DOCUMENT RESUME

ED 241 311 SE 044 004

TITLE Let Problem Solving Be the Focus for 1980's.
INSTITUTION Alberta Dept. of Education, Edmonton.
PUB DATE 83
NOTE 119p.
PUB TYPE Guides - General (050)

EDRS PRICE MF01/PC05 Plus Postage.
DESCRIPTORS Answer Keys; Calculators; Computers; Curriculum Development; Educational Objectives; Elementary Education; *Elementary School Mathematics; Instructional Materials; *Mathematics Curriculum; *Mathematics Instruction; Motivation; *Problem Sets; *Problem Solving; *Resource Materials; Teaching Methods; Worksheets
IDENTIFIERS Alberta (Edmonton)

ABSTRACT

This publication provides background information, ideas, and sample activities to help teachers in their efforts to aid students in developing problem solving skills. It provides an overview of the stages in the problem solving model recommended in the elementary mathematics program of studies. Several clusters of problem solving strategies are suggested for each stage of the model. Additional ideas on organization for instruction, the use of textbooks, and sources of problems and problem solving materials are included in a reference list. The document is divided into sections on problem solving as the focus of the curriculum, a general framework for problem solving, the problem solving model, planning for instruction in grades 1 and 2 and in grades 3 through 6, using the prescribed learning resources, and working through the model. Extensive classroom materials follow, such as sample problems for grades 1 through 6, calculator problems, computer problems, and challenge problems. (MNS)

LET PROBLEM SOLVING BE THE FOCUS FOR 1980's

1983

Curriculum

Alberta EDUCATION

TABLE OF CONTENTS

ACKNOWLEDGEMENTS

Alberta Education acknowledges with appreciation the contributions of the following members of the Problem Solving Committee operating under the direction of the Mathematics Coordinating Committee and Curriculum Policies Committee.

PROBLEM SOLVING MONOGRAPH COMMITTEE

Al Anderson, Consultant, Medicine Hat Public School Board
Rod Anderson, Supervisor, Edmonton Public School Board
Bill Bober, Supervisor, Edmonton Separate School Board
Hank Boer, Coordinator, Lethbridge School District
Marshall Bye, Supervisor, Calgary Board of Education
Joe Hazelzet, Supervisor of Applied Sciences, Calgary Catholic School Board
Don Nissen, Supervisor, Elementary Education, County of Strathcona
Richard Pawloff, Coordinator of Instruction of Computers and Mathematics, Red Deer Public School District
Garry Popowich, Associate Director of Curriculum, Curriculum Branch
Bruce Stonell, Consultant, Alberta Education
Henry Taschuk, Consultant, Edmonton Public School Board
Joan Worth, Professor, University of Alberta

PURPOSE

This monograph provides background information, ideas and sample activities to teachers in their efforts to assist students in developing problem solving skills. The problems have been selected to illustrate specific problem skills endemic to each grade level.

The monograph also provides an overview of the stages in the problem solving model recommended in the elementary mathematics program of studies. Several clusters of problem solving strategies are suggested for each stage of the model. Further ideas regarding organization for instruction, the use of textbooks, and sources of problems and problem solving materials are included in the references list.

LET PROBLEM SOLVING BE THE FOCUS FOR 1980's

INTRODUCTION

"Problem solving, the ability to reason and apply mathematics in problem situations, is considered an integral part of the basic skills required for mathematical literacy. The ability to solve problems increases with importance in light of the rapidly changing demands of today's technological society. Mathematics plays an important role in developing within each student the problem solving skills that will serve throughout life."[1]

Thus, one of the major goals of the elementary school mathematics program is to provide for the development of problem solving skills.

WHAT IS A PROBLEM?

Some problems have solutions that are readily obvious to the students because of their previous knowledge and experience. For other problem types, the solution is not immediately evident and requires the testing and application of one or more strategies.

A problem involves a situation in which an individual or group is required to perform a task and for which no immediate method of solution is evident.

Good mathematical problems should include some of the following characteristics:[2]

(1) they involve mathematics in some way.

(2) they are of interest to the child.

(3) they require the child to interpret and modify the solution process if necessary.

(4) they allow for several methods of solution.

(5) they allow the child to feel that he or she wants to and can solve the problem.

---

[1] Elementary Math Curriculum Guide 1982

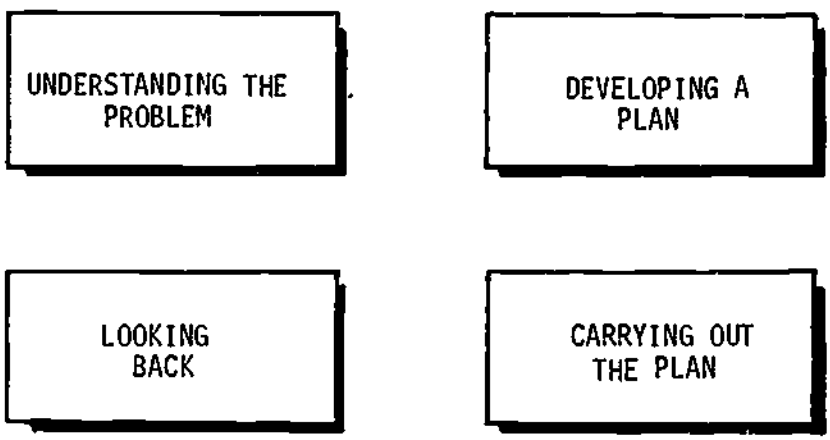
[2] NCTM 35th Year Book

A General Framework for Problem Solving

There is no single best strategy or method for either solving problems or teaching problem solving. However, there are strategies that can be learned which will improve the problem solving abilities of both children and adults. Research does indicate that problem solving performance is enhanced when students are taught to use a variety of strategies or heuristics, both general and specific. Elementary students, in particular, require an overall framework for solving problems and a repertoire of strategies and skills that may be used within that structure.

Several approaches and techniques have been proposed for problem solving. Polya's model provides the basis for the problem solving framework recommended in the elementary mathematics program of studies.

The framework consists of four stages:



1

Understanding the Problem

The major purpose of this stage is to involve the students in thinking about the problem before attempting a solution. Initially, the teacher asks questions and suggests strategies that will focus attention on the information and conditions in the problem.

Strategies in this stage of the process include:

- Using manipulatives
- Interpreting pictures
- Looking for patterns
- Identifying key words
- Acting it out
- Drawing diagrams
- Restating the problem in your own words
- Asking relevant questions
- Identifying wanted, given and needed information
- Identifying extraneous information
- Considering alternative interpretations

2

Developing a Plan

This is the planning stage during which students consider strategies for solving the problem. Students should be encouraged to choose alternative methods of solving problems. It is important that students consider and use strategies other than computation, and to learn to accept these as legitimate problem solving strategies.

Strategies in this stage of the process include:

- Acting it out
- Using manipulatives
- Collecting and organizing information (charts, graphs)
- Applying patterns
- Choosing and applying the appropriate operation
- Writing and solving a number sentence
- Guessing and checking
- Identifying and applying relationships
- Making diagrams and models
- Using a simpler problem
- Using logic or reason
- Constructing flow charts

3

Carrying out the Plan

This stage is very closely related to "developing a plan", but is listed separately in order to highlight the importance of the prior planning stage. Too often the whole focus of problem solving has been on this third stage, with emphasis on computing to get the right answer. In the general framework the "carrying out the plan" stage is merely doing what was planned in stage two.

Strategies in this stage of the process include:

- Acting it out
- Using manipulatives
- Collecting and organizing information (charts, graphs)
- Applying patterns
- Choosing and applying the appropriate operation
- Writing and solving a number sentence
- Guessing and checking
- Identifying and applying relationships
- Making diagrams and models
- Using a simpler problem
- Using logic or reason
- Constructing flow charts

4

Looking Back

This stage encourages the student to assess the effectiveness of the solution process. Students should learn to relate their answers to the question in the problem as one way of verifying that they have indeed solved the problem. Reflecting on the plans made and evaluating the strategies used assists students to become aware of the appropriateness of different strategies for a particular problem. This stage helps students to think through problems and to generalize the process for new situation.

Strategies in this stage of the process include:

- Stating an answer to the problem
- Restating the problem with the answer
- Checking the answer
- Determining the reasonableness of the answer
- Explaining the answer
- Reviewing the solution process
- Considering the possibility of other answers
- Looking for alternative ways to solve the problem
- Making and solving similar problems
- Generalizing solutions

Special Note:

It is important the students develop skills and strategies that can be applied to other problem solving situations. Using this framework as an overall procedure for attacking a problem provides students with a starting point and a way of organizing their efforts. The framework helps students and teachers to see problem solving as a process that consists of several interrelated stages or actions towards the solution of a problem, rather than a single action.

The framework should not be interpreted as a fixed sequence of stages and strategies that must be rigidly followed. Nor are the stages discrete or inseparable; their use will depend on the problem and on the individual student. Students may not always use all the stages or use them in the order given.

PROBLEM SOLVING MODEL

Understanding the Problem

- Using manipulatives
- Interpreting pictures
- Looking for patterns
- Identifying key words
- Acting it out
- Drawing diagrams
- Restating the problem in your own words
- Asking relevant questions
- Identifying wanted, given and needed information
- Identifying extraneous information
- Considering alternative interpretations

Looking Back

- Stating an answer to the problem
- Restating the problem with the answer
- Determining the reasonableness of the answer
- Explaining the answer
- Reviewing the solution process
- Considering the possibility of other answers
- Looking for alternative ways to solve the problem
- Making and solving similar problems
- Generalizing solutions

Developing and Carrying Out a Plan

- Acting it out
- Using manipulatives
- Collecting and organizing information (charts, graphs)
- Applying patterns
- Choosing and applying the appropriate operation
- Writing and solving a number sentence
- Guessing and checking
- Identifying and applying relationships
- Making diagrams and models
- Using a simpler problem
- Using logic or reason
- Constructing flow charts

PLANNING FOR INSTRUCTION

GRADES ONE AND TWO

At this level the general framework (page 2) is intended to be used primarily by the teacher to organize instruction. Students become acquainted with the framework indirectly as the teacher introduces and provides experiences involving the strategies in each stage. The framework, then, serves as a guide for conceptualizing the problem.

Children can think through and solve problems before they are able to read. Problem solving should not be set aside but should be developed as part of their language and number experiences. These early experiences provide a solid foundation and build confidence in solving problems.

Many everyday situations in the classroom can be used to create problem solving situations. For example: having students determine if there are enough chairs for a group. Discussing problems, posing questions, and setting tasks written in a problem context are effective unstructured techniques to be used on a regular basis.

Creative problem solving situations can be structured through the use of a variety of manipulatives. These experiences provided by the teachers through careful planning and questioning allow for the development of problem solving strategies and mathematical concepts.

Students should also develop the language of mathematics in their problem solving activities. This is assisted by experiences that require children to interpret problems by discussing, drawing, and acting.

GRADES THREE THROUGH SIX

Students at these levels can use the problem solving model directly. An introductory unit on problem solving will help students understand how to use the model.

The four stages of the framework could be put on a large chart and posted for reference. Several problems might be solved by working through each stage of the framework. This not only provides an introduction to the framework, but allows the teacher to model problem solving behavior. At each stage, the teacher asks questions or makes suggestions. These questions and suggestions could serve as a "checklist" for students as they practise solving problems.

