process of how skills are learned, namely through repeated practice over time, and set reasonable levels of expected mastery. We also suggest that you inform your students about how their work with the hm Program will be evaluated at the very beginning of their use of the Program.

USING THE hm MATH STUDY SKILLS PROGRAM: WHERE AND WHEN

We strongly suggest that the *hm Math Study Skills Program* be taught within the context of an ongoing mathematics course rather than in any separate setting. Only in the regular mathematics classroom can the teacher of the hm Program integrate the various math study skills with the curriculum of her or his course and show the student the immediate and long-term value of mastering the various skills.

The hm Program is designed for the pre-Algebra student. Our classroom testing has shown its applicability in grades 7–9. Of course, it may also be of value to students at other grade levels.

The *hm Math Study Skills Program,* as noted above, is most valuable to students when its units are integrated into an ongoing mathematics course. In our testing, we found some teachers who chose to devote one class period a week to the Program while others gave it two periods. We suggest that you discover the most beneficial way of integrating the Program into your curriculum through your own experience with its use.

TEACHING THE hm MATH STUDY SKILLS PROGRAM: A POTPOURRI OF HINTS AND SUGGESTIONS

Small Groups

Most of the units in the *hm Math Study Skills Program* are suitable for whole class or small group instruction. We highly recommend that you give your students the opportunity to work in groups of two, three, or four. If students work with a partner, we suggest that you pair students of roughly similar ability.

The interaction of students working on a common task can facilitate the learning of skills through shared problem solving. In this way students can learn from each other.

Small group processes offer a superb method for genuinely engaging students in an activity. Such processes help both to enhance motivation for learning and to increase interest in the content of the lesson, as they offer active participation to each and every student.

Individual Work in Study Skills

Individual work is of critical importance to the learning of skills. When a skill is introduced in a group setting, it becomes crucial to provide individual work with that skill through homework and/or other class activities.

Student Discussion and Learning

To learn study skills effectively and know how and when to use them, students need the opportunity to discuss their work. Their discussion must include not only the "right answer" but the process through which they arrived at the answer and their reasons for considering it correct. At this point in your students' development of study skills, the process is more important than the individual answer.

Reading Directions

Many students do not really listen to directions, preferring to ask questions afterwards. As a result of this habit, many students do not know how to read directions independently.

Much time is saved and good habits are developed when students read the directions to themselves. Thus, for the units in this Program, we have suggested that you consistently ask your students to read the directions to themselves first.

Once your students have read the directions, ask a student to explain what the class is going to do. If there are further questions, allow another student to respond to them.

If any individual still seems confused, ask that person to read the directions aloud to you. Usually that is sufficient.

Using Calculators

Students should have the opportunity to learn to use calculators wisely. One calculator for every two students is enough. If calculators are not available in the school, many students can bring them from home.

After you have distributed calculators to your students for a couple of classes, you need not continue this procedure. Rather, make the calculators available to your students so they can use them as they wish.

Using An Overhead Projector

Having students copy solutions to problems on the board so the class can discuss them is a helpful strategy. Yet it can be time consuming. Instead of using the board, you can give students a piece of acetate and have them work the problems on this surface. Then, their work can be shown to the class using the overhead projector.

Student Perceptions and Expectations

Sometimes students perceive new study skills as more time consuming than their unskilled learning behaviors. In a few cases, this is an accurate perception, but most often it is not.

You can help students gain a wider perspective about their own learning by telling them that any skill, by its very nature, takes more time to use when you are first learning how to do it. Then, as you become more competent in using the skill, it takes less and less time. Ask students to think of examples of this from their own experience. Or, give them a few examples which will illustrate this relationship between competence and time.

OTHER hm STUDY SKILLS PROGRAMS

The *hm Math Study Skills Program* is the third in a series which includes the *hm Study Skills Program* (Level I) for grades 5-7 and the *hm Study Skills Program* (Level II) for grades 8-10.

Forthcoming additions to the series are the *hm College Study Skills Program* and the *hm Science Study Skills Program* (middle school/junior high level).

The *hm Study Skills Program* (Level I) is also available in a Spanish language edition.

At the end of the Teacher's Guide on pages 57-60, you'll find "An Overview of Study Skills" which offers background material.

UNIT I: LISTENING IS A MATHEMATICAL SKILL

Listening means more than just hearing. Listening means hearing *and* understanding.

Too often we assume that our students already know how to listen effectively. Yet our daily experience in the classroom contradicts this assumption. Think about how many times each day you need to repeat something or ask students to listen more carefully. Why are our students such ineffective listeners? There are many contributing factors, but the most immediate and powerful one is simply that we have not taught them to listen skillfully.

The need for skilled listening is as great in mathematics as in any other subject area. In a mathematics class, the student must learn to listen carefully to directions and to descriptions of mathematical procedures. The student must learn to listen for information of various kinds. He or she must also learn to listen and translate from spoken word into mathematical symbol, and to listen and visualize what he or she is hearing about.

Unit I introduces your students to two critical concepts: listening is a skill; and listening is a mathematical skill. Its activities also engage your students in beginning to learn several specific listening skills.

Listening The First Time

For all of the exercises in this unit, we have recommended that you read the question, riddle, or problem only *once*. Learning to listen the first time is an important skill. Initially some students may find it difficult to attend to a single reading. However, when students know that you will only read the material once, they quickly learn to listen more carefully and become proud of their success in doing so.

In "Questions for Good Listeners" and, if necessary, in the other exercises, we suggest that you stop after the first several examples and discuss the answers. This practice will help students who are having difficulty.

A Note about Listening and Visualization

To visualize is to perceive mental images. Some people see mental "pictures" with their eyes closed. Others do so with their eyes open. Still other people don't see "pictures" as such when they visualize but do perceive language and/or feelings. Do you visualize? If you don't know, try this experiment: relax for a few moments and take a few deep breaths; now, think of a person you know well; can you see her or his face? (Try this experiment twice, first with your eyes closed and then with your eyes open.)

Visualization is a human ability which is accepted as something quite ordinary in many cultures. Yet we have tended to ignore it as a potential resource in the learning process. Research has shown that almost all children can see mental imagery. In their early teens, a large majority of people can still visualize. However, through their late teens and into their twenties, many people in our society lose this ability. Only about half of our adult population can see mental images clearly. As a vast majority of adults in some other cultures retain this ability throughout their lives, researchers have suggested that the loss of visualization is not an inevitable part of growing up. Rather, this loss may result simply from neglect of the visualization capacity. If you don't use it and value it, it goes away.

The abiiity to see mental imagery has been associated with imagination, creativity, and intuitive ways of knowing in many cultures. These associations have been supported by recent research in this field. We have included a listening and visualizing exercise in this unit because the use of mental imagery can be very helpful to students' learning. Visualizing can help a student gain a clearer sense of what she or he is hearing. The creation of mental images can also aid memory.

Even if you don't see mental images, most of your students probably do. Try the exercise in this unit with them, and see how it works! If you want to learn more about visualization, we suggest that you examine *Seeing With The Mind's Eye: The History, Techniques, and Uses of Visualization* by Mike Samuels, M.D. and Nancy Samuels (Random House: New York, 1975).

PLEASE NOTE: Suggestion can play a powerful role in visualization. Don't preface the listening and visualizing exercise by suggesting that some of your students will *not* be able to do it. Rather, ask your students to try it. After they have attempted the exercise, you may find that some students feel that they have not "seen" anything. At this point, you can explain to them that some people don't seem to see mental images clearly or at all. These people, however, often experience visualization in other ways, for example, by perceiving words or feelings.

Suggested Directions for Unit I

1. Give your students an overview of the *hm Math Study Skills Program* (what it will cover; what value you see in it; how you plan to use it in class; how their work with it will be evaluated). Then, pass out the Student Texts and have your students read the "Introduction" (pg. 1 in the Student Text). Discuss briefly.

   Approximate time: *5-10 minutes*

2. Have your students read "What Is Listening?" (pg. 2 in the Student Text). Discuss briefly; emphasize the concept of an active listener.

   5 minutes

3. Have your students read the directions for "Questions For Good Listeners" (pg. ..1 the Student Text). Explain to your students that you will read each question only once. Read each question slowly; then, give your students sufficient time to write their . .

   Stop after the first three questions and go over them. This will help students who are having difficulty.

When you have asked all the questions, go over the answers orally and discuss why people get wrong answers. (Many or most of your students will get some or many of the questions wrong.)

a. Stress the idea that errors result from poor listening, i.e., focusing on a single word rather than the entire question, or jumping to a conclusion based on expectations rather than what's actually said.

b. If the class is sufficiently mature, have students who got wrong answers raise their hands to show how widespread the errors are.

15 minutes

Please note: The ability to answer the "Questions for Good Listeners" correctly involves thinking skills as well as listening skills. However, many or most of the errors which students will make will result from ineffective listening. Exercises like these are very helpful in focusing students' attention on their own experience of listening.

QUESTIONS FOR GOOD LISTENERS

1. If it takes three minutes to boil an egg, how many minutes will it take to boil three eggs?
2. I have two United States coins. Together they total 55 cents. One is not a nickel. What are the coins?
3. Write thirteen thousand, thirteen hundred, and thirteen.
4. How many cubic feet of dirt can be taken from a hole in the ground four feet wide, five feet long, and ten feet deep?
5. An empty barrel weighs ten pounds. What can you put in the barrel to make it weigh nine pounds?
6. There are twelve one cent stamps in a dozen. How many two cent stamps are there in a dozen?
7. Nine crows sat on the limb of a tree. Farmer Jones shot three of them. How many crows remained on the tree?
8. How many gumdrops can you place in an empty bottle?
9. Jack had eight marbles. He lost all but three. How many marbles did he have left?
10. What is the difference between twice forty five and twice five and forty?
11. If you take three apples from five apples, how many do you have?

4. Have your students read "Becoming An Active Listener" (pg. 3 in the Student Text). Discuss for clarity and emphasis. Then, ask your students to read the directions for Exercise I (pg. 3 in the Student Text). Explain to your students that you will read each problem only once. Read the example, ask your students to write their answers, and then go over the correct answer. Read problems 1-6. Go over the answers and briefly discuss them.

   10 minutes

   PROBLEMS FOR EXERCISE I

   Example: A school trip cost $29,728. How much did it cost per student? What is missing?

   1. Alice drives at an average speed of 40 miles per hour. How many hours will it take her to drive from Newton to Springfield? What is missing?
   2. Sophia has enough money to buy three yards of fabric at $5.40 per yard. How much money will she have left after she makes her purchase? What is missing?
   3. An insurance agent travels 15,000 miles in a year. The car he uses averages 21 miles per gallon. Find the agent's yearly gas expense. What is missing?
   4. Two trucks started towards each other at the same time from 500 km apart. One truck traveled at a rate of 65 km per hour. After how many hours did they meet? What is missing?
   5. The sum of two numbers is 19. The second number is less than twice the first. Find the two numbers. What is missing?
   6. A parking lot charges $1.25 for the first hour and $ .60 for each additional hour or fraction of an hour. What will be the charge for a car that entered the lot at 8:30 in the morning? What is missing?

5. Have your students read "Listening Is A Mathematical Skill" (pg. 4 in the Student Text). Discuss for clarity and emphasis. Then, ask your students to read the directions for Exercise II (pg. 4 in the Student Text). Explain to your students that you will read each sentence only once. Read the example, ask your students to write their answers, and go over the correct answer. Then, read sentences 1-6. Go over the answers.

   10 minutes

   SENTENCES FOR EXERCISE II

   Example: $5 \times 10^6 = 5{,}000{,}000$

   1. $12 + 1.4 + .56 = n$
   2. $x - 2\frac{1}{4} = 5\frac{1}{2}$
   3. $7 \times -3 > 40$
   4. $10^3 \times 10^5 = 10^8$
   5. $.002 = 2 \times 10^n$
   6. $m + (m + 1) + (m + 2) = 3(m + 1)$
   7. $\frac{1}{4}x = 10$

6. Have your students read "Tips For Good Listeners" (pg. 5 in the Student Text). Discuss briefly for clarity and emphasis.

