DOCUMENT RESUME

ED 238 875 SP 023 648

AUTHOR Picus, Larry; And Others
TITLE Teaching Problem Solving: A Research Synthesis.
INSTITUTION Northwest Regional Educational Lab., Portland, Oreg.
SPONS AGENCY Alaska State Dept. of Education, Juneau.
PUB DATE Jun 83
NOTE 35p.; Product of the Goal Based Education Program.
PUB TYPE Information Analyses (070)

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS *Cognitive Processes; Comprehension; Concept Formation; Educational Research; Elementary Secondary Education; Knowledge Level; *Problem Solving; Skill Development; *Student Motivation; *Teacher Effectiveness; *Teaching Methods; Transfer of Training

ABSTRACT

This research synthesis identifies a number of important concepts regarding the teaching of problem solving skills to students in grades K-12. Studies investigated what problem solving skills are and whether they can be taught, as well as how problem solving can be organized in the curriculum and how it should be taught. Major findings from the research base include: (1) Students can learn to be better problem solvers through exposure to focused instruction; (2) Direct teaching of problem solving strategies improves problem solving skills in students; (3) Problem solving competence requires a knowledge base in the content area in which problems are posed; (4) Problem solving objectives are best taught through integration with existing curricula; (5) Transfer and use of problem solving strategies appears more likely when problems used in instruction are like those that will be routinely encountered later; (6) Successful instruction requires attention to student motivation, content knowledge, and problem solving skills and strategies; and (7) Effective instructional practices identified in this synthesis are consistent with the findings derived from the effective schooling research base. (JD)

TEACHING PROBLEM SOLVING:
A RESEARCH SYNTHESIS

Prepared For
The Alaska Department of Education

Prepared By

Larry Picus
Northwest Regional Educational Laboratory

Thomás P. Sachse
California State Department of Education

Ronald M. Smith
Northwest Regional Educational Laboratory

June 1983

Goal Based Education Program
Robert E. Blum, Director

Northwest Regional Educational Laboratory
300 S.W. Sixth Avenue
Portland, Oregon 97204

Goal Based Education Program

PREFACE

This paper is one of a series of research syntheses commmissioned by the Alaska Department of Education. As originally conceived, it was to have summarized and integrated the research literature related to the teaching and learning of thinking skills as a broad domain. After initial review of research sources, it became clear that the bulk of the applied research fell in the narrower, yet still important domain, of problem solving. Since the synthesis was intended to answer practical questions related to instruction rather than address theoretical issues, problem solving became the final focus.

As compared with other syntheses in the series, this work has a different format. The research base concerning the teaching of problem solving is small, relatively speaking. Yet the studies are diverse in approach and methodology. The unique character of the literature base led the authors to select a different synthesis format for this particular effort. The authors believe that the approach used here has resulted in a well balanced and accurate depiction of the research findings as they relate to key instructional issues.

Teaching Problem Solving Skills: A Research Synthesis was developed by the Goal Based Education Program of the Northwest Regional Educational Laboratory under contract to the Alaska Department of Education. The opinions expressed in this publication do not necessarily reflect the opinions of the agencies involved in its development, and no official endorsement should be inferred.

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RESEARCH HIGHLIGHTS

This document provides a review of recent literature on the topic of teaching problem solving skills to students in grades K-12. The major findings from the research base include the following:

- Students can learn to be better problem solvers through exposure to focused instruction.
- Direct teaching of problem solving strategies improves problem solving skills among students.
- Problem solving competence requires a knowledge base in the content area in which problems are posed.
- Problem solving objectives are best taught through integration with existing curricula.
- Transfer and use of problem solving strategies appears more likely when problems used in instruction are like those that will be routinely encountered later.
- Successful instruction requires attention to student motivation, content knowledge and problem solving skills and strategies.
- Effective instructional practices identified in this synthesis are consistent with the findings derived from the effective schooling research base.

