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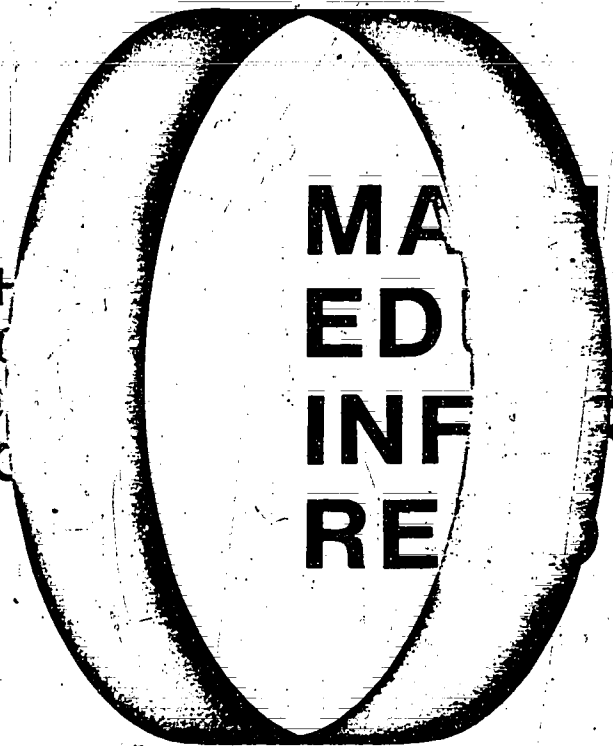
ABSTRACT

An editorial comment on the computer and the mathematics educator is included first. Then, abstracts and comments are presented for 11 articles. Studies included focus on mental addition, problem solving by sixth graders, mathematics anxiety, tutoring, direct instruction, a bilingual program, thinking strategies for multiplication basic facts, teacher education, cognitive style, problem solving at the college level, and motivating study in college mathematics. Research reported between January 1983 and March 1983 is also listed. (MNS)

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INVESTIGATIONS IN MATHEMATICS EDUCATION

Volume 16, Number 3 - Summer 1983

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INVESTIGATIONS IN MATHEMATICS EDUCATION

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An editorial comment...

The Computer and the Mathematics Educator

Kenneth J. Travers

University of Illinois at Urbana-Champaign

"How can the statistician control a computer, so that it becomes an extension of himself, as the piano is to the pianist?" (Francis J. Anscombe, Computing in Statistical Science through APL. New York: Springer-Verlag, 1981, page 2.)

Within the past few years, computers have become a part of our daily experience with startling rapidity. Few of even the most visionary among us predicted the pace of this development. The disturbing aspect of all this is whether, after all of the millions have been spent -- dollars on equipment and hours on training programs -- as far as education is concerned it will be business as usual.

To be sure, our past experience with video technology, and programmed instruction before that, offer little assurance that we have learned how to exploit the power of technology in pursuing significant new directions in curriculum and instruction. The introductory high school algebra course based on APL and developed by Iverson at IBM several years ago provides a case in point. The course moves quickly from a scalar to a vector-based algebra, thereby providing an enormous conceptual advance for dealing, say, with linear algebra and statistics. Yet there is scant public evidence that these materials have been taken seriously by curriculum developers or supervisors.

We hear on every side pleas for a mathematics curriculum which is more applicable to the "real world" -- a goal described by Hugh Burkhardt, in his just-released ERIC review of applications of school mathematics from an international perspective, as "the elusive Eldorado." Now we need to

see indications that the power of the computer has been turned loose and put in the hands of the teacher as a tool which can make a difference in the Kind of mathematics taught, and the way in which mathematics is taught.

Consider the case of instructional applications of the Monte Carlo Method. Through this method, concepts in physics dealing with heat flow and electronic circuitry can be handled without the advanced mathematical knowledge required by conventional approaches. Likewise, statistical ideas such as sampling distributions and confidence intervals are readily accessible to high school students of average ability. The method enables an experience-based, hands-on approach which greatly enhances instruction. All of this is made feasible and attractive by the growing availability of computers in classrooms.

One impediment to progress may be that school mathematics, and therefore mathematics educators, have been dominated by an arithmetic view of the world. (After all, until about 100 years ago, Harvard required only proficiency in arithmetic for admission!). So it may follow that the natural order of things has brought about a dominating view of the computer as only a fancy computing device. From this point of view, the French have a much better name for this awesome invention. They call the computer "ordinateur," which is a word whose roots mean "to order," "to arrange," "to tidy up."

In the field of research in mathematics education, we have an instructive example of how the non-computational role of the computer can change the nature of the enterprise. The First International Study of Mathematics, carried out in twelve countries some twenty years ago, signaled many firsts in educational research, the most obvious being that it pioneered large-scale survey research across a large number of language groups, educational systems, curriculums, teachers and students, and so forth. Not so well known is the fact that it was "state of the art" in terms of computer-based technology for file-building, data cleaning and

checking, and statistical analysis. In the report of the First IEA Mathematics Study, Wolf points out that the non-computational operations performed by the computer were "absolutely crucial" to the conduct of the project (Husén, 1967, Vol. 1, p. 205). The pioneering nature of the First IEA Study is exemplified by the fact that the IBM 1230 Optical Mark Scoring Reader was still under development in 1963. (For example, guidelines for the preparation of machine-readable answer sheets did not even exist at the time.) The end product of the data preparation phase of the study was the storage of some 50 million items of information on "slightly more than one half of a single reel of magnetic tape" (Wolf, *ibid*, p. 215).

In the Second International Mathematics Study there are exemplars of how computers continue to change the nature of research activity. Like the First International Study, the entire project, because of its scale and complexity, clearly could not have been done at all without computer assistance.

In the Second Study, data-based management techniques permit the manipulation of massive amounts of data in many, many data sets, for the purpose of cleaning, checking, filing, and, later on, for exploration and analysis. This technology in combination with computer networks across North America enables transmission of both raw data sets and computer output to researchers at other institutions engaged in the Study. Inter-continental networking through satellites is now being established, for in an international study one of the major problems is that of fast and effective communication between participants (which in the Second International Mathematics Study are found on every continent).

In what other ways might the computer have an impact on ways in which we do research? In experimental design, one could specify models for analysis given the purposes and conditions of a study. Then with simulated data the performance of the model, given various sample sizes and combinations of variables, could be explored. In the realm of exploratory



data analysis, it is essential that a quick and easy-to-read method of displaying various configurations of data be available. One wonders how much real exploration of data with sets of more than a few cases can be done without the aid of a computer. In secondary analyses, the capability of accessing data bases in a "user-friendly" mode is essential. It's a great pity that in many if not all of the large data collection activities which have been undertaken in the past, the data themselves have been greatly under-mined (that is, not mined to any where near their full "pay-dirt" capability). The computer is an essential tool in such endeavors.

As educators, we are dismayed when talent in our students remains undeveloped. "Johnny/Mary is not doing his/her best." The under-utilization of the resources of the computer is a cause of concern, too. We face enormous problems and challenges in the waning years of this century -- not the least of which is to help Johnny/Mary do his/her best. The computer has potentials for mathematics education which we have barely begun to realize. Are we creative enough to foster, nay, exploit its power?

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Ashcraft, Mark H. and Fierman, Bennett A. MENTAL ADDITION IN THIRD FOURTH, AND SIXTH GRADERS. Journal of Experimental Child Psychology 33: 216-234; April 1982.

Abstracts and comments prepared for I.M.E. by MERLYN J. BEHR, Northern Illinois University.

1. Purpose

The study sought: (1) to support the contention that children's processes for production of answers for basic addition facts changes developmentally from computation via counting algorithms to a memory retrieval process, and (2) to obtain information about the age, or age range, during which the shift from counting to memory retrieval occurs.

2. Rationale

A widely held view about children's mental addition process is that they obtain answers by reconstructing or computing using a counting algorithm. Models developed from several research efforts using response time (RT) support the so-called "min" model. This model suggests that a problem such as $3 + 4$ is solved by setting a "mental counter" to the maximum addend, 4, and incrementing by counting the value of the minimum addend, 3, up to the answer of 7. There are several lines of evidence however, to suggest a switch, sometime after first grade, from this reconstructive procedure to a memory retrieval process. The "min" model is suggested by research among young children which finds RT to be a linear function of the size of the minimum addend. Research among adults finds RT to be exponentially related to the size of the minimum addend; this, together with other research relating adult RT to addend and sum size, suggests that adults retrieve addition facts from memory. A shift from reconstructing to retrieval is suggested.

3. Research Design and Procedures

The subjects were 30 elementary school children randomly selected from grades 3, 4, and 6, resulting in a sample with mean age distribution

of 9.04, 9.63, and 11.72 years and male/female distribution of 6/4, 8/2, and 4/6 by grades, respectively.

The stimuli were 100 basic-fact addition problems (i.e., sums ≤ 18) presented in column form with the answer below the line. Fifty addition combinations were randomly selected to be given with correct answers; the remaining were presented with wrong answers randomly deviating (Split) from correct by $\pm 1, 5, 7$. Stimuli were randomly ordered for presentation except for two appropriate restrictions.

The procedures were that stimuli were projected onto a screen and subjects responded by pressing one of two buttons, randomly left or right, to indicate true or false. Response time was measured with appropriate apparatus. Subjects were tested individually in a darkened room. Among other instructions to the subject was that equal emphasis would be placed on accuracy and speed. Twenty practice items were given during which procedures were reviewed and emphasized.

4. Findings

Using "extremes - out" RT data (after an appropriate test of outliers) a $3 \times 2 \times 2 \times 3$ analysis of variance was conducted. This design was a mixed model with Grade (3,4,6) X Decision type (true, false) X Problem Size (small, sum ≤ 9 , large; $10 \leq$ sum ≤ 18) X Split ($\pm 1, 5, 7$) as variables. This analysis found all main effects to be significant: RT decreased across grade level, true problems were faster than false ones, Small problems were faster than large, and split 1 RTs were slower than 5 and 7. A Split by Decision Type interaction revealed a decline in RT to false problems as split increased. There was a significant four-way interaction showing that RT decreased from third to fourth to sixth grade, a consistent true/false effect was found at all grade levels, and large problems required more time than small.

The authors indicate that the interaction patterns suggest that third graders used different processes than fourth and sixth graders. Third graders exhibit a much larger problem-size effect than fourth and sixth graders. For this reason a more refined regression analysis to

examine the problem-size effect and what variables seem to account for it was conducted.

