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ABSTRACT

Enrichment activities for fifth-grade mathematics are presented. They are intended to be a continuation of the program started in the fourth grade. Some of the activities reinforce principles taught in the regular program; others introduce new concepts to challenge students. The activities are divided into the following categories: number pictures; tic-tac-toe word problems; logic puzzles; crossnumber puzzles; mathematical word search; metric measurements; coded computations; suppressed digits; magic squares; graphing; geometric shapes; spatial perception; word problems (advertising puzzles); finding the square root of a number; patterns; base ten; base five; base two; and number sense (multiplication and division). Answers to the puzzles and other activities are appended. (DC)

ED238682

MATHEMATICS ENRICHMENT

Grade 5

Curriculum Bulletin Number 237
Fort Worth Independent School District
Fort Worth, Texas

SE041782

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Foreword

These activities for the fifth grade were written as a continuation to the enrichment program ~~started~~ in the fourth grade. They are to be used throughout the year to create interest and enjoyment in mathematics. Some of the materials are for reinforcement of principles taught in the regular program, others introduce new concepts to challenge the pupils.

Patsy Johnston, curriculum writer/editor, wrote and edited these activities. The project was developed under the direction of Crawford Johnson, Program Director for Mathematics, and J. D. Shipp, Director of Elementary Schools. Appreciation is expressed for the creative work of these three as well as others who contributed to this project.

This publication was planned, written, edited, and published in the Department of Curriculum Development.

Dewey W. Mayer, Jr

Dewey W. Mayer, Jr.
Director of Curriculum Development

August, 1981

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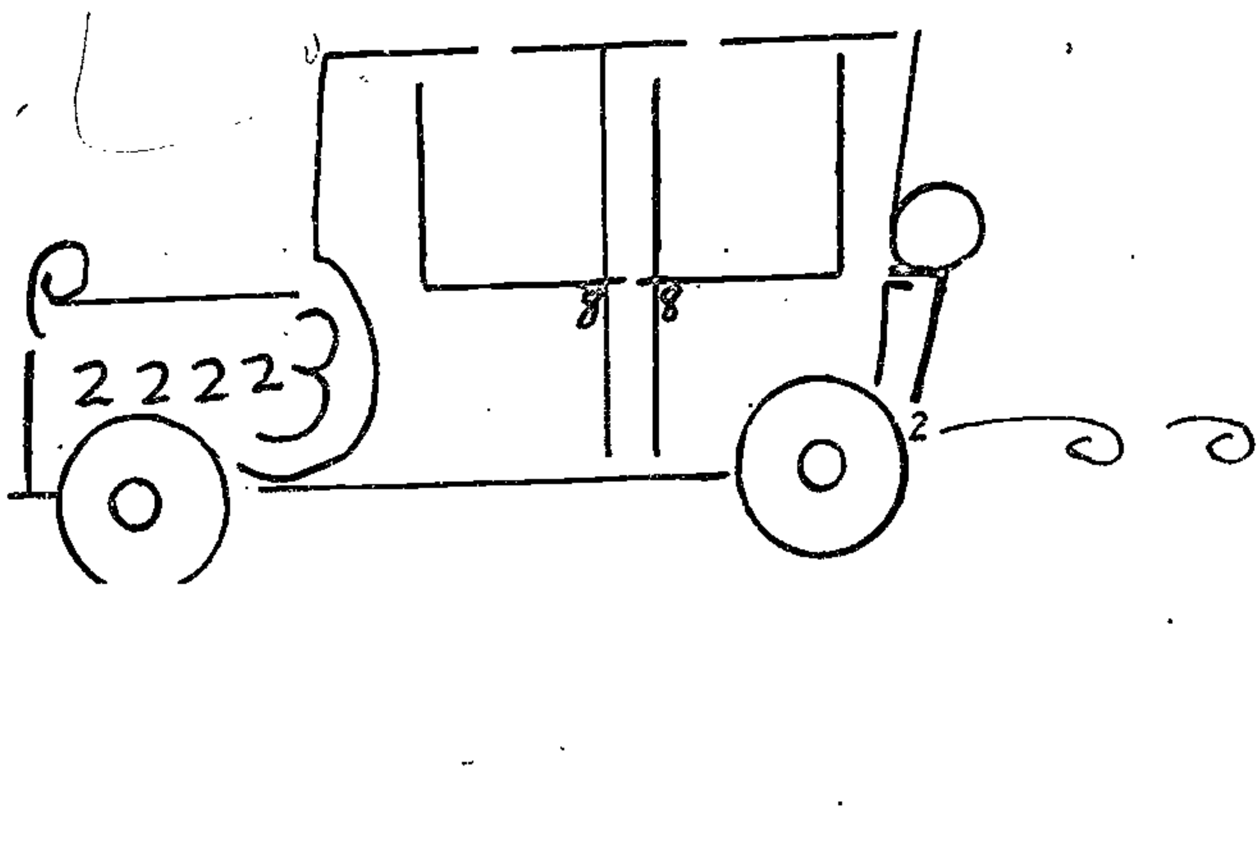
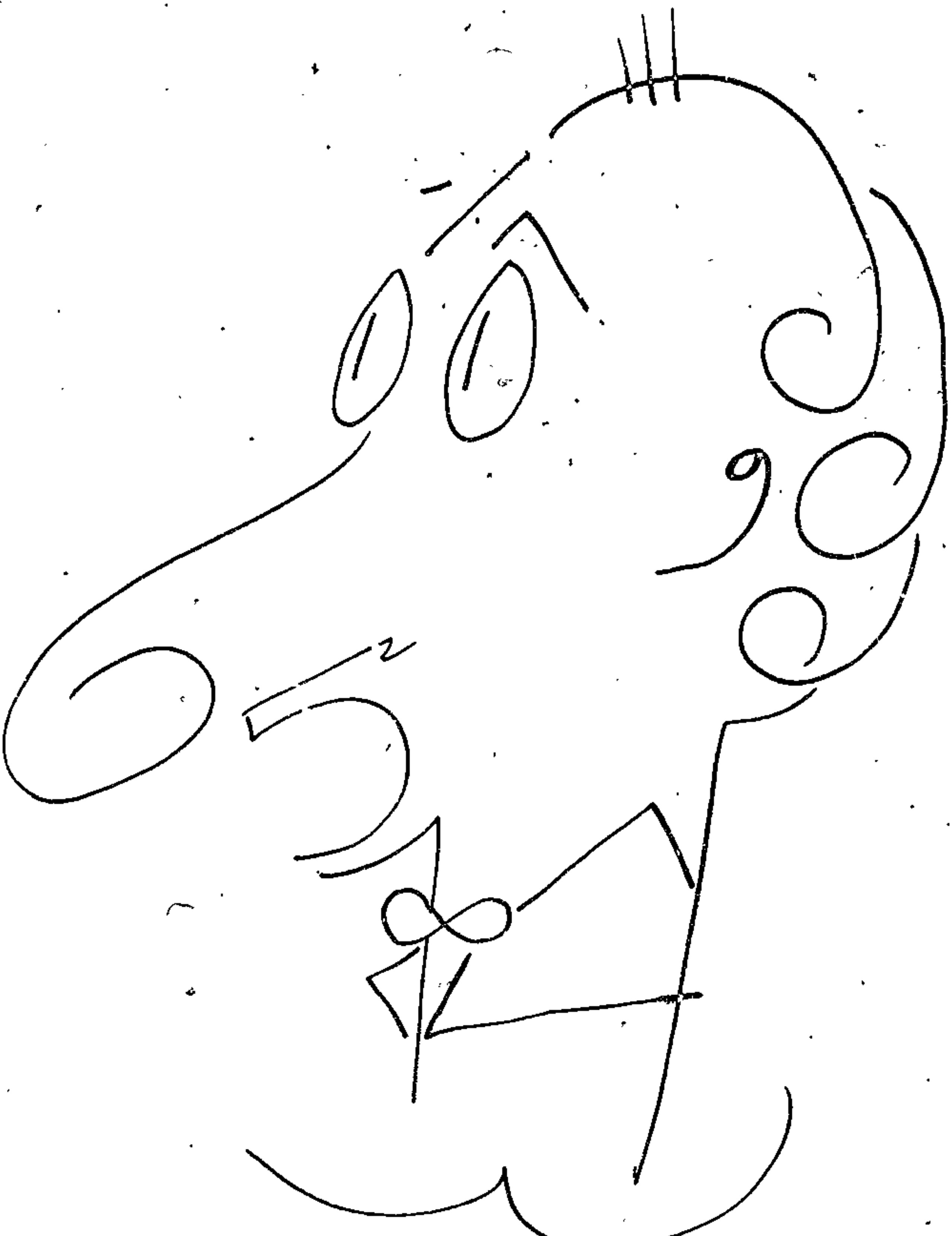
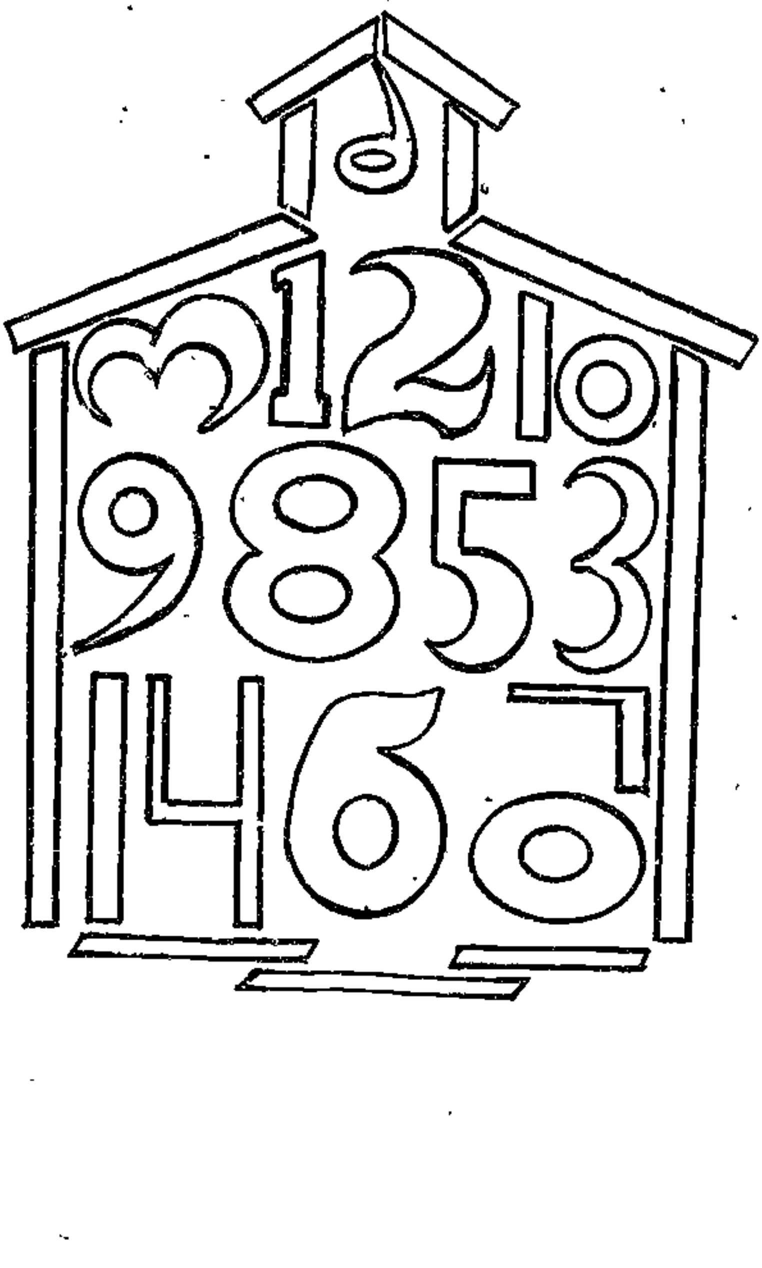
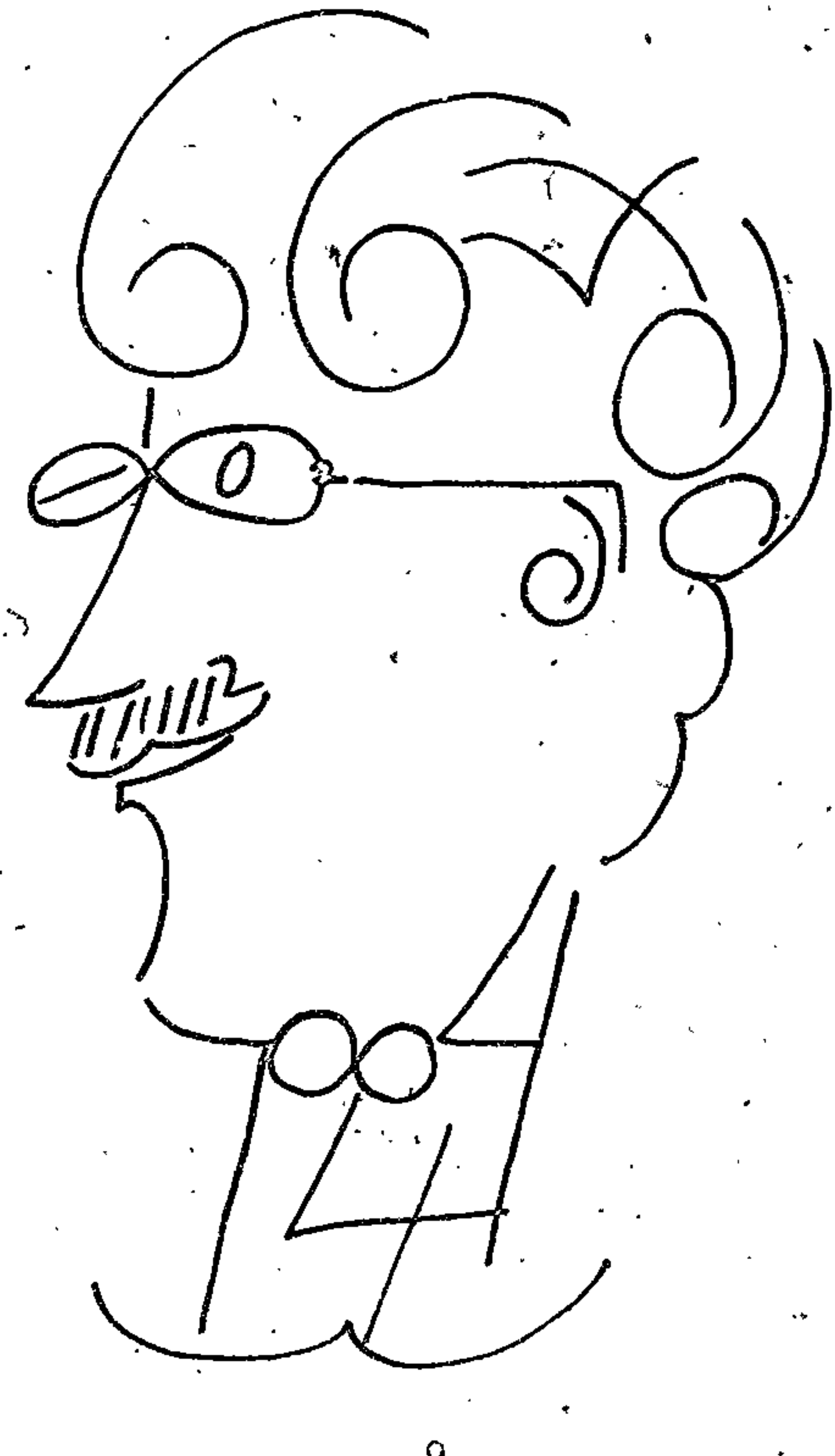
NUMBER PICTURES

PICTURES CAN BE MADE FROM NUMBERS. SEE IF YOU CAN MAKE ONE.









TIC-TAC-TOE

80	172	4
.4400	16	297.60
1656	28.71	71

CHOOSE ANY PROBLEM. SOLVE IT. PUT AN X OR A O ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WINS. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. BOB STROUD TRAVELS FROM BALTIMORE TO PHILADELPHIA, A DISTANCE OF 156 KM, ONCE A MONTH TO PURCHASE CLOTHING FOR HIS BOUTIQUE. A ROUND TRIP PLANE FARE COSTS $87.20. HE COULD DRIVE FOR ABOUT $.20 A KILOMETER. HOW MUCH COULD HENRY SAVE IN A YEAR BY DRIVING?

2. SILVIA IS GOING TO MAKE A PINE BOOKCASE. SHE NEEDS 4 SHELVES AND 2 SIDES. EACH SHELF WILL BE 80 CM LONG AND EACH SIDE 130 CM LONG. IF PINE BOARDS COST $4.95 A METER, HOW MUCH WILL SILVIA BE SPENDING?

3. IT TAKES A LUMBER COMPANY 42 HOURS TO CLEAR A TRACT OF LAND PLUS ANOTHER 68 HOURS TO REPLANT. HOW LONG WILL IT TAKE TO CLEAR AND REPLANT A TRACT OF LAND 40 TIMES AS LARGE?

4. LOGAN CAN TYPE 50 WORDS A MINUTE ACCURATELY. HOW MANY HOURS WILL IT TAKE LOGAN TO TYPE A 12,000 WORD RESEARCH PAPER?

5. ELIZABETH BOUGHT A SKI PARKA FOR $78.44 AND A TENNIS RACKET FOR $21.78. SHE CHARGED BOTH ITEMS ON HER NEW CHARGE ACCOUNT. SHE MADE TWO MONTHLY PAYMENTS OF $16 EACH. THE INTEREST CHARGES FOR THE TWO MONTHS WERE $2.78. IF ELIZABETH MAKES NO FURTHER CHARGES DURING THE TWO MONTHS, HOW MUCH DOES SHE OWE?

6. A MUSEUM RECEIVES ABOUT 3450 VISITORS EACH MONTH IN JUNE, JULY, AND AUGUST. IN SEPTEMBER, OCTOBER, APRIL, AND MAY, THE MUSEUM RECEIVES ABOUT 1400 VISITORS MONTHLY. IN EACH OF THE OTHER MONTHS, THE MUSEUM RECEIVES ABOUT 250 VISITORS MONTHLY. ABOUT HOW MANY HUNDREDS OF VISITORS ARE WELCOMED YEARLY?

7. IN THE MONTH OF MARCH THE SWEATER-SHOP SOLD 33 SWEATERS AT THE REGULAR PRICE OF $24 EACH AND ANOTHER 48 SWEATERS AT A CLEARANCE PRICE OF $18 EACH. WHAT WERE THE TOTAL SALES OF SWEATERS IN THE MONTH OF MARCH?

8. KEN IS REPAIRING THE AGEE'S COLOR TELEVISION. HIS REPAIR RATES ARE SHOWN BELOW. IT TOOK KEN 3 HOURS TO FIND THE TROUBLE AND WILL TAKE ANOTHER HOUR TO REPAIR THE SET. HOW MUCH WILL KEN CHARGE MR. AGEE FOR LABOR?

TYPE OF WORK	CHARGE FOR 1ST HOUR	EACH HOUR MORE
BLACK AND WHITE TV	$20	$15
COLOR TV	$26	$18

9. TWILA'S BASKETBALL TEAM WON THE CITY CHAMPIONSHIP. SHE WAS HIGH SCORER FOR THE GAME, SCORING 40 POINTS. SHE SCORED TWICE AS MANY FIELD GOALS (2 POINTS) AS FREE THROWS (1 POINT). HOW MANY FIELD GOALS DID TWILA SCORE IN THE CHAMPIONSHIP GAME?

TIC-TAC-TOE

		Fri.	Sat.	Sun.
2:00	Orchestra	10.00	10.00	9.00
	Balcony	6.50	6.50	6.00
8:00	Orchestra	14.00	14.00	X
	Balcony	9.50	9.50	

7.90	40	19.00
78	5	561
56	7767	2

CHOOSE ANY PROBLEM. SOLVE IT. PUT AN X OR A O ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WINS. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. MR. AND MRS. GOLDSTEIN PLAN TO SEE A PLAY AT THE COW PALACE. HOW MUCH WILL TWO BALCONY SEATS COST FOR A FRIDAY NIGHT PERFORMANCE?

2. THE SUM OF TWO NUMBERS IS 115. ONE NUMBER IS 3 LESS THAN THE OTHER NUMBER. WHAT IS THE SMALLER NUMBER?

3. ON JULY 1, WEST SIDE CAR SALES HAD 234 CARS ON THE LOT. BY JULY 15, $\frac{2}{3}$ OF THE CARS WERE LEFT. HOW MANY CARS HAD BEEN SOLD?

4. THE BOTTLING COMPANY HAS 2 BOTTLING MACHINES THAT BOTTLE 240 BOTTLES OF SOFT DRINKS A MINUTE. HOW MANY SODAS CAN BE BOTTLED BY ONE MACHINE IN ONE SECOND?

5. VIRGINIA WORKS 8 HOURS A DAY ON THE ASSEMBLY LINE AT DOWNTOWN MOTORS. FOR FIVE DAYS OF WORK, THE TOTAL OF THE DEDUCTIONS FROM HER WEEKLY PAYCHECK IS $78.42. HOW MANY HOURS A WEEK DOES VIRGINIA WORK?

6. WALTER BEGAN THE DAY WITH $100.52 IN HIS CASH DRAWER. DURING THE DAY, HE SOLD 48 ALBUMS AT $5.99 EACH, 14 ALBUMS AT $8.79 EACH, AND 2 COLLECTOR'S EDITIONS AT $24.95 EACH. HOW MUCH MONEY SHOULD WALTER HAVE IN HIS CASH DRAWER AT THE END OF THE DAY?

TIC-TAC-TOE (CONT'D)

7. THE THREE PLANETS CLOSEST TO THE SUN ARE MERCURY, VENUS, AND EARTH. THE DIAMETERS OF THESE THREE PLANETS ARE 4988 KM, 12,289 KM, AND 12,755 KM, RESPECTIVELY. HOW MUCH LARGER IS THE DIAMETER OF EARTH THAN THE DIAMETER OF MERCURY?

8. JAY DRIVES A TOTAL OF 84 KM A DAY TO AND FROM WORK. IF HIS CAR TRAVELS ABOUT 6 KM ON A LITER OF GASOLINE, IN HOW MANY DAYS WILL RANDY USE 70 LITERS OF GASOLINE?

9. LEON BOUGHT A 1.59 KG CHICKEN COSTING $3.15, 0.95 KG OF HAMBURGER COSTING $2.98, AND A BAG OF APPLES COSTING $1.77. WHAT WAS THE TOTAL COST OF LEON'S PURCHASES?

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TIC-TAC-TAC

59.80	88.08	24
63	1500	28
186	45	2135

CHOOSE ANY PROBLEM. SOLVE IT. PUT AN X OR A O ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WINS. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. THE MILLERS BUY A $487 STOVE AND A $736 REFRIGERATOR ON CREDIT. THE FINANCE CHARGE IS $289 IF THE LOAN IS PAID BACK IN 24 EQUAL MONTHLY PAYMENTS. HOW MUCH WILL EACH MONTHLY PAYMENT BE?

2. THE COUNTY ARENA HAS A SEATING CAPACITY OF 2000. TICKETS NORMALLY SELL FOR $4.00 A PIECE, BUT STUDENTS RECEIVE A $2.00 DISCOUNT. FOR THE HIGH SCHOOL TOURNAMENT BEING HELD THERE, THE MANAGER HAS MADE A SPECIAL EFFORT TO ALLOW AS MANY STUDENTS AS POSSIBLE TO ATTEND. FOR THE MANAGER TO MEET EXPENSES, $5000 WORTH OF TICKETS MUST BE SOLD. HOW MANY STUDENT TICKETS CAN HE AFFORD TO SELL AND STILL MEET EXPENSES?

3. MR. WILSON BUYS A $899 SOFA ON CREDIT. IN ADDITION TO THE SALES TAX OF $44.96, HE MUST PAY A FINANCE CHARGE OF $113. IF HE REPAYS THE LOAN IN 12 EQUAL MONTHLY PAYMENTS, WHAT IS THE AMOUNT OF EACH PAYMENT?

4. ALEX IS BUYING FENCING TO ENCLOSE A RECTANGULAR PLAY AREA FOR HIS DOG. THE VET RECOMMENDS THAT THE DOG HAVE A YARD AT LEAST 36 M^2. WHAT IS THE LEAST AMOUNT OF FENCING HE WILL NEED?

TIC-TAC-TOE (CONT'D)

5. THE POPULATION OF QUEBEC IN 1901 WAS 69,000. BY 1931, THE POPULATION HAD INCREASED BY 62,000. BY 1971, THE POPULATION HAD INCREASED ANOTHER 55,000. HOW MANY THOUSANDS WAS THE POPULATION OF QUEBEC IN 1971?

6. IN 1947, A TOWN BEGAN TAKING A POPULATION COUNT OF ITS CITIZENS. THE POPULATION THAT YEAR TOTALED 25,476. TEN YEARS LATER, THE THE POPULATION INCREASED ABOUT 8,500. THE LAST COUNT WAS TAKEN IN 1975 AND THE POPULATION WAS 42,789. HOW MANY YEARS PASSED BETWEEN THE TIME THE FIRST AND THE LAST COUNT WAS MADE?

7. AS AN ELECTRICIAN, SHERRY CHARGES $14.95 AN HOUR FOR HER SERVICES ON WEEKDAYS AND $22.95 AN HOUR ON WEEKENDS. IF SHERRY ESTIMATES THE JOB CAN BE FINISHED IN 4 HOURS, HOW MUCH WILL SHE EARN WORKING DURING THE WEEK?

8. MR. MAYS, AN APPLIANCE SALESPERSON, EARNS $55 FOR EACH DRYER, $90 FOR EACH REFRIGERATOR, AND $65 FOR EACH STOVE. IN A RECENT MONTH HE SOLD 12 STOVES, 12 REGRIGERATORS, AND 5 DRYERS. WHAT WERE HIS EARNINGS FROM THESE SALES?

9. THE SUM OF FOUR CONSECUTIVE NUMBERS IS 174. WHAT IS THE LARGEST OF THESE NUMBERS?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE ARE NEEDED TO WIN.

8	**11**	**12**
15	**16**	**10**
13	**9**	**14**

1. IF YOU MULTIPLY MY NUMBER BY 6 AND THEN DIVIDE BY 12, YOU GET 4. WHAT'S MY NUMBER?
2. IF YOU DIVIDE MY NUMBER BY 4 AND THEN MULTIPLY BY 10, YOU GET 40. WHAT'S MY NUMBER?
3. IF YOU DIVIDE MY NUMBER BY 7 AND THEN ADD 18, YOU GET 20. WHAT'S MY NUMBER?
4. IF YOU ADD 58 TO MY NUMBER AND THEN DIVIDE BY 7, YOU GET 10. WHAT'S MY NUMBER?
5. SEVEN TIMES MY NUMBER IS 20 MORE THAN 5 TIMES MY NUMBER. WHAT'S MY NUMBER?
6. IF YOU MULTIPLY MY NUMBER BY 9 AND SUBTRACT 49, YOU GET 50. WHAT'S MY NUMBER
7. MY NUMBER IS BETWEEN 10 AND 20. IF YOU DIVIDE IT BY 11, YOU GET A REMAINDER OF 4. WHAT'S MY NUMBER?
8. IF YOU MULTIPLY MY NUMBER TIMES ITSELF AND ADD 19, YOU GET 100. WHAT'S MY NUMBER?
9. IF YOU MULTIPLY MY NUMBER BY 5 AND ADD 10, YOU GET 75. WHAT'S MY NUMBER?

TIC-TAC-TOE

5	8	19
2	7	4
9	18	6

CHOOSE ANY PROBLEM. SOLVE IT. PUT AN X OR A O ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WINS. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

1. THE POSTAGE FOR 5 SMALL BOXES IS 95¢. HOW MUCH POSTAGE FOR EACH BOX?

3. ONE BOX HOLDS $\frac{1}{4}$ POUND OF LEMON DROPS. HOW MANY BOXES CAN BE FILLED WITH $1\frac{3}{4}$ POUNDS OF LEMON DROPS?

3. WHAT IS THE PERIMETER OF AN EQUILATERAL TRIANGLE WITH A SIDE 6 FT. LONG?

4. A BOX OF ASSORTED CHOCOLATES CONTAINS $\frac{1}{4}$ POUNDS OF EACH KIND. HOW MANY KINDS OF CHOCOLATES IN A $1\frac{1}{2}$ POUND BOX?

5. ROBERT EARNS $3.25 PER HOUR WORKING AT A GARAGE. ONE WEEK HE WORKED 15 HOURS AND THE NEXT WEEK HE WORKED 23 HOURS. HOW MANY MORE HOURS DID HE WORK THE SECOND WEEK?

6. THE CEMENT TRUCK HAS 8 CUBIC YARDS OF CONCRETE IN IT. THE DRIVER DUMPS $\frac{3}{4}$ OF IT FOR A DRIVEWAY. HOW MANY CUBIC YARDS ARE LEFT IN THE TRUCK?

7. WRITE THIS ROMAN NUMERAL IX AS AN ARABIC NUMERAL.

8. A BULLDOZER DRIVER GRADES $\frac{2}{3}$ OF A 15-ACRE LOT. HOW MANY ACRES DOES HE STILL HAVE TO GRADE?

9. HOW MANY SIDES DOES A RHOMBUS HAVE?

TIC-TAC-TOE

Choose any problem below. Solve it. Put an X or a 0 on the answer on the tic-tac-toe board. If two people play, three Xs or 0s in a line wins. If one person plays, two tic-tac-toes are needed to win.

6	**62**	**16**
48	**7**	**45**
32	**4**	**61**

1. A rug 9 ft. x 12 ft. costs \$34.56. This is how much per square foot?

2. If 30 oranges cost \$9.30, what do 2 oranges cost?

3. At 3 for 15¢, how much do 9 apples cost?

4. Peggy has 42 jacks, Beth has $\frac{1}{2}$ as many as Peggy, and Sue has $\frac{1}{3}$ as many as Peggy. How many more jacks does Beth have than Sue?

5. A boy walks 42 ft. in 7 seconds. This is how many ft./sec?

6. James ran 40 miles in 10 hours. This is how many miles per hour?

7. Write this Roman numeral LXI as an Arabic numeral.

8. How many inches are there in $\frac{1}{3}$ of 12 feet?

9. There are 64 beef and dairy cattle in the pasture. If $\frac{3}{4}$ of them are beef cattle, how many dairy cattle are there?

TIC-TAC-TOE

CHOOSE ANY PROBLEM BELOW. SOLVE IT. PUT AN X OR A O ON THE ANSWER ON THE TIC-TAC-TOE BOARD. IF TWO PEOPLE PLAY, THREE XS OR OS IN A LINE WIN. IF ONE PERSON PLAYS, TWO TIC-TAC-TOES ARE NEEDED TO WIN.

6	55	66
54	60	3
4	64	5

1. HOW MANY LEGS ON 12 COWS AND 9 CHICKENS?
2. I HAD \$20. I SPENT \$2.50, \$4.25, AND \$9.25. HOW MUCH IS LEFT?
3. HOW MUCH MONEY IS 44 DIMES AND 12 NICKELS?
4. ROLLER COASTER RIDES COST 75¢ EACH. FERRIS WHEEL RIDES COST 50¢ EACH. WHAT'S THE COST FOR 4 ROLLER COASTER RIDES AND 6 FERRIS WHEEL RIDES?
5. HOW MANY HOURS IN 2 DAYS AND 12 HOURS?
6. HOW MANY LEGS ON 12 DOGS AND 4 CATS?
7. I BOUGHT 4 RECORDS AT \$4.25 EACH I GAVE THE CLERK \$20. HOW MUCH CHANGE DID I RECEIVE?
8. THERE ARE 6 TEAMS OF FOURTH GRADERS AND 3 TEAMS OF FIFTH GRADERS. EACH TEAM HAS 6 PLAYERS. HOW MANY PLAYERS ARE THERE?
9. I HAD 47 COMIC BOOKS. I GAVE MY FRIEND 15 OF THEM AND HE GAVE ME 23 OF HIS. HOW MANY DO I HAVE NOW?

LOGIC PUZZLES

IT IS INTERESTING TO OBSERVE THAT PUZZLES OF THE PURELY LOGICAL TYPE ARE REPRESENTATIVE OF THE ENTIRE SCIENTIFIC PROCESS. AT THE BEGINNING ONE IS CONFRONTED WITH A MASS OF MORE OR LESS UNRELATED DATA. FROM THESE FACTS A FEW POSITIVE INFERENCES CAN BE SEEN IMMEDIATELY, BUT USUALLY IT IS NECESSARY TO MAKE ASSUMPTIONS TO GUIDE THE SEARCH FOR A SOLUTION. THE VALIDITY OF THESE ASSUMPTIONS MUST BE CHECKED BY TESTING FOR CONSISTENCY WITH THE ORIGINAL DATA. IF INCONSISTENCIES APPEAR, THE ASSUMPTIONS MUST BE REJECTED AND OTHERS TRIED UNTIL FINALLY A CONSISTENT SET OF CONCLUSIONS EMERGES.

THE SOLUTION OF LOGIC PUZZLES CANNOT BE REDUCED TO A FIXED PATTERN. NEVERTHELESS, THERE ARE SOME GENERAL SUGGESTIONS ON HOW TO ATTACK PUZZLES OF THIS SORT.

EXAMPLE:

JONES, SMITH, JOHNSON, AND MAYS ARE FOUR TALENTED ARTISTS, ONE A DANCER, ONE A PAINTER, ONE A SINGER, AND ONE A WRITER (NOT NECESSARILY IN THAT ORDER).

1) JONES AND JOHNSON WERE IN THE AUDIENCE THE NIGHT THE SINGER MADE HIS DEBUT ON THE CONCERT STAGE.
2) BOTH SMITH AND THE WRITER HAVE SAT FOR PORTRAITS BY THE PAINTER.
3) THE WRITER, WHOSE BIOGRAPHY OF MAYS WAS A BEST-SELLER, IS PLANNING TO WRITE A BIOGRAPHY OF JONES.
4) JONES HAS NEVER HEARD OF JOHNSON. WHAT IS EACH MAN'S ARTISTIC FIELD?