TEACHING PROBLEM SOLVING: A RESEARCH SYNTHESIS

Introduction

There is a growing interest throughout our nation in the quality of the education provided by the public schools. The release of the National Commission on Excellence in Education report, A Nation at Risk, has served to focus the attention of the president, presidential candidates and a growing segment of the population on the role of the schools in American life. This interest is directed primarily toward the declining performance of students as measured by standardized tests in the "basic skills" such as reading, writing and arithmetic.

Yet another important component of the educational process is to assure that students have the ability to use previously gained information in new situations. The National Commission indicated the importance of these problem solving skills when it stated, "Some worry that schools may emphasize such rudiments as reading and computation at the expense of other essential skills such as comprehension, analysis, solving problems, and drawing conclusions." (National Commission on Excellence in Education, 1983).

Results from the National Assessment of Educational Progress (NAEP) indicate that while recent performance in the basic skills is improving, students showed uniformly poor performance on exercises at the application or problem solving level (Carpenter et.al. 1980). The Education Commission of the States (1982) indicates that in the four learning areas tested by NAEP, reading, writing, mathematics and science, results "indicate that students may have acquired very few skills for examining ideas. Many are capable of preliminary interpretations, but few are taught to move on to extended comprehensive and evaluative skills."

These assessment results are indicative of the high level of interest in problem solving skills among educators. The assessment results also appear to indicate that it is possible to successfully teach problem solving skills in school, a claim that is generally backed by research on the topic. The purpose of this research synthesis is to review and report on the relevant research on teaching problem solving skills.

The literature existing on problem solving can be divided into three general areas: (1) conceptual/theoretical work; (2) experimental research; and (3) classroom applications research. Although the focus of this paper will be primarily on the third area, the teaching of problem solving skills, it is important to briefly review ideas from the other two areas as well.

Conceptual/theoretical Work. These papers are typically written by cognitive and instructional psychologists. Much of their content revolves around how students learn, and how that knowledge is applied to problem solving. Research into artificial intelligence has also added to this body of knowledge. Prominent conceptual frameworks are described in Gagne (1983), Greeno (1980), Mayer (1983), Resnick (1981), and Scandura (1977). Unfortunately, the work of learning theorists often does not get applied to changing the means by which students learn. The artificial intelligence community concentrates on using the results from studies of information processing to teach machines, but not students, to think.

Experimental Research. A second line of inquiry is experimental research in teaching problem solving skills. It differs from the classroom applications literature in that it occurs largely in the laboratory, and relies on the results of pre- and post-tests applied to a variety of controlled instructional interventions. Examples of this research include the work of Barratt (1975), Rosenthal (1979), Vos (1976) and Wollman and Lawson (1978). Here too, it is difficult to apply the findings from the laboratory to the classroom. Although the results are more applicable than those of the theorists, the conditions (one-on-one in the laboratory) are difficult to replicate in classrooms.

Classroom Applications Research. The third source of information on problem solving, classroom applications research, is the primary focus of this paper. Classroom applications research has looked at the process of teaching problem solving strategies and approaches to students. To describe the findings from the applications research, the balance of this paper is divided into five sections. The first section discusses how problem solving skills are defined in the research, and the steps or strategies that have been associated with successful problem solving. The second section of the paper addresses the teachability of problem solving skills. It also discusses existing research on teaching and measuring problem solving skills.

The third section of the paper discusses the literature on including problem solving in the curriculum. While problem solving is most often approached through instruction in mathematics, this section points out that problem solving strategies have been successfully taught in other subject areas as well.

The fourth section of the paper describes strategies that are successful in teaching problem solving skills to students. Because problem solving instruction is the primary focus of this research synthesis, this is the most detailed section of the paper.

What Are Problem Solving Skills?

It is helpful to define what is meant by a "problem" before attempting to define what problem solving is. Mayer (1983, p. 3) presents the generally accepted view that "a problem occurs when you are confronted with a given situation... and you want another situation...but there is no obvious way of accomplishing your goal." Problem solving, he states, is the process or series of mental operations used in moving from the present situation to the desired goal.