Following the presentation of the model and some beginning, specific strategies for use within the model, regularly scheduled problem-solving sessions should be provided for both the development and practice of new strategies. This not only allows students the time necessary to develop some proficiency in problem solving, but also indicates to them how important it is. During many of these practice sessions, students might work in pairs. This will help them to learn to express their own positions more clearly and to consider the opinions of others.

For presentation of more difficult problems, which take longer to solve, students could help prepare a problem solving bulletin board, A very simple format falls into three sections: one to display the problem; one to display hints; one to collect solutions. In the hints area suggestions for *understanding the problem and developing a plan* can be displaced. Solutions could be put into a big envelope and made available for other students to look at and read. Post the problem one day, then post hints, one or two at a time, over the next few days. The "looking back" discussion could then take place during the regular problem solving session.

Making up their own problems can provide students with considerable insight into the problem solving process. Their early efforts may often contain too much or not enough information and sometimes unrealistic numerical data, but all of this provides excellent opportunities for discussion. Students might start off with pictures from magazines or old texts and with articles and photos from newspapers. The resulting problems can be put on file cards or on stencils with the "author's" name prominently displayed.

USING THE PRESCRIBED LEARNING RESOURCES

The following suggestions illustrate some ways of making effective use of one of the major sources of activities - the textbook.

1. Assign only a few problems from the chapters which provide practice on a specific operation, on completion of those chapters. Then there will be problems available for later assignments from several different chapters, to mix the problems and operations.

2. Add another question to some problems so that students must use the answer to the first question to solve the second.

3. Modify textbook problems by presenting part of the data and the answers, and asking students to generate the missing data; or by removing a condition so that multiple answers may be generated.

4. Use the unusual, non-routine problems called "Challenges" or "Extras for Experts" in discussion sessions involving all students, not just the more capable.

5. Use the stages of the model (page 7) for discussion of textbook problems, to reinforce application of the stages in a problem solving situation.

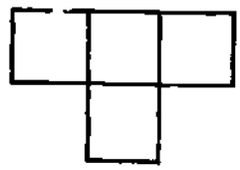
WORKING THROUGH THE MODEL

INTRODUCTION

Seven problems have been selected to illustrate how the strategies in the problem solving model may be applied instructionally. The strategies vary for each of the problems given and are presented with a number of related questions and activities that will assist the teacher in working a problem through the four problem solving steps. Teachers are encouraged to use the problems as a springboard for introducing problem solving activities in their classroom.

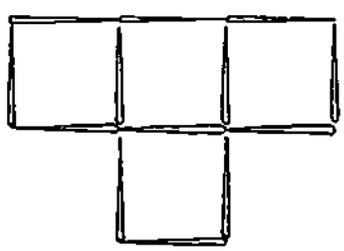
PROBLEM ONE:

The Smiths had 4 pigs on their farm. Each pig had its own pen. The pens were arranged as shown in the diagram below.



They decided to sell 1 pig and make 3 pens. In how many ways could they do this by removing the least number of fences?

Understanding the Problem
- Using manipulatives.



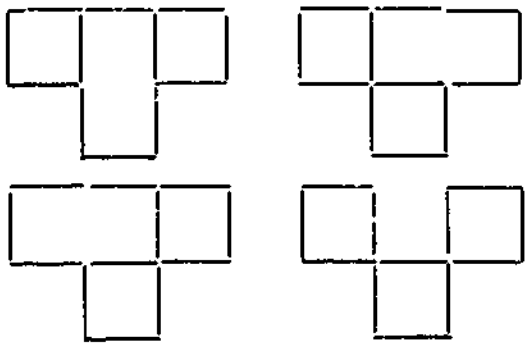
Suggestions:

- Initially students should discuss the problem to demonstrate understanding of what is required.
- The teacher should ask if the problem could be shown through the use of manipulatives, e.g., toothpicks, sticks, straws.
- Students build the four pig pens using the manipulative material.

Questions:

- What does the problem require?
- Can we use toothpicks to show the problem?
- Do the 3 new pens have to be the same size?

Developing and Carrying Out a Plan
- Using manipulatives.



Questions:

- How many ways are there to make 3 pens out of 4?
- How can you solve the problem using the toothpicks?
- Can you see a pattern in the fences which can be removed?

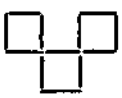
Suggestions:

- Using the "stick" pig pens they have constructed the students predict the possible arrangements of transforming 4 pens into 3 and test their predictions.
- The teacher should encourage the students to devise as many alternative arrangements as possible and draw diagrams to illustrate each alternative.

Looking Back
- Reviewing the solution process.

ragged fence arrangement

closed fence arrangement



Questions:

- Are there variations in the way the problem can be solved?
- Which fence arrangement would be the most practical?

Suggestions:

- The students should discuss their solutions for commonality and account for differences (e.g., ragged fence arrangements).
- The teacher should accept all solutions (closed, ragged arrangements).

EXTENDING THE PROBLEM

- What arrangements are possible if 2 fences are removed? 3 fences?
- Which arrangement would take up the least area if you remove 1 fence? 2 fences? 3 fences?
- What arrangements would give equal area for all 3 pens?
- Only 2 pens?

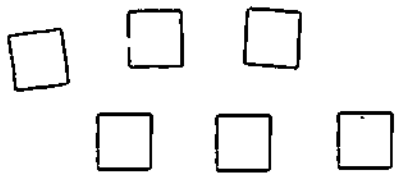
A CHALLENGE PROBLEM

Using 17 toothpicks, make 6 pens of *equal* size:

- Remove 1 toothpick to make 5 pens
- Remove 5 toothpicks to make 3 pens
- Remove 4 toothpicks to make 3 pens

PROBLEM TWO:

Kirk is planning his birthday party. He has 6 card tables. Only one person can be seated on each side of the table. If the tables are placed together, side against side, which table arrangement would seat the most people.



Suggestions:

- The students may approach the problem in small groups by posing a series of questions, e.g., the number of people per table, the kinds of table arrangements possible.
- These can be listed for discussion purposes.
- The teacher should guide the student groups in their efforts to ask questions which lead to ways and means of exploring different table arrangements.

UNDERSTANDING THE PROBLEM
- Asking relevant questions.

Questions to which information is given in the problem:

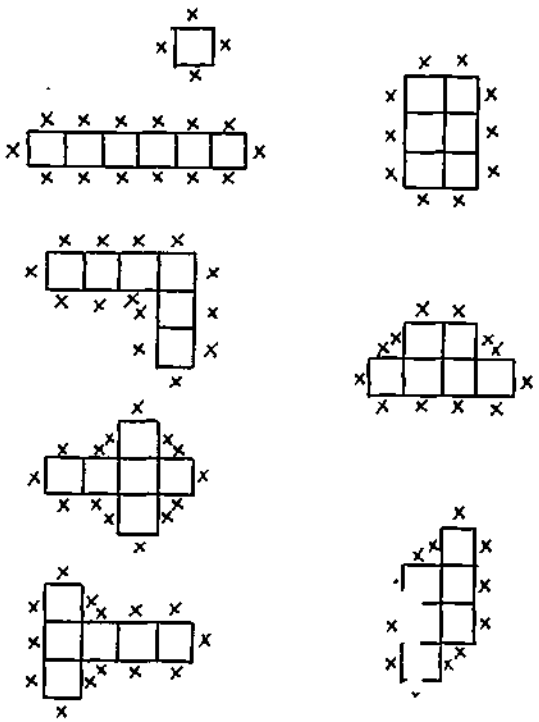
- How many tables are there?
- How many people per side?
- How are the tables to be arranged? (condition - side to side)

Questions to which information is unknown:

- Do all the tables have to line up in a straight line?
- Can the tables be joined at the corners? Mid-way on the joining side?
- Will certain arrangements seat more people than others?

20

DEVELOPING AND CARRYING OUT A PLAN
- Making diagrams or models.



Questions:

- What are some of the ways of solving this problem?
- Will a diagram help?

Suggestions:

- The students should discuss a way to solve the problem, drawing diagrams to illustrate possible solutions. This should lead to the students making various diagrams using 6 card tables, then discovering which arrangements will hold the most people.

LOOKING BACK
- Generalizing solutions.

There are 4 possible arrangements that seat 14 people. You must have 5 joining sides to have the maximum of 14 people.

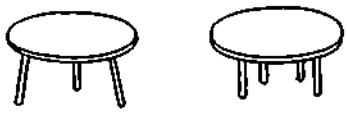
Suggestions:

- The students should consider Kirk.
- If Kirk saves himself a seat, one less friend can be seated.
- Can you make a rule?

PROBLEM THREE:

A classroom has several tables in it. Some have 3 legs and some have 4 legs. Altogether there are 31 legs. How many 3-legged and 4-legged tables are there?

UNDERSTANDING THE PROBLEM
- Drawing a diagram.



There would be 2 tables and 7 legs.

Suggestions:

- The students may best understand the problem by having them draw simple diagrams (3-legged table; 4-legged table).
- Have them count how many tables and how many legs.

DEVELOPING AND CARRYING OUT A PLAN
- Guessing.
- Checking.

Suggestions:

- The teacher should suggest to the students that a record be kept because there may be several guesses. Discuss.
- The students should determine that checking must be done to find the answer.

Questions:

Mary - How many of each would you say there are?

Jane - How many would you say there are?

Jerry- How can we tell if the answers are correct?

22

No. of 3-legged tables	No. of 4-legged tables	Total number of legs
1	0	3
5	6	(5x3)+(6x4)=39
3	4	(3x3)+(4x4)=25
4	4	(4x3)+(4x4)=28
5	4	(5x3)+(4x4)=31

- Discuss and devise a method for keeping a record and checking the suggested answers.
- After a few random guesses and checks, perhaps the students should develop a pattern

31÷3=10R1
31÷4=7R3

- The students should discover that all cannot be 3-legged tables or all 4-legged tables.

There are five 3-legged tables and four 4-legged tables.

LOOKING BACK

- Considering the possibility of other answers.

Suggestions:

- Discuss with the students a pattern of guessing that would include all possible answers.

3-legged	4-legged	Total legs
1	1	3+4=7
1	2	3+8=11
1	3	
1	4	
1	5	
1	6	3+24=27
1	7	3+28=31
1	8	3+32=35
2	1	
2	2	
2	3	
2	4	
3	1	9+4=13
3	2	9+8=17
3	3	9+12=21
3	4	9+16=25
3	5	9+20=29
3	6	9+24=33

- Have the students discover that the number of 3-legged tables must be odd.

- By now, suggest to the students which number of 3-legged tables will give an answer 31-9=22.

5	1		5 3-legged tables give
5	2		15 legs
5	3		31 - 15 = 16 legs.
5	4	15+16=31	

7	1		31-21=10
7	2		No solution because 10÷4=2R2.
	3		
	4		
	5		

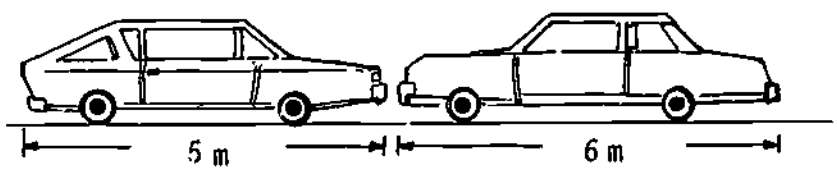
9	1	27+4=31	31-27=4

There are several solutions:

- one 3-legged table and seven 4-legged tables
- five 3-legged tables and four 4-legged tables
- nine 3-legged tables and one 4-legged table.

