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ABSTRACT

Experimenters sometimes wish to estimate for a particular dependent variable the proportion of total group variance that is associated with mean differences among fixed treatments or subject classifications. Hays (1981) represents this proportion by the parameter omega squared. A point estimate may be easily computed as a function of the number of treatments, the total sample size, and the mean squares between and within treatments. This paper presents tables which facilitate the construction of 90 or 95 percent confidence intervals for omega squared. (Author)

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Interval Estimation of ω^2, the Proportion of Variance Associated with a Set of Fixed Treatments

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Abstract

Experimenters sometimes wish to estimate for a particular dependent variable the proportion of total group variance that is associated with mean differences among fixed treatments or subject classifications. Hays (1981) represents this proportion by the parameter ω^2. A point estimate may be easily computed as a function of the number of treatments, the total sample size, and the mean squares between and within treatments. This paper presents tables which facilitate the construction of 90 or 95 percent confidence intervals for ω^2.

Introduction

Suppose a researcher has employed analysis of variance techniques to test the equality of treatment population means and has rejected this hypothesis. It is often of interest, as Hays states (1981, page 349), "to judge the extent to which the experimental treatments actually are accounting for variance in the dependent variable." Hays proposes the parameter ω^2 to describe the population proportion of variance associated with the treatments:

$$\omega^2 = \frac{\sum_j^k \frac{n_j}{N}(\mu_j - \mu)^2}{\sigma_Y^2} \quad . \tag{1}$$

In this expression μ_j is the mean for population j , $\mu = \Sigma n_j \mu_j / N$, n_j is the size of the sample under treatment j , $N = \Sigma n_j$, k is the number of treatments, and σ_Y^2 is the variance of the dependent variable for all treatment populations combined.

An estimate of ω^2 may be derived from consideration of the expected values of the mean squares for treatments (MS_A) and within treatments (MS_W). These expected values are

$$E[MS_A] = \sigma^2 + \Sigma n_j(\mu_j - \mu)^2/k-1$$

and

$$E[ms_w] = \sigma^2 \quad ,$$

where σ^2 is the variance of the dependent variable Y within all treatment populations. Thus, an unbiased estimate of the numerator of ω^2 is

$$\left(\frac{k-1}{N}\right)\left(MS_A - MS_W\right) \quad .$$

The denominator of ω^2 may be represented

$$\sigma_Y^2 = \sigma^2 + \sum \frac{n_j}{N}(\mu_j - \mu)^2 \quad ,$$

and may be estimated by

$$MS_W + \left(\frac{k-1}{N}\right)\left(MS_A - MS_W\right) \quad .$$

The ratio of these estimates may be taken as an estimate of ω^2 . Thus

$$\hat{\omega}^2 = \frac{MS_A - MS_W}{MS_A + \left(\frac{N-k+1}{k-1}\right) MS_W} \quad . \qquad (2)$$

This expression is algebraically equivalent to that given by Hays (1981, page 349).

Hays describes this statistic as a "rough sample estimate" of ω^2 , implying that it is susceptible to a considerable degree to sampling error. The purpose of this paper is to present tables which will facilitate the computation of confidence intervals for ω^2 . To those who find the point estimate of the proportion of variance a useful index, upper and lower limits on ω^2 should be valuable supplemental statistics.

Interval Estimation of ω^2

To obtain a 90% or 95% confidence interval for ω^2 we first relate this index to the non-centrality parameter associated with the *F* test of a fixed-effects analysis of variance. This parameter is usually represented and defined as

$$\delta^2 = \Sigma n_j(\mu_j - \mu)^2/\sigma^2 \quad , \tag{3}$$

where σ^2 is the expectation of the mean square within treatments. A common alternative definition of the non-centrality parameter, popularized by the Pearson-Hartley power function charts (1951), is

$$\phi^2 = \delta^2/k \quad . \tag{4}$$

The parameter ω^2 may be phrased as a function of δ^2 or ϕ^2 :

$$\omega^2 = \frac{\Sigma(n_j/N)(\mu_j - \mu)^2}{\sigma^2 + \Sigma(n_j/N)(\mu_j - \mu)^2} = \frac{\Sigma n_j(\mu_j - \mu)^2/\sigma^2}{N + \Sigma n_j(\mu_j - \mu)^2/\sigma^2}$$

$$= \frac{\delta^2}{N + \delta^2} = \frac{k\phi^2}{N + k\phi^2} \quad . \tag{5}$$

If *MSR* is used to symbolize the mean square ratio MS_A/MS_W , point estimates of ϕ^2 and ω^2 are as follows:

$$\hat{\phi}^2 = \left(\frac{k-1}{k}\right)\left(MSR - 1\right)$$

$$\hat{\omega}^2 = \frac{k\hat{\phi}^2}{N + k\hat{\phi}^2} = \frac{(k-1)(MSR - 1)}{N + (k-1)(MSR - 1)} \quad . \tag{6}$$

The latter expression is equivalent to that previously presented as equation (2).

Interval estimates of ϕ^2 , derived from the non-central *F* distribution, are presented in Tables 1-8. (The derivation is described in a later section.) Tables 1-4 assume a confidence coefficient of .90 for an interval bounded at both ends and .95 for an interval bounded at one end only. Tables 5-8 assume a coefficient of .95 for an interval bounded at both ends and .975 for an interval bounded at one end only. Upper and lower limits for ϕ^2, when substituted into (6), deliver upper and lower limits on ω^2.

Except for the fact that the confidence intervals are presented in tabular rather than graphical form, the present tables are analogous to the Clopper/Pearson (1934) nomographs for proportions. The appropriate table is entered with the sample statistic--in this case with the mean square ratio (*MSR*)--and the limits for ϕ^2 are obtained directly or by simple linear interpolation. Each table is specific to a given number of treatments and a given confidence coefficient (γ). Within each table the limits are presented for selected values of the *MSR* between 2.0 and 100.0, with ν_2 = 20, 40, 60, 120, or 240. Linear interpolation between these integral values of MSR and between the reciprocals of ν_2 involves relatively little error (Laubscher, 1965).

The selected values of ν_2 were chosen because they span the range that commonly occurs in experimental studies. The limits that apply when $\nu_2 > 240$ are very close to those for $\nu_2 = 240$. For example, if $\nu_1 = 4$, $\nu_2 = 400$, and the mean square ratio equals 9.0, the 90% limits on ϕ^2 are 3.2 and 11.1. If ν_2 is taken equal to 240, the limits read from Table 4 are 3.2 and 11.2. Thus, if the values for $\nu_2 = 240$ are used, even though a higher value of ν_2 actually holds, the resultant interval will have a slightly higher confidence coefficient than the nominal value of .90 or .95.