The National Council of Supervisors of Mathematics states that "problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations." (NCSM, 1977, p. 2). They go on to state that learning to solve problems is the principal reason for studying mathematics. Similarly, the National Council of Teachers of Mathematics (1980) states that "Problem Solving must be the focus of school mathematics in the 1980's." Huggins (1966, p. iv) defines problem solving as "a disposition toward inquiry which has as its goals the development of new ideas based upon older ones." And George Polya states that problem solving is "finding the unknown means to a distinctly conceived end." (In Krulik, 1980).

In How to Solve It Polya (1957) goes on to define four steps in problem solving. His four steps include:

1. Understanding the problem
2. Devising a plan for solving the problem
3. Carrying out the plan
4. Looking back or evaluating the solution

Although Polya's four steps are perhaps the most frequently quoted strategy for problem solving, they are by no means the only established approach to this issue. Dewey in How We Think (1909) postulated five steps in the problem solving process:

1. A felt difficulty
2. Its location and definition
3. Suggestion of possible solutions
4. Development by reasoning of the bearings of the suggestion
5. Further observation and experiment leading to its acceptance or rejection, that is, the conclusion of belief or disbelief.

The distinction between the four steps of Polya and the five of Dewey is in the preliminary art of understanding and defining the problem. In the remainder of this paper, the authors have adopted Polya's four step model for reasons of parsimony. But it is important to keep in mind the necessity of the problem definition step.

A number of other writers (Suydam and Weaver, 1977; Weiss et al., 1980; Shallcross, 1971; Merwin, 1977) have suggested other systems for approaching problem solving, ranging from three to as many as twelve steps, but Polya's four steps appear to be the most generally applicable and most frequently used in defining the major processes involved in problem solving.

From these problem solving steps, a number of strategies for solving problems emerge. LeBlanc (1977) suggests that there are two types of strategies that can be utilized in the problem solving process: 1) General strategies which help determine the overall plan to be used to help solve the problem, and 2) Helping strategies used to carry out the general strategy. Examples of general strategies include trial and error, simplification and working backwards. Examples of helping strategies include diagrams, lists and equations. These are discussed at more length in the teaching strategies section of this paper.

Can Problem Solving Be Taught?

Most of the research reviewed for this synthesis indicated that it is possible to improve students' problem solving capabilities through focused instructional means. While the research does not clearly lay out a single strategy that is more successful than others for solving problems, a number of successful approaches have been identified.

Early work in the field of problem solving illustrated the ability to distinguish good problem solvers from their less successful counterparts on the basis of specific skills. Bloom and Broder (1950), who studied the differences between good and poor problem solvers, found that good problem solvers pulled key ideas out of the problem, while poor problem solvers did not. They found that unsuccessful students tended to be mentally careless and superficial in solving problems while good problem solvers used careful thinking and sequential analysis to search for an answer to a problem.

More recent research has shown the ability to train specific problem solving skills through focused instruction. Using a training procedure based on principles of "programmed discovery," Barratt (1975) found that significant increases in mathematics skills requiring the student to combine more than one step to solve a problem were found in a test of adolescents aged 12 to 14 years. Rosenthal (1979) conducted a training study on 11 to 12 year-old girls. Using two different training methods, method training and dimension training, she found that both procedures significantly improved performance on posttests. Her findings included the fact that both training methods appeared to be equally effective, and also concluded through a second posttest that the improved performance was retained by the girls who participated in the treatment groups.

Using instructor designed instruments, Vos (1976) concluded that specific instruction in using problem solving behaviors increased the use of those behaviors. He also found that training in specific problem solving techniques was superior to repetition of problems in teaching secondary students to be successful problem solvers. In addition, Vos found that mathematical maturity was a definite factor in problem solving ability. He found that the higher level math classes performed better on all tests than did the students in the lower level math classes. Finally, Wollman and Lawson (1978) found that individualized procedures based on the use of physical materials are more effective than verbal textbook procedures in teaching students to solve problems involving the formal scheme of proportionality.