For a stepwise regression analysis, RT served as the dependent variable, and among the 15 predictor variables were: value of first addend, the correct sum, square of the correct sum, number of digits in sum, The results of this stepwise regression give support to the authors' contention that there is a process change between young and older children's performance. For third grade, only one predictor variable, number of digits in the sum, accounted for a sufficient amount of variance ($R^2 = 56.4$) to enter the equation. For fourth and sixth grade only the variable, correct sum squared, entered with $R^2 = 67.9$. No other variables added a significant increase in the variance accounted for in either case.

Each of these two predictors involved problem size, but a different aspect of it. A simple increase in R as a result of increased problem size (as indicated by number of digits in sum) is a benchmark in the research area, so the sum squared variable is taken to set the third grade apart from the fourth and sixth. The sum squared variable is interpreted as an index of memory search. Since the results suggest that a longer mental "distance" is traversed for larger problems, the authors indicate that fourth and sixth graders are using processes of mental retrieval similar to those employed by adults.

5. Interpretations

Since the counting model for processing basic fact additions fits the data for about half of the third-grade subjects, while data for the other half resembles more that from the fourth and sixth grade, it appears that third grade is a time of transition from counting to memory retrieval for obtaining answers to basic addition facts. Third-grade children apparently have numerical magnitude information stored as a mental representation. This is suggested by the observation that third graders (at least some) use memory retrieval for basic facts which requires a mental representation of arithmetical information. The switch from use of a counting model begins as early as the third grade and is apparently

complete by the sixth grade. Research results indicate that sixth graders are still less efficient in their basic fact addition performance than adults. This suggests a speculative hypothesis that "some of the relevant cognitive processes for addition ... shift from a slow, conscious process to a fast automatic one, as a function of mastery, overlearning, practice, and/or age" (p. 233).

Abstractor's Comments

This excellent article adds significantly to the knowledge base concerning young children's processing of basic addition facts. The study was carefully and logically conceptualized and conducted with scientific rigor. The research report gives adequate information for replication; also, it is carefully and succinctly written so that it is easily read by a knowledgeable reader.

As with other studies from which inferences are made about mental processes based on the goodness of fit between an hypothesized model and response time data, the conclusions of this study must be viewed as tentative. Corroborative evidence from clinical research using subject protocol data arising from "think-aloud" or "self-reporting" processes is necessary.

The article raises some interesting questions which remain to be investigated in the area of children's mental processing of basic addition facts.

Ballew, Hunter and Cunningham, James W. DIAGNOSING STRENGTHS AND WEAKNESSES IN SIXTH-GRADE STUDENTS IN SOLVING WORD PROBLEMS. Journal for Research in Mathematics Education 13: 202-221; May 1982.

Abstract and comments prepared for I.M.E. by IAN D. BEATTIE, University of British Columbia, Vancouver.

1. Purpose

The stated purposes of this study were: (a) to identify the main sources of difficulty that elementary school children have in solving word problems, and (b) to determine whether an efficient diagnosis of the main difficulty of an individual child is possible.

2. Rationale

The underlying assumption is that solving word problems is not a unidimensional process. The authors cite studies to support their contention that computational skills, reading ability, and ability to interpret the problem are important factors in solving word problems. They also assert that the ability to integrate these skills is an important factor, although this aspect has not been previously investigated. The authors believe that data derived from the study will improve instruction in solving word problems and facilitate research in the area.

3. Research Design and Procedures

The subjects in the study were all 244 sixth graders in two elementary schools in North Carolina. A basal mathematics text for grades 3-8 was used to develop three tests for each of these grade levels. The tests were constructed by taking the first three word problems in each word problem section and randomly assigning them to one of three groups. One group of problems was set up in computational form to be a measure of computational ability. Two-step problems contributed two items to the test. A second group of word problems was used to develop a problem interpretation test. The word problems

were read to the students, who were also given the written form of the problem. The students were required only to show what calculations would be necessary, not to complete the calculations. The third group of problems was presented in written form only, and students were required to show the necessary calculations (reading-problem interpretation) and to complete the calculations (reading-problem solving).

These three tests produced a profile of four scores for each student: a Computation score, a Problem Interpretation score, a Reading-Problem Interpretation score, and a Reading-Problem Solving score. A student's score on each test was defined to be the highest level at which 75% of the items were answered correctly. The tests were administered to all students. Complete data were obtained for 217 students. Areas of strength were identified by comparing computation, problem-interpretation, and reading-interpretation scores. An area of strength was defined to be one in which a student's score was one or more levels above his or her scores in the other areas. The procedure for finding areas of greatest need was more complicated. It included the reading-problem solving score and several "decision rules" to apply when scores were the same. These rules ensured that an area of greatest need was identified for each student.

4. Findings

Application of the decision rules to test scores identified the areas of greatest need as follows: computation (26%), problem interpretation (19%), reading-problem interpretation (29%), and integration of these areas (26%). Of the 217 students, 108 had an area of strength as follows: computation (75%), problem interpretation (21%), and reading-problem interpretation (4%). When comparison of pairs of scores was made, including those students with incomplete data, it was found that: (a) computation scores were generally higher (45% of the students) than problem interpretation scores (17% of the students), whilst the scores were tied 38% of the time; (b) computation scores were generally higher (60% of the students) than reading-problem interpretation scores

(12% of the students), whilst the scores were tied 28% of the time; and (c) problem interpretation scores were higher (44% of the students) than reading-problem interpretation (13% of the students), whilst the scores were tied 42% of the time.

5. Interpretations

The authors recognize certain limitations in the study. The word problems were selected from a single basal series and the sample was limited to sixth-grade students. Within these limitations, the authors conclude that problem-solving ability is composed of several component abilities, each of which can be a cause of difficulty in solving word problems, but suggest that mastery of the component abilities is not sufficient to guarantee success in solving word problems. They also suggest that reading ability is of greater importance in solving word problems than has been believed. Finally, the authors conclude that "... weaknesses and strengths in the major area of skills in solving word problems can be diagnosed through use of a comparatively simple procedure", and that "... individual students should receive treatment specified according to their areas of most immediate major need."

Abstractor's Comments

The authors are to be commended for undertaking a study to explore an area in which research has little to say of a practical nature to those concerned with diagnosis and remediation. Investigators are generally concerned with how to teach problem solving, not with determining sources of difficulty. It should be noted, too, that investigations in the area of diagnosis are not always amenable to standard research methods. The approach used by the authors is not only interesting but yields useful information. Undoubtedly the methodology used here will be refined and used in further studies. However, the report itself lacks clarity and omits a great deal of pertinent information that should have been included. Following are some concerns in those areas:

1) It was never made clear to the reader whether the purpose of the study was to develop a classification scheme and to verify it, or to use one to determine areas of difficulty. Discussion in the Problem and Design sections of the report suggest the former objective, but the Data Analysis and Discussion sections are devoted entirely to determining areas of difficulty. The investigation would have benefited from a pilot study where the focus was on the classification scheme and its verification. Then the second stated purpose, which should really be the first, could more properly be addressed.

2) The description of the test leaves many questions unanswered. The content and degree of difficulty of the items must have varied greatly over the span of grades 3-8. No information is given as to how many items were in each test at each grade level; what operations were involved and in what proportion; what concepts were covered (area, volume, etc.); whether the operations were on whole numbers, fractions, decimals, or integers; or how many problems were of the one-step or two-step type. It is possible that further analysis would reveal differences across these variables.

3) Procedures for the test administration are not clear. No information is given regarding time allotment, the order in which the three tests were administered, the interval between test administrations, how the scoring was done (e.g., for two-step problem-interpretation, was there partial marking for each of the two computations set up?), or whether all pupils started with the grade three level test. The order of administration, particularly, could have an effect on the results. The Reading-Problem Solving score, for example, could conceivably be enhanced by administration shortly after the Reading-Problem Interpretation test.

4) The decision rules are designed so that every student must have an area of difficulty. This seems unreasonable. Is it not possible that some students are equally able in each area? It could be argued, in fact, that such would be a desirable goal. The second decision rule includes the statement that "Students also cannot be expected

to interpret word problems when reading them if they are unable to interpret them when they hear them read aloud." The results show that 13% of the students did score higher when they did not hear the problems read aloud. The rules need to be rethought with these points in mind.

5) The authors compare the performance of students who have the problem read to them with their performance when they have to read the problem themselves. In fact, the comparison should be of performance when students have the problem read to them and have the opportunity to read it with performance when students can only read the problem.

Despite these limitations, the investigation is still of value. If the questions raised are addressed, perhaps using the data already gathered, the approach used should be followed by similar studies which will produce much needed information regarding the diagnosis and remediation of difficulties in solving word problems.

Bander, Ricki S.; Russell, Richard K.; And Zamostny, Kathy P. A COMPARISON OF CUE-CONTROLLED RELAXATION AND STUDY SKILLS IN THE TREATMENT OF MATHEMATICS ANXIETY. Journal of Educational Psychology 74: 96-103; February 1982.

Abstract and comments prepared for I.M.E. by JOANNE ROSSI BECKER, Virginia Polytechnic Institute and State University.

1. Purpose

This study examined the relative effectiveness of mathematics study skills training, cue-controlled relaxation, and a combined study skills and cue-controlled relaxation treatment of mathematics anxiety.

2. Rationale

The authors note that problems of mathematics avoidance and achievement have received increasing attention and that mathematics anxiety has been used as one explanation for both avoidance and poor performance. While some research has investigated relationships of affective variables to mathematics achievement, previous studies have provided little guidance for designing intervention programs to treat mathematics anxiety.

Because mathematics anxiety may function in a way similar to test anxiety, treatment approaches found effective with test anxiety might also be useful with mathematics anxiety. Therefore, cue-controlled relaxation was chosen as one of the treatments. Because improvement in mathematical skills might reduce anxiety, study skills training was chosen as the second treatment. A combined treatment was chosen because test anxiety research indicates that multicomponent programs may be more effective than single component ones.

3. Research Design and Procedures

The subjects were 36 college students who scored more than one standard deviation below the mean on the Mathematics Anxiety Scale. Pretreatment assessment included readministration of the Mathematics Anxiety Scale; the Anxiety Differential, a measure of state anxiety; the

Test Anxiety Scale; the Trait Form of the Strait-Trait Anxiety Inventory to measure trait anxiety; the Digit Symbol Test, a performance test; and the numerical sub-scale of the Differential Aptitude Test.

Subjects were randomly assigned to one of the three treatment groups (cue-controlled relaxation, study skills training, or a combination of these two), or to a no-treatment control group. Each treatment group met one hour per week for five consecutive weeks. The study skills training was designed to develop more effective study habits, and included solution of problems and discussion of underlying mathematical concepts. The cue-controlled relaxation treatment was designed to help people achieve relaxation in response to a self-induced cue word, "calm." The combination treatment included the same content as the two single-component treatments, but with less in-session practice.

