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ABSTRACT

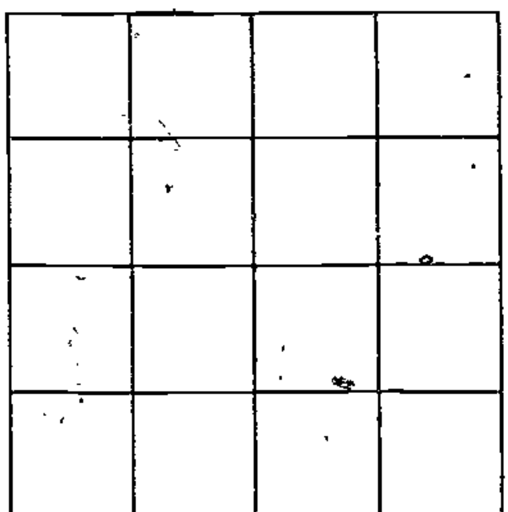
Several math activities designed to teach patterns which can be found in arrays of 16 squares organized into a four-by-four pattern are presented in this issue of "Student Math Notes." The activities include: (1) determining how many squares and rectangles are contained in a four-by-four array; (2) figuring out the least number of lines which can be removed from the array so that it contains no squares; (3) finding the shortest distance from one corner of the array to the other; (4) dividing the array into congruent halves; and (5) forming words out of letters which have been placed in each of the 16 squares of the array. Some activities based on a three-by-three array are also included. (DC)

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
STUDENT math notes
JANUARY 1984

4 × 4 Square Arrays

1. How many **squares** are in a 4 × 4 square array like the one shown below? One thing sure is that there are more than just one. Indeed, there are more than 16 squares. Just how many different squares of the same or different sizes can you find?

_____ squares

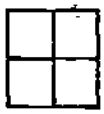
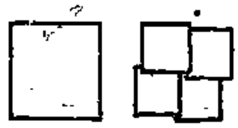
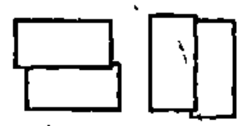
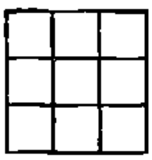
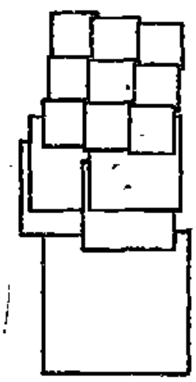
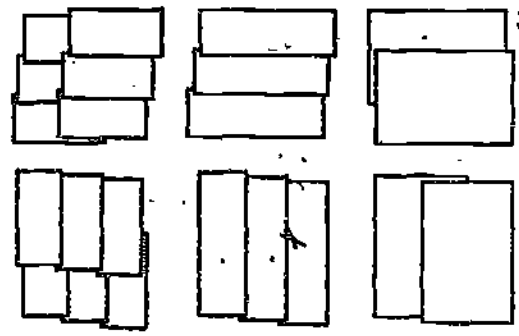


2. Here is another counting problem for a 4 × 4 square array. How many different **rectangles** of all sizes can you find? Count carefuly and systematically and look for patterns. Be sure to include all the squares you counted.

_____ rectangles

Patterns in Counting

You can find some interesting number patterns in counting squares and rectangles in square arrays. It s easiest to spot them by first reducing the problem to 3 × 3, 2 × 2, or even 1 × 1 arrays.

Array	Figure	Number of Squares	Number of all Rectangles including Squares
1×1		1 	1
2×2		5 	 9
3×3		14 	 36

The numbers 1, 5, and 14 are sums of squares.

1 × 1 array: $1 = 1 = 1^2$
2 × 2 array: $5 = 1 + 4 = 1^2 + 2^2$
3 × 3 array: $14 = 1 + 4 + 9 = 1^2 + 2^2 + 3^2$

The numbers 1, 9, and 36 are sums of cubes.

1 × 1 array: $1 = 1 = 1^3$
2 × 2 array: $9 = 1 + 8 = 1^3 + 2^3$
3 × 3 array: $36 = 1 + 8 + 27 = 1^3 + 2^3 + 3^3$

Use these patterns to predict the number of squares and rectangles of all sizes in a 4 × 4 array.