EXTENDING THE PROBLEM

- Grandson: Grandpa, how many geese and how many pigs do you have?
  Grandfather: I counted 30 heads and 100 feet this morning when I counted the geese and pigs. How many of each do I have?
- A city block is about 100 m long. If cars are parked bumper-to-bumper (a) what is the largest number of cars that can be parked? (b) what is the smallest number of cars? (c) if equal numbers of large and small cars are parked, how many would fit?



PROBLEM FOUR:

Dick is thinking of 2 numbers. The product of these 2 numbers is 48. The difference between them is 8. What are the 2 numbers?

UNDERSTANDING THE PROBLEM

- Restating the problem.

The 2 numbers multiplied together give 48. When the smaller number is subtracted from the larger, the answer is 8.

Suggestions:

- The students should identify and discuss the meaning of the key words - product and difference.
- The teacher should ask the students to describe the problem in their own words.

Questions:

- How many numbers are there?
- What does the word "product" mean?
- What do you do to get the product?
- What do you do to get the difference?

DEVELOPING AND CARRYING OUT A PLAN

- Guessing and checking.

6 x 8
1 x 48
2 x 24

Suggestions:

- The students should be encouraged to think of some products of 48 and keep a record of these.

6 x 8	8 - 6 = 2
1 x 48	48 - 1 = 47
2 x 24	24 - 2 = 22
3 x 16	16 - 3 = 13
4 x 12	12 - 4 = 8
8 x 6	8 - 6 = 2

- The students should then find the difference between the 2 numbers to see if the difference is 8.

Questions:

- Are there more possibilities for products of 48?
- Are there more with a difference of 8?

LOOKING BACK

- Restating the problem with the answer.

Dick is thinking of 4 and 12 because 4 x 12 = 48 and 12 - 4 = 8.

Suggestions:

- The students should discuss the solution and restate the problem with the solution.

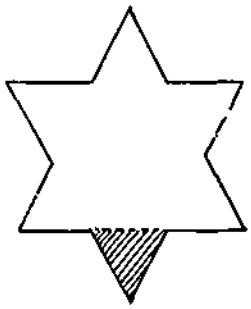
Questions:

- Can anyone make up similar problems? Product of 45 - difference of 4.

EXTENDING THE PROBLEM

- Students make up similar problems for their classmates to solve.
- Teacher can make up problems involving product and quotient - I am thinking of two numbers. Their product is 36. Their quotient is 4.
- Use problems involving sums and differences. The sum of two numbers is 10 and their difference is 4.

PROBLEM FIVE:



The area of the shaded region is 6 cm^2. What is the area of the total figure?

UNDERSTANDING THE PROBLEM

- Identifying wanted, given and needed information.

Suggestions:

- The teacher may ask a series of questions about the shapes, equivalent sides and meaning of the area.
- The discussion may continue on what is wanted, what is given and what is needed.

Questions:

- What shape is the whole figure?
- What is the shape of the shaded figure?
- What is the area of the shaded figure?
- What does the problem ask us to find?

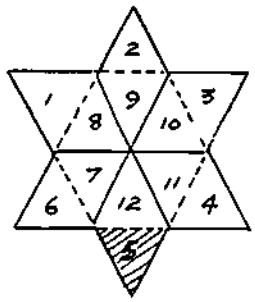
DEVELOPING AND CARRYING OUT A PLAN
- Using a simpler problem.

Suggestions:

- Further discussion should lead the students to see that the whole figure may be divided into triangles of the same shape and size.

Questions:

- Of which shapes can we find the area: Square? Rectangle?
- Triangle? Parallelogram?
- What is the shape of the shaded area?
- Can we divide the star into triangles, squares, rectangles and parallelograms?



- The students should be encouraged to divide the figure into various triangular shapes. Draw these figures on the blackboard.

$12 \times 6 = 72$

- Have the students count the number of such triangles and determine the operation used to find the area.

The area is 72 cm^2.

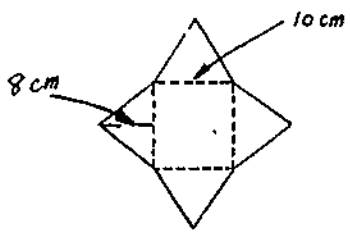
LOOKING BACK
- Explaining the answer.

Suggestions:

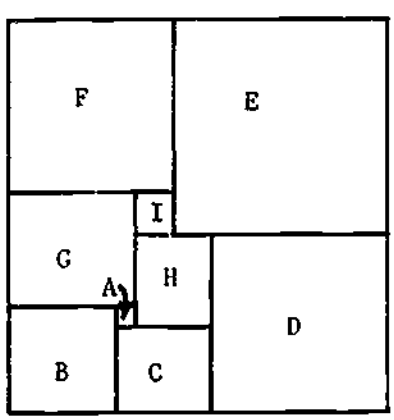
- Have the students explain that the area was found by dividing a complex figure into 12 congruent equilateral triangles. Then the area may be found by multiplying.

EXTENDING THE PROBLEM

- Give the students other examples where they have to divide a complex figure into simpler ones to find the area

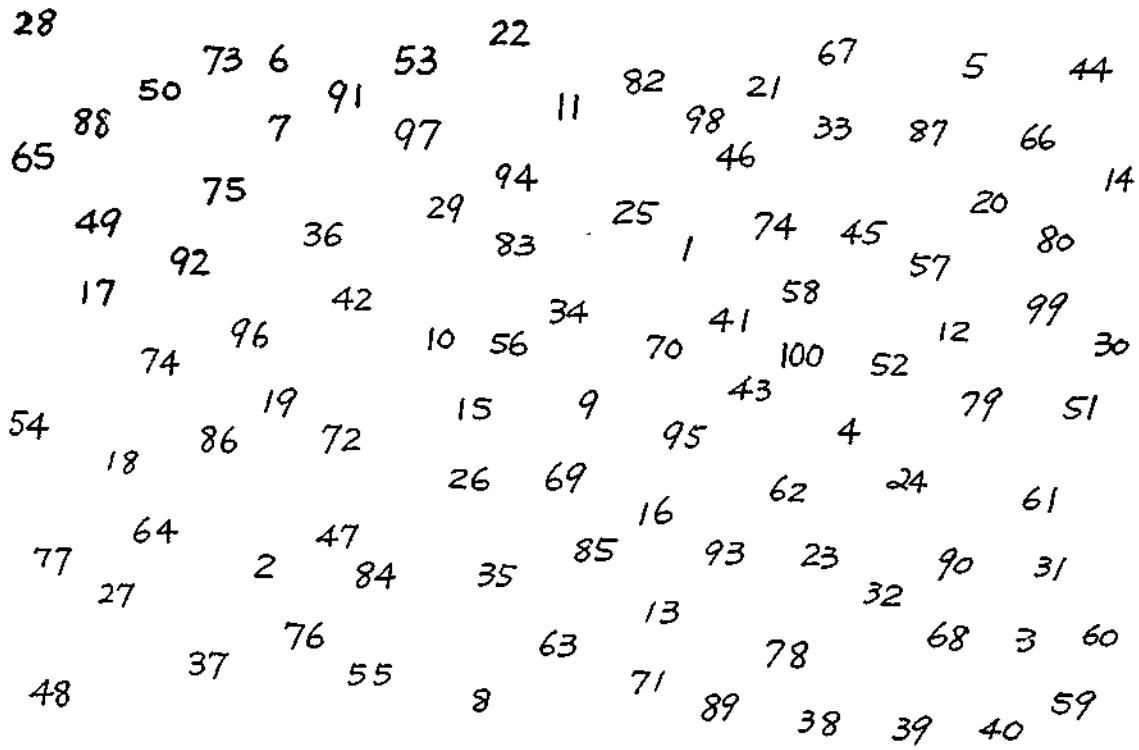


- Find the area of each square.



PROBLEM SIX:

Michael printed the number 1 to 100. However, one number was printed twice and one number was missing. Which number appears twice and which number is missing?

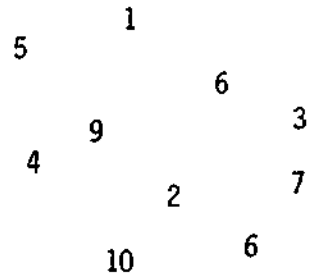


UNDERSTANDING THE PROBLEM
- Looking for patterns.

Suggestions:

- Initially the student should discuss the problem to demonstrate understanding of what is given and what is required.
- The teacher should do a simpler problem by writing numbers from 1 to 10, omitting 8 and writing 6 twice.

- What number is missing? What number appears twice?



DEVELOPING AND CARRYING OUT A PLAN - Collecting and organizing data.

1	11
2	12
3	13
4	
5	
6	
7	
8	
9	
10	100

Suggestions:

- The teacher should point out that the problem involves 100 numbers.
- The teacher should now discuss various methods for finding the missing number, and the number appearing twice.

Questions:

- Is it easy to find the number missing?
- Can we just cross out numbers, starting with one?
- What other method can we use?

- The students should arrive at several methods of finding the missing number.
- The teacher should point out the procedure, writing the numbers 1-100 and checking off the numbers at random.

Questions:

- How would we know what number is missing?
- How would we know what number was written twice?

LOOKING BACK
Stating an answer to the problem.

Suggestions:

- Let the students make statements to answer the two questions posed by the problem.

Extending the Problem

- The teacher can do a similar problem using multiples of 2, 5, 10, 25.

PROBLEM SEVEN:

An Increase in Allowance

Cathy is one year older than her brother Shaun. Cathy gets $3 allowance a week. Shaun gets $2.50.

Their mother agreed to increase their allowances by 20% every Christmas. Shaun liked the idea. He thought he might catch up or at least keep up with Cathy.

Find out how much allowance Cathy and Shaun will get for the next ten years. Round your answers to the nearest cent.

UNDERSTANDING THE PROBLEM

Identifying wanted, given and needed information.

Suggestions:

- Initially students should discuss the problem to find out what is wanted; what information is given; what is needed.

Questions:

- What calculations must be done?
- Can we use a calculator to do the computation?

DEVELOPING AND CARRYING OUT A PLAN:

Choosing and applying the appropriate operation.

- The teacher should suggest that a chart be developed to keep a record of the allowance.

	Start	1 year	2 years	3 years	4 years	5 years	6 years
Cathy	$3.00	(3.60)	(4.32)	____	____	____	____
Shaun	$2.50	(3.00)					
Difference	$0.50	0.60					

	7 years	8 years	9 years	10 years
Cathy	____	____	____	____
Shaun	____	____	____	____
Difference	____	____	____	____

Questions:

- How would you find Cathy's increase for the first year?
- What allowance would she be paid for that year?
- What would her increase be for the second year?
- What allowance would she be paid for the second year?

- The students should notice what is happening to the difference in the allowance.

LOOKING BACK

Generalizing Solutions.

3.00	3.60	4.32	5.18	6.22	7.46	8.95	10.74	12.89	15.47	18.56
2.50	3.00	3.60	4.32	5.18	6.22	7.46	8.95	10.74	12.89	15.57
0.50	0.60	0.72	0.86	1.04	1.24	1.49	1.79	2.15	2.58	3.09

SAMPLE PROBLEMS

Students need instruction and practice in using the skills of each stage, as well as in using the overall framework to help solve problems.

The problems which follow are provided as examples to illustrate the specific strategies within each stage of the general framework that could be used in the solution of a given problem.

Problems are deliberately varied in terms of setting; that is, they relate to many of the content and skill objectives in the Numeration, Operations and Properties, Measurement, Geometry and Graphing strands of the elementary mathematics curriculum. The problems also vary in difficulty and format. Some of the problems may be solved in a variety of ways, but are included as illustrations to help focus on particular strategies.