One week following the end of treatment sessions, the six dependent measures were administered to all groups. Three weeks later, a follow-up battery, consisting of the Mathematics Anxiety Scale, the Test Anxiety Scale, and the Differential Aptitude Test, was given to the treatment groups only. It was hypothesized that: the combined treatment would be the most effective in improving scores on the Mathematics Anxiety Scale, the Test Anxiety Scale, the Differential Aptitude Test, and the Digit Symbol Test; and the study skills and combination treatments would be significantly more effective than the other two conditions in reducing general trait and state anxiety.

A priori comparisons were done to determine main treatment effects by comparing the treatment groups to the control group, the combined treatment group to the average of the single component groups, and the relaxation group to the study skills group. Separate comparisons were done pre to post and post to follow-up for each variable available. Multivariate analyses of variance with repeated measures were used to compare the pre- to posttreatment assessments on the six variables for the four groups, and to compare the posttreatment and follow-up assessments (on the three variables given as follow-up) for the three treatment groups. Univariate analyses of variance were used with each appropriate dependent

variable. When Group X Time interactions were significant, Dunn's post hoc comparisons were performed.

4. Findings

For the first two testing times, significant effects were found on the MANOVA (six variables) for Time and the Group X Time interaction. For the posttreatment follow-up testings (three variables), significant differences also were found on the MANOVA for Time and the Group X Time interaction. No overall group differences were found.

Findings from the a priori comparisons, univariate ANOVAs, and post hoc comparisons are rather complex; these findings are summarized here in an attempt to provide a clearer picture of results on each variable.

1. Mathematics Anxiety Scale. The study skills treatment reduced anxiety significantly more than the other three treatments, which did not differ from pre to posttreatment. However, on the posttreatment to follow-up analysis, the study skills and combination treatments did not differ, and the relaxation treatment showed significantly greater improvement.

2. Test Anxiety Scale. Comparisons pre to post showed the relaxation and combination treatments to be superior to the study skills treatment and no treatment, which did not differ. However, the posttreatment to follow-up analyses showed only time significance; all groups tested improved.

3. Differential Aptitude Test. The study skills treatment was better than the relaxation treatment, pre to post. The posttreatment to follow-up analyses showed that the study skills treatment deteriorated over time, while the relaxation group improved on this variable; the study skills and combination groups were equal at follow-up.

4. Anxiety Differential and Trait Anxiety Scale. No significant differences were found on these two variables.

5. Digit Symbol. The study skills, relaxation, and no treatment groups were equivalent on this variable at the posttreatment assess-

ment, with the combined treatment more effective than these three in improving performance on this variable.

5. Interpretations

The authors state that the pattern of results pre to post suggests that the study skills treatment was most effective in reducing mathematics anxiety, but the relaxation treatment was best in improving general test anxiety. The combination treatment produced improvements on test anxiety and mathematics anxiety, but was superior to the individual components only on the Digit Symbol Test.

By the three-week follow-up, the relaxation treatment was superior to both the study skills and combination groups on the Mathematics Anxiety Scale and Differential Aptitude Test. The relaxation group continued to improve on all three dependent variables over the follow-up period. This continued improvement suggests the possible need for sufficient time to elapse for complete development of a conditioned relaxation response.

The authors conclude that cue-controlled relaxation is a potentially viable intervention strategy for treatment of mathematics anxiety.

Abstractor's Comments

The weaknesses of the design of the study, which the authors point out, make it difficult to draw any conclusions about the efficacy of one treatment of mathematics anxiety over another. Besides the small sample size, nonrepresentativeness of the sample, and lack of normative data on some scales, the most critical flaw was the failure to include the control group in the follow-up assessment. Familiarity and experience with the instruments might have caused the improvement over time on, for example, the Test Anxiety Scale and Differential Aptitude Test. Without follow-up testing of the control group, this alternate explanation cannot be excluded.

The authors stated they were interested in determining the relative efficacy of these intervention strategies for reducing mathematics anxiety and improving mathematics performance. A more realistic test of the treatments from this point of view might be to examine students' performance in

current or future mathematics course, or a simulated attempt to learn mathematics. The failure of the study skills group's improvement in anxiety to extend to the follow-up may be due to lack of practice of the skills outside the experiment. And the Digit Symbol Test seems like a poor measure of mathematics performance, albeit being anxiety-producing.

The design of the treatments also merits some comments. The combined treatment attempted to provide both cue-controlled relaxation and study skills in five hours. It is questionable if these skills were ever attained in that time. In fact, five hours seems a short time to learn either the relaxation or study skills alone. How do we know if the subjects ever attained either of these skills?

The data analyses were fairly complex, but it still seems that the findings could have been written more clearly and completely. For example, I could not find in the report the result of the univariate ANOVA for the Mathematics Anxiety Scale for the posttreatment to follow-up assessments. Also, some items on relative efficacy in the summary table for post hoc comparisons do not seem to be substantiated by the test. No discussion of the testing of assumptions for the MANOVA was included in the paper. And there is either an error in the descriptive statistics for the study skills group on the Test Anxiety Scale, or an error in the text.

To the authors' credit, they are cautious in their conclusions. Because of the limitations of the study, it is difficult to conclude that one treatment is better, or that single component treatments are better than multiple component ones. There is some indication that mathematics anxiety can be improved by intervention, and that relaxation techniques may be useful. But definitive answers must await further studies.

Bar-Eli, Nurit and Raviv, Amiram. UNDERACHIEVERS AS TUTORS. Journal of Educational Research 75: 139-143; January/February 1982.

Abstract and comments prepared for I.M.E. by GEORGE W. BRIGHT, Northern Illinois University, DeKalb.

1. Purpose

Underachieving fifth- and sixth-grade students tutored second-grade students in mathematics as a test of the following hypotheses: (a) The tutees improve in mathematics more than second-grade students who are not tutored. (b) The tutors show more improvement in mathematics, have fewer failing grades overall, and show a greater improvement in self-concept than underachieving fifth- and sixth-grade students who do not tutor.

2. Rationale

The research was conceptualized within the Learning Through Teaching paradigm which derives from a large body of peer teaching research. Recent research done within this paradigm has examined the effects of the tutoring on the tutors in addition to the effects on the tutees, and this study was a continuation of these recent efforts.

3. Research Design and Procedures

Underachievers among fifth and sixth graders were defined to be students with IQs of 110 or more, failing grades in mathematics, and failing grades in at least two more subjects (not including music, drawing, and gardening). Second graders viewed as appropriate for tutoring were those with mathematics scores of b or c on a scale of a, b, c. All subjects were boys "to rule out possible sex differences associated with the phenomenon of underachievement at different age-levels; and to prevent the concept of girls teaching boys and vice versa from interfering with the results of the program" (p. 140).

The fifth and sixth graders were selected as follows:

- a. Scores on a nationally administered achievement test had to be below 65%.
- b. Then, scores on a group-administered Israeli adaptation of the Large-Thorndike Intelligence Test had to be 90 or above.
- c. Then, scores on the WISC (individually administered) had to be 110 or above.

Thirty-six boys satisfied these criteria.

The second graders were selected as follows:

- a. Scores on a nationally administered achievement test had to be below 65%.
- b. Scores on MILCAN, "a new group intelligence test for the lower grades of primary school" (p. 140), had to be 90 or above. (No reference for this test was provided.)
- c. The classroom teacher and the school psychologist had to agree that the student exhibited no pathology.

Of the students meeting these criteria, the 36 with the highest IQs were selected as subjects.

(There is some lack of clarity about the number of pupils initially available for consideration. There is also no rationale given for the particular cut-off used for the various tests.)

The 36 fifth and sixth graders completed a self-concept questionnaire, and then the teaching project was explained to them. Lots were drawn to select the 18 tutors, with selections of experimental and control subjects matched on schools (there were three) and classes within schools. Two tutors and two control group subjects asked to change roles, and that request was granted. The 18 second graders to be tutored were selected at random, with matching again for schools and classes. Parents of all subjects gave permission for the tutoring program. The 18 tutors and 18 tutees were randomly paired.

The tutor/tutee pairs met three times per week for about four months. These sessions were in place of the regular mathematics classes. The experimenter visited each pair weekly and met alone with the tutor one afternoon per week. The tutor also met weekly with the second grader's teacher to plan for the tutoring sessions. The 'pay' for the tutor was extra help in learning English, which was viewed as one of the most

difficult subjects. The total number of sessions ranged from 26 to 40, and three of the tutors dropped out, reducing the number of experimental subjects to 30 (15 tutors and 15 tutees).

The standardized mathematics test was administered as a posttest; second graders took the same test that had been used in the selection process, and fifth and sixth graders took an alternative form of their pretest. The tutors were awarded certificates of completion in a public assembly, and the following day the fifth and sixth graders completed the self-concept questionnaire a second time.

Analysis of covariance was applied to each measure, with the corresponding pretest as the covariate. Comparison of number of failing grades resulted in significantly different regression slopes for the experimental and control groups, so a t-test was used to analyze these data.

4. Findings

The tutees showed more improvement on the standardized mathematics test (p less than .05) than the non-tutored second graders. There was no difference in the groups on class grades.

The tutors showed more improvement on the standardized mathematics test (p less than .01) and more improvement in class grades (p less than .05) than the non-tutoring fifth and sixth graders. The tutors also had a significant decrease in number of failing grades on report cards (p less than .05) when compared to the non-tutors. There was no significant difference in self-concept.

5. Interpretations

The findings support the assumption that peer tutoring benefits both the tutor and the tutee. The overall improvement in school work by the tutors is viewed as the most important finding of the study. Such overall improvement may increase the confidence of the tutors and improve the perceptions of teachers and parents about the likelihood of future achievement of the tutors. This may improve the motivation of the tutors and may alter the cycle of underachievement.

The lack of improvement of the tutees on class marks may be related to the relatively coarse scale used at that grade. Too, teachers may have

been reluctant to award improved marks to students who were absent from mathematics classes three days per week. Alternately, teachers may not have viewed any improvement in mathematics performance as permanent and thus may have acted conservatively in awarding marks.

The lack of effect on self-concept may be due to the relatively short duration of the experiment.

Abstractor's Comments

At a superficial level, the study seems to be fairly cleanly conducted, though exact replication of the research might be somewhat difficult. Of particular concern in this regard is the seemingly arbitrary cut-offs used for identifying the underachievers to be tutors and the second graders to be tutees. The specific definition of underachieving suggests that the definition of underachievement may have been created after the data for potential subjects were examined. Too, the uniquenesses of the Israeli educational system would prevent replication outside Israel.