Illustration

A study was made of the means and variability of undergraduate course grades awarded in five colleges of the University of Iowa: Business Administration, Education, Engineering, Nursing, and Pharmacy. A random sample of 5% of the fall semester grades awarded in each college was drawn, with the restriction that no student contributed more than one grade. The numbers of cases, means, and variances were as follows:

	n	M	S^2
Business	298	2.484	1.116
Education	137	3.235	.743
Engineering	80	2.777	1.196
Nursing	56	3.051	.734
Pharmacy	49	2.560	.818
Total Sample	620	2.745	1.083

The analysis of variance yielded $MS_A = 15.049$ (df = 4), $MS_W = .994$ (df = 615), and MSR = 15.14, which is statistically significant at the 5% level.

The point estimate of ω^2, via equation (6), is

$$\hat{\omega}^2 = \frac{(5-1)(15.14-1)}{620+(5-1)(15.14-1)} = .0836 \quad .$$

Thus, we estimate that approximately 8.4% of the variance of grades awarded in these five pre-professional colleges combined arises from differences among the means for the five colleges.

To place a 95% confidence interval around this estimate, we use Table 8 to obtain limits on ϕ^2 and substitute these limits in equation (6). For illustrative purposes, we take ν_2 equal to 240 rather than 615. Interpolation between *MSR* values of 15 and 16 yields a lower limit for ϕ^2 of 5.984; interpolation for the upper limit of ϕ^2 yields a value of 18.754. The limits on ω^2 then become

$$\omega_L^2 = \frac{5(5.984)}{620 + 5(5.984)} = .046$$

$$\omega_U^2 = \frac{5(18.754)}{620 + 5(18.754)} = .131 \quad .$$

Derivation of the Limits

For any given combination of ν_1 , ν_2 , and the observed mean square ratio (*MSR*), the limits on ϕ^2 are the values which render the following probability statements correct. If the interval is to have a confidence coefficient of γ , the upper limit (ϕ_U^2) is a value such that

$$P\left[F'_{\nu_1, \nu_2, \phi_U} > MSR \right] = \frac{1 + \gamma}{2} \quad ,$$

where *F′* is a non-central *F* variable. The lower limit (ϕ_L^2) is a value such that

$$P\left[F'_{\nu_1, \nu_2, \phi_L} > MSR \right] = \frac{1 - \gamma}{2} \quad .$$

For example, if $\nu_1 = 3$, $\nu_2 = 120$, and *MSR* = 20.0, the upper limit of the 90% confidence interval is a value of ϕ^2 for which $P[F'_{3,120,\ \phi_U} > 20.0] = .95$. The lower limit is the value of ϕ^2 that satisfies the condition $P[F'_{3,120,\ \phi_L} > 20.0] = .05$.

Unlike the analogous problem of obtaining limits on population means and proportions, the bounds for ϕ^2 cannot be stated as a simple function of sample statistics. The inverse function which determines the values of ϕ^2 that render the foregoing probability statements true is not readily derived. It is possible, however, to approximate ϕ_U^2 and ϕ_L^2 by an iterative procedure. This approach is probably best explained by its application to a specific example.

Consider the preceding illustration, in which

$$P[F'_{3,120,\ \phi_U} > 20.0] = .95$$

$$P[F'_{3,120,\ \phi_L} > 20.0] = .05$$

The first statement poses the problem: Which non-central *F* distribution has the value 20.0 for its fifth percentile? The second statement asks: Which non-central *F* distribution has 20.0 for its ninety-fifth percentile? A satisfactory solution to these problems may be obtained via the Tiku (1965) three-moment approximation of the non-central *F* distribution. This technique yields accurate estimates of P_5 and P_{95} for a given combination of ν_1, ν_2 and ϕ^2. By taking $\nu_1 = 3$, $\nu_2 = 120$, and $\phi^2 = 0.0$, 0.1, 0.2, 0.3, . . ., 60.0, it is possible to identify with suitable precision the non-central distribution for which $P_5 = 20.0$ and the distribution for which $P_{95} = 20.0$. The crucial values of ϕ^2 in this particular case are as follows:

ϕ^2	P_5	P_{95}
. . .	. . .	. . .
8.1		19.75
8.2		19.93
8.3		20.12
. . .	. . .	. . .
22.2	19.84	
22.3	19.95	
22.4	20.05	

By interpolation, it may be deduced that $P_{95} = 20.00$ when $\phi^2 = 8.24$; $P_5 = 20.00$ when $\phi^2 = 22.35$. These values, then, constitute the 90% confidence interval for ϕ^2 when *MSR* = 20.00. This example is portrayed pictorially in Figure 1.

Repetition of this basic process for $\nu_1 = 1, 2, 3,$ and 4; $\nu_2 = 20, 40, 60, 120,$ and 240; and $\phi^2 = 0.0, 0.1, \ldots, 60.0$ yielded 90% and 95% confidence intervals for ϕ^2 . A variety of checks, based on Renikoff and Lieberman's Tables of the Non-Central t-Distribution (1957), the Pearson-Hartley power function charts (1951), and values summarized or computed by Tiku (1965) confirm the accuracy of the intervals within $\pm$ 0.1.

The Tiku approximation involves the determination of a central *F*

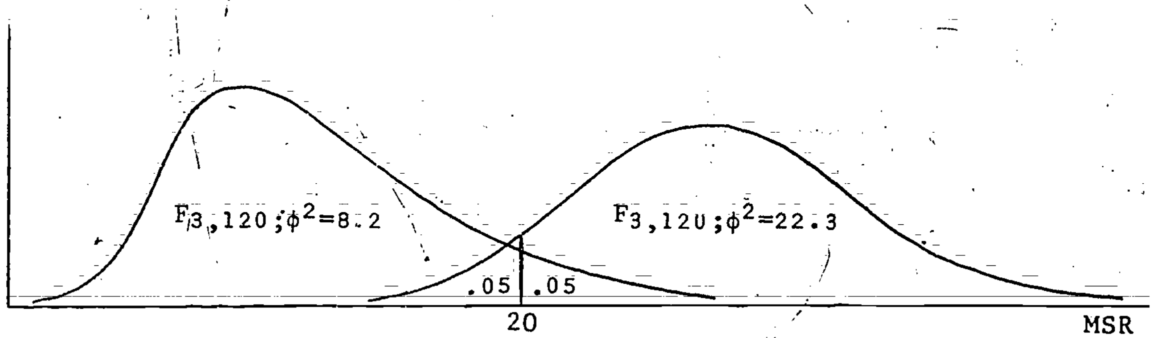

Figure 1. Non-Central F Distributions for Which P_5 and P_{95} = 20.0

variable which, when subjected to a linear transformation, results in a variable with the same first three moments as a given non-central F. In effect, the distribution of non-central $F'_{\nu_1, \nu_2, \phi}$ is approximated by $(h)F_{\tilde{\nu}_1, \nu_2} - c$. The constants h, $\tilde{\nu}_1$, and c are functions of ν_1, ν_2, and ϕ^2. The selected percentiles of $F_{\tilde{\nu}_1, \nu_2}$ are determined through the use of the inverse of the incomplete beta function and then transformed to the comparable percentiles of $F'_{\nu_1, \nu_2, \phi}$.

