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ABSTRACT

A committee composed of students, parents, teachers, administrators, college/university persons, and representatives from business and industry carefully studied the mathematics curriculum in North Carolina public schools and formulated the recommendations contained in this report. It contains ideas, suggestions, and information resulting from examination of the present mathematics program, including surveys, school visits, interviews, speakers, and position papers, as well as the literature of mathematics education. Chapter 1 outlines the organization of the study, while chapter 2 describes the background status of mathematics education. Chapter 3 presents 20 recommendations and rationales for the elementary school mathematics program; chapter 4, 15 recommendations for the middle school program; and chapter 5, 21 recommendations for the secondary school program. Each recommendation relates to one or more of the following areas: content, expectations, staffing, instruction, technology, and articulation. Appendices list personnel involved in the study and a summary of activities. (MNS)

ACKNOWLEDGEMENTS

The reward of a thing well done is to have done it. Ralph Waldo Emerson

This report is the result of countless hours of work on the part of many dedicated individuals who gave graciously and unselfishly of themselves for a cause. They envisioned, supported, and designed a better mathematics program which will benefit all the children in North Carolina when it is implemented. To this cause they gave their time, efforts, expertise, and total support. It is, therefore, appropriate that appreciation be expressed to all who worked and made this report possible.

Gratitude is expressed to the members of the study group for their devotion to the task and for their hard work. It has been a privilege to work with such outstanding individuals. Gratitude is also expressed to all the educators in the state who contributed to this report by responding to questionnaires, writing position papers, and attending open forums to discuss the recommendations.

Appreciation and recognition are extended to: State Superintendent A. Craig Phillips, Deputy State Superintendent Jerome Melton, Assistant State Superintendent for Instructional Services George Kahdy, and Deputy Assistant State Superintendent for Instructional Services Jerry Beaver for their encouragement and support; the Special Assistants in the Instructional Services Area, Wayne Dillon, Betty Moore, and Mary Purnell for their active participation and direction.

Special thanks and recognition go to Bob Jones, Director of the Mathematics Division, for his leadership, total commitment and contributions to every aspect of this project. Gratitude is also expressed to Cleo Meek and Kay Kemp, consultants for the Mathematics Division who diligently persevered and worked alongside the members of the Committee. The regional mathematics coordinators are also commended for their support in disseminating and collecting information from their regions. Finally, special thanks are extended to the Mathematics Division secretaries, Connie Hawthorne and Mildred Quick. The careful, efficient, and professional manner with which they handled the many tasks associated with the study contributed significantly to the quality of the final product.

Miriam A. Leiva, Chairman
Mathematics Curriculum Study Committee

CONTENTS

EXECUTIVE SUMMARY

GOALS OF MATHEMATICS EDUCATION

The State Mathematics Curriculum Study Committee has formulated specific recommendations which are listed and discussed in the following sections. Each of the specific recommendations relate to one or more of the following areas:

Content - That which is to be taught and learned.

Expectations - Setting of goals for mathematical attainment.

Staffing - Education, supervision and support of teachers.

Instruction - Methods used to impart the curriculum to the student.

Technology - Integration of computers and calculators into the mathematics programs of our schools.

Articulation - Coordination and cooperation between the elements in our educational system.

The Committee's recommendations are designed to support these goals. Each is stated below and followed by a brief rationale and delineation of related recommendations.

GOAL I: The mathematics curriculum should reflect: a) advances in the discipline, b) changes in society's need for mathematics, c) the growing need for a larger number of people to know more about mathematics than the rudiments of arithmetic, and d) advances in our knowledge about how and when children learn.

Rationale: Contrary to popular belief, mathematics as a discipline does change. New and powerful mathematics is being created at an increasing rate, and the relative emphasis of topics within mathematics shifts over time necessitating periodic revisions in the mathematics curriculum.

The mathematics curriculum needs adjustment to ensure access to mathematics beyond arithmetic for a larger proportion of students. Mathematics provides a "language" which facilitates communication in an increasing

number of fields of knowledge. As the demand for mathematics becomes more pervasive, the mathematics curriculum must be structured so that a greater number of students learn more mathematics.

To provide better mathematics for all students, recommendations have been formulated which call for revising and updating of course syllabi, restructuring some courses, and initiating new courses. Other recommendations address the crucial need to change the focus of the mathematics curriculum from a narrow emphasis on computational skills to a broader one which prepares students to use mathematics in everyday life and provides them with a foundation for further training.

GOAL II: Students should attain a higher level of mathematical competence.

Rationale: A quality mathematics curriculum will benefit all students who choose to take full advantage of it. Already the level of expectation is high for the most able students who take mathematics throughout high school, and there is evidence that these students attain a superior level of achievement. However, many students avoid serious mathematics study in high school and, as a result, do not achieve a useful level of mathematical literacy. Compounding this problem is the fact that many able students enter high school ill-prepared for formal mathematics study. Therefore, the expectations must also be raised in the mathematical performance of elementary and middle school students.

To ensure that students are ready to meet the expectations of a reasonable high school mathematics curriculum, it is recommended that the Developmental Mathematics Monitoring Program be established on the elementary level. This program is designed to identify students whose mathematical

ability is lagging. Students requiring remedial instruction will be identified early and given intensive instruction specially designed to prevent massive mathematical deficiencies later.

To ensure that more high school students are challenged to achieve acceptable levels of mathematical competency, it is recommended that the mathematics for ninth grade students, with some exceptions, include topics which are at least on the level of elementary algebra. Student mastery of this content would be tested as a part of the ninth grade Annual Testing Program. In addition, it is recommended that the Competency Test be administered during the tenth grade and expanded to test calculator proficiencies.

To foster a higher level of expectation and a more uniform measure of mathematical attainment, it is recommended that there be statewide achievement testing at the end of selected secondary mathematics courses. A further recommendation for three units of mathematics required for graduation complements the other recommendations stated, all of which are aimed towards a stronger mathematics program than is presently in existence in North Carolina's schools.

GOAL III: Each student should have the benefit of teachers who are qualified, competent and professional. Each teacher should have the respect, remuneration and professional status his/her training, expertise, and experience merit.

Rationale: Only well trained teachers can be expected to provide adequate instruction. There are no shortcuts available to increase the supply of capable mathematics teachers. It is imperative that those who enter the teaching profession be well trained and that those in the profession receive appropriate in-service training, adequate supervision,

and generous support. Therefore, recommendations have been made calling for revisions in the certification of teachers, improvements in both pre-service and in-service teacher education, and differential pay for mathematics teachers. Other recommendations deal with supervision and support for teachers in their day-to-day activities. It is felt that if proper conditions are provided for mathematics teachers, it will be reflected by attracting and retaining competent people in the teaching profession.

GOAL IV: The methods used to teach mathematics and the conditions under which the subject is taught should be chosen to maximize student learning and achievement.

Rationale: At all grade levels, effective mathematics teaching is enhanced by the use of appropriate teaching strategies and adequate educational resources. A number of recommendations have been made which deal with the allocation of time and its effective use, with the size of classes, and with the availability of educational resources. The implementation of these recommendations will promote the learning of mathematics at all grade levels.

GOAL V: Students should be taught to use calculators and computers within the context of the mathematics instructional program.

Rationale: Rapid advances in computer and calculator technology have made these useful and sophisticated tools readily available. Their impact on what mathematics should be taught and how it should be taught is significant. Students should still be able to carry out "paper and pencil" computations; however, there is no reason to continue to devote great amounts of time and effort to perfect paper and pencil computations when the key reason for doing this in the past, namely the absence of

alternatives, is no longer applicable. A number of recommendations have been formulated which, when implemented, will make calculators and computers integral parts of the program in mathematics at all levels.

The Committee recommends that calculators be used as tools in problem solving and for teaching concepts at all grade levels. Calculator proficiency is essential to mathematical literacy and, therefore, must be an integral part of the mathematics curriculum.

Greatly increased attention must be given to computers, especially microcomputers, in conjunction with the learning of mathematics. Computers and computer programming are in themselves appropriate topics of study; therefore, recommendations have been made relative to the availability of computer hardware and software. In addition, there are recommendations dealing with courses which provide instruction in programming and/or computer literacy. Because the use of computers as an instructional tool is increasing at a rapid rate, there are recommendations providing for staff development in the area of computers and their use. It is essential to have appropriately trained personnel to effectively implement programs utilizing computers in the classroom.

GOAL VI: A viable structure should be created to promote coordination and cooperation between elementary, secondary and post-secondary institutions.

Rationale: The coordination between educational levels is of great importance to the success of the major goal of more students learning more mathematics. Recommendations have been formulated which call for cooperation across educational levels for the purpose of understanding the content and levels of courses, defining appropriate credit for the various courses, analyzing the meaning of students' test score results, and working together to determine effective counseling programs for students.

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SUMMARY OF RECOMMENDATIONS

Elementary

RECOMMENDATION 1. We recommend that the elementary mathematics curriculum be strengthened to reflect an increasing emphasis on problem solving and the use of calculators and computers and that it be designed to encompass more than computational facility.

RECOMMENDATION 2. We recommend that an awareness of calculators and their uses be provided for K-2 students and that children in grades 3-6 be taught to use a four-function calculator.

RECOMMENDATION 3. We recommend that all elementary students have access to computers and appropriate courseware. The ratio of 1 computer per 24 students is recommended.

RECOMMENDATION 4. We recommend that all elementary children have the opportunity to explore mathematics through the use of a variety of manipulative materials and that there be an effort to effectively "bridge the gap" between manipulatives and the abstract levels of learning.

RECOMMENDATION 5. We recommend that additional state funds be appropriated on a per-pupil basis to purchase manipulative materials for use in mathematics at grade levels K-6. Particular emphasis should be placed on the purchase of materials necessary for a strong early childhood mathematics program.

RECOMMENDATION 6. We recommend that mathematics education at the kindergarten level be strengthened through:

A. In-service for kindergarten teachers;

B. A comprehensive teachers' manual which illustrates competency goals and performance indicators;

C. A specific per-pupil allotment to purchase manipulative materials in lieu of textbooks.

RECOMMENDATION 7. We recommend that each child have sixty (60) minutes of math daily with a balance between direct instruction, applications, use of manipulatives, maintenance of skills, and enrichment. Consideration needs to be given to the time of day when mathematics instruction occurs.

RECOMMENDATION 8. We recommend that students in grades 4-6 whose total mathematics scores fall below the 35th percentile on the third grade North Carolina Annual Testing Program be considered for the Developmental Mathematics Monitoring Program with instruction in groups of ten (10) or less by a teacher with mathematics training.

Elementary Recommendations (continued)

RECOMMENDATION 9. We recommend that the state guidelines for identification of exceptional children be restructured to enable school personnel to more effectively identify students who are mathematically gifted. We further recommend that the mathematics program for these students be designed to foster higher levels of cognitive thinking and to include an enrichment component.

RECOMMENDATION 10. We recommend that, if the North Carolina Annual Testing Program continues to be administered in the spring, norm referenced tests be used at all levels presently tested.

RECOMMENDATION 11. We recommend that the teacher/pupil class load in grades 4-6 be reduced so that one (1) teacher is responsible for no more than twenty-six (26) pupils.

RECOMMENDATION 12. We recommend that teacher training institutions include in their programs for prospective elementary teachers early internship experiences with mathematics instruction and that a mathematics competency examination be required of all candidates seeking elementary certification.

RECOMMENDATION 13. We recommend that an elementary mathematics specialist certification be established. Elementary teachers holding this certification should receive a salary differential.

RECOMMENDATION 14. We recommend that all levels of teacher training--pre-service, in-service, and graduate work--include instruction for using computers in teaching mathematics.

RECOMMENDATION 15. We recommend that staff development be offered for principals to support their leadership in elementary mathematics education.

RECOMMENDATION 16. We recommend that the following types of in-service programs for elementary school teachers be developed and funded to reach all teachers and aides within a five-year period:

A. An ongoing program consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, classroom management, efficient use of classroom time, and counseling strategies.

B. Special summer institutes to assist teachers in teaching mathematics. Consideration should be given to making the institutes permanent in order to foster continued upgrading of the competence level of teachers.

Funding should be provided for various financial incentives to allow teachers to participate.

Elementary Recommendations (continued)

RECOMMENDATION 17. We recommend that at least a third of the non-teaching continuing education units earned by a teacher for elementary certificate renewal be in the area of mathematics.

RECOMMENDATION 18. We recommend that the state, in conjunction with local education agencies, create mathematics coordinating teacher positions to provide continuous in-service via demonstration teaching, conducting workshops, and providing support and resource services. It is further recommended that:

A. The positions be phased in over a five-year period in all eight regions as teachers are identified and trained;

B. The coordinating teacher's contacts not exceed seventy-five classroom teachers and that each local education agency have at least one mathematics coordinating teacher;

C. The coordinating teacher receive a differential salary.

RECOMMENDATION 19. We recommend that every effort be made to enhance the teaching of mathematics in elementary schools by:

A. Providing appropriate scholarships for teachers to take mathematics or mathematics education courses;

B. Providing adequate reimbursement for expenses incurred at mathematics professional meetings;

C. Providing aides and/or clerical help in grades 4-6;

D. Identifying and recognizing outstanding elementary mathematics teachers;

E. Identifying and recognizing outstanding mathematics programs through the establishment of model programs.

RECOMMENDATION 20. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students who desire to pursue a mathematics specialist certification in grades K-6. We further recommend that the amount of each scholarship loan be increased to $2000/year.

SUMMARY OF RECOMMENDATIONS

Middle School

RECOMMENDATION 1. We recommend that, in order for students in grades 7 and 8 to have broad experiences in mathematics and to provide a strong base for further study, the mathematics curriculum in grades 7 and 8 be structured according to the following levels:

Grade 7

A. Comprehensive Mathematics broadly based in Pre-Algebra, informal plane and solid geometry, descriptive statistics and survey sampling, data analysis, consumer arithmetic, mental arithmetic and estimation, applications of mathematics and problem solving.