At a more detailed level, the study lacks important information; namely, the reader does not know what behaviors took place during the tutoring. Apparently the experimenter did not identify for the tutors what was expected of them. Further, the report contains no information about what the tutors actually did to try to help the tutees. Much more information about the range of behaviors occurring in the tutoring sessions would have been extremely helpful.

Of potentially deep concern about this study is that it doesn't seem to represent a significant extension of the theoretical model underlying it. The best conclusion that can be made is that mathematics tutoring can assist underachievers in improving academic performance. There was apparently no attempt to determine what aspects of the tutoring experience actually effected the observed changes. That seems to be the most important kind of information that is needed to expand and clarify the learning-through-teaching model. Additional short-term studies probably won't be very useful in providing this kind of information.

Too, follow-up is needed to find out if the effects that were observed are sustained. This relates to finding out what characteristics of tutoring cause the improvement among the tutors. If continual tutoring

is required to maintain the gains that the tutors displayed, then tutoring may not be a realistic instructional procedure for improving performance. By knowing what aspects of tutoring cause the improvement, teachers might be able to incorporate those aspects into regular classroom instruction.

Finally, the results arising out of the self-concept data are somewhat unexpected. The description of the recognition given to the tutors in the public assembly caused this reviewer to expect a strong self-concept improvement among the tutors. Further investigation of the reasons for the lack of this effect would seem to be called for. Certainly, the public assembly caused a confounding of the effect on self-concept. The effect of the tutoring on self-concept cannot be separated from the combined effect of tutoring with public recognition.

Becker, Wesley C. and Gersten, Russell. A FOLLOW-UP OF FOLLOW THROUGH THE LATER EFFECTS OF THE DIRECT INSTRUCTION MODEL ON CHILDREN IN FIFTH AND SIXTH GRADES. American Educational Research Journal 19: 75-92; Spring 1982.

Abstract and comments prepared for I.M.E. by LARRY LEUTZINGER, Area Education Agency 7, Cedar Falls, Iowa.

1. Purpose

This study investigated the long-range effects of Direct Instruction Follow Through. The programs being evaluated began in first grade and continued through third grade. The students tested were fifth and sixth graders who had been out of the Follow Through program for two or three years. The students were evaluated in the areas of reading, spelling, word knowledge, language, mathematics computation, mathematics concepts, mathematics problem solving, and science.

2. Rationale

When Direct Instruction Follow Through sites were evaluated after three years of instruction in the mid-1970s, the third-grade students performed significantly better in 80% of the tested items for mathematics, 50% in spelling, 60% in reading, and 100% in language. Whereas these data indicate that the Direct Instruction programs were successful, it was deemed important to evaluate the progress of the students involved in the program in later years to determine if their achievement levels remained high as compared to similar groups of students who had not been a part of Direct Instruction Follow Through.

3. Research and Design Procedures

In 1975, 624 fifth- and sixth-grade students who had been involved in Direct Instruction Follow Through in first through third grades were tested using the Metropolitan Achievement Test (MAT), Intermediate Level, and Levels I and II of the reading subtest of the Wide Range Achievement Test (WRAT). The students who participated were from five representative sites (East St. Louis, Illinois; Smithville, Tennessee; Uvalde, Texas;

Dayton, Ohio; Tupelo, Mississippi) which volunteered to be a part of the study. These students' test results were compared to those of 567 non-Follow Through students of similar backgrounds and abilities from the same five sites. In addition, the Follow Through students' results were compared to the national norm sample. The study was replicated in 1976 and involved 473 Follow Through students. In this study only four of the original five sites were involved.

An immense amount of data was gathered and scrutinized. The test results from all students involved were evaluated using a quasi-experimental design. For each Follow Through site, analysis of covariance was performed on each subtest of the WRAT and MAT. The sites were then classified as significant, suggestive of a trend, or nonsignificant. A meta-analysis technique where the P values on analysis of covariance for each site were changed to chi-square ratios and tested for significance was then used.

Finally, the average magnitude of effect in pooled standard deviation units for each subtest at each grade level was calculated. The method gives an estimate of the treatment effect that is not biased by the differential sample sizes at the various sites (page 81).

4. Findings

The ANCOVA comparisons for each fifth- and sixth-grade group on all the subtests of the WRAT and the MAT indicate that 56 of the site comparisons favor the Follow Through groups at the .15 level of significance and 102 comparisons are not statistically significant. Twenty additional comparisons favor the Follow Through groups at the .15 level of significance and 102 comparisons are not statistically significant. Of the 180 possible comparisons, only two favor the non-Follow Through groups, at the .15 level of significance.

On the mathematics section of the MAT, which includes concept, computation, and problem-solving subtests, the Follow Through groups were favored at the .15 level of significance or better in 5 of 11 comparisons on the computation subtest. On the concepts subtest, 5 of the 14 comparisons favored the Follow Through groups at the .05 level of significance. On the problem-

solving subtest, 7 of the 14 comparisons favored the Follow Through groups at the .15 level of significance or better. Of the 39 possible comparisons on the mathematics subtests of the MAT, in only one case were the non-Follow Through groups favored at the .15 level of significance.

On the meta-analysis using chi-square ratios for the pooled results, the Follow Through groups were favored in 8 of the 12 subtest categories for both Grades 5 and 6. On the mathematics problem-solving subtest for fifth grade, the chi-square analysis indicates a significance at the .05 level in favor of the Follow Through groups. At the sixth-grade level the results for each mathematics subtest favor the Follow Through groups at the .05 level of significance.

When the Follow Through groups are compared to the norm sample, the percentile rankings on the WRAT and MAT subtests indicate a dramatic decrease from third through sixth grade. The average of the percentile rankings for the sites involved on the mathematics subtest of the MAT is 62% at the end of third grade. By the end of fifth grade the average percentile ranking has fallen to 31% and by the end of the sixth grade the average is 27%.

5. Interpretations

Graduates of the Direct Instruction Follow Through Program perform better than similar children who were not a part of the program as measured by standardized achievement tests. The differences are most striking in WRAT reading and MAT mathematics problem solving and spelling. No outcomes favored the non-Follow Through groups at the .05 level of significance, while 31% of the comparisons favored the Follow Through groups.

While the Follow Through graduates outperform other low-income fifth and sixth graders in their communities, they fail to keep up with middle-income students. The failure appears to occur in computational skill development in mathematics and vocabulary development and reading comprehension.

"In order for these children to become fully literate adults, it appears they need high-quality instructional programs in the intermediate grade (and probably beyond)" (page 89).

Abstractor's Comments

Before beginning my critique of the article, a brief explanation of the Follow Through program is in order. Follow Through was designed as both a research and a demonstration project for disadvantaged children as a follow-up to the Head Start Program. Developers of innovative programs worked with local school districts or sites who volunteered to implement a particular sponsor's model. These models ranged from highly structured ones like Direct Instruction to open, loosely structured ones.

Follow Through was conceived in 1967 and is the largest educational experiment ever conducted, with over 500 million dollars invested in 15 years. The costs of evaluating the various programs alone were estimated at 50 million dollars.

The results of the evaluation of the Follow Through programs point to the superiority of the Direct Instruction Model to others used. However, questions were raised about the evaluation regarding the lack of comparable control groups and the reliability of using non-Follow Through data as the basis of comparison. The same questions can be raised regarding this study, but someone with a stronger statistical background than I should do so.

Not being a statistician by nature, one of two things usually happens when I come across masses of data. I either am greatly impressed by the insight and perseverance it took to complete and evaluate the information, or I naively question what it all means and if it was worth the effort.

For this study I was impressed by the amount of data. To do an analysis of covariance with as many variables as were involved is a monumental task, even for a computer. Then to analyze the scores of each subtest for significance is impressive.

What bothers me about the statistics, and this may be my naivety again, is the lack of consistency in the reporting of the results. Some of this inconsistency is due to misprints or deletions. These kinds of errors are understandable when this much information is reported. Heaven knows, there may be a misspelling or two in this very review.

On Table II it appears the numbers listed for the mathematics computation subtest in the 1976 study are misrepresented, since only three sites are listed -- not six. In Table III the degrees of freedom for the Grade 6 language subtest should be 14. On Table III the title indicates that there are seven Follow Through sites. While the number of sites used in the comparison varied for different analyses, at times being ten, eight, five, four, and three, seven is not a likely number and represents a misprint. The misprint aside, this inconsistency in the number of sites included in the various analyses raises questions regarding the intent of the statistics.

On Table I the sample sizes for the fifth grades are listed. But later in the study these numbers change for the Tupelo and Smithville sites on Table VI and for the Tupelo, Smithville, and Dayton sites on Table VII. No reason is listed for these discrepancies.

While this study professes to assess the later effects of the Direct Instruction Follow Through at five diverse sites, in none of the eight tables displaying information is a complete picture presented. By my count, in 64 cases data are missing or unavailable. On some of the tables nearly 20% of the data is missing. Even the tables which have no missing data are dependent on previous results where data were missing. It is difficult to draw valid conclusions from incomplete data.

When presenting information, usually the least massaged data appear first. In this study that is not the case. The unadjusted percentiles for the subtests on the MAT and WRAT appear near the end of the report. For those scores the fifth-grade Follow Through groups are statistically no different than the non-Follow Through groups in the MAT reading, total mathematics, spelling, and science subtests. From previous tables the conclusion was drawn that the Follow Through groups were clearly superior to the non-Follow Through groups. For the fifth-grade groups in 1975 this was not true.

Since percentile rankings are not the most sophisticated means of evaluating data, perhaps they should be replaced by more profound methods. On the other hand, it was the last sets of percentile scores that made the greatest impression on me. In those, the longitudinal analyses of the

percentiles for the two tests were displayed. Despite the fact that data were missing, a trend was apparent. The percentile scores on the subtests for the sixth-grade students were about one-half of the scores for the last year of the Follow Through program in third grade. This decline is precipitous and distressing. As cited in the article, "There is now a need to implement and evaluate instructional programs in the intermediate grades that systematically utilize principles of direct instruction, which include mastery learning, high level of feedback, and incremental steps to develop independent reading, writing and critical thinking" (page 89). If these suggestions are implemented, perhaps the drastic decline can be stemmed.

Despite its shortcomings, this article addresses a real need in educational research, that of longitudinal evaluation of programs and projects. The effort that was made in that regard is commendable. In the future, researchers should attempt to build in a data collection scheme which allows for the consistent, systematic evaluation of programs over a period of time. After all, learning is a change of behavior and the true measure of whether a change has occurred should be based on more than a one-time assessment.