Tiku (1965) has shown that this approach provides a very accurate approximation to the non-central F. This was confirmed in the present study except for $P_{2.5}$ and P_5 when ϕ^2 is near zero and $\nu_1 < 3$. These lower end percentiles were graphically approximated, using the fact that non-central percentiles approach central F percentiles as ϕ approaches zero. Except for these values, all computations, including evaluation of the inverse beta function integral, were performed via the IMSL program of G.M. Roe (1969) on the Prime computer.

It may be observed that when the mean square ratio is fairly large, say greater than 30.0, relationship of the limits to the *MSR* is practically linear. The cruder approximation of Patnaik (1949) sheds some light on this phenomenon. The Patnaik two-moment approximation of the non-central F is

$$F' = \left(\frac{\nu_1 + k\phi^2}{\nu_1} \right) F_{\tilde{\nu}_1, \nu_2} ,$$

where $\tilde{\nu}_1$ equals $\dfrac{(\nu_1 + k\phi^2)^2}{\nu_1 + 2k\phi^2}$. When the value of $k\phi^2$ is large, relative to ν_1, $\tilde{\nu}_1$ becomes quite large--greater than $.5k\phi^2 + .75\nu_1$. But the extreme percentiles of $F_{\tilde{\nu}_1, \nu_2}$ do not change rapidly as $\tilde{\nu}_1$ increases beyond 50. For example, the ninety-fifth percentiles of $F_{60,120}$, $F_{120,120}$, and $F_{200,120}$ are 1.43, 1.35 and 1.32, respectively. Had these values been absolutely constant, $_{.95}F'$ would be a precisely linear function of ϕ^2 , namely,

$$_{.95}F'_{\nu_1, \nu_2, \phi} = \left(\frac{k}{\nu_1}\right)\left(_{.95}F_{\tilde{\nu}_1, \nu_2}\right)\phi^2 + {}_{.95}F_{\tilde{\nu}_1, \nu_2} \quad .$$

Because the extreme percentiles of F' are almost linearly related to ϕ^2 , the limits on ϕ^2 are almost linearly related to the *MSR*. This property makes it possible to use linear interpolation within Tables 1-8 to approximate with negligible error the limits corresponding to any value of *MSR* less than 100.

Summary

Tables are presented which facilitate the determination of confidence intervals for ϕ^2 , the non-centrality parameter of the *F* distribution arising from a fixed-effects analysis of variance. The tables are entered with the observed value of the mean square ratio, the degrees of freedom for error, and the desired confidence coefficient (.90 or .95). With these limits on ϕ^2 one may construct confidence intervals for Hay's parameter ω^2 , the proportion of dependent variable variance associated with

the treatments. The intervals were obtained by the use of Tiku's three-parameter approximation of the non-central F distribution.

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Table 1. 90% Confidence Interval for ϕ^2 -- Two Treatments

MSR	ν_2 = 20		ν_2 = 40		ν_2 = 60		ν_2 = 120		ν_2 = 240	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1										
2		4.6		4.5		4.4		4.4		
3		5.8		5.7		5.6		5.6		
4		6.8		6.8		6.7		6.7	0.1	
5	0.1	7.8	0.1	7.7	0.1	7.6	0.1	7.6	0.2	
6	0.2	8.7	0.3	8.6	0.3	8.5	0.3	8.5	0.3	8.4
7	0.3	[illegible].6	0.4	9.5	0.5	9.4	0.5	9.3	0.5	9.3
8	0.5	10.5	0.6	10.3	0.7	10.2	0.7	10.1	0.7	10.1
9	0.7	11.4	0.8	11.1	0.9	11.0	0.9	10.9	0.9	10.9
10	0.9	12.3	1.0	11.9	1.1	11.8	1.1	11.7	1.1	11.6
11	1.0	13.2	1.2	12.8	1.3	12.6	1.3	12.5	1.4	12.4
12	1.2	14.1	1.4	13.6	1.5	13.4	1.6	13.2	1.6	13.1
13	1.4	14.9	1.7	14.4	1.7	14.2	1.8	14.0	1.9	13.9
14	1.6	15.7	1.9	15.2	2.0	14.9	2.1	14.7	2.2	14.6
15	1.8	16.6	2.1	16.0	2.3	15.7	2.4	15.5	2.4	15.3
16	2.0	17.4	2.4	16.7	2.5	16.5	2.6	16.2	2.7	16.1
17	2.3	18.2	2.6	17.5	2.8	17.2	2.9	16.9	3.0	16.8
18	2.5	19.1	2.9	18.2	3.1	17.9	3.2	17.7	3.3	17.5
19	2.7	19.9	3.2	19.0	3.3	18.7	3.5	18.4	3.6	18.2
20	2.9	20.7	3.4	19.8	3.6	19.4	3.8	19.1	3.9	18.9
21	3.2	21.5	3.7	20.5	3.9	20.2	4.1	19.8	4.2	19.6
22	3.4	22.4	4.0	21.3	4.2	20.9	4.4	20.5	4.5	20.3
23	3.6	23.2	4.2	22.0	4.5	21.6	4.7	21.2	4.8	21.0
24	3.9	24.0	4.5	22.8	4.8	22.3	5.0	21.9	5.2	21.6
25	4.1	24.8	4.8	23.5	5.1	23.1	5.3	22.6	5.5	22.3
26	4.3	25.6	5.1	24.3	5.4	23.8	5.7	23.3	5.8	23.0
27	4.6	26.4	5.4	25.0	5.7	24.5	6.0	24.0	6.1	23.7
28	4.8	27.2	5.6	25.7	6.0	25.2	6.3	24.6	6.5	24.4
29	5.1	28.0	5.9	26.5	6.3	25.9	6.6	25.3	6.8	25.0
30	5.3	28.8	6.2	27.2	6.6	26.6	6.9	26.0	7.1	25.7
32	5.8	30.5	6.8	28.7	7.2	28.0	7.6	27.4	7.8	27.0
34	6.3	32.1	7.4	30.1	7.8	29.4	8.3	28.7	8.5	28.3
36	6.8	33.7	8.0	31.6	8.4	30.8	8.9	30.1	9.2	29.6
38	7.3	35.3	8.6	33.0	9.1	32.2	9.6	31.4	9.9	30.9
40	7.8	36.9	9.2	34.5	9.7	33.6	10.3	32.7	10.6	32.2
42	8.3	38.5	9.8	35.9	10.3	35.0	11.0	34.0	11.3	33.5
44	8.8	40.1	10.4	37.4	11.0	36.4	11.7	35.4	12.1	34.8
46	9.3	41.7	11.0	38.8	11.6	37.8	12.4	36.7	12.8	36.1
48	9.9	43.3	11.6	40.3	12.3	39.2	13.1	38.0	13.5	37.4
50	10.4	44.8	12.2	41.7	13.0	40.5	13.8	39.3	14.2	38.7
75	16.9	64.6	20.1	59.5	21.4	57.5	22.9	55.4	23.7	54.3
100	23.6	84.4	28.1	77.2	30.0	74.4	32.2	71.4	33.5	69.7