TO KEEP TRACK MENTALLY OF THESE MANY FACTS AND THE HYPOTHESES

AND CONCLUSIONS BASED UPON THEM IS CONFUSING AND DIFFICULT. THEREFORE, MAKING A CHART WHICH SHOWS ALL POSSIBILITIES IS A GOOD WAY TO START YOUR SOLUTION.

	DANCER	PAINTER	SINGER	WRITER
JONES				
SMITH				
JOHNSON				
MAYS				

NOW FROM 1), IT IS KNOWN THAT NEITHER JONES NOR JOHNSON IS THE SINGER. PLACE AN X OPPOSITE THEIR NAMES IN THE COLUMN HEADED BY THE SINGER.

FROM 2), IT IS KNOWN THAT SMITH IS NEITHER THE PAINTER NOR THE WRITER. PLACE AN X OPPOSITE SMITH'S NAME IN THE TWO COLUMNS HEADED PAINTER AND WRITER.

FROM 3), THE WRITER IS NEITHER MAYS NOR JONES. PLACE AN X OPPOSITE THE NAMES MAYS AND JONES IN THE COLUMN HEADED WRITER.

THE CHART NOW LOOKS LIKE THE FOLLOWING:

	DANCER	PAINTER	SINGER	WRITER
JONES			X	X
SMITH		X		X
JOHNSON			X	
MAYS				X

IF A CLUE HAD STATED THAT SMITH WAS THE SINGER, A CHECK WOULD HAVE BEEN PLACED OPPOSITE SMITH'S NAME IN THE COLUMN HEADED SINGER.

However, in this problem, there were no direct facts given. But at this stage when the chart is examined, it is clear that Johnson is the writer because all other possibilities are marked off. So place a check opposite Johnson's name in the column headed writer and fill the remaining squares in his row with X's.

According to 2), Johnson has sat for the painter and in 4), Jones does not know Johnson. Therefore, Jones is not the painter. Place an X by Jones' name in the column headed painter. By elimination on the chart, Jones must be the dancer. Place X's in the other squares in the dancer column.

Observe that the singer must be Smith. Add a check and X's in the correct squares.

Finally, Mays must be the painter, and the solution is complete.

PROBLEMS

1) Shepard, Johnston, and Nichols make their living as carpenter, painter, and plumber, though not necessarily respectively. The painter recently tried to get the carpenter to do some work for him, but was told that the carpenter was out doing some remodeling for the plumber.

   The plumber makes more money than the painter. Johnston makes more money than Shepard. Nichols has never heard of Johnston. What is each man's occupation?

2) THE FOLLOWING IS A PART OF A REPORT SUBMITTED BY AN INVESTIGATOR FOR A MARKET ANALYSIS AGENCY WITH STANDARDS OF ACCURACY SO HIGH THAT IT BOASTS THAT AN EMPLOYEE'S FIRST MISTAKE IS HIS LAST.

NUMBER INTERVIEWED	100
NUMBER WHO DRINK COFFEE.	79
NUMBER WHO DRINK TEA	69
NUMBER WHO DRINK BOTH.	47

WHY WAS THE INTERVIEWER DISCHARGED?

3) MARTHA JUST GOT FOUR NEW STAMPS FOR HER COLLECTION. SHE IS CONFUSED ABOUT WHICH STAMP COMES FROM WHICH COUNTRY. CAN YOU SORT THE STAMPS?

1. THE STAMP WITH THE TRAIN ON IT IS PINK.
2. THE GERMAN STAMP HAS A PICTURE OF A RUNNER.
3. THE FLOWER IS NOT ON THE FRENCH STAMP.
4. THE SWEDISH STAMP IS NOT PINK.
5. THE PLANE IS NOT ON A YELLOW STAMP.
6. THE UNITED STATES STAMP IS BLUE.
7. THE FLOWER IS ON A VIOLET STAMP.

COUNTRY	____	____	____	____
COLOR	____	____	____	____
PICTURE	____	____	____	____

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4) This is the first basketball game Fred has ever seen. He's really mixed up about the players' names and positions. He doesn't even know who is captain of the team. Can you help him understand what's happening? Who is the captain of the team?

1. The right-guard was the low scorer.
2. Sam scored more than Ben but less than Ed.
3. The captain played left-forward.
4. Ed scored more than the captain.
5. Jack and Dave together scored less than the right-forward.
6. The center scored more than the guards but less than the forwards.
7. Dave played left-guard.
8. The right-forward scored more than the left-forward.

Points scored	7	16	25	10	15
Position played					
Player's name					

5) There are only three houses on Main Street. They are all on the same side of the street. The Wilsons, the Watsons, and the Jones live in the houses. Can you tell where each family lives and what kind of car each drives?

1. The Watsons live next to the Jones.
2. Mr. Wilson's son, Billy Bob, is friendly with Mr. Watson's son.
3. The family on the right does not drive a Ford.
4. The people who drive the Cadillac do not live next to the people who drive the Ford.
5. The family in the middle have no children.
6. Billy Bob thinks his dad's Cadillac is great.
7. One family owns a Plymouth.

6) A cat, a dog, a goat, and a horse are named Sandy, Blacky, Duke, and Rusty. Read the clues below to find each animal's name.

1. Duke is smaller than either the dog or Rusty.
2. The horse is younger than Sandy.
3. Blacky is the oldest and is a good friend of the dog.

7) Sally, Betty, Patsy, Bill, Sam, and Joe are six young persons who have been close friends from their childhood. They went through high school and college together, and when they finally paired off and became engaged nothing would do but a triple announcement party. Naturally they wanted to break the news to their friends in an unusual way.

At just the right moment during the party everyone was given a card bearing the information.

Who now are six will soon be three,
And gaily we confess it,
But how we've chosen you may know
No sooner than you guess it.

Joe, who is older than Sam is Sally's brother. Patsy is the oldest girl. The total age of each couple-to-be is the same although no two of us are the same age. Sam and Betty are together as old as Bill and Sally. What three engagements were announced at the party.

8) JACK DONAVAN WAS KILLED ON A LONELY ROAD TWO MILES FROM TRENTON AT 3:30 A.M. ON MARCH 17, 1933. SHORTY MALONE, TONY VERELLI, HANK RODGERS, JOEY FREIBERG, AND RED JOHNSON WERE ARRESTED A WEEK LATER AND QUESTIONED. EACH OF THESE MEN MADE FOUR SIMPLE STATEMENTS OF WHICH THREE WERE ABSOLUTELY TRUE AND ONLY ONE OF THEM FALSE. ONE OF THESE FIVE MEN KILLED DONAVAN. TO SOLVE THIS PUZZLE, PLACE A "T" OR "F" OVER EACH STATEMENT AS YOU DECIDE WHETHER IT IS TRUE OR FALSE. START WITH TONY'S STATEMENT, "ONE OF US IS GUILTY." THIS STATEMENT IS GIVEN TO BE TRUE. RED'S TWO STATEMENTS: "I DID NOT KILL DONAVAN" AND "SHORTY LIED WHEN HE SAID, 'I'M GUILTY,'" ARE EITHER BOTH TRUE OR BOTH FALSE. SINCE ONLY ONE STATEMENT CAN BE FALSE, BOTH OF THESE MUST BE TRUE.

<u>SHORTY:</u> "I WAS IN CHICAGO WHEN DONAVAN WAS MURDERED. I NEVER KILLED ANYONE. RED IS THE GUILTY MAN. JOEY AND I WERE PALS."

<u>HANK:</u> "I DID NOT KILL DONAVAN. I NEVER OWNED A REVOLVER IN MY LIFE. RED KNOWS ME. I WAS IN PHILADELPHIA THE NIGHT OF MARCH 17..."

<u>TONY:</u> "HANK LIED WHEN HE SAID HE NEVER OWNED A REVOLVER. THE MURDER WAS COMMITTED ON ST. PATRICK'S DAY. SHORTY WAS IN CHICAGO AT THAT TIME WHEN THE MURDER WAS COMMITTED. ONE OF US FIVE IS GUILTY."

<u>JOEY:</u> "I DID NOT KILL DONAVAN. RED HAS NEVER BEEN IN TRENTON. I NEVER SAW SHORTY BEFORE. HANK WAS IN PHILADELPHIA WITH ME THE NIGHT OF MARCH 17."

<u>RED:</u> "I DID NOT KILL DONAVAN. I HAVE NEVER BEEN IN TRENTON. I NEVER SAW HANK BEFORE NOW. SHORTY LIED WHEN HE SAID I'M GUILTY."

WHICH OF THE FIVE MEN KILLED DONAVAN?

9) IN A CERTAIN BANK THE POSITIONS OF CASHIER, MANAGER, AND TELLER ARE HELD BY BROWN, GREEN AND WHITE, THOUGH NOT NECESSARILY RESPECTIVELY. THE TELLER, WHO WAS AN ONLY CHILD, EARNS THE LEAST. WHITE, WHO MARRIED BROWN'S SISTER, EARNS MORE THAN THE MANAGER. WHAT POSITION DOES EACH MAN FILL?

10) IN THIS MURDER CASE YOU ARE TO FIND THE VICTIM, THE WITNESS, THE POLICEMAN, THE JUDGE, AND THE HANGMAN, AS WELL AS THE KILLER.

THE VICTIM DIED INSTANTLY OF A GUNSHOT WOUND INFLICTED AT CLOSE RANGE. THE WITNESS DID NOT SEE THE CRIME, BUT SWORE THAT HE HEARD AN ARGUMENT FOLLOWED BY A SHOT. AFTER A LENGTHY TRIAL, THE MURDERER WAS CONVICTED, SENTENCED TO DEATH, AND HANGED. THE SIX MEN INVOLVED IN THE CASE ARE CRAIG, DUNN, GROVER, HILL, MAYS, WILSON. YOU ALSO HAVE THE FOLLOWING FACTS.

1. MAYS HAD NOT KNOWN THE VICTIM OR THE MURDERER.
2. IN COURT THE JUDGE ASKED CRAIG TO GIVE HIS ACCOUNT OF THE SHOOTING.
3. WILSON WAS THE LAST PERSON TO SEE DUNN ALIVE.
4. HILL AND WILSON NEVER SAW EACH OTHER.
5. THE POLICEMAN TESTIFIED THAT HE PICKED UP GROVER NEAR THE PLACE WHERE THE BODY WAS FOUND.

IDENTIFY THE VICTIM, WITNESS, POLICEMAN, JUDGE, HANGMAN, AND MURDERER BY NAME. MAKE A CHART WITH NAMES DOWN ONE SIDE AND OCCUPATIONS ACROSS THE TOP. MAYS CANNOT BE THE VICTIM OR THE MURDERER.

11) THE ORGANIZATION OF THE BUSINESS OFFICE OF A CERTAIN COMPANY CONSISTS OF THE FOLLOWING: PRESIDENT, VICE-PRESIDENT, MANAGER, AUDITOR, CLERK, AND SECRETARY. THE NAMES OF THE OFFICE PERSONNEL IN ALPHABETICAL ORDER ARE: MR. BROWN, MR. CRAWFORD, MISS GREEN, MRS. JOHNSTON, MISS JONES, AND MR. SMITH.

1. THE VICE-PRESIDENT IS THE PRESIDENT'S GRANDSON.
2. THE MANAGER IS THE SECRETARY'S SON-IN-LAW.
3. THE AUDITOR IS MISS GREEN'S STEP-SISTER.
4. MR. BROWN IS A BACHELOR.
5. MR. CRAWFORD IS 25 YEARS OLD.
6. MR. SMITH IS THE PRESIDENT'S NEIGHBOR.

WHAT IS THE MANAGER'S NAME?

12) ONE AFTERNOON DAVID, JOE, AND SAM WITH THEIR WIVES, WHOSE NAMES IN ONE ORDER OR ANOTHER ARE SUE, JANE, AND BETTY, WENT OUT AND PLAYED EIGHTEEN HOLES OF GOLF TOGETHER.

1. BETTY, JANE, SUE, AND JOE SHOT 106, 102, 200, AND 94 RESPECTIVELY.
2. DAVID AND SAM SHOT A 98 AND A 96, BUT FOR SOME TIME THEY COULDN'T TELL WHO HAD MADE WHICH SINCE THEY HADN'T PUT THEIR NAMES ON THEIR SCORECARDS.
3. WHEN THE MEN FINALLY IDENTIFIED THEIR CARDS, IT TURNED OUT THAT TWO OF THE COUPLES HAD THE SAME SCORE.
4. JOE'S WIFE BEAT DAVID'S WIFE.

WHAT IS THE NAME OF EACH MAN'S WIFE, AND WHAT SCORES DID DAVID AND JOE MAKE?

13) Jimenez, Rodriguez, Martinez, and Contreras are four men whose occupations are baker, carpenter, postman, and police officer, though not necessarily respectively.

1. Jimenez and Rodriguez are neighbors and take turns driving each other to work.
2. Rodriguez makes more money than Martinez.
3. Jimenez beats Contreras regularly at backgammon.
4. The baker always walks to work.
5. The police officer does not live near the carptenter.
6. The only time the postman and the police officer ever met was when the police officer arrested the postman for speeding.
7. The police officer makes more money than the carpenter or the postman.

What is each man's occupation?

14) On an airplane flying from New York to San Francisco there are passengers named Bob, David, and Joe. The pilot, co-pilot, and navigator have the same first names, but not respectively.

1. The navigator lives halfway between New York and San Francisco.
2. Passenger Bob lives in New York.
3. The passenger who lives nearest to the navigator earns exactly three times as much a month as the navigator.
4. The passenger with the same name as the navigator lives in San Francisco.
5. Joe, a member of the crew, recently beat the co-pilot at handball.
6. Passenger David earns $200 a week.

What is the pilot's first name? 29

15) FIVE KIDS, RUTH, JANE, PHIL, MARK, AND JOE WENT FOR LUNCH. HERE'S WHAT THEY BOUGHT:

2 DOUBLEBURGERS. .	90¢ EACH	1 CHEESEBURGER. . . .	75¢ EACH
2 HAMBURGERS . . .	60¢ EACH	2 COLAS	20¢ EACH
2 SHAKES	35¢ EACH	1 MILK.	20¢ EACH
3 FRENCH FRIES . .	25¢ EACH		

UNFORTUNATELY EVERYTHING GOT MIXED UP ON THE TRAY. CAN YOU FIGURE OUT WHAT EACH KID SPENT AND JUST WHAT HE HAD FOR LUNCH?

1. EACH PERSON HAD ONE DRINK AND A BURGER OF SOME KIND.
2. THE PEOPLE WHO HAD DOUBLEBURGERS DIDN'T HAVE FRENCH FRIES.
3. THE CHEESEBURGER PERSON DRANK A COLA.
4. THE GIRLS HAD THE SAME THING TO DRINK.
5. PHIL DIDN'T HAVE FRENCH FRIES BUT HIS LUNCH COST THE MOST.
6. NO TWO PEOPLE HAD THE SAME LUNCH.
7. PHIL AND JOE DRANK THE SAME THING.
8. JANE'S LUNCH COST THE LEAST.

LUNCH	______	______	______	______	______
	______	______	______	______	______
	______	______	______	______	______
NAME	______	______	______	______	______
COST	$ ______	$ ______	$ ______	$ ______	$ ______

16) SIX HORSES RAN IN THE BIG RACE. THE JOCKEYS WORE SHIRTS OF DIFFERENT COLORS. CAN YOU FIGURE OUT THE ORDER IN WHICH THE HORSES FINISHED THE RACE AND THE COLOR OF THE SHIRT EACH JOCKEY WAS WEARING?

1. JOHNSTON'S DARLIN' WON THE RACE.
2. THE RED SHIRT CAME IN JUST BEHIND MARTIN'S FOLLY.
3. THE BLUE SHIRT WAS LAST.
4. THE YELLOW SHIRT CAME IN THIRD.
5. MARTIN'S FOLLY WAS SLOWER THAN THE YELLOW SHIRT.
6. THE JOCKEY ON JOHNSTON'S DARLIN' AND THE JOCKEY IN THE VIOLET SHIRT ARE BROTHERS.
7. THE ORANGE SHIRT CAME IN BETWEEN WINNER'S CIRCLE AND LUCKY LADY.
8. FIRE BALL CAME IN BEFORE THE YELLOW SHIRT.
9. THE VIOLET SHIRT BEAT WINNER'S CIRCLE.
10. LUCKY LADY HAS NEVER BEEN BEATEN BY DANCER.

COLOR	NAME	PLACE
RED	________	________
YELLOW	________	________
ORANGE	________	________
GREEN	________	________
BLUE	________	________
VIOLET	________	________

31

17) AFTER THE REGULAR SEASON HAD FINISHED, THE SCHOOL'S BASKETBALL COACH DECIDED TO HOLD A FREE THROW CONTEST. THE MANAGER HAD A LOT OF TIME, WHILE THE PLAYERS WERE TAKING THEIR SHOTS, SO HE MADE OUT THE FOLLOWING LIST OF STATEMENTS WHICH HE TURNED IN TO THE COACH AFTER THE PLAYERS HAD FINISHED SHOOTING:

1. THE FIVE WITH THE HIGHEST SCORES WERE LLOYD, CARLTON, RONALD, JOE, AND ROY, BUT NOT RESPECTIVELY.
2. THE PLAYER WITH THE HIGHEST SCORE AND THE PLAYER WHO FINISHED FIFTH HAD NEVER BEEN IN A SEASON GAME AT THE SAME TIME.
3. LLOYD AND ROY WERE BOTH STARTERS IN THE FINAL GAME OF THE SEASON.
4. THE PLAYER WITH THE HIGHEST SCORE AND THE PLAYER WHO FINISHED SECOND HAD NEVER PLAYED TOGETHER IN A GAME.
5. JOE FINISHED HIGHER IN THIS CONTEST THAN ROY DID EVEN THOUGH ROY HAD BEEN THE TEAM'S LEADING SCORER DURING THE SEASON.
6. CARLTON AND THE WINNER HAD BEEN STARTERS IN THE FIRST GAME OF THE SEASON.
7. THE PLAYER WHO FINISHED FOURTH HAD NOT PLAYED IN THE LAST FOUR GAMES OF THE SEASON DUE TO AN INJURY.
8. JOE HAD PLAYED AT SEPARATE TIMES DURING THE LAST GAME OF THE SEASON WITH BOTH THE HIGHEST SCORER AND THE RUNNER-UP.

USING THE FACTS PROVIDED, THE COACH WAS ABLE TO DETERMINE THE ORDER IN WHICH THESE FIVE FINISHED THE CONTEST.

IN WHAT ORDER DID THE PLAYERS FINISH?

1. ______ 2. ______ 3. ______ 4. ______ 5. ______

18) ANN, BETH, CLEO, DONNA, AND EDITH WERE CLOSE FRIENDS IN HIGH SCHOOL, AS WERE THE FELLOWS THEY DATED, NOT IN ORDER--LESTER, CALVIN, DICK, RALPH, AND WARREN. ALL TEN DECIDED TO APPLY FOR ADMISSION AT THE UNIVERSITY, AND ALL WERE ACCEPTED. THE GIRLS, WHO WERE ALL GOING TO MAJOR IN EDUCATION, DECIDED TO JOIN SOCIAL GROUPS, SINCE THEY HAD HEARD THAT THIS WAS A GOOD WAY TO MAKE NEW FRIENDS. EACH GIRL JOINED A DIFFERENT ORGANIZATION--ALPHA, BETA, DELTA, CHI, AND IOTA. THE BOYS EACH DECIDED ON A DIFFERENT COLLEGE MAJOR--PRE-MED, PRE-LAW, ENGLISH, ECONOMICS, AND HISTORY. FROM THE CLUES BELOW, DETERMINE WHICH GIRL EACH DATED AND WHICH GROUP SHE BELONGED TO AS WELL AS THE MAJOR SUBJECT EACH BOY DECIDED ON.

1. DICK DATED THE GIRL IN ALPHA, WHILE THE IOTA GIRL DATED THE PRE-MED STUDENT.
2. BETH'S USUAL ESCORT WAS THE PRE-MED STUDENT. LESTER DATED DONNA. ANN DATED THE ECONOMICS STUDENT.
3. THE FIRST WEEKS OF SCHOOL WERE HARD, FINANCIALLY, FOR CALVIN. HE COULDN'T AFFORD TO TAKE THE BETA GIRL OUT. SO THE TWO OF THEM STUDIED TOGETHER, AND SHE HELPED HIM WITH HIS HISTORY COURSES.
4. WARREN'S GIRL WAS A DELTA.
5. THE DELTA GIRL AND THE ECONOMICS MAJOR OFTEN DOUBLE-DATED WITH CLEO AND THE PRE-LAW STUDENT.

	ANN	BETH	CLEO	DONNA	EDITH
PERSON DATED	______	______	______	______	______
MAJOR SUBJECT	______	______	______	______	______
SOCIAL GROUP	______	______	______	______	______

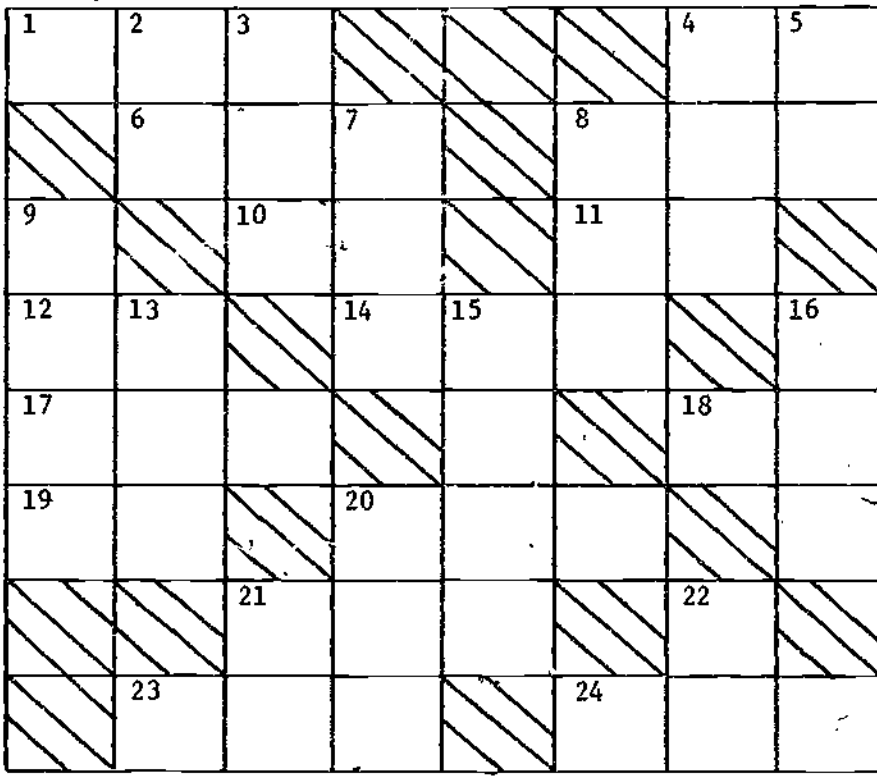
19) JUST BEFORE THE START OF THE HIGH SCHOOL'S BASEBALL SEASON THE CHEERLEADERS ASKED THE COACH WHAT THE LINE UP WOULD BE FOR THE FIRST GAME. THINKING THAT HE WOULD STUMP THE CHEERLEADERS, THE COACH GAVE THEM THE FOLLOWING STATEMENTS:

1. THE BATTING ORDER WILL BE ALTON, BILL, CARL, DENNIS, ERIK, FRANK, GEORGE, HENRY, AND JUSTIN.
2. THE CENTER FIELDER IS TALLER THAN THE RIGHT FIELDER.
3. ALTON IS NOT THE CATCHER THIS YEAR.
4. ERIK'S SISTER GOES STEADY WITH THE SECOND BASEMAN.
5. ERIK, FRANK, JUSTIN, THE RIGHT FIELDER, AND THE CENTER FIELDER ARE ALL SENIORS; THE REST ARE UNDERCLASSMEN.
6. ERIK AND THE OUTFIELDERS PLAYED BALL LAST SUMMER.
7. JUSTIN, BILL, AND THE PITCHER WERE ON THE ALL-CONFERENCE TEAM LAST YEAR.
8. HARRY AND THE THIRD BASEMAN RIDE TO SCHOOL IN THE SAME CAR.
9. ALTON, JUSTIN, AND THE SHORTSTOP ARE MATH CLUB MEMBERS.
10. ALL THE BATTERY AND THE INFIELD, EXCEPT BILL, ALTON, AND HARRY, ARE SHORTER THAN CARL.
11. THE PITCHER IS THE JUNIOR CLASS PRESIDENT.
12. THE CATCHER AND THE THIRD BASEMAN ARE CO-CHAIRMEN OF THE JUNIOR PROM DECORATIONS COMMITTEE THIS YEAR.
13. CARL IS THE ONLY JUNIOR IN SPANISH CLASS.
14. DENNIS, HARRY, JUSTIN, AND THE CATCHER ALL RIDE HOME EACH EVENING WITH THE SECOND BASEMAN.
15. FRANK IS TALLER THAN DENNIS, WHILE GEORGE IS SHORTER THAN DENNIS, EACH OF THEM IS HEAVIER THAN THE THIRD BASEMAN.
16. EITHER ALTON OR GEORGE WILL PLAY IN THE OUTFIELD, BUT NOT BOTH.
17. THE SHORTSTOP, THE THIRD BASEMAN, AND DENNIS ALL HAVE PART-TIME JOBS AFTER PRACTICE.

20) Recently a group of students from a nearby high school were discussing their school's offensive football team. They knew the names of the players, who were Bibb, Carr, Day, Finks, Harte, Jiminez, Kahler, Leggett, Mosley, Norwood, and Tarvin. No one in the group could tell which position each man played. They finally came up with a list of statements which enabled them to tell the position of each man on the team.

1. Carr, Bibb, Leggett, and all the backs were seniors.
2. Jiminez, Leggett, Tarvin, and the guards were in the same Spanish class.
3. Norwood and the halfbacks were taking calculus.
4. Bibb and Tarvin challenged the ends to a race.
5. Day, Mosley, and the tackles were juniors.
6. Kahler and the right halfback lived on the same street.
7. Tarvin, the backs, and the center were in the same English class.
8. Bibb and the center were senior class officers.
9. Tarvin and the right tackle were the only redheads on the team.
10. Jiminez, Mosley, and the center were the fastest linemen.
11. Leggett and the center were math club members.
12. Day, Leggett, and Tarvin, all linemen, played on the same side of the line.
13. The quarterback threw a pass to Mosley for a touchdown.
14. Finks, Norwood, and the fullback rode to school together.
15. Harte, Mosley, and the fullback were the team captains.
16. Finks usually lined up behind Jiminez.
17. The team usually lined up in a T formation.

What are the positions played by each man on the offensive team?

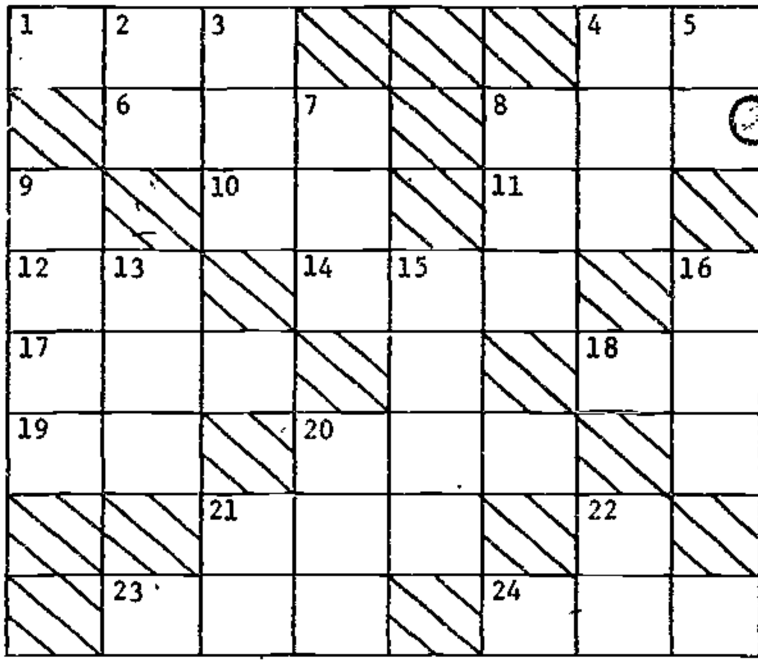


ACROSS

1) DCCXXV
4) XIV
6) CCXV
8) DIII
9) I
10) XC
11) XXI
12) LV
14) CDX
16) II
17) DXLI
18) XCV
19) XIII
20) CCCXLII
21) DCCLXI
22) V
23) CDLXII
24) DXXXV

DOWN

1) VII
2) XXII
3) DIX
4) CI
5) XLIII
7) DIV
8) DXX
9) MDLI
13) DXLIII
15) MDCXLI
16) CCLII
18) IX
20) CCCLXII
21) LXXVI
22) LIII
23) IV
24) V



ACROSS

1) AREA OF SQUARE: SIDE 16

4) L.C.M. OF 2, 8, 7

6) 46 x 17

8) 1239 FT. = ? YD.

9) NO. OF SIDES OF A RHOMBUS

10) NO. OF EGGS IN A DOZEN

11) 1032 IN. = ? FT.

12) $\frac{36}{35} \times \frac{175}{12}$

14) 39156 ÷ 52

16) NO. OF SIDES OF A QUADRILATERAL

17) L.C.M. OF 15, 16, 18

18) G.C.F. OF 45, 60

19) $\frac{1}{2} + \frac{1}{10} - \frac{5}{6} + \frac{2}{3}$

20) 1674 - 986

21) 17280 ÷ 36

23) 982.25 - 493.25

24) 324.58 + 196.42

DOWN

2) $\frac{1}{2} \times 114$

3) 227 YD. = ? FT.

4) 43 FT. = ? IN.

5) 9 x 7

7) $\frac{1}{3} \times 759$

8) 9236 - 8753

9) 237 + 1415 + 1519

13) 6872 - 6349

15) NO. OF FT. IN 1 MILE

16) 25 + 136 + 293

20) 25 + 237 + 163 + 264

21) 6 x 8

22) $\frac{3}{5} \times \frac{160}{3}$

23) NO. OF SIDES OF A PARALLELOGRAM

24) NO. OF SIDES OF A PENTAGON

MATHEMATICAL WORD SEARCH

WHEN USING WORD SEARCHES, YOU MIGHT WANT TO GIVE THE STUDENTS ONLY THE PUZZLES AND LET THEM MAKE LISTS OF THE MATHEMATICAL WORDS THAT THEY FIND. ANY WORD THAT IS IN THE MATHEMATICS DICTIONARY COUNTS. YOU MIGHT GIVE A PRIZE OR AN EXTRA A TO THE ONE WHO FINDS THE MOST WORDS. INSTEAD OF GIVING AWARDS, YOU MIGHT DIVIDE THE ROOM INTO TEAMS AND LET EACH TEAM WORK AS A GROUP ON ONE PUZZLE. SET A TIME LIMIT OF ONE OR TWO WEEKS FOR A CONTEST.

IF YOUR LIBRARY DOES NOT HAVE A MATHEMATICS DICTIONARY, CONTACT CRAWFORD JOHNSON, MATHEMATICS PROGRAM DIRECTOR, AND REQUEST ONE FOR YOUR SCHOOL.

GEOMETRYGRAM

THERE ARE AT LEAST 40 WORDS PERTAINING TO GEOMETRY IN THE PUZZLE BELOW.. THEY BE BE WRITTEN UP, DOWN, ACROSS, DIAGONALLY, BACKWARD, OR FORWARD. SOME WORDS MAY OVERLAP.