Cathcart, W. George. EFFECTS OF A BILINGUAL INSTRUCTIONAL PROGRAM ON CONCEPTUAL DEVELOPMENT IN PRIMARY SCHOOL CHILDREN. Alberta Journal of Education Research 8: 31-43; March 1982.

Abstract and comments prepared for I.M.E. by DOUGLAS H. CLEMENTS, Kent State University, Kent, Ohio.

1. Purpose

The purpose of the study was to investigate whether a bilingual instructional program would positively affect children's concept formation and cognitive functioning as measured by Piagetian conservation tasks and related verbal rationalizations.

2. Rationale

Evidence is reviewed that children who are fully bilingual seem to have some cognitive advantages over unilingual children, including increased "cognitive plasticity," achievement, and divergent thinking ability. Do bilingual instructional programs provide these advantages for students? Evaluations of bilingual programs in Canada provide some evidence that children enrolled in them perform higher on assessments of cognitive development, intelligence, and divergent thinking; however, it appears that a threshold of linguistic competence must be attained for these effects to manifest themselves. Evidence concerning effects of bilingual instruction on concept development as defined by Piagetian conservation tasks is sparse, but seems to indicate that children in the programs perform as well or better than their monolingual controls.

3. Research Design and Procedures

Subjects were 192 children: four boys and four girls randomly selected from each program (French/English bilingual or English) at each of grades 1, 2, and 3 in each of four schools. To measure concept formation, they were administered a 16-item test

assessing number and measurement conservation. Four number subtests involved numbers less than 10, numbers greater than 10, additive rearrangement, and quantity. Four measurement subtests included length, area, mass, and volume. Order of administration of the subtests was counterbalanced. In addition, as a measure of cognitive functioning, children's rationalizations for conservation were categorized as (a) operational identity, (b) substantive identity, (c) reversibility, (d) compensation, or (e) other. If a rationalization was given, a second was requested. The researcher and two graduate assistants tested all children individually. Adequate reliability of the conservation test was assumed based on previous research. Inter-rater reliability for the classification of the rationalizations was determined to be .88.

For the second- and third-grade students, percentile scores on the Metropolitan Readiness Test administered during the first grade were used as a covariate in a three-way analysis of covariance (grade x program x sex). No covariate was available for the first-grade students.

4. Findings

The adjusted means of the bilingual group were higher on five of the subtests, the reverse was true for one subtest, and the groups had identical means on one subtest. Results of the analysis of variance adjusted for readiness scores revealed that these differences were significant for only two subtests, additive rearrangement and length, in favor of the bilingual group.

The number of second rationalizations given was taken to be an indicator of cognitive flexibility. The difference in the unadjusted means was statistically significant in favor of the bilingual children; however, the adjusted means were not significantly different. Since there was a significant grade-by-program interaction, separate analyses were performed for the grade 2 and grade 3 samples. There was no difference at the

grade 2 level, but the adjusted mean of the bilingual grade 3 classes was significantly greater than the adjusted mean of the English classes. The adjusted mean of the children in the bilingual classes on the rationalization of reversibility was also significantly greater than the corresponding mean of the children in the English classes. No other means differed significantly.

There were also some grade differences. The mean of the grade 3 children was significantly greater than that of the grade 2 children on three number conservation subtests and on the number of second rationalizations given. There were no significant differences on any other items.

Since none of these results included grade 1 children, *t* tests done between the bilingual and the English classes were reported. There were no significant differences at grade 1 on the conservation subtests. There was a significant difference in favor of the bilingual group on the number of second rationalizations given at grades 1 and 3, but not at grade 2.

5. Interpretations

The author pointed out that at best the "study provides only weak support for the hypothesis that the concepts of number and measurement are more advanced in children who have been in a bilingual class for less than three years" (p. 42). However, the magnitude of the advantage for the bilingual group did increase as time in the program increased, offering support for the "threshold" theory, which maintains that a certain level of fluency needs must be attained before differences can be observed.

The author suggested that the results were more convincing with regard to cognitive functioning, as the number of second rationalizations and the use of reversibility were both at a higher level in children from the bilingual classes. He argued that this was not due to greater verbal ability, for controlling for readiness score should have controlled this variable to some extent.

It was concluded that the differences in favor of the bilingual classes provide justification for their continuation.

Abstractor's Comments

While the findings are not strongly in favor of bilingual instructional programs, they are suggestive, and certainly indicate that these programs do not have any harmful effects on cognitive development as measured by conservation tasks. Collaborative research, using these and other tasks, is needed before firm conclusions can be drawn.

The number of subjects, method of selection, and procedures used generally lend validity and reliability to the findings. However, several questions can still be raised. The author states that children "who experienced difficulty often transferred out of the bilingual program or never registered in it. Therefore, the 'better' pupils were in the bilingual program" (p. 36). To what degree could the finding that the children in the bilingual program increased their advantage from first to third grade be attributable to the attrition of those children who were not, for whatever reason, benefitting from the bilingual program? The author argues that the use of a readiness test as a covariate controlled for initial differences. But is it certain that controlling for variance in readiness controlled for any variance in development? Also, what of other differences that might have existed between the programs? It is not clear if the curricula were equivalent, and, possibly more important, if the teachers who choose to teach in the special bilingual program were themselves special in any way. However, to the extent that it can be assumed that the only difference between the experimental and control conditions was that the former involved bilingual instruction, the findings have significant educational implications. To this extent the study also highlights the importance of the role of

language in the development of logical-mathematical concepts. This role is frequently denied or minimized by some Piagetian-oriented researchers.

It is assumed that children's rationalizations are a measure of their level of cognitive functioning. Children in the bilingual program gave more reversibility-type arguments, the use of which was shown in previous research to be related to higher performance. It is interesting to speculate; does translation between languages promote the development of a reversibility mind-set? Other, possibly non-verbal, assessments of reversibility and cognitive functioning may provide interesting insights into these problems.

The study provides a useful review of the research and offers evidence that bilingual programs are not harmful, and may be beneficial, to children's cognitive development. It also raises interesting questions for future educational and developmental research.

Cook, Cathy J. and Dossey, John A. BASIC FACT THINKING STRATEGIES FOR MULTIPLICATION -- REVISITED. Journal for Research in Mathematics Education 13: 163-171; May 1982.

Abstract and comments prepared for I.M.E. by JOSEPH N. PAYNE, University of Michigan.

1. Purpose

The purpose was to compare results on multiplication fact instruction in grade 3 using two different approaches to the facts. One approach used thinking patterns based on special groups of related facts. The second approach was based on the size of the factors.

2. Rationale

Studies as early as 1935 were referenced to indicate interest in basic fact instruction. The study done by Thornton in 1978 was used as a model for the research by Cook and Dossey. Thornton's study dealt with addition, subtraction, multiplication, and division facts in grades 2 and 4. Cook and Dossey modified Thornton's study by claiming to remove the experimenter bias by having regular classroom teachers do the teaching. In addition, in-service was given to both treatment groups to equate drill-and-practice time to remove a problem they state may have existed in the Thornton study. Further, they lengthened the retention time and used grade 3 to assess early learning of the facts, suggesting that Thornton's work in grade 4 may have been on relearning facts.

3. Research Design and Procedures

The sample consisted of 220 (erroneously reported as 219 on page 164) grade 3 students from Schaumburg, Illinois. Schools and third-grade classes were chosen at random, but there was no indication whether treatments were assigned to schools randomly. Two of the original classes were dropped because one school did not follow the research design. The elimination of one school resulted in relatively unequal treatment groups, $n = 134$ for Thinking Strategies and $n = 86$ for Factor Size.

The Cognitive Abilities Test was used as an aptitude test, with no significant differences reported for the two treatment groups ($t = 0.07$; $df = 218$). Actual mean scores on the test were not included in the report.

The first in-service day (length of session not specified) dealt with the teaching schedule for the nine-week experimental period. On the second day (length again not specified), teachers in both groups constructed games and activities.

Instruction lasted for nine school weeks, for 20 minutes a day, beginning in mid-January. A one-minute timed quiz was given each week on the facts studied so far, but no indication was given on how it was scored and apparently no analysis was made of the results.

Dependent measures were the number of the 55 multiplication facts students got correct in two minutes. Students were told to do the easiest ones first. The facts tests were given as a pretest, at the end of the 3rd, 6th, and 9th week of instruction, and eight weeks later as a retention test, with facts in a different random order for each test. During the eight weeks before the retention test, the classes studied the concept of division, division facts, and other topics.

Means and standard deviations on each test are reported by treatments and for the entire sample. A one-way ANOVA was done for growth scores from pretest, and also for growth scores between successive tests. A two-way ANOVA was done using method x aptitude, with aptitude defined as "high" for score above 120 on the Cognitive Abilities Test and "low" if below 100. Differential effects of the two treatments were examined for 14 "hard facts" (6x7, 6x8, 6x9, 7x4, 7x6, 7x7, 7x8, 7x9, 8x6, 8x7, 9x4, 9x6, 9x7, and 9x8).

4. Findings

For the Thinking Strategies and Factor Size treatments the mean scores were, respectively: pretest, 12.29, 12.69; test 1, 28.39, 28.24; test 2, 39.00, 35.00; posttest, 46.60, 43.55; and retention test, 48.32, 45.27. For growth scores from the pretest, $p < .05$ for test 2, posttest, and

retention test in favor of the Thinking Strategies. For growth between successive tests, test 2 - test 1 favored thinking strategies, but no other difference was significant. (The article erroneously reports posttest - test 2 as significant; the table shows $p = .25$. Actual growth scores computed from another table show a growth of 7.60 for Thinking Strategies and 8.48 for Fact Size between test 2 and posttest.) No aptitude \times method interaction was significant, but the main factor of thinking strategies showed $p = 0.048$ for test 2 - test 1. High and low aptitude groups gained 8.92 and 9.75 respectively, in Thinking Strategies, but only 5.33 and 7.26, respectively, in Factor Size. A methods \times aptitude analysis of the results on the 14 hard facts showed aptitude as a significant factor; high aptitude mean was about 11 and low aptitude mean was about 6.

5. Interpretations

The data support the thinking strategies approach for teaching multiplication facts, although both groups achieved a high level mastery in the end. The investigators suggest that the rapid growth during the fourth through sixth weeks for the Thinking Strategies group "... allows for a shorter time to be spent on the facts and more time on the reviewing and relating these thinking strategies to the retention of the basic facts" (page 170). They note that teachers can be trained to teach thinking strategies in a short period (two days).