B. A course which encompasses the seventh and eighth grade comprehensive mathematics courses for those students who have shown superior mathematical ability and who will proceed to Algebra I in the eighth grade.

C. Developmental/remedial mathematics for students scoring below the 35th percentile on the sixth grade annual testing program in mathematics, taught in small groups (1:10) under the supervision of a teacher trained in diagnostic/prescriptive teaching of mathematics in a setting conducive to active participation in learning through problem-solving.

Grade 8

A. Continuation of Comprehensive Mathematics from seventh grade designed to further students' mathematical foundation in preparation for the course entitled Pre-Algebra in the ninth grade.

B. A more accelerated arithmetic/algebra course for the students who are qualified to handle a curriculum preparatory to Algebra I.

C. Algebra I for the students who are at that level and preferably were enrolled in accelerated mathematics in grade 7.

D. Continuation of developmental/remedial mathematics for those who continue to need assistance with the understanding of fundamental mathematics--taught in small groups (1:10) under the supervision of a teacher trained in diagnostic/prescriptive teaching of mathematics in a setting conducive to active participation in learning through problem solving.

RECOMMENDATION 2. We recommend that calculators be used as a teaching tool in grades 7 and 8 and that all students become proficient in using the four-function calculator with memory and percent.

Middle School Recommendations (continued)

RECOMMENDATION 3. We recommend that all middle school students be provided the opportunity to become computer literate and that an elective course in computer programming be developed and made available to interested students.

RECOMMENDATION 4. We recommend that there be 55 minutes of mathematics instruction daily with over 50 percent of the instructional time spent in teaching that develops content through interaction between the students and the teacher.

RECOMMENDATION 5. We recommend that a program be developed to assist guidance counselors, advisors, curriculum coordinators, principals, and teachers in advising middle school students. The intent of the program is to inform students about the quantity and quality of mathematics needed for life in a technological society.

RECOMMENDATION 6. We recommend that the state, in conjunction with local education agencies, create mathematics coordinating teacher positions to provide continuous in-service via demonstration teaching, conducting workshops, and providing support and resources services. It is further recommended that: (1) the positions be phased in over a five-year period in all eight regions as teachers are identified and trained; (2) the coordinating teacher's contacts not exceed seventy-five classroom teachers and that each local education agency have at least one mathematics coordinating teacher; (3) that there be a pay differential commensurate with the requirements of the position.

RECOMMENDATION 7. We recommend that the State Department of Public Instruction's Competencies and Guidelines for Approved Teacher Education Programs be amended for the 6-9 certificate in mathematics so that prospective teachers seeking a mathematics concentration be required to complete a minimum of 30 semester hours of which at least 15-21 semester hours be in mathematics and 9-15 semester hours be in mathematics education, exclusive of the student teaching experience. All teachers teaching mathematics in grades 7 and 8 must have this specialist certification.

RECOMMENDATION 8. We recommend that all components of teacher training--pre-service, in-service, graduate work--include instruction using computers for teaching mathematics.

RECOMMENDATION 9. We recommend that at least three units for certificate renewal of middle school mathematics teachers be in mathematics and/or mathematics education. We further recommend that these units be earned through college credits or through approved coursework with the same rigor in content and requirements as a college-level course.

Middle School Recommendations (continued)

RECOMMENDATION 10. We recommend that the following two types of in-service programs for middle school mathematics teachers be developed and funded:

A. An ongoing program with a curriculum consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, classroom management, efficient use of classroom time, and counseling strategies.

B. Special summer institutes to assist teachers in teaching middle school mathematics. Consideration should be given to making the institutes permanent to foster continued upgrading of the competence level of teachers.

Funding should be provided for various financial incentives to allow teachers to participate in either or both programs.

RECOMMENDATION 11. We recommend that microcomputers and appropriate courseware be made available to support the teaching of all mathematics courses in the 7-8 curriculum. A ratio of one (1) computer for every 28 students is recommended.

RECOMMENDATION 12. To attract competent, new mathematics teachers, we recommend the following:

A. Teaching assignments for first-year teachers involve contact with students of varying ability levels.

B. First-year teachers be offered a contract as early as possible.

C. First-year teachers be offered the opportunity of beginning employment as soon after their graduation as is practical.

RECOMMENDATION 13. We recommend that to attract and retain competent mathematics teachers a program of incentives be developed. Specifically, we recommend that:

A. Teaching loads in mathematics be restricted to no more than 26 students per class and 130 students per day.

B. Appropriate stipends be provided for summer study, curriculum development, staff development, etc.

C. Reimbursement be provided for expenses associated with attendance at professional mathematics meetings, including travel, lodging, registration fees, and substitute teacher pay.

Middle School Recommendations (continued)

RECOMMENDATION 14. We recommend that salaries be increased for appropriately certified mathematics teachers who have a majority of their teaching responsibilities in mathematics.

RECOMMENDATION 15. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students who desire to pursue a mathematics specialist certification in grades 7-8. We further recommend that the amount of each scholarship loan be increased to $2000/year.

SUMMARY OF RECOMMENDATIONS

Secondary

RECOMMENDATION 1. We recommend that steps be taken to ensure uniformity in the quality and depth of the various mathematics course offerings throughout the state. Specifically, we recommend the following:

A. A general course syllabus be developed for each secondary mathematics course.

B. A brief course outline be developed for each State-adopted secondary mathematics textbook.

RECOMMENDATION 2. To ensure that students are proficient in the use of a calculator, we recommend the following:

A. All students be taught to use a four-function calculator with memory and percent.

B. The mathematics section of the North Carolina Competency Test be revised to include a section testing calculator proficiency and to permit the use of the calculator on the Applications Section of the Test.

C. Students who plan to attend post-secondary schools be taught to use a scientific calculator.

D. Funding be appropriated from the state to the LEAs for purchasing, maintaining, and replacing calculators, and for providing supplementary materials.

E. Students be encouraged to use a calculator, whenever appropriate, in their mathematics classes.

RECOMMENDATION 3. We recommend that computers, at a ratio of 1 computer to 28 students, and appropriate courseware be made available for the following:

A. Use in the various secondary mathematics courses,

B. Teaching an introductory course in microcomputers and microcomputer programming, and

C. Use in student projects, homework, and independent study.

We further recommend that a computer programming course emphasizing applications in mathematics be offered to college-bound students who plan careers in mathematics, science, engineering or related fields. An alternative to this course would be Advanced Placement Computer Science.

Secondary Recommendations (continued)

RECOMMENDATION 4. We recommend that a course entitled Pre-Algebra be created. The course would be designed for the ninth grade student with below-average mathematical ability. Every ninth grade student not in the remedial program would be required to take either Pre-Algebra or a higher level course.

RECOMMENDATION 5. We recommend that the following courses replace the General Mathematics sequence. Each of these courses would have only a Pre-Algebra prerequisite and each would build on and develop the algebraic skills acquired in Pre-Algebra. Effective use of a calculator will be a point of emphasis in each of these courses.

A. Technical Mathematics: This course should prepare students for a post-graduate technical program. The course should include measurement, interpretation of graphs and charts, data analysis, intuitive geometry, formula manipulation, right angle trigonometry and other relevant topics.

B. Consumer Mathematics: This course would cover the topics in the current Consumer Mathematics course, but the coverage should be at a more sophisticated level. Moreover, there should be a much greater emphasis placed on solving the types of problems which consumers actually encounter, i.e., real world problems with "real" data.

C. Topics in Mathematics: This course would be aimed at the non-college bound student and would treat topics such as informal geometry, intuitive probability and statistics and further work in algebra.

RECOMMENDATION 6. We recommend that for students not in remedial courses, the scope of the ninth grade annual testing program be expanded to include algebraic skills at the level of the Pre-Algebra course.

RECOMMENDATION 7. We recommend that secondary schools strive to provide one or both of the following courses for those students who complete four years of college preparatory materials by the end of their junior year:

A. Calculus, at no less than the level prescribed for AB Calculus in the Advanced Placement syllabus.

B. A course, to be an extension of Advanced Mathematics, which is intended to develop pre-calculus skills and introduce new mathematical topics.

RECOMMENDATION 8. We recommend that problem solving become an integral part of each secondary mathematics course and that it be heavily emphasized in pre- and in-service training programs.

Secondary Recommendations (continued)

RECOMMENDATION 9. We recommend that the State Department of Public Instruction institute a program of statewide examinations to be administered at the end of certain college preparatory courses in mathematics--initially Algebra I, Geometry, and Algebra II. The expenses associated with administering these examinations should be assumed by the state. Colleges should be permitted to request a student's score on these tests.

RECOMMENDATION 10. We recommend that high school students be required to take the North Carolina Competency Test in the spring of their sophomore year. We further recommend that the Competency Testing Commission consider exempting students from the competency test requirement on the basis of superior performance on the mathematics section of the Ninth Grade Annual Testing Program.

RECOMMENDATION 11. We recommend that remedial mathematics classes be offered for all secondary students, grades 9-12, who are not ready to take the Pre-Algebra course or who have failed or are in danger of failing the Competency Test. Students at or below the 35th percentile on the mathematics section of the ninth grade annual testing program should be considered for remediation. The remediation should be conducted in a mathematics laboratory with a specially trained teacher, at least one instructional aide, and no more than 20 students per class period. Exit criteria should be developed for students to move from remedial classes into the Pre-Algebra course.

RECOMMENDATION 12. We recommend that, where possible, each of the courses, Algebra I, Geometry, Algebra II, be split into a two-semester course and that satisfactory performance in the first semester of each course be prerequisite for admission into the second semester. Moreover, we recommend that, insofar as it is possible, the first semester of each of these courses be offered in both the first and the second semester of each school year.

RECOMMENDATION 13. We recommend that three units of mathematics in grades 9-12 be required for graduation.

RECOMMENDATION 14. We recommend that those responsible for advising secondary students, i.e., guidance counselors, advisors, curriculum coordinators, principals and teachers, be made fully aware of the importance of mathematics to every member of our society. Those who advise secondary students should, in particular, be encouraging all students to:

A. take mathematics in their senior year, and

B. take courses at the most advanced level their backgrounds and abilities will allow.

Secondary Recommendations (continued)

RECOMMENDATION 15. We recommend that a mechanism be created to establish and maintain communication among the agencies involved with the teaching of mathematics. Specifically, we recommend that a committee representing the mathematics departments of public and private schools, colleges and universities (public and private), technical and community colleges, and the Mathematics Division of SDPI be created.

RECOMMENDATION 16. We recommend that the State Board of Education suggest to the post-secondary institutions of the state that:

A. Four-year college degree credit not be awarded for courses in which the level of content is equivalent to that of Algebra I or Geometry.

B. Colleges and universities devise and maintain more stringent entrance requirements in mathematics, utilizing high school grades and SAT scores.

C. If an institution of higher learning does admit students who are deficient in mathematics, then that institution should expect to offer remedial, non-credit mathematics courses.

RECOMMENDATION 17. We recommend that the State Board of Education suggest to the post-secondary institutions in the state that a placement examination be developed. This examination would be administered on a voluntary basis to high school juniors. It would be used to indicate the existing level of mathematical competence with regard to the background necessary for admission to the first credit course in mathematics at the college or university level.

RECOMMENDATION 18. To attract competent, new mathematics teachers, we recommend the following:

A. Teaching assignments for first-year teachers involve contact with students of varying ability levels.

B. First-year teachers be offered a contract as early as possible.

C. First-year teachers be offered the opportunity of beginning employment as soon after their graduation as is practical.

RECOMMENDATION 19. We recommend that no endorsement of certificates for teaching mathematics be permitted and that the only certificates in mathematics be those outlined by the Quality Assurance Program and which fully meet the program's guidelines.

RECOMMENDATION 20. We recommend that by the 1985-86 school year only those individuals appropriately certified in mathematics be allowed to teach mathematics in grades 7 through 12.

Secondary Recommendations (continued)

RECOMMENDATION 21. We recommend that for certificate renewal of secondary school mathematics teachers at least three units be in mathematics and/or mathematics education. We further recommend that these units be earned through college credits or through approved coursework with the same rigor in content and requirements as a college-level course.

RECOMMENDATION 22. We recommend that the following two types of in-service programs for secondary school mathematics teachers be developed and funded:

A. An ongoing program with a curriculum consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, computers, calculators, problem solving, classroom management, efficient use of classroom time, and counseling strategies.

B. Special summer institutes to assist teachers in teaching secondary level mathematics.

Funding should be provided for various financial incentives to allow teachers to participate in either or both programs.

RECOMMENDATION 23. We recommend that in-service be conducted on a continuing basis for administrators of each LEA concerning trends and issues in mathematics education.

RECOMMENDATION 24. We recommend that to attract and retain competent mathematics teachers a program of incentives be developed. Specifically, we recommend that:

A. Teaching loads in mathematics be restricted to no more than 26 students per class and 130 students per day.

B. Appropriate stipends be provided for summer study, curriculum development, staff development, etc.

C. Reimbursement be provided for expenses associated with attendance at professional mathematics meetings, including travel, lodging, registration fees, and substitute teacher pay.

RECOMMENDATION 25. We recommend that salaries be increased for appropriately certified mathematics teachers who have a majority of their teaching responsibilities in mathematics.

RECOMMENDATION 26. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students in mathematics education at the secondary level. We further recommend that the amount of each scholarship loan be increased to $2000/year.

RECOMMENDATION 27. We recommend that the present student allotment for textbook adoption be raised to $15.

ORGANIZATION OF THE STUDY

CHAPTER 1

ORGANIZATION OF THE STUDY

Formation

Dr. A. Craig Phillips, State Superintendent of Public Instruction, appointed the Mathematics Curriculum Study Committee in the late summer of 1981. The Committee was directed to study the status of all levels of mathematics instruction in North Carolina and to determine the desired goals of the mathematics program for the future.