O	S	R	A	D	I	U	S	N	E	S	U	T	B	O	O	O
R	E	C	T	A	N	G	L	E	W	E	K	S	N	I	P	A
E	G	O	R	A	F	A	A	A	P	A	T	H	G	I	R	P
T	M	N	I	E	I	P	R	I	S	M	E	E	E	N	O	C
E	E	G	A	M	N	R	E	T	E	M	A	I	D	L	T	P
M	N	R	N	U	I	I	T	C	I	I	Y	I	E	I	R	L
I	T	U	G	L	T	B	A	S	E	U	U	A	G	U	A	M
R	X	E	L	O	Y	P	L	A	A	A	L	A	R	R	C	E
E	E	N	E	V	S	E	I	N	T	E	R	S	E	C	T	R
P	T	T	I	I	E	E	R	I	L	P	I	T	E	E	O	O
E	R	O	X	O	T	A	D	L	O	O	A	O	S	R	R	E
L	E	A	A	U	D	U	A	U	P	L	A	N	E	E	W	H
G	V	E	C	I	A	R	U	A	A	A	A	A	D	H	F	T
N	R	A	U	E	A	S	Q	U	A	R	E	E	K	P	A	X
A	I	S	T	P	I	I	Q	C	R	I	C	L	E	S	C	I
I	O	E	E	R	G	E	D	I	S	O	S	C	E	L	E	S

GEOMETRYGRAM

ACUTE
ANGLE
AREA
ARC
AXIS

BASE

CIRCLE
CONE
CONGRUENT

DEGREE
DIAMETER

EQUAL

FACE

INFINITY
INTERSECT
ISOSCELES

LATERAL

METER

OBTUSE

PARALLEL

PATH
PERIMETER
PLANE
POINT
POLAR
POLE
PRISM
PROTRACTOR

QUADRILATERAL

RADIUS
RAY
RECTANGLE

SEGMENT
SET
SIX
SKEW
SPACE
SPHERE
SQUARE

THEOREM
TRIANGLE

VERTEX
VOLUME

WORD SEARCH

THERE ARE AT LEAST 85 WORDS PERTAINING TO MATHEMATICS IN THE PUZZLE BELOW. THEY MAY BE WRITTEN UP, DOWN, ACROSS, DIAGONALLY, BACKWARD, OR FORWARD. SOME WORDS MAY OVERLAP.

```
L T R A P E Z O I D Y R T E M O E G R E
I O N T M O N O T O N E Y A R D L R O V
N I N E N U L Z R U D I L O S N E O O E
E S C A L E N E L B Y I P L O T V U T N
S O U T H A Z A V L N S I I T O E P O T
T S L M O T V R E E E M T R I A N G L E
A C U T E O A I A Q I A L O B A R A P X
L E N G T H R R U T U S U B M O H R O I
A L I O E A Y A R Q U A M I N U E N D R
R E T L R F L W E A E D T Y R O E H T A
E S T I M A T E T D D D U E L I R H W N
T I A M B C H K E D T I Q D O D G X O E
A O N E A T P S M O H T U P I I S H B O
L E G T S O A M A T R I X S E V E N T U
I X E I E R R F I V E O R L I E I M U S
U C N A T G G O D E E N S A P R N D S U
Q E T P A R E U U C R O E N O G R A E N
E S P I L L E R A T E T A T W E L V E I
O S D E C I M A L O A E R A E N E D O M
A P E X T E R I O R L T C A R T B U S A
```

WORD SEARCH

ACUTE
ADD
ADDITION
ANGLE
APEX
ARC
AREA

BASE

DECIMAL
DIAGRAM
DIAMETER
DIVERGENT
DIVIDE
DOUBLE
DUAL
DUEL
DYNE

EIGHT
ELEVEN
ELLIPSE
EQUAL
EQUATE
EQUATION
EQUILATERAL
EQUIVALENT
ERG
ESTIMATE
EVENT
EXCESS
EXTERIOR
EXTRANEOUS

FACTOR
FIVE
FOUR

GEOMETRY
GRAPH
GROUP

INTEGER
ISOSCELES
ITEM

LEG
LENGTH
LIMIT
LINE
LINEAR
LOG
LUNE

MATRIX
MEAN
MINUEND
MINUS
MODE
MONOTONE
MULTIPLY

NINE

OBTUSE
ONE
OVAL

PAR
PARABOLA
PI
PLOT
POLE
POWER

RADIUS
RATE
RAY
REAL
RHOMBUS
RIGHT
ROOT

SCALENE
SEVEN
SIX
SKEW
SLANT
SOLID
SOUTH
SUBTRACT
SUM

TANGENT
TERM
THEORY
THREE
TON
TRAPEZOID
TRIANGLE
TWELVE
TWO

UNIT

VARY
VECTOR

YARD

ZERO

MATHEMATICAL WORD SEARCH

THERE ARE AT LEAST 90 WORDS PERTAINING TO MATHEMATICS IN THE PUZZLE BELOW. THEY MAY BE WRITTEN UP, DOWN, ACROSS, DIAGONALLY, BACKWARD, OR FORWARD. SOME WORDS MAY OVERLAP.

E	S	U	B	T	R	A	C	T	A	P	E	R	I	M	E	T	E	R	M
S	R	N	I	N	E	V	B	C	D	H	U	N	D	R	E	D	E	I	E
T	A	R	E	A	S	E	F	U	P	G	F	H	D	R	A	T	E	R	A
I	C	I	O	S	J	R	F	A	N	E	G	A	T	I	V	E	E	I	S
M	C	P	E	R	K	A	R	R	L	I	R	M	C	Z	G	G	N	A	U
A	U	L	D	I	A	G	O	N	A	L	T	C	O	T	E	I	P	N	R
T	R	A	I	Q	R	E	S	A	T	C	U	H	E	T	O	R	T	G	E
E	A	C	S	P	O	W	E	R	G	F	T	E	N	N	V	R	O	L	W
X	C	E	T	A	D	I	V	I	D	E	F	I	X	Y	T	Z	C	E	A
P	Y	V	A	R	B	C	T	T	D	E	O	G	O	W	P	R	I	M	E
O	F	A	N	A	G	A	H	H	I	J	U	H	B	N	I	K	L	D	M
N	N	L	C	L	R	N	U	M	B	E	R	T	A	C	O	D	D	P	Q
E	D	U	E	L	R	S	T	E	M	P	T	Y	S	E	T	O	T	A	L
N	I	E	L	E	M	E	N	T	U	O	U	V	E	Q	U	A	L	H	E
T	U	N	C	L	V	W	X	I	L	S	Y	R	V	U	U	Z	D	E	N
W	W	X	V	I	Y	Z	A	C	T	I	H	A	E	A	C	A	M	D	G
B	C	O	F	E	M	A	T	H	I	T	N	Y	N	T	X	U	R	D	T
E	N	U	M	E	R	A	L	F	P	I	G	E	H	I	L	I	B	E	H
S	P	H	E	R	E	S	L	J	L	V	K	L	S	O	M	N	O	E	P
G	R	E	A	T	E	R	E	Q	Y	E	R	S	V	N	P	O	I	N	T

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MATHEMATICAL WORD SEARCH

ACCURACY
ADD
ANGLE
ARC
AREA
ARITHMETIC
AVERAGE

BASE

CENT
CIRCLE
CUBE

DECIMAL
DIAGONAL
DIGIT
DISTANCE
DIVIDE
DUEL

EIGHT
EIGHTY
ELEMENT
EMPTY SET
END
EQUAL
EQUATION
ERROR
ESTIMATE
EVEN
EXPONENT

FACTOR
FEET
FIVE
FOUR
FRACTION

GRAPH
GREATER

HEIGHT
HUNDRED

INTEGER
INVERSE

JOIN
JOINT

LEG
LENGTH
LESS
LINE

MATH
MEASURE
METER
MULTIPLY

NEGATIVE
NET
NINE
NULL
NUMBER
NUMERAL

ODD
ONE

PARALLEL
PERCENT
PERIMETER
PI
PLACE VALUE
POINT
POSITIVE
POWER OF TEN
PRIME
PURE

RATE
RATIO
RAY
ROD

SET
SEVEN
SIX
SPHERE
SQUARE
SUBTRACT

TAX
TEN
TERM
THREE
TOTAL
TRIANGLE
TWO

UNIT

VALUE
VOLUME

WIDTH

ZERO

THE 105 MATHEMATICS WORD SEARCH

THERE ARE AT LEAST 105 WORDS PERTAINING TO MATHEMATICS IN THE PUZZLE BELOW. THEY MAY BE WRITTEN UP, DOWN, ACROSS, DIAGONALLY, BACKWARD, OR FORWARD. SOME WORDS MAY OVERLAP.

M	T	C	A	R	T	B	U	S	E	I	R	E	S	D	I	G	I	T	C
C	A	B	A	S	H	C	D	L	P	A	R	A	B	O	L	A	D	D	O
I	X	T	R	I	G	O	N	O	M	E	T	R	Y	P	S	E	V	E	N
G	I	E	H	N	I	N	E	P	A	R	A	L	L	E	L	R	I	N	G
O	O	H	M	E	E	T	A	E	X	A	I	N	T	E	G	E	R	F	R
L	M	R	P	I	M	R	P	E	I	D	N	U	A	H	P	A	R	G	U
A	L	G	E	B	R	A	A	L	M	I	V	M	N	P	F	I	R	E	E
L	M	M	R	Z	E	P	T	L	U	U	E	B	G	U	O	R	O	O	N
O	O	I	P	E	T	O	R	I	M	S	R	E	E	C	O	I	W	M	C
G	D	N	E	O	V	S	U	P	C	S	S	R	N	Q	R	T	N	E	E
A	U	F	N	Y	S	I	O	S	I	S	E	H	T	O	P	Y	H	T	C
R	L	I	D	N	T	T	F	E	T	Y	P	Y	M	R	C	R	Y	R	A
I	U	N	I	O	N	I	U	L	R	M	L	A	E	D	I	A	X	Y	L
T	S	I	C	D	O	V	N	L	I	M	A	T	R	I	X	E	B	U	C
H	B	T	U	E	I	E	C	U	A	E	N	H	O	N	P	N	U	E	U
M	A	E	L	C	T	R	T	P	N	T	E	R	E	A	T	I	M	I	L
E	S	E	A	I	A	E	I	U	G	R	E	E	H	T	O	L	P	U	U
T	E	P	R	M	U	H	O	O	L	I	M	E	T	E	R	A	U	Q	S
I	S	I	X	A	Q	P	N	R	E	C	U	R	S	I	V	E	L	O	P
C	I	R	C	L	E	S	G	G	E	Y	F	A	C	T	O	R	I	A	L

105 MATHEMATICAL WORD SEARCH

ADD
ALGEBRA
ANGLE
APEX
ARC
AREA
ARITHMETIC
AXIOM
AXIS

BAR
BASE

CALCULUS
CIRCLE
CONE
CONGRUENCE
CONTRAPOSITIVE
COORDINATE
CUBE

DECIMAL
DIGIT

EIGHT
ELLIPSE
EQUATION
EVEN

FACTOR
FACTORIAL
FIELD
FINITE
FIVE
FOUR
FUNCTION

GEOMETRY
GRAPH
GROUP

HYPOTHESIS

IDEAL
INFINITE
INTEGER
INVERSE

LIMIT
LINE
LINEAR

LOGARITHM
LOGIC

MAP
MATHEMATICS
MATRIX
MAXIMUM
MEAN
MEET
METER
METRIC
MODULUS

NET
NINE
NODE
NUMBER

ODD
ONE
ORDINATE

PARABOLA
PARALLEL
PERPENDICULAR
PI
PLANE
PLOT
PLUS
POINT
POLE
POSITIVE
POSTULATE
PRIME
PROOF

RADIUS
RATIO
RAY
REAL
RECURSIVE
RING
ROOT
ROW

SERIES
SET
SEVEN
SINE
SIX
SLOPE
SPHERE

SQUARE
SUBTRACT
SUM
SYMMETRIC

TANGENT
TEN
TERM
THEOREM
THREE
TON
TRIANGLE
TRIGONOMETRY
TWO

UNION
UNIT
UNITY

ZERO

MATHEMATICANS' HIDDEN NAMES

THE NAMES OF 41 MATHEMATICIANS ARE HIDDEN IN THE ARRAY OF LETTERS. THEY ARE ARRANGED HORIZONTALLY, VERTICALLY, DIAGONALLY--FORWARD AND BACKWARD.

```
H A L F S I O L A G S O D I R I C H L E T O F
T S L A R E A I X I A D I O P H A N T U S B E
I P O A S T D N E N D L L I N E S A I D S E W
R A B A C U S E S E H E C O N E I R E L U E E
E R O M A S T R M K G C U S T O N A D R A C I
L E I B N I Z O E I C N E R Y R E T I R G R E
L O C I T O I R R A H A A L A C S A P E L U R
O B O E O V E R T E X C A R S O E L I L A G S
H L B E R T E N F E S A R E I P A N N R U S T
E Y E E D P O S T E L K N A R E I R U O F E R
T R A C E T T R D I R K O H E R M I T T I N A
A E M O W E R O N P A M U M I N I M O S E T S
L T H E A B O O L E M O A K C I K L U E L E S
U S N I L A T T H E I G H T I C P H I V D N A
T E S T L B E N I R M I E S S U A G A L U C D
S V E C I B U C R C A Y L E Y S C A S Y C E I
O L S O S A E I A H C O M P O S I T E S Y O R
P Y T H A G O R A S C A K O S L O P E T A A A
R S R I B E G H T A L I N P N A L V I V X R M
I N A S O I T S U N A T N O M O I G E R I E Y
M P U R B O M C L A V I U S B E I N N E S D H
E R S B O T H E S I M P L E T I N N D A I R T
V E S F I Y E Q U A T E L E S S F S U M E O S
```

MATHEMATICIAN'S HIDDEN NAMES

AIKEN	GAUSS
ARCHIMEDES	GIBBS
BABBAGE	HARRIOT
BOOLE	HILBERT
CANTOR	HOLLERITH
CARDANO	KHAYYAM
CAUCHY	LEIBNIZ
CAYLEY	NAPIER
CLAVIUS	NEWTON
COPERNICUS	PACIOLI
DEMOIVRE	PASCAL
DESCARTES	PTOLEMY
DIOPHANTUS	PYTHAGORAS
DIRICHLET	REGIOMONTANUS
EUCLID	SYLVESTER
EULER	THYMARIDAS
FERMAT	VENN
FIBONACCI	VIETE
FOURIER	WALLIS
GALILEO	WEIERSTRASS
GALOIS	

SOME STUDENTS MAY ALSO DISCOVER MATHEMATICAL TERMS HIDDEN IN THE PUZZLE.

METRIC MEASUREMENTS

RULER MASTER AND METRIC PREFIXES

METRIC SYSTEM PREFIXES:

MILLI	=	0.001
CENTI	=	0.01
DECI	=	0.1
DEKA	=	10
HECTO	=	100
KILO	=	1000

METRIC UNITS OF LENGTH:

1000 MILLIMETERS	=	1 METER
100 CENTIMETERS	=	1 METER
10 DECIMETERS	=	1 METER
10 METERS	=	1 DEKAMETER
100 METERS	=	1 HECTOMETER
1000 METERS	=	1 KILOMETER

USEFUL METRIC MEASURES

LENGTH

1000 MILLIMETERS (MM)	=	1 METER (M)
100 CENTIMETERS (CM)	=	1 M
10 DECIMETERS (DM)	=	1 M
1000 M	=	1 KILOMETER

MASS

1000 MILLIGRAMS (MG)	=	1 GRAM (G)
1000 G	=	1 KILOGRAM (KG)
1000 KG	=	1 TONNE (T)

LAND AREA

100 SQUARE METERS (M^2)	=	1 ARE (A)
100 A	=	1 HECTARE (HA)
100 HA	=	1 SQUARE KILOMETER (KM^2)

VOLUME

1000 MILLILITERS (ML)	=	1 LITER (ℓ)
1000 CUBIC CENTIMETERS (CM^3)	=	1 ℓ
1000 CM^3	=	1 CUBIC DECIMETER (DM^3)
1000 ℓ	=	1 CUBIC METER (M^3)
1000 DM^3	=	1 M^3

MIXED MEASUREMENTS

THERE ARE AT LEAST SIXTY MEASUREMENT TERMS LISTED BELOW. THEY MAY BE WRITTEN UP, DOWN, ACROSS, DIAGONALLY, BACKWARD, OR FORWARD. SOME WORDS MAY OVERLAP.

D	E	F	A	T	H	O	M	C	O	R	D	F	M	D	M	O	N	T	H
R	O	D	O	O	R	M	I	L	C	E	L	A	A	N	R	O	E	H	O
A	M	Z	Y	G	T	Z	L	E	I	A	T	C	R	U	O	H	M	A	R
M	I	L	E	E	A	L	L	R	Y	M	A	R	G	O	T	C	E	H	S
I	L	G	E	N	R	S	I	R	N	I	U	E	A	P	I	N	T	E	E
L	O	F	T	A	I	L	L	A	O	L	D	E	C	A	D	E	E	C	P
L	T	E	A	U	M	I	I	B	L	L	A	T	E	H	R	D	R	T	O
I	A	A	S	H	R	E	T	I	L	I	C	E	D	I	M	E	X	O	W
M	C	A	L	O	R	I	E	M	A	E	E	L	L	O	T	G	R	M	E
E	T	R	A	U	Q	E	R	A	G	R	N	Y	Y	E	A	R	E	E	R
T	H	R	E	N	O	P	N	U	R	E	T	E	M	I	C	E	D	T	M
E	M	M	I	C	R	O	N	H	P	T	I	O	N	T	H	E	Y	E	O
R	I	E	N	E	W	T	O	N	E	Y	L	I	N	K	A	K	N	R	T
E	N	O	D	R	A	A	M	I	C	I	I	N	C	H	I	O	E	T	T
D	U	P	N	C	T	M	N	C	K	O	T	V	E	L	N	M	I	L	A
O	T	S	A	T	T	P	A	K	O	S	E	C	O	N	D	P	I	L	W
L	E	O	H	M	A	E	I	E	M	A	R	G	I	L	L	I	M	A	O
L	B	A	L	E	R	R	D	L	T	H	R	H	E	R	T	Z	R	A	L
A	M	J	O	U	L	E	A	S	P	A	N	C	E	N	T	U	R	Y	I
R	O	G	R	O	S	S	R	C	M	Y	R	I	A	M	E	T	E	R	K

MIXED MEASUREMENTS

ACRE
AMPERES
ARE

BALE
BARREL
BUSHEL

CALORIE
CARAT
CELSIUS
CENT
CENTILITER
CENTURY
CHAIN
CORD

DECADE
DECAGRAM
DECILITER
DECIMETER
DEGREE
DIME
DOLLAR
DOZEN
DRAM

ERG

FAHRENHEIT
FATHOM
FEET

GALLCN
GRAM
GROSS

HAND
HECTOGRAM
HECTOMETER
HERTZ
HORSEPOWER
HOUR

INCH

JOULE

KILOGRAM
KILOMETER
KILOWATT

LINK
LITER

METER
MICRON
MIL
MILE
MILL
MILLIER
MILLIGRAM
MILLILITER
MILLIMETTER
MINUTE
MONTH
MYRIAMETER

NEWTON
NICKEL

OHM

PECK
PINT
POUND

QUART

RADIAN
REAM
ROD
ROOD

SECOND
SPAN

TON

VOLT

WATT

YEAR

MIXED MEASUREMENTS

THERE ARE AT LEAST THIRTY-FIVE MEASUREMENT TERMS LISTED BELOW EITHER HORIZONTALLY OR VERTICALLY. CIRCLE AS MANY OF THESE TERMS AS YOU CAN FIND.

M	U	D	B	C	E	N	T	V	B	K	I	C	S	E	C	O	N	D	D
G	C	E	L	H	D	M	O	N	J	F	D	E	G	K	H	J	L	L	M
S	E	C	N	B	M	I	P	D	E	C	A	L	W	A	T	T	Q	R	S
E	D	I	M	E	R	L	U	R	K	E	Y	S	S	R	T	T	A	V	D
C	W	M	V	X	Y	L	G	M	I	L	L	I	M	E	T	E	R	Y	E
B	W	E	E	K	Z	C	R	A	L	L	T	U	I	M	B	G	A	R	G
R	E	T	D	O	G	E	A	B	O	R	H	S	L	I	T	E	R	H	R
T	L	E	S	D	T	N	M	R	G	R	R	R	L	L	C	E	N	E	E
O	L	R	C	E	N	T	I	G	R	A	M	S	I	L	X	C	A	C	E
N	T	C	H	K	R	U	L	H	A	H	A	H	G	I	D	E	B	T	T
N	W	E	L	A	L	R	L	O	M	Z	O	A	R	E	O	N	S	O	E
E	H	N	O	M	M	Y	I	H	B	O	Z	O	A	H	L	T	C	M	R
B	I	H	R	E	K	I	L	O	W	A	T	T	M	A	L	I	H	E	X
K	M	O	N	T	H	H	I	X	D	E	C	I	G	R	A	M	O	T	H
I	I	U	Y	E	H	I	T	H	E	R	E	Y	O	U	R	E	O	E	I
L	N	R	Z	R	B	H	E	C	T	A	R	E	C	D	B	T	L	R	A
O	U	X	M	E	T	E	R	H	Y	K	I	L	O	H	Y	E	A	R	G
S	T	A	N	D	Y	C	E	N	T	I	L	I	T	E	R	R	H	B	A
S	E	R	D	E	C	A	D	E	C	A	T	M	I	C	R	O	N	A	I
O	K	K	I	L	O	M	E	T	E	R	N	O	P	E	D	E	C	I	N

MIXED MEASUREMENTS

ARE
AREA

CELSIUS
CENT
CENTIGRAM
CENTILITER
CENTIMETER
CENTURY

DAY
DECADE
DECIGRAM
DECIMETER
DEGREE
DEKAMETER
DIME

EON

GRAM

HECTARE
HECTOMETER
HOUR

KILOGRAM
KILOMETER
KILOWATT

LITER

METER
MICRON
MIL
MILL
MILLIGRAM
MILLILITER
MILLIMETER
MINUTE
MONTH

SECOND

TON
TONNE

WATT
WEEK

YEAR

LINEAR MEASUREMENT

USING THE CODE BELOW, SHADE IN THE SQUARES WITH THE PROPER DESIGNS.

EXAMPLES:

1000 M = 1 KM = 

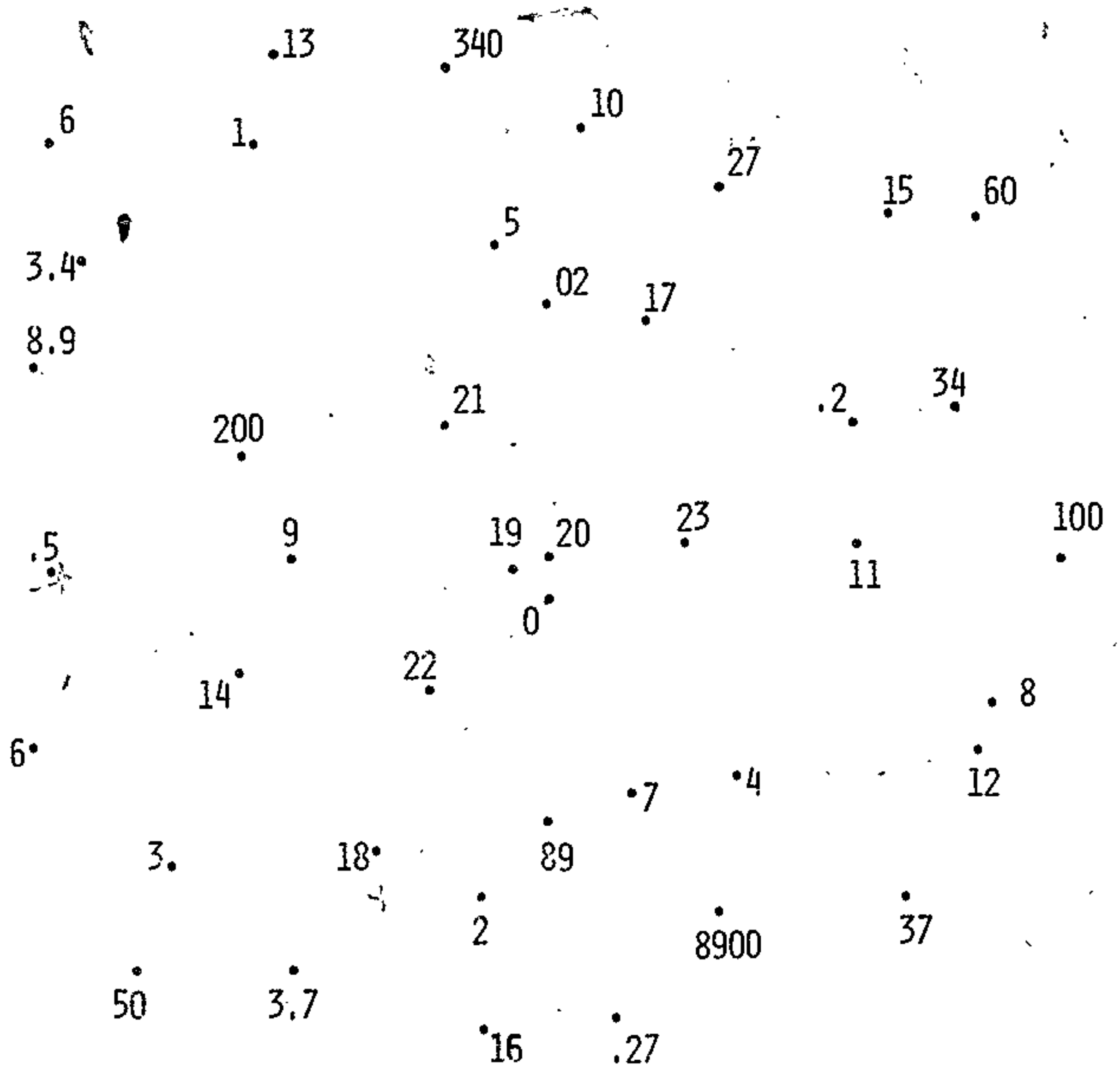
.001001 M = .001 M + .000001 M =

1 MM + 1 MICRON =

1 KM. 1 HM 1 DKM 1 M 1 DM 1 CM 1 MM 1 MICRON

1000m	100m	1000m	.11m	100m	1000m	.11m	100m	1000m	100m
.100001 m	.011m	1m	.001001 m	10m	1m	.001001 m	10m	.100001 m	.011m
1m	10m	1000m	0	.100001 m	.011m	0	100m	1m	10m
1000m	100m	0	10m	1m	10m	1m	0	1000m	100m
1m	10m	0	0	1000m	100m	0	0	1m	10m
1000 m	100m	0	0	1m	10m	0	0	1000m	100m
1m	10m	0	100m	1000m	100m	1000m	0	1m	10m
1000m	100m	1m	0	.100001 m	.011 m	0	10m	1000m	100m
.100001 m	.011 m	1000m	.11m	100m	1000m	.11m	100m	.100001 m	.011m
1m	10m	1m	.001001 m	10m	1m	.001001 m	10 10m	1m	10m

OPERATIONS WITH MASS MEASUREMENTS



CONNECT THE NUMBER OF EACH PROBLEM WITH THE ANSWER FOR THAT PROBLEM.

1. 9 G + 28 G = ____ G
2. 73 T - 46 T = ____ T
3. 15 KG + 19 KG = ____ KG
4. 8 G + 900 MG = ____ G
5. 8 G + 900 MG = ____ MG
6. 3 G - 2.9 G = ____ MG
7. 256 MG + 344 MG = ____ G
8. 454 G + 45 G = ____ KG
9. 1 KG - 800 G = ____ KG
10. 1 G - 730 MG = ____ G
11. .1 G + 100 MG = ____ MG
12. 2345 MG + 1055 MG = ____ G
13. 1 G - 911 MG = ____ MG
14. 59 G + 1000 MG = ____ G
15. 3 KG + 700 G = ____ KG
16. .1 T - 660 KG = ____
17. .042 G + 8 MG = ____ MG
18. 13 MG + 7 MG = ____ G

VOLUME

CUT OUT THE SQUARES. FIT THEM TOGETHER SO THAT THE TOUCHING EDGES NAME THE SAME VOLUMES.

.56 m^3 3.4 m^3 .034 ℓ 1 mℓ	.42 mℓ 340 mℓ 7ℓ 56 cm^3	2 m^3 1 cm^3 1ℓ 42 cm^3	560 mℓ 60 ℓ 1000 ℓ 2 dm^3
56 mℓ 2 mℓ 60 mℓ 18 .002 ℓ	370 mℓ 6ℓ 2000ℓ 1 m^3	1 cm^3 1 m^3 34 ℓ 2000ℓ	56 mℓ 1000 mℓ 3.4 cm^3 37ℓ
1000 dm^3 .7 mℓ 3400ℓ .56ℓ	5.6 mℓ 3.4 mℓ .6mℓ 42ℓ	37 mℓ 2000 mℓ .34ℓ 1ℓ	2 cm^3 3400 cm^3 7 mℓ .0056 ℓ
4.2ℓ 2 m^3 2ℓ 560ℓ	2000 cm^3 70 mℓ 001ℓ 3.7ℓ	5600 mℓ 34000 mℓ 3.4ℓ .056 ℓ	1 dm^3 34 cm^3 2 cm^3 5.6 ℓ

CODED COMPUTATIONS

PUZZLES CONSTRUCTED BY THE CODING OR SUPPRESSION OF DIGITS IN AN ARITHMETICAL CALCULATION REQUIRE NO MORE THAN ATTENTION TO OBVIOUS NUMERICAL FACTS. KEEP TRACK OF CLUES AND CONCLUSIONS IN AN ORDERLY WAY.

IN A CERTAIN MULTIPLICATION PROBLEM EACH DIGIT FROM 0 TO 9 WAS REPLACED BY A DIFFERENT LETTER, YIELDING THE FOLLOWING CODED CALCULATION.

```
      A L E
      R U M
    -------
    W I N E
  W U W L
E W W E
-----------
E R M P N E
```

FIND THE NUMBERS THAT CORRESPOND TO EACH LETTER. WRITE IN A ROW THE DIFFERENT LETTERS APPEARING IN THE PROBLEM:

A L E R U M W I N P

OVER EACH LETTER WRITE ITS NUMERICAL EQUIVALENT WHEN YOU DISCOVER IT. UNDER THE LETTERS RECORD CLUES AND GUESSES. IN PROBLEMS OF THIS SORT, THE DIGITS 0 AND 1 CAN OFTEN BE FOUND BY SIMPLE INSPECTION. FOR INSTANCE, 0 CAN NEVER OCCUR AS THE LEFT-MOST DIGIT OF AN INTEGER, AND WHEN ANY NUMBER IS MULTIPLIED BY ZERO THE RESULT CONSISTS ONLY OF ZEROES. WHEN ANY NUMBER IS MULTIPLIED BY 1, THE RESULT IS THAT NUMBER ITSELF. IN THE PRESENT PROBLEM, YOU CAN IDENTIFY 0 BY OBSERVING THAT N PLUS L EQUALS N WITH NOTHING CARRIED OVER FROM THE COLUMN ON THE RIGHT. THEREFORE, L MUST BE ZERO.

IN THE SEARCH FOR I ELIMINATE R, U, AND M SINCE NONE OF THESE, AS MULTIPLIERS, REPRODUCE A L E. ALSO E CANNOT BE I SINCE U TIMES E DOES NOT HAVE A PRODUCT ENDING IN U. NO FURTHER CLUES TO THE IDENTITY OF I.

NOW THE PARTIAL PRODUCT W U W L ENDS IN L, WHICH IS 0. THEREFORE, ONE OF THE TWO LETTERS U AND E MUST BE 5. SINCE M x E AND R x E ARE NUMBERS ENDING IN E, E MUST BE 5.

```
      A L E              A 0 5
      R U M              R U M
    ------             -------
    W I N E            W I N 5
  W U W L            W U W 0
E W W E            5 W W 5
-----------        ---------
E R M P N E        5 R M P N 5
```

NOTICE: R x A 0 5 = 5 W W 5. THEREFORE, R x 5 = W 5 AND R x A = 5 W. CHECK MULTIPLICATION FACTS; 9 x 5 = 45 AND 9 x 6 = 54. HENCE R = 9, A = 6, AND W = 4.

```
      6 0 5              6 0 5 9     4
      9 U M              A L E R U M W I N P
    -------
    4 I N 5
  4 U 4 0
5 4 4 5
-----------
5 9 M P N 5
```

NOTICE THAT U MUST BE EVEN AND M MUST BE ODD. ALSO M x 6 MUST BE 4 _?_; SO M IS EITHER 7 OR 8. SINCE M IS ODD, THEN M = 7. REPLACE M WITH 7 AND MULTIPLY TO FIND I AND N. THUS, N = 3 AND I = 2.

6059 7423

A L E R U M W I N P

IN CONCLUSION, U AND P MUST BE 1 AND 8; U IS AN EVEN NUMBER SO U = 8. THEREFORE, P = 1. SUBSTITUTE THE NUMERICAL EQUIVALENTS INTO THE PROBLEM AND CHECK THE ANSWERS BY MULTIPLICATION.

```
    605
    987
   ----
   4235
  4840
 5445
 ------
 597135
```

1) ADDITION

```
  XXXX
  YYYY
  ZZZZ
 -----
 YXXXZ
```

2) MULTIPLICATION

```
  PNX
   NX
  ---
  RNX
 NXS
 ----
 ZPNX
```

3) DIVISION

```
      HIL
    -----
 IL|PHIL
    IL
    --
     TI
     LS
     ---
     HIL
     HIL
     ---
```

4) DIVISION

```
     YFY
   ------
 AY|NELLY
    NLY
    ---
     PPL
     PNH
     ---
      NLY
      NLY
      ---
```

5) ADDITION

```
   HITLER
  GOERING
  -------
  HTTLLHHH
```

6) ADDITION

```
 AHAHA
  TEHE
 -----
 TEHAW
```

7) ADDITION

```
  TEN
  TEN
FORTY
-----
SIXTY
```

8) ADDITION

```
 ALGE
  BRA
   IS
-----
GREAT
```

9) SUBTRACTION

```
SEVEN
 NINE
-----
EIGHT
```

(TWO POSSIBLE SOLUTIONS)

10) DIVISION

```
         ABJ
    --------
ECA | FDBHJ
      CGG
      ----
      AGAH
      AAEA
      ----
       KDDJ
       KDBH
       ----
         AJ
```

PUZZLES NUMBERED 11 -- 18 CAN BE DECIPHERED WITH NO FURTHER CLUES. HOWEVER, WHEN THE LETTERS ARE ARRANGED IN THE ORDER OF THE NUMBERS THEY REPRESENT, THEY SPELL OUT A PHRASE.

11)

```
   USP
   RAP
   ---
  OHEP
 SSTS
OUNR
------
OEEOAP
```

12)

```
  ESP
  NRA
  ---
 TALP
NPYI
ESP
-----
PYILP
```

```
                  T U N
13)  H U B | T R A I T S
             N H P P
               U A I T
               R S I B
                 I H H S
                 I H I B
                     U N
```

```
                      H I E
14)  L O N E | B U N C H E D
               U N B C E
                 B C D U E
                 B B E C D
                   U L C E D
                   U D N H E
                     B C H E
```

```
                      C U D
15)  H I G H | D E S I G N
               H I G H
               C U C U G
               C E G L S
                 C C G G N
                 C C C L N
                       N E S
```

```
                     E R E
16)  A P E | T R I P L E
             T W R L
               R W S L
                 A P E
                 T E I E
                 T W R L
                   E W Y
```

```
                   E A R
17)  O D E | S M E A R
             M A E
             O O L A
             O L N N
               M R R
               A G L
               O N R
```

```
                     A T
18)  E A R | W R I T E
             W M R A
               I T M A
               I D U B
                 T U A
```

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SUPPRESSED DIGITS

CAN YOU RESTORE THE UNKNOWN DIGITS REPRESENTED BY STARS?