Teachers are encouraged to look beyond their own grade level(s) for examples of types of practice problems which might be modified to suit earlier or later grades.

SAMPLE PROBLEMS

CODE

Numeration - N
Operations and Properties - O
Measurement - M
Geometry - G
Graphing - Gr

(Example: Gr-2
Graphing - Objective 2)

37

GRADE 1

PROBLEM: 1.1
0-1

4 children were reading in a circle together. 5 more children joined them. How many children were in the reading circle then?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Acting it out	Stating an answer to the problem

GRADE 1

PROBLEM: 1.2
0-1

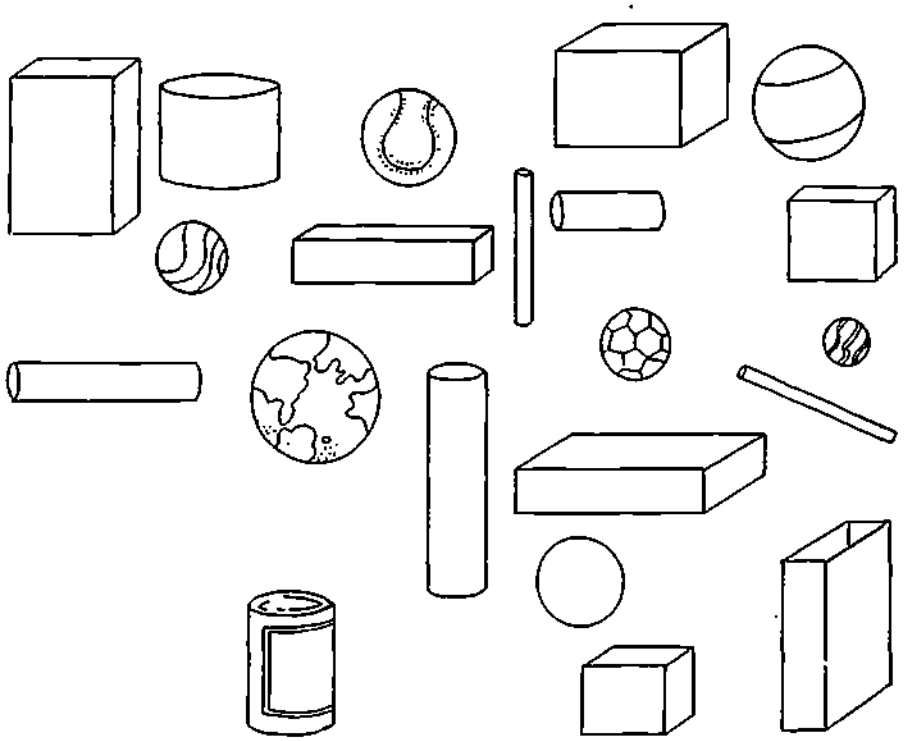
9  are on a flower.
4 fly away.
How many are left on the flower?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives	Using manipulatives Choosing and applying the appropriate operation	Restating the problem with the answer

GRADE 1

PROBLEM: 1.3
G-1 Gr-1

Which shape is there most of?

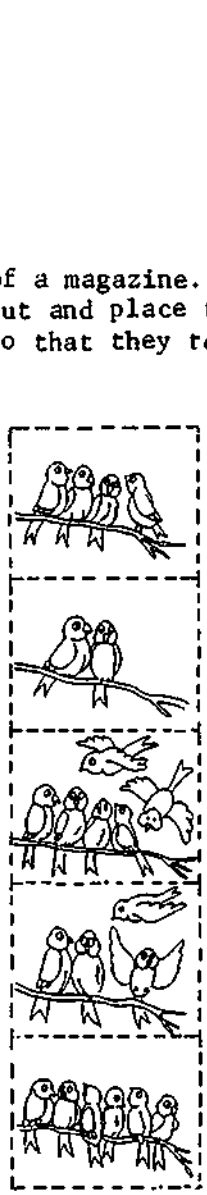


UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Interpreting pictures	Collecting and organizing information (charts, graphs) How can we keep track? (chart or table)	Restating the problem with the answer

GRADE 1

PROBLEM: 1.4
N-4

Jane cut these pictures out of a magazine. She got them mixed up. Cut out and place these pictures in the right order so that they tell a story.



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Interpreting pictures	Collecting and organizing the information (charts, graphs)	Restating the problem with the answer

GRADE 1

PROBLEM: 1.5
N-3

Mary Jane saw 5 rabbits. How many ears did those 5 rabbits have in all?

How many rabbits are there if you counted 12 ears?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Acting it out	Applying a pattern	Checking the answer

GRADE 1

PROBLEM: 1.6
O-2

8 camels were drinking.
2 camels walked away.
How many camels were still drinking?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Writing and solving a number sentence	Restating the problem with the answer

GRADE 1

PROBLEM: 1.7
Gr-1

What is the favourite pet of the children in the class?
Make a pictograph to show what you have found.

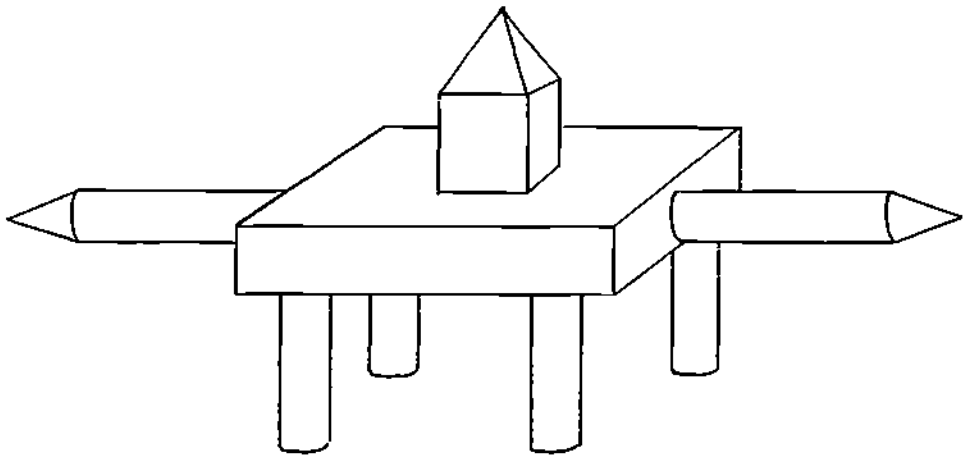
UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Collecting and organizing information (charts, graphs)	Reviewing the solution process

GRADE 1

PROBLEM: 1.8

G-1

Jim built a robot out of various 3-dimensional objects. Jane wanted to do the same. What objects would she have to use so that she can build the same kind of robot?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Interpreting pictures	Using manipulatives	Explaining the answer

GRADE 1

PROBLEM: 1.9
M-6 O-2

Jill has 6 pennies. Her mother gave her some more pennies. If she now has 13 pennies, how many did her mother give her?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives	Acting it out	Stating an answer to the problem

GRADE 1

PROBLEM: 1.10
0-2

At Jack's birthday party each of his friends were given 8 balloons. John, one of Jack's friends, burst 5 of his own. How many did John have left?

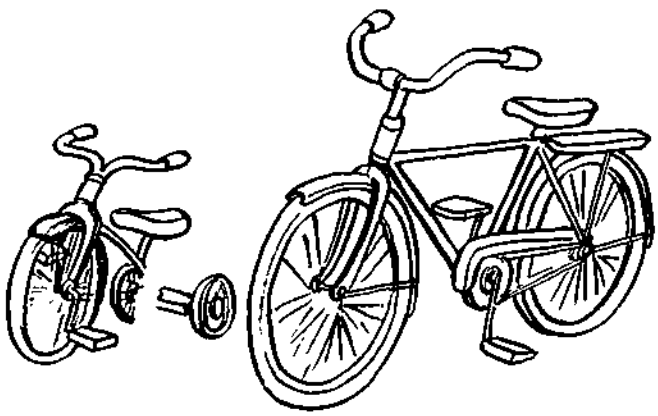
UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Writing and solving a sentence	Restating the problem with the answer

GRADE 2

PROBLEM: 2.1

M-6

If 7 bicycles and 3 tricycles went by you, how many wheels passed by?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Applying patterns	Checking the answer

GRADE 2

PROBLEM: 2.2
0-3, 0-5

A glass of punch costs 6 cents and a cookie 4 cents. How many of each did Jill buy if she spent exactly 22 cents?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Guessing and checking	Explaining the answer considering the possibility of other answers

GRADE 2

PROBLEM: 2.3

P-2, O-5

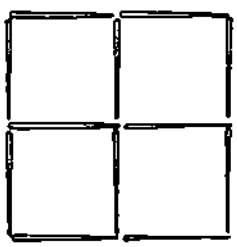
Where would you open your book so that the sum of the numbers on the two facing pages is 21? 43? 18?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives	Applying patterns	Checking the answer

GRADE 2

PROBLEM: 2.4
G-2

Which 2 toothpicks would you take away to make 2 squares?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives	Using manipulatives	Explaining the answer

GRADE 2

PROBLEM: 2.5
C-1

There were 4 parrots on the ship. Then 7 parrots were brought on board. Two of the parrots flew away. How many parrots were still on board the ship?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Writing and solving a number sentence	Restating the problem with the answer

GRADE 2

PROBLEM: 2.6
N-4

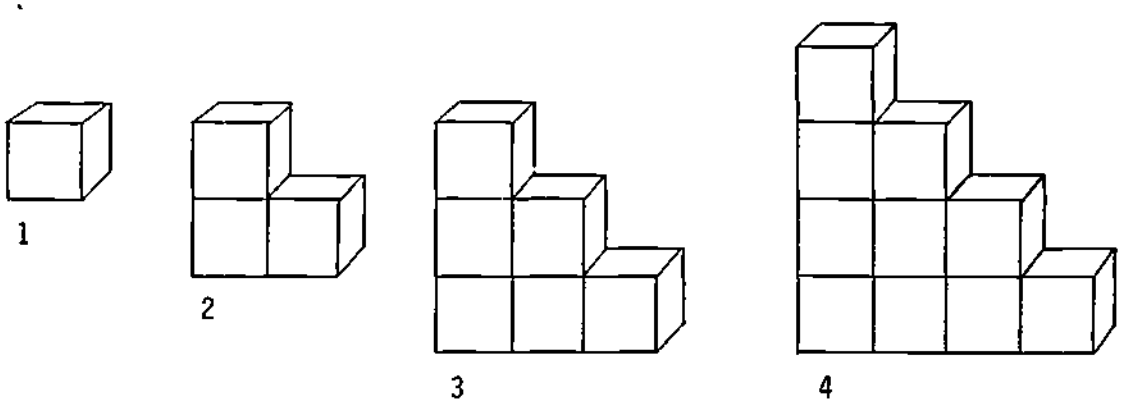
In a jungle parade, the tiger was behind the lion. The elephant was behind the gorilla. The tiger was in front of the gorilla. Which animal was first in the parade? Which animal was last?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Making diagrams and models	Stating an answer to the problem

GRADE 2

PROBLEM: 2.7
G-3

Jack was building stairways like those below, using his building blocks.