   5 minutes

7. Have your students read "Listening and Visualizing" (pg. 5 in the Student Text). Discuss briefly; answer any questions that your students raise about visualizing. Ask your students to read the directions for Exercise III (pg. 5 in the Student Text). Explain to your students that you will read each problem only once. Read the first problem aloud; give your students sufficient time to visualize and draw. Then, follow the same procedure with the second problem.

   To go over this exercise, use one or several of the following procedures: have several students discuss what they experienced when they visualized; have a number of students show their drawings to the class or make copies on the board; have students share their drawings with their neighbors; discuss how your students' drawings could help them solve the two problems; discuss how visualizing a math problem can help you solve it.

   10 minutes

   PROBLEMS FOR EXERCISE III

   1. A rectangular field is 100 meters long and 80 meters wide. What is its area?
   2. The Snow brothers took off from the same airport at the same time. They flew in opposite directions. Eks flew at 250 miles per hour. Why flew at 300 miles per hour. At the end of three hours, how many miles apart were they?

8. Ask your students to read the "Unit I Summary" (pg. 6 in the Student Text). Then, ask them to explain ways in which listening is a mathematical skill.

ANSWERS FOR EXERCISES IN UNIT I

Page 2: Questions For Good Listeners

1. 3 minutes. (Boil all the eggs together.)
2. A half dollar and a nickel. (*One* is not a nickel.)
3. 14,313 (13,000 + 1300 + 13)
4. None. (A hole is empty!)
5. Holes. (You need to put in enough holes to eliminate one pound.)
6. 12
7. None. (The others flew away.)
8. One. (After the first one, the bottle is no longer empty.)
9. 3
10. 40 $2 \times 45 = 90$ $(2 \times 5) + 40 = 50$
11. 3 (The three you take are what you have.)

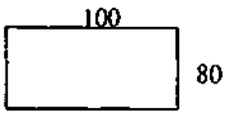
Page 3: Exercise I

Example: The number of students

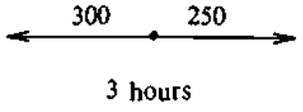
1. The distance from Newton to Springfield
2. The amount of money she has.
3. The cost of gas.
4. The speed of the second truck.
5. How much less the second number is than twice the first.
6. How long the car was parked in the lot.

Page 5: Exercise III

1.



2.



Additional Suggestions

1. Provide further practice in working problems with missing information by choosing exercises from the student textbook and omitting a necessary piece of information.
2. Listening skills will be sharpened when students realize that you will state directions only once. Have students repeat the directions in their own words when they have questions about them.

   Please note: If your students are accustomed to having you repeat directions several times, it will require some time, effort, and probably frustration before they will learn to listen carefully the first time.

3. Dictate two mathematical sentences to students on a regular basis. This activity can provide ongoing reinforcement but takes no more than a minute or two.

4. Have a student choose or make up a problem which presents a good activity for visualizing. Have her or him read it to the class, and ask your students to visualize the problem in some helpful way. This exercise requires no more than a few minutes and could easily develop into an interesting bulletin board display if you ask your students to draw their visualizations.

UNIT I SUMMARY: LISTENING IS A MATHEMATICAL SKILL

Listening is a skill. It takes effort and practice to learn how to be a good listener.

Listening is an important skill in mathematics. The better you can listen, the better you'll be able to learn in your math class.

A good listener is an *active listener*. An active listener hears words as they are spoken and thinks about what they mean.

How can you become an active listener?

1. Ask yourself questions about what the speaker is saying.
2. Try to connect what the speaker is saying with what you already know.
3. Try to listen "between the lines" for what the speaker is hinting at.
4. When you can, "picture" in your mind what the speaker is saying.
5. When you need to remember what the speaker is saying, take notes. Write down any information, diagrams, and examples which will help you remember.
6. Jot down any questions that you want to ask. Keep listening! Then, ask your questions when the speaker is ready to answer them.

In math class, a good listener can also:

1. Listen to a problem and make sense of it.
2. Translate spoken words into mathematical symbols.

UNIT II: PROBLEM S. LVING (1)

"The development of problem solving ability should direct the efforts of mathematics educators through the next decade. Performance in problem solving will measure the effectiveness of our personal and national possession of mathematical competence."[1]

American students performed well on the second National Assessment of Educational Progress "on exercises at the knowledge and skills level in many content areas. At all age levels, and in virtually every content area, performance was extremely low on exercises requiring problem solving or applications of mathematical skills. In general, respondents demonstrated a lack of the most basic problem solving skills. Rather than attempting to think through a problem and figure out what needed to be done to solve the problem, most respondents simply tried to apply a single arithmetic operation to the numbers in the problem."[2]

What Is Problem Solving?

Problem solving is a process which requires the ability to: (1) perceive, analyze, and understand a situation; (2) develop strategies for addressing the perceived problem; (3) select a strategy for action and follow it through; and (4) interpret the results of one's actions. Another way to think of problem solving is to see it as a group of related skills.

For students, the first issue is determining what the problem really is. Students often experience a problem as a situation in which they do not know what to do. Caught in this context, students usually seek to practice the procedure recently taught without consideration of its appropriateness to the problem.

The second issue involves the students' ability to choose a strategy for use in solving the problem. Too often students have developed no repertoire of problem solving strategies, nor even an awareness of their need for such. Impelled by a lack of confidence in their own ability, they do not pause to analyze a situation and select a useful strategy. Instead they quickly choose an operation to perform on the numbers at hand.

We need to help students learn that first they must discover what the problem really is. We must also teach them that solving problems involves choosing a strategy for use from a broad range of mathematical strategies and processes. "Mathematics programs should give students experience in the application of mathematics, in selecting and matching strategies to the situation at hand."[1]

We should also encourage students to invent t'.eir own strategies for solving problems. Even if these strategies are slow or awkward, they can help build the students' confidence in their own problem solving abilities. Through class discussion and analysis of alternative solutions suggested by the teacher and peers, students will gradually accumulate more elegant and efficient strategies.

---

1. *An Agenda for Action. Recommendations for School Mathematics of the 1980s.* National Council of Teachers of Mathematics: Reston, Virginia, 1980.
2. Carpenter, T.P. et al. "Results of the Second NAEP Mathematics Assessment: Secondary School." *Mathematics Teacher,* May 1980: 329-338.

When To Teach Problem Solving

The more opportunities that students have to solve problems, the more effective they will become as problem solvers. The teaching of problem solving skills requires considerable amounts of time, as does practice in solving problems. To cope with the limited time available, many mathematics teachers suggest the following:

1. A problem a day.
2. A problem a week. (These are problems which require a longer process for solution.)
3. One or two problem solving classes each week.
4. The homework problem for the day.

A combination of these approaches can help you shape a successful problem solving curriculum. Priority in the use of classroom time should be given to involving students in problem solving activities which they perceive as meaningful.

Suggestions For Teaching Problem Solving

1. Have students work in small groups. Instruct each student to develop her or his own strategy for solving the problem as a first step. Then, have the group discuss the strategies of its members and select which strategy and solution it wishes to present to the class.
2. Each student's *own* effective strategy, however slow or awkward, deserves recognition.
3. Class discussion of alternative strategies is crucial to the learning process. Students need to see a variety of effective strategies for the same problem and to learn that they are all acceptable.
4. You can use the bulletin board to display a collection of different and/or ingenious solutions to problems.

Please Note: Units II and V in this program can only introduce your students to a few basic strategies for problem solving. Also, the problems included are limited to those which can be solved within a short period of time.

Suggested Directions for Unit II

1. Organize your class into small groups of 2-3 students. Have the members of each group sit together.
2. Have your students read "What Is A Problem?" (pg. 7); remind them to answer the question at the end of the section. When they have done so, list all of the methods they suggest on the board. Then, tell them that you will return to this problem in a minute.

   Approximate time: *5 minutes*
3. Have your students read "Solving Problems: How To Begin" (pg. 7). Discuss briefly. Then, ask your students to read the directions for Exercise 1 (pg. 8) and to solve problem #1. When they have done so, go over the problem. Also, discuss the other strategies for solving this problem which your students suggested. Focus on the idea that there can be several or many good ways to solve the same problem.

   10 minutes

4. Have your students do problems #2 and #3 (pg. 9). Go over the problems.

   10 minutes

5. Have your students read "Making A Diagram" (pg. 9). Briefly discuss the example problem. Have your students read the directions for Exercise II (pg. 10), and do problem #1 in the exercise. When they have completed the problem, go over it. If students have different kinds of diagrams, have several students make copies of their diagrams on the board. Discuss.

   5 minutes

6. Have students do problems #2 and #3 (pg. 10). Go over the problems. If time permits, ask students to draw their diagrams on the board. Discuss the different ways of drawing a useful diagram.

   10 minutes

7. Have your students read "Creating A Simpler Problem" (pg. 11). Discuss briefly. Have your students read the directions for Exercise III (pg. 11), and do problem #1. Go over the problem. Then, have your students do problem #2 (pg. 12). Go over the problem step by step; discuss any alternative ways of solving it. Follow the same procedure with problem #3 (pg. 12).

   15 minutes

   (Some problems seem very complex. Others involve large quantities. To attempt a solution, students must learn that they can often break up a complex problem into manageable pieces by asking and answering simpler questions first. Students can also learn to insert smaller numbers into a problem and work with a simpler example as a start. Exercise III deals with these skills.)

ANSWERS FOR EXERCISES IN UNIT II

Page 8: Exercise I

1.

Number of folds	Parts
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Patterns: Answers will vary; accept any answer which shows a pattern. It's helpful to list a number of patterns so students can see ones which did not occur to them.

Examples: "The number of parts doubles each time another fold is made."

"It's based on a power of 2."

"If it is folded seven times, the number of parts is 2^7 or 128."

Page 9: Exercise I

2.

John	5	6	7	8	9	10	11	12			
Father	41	42	43	44	45	46	47	48			

Students' tables may vary. They need not use an increment of one.

3.

Day	1	2	3	4	5	6	7	8	27	28	29	30
Saving	$1	$2	$3	$4	$5	$6	$7	$8	$27	$28	$29	$30
Total	$1	$3	$6	$10	$15	$21	$28	$36	$378	$406	$435	$465

Some students will need to continue the table for each day. Others will see a pattern and be able to generalize. Either solution is acceptable.

Some students may see the solution as the sum of all numbers 1–30:

$$\overbrace{1 + 2 + 3 + 4 \ldots\ldots + 28 + 29}^{31} + 30 = 465$$

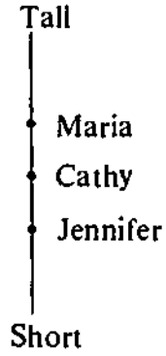
There are 15 sums of 31: $15 \times 31 = 465$. Do any students see that the answer can be stated as follows?

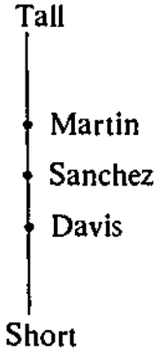
$$(n + 1)\frac{n}{2} \quad \text{or} \quad \frac{n(n+1)}{2}$$

However, at this stage the development of a formula should not be stressed. If students believe that a formula is the only acceptable solution, many will balk at trying to solve problems.

Page 10: Exercise II

1.





Answers: Cathy Sanchez
Jennifer Davis

2.

A K Q J	K A Q J	Q A K J	J A K Q
A K J Q	K A J Q	Q A J K	J A Q K
A Q K J	K Q J A	Q K A J	J K A Q
A Q J K	K Q A J	Q K J A	J K Q A
A J K Q	K J Q A	Q J A K	J Q A K
A J Q K	K J A Q	Q J K A	J Q K A

Answer: 24

If students draw other kinds of "cards," discuss how you can use different kinds of diagrams to solve a problem. Stress that all methods which produce the right answer are correct.