1) IN A CERTAIN PROBLEM IN LONG DIVISION, EVERY DIGIT EXCEPT 7 WAS SUPPRESSED. RESTORE THE MISSING DIGITS. START BY THINKING OF ALL THE MULTIPLICATION FACTS WHEN PRODUCTS END IN 7. THEY ARE 1 x 7 AND 3 x 9. SINCE ALL 7'S ARE KNOWN, USE 3 x 9. HENCE THE FIRST DIGIT IN THE QUOTIENT MUST BE 3 OR 9 AND THE LAST DIGIT IN THE DIVISOR MUST BE THE OTHER.

```
          * 7 *
       ----------
* *   | * * * * *
        * 7 7
        -----
          * 7 *
          * 7 *
          -----
            * *
            * *
          -----
```

2) IN THE FOLLOWING EXAMPLE OF MULTIPLICATION, MOST OF THE DIGITS HAVE BEEN SUPPRESSED. THOSE THAT REMAIN ARE NOT NECESSARILY ALL OF THE 4'S, 5'S, AND 6'S IN THE EXAMPLE. NOTICE THAT THE LAST DIGIT IN THE MULTIPLIER HAS TO BE 0 OR 1 BECAUSE THE PRODUCT IS A THREE DIGIT NUMBER.

```
        6 * *
        * * *
        -----
        * * *
      * * * *
    * 5 * 5
    ---------
    * * 5 * 4 *
```

3)

```
      * 7 *
      * 6 *
      -----
    * * 3 *
  * * 6 *
3 * * *
---------
* 1 * * 1 *
```

4)

```
              * * *
          ------------
* 4 *    | * 9 8 * * *
           * 7 * *
           -------
             * * 4 *
             * 7 * *
             -------
               3 * * *
               * * * *
               -------
```

5)
```
          9 * *
     3 * | * 4 * 9 *
           * 1 *
           -----
             * * *
             * 1 *
             -----
               2 * 5
               2 * 5
               -----
```

6)
```
        4 * *
          3 *
      -------
      3 6 * *
    * * 7 *
    ---------
    * * 3 * *
```

7)
```
              * * 8 * *
      * * | * * * * * * *
            * * *
            -----
                  * *
                  * *
                  -----
                    * * *
                    * * *
                    -----
```

8) ADDITION
```
   * 2 2 *
   1 * * 1
   -------
   3 4 8 9
```

9) ADDITION
```
     1 1 3
     6 * 4
     1 4 *
     * 2 6
    -------
   * 4 1 0
```

10) MULTIPLICATION
```
         * 7
         * *
       -----
       * * *
     * * 5
     -------
     * * 9 1
```

11) SUBTRACTION
```
   6 * 3 5
   * 8 2 *
   -------
     4 * 7
```

12) SUBTRACTION
```
   4 * * 2
   * 3 5 *
   -------
     1 2 1
```

13) MULTIPLICATION
```
          * * * 7
            * * *
        ---------
        * * * * 6
      * * 2 0 3
    * 3 7 * *
    -------------
    * * * * * * *
```

14)
```
            * * *
     * 6 | 6 * * *
           * 6
           ---
           * 9 *
           * * *
           -----
               3 *
               * 6
```

MAGIC SQUARES: SHEET 1

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF THE NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 15.

8	1	6
3	5	7
4	9	2

8	1	6	8	3	4	8	6
3	5	7	1	5	9	5	5
4	9	2	6	7	2	2	4
___	___	___	___	___	___	___	___

COMPLETE THESE MAGIC SQUARES USING THE SAME NUMBERS 1 THROUGH 9. BE SURE EACH ROW, COLUMN, AND DIAGONAL ADDS TO 15.

6		8
		4

4		
9		
	7	

2	9	
7		

8		4
	7	

6		
	3	4

4		
	5	
	1	

CAN YOU FIND ANOTHER MAGIC SQUARE USING THE SAME NUMBERS 1 THROUGH 9?

MAGIC SQUARES: SHEET 2

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF THE NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 12.

7	0	5
2	4	6
3	8	1

7	0	5	7	2	3	7	5
2	4	6	0	4	8	4	4
3	8	1	5	6	1	1	3

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

ADD 9

MULTIPLY BY 12

ADD $\frac{1}{2}$

MULTIPLY BY 10

ADD $\frac{1}{4}$

MULTIPLY BY 7

IF THE SAME NUMBER IS ADDED TO EACH ENTRY IN A MAGIC SQUARE, IS THE RESULT ANOTHER MAGIC SQUARE? ______________________

IF EACH ENTRY IN A MAGIC SQUARE IS MULTIPLIED BY THE SAME NUMBER, IS THE RESULT ANOTHER MAGIC SQUARE? ______________________

MAGIC SQUARES: SHEET 3

THIS ARRAY OF NUMBERS IS A 3 x 3 MAGIC SQUARE. SHOW THAT THE SUM OF NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 21.

10	3	8
5	7	9
6	11	4

10	5	6	10	3	8	10	8
3	7	11	5	7	9	7	7
8	9	4	6	11	4	4	6

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

ADD 3

SUBTRACT 3

ADD $\frac{1}{3}$

DIVIDE BY 2

SUBTRACT $\frac{2}{3}$

SUBTRACT $\frac{3}{4}$

MAGIC SQUARES: SHEET 4

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 45.

26	1	18
7	15	23
12	29	4

26	1	18	26	7	12	26	18
7	15	23	1	15	29	15	15
12	29	4	18	23	4	4	12
____	____	____	____	____	____	____	____

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

ADD 9

DIVIDE BY 2

ADD $\frac{1}{2}$

DIVIDE BY 3

SUBTRACT $\frac{3}{4}$

SUBTRACT $\frac{2}{3}$

MAGIC SQUARES: SHEET 5

FILL IN THE BLANK SPACES IN THE MAGIC SQUARE USING EACH OF THE FOLLOWING NUMBERS: 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

	1		
	5		
2	16	9	
3			

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

1			
8	4	9	
11			

	2	15	14
		5	
		4	

MAGIC SQUARES: SHEET 6

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

	2		
	11	10	8
	7		

	11		
	8		
12	13		

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 65.

		7		
		25		
	5	13	21	9
		1		

GRAPHING

ORDERED NUMBER PAIRS ARE USED TO DESCRIBE POINTS IN A PLANE. START WITH TWO SIGNED (+,-) NUMBER LINES, CALLED COORDINATE AXES, DRAWN AT RIGHT ANGLES TO EACH OTHER. THE HORIZONTAL (EAST-WEST) LINE IS CALLED THE X-AXIS. THE VERTICAL (NORTH-SOUTH) LINE IS CALLED THE Y-AXIS. IN A COORDINATE PLANE, THE POINT O AT WHICH THE TWO AXES INTERSECT (CROSS) IS CALLED THE ORIGIN.

THE X-AXIS AND THE Y-AXIS DIVIDE THE PLANE INTO FOUR REGIONS CALLED QUADRANTS. THESE QUADRANTS ARE NUMBERED I, II, III, AND IV IN A COUNTERCLOCKWISE ORDER.

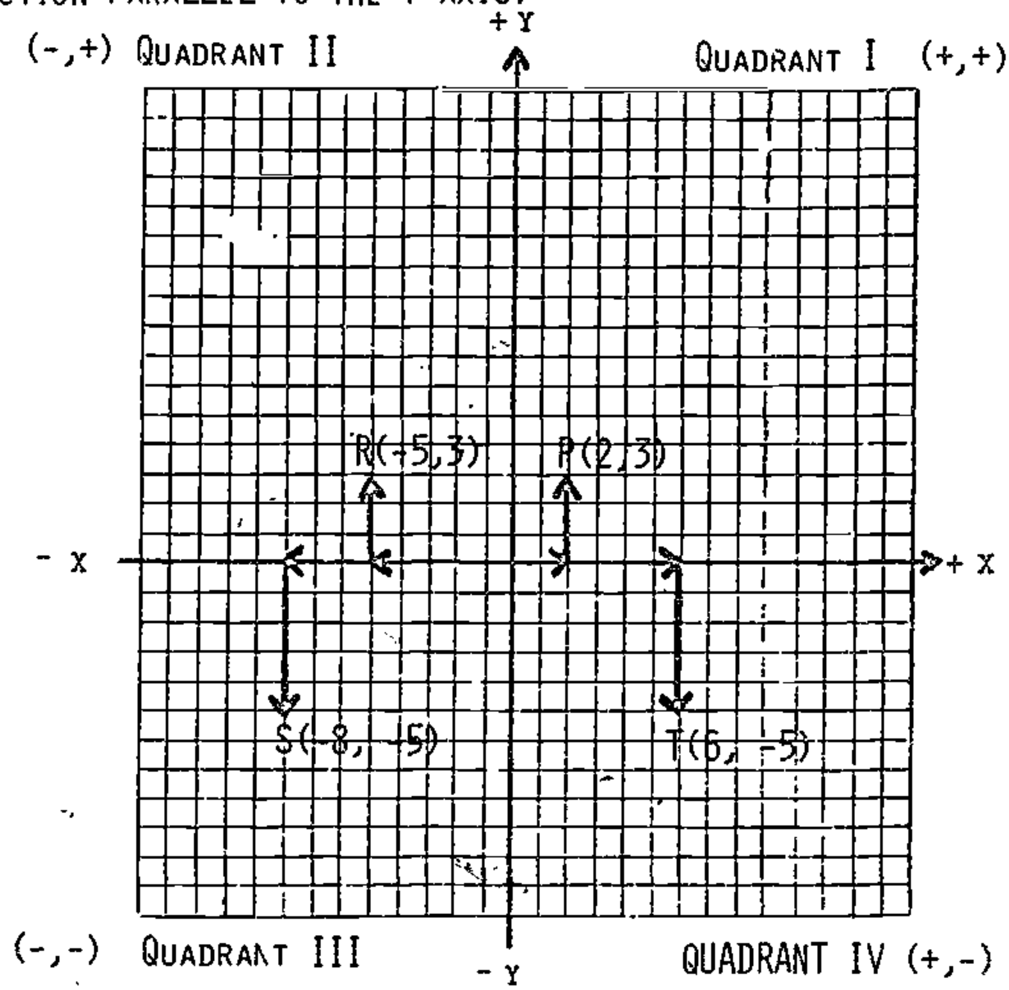
IT IS UNDERSTOOD IN ALL NUMBERS PAIRS THAT THE FIRST NUMBER ALWAYS REPRESENTS A DISTANCE ALONG THE X-AXIS; THE SECOND NUMBER ALWAYS REPRESENTS A DISTANCE ALONG THE Y-AXIS. FOR THIS REASON, IT IS NECESSARY <u>NOT</u> TO INTERCHANGE THE NUMBERS IN AN ORDERED PAIR.

DISTANCES MEASURED TO THE RIGHT OF Y-AXIS, ALONG THE X-AXIS OR ALONG A LINE PARALLEL TO THE X-AXIS, ARE CONSIDERED TO BE POSITIVE (+); DISTANCES MEASURED TO THE LEFT OF THE Y-AXIS ARE CONSIDERED TO BE NEGATIVE (-). DISTANCES MEASURED UPWARD FROM THE X-AXIS, ALONG THE Y-AXIS OR ALONG A LINE PARALLEL TO THE Y-AXIS, ARE CONSIDERED TO BE POSITIVE (+); DISTANCES MEASURED DOWNWARD FROM THE X-AXIS ARE CONSIDERED TO BE NEGATIVE (-). ALL NUMBERS ARE SIGNED NUMBERS. IF THERE IS NO SIGN IN FRONT OF THE NUMBER, THEN IT IS UNDERSTOOD TO BE POSITIVE.

THE DISTANCE OF A POINT FROM THE Y-AXIS, MEASURED EITHER ALONG THE X-AXIS OR ALONG A LINE PARALLEL TO IT, IS CALLED THE X-COORDINATE OR ABSCISSA. THE DISTANCE OF A POINT FROM THE

X-AXIS, MEASURED EITHER ALONG THE Y-AXIS OR ALONG A LINE PARALLEL TO IT, IS CALLED THE Y-COORDINATE OR ORDINATE. THE TWO NUMBERS WHICH ARE ASSOCIATED WITH ANY PARTICULAR POINT, THE ABSCISSA AND ORDINATE OF THE POINT, ARE CALLED THE COORDINATES OF THE POINT.

TO GRAPH THE POINT P, REPRESENTED BY THE ORDERED PAIR (2,3), START AT THE ORIGIN AND MOVE 2 UNITS (SQUARES) TO THE RIGHT ALONG THE X-AXIS, THEN MOVE 3 UNITS (SQUARES) UPWARD IN A DIRECTION PARALLEL TO THE Y-AXIS.



TO GRAPH R, REPRESENTED BY THE ORDERED PAIR (-5,3), START AT THE ORIGIN AND MOVE 5 UNITS TO THE LEFT ALONG THE X-AXIS, THEN MOVE 3 UNITS UPWARD IN A DIRECTION PARALLEL TO THE Y-AXIS.

To graph point S, represented by the ordered pair (-8, -5), start at the origin and move 8 units to the left along the X-axis, then move 5 units downward in a direction parallel to the Y-axis.

To graph point T, represented by the ordered pair (6,-5), start at the origin and move 6 units to the right along the X-axis, then move 5 units downward in a direction parallel to the Y-axis.

To graph the exercises, plot the first point, plot the second point, and then connect the two as you would in a dot to dot drawing. In the exercise start at the top of the first column of the first problem and read down. Then go to the second column of the first problem and read do'n Continue in the same manner for all problems. Continue p'oting the points but be careful to connect the points as you go. If you wait to connect them until you finish graphing all of the points, you may connect the wrong points. If you plot the points and connect them correctly, each graph will form a picture.

GRAPHING

1) (3, 2)
(3, 4)
(0, 4)
(0, 5)
(-2, 5)
(-2, 9)
(-1, 8)
(-2, 7)
(-2, 2)
(7, 2)
(9, 0)
(13, 0)
(10, -3)
(-9, -3)
(-12, 0)
(-8, 0)
(-4, 2)
(3, 2)

2) (3, 9)
(5, 9)
(5, 11)
(3, 11)
(3, 14)
(0, 16)
(-3, 14)
(3, 14)
(3, 11)
(-3, 11)
(-3, 14)
(-3, 11)
(-5, 11)
(-5, 9)
(3, 9)
(7, -11)
(-2, -11)
(-2, -6)
(2, -6)
(2, -11)
(-7, -11)
(-3, 9)

START NEW LINE
(1, 1)
(-1, 1)
(-1, -2)
(1, -2)
(1, 1)

START NEW LINE
(1, 5)
(1, 8)
(-1, 8)
(-1, 5)
(1, 5)

3) (0, -9)
(2, -9)
(2, -13)
(3, -13)
(-2, -13)
(-1, -13)
(-1, -9)
(0, -9)
(0, 10)
(-1, 10)
(-2, 13)
(3, 13)
(-2, 13)
($\frac{1}{2}$, 15)
(3, 13)
(2, 10)
(0, 10)
(1, 10)
(1, -9)

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GRAPHING

4) (2, -1)
(2, 12)
(0, 15)
(-2, 12)
(-2, -1)
(-1, -1)
(-1, -2)
(1, -2)
(1, -1)
(3, -1)
(3, -2)
(5, -2)
(5, -1)
(7, -1)
(7, 3)
(8, 3)
(8, 2)
(9, 2)
(9, 10)
(10, 9)
(9, 8)
(9, 3)
(10, 3)
(10, 2)
(11, 2)
(11, 3)
(12, 3)
(12, -11)
(7, -11)

(7, -1)
(7, -11)
(3, -11)
(3, -6)
(-3, -6)
(-3, -11)
(3, -11)
(-12, -11)
(-12, 3)
(-11, 3)
(-11, 2)
(-10, 2)
(-10, 3)
(-9, 3)
(-9, 10)
(-8, 9)
(-9, 8)
(-9, 2)
(-8, 2)
(-8, 3)
(-7, 3)
(-7, -11)
(-7, -1)
(-5, -1)
(-5, -2)
(-3, -2)
(-3, -1)
(-2, -1)
START NEW LINE
(10, -9)
(10, -7)
(9, -7)

(9, -9)
(10, -9)
START NEW LINE
(10, -3)
(10, -1)
(9, -1)
(9, -3)
(10, -3)
START NEW LINE
(1, 4)
(1, 6)
(-1, 6)
(-1, 4)
(1, 4)
START NEW LINE
(1, 9)
(1, 11)
(-1, 11)
(-1, 9)
(1, 9)
START NEW LINE
(-9, -1)
(-10, -1)
(-10, -3)
(-9, -3)
(-9, -1)
START NEW LINE
(-9, -7)
(-10, -7)
(-10, -9)
(-9, -9)
(-9, -7)

GRAPHING

5) (-2, 15)
(-4, 14)
(-6, 12)
(-7, 10)
(-8, 10)
(-8, -2)
(-11, -2)
(-11, -12)
(-8, -12)
(-8, -8)
(-6, -8)
(-6, -12)
(-8, -12)
(12, -12)
(12, 0)
(7, -2)
(7, 0)
(2, -2)
(2, 0)
(-3, -2)
(-3, -12)
(-3, -9)
(12, -9)
(12, -2)
(7, -2)
(7, -12)
(7, -2)
(2, -2)
(2, -12)
(2, -2)
(-3, -2)
(-8, -2)
(-6, -2)
(-6, 10)
(-7, 10)

6) (6, 2)
(7, 7)
(2, 6)
(6, 2)
(-5, -5)
(-3, -2)
(-6, -5)
(-7, -4)
(-3, 0)
(0, -3)
(-4, -7)
(-5, -6)
(-2, -3)
(-3, -2)
(2, 6)

7) (6, 14)
(4, 13)
(2, 9)
(1½, 10)
(1, 9)
(2, 9)
(2, 8)
(10, 14)
(8, 2)
(2, 5)
(2, 8)
(2, 4)
(8, 1)
(9, -5)
(2, 2)
(2, 4)
(2, -1)
(1, -1)
(1, 2)
(-6, -5)
(-5, 1)
(1, 4)
(1, 2)
(1, 5)
(-5, 2)
(-7, 14)
(1, 8)
(1, 5)
(1, 9)
(-1, 13)
(-3, 14)

GEOMETRIC SHAPES

DEFINITIONS

<u>CUBE</u> IS A SOLID BOUNDED BY SIX PLANES, WITH ITS TWELVE EDGES ALL EQUAL AND ITS FACE ANGLES ALL RIGHT ANGLES (DICE).

<u>PARALLELOGRAM</u> IS A QUADRILATERAL WITH ITS OPPOSITE SIDES PARALLEL.

<u>POLYGON</u> IS A SIMPLE CLOSED CURVE CONSISTING OF THREE OR MORE SEGMENTS, EACH PAIR OF ADJACENT SEGMENTS HAVING A COMMON END POINT.

<u>PRISM</u> IS A SOLID WITH TWO CONGRUENT, PARALLEL FACES WHICH ARE POLYGONS AND WITH THE REMAINING FACES PARALLELOGRAMS.

<u>PRISM WITH TRAPEZOIDAL BASES</u> IS A PRISM WHOSE TWO CONGRUENT, PARALLEL FACES ARE TRAPEZOIDS.

<u>PRISM WITH TRIANGULAR BASES</u> IS A PRISM WHOSE TWO CONGRUENT, PARALLEL FACES ARE TRIANGLES.

<u>QUADRILATERAL</u> IS A POLYGON WITH FOUR SIDES.

<u>RECTANGLE</u> IS A QUADRILATERAL WHOSE ANGLES ARE ALL RIGHT ANGLES. ALL SQUARES ARE RECTANGLES. ALL RECTANGLES ARE NOT SQUARES.

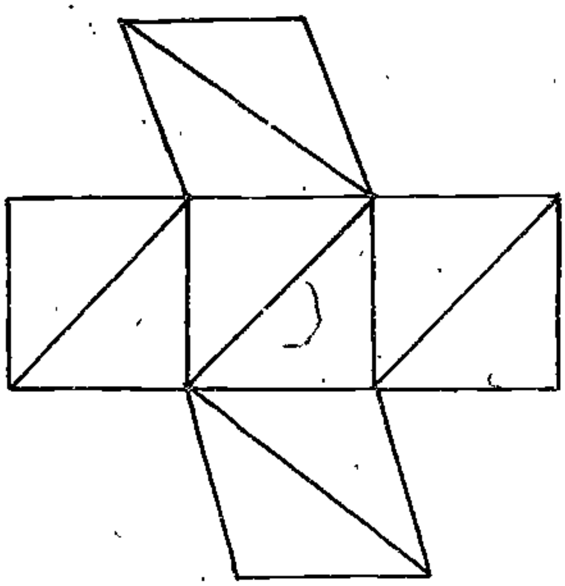
<u>RHOMBUS</u> IS A QUADRILATERAL WITH FOUR EQUAL SIDES.

<u>SQUARE</u> IS A QUADRILATERAL WITH EQUAL SIDES AND EQUAL ANGLES.

<u>TRAPEZOID</u> IS A QUADRILATERAL WITH EXACTLY TWO PARALLEL SIDES.

<u>TRIANGLE</u> IS A POLYGON WITH THREE SIDES.

GEOMETRIC SHAPES



____________ HOW MANY SQUARES?

____________ HOW MANY RECTANGLES?

____________ HOW MANY PARALLELOGRAMS?

____________ HOW MANY QUADRILATERALS?

____________ HOW MANY RHOMBUSES?

____________ HOW MANY TRIANGLES?

____________ HOW MANY TRAPEZOIDS?

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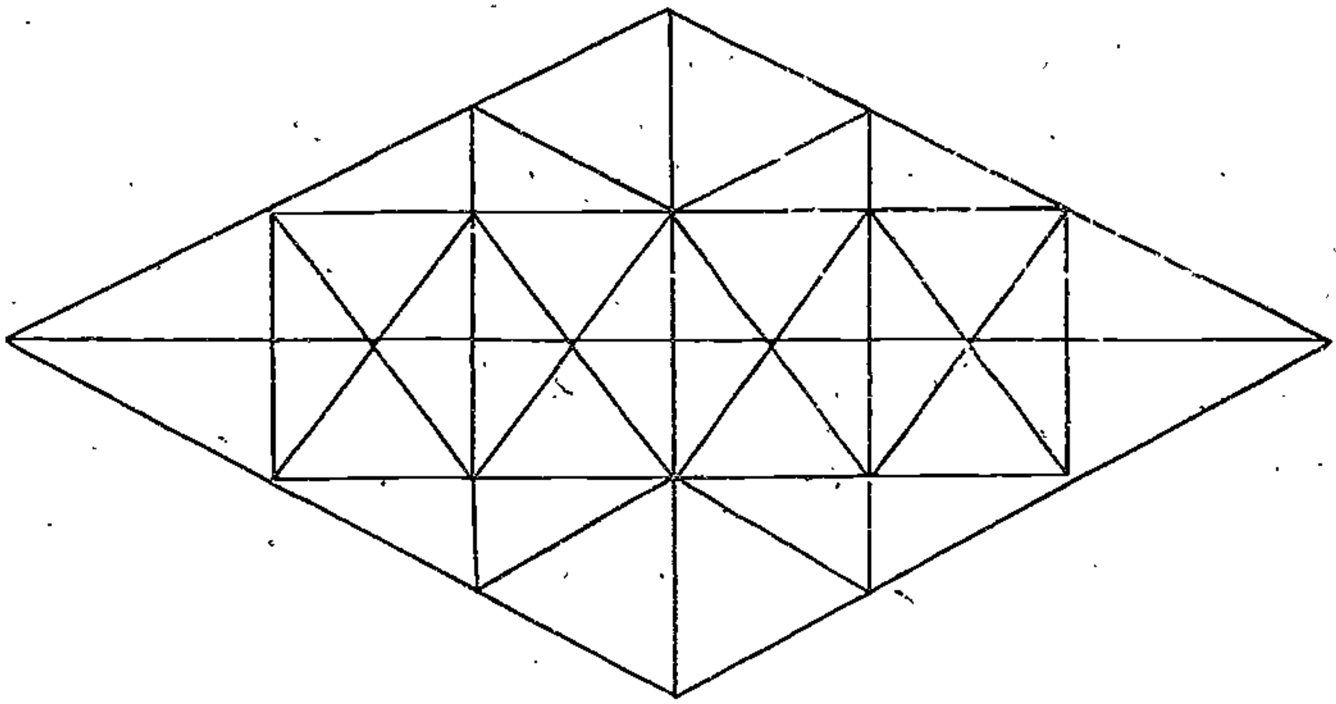
GEOMETRIC SHAPES



_______________ How many rectangles?

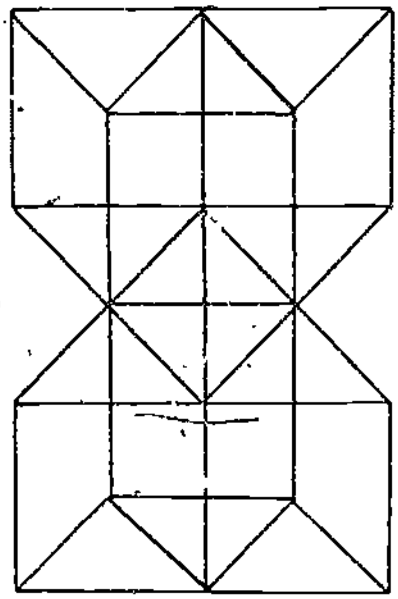
_______________ How many triangles?

_______________ How many parallelograms?

_______________ How many trapezoids?

_______________ How many quadrilaterals?

GEOMETRIC SHAPES



______ HOW MANY SQUARES?

______ HOW MANY RECTANGLES?

______ HOW MANY PARALLELOGRAMS?

______ HOW MANY TRIANGLES?

______ HOW MANY CUBES?

______ HOW MANY PRISMS WITH TRAPEZOIDAL BASES?

______ HOW MANY PRISMS?

______ HOW MANY TRAPEZOIDS?

______ HOW MANY TRIANGULAR PRISMS?

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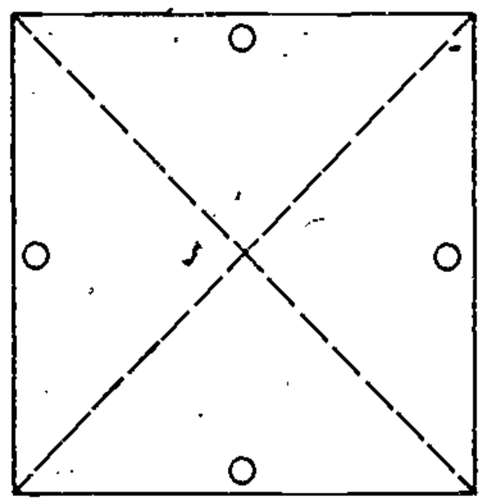
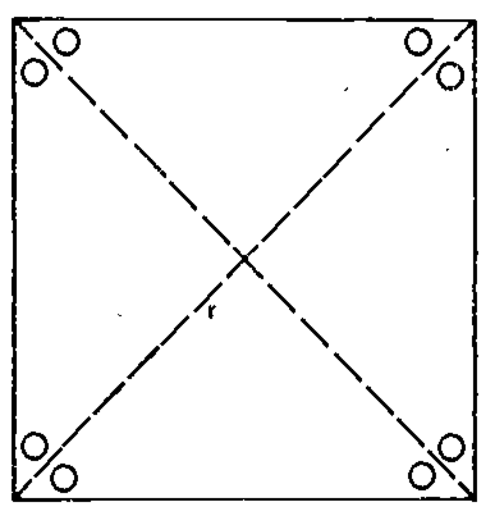
SPATIAL PERCEPTION

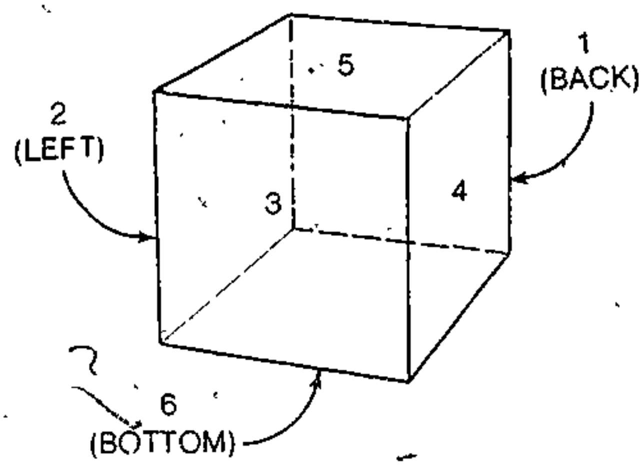
THESE ARE SQUARE SHEETS OF PAPER. THEY ARE TO BE FOLDED FLAT ALONG THE DOTTED LINES. HOW WILL EACH SHEET LOOK AFTER IT IS FOLDED?



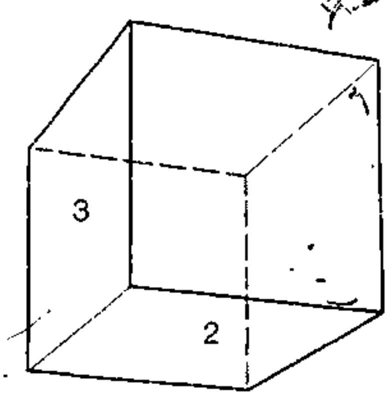


SPATIAL PERCEPTION

HERE IS A BLOCK ON WHICH SOMEONE HAS PAINTED NUMBERS. 1 IS ON THE BACK FACE. 2 IS ON THE LEFT FACE. 3 IS ON THE FRONT FACE. 4 IS ON THE RIGHT FACE. 5 IS ON THE TOP FACE. 6 IS ON THE BOTTOM FACE.



SOMEONE HAS MOVED THE BLOCK. 2 IS ON THE BOTTOM FACE. 3 IS ON THE LEFT FACE. THE FRONT FACE IS COVERED. WHAT NUMBER IS ON



A. THE TOP FACE?

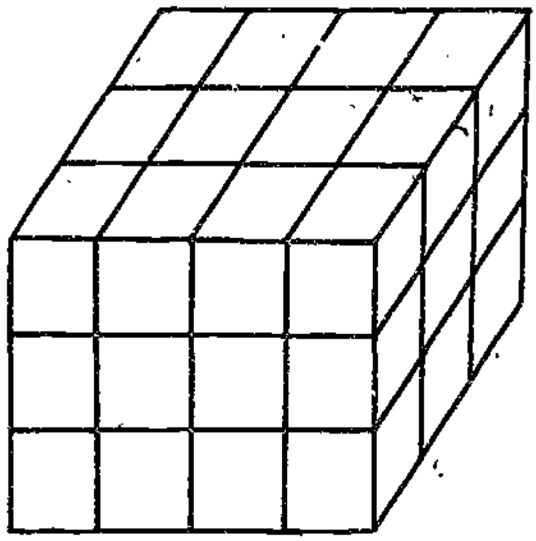
B. THE RIGHT FACE?

C. THE FRONT FACE?

D. THE BACK FACE?

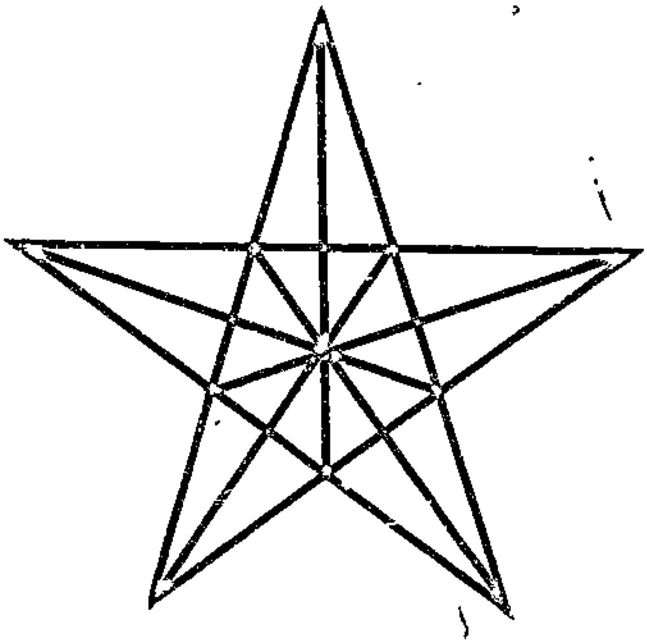
SPATIAL PERCEPTION

SOMEONE STACKED SUGAR CUBES TO MAKE THIS LARGE CUBE. THEN THIS PERSON PAINTED TOP AND BOTTOM OF THE LARGE CUBE RED AND ALL OF THE SIDES OF THE LARGE CUBE BLUE.

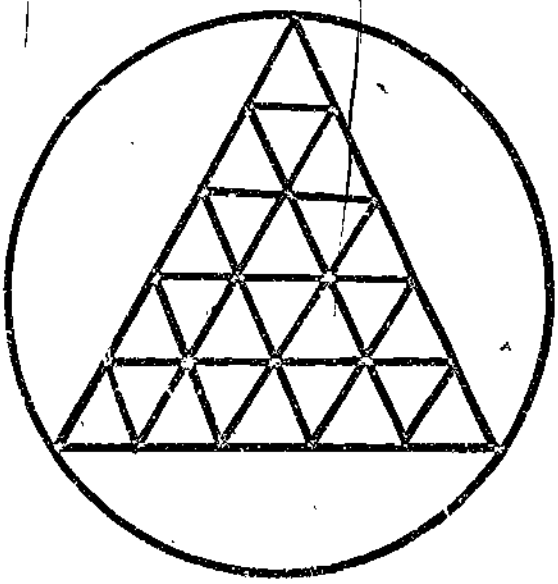


1. HOW MANY SUGAR CUBES ARE THERE?
2. HOW MANY HAVE A RED SIDE?
3. HOW MANY HAVE ONE BLUE SIDE?
4. HOW MANY HAVE TWO BLUE SIDES?
5. HOW MANY ARE ALL WHITE?
6. HOW MANY ARE RED, WHITE, AND BLUE?

