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ABSTRACT

The educational background of students termed "limited English proficient" (LEP) is discussed, with consideration of how that background might affect the LEP student's learning of arithmetic. Reasons why knowledge of background is important are first noted. Then examples of different ways to read and write numerals and differing subtraction and division algorithms are presented, to illustrate how LEP students might have learned to approach arithmetic in ways that differ from those typically taught. Implications of these differences and some specific suggestions for instruction are discussed. Finally, some general conclusions and recommendations are made. (MNS)

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THE EDUCATIONAL BACKGROUND OF LIMITED ENGLISH PROFICIENT STUDENTS: IMPLICATIONS FOR THE ARITHMETIC CLASSROOM

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This paper is based on numerous workshops given to teachers of limited English proficient students - bilingual, ESL, math and other mainstream teachers, their aides, and program administrators. These workshops were supported by the Office of Bilingual Education and Minority Languages Affairs, U.S. Department of Education through Grants # G008007106 and # G008200708. Any opinions, findings, conclusions and recommendations in this manuscript are those of the author and do not necessarily reflect the views of OBEMLA nor DOE.

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THE EDUCATIONAL BACKGROUND OF LIMITED ENGLISH PROFICIENT STUDENTS: IMPLICATIONS FOR THE ARITHMETIC CLASSROOM

ABSTRACT

A limited English proficient student's prior education, particularly that of a recent immigrant to the United States, can have implications for how that student should be taught arithmetic. Different educational systems can follow different conventions for reading and writing numbers. Students in those systems can learn algorithms for subtraction and for division which follow a different sequence of steps, employ different number skills and have different underlying thought processes than do the algorithms taught in the United States. Examples of these different conventions and algorithms and recommendations for modifying curricular objectives, instructional methods and testing of LEP students are given.

Running head: The Educational Background of LEP Students

THE EDUCATIONAL BACKGROUND OF LIMITED ENGLISH PROFICIENT STUDENTS: IMPLICATIONS FOR THE ARITHMETIC CLASSROOM

A student who has been exposed to a language other than English while growing up (called the "home language") and whose subsequent command of English (oral and academic skills) is less than that of his English-speaking age mates is considered to be limited English proficient (LEP) (Federal Register, 1980, 45 CFR 123.4). LEP students come from a variety of backgrounds: they may be native born Americans or immigrants; their native languages might be as common as Spanish, Arabic, Vietnamese, Thai, Lao, Hmong, or as rare as Hausa, or Malay are in the Midwest (Secada, 1982); their command of English might range from virtually nil to a superficially strong command of spoken English; they range from impoverished to upper class members of the elite; finally, and the concern of this paper, they might have little or a great deal of education prior to entering the American school system. All of these factors need to be considered in the LEP student's education. The purpose of this paper is to focus on one factor the student's educational background; and to describe how that factor might affect that student's learning of arithmetic.

There are many reasons why an arithmetic teacher should be concerned with a student's educational background. The first group of reasons have to do with specific ties between what a student knows and the curriculum, methods of instruction, or testing. A student might already know a specific topic and not need to relearn it; he might know it, but not as fully as desired.

In either case, the curriculum for that student could change. Furthermore, the student's knowledge of a topic might take a form different from what is commonly taught in the United States (see the examples below). In this case, the arithmetic teacher would need to modify her instructional style so as not to conflict with how the student is understanding and approaching the given topic. Further, testing should be modified to allow the student to show what he knows and the teacher's grading should not penalize the LEP student who "gets the right answer but did it the wrong way (Footnote 1)."

A second group of reasons for attending to a LEP student's educational background concerns a shifting of focus from what the student cannot do (i.e., his limitations in English) to what he can. The arithmetic teacher not only should think about what the student needs to learn, but also should plan on exploiting what the student already knows. The LEP student who may have learned some things differently will see this concern for him as a sign that he is a valued member of the class. His self-esteem can only rise. Furthermore, if he is asked to show other students how he approaches problems, he will become a resource for enrichment lessons in arithmetic and his competence in English will increase as he becomes more fully integrated into the class.

Finally, the LEP student's knowledge of arithmetic can be a major clue to other things about the student in general. If a recent immigrant from an Indochinese refugee camp displays a strong grasp of arithmetic, it is likely he received ongoing instruction from his extended family or that they went to great lengths to ensure that he continue/his educational progress.