Purpose

The purpose of the Mathematics Curriculum Study was to thoroughly study the present mathematics program and develop recommendations which, upon implementation, would serve as a blueprint to improve the quality of the mathematics program for all students in North Carolina.

Committee

The membership of the Mathematics Curriculum Study Committee consisted of fourteen male and thirteen female members, included members of ethnic minority origins, and was representative of the State's geographical regions. The professional backgrounds of the Committee membership included:

- Five classroom teachers
- Seven LEA supervisors
- Two principals
- Six university mathematics educators/teacher trainers
- One university mathematician
- One community college mathematics educator
- Two industry representatives
- One private consultant
- One college student
- One high school student.

Support Staff

Members of the State Department of Public Instruction who provided assistance to the Committee:

State Superintendent

Deputy State Superintendent

Assistant State Superintendent for Instructional Services

Deputy Asst. State Superintendent for Instructional Services

Special Assistants for Instructional Services

Elementary Education

Middle Grades Education

Secondary Education

Curriculum and Administration

Mathematics Division Staff

Director

Assistant Director

Consultants (9)

Secretaries (2).

Organization

Initially, the Committee members were assigned responsibilities on one of the subcommittees--curriculum, staff, or resources. The subcommittees addressed concerns and issues related to their specific topics. Summary reports from the subcommittees were presented to the total Committee. As the study progressed, it was deemed necessary to change the structure of the Committee. Its second organizational structure was composed of subcommittees by grade levels--elementary K-6, middle school 7-8, and secondary 9-12.

Procedure

The first task of the Committee was to ascertain the current status of the mathematics program in North Carolina. This was done by several methods:

1. Conducting surveys throughout the state. Over 1000 teachers and administrators responded to the questionnaire and/or attended regional hearings to discuss the issues. Summaries of the surveys were compiled by the regional staff of the Mathematics Division and submitted to the Committee Chairman. The Chairman presented a compilation of the summaries to the full Committee for its consideration.

2. Visiting schools and interviewing students, parents, teachers, principals, aides, and central office personnel.

3. Inviting representatives of various interest groups to speak before the Committee. The presenters focused on the status of the mathematics program as viewed by the membership of their respective groups.

4. Inviting teachers, principals, consultants, and university mathematics educators to write position papers on topics of particular concern to the Committee. Nineteen such papers were received and studied by the Committee.

As an understanding of the status of the mathematics program became clear, the Committee began to formulate recommendations designed to improve its quality for students at all grade levels. At each meeting the Committee met initially as a whole group to outline specific objectives to be accomplished, then dispersed into subcommittees for detailed deliberations. At the conclusion of each meeting, the subcommittees reported their progress to the full committee and delineated remaining tasks.

Tentative recommendations were formulated. They were formally presented to educators throughout the state as follows:

1. Presentation at the Name-of-Site Meeting of the National Council of Teachers of Mathematics in Charlotte in November, 1982.

2. Presentations at the three regional conferences of the North Carolina Council of Teachers of Mathematics in the spring of 1983.

3. Presentation at the Seminar on Issues in Mathematics Education sponsored by the Mathematics Division, January 31-February 1, 1983.

4. Distribution by mail to LEA supervisors and college mathematics educators in March, 1983. Over 1000 responses to this request for reactions were received and compiled by the regional staff of the Mathematics Division. Committee members participated in twenty regional meetings which were held in conjunction with this phase of the study.

A summary of all reactions to the tentative recommendations was prepared by the Mathematics Division staff and presented to the Committee. The reactions were carefully considered by the Committee before recommendations were finalized. The completed list of recommendations, their rationales, and related components of the study were presented to the State Board of Education at its August 3, 1983, meeting.

BACKGROUND STATUS OF MATHEMATICS EDUCATION

CHAPTER 2

BACKGROUND STATUS OF MATHEMATICS EDUCATION

Numerous national and world events in the 1950's reminded Americans that the United States was being challenged scientifically and technologically. The flurry of activity which followed for the next two decades to upgrade mathematics education was unprecedented in the history of American education.

Activity began with several conferences to explore the possibilities for improving mathematics education. These conferences were attended by research mathematicians, psychologists, university and pre-college level mathematics educators, and school administrators. The mission of the participants in the conferences was to suggest ways of making the United States second to no other nation in the development of human potential in the technological race.

The ultimate outcome of conference deliberations was to attack the problems of mathematics education on two fronts: 1) to improve the quality of the content of the K-12 mathematics curriculum, and 2) to develop the knowledge and skills of those who would teach the newly developed programs. The federal government, primarily through the National Science Foundation and the Office of Education, appropriated substantial funds for the development of mathematics curriculum projects at all levels. Additional monies to support these projects were made available through some of the large private foundations in this country. These curriculum development teams were comprised of research mathematicians, mathematics education specialists, classroom teachers, school administrators, psychologists, and learning theorists. During the same period of time, millions of dollars were spent

in an effort to prepare teachers to teach the newly developed programs. Cooperative efforts among the National Science Foundation, colleges and universities, and the elementary and secondary schools of this country led to the development of academic year institutes, summer institutes, and leadership development conferences. Weiss[60] notes that the attendance at National Science Foundation sponsored workshops and institutes generally increased with the higher grade level responsibility of the teacher, with more than one-third of all high school mathematics teachers having participated in at least one such activity.

Based upon past program developments and the high levels of financial support, one might conclude that large educational gains would occur. There are data available to support this. However, beginning in 1975, concerns were being raised that these gains were not being sustained. Then recent reports have revealed that when compared to Japan and Russia, this country's superiority in science and technology is rapidly declining.

A comparison of the American and Japanese educational systems reveals that the secondary educational system in Japan places a heavy emphasis on mathematics and science. There is a national guideline for lower secondary education (grades 7-9) which recommends that about 25 percent of the classroom time be devoted to mathematics and science courses. In secondary schools, nearly all of the college-bound students (roughly one-third) take four mathematics courses (algebra, geometry, calculus, or statistics) during their three-year high school career.[28]

Isaac Wirszup, an American authority on education in Russia, states that every Russian youngster who completes ten years of elementary and secondary schooling, as 98 percent reportedly do, has a thorough grounding in arithmetic, algebra, plane and solid geometry, and two years of calculus.[62]

By comparison, in this country the general requirements in grades K-6 in mathematics show that approximately 25 percent of the states and 40 percent of the local school districts have minimum instructional time for mathematics. Only 31 percent of recent high school graduates have taken Algebra II and 6 percent have completed calculus.(28)

Even though a considerable amount of money was spent during the post-Sputnik era on the improvement of mathematics education in the late 1950's and early 1960's, the enthusiasm for this effort began to wane in the late 1960's. By 1969 the United States had succeeded in putting a man on the moon. In many minds, this accomplishment indicated that this country's supremacy in science and technology could be taken as a given. Thus, the nation could shift its priorities to other societal goals.

Beginning in 1968, real-dollar federal investment in research and development in mathematics and science education began to decline. Concurrently, federal support for graduate fellowships in mathematics, science, and engineering declined sharply, while support for teacher-training institutes and curriculum development dropped sharply starting in the early 1970's and then virtually declined to a minimal level.(35)

A careful review of educational reports regarding the status of mathematics education has enabled the Committee to identify six critical problems in mathematics education. These are as follows:

1. There is a critical shortage of mathematics teachers at the secondary level.

In December 1982, the National Science Teachers Association(36) surveyed 600 colleges and universities that have teacher training programs. Year-by-year data were acquired for the ten-year period of 1971-80. The

survey indicated that nationally there has been a 77 percent decline in the number of mathematics teachers prepared to teach in the secondary schools. (In North Carolina, there has been a 79 percent decline in the production of mathematics teachers[45] during this interval of time.) Further, the fraction of those trained who do go into teaching has declined proportionately. The combined effect is an 80 percent reduction in newly employed mathematics teachers since 1971. As might be expected, the overall quality of instruction has declined. The most likely cause of this teacher shortage is the growing disparity between salaries inside and outside education. During 1978-79, for example, persons with a bachelor's degree in mathematics could expect to receive only three-quarters as much as a teacher as they could receive from private industry.

Other factors underlying the declining quality of secondary-level education are a lack of suitable classroom facilities and a decline in support for faculty development. Another factor is that the support system for teachers is eroding; there are relatively few persons available outside the classroom to provide quality control and assist teachers with pedagogical problems.[35]

2. There are a substantial number of unqualified persons teaching mathematics in the secondary schools.

The severe shortage of secondary school mathematics teachers has resulted in employing record numbers of unqualified persons to fill mathematics teaching positions because qualified teachers cannot be found. Studies have indicated that far too many of these people are drawn from the bottom quarter of the graduating high school and college students.

During the 1981-82 school year, 26 percent (4,070) of the mathematics classes, grades 7-12, in North Carolina were taught by inappropriately

certified teachers. This shortage was more profound in the 7th- and 8th-grade mathematics classes in that 48 percent (2,720) were taught by inappropriately certified teachers.(41)

3. Most certified mathematics teachers at the elementary and secondary levels are badly in need of in-service training.

Weiss(60) states that the typical mathematics teacher has been teaching approximately twelve years. She further notes that many of these teachers have not been involved during the past five years in any staff development activities to upgrade their backgrounds in mathematics.

A similar situation exists in North Carolina; the typical teacher has been teaching about eleven years. Many of these teachers have not elected to take additional courses since graduation. Instead, they have managed to renew their teaching certificates by participating in local staff development activities which rarely contain any mathematical content.

4. New sequences of mathematics courses and materials are needed which relate specifically to the students who will not be entering professional scientific and engineering careers.

A 1980 National Science Foundation and Department of Education report states the following:(35)

> The curricula programs developed in the 1950's and 1960's were an important strategy for improving science and mathematics teaching in the post-Sputnik era, and there is persuasive evidence that these programs had a long-term, salutary effect. Today there is a need for similar programs, but the target group is different. While programs in the 1950's and 1960's were aimed at developing textbooks for future science and engineering majors, today's desperate need is for curricula for students who are not interested in professional scientific and engineering careers.
>
> New curricula would provide students with a better basis for understanding and dealing with the science and technology they encounter as citizens, workers, and private individuals.

By stimulating interest in science and technology, they can also motivate students to take science and mathematics courses beyond 10th grade, thereby preserving their options to enter science and engineering careers.

This mismatch of current courses in mathematics and the need of a majority of students must be addressed.

5. Elementary and secondary schools need access to microcomputers, low-cost supplies, and other resources.

Through the National Defense Education Act (NDEA) of 1958, funds were made available to local school systems for equipping schools to improve the teaching of mathematics, science, and foreign languages. These funds could be used to remodel facilities on a minor scale and for purchasing equipment and modern teaching aids. A recent study concluded by Ohio State University[36] indicates that 66 percent of the elementary and 69 percent of the secondary school districts in the nation used these funds to purchase key support materials and upgrade mathematics and science facilities. Unfortunately, changes in national priorities caused these funds to be discontinued in the early 1970's.

Due to the lack of NDEA funds, rising costs of equipment and materials and cuts in educational budgets, many schools do not have funds available to obtain microcomputers and the software necessary for their proper usage. This prevents students from receiving instruction in the use of computers and keeps teachers from using them in the instructional program.

6. Students need to take more mathematics and should be encouraged to take higher level courses which will be challenging to them.

Only about one-sixth of all secondary school students currently take junior- and senior-year courses in mathematics. A large percentage of this group intends to pursue mathematics-related careers. This percentage

has remained almost constant for several years. Judged on the basis of nationwide test scores administered to high school seniors, the qualifications of those intent on mathematics-related careers have remained high. The achievement level for the remainder of the student population seems to be improving with regard to arithmetic skills, but continues to fall short in the more complex skills of problem solving and critical thinking.

Enrollment in secondary mathematics and science classes has declined at more than twice the rate of the general enrollment between 1977 and 1981. While this decline seems to have stabilized, enrollment after Grade 10 drops dramatically by about 55 percent in mathematics. This indicates that, given the choice, most high school students are not electing mathematics courses. Thus, in effect at the age of 16 (or earlier) many students deny themselves the opportunity to study many curricula offerings at the post-secondary level. The dropout rate from mathematics at the 10th-grade level is particularly severe for girls and minority students.

There appear to be several reasons for these declining participation rates, including low levels of achievement in mathematics and science, few requirements for graduation in most states, the reduction of college entrance requirements, a narrowing of the school curricula to focus on "basics," and a serious mismatch between most existing curricula and the interests and needs of those students who do not elect to pursue careers in mathematics, science, and engineering. Little or no vocational relevance is presented in such courses, and there is virtually no exposure to technology.

The challenge presented by other nations to the technological and scientific superiority of the United States citizenry is a critical reason for assessing the current status of mathematics education in the United States and particularly in North Carolina. The six major areas of concern listed above formed the base from which the Mathematics Curriculum Improvement Project launched its study of the mathematics curriculum in the state's public schools.

THE ELEMENTARY MATHEMATICS PROGRAM

CHAPTER 3

THE ELEMENTARY MATHEMATICS PROGRAM

PREFACE

Mathematics is the language of all sciences and is critical to the total development of the child. Education at the elementary level is the foundation upon which all future learning is built. Because the child's self-concept and his grasp of fundamental ideas influence subsequent academic success, the beginning years of formal instruction are critical. Attitudes toward mathematics are formed by experiences in elementary school; therefore, the influence of the elementary school teacher and the curriculum are paramount.

RECOMMENDATION 1. We recommend that the elementary mathematics curriculum be strengthened to reflect an increasing emphasis on problem solving and the use of calculators and computers and that it be designed to encompass more than computational facility.

Rationale: The elementary mathematics program must include appropriate goals and activities for problem solving, calculators and computers at each grade level to reflect the needs of students living in today's world, yet prepare students for the world with which they will be confronted.