SPATIAL PERCEPTION



HOW MANY TRIANGLES CAN YOU FIND IN THE DIAGRAM?



DRAW THIS FIGURE WITH ONE SINGLE STROKE OF THE PENCIL. YOU MAY NOT RETRACE LINES PREVIOUSLY DRAWN.

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WORD PROBLEMS

PROBLEM SOLVING

REMEMBER! TAKE TIME TO READ THE PROBLEM WITH CARE TO DECIDE:

1. WHAT IS GIVEN.
2. WHAT IS TO BE FOUND.
3. WHAT IS NEEDED TO SOLVE THE PROBLEM.
4. HOW YOU PLAN TO SOLVE THE PROBLEM.

EXAMPLES:

1. EXACT INFORMATION (JUST RIGHT AMOUNT)

COLOR PRINT & DEVEL. 12 Exposure ROLL	$1.79
COLOR PRINT & DEVEL. 20 Exposure ROLL	$2.79
COLOR PRINT & DEVEL. 24 Exposure ROLL	$3.29

IF DAVID NEEDS ONE 12 EXPOSURE ROLL, TWO 24 EXPOSURE ROLLS, AND THREE 20 EXPOSURE ROLLS, HOW MUCH WILL IT COST HIM? HOW MANY PICTURES CAN HE TAKE FROM THE ROLLS THAT HE BUYS?

ONE ROLL OF 12 EXPOSURE $1.79

TWO ROLLS OF 24 EXPOSURES

2 x $3.29 =6.58

THREE ROLLS OF 20 EXPOSURES

3 x $2.79 =8.37

$16.74

IT WILL COST DAVID $16.74. HE WILL BE ABLE TO TAKE 120 PICTURES FROM THE SIX ROLLS OF FILM.

ONE ROLL OF 12 EXPOSURE 12

TWO ROLLS OF 24 EXPOSURE 2 x 24 48

THREE ROLLS OF 20 EXPOSURE 3 x 20 60

120

2. <u>Extraneous Information</u> (too much)

Sue has \$2.25. She found \$5.00 on the way to the store. She spent \$1.50 for a notebook, 25¢ for an eraser, 50¢ for a pen. How much money did she spend?

She spent:

NOTEBOOOK	\$ 1.50
ERASER	.25
PEN	.50
	\$ 2.25

Knowing that Sue had \$2.25 and that she found \$5.00 is unneeded information for solving the problem. Be sure to shift all the known facts to find only what is needed.

3. <u>Insufficient Information</u> (not enough)

A. Bill has 17 marbles in 3 bags. He found 2 more bags of marbles. How many marbles does he have now?

This problem cannot be solved until it is determined how many marbles are in each of the 2 bags that Bill found. If you find a problem with insufficient information, state what piece of information is needed to solve the problem.

B. Mary is 5 years older than her brother. How old is Mary?

It is necessary to know how old Mary's brother is before you can determine Mary's age.

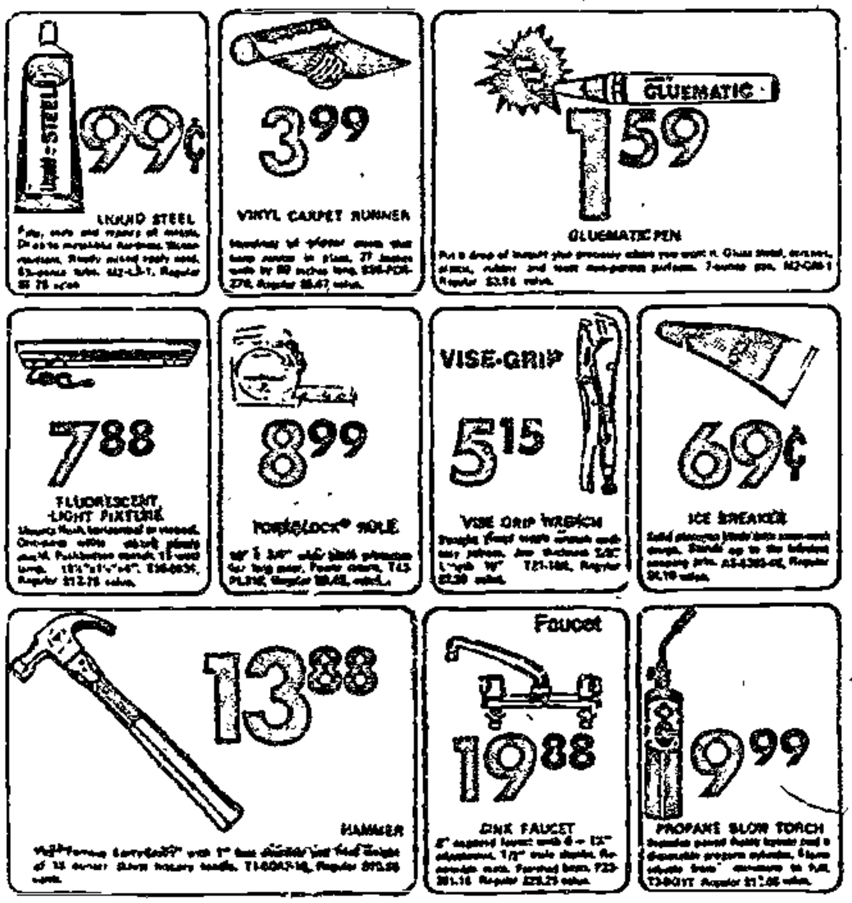
4. <u>No Numerical Solution</u>

William is going to buy a new car. William's father said that he must know all the expenses connected with owning a car before William has permission to buy the car. Besides the car payment, list some additional expenses for owning a car.

This type of problem does not have a numerical solution. However, it is a practical problem which needs solving.

ADVERTISING PUZZLES

(DISREGARD SALES TAX)



1. DAVID WENT TO THE STORE FOR HIS DAD. HE BOUGHT A SINK FAUCET, 3 ICE BREAKERS, AND A HAMMER. HOW MUCH DID DAVID SPEND?

2. MR. WOODS HAD PURCHASED 2 VISE-GRIPS, 4 GLUEMATIC PENS, AND A PROPANE BLOW TORCH. HOW MUCH MORE THAN $10.00 DOES HE NEED?

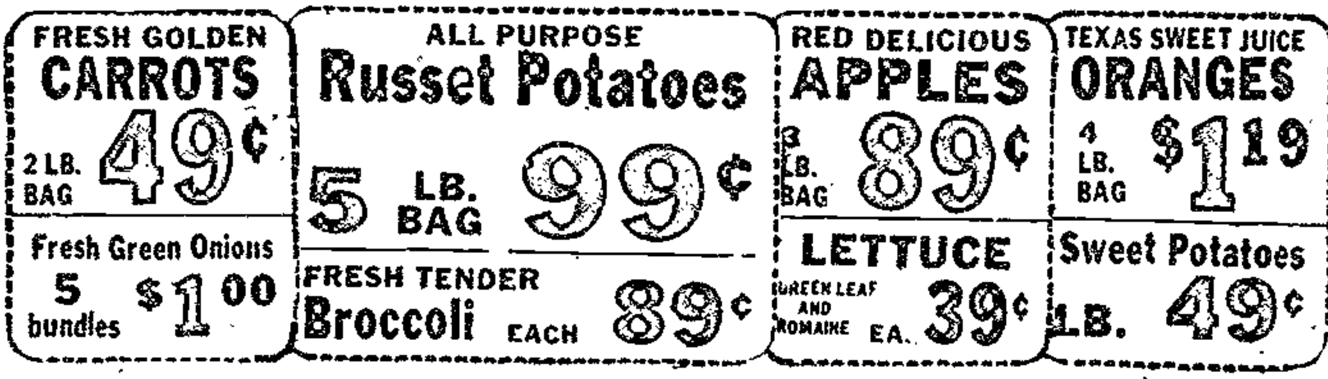
3. ANN BOUGHT 4 TUBES OF LIQUID STEEL, A VINYL CARPET RUNNER, AND A FLORESENT LIGHT FIXTURE. SHE RETURNED 2 TUBES OF LIQUID STEEL BECAUSE THEY HAD HOLES IN THEM. IF THE STORE REFUNDED THE FULL COST, HOW MUCH MONEY DID SHE RECEIVE?

4. IF MRS. MAYS BOUGHT A POWERLOCK RULE AND A VISE-GRIP WRENCH FOR HER HUSBAND AND PAID THE CLERK WITH A TWENTY DOLLAR BILL, HOW MUCH CHANGE SHOULD SHE RECEIVE?

5. SARAH WANTS TO BUY SOME ICE BREAKERS. HOW MUCH WILL THEY COST?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)



1. MARY NEEDED FIVE POUNDS OF CARROTS, 12 POUNDS OF RUSSET POTATOES, AND 2 BUNDLES OF ONIONS TO MAKE POTATOE SALAD FOR THE SCHOOL PICNIC. THE BAGS OF GROCERIES CANNOT BE SPLIT BUT THE BUNDLES OF ONIONS ARE SEPARATE. WHAT WILL THE COST OF THESE ITEMS BE?

2. JANE AND JOE ARE PLANNING A HALLOWEEN PARTY. THEY WANT 7 POUNDS OF APPLES AND 7 POUNDS OF ORANGES. REMEMBER THAT BAGS OF GROCERIES CANNOT BE SPLIT. WHAT WILL IT COST TO GET THE FRUIT THAT THEY WANT?

3. MRS. WATSON IS SERVING SNACKS FOR HER WEIGHT-WATCHERS CLUB MEETING. SHE WANTS ONE POUND OF CARROTS, 2 BROCCOLI, 5 POUNDS OF ORANGES, 2 POUNDS OF APPLES, AND 2 HEADS OF LETTUCE. HOW MUCH CHANGE WILL SHE GET FROM A TEN DOLLAR BILL?

4. MRS. MARTIN HAS 25 CHILDREN IN HER KINDERGARTEN CLASS. SHE IS GOING TO BUY ONE BAG OF RUSSET POTATOES FOR THEM TO GROW POTATO PLANTS IN CLASS. HOW MANY POUNDS ARE THERE IN A BAG OF POTATOES AT THIS STORE?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) BILL AND SUE ARE PLANNING TO GET MARRIED AND HAVE $500 TO SPEND ON A WASHER, DRYER, AND REFRIGERATOR. SINCE THEY DO NOT NEED THEM NOW, WHICH SHOULD THEY BUY TO SAVE THE MOST MONEY?

2) SUE'S MOTHER DECIDES THAT SHE WILL GIVE SUE $275 TO SPEND NOW. BILL'S FATHER SAYS THAT HE WILL GIVE THEM THE REST OF THE MONEY THAT THEY NEED TO BUY ALL THREE NOW. HOW MUCH WILL HE GIVE THEM?

3) WHAT IS THE TOTAL COST OF ALL THREE?

4) WHEN THEY GO TO BUY THEM, THE SALESMAN SAYS THAT THERE WILL BE A $35 CHARGE FOR STORAGE UNTIL THEY ARE MARRIED AND NEED THE APPLIANCES. SUE AND BILL DISCUSS WHETHER THEY SHOULD BUY THE APPLIANCES NOW, OR WAIT. THE SALE WILL BE OVER BY THE TIME THAT THEY ARE MARRIED. HOW MUCH WILL IT COST TO BUY NOW AND STORE UNTIL NEEDED?

5) WHAT WILL IT COST THEM TO WAIT AND BUY AFTER THE SALE IS OFF?

6) WHAT SHOULD THEY DO?

ADVERTISING PUZZLES

(DISREGARD SALES TAX)

1) JACK AND JILL ARE ON THE TRACK TEAM IN HIGH SCHOOL. THE COACH TELLS THEM THAT THEY SHOULD PRACTICE AS MUCH AS POSSIBLE. THEREFORE, THEY DECIDE TO PURCHASE THEIR OWN JOGGING SUITS TO BE USED ON WEEKENDS. JILL HAS $29 AND WANTS TO BUY THE MOST EXPENSIVE SUIT FOR THE LEAST AMOUNT OF MONEY. HOW MUCH WILL HER SUIT COST?

2) JACK WANTS AN ACRYLIC SUIT AND A PAIR OF SKI GLOVES FOR THE CHEAPEST PRICE. WHAT WILL IT COST HIM?

3) LOUISE IS GOING SKIING AND NEEDS GOGGLES, A JACKET, AND A PAIR OF GLOVES. SHE WANTS THE BEST APPAREL. SINCE SHE DOES NOT KNOW ABOUT SKI CLOTHES, SHE IS GOING TO ASSUME THAT THE ONES WHICH COST THE MOST ARE THE BEST. HOW MUCH WILL THEY COST?

4) BOB DECIDES TO GO WITH LOUISE AND ALSO NEEDS GOGGLES, A JACKET, AND A PAIR OF GLOVES. HE WANTS THE CHEAPEST APPAREL THAT HE CAN BUY. HOW MUCH MONEY WILL HE NEED?

FINDING A SQUARE ROOT OF A NUMBER

To square a number is to use it as a factor twice. For example, the square of $3 \times 3 = 9$.

Finding a square root of a number is to find one of its two equal factors. For example, a square root of 9, written $\sqrt{9}$, is 3 because $3 \times 3 = 9$. Finding a square root of a number is the inverse (opposite) operation of squaring.

To indicate a square root of a number, a radical sign, $\sqrt{}$, is used. The symbol $\sqrt{9}$ is called a radical; 9, the number under the radical sign, is called the radicand.

Numbers	Perfect Squares	Square Roots
1	1	1
2	4	2
3	9	3
4	16	4
5	25	5
6	36	6
7	49	7
8	64	8
9	81	9

COMPUTING THE SQUARE ROOT OF A PERFECT SQUARE

MODEL PROBLEMS

A. COMPUTE THE POSITIVE SQUARE ROOT OF 3136.

HOW TO PROCEED

SOLUTION

1. STARTING AT THE DECIMAL POINT AND MOVING TO THE LEFT, GROUP THE DIGITS OF THE NUMBER IN PAIRS OF TWO DIGITS. PLACE A DECIMAL POINT DIRECTLY ABOVE THE DECIMAL POINT IN THE NUMBER.

   $\sqrt{\overset{\frown}{31}\,\overset{\frown}{36}.}$

2. BELOW THE FIRST GROUP AT THE LEFT, WRITE THE LARGEST PERFECT SQUARE WHICH IS NOT MORE THAN THAT GROUP. WRITE THE SQUARE ROOT OF THE PERFECT SQUARE ABOVE THE FIRST GROUP.

   5 .

   $\sqrt{\overset{\frown}{31}\,\overset{\frown}{36}.}$

   25

3. SUBTRACT THE PERFECT SQUARE FROM THE FIRST GROUP AND BRING DOWN AND ANNEX THE NEXT GROUP TO THE REMAINDER.

   5

   $\sqrt{\overset{\frown}{31}\,\overset{\frown}{36}.}$

   25

   6 36

4. FORM A TRIAL DIVISOR BY DOUBLING (MULTIPLYING BY 2) THE PART OF THE ROOT ALREADY FOUND IN STEP 3 AND ANNEXING A 0.

   5 x 2 = 10

   TRIAL DIVSOR IS 100

   5

   $\sqrt{\overset{\frown}{31}\,\overset{\frown}{36}}$

   25

   100) 6 36

5. DIVIDE THE REMAINDER FOUND IN STEP 3 BY THE TRIAL DIVISOR FOUND IN STEP 4. ANNEX THE QUOTIENT TO THE PART OF THE ROOT ALREADY FOUND; ALSO, ADD IT TO THE TRIAL DIVISOR TO FORM THE COMPLETE DIVISOR.

$636 \div 100 = 6 +$

COMPLETE DIVISOR IS

$100 + 6 = 106$

```
       5   6
    √ 31  36.
      25
106|   6  36
```

6. MULTIPLY THE COMPLETE DIVISOR BY THE LAST DIGIT WHICH WAS PLACED IN THE ROOT, AND SUBTRACT THE PRODUCT FROM THE REMAINDER FOUND IN STEP 3. THE REMAINDER IS 0. THE REQUIRED ROOT IS 56.

```
       5   6
    √ 31  36.
      25
106|  636
      636
```

NOTE: WHEN NECESSARY, THE PROCEDURE GIVEN IN STEPS 4, 5, 6 IS REPEATED UNTIL THE REMAINDER IS ZERO.

B. COMPUTE: $\sqrt{817.96}$

1. STARTING AT THE DECIMAL POINT, MOVING FIRST TO THE LEFT AND THEN TO THE RIGHT, GROUP THE DIGITS IN PAIRS. THE FIRST GROUP ON THE LEFT MAY HAVE ONE DIGIT. IF THE LAST GROUP ON THE RIGHT HAS ONE DIGIT, ANNEX A 0 TO FORM A TWO-DIGIT GROUP. PLACE A DECIMAL POINT DIRECTLY ABOVE THE DECIMAL POINT IN THE NUMBER

```
          .
√ 8  17 .96
```

2. THE LARGEST PERFECT SQUARE NOT MORE THAN 8 IS 4. WRITE 4 BELOW 8. WRITE 2 ABOVE 8.

```
  2       .
√ 8  17 .96
  4
```

3. Subtract 4 from 8, obtaining 4. Bring down the next group, 17, and annex it to 4, forming 417.

```
  2
√8 17 . 96
 4
 417
```

4. Find the first divisor by doubling 2 and annexing a 0: $2 \times 2 = 4$; the trial divisor is 40.

5. Divide the remainder, 417, by the trial divisor, 40. The quotient is 8. Therefore, the complete first divisor is $40 + 8$, or 48.

```
      2  8 .
     √8 17 .96
      4
48|   4 17
      3 84
        33
```

6. $8 \times 48 = 384$. Subtract: $417 - 384 = 33$. Bring down the next group, 96, and annex it to 33, forming 3396.

7. Find the second trial divisor by doubling 28 and annexing a 0; $2 \times 28 = 56$; the trial divisor is 560.

```
        2  8.
       √8 17 . 96
        4
 48|    4 17
        3 84
560|      33  96
```

8. Divide the remainder, 3396, by the trial divisor, 560. The quotient is 6. Therefore, the complete divisor is $560 + 6 = 566$.

```
        2  8 .6
       √8 17 .96
        4
 48|    4 17
        3 84
566|      33  96
          33  96
```

9. $6 \times 566 = 3396$. Subtract: $3396 - 3396 = 0$. The required root is 28.6

COMPUTING THE APPROXIMATE SQUARE ROOT OF A NUMBER

FIND $\sqrt{48}$ CORRECT TO THE NEAREST TENTH.

1. IN ORDER TO APPROXIMATE $\sqrt{48}$ CORRECT TO THE NEAREST TENTH, CARRY THE WORK TO TWO DECIMAL PLACES AND THEN ROUND OFF THE RESULT TO THE NEAREST TENTH.

2. IN ORDER TO CARRY THE RESULT TO TWO DECIMAL PLACES, ANNEX TO 48 (AT THE RIGHT OF THE DECIMAL POINT) TWO GROUPS, EACH CONTAINING TWO ZEROES.

$\sqrt{48.\,00\,00}$

3. PERFORM THE COMPUTATION AND ROUND OFF THE ANSWER TO THE NEAREST TENTH. SINCE $6.92 \approx 6.9$, THE REQUIRED ROOT IS 6.9.

```
          6.  9  2
        √48. 00 00
         36
   129 | 12  00
         11  61
             39 00
  1382 |     27 64
             11 36
```

FIND $\sqrt{62}$ TO THE NEAREST HUNDREDTH.

1. IN ORDER TO CARRY THE WORK IN THE RESULT TO THREE DECIMAL PLACES, ANNEX THREE GROUPS, EACH CONTAINING TWO ZEROES, AT THE RIGHT OF 62.

2. THE FIRST TRIAL DIVISOR IS 140. SINCE 1300 ÷ 140 IS 8, THE COMPLETE DIVISOR IS 140 8, OR 148.

3. COMPLETE THE COMPUTATION AND ROUND OFF THE ANSWER TO THE NEAREST HUNDREDTH. SINCE $7.874 \approx 7.87$, THE REQUIRED ROOT IS 7.87.

```
           7.  8  7  4
         √62. 00 00 00
          14
   148 |  13  00
          11  84
           1  16 00
           1  09 69
               6 31 00
 15744 |       6 29 76
                  1 24
```

SQUARE ROOTS

PERFECT SQUARES:

1. $\sqrt{289}$ 2. $\sqrt{324}$ 3. $\sqrt{784}$ 4. $\sqrt{1296}$

5. $\sqrt{1.8496}$ 6. $\sqrt{11025}$ 7. $\sqrt{484}$ 8. $\sqrt{9.61}$

9. $\sqrt{56.25}$ 10. $\sqrt{129.96}$ 11. $\sqrt{90.25}$ 12. $\sqrt{57.76}$

13. $\sqrt{190.44}$ 14. $\sqrt{.9409}$ 15. $\sqrt{46.24}$ 16. $\sqrt{1.4884}$

TO THE NEAREST TENTH:

17. $\sqrt{29}$ 18. $\sqrt{.26}$ 19. $\sqrt{1.19}$ 20. $\sqrt{138}$

21. $\sqrt{147}$ 22. $\sqrt{5.2634}$ 23. $\sqrt{1.07}$ 24. $\sqrt{1.39}$

TO THE NEAREST HUNDRETH:

25. $\sqrt{59}$ 26. $\sqrt{129}$ 27. $\sqrt{43}$ 28. $\sqrt{149}$

29. $\sqrt{23.16}$ 30. $\sqrt{131}$ 31. $\sqrt{.83}$ 32. $\sqrt{1.08}$

PATTERNS

IN EACH CASE HOW ARE THE SUCCESSIVE NUMBERS BEING FORMED? WRITE THE NEXT 4 IN EACH SEQUENCE.

1) 2, 4, 6, 8, 10, ____, ____, ____, ____.

2) 1, 2, 4, 8, 16, ____, ____, ____, ____.

3) 1, 4, 9, 16, 25, ____, ____, ____, ____.

4) 1, 8, 27, 64, 125, ____, ____, ____, ____.

5) 2, 5, 8, 11, 14, ____, ____, ____, ____.

6) 2, 3, 5, 7, 11, 13, ____, ____, ____, ____.

7) 11, 121, 1331, 14641, ____, ____, ____, ____.

8) 1, 3, 6, 10, 15, ____, ____, ____, ____.

9) 1, 1, 2, 3, 5, 8, 13, 21, ____, ____, ____, ____.

10) 1, 3, 5, 7, ____, ____, ____, ____.

11) 1, 3, 9, 27, ____, ____, ____, ____.

12) 1, 4, 7, 10, ____, ____, ____, ____.

13) 9, 18, 36, 72, ____, ____, ____, ____.

14) 3, 9, 27, 81, ____, ____, ____, ____.

15) 2, 6, 18, 54, ____, ____, ____, ____.

PATTERNS

STUDY THESE SUMS OF CONSECUTIVE ODD NUMBERS:

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11 = 36 = 6^2$

1) WHAT DID YOU DISCOVER? ______________________________

____

2) WHAT IS THE SUM OF THE FIRST 7 ODD NUMBERS? (YOU SHOULD BE ABLE TO DO THIS IN YOUR HEAD IF YOU HAVE DISCOVERED THE PATTERN.) ______________________________

3) WHAT IS THE SUM OF THE FIRST 12 ODD NUMBERS? ______________

4) WHAT IS THE SUM OF THE FIRST 20 ODD NUMBERS? ______________

FIND SIX WAYS TO NAME 100 USING ALL THE DIGITS 1 THROUGH 9 INCLUSIVE.

EXAMPLE: $123 - 45 - 67 + 89 = 100$

$(12 \div 6) + 58 + 3 + (4 \times 7) + 9 = 100$

5) ______________________________

6) ______________________________

7) ______________________________

8) ______________________________

9) ______________________________

10) ______________________________

PATTERNS

STUDY THESE SUMS OF CONSECUTIVE NATURAL NUMBERS.

$1 = 1$

$1 + 2 = 3$

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 = 10$

$1 + 2 + 3 + 4 + 5 = 15$

$1 + 2 + 3 + 4 + 5 + 6 = 21$

COMPLETE THE FOLLOWING:

1) $1 + 2 + 3 + 4 + 5 + 6 + 7 =$ __________.

2) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 =$ __________.

3) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 =$ __________.

4) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 =$ __________.

5) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 =$ __________.

6) CAN YOU DISCOVER A PATTERN? ______________

7) CAN YOU WRITE A RULE FOR THE SUM OF THE FIRST N CONSECUTIVE NATURAL NUMBERS?

____

____

FIND THREE WAYS TO NAME 100 USING ALL THE DIGITS 1 THROUGH 9 INCLUSIVE.

EXAMPLES: $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$

$(12 \div 6) + 58 + 3 + (4 \times 7) + 9 = 100$

8) ____

9) ____

10) ____

BASE TEN

THE BASE TEN (DECIMAL) SYSTEM OF NUMERATION IS BASED UPON GROUPING BY TENS. IF MAN HAD ONLY FOUR FINGERS ON EACH HAND INSTEAD OF FIVE, PEOPLE MIGHT HAVE LEARNED TO GROUP BY EIGHTS INSTEAD OF TENS.

IT IS INTERESTING AND FUN TO REPRESENT NUMBERS USING BASES OTHER THAN TEN; BUT REMEMBER THAT REGARDLESS OF THE SYMBOLS USED OR THE WAY OBJECTS ARE GROUPED, THE NUMBERS ARE THE SAME. ONLY THE REPRESENTATIONS OF THE NUMBERS ARE DIFFERENT.

STUDYING OTHER BASES (NUMERATION SYSTEMS) WILL HELP ONE BECOME MORE FAMILIAR WITH EXPONENTS AND WILL GIVE A BETTER UNDERSTANDING OF PLACE VALUE IN BASE TEN.

A REVIEW OF BASE TEN WILL BE HELPFUL IN WORKING WITH OTHER BASES.

BASE TEN HAS TEN SYMBOLS: (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

THE PLACE VALUE IN BASE TEN IS BASED ON TENS AND POWERS OF TEN.

ONES PLACE	10^0 OR 1
TENS PLACE	10^1 OR 10
HUNDREDS PLACE	10^2 OR 100 (10 x 10)
THOUSANDS PLACE	10^3 OR 1000 (10 x 10 x 10)
TEN-THOUSANDS PLACE	10^4 OR 10,000 (10 x 10 x 10 x 10)
HUNDRED-THOUSANDS PLACE	10^5 OR 100,000 (10 x 10 x 10 x 10 x 10)
MILLIONS PLACE	10^6 OR 1,000,000 (10 x 10 x 10 x 10 x 10 x 10)

REMEMBER: IN THE POWER OF 10, 10 IS THE BASE AND THE RAISED NUMERAL IS THE EXPONENT.

$$\text{BASE} \longrightarrow 10^{4} \longleftarrow \text{EXPONENT}$$

IN THE NUMERAL 2,563,407 THE 7 IS IN THE ONES OR 10^0 PLACE,
THE 0 IS IN THE TENS OR 10^1 PLACE,
THE 4 IS IN THE HUNDREDS OR 10^2 PLACE,
THE 3 IS IN THE THOUSANDS OR 10^3 PLACE,
THE 6 IS IN THE TEN-THOUSANDS OR 10^4 PLACE,
THE 5 IS IN THE HUNDRED-THOUSANDS OR 10^5 PLACE,
THE 2 IS IN THE MILLIONS OR 10^6 PLACE.

THE PLACE VALUE OF EACH DIGIT IN THE NUMERAL 2,563,407 IS SHOWN BY THE CHART BELOW:

Millions	Hundred-Thousands	Ten-Thousands	Thousands	Hundreds	Tens	Ones
10^6	10^5	10^4	10^3	10^2	10^1	10^0
2,	5	6	3,	4	0	7

THE PLACE VALUE OF EACH DIGIT IN A BASE TEN NUMERAL CAN ALSO BE SHOWN BY USING EXPANDED NOTATION.

> EXPANDED NOTATION IS EXPRESSING A PLACE VALUE NUMERAL AS A SUM OF THE PRODUCTS FORMED WHEN EACH DIGIT IS MULTIPLIED BY ITS PLACE VALUE.

EXAMPLES OF EXPANDED NOTATION:

$$653 = (6 \times 10^2) + (5 \times 10^1) + (3 \times 10^0)$$

$$4702 = (4 \times 10^3) + (7 \times 10^2) + (0 \times 10^1) + (2 \times 10^0)$$

A DECIMAL NUMERAL WRITTEN IN EXPANDED NOTATION CAN EASILY BE CHANGED BACK TO THE NUMERAL IT REPRESENTS.

EXAMPLE:

$$(5 \times 10^2) + (3 \times 10^1) + (7 \times 10^0) =$$

$$(5 \times 100) + (3 \times 10) + (7 \times 1) =$$

$$500 + 30 + 7 =$$

$$537$$

BASE TEN

1. $5960 = (5 \times \_\_\_) + (\_\_\_ \times 10^2) + (\_\_\_ \times \_\_\_) + (0 \times 10^0)$

$= (5 \times 1000) + (\_\_\_ \times 100) + (\_\_\_ \times 10) + (0 \times 1)$

$= 5000 + \_\_\_ + \_\_\_ + \_\_\_$

$= 5960$

2. $23{,}579 = (2 \times 10^4) + (3 \times 10^3) + (5 \times \_\_\_) + (\_\_\_ \times 10^1) + (\_\_\_ \times 10^0)$

$= (2 \times \_\_\_) + (3 \times 1000) + (5 \times \_\_\_) + (\_\_\_ \times 10) + (\_\_\_ \times 1)$

$= \_\_\_ + 3000 + \_\_\_ + \_\_\_ + \_\_\_$

$= 23{,}579$

WRITE EACH BASE TEN NUMERAL IN EXPANDED NOTATION. SHOW ALL STEPS AS IN PROBLEM #1 ABOVE.

3. 5294 =

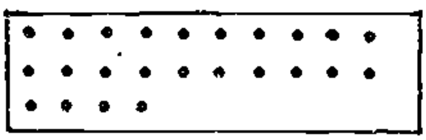
4. 36,058 =

CHANGING FROM BASE FIVE TO BASE TEN

PERHAPS IF MAN HAD ONLY ONE HAND, PEOPLE MIGHT BE USING THE BASE FIVE NUMERATION SYSTEM INSTEAD OF THE BASE TEN (DECIMAL) NUMERATION.

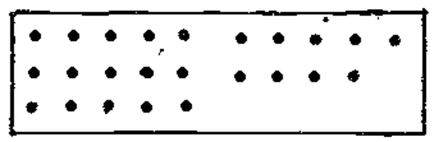
JUST AS BASE TEN USES TEN BASIC SYMBOLS (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) AND ITS PLACE VALUES ARE POWERS OF TEN, BASE FIVE USES FIVE SYMBOLS (0, 1, 2, 3, 4) AND ITS PLACE VALUES ARE POWERS OF FIVE.

GROUPING BY TENS



2 TENS + 4 ONES = 24_{TEN}

GROUPING BY FIVES



4 FIVES + 4 ONES = 44_{FIVE}

THE TEN AND FIVE ARE CALLED SUBSCRIPTS AND DENOTE THE BASE OF THE NUMERATION SYSTEM. <u>FROM THIS POINT ON, THE SUBSCRIPT FOR BASE TEN WILL NOT BE WRITTEN; HOWEVER, SUBSCRIPTS FOR THE OTHER BASES WILL BE WRITTEN</u>.

NUMBERS WRITTEN IN BASES OTHER THAN BASE TEN ARE READ DIFFERENTLY SINCE READING A BASE TEN NUMERAL INVOLVES PLACE VALUE. 3,462 IS READ "THREE THOUSAND, FOUR HUNDRED SIXTY-TWO." THEREFORE, WHEN READING NUMBERS IN OTHER BASES, SIMPLY READ EACH DIGIT STARTING AT THE LEFT AND THEN SAY THE BASE.

THE NUMERAL 42_{FIVE} IS READ,

"FOUR TWO, BASE FIVE."

IT MEANS 4 FIVES + 2 ONES.

FIVES → 4   ONES → 2_{FIVE}

BASE → FIVE

IN THE NUMERAL 2314_{FIVE}

THE 4 IS THE ONES OR 5^0 PLACE;

THE 1 IS IN THE FIVES OR 5^1 PLACE,

THE 3 IS IN THE TWENTY-FIVES OR 5^2 PLACE ($5^2 = 5 \times 5 = 25$);

THE 2 IS IN THE ONE HUNDRED TWENTY-FIVES OR 5^3 PLACE

($5^3 = 5 \times 5 \times 5 = 125$).

THE PLACE VALUE OF EACH DIGIT IN THE NUMERAL 2314_{FIVE} IS SHOWN BY THE CHART BELOW:

One Hundred Twenty-Fives	Twenty-Fives	Fives	Ones
5^3	5^2	5^1	5^0
2	3	1	4

NOTICE: BASE FIVE IS GROUPED BY ONES, FIVES, FIVE X FIVES, AND FIVE X FIVE X FIVES, ETC. ALSO, NOTICE THAT NO COMMAS ARE USED IN WRITING OTHER BASE NUMERALS. THE COMMAS ARE USED IN BASE TEN TO MAKE THE NUMERALS EASIER TO READ.