3.

train	30	60	90	120	135
car	20	40	60	80	90

Answer: 135 miles

Page 11: Exercise III

1.

Time	2 sec	1 min	1 hour	1 day
Blinks	3	90	5400	129,600

Page 12: Exercise III

2. "Simpler question": Answers will vary; accept any helpful question.

Sample question: How long would you have to wait in line if 60 people are ahead of you?

People	*Minutes* (total)
1-28	4
29-56	8
57-74	12

You have to wait 12 minutes before your group can enter.

Answer to original problem:

People	28	56	560	588
Wait (minutes)	4	8	80	84

Answer: 84 minutes

3. First question: How many liters of gas would the small car use?
   Answer: 35 liters

   Second question: How many liters of gas would the station wagon use?
   Answer: 60 liters

   Answer for original problem: 25 liters saved

Additional Suggestions

1. Having students copy problems and solutions on the board for class discussion is a helpful strategy. To save time, use an overhead projector instead of the board. Have a student in each group record the group's strategies and solutions on a piece of acetate. This can be done during the small group work and does not take time away from the large group discussion.

2. With the current emphasis on problem solving in mathematics, new materials are continually becoming available. The bibliography below reflects a sample of the problem solving materials currently on the market.

Auerbach, Bonnie and Chein, Orin. *Mathematics: Problem Solving Through Recreational Mathematics.* 1980: W. H. Freeman and Co.; San Francisco.

Gardner, Martin and Tappey, Robert. *The Aha! Book.* 1977: W. H. Freeman and Co.; San Francisco.

Greenes, C., Immerzeel, G., Ockenga, E., Schulman, L., and Spungin, R. *Techniques of Problem Solving (TOPS).* 1980: Dale Seymour Publishers; Palo Alto.

Greenes, C., Spungin, R. and Dombrowski, J. *Problem-Mathics.* 1977: Creative Publications; Palo Alto.

Harnadek, Anita. *Basic Thinking Skills.* 1977: Midwest Publications; Pacific Grove, California.

Maletsky, Evan and Hirsch, Christian. *Activities from The Mathematics Teacher.* 1981. National Council of Teachers of Mathematics; Reston, Virginia.

National Council of Teachers of Mathematics. *Problem Solving in School Mathematics.* 1980: NCTM; Reston, Virginia.

Ohio Department of Education. *Problem Solving: A Basic Mathematics Goal.* 1980: Ohio Department of Education; Columbus.

Pedersen, Jean and Armbruster, Franz. *A New Twist: Developing Arithmetic Skills Through Problem Solving.* 1979: Addison-Wesley; Menlo Park, California.

Price Laboratory School. *Iowa Problem Solving Cards.* 1979: Price Laboratory School; Cedar Falls, Iowa.

Seymour, Dale and Shedd, Margaret. *Finite Differences.* 1973: Creative Publications; Palo Alto.

Whimbey, Arthur and Lochhead, Jack. *Problem Solving and Comprehension.* 1979: The Franklin Institute Press; Philadelphia.

Wylie, C. R. Jr. *101 Puzzles in Thought and Logic.* 1957: Dover Publications; New York.

Polya, George. *How To Solve It.* 1957: Princeton University Press; Princeton, New Jersey. (A classic on problem solving!)

UNIT II SUMMARY: PROBLEM SOLVING (1)

Problem solving is a way of thinking that you can learn.

The first step in solving a problem is figuring out exactly what the question is. Read the problem carefully; then, tell yourself what the question is.

A second step in solving a problem is organizing the information in the problem. Sometimes when you organize the information in a problem, you'll find the solution. Two good ways to organize the information in a problem are:

1. Make a table or chart
2. Draw a diagram

When a problem seems unsolvable, try to find a simpler problem within the problem. Solve the simpler problem first. Then, use the new information to solve the original problem.

UNIT III: UNDERSTANDING THE LANGUAGE OF MATHEMATICS (1)

The language of mathematics presents difficulties to many students. They often do not even know that mathematics, like any other specialized activity, has its own language. When students cannot use the language of mathematics effectively, they learn to regard mathematics as mysterious or inpenetrable. It often makes little sense to them, and they come to believe that it isn't intended to be meaningful. Thus, it becomes a subject to be memorized, not understood.

One of the aims of this unit is to help students begin to see that mathematics has its own language. Then they can learn this language just as they do any other subject. Toward this end, the unit introduces the concept of "the language of mathematics." It engages students in looking at mathematical terms carefully, relating them to their own experience, and trying to make sense of them.

Unit III is also aimed at helping your students become more independent in their math work. To achieve this, students need to develop skills for using the math resources which are available to them. Several of the exercises in this unit involve your students in learning the meanings of mathematical terms and symbols by using their resources. Students are asked to learn what their math resources are, what kinds of information to expect in each, and which source would be most helpful in a particular situation.

Suggested Directions for Unit III

1. At the beginning of the period, organize your class into small groups of 2–3 students; have the students in each work group sit together. If possible, give each student a dictionary; at least, equip each group with a dictionary.

   You may wish to have your students do Exercises I–III in their groups or individually. We strongly recommend that you ask your students to complete Exercises IV–VI as group work.

2. Have your students read "The Language of Mathematics" (pg. 14); ask them to answer the question at the end of the section. When they have done so, go quickly around the room asking each student to say one of the words he or she has written. Briefly discuss the range of words offered by your students; emphasize the concept of "the language of mathematics."

   Approximate time: *5 minutes*

3. Have your students read "Using Your Resources" (pg. 14). Discuss the value of knowing what your resources are and how to use them. Then, have your students examine the Table of Contents in their textbooks. Discuss the ways in which the Table of Contents can serve as a resource.

   5 minutes

4. Ask your students to read "Sources For Meanings" (pgs. 14–16) and do Exercises 1–3 (pgs. 14–16). While your students work, you may wish to circulate around the room to provide direction and assistance as needed. Go over all three exercises. Then, discuss questions like the following:

   a. Which source has the most listings?
   b. Which one was the easiest to use? most helpful?

   15–20 minutes

5. Have your students read "Using Your Sources For Meanings" (pg. 16). Then, have your students do Exercise IV (pgs. 16-17) in their groups. When they are done, go over Exercise IV. Discuss which sources students find most useful and why. (You may wish to tell your students that there is no "right answer" to this question as preferences will vary.)

   10 minutes

6. Have your students do Exercise V (pg. 17). Then, go over the exercise; discuss what resources your students chose to use and why.

   10 minutes

7. Have your students read "Examples" (pg. 18). Discuss briefly. (You may want to tell your class that an example can be numbers, a drawing, or whatever is appropriate.) Then, ask your students to do Exercise VI (pg. 18); go over the exercise.

   10 minutes

ANSWERS FOR EXERCISES IN UNIT III

Answers for all exercises in this unit will vary according to the textbook and dictionary which your students use. The answers listed below are provided as "sample answers".

Page 15: Exercise I and Page 16: Exercise III

Answers will vary according to the textbook and dictionary which your students use. Some words may not be listed in one or either source.

Page 16: Exercise II

Answers will vary according to the textbook which your students use.

Pages 16-17: Exercise IV

1. glossary: If a, b, and c are whole numbers and $a \times b = c$, then a and b are called factors of c.

   index and text: $3 \times 4 = 12$ 3 and 4 are factors of 12

   dictionary: One of two or more quantities which, multiplied together, give a product.

   people: If 6 is the number, then 2 and 3 are factors.

2. glossary: The sum of the lengths of the sides of a polygon.

   index and text: The distance around a geometric figure.

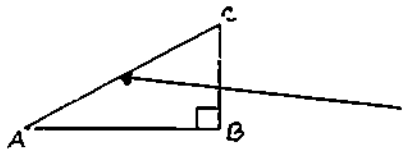
   dictionary: The outer boundary of a plane surface.

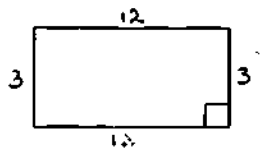
   people: The length around the outside.

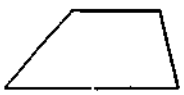
Page 17: Exercise V

1. A letter which can be replaced by numbers.
2. Two angles matched by any one-to-one correspondence between the angles of the two figures.
3. A natural number which is the product of two smaller numbers.
4. The product of all whole numbers from 1 to n is called factorial n, and is represented by n! For example, $3 \times 2 \times 1 = 6$.

Page 18: Exercise VI

1. The set of factors of 10 is $\{1, 2, 5, 10\}$.
2. 

hypotenuse
3. 

perimeter = 30 units
4. $|-3| = 3$
5. $-6 + 6 = 0$
6. 

Additional Suggestions

1. Reinforce the efficient use of the glossary, index and text, table of contents and dictionary by referring to these resources when students raise questions. Rather than answer a question yourself, ask the class to use their resources to find the answer.
2. Construct an exercise in which students work in groups and use their math resources. Structure it like a game. For example, divide your class into teams. Give each group a list of math terms and ask them to use their resources to find the definitions as quickly as possible. Have them record the definitions. Or, ask a group of students to create a game of this sort.

UNIT III SUMMARY: UNDERSTANDING THE LANGUAGE OF MATHEMATICS (1)

Like any other activity or subject, mathematics has its own special language. The more you know about this language, the more you'll be able to make sense of mathematics.

There are many resources which you can use to help you learn more about the language of mathematics.

1. In your textbook: the glossary
   the index and text
   the table of contents
2. The dictionary
3. Ask a friend, older sister or brother, or a parent

Finding examples of mathematical terms and looking carefully at these examples can also help you understand what the terms mean.

Using your resources to help you learn the meanings of mathematical terms is one useful study skill. You can also use these resources to answer other questions that come up in math.

UNIT IV: UNDERSTANDING THE LANGUAGE OF MATHEMATICS (2)

The activities of this unit are designed to continue your students' orientation to the language of mathematics. Students are introduced to "musical phrases" and mathematical symbols as elements of this special language. They work with examples of these elements and with a method for taking notes about new terms and symbols.

What we call "musical phrases," for example, least common denominator and greatest common factor, are mathematical terms which often present difficulty to students. Too often they learn these phrases by rote without developing a usable understanding of their meanings. Just as students can recite the Pledge of Allegiance for years without knowing what they are saying, they can also learn mathematical terms in the same uncomprehending way. Unfortunately this kind of learning leaves students helpless when they must understand a term and use it as a set of directions.

The aim of the "musical phrases" exercise in this unit is to help your students begin to see these phrases as parts of the language of mathematics which they can understand and use.

This unit also offers your students a method for taking useful notes about new terms and symbols. We strongly suggest that you stress effectiveness as the key concept in note taking. As students differ in their aptitude and learning style, the nature of their notes will also differ. Students need to learn to take notes which are effective for them as individuals. So, in Exercise III (pg. 24 in the Student Text). they should record only the information that they personally need to understand a new term or symbol.

Suggested Directions for Unit IV

1. At the beginning of the period, organize your class into small groups of 2-3 students. Have the students do the unit in groups, except for Exercise III.

2. Have your students read the "Introduction" (pg. 20) and "Musical Phrases" (pg. 20). Discuss for clarity and emphasis.

   Approximate time: *5 minutes*

3. Have your students do Exercise I (pg. 21). Go over the exercise.

   10 minutes

4. Ask your students to read "Mathematical Symbols" (pg. 22); remind them to list the symbols they know on the lines provided. When they have done so, have students write symbols and their meanings on the board. Involve as many students as possible. Discuss how many symbols your students already know and how they have learned them.

   5 minutes

5. Have your students read "Finding The Meanings Of Mathematical Symbols" (pg. 22) and do Exercise II (pg. 22). Go over the exercise. (If the text you use does not include a list of symbols, provide one for each student.)

   10 minutes

6. Ask your students to read "Keeping Track Of The Language Of Mathematics" (pg. 23) and "Why Take Notes In Math?" (pg. 23). Discuss for clarity and emphasis. Also, focus your discussion on the student's need to take notes which are effective for her or him as an individual.