Table 2. 90% Confidence Interval for ϕ^2 — Three Treatments

MSR	$\nu_2 = 20$		$\nu_2 = 40$		$\nu_2 = 60$		$\nu_2 = 120$		$\nu_2 = 240$	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1										
2		4.1		4.1		4.0		4.0		4.0
3		5.4		5.3		5.3		5.2		5.2
4	0.1	6.6	0.2	6.5	0.2	6.4	0.2	6.3	0.3	6.3
5	0.3	7.8	0.4	7.6	0.5	7.5	0.5	7.4	0.5	7.3
6	0.6	9.0	0.7	8.6	0.8	8.5	0.8	8.4	0.8	8.4
7	0.8	10.1	1.0	9.7	1.1	9.6	1.1	9.4	1.2	9.4
8	1.1	11.2	1.3	10.7	1.4	10.6	1.5	10.4	1.6	10.3
9	1.4	12.3	1.7	11.8	1.8	11.6	1.9	11.4	1.9	11.3
10	1.7	13.4	2.0	12.8	2.1	12.6	2.3	12.3	2.3	12.2
11	2.0	14.5	2.4	13.8	2.5	13.5	2.7	13.3	2.7	13.1
12	2.3	15.6	2.7	14.8	2.9	14.5	3.1	14.2	3.2	14.0
13	2.6	16.7	3.1	15.8	3.3	15.5	3.5	15.1	3.6	14.9
14	2.9	17.8	3.5	16.8	3.7	16.4	3.9	16.0	4.0	15.8
15	3.3	18.9	3.9	17.8	4.1	17.4	4.3	17.0	4.5	16.7
16	3.6	19.9	4.2	18.7	4.5	18.3	4.8	17.9	4.9	17.6
17	3.9	21.0	4.6	19.7	4.9	19.3	5.2	18.8	5.4	18.5
18	4.2	22.1	5.0	20.7	5.3	20.2	5.7	19.7	5.9	19.4
19	4.6	23.2	5.4	21.7	5.8	21.1	6.1	20.6	6.3	20.3
20	4.9	24.2	5.8	22.6	6.2	22.0	6.6	21.4	6.8	21.1
21	5.2	25.3	6.2	23.6	6.6	23.0	7.0	22.3	7.3	22.0
22	5.6	26.4	6.6	24.5	7.0	23.9	7.5	23.2	7.7	22.8
23	5.9	27.4	7.0	25.5	7.5	24.8	8.0	24.1	8.2	23.7
24	6.3	28.5	7.4	26.5	7.9	25.7	8.4	25.0	8.7	24.5
25	6.6	29.5	7.8	27.4	8.3	26.6	8.9	25.8	9.2	25.4
26	7.0	30.6	8.3	28.4	8.9	27.6	9.4	26.7	9.7	26.2
27	7.3	31.7	8.7	29.3	9.2	28.5	9.8	27.6	10.2	27.1
28	7.6	32.7	9.1	30.3	9.7	29.4	10.3	28.4	10.7	27.9
29	8.0	33.8	9.5	31.3	10.1	30.3	10.8	29.3	11.2	28.8
30	8.3	34.8	9.9	32.2	10.6	31.2	11.3	30.2	11.7	29.6
32	9.0	36.9	10.8	34.1	11.5	33.0	12.2	31.9	12.7	31.3
34	9.7	39.1	11.6	36.0	12.4	34.8	13.2	33.6	13.7	32.9
36	10.4	41.2	12.4	37.9	13.3	36.6	14.2	35.3	14.7	34.6
38	11.1	43.3	13.3	39.8	14.2	38.5	15.2	37.0	15.8	36.2
40	11.9	45.4	14.1	41.7	15.1	40.3	16.2	38.7	16.8	37.9
42	12.6	47.5	15.0	43.6	16.0	42.1	17.2	40.4	17.8	39.5
44	13.3	49.6	15.8	45.4	16.9	43.8	18.2	42.1	18.9	41.2
46	14.0	51.7	16.7	47.3	17.8	45.6	19.2	43.8	19.9	42.8
48	14.7	53.8	17.6	49.2	18.8	47.4	20.2	45.5	21.0	44.4
50	15.4	55.9	18.4	51.1	19.7	49.2	21.2	47.2	22.0	46.0
75	24.3	82.2	29.2	74.5	31.4	71.5	33.9	68.1	35.4	66.2
100	33.3	108.5	40.2	98.0	43.2	93.8	46.8	89.0	49.1	86.2