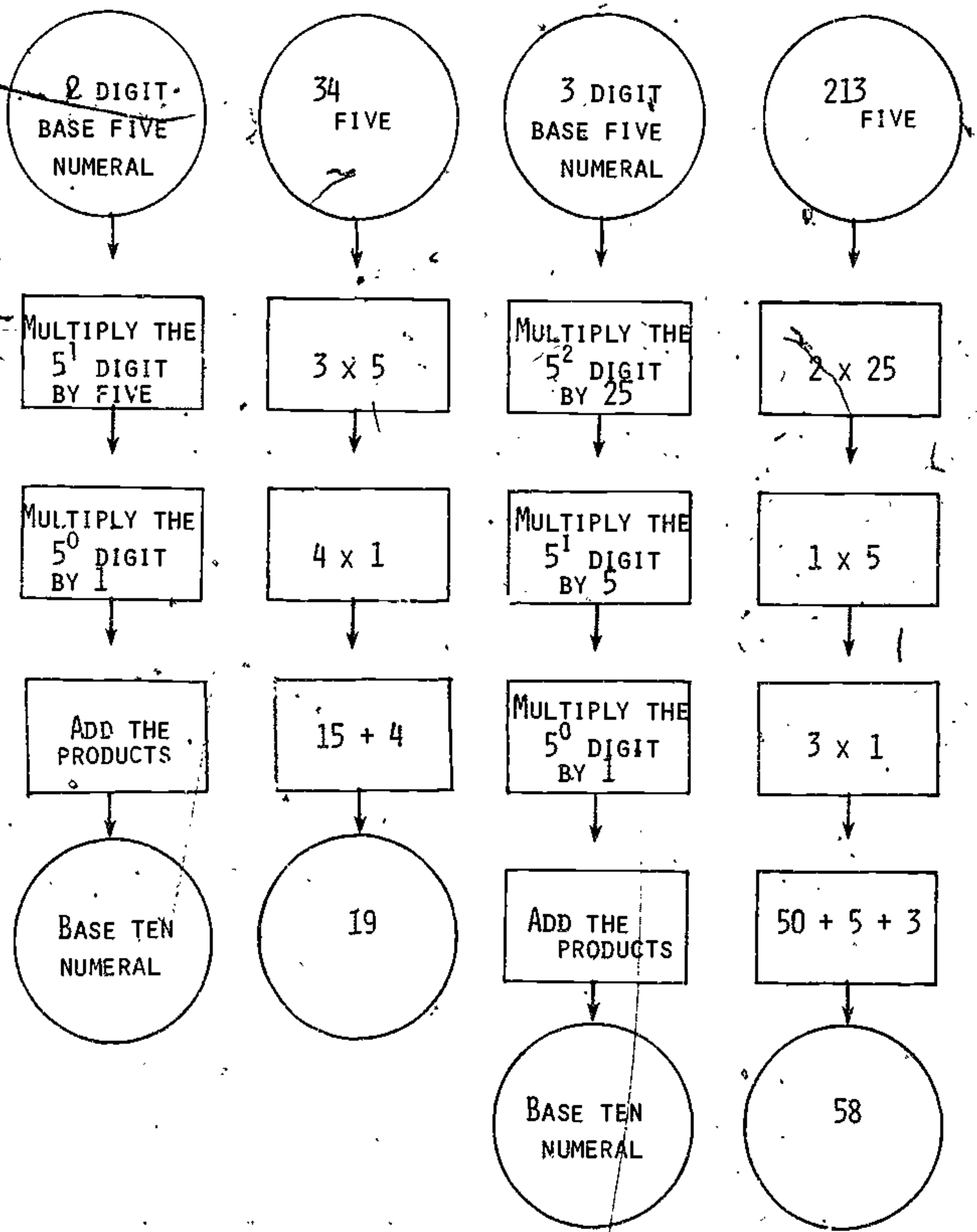
WHEN THE NUMERAL 2314_{FIVE} IS WRITTEN IN EXPANDED NOTATION AND SIMPLIFIED, THE RESULTING NUMERAL IS IN BASE TEN.

$$\begin{aligned} 2314_{\text{FIVE}} &= (2 \times 5^3) + (3 \times 5^2) + (1 \times 5^1) + (4 \times 5^0) \\ &= (2 \times 125) + (3 \times 25) + (1 \times 5) + (4 \times 1) \\ &= \quad 250 \quad + \quad 75 \quad + \quad 5 \quad + \quad 4 \\ &= 334 \end{aligned}$$

THEREFORE, $2314_{\text{FIVE}} = 334$ (BASE TEN).

TO CHANGE NUMBERS WRITTEN IN OTHER BASES TO BASE TEN, <u>WRITE IN EXPANDED NOTATION AND SIMPLIFY</u>.

CHANGING A NUMBER WRITTEN IN BASE FIVE TO A BASE TEN NUMERAL CAN BE SHOWN BY FLOW CHARTS.



CHANGING FROM BASE FIVE TO BASE TEN

WRITE A BASE FIVE NUMERAL FOR EACH SET.

1. + + + + +
   + + + + +
   + +

2. # # # # #
   # # # # #
   # # # # #
   # # # # #
   # #

3. * * * * * * *
   * * * * * * *
   * * * * * * *
   * *

HINT FOR NUMBER 3: FIRST REGROUP IN GROUPS OF FIVE.

4.

PLACE VALUE

BASE TEN	10^4	10^3	10^2	10^1	10^0
	10x10x10x10 10,000	10x10x10 ___	___x___ 100	OR ___	OR 1
BASE FIVE	5^4	5^3	5^2	5^1	5^0
	5x5x5x5 625	5x5x5 ___	5x5 ___	OR 5	OR ___

FILL IN THE BLANKS.

5. $132_{FIVE} = (1 \times \_\_) + (\_\_ \times 5^1) + (2 \times \_\_)$
   $= (1 \times \_\_) + (\_\_ \times 5) + (2 \times \_\_)$
   $= \_\_\_ + \_\_\_ + \_\_\_$
   $= \_\_\_$

6. $2034_{FIVE} = (\_\_ \times 5^3) + (0 \times \_\_\_) + (3 \times \_\_) + (\_\_ \times 5^0)$
   $= (\_\_ \times 125) + (0 \times \_\_) + (3 \times \_\_) + (\_\_ \times 1)$
   $= \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_$
   $= \_\_\_$

CHANGE EACH BASE FIVE NUMERAL TO BASE TEN BY USING EXPANDED NOTATION.

7. 43_{FIVE}

8. 1213_{FIVE}

9. 2132_{FIVE}

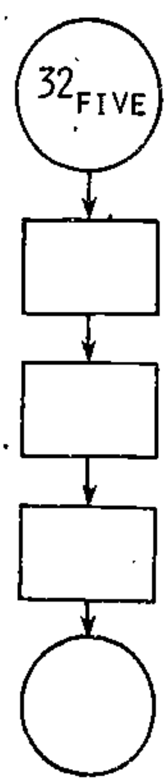
10. 444_{FIVE}

11. 4030_{FIVE}

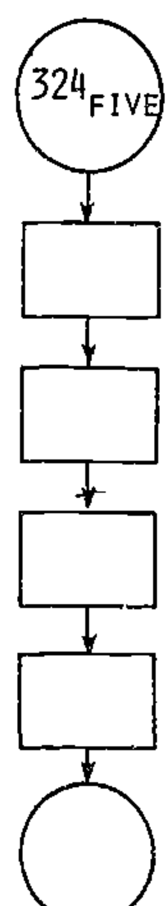
12. 11432_{FIVE}

COMPLETE THE FLOW CHARTS TO CHANGE EACH BASE FIVE NUMERAL TO BASE TEN.

13.



14.



CHANGING FROM BASE TEN TO BASE FIVE

TO CHANGE A BASE TEN NUMERAL TO BASE FIVE (REGROUPING IN GROUPS OF FIVE) DIVIDE BY POWERS OF BASE FIVE STARTING WITH THE LARGEST POWER OF FIVE JUST SMALLER THAN THE BASE TEN NUMERAL. THEN DIVIDE THE REMAINDER BY EACH SUCCESSIVE POWER OF FIVE LESS THAN THE FIRST DIVISIOR.

TO CHANGE 85 TO A BASE FIVE NUMERAL:

FIRST, LIST THE POWERS OF FIVE UNTIL A NUMBER LARGER THAN 85 IS OBTAINED.

$$5^0 = 1$$
$$5^1 = 5$$
$$5^2 = 25$$
$$\longleftrightarrow \quad 85$$
$$5^3 = 125$$

STOP, BECAUSE 125 IS GREATER THAN 85. THEREFORE, THE FIRST DIVISOR IS 25.

SECOND, REGROUP 85 INTO POWERS OF FIVE BY USING *SUCCESSIVE DIVISIONS. REMEMBER, THE FIRST DIVISOR IS 25.

$$\begin{array}{r} 3 \\ 25\overline{)85} \\ \underline{75} \\ 10 \end{array}$$

(THERE ARE 3 GROUPS OF 25 OR 5^2. THEREFORE, 3 WILL BE IN THE 5^2 PLACE.)

$$\begin{array}{r} 2 \\ 5\overline{)10} \\ \underline{10} \\ 0 \end{array}$$

(THERE ARE 2 GROUPS OF 5 OR 5^1. THEREFORE, 2 WILL BE IN THE 5^1 PLACE.)

110

$$\begin{array}{r} 0 \\ 1\overline{)0} \\ 0 \\ \hline 0 \end{array}$$

(There are 0 groups of 1 or 5^0. Therefore, 0 will be in the 5^0 place.)

*Successive Division--Dividing by numbers which are powers of the base beginning with largest possible power of the base, then dividing the successive remainders by the next largest power, and so on.

Study the chart showing the place value of each digit in the base five numeral.

Twenty-fives place 5^2	Fives place 5^1	Ones place 5^0
3	2	0

The answer is: $85 = 320_{IVE}$

A shorter way to write this computation is as follows:

Divisor ⟶ 25 | 85 | 3 ⟵ Quotient (3 x 25 = 75)

-75

5 | 10 | 2 (2 x 5 = 10)

-10

1 | 0 | 0 (0 x 1 = 0)

- 0

Therefore, $85 = 320_{FIVE}$

111

EXAMPLE: $348 = \underline{\ ?\ }_{\text{FIVE}}$

1. <u>LIST</u> THE <u>POWERS</u> OF FIVE

$5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$\longleftrightarrow$ 348

$5^4 = 625$

NOTICE: THE FIRST DIVISOR IS 125. THEREFORE, THE BASE FIVE NUMERAL WILL HAVE <u>FOUR</u> DIGITS, REQUIRING <u>FOUR</u> <u>DIVISIONS</u>.

2. <u>REGROUP</u> 348 INTO POWERS OF FIVE BY <u>SUCCESSIVE</u> <u>DIVISION</u>.

```
125 | 348 | 2
     -250
 25 |  98 | 3
      -75
  5 |  23 | 4
      -20
  1 |   3 | 3
       -3
        0
```

$2 \times 125 = 250$

$3 \times 25 = 75$

$4 \times 5 = 20$

$3 \times 1 = 3$

THEREFORE, $348 = 2343_{\text{FIVE}}$

CHANGING FROM BASE TEN TO BASE FIVE

1. COMPLETE THE DIVISION TO CHANGE 67 TO A BASE FIVE NUMERAL.

$5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$\longleftrightarrow$ 67

$5^3 = 125$

25 | 67 | 2

-50

| 17 |

1 | |

0

67 = ____ FIVE

CHANGE THE FOLLOWING BASE TEN NUMERALS TO BASE FIVE. SHOW ALL STEPS.

2. 58

5. 96

3. 25

6. 365

4. 124

7. 500

ADDITION, SUBTRACTION, AND MULTIPLICATION IN BASE FIVE

THE FOLLOWING BASE FIVE ADDITION TABLE CAN BE USED TO COMPUTE SUMS.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

$0 + 4 = 4$

$1 + 4 = 5 = (1 \times 5) + 0 = 10_{FIVE}$

$2 + 4 = 6 = (1 \times 5) + 1 = 11_{FIVE}$

$3 + 4 = 7 = (1 \times 5) + 2 = 12_{FIVE}$

$4 + 4 = 8 = (1 \times 5) + 3 = 13_{FIVE}$

ADDITION EXAMPLES:

A.

$$\begin{array}{r} {}^{1} \\ 13_{FIVE} \\ +\ 24_{FIVE} \\ \hline 42_{FIVE} \end{array}$$

ONES PLACE: 3 + 4 = 12; 1 FIVE, 2 ONES

REGROUP THE 1 FIVE IN THE FIVES PLACE

WRITE THE 2 IN THE ONES PLACE

FIVES PLACE: 1 + 1 + 2 = 4

B.

$$\begin{array}{r} 413_{FIVE} \\ +\ 222_{FIVE} \\ \hline 1140_{FIVE} \end{array}$$

ONES PLACE: 3 + 2 = 10; 1 FIVE, 0 ONES

FIVES PLACE: 1 + 1 + 2 = 4

FIVE^2 PLACE: 4 + 2 = 11; 1 FIVE, 1 ONE

FIVE^3 PLACE: 1

REFER TO THE <u>BASE FIVE ADDITION TABLE</u> TO <u>SUBTRACT</u> IN BASE FIVE.

TO SUBTRACT USING THE TABLE LOOK AT THE EXAMPLE TO THE RIGHT OF THE TABLE. FOUR AT THE TOP OF THE LAST COLUMN IS SUBTRACTED FROM EACH OF THE CIRCLED BASE FIVE NUMERALS TO OBTAIN THE BASE FIVE NUMERALS IN THE FIRST COLUMN.

+	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	10
2	2	3	4	10	11
3	3	4	10	11	12
4	4	10	11	12	13

→ 4 - 4 = 0

→ 10 - 4 = 1 (1 FIVE - 4 = 1)

→ 11 - 4 = 2 (1 FIVE + 1 = 6; 6 - 4 = 2)

→ 12 - 4 = 3 (1 FIVE + 2 = 7; 7 - 4 = 3)

→ 13 - 4 = 4 (1 FIVE + 3 = 8; 8 - 4 = 4)

SUBTRACTION EXAMPLES:

$^{3}\cancel{4}\,\boxed{^{1}\cancel{3}}_{FIVE}$ → <u>REMEMBER</u>!

$-2\ 4_{FIVE}$

14_{FIVE}

ONES PLACE: $\boxed{13_{FIVE}}$ = 1 FIVE + 3 ONES = 5 + 3 = 8 ONES

8 - 4 = 4

FIVES PLACE: 3 - 2 = 1

$^{0}\cancel{1}\,^{12}\cancel{3}\,^{1}2$

$-\ 4\ 4_{FIVE}$

$3\ 3_{FIVE}$

ONES PLACE: 12_{FIVE} = 1 FIVE + 2 ONES = 5 + 2 = 7 ONES

7 - 4 = 3

FIVES PLACE: 1 $FIVE^2$ + 2 FIVES = 5 + 2 = 7 FIVES (1 $FIVE^2$ = 5 FIVES) 7 - 4 = 3

Use the following base five multiplication table to compute products.

X	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	11	13
3	0	3	11	14	22
4	0	4	13	22	31

$\longrightarrow$ ($2 \times 4 = 8$; $8 \div 5 = 1$ R $3 = 13_{FIVE}$)

$\longrightarrow$ ($3 \times 4 = 12$; $12 \div 5 = 2$ R $2 = 22_{FIVE}$)

$\longrightarrow$ ($4 \times 4 = 16$; $16 \div 5 = 3$ R $1 = 31_{FIVE}$)

Multiplication Examples

A.

$$\begin{array}{r} {}^{1} \\ 3\,2_{FIVE} \\ \times\ \ 4_{FIVE} \\ \hline 2\,3\,3_{FIVE} \end{array}$$

(REGROUP)

Ones place: $4 \times 2 = 8$; $8 \div 5 = 1$ five, 3 ones

Fives place: $4 \times 3 = 12$; $12 + 1 = 13$, $13 \div 5 = 2$ R 3

Five^2 place: 2

B.

$$\begin{array}{r} {}^{1} \\ {}^{2} \\ 3\,4_{FIVE} \\ \times\ \ 2\,3_{FIVE} \\ \hline 2\,1\,2_{FIVE} \\ 1\,2\,3\phantom{_{FIVE}} \\ \hline 1\,4\,4\,2_{FIVE} \end{array}$$

(REGROUP)

Ones x ones place: $3 \times 4 = 12$; $12 \div 5 = 2$ fives, 2 ones

Ones x fives place: $3 \times 3 = 9$; $9 + 2 = 11$; $11 \div 5 = 2$ R 1

(REGROUP)

Fives x one place: $2 \times 4 = 8$; $8 \div 5 = 1$ R 3

Fives x fives place: $2 \times 3 = 6$; $6 + 1 = 7$; $7 \div 5 = 1$ R 2

OR,

$$
\begin{array}{r}
\scriptstyle 1 \phantom{\ 4_{FIVE}} \\
\scriptstyle 2 \phantom{\ 4_{FIVE}} \\
3\ 4_{FIVE} \\
\times\ \ 2\ 3_{FIVE} \\
\hline
2\ 1\ 2 \phantom{_{FIVE}} \\
1\ 2\ 3\ \phantom{2_{FIVE}} \\
\hline
1\ 4\ 4\ 2_{FIVE}
\end{array}
$$

3 x 4 = 22, WRITE 2, "CARRY" 2.

3 x 3 = 14 PLUS 2 (CARRIED) = 21.

1ST PARTIAL PRODUCT IS 212.

2 x 4 = 13, WRITE 3 UNDER 1, "CARRY" 1.

2 x 3 = 11 PLUS 1 (CARRIED) = 12, WRITE TO THE LEFT OF 3 (IN SECOND ROW). 2ND PARTIAL PRODUCT IS 1230.

SUM OF PARTIAL PRODUCTS IS 1442.

ADDITION, SUBTRACTION, AND MULTIPLICATION

COMPUTE EACH SUM IN BASE FIVE.

1. $\begin{array}{r} 33_{FIVE} \\ +\ 12_{FIVE} \\ \hline \end{array}$

2. $\begin{array}{r} 241_{FIVE} \\ +\ 334_{FIVE} \\ \hline \end{array}$

3. $\begin{array}{r} 223_{FIVE} \\ 411_{FIVE} \\ +\ 314_{FIVE} \\ \hline \end{array}$

COMPUTE EACH DIFFERENCE IN BASE FIVE.

4. $\begin{array}{r} 32_{FIVE} \\ -\ 23_{FIVE} \\ \hline \end{array}$

5. $\begin{array}{r} 141_{FIVE} \\ -\ 43_{FIVE} \\ \hline \end{array}$

6. $\begin{array}{r} 423_{FIVE} \\ -\ 124_{FIVE} \\ \hline \end{array}$

COMPUTE EACH PRODUCT IN BASE FIVE.

7. $\begin{array}{r} 43_{FIVE} \\ \times\ 2_{FIVE} \\ \hline \end{array}$

8. $\begin{array}{r} 32_{FIVE} \\ \times\ 42_{FIVE} \\ \hline \end{array}$

9. $\begin{array}{r} 242_{FIVE} \\ \times\ 34_{FIVE} \\ \hline \end{array}$

BASE TWO (BINARY NUMERATION SYSTEM)

AS A REVIEW, BASE TEN USES TEN SYMBOLS (0, 1, 2, 3, 4, 5, 6, 7, 8, 9), AND ITS PLACE VALUES ARE POWERS OF TEN. BASE FIVE USES FIVE SYMBOLS (0, 1, 2, 3, 4), AND ITS PLACE VALUES ARE POWERS OF FIVE.

BASE TWO, ALSO CALLED BINARY NUMERATION SYSTEM, USES TWO SYMBOLS (0, 1), AND ITS PLACE VALUES ARE POWERS OF TWO.

STUDY THE FOLLOWING TABLE OF PLACE VALUES IN BASE TWO.

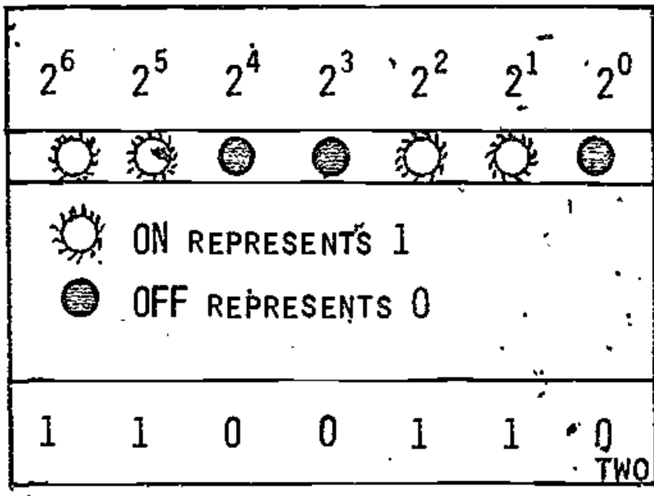
2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1

BASE TWO NUMERALS CAN ALSO BE WRITTEN IN EXPANDED NOTATION TO CHANGE THEM TO BASE TEN.

$$\begin{aligned} 1110_{TWO} &= (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) \\ &= (1 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1) \\ &= 8 + 4 + 2 + 0 \\ &= 14 \end{aligned}$$

THEREFORE, $1110_{TWO} = 14$.

SINCE THE BASE TWO SYSTEM OF NUMERATION USES ONLY THE DIGITS 0 AND 1, THE NUMERALS CAN EASILY BE SHOWN USING THE IDEA OF ELECTRICAL CURCUITS. IF A LIGHT IS ON IN A ROW OF LIGHTS, IT REPRESENTS 1 IN THAT POSITION. IF A LIGHT IS OFF, THE DIGIT IN THAT PLACE IS 0.



$1100110_{TWO} =$

$= (1 \times 2^6) + (1 \times 2^5) + (0 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$

$= (1 \times 64) + (1 \times 32) + (0 \times 16) + (0 \times 8) + (1 \times 4) + (1 \times 2) + (0 \times 1)$

$= 64 + 32 + 0 + 0 + 4 + 2 + 0$

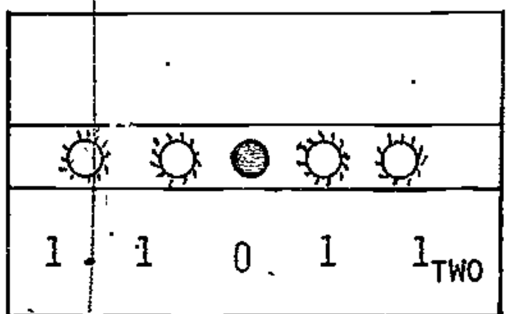
$= 102$

THEREFORE, $11001100_{TWO} = 102$.

NOTICE: MANY MORE DIGITS ARE REQUIRED TO WRITE A NUMBER IN BASE TWO. FOR THIS REASON ALONE IT WOULD NOT GENERALLY BE AS PRACTICAL TO USE AS BASE TEN.

EXAMPLE A:

$2^4 \quad 2^3 \quad 2^2 \quad 2^1 \quad 2^0$



$$
\begin{aligned}
&= (1 \times 2^4) + (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) \\
&= (1 \times 16) + (1 \times 8) + (0 \times 4) + (1 \times 2) + (1 \times 1) \\
&= \quad 16 \quad + \quad 8 \quad + \quad 0 \quad + \quad 2 \quad + \quad 1 \\
&= \quad 27
\end{aligned}
$$

THEREFORE, $110011_{TWO} = 27$.

BASE TEN NUMERALS CAN BE CHANGED TO BASE TWO NUMERALS BY SUCCESSIVE DIVISIONS USING AS THE FIRST DIVISOR THE LARGEST POWER OF TWO JUST SMALLER THAN THE BASE TEN NUMERAL. (REMEMBER BASE FIVE.)

$83 = \underline{\qquad}_{TWO}$

FIRST, LIST THE POWERS OF TWO UNTIL YOU OBTAIN A NUMBER LARGER THAN 83.

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64 \longleftrightarrow 83 \qquad 2^7 = 128$

SEE IF YOU CAN ANSWER THESE QUESTIONS BY STUDYING THE LIST OF POWERS OF TWO.

HOW MANY DIGITS WILL THE BASE TWO NUMERAL CONTAIN?

HOW MANY TIMES WILL YOU HAVE TO DIVIDE?

ANSWER--SEVEN

DIVISOR			QUOTIENT
64	83	1	
	-64		
32	19	0	(0 IS NECESSARY AS A PLACE HOLDER IN THE 2^2, 2^3, 2^5 PLACES. THEREFORE, IT IS NECESSARY TO DIVIDE BY 4, 8, AND 32.)
	- 0		
16	19	1	
	-16		
8	3	0	
	- 0		
4	3	0	
	- 0		
2	3	1	
	- 2		
1	1	1	
	- 1		
	0		

THEREFORE, $83 = 1010011_{TWO}$

ADDITION, SUBTRACTION, AND MULTIPLICATION EXAMPLES:

EXAMPLE B:

$$\begin{array}{r} {}^{1}\\ 11_{TWO} \\ +\ 10_{TWO} \\ \hline 101_{TWO} \end{array}$$

ONES PLACE: $1 + 0 = 1$

(REGROUP)

TWOS PLACE: $1 + 1 = 2;\ 2 \div 2 = 1\ R\ 0$

TWO^{2} PLACE: 1

EXAMPLE C:

$$\begin{array}{r} 1\ 1 \\ 1\ 1\ 1_{TWO} \\ +\ 1\ 0\ 1_{TWO} \\ \hline 1\ 1\ 0\ 0_{TWO} \end{array}$$

(REGROUP)

ONES PLACE: $1 + 1 = 2;\ 2 \div 2 = 1\ R\ 0$

TWOS PLACE: $1 + 1 = 2;\ 2 \div 2 = 2\ R\ 0$

TWO^{2} PLACE: $1 + 1 + 1 = 3;\ 3 \div 2 = 1\ R\ 1$

TWO^{3} PLACE: 1

EXAMPLE D:

$$\begin{array}{r} 10_{TWO} \\ -\ 1_{TWO} \\ \hline 1_{TWO} \end{array}$$

ONES PLACE: $10_{TWO} = 1$ TWO $+ 0$ ONES $= 2$ ONES;
$2 - 1 = 1_{TWO}$

EXAMPLE E:

$$\begin{array}{r} 0\ 10\ 1 \\ \cancel{1}\,\cancel{1}\,0_{TWO} \\ -\ 1\ 1_{TWO} \\ \hline 1\ 1_{TWO} \end{array}$$

ONES PLACE: 1 TWO + 0 ONES = 2 ONES; $2 - 1 = 1$

TWOS PLACE: 1 TWO^{2} + 0 TWOS = 2 TWOS; $2 - 1 = 1$
(1 TWO^{2} = 2 TWOS)

BASE TWO ADDITION TABLE

+	0	1
0	0	1
1	1	10

$1 + 1 = 2;\ 2 = (2 \times 1) + 0 = 10_{TWO}$

BASE TWO MULTIPLICATION TABLE

x	0	1
0	0	0
1	0	1

EXAMPLE F:

$$\begin{array}{r} 101_{TWO} \\ \underline{\times\ 11_{TWO}} \\ 101 \\ \underline{101} \\ 1111_{TWO} \end{array}$$

EXAMPLE G:

$$\begin{array}{r} 111_{TWO} \\ \underline{\times\ 101_{TWO}} \\ 111 \\ 000 \\ \underline{111} \\ 100011_{TWO} \end{array}$$

BASE TWO (BINARY NUMERATION SYSTEM)

1. WHAT IS ANOTHER NAME FOR BASE TWO?

2. WHY COULD THE NUMERAL 101121_{TWO} NOT BE A BASE TWO NUMERAL?

3. WITHOUT DIVIDING, HOW MANY DIGITS ARE REQUIRED TO WRITE 62 AS A BASE TWO NUMERAL?

WRITE EACH BASE TWO NUMERAL IN EXPANDED NOTATION AND CHANGE TO BASE TEN.

4. $1101_{TWO} = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$

$= (\quad\quad) + (\quad\quad) + (\quad\quad) + (\quad\quad)$

$= \_\_\_\_ + \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

$= \_\_\_\_$

5. 1010_{TWO}

6. 10101_{TWO}

7. 1001001_{TWO}

125

WRITE THE BASE TWO NUMERAL REPRESENTED BY EACH ROW OF LIGHTS.

8. 

9. 

10. 

CHANGE EACH BASE TEN NUMERAL TO BASE TWO. (SHOW ALL WORK.)

11. 38 = ____ TWO

$2^0 = 1$

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

$2^5 = 32$

$\longleftrightarrow$ 38

$2^6 = 64$

32 ⌋ 38

16 ⌋

8 ⌋

12. 23 = ____ TWO

13. 79 = ____ TWO

14. 157 = ____ TWO

PERFORM THE INDICATED OPERATIONS IN BASE TWO.

15. $\begin{array}{r} 101_{TWO} \\ \underline{+110_{TWO}} \end{array}$

16. $\begin{array}{r} 1111_{TWO} \\ \underline{+\ 101_{TWO}} \end{array}$

17. $\begin{array}{r} 1010_{TWO} \\ \underline{-\ 101_{TWO}} \end{array}$

18. $\begin{array}{r} 1011_{TWO} \\ \underline{-\ 111_{TWO}} \end{array}$

19. $\begin{array}{r} 101_{TWO} \\ \underline{\times\ 10_{TWO}} \end{array}$

20. $\begin{array}{r} 1010_{TWO} \\ \underline{\times\ 101_{TWO}} \end{array}$

21. $\begin{array}{r} 1010_{TWO} \\ \underline{+\ 110_{TWO}} \end{array}$

22. $\begin{array}{r} 1100_{TWO} \\ \underline{-101_{TWO}} \end{array}$

127

NUMBER SENSE

MULTIPLY BY 5

TAKE HALF OF THE NUMBER AND MULTIPLY BY 10.

36	THE NUMBER	36
18	HALF OF THE NUMBER	x5
180	MULTIPLY BY 10	180

47	THE NUMBER	47
23.5	HALF OF THE NUMBER	x5
235	MULTIPLY BY 10	235

MULTIPLY BY 15

MULTIPLY THE NUMBER BY 10 AND ADD HALF THE PRODUCT TO IT.

49	THE NUMBER	49
490	MULTIPLY BY 10	15
245	HALF THE PRODUCT	245
735	SUM OF PRODUCT AND	49
	HALF THE PRODUCT	735

MULTIPLY BY 9

MULTIPLY THE NUMBER BY 10 AND SUBTRACT THE NUMBER FROM IT.

93	THE NUMBER	93
930	MULTIPLY BY 10	x9
837	SUBTRACT THE NUMBER	837

MULTIPLY BY 99

MULTIPLY THE NUMBER BY 100 AND SUBTRACT FROM THIS PRODUCT THE ORIGINAL NUMBER.

236	THE NUMBER	236
23600	MULTIPLY BY 100	x99
23364	SUBTRACT THE NUMBER	2124
		2124
		23364

MULTIPLY BY 11

1. WRITE THE RIGHT-HAND DIGIT OF THE NUMBER AS THE RIGHT-HAND DIGIT OF THE ANSWER.
2. ADD TWO DIGITS AT A TIME STARTING AT THE RIGHT TO PRODUCE THE SUCCEEDING DIGITS OF THE ANSWER.
3. WRITE THE LEFT-HAND DIGIT OF THE NUMBER AS THE LEFT-HAND DIGIT OF THE ANSWER.

32415 x 11	32415
356565	x11
	32415
	32415
	356565

START ON THE RIGHT AND WORK LEFT.

WRITE THE RIGHT-HAND DIGIT (5).

ADD 5 + 1 = 6 FOR THE NEXT DIGIT.

ADD 1 + 4 = 5 FOR THE NEXT DIGIT.

ADD 4 + 2 = 6 FOR THE NEXT DIGIT.

ADD 2 + 3 = 5 FOR THE NEXT DIGIT.

WRITE THE LEFT-HAND DIGIT (3).

4. SOMETIMES IT IS NECESSARY TO CARRY.

257861 x 11 = 2836471

START ON THE RIGHT AND WORK LEFT.

WRITE THE RIGHT-HAND DIGIT (1).

ADD 1 + 6 = 7 FOR THE NEXT DIGIT.

ADD 6 + 8 =14; WRITE 4 AND CARRY 1.

ADD 8 + 7 =15; 15 + 1 = 16; WRITE 6 AND CARRY 1.

ADD 7 + 5 =12; 12 + 1 = 13; WRITE 3 AND CARRY 1.

ADD 5 + 2 = 7; 7 + 1 = 8; WRITE 8

WRITE 2 FOR THE LAST DIGIT ON THE LEFT.

MULTIPLY BY 12

1. DOUBLE THE RIGHT-HAND DIGIT OF THE NUMBER TO BE THE RIGHT-HAND DIGIT OF THE ANSWER.
2. DOUBLE EACH SUCCEEDING DIGIT AND ADD TO THE DIGIT ON THE RIGHT PLUS ANYTHING THAT IS CARRIED.
3. THE LEFT-HAND DIGIT OF THE NUMBER PLUS ANYTHING THAT IS CARRIED BECOMES THE LEFT-HAND DIGIT OF THE ANSWER.

32761 x 12 = 393132

DOUBLE 1, WRITE 2.

DOUBLE 6, ADD 1, WRITE 3, CARRY 1.

DOUBLE 7, ADD 6, ADD 1, WRITE 1, CARRY 2.

DOUBLE 2, ADD 7, ADD 2, WRITE 3, CARRY 1.

DOUBLE 3, ADD 2, ADD 1, WRITE 9.

WRITE 3.