While the research efforts indicate positive results for teaching problem solving, they tend to deal with specific instructional methodologies and focus on specific instructional issues such as the teaching of proportionality in mathematics. The research on teaching general task-knowledge (an approach which attempts to teach students a set of general problem solving techniques to use in a problem situation) is much more limited. One of the few cases in which someone has attempted to teach general task-knowledge is the work of A. H. Schoenfeld. As described by Smith and Bruce (1981), Schoenfeld taught college students some general strategies for solving integration problems in calculus. According to Smith and Bruce, Schoenfeld indicated that students benefited from this instruction, but a major problem was that students were frequently not sure of when a particular method should be applied.

Another potential problem with teaching general methods, identified by Smith and Bruce, is a concern that the knowledge taught is so broad that students have trouble implementing the appropriate technique when needed. For example, students might have difficulty breaking a problem into parts after being taught the heuristic "break the problem into parts." Shaughnessy (1983) suggests that there are a number of barriers or "derailers" that students can run into when attempting to solve problems. For instance, a potential derailer may be a "folklore paradigm," a belief about an event or relationship that is not true. Application of a false belief in a problem solving sequence may cause a failure in finding an acceptable solution. This derailer can be particularly significant when the false belief is centered on a key concept or relationship used in solving a broad set of problem types.

Shaughnessy suggests that approaches to teaching problem solving must take expected derailers into account if students are to be taught to solve problems successfully.

Related to the question of whether problem solving can be taught is the question of whether problem solving can be measured. There are a number of tests that have been developed to assess problem solving ability. Sachse (1981) identifies thirteen such instruments and provides a brief summary of the characteristics of each. Malone et al. (1980) propose a Rasch approach to measuring problem solving ability, and restates the Rasch model which measures an individual's performance on problems under the assumption that their performance is a function of two factors; the ability of the person and the difficulty of the problem. These measures are incorporated into an equation that assumes the greater the student's ability, the better his or her chance of success, and the more difficult the problem, the smaller the chance that the student will be able to solve it.

Schoen and Oehmke (in Krulik, 1980) describe the development of the Iowa Problem Solving Project's problem solving test. This test is designed to measure a student's ability in three of Polya's four stages: understanding the problem, applying the solution strategy chosen, and looking back at the solution (steps 1, 3 and 4).

Finally, Wasik (1974) compared the perceptual cues used by teachers to identify creative problem solving ability. Ratings of student creative problem solving ability from four teacher teams were related to the results of eight Structure of Intellect ability tests for 162 tenth grade students. Wasik concluded that the teacher teams used the same cues in rating students in terms of creative problem solving ability, and that students who were rated as good problem solvers exhibited high ability in the Structure of Intellect tests.

In summary, problem solving can be tested and existing measures do assess it with reasonable validity and reliability. However, it is necessary to be very clear about which skills are taught and which are tested, if existing measures are to be used effectively.

How Should Problem Solving be Organized in the Curriculum?

A review of the bibliography of this paper indicates that a majority of the literature on problem solving discusses the subject in terms of mathematics. Polya (in Krulik, 1980, p. 2) suggests that mathematics is the best subject area for teaching problem solving because it is "so much simpler than the other sciences." However, math is not the only subject that lends itself well to teaching problem solving. In addition to other logical curricular areas such as science and computer science, literature was found that suggested approaches to teaching problem solving in social studies (Weiss, et.al.), home economics (Dewald-Link and Wallace) and

English composition (Schiff). Since all curriculum areas present "problems" to students, problem solving as a content may cut across all curriculum areas.

The literature also suggests that problem solving is better taught as an integrated part of a curriculum area or areas, rather than as an independent subject. Dewald-Link and Wallace suggest that it is important for students to see all of their problem solving strategies as being related rather than isolated sets of techniques associated with a particular problem or a particular unit.

It seems that by looking for problem solving opportunities throughout the curriculum, teachers will be able to increase student interest in and success with problem solving. This is discussed in more detail in the next section of this paper dealing with teaching strategies for problem solving.