How many blocks does he need to build the sixth stairway? the eighth stairway?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Looking for patterns	Collecting and organizing information	Making a statement Stating an answer to the problem

GRADE 2

PROBLEM: 2.8
0-4

In how many different ways can you put 7 beans in 2 cups?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Collecting and organizing information (graphs, charts)	Explaining the answer

GRADE 2

PROBLEM: 2.9
N-6

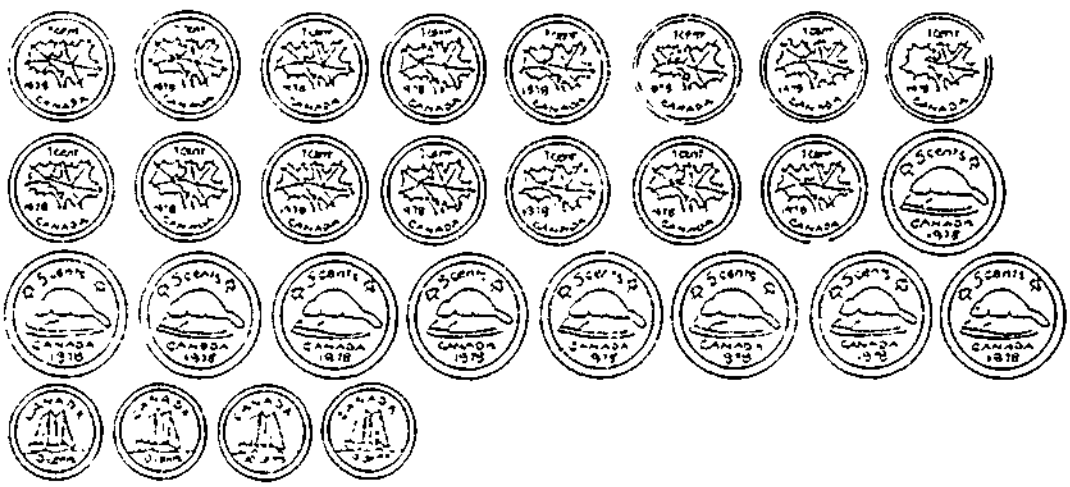
Jack shot 5 pucks at the goalkeeper every time his turn came. He had 6 turns. How many pucks did he shoot altogether?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying the key words	Applying patterns	Making a statement Stating an answer to the problem

GRADE 2

PROBLEM: 2.10
M-8, M-9

Tom opened his piggy bank. These are the coins he had. In how many ways can he give 25 cents to each of his 4 friends?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives	Acting it out	Explaining the answer

GRADE 3

PROBLEM: 3.1

N-1

John was using his calculator to multiply these numbers:

11 x 99 = 1089
22 x 99 = 2178
33 x 99 = 3267
44 x 99 = 4356

He decided to complete the rest of these without the calculator and without actually multiplying them out. Do these for John.

55 x 99 +
66 x 99 =
77 x 99 =
88 x 99 =
99 x 99 =

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Looking for patterns	Applying patterns	Checking the answers

GRADE 3

PROBLEM: 3.2
N-2

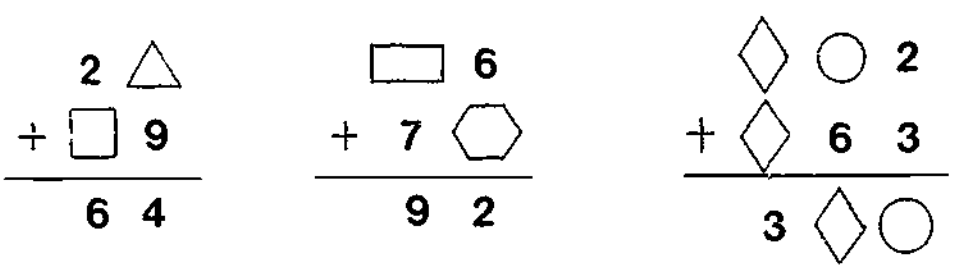
How many 5's would you use if you had to write all the numbers from 1 to 60?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Collecting and organizing the information (charts, graphs)	Explaining the answer

GRADE 3

PROBLEM: 3.3
0-2

Find the single digit you would put in each shape to make the question correct.



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Using logic or reason	Checking the answer

GRADE 3

PROBLEM: 3.4

M-6

You have a stick 8 dm long. You want to cut it into 8 equal pieces. It takes you one minute to cut one piece. How long would it take you to cut 8 pieces?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Making diagrams and models	Making and solving similar problems

GRADE 3

PROBLEM: 3.5
G-3

One rectangle has 4 sides. If you arrange/join 2 rectangles together, you will see 7 sides. If you join a third rectangle to the first two, you will see 10 sides. How many sides would you see if you joined 12 rectangles together?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING ON THE PLAN	LOOKING BACK
Drawing diagrams Looking for patterns	Applying patterns	Explaining the answer

GRADE 3

PROBLEM: 3.6
0-1, 0-2

There are 923 pages in Carol's book. There are 876 pages in Sam's book. Carol has already read 508 pages. How many pages does Carol have left to read in her book?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words Identifying extraneous information	Writing and solving a number sentence	Stating an answer to the problem

GRADE 3

PROBLEM: 3.7
M-2

Sue, Jan, Bob and Tom were born the same year. Sue's birthday falls on the sixth month of the year. Jan's birthday is 5 months before Sue's. Bob's birthday is 4 months after Jan's and one month before Sue's. Tom's birthday is exactly in between Jan's and Bob's.

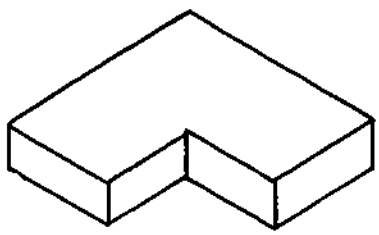
(a) What months of the year do Sue, Jan, Bob and Tom have their birthdays?
(b) Who is the oldest?
(c) Who is the youngest?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Identifying and applying relationships	Restating the problem with the answer

GRADE 3

PROBLEM: 3.8
G-1

How many faces, edges, and corners are there in the block below?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Interpreting pictures	Identifying and applying relationships	Reviewing the solution process

GRADE 3

PROBLEM: 3.9
Gr-4

Why does the fireman wear red suspenders?

10									
9									
8									
7									
6	V	X	Q	R	B	A	N	M	D	E
5	A	B	I	O	X	E	M	G	A	U
4	B	M	L	O	D	C	X	V	B	G
3	J	Z	V	S	R	V	H	P	F	M
2	L	T	M	R	K	Q	F	C	J	V
1	B	D	J	N	M	F	J	L	Q	O
	1	2	3	4	5	6	7	8	9	10

The answer must be deciphered by using these ordered pairs.

2,2 4,4

5,2 6,5 10,6 8,3

7,3 3,5 4,3

8,3 9,5 7,6 2,2 4,3

10,5 8,3

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying wanted, given and needed information	Collecting and organizing the information	Stating the answer to the problem Making and solving similar problems

GRADE 3

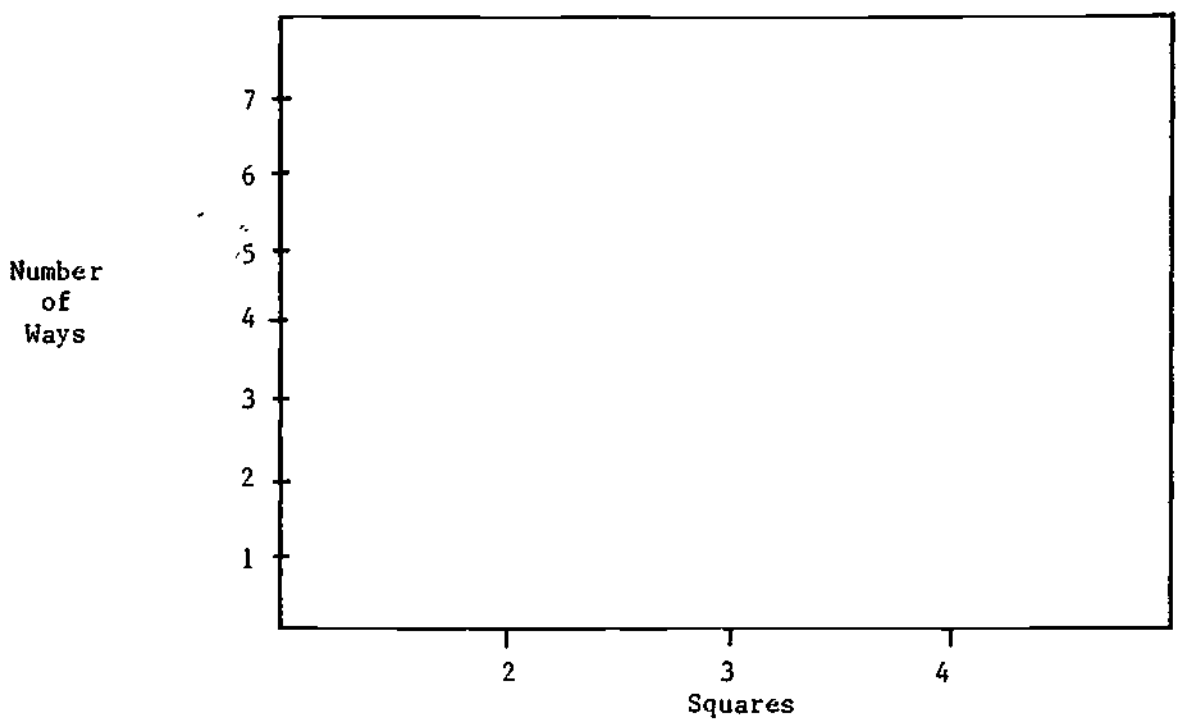
PROBLEM: 3.10
G-3, G-2

How many different shapes can you make using:

2 squares? 
3 squares? 
4 squares? 

(e.g.,  squares must be joined by a common edge.)

Show your findings on a graph.



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Using manipulatives Asking relevant questions	Acting it out	Explaining the answer Considering the possibility of other answers

GRADE 4

PROBLEM: 4.1

N-3

Find the largest 5-digit number which has 7 in the thousands place and a 0 with no 2 digits being the same.

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Using logic or reason	Generalizing the solution

GRADE 4

PROBLEM: 4.2
G-2

A pie is cut so that all cuts are through the centre.
How many pieces will there be after 10 cuts?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Applying patterns	Stating the answer to the problem

GRADE 4

PROBLEM: 4.3
N-6

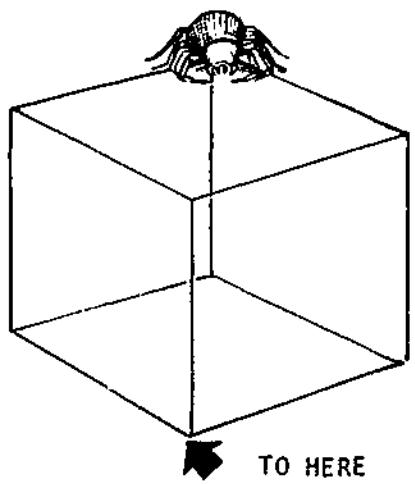
A hotel elevator went from the main floor to the 15th floor. Then it went down 8 floors, up 6, down 3, and up 1. What floor is the elevator now on?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Making diagrams and models	Stating an [illegible] to the lem

GRADE 4

PROBLEM: 4.4
G-1

Tony the spider is sitting on one corner of a cube. How many different paths can he take if he wants to travel to the opposite corner along an edge only?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Using manipulatives	Considering the possibility of other answers

GRADE 4

PROBLEM: 4.5
O-1, O-2

Jim counted 7 cycle riders and 19 cycle wheels travelling past his house Saturday morning. How many bicycles and how many tricycles passed his house?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying wanted, given and needed information	Guessing and checking	Explaining the answer

GRADE 4

PROBLEM: 4.6
M-1

How many times a day would the bird in a cuckoo clock cuckoo if it cuckoos on the hour?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Collecting and organizing information (charts, graphs)	Explaining the answer Looking for alternative ways to solve the problem

GRADE 4

PROBLEM: 4.7

M-5

You have 28¢ in your pocket. How many different combinations of coins could you have?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identify wanted, given and needed information	Collecting and organizing information (charts, graphs)	Making and solving similar problems

GRADE 4

PROBLEM: 4.8

M-3

Eva's mother told her that their gasoline tank holds 40 L. They bought 22 L to fill the tank. How many litres of gasoline were there in the tank before the "fill-up"?