MULTIPLY TWO-DIGIT NUMBER BY TWO-DIGIT-NUMBER

$\begin{array}{r} {}^{6}39 \\ \underline{\times 47} \\ 3 \end{array}$	1. START MULTIPLYING AS USUAL WITH THE ONES PLACES. 2. STOP USING THE REGULAR METHOD.
$\begin{array}{r} {}^{6}3\ 9 \\ \underline{\times 4\ 7} \end{array}$	3. NEXT, CROSS MULTIPLY.
$3 \times 7 = 21$ $9 \times 4 = \underline{36}$ 57	4. ADD THE PRODUCTS.
57 $\underline{6}$ 63	5. ADD THE ONES CARRIED.
$\begin{array}{r} {}^{6\ 6}39 \\ \underline{\times 47} \\ 33 \end{array}$	6. WRITE THE 3 IN THE TENS PLACE IN THE ANSWER, AND CARRY 6.
$3 \times 4 = 12$ $\begin{array}{r} {}^{6\ 6}39 \\ \underline{\times 47} \\ 1833 \end{array}$	7. MULTIPLY THE TENS PLACES, ADD THE ONES CARRIED, AND WRITE THIS TO COMPLETE THE ANSWER.

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MULTIPLY A THREE-DIGIT NUMBER BY A TWO-DIGIT NUMBER

$$\begin{array}{r} {}^{4} \\ 456 \\ \underline{\times 28} \\ 8 \end{array}$$

1. START MULTIPLYING AS USUAL WITH THE ONES PLACE.

2. STOP USING THE REGULAR METHOD.

$$\begin{array}{r} {}^{4} \\ 4\,5\,6 \\ \underline{\times\,2\,8} \\ 8 \end{array}$$

3. NEXT, CROSS MULTIPLY AS SHOWN.

$$\begin{array}{r} 5 \times 8 = 40 \\ 2 \times 6 + \underline{12} \\ 52 \\ \underline{4} \\ 56 \end{array}$$

4. ADD THE PRODUCTS.

5. ADD THE ONES CARRIED.

$$\begin{array}{r} {}^{54} \\ 456 \\ \underline{\times 28} \\ 68 \end{array}$$

6. WRITE THE 6 IN THE TENS PLACE IN THE ANSWER, AND CARRY 5.

$$\begin{array}{r} {}^{5\,4} \\ 4\,5\,6 \\ \underline{\times\,2\,8} \\ 6\,8 \end{array}$$

7. NEXT, CROSS MULTIPLY AS SHOWN.

$$\begin{array}{r} 2 \times 5 = 10 \\ 4 \times 8 = \underline{32} \\ 42 \\ \underline{5} \\ 47 \end{array}$$

8. ADD THE PRODUCTS.

9. ADD THE ONES CARRIED.

454
456
28
768

10. WRITE THE 7 IN THE HUNDREDS PLACE AND CARRY 4.

454
4 5 6
2 8
7 6 8

11. MULTIPLY THE HUNDREDS PLACE BY THE TENS PLACE.

$4 \times 2 = 8$

$\underline{4}$

12

12. ADD THE ONES CARRIED.

454
456
28
12768

13. WRITE THIS TO COMPLETE THE ANSWER.

MULTIPLY NUMBERS BY 25

DIVIDE THE NUMBER BY 4 AND MULTIPLY THE QUOTIENT BY 100.

25×48 DIVIDE 48 BY 4 → 12.

$\frac{100}{4} \times 48$ MULTIPLY 12 BY 100 → 1200.

100×12

1200

MULTIPLY NUMBERS BY 50

DIVIDE THE NUMBER BY 2 AND MULTIPLY THE QUOTIENT BY 100.

50×74 DIVIDE 74 BY 2 → 37.

$\frac{100}{2} \times 74$ MULTIPLY 37 BY 100 → 3700.

100×37

3700

MULTIPLY NUMBERS BY 75

DIVIDE THE NUMBER BY 4, MULTIPLY THE QUOTIENT BY 3, AND MULTIPLY THE PRODUCT BY 100.

75×28

$100 \times \frac{3}{4} \times 28$ DIVIDE 28 BY 4 → 7.

$100 \times 3 \times 7$ MULTIPLY 7 BY 3 → 21.

100×21 MULTIPLY 21 BY 100 → 2100.

2100

MULTIPLY NUMBERS CLOSE TO 100

93 x 97	1.	TAKE THE COMPLEMENTS TO 100 OF BOTH NUMBERS AND MULTIPLY THEM. 93--7 97--3 7 x 3 =21
_ _ 2 1	2.	WRITE 21 IN THE TEN'S AND ONE'S PLACES OF THE ANSWER.
93-3 = 90 OR 97-7 = 90 9 0 2 1	3.	SUBTRACT THE COMPLEMENT OF ONE NUMBER FROM THE OTHER NUMBER AND PLACE THE RESULT NEXT TO 2 1.

SQUARING A NUMBER

$(47)^2$	1.	SQUARE 7 TO GET 49.
9	2.	PUT THE 9 IN THE ONE'S COLUMN OF THE ANSWER AND CARRY THE 4.
	3.	TAKE 4 x 7 x 2 = 56 AND ADD THE 4 FROM STEP 2 TO GET 60.
09	4.	PUT THE 0 IN THE TEN'S COLUMN OF THE ANSWER AND CARRY THE 6.
	5.	SQUARE 4 TO GET 16 AND ADD THE 6 FROM STEP 4 TO GET 22.
2 2 0 9	6.	PLACE THIS RESULT NEXT TO 0 9.

SHORT CUT FOR SQUARING NUMBERS 51-59

$(53)^2$

1. WRITE 3^2 AS A TWO-DIGIT NUMBER. 0 9
2. SQUARE 5. 25
3. ADD 3. 3

   28
4. ANSWER 2 8 0 9

SHORT CUT FOR SQUARING A NUMBER ENDING IN 5

$(75)^2$

1. WRITE 5^2 IN ONE'S AND TEN'S PLACE. 2 5
2. MULTIPLY 7 BY ITS SUCCESSOR 8. 5 6
3. ANSWER 5 6 2 5

MULTIPLY BY DIFFERENCE OF 2 SQUARES METHOD

47 x 53 IS	1) SQUARE 50.	2500
(50-3) x (50+3)	2) SQUARE 3.	9
WHICH IS		2491
$50^2 - 3^2$	3) SUBTRACT.	

NUMBERS MUST "CENTER" AROUND A NUMBER LIKE 40, 50, 100, ETC.

$$74 \times 86 = (80 - 6)(80 + 6) = 80^2 - 6^2 = 6400 - 36 = 6364$$

Using Difference of 2 Squares for other problems

$$64^2 - 36^2 = (64+36)\ (64-36) = (100)\ (28) = 2800$$

Divisibility by 2

A number is divisible by 2 if the last digit is even (0, 2, 4, 6, or 8).

Divisibility by 3

A number is divisible by 3 if the sum of its digits is divisible by 3.

267 Sum of its digits: 2 + 6 + 7 = 15.

Since 15 is divisible by 3, then 267 is divisible by 3.

Divisibility by 4

A number is divisible by 4 if its last two digits form a number that is divisible by 4.

624 Since 24 is divisible by 4, then 624 is divisible by 4.

737 Since 37 is NOT divisible by 4, then 737 is not divisible by 4.

Divisibility by 5

A number is divisible by 5 if the last number is 0 or a 5.

Divisibility by 6

A number is divisible by 6 if it is divisible by 2 and by 3.

366 is divisible by 6 because it is divisible by 2 (the last digit 6 is even) and it is divisible by 3 (the sum of the digits is 15).

Divisibility by 7

Double the last digit of the number and subtract it from the remaining digits. If the remaining number is divisible by 7, the original number is divisible by 7. If the second number is still too large to divide by inspection, continue doubling the last digit and subtracting from the remaining digits until the answer is a number that is small enough to divide by inspection.

$$\begin{array}{r} 23\underline{8} \\ \underline{-16} \\ 7 \end{array}$$

Double 8 and subtract the result from the remaining digits. Since 7 is divisible by 7, then 238 is divisible by 7.

$$\begin{array}{r} 432\underline{7} \\ \underline{-14} \\ 41\underline{8} \\ \underline{16} \\ 25 \end{array}$$

Double 7 and subtract the results from the remaining digits.

Double 8 and subtract the results from the remaining digits.

Since 25 is not divisible by 7, then 4327 is not divisible by 7.

Divisibility by 8

A number is divisible by 8 if the last three digits are divisible by 8.

4<u>234</u> Since 234 is divisible by 8, then 4234 is divisible by 8.

62<u>311</u> Since 311 is not divisible by 8, then 62311 is not divisible by 8.

Divisibility by 9

A number is divisible by 9 if the sum of its digits is divisible by 9.

261	Sum of the digits: 2 + 6 + 1 = 9 which is divisible by 9. Therefore, 261 is divisible by 9.	

Divisibility by 10

A number is divisible by 10 if the last digit is 0.

Divisibility by 11

1. Starting with the one's place, place an "X" over every other digit in the number.
2. Add the digits with an "X" above them.
3. Add the digits without an "X" above them.
4. Find the difference of the two sums.
5. If the difference is divisible by 11, then the number is divisible by 11.

$\overset{X}{4}\,3,\overset{X}{5}\,5\,\overset{X}{6}$	$4 + 5 + 6 = 15$	This number is not
	$3 + 5 = \underline{8}$	divisible by 11.
	7	
$\overset{X}{9}\,8,\overset{X}{6}\,1\,\overset{X}{5}$	$9 + 6 + 5 = 20$	This number is divisible
	$8 + 1 = \underline{9}$	by 11.
	11	

Divisibility by 25

A number is divisible by 25 if the last two digits are 00, 25, 50, or 75.

Divisibility by 50

A number is divisible by 50 if the last two digits are 00 or 50.

NUMBER SENSE

MULTIPLICATION

1) 25 X 76 =	16) 50 X 24 =
2) 25 X 24 =	17) 50 X 80 =
3) 25 X 80 =	18) 50 X 284 =
4) 25 X 284 =	19) 50 X 328 =
5) 25 X 328 =	20) 50 X 456 =
6) 25 X 456 =	21) 50 X 96 =
7) 25 X 96 =	22) 11 X 96 =
8) 75 X 76 =	23) 11 X 456 =
9) 75 X 24 =	24) 11 X 4358 =
10) 75 X 80 =	25) 11 X 9276 =
11) 75 x 284 =	26) 11 X 3241 =
12) 75 X 328 =	27) 11 X 8273 =
13) 75 X 456 =	28) 10 X 85 =
14) 75 X 96 =	29) 100 X 63 =
15) 50 X 76 =	30) 1000 X 74 =

NUMBER SENSE

MULTIPLICATION

1) 49 X 15 =

2) 73 X 5 =

3) 734 X 11 =

4) 296 X 9 =

5) 956 X 99 =

6) 624 X 12 =

7) 90 X 25 =

8) 432 X 50 =

9) 592 X 75 =

10) 92 X 93 =

11) 525 X 11 =

12) 92 X 25 =

13) 36 X 15 =

14) 54 X 5 =

15) 95 X 96 =

16) 93 X 94 =

17) 1932 X 11 =

18) 500 X 25 =

19) 526 X 50 =

20) 48 X 75 =

21) 424 X 15 =

22) 196 X 5 =

23) 1095 X 11 =

24) 92 X 75 =

25) 96 X 97 =

26) 556 X 11 =

27) 2536 X 50 =

28) 44 X 15 =

29) 324 X 75 =

30) 250 X 50 =

NUMBER SENSE

MULTIPLICATION

1)	36	X	5	=	16)	91	X	99	=
2)	47	X	15	=	17)	169	X	11	=
3)	348	X	9	=	18)	32	X	25	=
4)	456	X	99	=	19)	96	X	50	=
5)	58	X	11	=	20)	32	X	75	=
6)	93	X	12	=	21)	926	X	15	=
7)	72	X	25	=	22)	96	X	5	=
8)	536	X	50	=	23)	9352	X	11	=
9)	912	X	75	=	24)	88	X	75	=
10)	95	X	97	=	25)	95	X	95	=
11)	862	X	11	=	26)	1095	X	11	=
12)	24	X	25	=	27)	1042	X	50	=
13)	18	X	15	=	28)	72	X	15	=
14)	94	X	96	=	29)	436	X	75	=
15)	48	X	5	=	30)	9254	X	50	=

SQUARING A NUMBER ENDING IN 5

1) $15^2 = 15 \times 15 =$

2) $25^2 = 25 \times 25 =$

3) $35^2 = 35 \times 35 =$

4) $45^2 = 45 \times 45 =$

5) $55^2 = 55 \times 55 =$

6) $65^2 = 65 \times 65 =$

7) $75^2 = 75 \times 75 =$

8) $85^2 = 85 \times 85 =$

9) $95^2 = 95 \times 95 =$

10) $105^2 = 105 \times 105 =$

11) $115^2 = 115 \times 115 =$

12) $125^2 = 125 \times 125 =$

13) $135^2 = 135 \times 135 =$

14) $145^2 = 145 \times 145 =$

15) $155^2 = 155 \times 155 =$

MULTIPLYING BY DIFFERENCE OF 2 SQUARES METHOD

1) $42 \times 48 = (45 - 3)(45 + 3) = 45^2 - 3^2 =$

2) $56 \times 54 =$

3) $92 \times 98 =$

4) $74 \times 76 =$

5) $81 \times 89 =$

6) $64 \times 66 =$

7) $71 \times 79 =$

8) $21 \times 29 =$

9) $32 \times 38 =$

10) $12 \times 18 =$

11) $13 \times 17 =$

12) $22 \times 28 =$

13) $34 \times 36 =$

14) $31 \times 39 =$

15) $33 \times 37 =$

DIVISIBILITY

PLACE AN "X" IN THE BOX IF THE NUMBER IN THE LEFT COLUMN IS DIVISIBLE BY THE NUMBER IN THE TOP ROW.

	2	3	4	5	6	7	8	9	10	11	25	50
7265												
432												
17526												
3400												
18243												
896												
15275												
9645												
3933												
4785												
19275												
23925												
9200												
5830												
10186												
763												
3113												
92564												
7250												
18725												
4900												

ANSWERS

MATHEMATICS ENRICHMENT
GRADE FIVE
ANSWERS

Page 17

1) Shepard--Carpenter
Johnston--Painter
Nichols--Plumber

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2) Of the 100, 79 (coffee drinkers) + 69 (tea drinkers) - 47 (number drinking both) = 101 (number of coffee only + the number of tea only drinkers). However, the investigator gave the total number interviewed as 100.

3)

County	Germany	France	U.S.	Sweden
Color	yellow	pink	blue	violet
Picture	runner	train	plane	flower

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4)

Points scored	7	16	25	10	15
Position played	R. G.	L. F.	R. F.	L. G.	Center
Player's name	Jack	Sam	Ed	Dave	Ben

Sam is the captain of the team.

5)

left house	center house	right house
Ford	Plymouth	Cadillac
Watson	Jones	Wilson

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6) cat--Duke
dog--Sandy
goat--Blacky
horse--Rusty

7) Sam--Sally
Bill--Patsy
Joe--Betty

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8) Hank

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9) Brown--manager
Green--teller
White--cashier

10) Hill--victim
Grover--witness
Craig--policeman
Mays--judge
Wilson--hangman
Dunn--murderer

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11) President--Mrs. Johnston
Vice-President--Mr. Brown
Manager--Mr. Crawford
Auditor--Miss Jones
Clerk--Miss Green
Secretary--Mr. Smith

12) David scored 98
Sam scored 96
David--Betty
Joe--Jane
Sam--Sue

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13) Jimenez--carpenter
Rodriguez--postman
Martinez--banker
Contreras--police officer

14) pilot--Joe

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15)

Jane		Ruth		Phil		Mark		Joe	
Ham	60¢	D. B.	90¢	D. B.	90¢	Cheese	75¢	Ham	60¢
Cola	20¢	Cola	20¢	Shake	35¢	Milk	20¢	Shake	35¢
Fries	25¢					Fries	25¢	Fries	25¢
	105¢		110¢		125¢		120¢		120¢

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16)

Color	Name	Place
Red	Lucky Lady	5th
Yellow	Winner's Circle	3rd
Orange	Martin's Folly	4th
Green	Johnston's Darlin'	1st
Blue	Dancer	6th
Violet	Fire Ball	2nd

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17) Ronald--1st
Lloyd--2nd
Joe--3rd
Carlton--4th
Roy--5th

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18) Lester, Donna, Chi, English
Calvin, Edith, Beta, History
Dick, Cleo, Alpha, Pre-law
Ralph, Beth, Iota, Pre-med
Warren, Ann, Delta, Economics

Lester	Calvin	Dick	Ralph	Warren
Donna	Edith	Cleo	Beth	Ann
Chi	Beta	Alpha	Iota	Delta
English	History	Pre-law	Pre-med	Economics

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19) Bill--catcher
Harry-pitcher
Erik--shortstop
Justin--1st
Frank--2nd
Alton--3rd
Carl--left field
Dennis--center field
George--right field

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20) Bibb--right guard
Carr--center
Day--left guard
Finks--right halfback
Harte--left halfback
Jiminez--right tackle
Kahler--full back
Leggett--left end
Mosley--right end
Norwood--quarter back
Tarvin--left tackle

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1 7	2 2	3 5				4 1	5 4
	6 2	1	7 5		8 5	0	3
9 1		10 9	0		11 2	1	
12 5	13 5		14 4	15 1	0		16 2
17 5	4	1		6		18 9	5
19 1	3		20 3	4	2		2
		21 7	6	1		22 5	
	23 4	6	2		24 5	3	5

[1] 2	[2] 5	[3] 6				[4] 5	[5] 6
	[6] 7	8	[7] 2		[8] 4	1	3
[9] 4		[10] 1	2		[11] 8	6	
[12] 1	[13] 5		[14] 7	[15] 5	3		[16] 4
[17] 7	2	0		2		[18] 1	5
[19] 1	3		[20] 6	8	8		4
		[21] 4	8	0		[22] 3	
	[23] 4	8	9		[24] 5	2	1

LINEAR MEASUREMENT

USING THE CODE BELOW, SHADE IN THE SQUARES WITH THE PROPER DESIGNS.

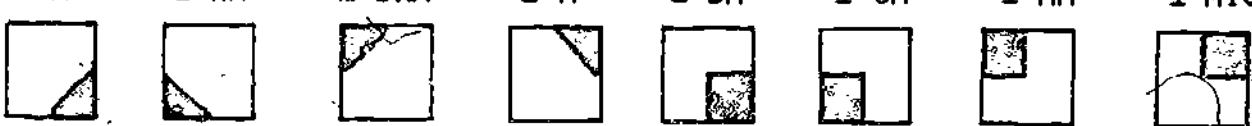
EXAMPLES:

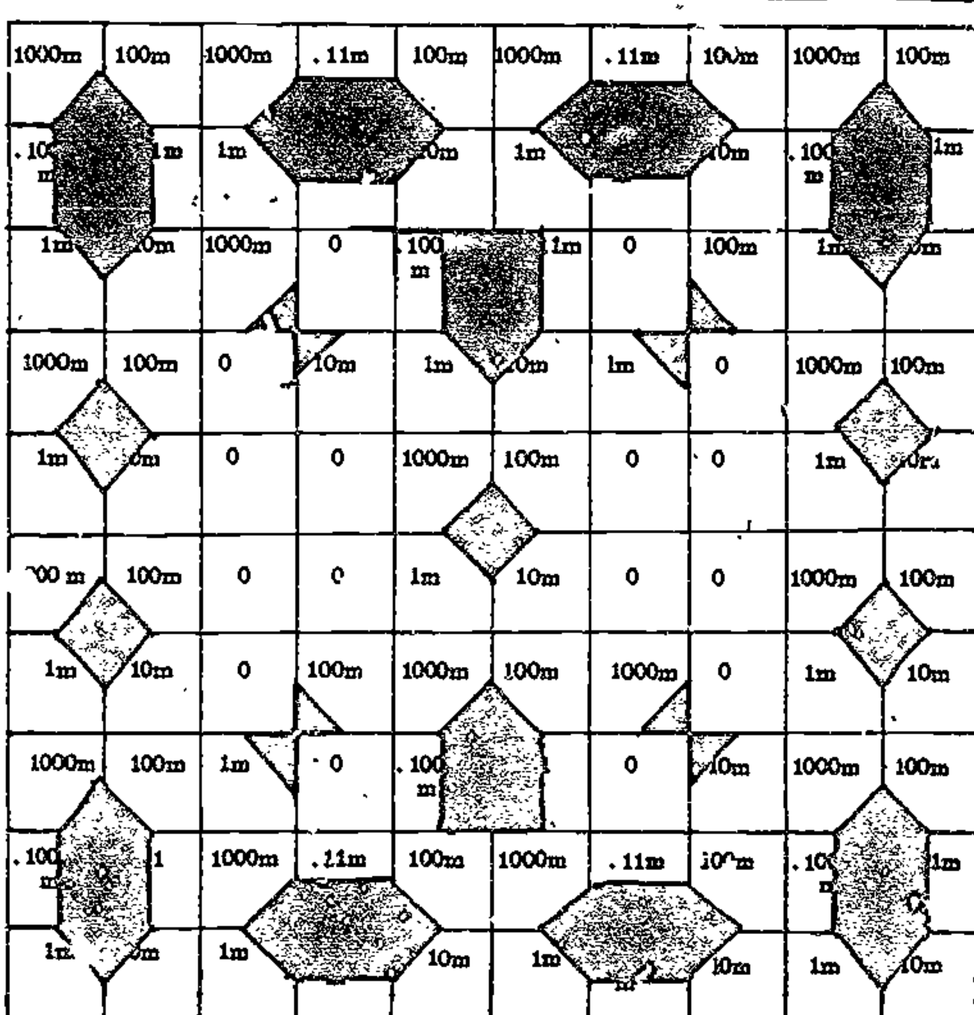
1000 M = 1 KM = 

.001001 M = .001 M + .000001 M =

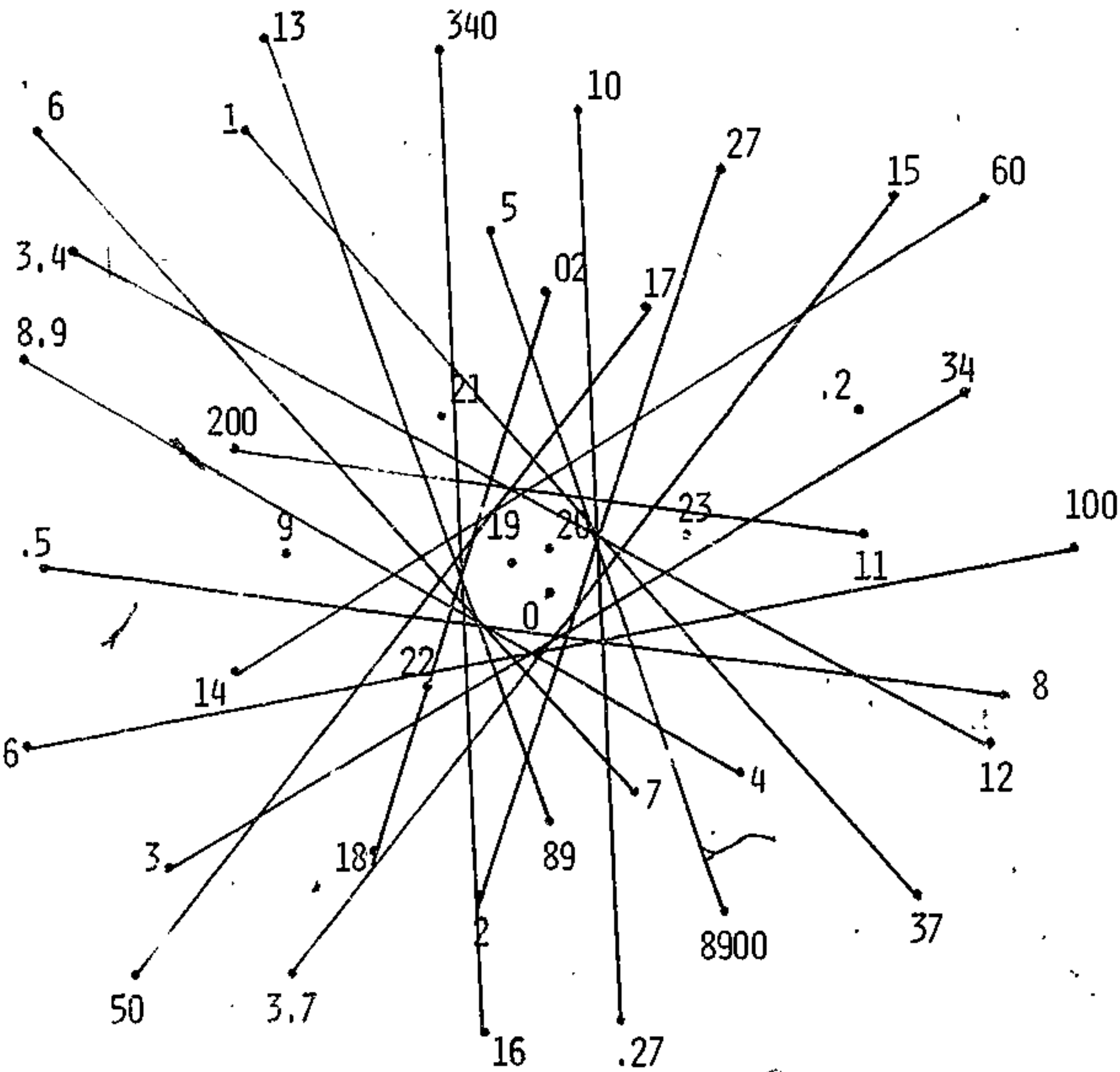
1 MM + 1 MICRON =

1 KM 1 HM 1 DKM 1 M 1 DM 1 CM 1 MM 1 MICRON





OPERATIONS WITH MASS MEASUREMENTS



CONNECT THE NUMBER OF EACH PROBLEM WITH THE ANSWER FOR THAT PROBLEM.

1. 9 G + 28 G = 37 G
2. 73 T - 46 T = 27 T
3. 15 KG + 19 KG = 34 KG
4. 8 G + 900 MG = 8.9 G
5. 8 G + 900 MG = 8900 MG
6. 3 G - 2.9 G = 100 MG
7. 256 MG + 344 MG = .6 G
8. 454 G + 45 G = .5 KG
9. 1 KG - 800 G = .2 KG
10. 1 G - 730 MG = .27 G
11. .1 G + 100 MG = 200 MG
12. 2345 MG + 1055 MG = 3.4 G
13. 1 G - 911 MG = 89 MG
14. 59 G + 1000 MG = 60 G
15. 3 KG + 700 G = 3.7 KG
16. 1 T - 660 KG = 340 KG
17. .042 G + 8 MG = 50 MG
18. 13 MG + 7 MG = .02 G

VOLUME

CUT OUT THE SQUARES. FIT THEM TOGETHER SO THAT THE TOUCHING EDGES NAME THE SAME VOLUMES.

(1) .56 M^3 (3) (4) 3.4 M^3 .034 ℓ (2) 1 Mℓ	(5) .42 Mℓ (7) (6) 340 Mℓ 7 ℓ (8) 56 CM^3	(9) 2 M^3 (11) (10) 1 CM^3 1 ℓ (12) 42 CM^3	(13) 560 Mℓ (14) (15) 60 ℓ 1000 ℓ (16) 2 DM^3
(8) 56 Mℓ (18) 2 Mℓ 60 Mℓ (17) (19) .002 ℓ	(20) 370 Mℓ 6 ℓ 2000 ℓ (21) (22) (23) 1 M^3	(2) 1 CM^3 1 M^3 34 ℓ (15) (24) (9) 2000 ℓ	(25) 56 Mℓ 1000 Mℓ 3.4 CM^3 (10) (26) (27) 37 ℓ
(23) 1000 DM^3 .7 Mℓ 3400 ℓ (28) (3) (13) .56 ℓ	(29) 5.6 Mℓ 3.4 Mℓ .6 Mℓ (26) (30) (31) .42 ℓ	(32) 37 Mℓ 2000 Mℓ .34 ℓ (33) (7) (34) 1 ℓ	(19) 2 CM^3 3400 CM^3 7 Mℓ (35) (36) (29) .0056 ℓ
(37) 4.2 ℓ 2 M^3 2 ℓ (22) (33) (1) 560 ℓ	(16) 2000 CM^3 70 Mℓ .001 ℓ (38) (11) (39) 3.7 ℓ	(40) 5600 Mℓ 34000 Mℓ 3.4 ℓ (24) (35) (25) .056 ℓ	(34) 1 DM^3 34 CM^3 2 CM^3 (4) (17) (40) 5.6 ℓ

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	1	2	3	4	5	6	7	8	9	
									or	
0		S	S	H	T		N	R		
1	Y	P	H	N	H		I	G	E	V
2		N	I	P	R	E	F	E	S	T
3		Z	P	A	L		S		T	G
4					N	A	X	B	N	E
5		X	L	Y		T	E	S	V	S
6		R	T	F	E	W	Y	T	I	N
7				L	I	H	R		H	I
8	Z				O		T	L	G	H
9	X			E	G		O	A		

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	10	11	12	13	14	15	16	17	18
0	H	N	I	F	D	S	W	L	D
1	K	O	N	A	O	C	R	O	U
2	A	S	T	I	U	H	I	N	M
3	E	U	E	N	B	E	T	G	B
4	G	R	R	T	L	D	E	D	W
5	J	E	P	B	E	U	P	R	A
6	B	P	L	R	C	L	L	E	I
7	C	A	A	U	H	I	A	A	T
8	D	T	Y	S	I	N	Y	M	E
9	F	H	S	H	N	G	S	S	R

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1.
```
        971
  53 ) 41463
       377
        376
        371
         53
         53
```

2.
```
   645
   721
   645
  1290
 4515
465045
```

3.
```
   478
   863
  1434
 2868
3824
412514
```

4.
```
         859
  347 ) 298053
        2776
         2045
         1735
          3103
          3103
```

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5.
```
        997
  35 ) 34895
       315
        339
        315
         245
         245
```

6.
```
  457
   38
 3656
1371
17366
```

7.
```
      90809
  12 ) 1089708
       108
         97
         96
          108
          108
```

8.
```
2228
1261
3489
```

9.
```
 113
 624
 147
 526
1410
```

10.
```
  47
  53
 141
235
2491
```

11.
```
4472
4351
 121
```

12.
```
   5467
    898
  43736
 49203
43736
4909366
```

13.
```
6235
5828
 407
```

14.
```
       181
  36 ) 6516
       36
       291
       288
         36
         36
```

MAGIC SQUARES: SHEET 1

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF THE NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 15.

8	1	6
3	5	7
4	9	2

8	1	6	8	3	4	8	6
3	5	7	1	5	9	5	5
4	9	2	6	7	2	2	4
15	15	15	15	15	15	15	15

COMPLETE THESE MAGIC SQUARES USING THE SAME NUMBERS 1 THROUGH 9. BE SURE EACH ROW, COLUMN, AND DIAGONAL ADDS TO 15.

6	1	8
7	5	3
2	9	4

4	3	8
9	5	1
2	7	6

2	9	4
7	5	3
6	1	8

8	3	4
1	5	9
6	7	2

6	7	2
1	5	9
8	3	4

4	9	2
3	5	7
8	1	6

CAN YOU FIND ANOTHER MAGIC SQUARE USING THE SAME NUMBERS 1 THROUGH 9?

4	3	8
9	5	1
2	7	6

MAGIC SQUARES: SHEET 2

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF THE NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 12.

7	0	5
2	4	6
3	8	1

7	0	5	7	2	3	7	5
2	4	6	0	4	8	4	4
3	8	1	5	6	1	1	3
12	12	12	12	12	12	12	12

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

16	9	14
11	13	15
12	17	10

ADD 9
TOTAL 39

84	0	60
24	48	72
36	96	12

MULTIPLY BY 12
TOTAL 144

$7\frac{1}{2}$	$\frac{1}{2}$	$5\frac{1}{2}$
$2\frac{1}{2}$	$4\frac{1}{2}$	$6\frac{1}{2}$
$3\frac{1}{2}$	$8\frac{1}{2}$	$1\frac{1}{2}$

ADD $\frac{1}{2}$
TOTAL $13\frac{1}{2}$

70	0	50
20	40	60
30	80	10

MULTIPLY BY 10
TOTAL 120

$7\frac{1}{4}$	$\frac{1}{4}$	$5\frac{1}{4}$
$2\frac{1}{4}$	$4\frac{1}{4}$	$6\frac{1}{4}$
$3\frac{1}{4}$	$8\frac{1}{4}$	$1\frac{1}{4}$

ADD $\frac{1}{4}$
TOTAL $12\frac{3}{4}$

49	0	35
14	28	42
21	56	7

MULTIPLY BY 7
TOTAL 84

IF THE SAME NUMBER IS ADDED TO EACH ENTRY IN A MAGIC SQUARE, IS THE RESULT ANOTHER MAGIC SQUARE? ____________________

IF EACH ENTRY IN A MAGIC SQUARE IS MULTIPLIED BY THE SAME NUMBER, IS THE RESULT ANOTHER MAGIC SQUARE? ____________________

MAGIC SQUARES: SHEET 3

THIS ARRAY OF NUMBERS IS A 3 X 3 MAGIC SQUARE. SHOW THAT THE SUM OF NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 21.

10	3	8
5	7	9
6	11	4

10	5	6	10	3	8	10	8
3	7	11	5	7	9	7	7
8	9	4	6	11	4	4	6
21	21	21	21	21	21	21	21

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

13	6	11
8	10	12
9	14	7

ADD 3

TOTAL 30

7	0	5
2	4	6
3	8	1

SUBTRACT 3

TOTAL 12

$10\frac{1}{3}$	$3\frac{1}{3}$	$8\frac{1}{3}$
$5\frac{1}{3}$	$7\frac{1}{3}$	$9\frac{1}{3}$
$6\frac{1}{3}$	$11\frac{1}{3}$	$4\frac{1}{3}$

ADD $\frac{1}{3}$

TOTAL 22

5	$1\frac{1}{2}$	4
$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$
3	$5\frac{1}{2}$	2

DIVIDE BY 2

TOTAL $10\frac{1}{2}$

$9\frac{1}{3}$	$2\frac{1}{3}$	$7\frac{1}{3}$
$4\frac{1}{3}$	$6\frac{1}{3}$	$8\frac{1}{3}$
$5\frac{1}{3}$	$10\frac{1}{3}$	$3\frac{1}{3}$

SUBTRACT $\frac{2}{3}$

TOTAL 19

$9\frac{1}{4}$	$2\frac{1}{4}$	$7\frac{1}{4}$
$4\frac{1}{4}$	$6\frac{1}{4}$	$8\frac{1}{4}$
$5\frac{1}{4}$	$10\frac{1}{4}$	$3\frac{1}{4}$

SUBTRACT $\frac{3}{4}$

TOTAL $18\frac{3}{4}$

MAGIC SQUARES: SHEET 4

THIS ARRAY OF NUMBERS IS A 3 x 3 MAGIC SQUARE. SHOW THAT THE SUM OF NUMBERS IN EACH ROW, COLUMN, AND DIAGONAL IS 45.