_____ squares _____ rectangles

Did you get the same answers you found when counting on page 1?

A checkerboard is an 8 × 8 array of squares. How many squares of all sizes do you think it contains?

_____ squares

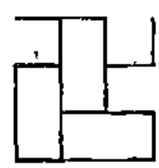
How many rectangles of all sizes do you think it contains?

_____ rectangles

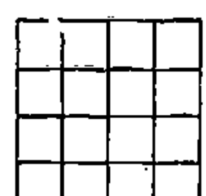
Removing 1 small segment of the figure makes a 1 × 1 array have no squares.

Removing 3 small segments can make a 2 × 2 array have no squares at all.

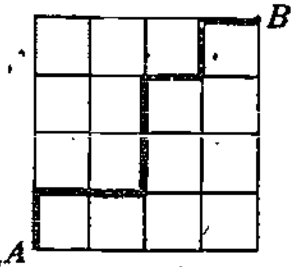
Removing 6 small segments can make a 3 × 3 array have no squares at all.



What is the least number of small segments that need to be removed from a 4 × 4 array so that it too has no squares at all? Show one solution by circling the segments you would like removed.



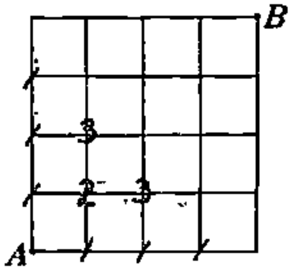
Taking a Walk



Imagine a 4 × 4 array of squares as representing 16 city blocks. There are many different ways to walk from A to B along the streets. One possible path from A to B is shown. How many blocks are in the shortest possible paths from A to B? ______ blocks

For the shortest walk, you must always move up or to the right for each block. How many different walks of this shortest length can you take from A to B? ______ walks

Here's one way to save time and avoid doing a lot of tedious counting in answering the same question. First, consider all possible 2-block walks, moving up or to the right. Mark each corner as shown below with the total number of ways you can get there.

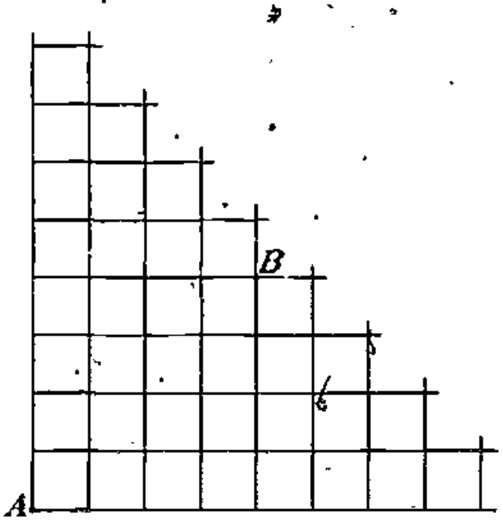


To fill in the number for all 3-block walks up and to the right, merely add the numbers of the preceding corners from which you must come.

Now continue this process, marking each corner in the 4 × 4 array with the appropriate number. How many different shortest walks can you take from A all the way to B? ______ walks

The total number of 1-block walks up or to the right from A is 1 + 1, or 2.
The total number of 2-block walks of the same kind from A is 1 + 2 + 1, or 4.
The total number of similar 3-block walks from A is 1 + 3 + 3 + 1, or 8.

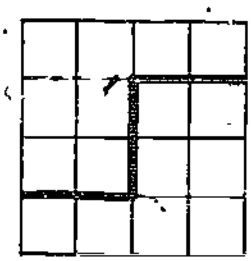
Study the sums above. Do you see a pattern? What do you think will be the total for all 4-block walks up and to the right from A? Check your answer.



