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ABSTRACT

Mathematics activities and facts related to pi are presented in this issue of "Student Math Notes." Included are: (1) an exercise based on Buffon's needle problem in which pieces of toothpicks are dropped onto a ruled surface; (2) a calculation of pi to 200 decimal places; (3) exercises related to Biblical and ancient Chinese approximations of pi; (4) exercises related to algebraic representations of pi including a computer program written in BASIC which approximates pi using the Leibnitz series; (5) mnemonic devices for remembering approximations of pi; (6) a computer program written in BASIC which simulates Buffon's needle problem; and (7) several additional miscellaneous questions and facts about pi. (DC)

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
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π: The Digit Hunt

Cut four 1-centimeter pieces from toothpicks. From a reasonable height, randomly drop the four sticks onto the ruled surface below. A "hit" occurs when a stick lands on or across one of the lines in the ruled surface. Count the number of hits on each of 25 drops of the four sticks. Keep an accurate record of the number of hits and drops in all.

Divide the total number of sticks dropped (100) by the total number of hits. What is the quotient? ________

Sum your hits with those of your classmates, and divide that number into the total number dropped by you and your class. What is the pooled result? ________

The Lure of Pi

Congratulations! You have just joined the long historical line of the digit hunters of the number π. Pi is defined as the ratio of the circumference of a circle to its diameter. The toothpick-tossing experiment you did is one way to compute a rough estimate of pi's value and is known as **Buffon's needle problem.** It is based on the expansion by Pierre Laplace (1749–1827) of an idea of Georges Louis Leclerc, Comte de Buffon (1707–1788). But, Buffon's needle problem came late in the quest for pi's digits.

Throughout the ages, many people have felt that the digits of pi would show some sort of pattern. And, they were determined to find these digits. But, in 1761, Johann Heinrich Lambert showed that pi is an irrational number and cannot be written as a repeating decimal. We can correctly compute pi to as many decimal places as we like, but there is no repetitive pattern to its digits. No fraction with integers for numerator and denominator can exactly equal pi, but there are many fractions that come close enough for practical applications.

Here is a calculation of pi that has been carried out to 200 decimal places. Impressive, isn't it?

$\pi \approx$ 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510
58209 74944 59230 78164 06286 20899 86280 34825 34211 70679
82148 08651 32823 06647 09384 46095 50582 23172 53594 08128
48111 74502 84102 70193 85211 05559 64462 29489 54930 38196

Now look back at your approximation of pi from page 1.
Do you think it is a good estimate of pi?
Is the pooled estimate of the class a better one?

Fractional Estimates of Pi

Throughout history, there have been many milestones along the road to finding the digits of pi. You may be surprised to learn that the Bible gives a value for pi. In I Kings 7:23, it is written:

> And He made a molten sea, ten cubits from one brim to the other: it was round all about, . . and a line of thirty cubits did compass it round about.

The passage is thought to refer to a large basin in Solomon's Temple.

Remembering the definition of pi, what value of pi is implied in this passage? __________

Around 240 B.C., Archimedes took a circle of diameter one unit. He computed the perimeter of the circumscribed and inscribed polygons of 96 sides and determined this inequality:

$$3\frac{10}{71} < \pi < 3\frac{1}{7}$$

Determine the first five places in the decimal equivalents for these two fractions. __________ __________

At what decimal place do they differ from the approximation of pi given above? __________

In the fifth century A.D., the Chinese astronomer Ch'ung-Chih found an interesting fractional approximation of pi.

Write the numerical representation of one hundred thirteen thousand, three hundred fifty-five. __________

Write a fraction using the first three of these digits as a three-digit denominator and the last three as a three-digit numerator. __________

This fraction gives pi to an accuracy of how many decimal places? __________

This fraction of Ch'ung-Chih is about as close an approximation of pi as is needed in most applications. But mathematicians were not satisfied. They were determined to unlock the exact value of pi.

Pi as an Infinite Product and an Infinite Sum

In 1592, about 1800 years after Archimedes' work with pi, the French mathematician François Viète expressed pi as an infinite sequence of algebraic operations. Here it is:

$$\pi = \frac{1}{\sqrt{1/2} \cdot \sqrt{1/2 + 1/2\sqrt{1/2}} \cdot \sqrt{1/2 + 1/2\sqrt{1/2 + 1/2\sqrt{1/2}}} \cdots}$$

The greater the number of terms used in the computation, the closer you get to the exact value of pi. Viète's work with pi is significant in that it was the first time the exact value of pi was expressed in a regular mathematical pattern. Note that the denominator is an infinite product of expressions of square roots.

In 1674, the German mathematician Gottfried Wilhelm von Leibnitz (one of two independent inventors of the calculus) used his invention to find an expression for pi as an infinite sum:

$$\pi = 4(1 - 1/3 + 1/5 - 1/7 + 1/9 - \ldots)$$

This formula presents pi as the limit of an infinite series of fractions whose denominators are odd numbers and whose signs alternate. The simplicity of the Leibnitz series makes it one of the best-known expressions for pi.