UNDERSTANDING THE PROBLEM	PLANNING AND CARRYING OUT THE PLAN	LOOKING BACK
Finding what is given and what is needed	Writing a number sentence	Explaining the solution with the correct answer

GRADE 4

PROBLEM: 4.9

N-1, O-2

A telephone list has an average of 92 names on each page. How many names will there be on 89 pages? The students wrote their answers on the blackboard. Which of these answers were reasonable?

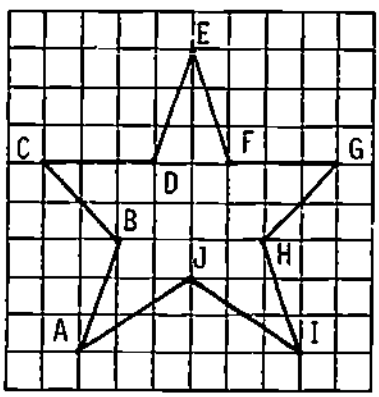
3 81 181 818 8188

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Choosing and applying the appropriate operation	Determining the reasonablness of the answer

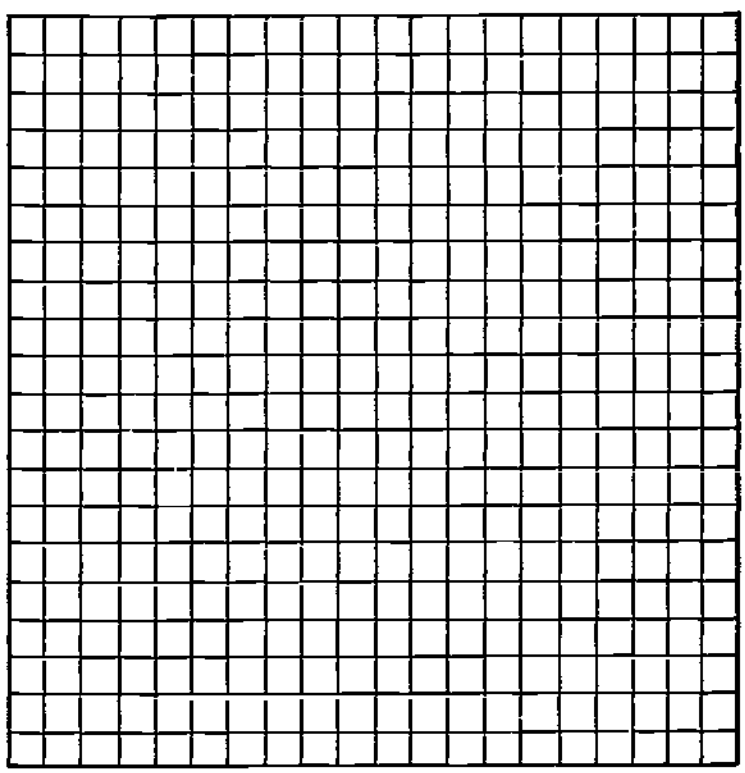
GRADE 4

PROBLEM: 4.10
Gr-3, Gr-4

You are required to make a double enlargement of this star by using the graph paper provided. List the coordinates.



A=(_,_)
B=(_,_)
C=(_,_)
D=(_,_)
E=(_,_)
F=(_,_)
G=(_,_)
H=(_,_)
I=(_,_)
J=(_,_)



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying wanted, given and needed information	Making diagrams and models	Making and solving similar problems

GRADE 5

PROBLEM: 5.1
M-10, M-11

A square garden plot has a perimeter of 12 m. What is its area? If a post is placed every metre, how many posts do you need to fence the garden plot?

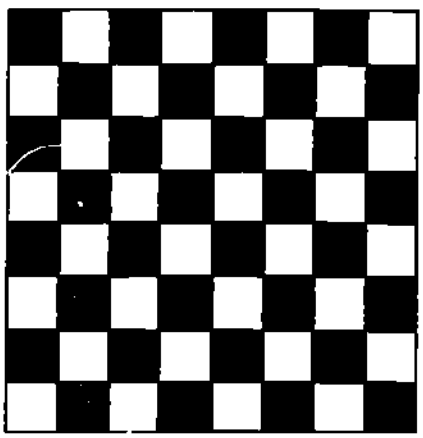
UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Making diagrams and models	Stating an answer to the problem

GRADE 5

PROBLEM: 5.2
G-1, G-2

How many rectangles are there in this figure?

How many squares are there in the checkerboard?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Using a simpler problem	Making and solving similar problems

GRADE 5

PROBLEM: 5.3
Gr-1

The marks from a math test are given. Organize the marks and show the information on a bar graph.

75	95	80	75	90	55	85
95	80	90	85	85	65	70
75	90	65	70	65	75	95
100	85	70	55	100	60	85

How many students took the test? What was the most common mark? Do you think the test was hard? How many problems were on the test?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Collecting and organizing information (charts, graphs)	Explaining the answer

GRADE 5

PROBLEM: 5.4

M-1, M-2

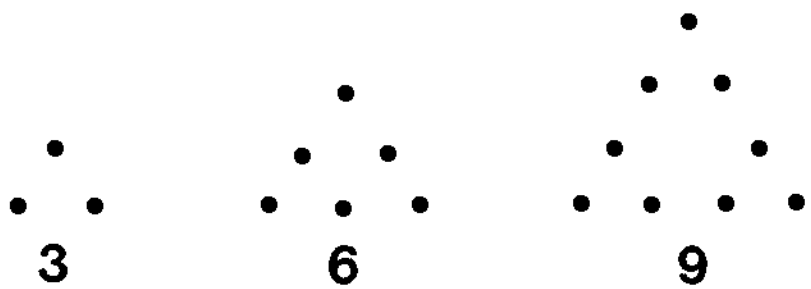
You have a clock that gains 1 minute and 30 seconds every hour. If the time is now 20:00, how should you set the clock so that it will show the correct time at 07:00?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying wanted, given and needed information	Choosing and applying the appropriate operation	Reviewing the solution process

GRADE 5

PROBLEM: 5.5
G-1

Triangle dot numbers are so named because that number of dots can be used to form a triangle with an equal number of dots on each side. What triangle dot number has 10 dots on each side?



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams Looking for patterns	Applying patterns	Stating an answer to the problem

GRADE 5

PROBLEM: 5.6
0-1, 0-3

Sue cannot resist doughnuts. She saw that plain doughnuts were on sale for 35¢ each and fancy doughnuts for 50¢ each. She bought a dozen doughnuts and paid $4.65. How many fancy doughnuts did she buy?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identify wanted, given and needed information	Guessing and checking	Restating the problem with the answer

GRADE 5

PROBLEM: 5.7
0-4

Mrs. Simons bought 7 dozen cookies for the 48 guests she invited to her party. How many cookies did each guest eat if each guest ate the same number of cookies.

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Writing and solving a number sentence	Checking the answer

GRADE 5

PROBLEM: 5.8
N-1

What year in the last 200 years reads the same backward, forward and upside down?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Using a simpler problem	Making and solving similar problems

GRADE 5

PROBLEM: 5.9
0-4

Find a number between 65 and 95, that is divisible by 2 and 9 but not by 5.

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Using logic or reason	Checking the answer

86

GRADE 5

PROBLEM: 5.10
0-3, 0-1

At the bank, there is a service charge of $1.00 per month and an extra charge of 25¢ for each cheque written. At another bank, there is a service charge of $2.00 per month and an extra charge of 10¢ for each cheque written. How many cheques would have to be written to get the better deal at the second bank?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Collecting and organizing information (charts, graphs)	Generalizing solutions

GRADE 6

PROBLEM: 6.1
0-3, 0-1

The planet Klepton has only creatures with 3 legs (triads) and 4 legs (tetrads). Astronaut Janice could not bear to look at these creatures. She kept her eyes on the ground. On her first day on Klepton she counted 31 legs. How many triads and tetrads did she meet?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identify wanted, given and needed information	Guessing and checking	Considering the possibility of other answers

GRADE 6

PROBLEM: 6.2
0-1

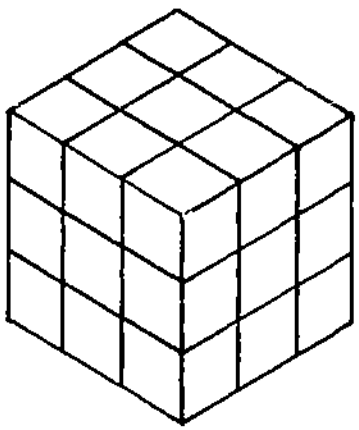
There are 8 people at a meeting. If every person shakes hands with every other person, how many handshakes would take place?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Acting it out	Generalizing solutions

GRADE 6

PROBLEM: 6.3
G-1

This cube was painted red. Then it was cut into 1 x 1 x 1 cubes. Organize a chart to show how many have 1 red face, 2 red faces, 3 red faces, no red faces.

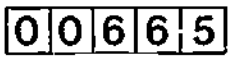


UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Do a simpler problem 2 x 2 x 2	Collecting data	Extension to 4 x 4 x 4

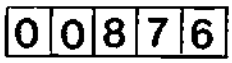
GRADE 6

PROBLEM: 6.4
N-1, O-1

Bill likes to watch the odometer on his father's car. When they started the trip the odometer looked like this:



Bogtown is 217 km farther from Bill's house than Smogtown. The odometer looked like this at Smogtown.



What would the odometer read at Bogtown?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Choosing and applying the appropriate operation	Explaining the answer

GRADE 6

PROBLEM: 6.5

0-4

You are the sports director in charge of setting up a soccer tournament which will be held during the last week in May. You have received 429 entries. You are planning a single elimination tournament (one loss and the team is out). How many games must you arrange for, to get the champion?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Drawing diagrams	Applying patterns	Stating an answer to the problem

GRADE 6

PROBLEM: 6.6

0-3

Each letter stands for a different digit. Rewrite the problem with numerals.

$$\begin{array}{r} V2V7 \\ \times \quad 2 \\ \hline X2X2Z \end{array} \qquad \begin{array}{r} YZ \\ \times \quad 2 \\ \hline XZZ \end{array} \qquad \begin{array}{r} YVZ \\ \times \quad V \\ \hline TTVZ \end{array}$$

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Restating the problem in your own words	Using logic or reason	Restating the problem with the answer

GRADE 6

PROBLEM: 6.7
N-8, O-3

Find a number that is less than 170, more than 90, and has 3, 5, and 9 as factors.