... ate a strong familial commitment to the student's ed... mbers might be asked to assist in that individual's a... ts' arithmetic instruction.

...n a limited background in arithmetic might have had (and ... to have) other pressing concerns which interfered with their .... Counselors and teachers involved in remedial work would need to t... e concerns into account; otherwise, the student might fall ...ively behind.

In the following sections, examples of different ways to read and write numbers as well as of different subtraction and division algorithms are presented. They are meant to illustrate how LEP students might have learned to approach basic arithmetic in ways that are qualitatively different from how it is taught in the regular classroom. The implications of these differences and some specific suggestions for instruction will be discussed.

These examples were collected at numerous workshops given throughout the Midwest on the teaching of math to LEP students. The commonly taught American algorithms would be contrasted to others in terms of their steps and underlying thought processes. Participants, particularly those taught in other educational systems, would be asked to identify those examples which they had learned or LEP had used or which they had seen LEP students using. Based on this ongoing informal survey of expert informants, the following examples have been identified to their respective countries.

In the final section of this paper, some general conclusions and recommendations are made.

Reading and Writing Numbers

Probably the most widespread difference between the United States and other countries for the writing of number lies in the uses of the comma(,) and the period (.). Whereas the comma is used to denote powers of a thousand in this country, the period serves that function in most of Europe, Latin America and parts of Asia using "Arabic" numerals. Alternately, the decimal point, which is denoted by a period in this country, is marked by a comma in those number systems. Thus, the number 2,500 might be understood as 2 1/2 by a LEP student; he might write 3.600 when meaning 3 thousand, 6 hundred. Teachers encountering such students should probe for the students' meanings when they read and write numbers and they should point out, to the student, that numbers are written differently in this country.

The numerals themselves show variation from country to country as well. In many countries outside the United States, the one is written like seven with a sharper top: e.g., 1 or even 1. Some students place an under scoring line: 1. The seven is distinguished from the one by a line crossing its middle: 7. It seems unnecessary to force a LEP student to follow American conventions in these cases; however, he should understand that the teacher and most of his classmates do write their numbers differently.

The true Arabic numerals are written differently than the ones we use (see figure 1). Not only must a student who uses these numerals learn the American version, but also he must keep the two sets of numerals from interfering with each other. For instance, a period at the end of a statement involving numbers might be confused for an added place: 5 + 3 = 8. (eight or eighty?). The student might understand the 3 as a backwards 4. He might write "$\mathcal{E}$" meaning "4." Once again, the simplest solution is to directly tell the student how numbers are written in this country and to give him some practice reading and writing numerals. A "translation" chart in the classroom might also prove helpful.

INSERT FIGURE 1 ABOUT HERE

The tens and units in Arabic are read and said in reverse order than in most other number systems. 43 is read as "three and forty," 543 is read as "five hundred, three and forty." A teacher might say a number and find out that an Arabic student has written the tens and unit digits in transposed order.

Arabic languages are read from right to left; Chinese, from top to down. These conventions might interfere with a student's reading a page full of numbers (as in the case of worksheets) or with situations in which the left to right direction implies the ordering of a set.

It is important to recognize, moreover, that the student who is following conventions such as the above ones shares much in common with the American student in the mainstream arithmetic classroom: knowledge of base 10, of place value, etc. Thus, knowledge of the student's educational background can help a teacher to pinpoint those areas which differ from what the student's peers already know and to help that student fill the gap between what he already knows about number and the American conventions for expressing that knowledge. The next examples provide instances of where a student's way of doing things is sufficient and additional work is not required.

The Subtraction Algorithm

Initial meanings for subtraction, range from take away to missing addends. These meanings, and the algorithms which are developed from them, vary from country to country around the world. The four subtraction algorithms presented in this section represent four different ways of writing the solution process, and also four different thought processes, each one embodied by a different algorithm.

INSERT FIGURES 2 TO 5 ABOUT HERE

The first algorithm is based on the commonly taught meaning of taking away (Figure 2). The bottom number is taken away from the top. If the lower digit in a given column is larger than the upper, a "10" is borrowed from the column to the left. This algorithm is the one commonly taught in the United States. It is also found in countries importing American texts or which have received American Peace Corps volunteers such as Jamaica.