Rather than detract from the learning of basic computational skills, the selective use of calculators and computers would encourage higher levels of cognitive thinking by allowing students to experience "real life" problems involving the organization and interpretation of data. It will allow teachers to spend more time with their students in application rather than drill. In its Third National Mathematics Assessment, the NAEP(25) states that "the technological innovations of the last

decade are transforming the ways in which people use mathematics. The mathematics needed today is not the mathematics that was needed a century ago, but for the most part, that is the mathematics still being taught in schools." Students must have more experiences which develop understandings and skills. "There are machines that can do the calculations," writes the NAEP report, "but it is still necessary to know what questions to ask and how to use the results."

RECOMMENDATION 2. We recommend that an awareness of calculators and their uses be provided for K-2 students and that children in grades 3-6 be taught to use a four-function calculator.

Rationale: To teach the use of calculators at these grade levels will enable children to apply and extend mathematical concepts beyond their computational abilities.

The public concern about the use of calculators in schools and fears that students would become so dependent on calculators that they would lose their ability to compute has motivated a number of research studies on the topic. Marilyn Suydam[56] summarized research on the effects of calculator use.

> Almost 100 studies on the effects of calculator use have been conducted during the past four or five years. . . . In all but a few instances, achievement scores are as high or higher when calculators are used for mathematics instruction (but not on tests) than when they are not used for instruction.

This recommendation supports the National Council of Teachers of Mathematics' An Agenda For Action[30] and would indicate that North Carolina is taking a leadership role in the application of technology in the classroom.

RECOMMENDATION 3. We recommend that all elementary students have access to computers and appropriate courseware. The ratio of 1 computer per 24 students is recommended.

Rationale: The computer must be one component of the organized, balanced, instructional program that provides a wide variety of experiences. Research shows that mathematics learning can be improved and that mathematics teaching can be facilitated through the use of technology.(8) Effective integration of computer-based education into the mathematics curriculum requires that sufficient numbers of microcomputers and/or terminals with appropriate courseware be available.

RECOMMENDATION 4. We recommend that all elementary children have the opportunity to explore mathematics through the use of a variety of manipulative materials and that there be an effort to effectively "bridge the gap" between manipulatives and the abstract levels of learning.

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RECOMMENDATION 5. We recommend that additional state funds be appropriated on a per-pupil basis to purchase manipulative materials for use in mathematics at grade levels K-6. Particular emphasis should be placed on the purchase of materials necessary for a strong early childhood mathematics program.

Rationale: Piaget presents a theory of intellectual development which states that children in the preoperational stage think quite differently from those at the concrete operational stage.(9) In elementary school, children are either in one of these stages or in transition between them. Children who are at the preoperational stage must have physical objects for manipulation in order to be able to reason about number and number relations, while the children at the concrete operational stage draw from previous experiences in performing these tasks.

The changes from preoperational to concrete operational occur as elementary pupils mature mentally as well as physically. These changes take place gradually, over a period of years. Children attain the concrete operational level in various situations at different times. During the time of transition the children are inconsistent--they will appear to have achieved concrete operational thought and then revert to preoperational thinking. It is therefore imperative that the instructional model of developing concepts from the concrete to the abstract must be utilized throughout the elementary school experience.

It is important that all elementary teachers be mindful of the significance of manipulatives and pictorial representations in the growth processes. Payne in FROM CONCRETE TO ABSTRACT(9) concludes that students can benefit from exposure to multiple representations of the same concept. It is critical not only that teachers be aware of different representations but also that they consistently make explicit the linking of steps in progression toward the symbolic representation.

In learning to mentally perform action on objects, students must have had many interactions with physical objects. This implies that the current abstract experiences in the middle and secondary schools are dependent on the foundations developed in the elementary school. Suydam and Higgins(57) note that students who had learned with the help of manipulative materials scored significantly higher on achievement tests than students whose instruction included no use of manipulatives.

RECOMMENDATION 6. We recommend that mathematics education at the kindergarten level be strengthened through:

A. *In-service for kindergarten teachers;*

B. *A comprehensive teachers' manual which illustrates competency goals and performance indicators;*

C. *A specific per-pupil allotment to purchase manipulative materials in lieu of textbooks.*

Rationale: While the state of North Carolina provides student texts and teachers' manuals for all students and teachers in grades one through twelve, nothing is provided for the kindergarten student or teacher. Kindergartens should receive funds for mathematics instruction in the form of allocations for manipulative materials rather than textbooks. Through in-service, classroom teachers must have opportunities to develop strategies for using these materials in their classrooms.

The Mathematics Division is developing a comprehensive kindergarten teacher's manual which will provide guidance for the teacher. Because providing specific suggestions for alternatives to abstract activities is not enough, the manual will provide information for the teachers on how to bridge from manipulatives to the more abstract ways of teaching mathematics.

RECOMMENDATION 7. We recommend that each child have sixty (60) minutes of math daily with a balance between direct instruction, applications, use of manipulatives, maintenance of skills, and enrichment. Consideration needs to be given to the time of day when mathematics instruction occurs.

Rationale: If elementary schools are to provide the solid foundation necessary for success in higher mathematics and to develop competent, discriminating consumers, mathematics must be included daily as a major part of the instructional program. Each day's lessons must include a

balance between developmental instruction, drill and practice, and experiences in applying mathematics' skills.(57)

Because research clearly indicates that time on task is directly correlated with children's levels of achievement, a total of sixty (60) minutes during every school day should be devoted to development of mathematical concepts.(9,11) Direct instruction should come during "prime teaching time" while the children are fresh and receptive, before they are tired from long hours of concentration and written work.

RECOMMENDATION 8. We recommend that students in grades 4-6 whose total mathematics scores fall below the 35th percentile on the third grade North Carolina Annual Testing Program be considered for the Developmental Mathematics Monitoring Program with instruction in groups of ten (10) or less by a teacher with mathematics training.

Rationale: Students' competence in mathematics at higher grade levels can be attributed to foundations developed in elementary school. When students have not mastered mathematical concepts and do not have a solid foundation on which to build, crisis situations arise in which remediation is needed to help students develop those understandings. Because it is more cost effective to remediate at the elementary level and because the estimated 7.6% dropout rate in North Carolina in the 1980-81 school year represents 26,709 students,(45) the logical alternative to more upper level remediation is to provide a developmental mathematics program at the elementary level.

At the fourth grade level most students move into larger classes without the assistance of an instructional aide. This means less individual attention for the children. The recommendation for small group instruction is based on studies which indicate that children

having frequent interaction with teachers in small groups have higher rates of success.(9)

Since North Carolina students take a standardized achievement test at the end of third grade, there is in place a measure for identifying children needing additional mathematics instruction in small groups.

The theories of mastery learning developed by Bloom emphasize the belief that every child can master the elementary school curriculum given appropriate instruction and sufficient time and support the need for expanded developmental instruction. The establishment of a Developmental Mathematics Monitoring Program would provide early intervention for students weak in mathematical skills and help to prevent the "math anxiety" that often keeps students from choosing to take more than the minimum mathematics requirements.

RECOMMENDATION 9. We recommend that the state guidelines for identification of exceptional children be restructured to enable school personnel to more effectively identify students who are mathematically gifted. We further recommend that the mathematics program for these students be designed to foster higher levels of cognitive thinking and to include an enrichment component.

Rationale: Under the present guidelines for identifying gifted children many students with special talents in mathematics do not qualify. Attention should be focused on recognizing these mathematically gifted students whether or not they exhibit giftedness in other areas.

Mathematics instruction must be provided on an appropriate level for all students; therefore, mathematically talented students must have the opportunity to develop their full potential.

RECOMMENDATION 10. We recommend that, if the North Carolina Annual Testing Program continues to be administered in the spring, norm referenced tests be used at all levels presently tested.

Rationale: While the Annual Testing Program is working well in assessing student achievement, there have been problems in selecting tests for the first and second grades. The major problem at these grades is that it is difficult to find a criterion-referenced test which has sufficient scope to measure the curriculum taught in North Carolina's first and second grades. An additional problem is that the converted criterion-referenced test scores to national norms lack validity. Thus we support the adoption of norm-referenced tests for grades one and two.

The current testing schedule does not permit the classroom teacher to utilize the test results. There can be no diagnostic/prescriptive utilization when the results are not available until the end of the year.

RECOMMENDATION 11. We recommend that the teacher/pupil class load in grades 4-6 be reduced so that one (1) teacher is responsible for no more than twenty-six (26) pupils.

Rationale: We continue to support the State Board of Education's efforts to hold the teacher/pupil class load at the primary levels to no more than twenty-six (26) pupils for one (1) teacher.

As children progress in elementary mathematics training, they need to work with manipulatives under the direct guidance of a teacher in the development of new concepts. At the same time, they also need individual attention to maintain their previously learned skills. A lower pupil/teacher ratio is necessary to accomplish this.

RECOMMENDATION 12. We recommend that teacher training institutions include in their programs for prospective elementary teachers early internship experiences with mathematics instruction and that a mathematics competency examination be required of all candidates seeking elementary certification.

Rationale: The Division of Certification and the Division of Mathematics of the State Department of Public Instruction have recently prescribed new competencies and skills that prospective mathematics teachers must acquire. As a part of the Quality Assurance Program for the teacher education institutions in North Carolina, the list of competencies is comprehensive and far-reaching; however, it does not encourage the establishment of an internship but it suggests early experiences in the classroom for prospective teachers.

The implementation of an internship program would enhance the preparation of the prospective teachers by providing early interaction with children and mathematical experiences. The internship program should be structured in a manner that will ensure a clearer understanding of children and how they learn mathematics. It would introduce prospective teachers to a variety of children's needs and to the resources available at the elementary level.

A passing score on a mathematics competency examination will indicate the candidate's knowledge of mathematics. This would also help to ensure uniformity and higher standards in teacher education programs in North Carolina.

RECOMMENDATION 13. We recommend that an elementary mathematics specialist certification be established. Elementary teachers holding this certification should receive a salary differential.

Rationale: To improve the quality of elementary mathematics instruction there must be teachers who are especially knowledgeable about mathematics content and methodology as well as child development. Few teachers in elementary school positions have completed as much as six hours of pre-service training in mathematics education. This certification would encourage teachers who already have K-4 or 4-6 certification and who have special interest in mathematics to pursue appropriate courses in order to become specialized in the area of mathematics.

Currently few institutions of higher learning in North Carolina are offering a degree program specifically for elementary mathematics. Teachers certified at the K-4 or 4-6 levels find few appropriate courses in elementary mathematics education at the graduate level. The existence of an elementary mathematics certification would encourage more colleges to establish the necessary courses.

RECOMMENDATION 14. We recommend that all levels of teacher training--pre-service, in-service, and graduate work--include instruction for using computers in teaching mathematics.

Rationale: Microcomputers have become commonplace in the home, school, and marketplace. The computer has tremendous potential for making competent, knowledgeable teachers more effective.

Positive experiences and the development of confidence are particularly important in the area of computers for teachers already in the profession who may be "technology-shy" due to their limited experience with this teaching tool. A coordinated program of staff development is essential for the implementation of this recommendation.

RECOMMENDATION 15. We recommend that staff development be offered for principals to support their leadership in elementary mathematics education:

Rationale: This recommendation encourages the principal to become more knowledgeable about the quantity and quality of mathematics instruction in elementary schools. As a result, the principal, as the instructional leader, will be better prepared to evaluate and plan for the mathematics program within his school.

According to the RAND Study the attitude of the principal is critical to the long-term results of a project.(50) "The support of the school principal for a special project is directly related to the likelihood that staff will continue to use project methods and materials after special training is withdrawn." Involvement in in-service training updates principals' classroom skills and knowledge about what is needed for a quality mathematics program.

This in-service will assist the principal to serve more effectively as the instructional leader by (1) observing mathematics lessons on a regular basis, (2) ensuring that mathematics is taught daily to all students by the best qualified teachers, (3) providing the format for a rotating schedule which will ensure some "prime teaching" time for mathematics, (4) encouraging flexible grouping for skills, (5) providing resources for obtaining ancillary materials, (6) recognizing sound methodology that provides opportunity for problem solving and divergent thinking.

RECOMMENDATION 16. We recommend that the following types of in-service programs for elementary school teachers be developed and funded to reach all teachers and aides within a five-year period:

A. *An ongoing program consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, classroom management, efficient use of classroom time, and counseling strategies.*

RECOMMENDATION 16 (continued)

B. *Special summer institutes to assist teachers in teaching mathematics. Consideration should be given to making the institutes permanent in order to foster continued upgrading of the competence level of teachers.*

Funding should be provided for various financial incentives to allow teachers to participate.

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RECOMMENDATION 17. We recommend that at least a third of the non-teaching continuing education units earned by a teacher for elementary certificate renewal be in the area of mathematics.

Rationale: Because the majority of elementary school children in North Carolina are being taught by teachers with about ten years of experience, changes must come through in-service of personnel already in the system. In the 1980-81 school year less than 2 percent of the teachers in North Carolina were new to the profession.(45)

When teachers do not know alternate methods of instruction, they rely on pencil and paper tasks for their mathematics program. An overwhelming majority of North Carolina elementary teachers surveyed recently by this study group indicated a lack of confidence in their mathematical skills. A study conducted by the National Science Foundation which surveyed K-6 teachers revealed that they felt a need for assistance in the following areas: (1) learning new teaching methods, (2) obtaining information about instructional materials, and (3) implementing discovery/inquiry and hands-on approaches in teaching.(60)

Intensive, positive experiences with mathematics activities are needed for teachers to gain skills for effective teaching and build confidence in their abilities. A model for such in-service might be the Primary Reading Institute which offers a format of intensive in-service

providing hands-on experiences and suggestions for teaching specific skills. The institutes have been effective agents for change as evidenced by increased achievement in reading for primary children in North Carolina.

Under the current guidelines, it is possible for elementary teachers to obtain certificate renewal through various experiences which are not related to mathematics and therefore do not contribute to the teacher's mathematics expertise.