Table 3. 90% Confidence Interval for ϕ^2 -- Four Treatments

MSR	$\nu_2 = 20$ Lower	$\nu_2 = 20$ Upper	$\nu_2 = 40$ Lower	$\nu_2 = 40$ Upper	$\nu_2 = 60$ Lower	$\nu_2 = 60$ Upper	$\nu_2 = 120$ Lower	$\nu_2 = 120$ Upper	$\nu_2 = 240$ Lower	$\nu_2 = 240$ Upper
1										
2		3.7		3.6		3.6		3.6		3.6
3		5.1	0.1	4.9	0.1	4.9	0.1	4.8		4.8
4	0.2	6.4	0.3	6.2	0.4	6.1	0.4	6.0	0.4	6.0
5	0.5	7.7	0.7	7.4	0.7	7.3	0.8	7.1	0.8	7.1
6	0.8	9.0	1.0	8.5	1.1	8.4	1.2	8.2	1.3	8.2
7	1.2	10.2	1.4	9.7	1.5	9.5	1.6	9.3	1.7	9.2
8	1.5	11.4	1.8	10.8	2.0	10.6	2.1	10.4	2.2	10.2
9	1.9	12.7	2.3	11.9	2.4	11.7	2.6	11.4	2.7	11.3
10	2.2	13.9	2.7	13.0	2.9	12.7	3.1	12.4	3.3	12.3
11	2.6	15.1	3.1	14.1	3.3	13.8	3.5	13.5	3.7	13.3
12	3.0	16.3	3.6	15.2	3.8	14.9	4.0	14.5	4.2	14.3
13	3.3	17.5	4.0	16.3	4.3	15.9	4.5	15.5	4.7	15.2
14	3.7	18.7	4.4	17.4	4.7	17.0	5.1	16.5	5.2	16.2
15	4.1	19.9	4.9	18.5	5.2	18.0	5.6	17.5	5.8	17.2
16	4.5	21.1	5.4	19.6	5.7	19.0	6.1	18.4	6.3	18.1
17	4.9	22.3	5.8	20.7	6.2	20.1	6.6	19.4	6.9	19.1
18	5.3	23.5	6.3	21.7	6.7	21.1	7.2	20.4	7.4	20.0
19	5.6	24.7	6.7	22.8	7.2	22.1	7.7	21.4	8.0	21.0
20	6.0	25.9	7.2	23.9	7.7	23.1	8.2	22.4	8.5	21.9
21	6.4	27.1	7.7	25.0	8.2	24.2	8.8	23.3	9.1	22.9
22	6.8	28.2	8.2	26.0	8.7	25.2	9.3	24.3	9.7	23.8
23	7.2	29.4	8.6	27.1	9.2	26.2	9.9	25.3	10.2	24.7
24	7.6	30.6	9.1	28.2	9.7	27.2	10.4	26.2	10.8	25.7
25	8.0	31.8	9.6	29.2	10.2	28.2	11.0	27.2	11.4	26.6
26	8.4	33.0	10.1	30.3	10.7	29.2	11.5	28.1	12.0	27.5
27	8.8	34.2	10.5	31.3	11.3	30.3	12.1	29.1	12.6	28.5
28	9.2	35.4	11.0	32.4	11.8	31.3	12.6	30.0	13.0	29.4
29	9.6	36.5	11.5	33.5	12.3	32.3	13.2	31.0	13.7	30.3
30	10.0	37.7	12.0	34.5	12.8	33.3	13.8	31.9	14.3	31.2
32	10.8	40.1	12.9	36.6	13.8	35.3	14.9	33.8	15.5	33.0
34	11.6	42.5	13.9	38.7	14.9	37.3	16.0	35.7	16.7	34.9
36	12.4	44.8	14.9	40.9	15.9	39.3	17.2	37.6	17.9	36.7
38	13.2	47.2	15.8	43.0	17.0	41.3	18.3	39.5	19.1	38.5
40	14.0	49.6	16.8	45.1	18.0	43.3	19.4	41.4	20.3	40.3
42	14.8	51.9	17.8	47.2	19.1	45.3	20.6	43.3	21.5	42.1
44	15.6	54.3	18.8	49.3	20.1	47.3	21.7	45.2	22.7	43.9
46	16.4	56.6	19.7	51.4	21.2	49.3	22.9	47.0	23.9	45.7
48	17.2	59.0	20.7	53.5	22.2	51.3	24.0	48.9	25.1	47.5
50	18.0	61.3	21.7	55.6	23.3	53.3	25.2	50.8	26.3	49.3
75	28.1	90.9	34.0	81.9	36.6	78.3	39.8	74.1	41.7	71.6
100	38.3	120.4	46.3	108.2	50.0	103.2	54.5	97.4	57.4	94.0

Table 4. 90% Confidence Interval for ϕ^2 -- Five Treatments

MSR	$\nu_2 = 20$		$\nu_2 = 40$		$\nu_2 = 60$		$\nu_2 = 120$		$\nu_2 = 240$	
	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
1										
2		3.5		3.4		3.3		3.3		3.3
3		4.9	0.1	4.7	0.2	4.6	0.2	4.6	0.2	4.5
4	0.3	6.3	0.5	6.0	0.5	5.9	0.6	5.8	0.6	5.7
5	0.7	7.6	0.9	7.2	0.9	7.1	1.0	6.9	1.1	6.9
6	1.0	8.9	1.3	8.4	1.4	8.2	1.5	8.1	1.6	8.0
7	1.4	10.2	1.7	9.6	1.9	9.4	2.0	9.2	2.1	9.1
8	1.8	11.5	2.2	10.8	2.4	10.5	2.5	10.3	2.6	10.1
9	2.2	12.8	2.7	12.0	2.9	11.7	3.1	11.3	3.2	11.2
10	2.6	14.1	3.1	13.1	3.4	12.8	3.6	12.4	3.7	12.2
11	3.0	15.4	3.6	14.3	3.9	13.9	4.1	13.5	4.3	13.3
12	3.4	16.7	4.1	15.4	4.4	15.0	4.7	14.5	4.9	14.3
13	3.8	17.9	4.6	16.6	4.9	16.1	5.3	15.6	5.5	15.3
14	4.2	19.2	5.1	17.7	5.4	17.2	5.8	16.6	6.1	16.3
15	4.6	20.5	5.6	18.9	6.0	18.3	6.4	17.7	6.7	17.3
16	5.1	21.8	6.1	20.0	6.5	19.4	7.0	18.7	7.3	18.3
17	5.5	23.0	6.6	21.2	7.0	20.5	7.6	19.7	7.9	19.3
18	5.9	24.3	7.1	22.3	7.6	21.6	8.2	20.8	8.5	20.3
19	6.3	25.5	7.6	23.4	8.1	22.6	8.7	21.8	9.1	21.3
20	6.7	26.8	8.1	24.6	8.7	23.7	9.3	22.8	9.7	22.3
21	7.2	28.1	8.6	25.7	9.2	24.8	9.9	23.8	10.3	23.3
22	7.6	29.3	9.1	26.8	9.8	25.9	10.5	24.8	11.0	24.3
23	8.0	30.6	9.6	28.0	10.3	27.0	11.1	25.8	11.6	25.3
24	8.4	31.9	10.2	29.1	10.9	28.0	11.7	26.9	12.2	26.2
25	8.9	33.1	10.7	30.2	11.4	29.1	12.3	27.9	12.8	27.2
26	9.3	34.4	11.2	31.4	12.0	30.2	12.9	28.9	13.5	28.2
27	9.7	35.7	11.7	32.5	12.6	31.3	13.5	29.9	14.1	29.1
28	10.1	36.9	12.2	33.6	13.1	32.3	14.1	30.9	14.7	30.1
29	10.6	38.2	12.7	34.7	13.7	33.4	14.8	31.9	15.4	31.1
30	11.0	39.4	13.3	35.9	14.2	34.5	15.4	32.9	16.0	32.0
32	11.9	42.0	14.3	38.1	15.3	36.6	16.6	34.9	17.3	34.0
34	12.7	44.5	15.3	40.4	16.5	38.7	17.8	36.9	18.6	35.9
36	13.6	47.0	16.4	42.6	17.6	40.9	19.0	38.9	19.9	37.8
38	14.4	49.5	17.4	44.8	18.7	43.0	20.3	40.9	21.2	39.7
40	15.3	52.0	18.5	47.1	19.8	45.1	21.5	42.9	22.5	41.6
42	16.2	54.5	19.5	49.3	21.0	47.2	22.8	44.9	23.8	43.5
44	17.0	57.1	20.6	51.6	22.1	49.4	24.0	46.9	25.1	45.4
46	17.9	59.6	21.6	53.8	23.2	51.5	25.2	48.9	26.4	47.3
48	18.7	62.1	22.7	56.0	24.4	53.6	26.5	50.8	27.7	49.2
50	19.6	64.6	23.8	58.3	25.5	55.7	27.7	52.8	29.1	51.1
75	30.4	96.1	36.9	86.2	39.8	82.3	43.4	77.6	45.7	74.7
100	41.2	127.6	50.1	114.3	54.1	108.7	59.2	102.6	62.3	98.9