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying key words	Using logic or reason	Looking for alternate ways to solve the problem

GRADE 6

PROBLEM: 6.8
0-3

A bicycle costs $150.00 cash. You can purchase it by putting 20% down and making 12 monthly payments of $12.50. How much would you save by paying cash for the bicycle?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identify wanted, given and needed information	Choosing and applying the appropriate operation	Stating an answer to the problem

GRADE 6

PROBLEM: 6.9
N-1

John decided to write the numbers 1 to 1 000 000. After writing 31 673 digits, he got tired and quit. How many numbers did he write? What number was his last?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Collecting and organizing information (charts, graphs)	Checking your answer

GRADE 6

PROBLEM: 6.10

0-1

In one season, the Edmonton Eskimos scored 315 points. They won the Grey Cup by 3 points. There were 79 points scored in the game. What was the score of the game?

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying extraneous information	Guessing and checking	Making and solving similar problems

CALCULATOR PROBLEMS

The calculator is an excellent tool for solving problems. This is particularly the case where large numbers are involved, where many calculations are necessary or when it is important to explore patterns or relationships in numbers. The following problems are examples of how the calculator can be used for such purposes.

PROBLEM: 5.11
0-1, 0-3, 0-4

A DC-10 airplane may not have a mass over 151 600 kg when landing. If its mass is more, it must burn off fuel by circling until it reaches the safe mass.

A DC-10 is flying from Winnipeg to Vancouver. While flying over Calgary, it has trouble with its air pressure system. It must land for repairs, but it still has too much fuel in its tanks. How long must it circle Calgary before it can land?

You will need the following information:

Number of passengers: 210
Average mass of a passenger: 75 kg
Average mass of a passenger's luggage: 20 kg
Mass of cargo: 13 800 kg
Mass of fuel at takeoff: 18 000 kg
Distance from Winnipeg to Calgary: 1 035 km

When cruising, the DC-10 burns fuel at the rate of 7 kg/km. It burns 6 350 kg/h while circling.

a) About how much is the mass of the passengers?
_____kg

b) About how much is the mass of the luggage?
_____kg

c) What was the total mass of plane, fuel, cargo, passengers and luggage when it left Winnipeg?
_____kg

d) How much fuel did the plane burn flying from Winnipeg to Calgary? _____kg

e) About how much is the mass of the loaded plane when it first gets to Calgary? _____kg

f) How much fuel must it burn before it can land? _____kg

g) How many kilograms of fuel does it burn each minute it circles? _____kg

h) How long, to the nearest minute, must it circle before it can land? _____min

UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Identifying wanted, given and needed information	Choosing and applying the appropriate operation	Determining the reasonableness of the answer

COMPUTER PROBLEMS

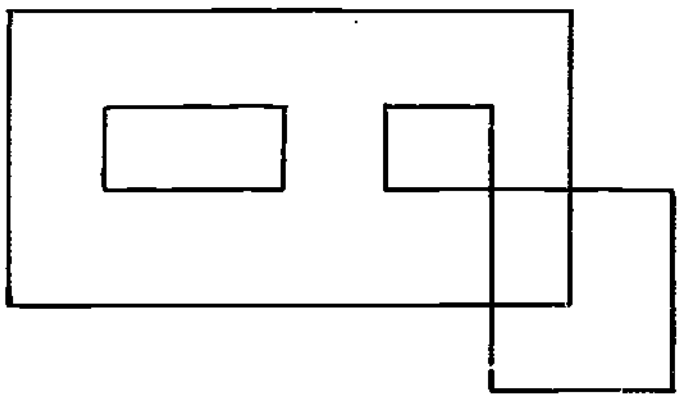
The computer may be used in a variety of ways to solve problems that require the setting up of computer procedures (commands), multiple step calculations and large amounts of data. The following questions are designed for LOGO use and illustrate problems involving the testing of procedures to construct and order geometric shapes.

Write separate procedures for drawing each of at least two different geometric shapes. Using these procedures, develop a set of LOGO instructions that will draw each shape in at least two different places on the screen.

Be sure to keep a list of the instructions for each procedure and for drawing with the procedures.

Develop a set of LOGO instructions to draw a square. Draw the square in at least four other locations and positions on the screen.

Develop a set of LOGO instructions to draw the following design:



103

CHALLENGE PROBLEMS

The following problems are provided for use as challenges to students at the Grade 3 to 6 levels. The problems are not presented in any order of difficulty, and may not involve mathematics directly.

The problems may be presented as "the question of the week" or at a mathematics interest station. It is intended that they be used with students seeking further challenge.

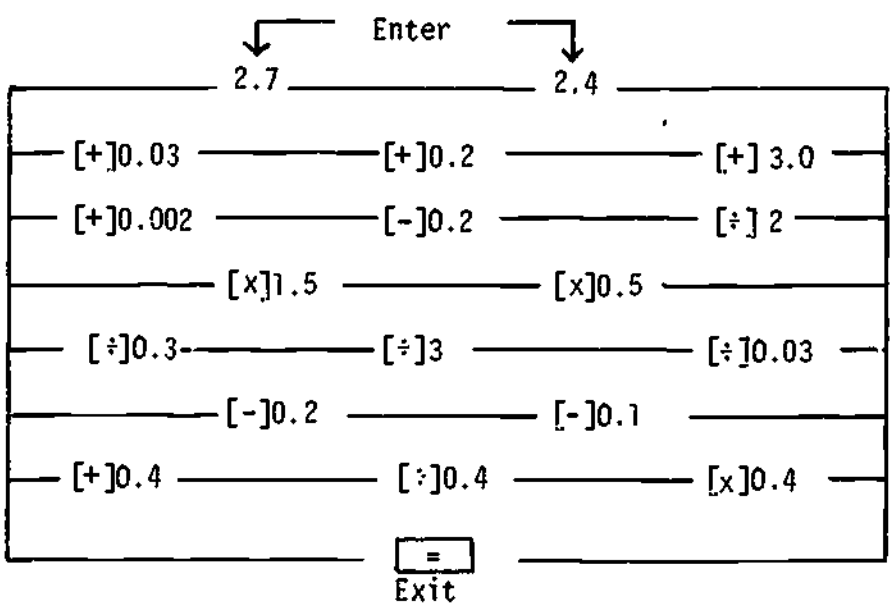
PROBLEM: 6.2

0-1, 0-3, 0-5

A house has seven rooms. Each room has two or three doors to choose from. Each time you go through a door into a room, do the computation shown. You may go into each room only once.

a) Try to go through the house so that you leave with the biggest number possible. Show your path.

b) Go through the house again. This time leave with the smallest number possible. Show your path.



UNDERSTANDING THE PROBLEM	DEVELOPING AND CARRYING OUT THE PLAN	LOOKING BACK
Asking relevant questions	Identifying and applying relationships	Reviewing the solution process

Find the sum of:

1 - 2 + 3 - 4 + 5 - 6 + ... + 99 - 100.

Find the sum of:

100 - 99 + 98 - 97 +96 - 95 + ... 4 - 3 + 2 - 1.

Answers: (-50), (+50)

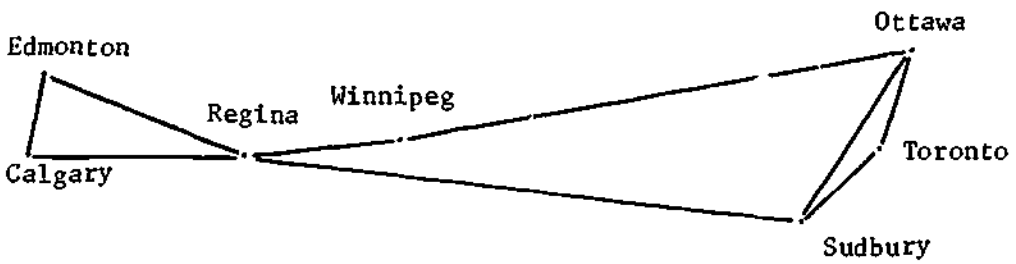
How many numbers less than 150 and greater than 0 do not contain the digit 3?

Answer: (116)

How can change for 75¢ be made using exactly 6 coins?

Answer: (3 nickels, 1 dime, 2 quarters)
(5 nickels, 1 fifty cents)
(5 dimes, 1 quarter)

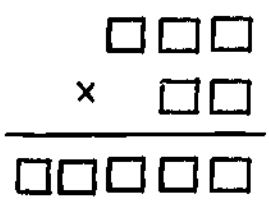
David, who lives in Edmonton, is preparing to visit his uncle in Ottawa. Using the most direct flight routes, he made the following diagram.



How many different direct routes can he fly to Ottawa?

Answer: (4 routes)

Arrange the numbers 2, 3, 4, 5, 6 in the boxes to get the largest possible product. Use each number once.



Answer: 542
x 63
34146

Susan delivers papers every day except Sunday. Every Saturday she collects $24.00. If each paper costs 25¢, how many customers does she have?

Answer: (16)

How many quarters can be placed around a given quarter so that all adjacent quarters touch the given quarter?

Answer: (6)

Find the area of a triangle whose vertices are A(-2,0), B(4.0) and C(6,5)

Answer: (15)

Two explorers set out on foot to explore the barren area of the north. They plan to go in a straight line and return by the same route. Each can carry 12 days of food supplies and travel 20 km a day. What is the farthest distance that can be explored?

Answer: (160 km)

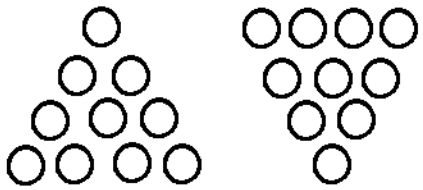
Steve, Michael, Sandra and Lesley are standing in line to buy tickets for a movie. In how many ways can they stand in line to buy their tickets?

Answer: (24)

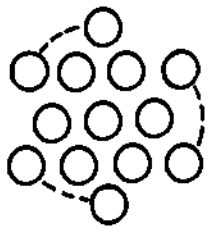
The fencing that encloses a square lot is attached to upright posts placed 5 m apart. If twenty posts are necessary to fence the lot, what is its area?

Answer: (625 m^2)

Move three circles in the figure on the left to make it like the figure on the right.



Answer:



These are three views of the same cube.

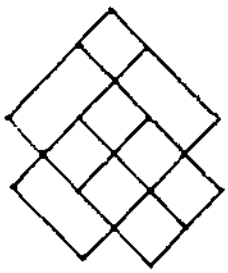
(a) Which side is opposite side 4?

(b) Which side is opposite side 5?

(c) Which side is opposite side 6?

Answer: (1, 2, 3)

How many squares are there in this figure?



Answer: (13)

One rectangle has 4 sides. If you arrange 2 rectangles together, you will see 7 sides. Join a third rectangle to the first two, and you will see 10 sides. How many sides would you see if you joined 12 rectangles together?

Answer: (37)

Sue can't resist donuts. She saw that plain donuts were on sale for 35¢ each and fancy donuts for 50¢ each. She bought a dozen donuts and paid $4.65. How many fancy donuts did she buy?

Answer: (3)

How many toothpicks are needed to make 5 triangles?

Answer: (15,11)

Licences for bicycles use the letters A, B, and C and the digits 1, 2, 3. All the licences start with the letter and are followed by a single letter or a digit. Make a diagram showing all the ways the licenses could occur.

Answer: (18 ways)

An artist receives 3¢ for each digit he prints on a page. If he earned $17.37 for a job, how many pages did he number?

Answer: (229 pages)

The numerals 333, 7777, and 88 all contain repeated, single digits. How many numerals between 11 and 999 999 contain repeated, single digits?

Answer: (43)

Using pennies, nickels, and dimes, in how many ways can you make 23¢?

Answer: (9)

You get on an elevator at the main floor.