   5 minutes

7. Have your students do Exercise III (pg. 24) individually. When they are done, ask them to compare their notes with their partner's. Then, go over the exercise and discuss.

   10 minutes

ANSWERS FOR EXERCISES IN UNIT IV

Page 21: Exercise I

1. greatest common factor: largest factor that is the same for all the fractions listed
2. least common multiple: smallest multiple that is the same for all the fractions listed
3. greatest possible error: number which is equal to 1/2 the unit of the measure used
4. prime factorization: one in which all the factors of a number are written as primes
5. perpendicular bisector: a line which is perpendicular to another line, separating it into two equal segments
6. Accept any reasonable or imaginative answer.

Page 22: Exercise II

1. absolute value of
2. is less than or equal to
3. square root
4. is congruent to
5. union of sets
6. measure of angle ABC

Page 24: Exercise III

	Term	Definition	Example	My description
1.	altitude	the line segment from any vertex of a triangle perpendicular to the opposite side		height which is perpendicular to the opposite side
2.	$\neq$	is not equal to	$5 \neq 3 + 1$	
3.	range	the difference between the highest and lowest numbers in a set	7, 10, 15, 8, 22, 19, range is 15	
4.	irrational numbers	a number that cannot be expressed in the form a/b when a and b are integers	$\sqrt{2}$	a number that cannot be written as a fraction

Additional Suggestions

1. Have your students play a game like "Guess the Math Term." Organize your class into small groups. In each group, a student states a definition, an example, or a description of a mathematics term. The other students try to guess what it is. When someone guesses, then she or he states a definition, example, or term, and so on. This is a good exercise to help students refine their definitions and descriptions.
2. Attach an envelope for "musical phrases" to the bulletin board. As students discover mathematical terms which they consider to be "musical phrases," ask them to place a card with the phrase and meaning in the envelope. Periodically take out the cards and discuss them in class. Or, ask students to examine the cards in the envelope when they have the opportunity.

UNIT IV SUMMARY: UNDERSTANDING THE LANGUAGE OF MATHEMATICS (2)

Two special parts of the language of mathematics are "musical phrases" and mathematical symbols.

A "musical phrase" is a group of words which have a specific meaning together. An example of a "musical phrase" is "least common denominator." When you come across a "musical phrase," look at each word carefully. Then, put all the words together to figure out the meaning of the phrase

A mathematical symbol is a sign or marking which has a special meaning. An example of a symbol is = which means equals.

A good way to keep track of new mathematical terms and symbols is to write them down in your notebook. Include the term or symbol, its definition, an example, and a description in your own words.

OVERVIEW: UNDERSTANDING THE LANGUAGE OF MATHEMATICS

When you come across a mathematical term that you don't understand, *look up its meaning* in:

1. The glossary of your textbook
2. The index and text of your textbook
3. The dictionary

Or you can *ask somebody else* about the term. Ask a friend, your teacher, or a member of your family.

When you find out what the term means, *write it down* in your notebook like this:

term	definition of the term	example of the term	description of the term in your own words

When you come across a mathematical symbol that you don't understand, look it up in the list of symbols at the back of your textbook. If you can't find the symbol there, ask your teacher about it.

UNIT V: PROBLEM SOLVING (2)

This unit continues the work begun in Unit II. Its activities engage your students in the use of graphs as a problem solving strategy and in learning more about the use of patterns for the same purpose. Students are also introduced to the idea that there are usually many effective ways to solve any particular problem in mathematics.

Suggested Directions for Unit V

1. Organize your class into small groups of 2-3 students. Have the members of each group sit together. Encourage each group to work cooperatively.

2. You may wish to begin with a quick review of the concepts introduced in Unit II: what is a problem; how to begin to solve a problem; strategies for solving problems — using a table, drawing a diagram, creating a simpler program.

3. Have your students read the "Introduction" (pg. 26) and "Using Graphs" (pg. 26). Then, ask your students to do the first half of Exercise I (pg. 27): questions 1-5. When they have answered the questions, go over the answers. Then, ask your students to think of other questions which could be answered by the data on the graph. Challenge them to think of as many questions as they can.

   Approximate time: *8-10 minutes*

4. Use the same procedure as above for the second half of Exercise I (pg. 28): questions 6-9.

   4-8 minutes

5. Ask your students to read the directions *and* the questions for Exercise II (pgs. 29-30). Then, have them do the exercise. Remind them to include a title and a key. When they are done, go over questions 1-4. Then, ask students to share all of their various responses to question 5; discuss the use of a graph as a means of picturing information and finding patterns.

   15 minutes

6. Have your students read "Finding Patterns" (pg. 30). Discuss the idea of patterns by asking your students to identify patterns in the room, i.e., tiles, clothes. Then, ask your students to do question 1 in Exercise III (pg. 31).

   Geoboards are not necessary for this exercise, but they can be helpful. Graph paper may also be used. Remind your students that 1 × 1 means one unit of length by one unit of length. It does not mean the number of nails.

   When your students have answered the questions, discuss the pattems which they have perceived. Find out how many students filled in part of the chart by seeing a pattern and how many found each square.

   10 minutes

7. Use the same procedure as above for questions 2 and 3 (pg. 32). (You may want to assign the question about the "fiftieth triangular number" for homework.)

   10 minutes

8. Have your students read "Different Strategies For The Same Problem" (pg. 33). Ask them to do question 1 in Exercise IV (pg. 33). When they have done so, list all of their suggestions on the board. Discuss the alternatives which offer the quickest solution and which are clever or unusual. Then, use the same procedure for question 2 in Exercise IV (pg. 33).

   12-15 minutes

9. Instruct your students to work on Exercise V (pg. 34) independently. Tell them that they are going to create a problem, solve it, and then share it with a partner. Have them do Exercise V. When they are done, ask them to copy their problems on separate sheets of paper. Have them exchange problems with a partner. Ask them to solve the problem they have received and then compare their strategies and solutions with those of their partner.

   10 minutes

ANSWERS FOR EXERCISES IN UNIT V

Pages 27-28: Exercise I

1. December
2. August
3. June, September
4. $2000
5. April, May
6. Accept any answer in the range between 440,000 and 460,000.
7. June, October
8. July, August
9. Answers will range between 90,000 and 110,000.

Pages 29-30: Exercise II

1. June, July
2. January, February, November
3. May
4. Answers will vary. Ask students to explain why they have chosen a particular answer.
5. Answers will vary.

Pages 31–32: Exercise III

1.

Dimensions of Squares

Size of Geoboard	1 × 1	2 × 2	3 × 3	4 × 4	Total
1 × 1	1				1
2 × 2	4	1			5
3 × 3	9	4	1		14
4 × 4	16	9	4	1	30

Answers will vary. For example: "The numbers are all square numbers." "They are the sum of successive square numbers." "If the square is 4 × 4, then the total number of squares would be:

$4^2 + 3^2 + 2^2 + 1^2$."

There are 385 squares on a 10 × 10 geoboard:

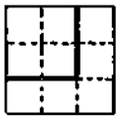
$10^2 + 9^2 + 8^2 \ldots$

1 × 1 

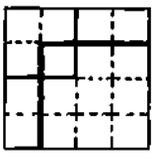
1 × 1 = 1

2 × 2 

1 × 1 = 4
2 × 2 = 1

3 × 3 

1 × 1 = 9
2 × 2 = 4
3 × 3 = 1

4 × 4 

1 × 1 = 16
2 × 2 = 9
3 × 3 = 4
4 × 4 = 1

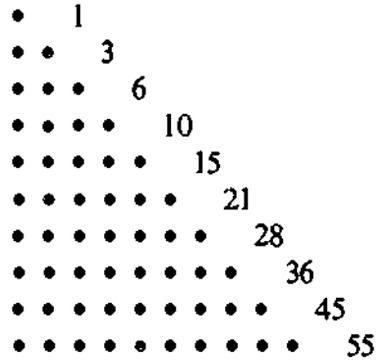
2.

inches	3/4	1 1/2	3	6	12	24	
miles	14	28	56	112	224	448	

Answers will vary. For example: "In this table I used a doubling pattern."

3. Triangular numbers

1st	1
2nd	3
3rd	6
4th	10
5th	15
6th	21
7th	28
8th	36
9th	45
10th	55



Answers will vary, as there are many patterns. For example: "The fourth triangular number is the sum of the third triangular number plus four. The fifth triangular number is the sum of the fourth triangular number plus five." "The fifth triangular number is the sum of 5 + 4 + 3 + 2 + 1. The sixth triangular number is the sum of 6 + 5 + 4 + 3 + 2 + 1." "For the fourth triangular number you can multiply 4 × 5, divide the answer by 2, and get 10. It also works for the sixth triangular number. 6 × 7 = 42. 42 ÷ 2 = 21."

The fiftieth triangular number is 1275.

Page 33: Exercise IV

1. Answers will vary. Some possible strategies are:

   Weighing. They could count out a certain number of beans and weigh them. They could then weigh the remaining coffee beans in the container. They must remember to find the weight of the container also.

   Counting. They could count the beans by placing them in piles of 50 or 100.

   Capacity. They could fill a small container with beans, count the capacity of that container, and then see how many times they could fill it with beans from the jar.

   Delegating responsibility or "passing the buck."
   They could ask their friend, Sandy, to solve the problem.

2. Answers will vary. Some possible strategies are: constructing a table
   looking for patterns
   guess and check

Additional Suggestions

1. The use of graphs and charts as a means of efficient presentation of information is more common than most students realize. Have students look through a daily newspaper, and bring to class all the graphs and charts which they find. Remind your students that some of the graphs and charts will be found in ads.

   Your discussion could focus on the number of graphs and charts, the kinds of graphs and charts, the ways in which these representations are used and their effectiveness, the more popular topics for graphs and charts, etc. You might ask your students to graph some of the information which you discover in your discussion.

2. On a regular basis, have students solve a problem involving a sequence. This takes only a few minutes and provides good practice in looking for patterns. You can also ask students to compose sequence problems and present them to the class.

3. As explained in Unit II, many students can visualize to help them solve math problems. You may find it valuable to work more with this skill. For example, students can often use visualization as a way of attacking a problem when they feel blocked or stuck. Suggest a procedure like the following one: "When you're trying to do a math problem and you feel stuck or you just don't know how to start, try this. First read the problem again carefully. Then, just relax and close your eyes for twenty or thirty seconds. See if any ideas or pictures come into your mind which can help you know what to do next."

   This method helps students to use their intuitive as well as their intellectual capacities in solving problems.

UNIT V SUMMARY: PROBLEM SOLVING (2)

Using a graph is a helpful strategy for solving problems in mathematics. Graphs show you a lot of information in a small space, and they can help you to understand that information.

Finding a pattern is another effective way to solve problems. Often you can make a table to help you find a pattern.

There are many correct ways to solve most problems in mathematics.

UNIT VI: LEARNING FROM YOUR HOMEWORK

Many students at the pre-Algebra level have never examined the ways in which they approach and complete their homework. As a result they tend to work haphazardly with little or no understanding of how they can learn most effectively.

This unit addresses this problem by engaging your students in an examination of their own study environment and behavior. It also introduces your students to several strategies for doing textbook chapter exercises and review exercises in an effective manner.

A key to the first part of this unit is that people learn best in their own personal ways. Usually, however, little instruction in school is directed towards the discovery of various aspects of the student's own *learning style*. This unit seeks both to foster self-awareness in the student about his or her own learning style and to suggest study behaviors and methods for the student to try.

The second part of the unit models ways of approaching and completing two common kinds of homework assignments: chapter exercises and review exercises in the math textbook. Many students do not understand the format of a textbook. Thus, they do not know how to use it as a learning tool. The activities in this part of the unit help students realize that the information they need to complete a homework assignment is usually available to them in their books. These activities engage students in learning more about how to use their texts and how to use effective strategies for doing their homework. They also seek to help students see that the purpose of homework is not only to earn a good grade but also to learn.