Table 5. 95% Confidence Interval for ϕ^2 -- Two Treatments

MSR	ν_2 = 20 Lower	ν_2 = 20 Upper	ν_2 = 40 Lower	ν_2 = 40 Upper	ν_2 = 60 Lower	ν_2 = 60 Upper	ν_2 = 120 Lower	ν_2 = 120 Upper	ν_2 = 240 Lower	ν_2 = 240 Upper
1										
2		5.9		5.9		5.9		5.9		5.9
3		7.1		7.0		7.0		7.0		7.0
4		8.2		8.1		8.0		8.0		8.0
5		9.3		9.1		9.0		9.0		8.9
6		10.3	0.1	10.1	0.1	10.0	0.1	9.9	0.1	9.9
7	0.1	11.3	0.2	11.0	0.2	10.9	0.2	10.8	0.2	10.7
8	0.2	12.3	0.3	11.9	0.3	11.8	0.4	11.7	0.4	11.6
9	0.3	13.2	0.4	12.8	0.5	12.7	0.5	12.5	0.5	12.4
10	0.5	14.2	0.6	13.7	0.6	13.5	0.7	13.4	0.7	13.3
11	0.6	15.1	0.7	14.6	0.8	14.4	0.9	14.2	0.9	14.1
12	0.7	16.1	0.9	15.4	1.0	15.2	1.1	15.0	1.1	14.9
13	0.9	17.0	1.1	16.3	1.2	16.0	1.3	15.8	1.3	15.7
14	1.0	17.9	1.3	17.1	1.4	16.9	1.5	16.6	1.5	16.6
15	1.2	18.8	1.5	18.0	1.6	17.7	1.7	17.4	1.8	17.2
16	1.4	19.7	1.7	18.8	1.8	18.5	1.9	18.1	2.0	18.0
17	1.5	20.6	1.9	19.6	2.0	19.3	2.2	18.9	2.3	18.7
18	1.7	21.5	2.1	20.4	2.3	20.1	2.4	19.7	2.5	19.5
19	1.9	22.4	2.3	21.3	2.5	20.8	2.7	20.4	2.8	20.2
20	2.1	23.3	2.6	22.1	2.7	21.6	2.9	21.2	3.1	21.0
21	2.3	24.2	2.8	22.9	3.0	22.4	3.2	21.9	3.3	21.7
22	2.5	25.1	3.0	23.7	3.2	23.2	3.5	22.7	3.6	22.4
23	2.7	26.0	3.3	24.5	3.5	24.0	3.7	23.4	3.9	23.1
24	2.9	26.9	3.5	25.3	3.7	24.7	4.0	24.1	4.2	23.9
25	3.1	27.8	3.7	26.1	4.0	25.5	4.3	24.9	4.5	24.6
26	3.3	28.7	4.0	26.9	4.3	26.2	4.6	25.6	4.8	25.3
27	3.5	29.6	4.2	27.7	4.5	27.0	4.9	26.3	5.1	26.0
28	3.7	30.4	4.5	28.5	4.8	27.8	5.2	27.1	5.4	26.7
29	3.9	31.3	4.7	29.2	5.1	28.5	5.5	27.8	5.7	27.4
30	4.1	32.2	5.0	30.0	5.3	29.3	5.7	28.5	6.0	28.1
32	4.5	33.9	5.5	31.6	5.9	30.8	6.3	29.9	6.6	29.5
34	4.9	35.7	6.0	33.2	6.5	32.3	6.9	31.3	7.2	30.9
36	5.3	37.4	6.5	34.7	7.0	33.7	7.6	32.7	7.8	32.2
38	5.8	39.2	7.1	36.3	7.6	35.2	8.2	34.1	8.5	33.6
40	6.2	40.9	7.6	37.8	8.2	36.7	8.8	35.6	9.2	35.0
42	6.6	42.7	8.1	39.4	8.8	38.2	9.4	36.9	9.8	36.3
44	7.1	44.4	8.7	40.9	9.3	39.6	10.1	38.3	10.5	37.6
46	7.5	46.1	9.2	42.4	9.9	41.1	10.7	39.7	11.2	39.0
48	8.0	47.9	9.8	44.0	10.5	42.6	11.4	41.1	11.8	40.3
50	8.4	49.6	10.3	45.5	11.1	44.0	12.0	42.5	12.5	41.6
75	14.1	71.2	17.5	64.5	18.9	62.0	20.5	59.3	21.4	57.9
100	19.9	92.8	24.8	83.5	26.8	79.9	29.3	76.1	30.7	74.0