It is not entirely clear at this point whether it is better to teach problem solving techniques to students and then have these techniques applied to a particular curriculum content area, or whether to help students solve problems in a content area, and then point out the problem solving techniques that they have used and therefore could continue to use in solving other problems. More specifically targeted research is needed to test the power of the deductive vs. inductive approach. It is important to note that such comparisons conducted in other content areas (e.g., teaching language arts skills and concepts) show a relative advantage for the deductive approach, that is, teaching the concept or skill directly, then applying the learning to a variety of situations (Rosenshine, 1983).

Polya's How to Solve It (1957) identified a number of heuristics that can be taught to students for application to a variety of problem solving situations. Dewald-Link and Wallace suggest that "the problem process be first taught as a concept and then applied in the solutions of problems related to the subject matter being taught." It may well be that some approaches to presenting curriculum may work better for sophisticated learners and other, different approaches work better for those experiencing learning difficulties or with limited basic skills proficiencies. The research is not yet clear on this point.

The following section discusses in more depth strategies that have been found to be successful in teaching problem solving to elementary and secondary school children.

How Should Problem Solving be Taught?

This section of the paper receives the greatest attention because teaching problem solving is essential to fostering higher-order thinking skills and because most teachers have little training or experience in this area. The topic is divided into two main subparts--what to teach and how to teach problem solving. As one would expect, there are many views on these subjects. This section reports only the content and techniques that receive consensus in the research literature. One final note is in order about the material presented in this section. The authors have attempted to report those ideas that teachers would find most useful in teaching problem solving in the classroom. Novel laboratory and other non-school techniques have been excluded from the following presentation.

What to teach. This section focuses on those things students should be taught that will make them better problem solvers. The discussion starts with a review of the characteristics displayed by good problem solvers. The general picture of the proficient problem solver is then dissected looking for teachable components.

In order to be good problem solvers, students need to have three qualities. First, they must be motivated to solve the problem. Innate intelligence and creative acumen are no replacement for the will to find a solution. Second, students must have knowledge in the area where the problem is set. The knowledge requirement is evident in both understanding (defining) the problem and in working (solving) the problem. Third, students must possess a repertoire of problem solving skills or strategies. As already mentioned, there are two general approaches to imparting these skills: 1) students are exposed to "correct" solution strategies and taught to apply them to problems, and 2) students are presented with different types of problems that embody an array of problem solving strategies from which they learn successful problem solving strategies. Each of these three qualities will be discussed in turn.

Clearly, motivation is a critical ingredient in all learning situations, but the evidence on the necessity of motivation is overwhelming in studies of problem solving (Polya, in Krulik, 1980; Moses, 1982; Davis and McKillip, 1980; Jacobsen, Lester and Stengel, 1980; Whimbey, 1980; Nelson, 1979; and Bloom and Broder, 1950).

Students that are highly interested in solving a particular problem persevere and succeed to a greater degree that those who are only moderately motivated. Teachers who want to promote an interest in problem solving are directed to model a sense of inquiry and excitement about problem solving and to provide students with opportunities to solve problems of their choice (Nickerson, 1981).

Modeling interest in problem solving is important because of the motivation it instills in students. Allowing students to work on problems they choose furthers their motivation to find solutions. Teachers can enhance this interest by providing a variety of problems that are relevant to the school day and life experiences of the students (Moses, 1982). Another suggestion for teachers is to present problems with colorful, graphic and highly concrete stimuli (Moses, 1980).

The second quality of good problem solvers deals with the knowledge base in the content area of the problem. As Bloom and Broder (1950) point out:

> It became clear that some specific information was necessary for the solution of examination problems and that a certain amount of background in the subject was indispensable. It became apparent that methods of problem solving, by themselves, could not serve as a substitute for basic knowledge of the subject matter.

As stated above, the knowledge component is required both to understand and to solve the problem. Not understanding the problem often leads to poor problem solving performance. In fact, there is a litany of research findings dealing with the importance of language and the need for fluency with it in problem solving (Barnett, Sowder and Vos, 1980; Kane, Byrne and Hater, 1974; Nesher and Teubl, 1974; Earp, 1970; Henney, 1971).