RECOMMENDATION 18. We recommend that the state, in conjunction with local education agencies, create mathematics coordinating teacher positions to provide continuous in-service via demonstration teaching, conducting workshops, and providing support and resource services. It is further recommended that:

A. *The positions be phased in over a five (5) year period in all eight regions as teachers are identified and trained;*

B. *The coordinating teacher's contacts not exceed seventy-five (75) classroom teachers and that each local education agency have at least one mathematics coordinating teacher;*

C. *The coordinating teacher receive a differential salary.*

Rationale: In keeping with North Carolina's current emphasis on mathematics, this recommendation addresses the long-term goal of improving the quality of instruction. By establishing teaching positions which would directly influence what happens within classrooms through contacts with students and through demonstration teaching, the state would establish continuous in-service and assistance for classroom teachers.(50)

The coordinating teachers will be involved in instructional activities which require extensive preparation time. In their roles as the mathematics experts in the schools, they will also serve as facilitators

of information related to mathematics curriculum materials and professional developments. They must continuously update their own training. Because of these added responsibilities and required training, the mathematics coordinating teachers must be paid an appropriate higher salary.

RECOMMENDATION 19. We recommend that every effort be made to enhance the teaching of mathematics in elementary schools by:

A. *Providing appropriate scholarships for teachers to take mathematics or mathematics education courses;*

B. *Providing adequate reimbursement for expenses incurred at mathematics professional meetings;*

C. *Providing aides and/or clerical help in grades 4-6;*

D. *Identifying and recognizing outstanding elementary mathematics teachers;*

E. *Identifying and recognizing outstanding mathematics programs through the establishment of model programs.*

Rationale: Financial incentives will provide assistance and motivation to teachers desiring to improve their knowledge of methods and content. To encourage elementary teachers to become more proficient in mathematics, scholarships must be available in mathematics. To encourage teachers to participate in professional organizations, reimbursements should be made for expenses at mathematics meetings.

Teachers in self-contained classes in grades four through six have tremendous responsibilities; they deal with large groups of students, all areas of the curriculum, and a wide range of abilities. Aides and/or clerical assistance are needed to free the teacher to concentrate maximum time on instruction.

The recognition and identification of quality mathematics instruction can promote interest and enthusiasm among elementary school teachers. The establishment of a model program will allow teachers to see first-hand a variety of techniques for teaching mathematics. Many teachers must "see to believe"; thus, demonstration lessons in a model setting provide a picture that no other type of in-service can offer.

RECOMMENDATION 20. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students who desire to pursue a mathematics specialist certification in grades K-6. We further recommend that the amount of each scholarship loan be increased to $2000/year.

Rationale: Under this scholarship/loan program, recipients of loans who teach in the public schools for the same number of years that financial assistance is received, have their debt cancelled proportionally. Thus, most recipients would have a definite commitment to enter and remain in teaching in North Carolina.

The current loan/scholarship of $1500 per year is inadequate to support the needs of students in view of rising college costs.

THE MIDDLE SCHOOL MATHEMATICS PROGRAM

CHAPTER 4

THE MIDDLE SCHOOL MATHEMATICS PROGRAM

PREFACE

The mathematics program at the middle grades level must respond to the needs of the student population. As the program is developed and revised, consideration must be given to characteristics of this age group.

Middle grades students are in a period of transition from childhood to adulthood. Physical, emotional, social and intellectual abilities are maturing and there is constant re-adjustment by the student to incorporate these changes into daily functioning.

Intellectual abilities are at different levels of development due to the varying rates of maturity.(19) Middle grades students have the widest range of ability levels when compared to other age levels. Some students are functioning at what Piaget(47) would call the concrete operational stage, while others are capable of more formal operational thought processes. It is recognized that a large percentage of the students do not function totally in either stage; rather, they move from one to the other depending on the nature of the tasks to be performed. The mathematics program must be structured so that it is capable of providing content and methodology appropriate to this wide range of developmental levels.

The middle grades mathematics program is an extension of the elementary program as outlined in the elementary recommendations and, as a result, is broader and more comprehensive than it has been in the past. Further, it is designed to prepare students for a more rigorous program of secondary mathematics.

RECOMMENDATION 1. We recommend that, in order for students in grades 7 and 8 to have broad experiences in mathematics and to provide a strong base for further study, the mathematics curriculum in grades 7 and 8 be structured according to the following levels:

Grade 7

A. Comprehensive Mathematics broadly based in Pre-Algebra, informal plane and solid geometry, descriptive statistics and survey sampling, data analysis, consumer arithmetic, mental arithmetic and estimation, applications of mathematics and problem solving.

B. A course which encompasses the seventh and eighth grade comprehensive mathematics courses for those students who have shown superior mathematical ability and who will proceed to Algebra I in the eighth grade.

C. Developmental/remedial mathematics for students scoring below the 35th percentile on the sixth grade annual testing program in mathematics, taught in small groups (1:10) under the supervision of a teacher trained in diagnostic/prescriptive teaching of mathematics in a setting conducive to active participation in learning through problem solving.

Grade 8

A. Continuation of Comprehensive Mathematics from seventh grade designed to further students' mathematical foundation in preparation for the course entitled Pre-Algebra in the ninth grade.

B. A more accelerated arithmetic/algebra course for the students who are qualified to handle a curriculum preparatory to Algebra 1.

C. Algebra 1 for the students who are at that level and preferably were enrolled in accelerated mathematics in grade 7.

D. Continuation of developmental/remedial mathematics for those who continue to need assistance with the understanding of fundamental mathematics--taught in small groups (1:10) under the supervision of a teacher trained in diagnostic/prescriptive teaching of mathematics in a setting conducive to active participation in learning through problem solving.

Rationale: Mathematics in the seventh and eighth grades is crucial to further mathematical development of students and does much to form their attitudes toward selection of high school courses. Therefore, it is imperative that instructional modes reflect the st pedagogical practices.

It is true that many seventh and eighth grade students are still in a "pre-logic" stage of development that precludes a deep understanding of some mathematical principles.(19) However, these students can benefit from the study of some very sophisticated mathematics concepts if they use concrete materials to model the concepts and are encouraged to work in an open-ended, inquiry mode.

Strong seventh and eighth grade mathematics courses are prerequisites for the successful study of Algebra I. However, mathematically adept students who are entering a "logic" stage of development should be able to complete this work in grades six and seven, thus allowing them the opportunity to begin their study of Algebra I in grade eight. Past experience has shown that students who are well-prepared and who begin Algebra I in grade eight successfully complete college-level work in mathematics before graduating from high school. In the past, such opportunities have been available primarily to students in large school systems. All students in North Carolina who qualify for accelerated study in mathematics should have the opportunity.

Because it is not cost-effective to have remediation at higher educational levels and because the estimated 7.6% dropout rate in North Carolina in the 1980-81 school year represents 26,709 students,(45) the logical alternative to more upper level remediation is to concentrate on providing an effective developmental and remedial mathematics program at the upper elementary level. Across North Carolina all students take the California Achievement Test at the end of the sixth grade. Thus, the Annual Testing Program is one standardized measure of a student's achievement in mathematics that should serve as a guideline for identifying

students for developmental/remedial mathematics. The theory of mastery learning advocated by Bloom[4,5] which emphasizes the belief that every student can master mathematical concepts given appropriate instruction and sufficient time lends support for expanded developmental instruction rather than later remedial assistance when students are even more deficient.

For the developmental/remedial program to have maximum effectiveness, the instruction in small groups must come under the supervision of a teacher with training in diagnostic/prescriptive teaching of mathematics. To accommodate the learning patterns, mental development stages, and interest of seventh and eighth grade students, instructional activities at each level must emphasize open-ended inquiry, exploratory exercises, and the use of visual and manipulative aids.

It is recommended that the 1980's middle school mathematics curriculum expand the definition of basic skills, use a diversity of instructional strategies, materials, and resources, while requiring more mathematics study and providing a more flexible curriculum with a greater range of options.[63]

RECOMMENDATION 2. We recommend that calculators be used as a teaching tool in grades 7 and 8 and that all students become proficient in using the four-function calculator with memory and percent.

Rationale: Research studies support the use of calculators at all grade levels, indicating that achievement and attitudes are enhanced when calculators are used as an instructional tool.[55,56] Research further indicates that the effective utilization of calculators results in higher scores when students are tested on paper-pencil tests not using calculators.[59]

Calculators should be used to help students explore mathematical topics beyond basic computational skills.(30) For example, while students quickly discover decimals as they experiment with calculators, they will encounter concepts and operations involving negative integers, exponents, square roots, scientific notation, large numbers--all topics in junior high mathematics. In addition, providing low-achieving students with calculators has the potential to open new areas of mathematics to these students and, at the same time, break down self-defeating attitudes.(23)

RECOMMENDATION 3. We recommend that all middle school students be provided the opportunity to become computer literate and that an elective course in computer programming be developed and made available to interested students.

Rationale: Familiarity with microcomputers and the knowledge and skills required to make intelligent use of them will be important for students in all subject areas, and, in particular, mathematics. Learning to operate and control a powerful problem-solving tool such as a microcomputer will equip students for dealing more effectively with the challenge of secondary school subjects, and for their future role as college students or as effective participants in the world of work.(30)

The recommended course should provide sufficient instruction in the use of an appropriate programming language to allow students to plan, develop, and refine simple computer programs. This course should also allow students to become familiar, through extensive hands-on experiences, with the wide range of microcomputer applications which would be of immediate use to them in their present role as students as well as in whatever future path they might pursue.

RECOMMENDATION 4. We recommend that there be 55 minutes of mathematics instruction daily with over 50 percent of the instructional time spent in teaching that develops content through interaction between the students and the teacher.

Rationale: Two very important variables that affect how well students achieve are the amount of time allocated to instruction and the use of instructional time by the teacher. Students' achievement in mathematics has been shown to be affected by these two variables. In grades 7 and 8, there must be enough time allocated to mathematics to ensure that the content of the courses are covered. Effective teachers of seventh and eiththh grade mathematics devote at least half of each class period to developing concepts and skills through lecture, demonstration, discussion, creative questioning, and appropriate modeling of the expected behavior.(9)

RECOMMENDATION 5. We recommend that a program be developed to assist guidance counselors, advisors, curriculum coordinators, principals, and teachers in advising middle school students. The intent of the program is to inform students about the quantity and quality of mathematics needed for life in a technological society.

Rationale: Research shows that the more mathematics a person knows, the more employment options he/she will have available.(12,52) There is a large discrepancy in the amount of mathematics training and skill acquired by those who are interested in science and engineering careers and those who are not. While there has always been such a discrepancy, the evidence indicates that in recent years it has been widening. The relatively few students who have a strong interest in science and engineering careers are studying as much mathematics as ever. However, the larger body of students are terminating their study of these subjects at an increasingly early stage and are performing less well on achievement measures.(35)

Because of peer pressure, elitism of upper level courses, past performance and unwillingness to work hard, many students are not exercising their options to study as much mathematics as they could. Teachers and guidance counselors, working closely together, can have a greater influence on student choices than they now do. Students must be encouraged to pursue more advanced mathematics and science courses by being informed of career opportunities which are available only to students with strong backgrounds in mathematics.

The need to encourage more females and minorities to take the mathematics and science courses necessary for careers depending on these academic areas is a recognized national problem. The educational environment of these groups must be altered to effect a change--that students should be seen as students first and be accepted without prejudgment and ready-made expectations due to sex, race, social class, or any other factor. The students' environment is composed of several significant groups of people including mathematics teachers, counselors, parents, and the target students themselves.(15)

RECOMMENDATION 6. We recommend that the state, in conjunction with local education agencies, create mathematics coordinating teacher positions to provide continuous in-service via demonstration teaching, conducting workshops, and providing support and resources services. It is further recommended that: (1) the positions be phased in over a five-year period in all eight regions as teachers are identified and trained; (2) the coordinating teacher's contacts not exceed seventy-five classroom teachers and that each local education agency have at least one mathematics coordinating teacher; (3) that there be a pay differential commensurate with the requirements of the position.

Rationale: In keeping with North Carolina's current emphasis on mathematics, this recommendation addresses the long-term goal of improving the

quality of instruction. By initiating teaching positions which would directly influence what happens within classrooms through contacts with students and through demonstration teaching, the state would establish continuous in-service and assistance for classroom teachers.(50) The statistics (40% of teachers teaching one or more mathematics classes in grades 7-12 in North Carolina do not have mathematics certification) are an indication of this need.(45)

The coordinating teachers will be involved in instructional activities which require extensive preparation time. In their role as the mathematics experts in the schools, they will also serve as facilitators of information related to mathematics curriculum, materials, and professional developments. They must continuously update their own training. Because of these added responsibilities and required training, the mathematics coordinating teachers must be paid an appropriate higher salary.

RECOMMENDATION 7. We recommend that the State Department of Public Instruction's Competencies and Guidelines for Approved Teacher Education Programs be amended for the 6-9 certificate in mathematics so that prospective teachers seeking a mathematics concentration be required to complete a minimum of 30 semester hours of which at least 15-21 semester hours be in mathematics and 9-15 semester hours be in mathematics education, exclusive of the student teaching experience. All teachers teaching mathematics in grades 7 and 8 must have this specialist certification.

Rationale: To successfully implement the kind of curriculum we envision in grades 7 and 8, the teacher must have the preparation in mathematics and teaching of mathematics implied by this recommendation and by the listed competencies in the Competencies and Guidelines for Approved Teacher Education Programs. A minimum number of semester hours necessary to meet these competencies needs to be stated at the 6-9 level since prospective teachers at this level may elect to have concentrations in several areas.

RECOMMENDATION 8. We recommend that all components of teacher training--pre-service, in-service, graduate work--include instruction using computers for teaching mathematics.