Suggested Directions for Unit VI

1. Tell your class that this unit is about how they do their homework. Ask your students to read "Doing Your Math Homework" (pg. 36) and answer the questions. Tell them to go ahead and read "Why Do Your Math Homework?" (pg. 37) when they finish the last question.

   Approximate time: *8-10 minutes*

2. Have your students read "Homework Is Studying" (pg. 37) and do Exercise I (pg. 38). When they have finished Exercise I, discuss what they see as "helpful" and "distracting" conditions in a study environment. Have several students write their lists on the board and discuss. Or, have your students discuss their lists in small groups. Find out which conditions of a study environment are most widely accepted by your students as "helpful," and list these on the board. Also, ask your students to compare the conditions they have listed as "helpful" and "distracting" with their answers to questions #2 and #3 on page 36.

   15 minutes

3. Have your students read "Tips For Doing Your Math Homework" (pg. 38). Discuss each tip briefly.

   10 minutes

4. Have your students read "Doing Chapter Exercises In Your Textbook" (pg. 40). When they are done, go over the four steps orally. Then, ask your students to do Exercises II and III (pgs. 41–43). Go over Exercise II. Then, ask several students to read their responses to Exercise III aloud. Discuss their responses. Emphasize the value of using an organized way of doing your homework, even if it feels a little uncomfortable at first.

   20 minutes

5. Ask your students to read "Doing Review Exercises In Your Textbook" (pg. 44). Discuss both of the suggested ways of learning how to do problems in review exercises. Then, have your students do Exercise IV (pg. 45). Go over the exercise.

   15 minutes

6. Have your students read "Two More Hints For Doing Homework" (pg. 46). Also, you may want them to read the "Unit Summary" (pg. 46) to reinforce the major concepts in the unit.

   8–10 minutes

ANSWERS FOR EXERCISES IN UNIT VI

Page 45: Exercise IV

1. 140–143
2. 144–147
3. 148–153
4. 154–159
5. 176–181
6. 164–169
7. 82–185
8. 170–175

Additional Suggestions

1. Some students will perceive the procedures suggested in this unit as more time consuming than their current ways of doing their homework. You can address these concerns as follows:

   (1) Explain to your students that learning and using any new skill takes more time when you first learn it than when you have mastered it. So, as they become more skilled in using the procedures taught in this unit, your students will be able to do them much more quickly.

   (2) When students use the procedures taught in this unit, they will learn more from doing their homework and will understand mathematics better. It does take time to learn. But these skills and procedures can help your students to use that time more efficiently.

2. Occasionally have your students complete a homework assignment in class. Use this period as an opportunity to evaluate your students' study skills, so you can provide further instruction for those who need it.

UNIT VI SUMMARY: LEARNING FROM YOUR HOMEWORK

Your *study environment* is everything that surrounds you when you are learning. Find out what makes a good study environment for you. Then, be sure to do your math homework in your good study environment.

Use the following tips for doing your homework:

1. Be sure to bring home from school whatever you need to do your homework.
2. Have all your materials, for example, pencil and paper, your textbook, and your notes, within reach when you start to work.
3. Work on your homework for 25–45 minutes. Then, take a break for 5 minutes or so before you start again.

One kind of homework assignment your teacher will often give you is *chapter exercises.* Organize the way you do these exercises like this:

1. First, read the textbook section before the exercises.
2. Be sure to try the sample questions and problems in the reading on your own. Check yourself.
3. Try a few of the odd numbered problems in the chapter exercises. Check yourself.
4. Now that you understand what you're doing, do your homework problems.

Another kind of assignment your teacher will give you is *review exercises.* When you're not sure how to do a problem in the review exercises, look up that kind of problem in the Table of Contents or the Index. Use your resources in your textbook to learn how to do it.

UNIT VII: USING FORMULAS

This unit is designed to help students learn about the nature of formulas, their value, and uses. Initially students work with the concept of area by counting unit squares. They are then introduced to area formulas as efficient tools and use them with a variety of problems, including finding the areas of simple and more intricate shapes and solving word problems.

The final exercise in this unit gives students the opportunity to apply their skills in using formulas to several new situations.

Snggested Directions for Unit VII

1. Have your students read the "Introduction" (pg. 47), "Area By Counting" (pg. 47), and the directions for Exercise I (pg. 47). Discuss the example briefly and ask your students to complete the exercise. When they are done, go over it. Discuss the relationships among some of the figures, i.e., A and B; C and D; C and D and E; etc..

   Approximate time: *10 minutes*

2. Ask your students to read "Area By Formula" (pg. 49). Answer any questions that arise. (You may want to provide your students with 3 x 5 cards so they can begin their 'formula collection' now.)

   8 minutes

3. Have your students do Exercise II (pg. 50). Remind them that each formula may be related to more than one figure. Go over the exercise.

   5 minutes

4. Have your students read the directions for Exercise III (pgs. 50-51). You may wish to discuss the meanings of base and height with your students. Then, have them complete the exercise. When they have done so, have students check each other's work.

   8-10 minutes

5. Ask your students to do Exercise IV (pg. 52); go over the exercise. Have your students do Exercise V (pg. 53). Go over it.

   15 minutes

6. Ask your students to read "Compound Figures" (pg. 54). Discuss briefly. Be sure to tell students that there is more than one way to dissect a compound figure. Have students do Exercise VI (pgs. 54-55). Remind them to write the formulas used next to the appropriate figures, as shown in the example. When your students have finished, go over the exercise. Ask a few students who have solved the problem in different ways to show their work.

   8-10 minutes

7. Have your students do Exercise VII (pgs. 55–56); go over the exercise. Then, have them do Exercise VIII (pg. 56). When they have drawn their figures, have them exchange books with a classmate and then find the area of their classmate's compound figure. Give students an opportunity to share their figures with the class.

   10–15 minutes

8. Ask your students to read "Solving Word Problems With Formulas" (pg. 57). Then, ask them to do Exercise IX (pg. 57). Go over the exercise.

   10 minutes

9. Have your students do Exercise X (pg. 58). Suggest that drawing diagrams may help them solve the problems. When they have finished, go over the exercise.

   10 minutes

ANSWERS FOR EXERCISES IN UNIT VII

Pages 47–48: Exercise I

A. 32 squares
   8 units length
   4 units length

B. 16 squares
   8 units base
   4 units height

C. 36 squares
   6 units side

D. 18 squares
   6 units base
   6 units height

E. 54 squares
   12 units $base_1$
   6 units $base_2$
   6 units height

F. 72 squares
   18 units $base_1$
   6 units $base_2$
   6 units height

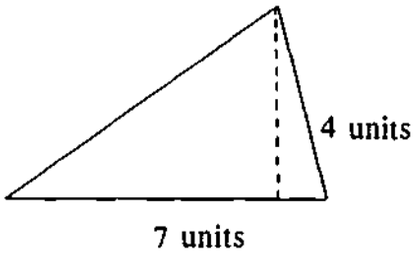
G. 72 squares
   12 units $base_1$
   12 units $base_2$
   6 units height

Page 50: Exercise II

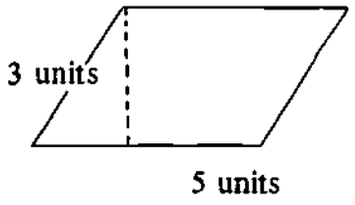
$A = lw$	A (C could also be correct.)
$A = \frac{1}{2}\ bh$	B, D, G
$A = s^2$	C
$A = \frac{1}{2}h\ (b_1 + b_2)$	E, F

Pages 50–51: Exercise III

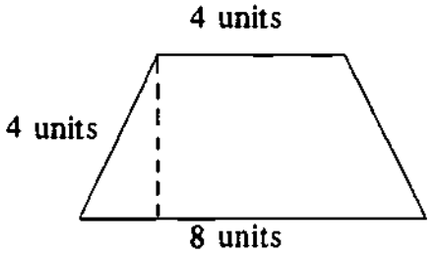
1.



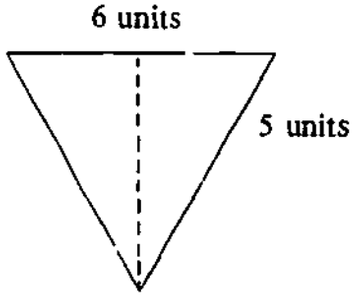
2.



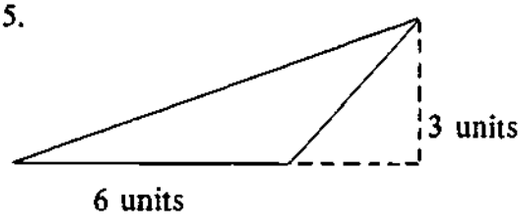
3.



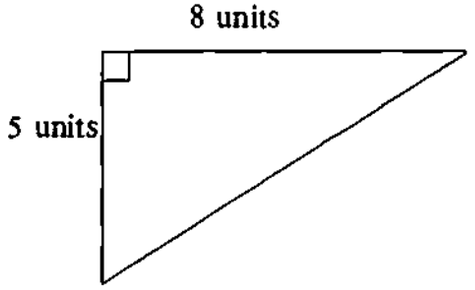
4.



5.



6.



Page 52: Exercise IV

1. Formula: $A = s^2$
   Equation: $A = 6^2$
   Area: 36 units ^{2}

2. Formula: $A = \frac{1}{2}h\ (b_1 + b_2)$
   Equation: $A = \frac{1}{2} \times 5\ (5 + 8)$
   Area: $32\frac{1}{2}$ units^2

3. Formula: $A = lw$
   Equation: $A = 11 \times 3$
   Area: 33 units^2

4. Formula: $A = \frac{1}{2}bh$
   Equation: $A = \frac{1}{2} \times 6 \times 5$
   Area: 15 units^2

5. Formula: $A = \frac{1}{2}h\ (b_1 + b_2)$
   Equation: $A = \frac{1}{2} \times 4\ (6 + 10)$
   Area: 32 units^2

6. Formula: $A = \frac{1}{2}bh$
   Equation: $A = \frac{1}{2} \times 6 \times 3$
   Area: 9 units^2

Page 53: Exercise V

1. Formula: $A = \frac{1}{2}bh$
   Equation: $A = \frac{1}{2} \times 50 \times 30$
   Area: $A = 750\ cm^2$

2. Formula: $A = \frac{1}{2}h\ (b_1 + b_2)$
   Equation: $A = \frac{1}{2} \times 2\ (6 + 2)$
   Area: $8\ m^2$

3. Formula: $A = \frac{1}{2}bh$
   Equation: $A = \frac{1}{2}\ (25 \times 15)$
   Area: $375\ cm^2$

4. Formula: $A = s^2$
   Equation: $A = 30^2$
   Area: $900\ m^2$

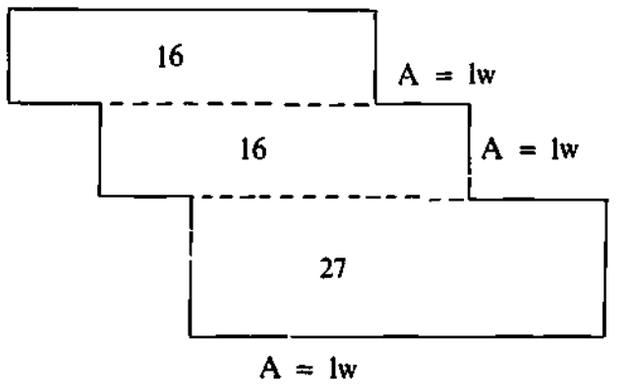
5. Formula: $A = \frac{1}{2}bh$
   Equation: $A = \frac{1}{2}\ (80 \times 60)$
   Area: $2400\ cm^2$

6. Formula: $A = \frac{1}{2}h\ (b_1 + b_2)$
   Equation: $A = \frac{1}{2} \times 18\ (42 + 30)$
   Area: $648\ cm^2$

Pages 54–55: Exercise VI

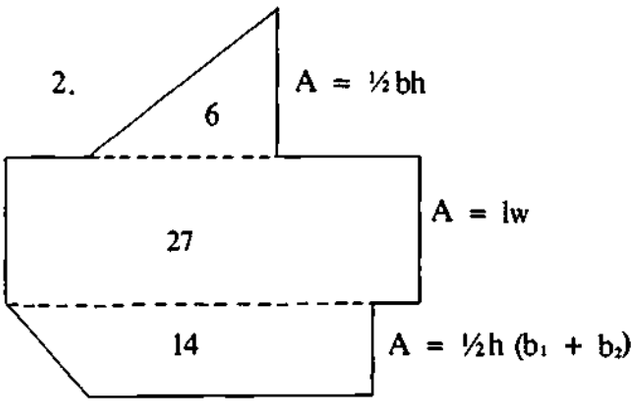
Each diagram below shows one possible solution.