Abstractor's Comments

The results from this study provide a helpful supplement to other fact studies and to the Thornton study especially. The information about the rapid growth of the Thinking Strategies group during the second three-week period of instruction is new, suggesting that students need three weeks for warm-up before the approach using thinking strategies begins to take hold. Evidently, the major heuristics that must develop for thinking strategies must be practiced a lot before their effectiveness can be recognized in the results.

It is something of a puzzle why the thinking strategies were not more effective for the harder facts, when Thornton found such startling differences. Could it be that this study did not utilize her strategies for the harder facts? Why was aptitude the more important factor here and not with the other facts?

There is no indication of the way the multiplication facts and thinking strategies were used in the subsequent work on division, taught in the posttest-to-retention-test period. Further, with such excellent results on multiplication, it would be valuable to have results on division. The results for multiplication and division must be related.

This study contained no interview data that proved so interesting in Thornton's study. What strategies did students use who were in the Factor Size group? Further, what did teachers do to help children find answers? Surely, they would have skip counted, added, or done something. Interviews and reports on what teachers actually did would have given more information on what happened in each treatment.

Comparisons with Thornton's study would have been easier if comparable tests and times had been used. Thornton used 100 fact tests, with three-minute intervals, and identified 49 facts as "hard," while Cook and Dossey used 55 facts, with two-minute intervals, and identified 14 "hard" facts.

Both treatments had a more carefully laid-out plan for fact learning than is usually found. Nine weeks of instruction were provided. Timed tests were given often. Games and practice were included. From all this classroom work, it should be clear to teachers and students that fact learning is important and that time must be provided both for development and practice. A planned program using thinking strategies over an extended period of time may be the message teachers and curriculum planners should draw from this study.

Galbraith, P.L. THE MATHEMATICAL VITALITY OF SECONDARY MATHEMATICS GRADUATES AND PROSPECTIVE TEACHERS: A COMPARATIVE STUDY. Educational Studies in Mathematics 13: 89-112; February 1982.

Abstract and comments prepared for I.M.E. by MAX STEPHENS, State Education Department of Victoria, Australia.

1. Purpose

The study is intended to link two areas of current interest in mathematics education: the "mathematical characteristics of prospective teachers, and the notion of levels of understanding" (p. 89).

2. Rationale

Those who have sought to explain the gap between the intentions of curriculum reformers and what has been achieved in the classroom have viewed the mathematical characteristics of teachers as a possible source of explanation. Mathematics educators have marvelled at the fact that many students who have undertaken extensive studies in mathematics, either at high school or in undergraduate courses, still seem to approach mathematics as though it were a matter of applying set rules to produce routine answers to standard textbook questions. Many of these complaints are anecdotal, and have not been supported by reliable evidence establishing the nature and extent of students' misunderstandings and misconceptions.

Respected writers such as Howson (1975), Gray (1975), and Buckland (1969) have argued that mathematics graduates, especially those who are preparing to become teachers, often lack a lively understanding of and sound competence in mathematical inquiry. These writers echo a widespread fear that deficits in the present crop of prospective teachers will be transmitted to the next generation of students in elementary and high school.

In this study, the researcher proposes to investigate the mathematical vitality of mathematics students who are among those currently enrolled in the first year of an undergraduate program, and those who are undertaking a course of teacher preparation after completion of an undergraduate degree.

3. Research Design and Procedures

The author asks what kinds of attributes would one expect to find in a "mathematically aware student". These attributes, which the author chooses to refer to under an umbrella term "mathematical vitality", are those which he expects to be derived from a range of mainstream courses in mathematics and not from any particular course. His chosen criterion attributes include:

- (1) Appreciation of logical form including the ability to
 - (a) write and interpret inferential statements,
 - (b) distinguish between necessary and sufficient conditions and between implication and equivalence,
 - (c) understand the relation between statement and converse,
 - (d) use counter examples effectively,
 - (e) evaluate the validity of extended chains of reasoning such as proofs.
- (2) A knowledge of the major conventions such as $|x|$ and \sqrt{x} and the capacity to interpret them in context and with consistency.
- (3) The ability to interpret the precise meaning of statements, e.g. definitions, and judge whether specific criteria are adequate or have been met.
- (4) An understanding of fundamental notions of analysis such as the distinction between the limit and value of a function, and between continuous and differentiable.
- (5) The capacity not only to apply rules, e.g. for derivatives, but habitually to consider when it is legitimate to do so.
- (6) Awareness of possibilities and impossibilities, e.g. that area should have the dimension of $(\text{length})^2$
- (7) Awareness of the arbitrary meaning of a definition in mathematics, and the capacity to explore properties of a mathematical system using a defined operation such as $a * b = 2(a + b)$.
- (8) Awareness of the nature of the domain of a function and its importance in a variety of contexts. (pp. 91-92)

These attributes have been incorporated into an 18-item test utilizing multiple-choice responses.

In order to check whether the test items were reliable, the author tried them out over a four-year period in one university on students who were completing a course of teacher training after an initial degree which included a major or a minor study in mathematics. The author reports that the distribution of response patterns was relatively stable over the period of trials.

The study was then carried out on two groups of students in universities in three Australian states. One group replicated the characteristics of the original trial group. The other group comprised students who had just completed high school and who are studying a "typical" first-year course in mathematics.

Since no significant difference emerged among states for each group, the author collapsed the six sample groups into an undergraduate and post-graduate sample. Within each group a t-test was carried out in order to determine whether the amount of prior study in mathematics was a significant factor affecting performance on the test. Cross tabulations were also carried out on groups of items which were expected to measure the same attribute to ascertain whether success on any items was related to success on other items.

4. Findings

The t-tests did not show any significant difference in either group according to the amount of prior study at high school or at university. Although the proportion of correct responses is usually higher at the post-graduate level, the author argues that

in general there is no tendency for the responses of the post-graduate group to close in on the correct alternative in comparison with the corresponding undergraduate patterns. (p. 102)

Cross tabulations did not reveal any discernible hierarchy among items which embodied similar mathematical tasks.

On the 18 items of the test, the postgraduate group achieved a proportion of correct responses in excess of 40% on seven items only. The postgraduate group scored between 21% and 40% correct on nine items, whereas the undergraduate group had 11 items with the same band of correct responses, and a lower response rate on four items.

5. Interpretations

From the low proportion of correct responses, the author infers that some mathematical concepts are genuinely difficult to acquire. From the distribution of correct responses the researcher proposes to identify and link gaps in students' mathematical performance, and seeks to explain these deficiencies in terms of underlying problems rather than looking only at the proportion of right and wrong responses in each group.

The author argues that the level of mathematical vitality achieved by undergraduate students is "independent of local contexts with regard to precise syllabus content and curriculum emphasis" (p. 107). For postgraduate students, mathematical vitality is independent of the particular tertiary institution and of specific units studied. Moreover, he argues that the misconceptions revealed by the test tend to remain and are unaffected despite further courses in mathematics. He contends that mathematical vitality is not improved by studying more mathematics at university.

In order to explain the poor performance of both groups on many items, the author invokes a distinction between "relational" and "instrumental" understanding (cf. Skemp, 1976). An instrumental understanding of mathematics consists of mastering a collection of rules and procedures which are to be applied in isolation from other elements of mathematical understanding. On the other hand, a striving for consistency, coherence, and a sense of interrelationships among fundamental mathematical ideas characterize "relational mathematics".

The author posits a relationship between these two types of understanding and the contexts in which mathematics is taught and learned. Even where one would assume that courses at tertiary level have been taught with "relational intent", instrumental learning can still result:

It seems that relational intent on the part of the instructor can be defeated when "success" can be achieved by retreating to and performing at an instrumental level, e.g. by learning and reproducing content and techniques for examination purposes (p. 108).

If success in high school and university mathematics can be achieved by opting for a simpler instrumental approach, then those who become teachers are likely to perpetuate instrumental patterns of learning in their own classrooms. Previous research on links between teacher knowledge and student performance has tended to concentrate on teachers' knowledge of specific mathematical content. However, mathematical vitality as a measure of "relational mathematics" is quite different from "mere content knowledge". It is recommended that further research on this characteristic of teachers and the mathematical performance of their students be undertaken.

Abstractor's Comments

In this section, I ask about the author's interpretation of test results, offer some explanatory remarks about the Australian context in which the study was conducted, and discuss the attribute of "mathematical vitality" which the test is presumed to measure. Then, I attend to the distinction which the author makes between "relational" and "instrumental", and relate this distinction to some limitations of the conceptual framework of the study itself.

The author does leave unexplained the wide range of correct responses, a feature common to both groups. For the undergraduate sample, the proportion of correct responses is as high as 94% and as low as 5%. Is it possible that some items are more closely related to the specific content of mathematics courses than others? Readers would be interested to know what explanation the author might offer for this feature. Nor does the author explain why the performance of the postgraduate sample is markedly better on many items than that of the undergraduate group. It may be true that some elements of the former group cannot be distinguished from other elements according to the number of units of mathematics studied at

tertiary level. However, its performance can be distinguished as generally more successful on many items than that of the undergraduate group. The performance of the undergraduate group outstrips that of the postgraduate group on three items only. Given the author's contention that "mathematical vitality is not enhanced by the mere process of studying more mathematics at the tertiary level" (p. 107), any discrepancy in performance between the two groups should be addressed.

The author's reasons for treating his two sample groups from a national perspective are well supported by an analysis of results from state to state. Many Australian readers of the study would support these reasons from their own knowledge and experience of Australian courses in mathematics. However, an overseas reader may need to be aware that mathematics courses in the final year of high school in Australia are usually academically oriented, with most courses containing solid components of calculus and statistics. Those who in the undergraduate sample were enrolled in "typical first-year mathematics courses at university" had, on the basis of their high school results, successfully gained entry to university as distinct from a college of advanced education. It is also likely that those typical first-year courses had as prerequisites certain mathematics courses at year 12, which in their turn required a high level of performance in mathematics courses in the preceding years. Thus, the first-year undergraduate sample would contain a specially selected group of students.

The author's conclusions do cast a gloomy shadow over the claims of those who promised a revival of mathematical understanding following the introduction of new courses in mathematics during the 1960s and 1970s. An initial response to the items of Galbraith's test might be to ask whether they do indeed measure mathematical vitality. Clearly the items require students to exhibit a degree of mathematical insight and logical discrimination, but among the characteristics of mathematical vitality I would expect to see an understanding of the interaction between mathematics and reality; for example, the ability to use a mathematical model "to help raise or answer questions about physical reality, as well as techniques

for exploring the behavior of the models themselves" (Buck and Buck, 1965). Given Galbraith's emphasis on relational understanding, one would expect that an ability to explore the relationships between mathematics and physical reality would be prominent among the attributes to be investigated by his study.