Rationale: Microcomputers are quickly becoming commonplace in the home, school, and marketplace. In school, in particular, the computer is a tool which has tremendous potential to make a competent, knowledgeable teacher more effective. Teacher education programs for all levels of mathematics must include computer literacy, some computer programming, and strategies for effective use of computers in the curriculum. A coordinated program of staff development and other types of support are essential in order to fulfill this potential.(30)

RECOMMENDATION 9. We recommend that at least three units for certificate renewal of middle school mathematics teachers be in mathematics and/or mathematics education. We further recommend that these units be earned through college credits or through approved coursework with the same rigor in content and requirements as a college-level course.

Rationale: Under the current guidelines, it is possible for mathematics teachers to obtain certificate renewal through various experiences which are not related to mathematics and which do not contribute to the teacher's mathematics expertise. This recommendation is directed toward improving the quality of our teachers and through them, the education of our children.

RECOMMENDATION 10. We recommend that the following two types of in-service programs for middle school mathematics teachers be developed and funded:

A. *An ongoing program with a curriculum consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, classroom management, efficient use of classroom time, and counseling strategies.*

RECOMMENDATION 10 (continued)

B. Special summer institutes to assist teachers in teaching middle school mathematics. Consideration should be given to making the institutes permanent to foster continued upgrading of the competence level of teachers.

Funding should be provided for various financial incentives to allow teachers to participate in either or both programs.

Rationale: Teachers are the most important educational influence on students' learning of mathematics.(13) The 1981-82 North Carolina Mathematics Teacher Profile(41) reports that 58% of the mathematics teachers in grades 7 and 8 are not properly certified to teach mathematics at this level. It is generally accepted and documented by research that teachers who possess a sound knowledge of their subject (mathematics content and structure), as well as expertise in methodology, can project a more positive influence on students, thus enhancing and/or assuring continued growth in the level of achievement.

Many of the teachers presently in the field have not had pre-service opportunities which include instruction and activities as described in this recommendation.

RECOMMENDATION 11. We recommend that microcomputers and appropriate courseware be made available to support the teaching of all mathematics courses in the 7-8 curriculum. A ratio of one (1) computer for every 28 students is recommended.

Rationale: Microcomputers have obvious appeal for all students and their effectiveness as a tool for promoting mathematics instruction has been well documented. The NCTM Agenda For Action had recommended that integration of technology, in particular, computers and hand calculators, into the mathematics curriculum at all levels be a major goal of mathematics education for the 1980's.

Applications of microcomputers in the mathematics curriculum for grades 7 and 8 should be broad-based[13] and should be included in all middle school mathematics courses. These applications include:

- traditional computer-assisted instruction, such as drill and practice, review lessons, and testing of knowledge of facts and acquisition of skills;
- simulations, the modeling of processes and events which are too dangerous, too expensive, or too complex to demonstrate in the classroom;
- data analysis of real life problems with real life data, which would be too long and tedious to calculate without the aid of technology;
- computer-managed instruction, wherein the computer is used to determine the appropriate level and type of instruction (which may or may not be computer-based instruction) for a particular student, as well as to maintain an ongoing record of each student's activities and progress.

In addition, microcomputers serve as a powerful tool for enhancing children's problem solving and other higher-level thinking skills by promoting the application of a diversity of problem solving strategies.

To optimize the use of the microcomputer as an instructional tool, microcomputer hardware must be available in sufficient numbers that all teachers and their classes have easy access to them whenever their use would promote the instructional program.

RECOMMENDATION 12. To attract competent, new mathematics teachers, we recommend the following:

A. *Teaching assignments for first-year teachers involve contact with students of varying ability levels.*

B. *First-year teachers be offered a contract as early as possible.*

C. *First-year teachers be offered the opportunity of beginning employment as soon after their graduation as is practical.*

Rationale: See Rationale for Secondary Recommendation 18, pp. 90-91.

RECOMMENDATION 13. We recommend that to attract and retain competent mathematics teachers a program of incentives be developed. Specifically, we recommend that:

A. *Teaching loads in mathematics be restricted to no more than 26 students per class and 130 students per day.*

B. *Appropriate stipends be provided for summer study, curriculum development, staff development, etc.*

C. *Reimbursement be provided for expenses associated with attendance at professional mathematics meetings, including travel, lodging, registration fees, and substitute teacher pay.*

Rationale: See Rationale for Secondary Recommendation 24, pp. 94-95.

RECOMMENDATION 14. We recommend that salaries be increased for appropriately certified mathematics teachers who have a majority of their teaching responsibilities in mathematics.

Rationale: See Rationale for Secondary Recommendation 25, pp. 95-96.

RECOMMENDATION 15. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students who desire to pursue a mathematics specialist certification in grades 7-8. We further recommend that the amount of each scholarship loan be increased to $2000/year.

Rationale: See Rationale for Secondary Recommendation 26, p. 96.

CHAPTER 5

THE SECONDARY MATHEMATICS PROGRAM

PREFACE

Secondary mathematics provides the conceptual framework for all the mathematics which precedes it, and, at the same time, secondary mathematics is the foundation for all advanced work in mathematics, science and technology. A successful secondary mathematics program requires two interwoven components--a comprehensive up-to-date curriculum and dedicated, well-prepared teachers. The secondary mathematics program in North Carolina has many strengths, but it has problems which must be addressed. Our concerns about the program reflect problems which are national in scope. The dedicated efforts of those responsible for instruction in mathematics have not overcome inadequate support for professional development and for educational resources. Moreover, inadequate salaries and benefits for teachers, coupled with an increasing demand from industry, government, etc., for persons with mathematical training, have created a critical, well documented, shortage of trained mathematics teachers. The shortage of teachers and resources is compounded by a curriculum which has yet to respond fully to technological change. In particular, computers and calculators have not been completely integrated into the secondary mathematics curriculum in a comprehensive fashion.

Several years ago, those national professional organizations which are concerned primarily with mathematics education began to work towards an understanding and an eventual solution of the problems facing secondary school mathematics. Two of the organizations, the National Council of Teachers of Mathematics (NCTM) and the Mathematical Association of America

(MAA), produced documents which are of particular importance.[22,30] These are as follows:

(1) Recommendations for the Preparation of High School Students for College Mathematics Courses, a joint 1980 publication of the MAA and the NCTM.

(2) An Agenda For Action: Recommendations for School Mathematics in the 1980's, a 1980 publication of the NCTM.

The reader who is familiar with these publications will discover that they motivated many of the recommendations included in this report. This emphasizes that the problems faced in North Carolina are not unique to this state or even to this region. The problems are truly national in extent.

RECOMMENDATION 1. We recommend that steps be taken to ensure uniformity in the quality and depth of the various mathematics course offerings throughout the state. Specifically, we recommend the following:

A. *A general course syllabus be developed for each secondary mathematics course.*

B. *A brief course outline be developed for each State-adopted secondary mathematics textbook.*

Rationale: The use of syllabi in secondary mathematics courses will provide the basis for uniformity in student exposure to concepts and techniques. Syllabi will also provide continuing guidelines for State textbook adoptions. Since textbooks are the most available, and, therefore, the most widely used teaching tools, the use of syllabi, coupled with the use of course outlines will provide much needed guidance to the inexperienced teacher of mathematics.

Even experienced mathematics teachers need guidance if, as recommended in An Agenda For Action, computers are to be fully utilized at all grade levels. Many secondary teachers also need help in making appropriate use of models and manipulative materials in their teaching. It is important

then that the course syllabi contain suggestions for using models, manipulatives and computers. In particular, the syllabi should contain recommendations about computer software and hardware.

RECOMMENDATION 2. To ensure that students are proficient in the use of a calculator, we recommend the following:

A. *All students be taught to use a four-function calculator with memory and percent.*

B. *The mathematics section of the North Carolina Competency Test be revised to include a section testing calculator proficiency and to permit the use of the calculator on the Applications Section of the Test.*

C. *Students who plan to attend post-secondary schools be taught to use a scientific calculator.*

D. *Funding be appropriated from the state to the LEAs for purchasing, maintaining, and replacing calculators, and for providing supplementary materials.*

E. *Students be encouraged to use a calculator, whenever appropriate, in their mathematics classes.*

Rationale: A four-function calculator with memory and percent is an extremely useful tool in a wide variety of everyday activities. Students who are not taught to use this tool will be handicapped at work and will be unnecessarily burdened with paper and pencil arithmetic. Students will be more likely to learn to use calculators effectively, if calculator proficiency is taught in school and the use of calculators is required in the Competency Testing Program.

Recommendation 3 of An Agenda For Action points out that if the arithmetic drudgery is removed from mathematics courses at the seventh grade level and above, teachers can focus on building understanding and developing problem solving skills. Certainly, more sophisticated problems can be assigned if the use of a calculator is encouraged. It is also

possible that more students may elect to take additional and more advanced mathematics courses if the emphasis on paper and pencil computation is removed from these courses and their prerequisites.

The use of calculators on the Applications Section of the Competency Test will allow more realistic problems to be included on the Test. This will make this section of the Test a more reliable measure of the students' conceptual skills as opposed to their purely computational skills.

The ability to use a scientific calculator effectively is a skill prerequisite to most post-secondary courses in science and mathematics. The student who can effectively use a scientific calculator to solve exponential equations, for example, will have an advantage over the student who does not possess this skill.

It is clear that teaching a class to use a calculator will be easier if each student in the class is using the same model calculator. Thus, to encourage the teaching and use of calculators, the state should provide funds for the initial purchase as well as for the maintenance and replacement of calculators. The state should also provide funds for the necessary supplementary materials.

RECOMMENDATION 3. We recommend that computers, at a ratio of 1 computer to 28 students, and appropriate courseware be made available for the following:

A. *Use in the various secondary mathematics courses,*

B. *Teaching an introductory course in microcomputers and microcomputer programming, and*

C. *Use in student projects, homework, and independent study.*

-and-

RECOMMENDATION 3 (continued)

We further recommend that a computer programming course emphasizing applications in mathematics be offered to college-bound students who plan careers in mathematics, science, engineering or related fields. An alternative to this course would be Advanced Placement Computer Science.

Rationale: As noted elsewhere in this report, microcomputers are rapidly becoming commonplace in the home, in the business world and in schools. If secondary mathematics teachers are to take advantage of the power of the microcomputer as an instructional tool, microcomputers must be available in numbers which allow students and teachers easy access to them whenever their use would promote the instructional program.

Formal instruction in microcomputers and microcomputer programming will prepare secondary students for a world in which the ability to use computers will become increasingly important. The introductory course recommended would offer students the hands-on experiences they need to become familiar with a wide range of useful applications. This course could be offered as a semester or as a year course.

As students become familiar with microcomputers and their capabilities, they should be permitted and encouraged to make use of microcomputers for homework assignments, class projects and individual study. Secondary schools should provide students with access to microcomputers during the school day and should encourage teachers, in all fields, to design homework and project assignments which involve the use of a microcomputer.

It is the special responsibility of those who teach secondary mathematics to keep the mathematics curriculum up-to-date. Therefore, these teachers should treat in their courses mathematical procedures which make use of computer technology as often as is appropriate.

Familiarity with microcomputers and the knowledge and skills required to make intelligent use of them will be especially important for students who intend to pursue careers in mathematics, science or related fields. These occupations will continue to lead others in making intensive use of sophisticated computer programs. Students who pursue scientific or technical careers need to be as familiar with computers as earlier generations of students were with slide rules. The computer programming course we recommend will meet this need. The course could be offered in either a year or a semester format and should have an Algebra II prerequisite.

We strongly recommend that students who take the programming course take at least four years of "bona fide" high school mathematics as well. We feel that the programming course has maximum value when it supplements the standard, four-year sequence of college preparatory mathematics courses.

RECOMMENDATION 4. We recommend that a course entitled Pre-Algebra be created. The course would be designed for the ninth grade student with below-average mathematical ability. Every ninth grade student not in the remedial program would be required to take either Pre-Algebra or a higher level course.

-and-

RECOMMENDATION 5. We recommend that the following courses replace the General Mathematics sequence. Each of these courses would have only a Pre-Algebra prerequisite and each would build on and develop the algebraic skills acquired in Pre-Algebra. Effective use of a calculator will be a point of emphasis in each of these courses.

A. *Technical Mathematics: This course should prepare students for a post-graduate technical program. The course should include measurement, interpretation of graphs and charts, data analysis, intuitive geometry, formula manipulation, right angle trigonometry and other relevant topics.*

B. *Consumer Mathematics: This course would cover the topics in the current Consumer Mathematics course, but the coverage should be at a more sophisticated level. Moreover, there should be a much greater emphasis placed on solving the types of problems which consumers actually encounter, i.e., real world problems with "real" data.*

RECOMMENDATION 5 (continued)

C. *Topics in Mathematics: This course would be aimed at the non-college bound student and would treat topics such as informal geometry, intuitive probability and statistics and further work in algebra.*

-and-

RECOMMENDATION 6. We recommend that for students not in remedial courses, the scope of the ninth grade annual testing program be expanded to include algebraic skills at the level of the Pre-Algebra course.

Rationale: Mathematics educators agree that almost all high school students should have an exposure to algebra. Algebra is, in essence, symbolic representation and manipulation and as such it is the underlying language of all mathematics. Moreover, symbolic manipulation and the use of variables are at the heart of computer programming which implies that facility with algebraic concepts and manipulations is essential for today's high school student.

Currently, minority students and women[52] are often not encouraged to take introductory courses in algebra. Large numbers of potentially able students are thereby precluded from taking the higher level mathematics and science courses which lead to productive and personally rewarding careers in scientific and technical fields. The Committee examined testimony from many mathematics educators, including members of minority groups, who, cognizant of this implicit discrimination and aware of the intrinsic value offered by a course in algebra, recommended that all students be required to take a course in algebra. There was, however, conflicting testimony, and we realize that, even without a "universal" requirement in algebra, the most often failed course in high school in Algebra I. It seems then that for some students, the existing courses in algebra are

overly rigorous. Thus, there is a genuine need for a Pre-Algebra course. Indeed, there are LEAs which already offer, with success, such a course.