In order to use these research findings in the classroom, teachers must help students select problems that do not have linguistic or subject matter complexity beyond the child's present ability level. In fact, teachers may wish to simplify the language or knowledge requirements to enable the students to meet with success in problem solving. Ascertaining the knowledge requirement of a problem is facilitated by having the student read and discuss the problem aloud. Linguistic understanding can be assessed by having the student restate or translate the problem. As teachers move toward imparting problem solving and other higher order thinking skills, it is important to consider the knowledge requirements inherent in the problem at hand. In other words, don't start posing problems for students until they have a modicum of basic skills. In the long-run, competent problem solving is built on a knowledge base. The larger, more sophisticated and more integrated the base, the more effective the problem solving effort will be.

Keeping in mind the motivation and knowledge requirements for problem solving, the third quality of successful problem solvers is that they possess the set of strategies or techniques for problem solving, and a process by which they can be usefully applied. The process and strategies are discussed in much of the literature on problem solving and each author poses a different but somewhat similar typology. The research does not support one specific list over any other. The more generic approaches, such as Polya's (1957) seem to capture the major elements which may be elaborated upon or expanded by other researchers. Because of its generality and prominence in the literature, Polya's model will be used to illustrate the derivation of teachable skills.

As noted earlier, in Polya's model there are four steps which include:

1. Understanding the problem
2. Devising a plan
3. Carrying out the plan
4. Looking back

Of these four steps, emphasis will be placed on skills associated with understanding the problem and devising a plan. Not suprisingly, these steps pose the greatest challenge to would be problem solvers. Steps 3 and 4 while not trivial are fairly mechanistic applications of subject matter knowledge.

In Polya's step 1, the problem gets defined or understood. Four generalized procedures have been identified by which the problem can be clarified. The first procedure involves carefully reading and analyzing the situation as presented. As mentioned above, language facility is a critical ingredient needed to understand the givens, unknowns and setting in which the problem is cast. In many cases, the student must first perceive that a problem exists. This perception then triggers additional analysis. As a practical consequence, teachers need to create situations where students sense that a problem exists, and then create a definition of the problem to be solved. Practice in writing or verbalizing a problem may help learners comprehend problems written by others.

A second procedure for developing understanding of a problem involves listing the important data and extraneous information in a problem statement (Nickerson, 1981). One suggestion for identifying the useful facts in word problems involves crossing out those words and numbers which are unnecessary (Moses, 1982). Other authors including Polya (in Krulik, 1980), suggest that students make lists of important information to gain a more complete understanding of the problem and the data at hand.

A third procedure involves restating or translating the problem. Teachers can ask a number of students to rephrase the problem in order to show appreciation for different perspectives on understanding the problem. The translation procedure works well when reformulating word problems into numbers or number problems into story problems. It also works well for understanding complex problems that involve many "facts" of varying levels of importance (e.g., finding cause and effect related to an historical event).

A fourth procedure for understanding problems is reconceptualizing the problem in visual or graphic forms. Several authors have found improvement in students' problem solving ability when the problems are diagrammed in some way (Wicker, et al., 1970; LeBlanc, 1977; Kaufmann, 1979). When problems are represented visually (or by diagrams, tables or graphs) the would-be problem solver "sees" the problem more clearly.

In summary, Polya's first step in the problem solving process requires getting an accurate perspective on the nature of the problem. The problem solver usually begins by reading or talking about the problem and then proceeds by focusing on salient features, translating the information or portraying it.

Polya's second step requires devising a plan to solve the problem. Since different problems require different strategies for their solution, success in problem solving demands a repertoire of strategies or game plans. There are many strategies for solving problems. The list below is a compilation from numerous authors (Polya, 1957, in Krulick, 1980; Nickerson, 1981; LeBlanc, 1977; Moses, 1982; Musser and Shaughnessey, 1980; Kaufman, 1979; Schoenfeld, 1980).