Table 6. 95% Confidence Interval for ϕ^2 -- Three Treatments

MSR	ν_2 = 20 Lower	Upper	ν_2 = 40 Lower	Upper	ν_2 = 60 Lower	Upper	ν_2 = 120 Lower	Upper	ν_2 = 240 Lower	Upper
1										
2		5.0		4.9		4.9		4.9		4.9
3		6.4		6.3		6.2		6.1		6.1
4		7.8		7.5		7.4		7.3		7.3
5	0.1	9.0	0.2	8.7	0.2	8.6	0.2	8.5	0.3	8.4
6	0.3	10.3	0.4	9.9	0.4	9.7	0.5	9.6	0.5	9.5
7	0.5	11.5	0.6	11.0	0.7	10.8	0.8	10.6	0.8	10.5
8	0.7	12.8	0.9	12.1	1.0	11.9	1.1	11.7	1.1	11.6
9	0.9	14.0	1.2	13.2	1.3	13.0	1.4	12.7	1.4	12.6
10	1.1	15.2	1.5	14.3	1.6	14.0	1.7	13.7	1.8	13.6
11	1.4	16.4	1.8	15.4	1.9	15.1	2.1	14.7	2.1	14.5
12	1.6	17.6	2.1	16.5	2.2	16.1	2.4	15.7	2.5	15.5
13	1.9	18.7	2.4	17.5	2.6	17.1	2.8	16.7	2.9	16.5
14	2.2	19.9	2.7	18.6	2.9	18.1	3.2	17.7	3.3	17.4
15	2.4	21.1	3.1	19.6	3.3	19.1	3.6	18.6	3.7	18.3
16	2.7	22.3	3.4	20.7	3.7	20.1	4.0	19.6	4.1	19.3
17	3.0	23.4	3.7	21.7	4.0	21.1	4.4	20.5	4.5	20.2
18	3.3	24.6	4.1	22.8	4.4	22.1	4.8	21.5	5.0	21.1
19	3.6	25.8	4.4	23.8	4.8	23.1	5.2	22.4	5.4	22.0
20	3.9	26.9	4.8	24.8	5.2	24.1	5.6	23.3	5.8	22.9
21	4.1	28.1	5.1	25.9	5.6	25.1	6.0	24.3	6.3	23.8
22	4.4	29.2	5.5	26.9	5.9	26.1	6.4	25.2	6.7	24.7
23	4.7	30.4	5.9	27.9	6.3	27.0	6.9	26.1	7.2	25.6
24	5.0	31.6	6.2	28.9	6.7	28.0	7.3	27.0	7.6	26.5
25	5.3	32.7	6.6	30.0	7.1	29.0	7.7	27.9	8.1	27.4
26	5.6	33.9	7.0	31.0	7.5	29.9	8.2	28.8	8.5	28.3
27	5.9	35.0	7.3	32.0	7.9	30.9	8.6	29.8	9.0	29.1
28	6.2	36.2	7.7	33.0	8.3	31.9	9.1	30.7	9.5	30.0
29	6.5	37.3	8.1	34.0	8.8	32.8	9.5	31.6	9.9	30.9
30	6.8	38.5	8.5	35.1	9.2	33.8	9.9	32.5	10.4	31.8
32	7.4	40.8	9.2	37.1	10.0	35.7	10.8	34.3	11.3	33.5
34	8.0	43.1	10.0	39.1	10.8	37.6	11.8	36.1	12.3	35.2
36	8.6	45.4	10.8	41.1	11.6	39.5	12.7	37.8	13.3	37.0
38	9.3	47.7	11.5	43.1	12.5	41.4	13.6	39.6	14.2	38.7
40	9.9	49.9	12.3	45.1	13.3	43.3	14.5	41.4	15.2	40.4
42	10.5	52.2	13.1	47.1	14.2	45.2	15.5	43.2	16.2	42.1
44	11.1	54.5	13.9	49.2	15.0	47.1	16.4	44.9	17.2	43.8
46	11.7	56.8	14.6	51.2	15.9	49.0	17.3	46.7	18.2	45.5
48	12.4	59.1	15.4	53.2	16.7	50.9	18.3	48.5	19.2	47.1
50	13.0	61.4	16.2	55.2	17.6	52.8	19.2	50.2	20.2	48.8
75	20.8	90.0	26.1	80.2	28.4	76.3	31.3	72.1	33.0	69.6
100	28.8	118.6	36.1	105.2	39.5	99.9	43.6	93.9	46.1	90.5

Table 7. 95% Confidence Interval for ϕ^2 -- Four Treatments

MSR	ν_2 = 20 Lower	ν_2 = 20 Upper	ν_2 = 40 Lower	ν_2 = 40 Upper	ν_2 = 60 Lower	ν_2 = 60 Upper	ν_2 = 120 Lower	ν_2 = 120 Upper	ν_2 = 240 Lower	ν_2 = 240 Upper
1										
2		4.5		4.4		4.3		4.3		4.3
3		6.0		5.8		5.7		5.6		5.6
4	0.1	7.5	0.1	7.1	0.2	7.0	0.2	6.9	0.2	6.8
5	0.3	8.8	0.4	8.4	0.5	8.2	0.5	8.1	0.5	8.0
6	0.5	10.2	0.7	9.6	0.8	9.4	0.9	9.2	0.9	9.1
7	0.8	11.5	1.0	10.9	1.1	10.6	1.2	10.4	1.3	10.2
8	1.0	12.9	1.4	12.1	1.5	11.8	1.6	11.5	1.7	11.3
9	1.3	14.2	1.7	13.3	1.9	12.9	2.0	12.6	2.1	12.4
10	1.6	15.5	2.1	14.5	2.3	14.1	2.5	13.7	2.6	13.5
11	1.9	16.9	2.5	15.6	2.7	15.2	2.9	14.7	3.0	14.5
12	2.2	18.2	2.9	16.8	3.1	16.3	3.4	15.8	3.5	15.5
13	2.6	19.5	3.2	18.0	3.5	17.4	3.8	16.9	4.0	16.6
14	2.9	20.8	3.6	19.1	4.0	18.5	4.3	17.9	4.5	17.6
15	3.2	22.1	4.1	20.3	4.4	19.6	4.8	18.9	5.0	18.6
16	3.5	23.4	4.5	21.4	4.8	20.7	5.3	20.0	5.5	19.6
17	3.9	24.7	4.9	22.6	5.3	21.8	5.7	21.0	6.0	20.6
18	4.2	26.0	5.3	23.7	5.7	22.9	6.2	22.0	6.5	21.6
19	4.6	27.3	5.7	24.9	6.2	24.0	6.7	23.1	7.0	22.6
20	4.9	28.6	6.1	26.0	6.6	25.1	7.2	24.1	7.6	23.5
21	5.2	29.9	6.6	27.2	7.1	26.2	7.7	25.1	8.1	24.5
22	5.6	31.2	7.0	28.3	7.6	27.2	8.3	26.1	8.6	25.5
23	5.9	32.5	7.4	29.4	8.0	28.3	8.8	27.1	9.2	26.5
24	6.3	33.8	7.8	30.6	8.5	29.4	9.3	28.1	9.7	27.4
25	6.6	35.1	8.3	31.7	9.0	30.5	9.8	29.1	10.3	28.4
26	7.0	36.3	8.7	32.8	9.5	31.5	10.3	30.1	10.8	29.4
27	7.3	37.6	9.1	34.0	9.9	32.6	10.8	31.1	11.4	30.3
28	7.6	38.9	9.6	35.1	10.4	33.7	11.4	32.1	11.9	31.3
29	8.0	40.2	10.0	36.2	10.9	34.7	11.9	33.1	12.5	32.2
30	8.3	41.5	10.5	37.4	11.4	35.8	12.4	34.1	13.0	33.2
32	9.0	44.1	11.3	39.6	12.3	37.9	13.5	36.1	14.1	35.1
34	9.7	46.6	12.2	41.9	13.3	40.0	14.5	38.1	15.3	37.0
36	10.4	49.2	13.1	44.1	14.2	42.2	15.6	40.0	16.4	38.9
38	11.1	51.8	14.0	46.4	15.2	44.3	16.7	42.0	17.6	40.7
40	11.9	54.4	14.9	48.6	16.2	46.4	17.8	43.9	18.7	42.6
42	12.6	56.9	15.8	50.8	17.2	48.5	18.9	45.9	19.9	44.5
44	13.3	55.5	16.7	53.1	18.1	50.6	19.9	47.9	21.0	46.3
46	14.0	58.1	17.6	55.3	19.1	52.7	21.0	49.9	22.2	48.2
48	14.7	60.6	18.5	57.6	20.1	54.8	22.1	51.8	23.3	50.1
50	15.4	63.2	19.3	59.8	21.1	56.9	23.2	53.7	24.5	51.9
75	24.3	99.3	30.6	87.8	33.5	83.3	37.1	78.0	39.3	75.0
100	33.3	131.5	42.0	115.8	46.0	109.6	51.1	102.3	54.4	98.0