Recommendation 6 in An Agenda For Action calls for a greater range of options to accommodate the diverse needs of our student population. At present, the options available to the non-college bound student are often limited. The introduction of the courses we recommend in Recommendation 5 will provide attractive and useful alternatives to the student who successfully completes Pre-Algebra, Algebra I-A, or Algebra I. The Pre-Algebra prerequisite and the use of calculators will allow these new courses to concentrate on problem solving in an applied context rather than on drill in arithmetic skills. Indeed, solving realistic problems with "real" data will be a point of emphasis in both the Consumer Mathematics and the Technical Mathematics courses. Note that work in data analysis with real data is strongly recommended in An Agenda For Action and by the National Science Board's Commission on Precollege Education in Mathematics, Science and Technology.(34)

If these courses are established and if remedial mathematics is offered in every LEA, then there will no longer be a need for General Mathematics, especially as it is currently taught in many LEAs, i.e., as a remedial course in arithmetic. Indeed there is good reason not to offer General Mathematics if it serves, in fact, as a remedial course and siphons students, who are capable of performing at a higher level, into a course which offers them nothing which is new, challenging or even particularly useful.

RECOMMENDATION 7. We recommend that secondary schools strive to provide one or both of the following courses for those students who complete four years of college preparatory mathematics by the end of their junior year:

A. *Calculus, at no less than the level prescribed for AB Calculus in the Advanced Placement syllabus.*

B. *A course, to be an extension of Advanced Mathematics, which is intended to develop pre-calculus skills and introduce new mathematical topics.*

Rationale: Unless students are ready for a full-fledged course in calculus, they should concentrate on pre-calculus skills. Testimony from college and university mathematics educators indicates that many students who have taken high school calculus enter college with inadequate pre-calculus skills. Areas of particular weakness are analytic geometry, solid geometry, algebra and trigonometry. Genuine mastery of calculus depends heavily upon these skill areas.(22,30)

On the other hand, a calculus course, offered at the level recommended, is potentially the most challenging and intellectually stimulating mathematics course offered at the high school level, and students, who have successfully completed four years of high school mathematics and who are prepared for such a calculus course, truly benefit from it. Moreover, students who plan to take advanced work in mathematics or science in college can accelerate their mathematical and scientific development by taking calculus in high school.

Many students who have completed four years of high school mathematics are not ready for a calculus course at the level recommended. In many LEAs, there is no desirable senior year mathematics course for them. Students who take the course recommended in Recommendation 7B will further develop their pre-calculus skills, preparing thereby to take calculus in college, and learn some new and intellectually stimulating mathematics. The course

should include a set of designated topics such as: analytic geometry, solid geometry, theory of equations, sequences, series, probability and statistics.

RECOMMENDATION 8. We recommend that problem solving become an integral part of each secondary mathematics course and that it be heavily emphasized in pre- and in-service training programs.

Rationale: A primary recommendation in An Agenda For Action is that problem solving be the focus of school mathematics of the 1980's. It becomes clear upon reading the specific suggestions for action that the term "problem solving," as it is used in An Agenda For Action, includes everything from the simplest application to the most thought provoking problem solving situation, one which requires higher cognitive thought processes.(5)

If all levels of problem solving are important to the mathematics curriculum, then each level should be specified and related to the curriculum accordingly. What follows are some possible levels which might be identified:

1. Simple Translation Problems: These are routine applications which all students should master. For example, if I spent $9.75 from a $20 bill, how much do I have left?

2. Complex Translation Problems: These problems might also be termed routine applications and might include applications similar to these from the competency examination: distance-rate-time problems, gas mileage problems, investment problems, problems involving perimeters and areas, and other formula related problems.

3. Higher Order Applications: These problems are characterized as multi-step problems, the solutions to which require the use of higher cognitive skills, but which can still be solved using step by step procedures. These problems reduce to "involved" routine applications. Problems in chemistry which require the use of direct and inverse variation concepts are good examples of this type of problem.

4. "Real" Problem Solving: This is the highest order of problem solving and is characterized by its non-routine nature. Students are involved in real problem solving when they confront a problem they do not already know how to solve. They possess the prerequisite knowledge and skills to solve the problem, but "figuring out" how to apply their previous learnings to the problem at hand, thereby sparking the insights necessary to produce a solution, is the heart of the process for real problems. Such problems include proofs, puzzles and logical expressions.

All of these levels of problem solving are important; thus, teachers must ensure that the higher level of problem solving skills are indeed taught.

Implementation of this recommendation should receive top priority for funds, time and energy from the North Carolina mathematics community at all levels and should be put on a five-year implementation schedule.

If North Carolina is to do its part in implementing Recommendation 1 of An Agenda For Action, i.e., making problem solving the focus of school mathematics, then a concerted statewide effort is needed. Effective implementation of this recommendation will require intensive in-service training of all mathematics teachers.

RECOMMENDATION 9. We recommend that the State Department of Public Instruction institute a program of statewide examinations to be administered at the end of certain college preparatory courses in mathematics--initially Algebra I, Geometry, and Algebra II. The expenses associated with administering these examinations should be assumed by the state. Colleges should be permitted to request a student's score on these tests.

Rationale: Statewide examinations will provide a uniform measure of achievement for courses in which there is currently wide variation in content and expectation. Students, teachers and colleges will all benefit from such a uniform measure of accomplishment.

RECOMMENDATION 10. We recommend that high school students be required to take the North Carolina Competency Test in the spring of their sophomore year. We further recommend that the Competency Testing Commission consider exempting students from the Competency Test requirement on the basis of superior performance on the mathematics section of the Ninth Grade Annual Testing Program.

Rationale: Students who fail the mathematics section of the Competency Test in the spring of their sophomore year can be scheduled into an appropriate remediation program at the beginning of their junior year. Thus, the time available for remediation will be increased and scheduling will be simplified.

Time and money both will be saved if students with appropriately high scores in mathematics in the Ninth Grade Annual Testing Program are exempted from the mathematics section of the Competency Test. Moreover, the prospect of exemption from the Competency Test will provide an incentive for improved performance on the ninth grade test.

RECOMMENDATION 11. We recommend that remedial mathematics classes be offered for all secondary students, grades 9-12, who are not ready to take the Pre-Algebra course or who have failed or are in danger of failing the Competency Test. Students at or below the 35th percentile on the mathematics section of the Ninth Grade Annual Testing Program should be considered for remediation. The remediation should be conducted in a mathematics laboratory with a specially trained teacher, at least one instructional aide, and no more than 20 students per class period. Exit criteria should be developed for students to move from remedial classes into the Pre-Algebra course.

Rationale: The minimal goal of the remedial mathematics program is to prepare students to pass the North Carolina Competency Test. We hope that most students in the remedial program would develop skills that would enable them to move into the Pre-Algebra course. (See Recommendation 4.) For the remedial program to have maximum effectiveness, instruction must occur in small groups under the supervision of a teacher

with training in the diagnostic and prescriptive mode of instruction. To keep the pupil to teacher ratio at the desired 10:1 level and to facilitate individualized instruction, these remedial classes must employ an instructional aide along with the appropriately trained teacher.

RECOMMENDATION 12. We recommend that, where possible, each of the courses, Algebra I, Geometry, Algebra II, be split into a two-semester course and that satisfactory performance in the first semester of each course be prerequisite for admission into the second semester. Moreover, we recommend that, insofar as it is possible, the first semester of each of these courses be offered in both the first and the second semester of each school year.

Rationale: The courses named in this recommendation are each very much hierarchical in nature. Thus, students must master the material which is introduced at each stage of the course if they are to master the material which will be introduced later. Students who have failed the first half of one of these courses have virtually no chance of successfully completing the course as a whole. Such students are, moreover, likely to become discipline problems if they are forced to remain in a class where their potential for success, even on the most limited of scales, is virtually nonexistent. These students could much more effectively spend their time and their academic energies repeating the work they failed to master initially. They could then successfully complete the second semester of the course during summer school and be ready for the next course in the Algebra-Geometry sequence at the beginning of the next school year.

This recommendation, which has been warmly supported by many secondary teachers, is indeed capable of implementation in many, if not most,

secondary schools. It is the prevailing mode in secondary schools which operate on a semester-course, rather than a year-course, calendar. Some smaller secondary schools will have difficulty fully implementing this recommendation. However, we urge that every secondary school attempt to implement our recommendation for as many of the three courses specified as is practical, given the enrollment and other constraints under which the individual school must operate.

RECOMMENDATION 13. We recommend that three units of mathematics in grades 9-12 be required for graduation.

Rationale: Students with higher levels of mathematical skills and understanding will have increasing advantage as society becomes more technologically complex. Reaching these levels of mathematical competence will require more than two units of mathematics in grades 9-12.(30)

Note that the implementation of Recommendations 4 and 5 will provide four attractive and useful course options to the non-college bound student. College-bound students should be taking three, if not four, courses in mathematics already.

RECOMMENDATION 14. We recommend that those responsible for advising secondary students, i.e., guidance counselors, advisors, curriculum coordinators, principals and teachers, be made fully aware of the importance of mathematics to every member of our society. Those who advise secondary students should, in particular, be encouraging all students to:

A. take mathematics in their senior year, and

B. take courses at the most advanced level their backgrounds and abilities will allow.

Rationale: Often the mathematical shortcomings students experience after graduation are as much of their advisors' making as of their own. Some

advisors are not sufficiently aware of the importance of proper and adequate mathematical training in high school. (See Recommendation 6 of An Agenda For Action.) Some advisors feel that only students who intend to go on for a bachelor's degree in mathematics, science or engineering need to emphasize the study of mathematics in high school. Women and minority students, especially, are not encouraged to take elective courses in mathematics. The discouragement may be more implicit than explicit; an advisor who would urge a white male student to take more mathematics because "You might need it" might not similarly encourage a woman or a minority student.

Those who are responsible for advising and registering students must be made aware of the increasing importance of mathematics so that they can adequately communicate this importance to those whom they advise. This can be accomplished through in-service training for guidance counselors and other educational leaders who advise students.

Of particular concern to many college and university educators are those students who do not take a mathematics course during their senior year in high school. It has been clearly demonstrated that mathematical skills deteriorate if they are not used. Thus, students who take no mathematics in the junior and senior years of high school often find that they are not ready for the mathematical demands that a job or a post-secondary educational program places on them. These students learn that they had too little mathematics, too long ago, and that they are disadvantaged in the job market or are relegated to remedial courses in the college of their choice. (See Recommendation 8 of the joint 1980 report of the MAA and the NCTM.)

The technological nature of the era in which we live dictates that talented students be provided opportunities to realize their full potential in mathematics. Students with outstanding mathematical ability are a precious national resource, sorely needed to maintain leadership in a technologically evolving world. (See Recommendation 6 of An Agenda For Action.) We must guarantee that these special students fully realize their potential. Indeed, we must strongly encourage them to do so.

RECOMMENDATION 15. We recommend that a mechanism be created to establish and maintain communication among the agencies involved with the teaching of mathematics. Specifically, we recommend that a committee representing the mathematics departments of public and private schools, colleges and universities (public and private), technical and community colleges, and the Mathematics Division of SDPI be created.

Rationale: Since a recommendation very much like this one appears in the joint 1980 report of the MAA and the NCTM, it is felt that the problems addressed by this recommendation are national in scope. Teachers, school officials, counselors and parents of students at all levels, need direction in determining the mathematics programs which are suitable for a particular school system. Colleges and universities should cooperate with the public schools to devise programs for the gifted mathematics student as well as programs for the student who is learning disabled. It will benefit teachers in grades K-12 to share ideas and to receive recommendations from educators at the college level. Techniques, theories, points of emphasis and priorities need to be updated as college, university and technical school programs change. It is essential that those in the public schools who are responsible for instruction in mathematics be kept informed about these changes. It is likewise essential that college and university educators, especially those responsible for teacher training, be informed about what is happening in the public schools.

RECOMMENDATION 16. We recommend that the State Board of Education suggest to the post-secondary institutions of the state that:

A. *Four-year college degree credit not be awarded for courses in which the level of content is equivalent to that of Algebra I or Geometry.*

B. *Colleges and universities devise and maintain more stringent entrance requirements in mathematics, utilizing high school grades and SAT scores.*

C. *If an institution of higher learning does admit students who are deficient in mathematics, then that institution should expect to offer remedial, non-credit mathematics courses.*

Rationale: As long as colleges, universities and technical schools award credit for introductory level courses which have been traditionally taught at the high school level (i.e., Algebra I and Geometry), many college-bound students will not take these courses in high school. Thus, offering remedial work for college credit encourages students not to do this work in high school where it should, in fact, be done and expands thereby the demand for remedial work at the college level. Fewer students will opt out of mathematics early in high school if our institutions of higher learning adopt the suggestions in this recommendation. It has been documented that students who discontinue the study of mathematics early in high school severely limit their subsequent educational and vocational options. Many doors, both in college-level programs and in vocational training, are closed to those students.(12,30)

RECOMMENDATION 17. We recommend that the State Board of Education suggest to the post-secondary institutions in the state that a placement examination be developed. This examination would be administered on a voluntary basis to high school juniors. It would be used to indicate the existing level of mathematical competence with regard to the background necessary for admission to the first credit course in mathematics at the college or university level.

Rationale: Many high school graduates today are accepted by colleges and universities and are surprised to discover that placement test results

require their enrollment in non-credit, remedial mathematics courses or programs. High school juniors, who take this examination may discover that their mathematical deficiencies will prevent enrollment in the college-level mathematics program at the school of their choice. This discovery would be made early enough so that the problem could be resolved prior to graduation from high school. The examination will encourage some students to take additional mathematics courses to ensure that they will be adequately prepared for college-level courses and will not be forced into remedial programs. A testing program, similar to the one we recommend here, has been implemented by Ohio State University.(46) This program has yielded the desirable outcomes outlined above.