The next algorithm, missing addend substraction, is based on the addition meaning of subtraction (Figure 3). Rather than thinking in terms of take away, the student thinks in terms of adding a number to the bottom digit (in a given column) in order to get the digit in the top row. As can be seen, when the sum -- the subtrahend plus the (partial) difference -- exceeds 10, the 10 is "carried" to the next column on the left. In more advanced forms of this algorithm, the 10 is carried mentally and not written down. This algorithm seems most common in countries that adopted the French educational system while they were its colonies: Vietnam, Thailand, Laos. Though not agreeing universally among themselves, teachers and aides educated in Hong Kong, Mexico and other countries in Latin America also recognize this algorithm as being taught in their native educational systems.

The equal addition form of subtraction is a mixture of two meanings: take away and related addition (Figure 4). When the bottom digit of a given column is larger than the upper digit, the former is converted to a ten by adding a suitable number to it. The 10 is carried to the next column; simultaneously, the number added to the bottom must be added to the top digit to keep the problem the same. But this number can be brought down immediately since zero from anything is that number. Though less common than either of the first two algorithms, some teachers from this country as well as from Latin America say they learned to use equal additions.

The final subtraction algorithm which uses negative number meanings for its execution seems to be used in Eastern Europe and in the Soviet Union (Figure 5). It is based on understanding that a larger from a smaller digit yields a negative digit and that the final answer can be obtained by adding together all the positive and negative numbers obtained from each individual column's substep.

These four algorithms differ in their written forms; in the thought processes and understandings which they embody; and finally, in the related skills and abilities required for their successful execution. The teacher accustomed to the first, take-away, algorithm will probably be confused when seeing any of the other three. Furthermore, the second (missing addend) and third (equal addition) algorithms are remarkably similar in their written forms: in their most advanced forms, only the answers to the algorithms are written; in their intermediate forms, 10's are "carried" along the bottom row. Thus a teacher who suspects that a student is using one of these alternative algorithms will need to ask the student about how he solves subtraction problems. Furthermore, students using the latter three algorithms are encouraged to do most of their computations in their heads. This might lead a teacher to suspect cheating when a LEP student can get the right answers but fails to show his work. The student's inability to express himself in English might reinforce that impression since he will find it difficult to explain a complicated process (which he is just mastering) to someone who doesn't understand it (Footnote 2). Needless to say, much care and sensitivity is needed in a situation such as this.

These four algorithms represent qualitatively different basic understandings of subtraction: take away, related addition, facts up to 10, and negative numbers respectively for each algorithm. Teachers who insist that all students take away and borrow risk confusing their students needlessly and causing conflicts at home where the students' families are probably trying to help by teaching their children the algorithms as taught in their home countries.

The prerequisite skills for each algorithm also differ: sub traction number facts, related addition facts, addition facts to 10, and negative numbers. Students who are learning the missing addend algorithm might have difficulty memorizing their subtraction facts unless they are presented in that format. Moreover, these students probably will not need to spend much time learning subtraction number facts since they are taught solely for use in the take-away algorithm. Since the students should already know the related addition facts, subtraction facts would be redundant.

Students who have already learned one of the three alternate subtraction algorithms do not need to learn the one based on take away meanings. Nor, should they be penalized for using that different algorithm. The subskills associated to take away, including subtraction facts, are superfluous for these students and they should not be tested on those skills.

A LEP student's knowledge of an alternate algorithm could be the basis for an enrichment lesson. The teacher could explain that there are other ways of doing subtraction to the class and she could ask the student to demonstrate one of those ways. After the student has made his presentation, the teacher should be ready to provide additional explanations and examples to the class.

The problem of teaching students who do not have perfect mastery of an alternate algorithm remains. If a teacher can diagnose the source of a minor error, he should remediate it in a manner that is consistent with what the student knows and consistent with the underlying thought processes. For instance, if a student is forgetting to carry a "ten" in the missing addend algorithm, his teacher should use the termi logy of addition (not take away) when explaining to the student why and how to carry the 10.

If a teacher does not understand the algorithm being used by the student, she should say so to the student and see if another student or member of the student's family could help the student. Failing in these options or determining that the student really does not know what he is doing, a teacher should explain that she does not understand what the student is trying but that she can show him a <u>different</u> way of solving the problem. The teacher should be alert in case bugs arise in the new algorithm due to interference from the old one.