Table 8. 95% Confidence Interval for ϕ^2 -- Five Treatments

MSR	$\nu_2 = 20$ Lower	$\nu_2 = 20$ Upper	$\nu_2 = 40$ Lower	$\nu_2 = 40$ Upper	$\nu_2 = 60$ Lower	$\nu_2 = 60$ Upper	$\nu_2 = 120$ Lower	$\nu_2 = 120$ Upper	$\nu_2 = 240$ Lower	$\nu_2 = 240$ Upper
1										
2		4.1		4.0		3.9		3.9		3.9
3		5.7		5.4		5.3		5.2		5.2
4	0.1	7.2	0.2	6.8	0.3	6.7	0.4	6.5	0.4	6.5
5	0.4	8.6	0.6	8.1	0.6	7.9	0.7	7.8	0.8	7.7
6	0.7	10.1	0.9	9.4	1.0	9.2	1.1	9.0	1.2	8.8
7	1.0	11.5	1.3	10.7	1.4	10.4	1.6	10.1	1.7	10.0
8	1.3	12.9	1.7	12.0	1.9	11.6	2.0	11.3	2.1	11.1
9	1.6	14.3	2.1	13.2	2.3	12.8	2.5	12.4	2.6	12.2
10	2.0	15.7	2.5	14.5	2.8	14.0	3.0	13.5	3.2	13.3
11	2.3	17.1	3.0	15.7	3.2	15.2	3.5	14.6	3.7	14.4
12	2.7	18.5	3.4	16.9	3.7	16.4	4.0	15.8	4.2	15.5
13	3.0	19.9	3.8	18.2	4.2	17.5	4.6	16.9	4.8	16.5
14	3.4	21.3	4.3	19.4	4.7	18.7	5.1	18.0	5.3	17.6
15	3.7	22.7	4.7	20.6	5.1	19.8	5.6	19.0	5.9	18.6
16	4.1	24.0	5.2	21.8	5.6	21.0	6.2	20.1	6.5	19.7
17	4.5	25.4	5.6	23.0	6.1	22.1	6.7	21.2	7.0	20.7
18	4.8	26.8	6.1	24.2	6.6	23.3	7.2	22.3	7.6	21.7
19	5.2	28.2	6.6	25.5	7.1	24.4	7.8	23.3	8.2	22.8
20	5.6	29.5	7.0	26.7	7.6	25.6	8.4	24.4	8.8	23.8
21	5.9	30.9	7.5	27.9	8.1	26.7	8.9	25.5	9.4	24.8
22	6.3	32.3	7.9	29.1	8.6	27.9	9.5	26.5	9.9	25.8
23	6.7	33.7	8.4	30.3	9.2	29.0	10.0	27.6	10.5	26.8
24	7.0	35.0	8.9	31.5	9.7	30.1	10.6	28.6	11.1	27.8
25	7.4	36.4	9.4	32.7	10.2	31.3	11.2	29.7	11.7	28.9
26	7.8	37.8	9.8	33.9	10.7	32.4	11.7	30.8	12.3	29.9
27	8.2	39.2	10.3	35.1	11.2	33.5	12.3	31.8	13.0	30.9
28	8.5	40.5	10.8	36.3	11.7	34.6	12.9	32.9	13.6	31.9
29	8.9	41.9	11.2	37.5	12.2	35.8	13.5	33.9	14.2	32.9
30	9.3	43.3	11.7	38.7	12.8	36.9	14.0	34.9	14.8	33.9
32	10.0	46.0	12.7	41.1	13.8	39.2	15.2	37.0	16.0	35.9
34	10.8	48.8	13.6	43.5	14.9	41.4	16.4	39.1	17.2	37.9
36	11.6	51.5	14.6	45.8	15.9	43.7	17.5	41.2	18.5	39.8
38	12.3	54.2	15.5	48.2	17.0	45.9	18.7	43.3	19.7	41.8
40	13.1	57.0	16.5	50.6	18.0	48.1	19.9	45.3	21.0	43.8
42	13.8	59.8	17.4	53.0	19.1	50.4	21.0	47.4	22.2	45.7
44	14.6	62.6	18.4	55.4	20.1	52.6	22.2	49.5	23.5	47.7
46	15.3	65.4	19.4	57.8	21.2	54.9	23.4	51.6	24.7	49.7
48	16.1	68.2	20.3	60.2	22.2	57.1	24.6	53.6	26.0	51.6
50	16.9	71.0	21.3	62.6	23.3	59.3	25.8	55.7	27.3	53.6
75	26.4	104.9	33.4	92.3	36.6	87.3	40.7	81.4	43.3	78.0
100	36.0	139.2	45.6	122.1	50.0	115.3	55.8	107.2	59.5	102.4