1.



$A =$ 59 units^2

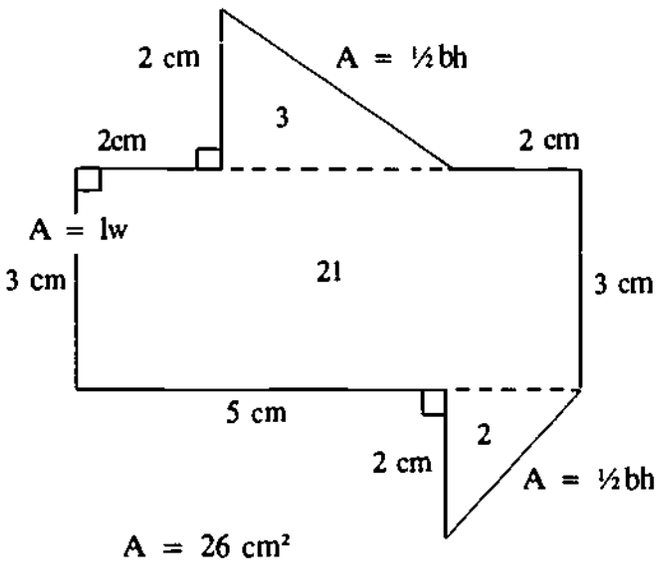
2.



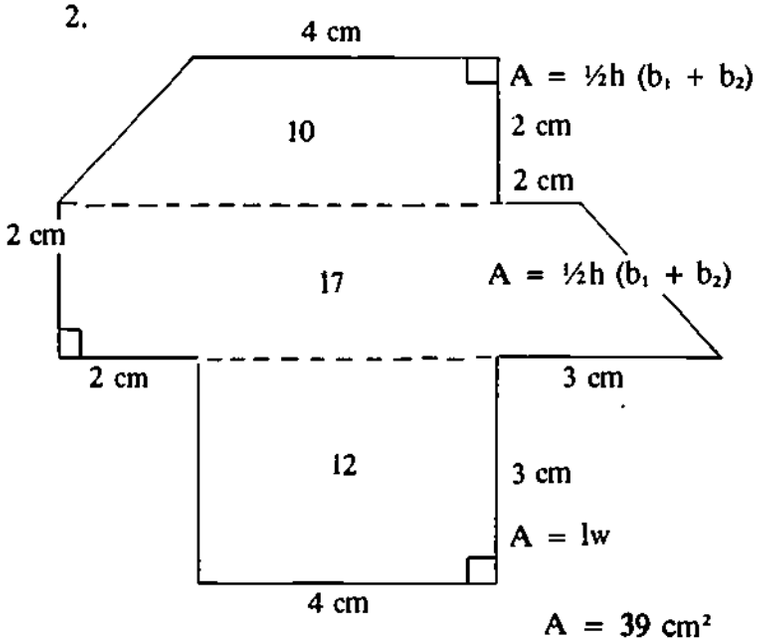
$A =$ 47 units^2

Pages 55–56: Exercise VII

1.



2.



Page 56: Exercise VIII

Answers will vary.

Page 57: Exercise IX

1. 150 m²
2. 200 m²
3. 9 m²
4. 225 m²
5. 275 m²

Page 58: Exercise X

1. 105 miles
2. 38.4 m
3. 60 m
4. .429
5. .444

Additional Suggestions

1. Have students write formulas for common tasks, i.e., averaging test scores for a marking period; computing scoring averages for the school basketball team; etc.
2. Begin a collection of formulas from the sciences which can give students additional practice in developing skills for using formulas.
3. Have students draw compound figures on oaktag and place sample area solutions on the back. Let students work on finding the areas of these figures when they have completed their class work.

UNIT VII SUMMARY: USING FORMULAS

A formula is a general rule which you can use to solve practical problems.

Formulas are written as equations using symbols.

To use formulas, you need to:

1. Know the right formula to use for a particular problem.
2. Know what each symbol in the formula means.
3. Know how to replace the symbols or letters with the right numbers from the problem.

Write each formula that you need to learn on the front of a 3 x 5 card. Write a sample problem using that formula on the back of the card and solve it. Use your cards to learn formulas and to study for tests.

UNIT VIII: ESTIMATION

Estimation is a fundamental mathematical skill which is effectively learned only through ongoing practice. "Teachers should incorporate estimation activities into all areas of the program on a regular and sustaining basis, in particular, encouraging the use of estimating skills to pose and select alternatives and to assess what a reasonable answer may be."[1]

We can daily see the importance of estimation when we correct students' math papers and find answers that make no sense at all. "Estimation should be the first step in any serious calculation..."[2] When students learn to estimate to check the reasonableness of their answers before they calculate, they make far fewer mistakes.

Another critical value of estimation skills involves the use of the calculator. As students will be working with calculators for the rest of their lives, it's important that they learn to use them intelligently. One example of such use is employing estimation with the calculator to test for a reasonable answer. A second use is illustrated by the exercise in this unit where students see that mental arithmetic can sometimes be faster than using a calculator. This kind of experience can help students learn to assess the calculator as a tool which is more useful in some situations than others.

The activities of this unit engage your students in learning what the basic estimation skills are and in practicing several ways to use these skills. Some of the exercises in this unit call for patience on your part. Students are accustomed to answers which are right or wrong. You may need to help them understand that a close answer is all that is needed in some situations and that an educated guess is a good strategy for solving some problems.

Please Note: For Exercises II and III (pgs. 63, 65) you will need enough calculators for at least half of your class.

THE JELLY BEAN: ANOTHER CHANCE is on page 68.

Suggested Directions for Unit VIII

1. Ask your students to read the directions for "The 'Educated Guessing' Game" (pg. 60) but not to begin until you give the word. (If your students are not familiar with the term "educated guess," you may want to explain it to them.) Then, give them a signal to begin and another signal to stop after 20 seconds. Go over the problem by having a student explain his/her solution. If others solved the problem in different ways, ask them to explain their procedures.

   Approximate time: *3-5 minutes*

1. *An Agenda for Action: Recommendations for School Mathematics of the 1980s.* National Council of Teachers of Mathematics: Reston, Virginia, 1980.

2. *Project Torque. A New Approach to the Assessment of Children's Mathematical Competence.* Educational Development Center, Inc.: Newton, Mass., 1976.

2. Have your students do "Another 'Educated Guessing' Game" (pg. 60) in the same way as the previous game. Go over the problem. Then, ask your students to read "Estimation" (pg. 61). When they have done so, discuss what estimation is and how they either used it or could have used it in the two games.

   3-5 minutes

3. Have your students read "Rounding" (pg. 61). Discuss what "rounding to the highest place value" means. Then, ask your students to do Exercise I (pg. 61). Go over the answers.

   10 minutes

4. Ask your students to read "Powers and Multiples of 10" (pg. 62) and to answer the question at the bottom. Have your students discuss any patterns which they observed and any "tricks" which they discovered.

   8-10 minutes

5. Have your students read the directions for Exercise II (pg. 63). Divide the class into two groups, and equip Group A with calculators. Then, ask your students to begin the exercise. When students raise their hands to show that they are done, tell them the number of seconds it took them to complete the exercise. Go over the answers; then, discuss why this activity can be accomplished more quickly with mental arithmetic than with a calculator.

   10 minutes

6. Ask your students to read "Dividing with Powers and Multiples of 10" (pg. 64) and to answer the question at the end of the section. Have your students discuss any patterns which they observed and any "tricks" which they discovered. Then, have students in Group A give their calculators to people in Group B; have your students do Exercise III (pg. 65) in the same way that they did Exercise II.

   15 minutes

7. Ask your students to read the directions to "The Jellybean Game" (pg. 66). Then, give them a signal to begin. When they have finished, go over the answers and ask them to compute their scores.

   (You may want to set a time limit for this game to emphasize the need for the use of estimation rather than calculation. For example, call "time" when about 75% of the students are done.)

   10 minutes

8. Follow the same procedures for "The Jellybean Game: Another Chance" (pg. 68). Then, have students discuss how and why they worked differently the second time through.

   (Many students will have some difficulty and will proceed slowly the first time they play the game. On the second round, most will work much more quickly and will probably finish within the time limit you set for the first round. If possible, allow all students to finish in the second game.)

   10 minutes

9. Have your students read "Numbers between 0 and 1" (pg. 67) and answer the question at the end of the section. Discuss the patterns which they discovered.

   5 minutes

10. Ask your students to read "When Do You Estimate?" (pg. 69) and do Exercise IV (pg. 69). Go over the answers; ask your students to explain how they judged an answer as reasonable or not.

    10 minutes

11. Have your students read "Another Use for Estimation" (pg. 70) and do Exercise V (pg. 70). Go over the answers.

    10 minutes

12. Ask your students to read "Estimation and Problem Solving" (pg. 70) and do Exercise VI (pg. 71). Go over the answers.

    10 minutes

13. Organize your class into pairs so that each pair has at least one calculator. Ask your students to read "Guess and Check" (pg. 71) and complete Exercise VII (pg. 72) with their partners. Go over the answers; discuss the ways in which the guess and check strategy can be useful.

    (You may want to explore other patterns associated with division by 11. Some students will quickly see that the pattern is dependent on the remainder and that the remainder is related to the multiplication table of nines.)

    15 minutes

ANSWERS FOR EXERCISES IN UNIT VIII

Page 60: The "Educated Guessing" Game

Yes

Page 60: Another "Educated Guessing" Game

Yes

Page 61: Exercise I

1. 5000
2. 300,000
3. 60,000
4. 10,000
5. 5,000,000
6. 30
7. $80.00
8. $400.00

Page 62: Powers and Multiples of 10

Answers will vary. For example: "If there are two zeroes in the first factor and three zeroes in the second factor, there will be five zeroes in the answer, unless the two numbers to be multiplied, like 5×6, have a zero. Then, there will be six zeroes in the answer.

Page 63: Exercise II

a. 1500
b. 400,000
c. 18,000,000
d. 800,000
e. 32,000,000
f. 2,500,000
g. 40,000,000
h. 90,000,000

Page 64: Dividing with Powers and Multiples of 10

Answers will vary. For example: "You subtract the number of zeroes in the number you're dividing by from the number of zeroes in the number you are dividing. The exception is when one of the zeroes is needed for the division, as in 200,000 divided by 500."

Page 65: Exercise III

a. 9000
b. 40
c. 600
d. 50
e. 20
f. 400
g. 1000
h. 7000

Page 66: The Jellybean Game

Each answer which falls within the range specified below warrants 1000 jellybeans. A range is allowed because students may round in different ways. For example, in the first problem, a student may compute 9000×1000 or 9000×1100 or 9000×1200.

a. 9,000,000 - 11,000,000
b. 54,000,000 - 65,000,000
c. 47,000,000 - 48,000,000
d. 35 - 43
e. 4 - 6
f. 23,000,000 - 27,000,000
g. 43 - 50
h. 22,000,000 - 26,000,000
i. 49 - 57
j. 45,000,000 - 53,000,000

Page 67: Numbers between 0 and 1

Answers will vary. For example: "When you multiply by tenths, it's the same as crossing off one zero. When you divide by tenths, it's the same as adding one zero."