Every 8-block walk from A up and to the right on the 4 × 4 array shown above will take you to B. But imagine extending the boundary so that there are a great many square blocks up and to the right of A. What would be the total number of 8-block walks possible starting at A and always moving up or to the right at each corner? ______ walks

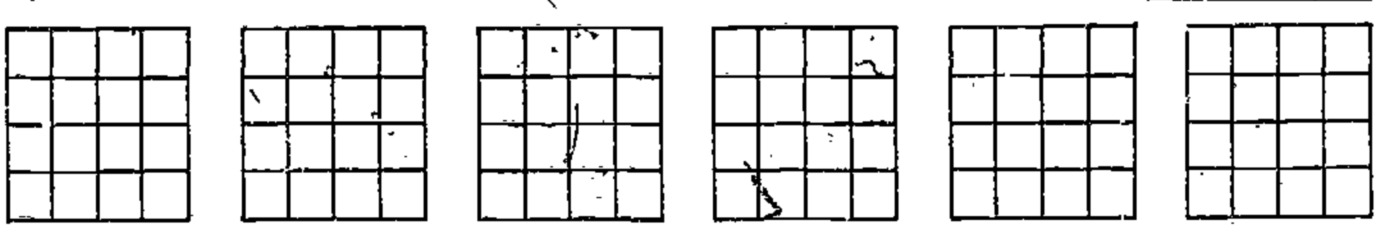
If you chose one of these many 8-block walks at random, what would be your chance of arriving at B? (*Hint:* Compare the number of 8-block walks that end at B to all such possible walks.) ______

Cutting Up



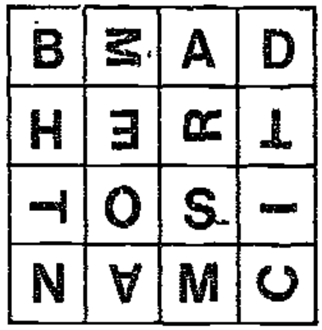
Some paths on a 4 × 4 array are of particular interest. For example, this path across the array cuts the array into two congruent halves.

Show as many other paths as you can find that cut the 4 × 4 array of squares into two congruent halves. Don't count paths that cut the array into the same shapes as those already counted. And remember, stay on the lines.



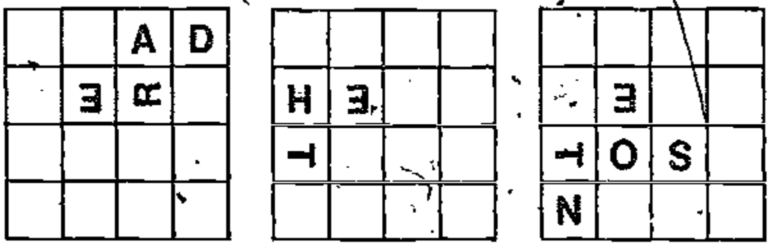
Study all the paths you have found. Do they all pass through the center point of the array? Are they all symmetric about that point? Explain why the answers to these questions must be yes.

4 × 4 Word Search



Sixteen letters have been placed in this 4 × 4 square array. The array contains many words formed using letters in squares that are connected either by an edge or a vertex. No letter in a given position can be used more than once in the same word.

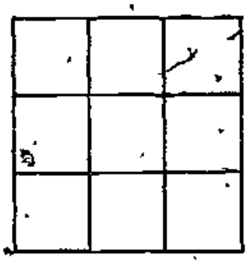
Here are some examples:



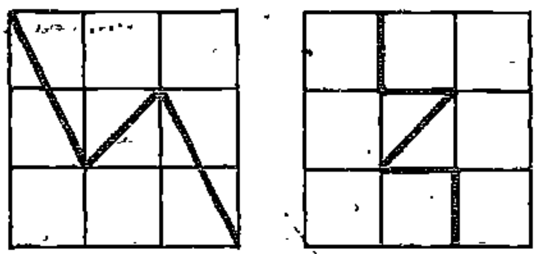
READ THE NOTES

See how many more words you can find. One has 11 letters!

Bet you can't answer these . . .



- Can you cut a 3 × 3 square array into two congruent halves cutting only on grid lines?



- There are 13 ways to cut a 3 × 3 square array into two congruent halves by cutting along a polygonal path with vertices located on the grid. Here are two examples. Try to find the rest. Remember, don't count paths in different locations that give the same shaped halves.
- Find the length of each of the different paths found in the preceding problem assuming the original array measured 3 cm × 3 cm.
- Read about the famous French mathematician and philosopher Blaise Pascal (1623–1662). Find out what he has to do with the random walk problem given on page 3.

Our thanks to the readers who noted the error in Viète's expression for π as given in the November issue. The numerator should have been 2 and not 1.

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