At issue is the way in which the terms "relational" and "instrumental" are employed by Galbraith in interpreting his results. His use of these adjectives in three quite different contexts needs far more unpacking if they are to carry the weight of interpretation which he hopes for. I doubt whether Skemp's original distinction between relational and instrumental understanding can be maintained clearly and consistently even with relatively simple cognitive performances. I find this distinction between relational and instrumental understanding still more implausible when it is extended to the kinds of complex mathematical performances which are embodied in Galbraith's test items. Even those students who know how to differentiate know, in a possibly rudimentary way, when it is appropriate to do so, and they are also likely to have a sense of what constitutes an appropriate result when that operation is applied. In this respect they seem to display a measure of relational understanding, and if so one cannot simply classify their performance as solely instrumental. Further difficulties arise when Galbraith attaches these adjectives to mathematics. His references to "relational mathematics" and "instrumental mathematics" give an impression that he is referring to some consistently identifiable attribute of mathematical performance. However, his intended meaning is far from clear. Similarly, his references to "relational teaching" and "instrumental teaching" are further extensions from Skemp's original distinction. Unfortunately, Galbraith draws his readers into these uncertain pedagogical distinctions with very few guideposts. Is it intended that the links between relational/instrumental teaching, relational/instrumental mathematics, and relational/instrumental understanding are causal? If they are, then the causal links need more interpretation and elucidation. They need to be discussed and illustrated rather than left to the reader's imagination as one slips from teaching to mathematics, and from mathematics to understanding.

I suspect that Galbraith intends to use these terms in order to orient the reader to examine the beliefs, purposes, and values which underpin different contexts of teaching and learning mathematics. He does allude to the kinds of beliefs, purposes, and values which might have influenced the poor performance of students on the test. But these conjectures are post hoc rationalizations of his test results, and they draw attention to the underlying inability of his conceptual framework to illuminate our understanding of teaching and learning mathematics.

Galbraith's study has sought to use psychological explanations of teaching and learning mathematics. His use of a distinction between relational and instrumental understanding, and his extension of that distinction to mathematics and teaching is typical of a psychological reductionism which has dominated many studies of teaching and learning.

The conceptual framework of his study with its inherent psychological reductionism conceals and precludes a fuller and more complete description of the social context of teaching. That context should include reference to the beliefs, purposes, and values, and to patterns of work and knowledge which govern what happens in classrooms. Galbraith suggests a link between certain patterns of teaching and certain kinds of learning outcomes. But his conceptual framework is bound to ignore important questions of how and why certain approaches to teaching and learning predominate in the mathematics classroom. Galbraith quite rightly refuses to explain such outcomes in terms of teachers' knowledge, but his alternative explanations are outlined in a few tantalizingly brief sentences. In order to investigate these alternatives, Galbraith would need to throw off the blinkers of a categorization of teaching and learning which has become totally dependent upon psychological classificatory systems. His own conceptual framework does not allow him to explore the possibility that students, whether in school or university, learn not only the subject matter of mathematics, but through their work they are taught the appropriate forms in which to cast their knowledge. This notion of work as a social and ethical construct is necessary in order to portray effectively the social dimension of the acquisition and application of mathematical knowledge.

A psychological reductionist model of teaching and learning distorts the nature of mathematics itself in treating it as an isolated intellectual enterprise. Mathematics is a human activity whose development cannot be understood without reference to an historically situated community of scholars. The development and vitality of mathematical knowledge are not well portrayed by the crystallized forms which are present in Galbraith's test.

Mathematics, as Buck (1965) argues, is

marked by inventions; discoveries, guesses both good and bad, andthe frontier of its growth is covered by interesting unanswered questions. (p. 951).

Galbraith's notion of mathematical vitality bears only a slight resemblance to this picture of mathematics as an intellectual craft carried out in a community of other vital minds. His notion of mathematical vitality is confined and attenuated under the influence of his own conceptual framework. That framework prevents us from exploring important social dimensions of mathematical knowledge, and, in particular, the beliefs, purposes, and values which influence how mathematics is taught and learned.

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Khoury, Helen Adi and Behr, Merlyn. STUDENT PERFORMANCE, INDIVIDUAL DIFFERENCES, AND MODES OF REPRESENTATION. Journal for Research in Mathematics Education 13: 3-15; January 1982.

Abstract and comments prepared for I.M.E. by J. PAUL McLAUGHLIN, Purdue University Calumet, Hammond, Indiana.

1. Purpose

Potential sources of variability on tasks which require an interplay between representational modes are of interest. This study investigated the relationship of two variables -- field dependence/independence and spatial visualization ability -- to performance of college students on retention tests involving problems presented in (a) pictorial mode only, (b) symbolic mode only, and (c) mixed symbolic/pictorial modes. "The study also investigated the extent to which the four variables -- field dependence/independence, spatial visualization ability, symbolic mode retention test performance, and pictorial mode retention test performance -- account for the variability in the retention test performance on tasks requiring an interplay between the symbolic and the pictorial modes" (p. 3), i.e., performance on the mixed symbolic/pictorial modes retention tests.

2. Rationale

"Field independence refers to a predisposition to perceive the environment in analytic terms, or differentiated fashion, and field dependence refers to a predisposition to perceive the environment in a global and undifferentiated fashion" (p. 4). Subjects which tend toward field independence should perform tasks involving a mixture of representational modes more easily than those who tend toward field dependence. A field dependent subject would be more likely to translate all tasks to the mode with which he, or she feels more comfortable. A subject who tends to field independence would be more likely to work with the representational mode used in the task presentation.

It was hypothesized that field dependent subjects with high spatial visualization ability would perform as well or better than field independent subjects with low spatial visualization ability and field independent students with high spatial visualization ability would score better on mixed mode tasks than field dependent students with low spatial visualization ability.

3. Research Design and Procedures

Subjects were 96 students in two intact classes of a university course in methods of teaching elementary school mathematics. There were 82 females and 14 males. All were preservice elementary school teachers.

Two pencil-and-paper tests -- Gottschaldt Hidden Figures Test (HFT) to determine field dependence/independence and the Purdue Spatial Visualization Test (SPV) to assess spatial visualization ability -- were taken by all subjects.

This testing was followed by one week of instruction on whole number addition algorithms using counting sticks and bundles of counting sticks for the manipulative mode, pictures representing the sticks and bundles of sticks for the pictorial mode, and the horizontal equation algorithm for the symbolic mode. The steps in symbolic algorithms are illustrated by the following example.

$$12 + 13 = n$$

$$(10 + 2) + (10 + 3) = n$$

$$(10 + 10) + (2 + 3) = n$$

$$(10 + 10) + 5 = n$$

$$20 + 5 = n$$

$$25 = n$$

A post-treatment test was administered immediately following the treatment, and a retention test was given three weeks after the treatment. Scores on the retention test were used in the analysis. (Scores on

the post-treatment test showed very small variance.) The retention test consisted of six pictorial mode problems, six symbolic mode problems, and six mixed mode problems. In each case students were to select from among five possibilities the correct "statement" or "picture" at each stage of a six- or eight-step addition problem. On the mixed mode problems, the representational mode was alternated from step to step in the solution.

4. Findings.

With each of the three parts on the retention test considered separately, the results of the three retention tests, the HFT, and the SPV were:

	Mean	Maximum Possible	Std. deviation
Pictorial Mode:	3.58	6.00	1.80
Symbolic Mode:	5.04	6.00	1.55
Mixed Mode:	4.59	6.00	1.63
HFT Score:	8.41	20.00	5.01
SPV Score:	54.89	80.00	9.89

Correlations between the various modes of representation (pictorial, symbolic, and mixed) ranged from 0.64 to 0.83. Correlations between the SPV (spatial visualization) and the three modes of representation ranged from 0.18 to 0.21. Correlations between the HFT (field dependent/independent) and the three modes of representation ranged from 0.27 to 0.33. The correlation between the SPV scores and the HFT measures was 0.48.

Stepwise regression analysis using the HFT and SPV scores as independent variables indicated that scores on the HFT accounted for less than 11% of the variance in each of the retention tests and the SPV accounted for less than 1% of the variance.

Stepwise regression analysis using the mixed mode retention test score as the dependent variable and the other four test scores as independent variables indicated that the Symbolic Mode Retention test

score accounted for 68.6% of the variance in the scores on the Mixed Mode Retention test; the Pictorial Mode score, 0.8%; the SPV score, 0.2%; and HFT score, 0.02%.

Using 2x2x3 factorial analysis of variance (High-Low SPV by High-Low HFT by Pictorial-Symbolic-Mixed Mode), a significant within-subject main effect due to the retention test mode was observed, but no other within-subject effect was significant. Analysis of variance also indicated a significant between-subject main effect due to spatial visualization ability. No significant main effect due to HFT (field dependence/independence) was indicated.

5. Interpretations

While scores on the HFT correlated somewhat higher than the spatial visualization scores with each of the three retention test modes, the HFT score accounted for less than 1% of the variability in the performance test scores. Scores on the symbolic mode retention test accounted for 68.6% of the variance in the mixed mode test results, with the other three tests combined accounting for 1%. Of the retention tests the symbolic mode was easiest for students, the pictorial mode the most difficult. A significant interaction effect between spatial visualization and the retention test mode was found. The greatest difference in group means was on the pictorial test between the high spatial visualization group and the low spatial visualization group.

Further research on the effects of field dependence/independence and spatial visualization on mixed representational mode performance is suggested.

Studies should use students during their early encounters with the subject matter. Subjects in this study were familiar with the content except possibly for the manipulative and pictorial representational mode. Studies in which the subject matter is new or unfamiliar to the students involved should produce greater variance.

Abstractor's Comments

I felt the rationale was incomplete. Some discussion of how a tendency toward field dependence might affect a student's performance on the tasks used in this study would have been helpful. Also, is spatial visualization ability independent of field dependence/independence? Why would only three of 61 students fall in the "field dependent-high spatial visualization" group in the High-Low crossing of these two groups? There was no hypothesis on how high spatial visualization ability-field independent subjects would compare to high spatial visualization ability-field dependent subjects. Nor was there a hypothesis on how low spatial visualization ability-field independent subjects would compare to low spatial visualization ability-field dependent subjects.

The use of subject matter which students are already expected to have mastered, with the only new element being the mode of representation, would appear to limit the usefulness of the results. Perhaps this could have been alleviated somewhat by using some number base other than ten in the teaching and testing.