Trial and error--applying possible operations or examples to the information at hand

Organized listing--generating and clustering ideas that represent possible solutions

Experimentation--establishing a hypothetical solution and testing the hypothesis with sample data

Simulation--using experimental methods on artificial data

Simplification--solving a special case (or a simpler version) of the more complicated problem

Generalization--Creating a larger, more tractable problem and then simplifying that procedure for the case in question

Deduction--breaking the larger problem into more manageable supports

Induction--summing individual observations to form more general principles

Searching for a pattern--finding a relationship among properties and exploiting the similarity to find the answer

Analogies (visual or word)--using similarities among objects (or propositions) to establish relationships among the known and unknown

Working backwards--beginning with the desired result, step back and work towards the initial problem state

Computation--employing arithmetic, algebraic and geometric techniques to solving word or number problems

In looking over the above list, it is important to recognize that there is overlap in some of the strategies like simplication/deduction and searching for a pattern/analogies. But whatever term is used to describe such techniques, the research is clear (Suydan and Weaver, 1977; Nickerson, 1981; Reidel, 1969) that seeing and practicing the use of strategies improves learners problem solving ability. Teachers should receive training in and examples of problems solved using the strategies defined above. Such training is essential as an inservice mode, but would be also important for institutions of higher education to use in the preservice training of teacher candidates.

The activity of devising a plan requires both experience in knowing various strategies as well as knowing which to select. Fortunately, for many problems, more than one strategy will work; unfortunately, there is no rule of thumb for selecting a first strategy. Students should be given opportunities to select and implement a variety of strategies in a variety of (subject matter) contexts.

Polya's third and fourth steps are to carry out the plan (or implement the strategy) and looking back (or check the answer). These steps are fairly procedural, and for students that have the requisite knowledge, the implementation and verification steps are generally straightforward. One consideration for teachers fostering problem solving skill is the need to have students check their own (or a peer's) solution to a problem. Oftentimes, the fourth step checking activity is done by the teacher for the purpose of assigning a grade. Students should be encouraged to validate their answer before submitting an assignment for a grade.

To recap, remember that necessary precursors to problem solving are the qualities of motivation and knowledge. When students can demonstrate these characteristics, teachers can pose problems that students will find challenging and meaningfull. In learning problem solving skills, it is important for students to learn to understand and define the problem before they develop a plan for solving it. After the plan is developed and carried out, the answer needs to be checked without the aid of an answer book or teacher.

How to teach. This section focuses on the instructional strategies used to teach problem solving skills. Research suggests that some instructional approaches are more effective than others in developing problem solving proficiency.

Teaching problem solving skills is hard work because it requires considerable planning time and careful monitoring of students' progress. The first step in teaching problem solving requires creating the problems for students to solve. In order to maintain students' interest, the problems must have variety and, at the same time, relevance to the learners' daily environment (Suydan and Weaver, 1977; Charles, 1983). The problems must demonstrate varying levels of difficulty and many different settings. The variation of difficulty level is itself an interesting conundrum in that the literature demands varying difficulty levels while maintaining appropriateness to the student's ability level. At first blush, this may sound internally inconsistent, but in fact requires that problems be posed in a band from below to slightly above the ability range. For group problem solving (discussed below), the variation in difficulty is an easier matter. Varying the setting or context for the problem is also workable with some amount of creative planning. The setting of the problem can be one which students know very well (e.g., their own immediate environment or a larger, yet still meaningful context) or one which they would like to think about (e.g., Disneyland, Star Wars or Little House on the Prairie). Varying the problem also requires changing the strategies that will yield successful solutions. Especially in math, teachers create problem sets that all use the same arithmetic operation (Butts, 1980). One suggestion for creating problems requiring different strategies is to start with mathematical sentences and work back to develop a word problem that restates that math sentence (Riedesel, 1969).

Problems, in puzzle format, have been examined as an instructional medium for improving problem solving. There seems to be some difference of opinion concerning their effectiveness. Some authors encourage their use (Polya, 1957; Moses, 1980) while others discourage using puzzles (Brownell, 1942). The distinction between these perspectives may be in the degree to which generalizations can be drawn from the puzzles at hand. Some books on puzzles are especially well suited to learning a larger concept from solving special-case puzzles (Gardner, 1978). In situations where puzzles are used in isolation, teachers need to introduce and reinforce the generalizations which the student is supposed to learn.