26	1	18
7	15	23
12	29	4

26	1	18	26	7	12	26	18
7	15	23	1	15	29	15	15
12	29	4	18	23	4	4	12
45	45	45	45	45	45	45	45

PERFORM THE INDICATED OPERATIONS ON EACH NUMBER IN THE MAGIC SQUARE ABOVE. ENTER THE RESULTS BELOW. THEN SEE IF EACH NEW ARRAY IS ALSO A MAGIC SQUARE.

35	10	27
16	24	32
21	38	13

ADD 9

TOTAL 72

13	$\frac{1}{2}$	9
$3\frac{1}{2}$	$7\frac{1}{2}$	$11\frac{1}{2}$
6	$14\frac{1}{2}$	2

DIVIDE BY 2.

TOTAL $22\frac{1}{2}$

$26\frac{1}{3}$	$1\frac{1}{3}$	$18\frac{1}{3}$
$7\frac{1}{3}$	$15\frac{1}{3}$	$23\frac{1}{3}$
$12\frac{1}{3}$	$29\frac{1}{3}$	$4\frac{1}{3}$

ADD $\frac{1}{3}$

TOTAL 46

$8\frac{2}{3}$	$\frac{1}{3}$	6
$2\frac{1}{3}$	5	$7\frac{2}{3}$
4	$9\frac{2}{3}$	$1\frac{1}{3}$

DIVIDE BY 3

TOTAL 15

$25\frac{1}{4}$	$\frac{1}{4}$	$17\frac{1}{4}$
$6\frac{1}{4}$	$14\frac{1}{4}$	$22\frac{1}{4}$
$11\frac{1}{4}$	$28\frac{1}{4}$	$3\frac{1}{4}$

SUBTRACT $\frac{3}{4}$

TOTAL $42\frac{3}{4}$

$25\frac{1}{3}$	$\frac{1}{3}$	$17\frac{1}{3}$
$6\frac{1}{3}$	$14\frac{1}{3}$	$22\frac{1}{3}$
$11\frac{1}{3}$	$28\frac{1}{3}$	$3\frac{1}{3}$

SUBTRACT $\frac{2}{3}$

TOTAL 43

MAGIC SQUARES: SHEET 5

FILL IN THE BLANK SPACES IN THE MAGIC SQUARE USING EACH OF THE FOLLOWING NUMBERS: 4, 6, 7, 8, 10, 11, 12, 13, 14, 15, SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

14	1	8	11
15	5	4	10
2	16	9	7
3	12	13	6

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

14	15	2	3
1	5	16	12
8	4	9	13
11	10	7	6

3	2	15	14
12	16	5	1
13	9	4	8
6	7	10	11

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MAGIC SQUARES: SHEET 6

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 34.

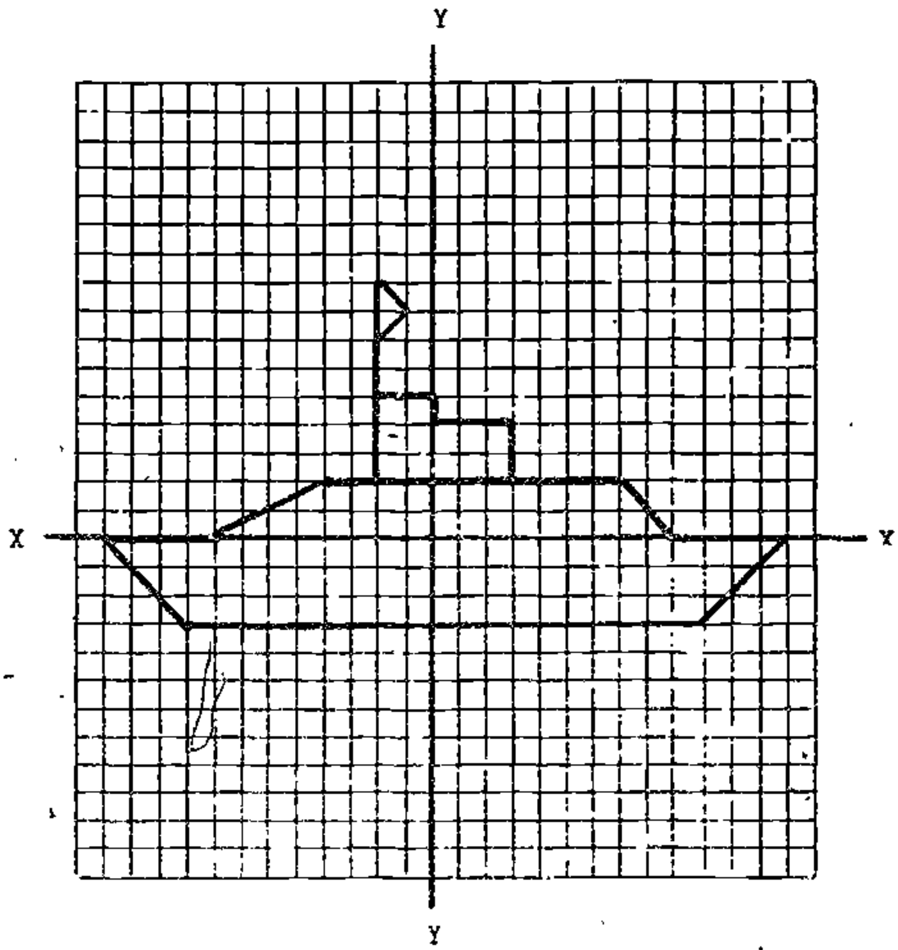
16	2	3	13
5	11	10	8
9	7	6	12
4	14	15	1

14	11	5	4
1	8	10	15
12	13	3	6
7	2	16	9

FILL IN THE BLANK SPACES IN THE MAGIC SQUARES SO THAT THE SUM OF THE INTEGERS IN EACH COLUMN, ROW, AND DIAGONAL IS 65.

11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

1)



16

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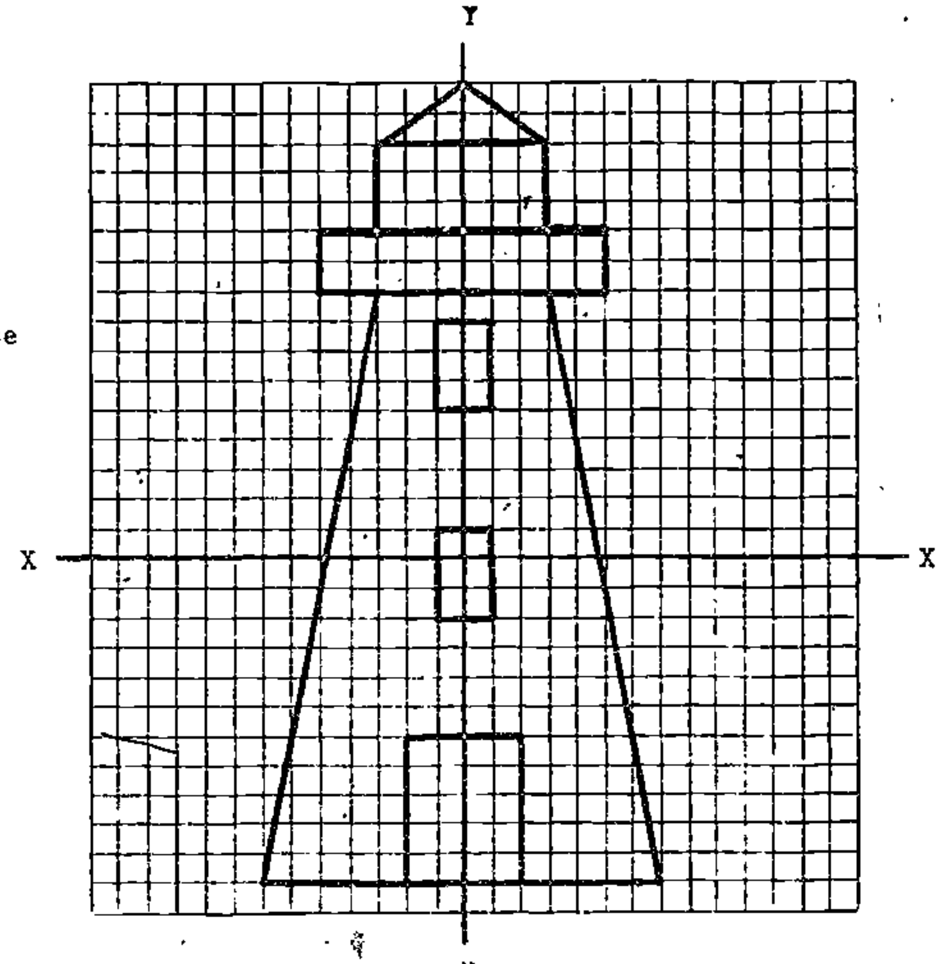
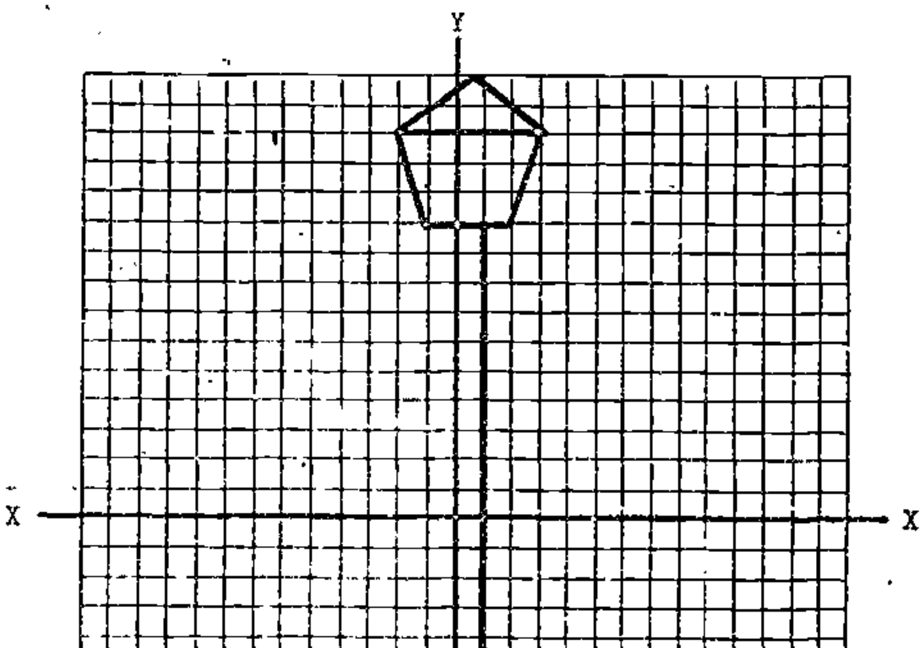
2)

Lighthouse



3)

Lantern



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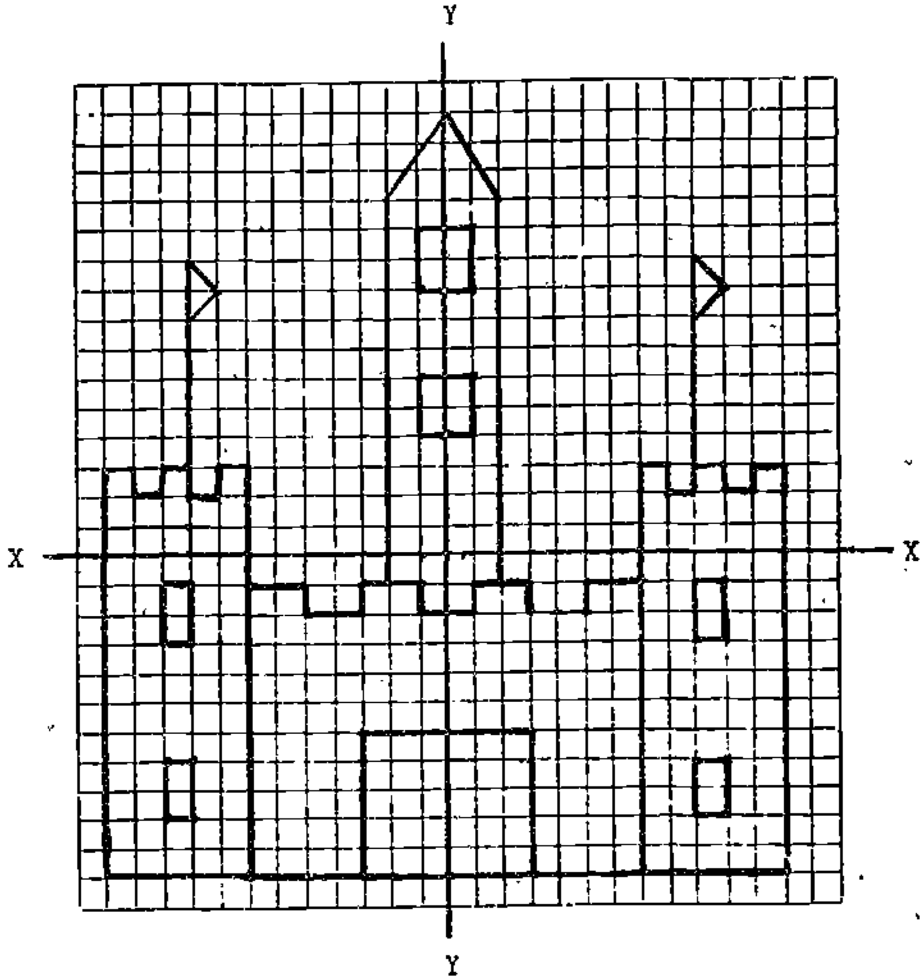
4)

Castle



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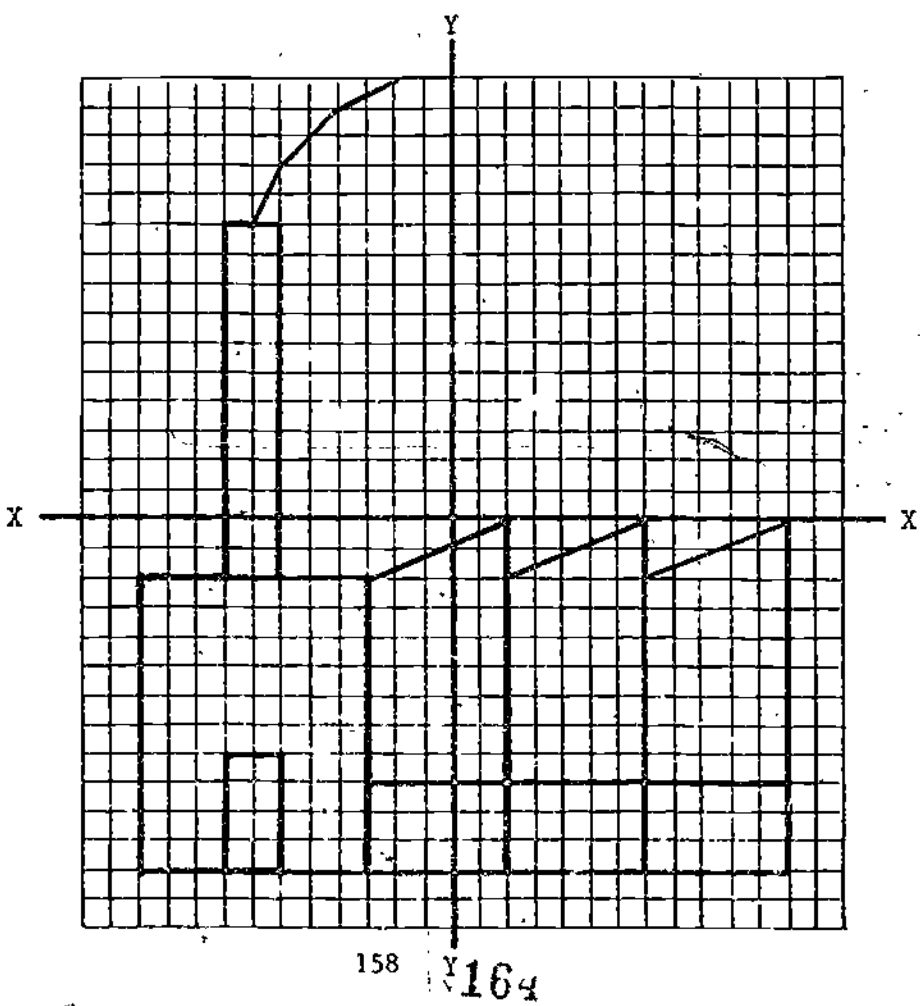
5)

Factory



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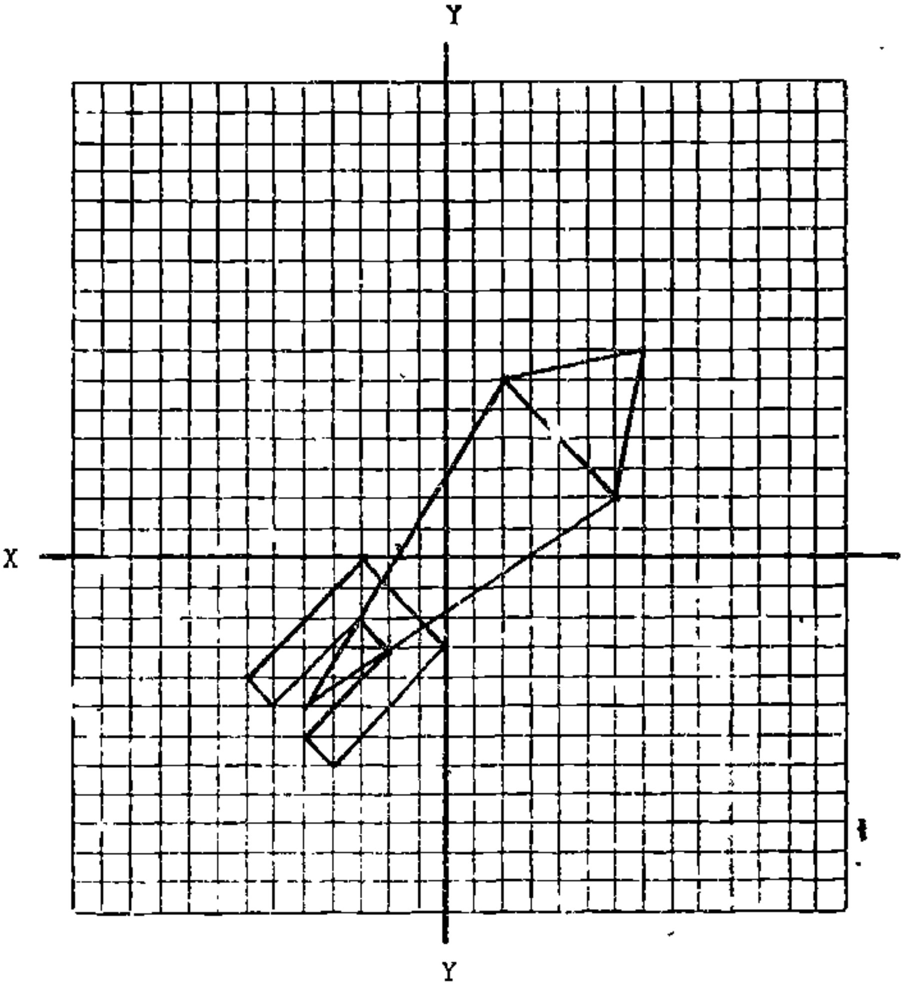
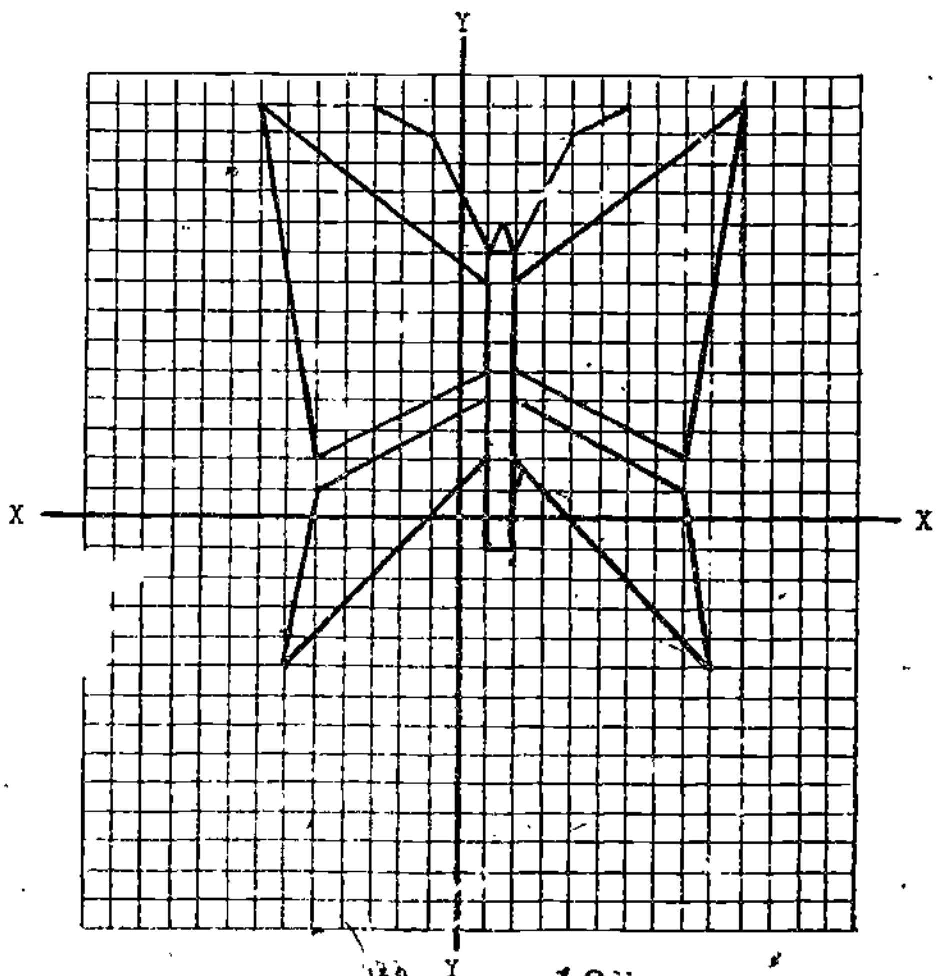
6)

Rocket



7)

Butterfly



Page 72

	At least
squares	3
rectangles	6
parallelograms	11
quadrilaterals	17
rhombuses	5
triangles	10
trapezoids	4

Page 73

	At least
rectangles	28
triangles	96
parallelograms	42
trapezoids	72
quadrilaterals	121

Page 74

	At least
squares	12
rectangles	21
parallelograms	39
triangles	26
cubes	4
prisms with trapezoidal bases	6
prisms	16
trapezoids	32
triangular prisms	6

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Page 76

A. 4
B. 1
C. 6
D. 5

Page 77

1. 36
2. 24
3. 18
4. 12
5. 2
6. 8

Page 78

triangles 100

Page 82

1. $35.83
2. $16.65
3. $1.98
4. $5.86
5. You need to know how many ice breakers Sarah wants.

Page 83

1. $4.84
2. $5.05
3. $3.68
4. 5 pounds

Page 84

1. refrigerator
2. $ 224.90
3. $999.90
4. $1034.90
5. $1189.90
6. Buy now

Page 85

1. $ 24.95
2. $ 41.90
3. $159.85
4. $ 59.85

Page 91

1. 17
2. 18
3. 28
4. 36
5. 1.36
6. 105
7. 22
8. 3.1
9. 7.5
10. 11.4
11. 9.5
12. 7.6
13. 13.8
14. .97
15. 6.8
16. 1.22
17. 5.4
18. .5
19. 1.1
20. 11.7
21. 12.1
22. 2.3
23. 1.0
24. 1.2
25. 7.68
26. 11.36
27. 6.56
28. 12.21
29. 4.81
30. 11.45
31. .91
32. 1.04

Page 92

1. 12, 14, 16, 18
2. 32, 64, 128, 256
3. 36, 49, 64, 81
4. 216, 343, 512, 729
5. 17, 21, 24, 27
6. 17, 19, 23, 29
7. 161051, 1771561, 19487171, 214358881
8. 21, 28, 36, 45
9. 34, 55, 89, 144
10. 9, 11, 13, 15
11. 81, 243, 729, 2187
12. 13, 16, 19, 22
13. 144, 288, 576, 1152
14. 243, 729, 2187, 6561
15. 162, 486, 1458, 4374

Page 93

1. The sum of a specific number of consecutive odd numbers is that specific number squared.
2. 49
3. 144
4. 400

There are a number of different answers. Here are some possible answers for 5-10.

5. $1 + 2 + 3 + 4 + 5 + 6 + 7 + (8 \times 9) = 100$
6. $(4 \times 5) + (7 \times 8) + (3 \times 6) + 9 - 2 - 1 = 100$
7. $(5 \times 7) + 8 - 4 + (3 \times 2) + (9 \times 6) + 1 = 100$
8. $(7 \times 8) + 45 + 1 - (36 \div 9) + 2 = 100$
9. $72 + 38 - (45 \div 9) + 1 - 6 = 100$
10. $92 + (56 \div 7) - 3 - 4 + 8 - 1 = 100$

Page 94

1. 28
2. 36
3. 45
4. 55
5. 66
6. yes
7. $\frac{n}{2}(1 + n)$

There are a number of different answers. Here are some possible ones for 8-10.

8. $123 - 45 - 67 + 89 = 100$
9. $79 + 56 - 32 + 1 - 8 + 4 = 100$
10. $89 - 62 + 71 + 4 - 5 + 3 = 100$

Page 98

1. $5960 = (5 \times 10^3) + (9 \times 10^2) + (6 \times 10^1) + (0 \times 10^0)$

   $= (5 \times 100) + (9 \times 100) + (6 \times 10) + (0 \times 1)$

   $= (5000 + 900 + 60 + 0$

   $= 5960$

2. $23,579 = (2 \times 10^4) + (3 \times 10^3) + (5 \times 10^2) + (7 \times 10^1) + (9 \times 10^0)$

   $= (2 \times 10000) + (3 \times 1000) + (5 \times 100) + (7 \times 10) + (9 \times 1)$

   $= 20000 + 3000 + 500 + 70 + 9$

   $= 23,579$

3. $5294 = (5 \times 10^3) + (2 \times 10^2) + (9 \times 10^1) + (4 \times 10^0)$

   $= (5 \times 1000) + (2 \times 100) + (9 \times 10) + (4 \times 1)$

   $= (5000 + 200 + 90 + 4)$

   $= 5294$

4. $36,058 = (3 \times 10^4) + (6 \times 10^3) + (0 \times 10^2) + (5 \times 10^1) \quad \times 10^0)$

   $= (3 \times 10000) + (6 \times 1000) + (0 \times 100) + (5 \times 10) + (8 \times 1)$

   $= 30000 + 6000 + 0 + 50 + 8$

   $= 36058$

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1. 22_{five} 2. 42_{five} 3. 43_{five}

4.

Place Value

	10^4	10^3	10^2	10^1	10^0
Base Ten	10x10x10x10 10,000	10x10x10 1000	10 x 10 100	or 10	or 1
Base Five	5^4	5^3	5^2	5^1	5^0
	5x5x5x5 625	5x5x5 125	5x5 25	or 5	or 1

5. $132_{five} = (1 \times 5^2) + (3 \times 5^1) + (2 \times 5^0)$

   $= (1 \times 25) + (3 \times 5) + (2 \times 1)$

   $= 25 + 15 + 2$

   $= 42$

6. $2034_{five} = (2 \times 5^3) + (0 \times 5^2) + (3 \times 5^1) + (4 \times 5^0)$

$= (2 \times 125) + (0 \times 25) + (3 \times 5) + (4 \times 1)$

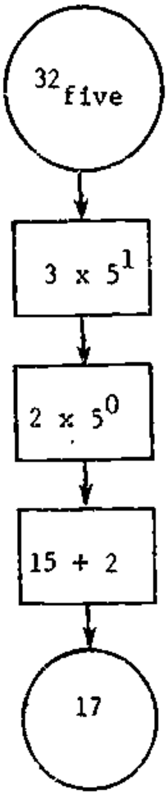
$= 250 \quad + \quad 0 \qquad 15 \quad + \quad 4$

$= 269$

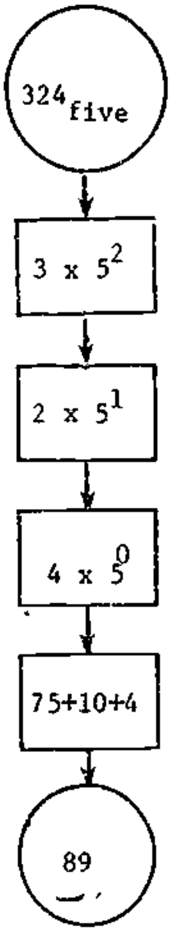
Page 104

7. 23
8. 183
9. 292
10. 124
11. 515
12. 867

13.



14.



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1. $5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$\longleftarrow\ \longrightarrow$ 67

$5^3 = 125$

$67 = 232_{five}$

$$25 \overline{)67} \quad 2$$
$$-50$$
$$5 \overline{)17} \quad 3$$
$$15$$
$$1 \overline{)2} \quad 2$$
$$\frac{2}{0}$$

2. 213_{five}

3. 100_{five}

4. 444_{five}

5. 341_{five}

6. 2430_{five}

7. 4000_{five}

Page 113

1. 100_{five}
2. 1130_{five}
3. 2003_{five}
4. 4_{five}
5. 43_{five}
6. 244_{five}
7. 141_{five}
8. 2444_{five}
9. 20433_{five}

Page 120

1. Binary Numeration System
2. There are no 2's in base two.
3. Six
4. $1101_{two} = (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
   $= (1 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$
   $= 8 + 4 + 0 + 1$
   $= 13$
5. $1010_{two} = (1 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0)$
   $= (1 \times 8) + (0 \times 4) + (1 \times 2) + (0 \times 1)$
   $= 8 + 0 + 2 + 0$
   $= 10$
6. $10101_{two} = (1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0)$
   $= (1 \times 16) + (0 \times 8) + (1 \times 4) + (0 \times 2) + (1 \times 1)$
   $= 16 + 0 + 4 + 0 + 1$
   $= 21$
7. $1001001_{two} = (1 \times 2^6) + (1 \times 2^3) + (1 \times 2^0)$
   $= (1 \times 64) + (1 \times 8) + (1 \times 1)$
   $= 64 + 8 + 1$
   $= 73$

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8. 1001_{two}

9. 10110_{two}

10. 10100_{two}

11. 100110_{two}

12. 10111_{two}

13. 1001111_{two}

14. 10011101_{two}

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15. 1011_{two}

16. 10100_{two}

17. 101_{two}

18. 100_{two}

19. 1010_{two}

20. 110010_{two}

21. 10000_{two}

22. 111_{two}

Page 135

1. 1900
2. 600
3. 2000
4. 7100
5. 8200
6. 11400
7. 2400
8. 5700
9. 1800
10. 6000
11. 21300
12. 24600
13. 34200
14. 7200
15. 3800

Page 135 (cont'd)

16. 1200
17. 4000
18. 14200
19. 16400
20. 22800
21. 4800
22. 1056
23. 5016
24. 47938
25. 102036
26. 35651
27. 91003
28. 850
29. 6300
30. 74000

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1. 735
2. 365
3. 8074
4. 2951
5. 95501
6. 7488
7. 2250
8. 21600
9. 44400
10. 8556
11. 6075
12. 2300
13. 540
14. 270
15. 9120
16. 8742
17. 21252
18. 12500
19. 26300
20. 3600
21. 6360
22. 980
23. 12045
24. 6900
25. 9312
26. 6116
27. 126800
28. 660
29. 61800
30. 12500

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1. 180
2. 705
3. 3471
4. 45501
5. 638
6. 1116
7. 1800
8. 26800
9. 68400
10. 9215
11. 9482
12. 600
13. 270
14. 9024
15. 240
16. 9009
17. 1859
18. 800
19. 4800
20. 2400
21. 13890
22. 480
23. 102872
24. 6600
25. 9025
26. 12045
27. 52100
28. 1080
29. 32700
30. 462700

Page 138

1. 225
2. 625
3. 1225
4. 2025
5. 3025
6. 4225
7. 5625
8. 7225
9. 9025
10. 11025
11. 13225
12. 15625
13. 18225
14. 21025
15. 24025

Page 139

1. 2016
2. 3024
3. 9016
4. 5624
5. 7209
6. 4224
7. 5609
8. 609
9. 1216
10. 216
11. 221
12. 616
13. 1224
14. 1209
15. 1221

DIVISIBILITY

	2	3	4	5	6	7	8	9	10	11	25	50
7265				X								
432	X	X	X		X		X	X				
17526	X	X			X							
3400	X			X			X		X		X	X
18243		X						X				
896	X		X			X	X					
15275				X							X	
9245				X								
3933		X						X				
4785		X		X						X		
19273												
23925		X		X						X	X	
9200	X		X	X			X		X		X	X
5830	X			X					X	X		
10186	X									X		
763						X						
3113										X		
92564	X		X									
7250	X			X					X		X	X
18725				X		X					X	
4900	X		X	X		X			X		X	X