These and other subtraction algorithms are discussed in Beattie (1979); Bell, Fuson & Lesh (1976); Leutzinger & Nelson (1979); Musser (1982); and Sherrill (1979). Teachers should be able to recognize when their LEP students are using alternate algorithms. In such cases, changes in the student's curriculum, instruction and testing are indicated.

The Division Algorithm

Unlike the subtraction algorithm, for which there seem to be many variants taught around the world, there seem to be two main division algorithms. The first is the one commonly taught in this country as long division (Figure 6). The dividend is placed to the right of the divisor with the short end of a sideways "L" separating the two. The thought process for the algorithm is based on dividing the quotient "into" groups the size of divisor. The subskills required for division this way include the ability to make a good initial estimate as to how many times the divisor will fit into the dividend, multiplication, and subtraction. Other division algorithms have been suggested as transitional, due to the difficulty of the one followed in this country (Bell, Fuson & Lesh, 1976; Lang & Mayer, 1982).

INSERT FIGURE 6 ABOUT HERE

The alternate algorithm whose intermediate and advanced forms are seen in Figure 7 is taught in France, Spain and their former colonies which retained their educational systems: Latin America, Indochina (Footnotes 3 and 4). In this algorithm, the dividend is written to the left of the divisor; while the quotient is written underneath. In its intermediate form, the results of the multiplication step are written above the dividend;

those of the subtraction, below. In the advanced forms of this algorithm, the multiplication and subtraction steps are done mentally with only the remainders being written below the dividend (see bottom of Figure 7).

INSERT FIGURE 7 ABOUT HERE

The thought processes followed by this alternate algorithm stress the relationship between multiplication and division. The student is taught to ask himself what number times the divisor yields an approximation to the dividend. The use of mental computations requires that the students have well developed multiplication and subtraction reflexes.

The relationship between division and multiplication is also stressed by students being taught to check their answers. Hispanic students are taught to do so by working the related multiplication problem; if there was a remainder from the original division problem, that number gets added to the result of multiplying the divisor by the quotient. Indochinese and other students who have gone through French based educational systems, will check their answers by <u>casting-out-nines</u> for the related multiplication problem.

INSERT FIGURE 8 ABOUT HERE

In figure 8 are outlined two examples of casting-out-nines for multiplication. A large X is drawn to the side of a problem. For each multiplier, the digits are added together until a single digit remains. The top multiplier's digit goes atop the "X"; the second at the bottom. These two digits get multiplied and the resulting number undergoes a similar reduction process until a single digit remains. This number gets placed to the right of the cross. From the original multiplication problem, the product undergoes the same reduction process of adding all its digits until a single number remains. This goes in the left side of the "X". If the two digits to the right and to the left of the "X" match, the answer is correct (up to the factor of 9).

Casting-out-nines can be used to check all of the basic operations. It is described in greater detail in Bell, Fuson & Lesh (1976). It is given that name because nines can be dropped immediately from the reduction process. For instance, 93 reduces to a 3: 9 + 3 = 12; 1 + 2 = 3. However if the 9 is dropped or cast out, 3 (the answer) remains.

INSERT FIGURE 9 ABOUT HERE

How a student would check the division problem of Figure 7 is outlined in Figure 9. The related multiplication for the sample problem of Figure 8 is 24 x 78 = 1872. The student works out the casting out nines check for this problem.

In division problems where a remainder is left over, the remainder gets added to the reduced digit at the right of the "X". This new number gets reduced via casting out nines; the new result then must match the reduced dividend.

Not only is the alternate division algorithm taught differently than in American schools, but also it requires a different set-up, different thought processes and more advanced mental computational skills than does its American counterpart. These demands and the practice for mental computations afforded by the alternative algorithm and its casting-out-nines checking method, might help explain why teachers report that some of their Indochinese students are well ahead of their other students in computational arithmetic. Indeed, for some Indochinese students, our algorithm may be the "low stress" alternative.

The points already made about curriculum, instruction and testing vis-a-vis the subtraction algorithms also apply for division. Moreover, teachers should be careful in their tests that division problems are not set up only as in Figure 6. Students used to the alternative algorithm might read the problem as 24 ÷ 1872 rather than the reverse. An alternative would be to write problems in their universally understood format (1872 ÷ 24) as well as having both set ups from Figures 6 and 7.