It would seem appropriate to have one test involving the manipulatives as the representational mode. In fact, the use of pencil-and-paper, multiple-choice tests does not appear to be appropriate when one is examining the relation between field dependence/independence and representational mode. An interview format or written comment format in which students indicate their thinking would yield useful information. Such results would be much more difficult to analyze and interpret; however, the desirability of having such information is implied in the conclusion of the report, where some student comments are included.

Just how good is the match between manipulatives that are used, the pictures which we draw to represent steps in an algorithmic process, and the steps we write in the symbolic algorithm?

How much work with manipulatives and pictorial representations is necessary or appropriate? For whom is it appropriate? What information about a child's cognitive style would guide the teacher in making decisions (answering these questions) for one child or a group of children? The answers to these questions would be useful to teachers and to those in mathematics education who prepare and/or work with teachers and future teachers. Research of the type of this report would help provide some of these answers.

Schoenfeld, Alan H. MEASURES OF PROBLEM-SOLVING PERFORMANCE AND OF PROBLEM-SOLVING INSTRUCTION. Journal for Research in Mathematics Education 13: 31-49; January, 1982.

Abstract and comments prepared for I.M.E. by ARTHUR F. COXFORD, The University of Michigan.

1. Purpose

Two purposes were central: (1) "to document the results of a month-long intensive problem-solving course on students' performance when solving nonroutine college level mathematics problems"; (2) to present and discuss the easily graded paper-and-pencil tests that focus on problem-solving processes used to measure the problem solving of the subjects.

2. Rationale

Professional and research groups suggest that problem-solving in mathematics is a major goal of mathematics instruction. Yet problem-solving is notoriously difficult to teach. That heuristics provide a basis for improved problem-solving performance has been supported in a variety of recent investigations. Additional support would be useful in the development of research directions. The usual procedure in such work is protocol analysis. This method is time-inefficient and costly. The development of cost-efficient alternatives is desirable.

The research itself is based upon three working assumptions.

These are that necessary conditions for success in problem solving are:

- 1) an adequate mathematical knowledge base of facts and principles;
- 2) a mastery of basic problem-solving techniques -- heuristics similar to those described by Polya; and
- 3) a managerial strategy which is used to select appropriate techniques and terminate inappropriate ones.

The author points out that the complexity and subtlety of the use of heuristics should not be underestimated (p. 32).

3. Research Design and Procedures

The basic design was pretest-posttest with a "control" group. The experimental group consisted of 11 students at a small liberal arts college who enrolled in an intensive month-long problem-solving course (2-½ hours of class work for 18 consecutive weekdays, plus homework). This was the only course elected, and all the time was spent examining relevant heuristics and solving problems in as many ways as possible.

The "control" group was a group of eight similar students enrolled in a month-long course in structured programming "designed to teach a structured, orderly way to approach problems." As the author recognized, the "control" group was of limited use because it did not deal with mathematical problem solving.

Three pretest-posttest assessments were taken. Measure 1, five items with 20 minutes allowed for each, assessed the subjects' problem-solving skill and the successful and unsuccessful strategies tried. This measure was scored in two ways: multiple count and best approach scoring. In the former, credit was given for all work, whereas in the latter only the best effort was evaluated. Measure 2, six items with 4 minutes' response time given after each item on Measure 1, assessed the subjects' self perceptions of their problem solving. Measure 3 assessed heuristic fluency and transfer. It included nine items: three each of problems related, somewhat related, and not related to the instructional problems. The students were given one hour to present a plan for a solution to each item. (Finished solutions were not requested.) The three pretests were given on day 1 of instruction; the posttests were given on the last day of class.

4. Findings

Measure 1: Using the "multiple count" scoring, the "control" group exhibited only minor changes in pretest to posttest performance. The experimental group, which was similar to the control group on the pretest, showed substantial improvement in posttest scores. For example, they solved only .27 problems per student initially, while solving 2.64 in the posttest.

Using the "best approach" scoring with a range of 0-100, the "control" group increased from 14% to 24%, while the experimental group increased from 20.8% to 72.2%.

Measure 2: The two groups were roughly comparable on the pretest. On the posttest, the "control" group showed moderate change in their self-perception, while the experimental group showed increased planning and thinking about the solution before actually beginning the process.

Measure 3: The "control" group showed essentially no change from the pre- to the posttest measures. On the pretest, the treatment group was unexplainably superior to the "control" group. The treatment group also showed evidence of fluency of heuristic use in the "somewhat related" problems and actually completely solved some "closely related" problems in the six or seven minutes available for each problem, even though asked only to plan the solution.

5. Interpretations

The author concluded that the measures used were reliable and informative. The scoring was consistent across trained scorers. Additionally, the self-perceptions of the experimental group with regard to planning and to organization were accurate. Similarly, the "control" group showed no such improvement in planning and organizing solutions. The author suggested that without some heuristics to manage, a manager will not be able to do much. No specific conclusions could be made from the data regarding managerial strategies, but the author opined that "...more riches lie in a better understanding of how good problem solvers perceive the problems they work on and select various approaches to them" (p. 48).

Abstractor's Comments

It would be easy for a reviewer to be critical of the design of this investigation, for the subjects were not randomly assigned to treatment, the N's were prohibitively small, and the subjects were drawn

from a select population. However, for this study on this topic at this time in the development of research in mathematical education, such criticism would be petty. Problem solving is the objective of mathematics instruction and little reliable information is available for the practitioner. The work reported here exhibits promise of both theoretical and practical relevance.

Heuristics have been touted as appropriate tools for the problem solving for years. . Yet little has been said about how one learns to employ the myraid of heuristics. The concept of a "managerial system" seems a viable one which should be investigated further. The practitioner and the theoretician need information on the manner in which the heuristics were taught, which ones were taught, how students were taught to "manage" them, what clues were used in heuristic solution, etc. As the author concludes, this is a complex topic -- one that could certainly profit from a collaboration of mathematics educators and cognitive psychologists interested in high level-human functioning.

Wiebe, James H. USING GRADED QUIZZES, HOMEWORK, AND ATTENDANCE FOR MOTIVATING STUDY IN A COLLEGE MATH CLASS. Mathematics and Computer Education 16: 24-28; Winter 1982.

Abstract and comments prepared for I.M.E. by JOE DAN AUSTIN, Rice University.

1. Purpose

The purpose of this study was to determine the effect on college mathematics students "of attempts by instructors to make sure that students properly manage their study time through the use of graded quizzes, homework and attendance" (p. 25).

2. Rationale

At the college level teachers often assume that students are self-motivated and responsible for deciding the frequency of their home study and class attendance. In such situations instructors often use nongraded quizzes and nongraded homework to provide feedback to students. Other instructors attempt to require students to study regularly by using graded homework and graded quizzes.

Studies on the effects of homework, quizzes, and tests have been mixed. There is evidence that at the high school level homework, frequent tests, and quizzes can improve achievement in mathematics. However, these studies have not seemed to generalize as well to the college level. Also, the studies have considered separately the effects of homework, tests, and quizzes. No studies were found that attempted to study the effects of homework, graded quizzes, and required attendance as a unified procedure. This study attempted to study the effect of the unified procedure of graded homework, graded quizzes, and required attendance on college students' attitudes and achievement scores.

3. Research Design and Procedures

All students in six classes of Mathematics 180 (Theory of Arithmetic) were involved in this study. About two-thirds of the students were

elementary or special education majors. About one-sixth were in other areas of education. The remainder were in business or liberal arts. The study lasted two months.

Three instructors taught two sections each of Mathematics 180. For each instructor one class was randomly selected to serve as an experimental class, while the other class of each instructor served as a control class. In the three experimental classes homework was assigned, collected, and graded; unannounced quizzes were given at least once a week; and attendance was checked. The final grade was based on attendance and scores on examinations, quizzes, and homework. The three control classes had homework assigned and discussed in class, but not collected. The quizzes given to the experimental classes were handed out as study guides in the control classes and discussed. The final grade was based only on examination grades.

All students took a multiple choice (arithmetic skills) pretest and an (achievement) posttest. Each completed at the end of the study a questionnaire on attitude toward mathematics, toward Theory of Arithmetic, and toward classroom grading procedures.

The two groups -- experimental classes and control classes -- were compared using posttest scores, attitude toward mathematics, attitude toward classroom grading procedures, and drop-out rates.

4. Findings

Using the posttest achievement data, "a statistical analysis showed no significant differences" (p. 26) between groups. "No significant differences were found between the two groups in attitude toward mathematics or the methods used in the course which did not relate to the independent variables" (p. 26). Significant differences were found between the two groups on attitudes toward grading techniques used in the study. Each group favored the grading technique that had actually been used with it over the other grading technique. "Differences in dropout rates did not appear to be significant" (p. 26).

5. Interpretations

The results of this study suggest that for college mathematics students the effort required to collect and grade homework, to give and correct quizzes, and to check attendance does not result in increased achievement or better student attitudes toward mathematics. However, the study does not imply that instructors simply lecture and give examinations, as both groups had regular feedback. The control group had review sheets and homework problems that were discussed but not collected. The results of the analysis of student attitudes on grading procedures suggest that students can become accustomed to either grading policy used in the study.

Studies with students in other majors and in other college mathematics courses are needed to determine whether the results of this study generalize to other populations and to other mathematics courses.

Abstractor's Comments

This is an interesting study that addresses a question that seems important. The combined effects of homework, quizzes, and required attendance is a reasonable treatment to study. The study is well designed. It is a plus that each instructor taught a class in each treatment group and that classes were randomly assigned to treatment groups. The article was particularly readable. Finally, the author is careful that the interpretations are consistent with reported results.

In spite of the many positive aspects of the study, a number of important questions exist. These questions include the following:

1. How many students were in the six Mathematics 180 classes?
2. What were the reliabilities of the two tests and questionnaires?
3. Why were no mean scores for either group given?
4. What statistical analyses were used to decide whether there were significant differences? What alpha level was used?
5. Why was no statistical test made on the drop-out rates? What were the drop-out rates for the two groups?

6. Were the pretest scores used in any analysis? Were attitudes toward Theory of Arithmetic analyzed?
7. What was the attendance rate for the experimental classes?

This reviewer feels that this is an incredible list of basic questions that cannot be answered. For some questions it is hard not to fault also the editor. For example, in question one the course number is given but not the number of students in the study! These questions seem so basic and extensive that one has no idea how valid the results are likely to be. Most of the questions could probably be answered. In fact, the author indicates that a more detailed report is available from him. However, even a shortened report should briefly address some if not all of these questions.

In summary, it is not possible to decide whether the results are valid or not. Too much important information is simply missing.

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