Puzzles can vary dramatically in their complexity as can the problems teachers pose for students. Research on the difficulty level of problem solving (Suydam and Weaver, 1977) shows the following principles:

- Problems which are (or can be) represented visually are easier to solve.
- The inclusion of irrelevant data makes problems more difficult.
- Problems requiring multiple, small steps are easier than single-step problems, where the step is large and undifferentiated.
- Problem difficulty increases with the readability level of the passage.

Although these ideas are fairly logical, teachers need to be made aware of the interaction effect (e.g., two-step problems with illustrations may be easier than a single-step problem posed in abstract prose that uses sophisticated vocabulary).

In developing lessons to teach problem solving, most emphasis should be placed on understanding the problem and devising the plan to solve it. Translation techniques such as paraphrasing, visual or dramatic representation, or shifting from verbal to numerical formats seem to be particularly useful as instructional activities.

There seems to be growing interest and agreement in the power of group problem solving activities in the classroom. Although the quality and efficiency in searching for an answer does not improve markedly (Suydam and Weaver, 1977), there are other benefits associated with group problem solving activities. Noddings (1983) found five benefits to group problem solving activities including (1) students are engaged, (2) information is shared, (3) faulty heuristics (strategies) are challenged, (4) new strategies are acquired and (5) students like working together. In using groups for problem solving, the researchers note that it is important not to let one student dominate the process (because of intelligence, assertiveness, etc.). Each student should take a turn as "leader." Teachers should monitor the group's work to keep the group actively working toward a solution and to reinforce the strategies and generalizations coming out of a group discussion (Noddings, 1983; Charles, 1983).

As discussed above, creating a variety of problems is important to maintaining learners' interest and encouraging practice and skill in problem solving. One way to create the variation in problems and strengthen the expectation for problem solving is to imbed problem solving activities throughout the school curriculum. Through problem solving research has traditionally been math oriented, the actual teaching of problem solving skills needs to be more widely based. Problems of varying types are presented to students in all areas of the cirriculum. Staff could mutually support the development of problem solving proficiency by providing consistency in approach to problems (the generic problem solving steps) and by providing high quality feedback to students regarding their efforts as problem solvers.

Problem solving can and should be reinforced in non-school contexts as well. As a first step, the utility of problem solving strategies can be demonstrated in school council and other school related forums.

Negotiation techniques can be shown to be examples of problem solving strategies. The application of problem solving techniques to life problems is endless. To the extent students learn to deal with problems more effectively, they will lead fuller and more productive lives.

Summary

This research synthesis has identified a number of important concepts regarding the teaching of problem solving skills to students in grades K-12. In general, it has shown that problem solving skills and strategies can be taught to students.

Although different writers propose different models for problem solving, George Polya's four steps to problem solving appeared to the authors of this synthesis as the best, and most widely used, model for consideration. Polya's steps include:

1. Understanding the problem
2. Devising a plan for solving the problem
3. Carrying out the plan
4. Looking back or evaluating the solution

While the research indicates that all four steps are important in problem solving, it also indicates that most emphasis should be placed on the first two. In understanding the problem, recognition and analysis techniques are important. In devising a plan, a broad repertoire of strategies is helpful along with criteria for knowing which to apply and when to apply them.

The research also supports the importance of having an explicit instructional focus on problem solving skills, though that focus should be imbedded in and spread through the regular curriculum, rather than expressed as a separate course or as a solitary unit of instruction. By encouraging teachers throughout a school or a school district to strengthen problem solving skills at all levels and in all subject areas, it will be possible to engage students in more complex problem solving activities as grade levels advance, leading to graduates who can successfully approach and solve a wide range of academic and life experience problems.

In most schools, problem solving is considered an important student learning goal. In addition to solutions of academic problems, the application of problem solving techniques to real world situations is a skill that all students should ideally have. The evidence is clear that these skills not only can be taught, but that they can be taught effectively and as part of an already existing curriculum.

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