Some General Conclusions and Recommendations

These examples were meant to illustrate the point that LEP students' educational backgrounds may have implications for the arithmetic classroom. This background can influence how the students read, write and understand numbers, how they perform basic operations and other related areas of concern to the arithmetic teacher.

When preparing to introduce a new topic, a teacher should ask herself if that topic's presentation should be modified in order to accommodate her LEP students' prior education. The teacher might interview the student himself, older students, members of the student's family or other adults (e.g. aides). She should then check to see how far the student has developed his understanding and skills as related to that topic. If the student already knows the topic, but in a different way, he might be given other work or he might participate in the lessons as a form of enrichment. If the student is enroute to a full mastery of a given point, a careful analysis of the topic is necessary for adapting curriculum, instruction and testing to fit the student's level of mastery.

Care is needed to build upon what a student already knows and to avoid confusing him with extraneous information. Even a teacher determines that a LEP student lacks any background experiences in a specific topic, she still should take into consideration the student's linguistic and cultural background. These other factors are discussed, in part, by Castellanos (1980) and by Lovett (1979).

19

FIGURE 1: ARABIC AND ENGLISH NUMERALS

Arabic:	١٠	٩	٨	٧	٦	٥	٤	٣	٢	١	٠
English:	(10)	(9)	(8)	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)

FIGURE 2: TAKE AWAY SUBTRACTION

<u>Written Form</u>	<u>Thought Process</u>
$\begin{array}{r} 542 \\ -\ \underline{289} \end{array}$	
$\begin{array}{r} {}^{3}{}_{1}\ \\ 5\cancel{4}2 \\ -\ \underline{289} \end{array}$	I can't take 9 from 2. Borrow a 10 to make it 12. 4 becomes 3.
$\begin{array}{r} {}^{3}{}_{1}\ \\ 5\cancel{4}2 \\ -\ \underline{289} \\ 3 \end{array}$	9 from 12 is 3
$\begin{array}{r} {}^{4}{}_{1}{}^{3}{}_{1}\ \\ \cancel{5}\cancel{4}2 \\ -\ \underline{289} \\ 3 \end{array}$	I can't take 8 from 3. Borrow 1 from the 5 to make 3 a 13. 5 becomes 4.
$\begin{array}{r} {}^{4}{}_{1}{}^{3}{}_{1}\ \\ \cancel{5}\cancel{4}2 \\ -\ \underline{289} \\ 53 \end{array}$	8 from 13 is 5.
$\begin{array}{r} {}^{4}{}_{1}{}^{3}{}_{1}\ \\ \cancel{5}\cancel{4}2 \\ -\ \underline{289} \\ 253 \end{array}$	2 from 4 is 2

21

FIGURE 3: MISSING ADDEND SUBTRACTION

Written Form	Thought Process
$\begin{array}{r} 542 \\ -\ 289 \\ \hline \end{array}$	9 plus what number is 12? 3. Write the 3.
$\begin{array}{r} 542 \\ -\ 289 \\ \hline 3 \end{array}$	Since 9 + 3 = 12, not 2, I need to carry a "1" to the next column.
$\begin{array}{r} 542 \\ -\ 289 \\ \hline 3 \end{array}$	8 + 1 = 9. 9 plus what number is 14? 5. Write the 5.
$\begin{array}{r} 542 \\ -\ 289 \\ \hline 53 \end{array}$	Now, 8 + 1 + 5 = 14, not 4; so carry 1 to the next column.
$\begin{array}{r} 542 \\ -\ 289 \\ \hline 53 \end{array}$	2 + 1 = 3. 3 plus what number is 5? 2. Write the 2.
$\begin{array}{r} 542 \\ -\ 289 \\ \hline 253 \end{array}$	check 253 + 289 = 542, so 253 is my answer.

FIGURE 4: EQUAL ADDITION SUBTRACTION

<u>Written Form</u>	<u>Thought Process</u>
$\begin{array}{r} 542 \\ -\underline{289} \end{array}$	I can't take 9 from 2. If I add 1 to 9, it becomes a 10, and I can take zero away from anything. But, if I add 1 to the 9, I also must add 1 to the 2 to keep the problem the same. 2 + 1 is 3, and 0 (in the 10) from 3 is 3.
* $\begin{array}{r} 542 \\ -\underline{289} \\ 3 \end{array}$	The 8 is now a 9 since I added 1 to 89. I can't take 9 from 4, so I add 1 to it to make it a "10." 1 + 4 is 5.
* $\begin{array}{r} 542 \\ -\underline{289} \\ 53 \end{array}$	The 2 is now a 3 since I have added 11 to 89 (or 10 to 90, or 1 to 9). 3 from 5 is 2.
* $\begin{array}{r} 542 \\ -\underline{289} \\ 253 \end{array}$	

* An intermediate form includes "carries" of the number added:

$\begin{array}{r} 542 \\ -\underline{289} \\ 253 \end{array}$	This reminds the student that the 8 has become 8 + 1 = 9, and the 2 has become 2 + 1 = 3.