Page 68: The Jellybean Game: Another Chance

a. 18 - 22
b. 7.5 - 9.0
c. 22,000,000 - 25,000,000
d. 46 - 52
e. 36,000,000 - 42,000,000
f. 200 - 220
g. 7,000,000 - 9,000,000
h. 18,000,000 - 20,000,000
i. 13,000,000 - 16,000,000
j. 45 - 50

Page 69: Exercise IV

Incorrect or "unreasonable" answers: c, d, e, g

Page 70: Exercise V

Examples in which the solution is a number between 50,000 and 90,000: a, b, g

Page 71: Exercise VI

1. 18,900
2. a, b, d, e

Page 72: Exercise VII

1. The intermediate guesses will vary. Cindy is 27 years old.
2. The intermediate guesses will vary. The numbers are 203 and 197.
3. 3.272727
   5.272727
   25

Additional Suggestions

1. Have students write additional Jellybean Games for the class. Have them write the problems on a duplicating master. Remind them to attach an answer sheet.

   At this stage, the range for correct answers should be between 10% more and 10% less than the exact solution. Students should use a calculator.

   This activity gives your student practice with using a calculator and figuring percents as well as providing you with additional estimation activities for the whole class.

2. Have your students write an estimate for every math problem that they do. This is good practice for estimation, and it makes your students think about what they are doing.

UNIT VIII SUMMARY: ESTIMATION

When you estimate, you figure out approximately what the answer is. An estimation is not an exact answer. Rather, it gives you an idea of about what the answer is.

To become a good estimator, you need to practice:

1. Rounding off numbers
2. Computing with numbers which have been rounded
3. Mental arithmetic, or computing in your head

You can use estimation to check answers that you've gotten with your calculator or through computation. A quick estimate will tell you if your answer is reasonable or not.

Estimation is also a useful skill for problem solving. One special way of using estimation to solve problems is the guess and check strategy. You read the problem, make a good guess about the answer, and check your guess. Then you use the information you've gained from the checking to make a better guess. You repeat these steps until you find the answer.

UNIT IX: PREPARING FOR A TEST

The ability to organize one's preparation for a test and to follow through with one's planning is a highly rewarding skill which many pre-Algebra students have not mastered. Students' inability to act effectively in this way is also one of the prime sources of test anxiety.

This unit introduces students to the idea that their own anxious feelings impair their capacity to learn and that they can alleviate their anxiety by planning ahead. The exercises in the unit familiarize students with a procedure for test preparation which they can adapt to their own needs.

The best time to present this unit is at the completion of a chapter when an actual test is imminent. Students can learn about this procedure and practice the steps in a real-life situation. This experience will help them to see clearly the benefits of planning and working ahead.

Test anxiety is addressed early in the unit because it can be so debilitating. Many students not only do not understand it but also believe it is unique to their own experience. Through class discussion, you can emphasize the idea that this kind of anxiety is a natural and common experience.

The procedure for test preparation which this unit introduces emphasizes planning ahead, studying over a period of several days, and reviewing actively. Students need to learn that simply reading mathematics material is not sufficient for effective reviewing in most cases. Review must be active, with pencil in hand.

The unit closes with the introduction of a simple relaxation and focusing technique. Through this method, most students can learn to calm and focus their energies when they need to do so.

Suggested Directions for Unit IX

1. You will need a supply of 3 by 5 cards for the exercises in this unit.

2. Have your students read "How Do <u>You</u> Prepare For A Test?" (pg. 74) and answer the question at the end. Tell them that when they have finished writing, they can go ahead and read "Getting Started. What Gets In The Way?" (pgs. 75–76). When they have all completed this section, discuss and clarify test anxiety, how and when people experience it, and the idea that it can be alleviated by planning and working ahead, i.e., if you feel prepared, then you won't feel so anxious. Give your students an opportunity to talk about test anxiety in ways that feel appropriate for them.

   Approximate time: <u>*15 minutes*</u>

Please note: You may wish to organize your class into pairs and have your students do the remainder of this unit cooperatively.

3. Have your students read "Preparing For A Math Test" (pg. 76) and the directions for Exercise I (pg. 77). Emphasize that for Steps 1-5, they are to read all of the instructions but only follow the underlined ones during this class.

   5 minutes

4. Ask your students to read the first instruction in Step 1. Then, tell them which chapter they will be studying and when the test will be. Ask them to complete Step 1 (pgs. 77-78). When they are done, go over the topics listed and starred. If students have variations in their lists, allow them to discuss why they have chosen as they have. If necessary, help them learn to see which topics are most important.

   10-15 minutes

5. Have your students read and complete Step 2 (pg. 78). When they have finished, have them share their cards with a neighbor or two. You can circulate to be sure that everyone is following the correct procedure.

   10 minutes

6. Have your students read and complete Step 3 (pg. 79). When they are done, ask your students to discuss the suggestions which they have checked and the priorities which they have assigned. It is useful for students to see that there will not necessarily be agreement on the assignment of priorities, that, in some cases, this is a matter of individual learning style.

   10 minutes

7. Ask your students to read and complete Steps 4 and 5 (pgs. 80-82). Then, go over the rules, principles and formulas which are important in this chapter. Also, give your students an opportunity to share some of the questions which they have created with each other. This can be accomplished with the whole class or in small groups. Finally, have your students read "The Night Before The Test" (pg. 82). Discuss briefly.

   15-20 minutes

8. Ask your students to read "What Can You Do When You Feel Nervous?" (pg. 83). Then, guide them through the "Relaxation Exercise" (pgs. 83-84). You'll find the "script" for the exercise reproduced below. If you're not familiar with this kind of exercise, please read the discussion below the "script" in the Teacher's Guide. When you have finished the relaxation exercise, give your students an opportunity to discuss their experience. Then, have them read "A Few Notes About Relaxing" (pg. 83). You may also want to go over the "Unit Summary" (pg. 84) with them.

   10-15 minutes

RELAXATION EXERCISE: "SCRIPT"

Sit in a comfortable position.

Close your eyes and take a couple of smooth, deep breaths. Feel the air come in all the way down to your abdomen, then feel the air go out again ---

Now, sit quietly and just be aware of your breathing for a little while --- Don't try to control it. Just be aware of your breath coming in, your breath going out ---

Now, with your inner eye, imagine a peaceful scene that you like. See this peaceful scene as clearly as you can --- What do you see? --- What do you hear? --- What do you feel? --- Make this scene very, very peaceful ---

Do you see yourself in your peaceful scene? If you're not already there, put yourself there if you can. And feel the relaxation of this peaceful, calm place --- Feel yourself very relaxed, and wide awake ---

Now, with your inner voice, tell yourself: I am relaxed and awake and ready to learn. I am relaxed and awake and ready to learn. Say this two or three times more ---

Now, gently open your eyes. You're ready to begin your learning!

RELAXATION EXERCISE: DISCUSSION

The ability to relax, reduce stress, and focus one's energies can be a very helpful tool. We all know from our own experience that high stress and over-anxiety inhibit learning. When students know how to address their own anxiety both by planning and studying ahead and by relaxing and focusing themselves in the moment, they have at hand a pair of powerful skills for enhancing their own learning and performance.

If you have not guided an exercise like this one before, practice a few times before you try it with a class. A friend, your own children, someone in your family, or a colleague would certainly be interested in learning this skill. You can also tape yourself and then listen to the tape.

Speak slowly and calmly with a peaceful tone of voice. When you are asking the listener to act in some way, give her or him sufficient time to complete the action. The dashes in the script above suggest places where you may want to pause.

When you are doing this exercise in class, try to be sure that you will not be interrupted by sources outside your room.

If you are interested in learning more about relaxation and focusing exercises, you may want to look at *The Relaxation Book* by Gerald Rosen (1977: Prentice-Hall) and *Guide To Stress Reduction* by L. John Mason (1980: Peace Press). Another interesting resource on this subject is *Holistic Health* by Patricia Randolph Flynn (1980: Robert J. Brady Co.).

ANSWERS FOR EXERCISES IN UNIT IX

Answers for most of the exercises will depend on the chapter with which you are working.

Answers for Step 3 (pg. 79) will vary. It is important, however, for students to realize that suggestions like "turn on the T.V." and "plan to do the hard problems tomorrow" are not constructive or helpful.

Additional Suggestions

1. After the test, go over the questions which your students created in Step 5: Final Review. Have your students examine how well they did in projecting what would be on the test. Discuss how they could learn to do this more effectively.
2. Share your own tips and "tricks" about taking tests with your students.
3. Suggest to students that they may want to illustrate their review cards as a way of making them more useful. For example: line drawings, charts and graphs, cartoons, etc.
4. Several weeks after you've taught this unit, have your students bring their review cards to class for a test which is imminent. Have them work in small groups and share review cards. This will give students an opportunity to compare cards and will help them assess their skills in this area.

UNIT IX SUMMARY: PREPARING FOR A TEST

Anxiety or nervousness about a test is a very natural feeling. Everyone feels this way at times. But too much anxiety can get in your way when you're studying for a test.

One good way to feel calmer is to have a plan for preparing for the test. Then, follow your plan.

Begin your studying a few days before the test. Even 20 minutes a night can be enough time.

List the major topics on which you'll be tested.

Make a review card for each major topic. A review card includes the topic and a sample problem for that topic on the front of the card, and the solution to the problem on the back.

Also, make review cards for any rules, formulas, and principles that you need to know. Use the cards to help you memorize the formulas, rules, and principles.

Use all of your review cards to do a final review before the test.

If you feel too nervous or anxious to study, relax yourself by using a relaxation exercise like the one in this unit.

Be sure to get a good night's sleep before the test!

UNIT X: TAKING A MATH TEST

By the middle school/junior high years, students have been taking mathematics tests for years, but most have never had the opportunity to gain insight into the nature of test questions. When students begin to understand how test questions are structured, they learn to assess their math material in more realistic ways. In particular, they gain skills for deciding what elements of their math material are most important.

The exercises in this unit engage your students in a simple analysis of the most common kinds of math test questions. In these activities, students are asked to examine test questions from the test writer's point of view. They are also offered tips for answering test questions more effectively.

The first part of this unit connects these exercises with those in Unit IX. Students are asked to reflect on their own experience of math tests as threats or challenges. The aim of this activity is to give students an opportunity to re-frame their attitudes towards tests in more positive ways.

Please note: As with Unit IX, this unit will be of most value when it is taught in relation to an actual test. The goal-setting exercise on page 85, in fact, calls for students to set a goal for a real-life test.

However, we suggest that you teach this unit in conjunction with a later test than the one you use with Unit IX, as students may experience both units for one test as an "overload."

Suggested Directions for Unit X

1. Let your students know when they will have the test for which this unit will help to prepare them and what will be covered.

2. Ask your students to read "Threat or Challenge?" (pg. 85) and answer the question at the end. Have several students share their descriptions of the difference between a threat and a challenge with the class; then, discuss for clarity and understanding.

   Approximate time: *5 minutes*

3. Have your students read "Challenging Yourself In Math" (pg. 85) and set goals for themselves. You may want to discuss the nature and value of goal-setting in more detail. Stress that goals must be both realistic and challenging to be useful. You may also want to give students the chance to discuss their goals with each other in small groups for a few minutes.

   5 minutes

4. Ask your students to read "Who Makes Up Tests?" (pg. 86). It would be helpful to the students for you to relate how you plan and develop the tests in your classes. This discussion could also include reasons for giving tests.

   5 minutes

   Please note: You may want to have students work some or all of the exercises in this unit in pairs or small groups. The kinds of discussion that would take place about the exercises would probably be very useful to students' learning.

5. Have your students read "Multiple Choice" (pg. 86) and do the example. Go over the example and discuss it. Then, ask your students to complete Exercise I (pg. 87). Go over the exercise and discuss it. Or, you could have students exchange papers and discuss the "errors" in pairs.