What is Leibnitz's approximation for pi for six terms of the series? __________

For 11 terms? __________

These approximations for pi may not impress you. It is only when the number of terms becomes extremely large that the series becomes a powerful tool for approximating pi.

Here is a computer program written in BASIC for approximating pi with Leibnitz's series. Experiment with it, using different values for the number of terms.

```
10  PRINT "LEIBNITZ'S SERIES USED TO
       APPROXIMATE PI"
20  INPUT "NUMBER OF TERMS="; N
30  PI = 1
40  A = 3
50  B = -1
60  FOR C = 2 TO N
70  PI = PI + B/A
80  B = B*(-1)
90  A = A + 2
100 NEXT C
110 PI = 4*PI
120 PRINT "PI IS APPROXIMATELY EQUAL TO "; PI
130 GOTO 20
140 END
```

The Search Goes On

With the advent of the computer, a programmer can easily calculate the value of pi to thousands, even millions, of decimal places. In fact, the task of calculating the decimal places of pi is used to determine if new computers are functioning properly.

Piet Hein wrote:

A number will find
fulfillment enough
in knowing its mind
and doing its stuff.

Even today, pi continues to find fulfillment in "doing its stuff" to lure the digit hunters to keep up the endless search for its digits.

Mnemonics for Pi

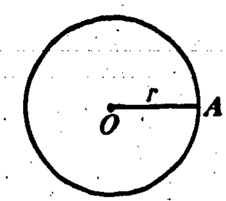
The 1983 *Guinness Book of World Records* lists Rajan Mahadevan as having recited 31 811 places of pi from memory. For those of you who need help remembering some of the digits, here are some memory aids. Do you see the relationship between the number of letters in the words and the approximation of pi?

Yes, I know a digit.
May I have a large container of coffee?

See, I have a rhyme assisting
My feeble brain, its tasks ofttimes resisting.

Bet you can't answer these . . .

- Make up a mnemonic that will give the first 10 digits of pi.
- The digits of pi are thought to be distributed randomly. So each digit should occur about 1/10 of the time. In the sample of 200 decimal places of pi, about how many times should each digit appear? ________ Do a frequency count of the 10 digits to check your conjecture.
- On this circle of radius *r*, draw a square having $\overline{OA}$ as one of its sides. What is the ratio of the area of circle O to the area of the square? ________



Bet you didn't know that . . .

- in 1897, Indiana's General Assembly considered Bill No. 246 to legislate the value of pi. The value under discussion was incorrect.
- the millionth decimal place of pi is 1; the two-millionth is 9.
- in 1610, German Ludolph van Ceulen completed his calculation of pi to 35 decimal places. This approximation was engraved on his tombstone, and even today, pi is referred to in Germany as the Ludolphine number.
- the Greek letter π is the first letter of the Greek word **perimetron**, meaning "the measurement around." The use of π to represent the ratio of a circle's circumference to its diameter became widespread in 1737 when mathematician Leonhard Euler began using it.
- Buffon's needle problem is an example of the Monte Carlo method of probability, in which a numerical value is found by conducting and observing a random event many times. This technique has a wide field of applications.

You might enjoy using this computer simulation of Buffon's needle problem. The program is written in BASIC for an Apple computer. It is a modified version of a program first published by Ronald J. Carlson and his Plymouth-Canton (Mich.) High School computer class (*Mathematics Teacher*, November 1981, p. 639). You'll notice in the graphic display that the needles are the same length as the distance between rules. The adjustment for this change has been made with the insertion of the 2 in the formula given in line 250.

```
10  REM COUNT OF BUFFON'S ESTIMATION OF PI
20  INPUT "HOW MANY NEEDLES TO DROP?"; N
30  HOME
40  HGR
50  HCOLOR = 2
60  FOR Y = 0 TO 159 STEP 10
70  HPLOT 0, Y TO 279, Y
80  NEXT Y
90  HIT = 0
100 HCOLOR = 3
110 FOR Z = 1 TO N
120 X = INT (RND (Z) * 280)
130 Y = INT (RND (Z) * 160)
140 ANGLE = RND (Z) * 1000
150 X1 = X + 10 * COS (ANGLE)
160 Y1 = Y + 10 * SIN (ANGLE)
170 IF X1 < 0 OR X1 > 279 THEN 200
180 IF Y1 < 0 OR Y1 > 159 THEN 200
190 HPLOT X, Y TO X1, Y1
200 IF Y = Y1 AND INT (Y / 10) = Y / 10 THEN HIT = HIT +
      1
210 IF Y / 10 = INT (Y / 10) AND Y1 / 10 = INT (Y1 / 10)
      THEN 230
220 IF INT (Y / 10) < > INT (Y1 / 10) THEN HIT = HIT + 1
230 NEXT Z
240 VTAB 21
250 PRINT "PI ESTIMATE = "; 2 * N / HIT
260 END
```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