23

FIGURE 5: USING NEGATIVE NUMBERS FOR SUBTRACTION

Written Form	Thought Process
542 - 289	9 from 2 is minus 7.
542 - 289 -7	8 from 4 is minus 4.
542 - 289 -4-7	2 from 5 is 3.
542 - 289 3-4-7	300 - 40 is 260; 260 - 7 is 253.
542 - 289 3-4-7 253	

FIGURE 6: LONG DIVISION

Written Form	Thought Process
24 ⟌ 1872	2 can't go into 1. 2 into 18 is 9; but 9 times 4 is 36 so it's too big. Try 8.
8 24 ⟌ 1872 2	8 times 4 is 32. Put down your 2, carry on 3.
8 24 ⟌ 1872 2	8 times 2 is 16; 16 + 3 is 19; 8 is still too big. Erase and try a 7.
7 24 ⟌ 1872	7 times 4 is 28. Put down your 8, carry a 2.
7 24 ⟌ 1872 4	7 times 2 is 14; 14 + 2 is 16.
7 24 ⟌ 1872 168 192	8 from 17 is 9; 6 from 7 is 1. Bring down the 2.
7 24 ⟌ 1872 168 192	2 into 19 is 9; but 9 times 4 is 36, so 9 is too big. It can't be 7 because 7 times 24 is 168. Try 8.
78 24 ⟌ 1872 168 192	8 times 4 is 32, put down a 2, carry your 3. 8 times 2 is 16; 16 plus 3 is 19.
78 24 ⟌ 1872 168 192 192	The amount is 78.

FIGURE 7: INTERMEDIATE FORM OF THE ALTERNATE DIVISION ALGORITHM

<table>
<tr><th>Written Form</th><th>Thought Process</th></tr>
<tr><td>1872 | 24</td><td>What number times 24 is close to 187? It can't be 8, because 8 times 4 is 32, and 16 + 3 is 19. So it's 7.</td></tr>
<tr><td>1872 | 24
7</td><td>7 times 24; 7 times 4 is 28; 7 times 2 is 14; 14 plus 2 is 16; so, 168.</td></tr>
<tr><td>168
* 1872 | 24
7</td><td>187 less 168 is 19, bring down the 2.</td></tr>
<tr><td>168
* 1872 | 24
192 7</td><td>What number times 24 is close to 192? 8.</td></tr>
<tr><td>* 168
1872 | 24
192 78</td><td>8 times 24. 8 times 4 is 32. 8 times 2 is 16; 16 + 3 is 19.</td></tr>
<tr><td>* 192
* 168
1872 | 24
192 78</td><td>192 less 192 is 0</td></tr>
<tr><td>192
168
1872 | 24
192 78
0</td><td>The answer is 78; now check the related multiplication problem.</td></tr>
</table>

* Most students are taught to carry out the multiplication and subtraction steps in their heads. Thus the numbers with an * are not written in the advanced, more mature form:

```
1872 | 24
 192   78
   0
```

FIGURE 8: CASTING OUT NINES FOR MULTIPLICATION

Example 1:

2 + 1 = 3

```
  21
  13
  63
 21
 273
```

2 + 7 + 3 = 12; 1 + 2 = 3
2 + 7 = 9; 3 remains

1 + 3 = 4

3 X 4 = 12; 1 + 2 = 3

Example 2:

2 + 3 + 7 = 12; 1 + 2 = 3
2 + 7 = 9; 3 remains

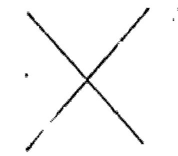
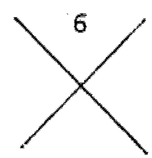
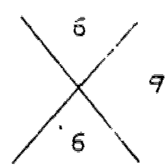
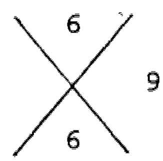
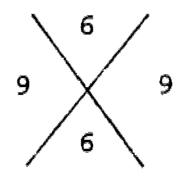
```
   237
   322
   474
  474
 711
 76314
```