RECOMMENDATION 18. To attract competent, new mathematics teachers, we recommend the following:

A. *Teaching assignments for first-year teachers involve contact with students having ability levels.*

B. *First-year teachers be offered a contract as early as possible.*

C. *First-year teachers be offered the opportunity of beginning employment as soon after their graduation as is practical.*

Rationale: Examination of the North Carolina Mathematics Teacher Profile, 1982, shows that a large number of first-year teachers leave teaching. The two reasons most commonly given for this are, first, that new teachers received unattractive assignments and, second, that the monetary rewards were not adequate to compensate for the strenuous mental and physical demands of a teaching career.

Many college students graduate in May and would like to begin employment at once. However, in most school systems, first-year teachers begin work in mid to late August, and receive their first paycheck in

September. This situation so frustrates many prospective teachers that they take a job in a business or industry and never enter the teaching profession. Even worse for the prospective teacher is the fact that many LEAs do not even offer contracts to first-year teachers until August. A capable, ambitious graduate wants to know as early as possible where he or she will begin his or her career. The prospective teacher sees all too clearly that students who plan to enter careers in business or industry are offered contracts well before graduation.

We note that if beginning teachers could be placed on the payroll effective July 1 then they would receive a paycheck in August rather than September. More importantly, perhaps, they could begin to prepare for their initial teaching assignment in the less hectic summer months.

RECOMMENDATION 19. We recommend that no endorsement of certificates for teaching mathematics be permitted and that the only certificates in mathematics be those outlined by the Quality Assurance Program and which fully meet the program's guidelines.

Rationale: The State of North Carolina has a set of minimum qualifications that one must meet to receive a license to teach mathematics. These qualifications are enumerated as competencies for each level of certification in the Quality Assurance Program. These requirements are the minimums set; there is not any set of qualifications less than these which should be accepted for licensing or recognized as legitimate. While persons not properly certified might be utilized in temporary, emergency situations, standards for certifications should never be diluted.

RECOMMENDATION 20. We recommend that by the 5-86 school year only those individuals appropriately certified in mathematics be allowed to teach mathematics in grades 7 through 12.

Rationale: This recommendation is aimed at strengthening the quality of education in our schools by ensuring that only qualified and certified teachers be allowed to teach mathematics to our high school students. Mathematics is a language, and, as such, it is as difficult to teach mathematics as it is to teach Swahili or any other foreign language. The state of North Carolina has already embarked on a vigorous program to train teachers who are currently teaching mathematics out of field. We commend this program and believe it will produce better trained teachers who have an appropriate certification in mathematics. The continuation of these efforts and the implementation of this recommendation will ensure that, by the school year 1985-86, all secondary students across the state will be taught mathematics by properly certified mathematics teachers.

RECOMMENDATION 21. We recommend that for certificate renewal of secondary school mathematics teachers at least three units be in mathematics and/or mathematics education. We further recommend that these units be earned through college credits or through approved coursework with the same rigor in content and requirements as a college-level course.

Rationale: Under the current guidelines, it is possible for mathematics teachers to obtain certification renewal with experiences which are not related to mathematics and which do not contribute to the teachers' mathematical expertise. This recommendation is directed toward improving the professional qualifications of our teachers and, through them, the education of our children.

RECOMMENDATION 22. We recommend that the following two types of in-service programs for secondary school mathematics teachers be developed and funded:

A. *An ongoing program with a curriculum consisting of mathematics content, learning theory, evaluation techniques, effective use of manipulatives, computers, calculators, problem solving, classroom management, efficient use of classroom time, and counseling strategies.*

B. *Special summer institutes to assist teachers in teaching secondary level mathematics.*

Funding should be provided for various financial incentives to allow teachers to participate in either or both programs.

-and-

RECOMMENDATION 23. We recommend that in-service be conducted on a continuing basis for administrators of each LEA concerning trends and issues in mathematics education.

Rationale: High priority must be given to providing quality in-service programs to all mathematics teachers. The Committee feels strongly that in-service credit should not be given to mathematics teachers for courses that are not related to their teaching assignments; in-service programs should emphasize continuous, professional growth.

High priority must also be given to keeping administrators up-to-date on trends and issues in mathematics education.[21] Unless this is done, funds and personnel will not be allocated in the most productive fashion. Moreover, administrators must communicate to students and faculty alike the relative importance of mathematics in the overall curriculum.

Good in-service programs will reduce the rate at which good, new teachers are lost to industry. Note that research shows that more mathematics teachers leave the profession after one to three years than at any other time.

Mathematics teachers must be able to deal with computers both as objects of study and as teaching tools. In-service programs providing

training in the use of computers will be necessary if this is to happen. Teachers will also need guidance in implementing our recommendations on calculators. Finally, as noted in Recommendation 8, since problem solving is a vital area in mathematics education, in-service programs should reflect this importance and help teachers develop appropriate problem solving activities.

RECOMMENDATION 24. We recommend that to attract and retain competent mathematics teachers a program of incentives be developed. Specifically, we recommend that:

A. *Teaching loads in mathematics be restricted to no more than 26 students per class and 130 students per day.*

B. *Appropriate stipends be provided for summer study, curriculum development, staff development, etc.*

C. *Reimbursement be provided for expenses associated with attendance at professional mathematics meetings, including travel, lodging, registration fees, and substitute teacher pay.*

Rationale: Students in a typical classroom today come from diverse educational and social backgrounds making it difficult for a teacher to work with each of them on an individualized basis. A teacher can come closer to providing individualized instruction when the pupil-teacher ratio is kept at a reasonable level.(10)

The mathematics curriculum is changing rapidly, and if teachers are to remain abreast of these changes, ongoing self-improvement must occur. In-service is not enough. Activities such as summer study at a university and attendance at professional meetings are also necessary. These activities are expensive, and the costs may prevent some teachers from pursuing them. It is in the state's interest to encourage these professional activities, and the state, therefore, should provide the

financial support to ensure that teachers receive the ongoing professional development they require.

The teaching profession will be strengthened and the secondary mathematics program will benefit if teachers are offered the option of twelve-month employment. Currently teachers are employed for ten months and, for financial reasons, may seek other employment during the summer. By extending the term of employment, we will provide these teachers with some needed income and thereby make their profession more attractive. During the summer months, employed teachers could pursue further training in mathematics, develop new courses, enrich existing curriculum by developing materials and techniques, organize materials and resources, plan model programs, or aid in staff development activities.

RECOMMENDATION 25. We recommend that salaries be increased for appropriately certified mathematics teachers who h rity of their teaching responsibilities in mathematics.

Rationale: Steps must be taken to retain com . mathematics teachers and to make mathematics teaching more attractive to prospective teachers. There is a severe shortage of certified mathematics teachers across the nation. In 1982, mathematics had the most severe teacher shortage of any subject area in the country.(28)

A review of the pay scales in effect at our colleges and universities will show that in order to retain competent faculty in disciplines such as medicine, law, applied mathematics and computer science, differential levels of pay are common practice. It is commonly accepted that many competent mathematics teachers are being attracted to higher paying jobs in business and industry. This situation will probably worsen in the

next few years as salaries paid to employees in business and industry are approximately fifty to sixty percent higher than those paid to teachers with comparable training and experience. This disparity in salaries certainly leads many dedicated professionals to review their career goals.

RECOMMENDATION 26. We recommend that substantial additional funds be provided for the Prospective Teacher Scholarship Loan Program to support undergraduate students in mathematics education at the secondary level. We further recommend that the amount of each scholarship loan be increased to $2000/year.

Rationale: The shortage of trained mathematics teachers referred to above can be fully alleviated only by attracting capable, young people into the profession. The implementation of this recommendation will make mathematics teaching more attractive to college students with an interest in mathematics. Under this scholarship/loan program, recipients of loans who teach in the public schools for the same number of years that financial assistance is received have their debt cancelled proportionally. Thus, most recipients would have a definite commitment to enter and remain in North Carolina in teaching. The current scholarship/loan of $1500 per year is inadequate in view of rising college costs.

RECOMMENDATION 27. We recommend that the present student allotment for textbook adoption be raised to $15.

Rationale: The present per student, per year allotment for textbooks is not sufficient to purchase the needed textbooks at the high school level. High school mathematics textbooks cost as much as $13.80, and some textbooks in other subjects cost even more. If a school system is

to provide up-to-date textbooks, local funds must be used to make up the difference between the monies that are allotted by the state and the actual cost of the books. Some school systems do not have these funds; thus, they continue to use previously adopted textbooks which may present outdated mathematical techniques and applications.

APPENDICES

APPENDIX A

Membership of the Mathematics Curriculum Study Committee

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APPENDIX B

Consultants to the Mathematics Curriculum Study Committee

The Committee invited those professional organizations with an interest in any aspect of the mathematics program to present their views and concerns by way of position papers and/or verbal presentations at its March 16-17, 1982, meeting. The following organizations responded positively to the Committee's invitation:

N.C. Council of Teachers of Mathematics - Katharine W. Hodgin

N.C. Association for the Gifted - Linda Weiss

Professional Educators of North Carolina - Sue Sams

N.C. Federation of Teachers - Robert Joyner

N.C. League of Middle/Junior High Schools - Pat Knight

Minorities in the Mathematics Program - Larry Clark

N.C. Association of School Administrators - Raymond Surbaugh

N.C. Principals/Asst. Principals Association - Joe Bost

The UNC Mathematics Department Chairpersons - E. E. Bronston

Association for Supervision and Curriculum Development - Lula Monds
Hazel Lamm

N.C. Congress of Parents and Teachers, Inc. - Lola Taylor

N.C. Association for the Gifted and Talented - Smith Goodrum

Position papers on topics of parti[illegible]rn to the Committee were solicited from selected mathematic[illegible]. As a result, nineteen such papers were submitted to the Committee. They proved to be of great value as the study progressed. Special appreciation is expressed to the following individuals who gave of their time and expertise to prepare papers for the Committee:

Edward G. Blakeway, Private Consultant, Raleigh

Roger Bruhwel, West Charlotte High School, Charlotte

Grace M. Burton, UNC-Wilmington, Wilmington

Richard E. Cowan, Roanoke Rapids City Schools, Roanoke Rapids

Ralph DeVane, Western Carolina University, Cullowhee

David L. Green, Jordan High School, Durham

Neal Hayes, Burlington City Schools, Burlington

Ron Marshall, Western Carolina University, Cullowhee

William Morgan, St. Andrews College, Laurinburg

Fletcher R. Norris, UNC-Wilmington, Wilmington

William F. Palmer, Catawba College, Salisbury

Annie Puett, East Mecklenburg High School, Charlotte

Janice Richardson, Alamance County Schools, Graham

Ann F. Roach, Northwood High School, Pittsboro

Harvey E. Sadoff, UNC-CH, Chapel Hill

Julie S. Schilawski, Cary Senior High School, Cary

William Smith, UNC-CH, Chapel Hill

Linda H. Sullivan, Scotland County Schools, Laurinburg

Betty C. Williamson, Richmond Senior High School, Rockingham

APPENDIX C

SDPI Staff for Mathematics Curriculum Study Committee

Many members of the State Education Agency assisted the Mathematics Curriculum Study Committee with their work.

Instructional Services Area

Jerry Beaver, Deputy Asst. Superintendent

Wayne Dillon, Special Assistant
Middle Grades Education

William McMillan, Special Assistant
Secondary Education

Betty Moore, Special Assistant
Curriculum and Administration

Mary Purnell, Special Assistant
Elementary Education

Mathematics Division

Robert R. Jones, Director
Coordinator - Mathematics Curriculum Study Committee

Cleo M. Meek, Assistant Director
Liaison Person - Secondary Subcommittee

Kay Kemp, Consultant
Liaison Person - Elementary Subcommittee

Barbara Hardison, Regional Mathematics Coordinator
Northeast Regional Education Center

Barbara Leland, Regional Mathematics Coordinator
Southeast Regional Education Center

John Ogle, Regional Mathematics Coordinator
Central Regional Education Center
Liaison Person - Middle School Subcommittee

Gloria Dantzler, Regional Mathematics Coordinator
South Central Regional Education Center

Jeanette Gann, Regional Mathematics Coordinator
North Central Regional Education Center

Mathematics Division (continued)

Sandi Lee, Regional Mathematics Coordinator
Southwest Regional Education Center

Emery Partee, Regional Mathematics Coordinator
Northwest Regional Education Center

Doug Barker, Regional Mathematics Coordinator
Western Regional Education Center

Mildred Quick, Secretary
Secretary for Committee

Connie Hawthorne, Secretary
Secretary for Committee

APPENDIX D

Summary of Activities of the Mathematics Curriculum Study Committee

November 4-5, 1981
- Total committee
  - Organization, purpose, objectives of the Study

January 12-13, 1982
- Curriculum subcommittee
  - Identification of concerns

March 16-17, 1982
- Total committee
  - Presentations by representatives of professional organizations

July 22-23, 1982
- Secondary subcommittee

September 23-24, 1982
- Total committee
  - Presentation of elementary and middle grades recommendations

October 15, 1982
- Secondary subcommittee

November 30-December 1, 1982
- Total committee
  - Computer subcommittee report

January 31-February 1, 1983
- Total committee
  - Preparation of tentative recommendations and rationales

February 2, 1983
- Total committee
  - Seminar on Issues in Mathematics Education
  - Presentation of tentative recommendations and rationales

April 28-29, 1983
- Editing committee
  - Review of summaries of surveys and hearings related to the tentative recommendations

May 17-18, 1983
- Total committee
  - Review of draft document

May 31-June 2, 1983
- Total committee
  - Finalize all components of the Study

In addition to the twelve formal meetings of the total committee or its subcommittees, numerous small work groups functioned on a regular basis to address specific tasks identified by the Committee as a whole.

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