   15 minutes

6. Ask your students to read the "Tips For Multiple Choice Questions" (pg. 88). Discuss for clarity and emphasis. You may also want to have your students work some multiple choice questions of your own design at this point, so they can practice the "tips" at once.

   5 minutes

7. Have your students read "Matching" (pg. 88). Answer any questions which students raise. Then, have them do Exercise II (pg. 89). Go over the exercise and have students discuss and evaluate the various "suggestions" and "reasons."

   10 minutes

8. Ask your students to read "True/False" (pg. 89). Answer any questions which they raise. Then, ask them to do Exercise III (pg. 90). Go over the exercise.

   10 minutes

9. Ask your students to read " 'Show Your Work' Problems" (pg. 91). Discuss briefly for clarity and emphasis. Then, have your students do Exercise IV (pgs. 91-92). Have students discuss the number of points which they gave for each solution and the reasons why they did so.

   10 minutes

10. Ask your students to recall occasions when they had to fill in computer forms. Ask them to share any difficulties or problems which they experienced. Then, have your students read "Tests And Computer Cards" (pg. 92) and the directions for Exercise V (pg. 92). Answer any questions which they may raise. Then, ask them to do Exercise V (pgs. 92-94). You may want to emphasize that students are to choose answers which are most correct for them as individual learners. When your students have finished the exercise, have them discuss their answers in small groups. Or, go over the questions with the whole class, asking several students to share their answers for each question.

    15 minutes

ANSWERS FOR EXERCISES IN UNIT X

Page 85: Threat or Challenge?

A threat is something that may do harm to you.
A challenge is an opportunity to test yourself, to see how well you can do.

Page 86: Example

14 + 8 = 8 + 14

a. 6: ignored the 8 on the left side of the equation, added 8 + 6 to get 14

b. 16: added incorrectly, did not see the identity between both sides of equation

c. 22: added 14 + 8, ignored the 8 on the right side of the equation

Page 87: Exercise I

Answers will vary. Be sure that students offer good reasons for the "mistakes."

Page 89: Exercise II

7
1
6
3
2
5
4

Page 90: Exercise III

1.	false	(all)
2.	true	(none)
3.	true	(most)
4.	true	(always)
5.	true	(some)
6.	false	(all)
7.	false	(all)

Pages 91-92: Exercise IV

Answers will vary for points assigned. The correct solution is $.89.

Pages 92-94: Exercise V

Answers will vary for questions 1-9.

Additional Suggestions

1. As a homework assignment or a class lesson, ask students to develop tests for a chapter or unit. Remind them to provide an answer sheet for each test. Students can then work in small groups to evaluate the various individual tests and compile one test for their group. Remind them to set parameters for scoring the tests.

2. Collect math tests from a variety of sources: other math texts, workbooks, standardized tests which are not used in your school system. Have your students take these tests or parts of them with the understanding that they will not be graded on them. Students can then correct their own tests. This experience gives them the opportunity to practice test taking in a non-threatening situation.

UNIT X SUMMARY: TAKING A MATH TEST

Learn to see a math test as a challenge, not a threat, by:

1. Planning your study and following your plan, so you'll be ready for the test and feel less threatened by it.
2. Setting a goal for the test. Be sure that your goal is realistic and challenging.
3. Understanding how math tests work. The more you know about how test questions are made, the more you'll know about how to answer them.

The kind of questions used most often in math tests are:

1. Multiple choice	Read the question carefully. See if you can figure out the answer before you look at the choices. Then, read all of the choices and pick the best one. If you don't know which answer is right, cross out all of the ones which you know are wrong. Then, pick the best answer from the remaining choices.
2. Matching	Read the directions carefully. Use a process of elimination to match the items on the two lists. Do the ones you know first. Then, do the best you can with the items left.
3. True/false	Read the question carefully. If any part of the statement is false, it's a false statement. Watch for key words like always, only, all, never, often, usually, and none. Think about what these words mean in the statement.
4. 'Show your work' problems	Show all of your work. Organize your work on your paper as neatly as you can. Use a pencil so you can erase.

When you record your answers for a test on a computer card, you need to be very careful. Be sure that you mark only the boxes that you want to mark.

An Overview of Study Skills

During the past decade, many educators have defined basic skills primarily in terms of communication and computation skills. Study skills, though often neglected or ignored, are also basic skills. These skills for learning are at the core of the educative process.

WHAT ARE STUDY SKILLS?

Study skills are learned abilities for acquiring knowledge and competence. The table of contents of a study skills handbook or text usually includes, among others, the following skill areas: listening; textbook reading; note taking; planning your time; study behavior and environment; vocabulary skills; test taking. These skills and others which fit into the category of study skills directly relate to many of the activities in which students are engaged while they are in school or doing schoolwork.

A broader definition of study skills or learning skills views them as processes for learning. When one learns a study skill, one is learning more than a specific technique. One is learning a way of problem solving, a method which can be used in any relevant contest. One is also learning more about how to leam effectively and how to be in charge of one's own leaming. It is this perception of study skills as transferable processes for learning which reveals the critical importance of these skills.

A review of nearly 80 years of study skills literature indicates that, at least throughout this century, the nature of study skills has remained relatively constant. Study skills in 1940 were much like those in 1920, in 1960, and even in 1980. There have been variations of emphasis as well as some genuinely innovative developments over the years, but the strongest impression is one of continuity.

This continuity is important to keep in mind in view of the trendy nature of American education which often seems to produce a good deal of ill-considered change. Even though an element of the curriculum is found to be useful, we still often choose to eliminate or ignore it whenever the next new trend appears. Because study skills have been relatively impervious to change, they have been periodically discarded over the years, only to be rediscovered much later. This pendulum effect has had damaging results because the importance of learning how to learn never diminishes.

HOW HAVE STUDY SKILLS BEEN TAUGHT?

The most common approach to study skills instruction during the first few decades of this century was essentially the preaching of morality. This approach related good study skills to what was called "high morals," indicating that students who displayed the correct moral values would be the ones with good study skills.

This moralistic approach to the teaching of study skills peaked before World War II, at least if the literature is an accurate guide, and was replaced by a focus on techniques and formulae. The most famous of these formulaic methods is SQ3R, an excellent method for reading a textbook which was developed by F. P. Robinson.

The chief characteristic of a formulaic approach to study skills is the learning of a specific series of behaviors. For example, using SQ3R, one learns to survey, question, read, recite, and review. Unfortunately the use of such formulae often descends to the level of gimmickry. Students are taught to use a particular technique as if it involved an act of magic; that is, something beyond their active and conscious participation, comprehension, and control. Although formulaic methods attempt to create understanding and involvement in the student, the mere fact of their use guarantees neither of these necessary outcomes.

While formulaic techniques can be very useful, they must be employed within a framework for the teaching of study skills which engages the student not only in learning a particular skill but also in learning about what happens within oneself when one learns and uses a study skill. It is important for the student to learn why a skill works and why it is valuable as he or she is learning how to do it. A central part of this learning involves the student's initiative in the exercise of judgment and choice.

A third method of teaching study skills is the handbook approach. While this method claims to understand and appreciate the value of study skills, it argues that no class time is available for study skills instruction. Thus, this approach relies on the handbook to elicit involvement and learning. Students are given written descriptions of various study skills and are expected somehow to translate verbal abstractions into behavior. A few students can and will make this leap, but the vast majority cannot, because they lack the motivation, self-discipline, and conceptual ability required to learn study skills independently. Though not uncommon, the handbook approach is not particularly helpful. Although it is an effort to address the need for teaching study skills, its use results in a minimum of learning while simultaneously creating frustration and resistance in the student.

STATUS OF STUDY SKILLS TEACHING

The overview of study skills instruction presented below is primarily drawn from two sources: the authors' observations in six eastern states over the past nine years, and an extensive national survey conducted by one of their colleagues. While this overview is a generalization to which specific contradictions can certainly be cited, the larger picture which it offers seems accurate.

During the past several years, the "back to basics" movement has led to a greater emphasis on basic skills during the middle school years. However, this movement has not addressed the lack of effective study skills instruction in most elementary, middle, and junior high school curricula. Despite the renewed concern for skills education, the widespread failure to regard study skills as basic skills has resulted in a continuing lack of emphasis on study skills instruction.

Another problem involves the nature of study skills teaching which does take place. Though many teachers do include some study skills instruction in their curricula, they tend to do so with a lack of focus on precisely what skills they wish to teach and on how their students' learning is going to occur. This absence of focus results in a lack of coordination of their efforts towards the teaching of study skills in any given class over the course of the year. Rather than a well-planned, highly coordinated effort, what often results is haphazard collection of insufficiently related lessons.

A third shortcoming is the lack of coordination of efforts in teaching study skills among teachers of different subjects at the same grade level. While there is some uncertainty and conflict in regard to the responsibility for teaching other basic skills, this confusion is particularly intense in regard to study skills. For example, is the English teacher the only one who is responsible for teaching the various communication skills which are also study skills? Or should every teacher deal with these in some way relevant to his or her own subject? The reality is that schools often fail to delineate the areas of responsibility for the teaching of study skills. It is no wonder that confusion arises among teachers; ultimately, though, it is the students who suffer most.

A fourth problem is an extension of the previous one: not only do schools fail to coordinate and organize the teaching of study skills in any particular grade, but the same phenomenon also takes place on a system-wide basis throughout a student's academic career. Teachers often assume that somebody else has taught certain skills to their students or will teach them later on. But often no one ever gets around to teaching them because there is no clear assignment of responsibility.

HOW DO WE TEACH STUDY SKILLS?

People learn skills through processes of repeated trial and error. One key to effective study skills teaching, then, is providing the student with sufficient opportunity for practice of the skills to be learned. Each **hm Study Skills Program** includes only the initial practice for each skill which is introduced. If your students are to master the various study skills presented in this Program, it is essential that you provide them with structured opportunities for practice of the various skills.

Of course, there is an inevitable tension between providing students with trial and error practice of a new study skill and helping students to maintain their interest in learning the skill in the face of the necessary repetition. While this tension cannot be removed, it can be minimized by acknowledging the tension as a natural part of learning something new, and through the use of varied and imaginative instructional design.

A second key to study skills teaching is the recognition that learning a study skill requires the learner to err before he or she can succeed. We learn skills by trying to use a new skill, making mistakes, identifying our mistakes and learning from them, and then correcting our mistakes. This understanding creates several responsibilities for the teacher:

1. The teacher provides a space within the learning process where the student can try out a new skill, experience a good deal of error, but not feel that he or she has failed or is a failure.
2. The teacher provides usable feedback to the student about the effectiveness of the student's use of the new study skill.
3. The teacher rewards the student for what he or she has done well in using the new study skill. With such recognition, the student experiences success in the learning process and will be more motivated to continue the development of mastery in the new study skill. Recognition involves the acknowledgement of the effort to risk in the face of possible failure.

In addition to these two key understandings, a program of effective study skills instruction would be based on the following underlying values:

1. Study skills need to be defined as processes for learning.
2. Study skills need to be included within the cluster of basic skills.
3. An important part of learning study skills involves learning more about how one learns. Thus, instruction in study skills engages the student in an active participation in his or her own learning. The student is encouraged and provided with the opportunity to develop the ability to exercise his or her own judgment in regard to the learning and use of study skills.
4. Learning by doing is the best way to learn study skills.
5. A considerable part of study skills instruction ought to take place during class time. Such instruction should also be integrated with the regular curriculum of the course.
6. The learning of study skills offers a transfer effect; a skill learned in relation to one subject can be applied to any other relevant context.
7. The work of various developmental psychologists has shown us that there is a continuum of cognitive development throughout the years of childhood and adolescence, and that people are only able to deal successfully with learning tasks which are appropriate for their level of development. It is crucial to relate this insight to the teaching of study skills and to ask a student to learn only what is within the realm of his or her cognitive ability at that level of development.