7 + 6 + 3 + 1 + 4 = 21; 2 + 1 = 3
6 + 3 = 9; 7 + 1 + 4 = 12; 1 + 2 = 3

3 + 2 + 2 = 7

3 X 7= 21; 2 + 1= 3

FIGURE 9: CHECKING DIVISION BY CASTING OUT NINES

Written Problem	Casting-out-Nines	Thought Process
1872 ⌊ 24 192 78 0		24 is the divisor. 2 + 4 is 6. Write that a top of the "X"
1872 ⌊ 24 192 78 0		78 is the quotient. 7 + 8 = 15; 1 + 5 = 6. Write that on the bottom of the "X".
1872 ⌊ 24 192 78 0		6 X 6 = 36; 3 + 6 = 9. Write that to the right of the "X".
1872 ⌊ 24 192 78 0		1872 is the dividend (or, the product of 24 x 78). 1 + 8 = 9; drop them. 7 + 2 = 9. Write 9 to the left of the "X".
1872 ⌊ 24 192 78 0		The two 9's match; the answer is correct. So, 1872 ÷ 24 = 78.

FOOTNOTES

1. In workshops I have given throughout the Midwest on teaching mathematics to LEP students, I have heard this or a similar refrain enough times to become concerned that many classroom teachers rigidly adhere to a single way of doing mathematics. This unyielding rigidity is immediately harmful to LEP students who may have learned different algorithms or approaches to certain topics, but whose grades unfairly drop. It is also harmful to all students who pick up similar attitudes about mathematics.

2. When foreign educated teachers or aides have attempted to present alternate subtraction and division algorithms to their peers at workshops, I have been impressed by how difficult it is for other teachers to understand the new algorithms. Commonly, the presenters work the problems while providing only a few comments to explain the steps and their underlying thought processes. Also, teachers in the audience try to interpret the algorithms in terms of the "right way". Given the difficulty educated adults have in explaining how they execute these algorithms, we should understand the enormous difficulties facing LEP students as they attempt similar tasks.

3. In Hong Kong the algorithm and thought processes are taught identically to those outlined in Figure 4. However, the initial set-up differs in that the dividend is to the right of the divisor: $\underline{24|}$ 1872.

4. A totally different written algorithm is used in Germany (Kulm, 1979) and probably would be taught in its former colonies. However, it also stresses the relationship of multiplication to division.

2.)

References

Beattie, I.D. Children's strategies for solving subtraction fact combinations. Arithmetic Teacher, Volume 27, Number 1, September 1979, 14-15.

Bell, M.S.; Fuson, K. C. ; and Lesh, R. A. Algebraic and Arithmetic Structures: A Concrete Approach for Elementary School Teachers. New York: The Free Press, 1976.

Castellanos, G.C. Mathematics and the Spanish speaking student. Arithmetic Teacher, Volume 28, Number 3, November 1980, 16.

Federal Register, 1980, 45 (67), 23211-23228.

Kulm, G. Multiplication and division algorithms in German schools. Arithmetic Teacher, Volume 27, Number 9, May 1979, 26-27.

Laing, R.A. and Meyer, R.A. Transitional division algorithms. Arithmetic Teacher, Volume 29, Number 9, May 1982, 10-12.

Lentzinger, L.P. and Nelson, G. Let's do it: using addition facts to learn subtraction facts. Arithmetic Teacher, Volume 27, Number 4, December 1979, 8-13.

Lovett, J.C. Bilingual education: What role for mathematics teaching. Arithmetic Teacher, Volume 27, Number 8, April 1979, 14-17.

Musser, G.L. Let's teach mental algorithms for addition and subtraction. Arithmetic Teacher, Volume 29, Number 8, April 1982, 40-42.

Secada, W. G. and the staff of the Bilingual Education Service Center. DSA VI: A profile of bilingual education in the midwest. Arlington Heights, IL: Northwest Educational Cooperative, 1982.

Sherrill, J.M. Subtraction: Decomposition versus equal addends. Arithmetic Teacher, Volume 27, Number 1, September 1979, 16-17.