Go up 7 floors.
Go down 4 floors.
Go up 8 floors.
Go down 3 floors.
Go down 6 floors.
Go up 2 floors.

Get off the elevator. What floor are you on?

Answer: (5)

Two neighboring families each have three children. The product of the children's ages in each family is 72 and the sum of the ages is the same for both families, yet the children in one family are of different ages than the children in the other family. Find the ages of the six children.

Answers: (2,6,6)
(3,3,8)

During a football game, your team scored 18 points. In how many different ways could your team have made this score?

touchdown	6 pt
point after	1 pt
field goal	3 pt
safety	2 pt

Answer: (14 ways)

One of seven objects, identical in appearance, is slightly heavier than the rest. With only an equal arm balance available, how can the heavier object be determined in just two weighings?

Answer:
$$\begin{pmatrix} 3 & 3 & \text{balance} \\ 1 & & \text{(heavier)} \\ \\ 1 & 1 & \text{balance} \\ 1 & & \text{(heavier)} \end{pmatrix}$$

Eight men and two boys want to cross a river using a small canoe. The canoe can carry two boys or one man. How many times must the canoe cross the river to get everyone to the other side?

Answer: (33 trips)

A can with 40 marbles in it has a mass of 135 g. The same can with 20 marbles has a mass of 75 g. What is the mass of the can?

Answer: (15 g)

A man, fox, goose, and some corn are on one side of a river. The man wishes to get himself, the animals, and the corn across the river using a boat which will carry him and only one other thing. The fox will eat the goose if left alone together, and the goose will eat the corn if left alone with it. How can the crossing be made?

Answer: (7 trips)

$$\begin{array}{r} A\ B\ A \\ -\ C\ A \\ \hline A\ B \end{array}$$

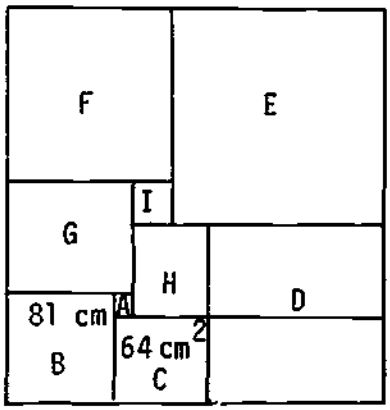
In the above subtraction problem, each digit has been replaced by the given letter. Can you determine the original problem?

Answer: $\begin{array}{r} 101 \\ -\ 91 \\ \hline 10 \end{array}$

If 6 boys fill 6 notebooks in 6 weeks and 4 girls fill 4 notebooks in 4 weeks, how many notebooks will a class of 12 boys and 12 girls fill in 12 weeks?

Answer: (60)

From the information given, find the areas of the square regions below.

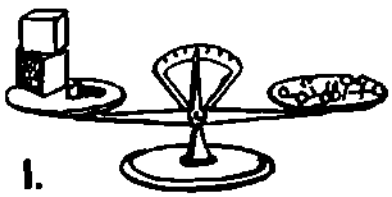


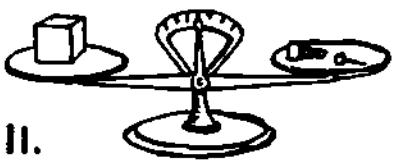
Answer: A - 1 cm^2
D - 256
E - 324
F - 196
G - 100
H - 49
I - 16

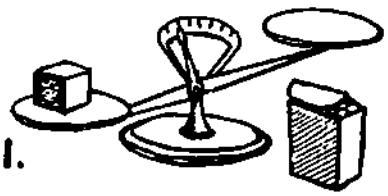
There are only two rectangles whose sides are whole numbers and whose area and perimeter are the same number. What are their dimensions?

Answers: (4 x 4)
(3 x 6)

PROBLEM


I.

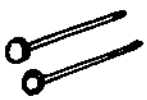

II.


III.

How many nails will balance one cube?

 - block

 - bolt

 - nails

Answer: (4)

A rubber ball is dropped from a height of 16 m. Each time it lands it bounces to a height which is half the distance from which it last fell. The ball is finally caught when it bounces to a height of 1 m. Find the total distance the ball travelled.

Answer: (45 m)

ANSWER KEY

PROBLEM SET 1:

Problem 1.1	9 children
Problem 1.2	5 butterflies
Problem 1.3	7 cylinders
Problem 1.4	2, 4, 1, 3, 5
Problem 1.5	10 ears, 6 rabbits
Problem 1.6	6 camels
Problem 1.7	pictograph results will vary
Problem 1.8	2 cones, 6 cylinders, 1 pyramid, 1 cube and 1 rectangular box or prism
Problem 1.9	7 pennies
Problem 1.10	3 balloons

PROBLEM SET 2:

Problem 2.1	skip count: 2, 4, 6, 8, 10, 12, 14, 17, 20, 23
Problem 2.2	1 glass of Punch and 4 cookies 3 glasses of punch and 1 cookie
Problem 2.3	Use the math text. 10-11, 18-19 not possible as pages are not facing. Sum is not an odd number.
Problem 2.4	-
Problem 2.5	9 parrots
Problem 2.6	First-lion, Last-elephant
Problem 2.7	6th-21 blocks, 8th-36 blocks
Problem 2.8	8 different ways
Problem 2.9	5, 10, 15, 20, 25, 30 / 30 pucks
Problem 2.10	Take all the combinations that make up 25 cents. The combinations must add up to $1.00.

PROBLEM SET 3:

Problem 3.1	5445, 6534, 7623, 8712, 9801
Problem 3.2	16-5's
Problem 3.3	5 6 3 1 1 5
Problem 3.4	7 minutes
Problem 3.5	37 sides
Problem 3.6	415 Pages
Problem 3.7	Sue-June, Jan-January, Bob-May, Tom-March, Jan is the oldest. Sue is the youngest.
Problem 3.8	8 faces, 18 edges, 12 corners
Problem 3.9	To keep his Pants up.
Problem 3.10	For 2 squares-1, for 3 squares-2, for 4 squares-5.

PROBLEM SET 4:

Problem 4.1	97860
Problem 4.2	20 pieces
Problem 4.3	11th floor
Problem 4.4	6 paths
Problem 4.5	5 tricycles, 2 bicycles
Problem 4.6	156 times
Problem 4.7	1 quarter + 3 pennies 2 dimes + 1 nickel + 3 pennies 2 dimes + 8 pennies 1 dime + 3 nickels + 3 pennies 1 dime + 2 nickels + 8 pennies 1 dime + 1 nickel + 13 pennies 5 nickels + 3 pennies 4 nickels + 8 pennies 3 nickels + 13 pennies 2 nickels + 18 pennies 1 nickel + 23 pennies 28 pennies
Problem 4.8	18 litres
Problem 4.9	8188
Problem 4.10	(follow directions as specified)

PROBLEM SET 5:

Problem 5.1	9 m^2 / 12 posts
Problem 5.2	28 rectangles, 204 squares
Problem 5.3	28 students, most common mark: 85 The test was not difficult. There were 20 problems.
Problem 5.4	19:43:30
Problem 5.5	27
Problem 5.6	3 doughnuts
Problem 5.7	4 cookies each and 12 left over
Problem 5.8	1881
Problem 5.9	72
Problem 5.10	At least 7.

PROBLEM SET 6:

Problem 6.1	9 triads, 1 tetrad 1 triad, 7 tetrads 5 triads, 4 tetrads
Problem 6.2	28
Problem 6.3	1-6, 2-12, 3-8, 0-1
Problem 6.4	01093
Problem 6.5	428 games
Problem 6.6	X=1 Y=5 Z=0 B=6 I=3 Horizontal form
Problem 6.7	135
Problem 6.8	$30.00
Problem 6.9	8195 numbers 8195 is the last number
Problem 6.10	Edmonton 41 / Other 38

PROBLEM SOLVING

References

Anderson, A. L. *Problem Solving in the 80's: A Mathematics Workbook.* D. C. Heath Canada Limited. 1983.

Greenes, C., J. Gregory and D. Seymour. *Successful Problem Solving Techniques.* Palo Alto, California: Creative Publications, Inc., 1977.

Greenes, C., G. Immerzeel, E. Ockenga, L. Schulman and R. Spungin. *Techniques of Problem Solving: Problem Decks A, B, C, D.* Palo Alto, California: Dale Seymour Publications, 1980.

Greenes, C., G. Immerzeel, L. Schulman and R. Spungin. *Techniques of Problem Solving (TOPS) Beginning Problem Solving Kit, K-1.* Palo Alto, CA: Dale Seymour Publications, 1982.

Harvey, L., and A. Roper. *Pattern Blocks Problems for Primary People.* Palo Alto, CA: Creative Publications, 1979.

Houghton Mifflin Mathematics Problem Solving Activities. Houghton Mifflin Canada Limited, 1982.

Immerzeel, G., E. Ockenga, J. Duea and J. Tarr. Iowas Problem Solving Project: *Resource Decks.* Cedar Falls, Iowa: Price Laboratory School, 1978.

Jacobson, Marilyn H., Frank K. Lester, Jr., and Arthur Stengel. Making problem solving come alive in the intermediate grades. In S. Krulik and R. Reys (eds.), *Problem Solving in School Mathematics:* 1980 Yearbook of the National Council of Teachers of Mathematics, 127-135. Reston, Virginia: The Council, 1980.

Kilpatrick, Jeremy. Base the curriculum on problem solving? How? Why? Address given at NCTM Annual Meeting, Seattle, 1980.

Krulik, Stephen and Jesse Rudnick. *Problem Solving: A Handbook for Teachers.* Boston: Allyn & Bacon, 1980.

Krulik, Stephen and Robert E. Reys (eds.), *Problem Solving in School Mathematics.* 1980 Yearbook of the National Council of Teachers of Mathematics. Reston, Virginia: The Council, 1980.

LeBlanc, John F. You can teach problem solving. *Arithmetic Teacher* 25. (November, 1977): 16-20.

Lester, F. Ideas About Problem Solving: A Look at Some Psychological Research, *Arithmetic Teacher,* 25 (November, 1977): 12-14.

Meiring, Steven P. *Problem Solving - A Basic Mathematics Goal: Book 1, Becoming A Better Problem Solver, and Book 2, A Resource for Problem Solving.* Inservice Education, Ohio Department of Education, Columbus, Ohio, 1980.

Morris, Janet. *How to Develop Problem Solving Using a Calculator.* Reston, Virginia: National Council of Teachers of Mathematics, 1981.

Nelson, L. D., and J. Kirkpatrick "Problem Solving: in J. Payne (ed.). *Mathematics Learning in Early Childhood.* 37th Yearbook of the National Council of Teachers of Mathematics, 69-75. Reston, Virginia: The Council, 1975.

Polya, G. *How to Solve It.* 3rd ed. Princeton, N.J.: Princeton University Press, 1973.

Porter, Richard D. *Project-a-puzzle.* Reston, Virginia: National Council of Teachers of Mathematics, 1978.

Roper, A., and L. Harvey. *The Pattern Factory: Elementary Problem Solving Through Patterning.* Palo Alto, CA: Creative Publications, 1980.

Seymour, Dale. *Favorite Problems.* (Poster problems and reproducible worksheets). Palo Alto, CA: Dale Seymour Publications, 1982.

Suydam, Marilyn N. Untangling clues from research on problem solving. In S. Krulik and R. Reys (eds.), *Problem Solving in School Mathematics:* 1980 Yearbook of the National Council of Teachers of Mathematics, 34-50. Reston, Virginia: The Council, 1980.